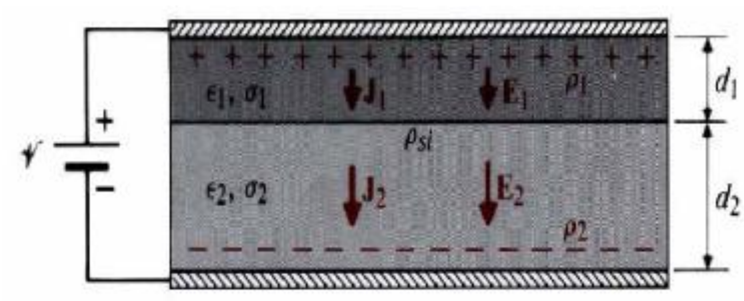
Example: static charge & steady current



Parallel-plate capacitor with lossy dielectrics

Find:

(a) Current density between the plates

(b) Electric Field intensities in both dielectrics, and

(c) Surface charge densities on the plates and at the interface.

$$= \frac{\sigma_1 \sigma_2 V}{\sigma_2 d_1 + \sigma_1 d_2} \left(A/m^2 \right)$$

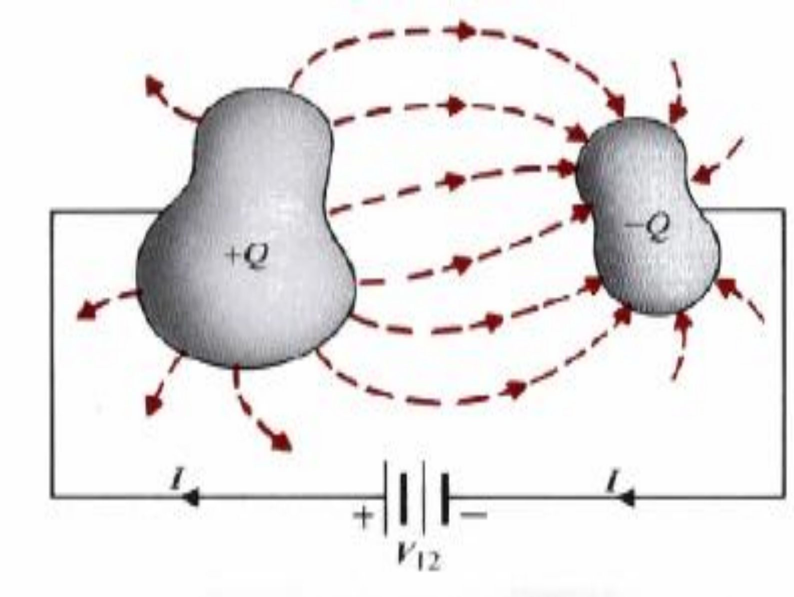
$$E_1 = \frac{\sigma_2 V}{\sigma_2 d_1 + \sigma_1 d_2} \left(\frac{V}{m} \right)$$

$$E_2 = \frac{\sigma_1 V}{\sigma_2 d_1 + \sigma_1 d_2} \left(\frac{V}{m} \right)$$

$$\rho_{Si} = (\varepsilon_2 \frac{\sigma_1}{\sigma_2} - \varepsilon_1) \frac{\sigma_2 V}{\sigma_2 d_1 + \sigma_1 d_2}$$

$$= \frac{\varepsilon_2 \sigma_1 - \varepsilon_1 \sigma_2}{\sigma_2 d_1 + \sigma_1 d_2} V$$

Resistance Calculation



$$C = \frac{Q}{V} = \frac{\oint_S \vec{D}\cdot d\vec{S}}{-\int_L \vec{E}\cdot d\vec{l}} = \frac{\int_S \varepsilon\vec{E}\cdot d\vec{S}}{-\int_L \vec{E}\cdot d\vec{l}} \ ;$$

