Electromagnetic Theory

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BITS Pilani
Pilani Campus

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Course No. EEE F212/INSTR F212

Lecture-1: Introduction

The Course Handout

Course Description

- ***Course No.*** : EEE F212/INSTR F212
- ***Course Title*** : **Electromagnetic Theory**
- ***Instructor-in-charge*** : **Dr. Navneet Gupta**

  (email: ngupta@pilani.bits-pilani.ac.in)

  (Chamber: 2210-H, FD-II)
- ***Instructors*** : sec-1: Mahesh Angira

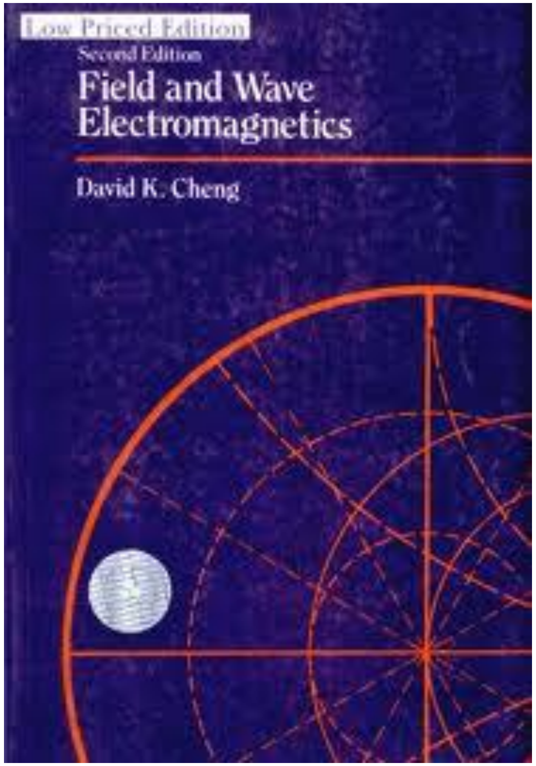
  sec-2: Ashish Kumar Sharma,

  sec-3: Priyanka Choudhary

  sec-4: Rajneesh Kumar

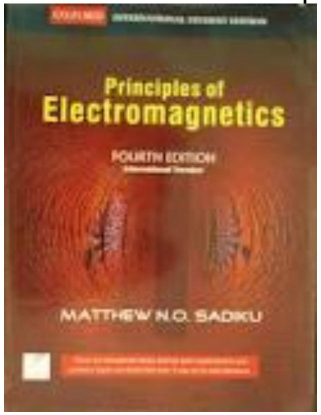
Text Book

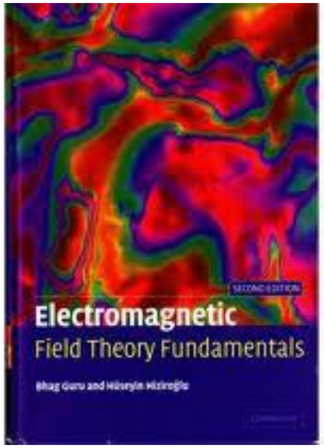
David K.Cheng, "**Field and Wave Electromagnetics**" 2nd ed. Pearson Education, New Delhi, 2009



Reference Books

- Matthew N.O.Sadiku, "**Principles of Electromagnetics**" 4th ed. Oxford University Press, New Delhi, 2009.
- Bhag Guru and Huseyin Hiziroglu, "**Electromagnetic Field Theory Fundamentals**" Cambridge University Press., United Kingdom.





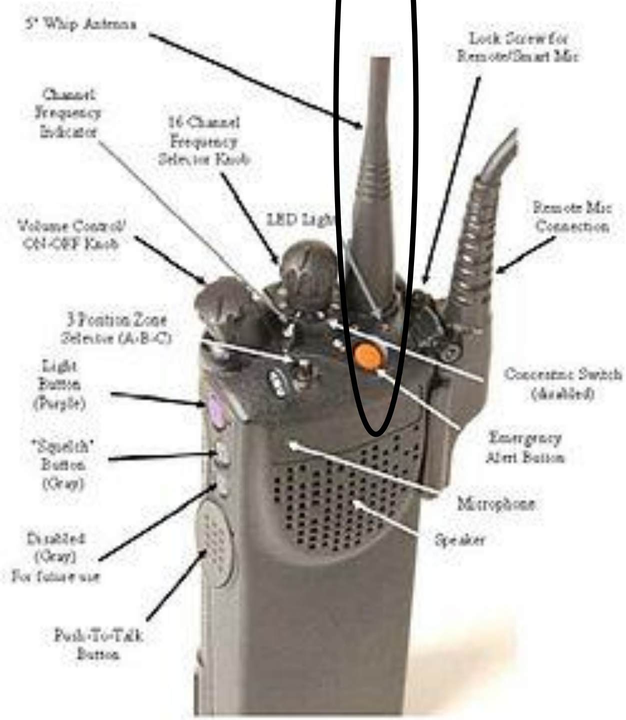
Evaluation Scheme

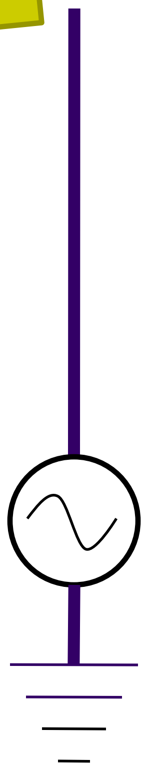
Component	Duration	Marks (200)	Weightage	Date & Time	Evaluation type
Mid-term Test	90 min	60	30%	27/9/2013 4:00-5:30 pm	Closed Book
Assignment	--	10	5%	--	Open Book
Surprize Quizzes	10 min	40	20%	During Tutorial Hour	Closed Book
Quiz	30 min			Will be announced in class	Closed Book
Compre.Exam.	3 hours	90	45%	3/12/2013 AN	OB + CB

Objective of the course:

- The objective of this course is to provide basic theory of electromagnetic fields which is required to understand various applications.
- Electromagnetics is important because it provides a real-world, three-dimensional understanding of electricity and magnetism.

Communication



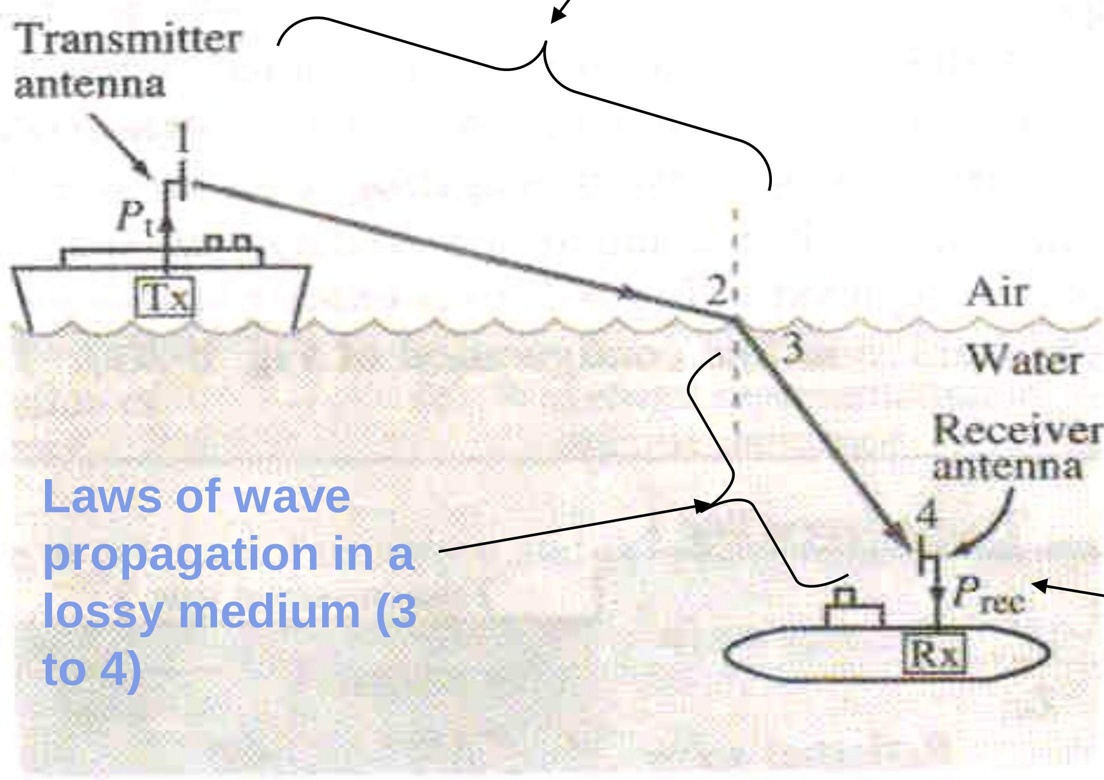


A monopole antenna

Communication:

Wave propagation in lossless medium (1 to 2)

