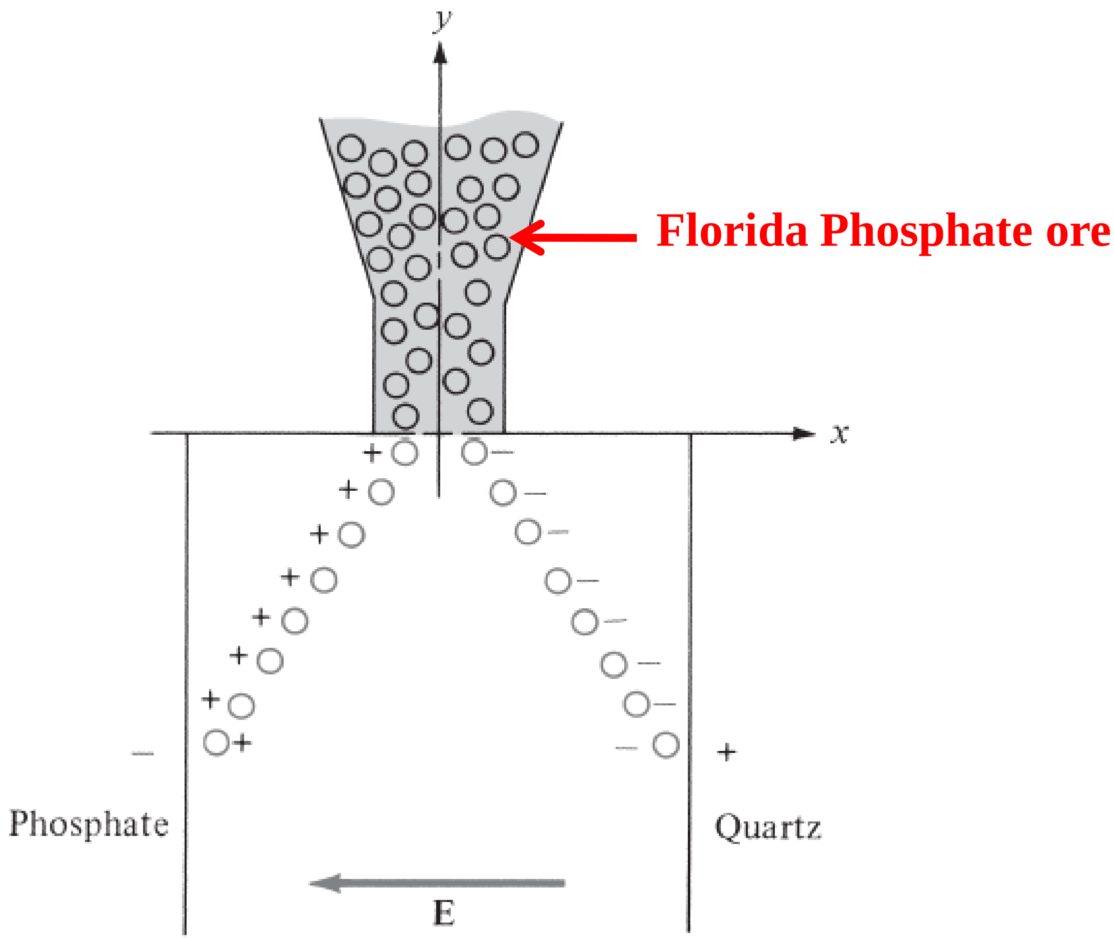
Practical Application:

