

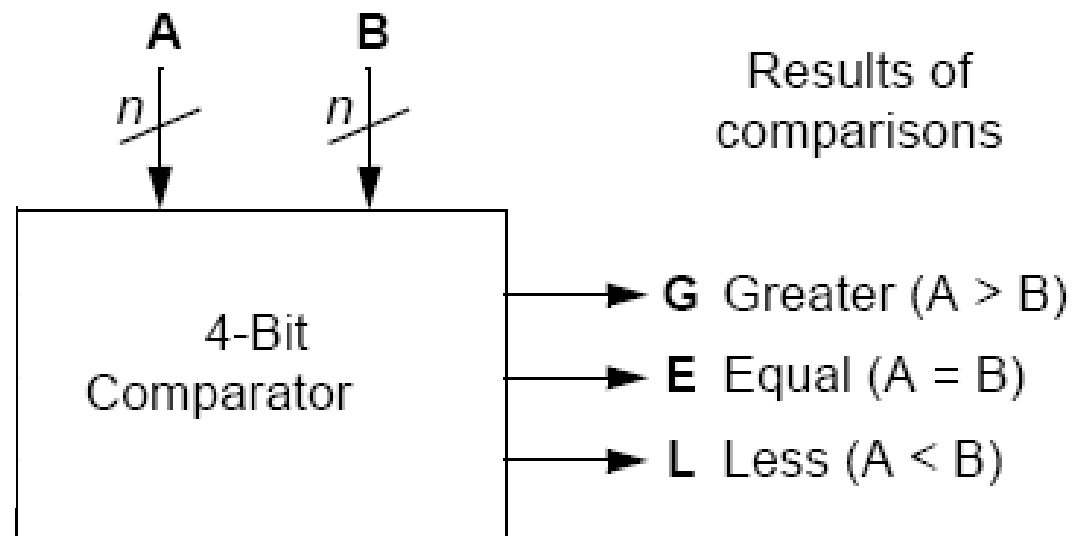
**Back to Combinational Logic Ckt design**

# COMPARATORS

## MAGNITUDE COMPARATOR

- Given two  $n$ -bit magnitudes,  $A$  and  $B$ , a comparator indicates whether
  - $A = B$ ,  $A > B$ , or  $A < B$

$n$ -bit input magnitudes



- The approach is to use the XNOR function (equivalence) on each of the  $n$ -bits as follows

$$x_i = \mathbf{A}_i \mathbf{B}_i + \overline{\mathbf{A}_i} \overline{\mathbf{B}_i} = \overline{\mathbf{A}_i \oplus \mathbf{B}_i}$$

- The Boolean functions for a 4-bit magnitude comparator is as follows
  - $(\mathbf{A} = \mathbf{B}) = x_3 x_2 x_1 x_0$
  - $(\mathbf{A} > \mathbf{B}) = \mathbf{A}_3 \overline{\mathbf{B}_3} + x_3 \mathbf{A}_2 \overline{\mathbf{B}_2} + x_3 x_2 \mathbf{A}_1 \overline{\mathbf{B}_1} + x_3 x_2 x_1 \mathbf{A}_0 \overline{\mathbf{B}_0}$
  - $(\mathbf{A} < \mathbf{B}) = \overline{\mathbf{A}_3} \mathbf{B}_3 + x_3 \overline{\mathbf{A}_2} \mathbf{B}_2 + x_3 x_2 \overline{\mathbf{A}_1} \mathbf{B}_1 + x_3 x_2 x_1 \overline{\mathbf{A}_0} \mathbf{B}_0$

Note:  $\mathbf{A}_i \overline{\mathbf{B}_i}$  indicates whether  $\mathbf{A}_i > \mathbf{B}_i$ ,  $\overline{\mathbf{A}_i} \mathbf{B}_i$  indicates whether  $\mathbf{A}_i < \mathbf{B}_i$ , and  $x_i$  indicates whether  $\mathbf{A}_i = \mathbf{B}_i$ .

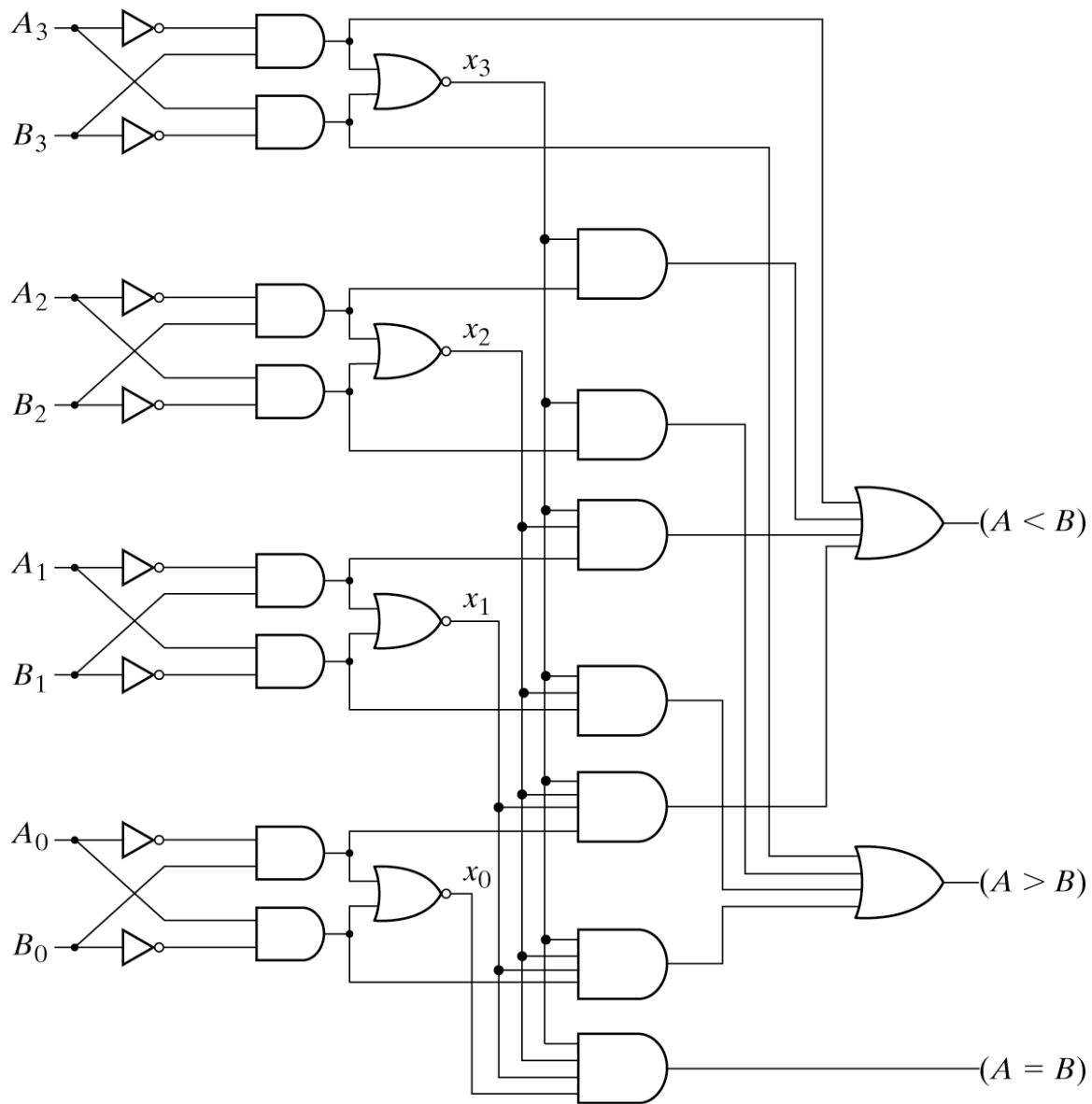
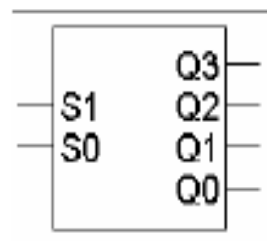


Fig. 4-17 4-Bit Magnitude Comparator

# DECODERS

## What a decoder does

- A  **$n$ -to- $2^n$  decoder** uses its  $n$ -bit input to determine which of  $2^n$  outputs will be uniquely activated.



| S1 | S0 | Q0 | Q1 | Q2 | Q3 |
|----|----|----|----|----|----|
| 0  | 0  | 1  | 0  | 0  | 0  |
| 0  | 1  | 0  | 1  | 0  | 0  |
| 1  | 0  | 0  | 0  | 1  | 0  |
| 1  | 1  | 0  | 0  | 0  | 1  |

- Here is a block diagram and truth table for a **2-to-4 decoder**.
  - The two-bit input is called **S1S0**, and the four outputs are **Q0-Q3**.
  - If the input is the binary number  $i$ , then output  $Q_i$  alone will be true.
- This circuit “decodes” a binary number into a “one-of-four” code.

## Building a decoder

---

- We can use the truth table to derive minimal sum of products equations for each of the four outputs (Q0-Q3), based on the two inputs (S0-S1).

| S1 | S0 | Q0 | Q1 | Q2 | Q3 |
|----|----|----|----|----|----|
| 0  | 0  | 1  | 0  | 0  | 0  |
| 0  | 1  | 0  | 1  | 0  | 0  |
| 1  | 0  | 0  | 0  | 1  | 0  |
| 1  | 1  | 0  | 0  | 0  | 1  |

- In this case there's not much to be simplified. Here are the equations:

$$Q0 = S1'S0'$$

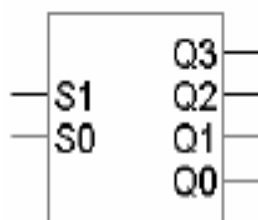
$$Q1 = S1'S0$$

$$Q2 = S1S0'$$

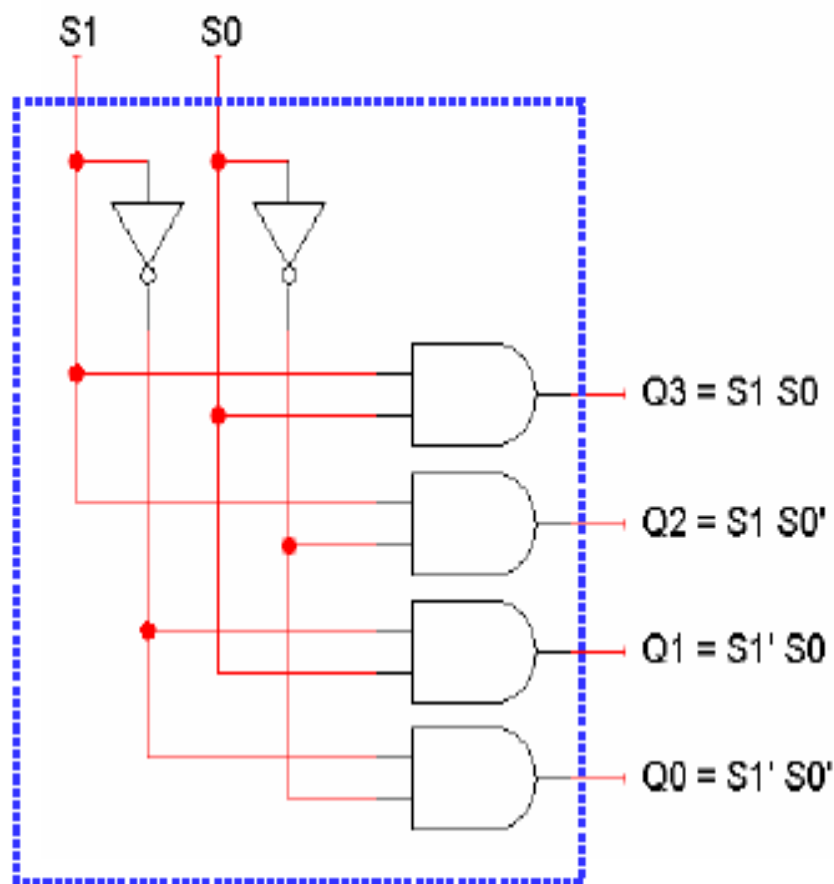
$$Q3 = S1S0$$

## Decoder circuit diagram

- Here is an implementation of a 2-to-4 decoder.