Transmission step-1



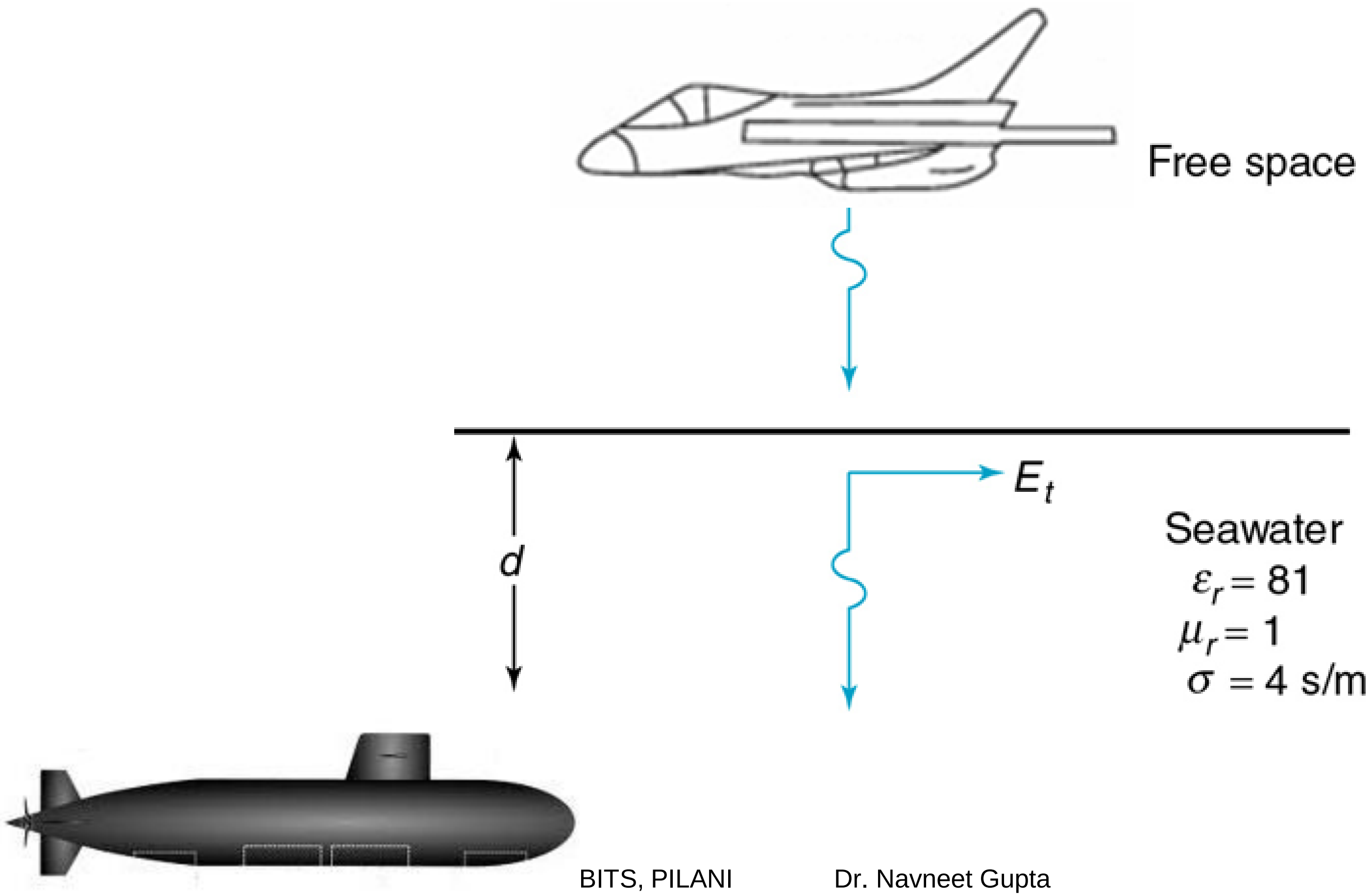
Laws of wave propagation in a lossy medium (3 to 4)

Receiving step

Figure 8-1: Signal path between a shipboard transmitter (Tx) and a submarine receiver (Rx).

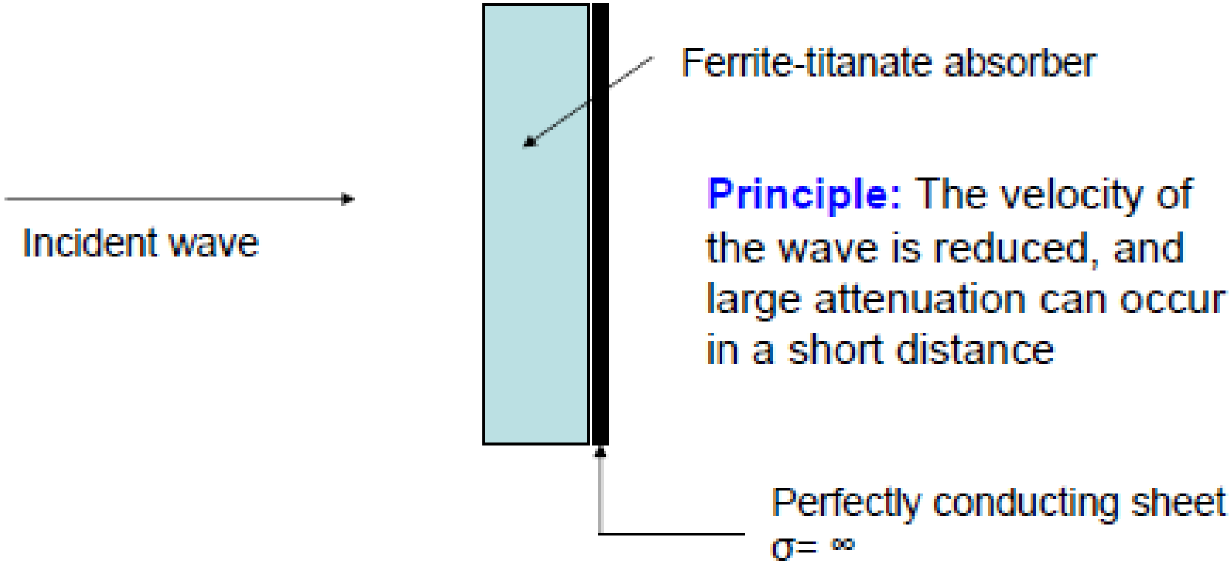
Ocean penetration:

How deep submarine can be submerged and still be reached by airplane?



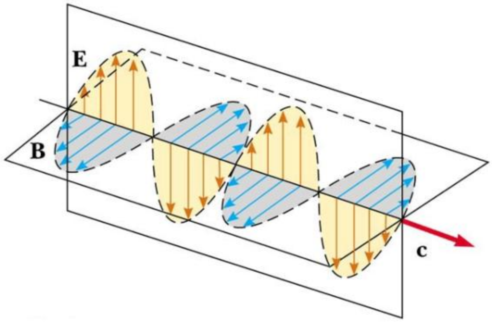
Radar absorbent material:

F-117A Nighthawk, nicknamed "The Black Jet", is the world's first operational aircraft completely designed on **stealth technology**



Common use:

Introduction to Electromagnetic Fields



Electromagnetics is the study of the effect of charges at rest and charges in motion.

Electromagnetics

Special cases

Electro-statics

Magneto-statics

Electro-magnetic waves

Geometric Optics

Statics: $\frac{\partial}{\partial t} \equiv 0$

Transmission Line Theory

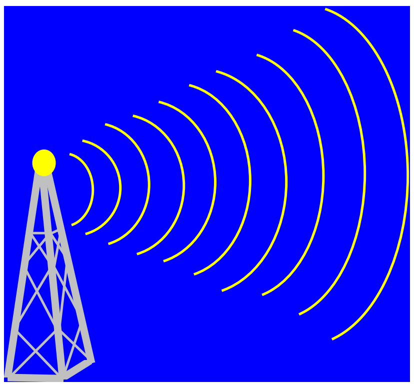
Kirchoff's Laws

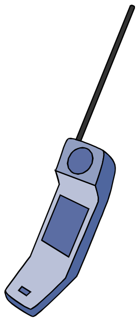
$d << \lambda$

Input from other disciplines

Circuit Theory

Introduction to Electromagnetic Fields



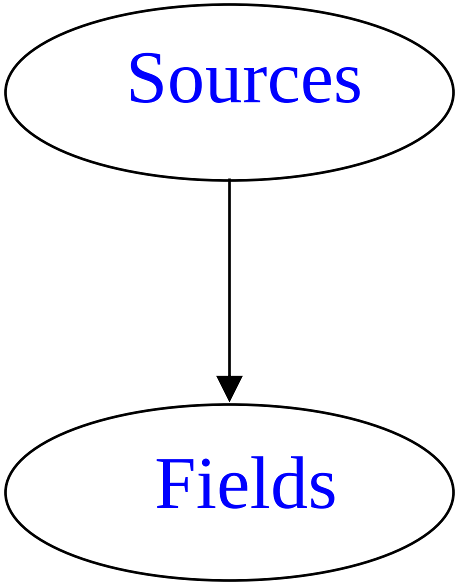


- transmitter and receiver are connected by a “field.”

Introduction to Electromagnetic Fields

- When an event in one place has an effect on something at a different location, we talk about the events as being connected by a "field".
- A ***field*** is a spatial distribution of a quantity; in general, it can be either *scalar* or *vector* in nature.