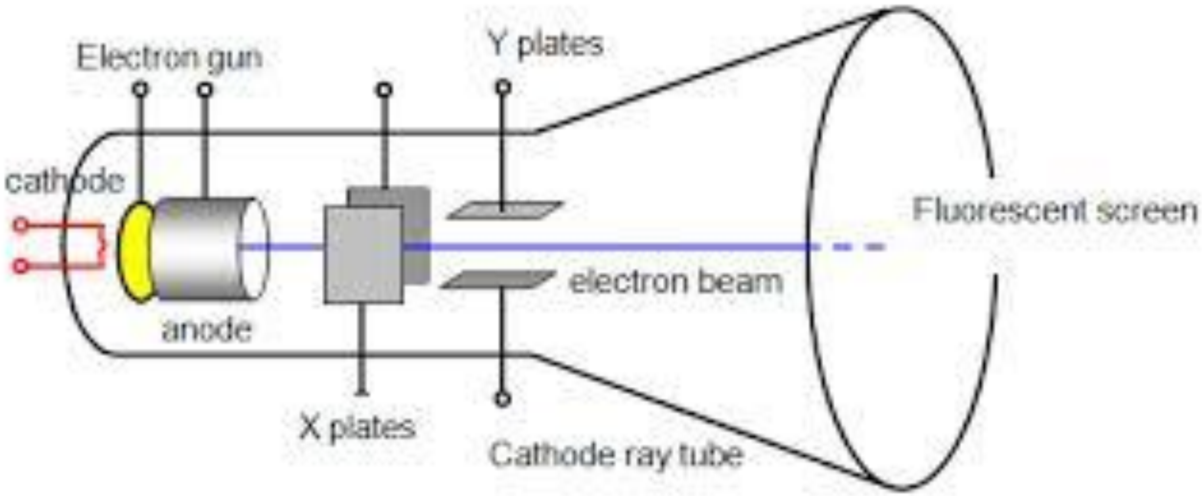
Lecture-5

1. Electrostatic separation of solids

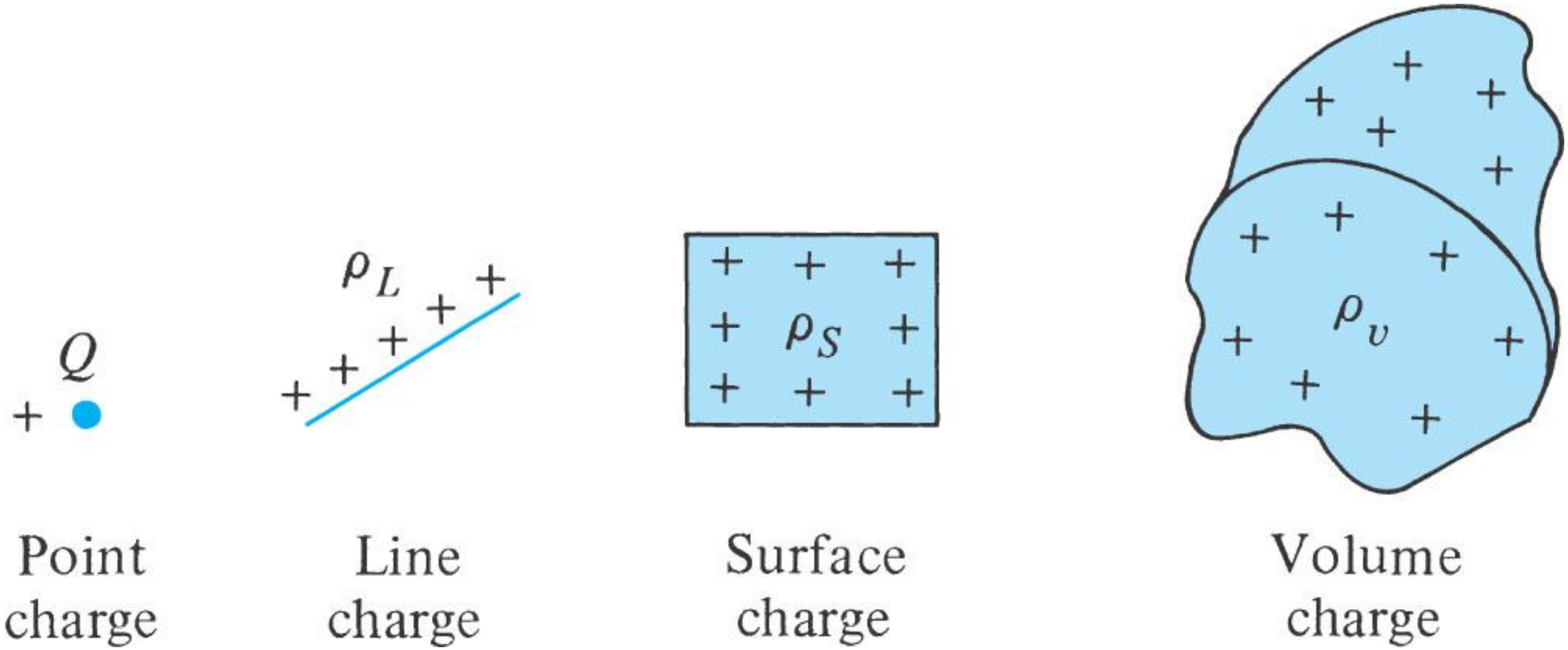


Practical Application:

2. Cathode Ray oscilloscope



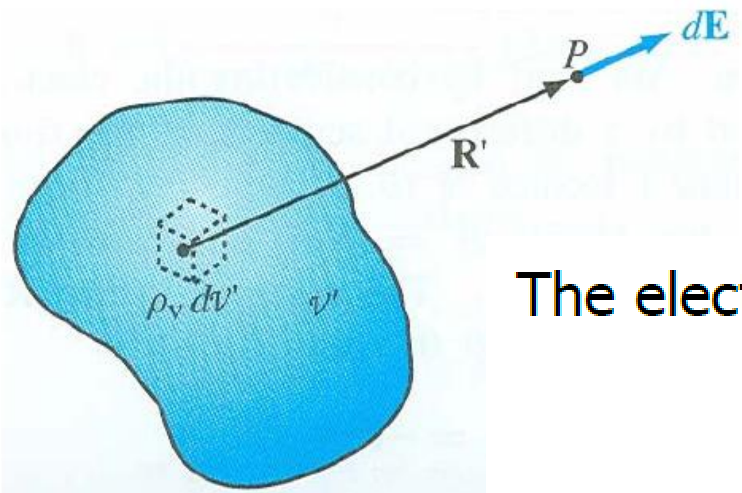
Electric Field Due to continuous charge distribution



Electric field due to a volume charge distribution

$$d\mathbf{E} = \mathbf{a}_{R'} \frac{dq}{4\pi\varepsilon R'^2} = \mathbf{a}_{R'} \frac{\rho_v dv'}{4\pi\varepsilon R'^2} \quad V/m$$

$$\Rightarrow \mathbf{E} = \int_{v'} d\mathbf{E} = \frac{1}{4\pi\varepsilon}\int_{v} \mathbf{a}_{R'} \frac{\rho_v dv'}{R'^2}$$



The electric field due to charges on a surface is

$$\mathbf{E} = \frac{1}{4\pi\varepsilon}\int_{S'} \mathbf{a}_{R'} \frac{\rho_s ds'}{R'^2}$$

The electric field due to charges on a line is

$$\mathbf{E} = \frac{1}{4\pi\varepsilon}\int_{l'} \mathbf{a}_{R'} \frac{\rho_l dl'}{R'^2}$$

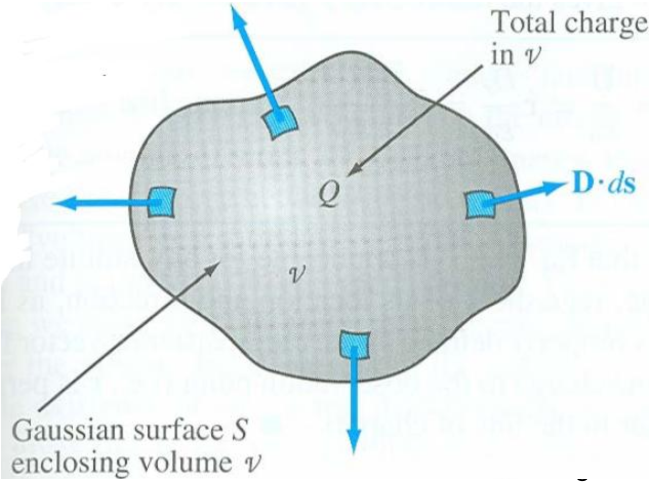
Gauss's Law and Applications

- Total outward flux of the E-field over any closed surface in free space is equal to the total (net) charge enclosed in the surface divided by ε_0.
- Concept of Electric flux density

Gauss's Law states that the outward flux of $\mathbf{D}$ through a surface is proportional to the enclosed charged Q.

