Quine-McCluskey Method

K-Maps Difficult in Visualization for six variables and above

Not software adaptable

Quine-McCluskey Method
An Example

1. Find all the prime implicants

$$f(a,b,c,d) = \sum m(0,1,2,5,6,7,8,9,10,14)$$

group 0	0	0000
group 1	1	0001
	2	0010
	8	1000
group 2	5	0101
	6	0110
	9	1001
	10	1010
group 3	7	0111
	14	1110

Group the minterms according to the number of 1s in the minterm.

This way we only have to compare minterms from adjacent groups.

Quine-McCluskey Method
An Example

Column I | Column II

Combining group 0 and group 1:

group 0	0	0000
group 1	1	0001
	2	0010
	8	1000
group 2	5	0101
	6	0110
	9	1001
	10	1010
group 3	7	0111
	14	1110

Quine-McCluskey Method
An Example

	Column I		Column II
group 0	0	0000	0,1 000-
group 1	1	0001	
	2	0010	
	8	1000	
group 2	5	0101	
	6	0110	
	9	1001	
	10	1010	
group 3	7	0111	
	14	1110	

Combining group 0 and group 1:

Quine-McCluskey Method
An Example

	Column I		Column II
group 0	0	0000	0,1 000-
group 1	1	0001	0,2 00-0
	2	0010	
	8	1000	
group 2	5	0101	
	6	0110	
	9	1001	
	10	1010	
group 3	7	0111	
	14	1110	

Combining group 0 and group 1:

Quine-McCluskey Method
An Example

Does it make sense to combine group 0 with group 2 or 3?

No, there are at least two bits that are different.

	Column I		Column II	
group 0	0	0000	0,1	000-
group 1	1	0001	0,2	00-0
	2	0010	0,8	-000
	8	1000		
group 2	5	0101		
	6	0110		
	9	1001		
	10	1010		
group 3	7	0111		
	14	1110		

Quine-McCluskey Method
An Example

Does it make sense to no combine group 0 with group 2 or 3?

No, there are at least two bits that are different.

Thus, next we combine group 1 and group 2.

	Column I		Column II	
group 0	0	0000	0,1	000-
group 1	1	0001	0,2	00-0
	2	0010	0,8	-000
	8	1000		
group 2	5	0101		
	6	0110		
	9	1001		
	10	1010		
group 3	7	0111		
	14	1110		

Quine-McCluskey Method
An Example

	Column I		Column II
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	8	1000	1,5 0-01
group 2	5	0101	
	6	0110	
	9	1001	
	10	1010	
group 3	7	0111	
	14	1110	

Combine group 1 and group 2.

Quine-McCluskey Method
An Example

	Column I		Column II
group 0	0	0000	0,1 000-
group 1	1	0001	0,2 00-0
	2	0010	0,8 -000
	8	1000	1,5 0-01
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	6	0110	
	9	1001	
	10	1010	
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Combine group 1 and group 2.

Quine-McCluskey Method
An Example

	Column I		Column II
group 0	0	0000	0,1 000-
group 1	1	0001	0,2 00-0
	2	0010	0,8 -000
	8	1000	1,5 0-01
group 2	5	0101	1,9 -001
	6	0110	
	9	1001	
	10	1010	
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Combine group 1 and group 2.

Quine-McCluskey Method
An Example

	Column I		Column II
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Combine group 1 and group 2.

