### **Quine-McCluskey Method**

### K-Maps Difficult in Visualization for six variables and above

Not software adaptable

1. Find all the prime implicants

$$f(a,b,c,d) = \sum m(0,1,2,5,6,7,8,9,10,14)$$

$$\begin{array}{c|cccc}
\text{group 0} & \underline{0} & \underline{0000} \\
1 & \underline{0001} \\
2 & \underline{0010} \\
8 & \underline{1000} \\
\end{array}$$

$$\begin{array}{c|cccc}
5 & \underline{0101} \\
6 & \underline{0110} \\
9 & \underline{1001} \\
\underline{10} & \underline{1010} \\
\end{array}$$

$$\begin{array}{c|cccc}
7 & \underline{0111} \\
\underline{14} & \underline{1110} \\
\end{array}$$

Group the minterms according to the number of 1s in the minterm.

This way we only have to compare minterms from adjacent groups.

### Column I

### Column II

Combining group 0 and group 1:

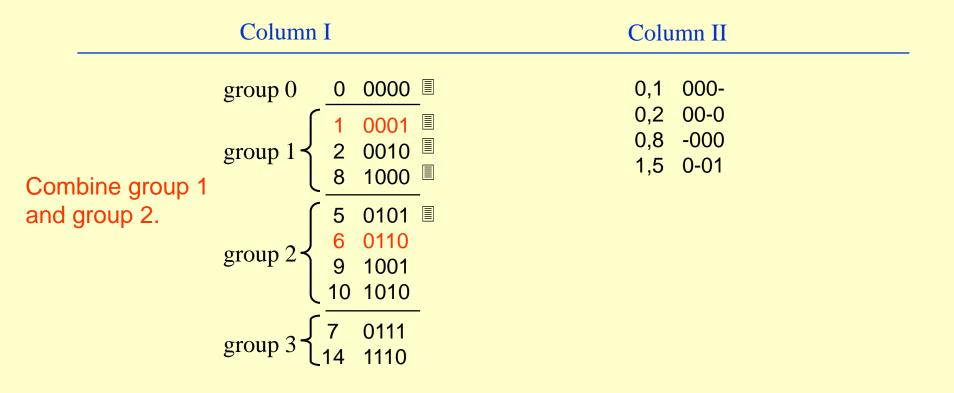
```
\begin{array}{c|cccc}
\text{group 0} & 0 & 0000 \\
\hline
\text{group 1} & 1 & 0001 \\
2 & 0010 \\
8 & 1000 \\
\hline
\\
\text{group 2} & 5 & 0101 \\
6 & 0110 \\
9 & 1001 \\
10 & 1010 \\
\hline
\\
\text{group 3} & 7 & 0111 \\
14 & 1110 \\
\hline
\end{array}
```

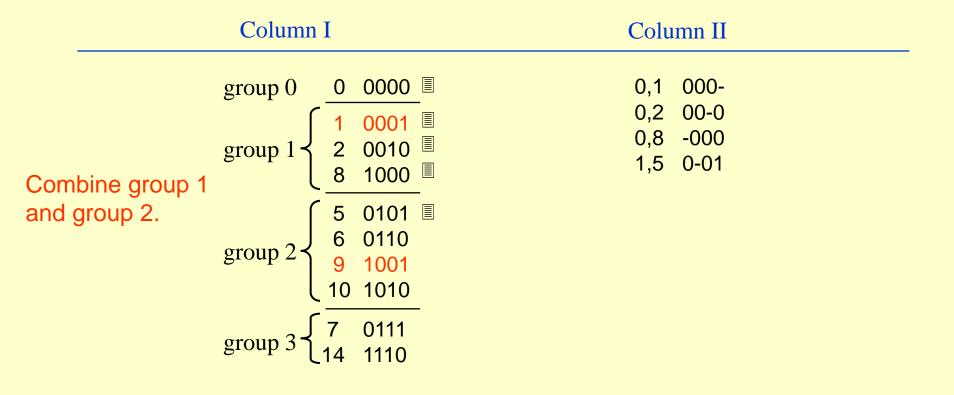
	Column I	Column II
Combining	group 0 0000 $\blacksquare$ group 1 $\left\{ \begin{array}{ccc} 0 & 0000 & \blacksquare \\ 1 & 0001 & \blacksquare \\ 2 & 0010 & 8 & 1000 \end{array} \right.$	0,1 000-
group 0 and group 1:	$     \text{group 2} \begin{cases}       5 & 0101 \\       6 & 0110 \\       9 & 1001 \\       10 & 1010     \end{cases} $	
	group $3\begin{cases} 7 & 0111\\ 14 & 1110 \end{cases}$	

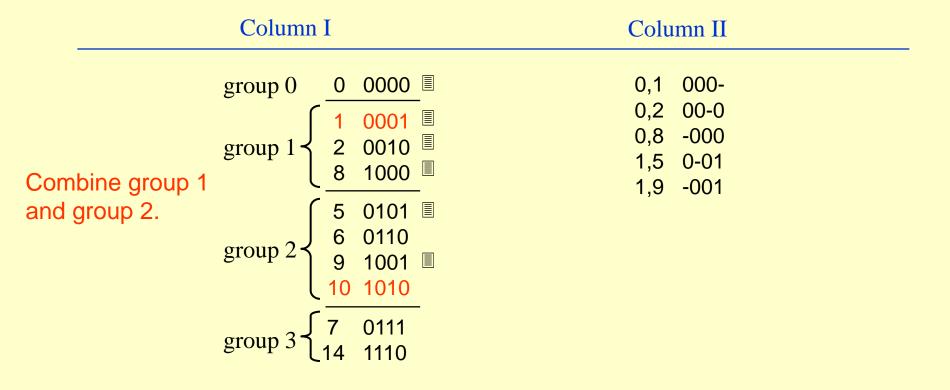
	Column I	Column II
Combining group 0 and group 1:	$ \begin{array}{c cccc} \text{group 0} & \underline{0} & \underline{0000} & \underline{\mathbb{I}} \\ 1 & \underline{0001} & \underline{\mathbb{I}} \\ 2 & \underline{0010} & \underline{\mathbb{I}} \\ 8 & \underline{1000} & \underline{\mathbb{I}} \\ \text{group 2} & \begin{cases} 5 & \underline{0101} \\ 6 & \underline{0110} \\ 9 & \underline{1001} \\ 10 & \underline{1010} \\ \end{array} $	0,1 000-0,2 00-0
	group $3 \begin{cases} 7 & 0111 \\ 14 & 1110 \end{cases}$	

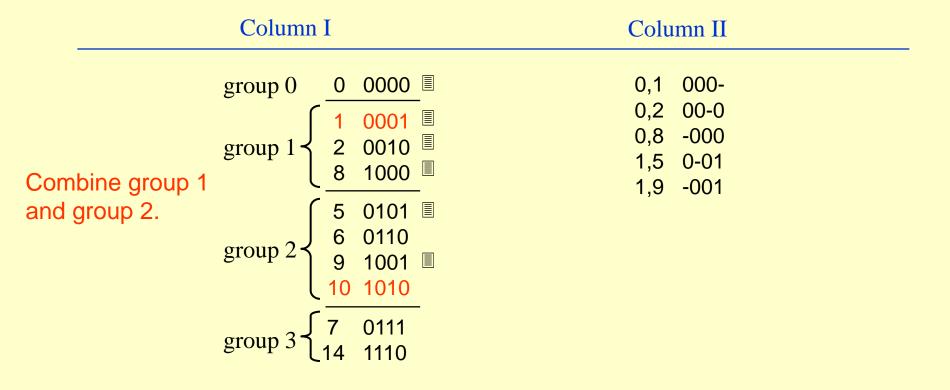
	Column I	Colu	mn II
Does it make sense to combine group 0 with group 2 or 3?	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	00	000- 00-0 -000
No, there are at least two bits that are different.	$     \text{group 2} \begin{cases}       5 & 016 \\       6 & 019 \\       9 & 106 \\       10 & 109     \end{cases} $		
	group $3 \begin{cases} 7 & 017 \\ 14 & 117 \end{cases}$	11 10	