| S1 | S0 | Q0 | Q1 | Q2 | Q3 |
|----|----|----|----|----|----|
| 0  | 0  | 1  | 0  | 0  | 0  |
| 0  | 1  | 0  | 1  | 0  | 0  |
| 1  | 0  | 0  | 0  | 1  | 0  |
| 1  | 1  | 0  | 0  | 0  | 1  |

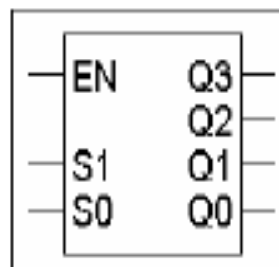




## Enable inputs

decoders can include **enable inputs**.

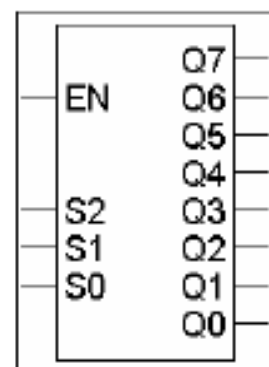
- **EN=0** disables the decoder, which by convention means that all of the decoder's outputs are 0.
- **EN=1** enables the decoder so that it behaves as specified earlier, with exactly one of the outputs being 1.



| EN | S1 | S0 | Q0 | Q1 | Q2 | Q3 |
|----|----|----|----|----|----|----|
| 0  | x  | x  | 0  | 0  | 0  | 0  |
| 1  | 0  | 0  | 1  | 0  | 0  | 0  |
| 1  | 0  | 1  | 0  | 1  | 0  | 0  |
| 1  | 1  | 0  | 0  | 0  | 1  | 0  |
| 1  | 1  | 1  | 0  | 0  | 0  | 1  |

## A 3-to-8 decoder

- Larger decoders are similar. Here is a 3-to-8 decoder.
  - There are three selection inputs **S2S1S0**, which activate one of eight outputs, **Q0-Q7**.
  - Again, only one output will be true for any input combination.
- A truth table and output equations for a 3-to-8 decoder (without EN) are given below.



| S2 | S1 | S0 | Q0 | Q1 | Q2 | Q3 | Q4 | Q5 | Q6 | Q7 |
|----|----|----|----|----|----|----|----|----|----|----|
| 0  | 0  | 0  | 1  | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 0  | 0  | 1  | 0  | 1  | 0  | 0  | 0  | 0  | 0  | 0  |
| 0  | 1  | 0  | 0  | 0  | 1  | 0  | 0  | 0  | 0  | 0  |
| 0  | 1  | 1  | 0  | 0  | 0  | 1  | 0  | 0  | 0  | 0  |
| 1  | 0  | 0  | 0  | 0  | 0  | 0  | 1  | 0  | 0  | 0  |
| 1  | 0  | 1  | 0  | 0  | 0  | 0  | 0  | 1  | 0  | 0  |
| 1  | 1  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 1  | 0  |
| 1  | 1  | 1  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 1  |

$$Q0 = S2'S1'S0'$$

$$Q1 = S2'S1'S0$$

$$Q2 = S2'S1S0'$$

$$Q3 = S2'S1S0$$

$$Q4 = S2S1'S0'$$

$$Q5 = S2S1'S0$$

$$Q6 = S2S1S0'$$

$$Q7 = S2S1S0$$

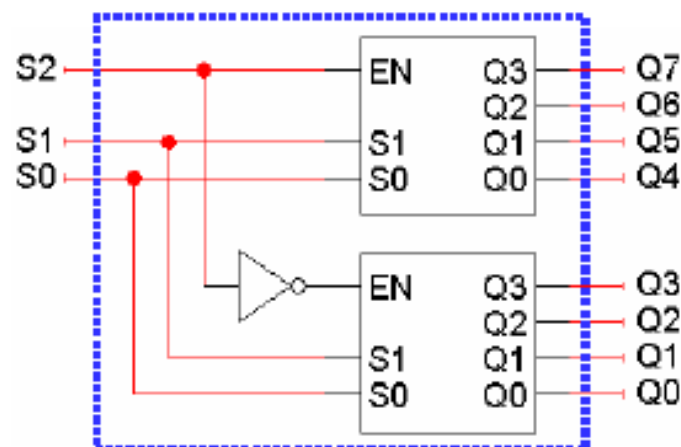
## Building a 3-to-8 decoder

- You could build a 3-to-8 decoder from the truth table and MSP equations below, just like we built the 2-to-4 decoder earlier.
- Another way to design a decoder is to break it into smaller pieces.
- Notice some patterns in the table below:
  - When  $S_2 = 0$ , outputs  $Q_0$ - $Q_3$  are generated as in a 2-to-4 decoder.
  - When  $S_2 = 1$ , outputs  $Q_4$ - $Q_7$  are generated as in a 2-to-4 decoder.

| S2 | S1 | S0 | Q0 | Q1 | Q2 | Q3 | Q4 | Q5 | Q6 | Q7 |
|----|----|----|----|----|----|----|----|----|----|----|
| 0  | 0  | 0  | 1  | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 0  | 0  | 1  | 0  | 1  | 0  | 0  | 0  | 0  | 0  | 0  |
| 0  | 1  | 0  | 0  | 0  | 1  | 0  | 0  | 0  | 0  | 0  |
| 0  | 1  | 1  | 0  | 0  | 0  | 1  | 0  | 0  | 0  | 0  |
| 1  | 0  | 0  | 0  | 0  | 0  | 0  | 1  | 0  | 0  | 0  |
| 1  | 0  | 1  | 0  | 0  | 0  | 0  | 0  | 1  | 0  | 0  |
| 1  | 1  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 1  | 0  |
| 1  | 1  | 1  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 1  |

## Decoder expansion

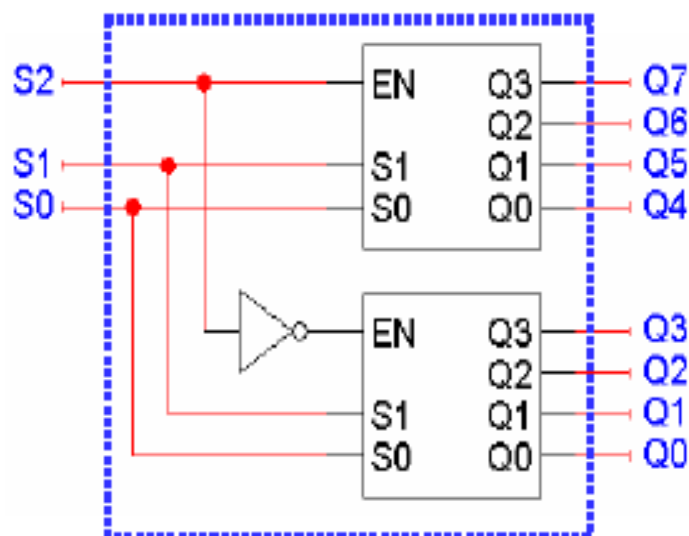
- Here's a 3-to-8 decoder built from two smaller 2-to-4 decoders.
- When  $S2=0$ , the bottom 2-to-4 decoder is enabled and generates a 1 for one of outputs  $Q0$ ,  $Q1$ ,  $Q2$  or  $Q3$ .
- When  $S2=1$ , the top 2-to-4 decoder is enabled instead, and a 1 will be output for either  $Q4$ ,  $Q5$ ,  $Q6$  or  $Q7$ .



| S2 | S1 | S0 | Q0 | Q1 | Q2 | Q3 | Q4 | Q5 | Q6 | Q7 |
|----|----|----|----|----|----|----|----|----|----|----|
| 0  | 0  | 0  | 1  | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 0  | 0  | 1  | 0  | 1  | 0  | 0  | 0  | 0  | 0  | 0  |
| 0  | 1  | 0  | 0  | 0  | 1  | 0  | 0  | 0  | 0  | 0  |
| 0  | 1  | 1  | 0  | 0  | 0  | 1  | 0  | 0  | 0  | 0  |
| 1  | 0  | 0  | 0  | 0  | 0  | 0  | 1  | 0  | 0  | 0  |
| 1  | 0  | 1  | 0  | 0  | 0  | 0  | 0  | 1  | 0  | 0  |
| 1  | 1  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 1  | 0  |
| 1  | 1  | 1  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 1  |

# Modularity

- You could verify that this circuit is a 3-to-8 decoder, by using equations for the 2-to-4 decoders to derive equations for the 3-to-8.



## So what good is a decoder?

---

- Do the truth table and equations look familiar?

| S1 | S0 | Q0 | Q1 | Q2 | Q3 |               |
|----|----|----|----|----|----|---------------|
| 0  | 0  | 1  | 0  | 0  | 0  | $Q0 = S1'S0'$ |
| 0  | 1  | 0  | 1  | 0  | 0  | $Q1 = S1'S0$  |
| 1  | 0  | 0  | 0  | 1  | 0  | $Q2 = S1 S0'$ |
| 1  | 1  | 0  | 0  | 0  | 1  | $Q3 = S1 S0$  |

- Decoders are sometimes called **minterm generators**.
  - For each input combination, exactly one output is true.
  - Each output equation contains all of the input variables.
- This means that you can easily use a decoder, or a minterm generator, to implement any sum of minterms expression.

