Lecture - 32

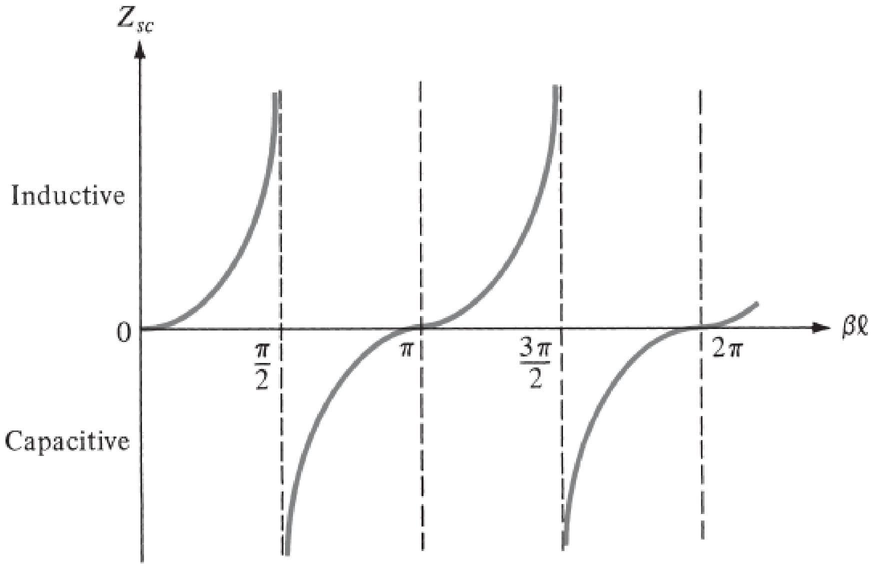
Transmission Lines as circuit elements

For a lossless line: $$Z_{in} = Z_o \left[\frac{Z_L + jZ_o \tan \beta l}{Z_o + jZ_L \tan \beta l} \right]$$

Transmission Lines terminated in short or open circuits are commonly used as tuning elements.

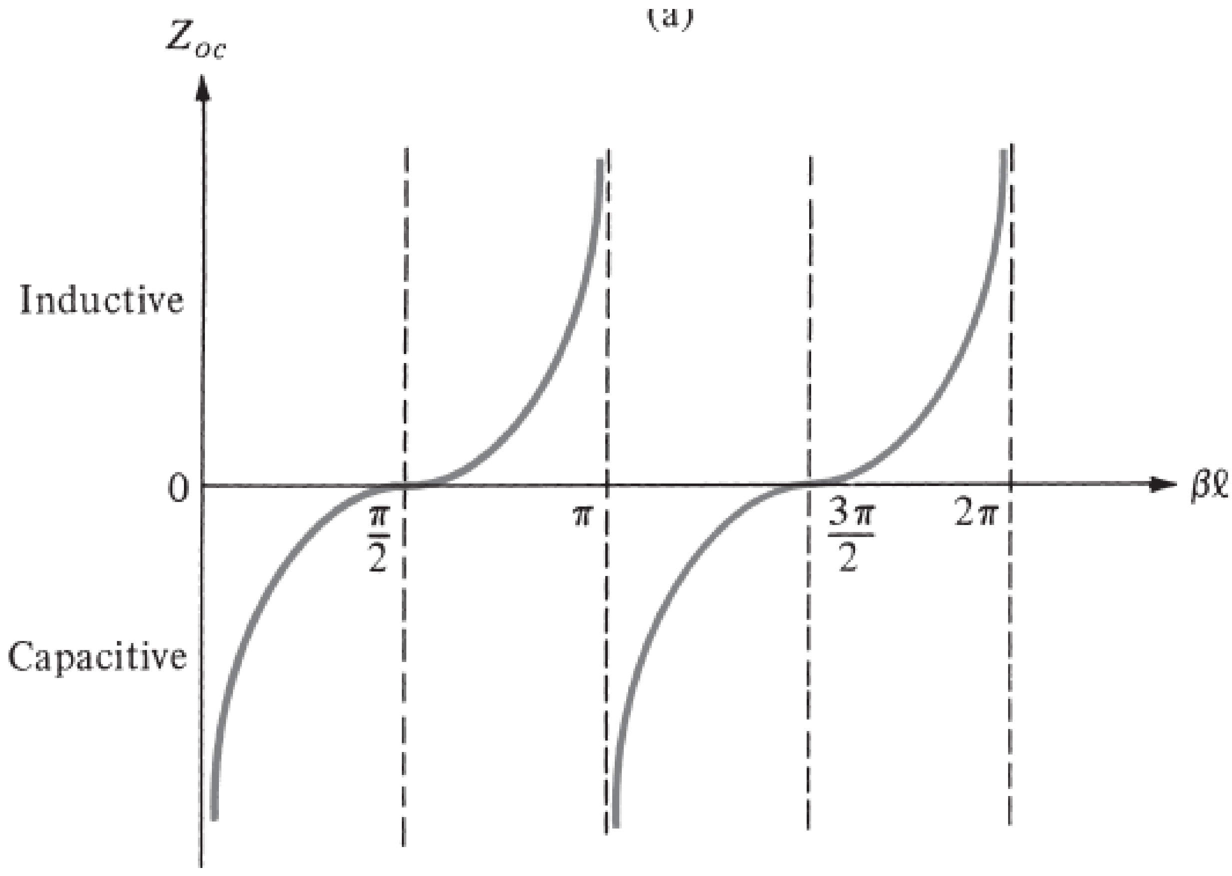
If the line is **short-circuited**: $Z_L = 0$

$$\boxed{Z_{SC} = jZ_o \tan \beta l}$$



(a)

For a lossless line: open-circuited: Z_L = infinity



(b)

$$Z_{in} = -jZ_o \cot \beta l$$

$$Z_o = \sqrt{Z_{OC} Z_{SC}}$$

$$\gamma = \frac{1}{l} \tanh^{-1} \sqrt{\frac{Z_{SC}}{Z_{OC}}}$$

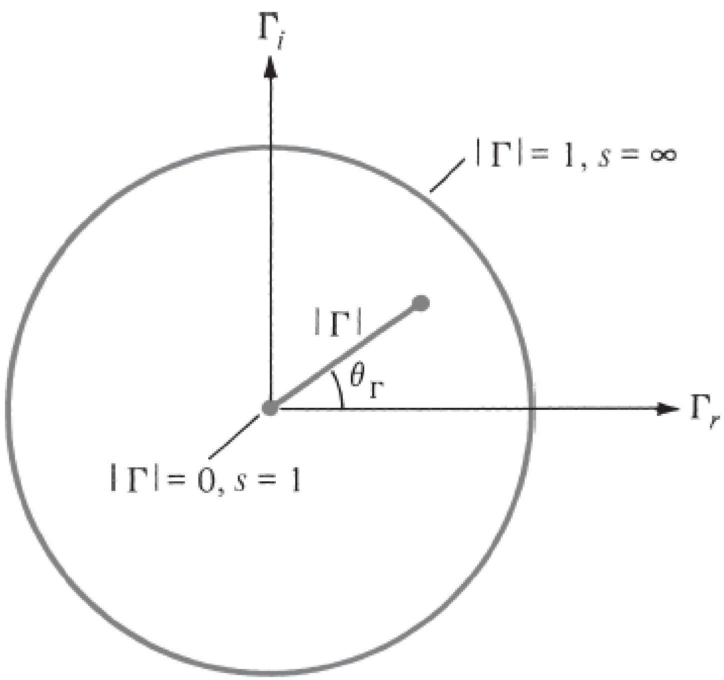
Short television antenna lead-in wire

- The open-circuit and short-circuit impedances measured at the input terminals of a TV antenna lead-in wire of length 1.5 (m), which is less than a quarter wavelength, are –j54.6 ohm and j103 ohm, respectively. (a) Find Z_o and propagation constant of the line. (b) Without changing the operating frequency, find the input impedances of a short-circuited line that is twice the given length.
- **Answers:** (a) 75 ohm, j0.628 rad/m; (b) –j231 ohm

