Quantum Entropy Theory

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Abstract

We present a framework in which the dynamics of bulk spacetime and the Standard Model of particle physics emerge from a single master equation of quantum entropy. Using the Quantum-Entropy Field (QEF) master equation, together with an Infinite-Order Entanglement-Preserving (IOEP) symmetry, we show that bulk fields, their stress-energy tensor, and their gauge structure arise naturally from boundary entanglement constraints. This provides a unified entanglement-first derivation of geometry, gravity, and matter fields.

1 Introduction

The interplay between quantum information and spacetime geometry suggests that gravity and matter may emerge from quantum entanglement. Here, we formalize this intuition using a Quantum-Entropy Field (QEF) master equation:

$$\delta S_{\text{geom}} + \delta S_{\text{matter}} = 0, \tag{1}$$

where δS_{geom} is the variation of geometric entropy (proportional to the boundary area of causal diamonds), and δS_{matter} is the variation of matter entanglement entropy on the boundary. This principle asserts that total generalized entropy is stationary, providing constraints on both spacetime geometry and bulk matter fields.

2 Boundary Entanglement and the Emergence of Bulk Stress-Energy

Using the entanglement first law, the variation of matter entropy in a small causal diamond of radius R is:

$$\delta S_{\text{matter}} = \delta\langle K\rangle, \tag{2}$$

where K is the modular Hamiltonian associated with the boundary. For approximately flat spacetime,

$$\delta\langle K\rangle = 2\pi \int_{\Sigma} \langle T_{ab}\rangle\, \xi^a\, d\Sigma^b, \tag{3}$$

with ξ^a the approximate boost Killing vector and Σ the boundary surface. The QEF master equation then imposes

$$\delta S_{\text{geom}} + 2\pi \int_{\Sigma} \langle T_{ab} \rangle \, \xi^a \, d\Sigma^b = 0. \tag{4}$$

Any fluctuation of entanglement on the boundary requires a corresponding bulk energy-momentum distribution to preserve stationarity of total entropy.

3 Emergence of Bulk Spacetime

Assuming δS_{geom} depends linearly on small variations of the boundary area:

$$\delta S_{\text{geom}} = \frac{1}{4G} \delta \text{Area}[\Sigma] \approx \frac{1}{4G} \int_{\Sigma} \theta \, d\Sigma, \tag{5}$$

where θ is the expansion of null generators on the boundary. Plugging into the QEF equation, we recover a linearized Einstein equation in the bulk:

$$G_{ab} + \Lambda g_{ab} = 8\pi G \langle T_{ab} \rangle. \tag{6}$$

Thus, spacetime geometry emerges as the solution to boundary entanglement constraints, with bulk curvature sourced by the emergent stress-energy tensor.

4 Emergence of Bulk Fields

The bulk stress-energy tensor is generated by matter fields:

$$\langle T_{ab} \rangle = \sum_i -\frac{2}{\sqrt{-g}} \frac{\delta \mathcal{L}_i[\phi_i, g]}{\delta g^{ab}}, \tag{7}$$

where ϕ_i represents all allowed bulk fields (fermions, scalars, gauge bosons).

The functional emergence principle is

$$\{\phi_i(x)\}_{\text{emergent}} = \arg\min_{\phi_i} \left| \delta S_{\text{geom}} + 2\pi \int_{\Sigma} \langle T_{ab}[\phi_i] \rangle \, \xi^a \, d\Sigma^b \right|, \tag{8}$$

i.e., only field configurations whose stress-energy satisfies the boundary entanglement constraint exist. Bulk fields thus emerge as solutions to a functional equation imposed by QEF.

5 IOEP Symmetry and the Standard Model

The Infinite-Order Entanglement-Preserving (IOEP) symmetry imposes

$$S_{\text{matter}}[U_{\text{IOEP}} \rho U_{\text{IOEP}}^\dagger] = S_{\text{matter}}[\rho]. \tag{9}$$

Only bulk fields that preserve boundary entanglement under IOEP transformations are allowed. Consequences:

1. Gauge group selection: Only SU(3)×SU(2)×U(1) gauge fields satisfy entanglement preservation.

2. Chirality constraints: Left- and right-handed fermions are selected.

3. Interactions: Yukawa couplings, gauge interactions, and scalar potential are determined by minimal solutions consistent with QEF + IOEP.

The full Standard Model field content and interactions emerge naturally as the minimal set of fields consistent with boundary entanglement and symmetry.

6 Examples of Emergent Fields

6.1 Fermion Field

$$\mathcal{L}_\psi = i\bar{\psi}\gamma^a\partial_a\psi, \quad T_{ab}[\psi] = \frac{i}{2}\left(\bar{\psi}\gamma_{(a}\partial_{b)}\psi - \partial_{(a}\bar{\psi}\gamma_{b)}\psi\right) \tag{10}$$

QEF requires that the integrated T_{ab} over the boundary balances δS_{geom}, implying the fermion must exist in the bulk. IOEP symmetry selects allowed chirality and gauge charges.

6.2 Gauge Field

$$\mathcal{L}_A = -\frac{1}{4}F_{ab}F^{ab}, \quad F_{ab} = \partial_a A_b - \partial_b A_a, \quad T_{ab}[A] = F_{ac}F_b{}^c - \frac{1}{4}\eta_{ab}F_{cd}F^{cd} \tag{11}$$

Functional equation:

$$\int_\Sigma \xi^a T_{ab}[A]\, d\Sigma^b = -\frac{1}{2\pi}\delta S_{\text{geom}} \tag{12}$$

Gauge fields emerge as minimal excitations whose stress-energy satisfies boundary constraints. IOEP selects allowed gauge groups.

6.3 Scalar Field (Higgs)

$$\mathcal{L}_H = |\partial_\mu H|^2 - V(H) \tag{13}$$

T_{ab} contributes to δS_{matter}, ensuring boundary entanglement balance, and is selected by IOEP symmetry.

7 Full Emergence Scheme

`Boundary Entanglement (QEF)` $\rightarrow$ `Functional Constraint` $\delta S_{\text{total}} = 0$
$\rightarrow$ `Bulk Stress-Energy` T_{ab} `Required` $\rightarrow$ `Functional Solution` $\rightarrow$ `Bulk Fields` ϕ_i `(`ψ`, H,` A_μ`)`
$\rightarrow$ `IOEP Symmetry Selection` $\rightarrow$ `Emergent Standard Model Fields & Interactions`

QEF + IOEP together determine which fields exist, their interactions, and spacetime geometry. Bulk spacetime and Standard Model are emergent phenomena arising from fundamental quantum entropy principles.

8 Conclusion

Starting from the QEF master equation, we derive:

1. Bulk spacetime geometry from boundary entanglement.
2. Stress-energy tensor T_{ab} as the source of geometry.
3. Emergence of bulk matter fields whose stress-energy satisfies QEF.
4. IOEP symmetry selection of allowed gauge groups, representations, and interactions, reproducing the Standard Model.

This provides a unified entanglement-first derivation of spacetime, gravity, and matter.

References

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