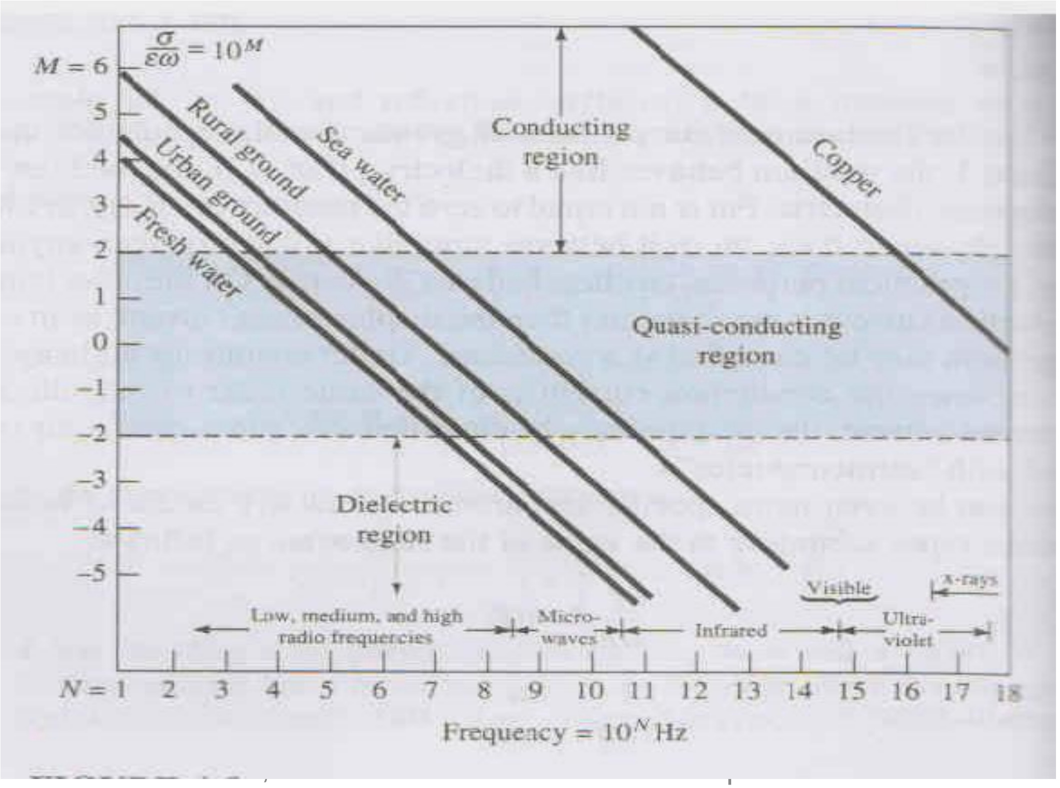
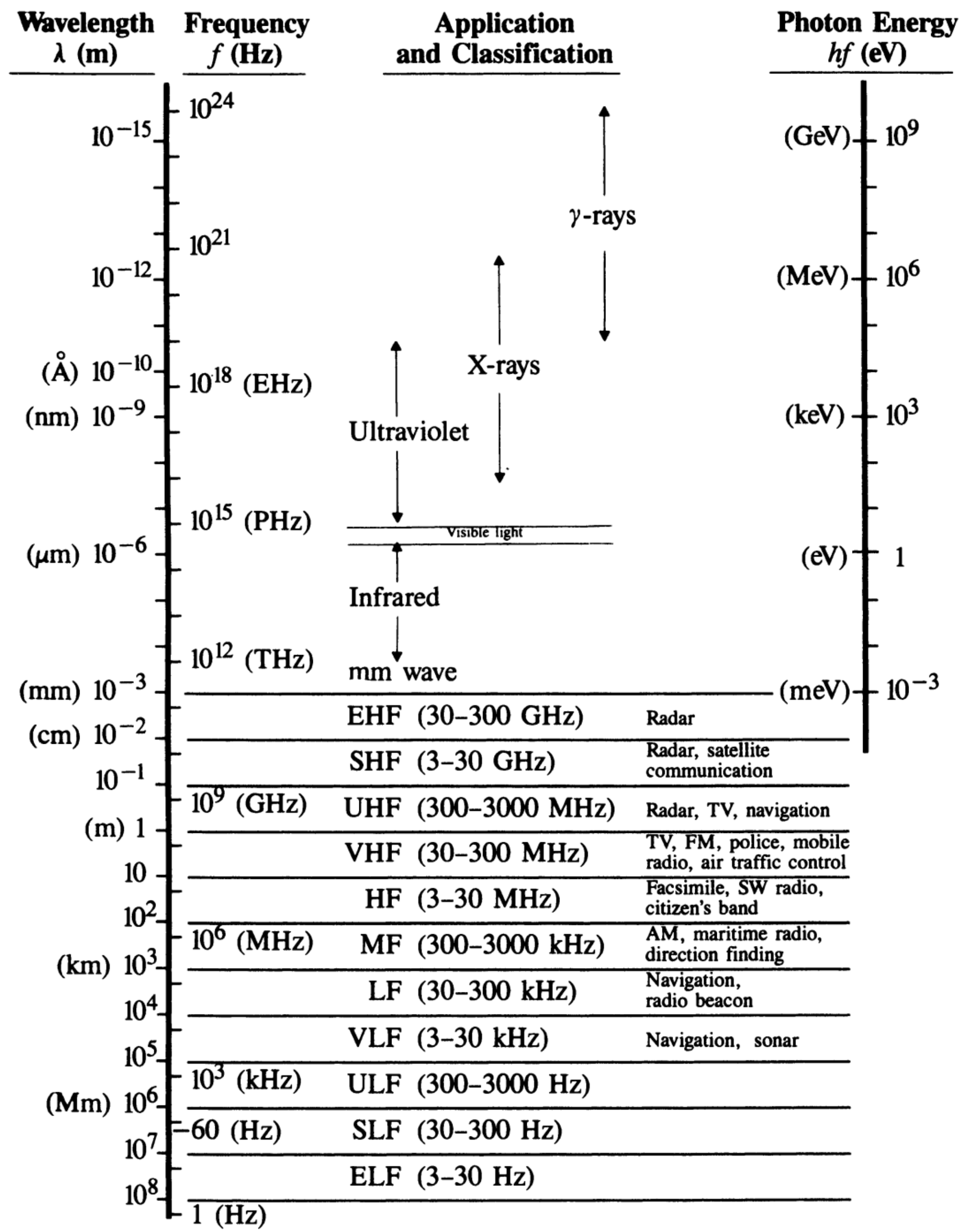
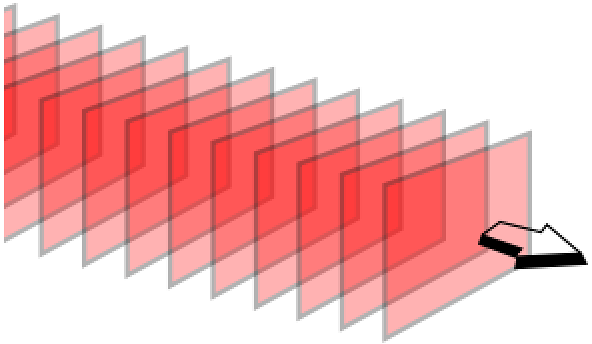
- Importance of loss tangent:
  - Measure of the power loss in the medium
  - Also used in deciding the type of medium (at operating frequency)
  - **Example:** A sinusoidal electric field intensity of amplitude 250 V/m and frequency 1 GHz exists in a lossy dielectric medium that has a relative permittivity of 2.5 and a loss tangent of 0.001. Find the average power dissipated in the medium per cubic meter.