SMITH CHART

- Transmission line calculation-such as the determination of input impedance, reflection coefficient and load impedance often involve tedious manipulations of complex numbers.
- Graphical method of solution → **Smith Chart**
- **Smith Chart** was devised by P.H.Smith (in 1930).
- Smith Chart is a graphical plot of normalized resistance and reactance functions in the reflection-coefficient plane

Introduction



The normalized impedance is represented on the Smith chart by using families of curves that identify the normalized resistance r (real part) and the normalized reactance x (imaginary part)

- If a lossless line of Z_0 is terminated with Z_L, $z_L = Z_L/Z_0$ (normalized load impedance),

$$\Gamma = \frac{z_L - 1}{z_L + 1} = |\Gamma| e^{j\theta} \Rightarrow z_L = \frac{1 + |\Gamma| e^{j\theta}}{1 - |\Gamma| e^{j\theta}}$$

- Let $\Gamma = \Gamma_r + j\Gamma_i$, and $z_L = r_L + jx_L$.

$$r_L + jx_L = \frac{(1+\Gamma_r) + j\Gamma_i}{(1-\Gamma_r) - j\Gamma_i} \Rightarrow r_L = \frac{1 - \Gamma_r^2 - \Gamma_i^2}{(1-\Gamma_r)^2 + \Gamma_i^2} \quad x_L = \frac{2\Gamma_i}{(1-\Gamma_r)^2 + \Gamma_i^2}$$

$$\left(\Gamma_r - \frac{r_L}{1+r_L}\right)^2 + \Gamma_i^2 = \left(\frac{1}{1+r_L}\right)^2,$$

$$(\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{x_L}\right)^2 = \left(\frac{1}{x_L}\right)^2$$

The real part gives

Add a quantity equal to zero

$$r=\frac{1-\mathrm{Re}^2(\Gamma)-\mathrm{Im}^2(\Gamma)}{(1-\mathrm{Re}(\Gamma))^2+\mathrm{Im}^2(\Gamma)}$$

$$r(\mathrm{Re}(\Gamma)-1)^2+\left(\mathrm{Re}^2(\Gamma)-1\right)+r\,\mathrm{Im}^2(\Gamma)+\mathrm{Im}^2(\Gamma)+\underbrace{\frac{1}{1+r}-\frac{1}{1+r}}_{=0}=0$$

$$\left[r(\mathrm{Re}(\Gamma)-1)^2+\left(\mathrm{Re}^2(\Gamma)-1\right)+\frac{1}{1+r}\right]+(1+r)\,\mathrm{Im}^2(\Gamma)=\frac{1}{1+r}$$

$$(1+r)\left[\mathrm{Re}^2(\Gamma)-2\,\mathrm{Re}(\Gamma)\frac{r}{1+r}+\frac{r^2}{(1+r)^2}\right]+(1+r)\,\mathrm{Im}^2(\Gamma)=\frac{1}{1+r}$$

$$\Rightarrow\qquad\left[\mathrm{Re}(\Gamma)-\frac{r}{1+r}\right]^2+\mathrm{Im}^2(\Gamma)=\left(\frac{1}{1+r}\right)^2$$

Equation of a circle

The imaginary part gives

Multiply by x and add a quantity equal to zero

$$x=\frac{2\,\mathrm{Im}(\Gamma)}{(1-\mathrm{Re}(\Gamma))^2+\mathrm{Im}^2(\Gamma)}$$

$$x^2\left[(1-\mathrm{Re}(\Gamma))^2+\mathrm{Im}^2(\Gamma)\right]-2x\,\mathrm{Im}(\Gamma)\overbrace{+1-1}^{=0}=0$$

$$\left[(1-\mathrm{Re}(\Gamma))^2+\mathrm{Im}^2(\Gamma)\right]-\frac{2}{x}\mathrm{Im}(\Gamma)+\frac{1}{x^2}=\frac{1}{x^2}$$

$$(1-\mathrm{Re}(\Gamma))^2+\left[\mathrm{Im}^2(\Gamma)-\frac{2}{x}\mathrm{Im}(\Gamma)+\frac{1}{x^2}\right]=\frac{1}{x^2}$$

$$\Rightarrow \qquad (\mathrm{Re}(\Gamma)-1)^2+\left[\mathrm{Im}(\Gamma)-\frac{1}{x}\right]^2=\frac{1}{x^2}$$

Equation of a circle

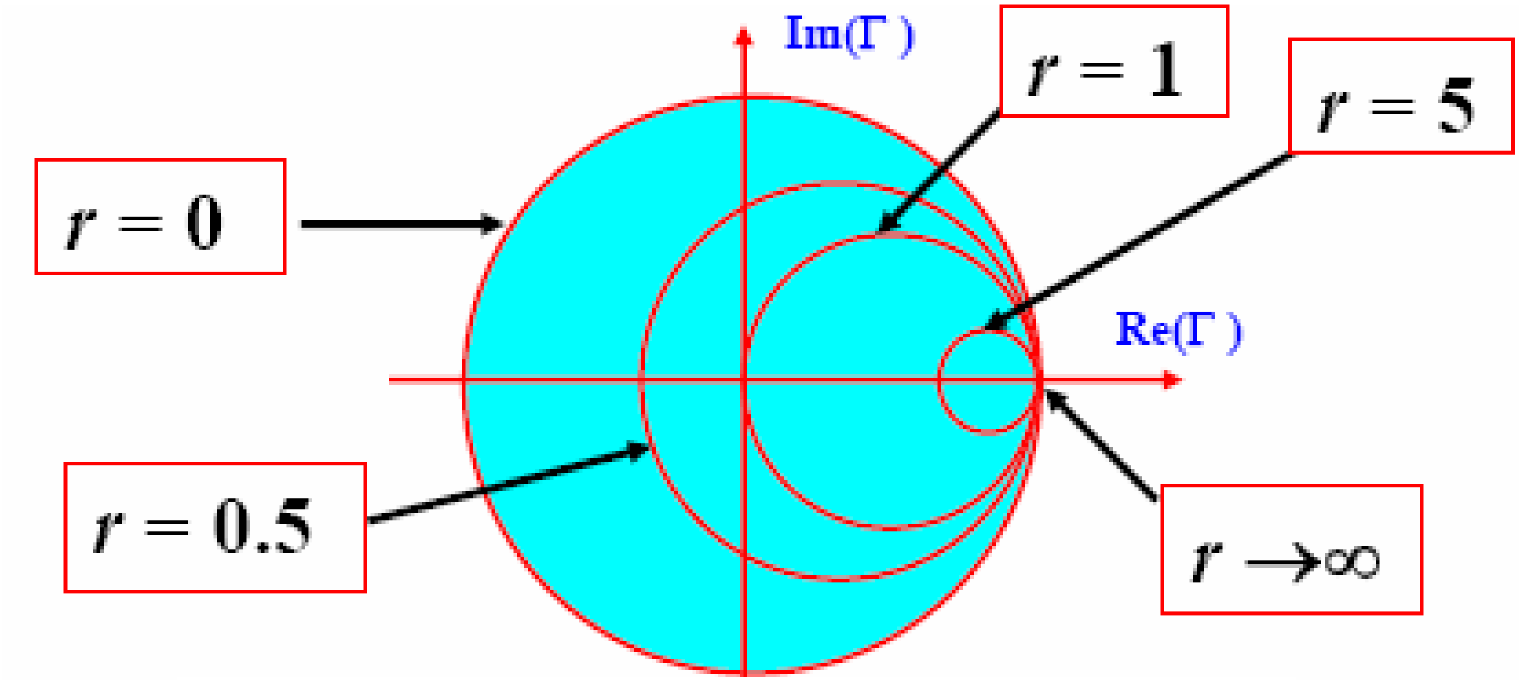
$$\left(\Gamma_r - \frac{r_L}{1+r_L}\right)^2 + \Gamma_i^2 = \left(\frac{1}{1+r_L}\right)^2,$$