$$\nabla \cdot \mathbf{D} = \rho_v \qquad \text{Differential form}$$

$$\oint_S \mathbf{D} \cdot d\mathbf{S} = Q \qquad \text{Integral form}$$



Example: point charge

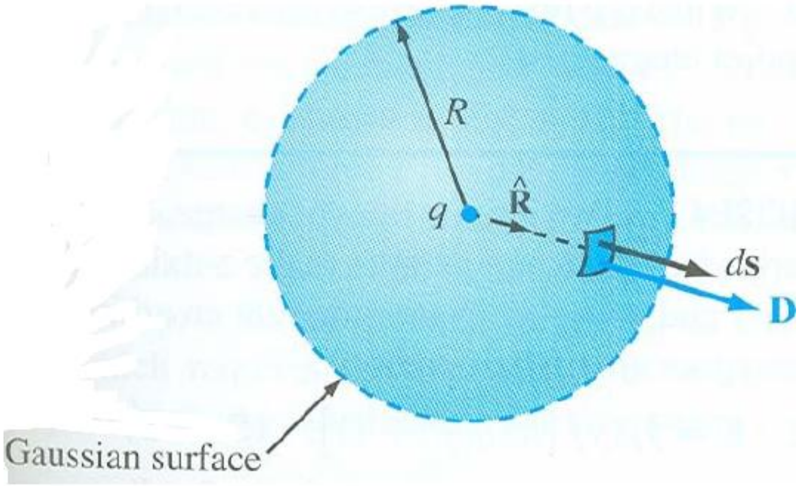
Construct a spherical surface with centre at q and radius R. Electric field is the same everywhere on the surface. Applying integral form of Gauss's law gives

$$\oint_s \mathbf{D}\cdot d\mathbf{s} = \oint_s \mathbf{a}_{R'} D_R \cdot d\mathbf{s}$$

$$= \oint_s D_R ds$$

$$= 4\pi R^2 D_R = q$$

$$\therefore \mathbf{E} = \mathbf{D}/\varepsilon = \frac{q}{4\pi\varepsilon R^2}\mathbf{a}_{R'}$$



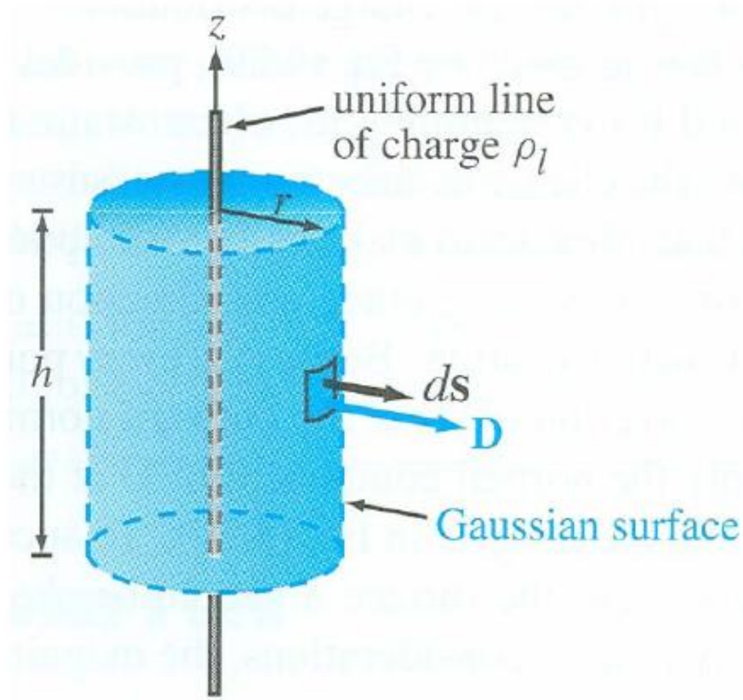
Example: infinite long line of charge

Since the line of charge is infinite in extent and is along the z-axis, $\mathbf{D}$ must be in the radial r-direction and must not depend on or z.

$$\oint_s \mathbf{D} \cdot d\mathbf{s} = \int_{z=0}^{h} \int_{\varphi=0}^{2\pi} \mathbf{a}_r D_r \cdot \mathbf{a}_r r\, d\varphi\, dz$$

$$= 2\pi h D_r r = \rho_l h$$

$$\therefore \mathbf{E} = \mathbf{D} / \varepsilon = \frac{\rho_l}{2\pi\varepsilon r} \mathbf{a}_r$$



Example: dc transmission line

- Two long parallel conductors of a dc transmission line separated by 2 m have charges of $\rho_L = 5\mu C/m$ of opposite sign. Both lines are 8 m above ground. What is the magnitude of the electric field 4 m directly below one of the wires? $\varepsilon_r=1$
- Answer = 1.61 kV/m

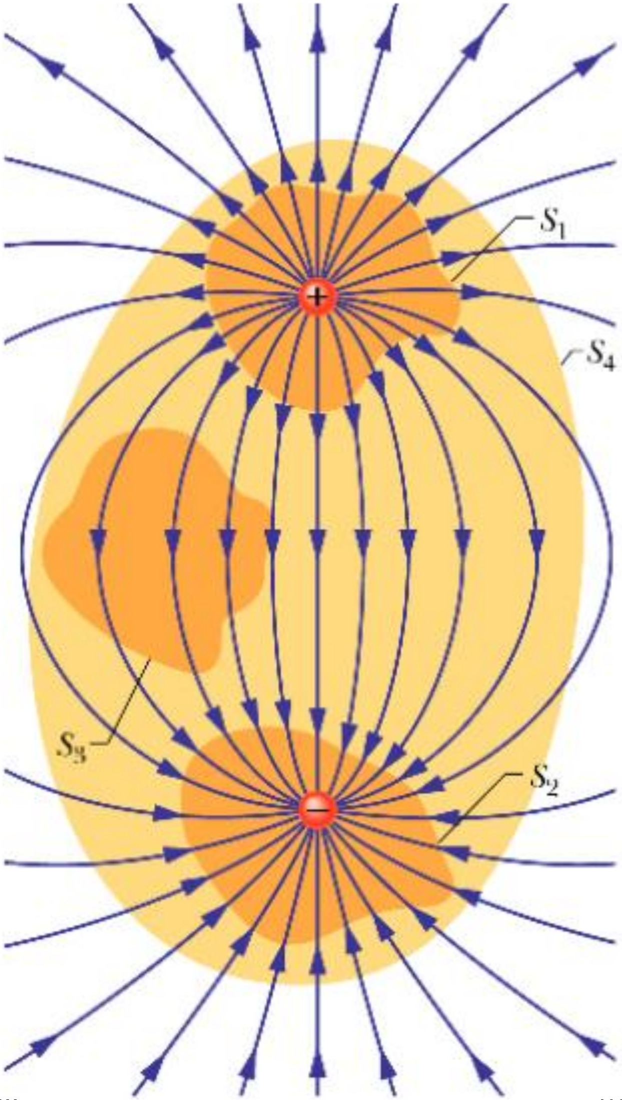
Recap..

