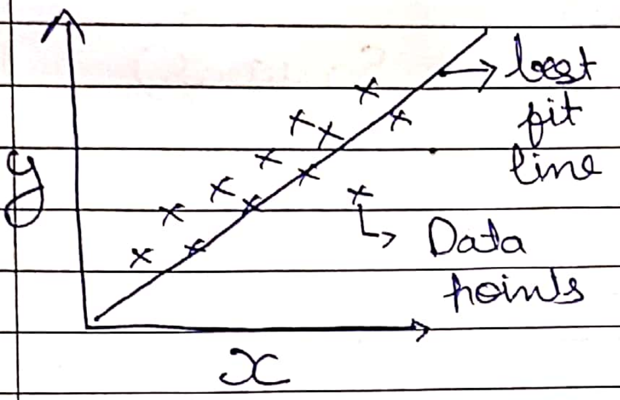
Linear Regression



Simplest Linear regression is of the form

$$\boxed{\underset{\text{target}}{y} = \underset{\text{slope or weight}}{m}\,\underset{\text{feature}}{x} + \underset{\text{intercept/bias}}{b}}$$

This was for a dataset having one feature, one target. Such datasets are rarely used in practice (Time series is similar but different).

So let expand our intuition when we have multiple features.

$$\underset{\text{target}}{y} = \theta_1 x_1 + \theta_2 x_2 \;\cdots\; \theta_n x_n = \underbrace{X^T\theta}_{\text{vectorized implementation}}$$

slope/ weights — θ_1, θ_2

features — $x_2 \cdots x_n$

$x_1 = \cancel{\theta}\ 1$

↳ bias term

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Two ways to update weights and biases are:

1) OLS (Oridinary Least Squares)
2) Gradient Descent.

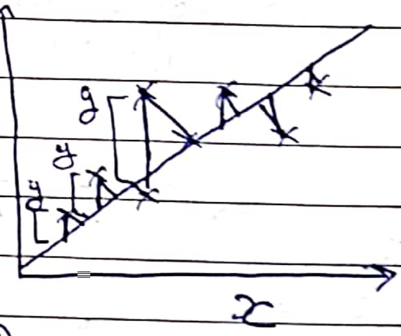
Ordinary Least Squares

Slope $\boxed{m = \dfrac{\sum_{i=0}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=0}^{n}(x_i - \bar{x})^2}}$

bias $\boxed{b = \bar{Y} - m\bar{X}}$ where $\bar{x}, \bar{y}$ = mean

Lets now prove these:

$$\text{error} = (y_1 - \hat{y})^2 + (y_2 - \hat{y})^2 \cdots + (y_n - \hat{y})^2$$



$$\text{error} = \sum_{i=1}^{n}(y_i - \hat{y})^2 \quad \text{—} (1)$$

But we know $\hat{y}_i = mx_i + b_i$ —(2)

Using (2) in (1)

$$\text{error} = \sum_{i=1}^{n}(y_i - (mx_i + b_i))^2$$

To reduce this error, we have to differentiate the equation wrt to m and b (as they can only reduce the error)

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"It is the prime responsibility of every citizen to feel that his country." —Sardar Vallabhai Patel

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So

$$\frac{d}{db}\sum(y_i-(mx_i+b_i))^2=0$$

and

$$\frac{d}{dm}\sum(y_i-(mx_i+b_i))^2=0$$

Lets solve this

$$\sum 2(y_i-(mx_i+b_i))(-1)=0$$

~~we~~

$$\sum(y_i-(mx_i+b_i))=0$$

this is just nuber of features × bias

$\sum b_i = nb$ — (3)

using (3) we get

$$\sum y_i-(mx_i+nb)=0$$

$$\sum y_i-\sum mx_i - nb=0$$

Divide by n we get

$$\frac{\sum y_i}{n} - m\frac{\sum x_i}{n} - b = 0$$

$$\bar{y} - m\bar{x} - b = 0$$

$$\boxed{b=\bar{y}-m\bar{x}}$$

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Now,

$$\frac{d}{dm} \Sigma (y_i - (mx_i + b_i))^2 \quad -①$$

Now from previous we know

$$b_i = \bar{y} - m\bar{x} \quad -②$$

Lets use ② in ①

$$\frac{d}{dm} \Sigma (y_i - (mx_i + \bar{y} - m\bar{x}))^2 = 0$$

$$\Sigma 2(y_i - (mx_i + \bar{y} - m\bar{x}) \cdot (x_i - \bar{x})) = 0$$

$$\downarrow$$

$$\Sigma (y_i - (mx_i + \bar{y} - m\bar{x}) (x_i - \bar{x}) = 0$$

$$\Sigma (y_i - \bar{y}) - m(x_i - \bar{x})) (x_i - \bar{x}) = 0$$

$$\downarrow$$

$$\Sigma (y_i - \bar{y})(x_i - \bar{x}) = m \Sigma (x_i - \bar{x})^2$$

$$\downarrow$$

$$\boxed{m = \frac{\sum_{i=0}^{n} (y_i - \bar{y}) (x_i - \bar{x})}{\sum_{i=0}^{n} (x_i - \bar{x})^2}}$$