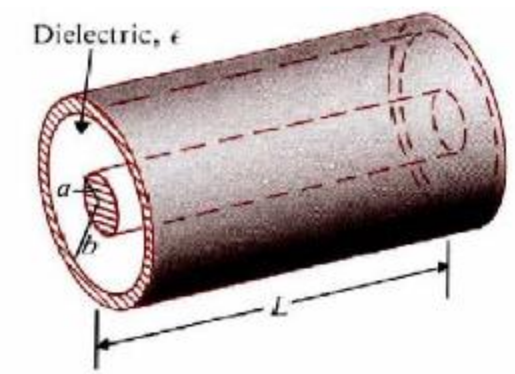
$$R = \frac{V}{I} = \frac{-\oint_L \vec{E}\cdot d\vec{l}}{\int_S \vec{J}\cdot d\vec{S}} = \frac{-\int_L \vec{E}\cdot d\vec{l}}{\int_S \sigma\vec{E}\cdot d\vec{S}} \ ;$$

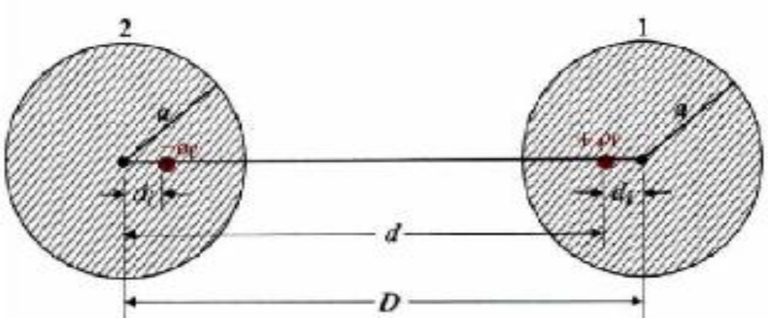
$$RC = \frac{C}{G} = \frac{\varepsilon}{\sigma}$$

Only when medium is homogeneous

If the capacitance between two conductors is known, the resistance (or conductance) can be obtained directly from the ε/σ ratio without recomputation

Example:





$$C_1 = \frac{2\pi\varepsilon}{\ln\left(\frac{b}{a}\right)} \left(F/m\right)$$

$$C_1' = \frac{\pi\varepsilon}{\cosh^{-1}\left(\frac{D}{2a}\right)} \left(F/m\right)$$

$$R_1 = \frac{\varepsilon}{\sigma}\left(\frac{1}{C_1}\right) = \frac{1}{2\pi\sigma}\ln\left(\frac{b}{a}\right)(\Omega-m)$$

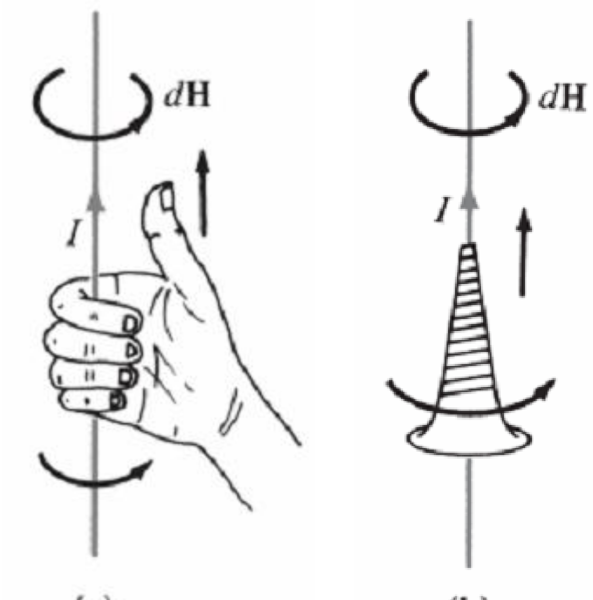
$$R_1' = \frac{\varepsilon}{\sigma}\left(\frac{1}{C_1'}\right) = \frac{1}{\pi\sigma}\cosh^{-1}\left(\frac{D}{2a}\right)$$

$$= \frac{1}{\pi\sigma}\ln\left[\frac{D}{2a} + \sqrt{\left(\frac{D}{2a}\right)^2 - 1}\right](\Omega-m)$$

Static Magnetic Fields

- **A magnetostatic field is produced by a constant current flow.**

- Two important laws governing magnetostatic fields
  - **Biot-Savart Law** and **Ampere's Circuital Law**



(a) (b)

$$d\mathbf{H} = \frac{I\,d\mathbf{l} \times \mathbf{a}_R}{4\pi R^2} = \frac{Id\mathbf{l} \times \mathbf{R}}{4\pi R^3}$$

Different Current Distribution

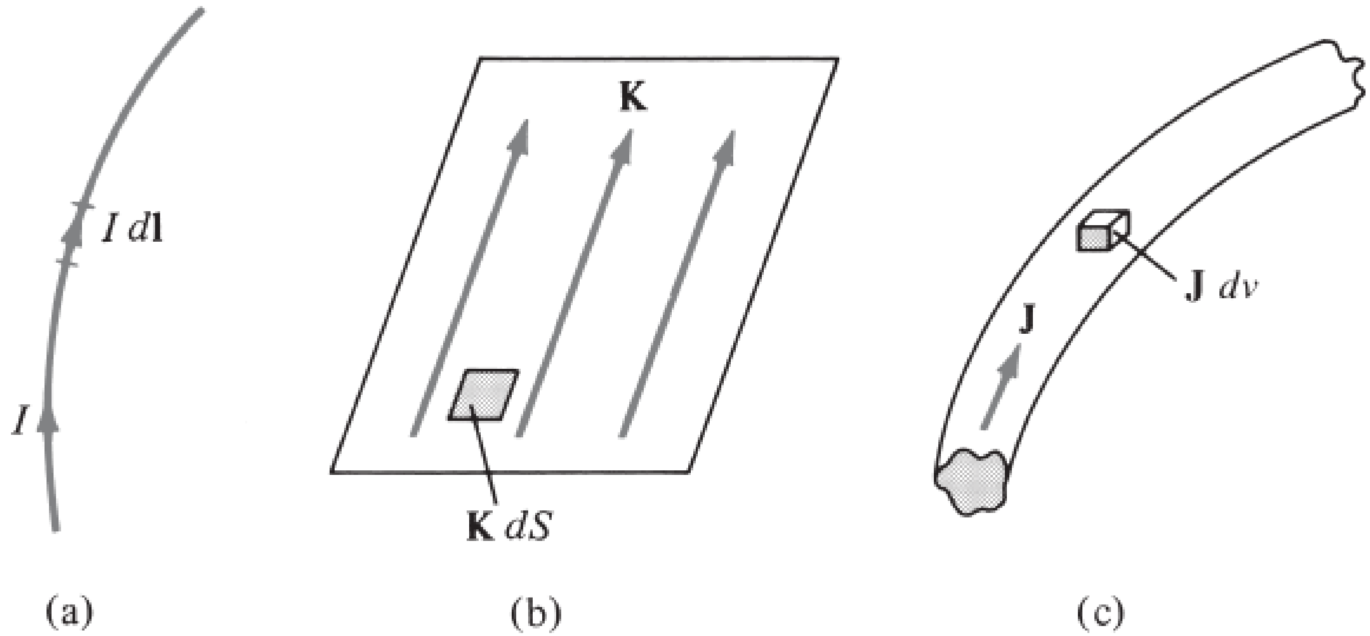


Figure 6.4 Current distributions: **(a)** line current, **(b)** surface current, **(c)** volume current.

Example: Finite straight current filament

$$\boxed{\mathbf{H} = \frac{I}{4\pi r}\left(\cos\alpha_2 - \cos\alpha_1\right)\mathbf{a}_\phi}$$

This expression can be used as a general expression which can be used to find the magnetic field due any straight filamentary conductor of finite length.

Special Cases:

Infinite long straight filament: $\mathbf{H} = \frac{I}{2\pi r}\mathbf{a}_\phi$

Semi-infinite straight filament: $\mathbf{H} = \frac{I}{4\pi r}\mathbf{a}_\phi$

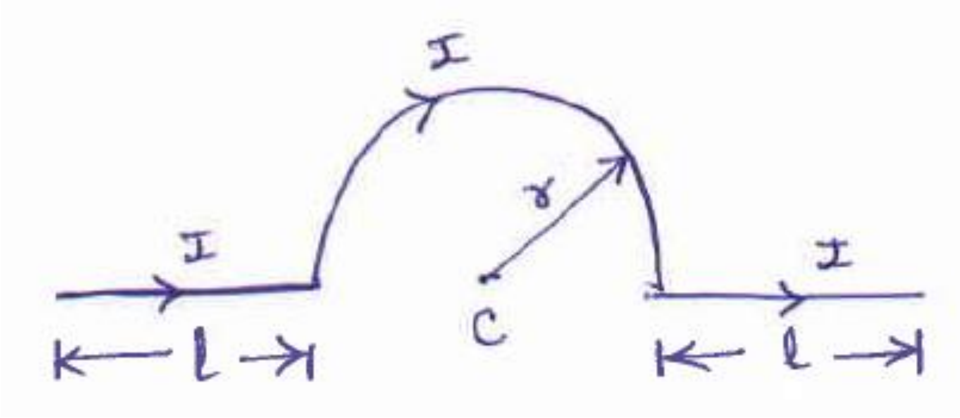
Example: Square current carrying loop.

- Find the magnetic field intensity at the centre of square current loop, with side *a* carrying current *I*.

$$\mathbf{H} = \frac{2\sqrt{2}I}{\pi a}\mathbf{a}_z$$

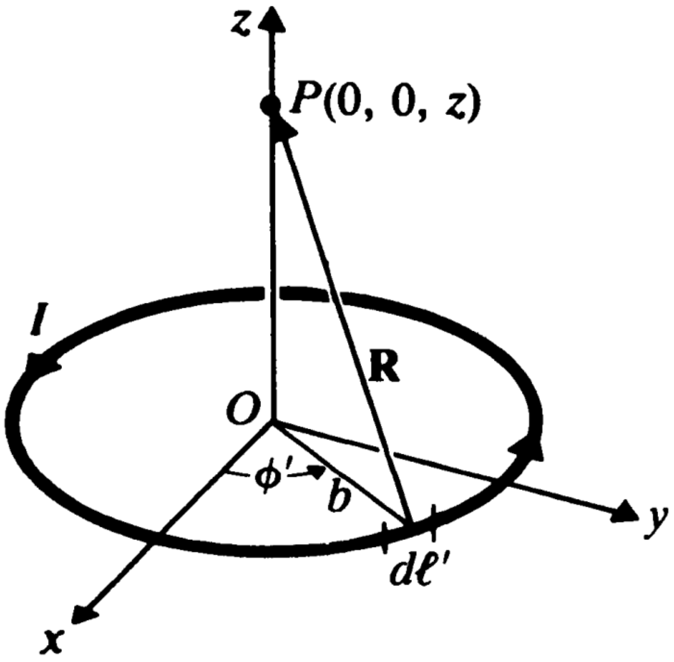
Example: Semi-Circular Loop

- A long wire having a semi-circular loop of radius *r* carries a current *I*, in Figure. Find the magnetic field at the centre *C* due to the entire wire.



Example: Circular loop of current:

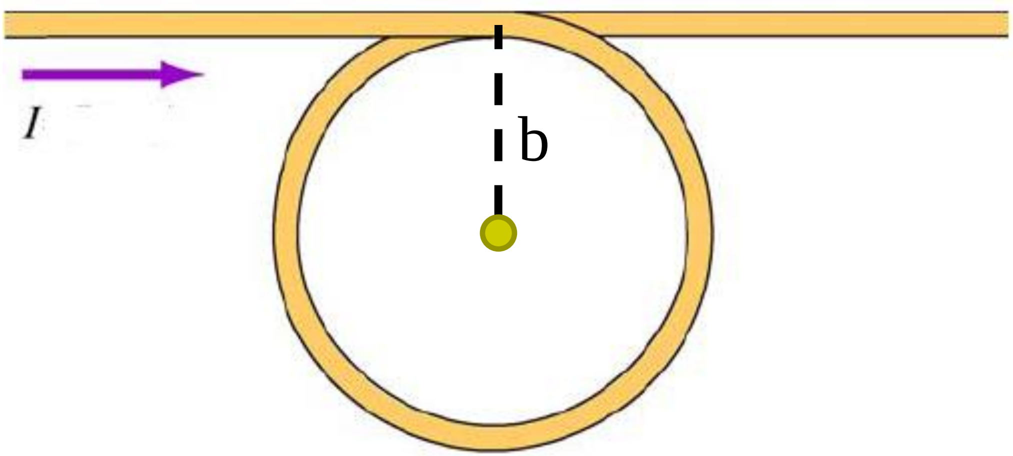
- Calculate the magnetic field on the axis of a circular current carrying loop.



$$\mathbf{H} = \frac{Ib^2}{2\left(b^2 + z^2\right)^{3/2}} \mathbf{a}_z$$

Example:

Determine the magnitude and direction of the magnetic field at the center of the loop.

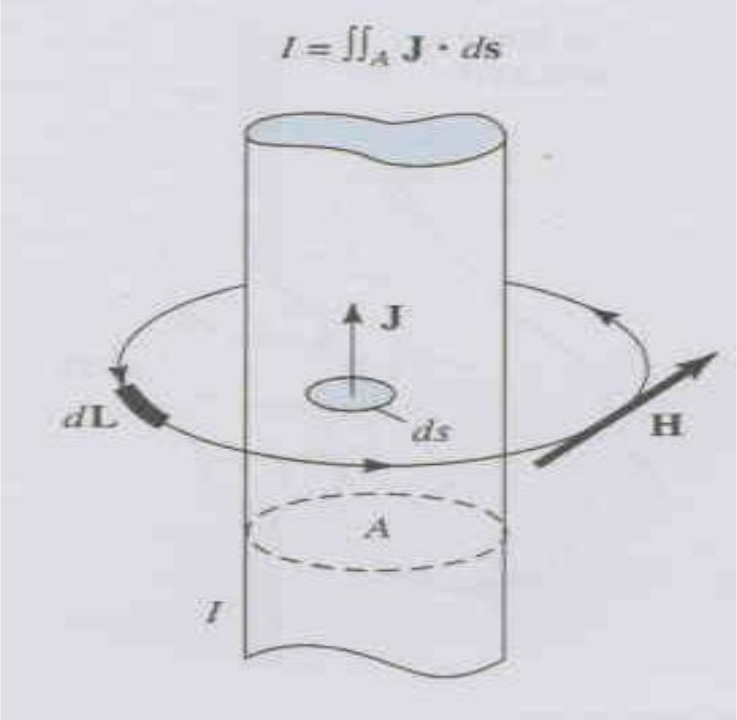


Ampere’s circuital law

$$\oint_C \vec{H}.d\vec{l} = I$$ **Integral Form**

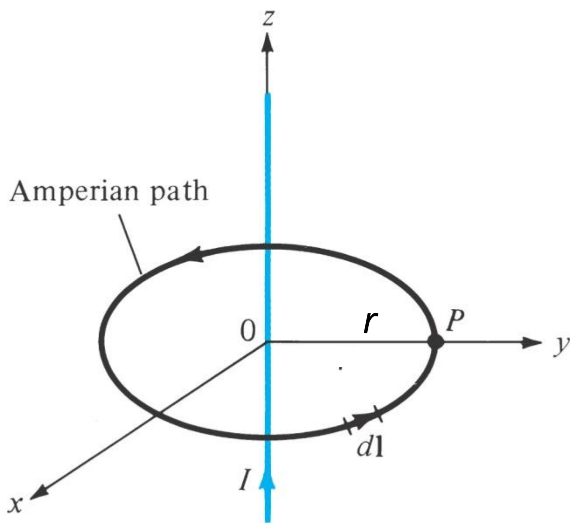
$$\nabla \times \vec{H} = \vec{J}$$ **Differential Form**

Magnetostatic Field is not conservative

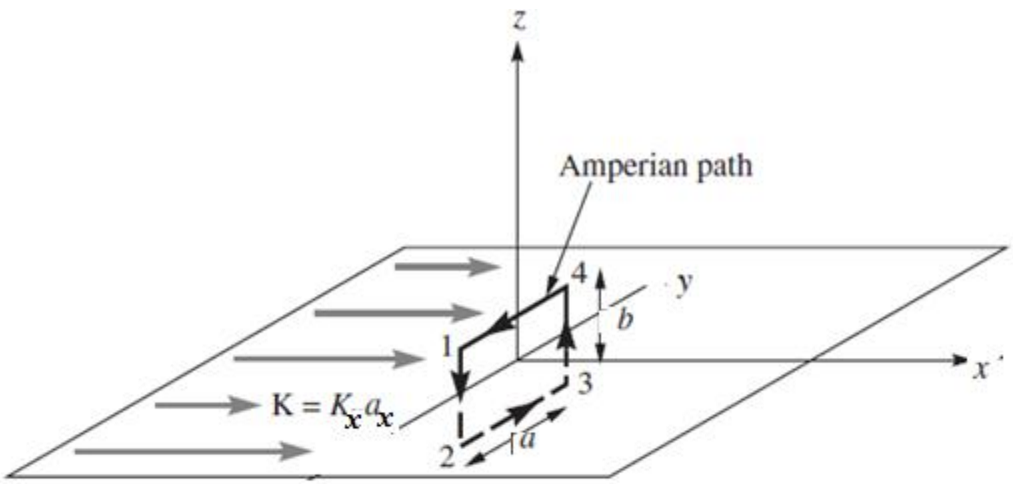


Applications of Ampere's Law

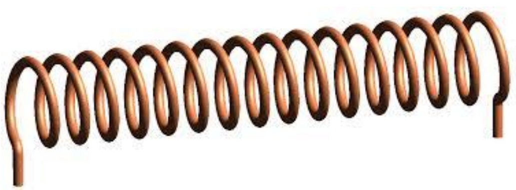
Infinite Line Current

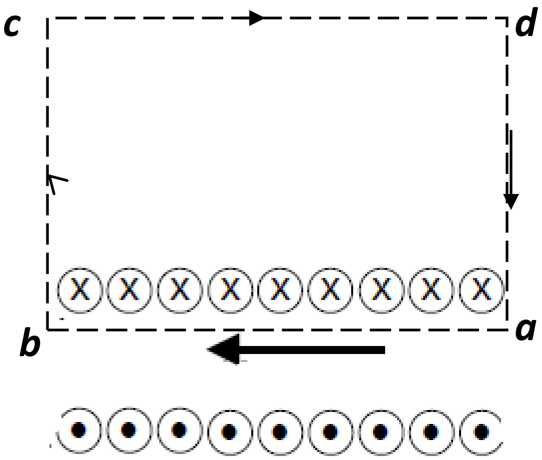


Infinite Sheet of Current



Solenoid





$$\oint_{abcd} \mathbf{H} \cdot d\mathbf{l} = I_{enclosed}$$

$$\oint_{abcd} \mathbf{H} . d\mathbf{l} = \int_a^b \mathbf{H} . d\mathbf{l} + \int_b^c \mathbf{H} . d\mathbf{l} + \int_c^d \mathbf{H} . d\mathbf{l} + \int_d^a \mathbf{H} . d\mathbf{l}$$

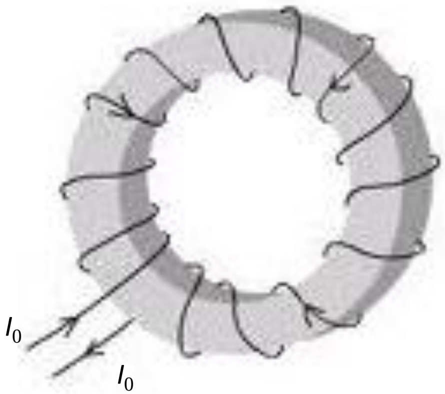
$$\oint_{abcd} \mathbf{H} . d\mathbf{l} = H \int_a^b dl \cos 0^0 = I_{enclosed}$$

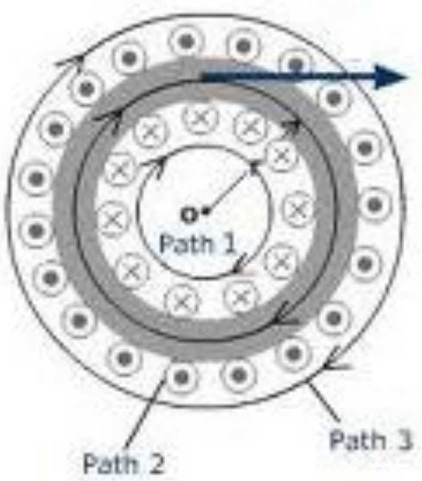
n is the number of turns per unit length of the solenoid = N/L

Net current enclosed within the rectangle *abcd* is $I_{enclosed} = nxI_0$

$$\boxed{H = nI_0 = \frac{NI_0}{L}}$$

Endless Solenoid (or Toroid)