	Column I	Column II
Does it make sense to no combine group 0 with group 2 or 3?	group 0 $\begin{array}{c c} 0 & 0000 \\ \hline group 1 \left\{ \begin{array}{ccc} 1 & 0001 \\ 2 & 0010 \\ 8 & 1000 \end{array} \right. \end{array}$	0,2 00-0
No, there are at least two bits that are different.	$ \operatorname{group} 2 \begin{cases} 5 & 0101 \\ 6 & 0110 \\ 9 & 1001 \\ 10 & 1010 \end{cases} $	
Thus, next we combine group 1	group $3\begin{cases} 7 & 0111\\ 14 & 1110 \end{cases}$	

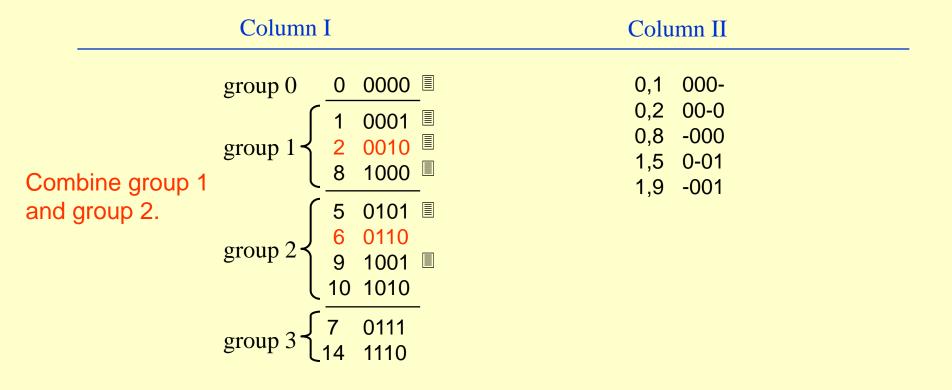


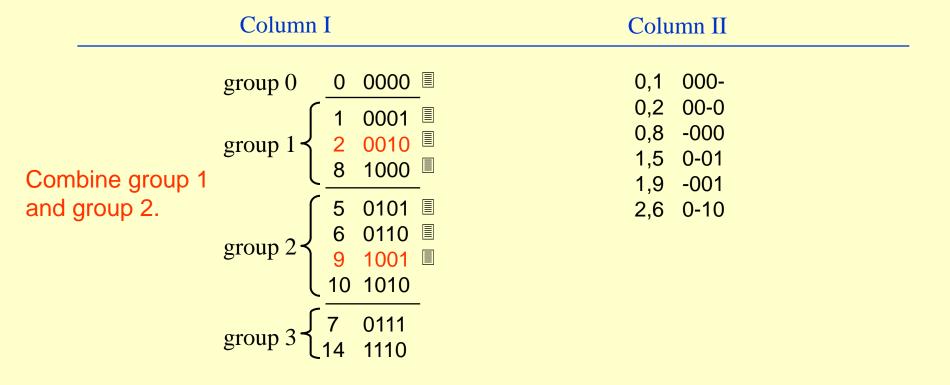


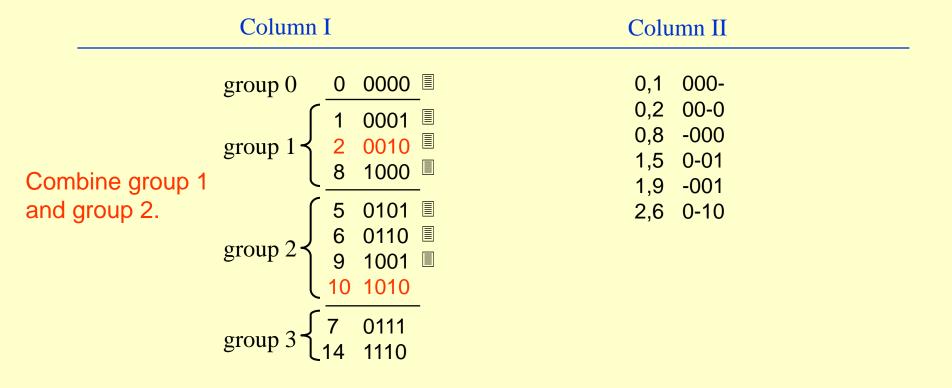




	Column I	Column II
Combine group 1 and group 2.	group 0 0 0000 group 1 $\begin{cases} 1 & 0001 \\ 2 & 0010 \\ 8 & 1000 \end{cases}$	0,1 000- 0,2 00-0 0,8 -000 1,5 0-01 1,9 -001
5 1	group 2	
	group $3\begin{cases} 7 & 0111 \\ 14 & 1110 \end{cases}$	

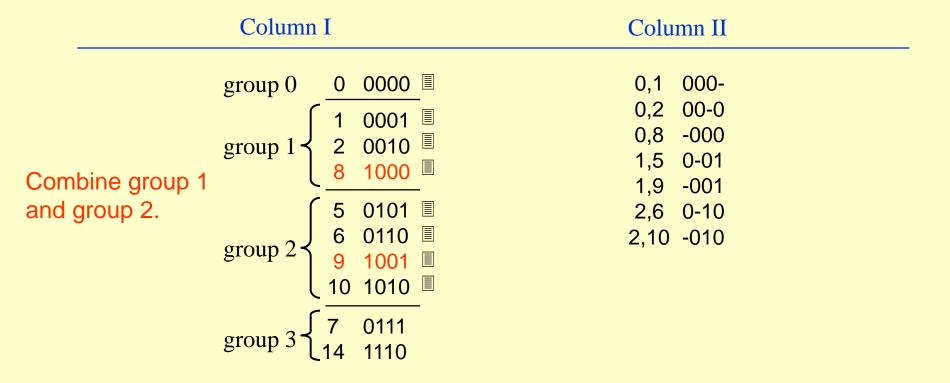






	Column I	Column II
Combine group 1	group 0 0 0000 $\blacksquare$ group 1 $\left\{\begin{array}{cccc} 1 & 0001 & \blacksquare \\ 2 & 0010 & \blacksquare \\ 8 & 1000 & \blacksquare \end{array}\right.$	0,1 000- 0,2 00-0 0,8 -000 1,5 0-01 1,9 -001
and group 2.	group 2 \begin{cases} 5 & 0101 \\ 6 & 0110 \\ 9 & 1001 \\ 10 & 1010 \end{cases} \\ \end{cases}	2,6 0-10 2,10 -010
	group $3\begin{cases} 7 & 0111\\ 14 & 1110 \end{cases}$	

### 



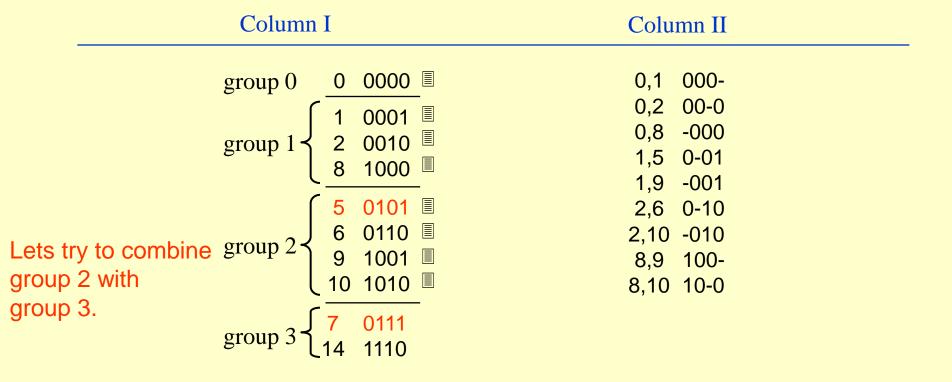
### Column I Column II group 0 0000 🗏 0,1 000group 1 \begin{cases} 1 & 0001 \\ 2 & 0010 \\ 8 & 1000 \end{cases} \end{cases} 0,2 00-0 0,8 -000 1,5 0-01 Combine group 1 1,9 -001 group 2 $\begin{cases} 5 & 0101 \\ 6 & 0110 \\ 9 & 1001 \\ 10 & 1010 \end{cases}$ and group 2. 2,6 0-10 2,10 -010 8,9 100group 3 \ \frac{7}{14} \ \ \frac{0111}{110}

