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Note about Optimal Substructure

Chapters 5 of Dasgupta *et al.*

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Optimal substructure

Let S' be the remaining sub-problem after the greedy choice.

A' is an optimal solution for S'
$\Leftrightarrow$
$A = A' \cup \{greedy\}$ is an optimal solution for S

Proofs in class:

- Activity selection (both $\Leftarrow$ and $\Rightarrow$)
- Fractional knapsack ($\Leftarrow$)

Homework/exams: it is sufficient to prove $\Rightarrow$

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Activity selection ($\Longrightarrow$)

Claim. If *A'* is an optimal solution for *S'* = {*i* in *S*: $s_i \geq f_1$ } then *A'* $\cup$ {*1*} is an optimal solution for *S*.

Proof (by contradiction). If *A=A'*$\cup${*1*} is not optimal for *S*, then there is a solution *B=B'*$\cup${*1*} for *S* with more activities than *A*. *B'* is a solution for *S'* because the activities in *B'* are non overlapping. But |*B'*|>|*A'*|, which contradicts the optimality of *A'*.

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Activity selection ($\Longleftarrow$)

Claim. If *A* is optimal to *S* and *A* contains task *1*, then *A'* = *A* – {*1*} is optimal to *S'*={*i* in *S*: $s_i \geq f_1$}

Proof (by contradiction). If *A'* is not optimal for *S'*, then there is a solution *B'* to *S'* with more activities than *A'*. Adding activity 1 to *B'* would yield a solution *B* to *S* with more activities than *A* contradicting the optimality of *A*.

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