Introduction to Electromagnetic Fields



Conservation of electric charge

$$\rho_v = \lim_{\Delta v \to 0} \frac{\Delta q}{\Delta v} \quad \mathrm{C/m^3}$$

Introduction to Electromagnetic Fields

- Fundamental vector field quantities in electromagnetics
  - Electric field intensity (**E**)

    Volts/meter
  - Electric flux density (Electric Displacement) (**D**)

    Coulombs / meter^2
  - Magnetic flux density (**B**)

    Tesla = Webers / meter^2
  - Magnetic field intensity (**H**)

    Amps per meter (A/m)

Introduction to Electromagnetic Fields

- In time-varying electromagnetics, we consider ***E*** and ***H*** to be the "primary" responses, and attempt to write the "secondary" responses ***D*** and ***B***, in terms of ***E*** and ***H***.
- The relationships between the "primary" and "secondary" responses depends on the *medium* in which the field exists.
- The relationships between the "primary" and "secondary" responses are called ***constitutive relationships***.

Introduction to Electromagnetic Fields

- Universal constants in electromagnetics:
  - Velocity of an electromagnetic wave (e.g., light) in free space (perfect vacuum)

$$c \approx 3\times 10^{8} \text{ m/s}$$

  - Permeability of free space $\mu_0 = 4\pi\times 10^{-7} \text{ H/m}$
  - Permittivity of free space: $\varepsilon_0 \approx 8.854\times 10^{-12} \text{ F/m}$
  - Intrinsic impedance of free space: $\eta_0 \approx 120\pi\,\Omega$

Introduction to Electromagnetic Fields

- Relationships involving the universal constants:

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \qquad \eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}$$

Course Plan

- Total Lectures: 40
- Broad Topics:
  - Mathematical Tools for electromagnetics
  - Electrostatics
  - Magnetostatics
  - Maxwell's Equations/Time varying Fields
  - Plane EM Wave
  - Transmission lines

Mathematical tools for Electromagnetics

- A physical quantity that can be completely described by its magnitude is called a ***scalar.***

  Examples: voltage, current, charge and energy

- A physical quantity having magnitude as well as direction is called a ***vector***.

  Examples: electric and magnetic fields

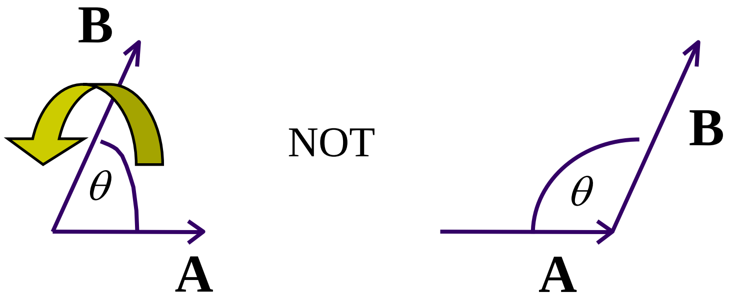
Vector operations

- Vector representation/notation
- Unit vector: Vector with unit magnitude .
- Vector addition:
  - Sum of two vectors is a vector
  - Commutative Law: **A** + **B** = **B** + **A**
  - Associative law: **A** + (**B** + **C**) = (**A** + **B**) + **C**
- Vector Subtraction
- Position and Distance Vectors

Vector multiplication

- Dot Product

$$\mathbf{A}\bullet\mathbf{B} = |\mathbf{A}|\,|\mathbf{B}|\cos\theta$$



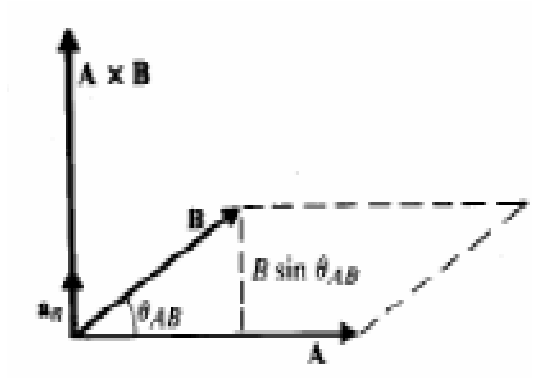
Note cos(90°) = 0, so for perpendicular vectors the dot product is zero.

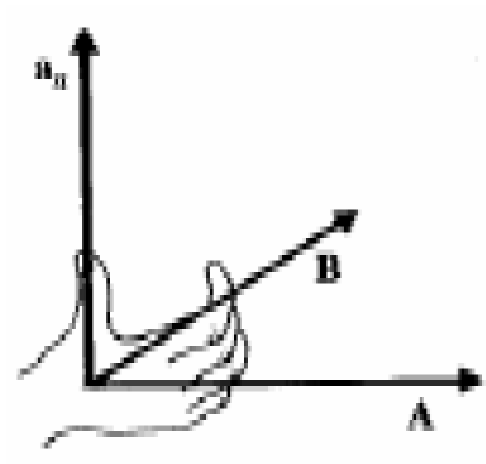
Commutative Law: **A.B** = **B.A**

Distributive Law: **A.**(**B** + **C**) = **A.B** + **A.C**

A.A = A^2 (cos 0° = 1)

Cross Product





Cross product: $|\mathbf{A}\text{x}\mathbf{B}| = \hat{a}_n \left|AB \sin\theta\right|$

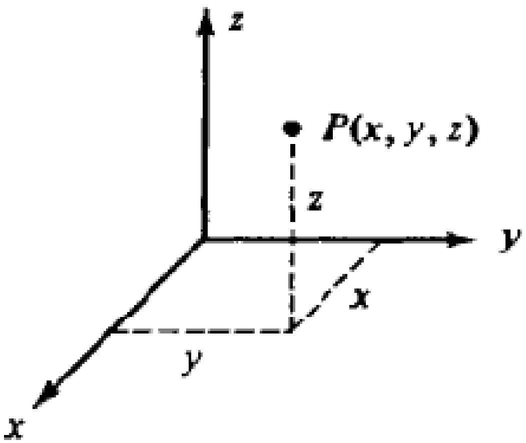
Not Commutative Law

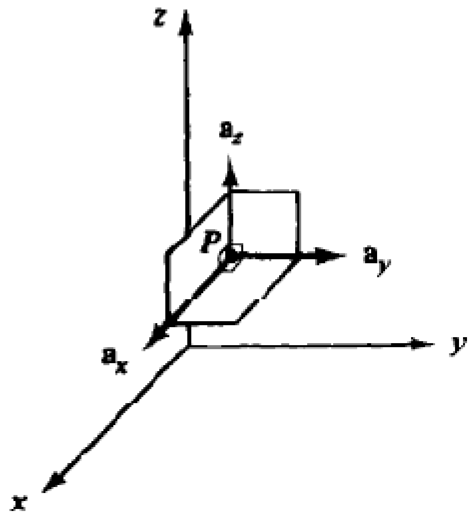
Distributive Law: $\mathbf{A} \mathbf{X} (\mathbf{B} + \mathbf{C}) = \mathbf{A} \mathbf{X} \mathbf{B} + \mathbf{A} \mathbf{X} \mathbf{C}$