$$\left(\Gamma_r - 1\right)^2 + \left(\Gamma_i - \frac{1}{x_L}\right)^2 = \left(\frac{1}{x_L}\right)^2$$

The result for the real part indicates that on the complex plane with coordinates (Re(Γ), Im(Γ)) all the possible impedances with a given normalized resistance r are found on a circle with

$$\text{Center} = \left\{\frac{r}{1+r}, 0\right\} \qquad \text{Radius} = \frac{1}{1+r}$$

As the normalized resistance r varies from 0 to ∞, we obtain a family of circles completely contained inside the domain of the reflection coefficient $|\Gamma| \le 1$.

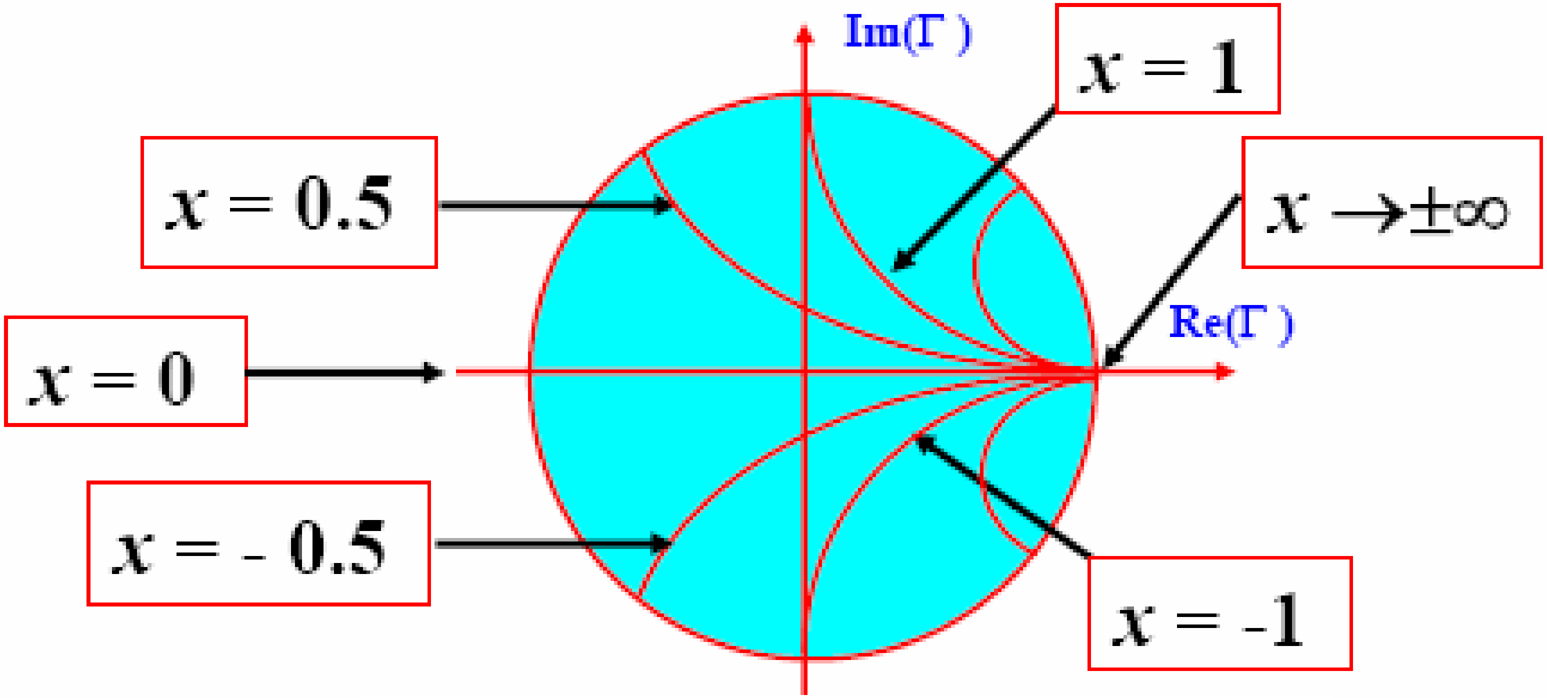


$$\text{Center} = \left\{ \frac{r}{1+r}, 0 \right\} \qquad \text{Radius} = \frac{1}{1+r}$$

The result for the imaginary part indicates that on the complex plane with coordinates (Re(Γ), Im(Γ)) all the possible impedances with a given normalized reactance x are found on a circle with