## Example: addition

---

- A truth table and sum of minterm equations for  $C$  and  $S$  are shown below.

| X | Y | Z | C | S |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 |
| 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 |

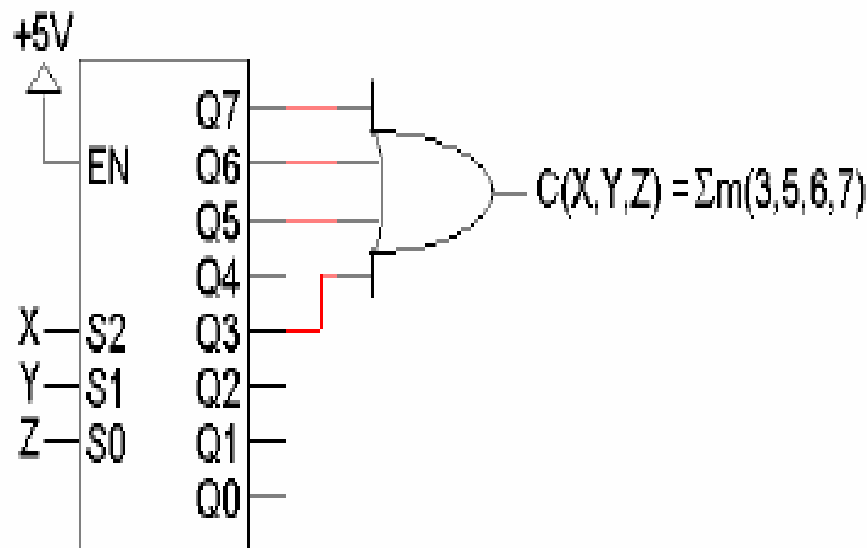
$$C(X,Y,Z) = \sum m(3,5,6,7)$$

$$S(X,Y,Z) = \sum m(1,2,4,7)$$

- Today we'll implement these two functions using 3-to-8 decoders.

# Implementing functions with decoders

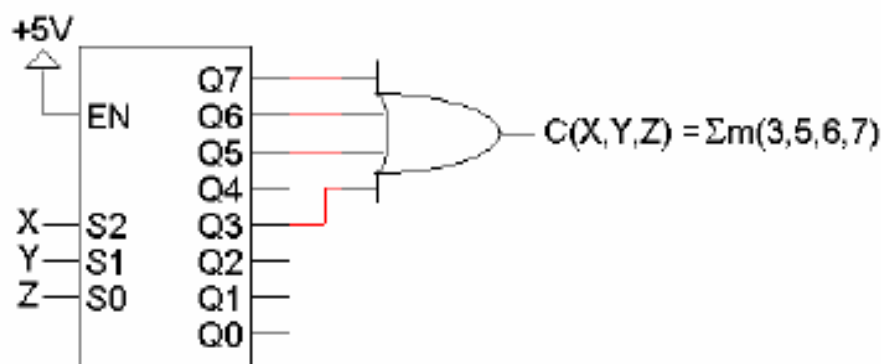
- Here, a 3-to-8 decoder implements  $C$  as a sum of minterms.



- If  $XYZ$  is 011, 101, 110 or 111, then one of the decoder outputs  $Q_3, Q_5, Q_6$  or  $Q_7$  will be true, and the output  $C(X,Y,Z)$  will also be true.



## Verifying our adder



- C is the sum of decoder outputs Q3, Q5, Q6 and Q7.

$$C = Q3 + Q5 + Q6 + Q7$$

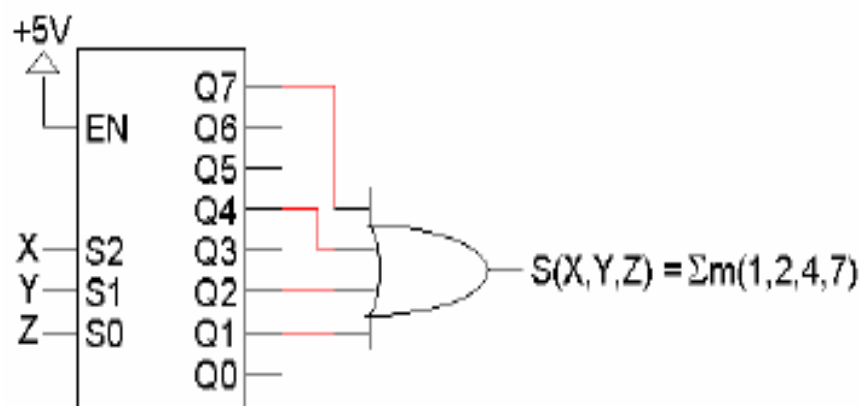
- We know how the decoder's outputs are generated from its inputs.

$$C = S2'S1S0 + S2S1'S0 + S2S1S0' + S2S1S0$$

- We can substitute XYZ for S2S1S0, resulting in a sum of minterms

$$C = X'YZ + XY'Z + XYZ' + XYZ$$

## Decoder-based sum

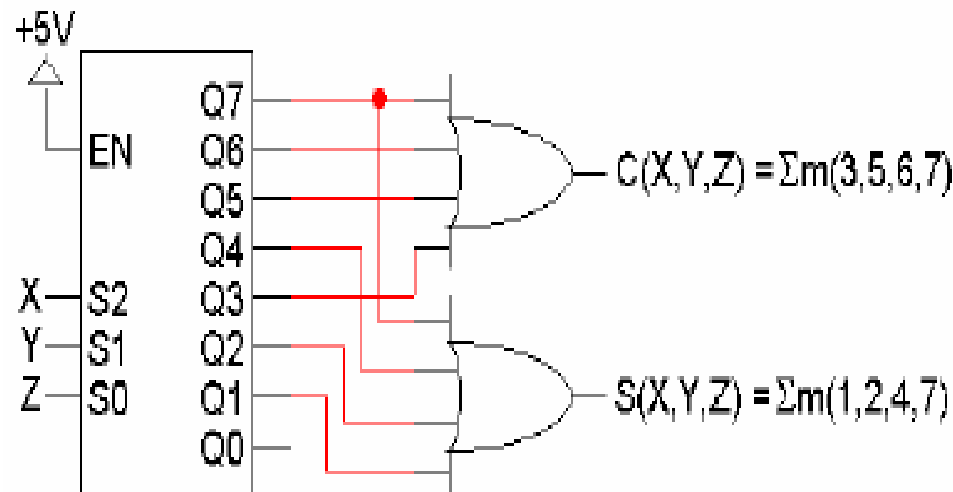


- If XYZ is 001, 010, 100 or 111, then one of decoder outputs Q1, Q2, Q4 or Q7 will be true, and  $S(X,Y,Z)$  will be true as well.
- We can verify this circuit algebraically, just like before.

$$\begin{aligned} S(X,Y,Z) &= Q1 + Q2 + Q4 + Q7 \\ &= S2'S1'S0 + S2'S1S0' + S2S1'S0' + S2S1S0 \\ &= X'Y'Z + X'YZ' + XY'Z' + XYZ \\ &= m_1 + m_2 + m_4 + m_7 \end{aligned}$$

# Using just one decoder

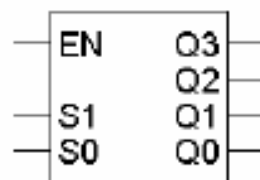
- Since the two functions  $C$  and  $S$  both have the same inputs, we could use just one decoder instead of two.



- Decoder output  $Q_0$  is unused, while  $Q_7$  is used multiple times. In general, you can always use circuit outputs as many or as few times as you need.

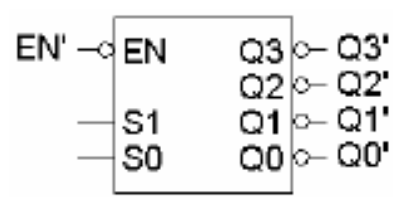
## A variation of the standard decoder

- The decoders we've seen so far are **active-high** decoders.



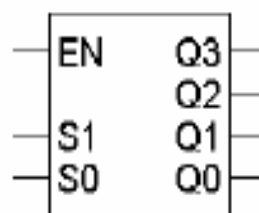
| EN | S1 | S0 | Q0 | Q1 | Q2 | Q3 |
|----|----|----|----|----|----|----|
| 0  | x  | x  | 0  | 0  | 0  | 0  |
| 1  | 0  | 0  | 1  | 0  | 0  | 0  |
| 1  | 0  | 1  | 0  | 1  | 0  | 0  |
| 1  | 1  | 0  | 0  | 0  | 1  | 0  |
| 1  | 1  | 1  | 0  | 0  | 0  | 1  |

- An **active-low decoder** is similar, but with an inverted EN input *and* inverted outputs—exactly one of the outputs will be *false*.



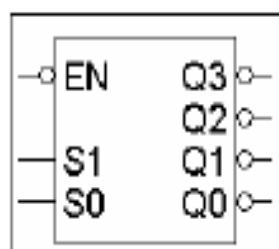
| EN' | S1 | S0 | Q0' | Q1' | Q2' | Q3' |
|-----|----|----|-----|-----|-----|-----|
| 0   | 0  | 0  | 0   | 1   | 1   | 1   |
| 0   | 0  | 1  | 1   | 0   | 1   | 1   |
| 0   | 1  | 0  | 1   | 1   | 0   | 1   |
| 0   | 1  | 1  | 1   | 1   | 1   | 0   |
| 1   | x  | x  | 1   | 1   | 1   | 1   |

- Active-high decoders generate minterms, as we've already seen.



$$\begin{aligned} Q3 &= S1 S0 \\ Q2 &= S1 S0' \\ Q1 &= S1' S0 \\ Q0 &= S1' S0' \end{aligned}$$

- The output equations for an active-low decoder are mysteriously similar, yet somehow different.

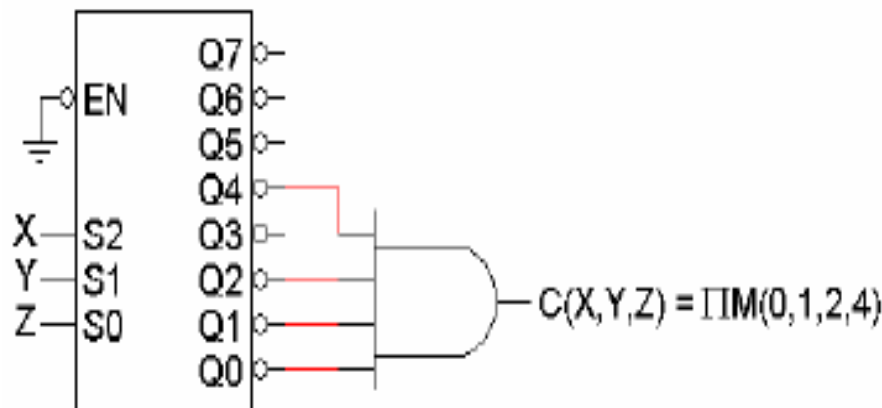


$$\begin{aligned} Q3' &= (S1 S0)' = S1' + S0' \\ Q2' &= (S1 S0')' = S1' + S0 \\ Q1' &= (S1' S0)' = S1 + S0' \\ Q0' &= (S1' S0')' = S1 + S0 \end{aligned}$$

- It turns out that active-low decoders generate **maxterms**, so they can implement functions as sums of maxterms.