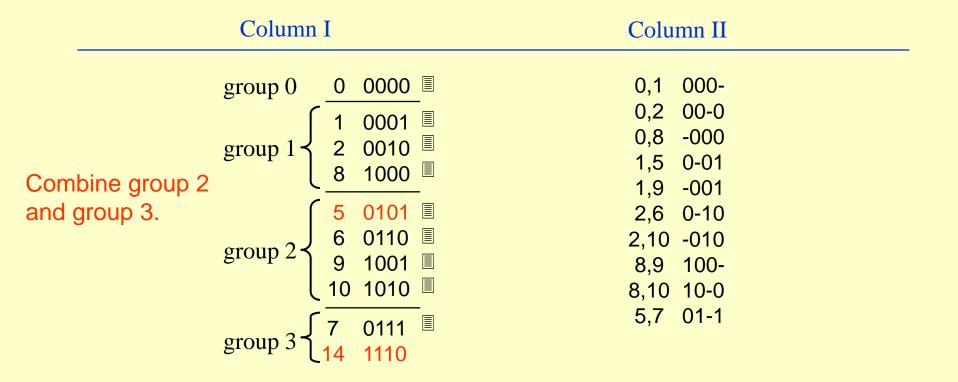
### Column I

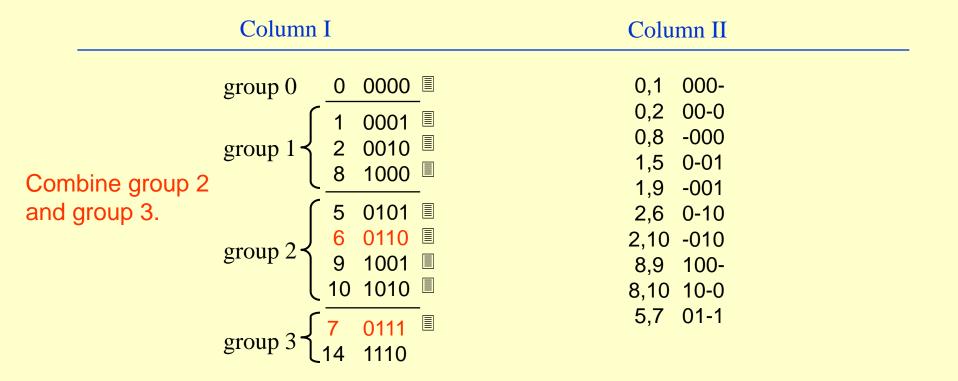
# group 0 0 0000 $\blacksquare$ group 1 $\left\{\begin{array}{cccc} 1 & 0001 & \blacksquare \\ 2 & 0010 & \blacksquare \\ 8 & 1000 & \blacksquare \end{array}\right.$ group 2 $\left\{\begin{array}{ccccc} 5 & 0101 & \blacksquare \\ 6 & 0110 & \blacksquare \\ 9 & 1001 & \blacksquare \\ 10 & 1010 & \blacksquare \end{array}\right.$ group 3 $\left\{\begin{array}{ccccc} 7 & 0111 \\ 14 & 1110 & \blacksquare \end{array}\right.$

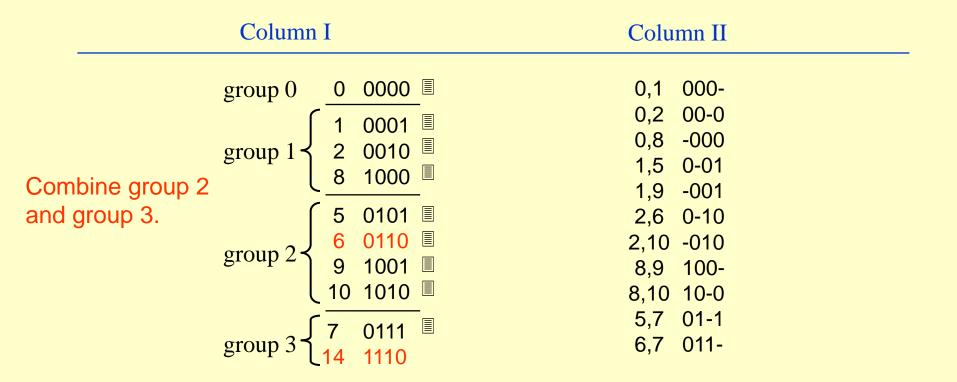
### Column II

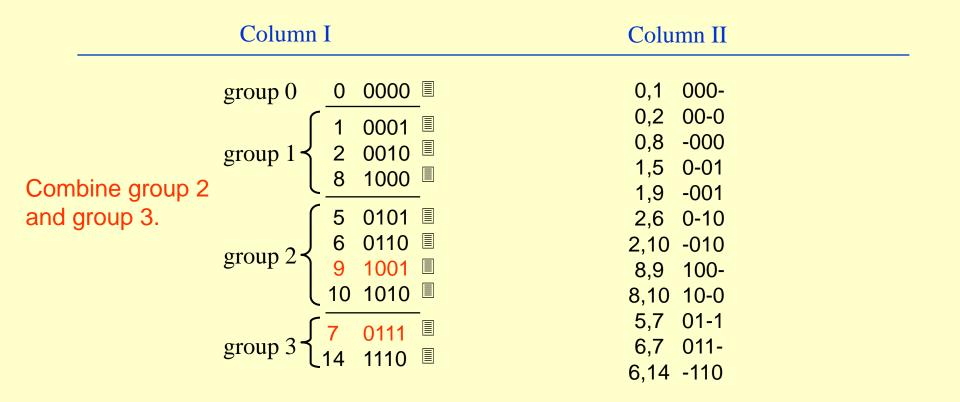
```
0,1 000-
0,2 00-0
0,8 -000
1,5 0-01
1,9 -001
2,6 0-10
2,10 -010
8,9 100-
8,10 10-0
```

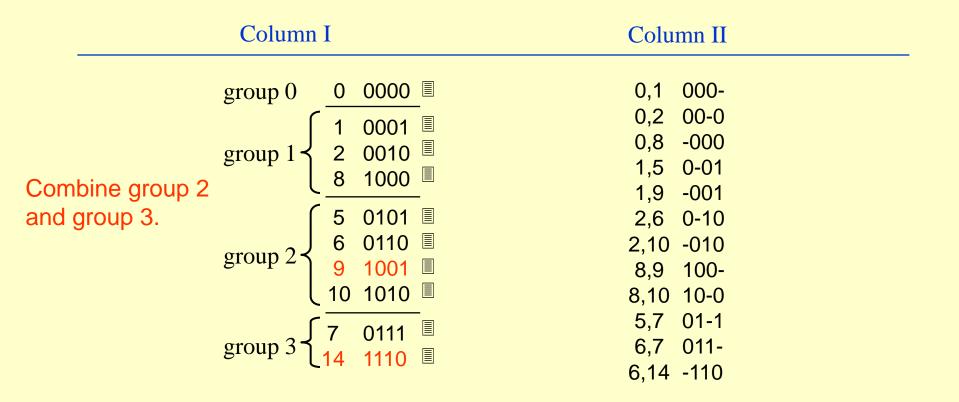




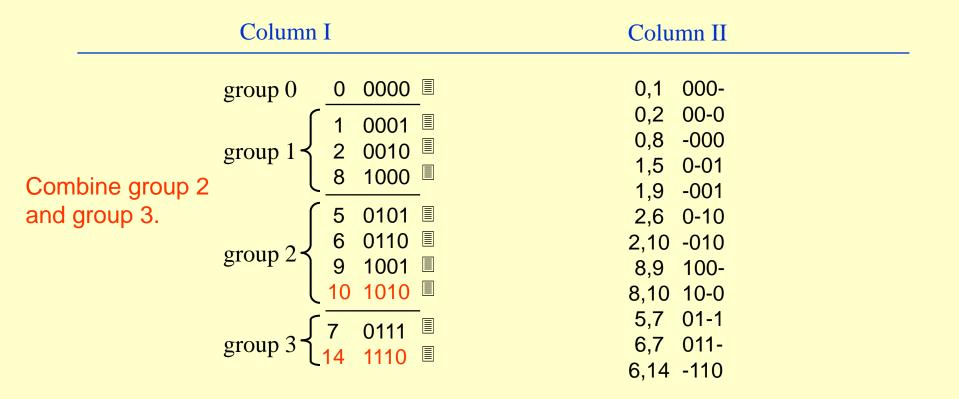








	Column I	Column II
Combine group 2 and group 3.	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	0,1 000- 0,2 00-0 0,8 -000 1,5 0-01 1,9 -001 2,6 0-10 2,10 -010 8,9 100- 8,10 10-0 5,7 01-1 6,7 011- 6,14 -110



Column I

Column II

We have now completed the first step. All minterms in column I were included.