- The meaning of flux is just the number of field lines passing through the surface.
- Gauss's law: The outward flux of the electric field through any closed surface equals the net enclosed charge divided by ε_0

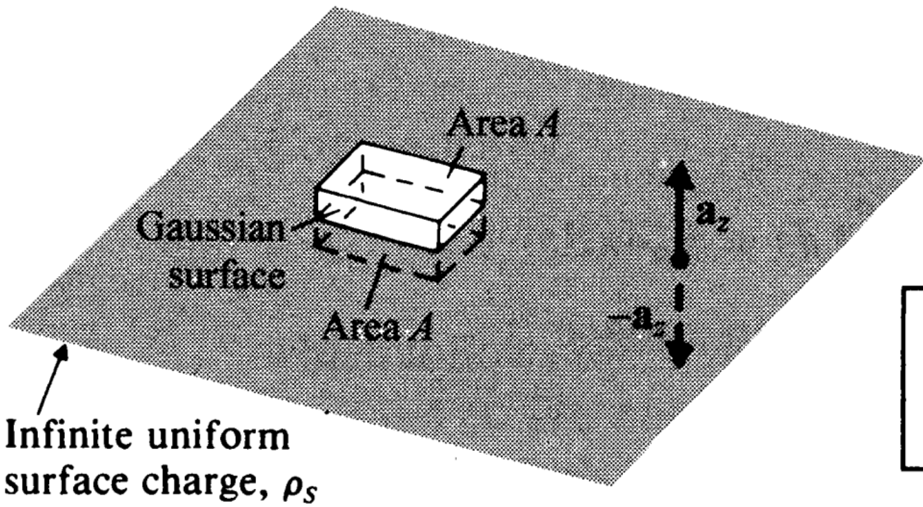
Example:

- **Assume two charges, +q and –q. Find fluxes through surfaces.**

$$\Phi_1 = +q/\varepsilon_0$$
$$\Phi_2 = -q/\varepsilon_0$$
$$\Phi_3 = 0$$
$$\Phi_4 = (q - q)/\varepsilon_0 = 0$$



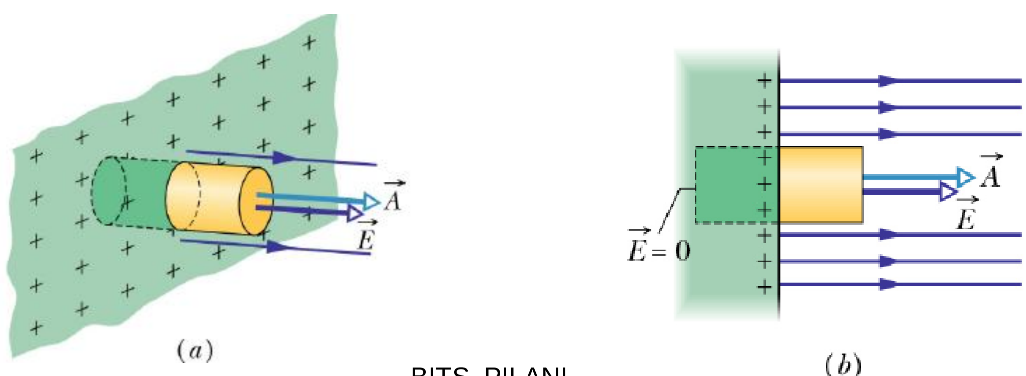
Example: Infinite Planar Charge with a uniform surface charge density



$$\boxed{\mathbf{E} = \mathbf{a}_z E_z = \mathbf{a}_z \frac{\rho_s}{2\epsilon_0}, \qquad z > 0,}$$

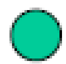
$$\boxed{\mathbf{E} = -\mathbf{a}_z E_z = -\mathbf{a}_z \frac{\rho_s}{2\epsilon_0}, \qquad z < 0.}$$