## Active-low decoder example

- Here is the carry function  $C$  implemented with an active-low decoder.
  - Recall from our algebra lecture that  $\Sigma m(3,5,6,7) = \Pi M(0,1,2,4)$ .
  - We need an AND gate to generate a *product* of sums.

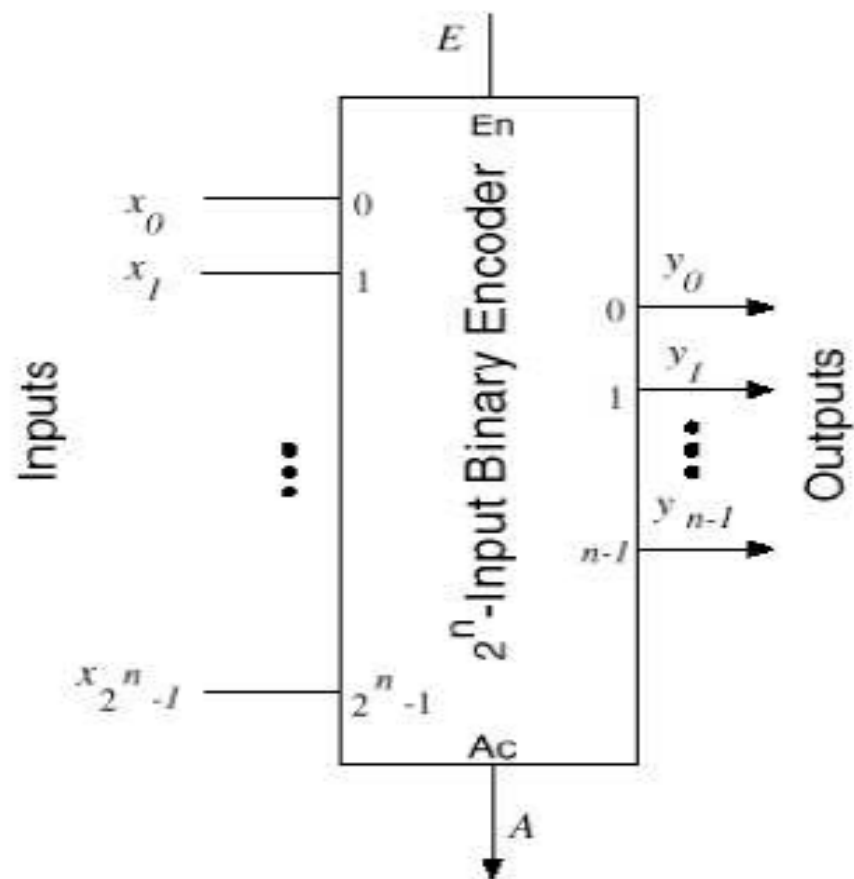


- The "ground" symbol connected to the EN input represents logical 0, so this decoder is always enabled.

# Encoders

- An encoder is a digital circuit that performs the inverse operation of a decoder. An encoder has  $2^n$  input lines and  $n$  output lines.
- The output lines generate the binary equivalent of the input line whose value is 1.

# Encoders (cont.)





# Encoder Example

- Example: 8-to-3 binary encoder (octal-to-binary)

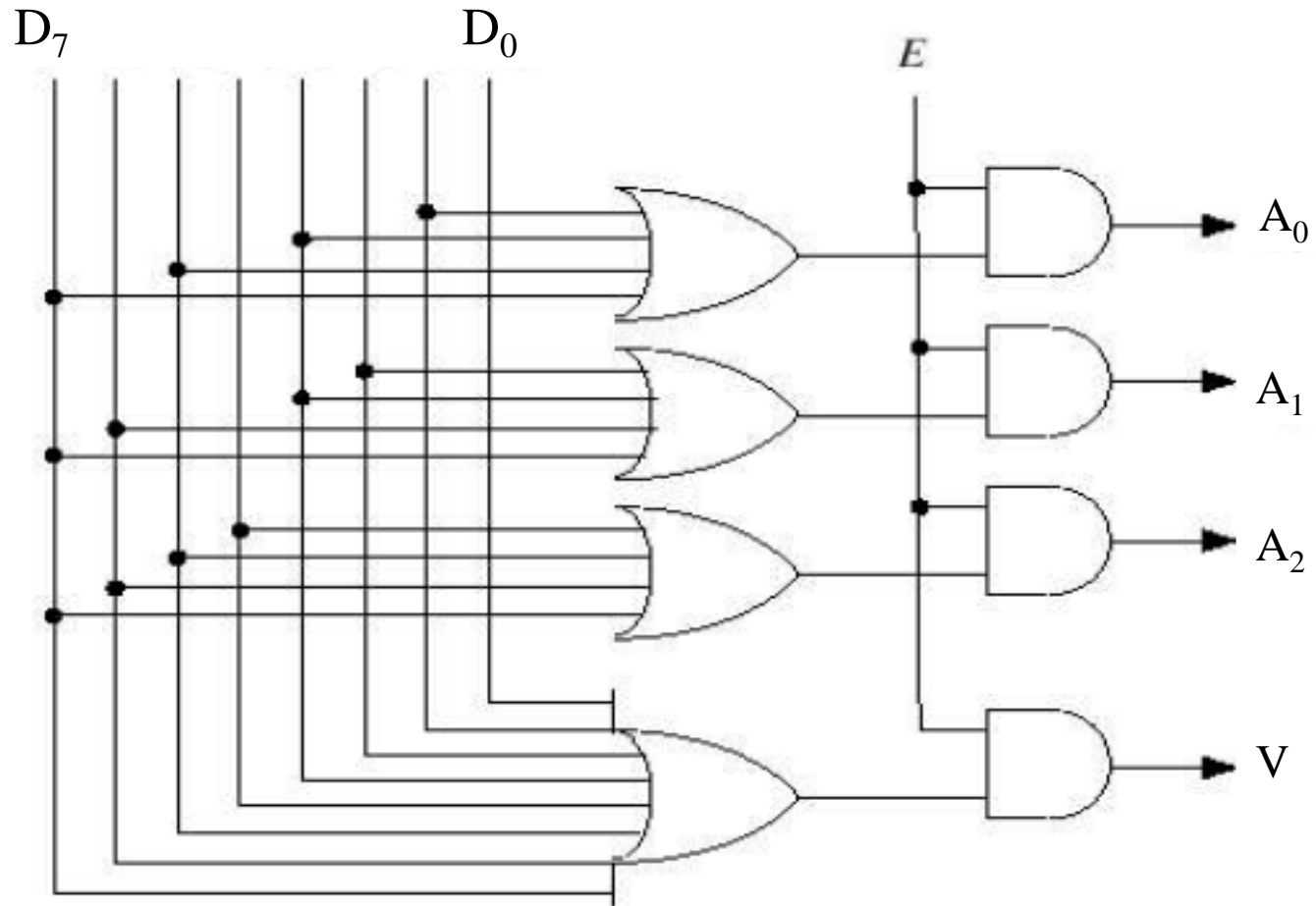
| Inputs         |                |                |                |                |                |                |                | Outputs        |                |                |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| D <sub>7</sub> | D <sub>6</sub> | D <sub>5</sub> | D <sub>4</sub> | D <sub>3</sub> | D <sub>2</sub> | D <sub>1</sub> | D <sub>0</sub> | A <sub>2</sub> | A <sub>1</sub> | A <sub>0</sub> |
| 0              | 0              | 0              | 0              | 0              | 0              | 0              | 1              | 0              | 0              | 0              |
| 0              | 0              | 0              | 0              | 0              | 0              | 1              | 0              | 0              | 0              | 1              |
| 0              | 0              | 0              | 0              | 0              | 1              | 0              | 0              | 0              | 1              | 0              |
| 0              | 0              | 0              | 0              | 1              | 0              | 0              | 0              | 0              | 1              | 1              |
| 0              | 0              | 0              | 1              | 0              | 0              | 0              | 0              | 1              | 0              | 0              |
| 0              | 0              | 1              | 0              | 0              | 0              | 0              | 0              | 1              | 0              | 1              |
| 0              | 1              | 0              | 0              | 0              | 0              | 0              | 0              | 1              | 1              | 0              |
| 1              | 0              | 0              | 0              | 0              | 0              | 0              | 0              | 1              | 1              | 1              |

$$A_0 = D_1 + D_3 + D_5 + D_7$$

$$A_1 = D_2 + D_3 + D_6 + D_7$$

$$A_2 = D_4 + D_5 + D_6 + D_7$$

# Encoder Example (cont.)



# Encoder Design Issues

- There are two ambiguities associated with the design of a simple encoder:
  1. Only one input can be active at any given time. If two inputs are active simultaneously, the output produces an undefined combination (for example, if  $D_3$  and  $D_6$  are 1 simultaneously, the output of the encoder will be 111).
  2. An output with all 0's can be generated when all the inputs are 0's, or when  $D_0$  is equal to 1.

# Priority Encoders

- Solves the ambiguities mentioned above.
- Multiple asserted inputs are allowed; one has priority over all others.
- Separate indication of no asserted inputs.

