# On Horizon Duality

Tom Chen

September 10, 2025

**Abstract**

We propose that the event horizon is the fundamental unit of spacetime structure, unifying black holes and the universe through a deep duality. A black hole horizon and the cosmic horizon are inside–out images of each other: the black hole interior void corresponds to the universe exterior void, while the spacetime bulk is interchanged under inversion. This **Inside–Out Equivalence Principle (IOEP)** demands invariance of all bulk dynamics under O(3) rotations and inversion symmetry, eliminating the cosmological constant and constraining horizon evolution. In this framework, cosmic expansion is the inside–out analogue of black hole accretion, and post-heat-death horizon shrinkage mirrors Hawking evaporation. Using geometric algebra, we formulate IOEP mathematically as an inversion acting on curvature and horizon generators, showing how horizon duality constrains Einstein's equations and yields testable cosmological implications.

## 1 Introduction

The origin of spacetime from quantum information is one of the central questions in fundamental physics. Wheeler's "It from Bit" paradigm suggests that spacetime emerges from quantum information encoded on boundaries, particularly horizons [1]. Developments in the AdS/CFT correspondence reinforce this idea: horizons act holographically, encoding bulk information in boundary quantum states [2, 4].

We argue that the cosmic horizon and black hole horizon are dual to one another under inversion. In a black hole, bulk spacetime collapses inward toward the horizon and the interior void. In the universe, bulk spacetime expands outward toward the horizon and the exterior void. This inside–out relation defines the **Inside–Out Equivalence Principle (IOEP)**, which treats the two horizons as mirror images of the same physical law.

## 2 Horizon as the Source of Spacetime and Curvature

Let $\mathcal{H}$ denote a horizon supporting a quantum state $|\Psi_{\mathcal{H}}\rangle$. Bulk geometry $(g_{\mu\nu}, R_{\mu\nu\rho\sigma})$ emerges from $|\Psi_{\mathcal{H}}\rangle$, separating spacetime bulk from void. The extrinsic curvature vector $\vec{K}$ points from bulk to void. For a black hole, $\vec{K}$ points inward; for the universe, $\vec{K}$ points outward. This reversal establishes the duality: black hole and universe are identical structures related by inversion.

# 3 Inside–Out Equivalence Principle (IOEP)

The IOEP states that all dynamical laws in the bulk are invariant under:

1. **O(3) rotations**, expressing isotropy of the horizon.
2. **Inversion symmetry**, exchanging bulk and void.

For any bulk dynamical operator $\mathcal{O}[g_{\mu\nu}, T_{\mu\nu}]$, IOEP requires

$$\mathcal{O}[g_{\mu\nu}, T_{\mu\nu}] = \mathcal{O}[R g_{\mu\nu} R^T, T_{\mu\nu}] = \mathcal{O}[g^{\text{inv}}_{\mu\nu}, T^{\text{inv}}_{\mu\nu}], \tag{1}$$

for all $R \in O(3)$. This symmetry forbids a cosmological constant, since it would not remain invariant under inversion.

# 4 Einstein Equations under IOEP

Starting from the Einstein field equations without a cosmological constant,

$$G_{\mu\nu} \equiv R_{\mu\nu} - \tfrac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu}, \tag{2}$$

the IOEP requires invariance under the inversion $g_{\mu\nu} \to g^{\text{inv}}_{\mu\nu}$. This enforces duality between black hole dynamics and cosmological dynamics:

- Universe expansion corresponds to bulk curvature flowing outward toward the cosmic void, the inside–out analogue of black hole accretion.
- After thermal equilibrium (heat death), the cosmic horizon shrinks by quantum evaporation, the analogue of Hawking radiation in black holes [3].

# 5 Geometric Algebra Formulation of IOEP

We now reformulate the IOEP in the language of geometric algebra (GA), which makes the inversion symmetry explicit.

## 5.1 Spacetime and Curvature in GA

Let spacetime be modeled by a 4D vector space $V$ with basis $\{\gamma_\mu\}$ and signature $(+,-,-,-)$. The event horizon $\mathcal{H}$ is represented by a null vector $n$ satisfying $n^2 = 0$. Curvature is expressed as a bivector field

$$R = \frac{1}{2} R_{\mu\nu}\, \gamma^\mu \wedge \gamma^\nu. \tag{3}$$

## 5.2 Inversion Map

The inversion acting on any multivector $X$ is defined as

$$X^{\text{inv}} = \frac{x}{x^2}\, X\, \frac{x}{x^2}, \qquad x \in V. \tag{4}$$

For the curvature bivector $R(x)$ this yields

$$R^{\text{inv}}(x) = \frac{1}{x^4} R(x), \tag{5}$$

demonstrating that curvature near a black hole horizon maps to curvature at cosmic scales under inversion.

### 5.3 Horizon Generators

The horizon generator is encoded by the bivector

$$H = n \wedge t, \tag{6}$$

where $t$ is a timelike vector tangent to $\mathcal{H}$. IOEP requires

$$\langle H \rangle_{\text{universe}} = -\langle H \rangle^{\text{inv}}_{\text{black hole}}. \tag{7}$$

Thus the extrinsic curvature vector points outward in the universe case and inward in the black hole case.

### 5.4 Einstein Equations in GA

In GA form the Einstein equation can be written

$$R(a) - \tfrac{1}{2} a R = 8\pi G\, T(a), \tag{8}$$

where $R(a)$ is the Ricci operator acting on a vector $a$. The IOEP requires invariance:

$$R^{\text{inv}}(a) - \tfrac{1}{2} a R^{\text{inv}} = 8\pi G\, T^{\text{inv}}(a). \tag{9}$$

This condition rules out a cosmological constant term, since a constant curvature contribution would not remain covariant under inversion.

## 6 Cosmological Implications

The IOEP yields several consequences:

1. **Dark energy:** Accelerated expansion is a manifestation of horizon curvature dynamics, not a cosmological constant.

2. **Horizon evolution:** The universe horizon grows with accumulated bulk; after heat death it evaporates, paralleling black hole evaporation.

3. **Black hole–universe duality:** The two horizons represent the same law under inversion, providing a unified view of gravitational collapse and cosmological expansion.

## 7 Conclusion

Horizons are the primary source of spacetime and curvature, and the IOEP elevates their duality into a principle. Black holes and the universe are inside–out realizations of the same horizon dynamics. This principle eliminates the cosmological constant, accounts for cosmic expansion and late-time evaporation, and links thermodynamics of black holes with cosmology. Future work includes quantitative GA models of horizon-induced curvature, predictions for primordial gravitational waves, and new tests of horizon duality in cosmology.

## References

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