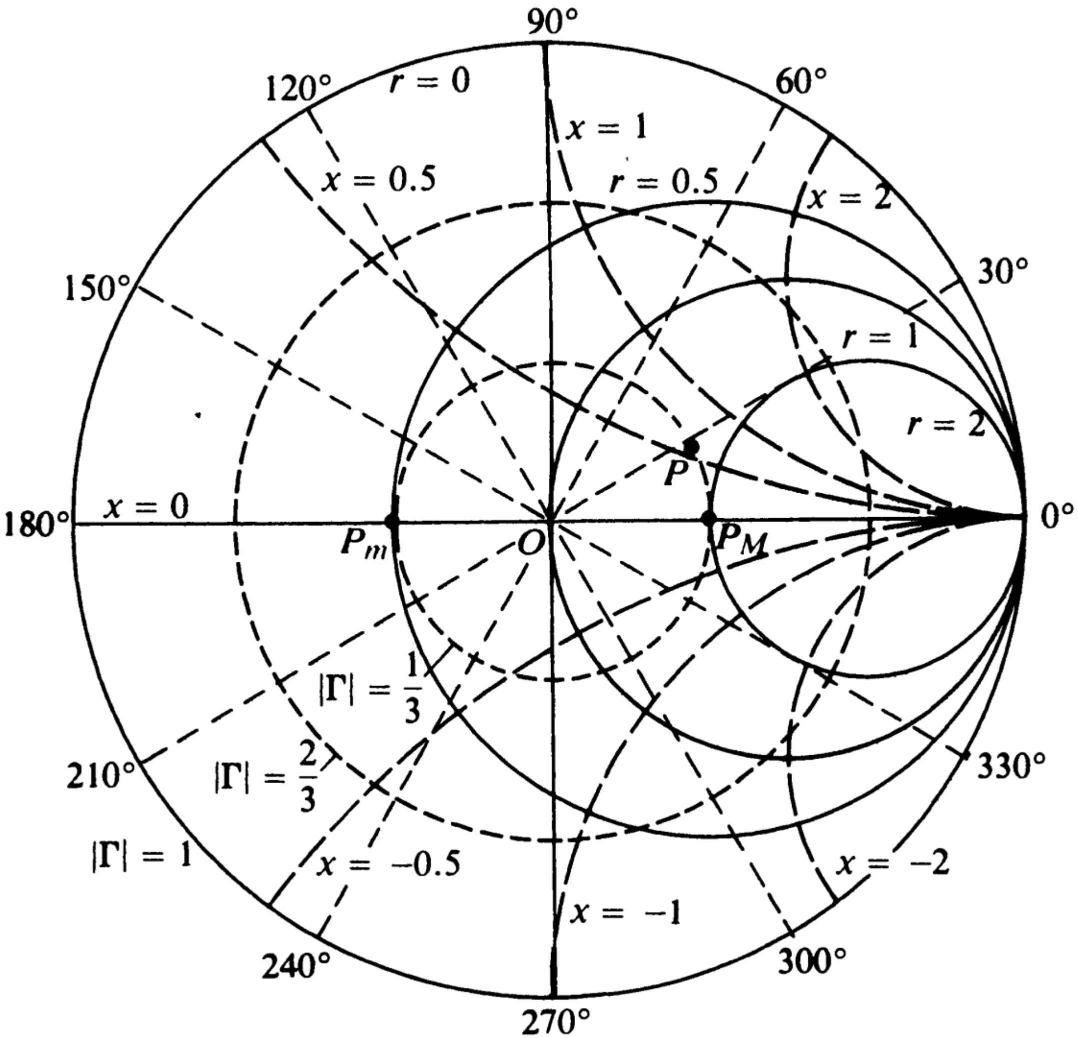
$$\text{Center} = \left\{ 1, \frac{1}{x} \right\} \qquad \text{Radius} = \frac{1}{x}$$

As the normalized reactance x varies from $-\infty$ to ∞ , we obtain a family of arcs contained inside the domain of the reflection coefficient $|\Gamma| \leq 1$.



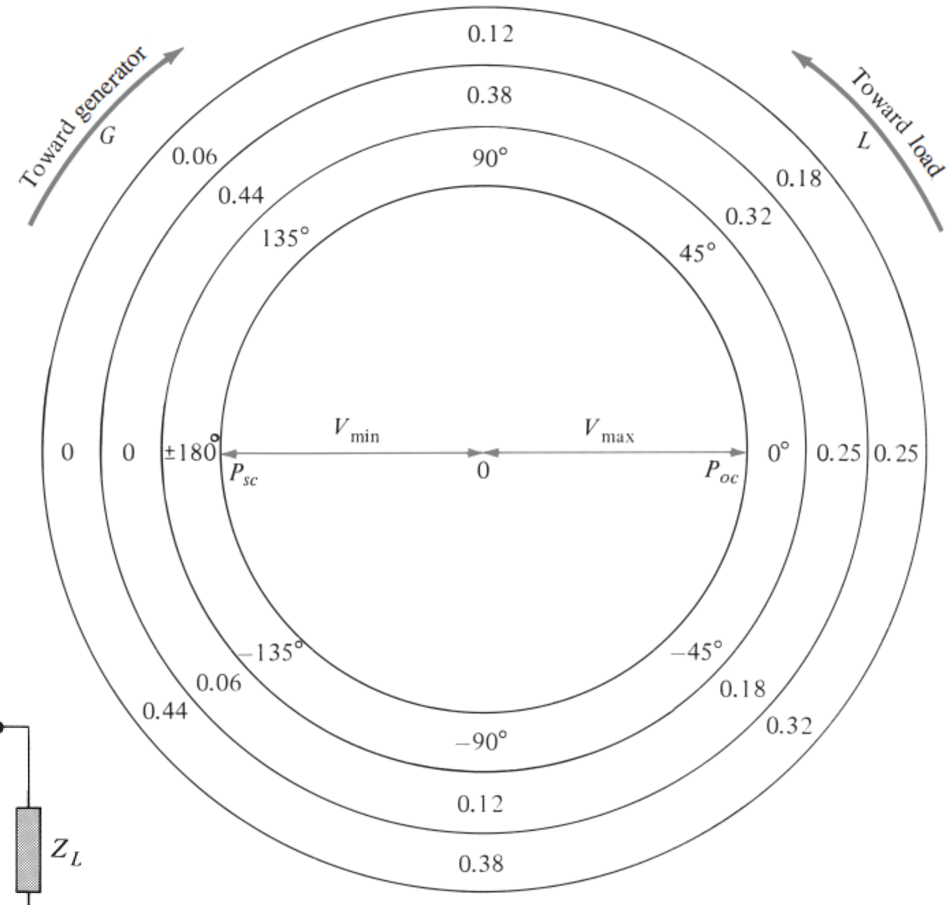
Center = $\left\{ 1, \dfrac{1}{x} \right\}$ Radius = $\dfrac{1}{x}$

Superimpose the two circles families

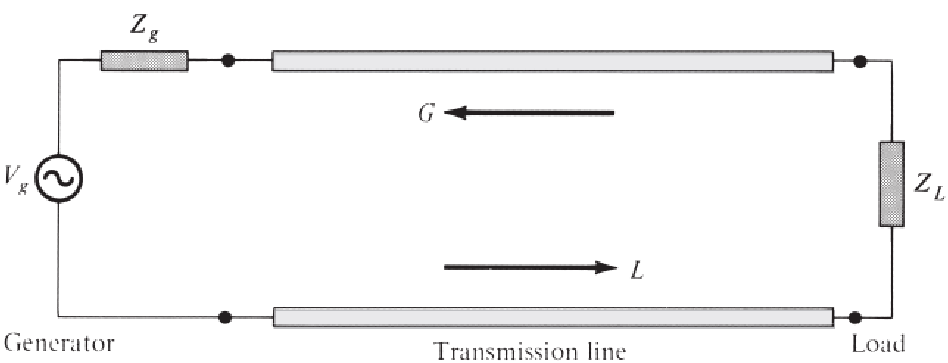


The intersection of an r-circle and an x-circle defines a point that represents a normalized load impedance

Important points about Smith chart



(a)



(b)