We can divide column II into groups.

group 0	0	0000	
	1	0001	
group 1	2	0010	
	8	1000	
	5	0101	
	6	0110	
group 2	9	1001	
L	10	1010	
<u></u>	7	0111	
group 3 \( \)	14	1110	

0,1 0000,2 00-0
0,8 -000
1,5 0-01
1,9 -001
2,6 0-10
2,10 -010
8,9 1008,10 10-0
5,7 01-1
6,7 0116,14 -110
10,14 1-10

Column I	Column II
group 0 0 0000 $\blacksquare$ group 1 $\left\{ \begin{array}{ccc} 1 & 0001 & \blacksquare \\ 2 & 0010 & \blacksquare \\ 8 & 1000 & \blacksquare \end{array} \right.$	0,1 000- 0,2 00-0 0,8 -000 1,5 0-01 1,9 -001
$ \operatorname{group} 2 \begin{cases} 5 & 0101 \\ 6 & 0110 \\ 9 & 1001 \\ 10 & 1010 \end{cases} $	2,6 0-10 2,10 -010 8,9 100- 8,10 10-0
group $3 \begin{cases} 7 & 0111 \\ 14 & 1110 \end{cases}$	5,7 01-1 6,7 011- 6,14 -110 10,14 1-10

Column I	Column II	Column III
group 0 0 0000 $\blacksquare$ group 1 $\left\{ \begin{array}{cccc} 1 & 0001 & \blacksquare \\ 2 & 0010 & \blacksquare \\ 8 & 1000 & \blacksquare \end{array} \right.$ group 2 $\left\{ \begin{array}{ccccc} 5 & 0101 & \blacksquare \\ 6 & 0110 & \blacksquare \\ 9 & 1001 & \blacksquare \\ 10 & 1010 & \blacksquare \end{array} \right.$	0,1 000- 0,2 00-0 0,8 -000 1,5 0-01 1,9 -001 2,6 0-10 2,10 -010 8,9 100-	
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	8,10 10-0 5,7 01-1 6,7 011- 6,14 -110 10,14 1-10	

Column II

Column III

Column I

Column I	Column II	Column III
group 0 0000 🗉	0,1 000- 0,2 00-0	
group 1 { 1 0001	0,8 -000 1,5 0-01 1,9 -001	
group 2	2,6 0-10 2,10 -010 8,9 100- 8,10 10-0	
group $3\begin{cases} 7 & 0111 \\ 14 & 1110 \end{cases}$	5,7 01-1 6,7 011- 6,14 -110 10,14 1-10	

Column II

Column III

Column I

Column I	Column II	Column III
group 0 0000 🗏	0,1 000- 0,2 00-0	
group 1 { 1 0001	0,8 -000 1,5 0-01 1,9 -001	
group 2	2,6 0-10 2,10 -010 8,9 100- 8,10 10-0	
group $3 \begin{cases} 7 & 0111 \\ 14 & 1110 \end{cases}$	5,7 01-1 6,7 011- 6,14 -110 10,14 1-10	

Column I	Column II	Column III
group 0 0 0000 $\blacksquare$ group 1 $\left\{ \begin{array}{ccc}                                   $	0,1 000- 0,2 00-0 0,8 -000 1,5 0-01 1,9 -001	
group 2	2,6 0-10 2,10 -010 8,9 100- 8,10 10-0	
group 3 { 7 0111	5,7 01-1 6,7 011- 6,14 -110 10,14 1-10	

Column I	Column II	Column III
group 0 0 0000 $\blacksquare$ group 1 $\left\{ \begin{array}{ccc} 0 & 0000 & \blacksquare \\ 1 & 0001 & \blacksquare \\ 2 & 0010 & \blacksquare \\ 8 & 1000 & \blacksquare \end{array} \right.$	0,1 000- 0,2 00-0 0,8 -000 1,5 0-01 1,9 -001	
group $2\begin{cases} 5 & 0101 \\ 6 & 0110 \\ 9 & 1001 \\ 10 & 1010 \end{cases}$ group $3\begin{cases} 7 & 0111 \\ 14 & 1110 \end{cases}$	2,6 0-10 2,10 -010 8,9 100- 8,10 10-0 5,7 01-1 6,7 011-	
S <sup>10up 3</sup>	6,14 -110 10,14 1-10	

Column I	Column II	Column III
group 0 0 0000 $\blacksquare$ group 1 $\left\{ \begin{array}{ccc} 0 & 0000 & \blacksquare \\ 1 & 0001 & \blacksquare \\ 2 & 0010 & \blacksquare \\ 8 & 1000 & \blacksquare \end{array} \right.$	0,1 000- <sup>□</sup> 0,2 00-0 0,8 -000 1,5 0-01 1,9 -001	0,1,8,9 -00-
$     \text{group 2} \begin{cases}       5 & 0101 \\       6 & 0110 \\       9 & 1001 \\       \hline       10 & 1010     \end{bmatrix}     $ $     \text{group 3} \begin{cases}       7 & 0111 \\       \hline       14 & 1110     \end{bmatrix}   $	2,6 0-10 2,10 -010 8,9 100- 8,10 10-0 5,7 01-1 6,7 011- 6,14 -110 10,14 1-10	

Column I	Column II	Column III
group 0 0 0000 $\blacksquare$ group 1 $\left\{ \begin{array}{ccc} 0 & 0000 & \blacksquare \\ 1 & 0001 & \blacksquare \\ 2 & 0010 & \blacksquare \\ 8 & 1000 & \blacksquare \end{array} \right.$	0,1 000-	0,1,8,9 -00-
$ \begin{array}{c}                                     $	2,6 0-10 2,10 -010 8,9 100- 8,10 10-0 5,7 01-1 6,7 011- 6,14 -110 10,14 1-10	

Column I	Column II	Column III
group 0 0 0000 $\blacksquare$ group 1 $\left\{ \begin{array}{ccc} 1 & 0001 & \blacksquare \\ 2 & 0010 & \blacksquare \\ 8 & 1000 & \blacksquare \end{array} \right.$	0,1 000-   0,2 00-0 0,8 -000 1,5 0-01	0,1,8,9 -00-
$ \begin{array}{c}                                     $	1,9 -001 2,6 0-10 2,10 -010 8,9 100- 8,10 10-0 5,7 01-1 6,7 011- 6,14 -110 10,14 1-10	

Column I	Column II	Column III
group 0 0 0000 $\blacksquare$ group 1 $\left\{ \begin{array}{ccc} 1 & 0001 & \blacksquare \\ 2 & 0010 & \blacksquare \\ 8 & 1000 & \blacksquare \end{array} \right.$	0,1 000-	0,1,8,9 -00-
group $2\begin{cases} 5 & 0101 \\ 6 & 0110 \\ 9 & 1001 \\ 10 & 1010 \end{cases}$ group $3\begin{cases} 7 & 0111 \\ 14 & 1110 \end{cases}$	2,6 0-10 2,10 -010 8,9 100- 8,10 10-0 5,7 01-1 6,7 011- 6,14 -110 10,14 1-10	