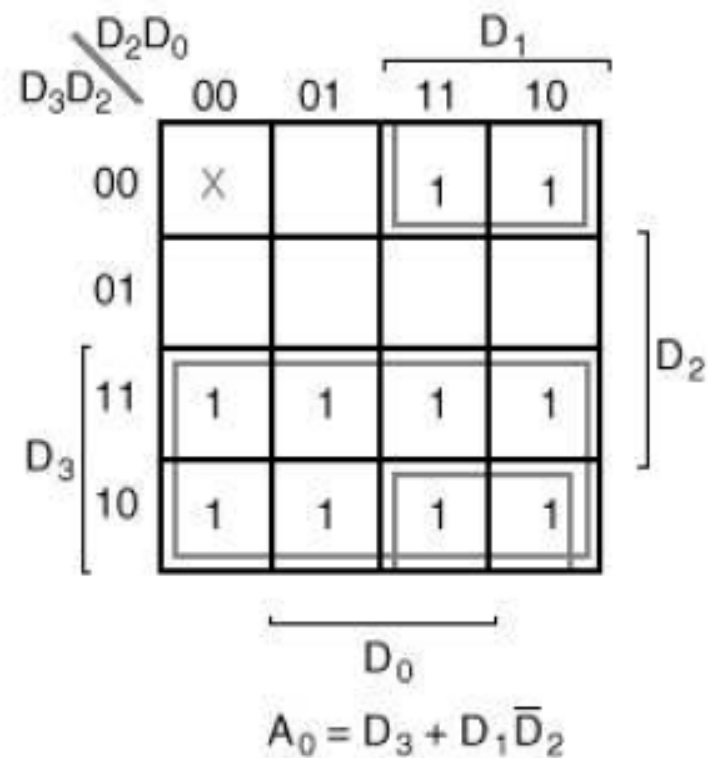
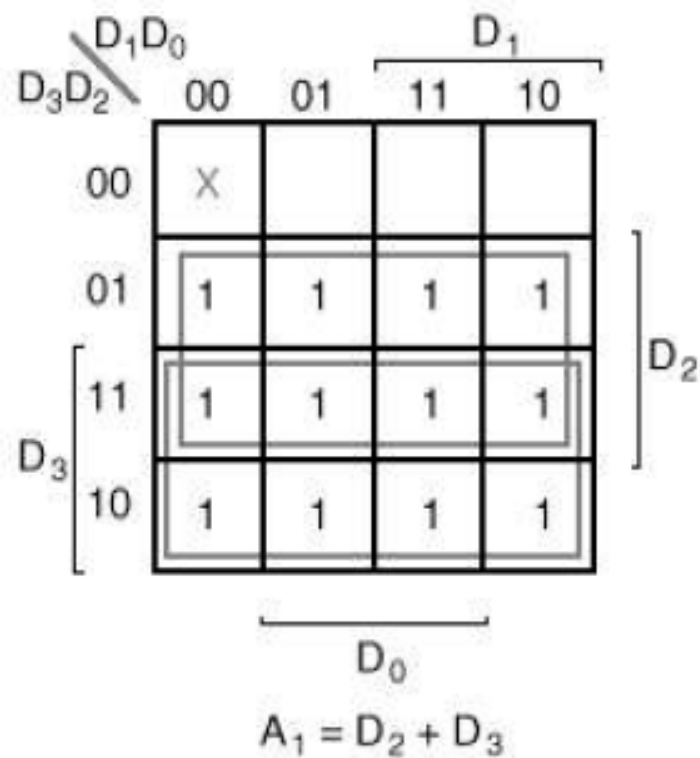
## 4-to-2 Priority Encoder (cont.)

- The operation of the priority encoder is such that:
- If two or more inputs are equal to 1 at the same time, the input in the highest-numbered position will take precedence.
- A *valid output indicator*, designated by  $V$ , is set to 1 only when one or more inputs are equal to 1.  $V = D_3 + D_2 + D_1 + D_0$  by inspection.

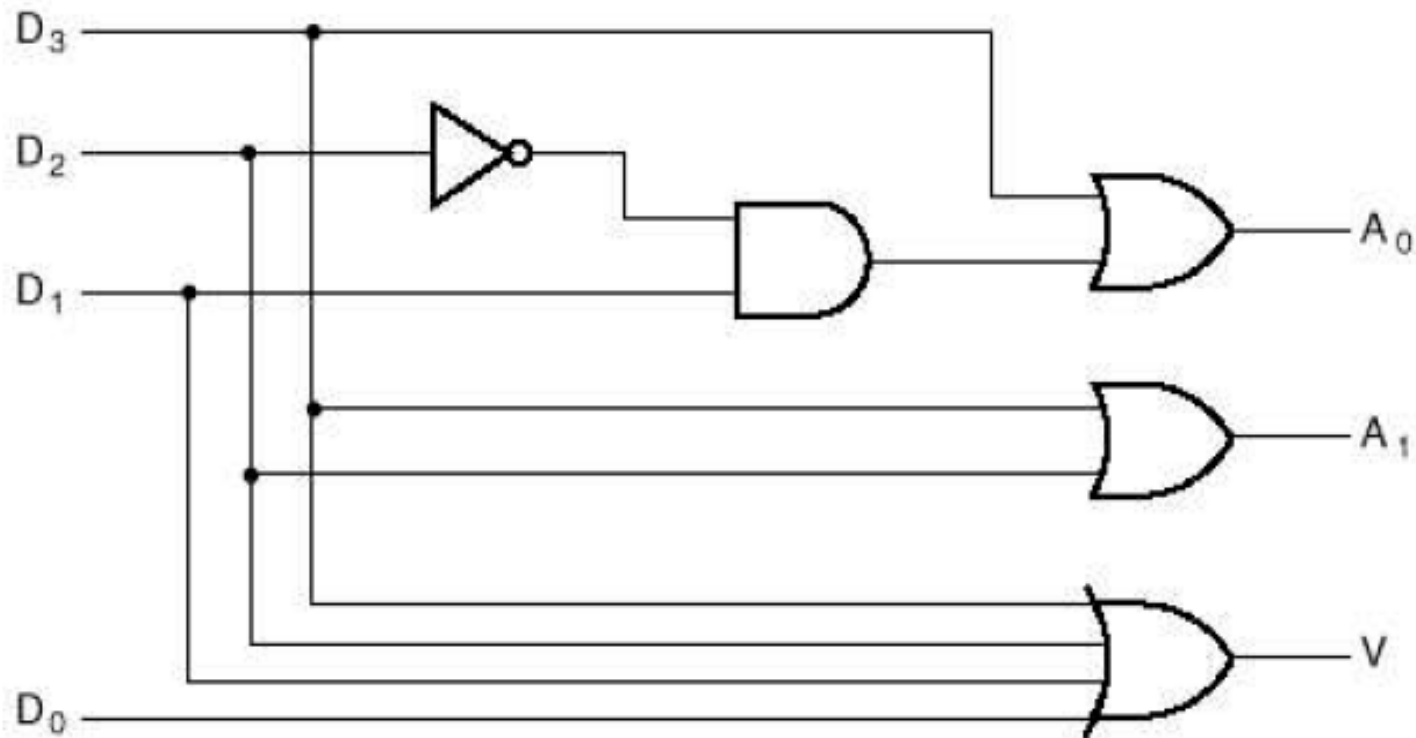
# Example: 4-to-2 Priority Encoder Truth Table

| Inputs |       |       |       | Outputs |       |     |
|--------|-------|-------|-------|---------|-------|-----|
| $D_3$  | $D_2$ | $D_1$ | $D_0$ | $A_1$   | $A_0$ | $V$ |
| 0      | 0     | 0     | 0     | X       | X     | 0   |
| 0      | 0     | 0     | 1     | 0       | 0     | 1   |
| 0      | 0     | 1     | X     | 0       | 1     | 1   |
| 0      | 1     | X     | X     | 1       | 0     | 1   |
| 1      | X     | X     | X     | 1       | 1     | 1   |