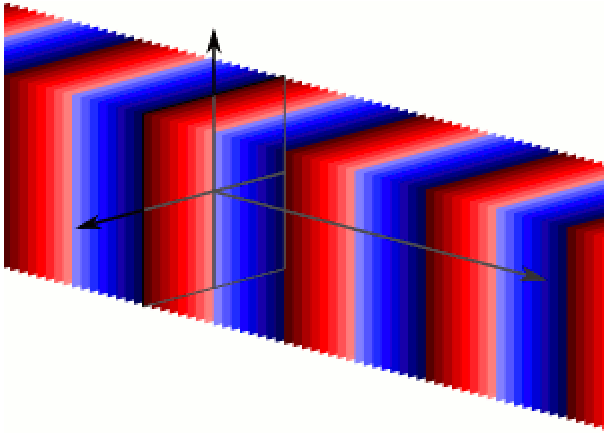

Wavelength λ (m)
Frequency f (Hz)
Application and Classification
Photon Energy hf (eV)
10^24
10^−15
(GeV) 10^9
γ-rays
10^21
10^−12
(MeV) 10^6
X-rays
(Å) 10^−10
10^18 (EHz)
(nm) 10^−9
(keV) 10^3
Ultraviolet
10^15 (PHz)
Visible light
(μm) 10^−6
(eV) 1
Infrared
10^12 (THz)
mm wave
(mm) 10^−3
(meV) 10^−3
EHF (30–300 GHz)
Radar
(cm) 10^−2
SHF (3–30 GHz)
Radar, satellite communication
10^−1
10^9 (GHz)
UHF (300–3000 MHz)
Radar, TV, navigation
(m) 1
VHF (30–300 MHz)
TV, FM, police, mobile radio, air traffic control
10
HF (3–30 MHz)
Facsimile, SW radio, citizen's band
10^2
10^6 (MHz)
MF (300–3000 kHz)
AM, maritime radio, direction finding
(km) 10^3
LF (30–300 kHz)
Navigation, radio beacon
10^4
VLF (3–30 kHz)
Navigation, sonar
10^5
10^3 (kHz)
ULF (300–3000 Hz)
(Mm) 10^6
60 (Hz)
SLF (30–300 Hz)
10^7
ELF (3–30 Hz)
10^8
1 (Hz)

Chapter 8: Plane Electromagnetic Waves

A **plane wave** is a constant-frequency wave whose wavefronts (surfaces of constant phase) are infinite parallel planes of constant peak-to-peak amplitude normal to the phase velocity vector.



The wavefronts of a plane wave traveling in 3-space



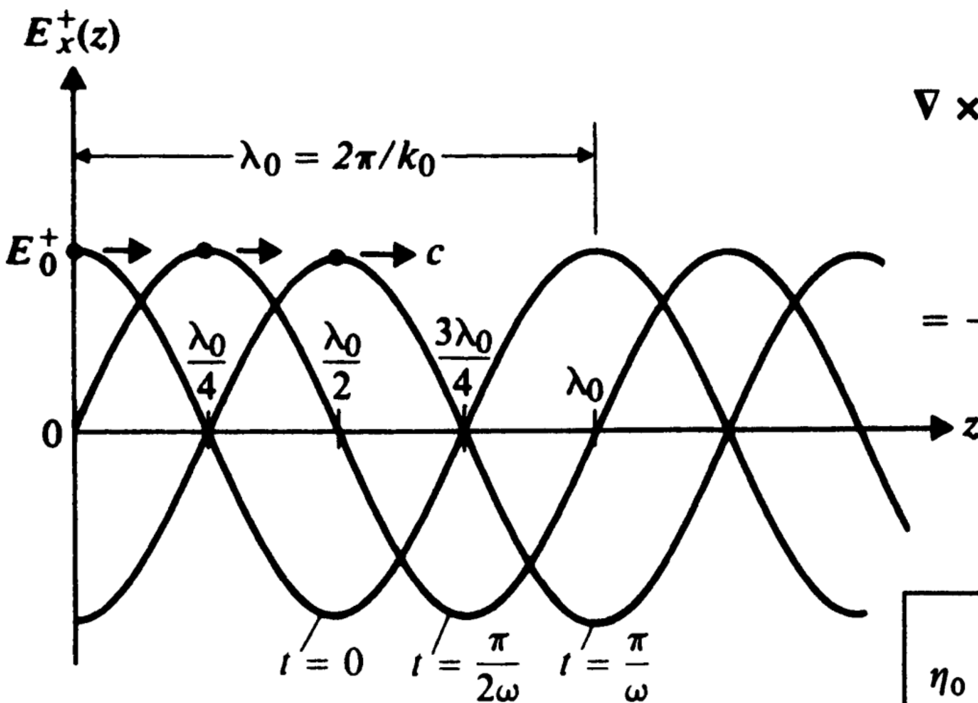
Plane Waves in Lossless Media

- Considering the source free wave equation for free space:

$$\boxed{\nabla^2 \mathbf{E} + k_0^2 \mathbf{E} = 0,}$$

$$\boxed{k_0 = \omega\sqrt{\mu_0 \epsilon_0} = \frac{\omega}{c} \qquad \text{(rad/m).}}$$

