Birla Institute of Technology & Science, Pilani (Raj.)

Second Semester 2015-2016, MATH F113 (Probability and Statistics)

Comprehensive Examination, Open Book

Time: 180 Min. **Date: 10 May, 2016 (Tuesday)** **Max. Marks: 135**

1. There are five questions in all.
2. Define events and random variables as and when required.
3. Start new question on fresh page. Answer each subpart of a question in continuation.
4. **Attempt questions of Part A and Part B in two separate answer books. Write Part A and Part B on the top right corner of the answer book.**

Part A

.1 (a) Two electrons are moving independently in an electron accelerator. Their total kinetic energy E has a gamma distribution with mean 6 MeV and whose density is maximum at $E = 4$ MeV. (i) If the electron accelerator switches off when E exceeds 7.84 MeV, find the probability that the accelerator is not switched off. (ii) If the electron accelerator is required to be switched off 5% of the times, after which value of E should it switch off? [8]

(b) Among the tube lights manufactured by Philips, on an average 2 tube lights fuse every year. A batch of tube lights manufactured by Philips is trashed, immediately dispatched for sale, sent for further testing if the life of a randomly chosen tube light from it is, respectively, less than 1 year, more than 2 years, and between 1 and 2 years. (i) Find the probability that a batch of tube lights manufactured by Philips is sent for further testing. (ii) Find the probability that among 400 batches of tube lights manufactured by Philips, the number of batches sent for further testing is at least 80 and at most 100. [12]

(c) A continuous random variable X has the moment generating function $m_X(t)$ given by

$$m_X(t) = \frac{e^t}{1-t^2}.$$

Using Chebychev's inequality, find an upper bound on $P[X^2 - 2X > 2]$. [8]

.2 (a) The joint pdf of bivariate random variable (X, Y) is given by

$$f(x,y) = \begin{cases} kxy^2; & 0 < x < y < 1 \\ 0; & \text{otherwise.} \end{cases}$$

(i) Find the value of constant k.

(ii) Find the marginal densities for X and Y. [2 + 4 +

(iii) Find $P[Y > \frac{1}{2} \,|X = \frac{1}{4}]$.

(b) Let X be a continuous random variable with the density function

$$f(x) = \begin{cases} \frac{1}{8}; & 0 \le x < 2 \\ \frac{3}{8}; & 2 \le x < 4 \\ 0; & \text{elsewhere.} \end{cases}$$

Simulate the two values of X by using the random number 20 and 64.

(c) The cumulative distribution function of a discrete random variable X is

$$F(x) = \begin{cases} 0; & x < 0 \\ \frac{1}{4}; & 0 \le x < 1 \\ \frac{1}{3}; & 1 \le x < 2 \\ \frac{1}{2}; & 2 \le x < 3 \\ 1; & x \ge 3. \end{cases}$$

Find probability density function of X and hence find $E[X]$.

Q.3 (a) The probability of detecting tuberculosis in X-ray examination of a person suffering from the disease is $1-b$. The probability of diagnosing a healthy person as tubercular is a. If the ratio of tubercular patients to the whole population is c, find the probability that a person is healthy if after examination he is diagnosed as tubercular. [9]

(b) For two random variables X and Y, show that $\rho(aX+b,\, cY+d) = \frac{ac}{|a||c|}\rho_{X,Y}$ where a, b, c, d are real numbers and $a \neq 0$, $c \neq 0$. [7]

(c) A marketing company claims that it receives 4% responses from its mailing. To test this claim, a random sample of size 500 were surveyed and 25 responses were obtained. For a 5% significance level, test the null hypothesis. [11]

Part B

Q.4 (a) Let X_i be a discrete random variable with the following pdf

X_i	1	−1
$f(x_i)$	p	q

Consider $X = \sum_{i=1}^{n} X_i$, find the moment generating function of $(X+n)/2$, by using moment generating function of X and hence determine the distribution of $(X+n)/2$ with parameters. [11]

(b) A continuous random variable X has following probability density function:

$$f(x) = \begin{cases} \dfrac{x^3}{6\beta^4}\exp\left(\dfrac{-x}{\beta}\right); & \text{if } 0 \le x < \infty,\ \beta > 0 \\ 0; & \text{otherwise.} \end{cases}$$

Find the maximum likelihood estimator for β based on a random sample of size n from the distribution. [8]

(c) Assume that a single digit random number is generated for 100 trials. Let X denote the number generated per trial. Suppose that $\bar{x}$ assumed the value 3.85 for these 100 trials. Find a 94% confidence interval for mean of X. [8]

Q.5 (a) The manager of an industrial plant is planning to buy a machine of either type A or type B. For each day's operation, the number X of repairs that the machine A needs, is a Poisson random variable with mean 0.96. The daily cost of operating A is $C_A = 160 + 40X^2$. For machine B, let Y be Poisson random variable indicating the number of daily repairs, which has mean 1.12, and the daily cost of operating B is $C_B = 128 + 40Y^2$. Assume that the repairs take negligible time and each night the machines are cleaned so that they operate like new machine at the start of each day. Which machine minimizes the expected daily cost ? [7]

(b) A manufacturer claims that the thickness of the spearmint gum it produces is 7.5 one-hundredths of an inch. A quality control specialist regularly checks this claim. On one production run, he took a random sample of $n = 10$ pieces of gum and measured their thickness. He obtained:
7.65 7.60 7.65 7.70 7.55 7.55 7.40 7.40 7.50 7.50 (Assume these data have come from normal population)

(i) Set up the null and alternative hypothesis, choose a suitable test statistic to find the critical region at 0.05 level of significance, and use the same to describe the specialist's conclusion from the test. [14]

(ii) Find the P value of the test. Using the P value, describe the specialist's conclusion at the 0.05 level of significance. Does the conclusion differ from the one in part (i). [6]

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