# Example: 4-to-2 Priority Encoder K-Maps



# Example: 4-to-2 Priority Encoder Logic Diagram



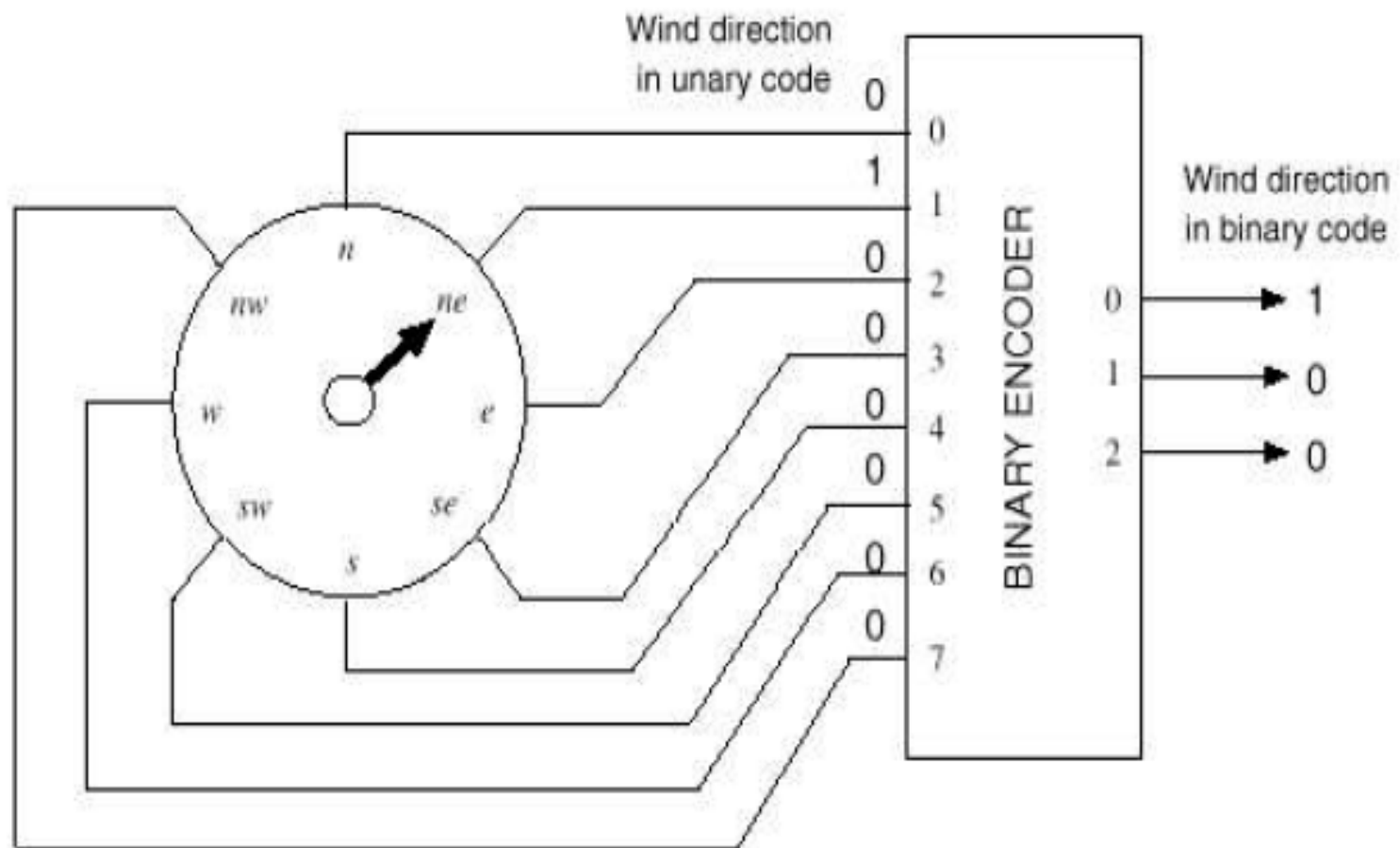


## 8-to-3 Priority Encoder

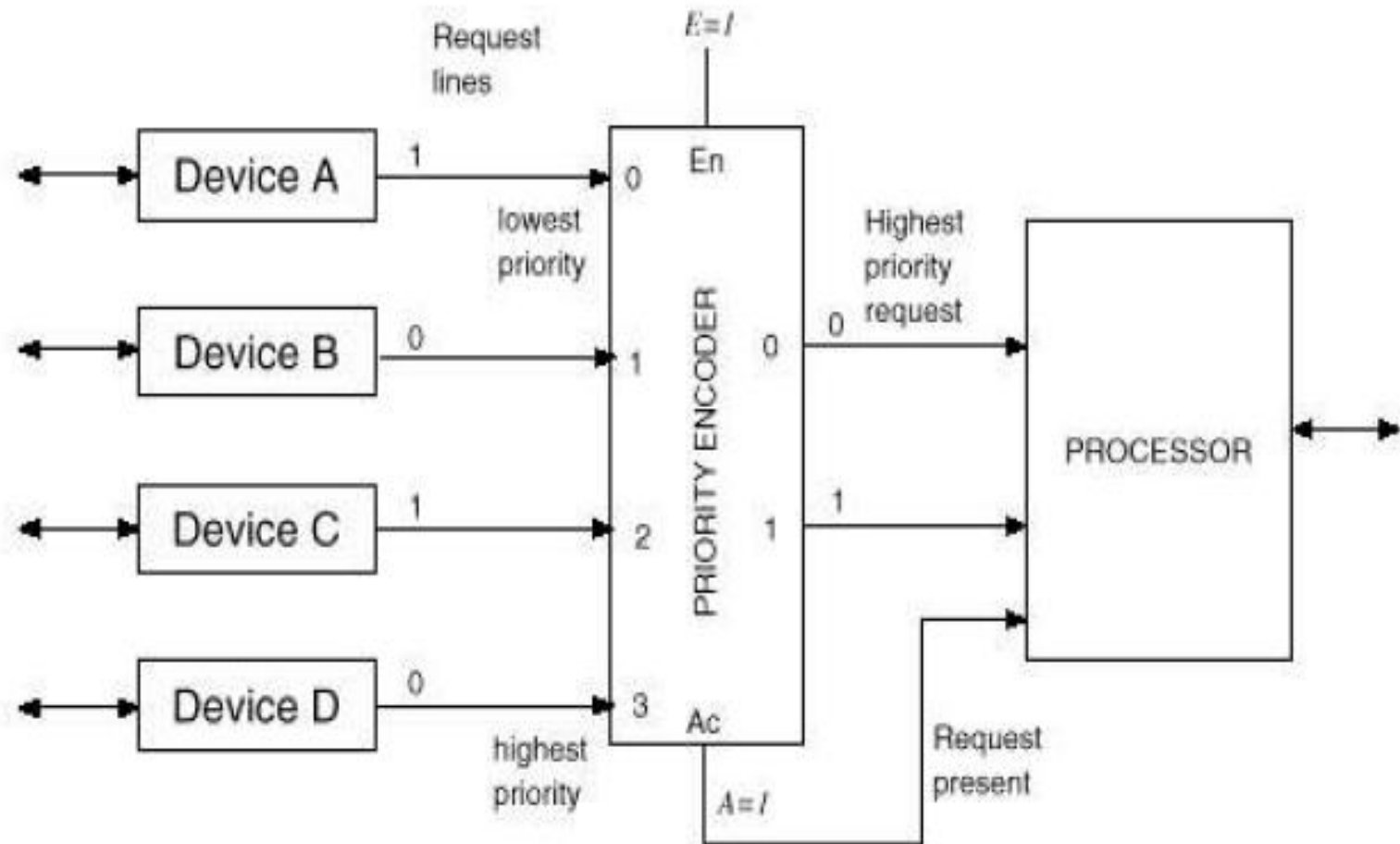
**Table 4.6** A priority encoder.

[illegible]

# Uses of binary Encoders

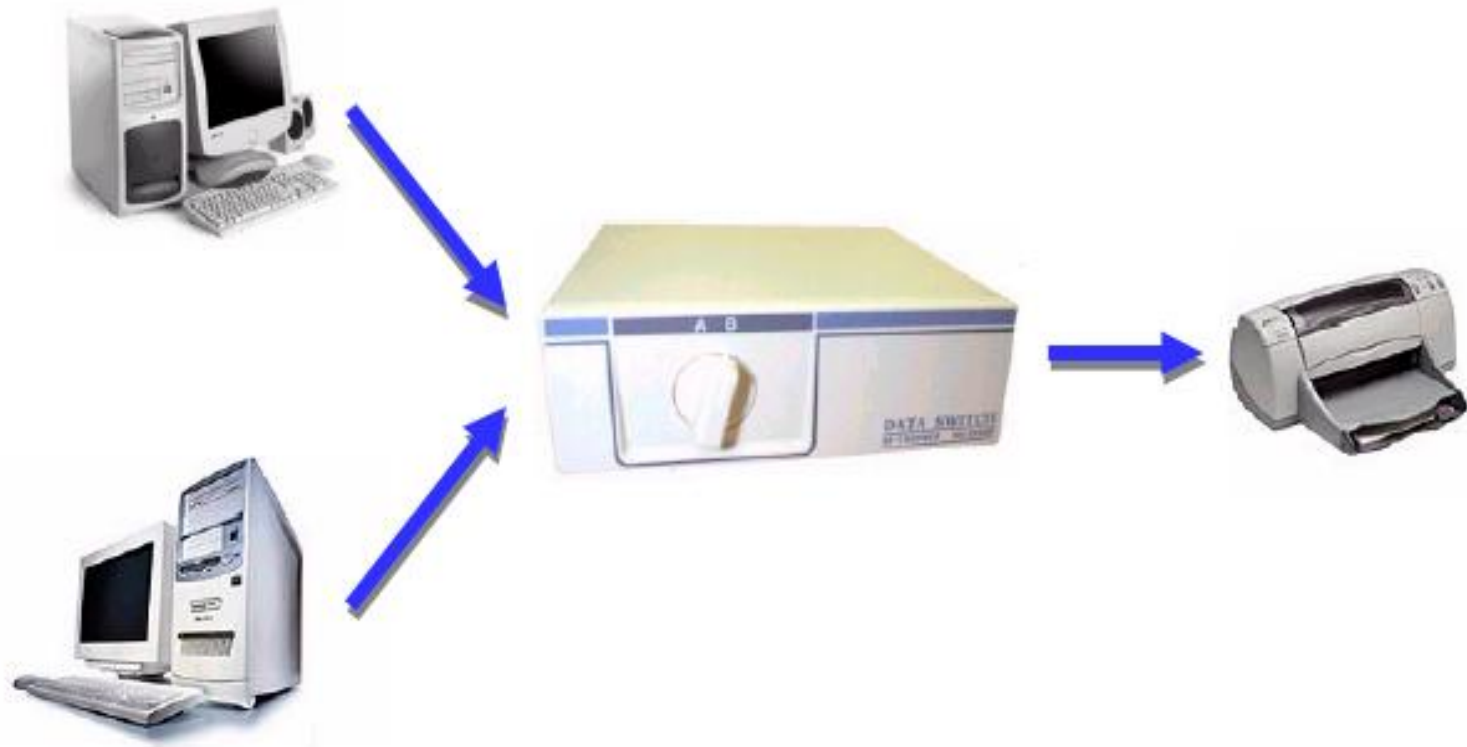


# Uses of priority encoders (cont.)



# Multiplexers

- Let's think about building another circuit, a **multiplexer**.
- In the old days, several machines could share an I/O device with a **switch**.

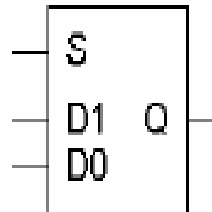


- The switch allows one computer's output to go to the printer's input.

## A 2-to-1 multiplexer

---

- Here is the circuit analog of that printer switch.



- This is a 2-to-1 multiplexer, or mux.
  - There are two data inputs D0 and D1, and a select input called S.
  - There is one output named Q.
- The multiplexer routes one of its data inputs (D0 or D1) to the output Q, based on the value of S.
  - If  $S=0$ , the output will be D0.
  - If  $S=1$ , the output will be D1.

## Building a multiplexer

- Here is a truth table for the multiplexer, based on our description from the previous page:

The multiplexer routes one of its data inputs (D0 or D1) to the output Q, based on the value of S.

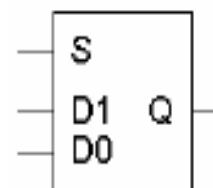
- If  $S=0$ , the output will be D0.
- If  $S=1$ , the output will be D1.

| S | D1 | D0 | Q |
|---|----|----|---|
| 0 | 0  | 0  | 0 |
| 0 | 0  | 1  | 1 |
| 0 | 1  | 0  | 0 |
| 0 | 1  | 1  | 1 |
| 1 | 0  | 0  | 0 |
| 1 | 0  | 1  | 0 |
| 1 | 1  | 0  | 1 |
| 1 | 1  | 1  | 1 |

- You can then find an MSP for the mux output Q.

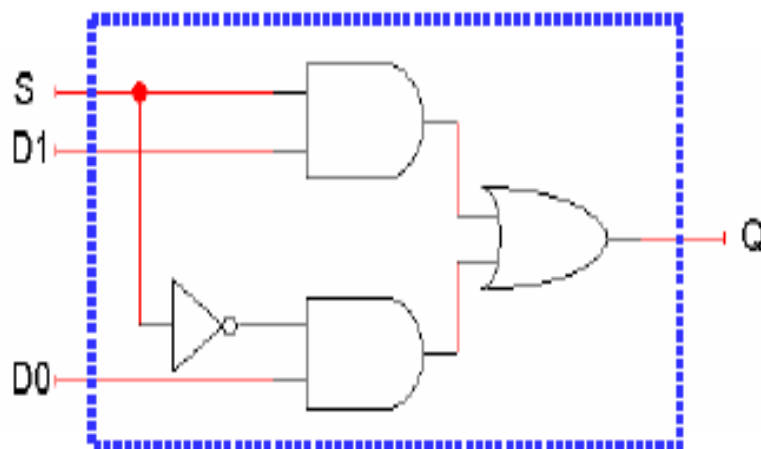
$$Q = S'D0 + S D1$$

- Note that this corresponds closely to our English specification above—sometimes you can derive an expression without first making a truth table.



## Multiplexer circuit diagram

- Here is an implementation of a 2-to-1 multiplexer.

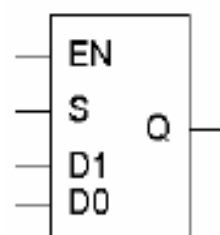


$$Q = S'D0 + S D1$$

- Remember that a minimal sum of products expression leads to a minimal two-level circuit.