Traveling Wave



$$\nabla \times \mathbf{E} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ 0 & 0 & \dfrac{\partial}{\partial z} \\ E_x^+(z) & 0 & 0 \end{vmatrix}$$

$$= -j\omega\mu_0(\mathbf{a}_x H_x^+ + \mathbf{a}_y H_y^+ + \mathbf{a}_z H_z^+),$$

$$\boxed{\eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} \cong 120\pi \cong 377 \qquad (\Omega),}$$

EXAMPLE 8–1 A uniform plane wave with $\mathbf{E} = \mathbf{a}_x E_x$ propagates in a lossless simple medium ($\epsilon_r = 4$, $\mu_r = 1$, $\sigma = 0$) in the $+z$-direction. Assume that E_x is sinusoidal with a frequency 100 (MHz) and has a maximum value of $+10^{-4}$ (V/m) at $t = 0$ and $z = \frac{1}{8}$ (m).

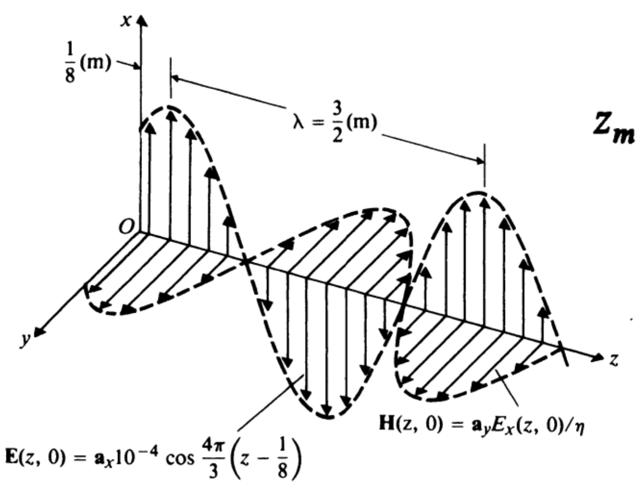
a) Write the instantaneous expression for $\mathbf{E}$ for any t and z.

b) Write the instantaneous expression for $\mathbf{H}$.

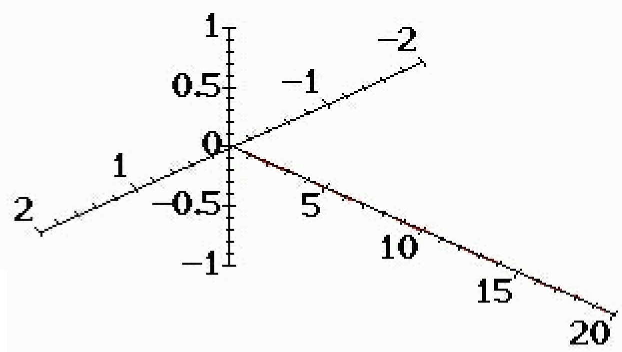
c) Determine the locations where E_x is a positive maximum when $t = 10^{-8}$ (s).

$$\mathbf{E}(z, t) = \mathbf{a}_x 10^{-4} \cos\left[2\pi 10^8 t - \frac{4\pi}{3}\left(z - \frac{1}{8}\right)\right] \quad \text{(V/m)}.$$

$$\mathbf{H}(z, t) = \mathbf{a}_y \frac{10^{-4}}{60\pi} \cos\left[2\pi 10^8 t - \frac{4\pi}{3}\left(z - \frac{1}{8}\right)\right] \quad \text{(A/m)}.$$