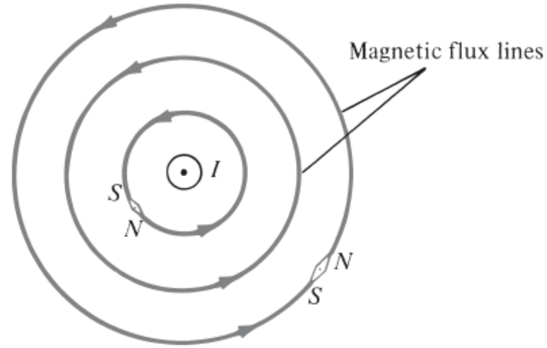


$$\oint_{circle} \mathbf{H}.d\mathbf{l} = \int H \; dl \; = H \int dl \qquad (\therefore \mathbf{H} \text{ and } d\mathbf{l} \text{ are parallel})$$

$$H \int dl = H(2\pi r) = NI_0 \qquad H = \frac{NI_0}{2\pi r} \qquad H = \frac{NI_0}{l}$$

Magnetic Flux Density

$$\boxed{\psi = \int_S \vec{B}.d\vec{S}}$$



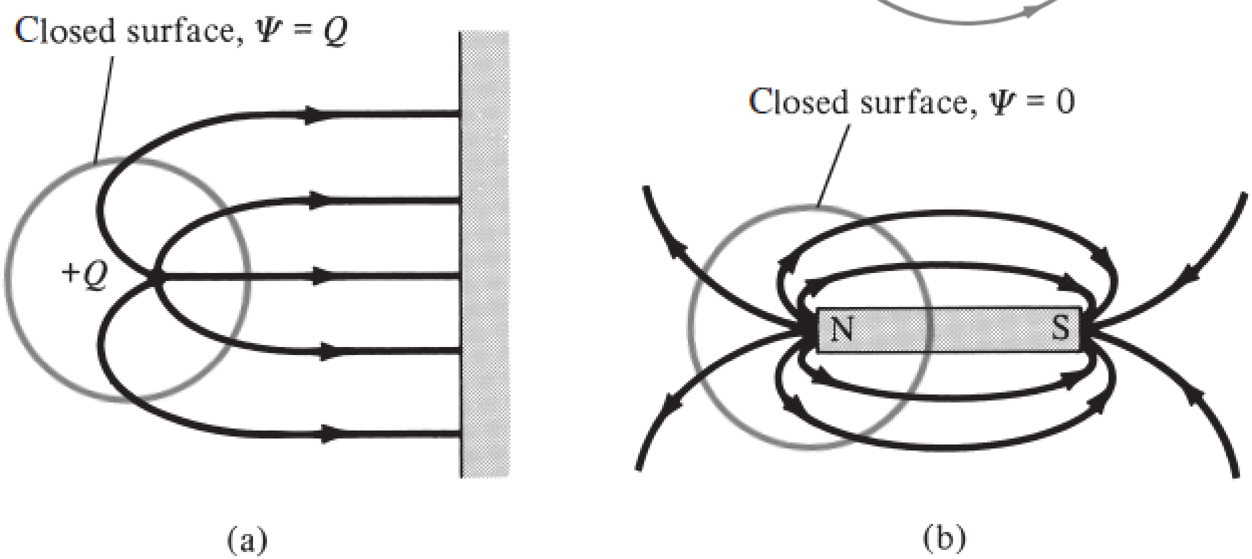


Figure 6.17 Flux leaving a closed surface due to **(a)** isolated electric charge $\Psi = \oint_S \mathbf{D} \cdot d\mathbf{S} = Q$, **(b)** magnetic charge, $\Psi = \oint_S \mathbf{B} \cdot d\mathbf{S} = 0$.

It is not possible to have isolated magnetic poles

Gauss's Law for magnetic fields

- For a closed surface

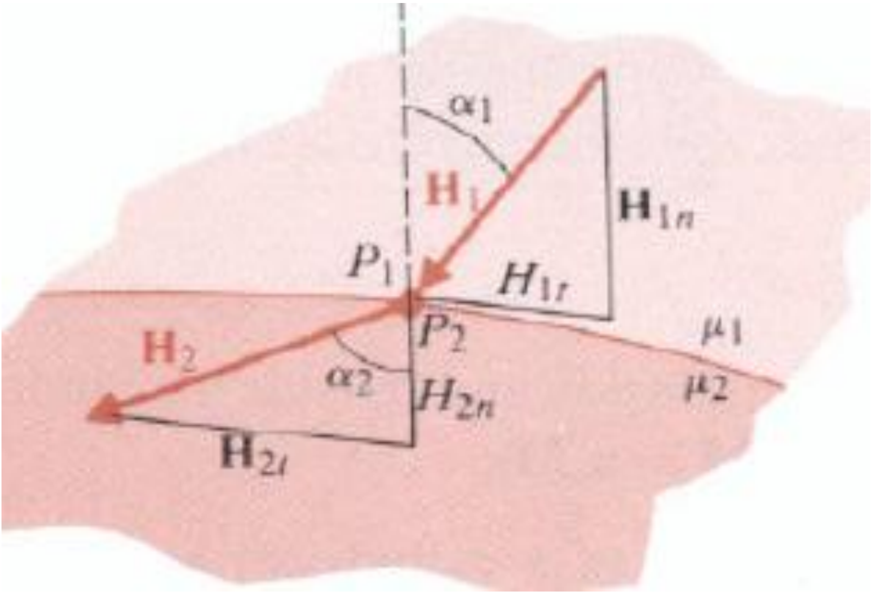
$$\boxed{\begin{array}{l} \oint_{c} \vec{B}.d\vec{S} = 0 \\ \nabla.\vec{B} = 0 \end{array}}$$

Maxwell's Equation for Static electric and magnetic fields

Boundary conditions for magnetostatic field

- **Normal component**:
  - The **normal component** of **B** is continuous across an interface
  - The **normal component** of **H** is discontinuous by the ratio μ_2/μ_1.
- **Tangential component**:
  - $H_{t1} - H_{t2} = K$
  - If the surface current density is zero. $H_{t1} = H_{t2}$

$$\boxed{\frac{\tan \alpha_2}{\tan \alpha_1} = \frac{\mu_2}{\mu_1}}$$



$$H_2 = \sqrt{H_{2t}^2 + H_{2n}^2} = \sqrt{(H_2 \sin \alpha_2)^2 + (H_2 \cos \alpha_2)^2}.$$

$$\boxed{H_2 = H_1\left[\sin^2 \alpha_1 + \left(\frac{\mu_1}{\mu_2} \cos \alpha_1\right)^2\right]^{1/2}.}$$

Vector magnetic potential

Since $\nabla \cdot \mathbf{B} = 0$ ⟹ $\mathbf{B} = \nabla \times \mathbf{A}$

vector magnetic potential.

$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$ yields $\nabla \times \nabla \times \mathbf{A} = \mu_0 \mathbf{J}$

$\nabla \times \nabla \times \mathbf{A} = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$ we have $\nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A} = \mu_0 \mathbf{J}$

For the purpose of simplicity, we choose $\nabla \cdot \mathbf{A} = 0$ therefore $\nabla^2 \mathbf{A} = -\mu_0 \mathbf{J}$

$$\nabla^2 \mathbf{A} = \mathbf{a}_x \nabla^2 A_x + \mathbf{a}_y \nabla^2 A_y + \mathbf{a}_z \nabla^2 A_z$$

$$\nabla^2 V = -\rho / \varepsilon_0$$

$$V = \frac{1}{4\pi\varepsilon_0} \int_{v'} \frac{\rho}{R} dv'$$

⟺

$$\nabla^2 \mathbf{A} = -\mu_0 \mathbf{J}$$

$$\mathbf{A} = \frac{\mu_0}{4\pi} \int_{v'} \frac{\mathbf{J}}{R} dv'$$

Scalar magnetic potential

scalar electric potential V ⟹ $\mathbf{E} = -\nabla V$

$$\vec{\mathrm{H}} = -\nabla V_m \ \text{if}\ \vec{\mathrm{J}} = 0$$

$$\mathbf{A} = -r^2/8\mathbf{a}_z$$

Find out the total magnetic flux crossing the surface

$$\phi = \frac{\pi}{2},\ 1 \le r \le 3\text{m},\ 0 \le z \le 4\text{m}.$$