## Enable inputs

- Many devices have an additional **enable input**, which “activates” or “deactivates” the device.
- We could design a 2-to-1 multiplexer with an enable input that’s used as follows.
  - EN=0 disables the multiplexer, which forces the output to be 0. (It does *not* turn off the multiplexer.)
  - EN=1 enables the multiplexer, and it works as specified earlier.
- Enable inputs are especially useful in combining smaller muxes together to make larger ones, as we’ll see later today.



| EN | S | D1 | D0 | Q |
|----|---|----|----|---|
| 0  | 0 | 0  | 0  | 0 |
| 0  | 0 | 0  | 1  | 0 |
| 0  | 0 | 1  | 0  | 0 |
| 0  | 0 | 1  | 1  | 0 |
| 0  | 1 | 0  | 0  | 0 |
| 0  | 1 | 0  | 1  | 0 |
| 0  | 1 | 1  | 0  | 0 |
| 0  | 1 | 1  | 1  | 0 |
| 1  | 0 | 0  | 0  | 0 |
| 1  | 0 | 0  | 1  | 1 |
| 1  | 0 | 1  | 0  | 0 |
| 1  | 0 | 1  | 1  | 1 |
| 1  | 1 | 0  | 0  | 0 |
| 1  | 1 | 0  | 1  | 0 |
| 1  | 1 | 1  | 0  | 1 |
| 1  | 1 | 1  | 1  | 1 |



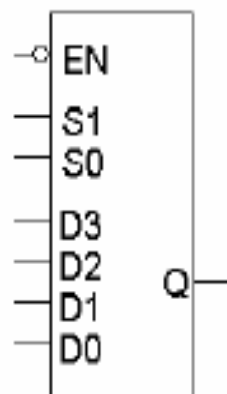
# A KVM switch

- This **KVM switch** allows four computers to share a single keyboard, video monitor, and mouse.



## A 4-to-1 multiplexer

- Here is a block diagram and abbreviated truth table for a 4-to-1 mux, which directs one of four different inputs to the single output line.
  - There are four data inputs, so we need *two* bits, **S1** and **S0**, for the mux selection input.

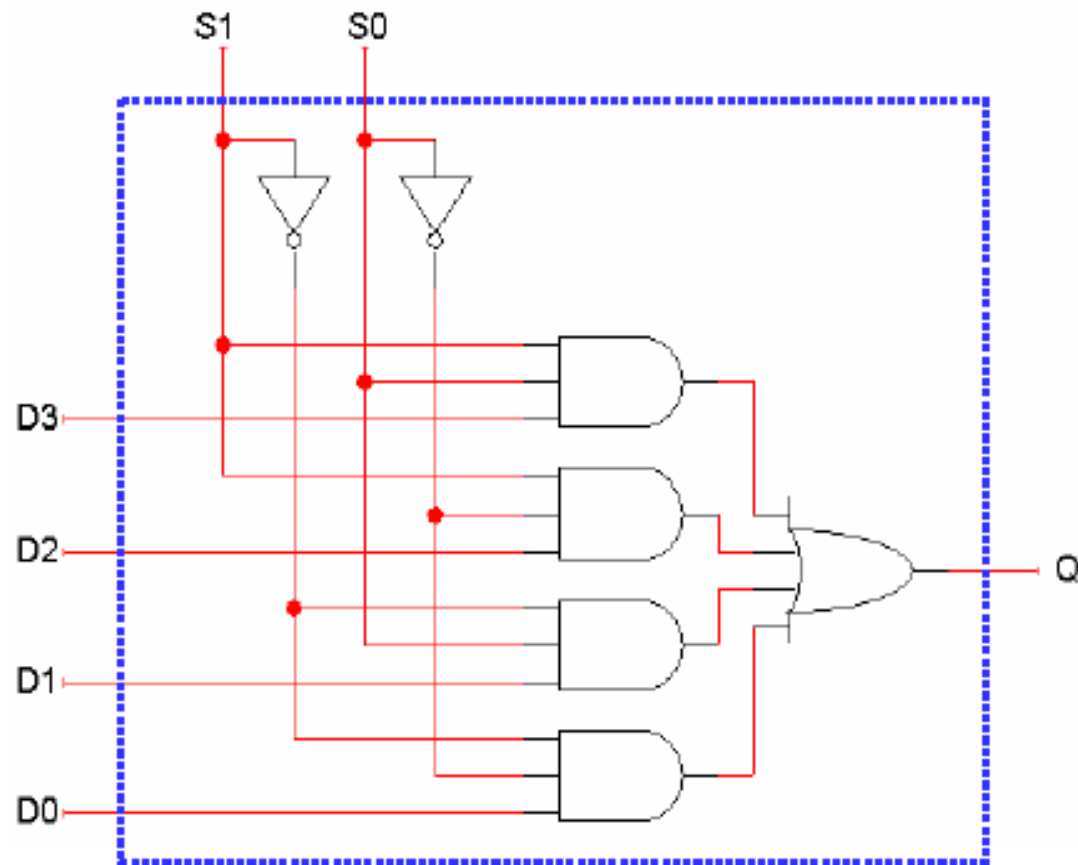


| EN' | S1 | S0 | Q  |
|-----|----|----|----|
| 0   | 0  | 0  | D0 |
| 0   | 0  | 1  | D1 |
| 0   | 1  | 0  | D2 |
| 0   | 1  | 1  | D3 |
| 1   | x  | x  | 1  |

$$Q = S1'S0'D0 + S1'S0 D1 + S1 S0'D2 + S1 S0 D3$$

## A 4-to-1 multiplexer implementation

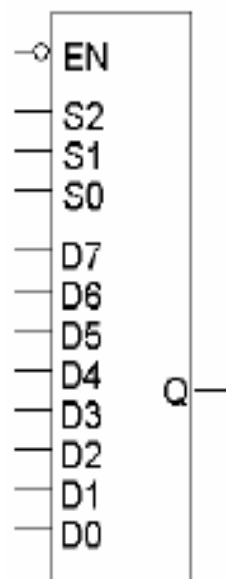
- Again we have a minimal sum of products expression, which leads to a minimal two-level circuit implementation.



$$Q = S1'S0'D0 + S1'S0 D1 + S1 S0'D2 + S1 S0 D3$$

## $2^n$ -to-1 multiplexers

- You can make even larger multiplexers, following the same pattern.
- A  $2^n$ -to-1 multiplexer routes one of  $2^n$  input lines to the output line.
  - There are  $2^n$  data inputs, so there must also be  $n$  select inputs.
  - The output is a single bit.
- Here is an 8-to-1 multiplexer, probably the biggest we'll see in this class.



## Example: addition

- Multiplexers can sometimes make circuit design easier.
- As an example, let's make a circuit to add three 1-bit inputs  $X$ ,  $Y$  and  $Z$ .
- We'll need two bits to represent the total.
  - The bits will be called  $C$  and  $S$ , standing for "carry" and "sum."
  - These are two separate functions of the inputs  $X$ ,  $Y$  and  $Z$ .
- A truth table and sum of minterm equations for  $C$  and  $S$  are shown below.

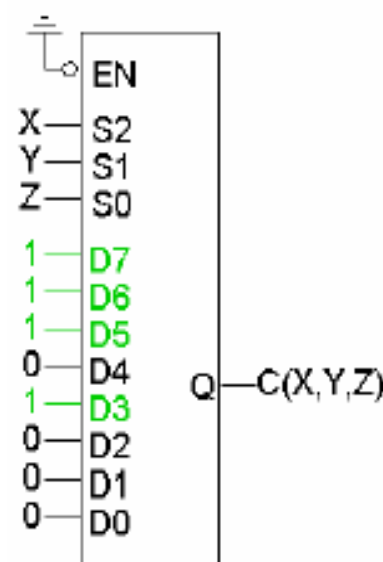
|                              | $X$ | $Y$ | $Z$ | $C$ | $S$ |
|------------------------------|-----|-----|-----|-----|-----|
|                              | 0   | 0   | 0   | 0   | 0   |
|                              | 0   | 0   | 1   | 0   | 1   |
|                              | 0   | 1   | 0   | 0   | 1   |
| $0 + 1 + 1 = 10 \rightarrow$ | 0   | 1   | 1   | 1   | 0   |
|                              | 1   | 0   | 0   | 0   | 1   |
|                              | 1   | 0   | 1   | 1   | 0   |
|                              | 1   | 1   | 0   | 1   | 0   |
| $1 + 1 + 1 = 11 \rightarrow$ | 1   | 1   | 1   | 1   | 1   |

$$C(X,Y,Z) = \sum m(3,5,6,7)$$
$$S(X,Y,Z) = \sum m(1,2,4,7)$$

## Implementing functions with multiplexers

- We could implement a function of  $n$  variables with an  $n$ -to-1 multiplexer.
  - The mux select inputs correspond to the function's input variables, and are used to select one row of the truth table.
  - Each mux data input corresponds to one output from the truth table. We connect 1 to data input  $D_i$  for each function minterm  $m_i$ , and we connect 0 to the other data inputs.
- For example, here is the carry function,  $C(X,Y,Z) = \Sigma m(3,5,6,7)$ .

| X | Y | Z | C |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |



## Partitioning the truth table

- We can actually implement  $C(X,Y,Z) = \Sigma m(3,5,6,7)$  with just a 4-to-1 mux, instead of an 8-to-1.
  - Instead of using three variables to select one row of the truth table, we'll use two variables to pick a *pair* of rows in the table.
  - The multiplexer data inputs will be functions of the remaining variable, which distinguish between the rows in each pair.
- First, we can divide the rows of our truth table into pairs, as shown on the right. X and Y are constant within each pair of rows, so C is a function of Z only.
  - When  $XY=00$ ,  $C=0$
  - When  $XY=01$ ,  $C=Z$
  - When  $XY=10$ ,  $C=Z$
  - When  $XY=11$ ,  $C=1$

| X | Y | Z | C |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

## A more efficient adder

- All that's left is setting the multiplexer inputs.
  - The two input variables  $X$  and  $Y$  will be connected to select inputs  $S1$  and  $S0$  of our 4-to-1 multiplexer.
  - The expressions for  $C(Z)$  are then connected to the data inputs  $D0$ - $D3$  of the multiplexer.