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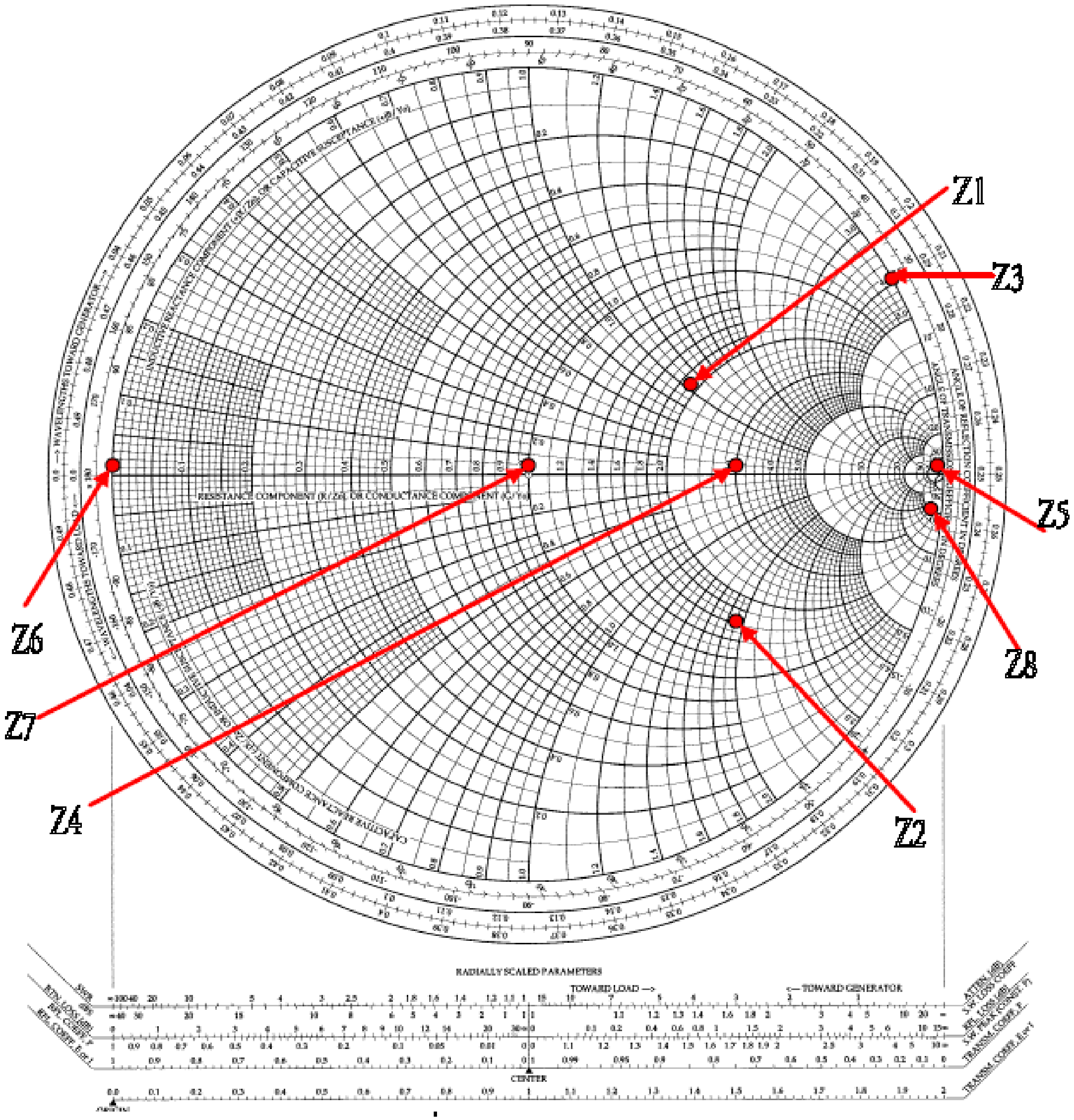
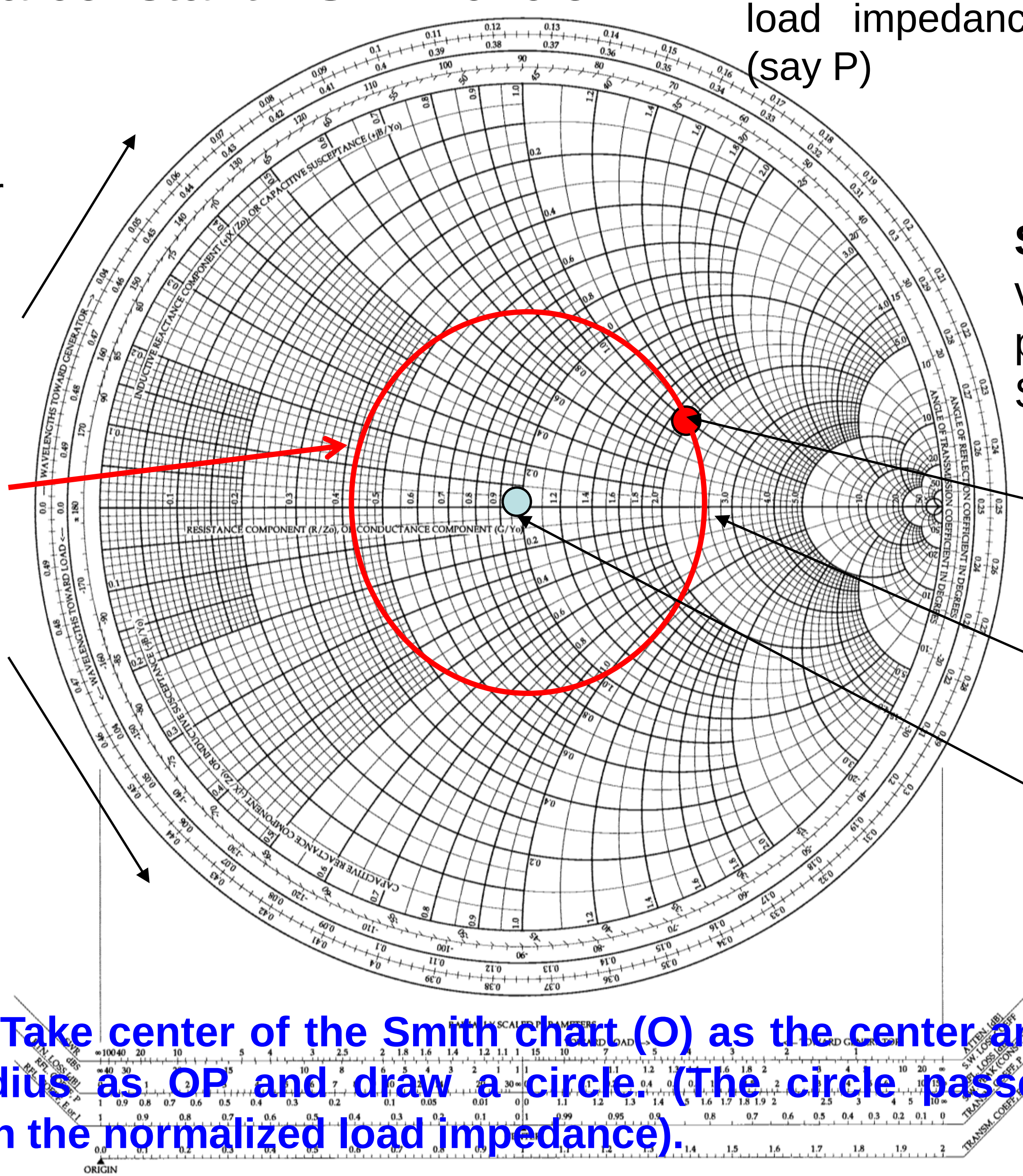
- Impedance divided by line impedance (50 Ohms)
  - Z1 = 100 + j50
  - Z2 = 75 -j100
  - Z3 = j200
  - Z4 = 150
  - Z5 = infinity (an open circuit)
  - Z6 = 0 (a short circuit)
  - Z7 = 50
  - Z8 = 184 -j900

- Then, normalize and plot. The points are plotted as follows:
  - z1 = 2 + j
  - z2 = 1.5 -j2
  - z3 = j4
  - z4 = 3
  - z5 = infinity
  - z6 = 0
  - z7 = 1
  - z8 = 3.68 -j18S



Plotting a constant VSWR circle

Step1: Locate the normalized load impedance on the chart (say P)

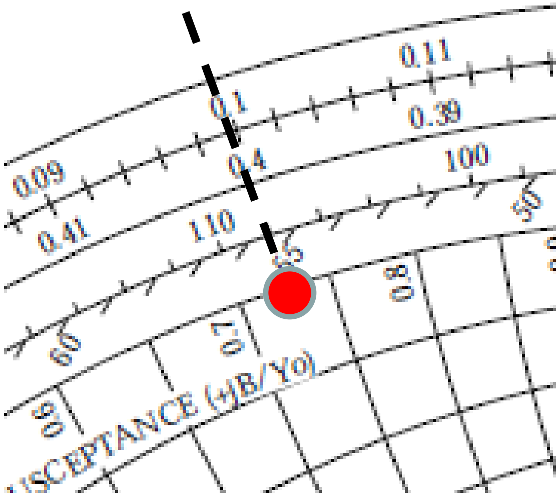


Step3: The value of r at this point gives you SWR ‘s’

Step2: Take center of the Smith chart (O) as the center and the radius as OP and draw a circle. (The circle passes through the normalized load impedance).

Example

- Find the input impedance of a section of a 50 Ω loss-less transmission line that is 0.1λ long and is terminated in a short circuit.



Applications of Smith Chart

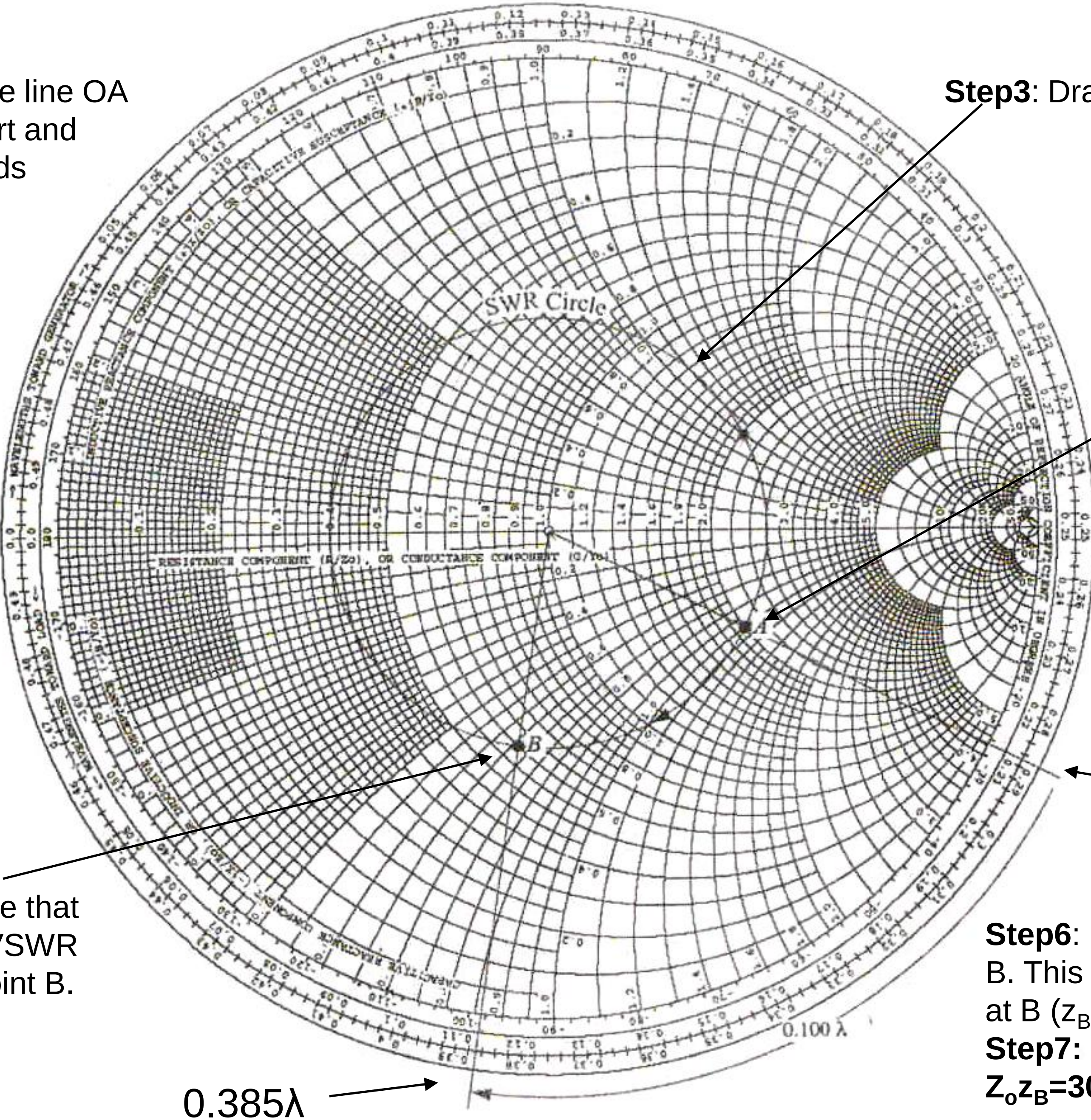
- **To find input impedance using a known load**
- Given that: Z_L = 100-j50 Ohm; Z_o = 50 Ohm; length of 0.1λ
- **Step1**: Find normalized impedance
- **Step2**: Locate r and x intersection Point
- **Step3**: Draw VSWR circle
- **Step4**: Extend the line OA at the end of chart and move 0.1λ towards generator.
- **Step5**: The line that intersect the VSWR circle gives point B.
- **Step6**: Find coordinates of B. This gives impedance at B (Z_B)
- **Step7**: Calculate $Z_{in} = Z_o Z_B$=30-j33

Step4: Extend the line OA at the end of chart and move 0.1λ towards generator

Step3: Draw VSWR circle

Step1: Find normalized impedance

Step2: Locate r and x intersection Point



0.285λ

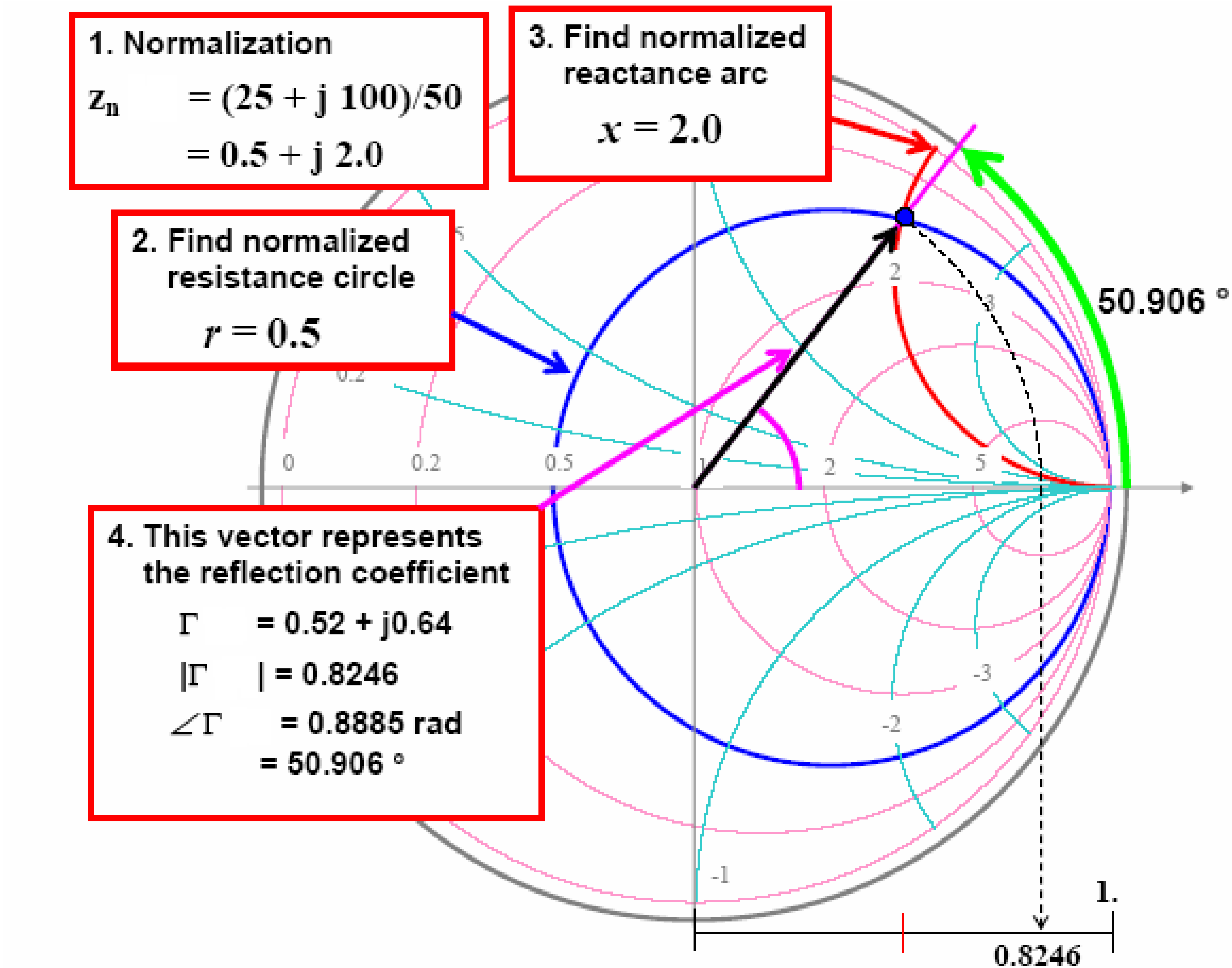
Step5: The line that intersect the VSWR circle gives point B.

Step6: Find coordinates of B. This gives impedance at B (z_B)

Step7: Calculate $Z_{in} = Z_o z_B$=30-j33

0.385λ

To find the reflection coefficient for a known load



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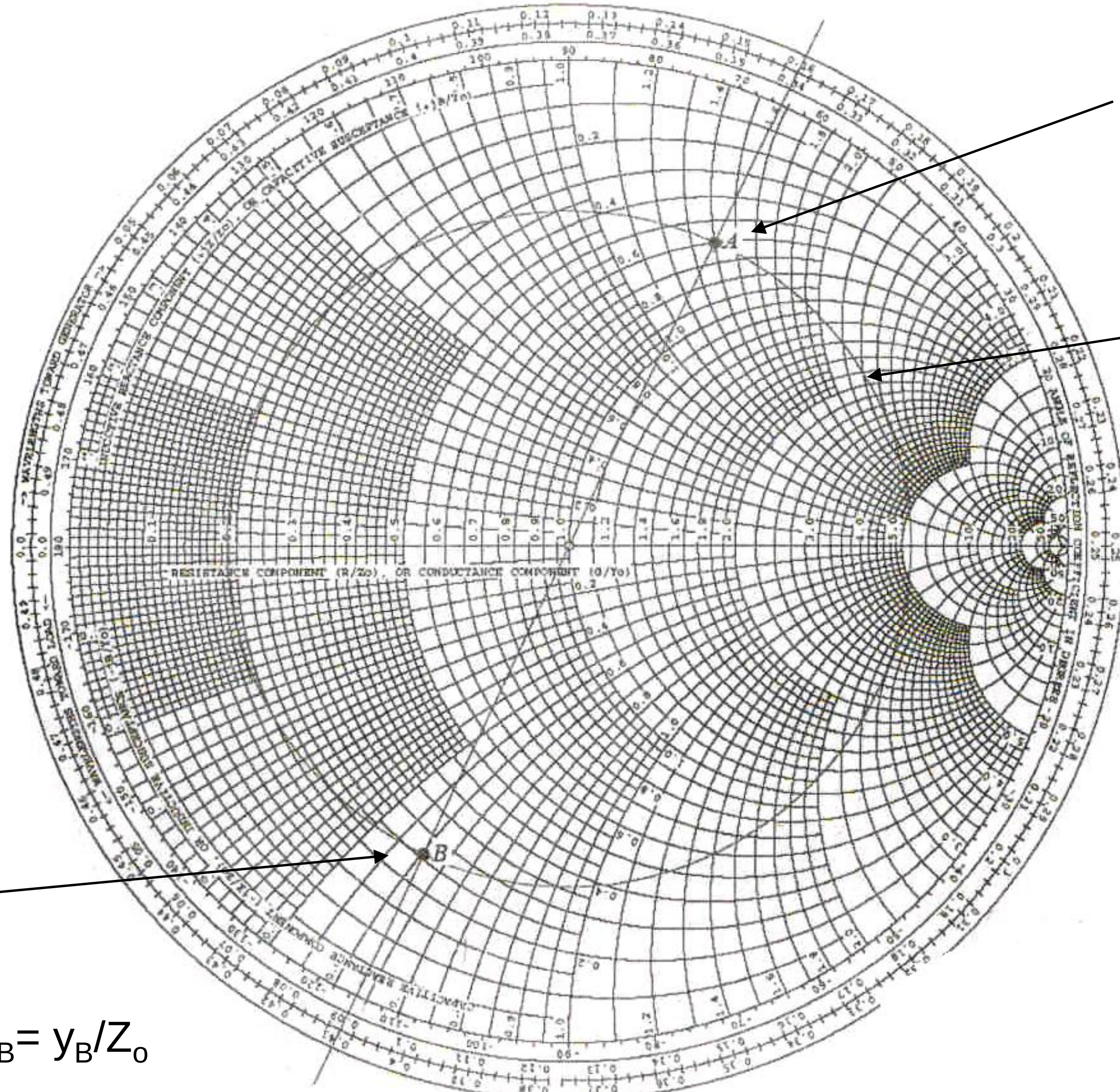
To find the admittance for a known load

$Z_L = 30 + j70$



Step1:

Normalized impedance

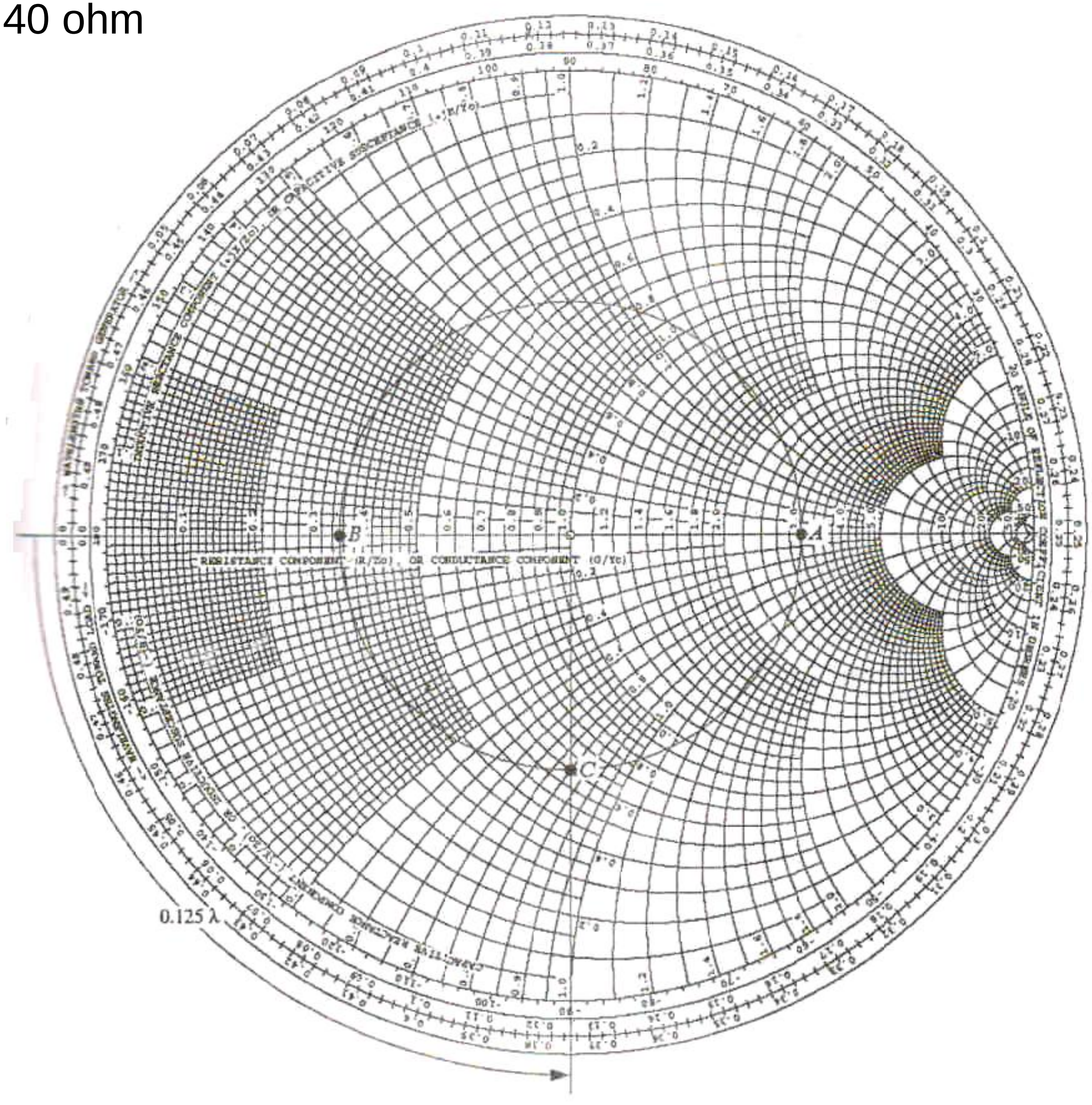
step2

y_B

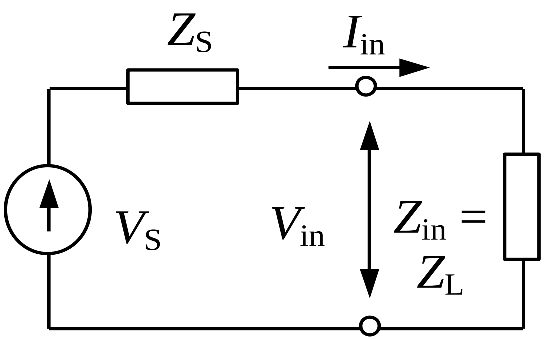
$Y = Y_0 y_B = y_B / Z_o$

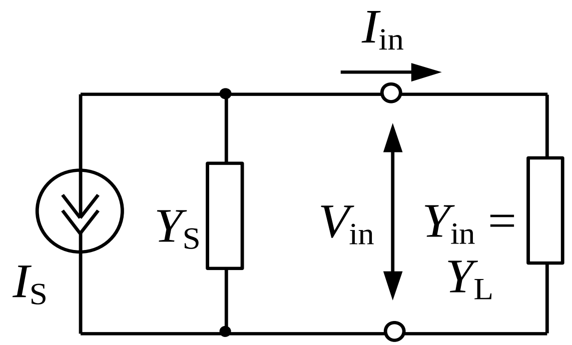
To find the location of V_{max} and V_{min} on a line from a known load

Z_L = 30 – j40 ohm



Maximum Power transfer to the Load





$Z_S = R_S + jX_S$ - source impedance

$Z_L = R_L + jX_L$ - load impedance

$$P = \frac{1}{2} V_{in}^2 \,\mathrm{Re}\left(\frac{1}{Z_L}\right) = \frac{1}{2} V_S^2 \left|\frac{Z_L}{Z_S + Z_L}\right|^2 \mathrm{Re}\left(\frac{1}{Z_L}\right)$$

- power delivered to load

(substitution of real and imaginary parts of source and load impedances)

$$P = \frac{1}{2} V_S^2 \frac{R_L}{(R_S + R_L)^2 + (X_S + X_L)^2}$$

- power delivered to load as function of circuit parameters

For fixed source impedance Z_S, to maximize output power

$$\frac{\partial P}{\partial R_L} = 0 \qquad \frac{\partial P}{\partial X_L} = 0$$

$$P = \frac{1}{2} V_S^2 \frac{R_L}{(R_S + R_L)^2 + (X_S + X_L)^2}$$

$$\boxed{P = \frac{V_S^2}{8R_S}}$$

- maximum power delivered to load

$$\begin{cases} R_S^2 - R_L^2 + (X_L + X_S)^2 = 0 \\ X_L(X_L + X_S) = 0. \end{cases}$$

$$\begin{cases} R_S = R_L \\ X_L = -X_S \end{cases} \qquad Z_L = Z_S^*$$

- impedance conjugate matching conditions

$$\begin{cases} G_S = G_L \\ B_L = -B_S \end{cases} \quad \textit{or} \quad Y_L = Y_S^*$$

- admittance conjugate matching conditions

Impedance Matching Techniques

Impedance matching is necessary to provide maximum delivery of RF power to load from source

Impedance matching using :

- Lumped elements
- **Quarter-Wave transformer**
- Single-Stub Tuner
- Double-Stub Tuner