Column I	Column II	Column III
group 0 0000 🗏	0,1 000-	0,1,8,9 -00-
group 1 { 1 0001	0,2 00-0 0,8 -000 1,5 0-01 1,9 -001	
group 2	2,6 0-10 2,10 -010 8,9 100-  8,10 10-0	
group $3 \begin{cases} \frac{1}{7} & 0111 \\ 14 & 1110 \end{cases}$	5,7 01-1 6,7 011- 6,14 -110 10,14 1-10	

Column I	Column II	Column III
group 0 0000 🗉	0,1 000-	0,1,8,9 -00-
group $1 \begin{cases} 1 & 0001 \\ 2 & 0010 \\ 8 & 1000 \end{cases}$	0,2 00-0 0,8 -000 1,5 0-01 1,9 -001	
group $2 \begin{cases} 5 & 0101 \\ 6 & 0110 \\ 9 & 1001 \\ 10 & 1010 \end{cases}$	2,6 0-10 2,10 -010 8,9 100- 10 8,10 10-0	
group $3 \begin{cases} 7 & 0111 \\ 14 & 1110 \end{cases}$	5,7 01-1 6,7 011- 6,14 -110 10,14 1-10	

Column I	Column II	Column III
group 0 0000 🗏	0,1 000-	0,1,8,9 -00-
group $1 \begin{cases} 1 & 0001 \\ 2 & 0010 \\ 8 & 1000 \end{cases}$	0,2 00-0 0,8 -000 1,5 0-01 1,9 -001	
group 2 5 0101 6 0110 9 1001 10 1010 10 1010	2,6 0-10 2,10 -010 8,9 100- <b>8</b>	
group $3 \begin{cases} \frac{1}{7} & 0111 \\ 14 & 1110 \end{cases}$	5,7 01-1 6,7 011- 6,14 -110 10,14 1-10	

Column I	Column II	Column III
group 0 0 0000 $\blacksquare$ group 1 $\left\{ \begin{array}{ccc} 1 & 0001 & \blacksquare \\ 2 & 0010 & \blacksquare \\ 8 & 1000 & \blacksquare  \end{array} \right.$	0,1 000-   0,2 00-0   0,8 -000 1,5 0-01 1,9 -001 2,6 0-10 2,10 -010	0,1,8,9 -00- 0,2,8,10 -0-0
group $2\begin{cases} 5 & 0101 \\ 6 & 0110 \\ 9 & 1001 \\ 10 & 1010 \end{cases}$ group $3\begin{cases} 7 & 0111 \\ 14 & 1110 \end{cases}$	8,9 100- 8,10 10-0 5,7 01-1 6,7 011- 6,14 -110 10,14 1-10	

Column I	Column II	Column III
group 0 0 0000 $\blacksquare$ group 1 $\left\{ \begin{array}{ccc} 0 & 0000 & \blacksquare \\ 1 & 0001 & \blacksquare \\ 2 & 0010 & \blacksquare \\ 8 & 1000 & \blacksquare \end{array} \right.$	0,1 000-	0,1,8,9 -00- 0,2,8,10 -0-0
group $2\begin{cases} 5 & 0101 \\ 6 & 0110 \\ 9 & 1001 \\ \hline 10 & 1010 \\ \hline \end{cases}$ group $3\begin{cases} 7 & 0111 \\ \hline 14 & 1110 \\ \hline \end{cases}$	2,6 0-10 2,10 -010 8,9 100- 8,10 10-0 5,7 01-1 6,7 011- 6,14 -110 10,14 1-10	

Column I	Column II	Column III
group 0 0 0000 $\blacksquare$ group 1 $\left\{ \begin{array}{ccc} 1 & 0001 & \blacksquare \\ 2 & 0010 & \blacksquare \\ 8 & 1000 & \blacksquare \end{array} \right.$	0,1 000-	0,1,8,9 -00- 0,2,8,10 -0-0 0,8,1,9 -00-
	2,6 0-10 2,10 -010 8,9 100- 8,10 10-0 5,7 01-1 6,7 011- 6,14 -110 10,14 1-10	

Column I	Column II	Column III
group 0 0 0000 $\blacksquare$ group 1 $\left\{ \begin{array}{ccc} 1 & 0001 & \blacksquare \\ 2 & 0010 & \blacksquare \\ 8 & 1000 & \blacksquare \\ \end{array} \right.$ $\left\{ \begin{array}{cccc} 5 & 0101 & \blacksquare \\ \end{array} \right.$	0,1 000-   0,2 00-0   0,8 -000   1,5 0-01   1,9 -001   2,6 0-10	0,1,8,9 -00- 0,2,8,10 -0-0 0,8,1,9 -00-
group 2 \begin{cases} 5 & 0101 \\ 6 & 0110 \\ 9 & 1001 \\ 10 & 1010 \end{cases} \\ \frac{10}{10} & 1010 \end{cases} \end{cases}	2,10 -010 8,9 100- 8,10 10-0 5,7 01-1	
group $3 \begin{cases} 7 & 0111 \\ 14 & 1110 \end{cases}$	6,7 011- 6,14 -110 10,14 1-10	

Column I	Column II	Column III
group 0 0000	0,1 000-   0,2 00-0   0,8 -000   1,5 0-01   1,9 -001	0,1,8,9 -00- 0,2,8,10 -0-0 0,8,1,9 -00- 0,8,2,10 -0-0
group 2 \begin{cases} 5 & 0101 \\ 6 & 0110 \\ 9 & 1001 \\ 10 & 1010 \end{cases} \]	2,6 0-10 2,10 -010 <b>1</b> 8,9 100- <b>1</b> 8,10 10-0 <b>1</b>	
group $3 \begin{cases} 7 & 0111 \\ 14 & 1110 \end{cases}$	5,7 01-1 6,7 011- 6,14 -110 10,14 1-10	

Column I	Column II	Column III
group 0 0 0000 $\blacksquare$ group 1 $\left\{ \begin{array}{ccc} 1 & 0001 & \blacksquare \\ 2 & 0010 & \blacksquare \\ 8 & 1000 & \blacksquare \end{array} \right.$	0,1 000-   0,2 00-0   0,8 -000   1,5 0-01   1,9 -001   2,6 0-10	0,1,8,9 -00- 0,2,8,10 -0-0 0,8,1,9 -00- 0,8,2,10 -0-0
group $2\begin{cases} 5 & 0101 \\ 6 & 0110 \\ 9 & 1001 \\ 10 & 1010 \end{cases}$ group $3\begin{cases} 7 & 0111 \\ 14 & 1110 \end{cases}$	2,10 -010	

Column I	Column II	Column III
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	0,1 000-	0,1,8,9 -00- 0,2,8,10 -0-0 0,8,1,9 -00- 0,8,2,10 -0-0
group 3 { 7 0111	5,7 01-1 6,7 011- 6,14 -110 10,14 1-10	

Column I	Column II	Column III
group 0 0 0000 $\blacksquare$ group 1 $\left\{ \begin{array}{ccc} 1 & 0001 & \blacksquare \\ 2 & 0010 & \blacksquare \\ 8 & 1000 & \blacksquare \end{array} \right.$	0,1 000-	0,1,8,9 -00- 0,2,8,10 -0-0 0,8,1,9 -00- 0,8,2,10 -0-0
group $2\begin{cases} 5 & 0101 \\ 6 & 0110 \\ 9 & 1001 \\ 10 & 1010 \end{cases}$ group $3\begin{cases} 7 & 0111 \\ 14 & 1110 \end{cases}$	2,6 0-10 2,10 -010	