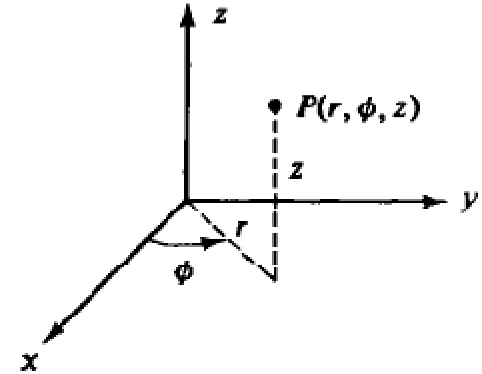
Not Associative

Coordinate Systems

Lecture - 2

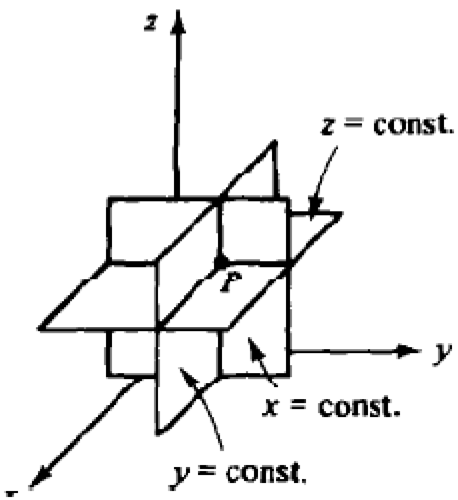


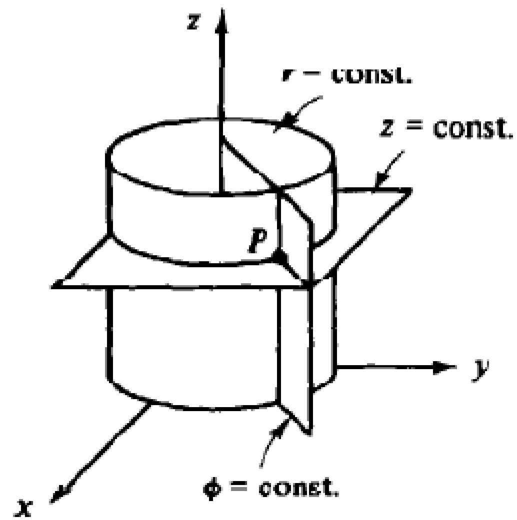


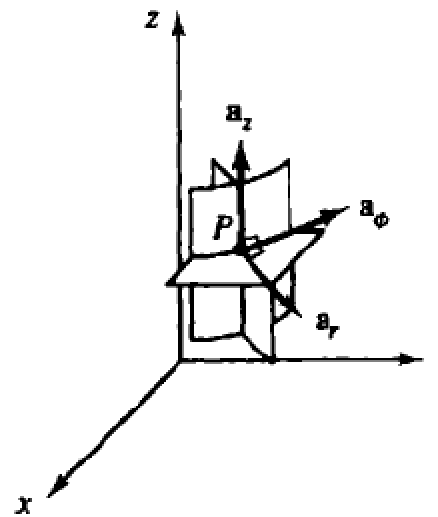


Cartesian

Cylindrical

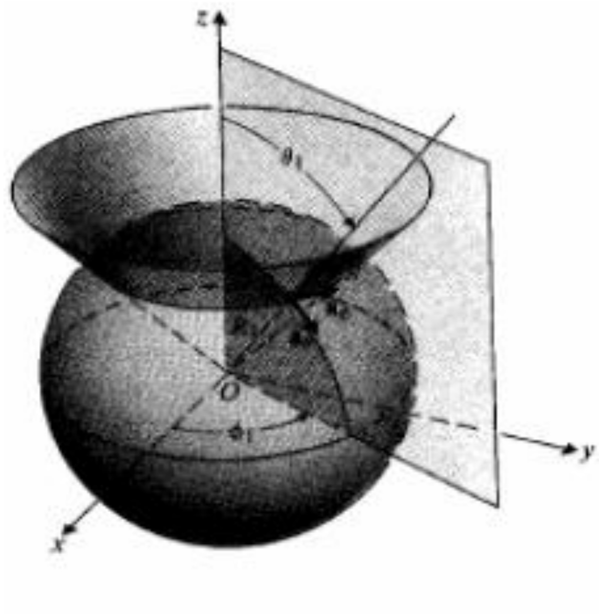






Coordinate Systems

Spherical

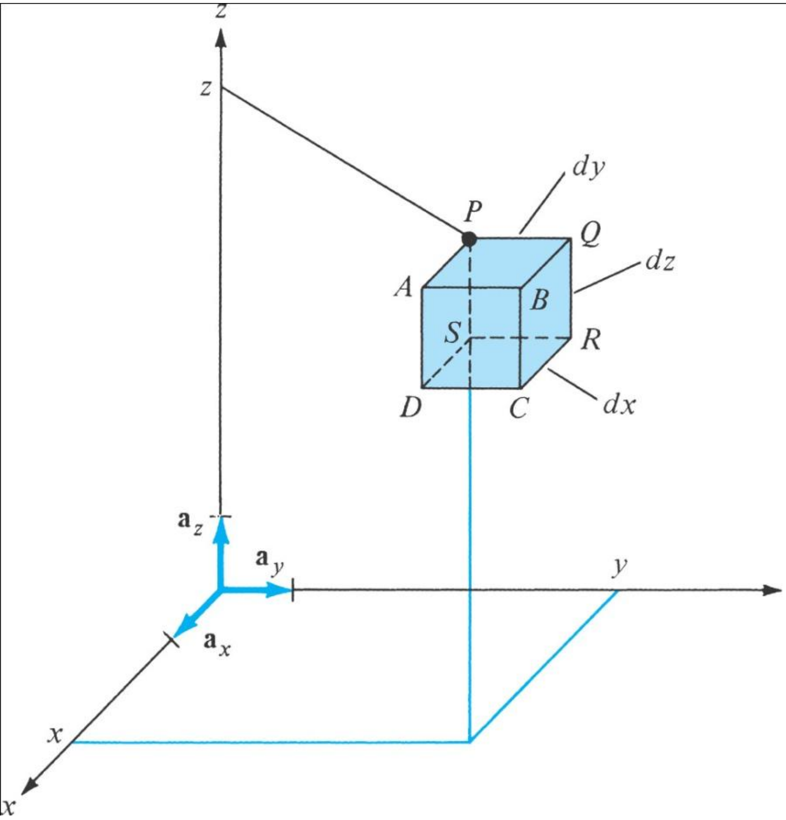


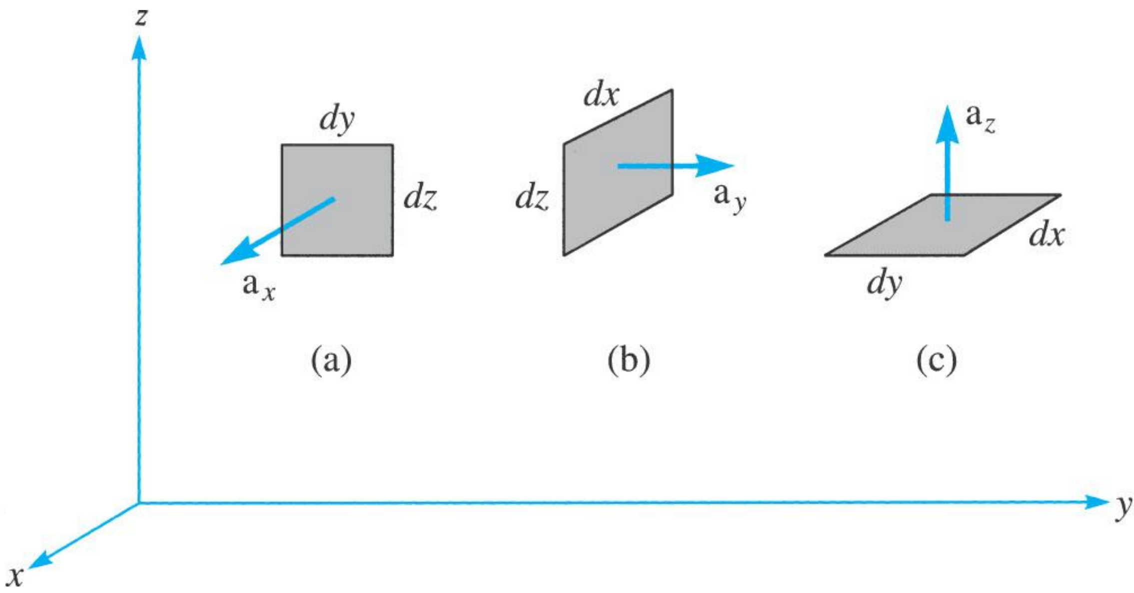
$$\boxed{\mathbf{A} = \mathbf{a}_R A_R + \mathbf{a}_\theta A_\theta + \mathbf{a}_\phi A_\phi .}$$

$$\mathbf{a}_R \times \mathbf{a}_\theta = \mathbf{a}_\phi$$
$$\mathbf{a}_\theta \times \mathbf{a}_\phi = \mathbf{a}_R,$$
$$\mathbf{a}_\phi \times \mathbf{a}_R = \mathbf{a}_\theta.$$

