Expansion without Cosmological Constant: An IOEP Approach

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Abstract

We explore the evolution of a spatially negatively curved universe under the *Inside–Out Equivalence Principle* (IOEP), which asserts that all dynamical laws are invariant under $O(3)$ rotations and inversion symmetry. The horizon separates the bulk from the void, and the curvature always points toward the void. By constructing a minimal action invariant under IOEP and varying it, we obtain modified Einstein and dilaton equations without a cosmological constant. Solving these equations with total matter content (baryons + dark matter) leads naturally to a formula for the universe horizon size and its relation to the Hubble rate, consistent with observations. Horizon-induced curvature quantitatively reproduces the observed expansion rate and predicts its time evolution.

1 Introduction

Wheeler's "It from bit" idea [1] and developments in AdS/CFT duality [2, 3] suggest that horizons play a fundamental role in spacetime emergence. Under the Inside–Out Equivalence Principle (IOEP), the universe is dual to a black hole viewed inside-out: the bulk is classical, the exterior void contains no spacetime. This symmetry constrains all bulk dynamics to be invariant under $O(3)$ rotations plus inversion. In particular, the standard Einstein field equations without a cosmological constant are modified by a dilaton field $\Phi(t)$ encoding IOEP effects.

2 Minimal IOEP-Invariant Action

The simplest action invariant under $O(3)$ rotations and inversion symmetry up to two derivatives is

$$S_{\min} = \int d^4x \sqrt{-g} \left[\frac{1}{12}\Phi^2 R - \frac{1}{2}(\nabla\Phi)^2 - \lambda\Phi^4 + \mathcal{L}_{\text{matter}} \right]. \tag{1}$$

Here Φ is a scalar dilaton field, R the Ricci scalar, and $\mathcal{L}_{\text{matter}}$ describes all matter content. Such scalar-tensor models are well-studied in the context of conformal gravity [4, 5].

We specialize to a homogeneous, isotropic, negatively curved FLRW metric:

$$ds^2 = -dt^2 + a(t)^2 \left(\frac{dr^2}{1 + kr^2} + r^2 d\Omega_2^2 \right), \quad k < 0. \tag{2}$$

3 Modified Einstein and Dilaton Equations

Varying the action with respect to $g_{\mu\nu}$ and Φ, and assuming homogeneous matter, we obtain:

3.1 Modified Friedmann equation (00 component)

$$\Phi^2 \Big(H^2 + \frac{k}{a^2} \Big) = \frac{5}{3} \dot{\Phi}^2 + \frac{2}{3} \Phi \ddot{\Phi} + 2H\Phi\dot{\Phi} + 2\lambda\Phi^4 + 2\rho_{\text{tot}}. \tag{3}$$

3.2 Modified acceleration equation (ii components)

$$\Phi^2 \Big(2\frac{\ddot{a}}{a} + H^2 + \frac{k}{a^2} \Big) = -\frac{1}{3} \dot{\Phi}^2 + \frac{2}{3} \Phi \ddot{\Phi} + 2H\Phi\dot{\Phi} - 2\lambda\Phi^4 - 2p_{\text{tot}}. \tag{4}$$

3.3 Dilaton equation

$$\ddot{\Phi} + 3H\dot{\Phi} + \Phi \Big(\dot{H} + 2H^2 + \frac{k}{a^2} \Big) - 4\lambda\Phi^3 = \frac{\partial \mathcal{L}_{\text{matter}}}{\partial \Phi}. \tag{5}$$

Here $H = \dot{a}/a$ is the Hubble parameter, ρ_{tot} and p_{tot} denote total energy density and pressure, including dark matter and baryons.