Column I	Column II	Column III
group 0 0 0000 $\blacksquare$ group 1 $\left\{\begin{array}{cccccccccccccccccccccccccccccccccccc$	0,1 000- 0,2 00-0  0,8 -000  1,5 0-01  1,9 -001  2,6 0-10  2,10 -010  8,9 100-  8,10 10-0  5,7 01-1  6,7 011-  6,14 -110	0,1,8,9 -00- 0,2,8,10 -0-0 0,8,1,9 -00- 0,8,2,10 -0-0
	10,14 1-10	

Column I	Column II	Column III
group 0 0 0000 $\blacksquare$ group 1 $\left\{ \begin{array}{ccc} 1 & 0001 & \blacksquare \\ 2 & 0010 & \blacksquare \\ 8 & 1000 & \blacksquare \end{array} \right.$	0,1 000-   0,2 00-0   0,8 -000   1,5 0-01   1,9 -001	0,1,8,9 -00- 0,2,8,10 -0-0 0,8,1,9 -00- 0,8,2,10 -0-0 2,6,10,1410
group $2\begin{cases} 5 & 0101 \\ 6 & 0110 \\ 9 & 1001 \\ 10 & 1010 \end{cases}$ group $3\begin{cases} 7 & 0111 \\ 14 & 1110 \end{cases}$	2,6 0-10	

Column I	Column II	Column III
group 0 0000	0,1 000-   0,2 00-0   0,8 -000   1,5 0-01   1,9 -001	0,1,8,9 -00- 0,2,8,10 -0-0 0,8,1,9 -00- 0,8,2,10 -0-0 2,6,10,1410
group 2	2,6 0-10 = 2,10 -010 = 8,9 100- = 8,10 10-0 = 100-	2,10,6,1410
group $3 \begin{cases} 7 & 0111 \\ 14 & 1110 \end{cases}$	5,7 01-1 6,7 011- 6,14 -110 = 10,14 1-10 =	

Column I	Column II	Column III
group 0 0 0000 $\blacksquare$ group 1 $\left\{ \begin{array}{ccc} 1 & 0001 & \blacksquare \\ 2 & 0010 & \blacksquare \\ 8 & 1000 & \blacksquare  \end{array} \right.$	0,1 000-	0,1,8,9 -00- 0,2,8,10 -0-0 0,8,1,9 -00- 0,8,2,10 -0-0 2,6,10,1410
group $2\begin{cases} 5 & 0101 \\ 6 & 0110 \\ 9 & 1001 \\ 10 & 1010 \end{cases}$ group $3\begin{cases} 7 & 0111 \\ 14 & 1110 \end{cases}$	2,6 0-10 = 2,10 -010 = 8,9 100- = 8,10 10-0 = 5,7 01-1 6,7 011-6,14 -110 = 10,14 1-	2,10,6,1410

Column I	Column II	Column III
group 0 0 0000 $\blacksquare$ group 1 $\left\{ \begin{array}{ccc} 1 & 0001 & \blacksquare \\ 2 & 0010 & \blacksquare \\ 8 & 1000 & \blacksquare \end{array} \right.$	0,1 000-   0,2 00-0   0,8 -000   1,5 0-01   1,9 -001	0,1,8,9 -00- 0,2,8,10 -0-0 0,8,1,9 -00- 0,8,2,10 -0-0 2,6,10,1410
	2,6 0-10	2,10,6,1410

Column I	Column II	Column III
group 0 0 0000 $\blacksquare$ group 1 $\left\{ \begin{array}{ccc} 0 & 0000 & \blacksquare \\ 1 & 0001 & \blacksquare \\ 2 & 0010 & \blacksquare \\ 8 & 1000 & \blacksquare \end{array} \right.$	0,1 000-	0,1,8,9 -00- 0,2,8,10 -0-0 0,8,1,9 -00- 0,8,2,10 -0-0 2,6,10,1410
$         \text{group 2} \begin{cases}             5 & 0101 \\             6 & 0110 \\             9 & 1001 \\             10 & 1010 \\             \hline             \hline          $	2,6 0-10	2,10,6,1410  No more combinations are possible, thus we stop here.

Column I	Column II	Column III
group 0 0 0000 $\blacksquare$ group 1 $\left\{\begin{array}{cccc} 1 & 0001 & \blacksquare \\ 2 & 0010 & \blacksquare \\ 8 & 1000 & \blacksquare \\ \end{array}\right.$ group 2 $\left\{\begin{array}{ccccc} 5 & 0101 & \blacksquare \\ 6 & 0110 & \blacksquare \\ 9 & 1001 & \blacksquare \\ 10 & 1010 & \blacksquare \\ \end{array}\right.$	0,1 000-   0,2 00-0   0,8 -000   1,5 0-01   1,9 -001   2,6 0-10   2,10 -010   8,9 100-	0,1,8,9 -00- 0,2,8,10 -0-0 
group $3 \begin{cases} 10 & 1010 \\ \hline 7 & 0111 \\ \hline 14 & 1110 \end{cases}$	8,10 10-0 5,7 01-1 6,7 011- 6,14 -110 1 10,14 1-10 1	We can eliminate repeated combinations

Column I	Column II	Column III
group 0 0 0000 $\blacksquare$ group 1 $\left\{ \begin{array}{cccc}                                  $	0,1 000-   0,2 00-0   0,8 -000   1,5 0-01   1,9 -001   2,6 0-10	0,1,8,9 -00- 0,2,8,10 -0-0 2,6,10,1410 Now we form f with the terms not checked
group $2 \begin{cases} 5 & 0101 \\ 6 & 0110 \\ 9 & 1001 \\ \hline 10 & 1010 \end{cases}$ group $3 \begin{cases} 7 & 0111 \\ 14 & 1110 \end{cases}$	2,10 -010	f = a'c'd

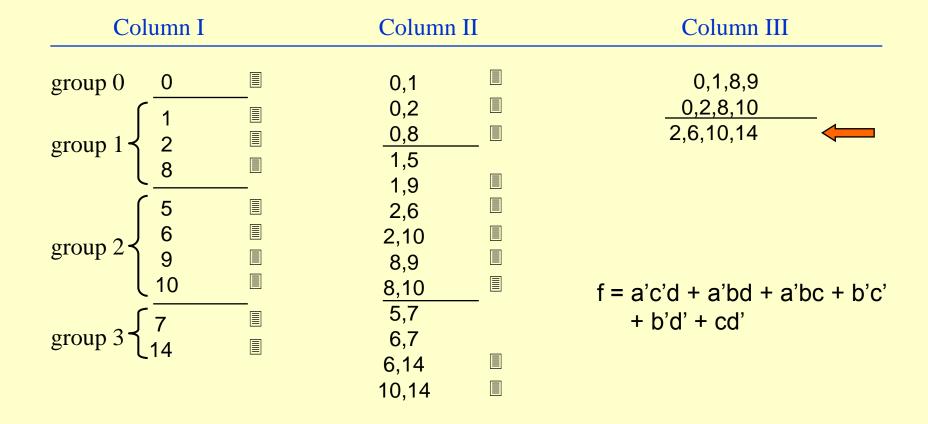
Column I	Column II	Column III
group 0 0 0000 $\blacksquare$ group 1 $\begin{cases} 1 & 0001 & \blacksquare \\ 2 & 0010 & \blacksquare \\ 8 & 1000 & \blacksquare \end{cases}$ $\begin{cases} 5 & 0101 & \blacksquare \\ 6 & 0110 & \blacksquare \end{cases}$	0,1 000-   0,2 00-0   0,8 -000   1,5 0-01   1,9 -001   2,6 0-10   2,10 -010	0,1,8,9 -00- 0,2,8,10 -0-0 2,6,10,1410
group $2 \begin{cases} 5 & 0101 \\ 6 & 0110 \\ 9 & 1001 \\ \hline 10 & 1010 \end{cases}$ group $3 \begin{cases} 7 & 0111 \\ \hline 14 & 1110 \end{cases}$	8,9 100- 8,10 10-0 5,7 01-1 6,7 011- 6,14 -110 10,14 1-10 8	f = a'c'd + a'bd

Column I	Column II	Column III
$ \begin{array}{c cccc} group 0 & 0 & 0000 & \\ \hline 1 & 0001 & \\ 2 & 0010 & \\ 8 & 1000 & \\ \hline \end{array} $ $ \begin{array}{c cccc} group 2 & 5 & 0101 & \\ 6 & 0110 & \\ 9 & 1001 & \\ 10 & 1010 & \\ \hline \end{array} $	0,1 000-	0,1,8,9 -00- 0,2,8,10 -0-0 2,6,10,1410
group $3 \begin{cases} 9 & 1001 \\ 10 & 1010 \end{cases}$	8,9 100- 8,10 10-0 5,7 01-1 6,7 011- 6,14 -110 10,14 1-10 10	f = a'c'd + a'bd + a'bc

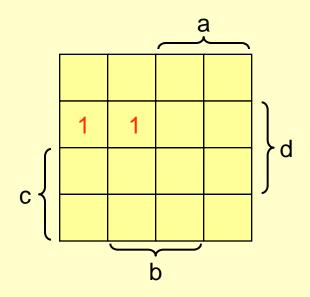
Column I	Column II	Column III
group 0 0 0000 g  group 1 1 0001 2 2 0010 3 1000 3 1001 3 10010 3 1001 5 10 1010 5 1010 5	0,1 000-   0,2 00-0   0,8 -000   1,5 0-01   1,9 -001   2,6 0-10   2,10 -010   8,9 100-   8,10 10-0   5,7 01-1   6,7 011-   6,14 -110	$0,1,8,9 -00-\frac{0,2,8,10 -0-0}{2,6,10,1410}$ $f = a'c'd + a'bd + a'bc + b'c'$
	10,14 1-10 🗏	

Column I	Column II	Column III
group 0 0 0000 $\blacksquare$ group 1 $\left\{ \begin{array}{ccc} 0 & 0000 & \blacksquare \\ 1 & 0001 & \blacksquare \\ 2 & 0010 & \blacksquare \\ 8 & 1000 & \blacksquare \end{array} \right.$	0,1 000-   0,2 00-0   0,8 -000   1,5 0-01   1,9 -001	0,1,8,9 -00- 0,2,8,10 -0-0 2,6,10,1410
group $2\begin{cases} 5 & 0101 \\ 6 & 0110 \\ 9 & 1001 \\ \hline 10 & 1010 \end{cases}$ group $3\begin{cases} 7 & 0111 \\ 14 & 1110 \end{cases}$	2,6 0-10	f = a'c'd + a'bd + a'bc + b'c' + b'd'

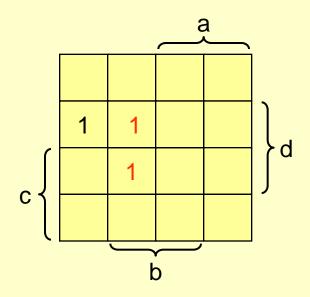
Column I	Column II	Column III
group 0 0 0000 $\blacksquare$ group 1 $\left\{ \begin{array}{ccc} 0 & 0000 & \blacksquare \\ 1 & 0001 & \blacksquare \\ 2 & 0010 & \blacksquare \\ 8 & 1000 & \blacksquare \end{array} \right.$	0,1 000-   0,2 00-0   0,8 -000   1,5 0-01   1,9 -001	0,1,8,9 -00- 0,2,8,10 -0-0 2,6,10,1410
	2,6 0-10 = 2,10 -010 = 8,9 100- = 8,10 10-0 = 5,7 01-1 6,7 011-6,14 -110 = 10,14 1-	f = a'c'd + a'bd + a'bc + b'c' + b'd' + cd'



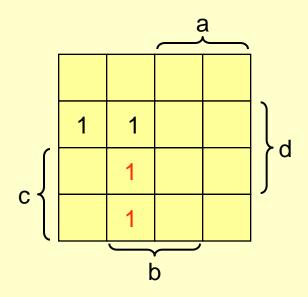
$$f = a'c'd + a'bd + a'bc + b'c' + b'd' + cd'$$



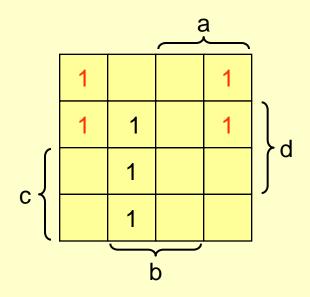
$$f = a'c'd + a'bd + a'bc + b'c' + b'd' + cd'$$



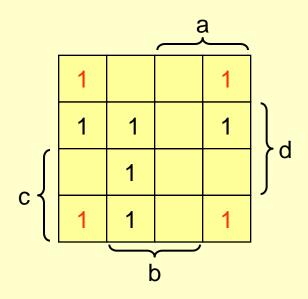
$$f = a'c'd + a'bd + a'bc + b'c' + b'd' + cd'$$



$$f = a'c'd + a'bd + a'bc + b'c' + b'd' + cd'$$



$$f = a'c'd + a'bd + a'bc + b'c' + b'd' + cd'$$



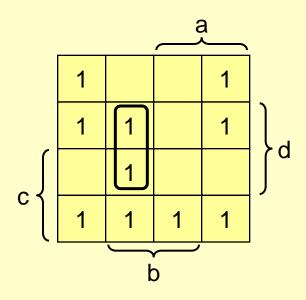
$$f = a'c'd + a'bd + a'bc + b'c' + b'd' + cd'$$

$$f(a,b,c,d) = \sum m(0,1,2,5,6,7,8,9,10,14)$$

$$c \left\{ \begin{array}{c|c} 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array} \right\} d$$

$$f = a'c'd + a'bd + a'bc + b'c' + b'd' + cd'$$

$$F = a'bd$$

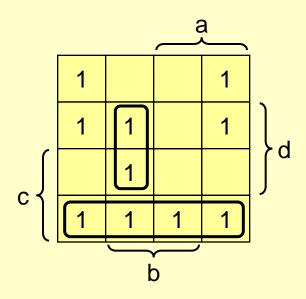


#### Quine-McCluskey Method An Example

But, the form below is not minimized, using a Karnaugh map we can obtain:

$$f = a'c'd + a'bd + a'bc + b'c' + b'd' + cd'$$

$$F = a'bd + cd'$$

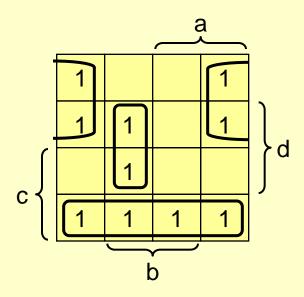


#### Quine-McCluskey Method An Example

But, the form below is not minimized, using a Karnaugh map we can obtain:

$$f = a'c'd + a'bd + a'bc + b'c' + b'd' + cd'$$

$$F = a'bd + cd' + b'c'$$



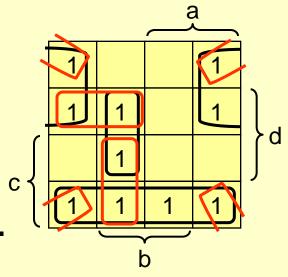
#### Quine-McCluskey Method An Example

What are the extra terms in the solution obtained with the Quine-McCluskey method?

$$f = a'c'd + a'bd + a'bc + b'c' + b'd' + cd'$$

$$F = a'bd + cd' + b'c'$$

Thus, we need a method to eliminate this redundant terms from the Quine-McCluskey solution.



#### **The Prime Implicant Chart**

The prime implicant chart is the second part of the Quine-McCluskey procedure.

It is used to select a minimum set of prime implicants.

Similar to the Karnaugh map, we first select the essential prime implicants, and then we select enough prime implicants to cover all the minterms of the function.

		minterms									
		0	1	2	5	6	7	8	9	10	14
(0,1,8,9)	b'c'	X	X					X	X		
(0,2,8,10)	b'd'	X		X				X		X	
(2,6,10,14)	cď'			X		X				X	X
(1,5)	a'c'd		X		X						
(5,7)	a'bd				X		X				
(6,7)	a'bc					X	X				
	(0,2,8,10) (2,6,10,14) (1,5)	(0,1,8,9) b'c' (0,2,8,10) b'd' (2,6,10,14) cd' (1,5) a'c'd (5,7) a'bd (6,7) a'bc				0 1 2 5	0 1 2 5 6	0 1 2 5 6 7	0 1 2 5 6 7 8	0 1 2 5 6 7 8 9	0 1 2 5 6 7 8 9 10

Question: Given the prime implicant chart above, how can we identify the essential prime implicants of the function?