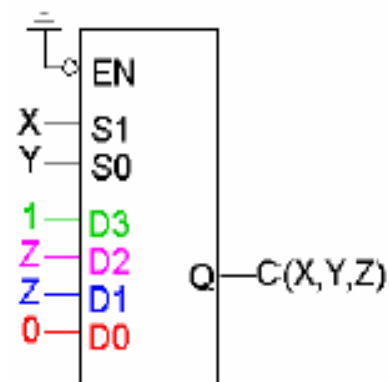
| X | Y | Z | C |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

When  $XY=00$ ,  $C=0$

When  $XY=01$ ,  $C=Z$

When  $XY=10$ ,  $C=Z$

When  $XY=11$ ,  $C=1$



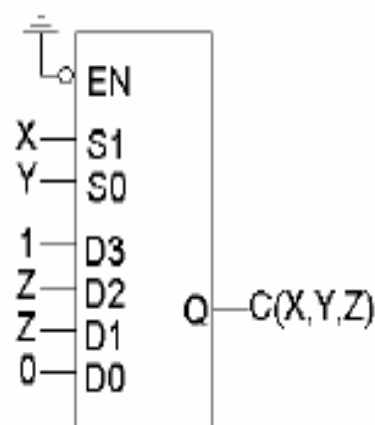


## Verifying our adder

- Don't believe that this works? Start with the equation for a 4-to-1 multiplexer

$$Q = S1'S0'D0 + S1'S0 D1 + S1 S0'D2 + S1 S0 D3$$

- Then just plug in the actual inputs to our circuit, as shown again on the right:  $S1S0 = XY$ ,  $D3 = 1$ ,  $D2 = Z$ ,  $D1 = Z$ , and  $D0 = 0$ .



$$\begin{aligned}C &= X'Y' \cdot 0 + X'YZ + XY'Z + XY \cdot 1 \\&= X'YZ + XY'Z + XY \\&= X'YZ + XY'Z + XY(Z' + Z) \\&= X'YZ + XY'Z + XYZ' + XYZ\end{aligned}$$

- So the multiplexer output really is the carry function,  $C(X,Y,Z) = \Sigma m(3,5,6,7)$ .

## Multiplexer-based sum

- Here's the same thing for the sum function,  $S(X,Y,Z) = \Sigma m(1,2,4,7)$ .

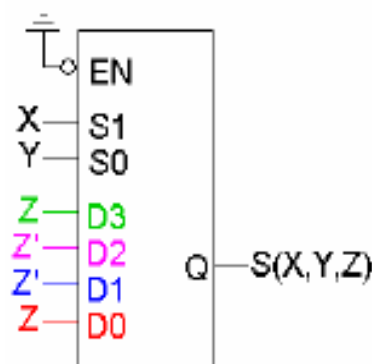
| X | Y | Z | S |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 |

When  $XY=00$ ,  $S=Z$

When  $XY=01$ ,  $S=Z'$

When  $XY=10$ ,  $S=Z'$

When  $XY=11$ ,  $S=Z$

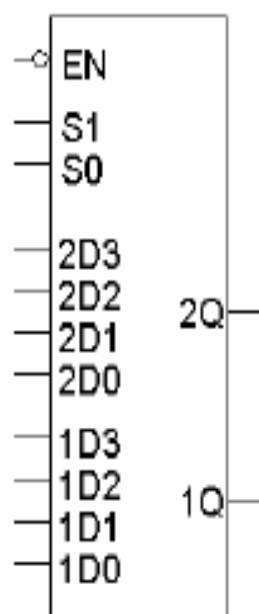


- Again, we can show that this is a correct implementation.

$$\begin{aligned} Q &= S1'S0'D0 + S1'S0 D1 + S1 S0'D2 + S1 S0 D3 \\ &= X'Y'Z + X'YZ' + XY'Z' + XYZ \\ &= \Sigma m(1,2,4,7) \end{aligned}$$

# Dual multiplexers

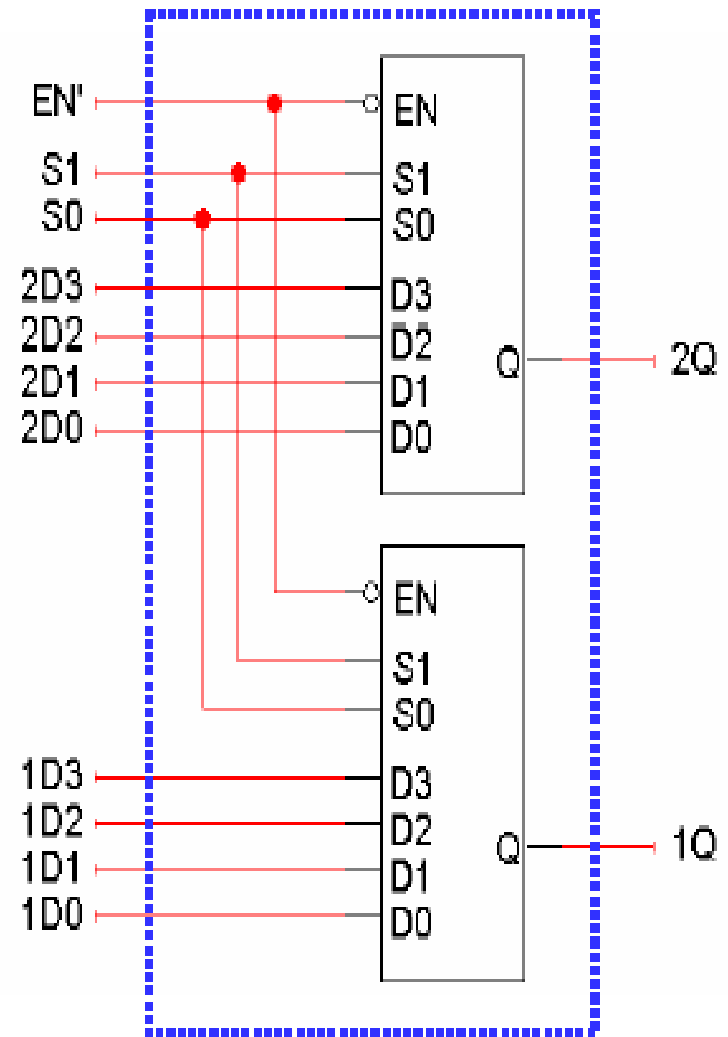
- A **dual 4-to-1 mux** allows you to select from one of four 2-bit data inputs.
  - The two output bits are 2Q 1Q, and S1-S0 select a *pair* of inputs.



| EN' | S1 | S0 | 2Q  | 1Q  |
|-----|----|----|-----|-----|
| 0   | 0  | 0  | 2D0 | 1D0 |
| 0   | 0  | 1  | 2D1 | 1D1 |
| 0   | 1  | 0  | 2D2 | 1D2 |
| 0   | 1  | 1  | 2D3 | 1D3 |
| 1   | x  | x  | 1   | 1   |

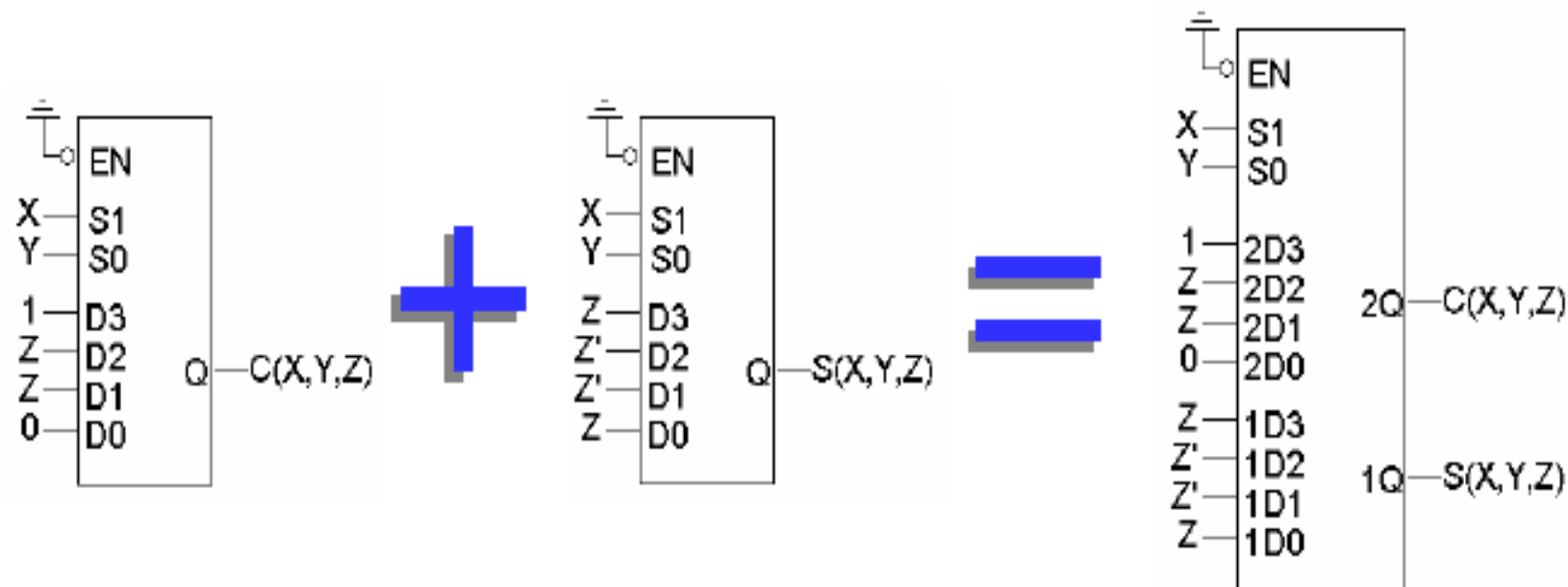
# Dual muxes in more detail

- You could build a dual 4-to-1 mux from its truth table and our familiar circuit design techniques.
- It's also possible to combine smaller muxes together to form larger ones.
- You can build the dual 4-to-1 mux just by using two 4-to-1 muxes.
  - The two 4-to-1 multiplexers share the same  $EN'$ ,  $S1$  and  $S0$  signals.
  - Each smaller mux produces one bit of the two-bit output  $2Q$   $1Q$ .
- This kind of hierarchical design is very common in computer architecture.



## Dual multiplexer-based adder

- We can use this dual 4-to-1 multiplexer to implement our adder, which produces a two-bit output consisting of C and S.



- That KVM switch from earlier is really a "tri 4-to-1 multiplexer," since it selects from four sets of three signals (keyboard, video and mouse).

# De Multiplexers

**A De-multiplexer is a logic circuit that transmit Information on a single line on one of  $2^n$  output lines**

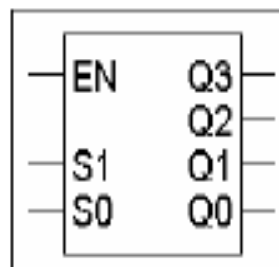
**Selection of output line depends on the value of n select lines**

**A Decoder with enable input can function as a De-multiplexer**

## Enable inputs

decoders can include **enable inputs**.

- **EN=0** disables the decoder, which by convention means that all of the decoder's outputs are 0.
- **EN=1** enables the decoder so that it behaves as specified earlier, with exactly one of the outputs being 1.



| EN | S1 | S0 | Q0 | Q1 | Q2 | Q3 |
|----|----|----|----|----|----|----|
| 0  | x  | x  | 0  | 0  | 0  | 0  |
| 1  | 0  | 0  | 1  | 0  | 0  | 0  |
| 1  | 0  | 1  | 0  | 1  | 0  | 0  |
| 1  | 1  | 0  | 0  | 0  | 1  | 0  |
| 1  | 1  | 1  | 0  | 0  | 0  | 1  |