Field at the Surface of a Conductor



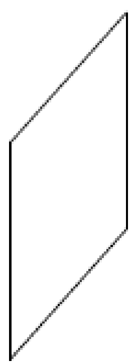
Summary for different dimensions

$$\boxed{E \propto \frac{1}{r^{2-d}}}$$



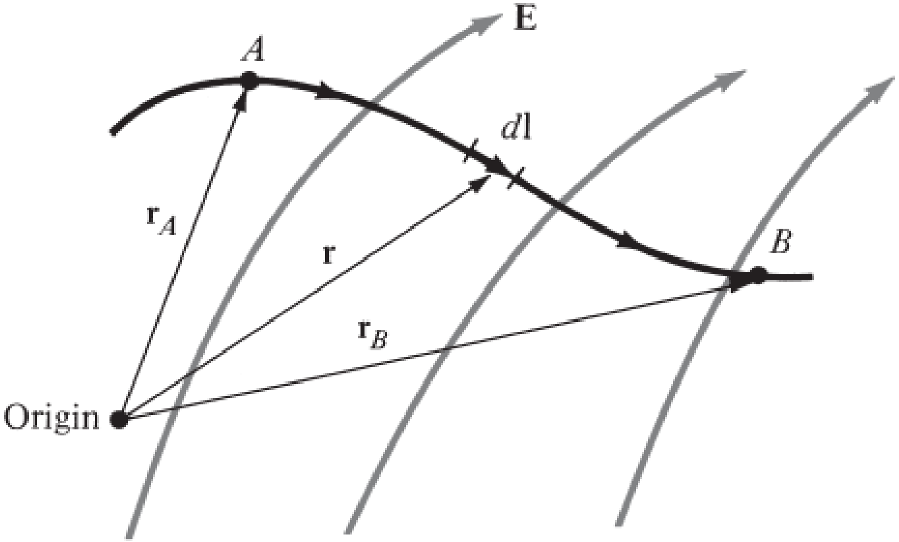
Point charge, d=0

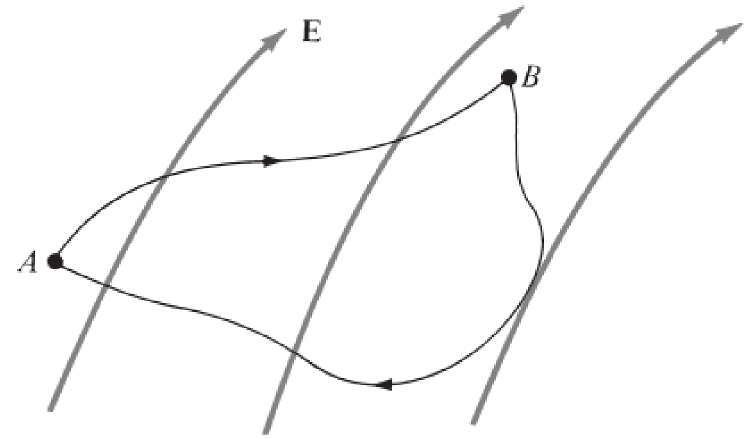
Line charge, d=1



Surface charge, d=2

Electric Potential V





$$V = -\int_{\infty}^{r} \mathbf{E} \cdot d\mathbf{l}$$

The integral is independent of the path taken

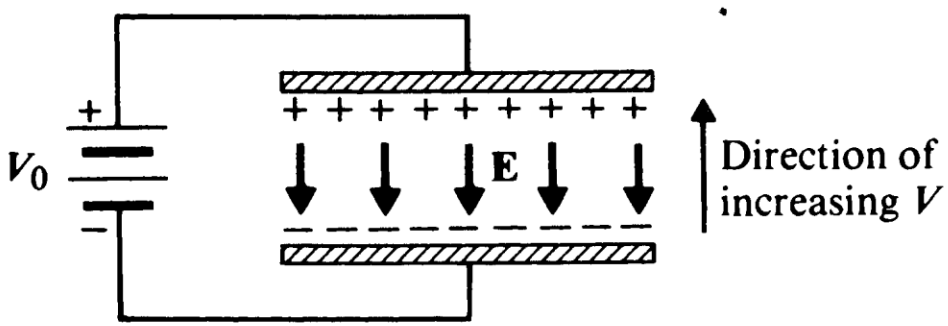
No net work is done in moving a charge along a closed path in an electrostatic field.

$$\nabla \times \mathbf{E} = 0$$

The electrostatic field is irrotational or conservative

$\mathbf{E} = -\nabla V$

-ve sign shows that direction of E is opposite to the direction in which V increases.



According to vector identity, for any scalar V

$$\nabla \times (\nabla V) \equiv 0$$

Electric Potential due to charge distribution

- Point charges:
  - The total electric potential at a point is the algebraic sum of the individual potentials at the point.
- Example: Electric Dipole Potential

An ***electric dipole*** is formed when two point charges of equal magnitude but opposite sign are separated by a small distance.

Electric Dipole

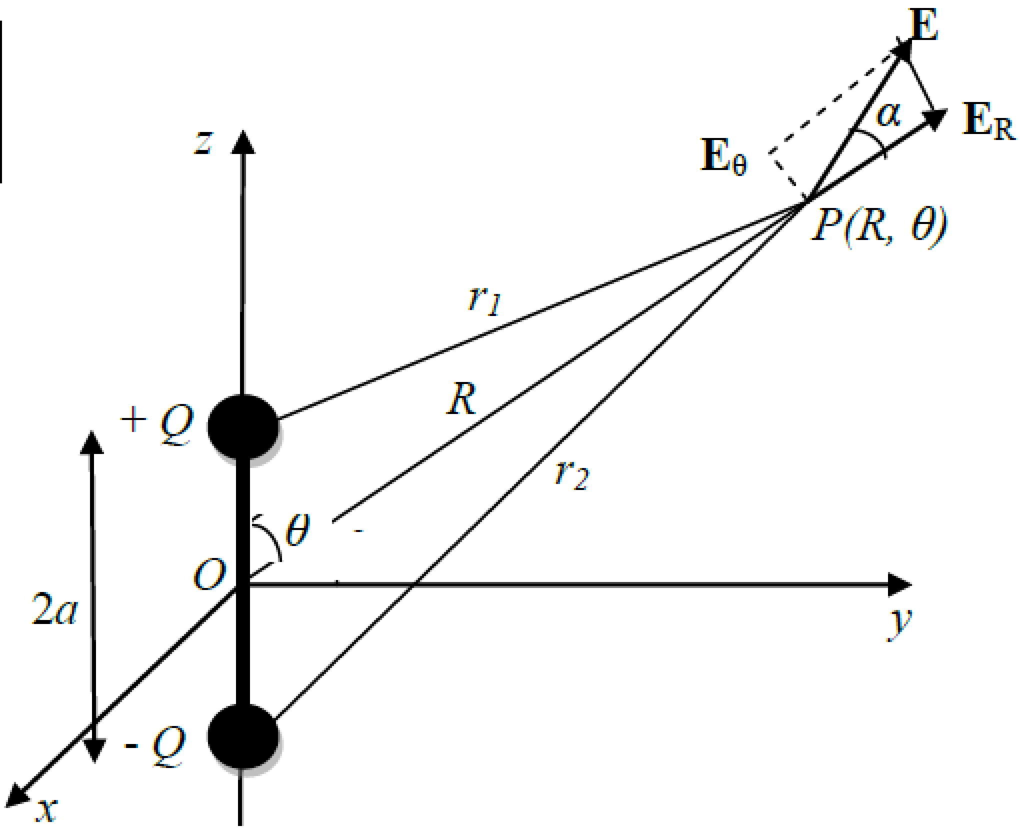
$$V_p = \frac{1}{4\pi\varepsilon_0}\left[\frac{\mathbf{p.R}}{R^3}\right]$$

$$\mathbf{E} = -\nabla V = -\left(\frac{\partial V}{\partial R}\mathbf{a}_R + \frac{1}{R}\frac{\partial V}{\partial \theta}\mathbf{a}_\theta + \frac{1}{R\sin\theta}\frac{\partial V}{\partial \phi}\mathbf{a}_\phi\right)$$

$$\boxed{\mathbf{E} = \frac{p}{4\pi\varepsilon_0 R^3}\left(\hat{a}_R 2\cos\theta + \hat{a}_\theta \sin\theta\right)}$$

$$E = \sqrt{E_R^2 + E_\theta^2}$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{p}{R^3}\sqrt{1+3\cos^2\theta}$$