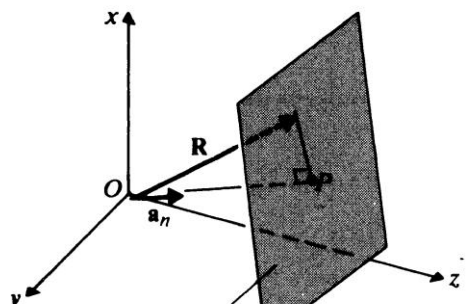


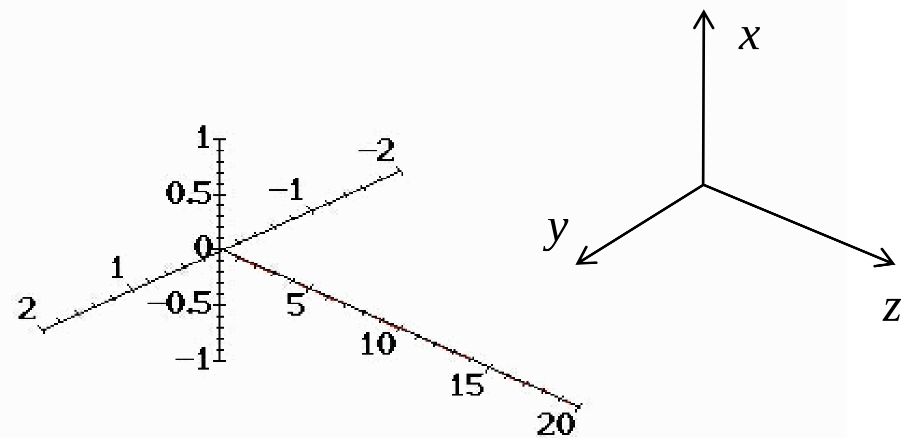
$$z_m = \frac{13}{8} \pm \frac{3}{2} n \quad \text{(m)},$$



Transverse Electromagnetic Waves (TEM)

E and **H** are perpendicular to each other, and both are transverse to the direction of propagation.





$$\boxed{\mathbf{E}(\mathbf{R}) = \mathbf{E}_0 e^{-j\mathbf{k}\cdot\mathbf{R}} = \mathbf{E}_0 e^{-jk\mathbf{a}_n\cdot\mathbf{R}} \qquad (\mathrm{V/m}),}$$

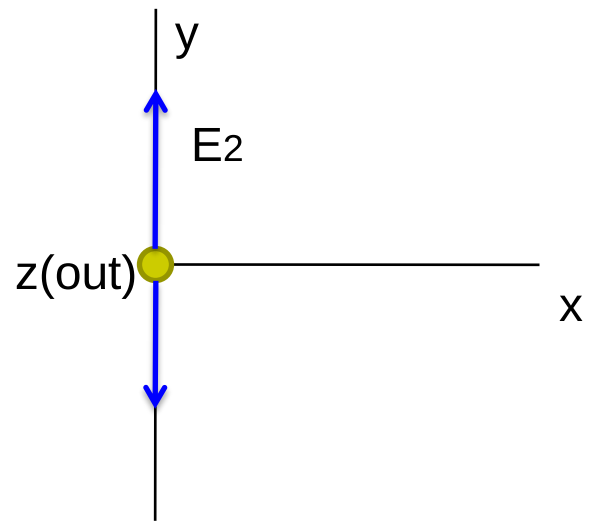
$$\boxed{\mathbf{H}(\mathbf{R}) = \frac{1}{\eta}\mathbf{a}_n \times \mathbf{E}(\mathbf{R}) \qquad (\mathrm{A/m}),}$$

$$\mathbf{H}(\mathbf{R}) = -\frac{1}{j\omega\mu}\nabla \times \mathbf{E}(\mathbf{R})$$

Polarization of Plane Waves

Polarization of a uniform plane wave describes the time-varying behaviour of the electric field intensity vector at a given point in space.

Wave polarization describes the **shape** and **locus of tip** of the vector **E** at a given point in space as a function of time.



