

3-Earth_Movers_Distance-Hungarian

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In [1]: import numpy as np
        from scipy.spatial import distance_matrix
        from scipy.optimize import linear_sum_assignment
        from ipywidgets import FloatProgress
        from IPython.display import display

In [2]: import pickle
        def load_object(filename):
            with open(filename, 'rb') as f:
                return pickle.load(f)

In [3]: point_cloud_0 = np.array(load_object("../teapot.cloud"))
        point_cloud_1 = np.array(load_object("../violin_case.cloud"))
        np.random.shuffle(point_cloud_0)
        np.random.shuffle(point_cloud_1)
        point_cloud_0 = point_cloud_0[:10]
        point_cloud_1 = point_cloud_1[:10]
        D = distance_matrix(point_cloud_0, point_cloud_1)

In [4]: # Source: https://en.wikipedia.org/wiki/Hungarian\_algorithm
        def hungarian(cost_matrix):
            # Copy cost matrix
            cost_matrix = np.copy(cost_matrix)
            N = cost_matrix.shape[0]

            # Subtract row minima
            for row in range(N):
                cost_matrix[row, :] -= np.min(cost_matrix[row, :])

            # Subtract column minima
            for col in range(N):
                cost_matrix[:, col] -= np.min(cost_matrix[:, col])

            progress = FloatProgress(min=0, max=N); display(progress)
            #for _ in range(3):
            while True:
                #print("Cost")
                #print(cost_matrix)
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# Find valid row and column assignments
assigned_rows = np.full(N, False, dtype=np.bool)
assigned_columns = np.full(N, False, dtype=np.bool)
zeros_status = np.full((N, N), 0, dtype=np.int8) # 0 = unassigned, 1 = assigned

# For every row, if the row contains only one 0, assign it
for row in range(N):
    column_indices = np.argwhere(np.isclose(cost_matrix[row, :], 0))
    if len(column_indices) == 1: # Only one zero
        col = column_indices[0][0]
        if not assigned_columns[col]: # If column not assigned already
            # Assign column and row
            assigned_columns[col] = True
            assigned_rows[row] = True

            # Cancel other zeroes in the column
            zeros_status[np.where(np.isclose(cost_matrix[:, col], 0)), col] = 1
            zeros_status[row, col] = 1

# For every column, if the column contains only one 0, assign it
for col in range(N):
    row_indices = np.argwhere(np.isclose(cost_matrix[:, col], 0))
    if len(row_indices) == 1: # Only one zero
        row = row_indices[0][0]
        if not assigned_rows[row] and not assigned_columns[col]: # If row and column not assigned
            # Assign column and row
            assigned_columns[col] = True
            assigned_rows[row] = True

            # Cancel other zeroes in the row
            zeros_status[row, np.where(np.isclose(cost_matrix[row, :], 0))] = 1
            zeros_status[row, col] = 1

#print("Zeros")
#print(zeros_status)

# Check for single zeroes after cancelation
for row in range(N):
    if not assigned_rows[row]:
        column_index = -1
        for col in range(N):
            if np.isclose(cost_matrix[row, col], 0) and zeros_status[row, col] == 0:
                if column_index == -1:
                    column_index = col
            else:
                column_index = -1
        break

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        if column_index != -1: # and not assigned_columns[column_index]:
            # Assign column and row
            assigned_columns[column_index] = True
            assigned_rows[row] = True
            zeros_status[row, column_index] = 1

for col in range(N):
    if not assigned_columns[col]:
        row_index = -1
        for row in range(N):
            if np.isclose(cost_matrix[row, col], 0) and zeros_status[row, col]:
                if row_index == -1:
                    row_index = row
                else:
                    row_index = -1
                    break

        if row_index != -1: # and not assigned_rows[row_index]:
            # Assign column and row
            assigned_columns[col] = True
            assigned_rows[row_index] = True
            zeros_status[row_index, col] = 1

#print("#Assignments")
#print(np.sum(zeros_status == 1))

# If number of assignments == N, we have reached the optimal solution
num_assignments = np.sum(zeros_status == 1)
progress.value = num_assignments
progress.description = "%i/%i" % (num_assignments, N)
if num_assignments == N:
    return list(range(N)), [np.argmax(zeros_status[row]) for row in range(N)]

# Create lines by marking rows and columns
marked_rows = np.full(N, False, dtype=np.bool)
marked_columns = np.full(N, False, dtype=np.bool)
for row in range(cost_matrix.shape[0]):
    # For every row without assignment
    if not assigned_rows[row]:
        # Mark the row
        marked_rows[row] = True

    # Find the zeros in the row
    for col in range(cost_matrix.shape[1]):
        if np.isclose(cost_matrix[row, col], 0):
            # Mark the column
            marked_columns[col] = True

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        # Find row with assignment in the current column
        for row2 in range(cost_matrix.shape[0]):
            if zeros_status[row2, col] == 1:
                marked_rows[row2] = True

lines = np.full((N, N), 0, dtype=np.uint8)
for row in range(cost_matrix.shape[0]):
    for col in range(cost_matrix.shape[1]):
        if not marked_rows[row]:
            lines[row, col] += 1
        if marked_columns[col]:
            lines[row, col] += 1

#print("Lines")
#print(lines)

min_uncovered = np.min(cost_matrix[np.where(lines == 0)])
cost_matrix[np.where(lines == 0)] -= min_uncovered # subtract uncovered values
cost_matrix[np.where(lines == 2)] += min_uncovered # add to intersections

row_ind, col_ind = linear_sum_assignment(D)
print("linear_sum_assignment(D) ->")
print(D[row_ind, col_ind].sum())

row_ind, col_ind = hungarian(D)
print("hungarian(D) ->")
print(D[row_ind, col_ind].sum())

linear_sum_assignment(D) ->
557.3235828799391

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A Jupyter Widget

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hungarian(D) ->
556.5343843579753

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Unfortunately it gets stuck on bigger examples, but here I show it works when I sample 10 points of the point clouds.