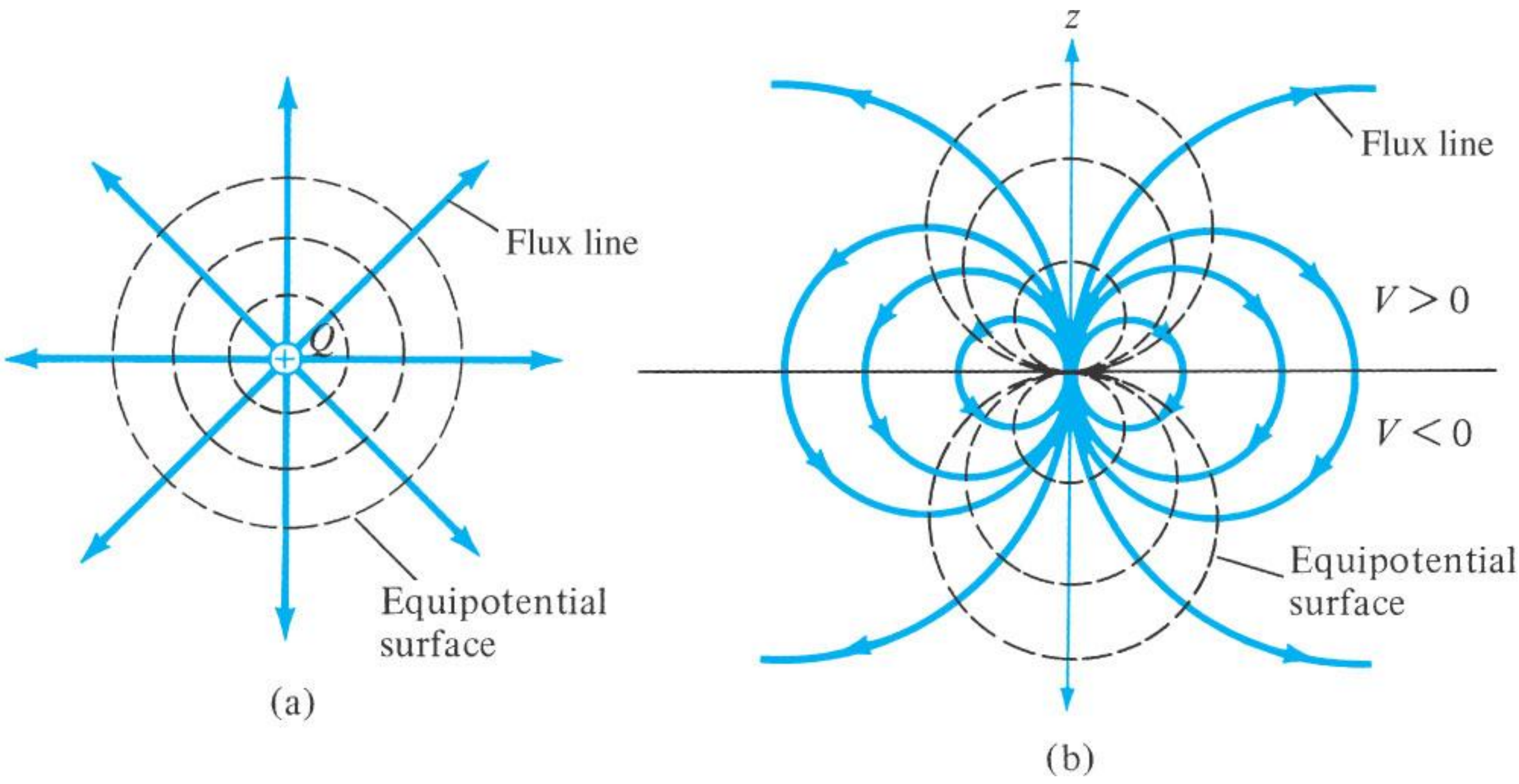
Example:

- Two dipoles with dipole moments -5$\mathbf{a}_z$ nC/m and 12$\mathbf{a}_z$ nC/m are located at points (0,0,-3) and (0,0,2), respectively. Compute the potential at the origin.

$$V = \sum_{k=1}^{2} \frac{\mathbf{p}_k.\mathbf{R}_k}{4\pi\varepsilon_0 R_k^{\,3}}$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{\mathbf{p}_1.\mathbf{R}_1}{R_1^{\,3}} + \frac{\mathbf{p}_2.\mathbf{R}_2}{R_2^{\,3}}\right]$$

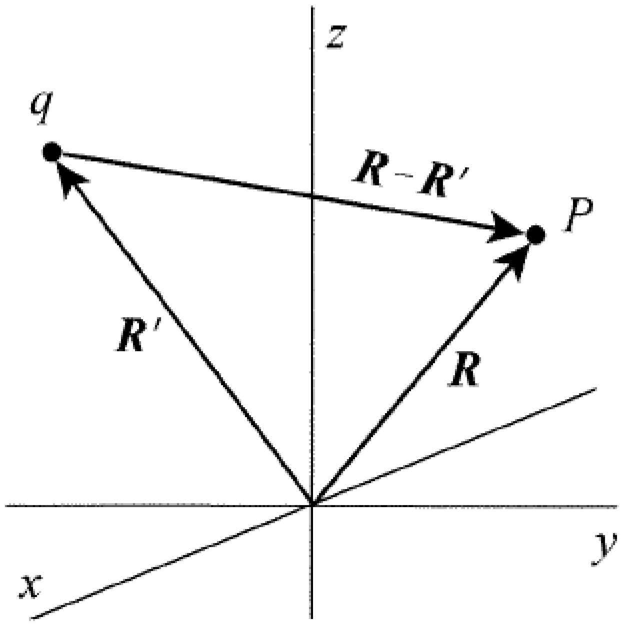
Graphical Representation of Potential: Equipotential lines



<u>Point Charge</u>

$$V(\boldsymbol{R}) = \frac{q}{4\pi\epsilon_o R_o}$$

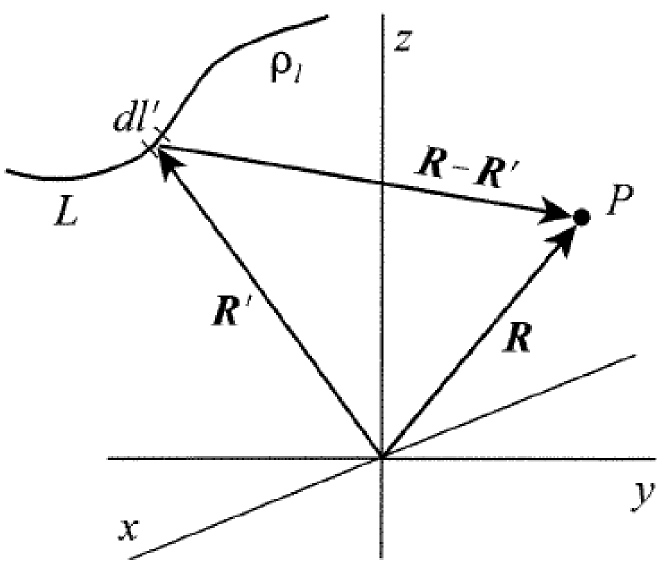
$$R_o = \left| \boldsymbol{R} - \boldsymbol{R}' \right|$$