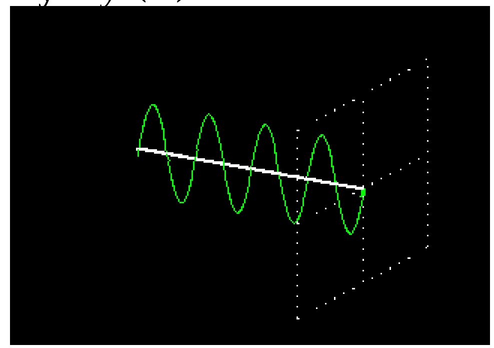
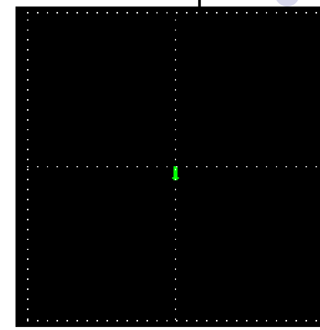
- A uniform plane wave traveling in the +z direction **<u>may have</u>** x- and y- components. $\mathbf{E}(z)=\hat{a}_{\mathbf{x}}E_x(z)+\hat{a}_{\mathbf{y}}E_y(z)$

$$E_x = E_1 \sin(\omega t - \beta z)$$

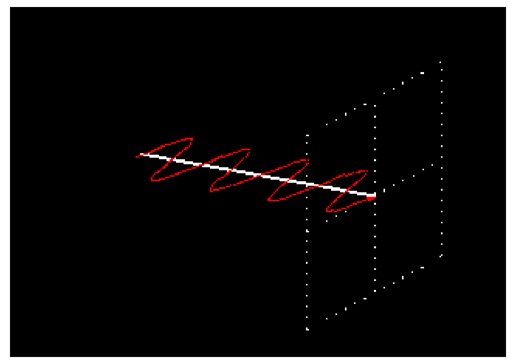
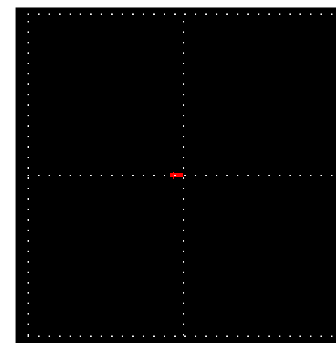
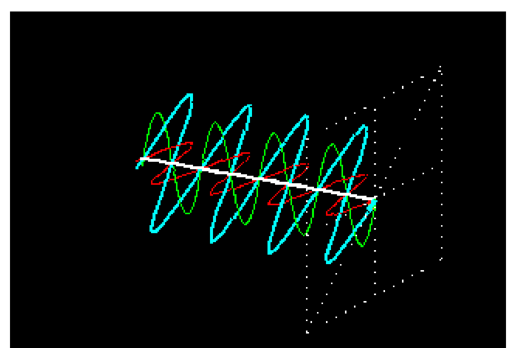
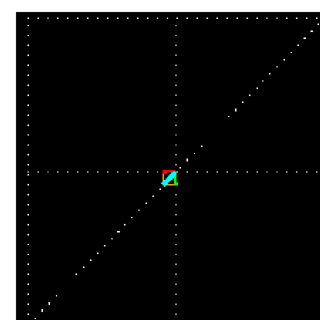
$$E_y = E_2 \sin(\omega t - \beta z + \delta)$$

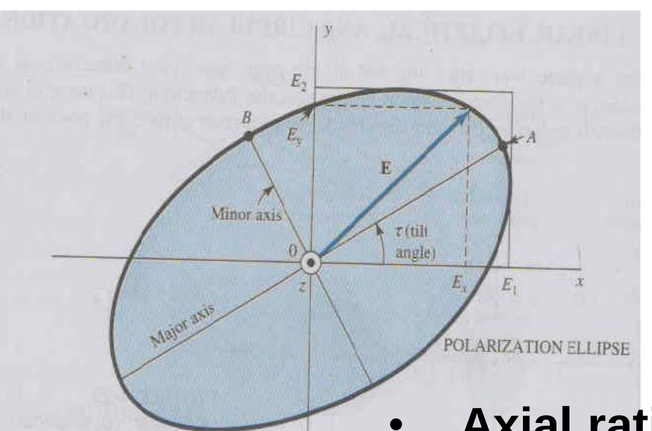
Generally an Elliptically polarized wave

$$\boxed{\frac{E_y^{\,2}}{E_2^{\,2}} - \frac{2E_xE_y}{E_1E_2}\cos\delta + \frac{E_x^{\,2}}{E_1^{\,2}} = \sin^2\delta}$$