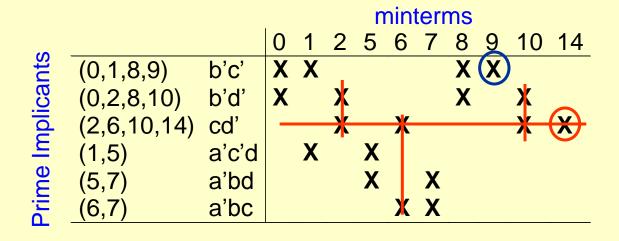
		minterms									
		0	1	2	5	6	7	8	9	10	14
(0,1,8,9)	b'c'	X	X					X	X		
(0,2,8,10)	b'd'	X		X				X		X	
(2,6,10,14)	cď'			X		X				X	<b>(X)</b>
(1,5)	a'c'd		X		X						
(5,7)	a'bd				X		X				
(6,7)	a'bc					X	X				
	(0,2,8,10) (2,6,10,14) (1,5) (5,7)	(0,2,8,10) b'd' (2,6,10,14) cd' (1,5) a'c'd (5,7) a'bd	(0,2,8,10) b'd' X (2,6,10,14) cd' (1,5) a'c'd (5,7) a'bd	(0,1,8,9) b'c' X X (0,2,8,10) b'd' X (2,6,10,14) cd' (1,5) a'c'd X (5,7) a'bd	(0,1,8,9)       b'c'       X       X         (0,2,8,10)       b'd'       X       X         (2,6,10,14)       cd'       X         (1,5)       a'c'd       X         (5,7)       a'bd       X	(0,1,8,9)       b'c'       X       X       X         (0,2,8,10)       b'd'       X       X       X         (2,6,10,14)       cd'       X       X       X         (1,5)       a'c'd       X       X       X         (5,7)       a'bd       X       X	(0,1,8,9)       b'c'       X       X       X       X         (0,2,8,10)       b'd'       X       X       X       X         (2,6,10,14)       cd'       X       X       X       X         (1,5)       a'c'd       X       X       X       X         (5,7)       a'bd       X       X       X       X	(0,1,8,9)       b'c'       X <t< th=""><th>(0,1,8,9)       b'c'       X       X       X         (0,2,8,10)       b'd'       X       X       X         (2,6,10,14)       cd'       X       X       X         (1,5)       a'c'd       X       X       X         (5,7)       a'bd       X       X       X</th><th>(0,1,8,9)       b'c'       X       <t< th=""><th>(0,1,8,9)       b'c'       X       <t< th=""></t<></th></t<></th></t<>	(0,1,8,9)       b'c'       X       X       X         (0,2,8,10)       b'd'       X       X       X         (2,6,10,14)       cd'       X       X       X         (1,5)       a'c'd       X       X       X         (5,7)       a'bd       X       X       X	(0,1,8,9)       b'c'       X <t< th=""><th>(0,1,8,9)       b'c'       X       <t< th=""></t<></th></t<>	(0,1,8,9)       b'c'       X <t< th=""></t<>

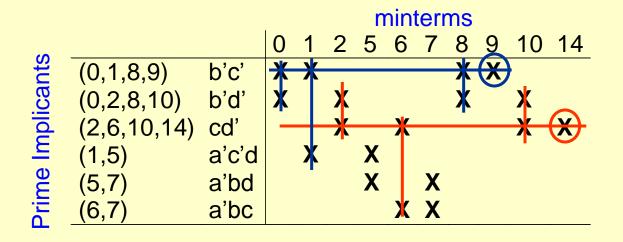
Similar to the Karnaugh map, all we have to do is to look for minterms that are covered by a single term.

		minterms									
		0	1	2	5	6	7	8	9	10	14
(0,1,8,9)	b'c'	X	X					X	X		
(0,2,8,10)	b'd'	X		X				X		X	
(2,6,10,14)	cď'			X		X				X	<b>(X)</b>
(1,5)	a'c'd		X		X						
(5,7)	a'bd				X		X				
(6,7)	a'bc					X	X				
	(0,2,8,10) (2,6,10,14) (1,5) (5,7)	(0,2,8,10) b'd' (2,6,10,14) cd' (1,5) a'c'd (5,7) a'bd	(0,2,8,10) b'd' X (2,6,10,14) cd' (1,5) a'c'd (5,7) a'bd	(0,1,8,9) b'c' X X (0,2,8,10) b'd' X (2,6,10,14) cd' (1,5) a'c'd X (5,7) a'bd	(0,1,8,9) b'c' X X (0,2,8,10) b'd' X X (2,6,10,14) cd' X (1,5) a'c'd X (5,7) a'bd	0 1 2 5 	0 1 2 5 6 	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	(0,1,8,9)       b'c'       X       X       X       X         (0,2,8,10)       b'd'       X       X       X       X         (2,6,10,14)       cd'       X       X       X         (1,5)       a'c'd       X       X       X         (5,7)       a'bd       X       X       X	(0,2,8,10) b'd' X X X X (2,6,10,14) cd' X X X (1,5) a'c'd X X X (5,7) a'bd X X X	(0,1,8,9)       b'c'       X <t< th=""></t<>

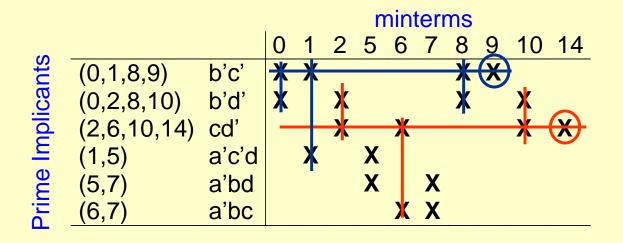
Once a term is included in the solution, all the minterms covered by that term are covered.

Therefore we may now mark the covered minterms and find terms that are no longer useful.

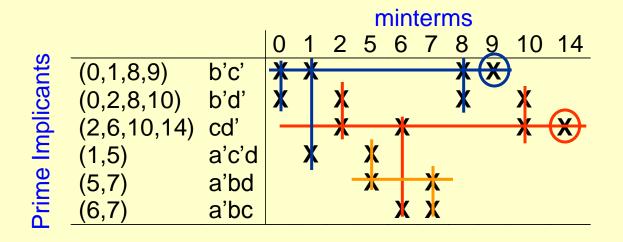




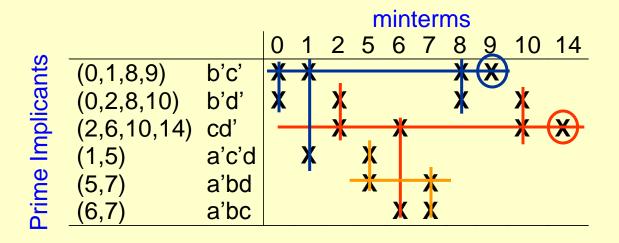
As we have not covered all the minterms with essential prime implicants, we must choose enough non-essential prime implicants to cover the remaining minterms.



What strategy should we use to find a minimum cover for the remaining minterms?



We choose first prime implicants that cover the most minterms.



Therefore our minimum solution is:

$$f(a,b,c,d) = b'c' + cd' + a'bd$$

#### To Simplify in POS Form:

- Take 0's as initial list of minterms
- Simplify to get the compliment of the function in SOP form
- Compliment to get the function in POS form

#### Function with Don't care inputs

- Don't cares included while computing Prime implicants

-In the Selection of Essential Prime implicants don't cares not used.

#### Simplify Using QM Method

$$F(A,B,C,D) = \Sigma(6,7,14)$$
  
 $d(A,B,C,D) = \Sigma(0,8,15)$