

- Birla Institute of Technology and Science, Pilani,  
 Comprehensive Examination: EEE F243 / INSTR F243: Signals and Systems  
 Marks: 90 Date: 11-May-2017, Thursday  
 Time: 180 minutes Pages: 02  
 Note: Neat and legible figures must be drawn wherever mentioned with all credentials.  
 AY: 2016-17, Semester: II CLOSE BOOK
- a. Sketch the following signal    1.  $x[n] = \delta \left[ \cos \frac{\pi}{6} n \right]$     2.  $x(t) = u \left( \sin \frac{\pi}{T} t \right) - u \left( -\sin \frac{\pi}{T} t \right)$     15
- b. Consider the system shown in figure 1a with  $x(t)$  as input and  $y(t)$  as output. Determine whether it is (1) memoryless, (2) causal, (3) linear, (4) time-invariant or (5) stable. Justify your answer.
- c. Consider the feedback system shown in figure 1b. Assume that  $y[n] = 0$  for  $n < 0$ . Sketch the output  $y[n]$ , when  $x[n] = \delta[n]$  and  $x[n] = u[n]$ .

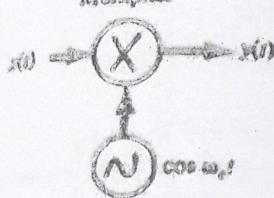


Figure 1a

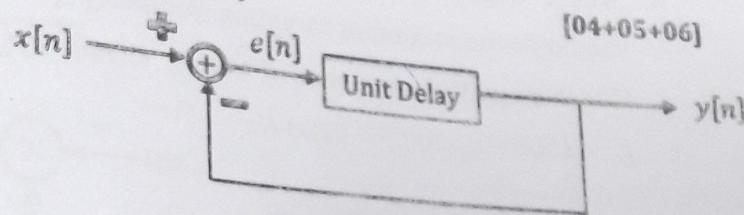


Figure 1b

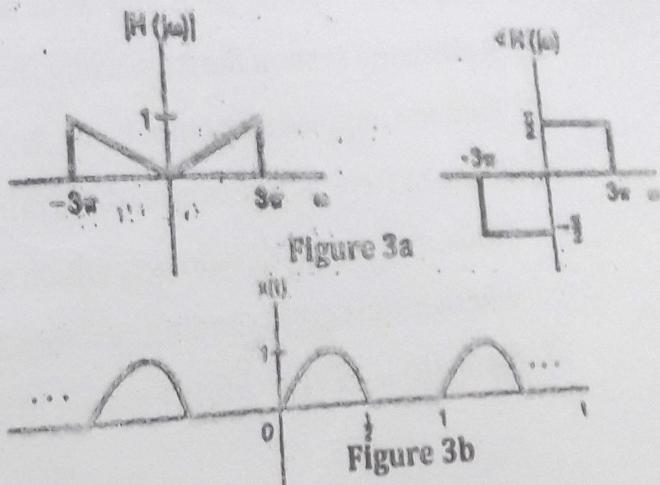
- Consider the signal  $x[n] = \alpha^n u[n]$ . Determine the signal  $g[n] = x[n] - \alpha x[n-1]$ . Use 15 this result in conjunction with the properties of convolution to determine a sequence  $h[n]$  such that  $x[n] * h[n] = \left(\frac{1}{2}\right)^n [u[n+2] - u[n-2]]$ . Where \* denote convolution. Perform all operations in time domain only.
- Using graphical convolution method, compute and sketch the output  $y(t)$  for a continuous-time LTI system whose impulse response is  $h(t) = e^{-3t}u(t)$  and the input is  $x(t) = u(t-3) - u(t-5)$ .    [08+07]

Figure 3a shows the frequency response  $H(j\omega)$  of a continuous-time system (filter). For each 15 of the input signals  $x(t)$  below, determine the system output  $y(t)$ .

- a.  $x(t) = \cos(2\pi t + \theta)$
- b.  $x(t) = \cos(4\pi t + \theta)$
- c.  $x(t) = \begin{cases} \sin 2\pi t, & m \leq t \leq \left(m + \frac{1}{2}\right) \\ 0, & \left(m + \frac{1}{2}\right) \leq t \leq m \end{cases}$

For any integer  $m$

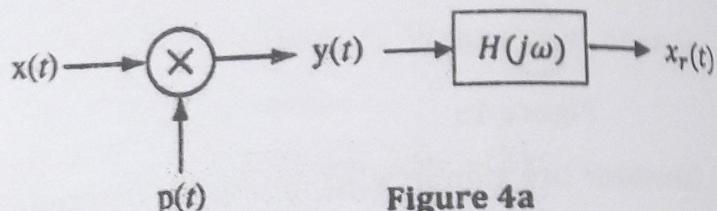
See figure 3b for  $x(t)$     [05+03+07]



Q4 Figure 4a show the sampling and reconstruction process.

$x(t) = 10 \cos(600\pi t) \cos^2(1600\pi t)$  is an input signal,  $p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$  is an impulse train,  $f_s = 4000$  Hz is sampling frequency,  $n$  is an integer,  $H(j\omega)$  is the frequency response of a low-pass filter with cut-off frequency  $f_c$  Hz. Precisely mention values of frequencies ( $f$ ) in Hz, and amplitude/magnitude on the sketch.

- Find the range of cut-off frequencies  $f_c$  in Hz of low-pass filter such that  $x_r(t) = x(t)$ .
- Sketch magnitude response of  $X(j\omega)$ ,  $Y(j\omega)$ ,  $H(j\omega)$ ,  $X_r(j\omega)$  and find  $x_r(t)$ , what should be the minimum sampling frequency  $f_s$  for  $x_r(t) = x(t)$ .
- Sketch magnitude response of  $Y(j\omega)$ ,  $H(j\omega)$ ,  $X_r(j\omega)$  and find  $x_r(t)$  for  $f_s = 2500$  Hz and  $f_c = 1250$  Hz



[02+08+05]

Figure 4a

Q5 a) The signal  $y(t) = e^{-2t}u(t)$  is the output of a causal all-pass system for which the system function is  $H(s) = \frac{s-1}{s+1}$ . Find at least two possible input  $x(t)$  that could produce  $y(t)$  and

sketch the respective ROC. What is the input  $x(t)$  if it is known that  $\int_{-\infty}^{\infty} x(t)dt < \infty$ .

b) Using Laplace transform find the transfer function and frequency response of

1. Ideal delay of T sec,
2. Ideal differentiator, and
3. Ideal integrator

Plot the magnitude and phase responses

[09+06]

Q6 A causal LTI system is given by the difference equation  $y[n] - 3y[n-1] + 2y[n-2] = x[n]$

- Find  $H(z)$ . Plot the poles and zeros and sketch the ROC.
- Realize the system function using Direct-I form.
- Find the impulse response. Is the system stable? Justify your answer.
- If the system is not causal, determine the all possible system functions, associated ROCs, and impulse responses which satisfy the preceding difference equation. Specify whether the corresponding systems are stable.
- Find  $y[n]$  if  $x[n] = 3^n u[n]$ , sketch ROC.

[03+02+03+04+03]