BIRLA INSTITUTE OF TECHNOLOGY & SCIENCE, PILANI

I SEMESTER 2012-2013

ES C233 - Logic in Computer Science

08th October, 2012 — Mid-semester Test (closed book) — Weightage: 25%

1. Without using the natural laws of deduction, determine whether the sequent given below is valid or not.

$$(p \to q) \wedge (r \to \neg p) \wedge r \ \vdash \neg q$$

[3]

p	q	r	$p \to q$	$\bar{p}$	$r \to \bar{p}$	$(p \to q) \wedge (r \to \bar{p}) \wedge r$	$\bar{q}$	$((p \to r) \wedge (r \to \bar{p}) \wedge r) \to \bar{q}$
T	T	T	T	F	F	F	F	T
T	T	F	T	F	T	F	F	T
T	F	T	F	F	F	F	T	T
T	F	F	F	F	T	F	T	T
F	T	T	T	T	T	T	F	F
F	T	F	T	T	T	F	F	T
F	F	T	T	T	T	T	T	T
F	F	F	T	T	T	F	T	T

As it follows from the truth table, $((p \to q) \wedge (r \to \bar{p}) \wedge r) \to \bar{q}$ is not a tautology, so the argument $(p \to q) \wedge (r \to \bar{p}) \wedge r \vdash \bar{q}$ is not valid. In particular a counter example for it is when p, q and r are false, true and true correspondently.

2. Find a conjunctive normal form (CNF) equivalent to the formula

$$((p \to q) \wedge \neg q) \to \neg p$$

Determine, without using truth table, whether the formula in CNF is a tautology or not.

[3]

$$\begin{aligned}
&((p \to q) \wedge \neg q) \to \neg p \\
\rightsquigarrow\ &\neg((\neg p \vee q) \wedge \neg q) \vee \neg p \\
\rightsquigarrow\ &(p \wedge \neg q) \vee q \vee \neg p \\
\rightsquigarrow\ &(p \vee q \vee \neg p) \wedge (\neg q \vee q \vee \neg p)
\end{aligned}$$

Yes, the formula is a tautology since both disjunctions include a pair $x, \neg x$.

3. Show that

$$(p \to q) \leftrightarrow (\neg q \to \neg p)$$

is a tautology, where $p \leftrightarrow q$ means $(p \to q) \wedge (q \to p)$

[3]

p	q	$\bar{q}$	$\bar{p}$	$p \to q$	$\bar{q} \to \bar{p}$	$(p \to q) \leftrightarrow (\bar{q} \to \bar{p})$
T	T	F	F	T	T	T
T	F	T	F	F	F	T
F	T	F	T	T	T	T
F	F	T	T	T	T	T

All truth values of $(p \to q) \leftrightarrow (\bar{q} \to \bar{p})$ in the truth table are true no matter what the truth values of its simple components. So this proposition is a tautology by definition.

4. *Let ϕ be a formula of propositional logic. Then ϕ is satisfiable iff $\neg\phi$ is not valid*.
   Explain the usefulness of the above result and give proper justifications for your answer.
   We need a decision process for only one of the concepts (Validity or Satisfiability). [2 marks]
   Give flow charts for converting a decision process to check Validity into a decision process for Satisfiability and vice-versa [1.5+1.5]. Refer to class notes, date 03rd September, 2012.

[5]

5. Show that every HORN formula can be converted into CNF.
   Horn formulas are already of the form $\psi_1 \wedge \psi_2 \wedge \psi_3 \ldots \wedge \psi_n$, where each ψ_i is of the form $p_1 \wedge p_2 \wedge p_3 \ldots \wedge p_{i_k} \rightarrow q_i$, which is equivalent to $\neg(p_1 \wedge p_2 \wedge p_3 \ldots \wedge p_{i_k}) \vee q_i$, which in turn is equivalent to $\neg p_1 \vee \neg p_2 \vee \neg p_3 \ldots \vee \neg p_{i_k} \vee q_i$. Thus we may convert any Horn formula into a CNF where each conjunction clause has at most one positive literal in it.

[3]

6. Translate the following into symbolic form using Predicate Logic:
   (i) Somebody cried out for help and called the police
   (ii) Nobody can ignore her
   Clearly define the predicate and function symbols you plan to use.
   UoD = all human beings.
   $(\exists x)[H(x) \wedge P(x)]$, where $H(x)$ - *x cried out for help* and $P(x)$ - *x called the police*
   $\sim (\exists x)I(x)$ or $(\forall x)[\sim I(x)]$, where $I(x)$ - *x can ignore her*

[2]

7. Give one example each for function symbols with arity 2 & 3.
   Grade (student, course)
   Temperature (longitude, latitude, time)

[2]

8. Consider the following predicates:
   P(x; y) : x > y
   Q(x; y) : x ≤ y
   R(x) : x - 7 = 2
   S(x) : x > 9

PTO ➔

If the universe of discourse is the real numbers, give the truth value of each of the following:

$$\begin{aligned}
&\text{(i)}(\exists x)R(x) \\
&\text{(ii)}(\forall y)[\sim S(y)] \\
&\text{(iii)}(\forall x)(\exists y)P(x,y) \\
&\text{(iv)}(\exists y)(\forall x)Q(x,y)
\end{aligned}$$

$$\begin{aligned}
&\text{(v)}(\forall x)(\forall y)[P(x,y) \vee Q(x,y)] \\
&\text{(vi)}(\exists x)S(x) \wedge \sim (\forall x)R(x) \\
&\text{(vii)}(\exists y)(\forall x)[S(y) \wedge Q(x,y)] \\
&\text{(viii)}(\forall x)(\forall y)[\{R(x) \wedge S(y)\} \rightarrow Q(x,y)]
\end{aligned}$$

Give proper reasons for your answer. Just writing T or F will not fetch you any marks.

(i) T, $\exists x, x = 9$, that $R(x)$ is true

(ii) F, counter example $y = 10$

(iii) T, for any real number always exists another real number that is less then it.

(iv) F, there is no such real number that is grater or equal to all other real numbers.

(v) T, any two real numbers x and y are either $x > y$ or $x \leq y$.

(vi) T, there exist real numbers that are grater than 9, and not all real numbers are equal to 9

(vii) F, there is no such real number that is grater or equal to all other real numbers, even if this number is grater than 9.

(viii) T, this follows from the fact that $(\forall x)R(x)$ is false. Therefore $(\forall x)(\forall y)[R(x) \wedge S(y)]$ is also false, so $(\forall x)(\forall y)[\{R(x) \wedge S(y)\} \rightarrow Q(x,y)]$ is true.

[0.5*8=4]