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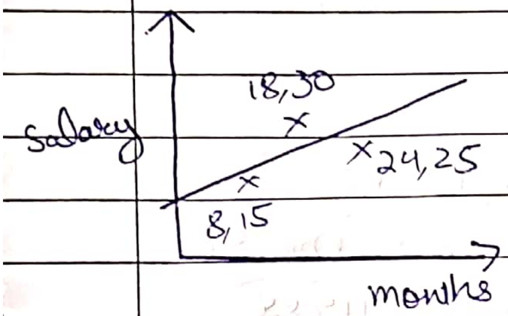
Gradient Descent

- It is an optimization technique

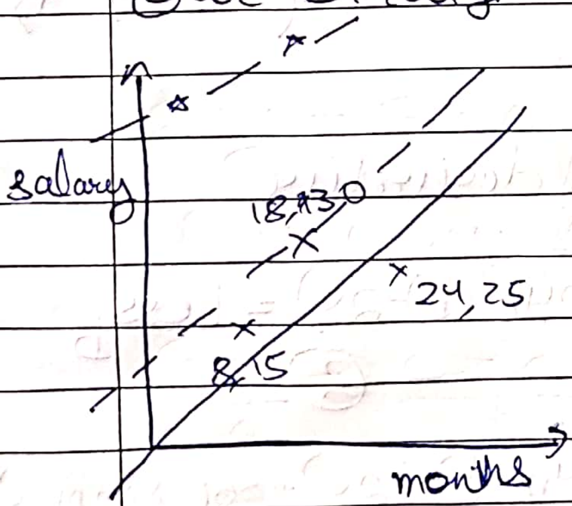


we have to find the line ($y=mx+b$) which reduces the residual error sum.

For now lets assume m is fixed ; $\boxed{m=1}$

$$error = (y_1-\hat{y}_1)^2 + (y_2-\hat{y}_2)^2 + (y_3-\hat{y}_3)^2 \quad -(1)$$

Our straight line equation at $m=1$ is $y=x+b$



-*- $m=1, b=0$

--- $m=1, b=5$

—— $m=1, b=20$

b	error
0	194
5	69
⋮	
20	500

we calculated error using (1) since we know both m and b and actual $\hat{y}$

"Education is the manifestation of perfection already in man." —Swami Vivekananda

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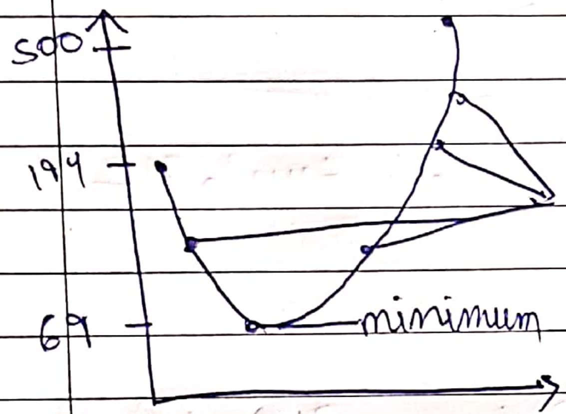
If we plot this bias and error (Loss) we get



values we get if we solve for more we get this curve

Lets do some math!

$$Loss = \sum_{i=1}^{n}(y_i - \hat{y}_i)^2$$

So for our salary graph:

$$Loss = ((8m+b)-15)^2 + (24m+b)-25)^2 + ((18m+b)-30)^2$$

we start filling some values

Lets calculate the partial derivative

$$\frac{df(m,b)}{db} = 2((8m+b)-15) \times (1) + 2((24m+b)-25) + 2((18m+b)-30) = Loss_b \quad -(2)$$

$$\frac{df(m,b)}{dm} = 2((8m+b)-15)\cdot 8 + 48((24m+b)-25) + 36((18m+b)-30) \quad -(3)$$

$$= Loss_m$$

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Initialise m, and b with some value say

$m=1$, $b=2$

and suppose we get $Loss_b = -20$

$Loss_m = -13$

I multiply $Loss_b$ and Loss m with α (Learning rate), Let α be 0.01

$Step_b = -20 \times 0.01 = -0.02$

$Step_m = -13 \times 0.01 = -0.013$

we get new value for b and m

$n_b = old_b - Step_b = 2 - (-0.02) = 2.02$

$n_m = old_m - Step_m = 1 - (-0.013) = 1.013$

we keep updating this to reach the min