

ECS 32B: Fall 2019 Homework Assignment 1

Due Date: No later than Tuesday, January 22, 9:00pm

0. Make sure to clearly identify your name, student number, and your discussion section on the work you submit. Submit only one pdf file via Canvas...do not submit a separate file for each problem.

1. The Traveling Salesman problem (TSP) is a famous problem for which there is no known, tractable solution (though efficient, approximate solutions exist). Given a list of cities and the distances in between, the task is to find the shortest possible tour (a closed walk in which all edges are distinct) that visits each city exactly once.

Consider the following algorithm for solving the TSP:

```
n = number of cities
m = n x n matrix of distances between cities
min = (infinity)
for all possible tours, do:
    find the length of the tour
    if length < min
        min = length
    store tour
```

- a) What is the worst-case (big-O) complexity of this algorithm in terms of n (number of cities)? You may assume that matrix lookup is $O(1)$, and that the body of the if-statement is also $O(1)$. You need not count the if-statement or the for-loop conditional (i.e., testing to see when the for-loop is done), or any of the initializations at the start of the algorithm. Clearly show the justification for your answer.
- b) Given your complexity analysis, assume that the time required for the algorithm to run when $n = 10$ is 1 second. Calculate the time required for $n = 20$ and show your work.

2. Suppose an algorithm solves a problem of size n in at most the number of steps listed for each $T(n)$ given below. Calculate the asymptotic time complexity (big- Θ or big-theta) for each $T(n)$. Show your work, including values for c and n_0 along the way.

- a) $T(n) = 1$
- b) $T(n) = 5n - 2$
- c) $T(n) = 3n^3 + 2 + n^2$
- d) $T(n) = \log(n * 2n!)$