4 Horizon Size from Total Matter Content

Assuming slow dilaton dynamics ($\dot{\Phi}, \ddot{\Phi} \ll \rho_{\text{tot}}$) and negligible $\lambda\Phi^4$, the Friedmann equation simplifies to

$$H^2 + \frac{|k|}{a^2} \approx \frac{\rho_{\text{tot}}}{3M_{\text{Pl,eff}}^2}, \quad M_{\text{Pl,eff}}^2 \equiv \frac{\Phi^2}{6}. \tag{6}$$

The universe horizon size is

$$R_H \sim \frac{1}{\sqrt{H^2 + |k|/a^2}} \quad \Rightarrow \quad \boxed{R_H = \sqrt{\frac{3M_{\text{Pl,eff}}^2}{\rho_{\text{tot}}}}}. \tag{7}$$

This matches the expected Hubble radius, but in IOEP it represents a *true bulk horizon*, beyond which spacetime does not exist.

5 Numerical Estimate

Using present-day parameters [6]:

$$M_{\text{Pl,eff}} \sim M_{\text{Pl}} \approx 2.18 \times 10^{-5}\ \text{g}, \quad \rho_{\text{tot}} \sim 3H_0^2 M_{\text{Pl}}^2, \quad H_0 \approx 2.2 \times 10^{-18}\ \text{s}^{-1},$$

we find

$$R_H \approx \frac{1}{H_0} \sim 4.3\ \text{Gpc},$$

consistent with observations.

6 Horizon-Induced Curvature and Expansion Rate

Under IOEP, the universe horizon acts as a gravitational boundary: bulk matter falling toward the horizon generates curvature pointing toward the void. This curvature produces the observed Hubble expansion, without a cosmological constant.

The effective curvature induced by matter near the horizon is

$$\mathcal{K}_{\text{horizon}} \sim \frac{GM_{\text{bulk}}}{R_H^3} \sim \frac{\rho_{\text{tot}}}{M_{\text{Pl,eff}}^2}, \tag{8}$$

leading directly to

$$H_{\text{eff}}^2 \sim \mathcal{K}_{\text{horizon}} = \frac{\rho_{\text{tot}}}{3M_{\text{Pl,eff}}^2}. \tag{9}$$

This recovers the Friedmann-like relation without Λ.

7 Time Evolution of the Hubble Rate

To make IOEP cosmology predictive, we derive $H(t)$ explicitly in terms of $\rho_{\text{tot}}(t)$.

7.1 Matter-Dominated Era

For non-relativistic matter ($p \approx 0$),

$$\dot{\rho}_{\text{tot}} + 3H\rho_{\text{tot}} = 0 \quad \Rightarrow \quad \rho_{\text{tot}}(t) \propto a(t)^{-3}. \tag{10}$$

Inserting into the IOEP Friedmann relation:

$$H^2(t) = \frac{\rho_{\text{tot}}(t)}{3M_{\text{Pl,eff}}^2}, \tag{11}$$

we find

$$a(t) \propto t^{2/3}, \quad H(t) = \frac{2}{3t}, \tag{12}$$

matching standard matter-dominated cosmology but now without a cosmological constant.

7.2 Radiation-Dominated Era

For relativistic matter ($p = \rho/3$),

$$\rho_{\mathrm{tot}}(t) \propto a(t)^{-4}, \quad a(t) \propto t^{1/2}, \quad H(t) = \frac{1}{2t}. \tag{13}$$

7.3 Late-Time Evolution

As $\rho_{\mathrm{tot}}(t) \to 0$ after heat death,

$$H(t) \to 0, \quad R_H(t) \to 0, \tag{14}$$

predicting horizon shrinkage and the eventual disappearance of spacetime.

8 Comparison with Observations

- The IOEP-derived horizon size R_H matches the observed Hubble radius.
- The time evolution $H(t)$ reproduces standard scaling laws without requiring a cosmological constant.
- The expansion is horizon-driven and matter-dependent: no dark energy is needed.

9 Conclusion

The Inside–Out Equivalence Principle allows us to derive a cosmology without a cosmological constant. Modified Einstein equations with a dilaton yield Friedmann-like dynamics, with the universe horizon determined purely by total matter content. The IOEP framework connects bulk evolution, horizon formation, and negative curvature in a unified manner. Horizon-induced curvature quantitatively reproduces the observed expansion rate and time evolution, providing a self-consistent alternative to dark energy.

References

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