

$$H_1^0 = \begin{bmatrix} \cos \theta_1 & -\sin \theta_1 & 0 \\ \sin \theta_1 & \cos \theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$H_2^1 = \begin{bmatrix} 1 & 0 & l_1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$H_3^2 = \begin{bmatrix} \cos \theta_2 & -\sin \theta_2 & 0 \\ \sin \theta_2 & \cos \theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$H_4^3 = \begin{bmatrix} 1 & 0 & l_2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$H_4^0 = H_1^0 \cdot H_2^1 \cdot H_3^2 \cdot H_4^3$$

$$H_4^0 = \begin{bmatrix} \cos \theta_1 & -\sin \theta_1 & 0 \\ \sin \theta_1 & \cos \theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & l_1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos \theta_1 & -\sin \theta_1 & l_1 \cos \theta_1 \\ \sin \theta_1 & \cos \theta_1 & l_1 \sin \theta_1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$H_3^0 = \begin{bmatrix} \cos \theta_1 & -\sin \theta_1 & l_1 \cos \theta_1 \\ \sin \theta_1 & \cos \theta_1 & l_1 \sin \theta_1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \theta_2 & -\sin \theta_2 & 0 \\ \sin \theta_2 & \cos \theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos \theta_1 \cdot \cos \theta_2 + (-\sin \theta_1 \cdot \sin \theta_2) \\ \cos \theta_1 \cdot \sin \theta_2 - \sin \theta_1 \cdot \cos \theta_2 \\ \sin \theta_1 \cdot \cos \theta_2 + \cos \theta_1 \cdot \sin \theta_2 \\ \sin \theta_1 \cdot \sin \theta_2 + \cos \theta_1 \cdot \cos \theta_2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos \theta_1 \cdot \cos \theta_2 + (-\sin \theta_1 \cdot \sin \theta_2) & -\sin \theta_2 \cos \theta_1 + (-\sin \theta_1 \cos \theta_2) & 1, \cos \theta_1 \\ \sin \theta_1 \cdot \cos \theta_2 + \cos \theta_1 \cdot \sin \theta_2 & -\sin \theta_1 \sin \theta_2 + \cos \theta_1 \cos \theta_2 & 1, \sin \theta_1 \\ 0 & 0 & 1 \end{bmatrix}$$

are identical

$$= \begin{bmatrix} \cos(\theta_1 + \theta_2) & -\sin(\theta_1 + \theta_2) & 1, \cos \theta_1 \\ \sin(\theta_1 + \theta_2) & \cos(\theta_1 + \theta_2) & 1, \sin \theta_1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\sin(\theta_1 + \theta_2) = \sin \theta_1 \cos \theta_2 + \sin \theta_2 \cos \theta_1$$

$$\cos(\theta_1 + \theta_2) = \cos \theta_1 \cos \theta_2 - \sin \theta_1 \sin \theta_2$$

$$H_u^0 = \begin{bmatrix} \cos(\theta_1 + \theta_2) & -\sin(\theta_1 + \theta_2) \\ \sin(\theta_1 + \theta_2) & \cos(\theta_1 + \theta_2) \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} l_2 \cos(\theta_1 + \theta_2) + l_1 \cos \theta_1 \\ l_2 \sin(\theta_1 + \theta_2) + l_1 \sin \theta_1 \\ 1 \end{bmatrix}$$

Substituting

$$H_u^0 = \begin{bmatrix} \cos(\theta_1 + \theta_2) & -\sin(\theta_1 + \theta_2) \\ \sin(\theta_1 + \theta_2) & \cos(\theta_1 + \theta_2) \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} l_1 \cos \theta_1 + l_2 \cos(\theta_1 + \theta_2) \\ l_1 \sin \theta_1 + l_2 \sin(\theta_1 + \theta_2) \\ 1 \end{bmatrix}$$