<u>Line Charge</u> $(\rho_l\, dl' \Leftrightarrow Q)$

$$dV(\boldsymbol{R}) = \frac{\rho_l\, dl'}{4\pi\epsilon_o R_o}$$

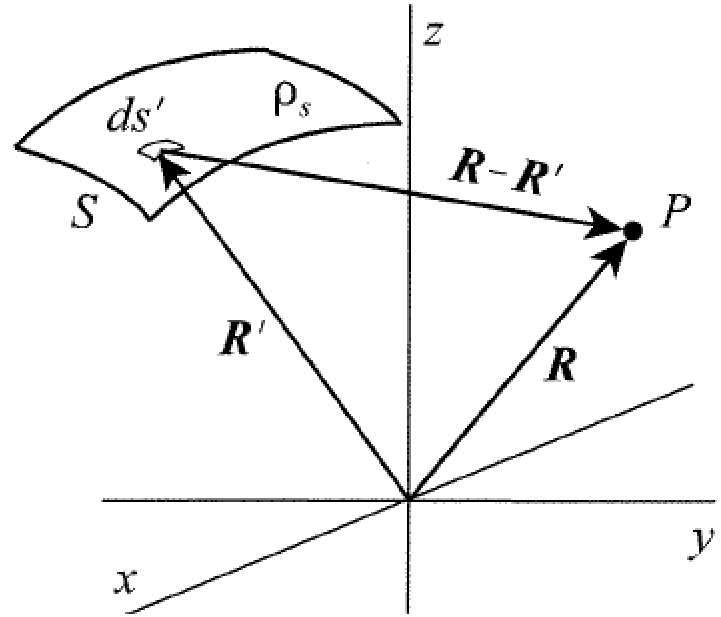
$$V(\boldsymbol{R}) = \int_L dV(\boldsymbol{R}) = \frac{1}{4\pi\epsilon_o} \int_L \frac{\rho_l}{R_o}\, dl'$$



Surface Charge ($\rho_s\, ds' \Leftrightarrow Q$)

$$dV(\boldsymbol{R}) = \frac{\rho_s\, ds'}{4\pi\epsilon_o R_o}$$

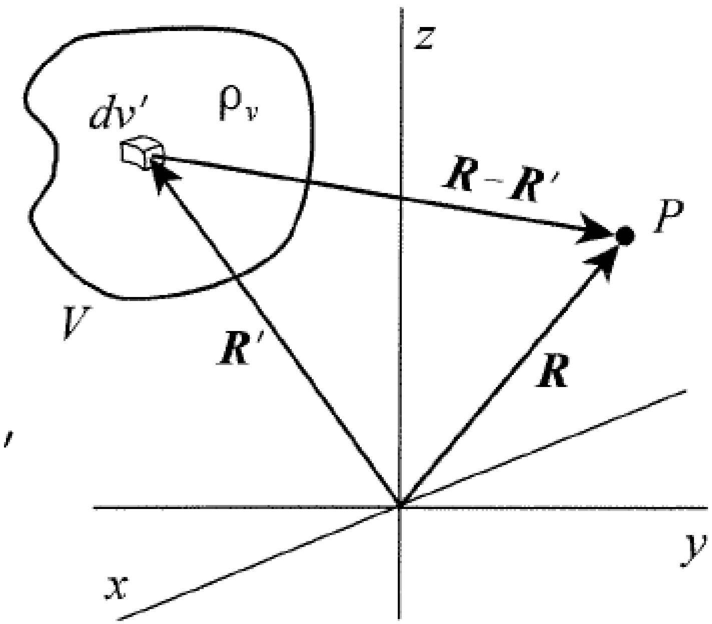
$$V(\boldsymbol{R}) = \iint_S dV(\boldsymbol{R}) = \frac{1}{4\pi\epsilon_o}\iint_S \frac{\rho_s}{R_o}\, ds'$$



Volume Charge ($\rho_v\, dv' \Leftrightarrow Q$)

$$dV(\boldsymbol{R}) = \frac{\rho_v\, dv'}{4\pi\epsilon_o R_o}$$

$$V(\boldsymbol{R}) = \iiint_V dV(\boldsymbol{R}) = \frac{1}{4\pi\epsilon_o}\iiint_V \frac{\rho_v}{R_o}\, dv'$$

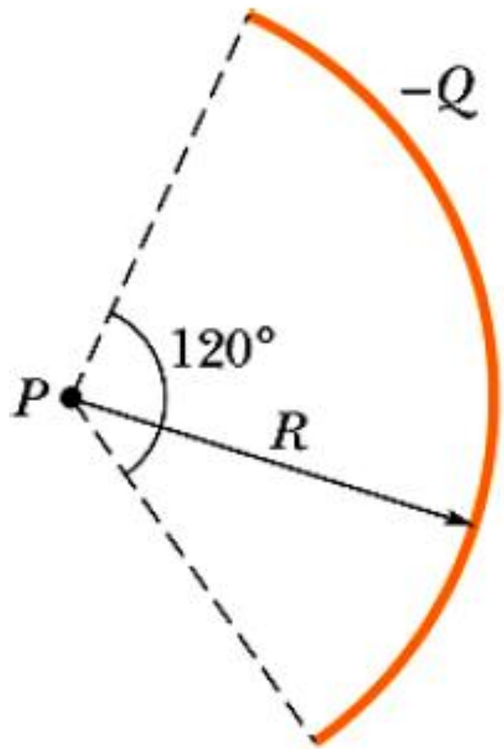


Check:

- **Uniformly charged rod with charge of –Q bent into arc of 120° with radius R.**

  What is V(P), the electric potential at the center?

$$-\frac{kQ}{R}$$



Example:

- Find out the electric field intensity along the axis of a uniform line charge of length L. The uniform line-charge density is ρ_l

Line Charge $(\rho_l \, dl' \Leftrightarrow Q)$

$$dV(\boldsymbol{R}) = \frac{\rho_l \, dl'}{4\pi\epsilon_o R_o}$$

$$V(\boldsymbol{R}) = \int_L dV(\boldsymbol{R}) = \frac{1}{4\pi\epsilon_o}\int_L \frac{\rho_l}{R_o}\, dl'$$

Electric Field in Material Space

The electromagnetic constitutive parameters of a material medium are

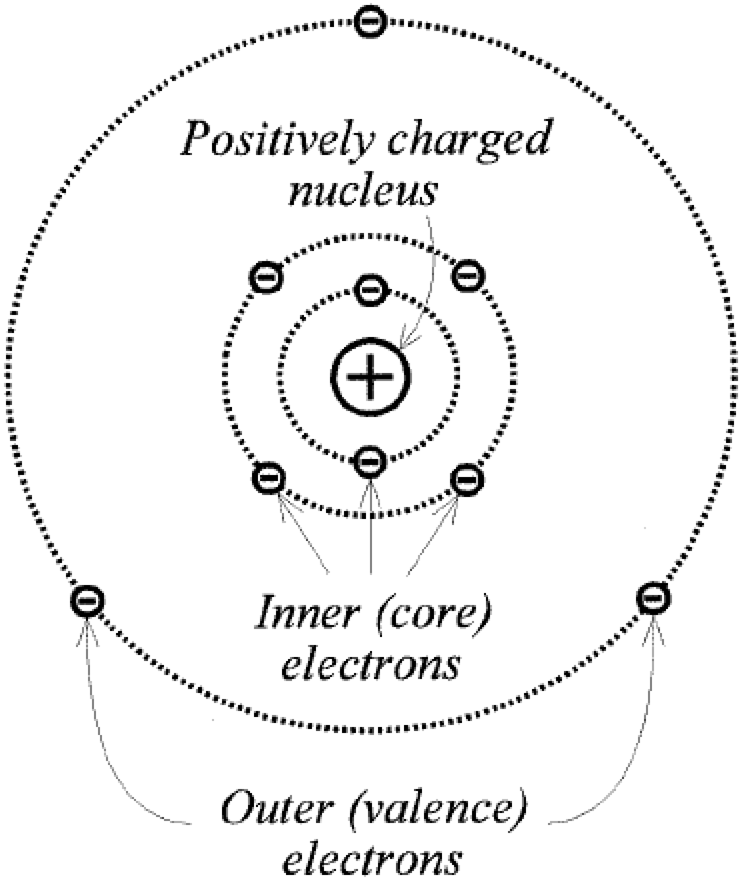
- Electrical permittivity ε
- Electrical permeability μ
- Conductivity σ