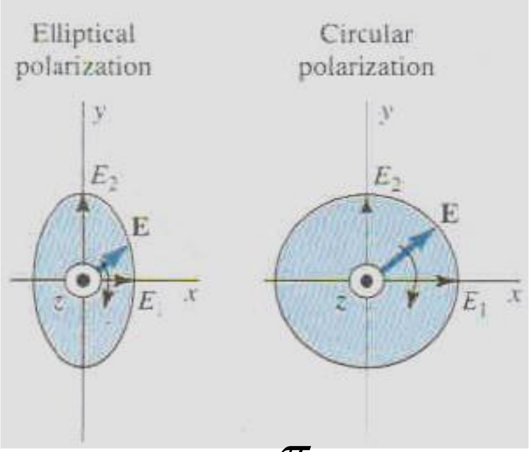
$$\delta = 0$$

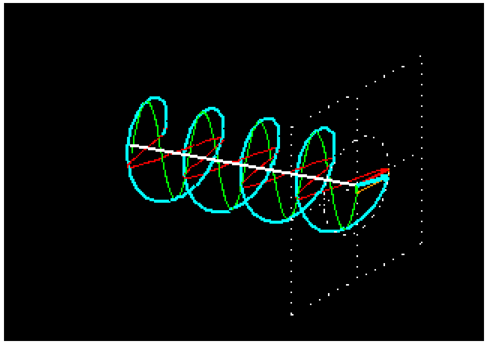


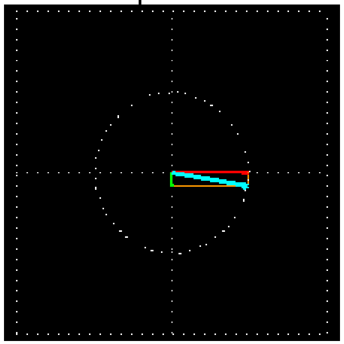
- **Axial ratio=OA/OB**
- **Tilt angle: τ**

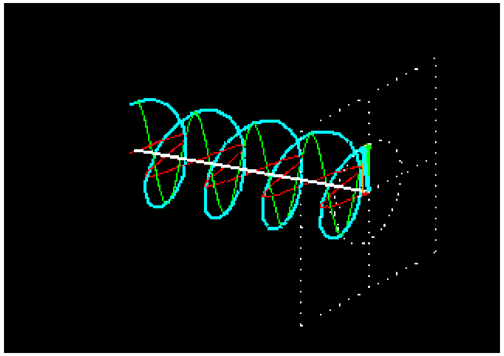
$$\frac{E_y^{\;2}}{E_2^{\;2}} - \frac{2E_xE_y}{E_1E_2}\cos\delta + \frac{E_x^{\;2}}{E_1^{\;2}} = \sin^2\delta$$

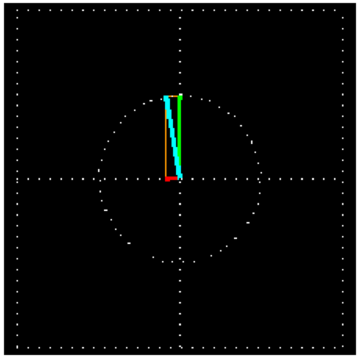


$E_1 \neq E_2$ $\delta = \pm\frac{\pi}{2}$ $E_1 = E_2$









Example

- Determine the polarization of the wave if the electric field is given by $\vec{\mathrm{E}} = (3\hat{a}_x + j4\hat{a}_y)e^{-0.2z}e^{-j0.5z}$

Example

- A uniform plane wave, propagating in the z–direction in vacuum, has the following electric field:

$$\mathbf{E}(z,t) = 2\hat{\mathbf{x}}\mathrm{Cos}(\omega t - kz) + 4\hat{\mathbf{y}}\mathrm{Sin}(\omega t - kz)$$

- Determine the vector phasor representing **E(z, t) in the complex form** $\mathbf{E} = \mathbf{E}_0 e^{j\omega t - jkz}$
- Determine the polarization of this electric field (linear, circular, elliptic, left–handed, right–handed).
- Determine the magnetic field **H(z, t) in its real–valued form.**