Differential Length, Area and Volume

Cartesian System

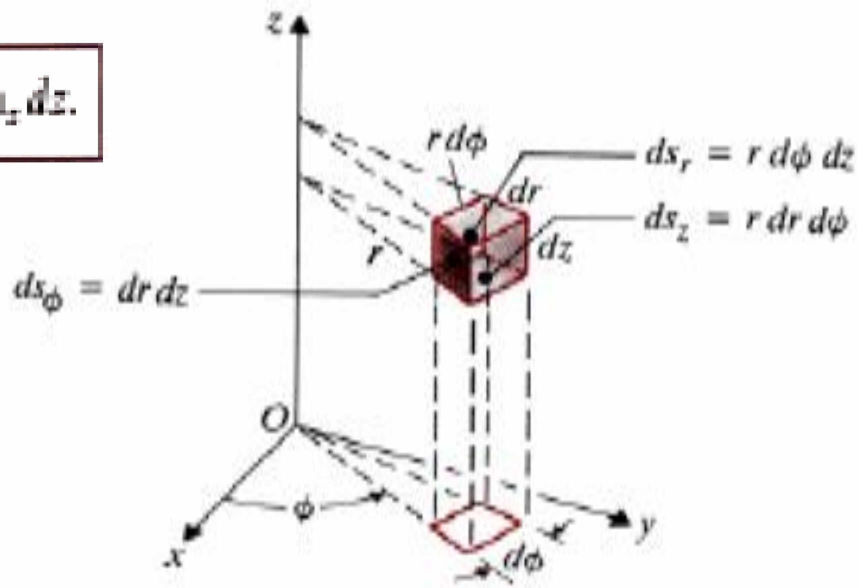




Differential Length, Area and Volume

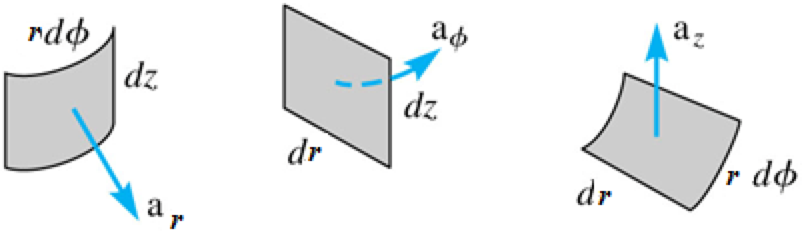
Cylindrical System

$$d\ell = \mathbf{a}_r\,dr + \mathbf{a}_\phi\,r\,d\phi + \mathbf{a}_z\,dz.$$



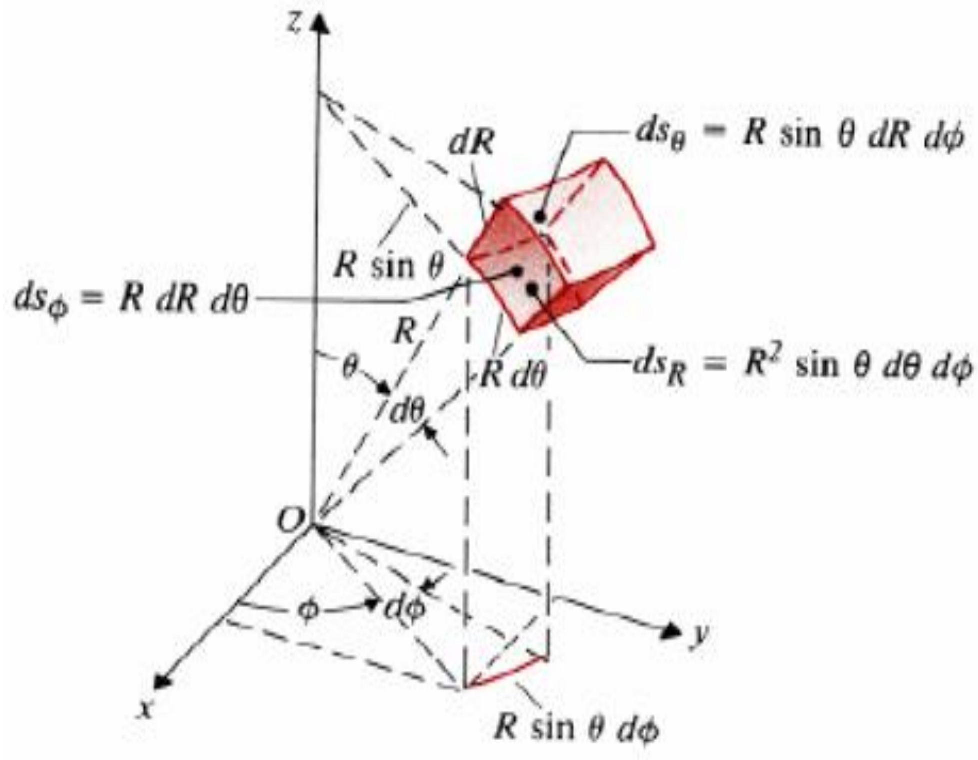
$$ds_r = r\,d\phi\,dz,$$
$$ds_\phi = dr\,dz,$$
$$ds_z = r\,dr\,d\phi,$$

$$dv = r\,dr\,d\phi\,dz.$$



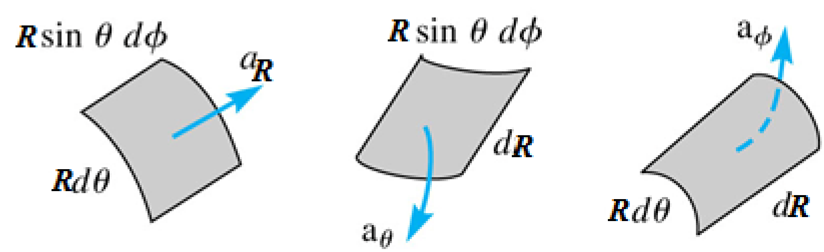
Differential Length, Area and Volume

Spherical System



$$ds_R = R^2 \sin\theta \, d\theta \, d\phi,$$
$$ds_\theta = R \sin\theta \, dR \, d\phi,$$
$$ds_\phi = R \, dR \, d\theta,$$

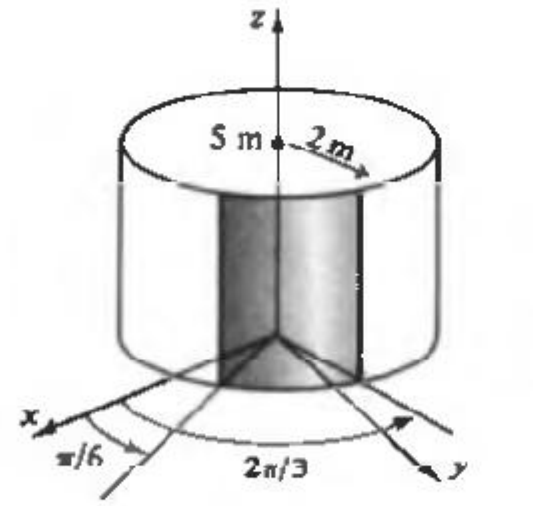
$$dv = R^2 \sin\theta \, dR \, d\theta \, d\phi.$$



Example-1

Use cylindrical coordinate system to find the area of the curved surface of a right circular cylinder, where r = 2 m, h = 5 m,

$$30^0 \le \phi \le 120^0$$



Line, Surface and Volume Integral

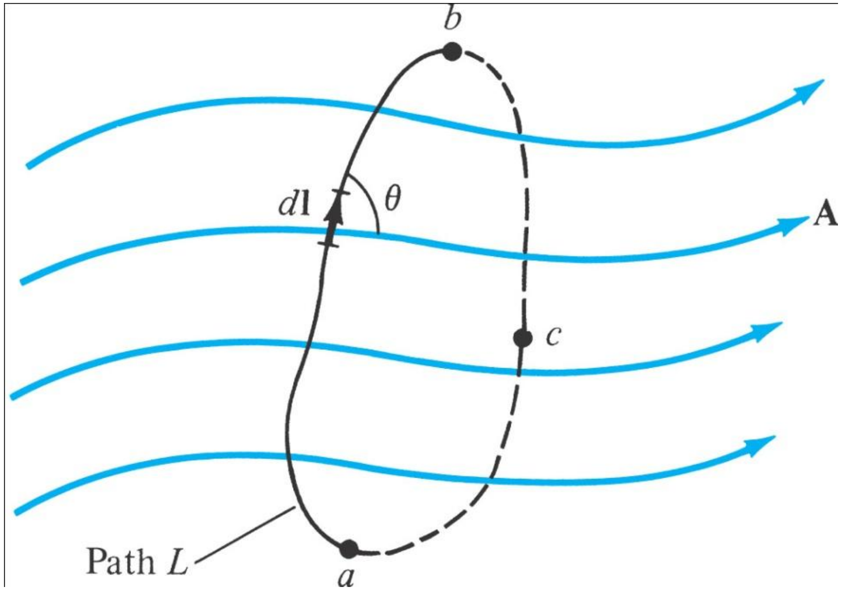


Figure Path of integration of vector field **A**.

Line, Surface and Volume Integral

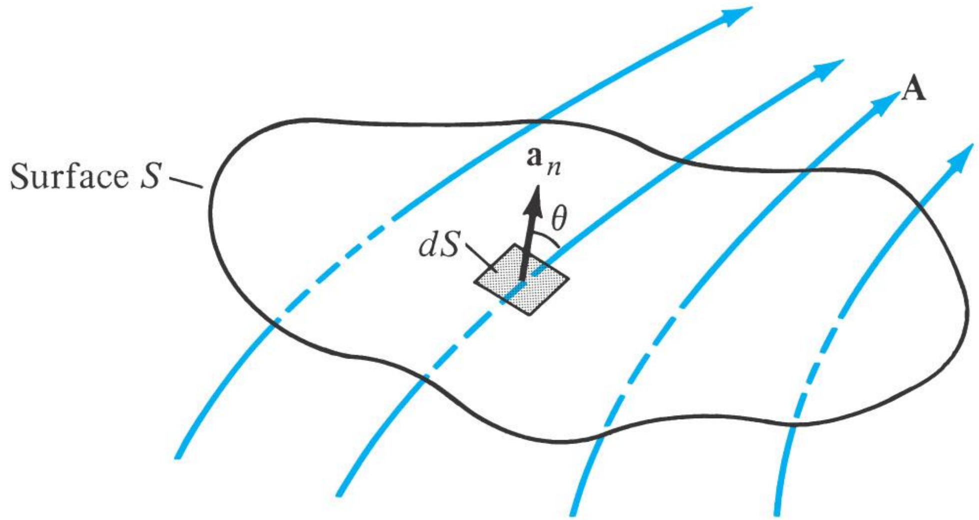
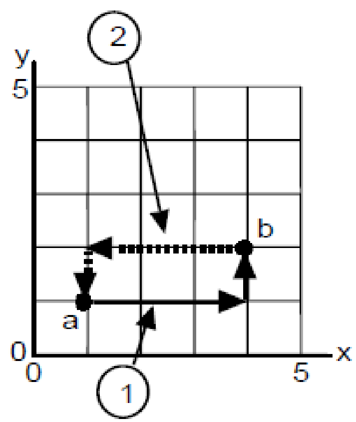


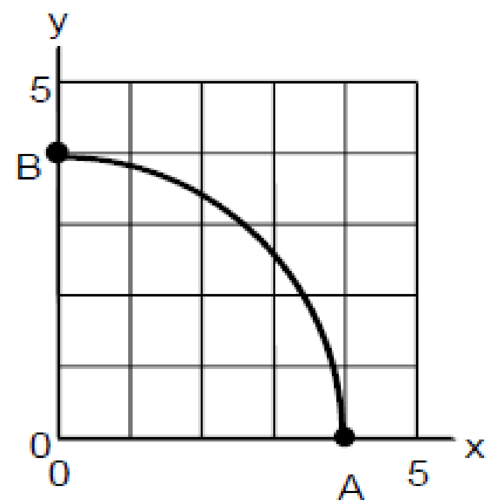
Figure The flux of a vector field **A** through surface *S*.

Example:

- **Concept of line-integral and Conservative Fields:**
- Calculate the work required to move the cart along the path 1 indicated in Figure 1 against a force field **F** where **F** = 3xy $\mathbf{a}_x$ + 4xy $\mathbf{a}_y$.
- Calculate the work ΔW required to move the cart along the closed path (Figure-1) if the force field is **F** = 3$\mathbf{a}_x$ + 4$\mathbf{a}_y$.

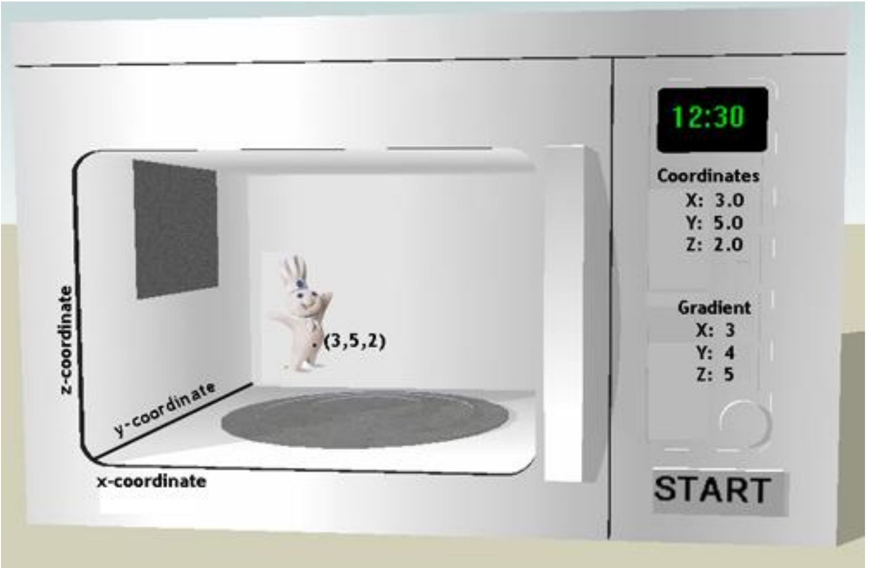
  Calculate the work ΔW required to move the cart along the circular path from point A to point B (Figure-2) if the force field is **F** = 3xy $\mathbf{a}_x$ + 4x $\mathbf{a}_y$.





Gradient of a scalar field

- Represents both the magnitude and the direction of the maximum space rate of increase of *V*.



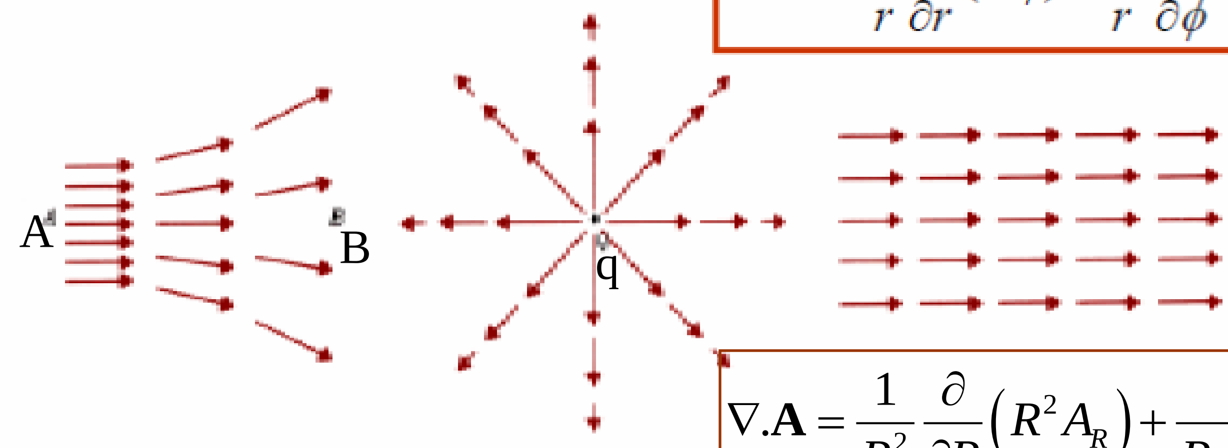
Divergence of a vector field

The net outward flux per unit volume as volume shrinks to zero

$$\text{div}\,\mathbf{A} \triangleq \lim_{\Delta v \to 0} \frac{\oint_S \mathbf{A} \cdot d\mathbf{s}}{\Delta v}.$$

$$\text{div}\,\mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}.$$

$$\nabla \cdot \bar{A} = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{1}{r}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$



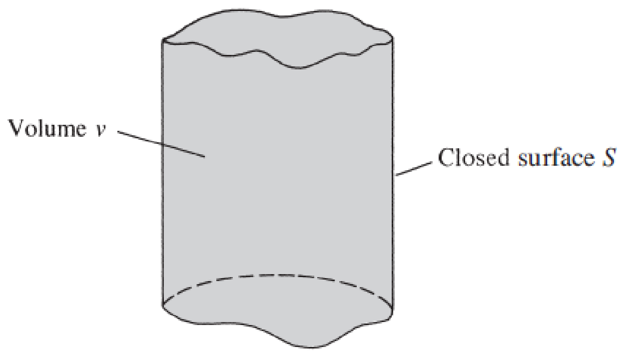
$$\nabla.\mathbf{A} = \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2 A_R\right) + \frac{1}{R\sin\theta}\frac{\partial}{\partial \theta}\left(A_\theta \sin\theta\right) + \frac{1}{R\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

Divergence Theorem

$$\boxed{\operatorname{div} \mathbf{A} \triangleq \lim_{\Delta v \to 0} \frac{\oint_S \mathbf{A} \cdot d\mathbf{s}}{\Delta v}.}$$

$$\boxed{\int_V \nabla \cdot \mathbf{A}\, dv = \oint_S \mathbf{A} \cdot d\mathbf{s}.}$$

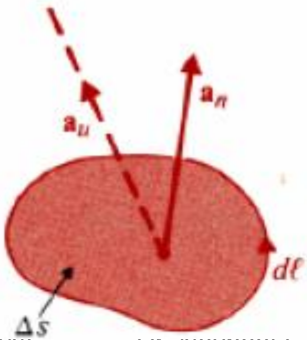
The volume integral of the divergence of a vector field equals the total outward flux of the vector through the surface that bound the volume



Curl of a vector field

$$\text{curl } \mathbf{A} \equiv \nabla \times \mathbf{A}$$
$$\triangleq \lim_{\Delta s \to 0} \frac{1}{\Delta s} \left[\mathbf{a}_n \oint_C \mathbf{A} \cdot d\boldsymbol{\ell} \right]_{\max}.$$

The **curl** of a vector field **A**, denoted by curl **A** or $\nabla \times \bar{A}$, is a vector whose magnitude is the maximum net circulation of **A** per unit area as the area tends to zero and whose direction is the normal direction of the area when the area is oriented to make the net circulation maximum.



In Cartesian coordinate system

$$\boxed{\nabla \times \mathbf{A} = \mathbf{a}_x\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \mathbf{a}_y\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \mathbf{a}_z\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right).}$$

or

$$\boxed{\nabla \times \mathbf{A} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}.}$$

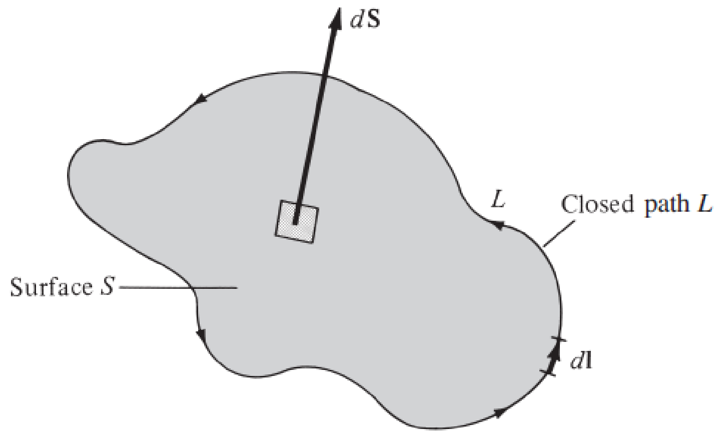
In Cylindrical coordinate system

$$\boxed{\nabla \times \mathbf{A} = \frac{1}{r}\begin{vmatrix} \mathbf{a}_r & \mathbf{a}_\phi r & \mathbf{a}_z \\ \dfrac{\partial}{\partial r} & \dfrac{\partial}{\partial \phi} & \dfrac{\partial}{\partial z} \\ A_r & rA_\phi & A_z \end{vmatrix},}$$

In Spherical coordinate system

$$\nabla \times \mathbf{A} = \frac{1}{R^2 \sin\theta} \begin{vmatrix} \mathbf{a}_R & \mathbf{a}_\theta R & \mathbf{a}_\phi R \sin\theta \\ \dfrac{\partial}{\partial R} & \dfrac{\partial}{\partial \theta} & \dfrac{\partial}{\partial \phi} \\ A_R & RA_\theta & R\sin\theta A_\phi \end{vmatrix},$$

Stokes's Theorem

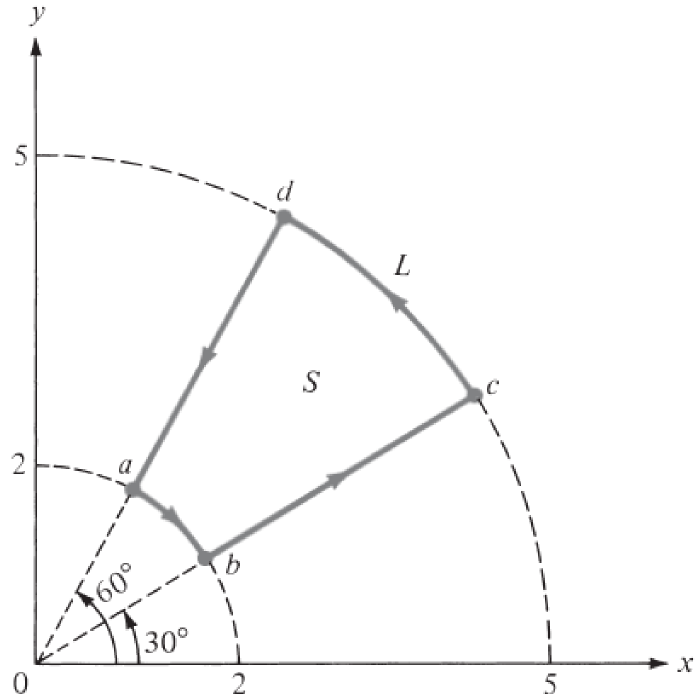


$$\int_S (\nabla \times \mathbf{A}) \cdot d\mathbf{s} = \oint_C \mathbf{A} \cdot d\boldsymbol{\ell},$$

which states that ***the surface integral of the curl of a vector field over an open surface is equal to the closed line integral of the vector along the contour bounding the surface.***

Example

$$\vec{A} = r\cos\phi\,\hat{a}_r + \sin\phi\,\hat{a}_\phi$$



Laplacian

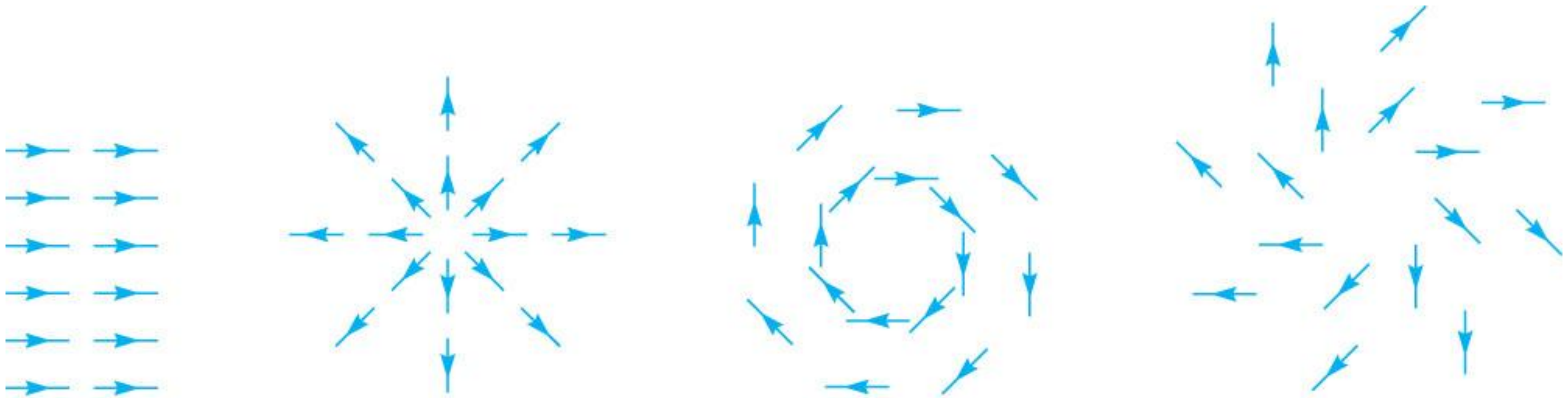
- Laplacian of a scalar: divergence of gradient of V

- Laplacian of a vector:

$$\nabla^2 \vec{A} = \nabla\left(\nabla . \vec{A}\right) - \nabla \times \nabla \times \vec{A}$$

Classification of Fields

A divergenceless field is **solenoidal** and a curl-free field is **irrotational**



(a) (b) (c) (d)

Two Null Identities

Identity 1

$$\nabla \times (\nabla V) \equiv 0$$

The curl of the gradient of any scalar field is identically zero.

This is because

$$\int_S [\nabla \times (\nabla V)] \cdot ds \overset{\text{Stokes's Theorem}}{=} \oint_C (\nabla V) \cdot dl \overset{dV = (\nabla V) \cdot dl}{=} \oint_C dV = 0$$

If a vector field is curl-free, then it can be expressed as the gradient of a scalar field. Let a vector field be $\boldsymbol{E}$, then if $\nabla \times \mathbf{E} = 0$ We can define a scalar field V such that

$$\mathbf{E} = -\nabla V$$

Note that the negative sign here is unimportant as far as Identity 1 is concerned.

Two Null Identities

Identity 2

$$\nabla \cdot (\nabla \times \mathbf{A}) \equiv 0$$

The divergence of the curl of any vector field is identically zero.

$$\int_V [\nabla \cdot (\nabla \times \mathbf{A})] \cdot dv \overset{\text{Divergence Theorem}}{=} \oint_S (\nabla \times \mathbf{A}) \cdot ds = \int_{S_1} (\nabla \times \mathbf{A}) \cdot \mathbf{a}_{n1} ds + \int_{S_2} (\nabla \times \mathbf{A}) \cdot \mathbf{a}_{n2} ds$$

$$= \oint_{C_1} \mathbf{A} \cdot dl + \oint_{C_2} \mathbf{A} \cdot dl = 0$$

If a vector field is divergenceless, then it can be expressed as the curl of another vector field. Let a vector field be ***B***, then if $\nabla \cdot \mathbf{B} = 0$ We can define a vector field ***A*** such that

$$\mathbf{B} = \nabla \times \mathbf{A}$$

It will be studied in later chapter that if magnetic flux density ***B*** is solenoidal, ***B*** is called magnetic vector potential ***A***.

Static Electric Fields

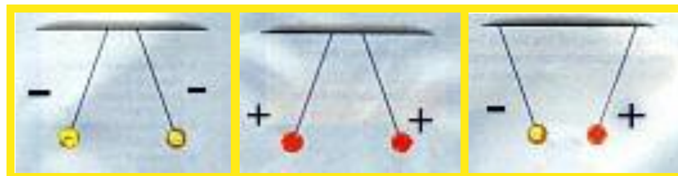
- Importance of study (in terms of its applications)
  - Electrical and Electronic devices
  - Computer peripheral devices
  - Medical devices
  - Industry
- Fundamental laws:
  - Coulomb’s Law
  - Gauss’s Law

(both these laws are based on experimental studies and are interdependent)

Coulomb's Law

Coulomb's law describes the interaction between bodies due to their charges

$$F = \frac{kQ_1Q_2}{R^2}$$

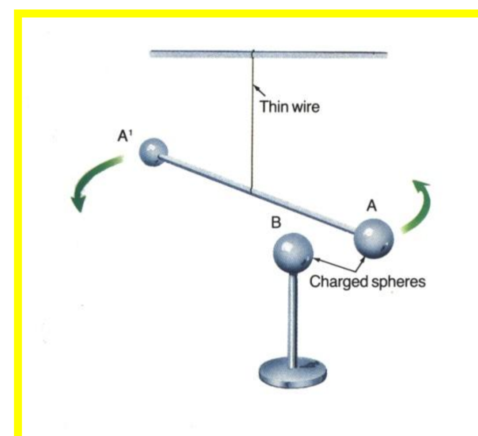


$$k = (4\pi\varepsilon_0)^{-1} = 9.0 \text{ x } 10^9 \text{ Nm}^2/\text{C}^2$$

ε_0 = permittivity of free space

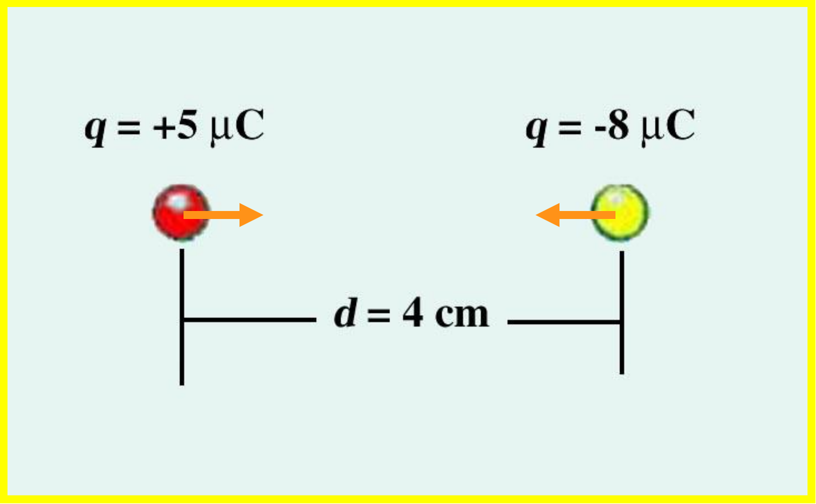
$= 8.854 \text{ x } 10^{-12}\, \text{C}^2/\text{Nm}^2$

$$\vec{F}_{12} = \frac{Q_1Q_2}{4\pi\varepsilon_0 R^2}\hat{a}_{R_{12}}$$



The Electrostatic Force

EXAMPLE 1 - Find the force between these two charges

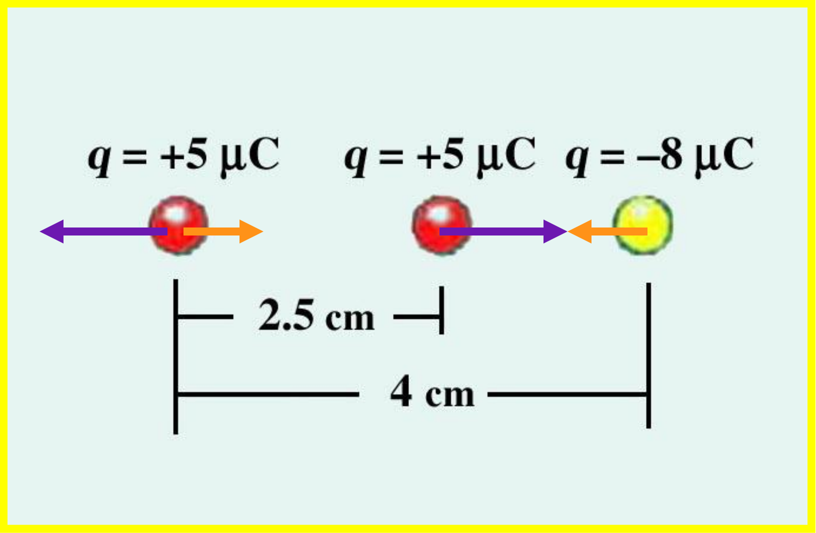


$$F_e = \frac{(9.0\times10^9)(5\times10^{-6}\ \text{C})(-8\times10^{-6}\ \text{C})}{(0.04\ \text{m})^2}$$

$$F_e = -225\ \text{N}$$

The negative signs means force of attraction.

EXAMPLE 2 - Find the net force on the left charge



$$F_e = \frac{(9.0\times10^9)(5\times10^{-6}\ \text{C})(5\times10^{-6}\ \text{C})}{(0.025\ \text{m})^2}$$

$$F_e = 360\ \text{N} \qquad \text{(force of repulsion)}$$

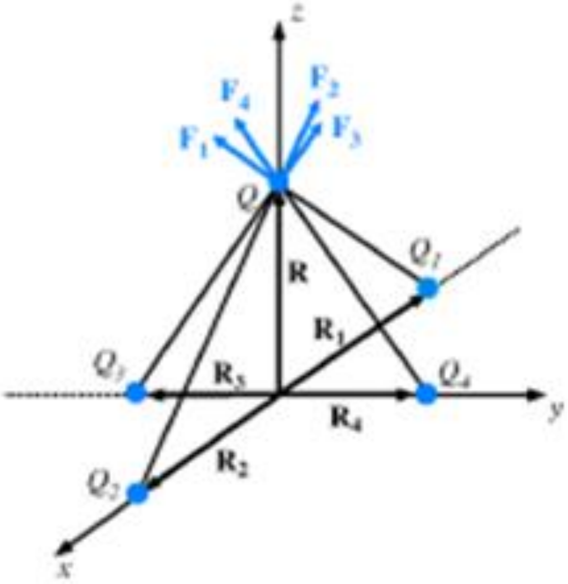
$$F_{net} = F_{left} - F_{right}$$

$$F_{net} = 360\ \text{N} - 225\ \text{N} = 135\ \text{N}, \text{ to the left}$$

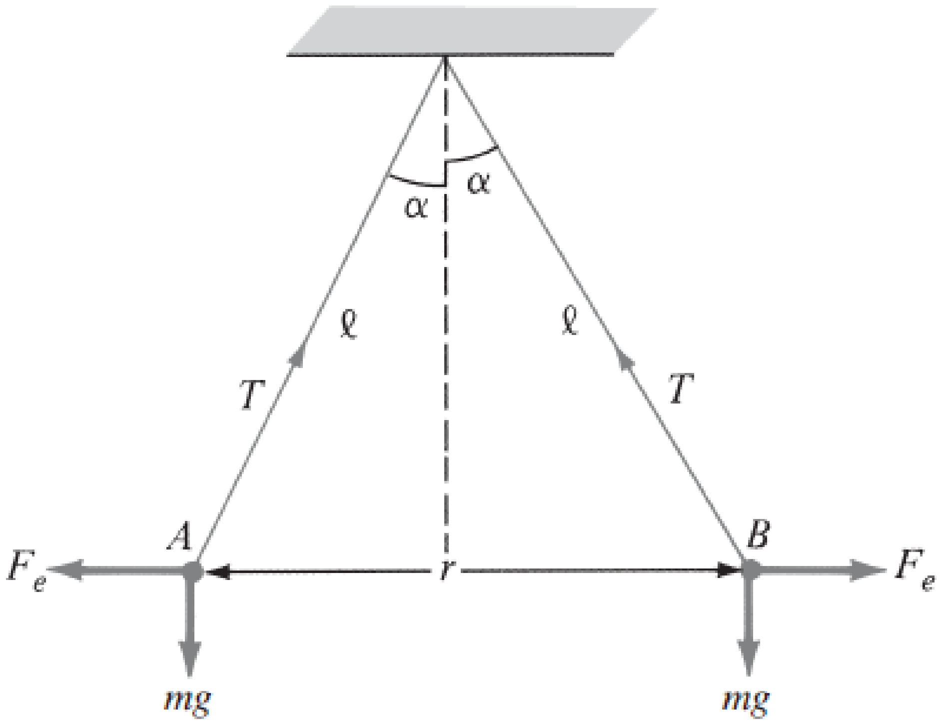
- For more than two point charges→ use principle of superposition

$$\vec{F} = \frac{Q}{4\pi\varepsilon_0} \sum_{k=1}^{N} \frac{Q_k(\vec{r} - \vec{r}_k)}{\left|\vec{r} - \vec{r}_k\right|^3}$$

- Four charges of 10 mC each are located in free space at points with Cartesian coordinates (-3;0;0), (3;0;0), (0;-3;0), and (0;3;0). Find the force on a 20-mC charge located at (0;0;4). All distances are in meters.



Example:

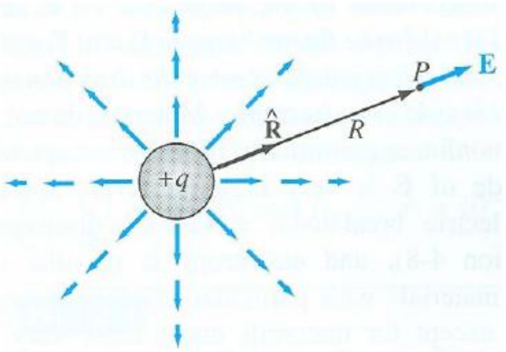


Electric Field Intensity

Electric Field Intensity = **F**/Q

1. An isolated charge q induces an electric field $\mathbf{E}$ at every point in space and is given by

$$\mathbf{E} = \mathbf{a}_R \frac{q}{4\pi\varepsilon R^2} \qquad V/m$$



The expression for the electric field due to a single charge can be extended to find the field due to multiple point charges as well as continuous charge distributions.

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \sum_{k=1}^{N} \frac{Q_k(\vec{r} - \vec{r}_k)}{\left|\vec{r} - \vec{r}_k\right|^3}$$

Practical Application:

Electrostatic separation of solids