Material Classification Based on Conductivity

Positive nucleus charge = Total negative electron charge

The atom is electrically neutral. $(\rho_v = 0, V = 0, \boldsymbol{E} = \mathbf{0})$

Simple atomic model



Materials are classified based on the strength of bonds between the valence electrons and the atom nucleus

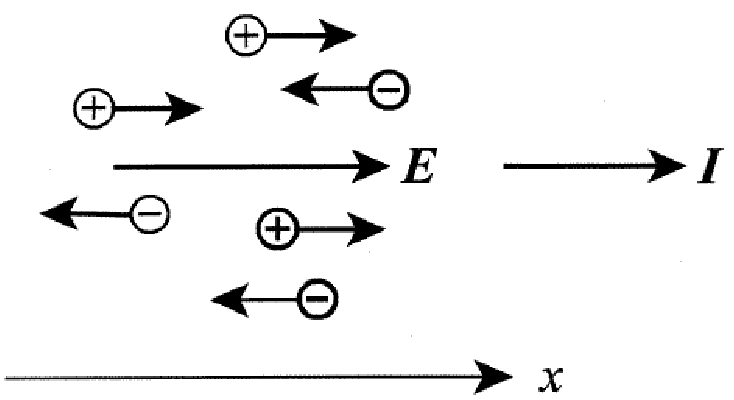
<u>Ideal material characteristics</u>

Perfect Insulator $(\sigma = 0)$

Perfect conductor $(\sigma = \infty)$

Conductors in Static Electric Field

A conductor has a large number of loosely attached electrons in the outermost shells of the atoms.



Perfect dielectric:

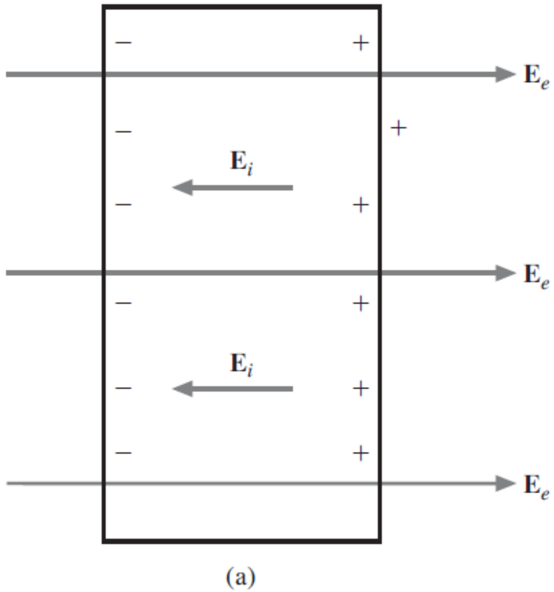
σ=0 and then $\mathbf{J}$ = 0 regardless of $\mathbf{E}$

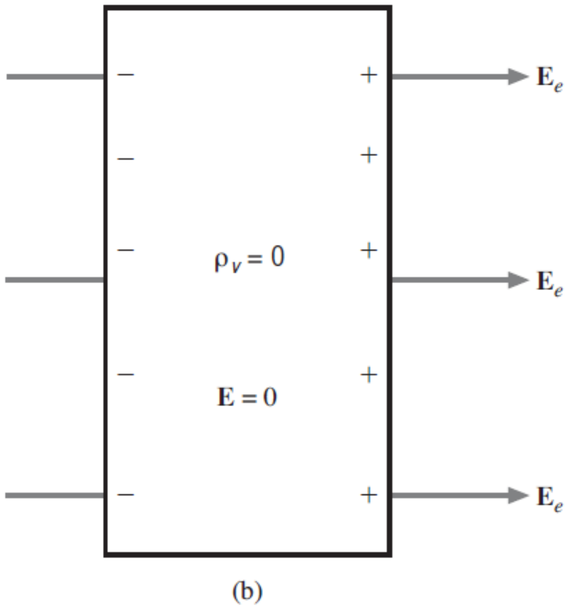
Perfect conductor:

$\sigma=\infty$ and then $\mathbf{E}$ = 0 regardless of $\mathbf{J}$

Their movement gives rise to a conduction current

$$\mathbf{J} = \sigma\mathbf{E}$$





Now let us check the situation on the surface of the conductor

Consider a conductor-free space interface

- $E_t=0$
- $E_n= \rho_s/\varepsilon_0$
- Application = Electrostatic screening/shielding

Example: Conducting spherical shell

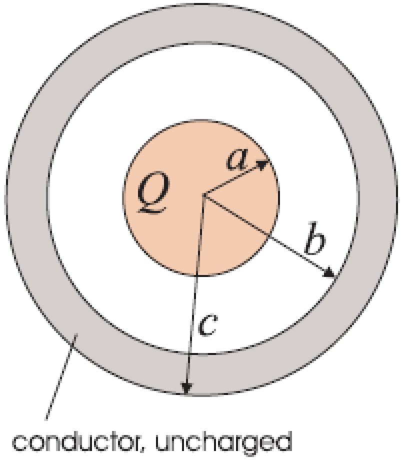
- A solid insulating sphere of radius *a* is uniformly charged with a total charge Q. *It* is surrounded by an concentric uncharged conducting spherical shell with an inner radius *b* and an outer radius c.
- Find the electric field at *r:*

  (i) inside the insulating sphere ($r<a$)

  (ii) between the insulating sphere and the conducting shell ($a<r<b$)

  (iii) within the wall of the conducting shell ($b<r<c$)

  (iv) outside of the conducting shell ($r>c$)



Solution: