Student Information

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Answer 1

Before we even begin solving the questions, it is safe to first state our groups and related parameters in a more formal way so that it gets easier to follow along.

Our first group, X, is people with age 40 and above:
The number of people in this group is 19, such that $n_X = 19$,
The mean of the answers of this group is 3.375, such that $\bar{X} = 3.375$,
The standard deviation of the answers of this group is 0.96, such that $s_X = 0.96$.

Our first group, Y, is people with age under 40:
The number of people in this group is 15, such that $n_Y = 15$,
The mean of the answers of this group is 2.05, such that $\bar{Y} = 2.05$,
The standard deviation of the answers of this group is 1.12, such that $s_Y = 1.12$.

While we are looking to T table, with degrees of freedom v and confidence interval (1-α)100%, we will use the following representation: $T_{(v,\alpha)}$. For part **(a)** and **(b)** we will need to use the *Satterthwaite approximation* to approximate the degree of freedom since we have unequal variances.

$$v = \frac{(\frac{s_X^2}{n_X} + \frac{s_Y^2}{n_Y})^2}{\frac{s_X^4}{n_X^2(n_X-1)} + \frac{s_Y^4}{n_Y^2(n_Y-1)}}$$

$$= \frac{(\frac{0.96^2}{19} + \frac{1.12^2}{15})^2}{\frac{0.96^4}{19^2(19-1)} + \frac{1.12^4}{15^2(15-1)}} = 27.702$$

We have found a non-integer value for v. To use the T-table we just take the closest v that is given in that table, which in this case 28.

a)

Since we are dealing with means, particularly differences in means, we are going to use T intervals. We are asked to use 95% confidence interval which corresponds to $\alpha = 0.05$ but since this is a two-tailed probability we will use $\frac{\alpha}{2} = 0.025$ and we have found the degrees of freedom in solution definition part such that $v = 28$. Our confidence interval will have the form:

$$(\bar{X} - \bar{Y}) \pm T_{(28,0.025)} \sqrt{\frac{s_X^2}{n_X} + \frac{s_Y^2}{n_Y}}$$

$$(3.375 - 2.050) \pm (2.048) \sqrt{\frac{0.96^2}{19} + \frac{1.12^2}{15}}$$

$$(1.325) \pm (2.048) \sqrt{\frac{0.9216}{19} + \frac{1.2544}{15}}$$

$$(1.325) \pm (2.048) \sqrt{0.1322}$$

$$(1.325) \pm (2.048)(0.3635)$$

$$1.325 \pm 0.7445 = [0.5805, 2.0695]$$

b)

Since we are dealing with means, particularly differences in means, we are going to use T intervals. We are asked to use 90% confidence interval which corresponds to $\alpha = 0.10$ but since this is a two-tailed probability we will use $\frac{\alpha}{2} = 0.05$ and we have found the degrees of freedom in solution definition part such that $v = 28$. Our confidence interval will have the form:

$$(\bar{X} - \bar{Y}) \pm T_{(28,0.05)} \sqrt{\frac{s_X^2}{n_X} + \frac{s_Y^2}{n_Y}}$$

$$(3.375 - 2.050) \pm (1.701) \sqrt{\frac{0.96^2}{19} + \frac{1.12^2}{15}}$$

$$(1.325) \pm (1.701) \sqrt{\frac{0.9216}{19} + \frac{1.2544}{15}}$$

$$(1.325) \pm (1.701)\sqrt{0.1322}$$

$$(1.325) \pm (1.701)(0.3635)$$

$$1.325 \pm 0.6183 = [0.7067, 1.9433]$$

c)

Based on the information provided, the significance level is $\alpha = 0.05$, and the critical value for a right-tailed test is $\mathrm{t}_c = 1.734$. The rejection region for this left-tailed test is $\boldsymbol{R} = [\ 1.734\ ,\ \infty)$ The t-statistic is computed as follows:

$$t = \frac{\bar{X} - 3}{\frac{s_X}{\sqrt{n_X}}}$$

$$= \frac{3.375 - 3}{\frac{0.96}{\sqrt{19}}} = 1.703$$

Since it is observed that $t = 1.703 \leq \mathrm{t}_c = 1.734$, it is then concluded that, we **can** say people with age 40 and above supports BREXIT with 95% confidence level.

Answer 2

Before we even begin solving the questions, it is safe to first state our group and related parameters in a more formal way so that it gets easier to follow along.

$$\mu_0 = 20.00;\ \bar{X} = 20.07;\ s_X = 0.07;\ n_X = 11$$

a)

$$H_0 : \mu = \mu_0$$

b)

$$H_A : \mu \neq \mu_0$$

c)

Let us test

$$H_0 : \mu = \mu_0 \qquad \text{vs} \qquad H_A : \mu \neq \mu_0$$

at a significance level $\alpha = 0.01$. This corresponds to a two-tailed test, then we will need to take $\frac{\alpha}{2} = 0.005$. We have sample statistics:

$$\bar{X} = 20.07 \text{ and, } s_X = 0.07$$

Compute the T-statistic,

$$t = \frac{\bar{X} - \mu_0}{\frac{s_X}{\sqrt{n_X}}} = \frac{20.07 - 20.00}{\frac{0.07}{\sqrt{11}}}$$

$$= \frac{0.07}{\frac{0.07}{\sqrt{11}}} = \sqrt{11} = 3.3166$$

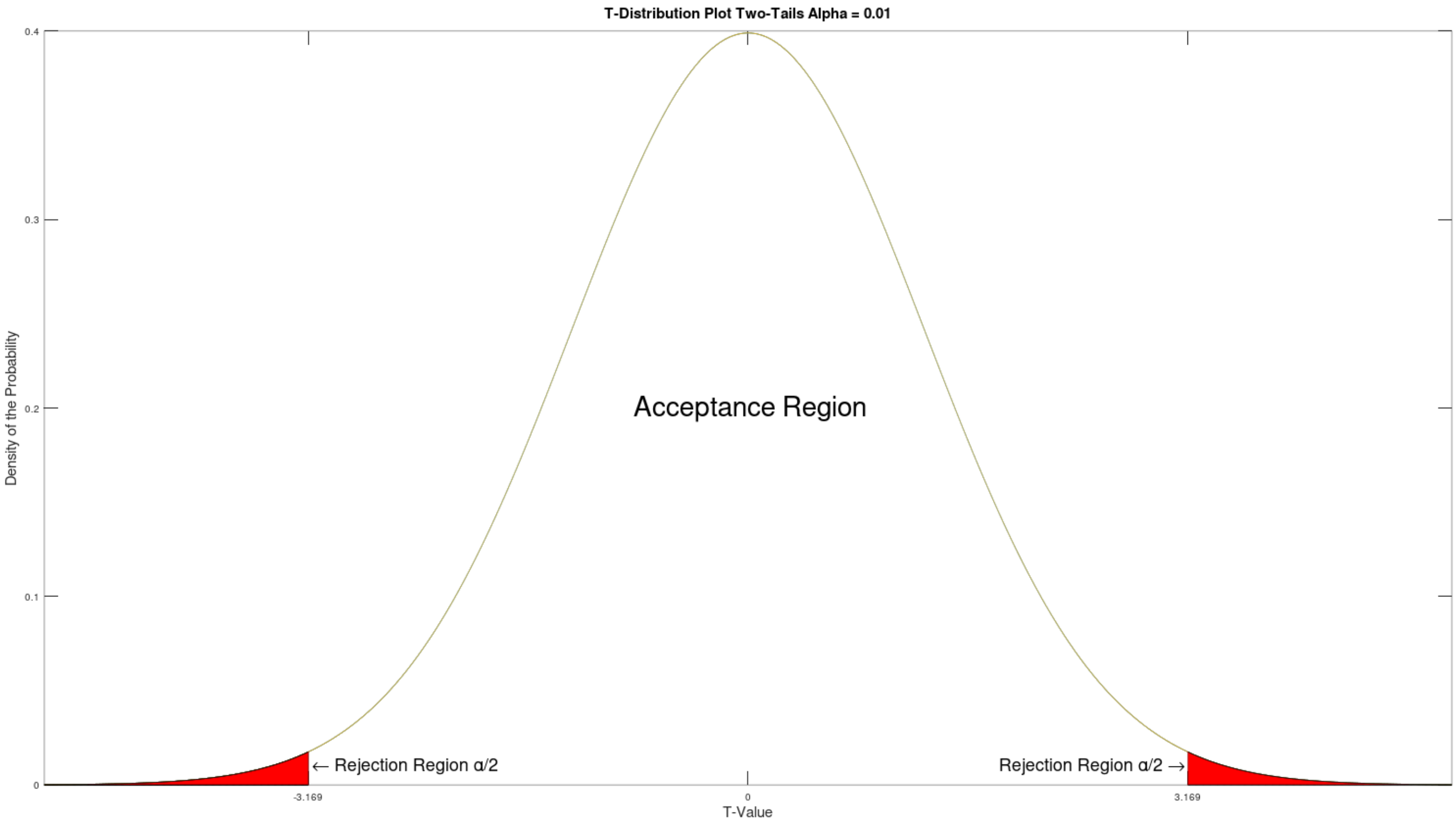
Now, let us examine the rejection region which relay on the T-table value $\mathrm{T}_{(DF,SL)}$ where DF is the degree of freedom which is 10, 11-1 = 10, in our case and SL is the significance level as percentage which is 1% in our case($\frac{\alpha}{2} = 0.005$). Then we look for the critical value of T from the T-table.

$$T_{(10,0.005)} = 3.1690$$

Therefore, rejection region is:

$$\mathcal{R} = (-\infty, -3.1690] \cup [3.1690, \infty)$$

Since t $\in \mathcal{R}$, we rejecct the null hypothesis and conclude that there is a significant evidence so **that they should stop producing and check the production line**.



Above figure shows us the acceptance and rejection regions of Question-2.

Answer 3

Before we even begin solving the questions, let us define a formal representation for our given parameters in a more formal way so that it gets easier to follow along. We have 2 cases and we assume that the samples are independently collected and the sample sizes are the same, $n_X = n_Y = 68$.

Our first case, X, is the painkillers existing in the market:
Reduce the headache in 3 minutes, such that $\bar{X} = 3$,
The standard deviation of the time required for reducing the headache is 1.4 minute, $\sigma_X = 1.4$.
Our second case, Y, is the new painkiller:
On average reduce the headache in 2.8 minutes, such that $\bar{Y} = 2.8$,

The standard deviation of the time required for reducing the headache is 1.7 minute, $\sigma_Y = 1.7$.

a)

$$H_0 : \mu_Y = \mu_X$$

b)

The two-sided alternative of the null hypothesis H_A: $\mu_y \neq \mu_X$. However, if we worry about longer time required to reduce the headache only, we can conduct a one-sided test of:

$$H_A : \mu_Y < \mu_X$$

c)

This corresponds to a right-tailed test, for which a Z-test for two population means, with known population standard deviations will be used. Based on the information provided, the significance level is $\alpha = 0.05$, and the critical value for a right-tailed test is $\mathrm{Z}_c = 1.64$. The rejection region for this right-tailed test is:

$$\mathcal{R} = [1.640, \infty)$$

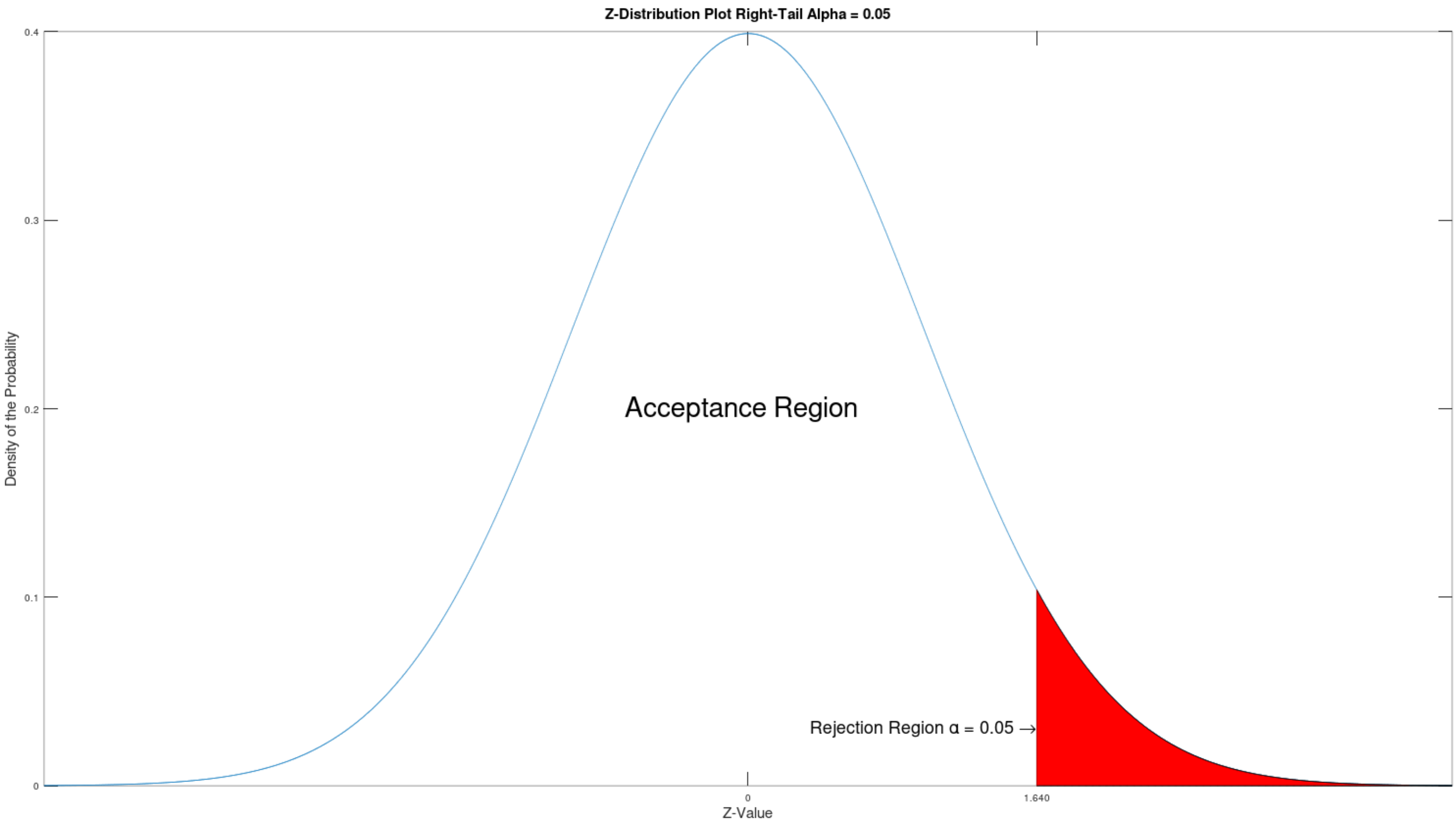
The z-statistic is computed as follows:

$$z = \frac{\bar{X} - \bar{Y}}{\sqrt{\frac{\sigma_X^2}{n_X} + \frac{\sigma_Y^2}{n_Y}}}$$

$$= \frac{3 - 2.8}{\sqrt{\frac{1.4^2}{68} + \frac{1.7^2}{68}}}$$

$$= 0.749$$

Since it is observed that $z = 0.749 \leq \mathrm{Z}_c = 1.64$, calculated z value is in the '*Acceptance Region*', it is then concluded that there is not sufficient evidence to reject the null hypothesis. Therefore, we can state that, **with 5% significance level we do not have sufficient evidence to say the new painkiller really produce better results**.



Above figure shows us the acceptance and rejection region of Question-3.