Quine-McCluskey Method
An Example

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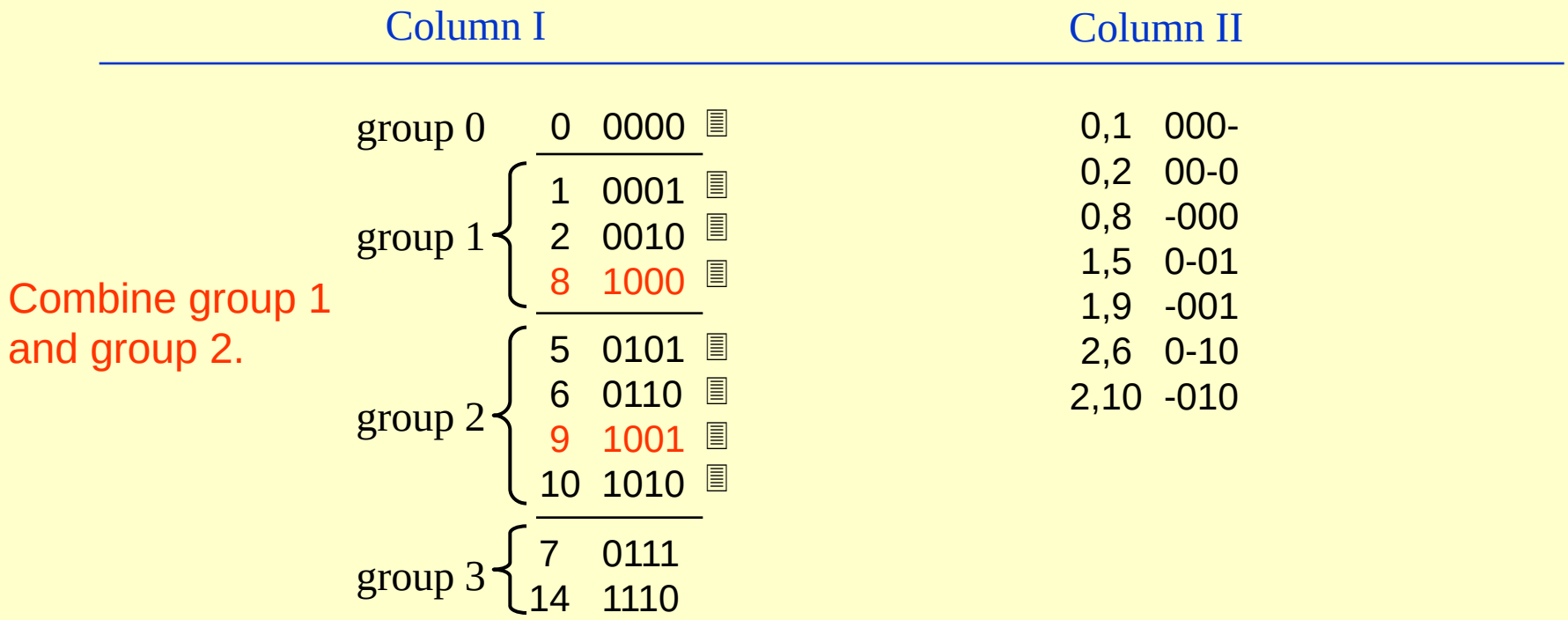
	6	0110	2,6	0-10
	9	1001	2,10	-010
	10	1010		
group 3	7	0111		
	14	1110		

Quine-McCluskey Method
An Example



Quine-McCluskey Method
An Example

Combine group 1 and group 2.

	Column I		Column II	
group 0	0	0000	0,1	000-
group 1	1	0001	0,2	00-0
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group 3	7	0111		
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	Column I		Column II	
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			0,2	00-0
group 1	1	0001	0,8	-000
	2	0010	1,5	0-01
	8	1000	1,9	-001
			2,6	0-10
group 2	5	0101	2,10	-010
	6	0110	8,9	100-
	9	1001	8,10	10-0
	10	1010		
group 3	7	0111		
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	Column I		Column II	
group 0	0	0000	0,1	000-
group 1	1	0001	0,2	00-0
	2	0010	0,8	-000
	8	1000	1,5	0-01
			1,9	-001
group 2	5	0101	2,6	0-10
	6	0110	2,10	-010
	9	1001	8,9	100-
	10	1010	8,10	10-0
group 3	7	0111		
	14	1110		

Lets try to combine group 2 with group 3.

Quine-McCluskey Method
An Example

	Column I		Column II
group 0	0	0000	0,1 000-
group 1	1	0001	0,2 00-0
	2	0010	0,8 -000
	8	1000	1,5 0-01
			1,9 -001
group 2	5	0101	2,6 0-10
	6	0110	2,10 -010
	9	1001	8,9 100-
	10	1010	8,10 10-0
			5,7 01-1
group 3	7	0111	
	14	1110	

Combine group 2 and group 3.

Quine-McCluskey Method
An Example

Combine group 2 and group 3.

	Column I		Column II	
group 0	0	0000	0,1	000-
group 1	1	0001	0,2	00-0
	2	0010	0,8	-000
	8	1000	1,5	0-01
group 2	5	0101	1,9	-001
	6	0110	2,6	0-10
	9	1001	2,10	-010
	10	1010	8,9	100-
group 3	7	0111	8,10	10-0
	14	1110	5,7	01-1

Quine-McCluskey Method
An Example

Combine group 2 and group 3.

	Column I		Column II	
group 0	0	0000	0,1	000-
group 1	1	0001	0,2	00-0
	2	0010	0,8	-000
	8	1000	1,5	0-01
			1,9	-001
group 2	5	0101	2,6	0-10
	6	0110	2,10	-010
	9	1001	8,9	100-
	10	1010	8,10	10-0
group 3	7	0111	5,7	01-1
	14	1110	6,7	011-

Quine-McCluskey Method
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Combine group 2 and group 3.

	Column I		Column II	
group 0	0	0000	0,1	000-
group 1	1	0001	0,2	00-0
	2	0010	0,8	-000
	8	1000	1,5	0-01
group 2	5	0101	1,9	-001
	6	0110	2,6	0-10
	9	1001	2,10	-010
	10	1010	8,9	100-
group 3	7	0111	8,10	10-0
	14	1110	5,7	01-1
			6,7	011-
			6,14	-110

Quine-McCluskey Method
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Combine group 2 and group 3.

	Column I		Column II	
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	6	0110	2,6	0-10
	9	1001	2,10	-010
	10	1010	8,9	100-
group 3	7	0111	8,10	10-0
	14	1110	5,7	01-1
			6,7	011-
			6,14	-110

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	6	0110	2,10 -010
	9	1001	8,9 100-
	10	1010	8,10 10-0
group 3	7	0111	5,7 01-1
	14	1110	6,7 011-
			6,14 -110

Combine group 2 and group 3.

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group 2	5	0101	2,6	0-10
	6	0110	2,10	-010
	9	1001	8,9	100-
	10	1010	8,10	10-0
group 3	7	0111	5,7	01-1
	14	1110	6,7	011-
			6,14	-110

Combine group 2 and group 3.

Quine-McCluskey Method
An Example

We have now completed the first step. All minterms in column I were included.

We can divide column II into groups.

	Column I		Column II
group 0	0	0000	0,1 000-
			0,2 00-0
group 1	1	0001	0,8 -000
	2	0010	1,5 0-01
	8	1000	1,9 -001
			2,6 0-10
group 2	5	0101	2,10 -010
	6	0110	8,9 100-
	9	1001	8,10 10-0
	10	1010	5,7 01-1
			6,7 011-
group 3	7	0111	6,14 -110
	14	1110	10,14 1-10

Quine-McCluskey Method
An Example

	Column I		Column II	
group 0	0	0000	0,1	000-
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group 1	1	0001	0,8	-000
	2	0010	1,5	0-01
	8	1000	1,9	-001
group 2	5	0101	2,6	0-10
	6	0110	2,10	-010
	9	1001	8,9	100-
	10	1010	8,10	10-0
group 3	7	0111	5,7	01-1
	14	1110	6,7	011-
			6,14	-110
			10,14	1-10

Quine-McCluskey Method
An Example

	Column I	Column II	Column III
group 0	0 0000	0,1 000-	
		0,2 00-0	
group 1	1 0001	0,8 -000	
	2 0010	1,5 0-01	
	8 1000	1,9 -001	
group 2	5 0101	2,6 0-10	
	6 0110	2,10 -010	
	9 1001	8,9 100-	
	10 1010	8,10 10-0	
group 3	7 0111	5,7 01-1	
	14 1110	6,7 011-	
		6,14 -110	
		10,14 1-10	

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	Column I	Column II	Column III
group 0	0 0000	0,1 000-	
		0,2 00-0	
group 1	1 0001	0,8 -000	
	2 0010	1,5 0-01	
	8 1000	1,9 -001	
group 2	5 0101	2,6 0-10	
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	9 1001	8,9 100-	
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	14 1110	6,7 011-	
		6,14 -110	
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	Column I		Column II		Column III
group 0	0	0000	0,1	000-	
			0,2	00-0	
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group 1	1	0001	1,5	0-01	
	2	0010	1,9	-001	
	8	1000	2,6	0-10	
			2,10	-010	
group 2	5	0101	8,9	100-	
	6	0110	8,10	10-0	
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group 3	7	0111	6,14	-110	
	14	1110	10,14	1-10	

Quine-McCluskey Method
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	Column I	Column II	Column III
group 0	0 0000	0,1 000-	
		0,2 00-0	
group 1	1 0001	0,8 -000	
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	9 1001	8,9 100-	
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	Column I	Column II	Column III
group 0	0 0000	0,1 000-	0,1,8,9 -00-
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	2 0010	0,8 -000	
	8 1000	1,5 0-01	
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	6 0110	2,6 0-10	
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		6,14 -110	
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	Column I		Column II		Column III	
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group 1	1	0001	0,2	00-0	0,2,8,10	-0-0
	2	0010	0,8	-000		
	8	1000	1,5	0-01		
group 2	5	0101	1,9	-001		
	6	0110	2,6	0-10		
	9	1001	2,10	-010		
	10	1010	8,9	100-		
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	14	1110	5,7	01-1		
			6,7	011-		
			6,14	-110		
			10,14	1-10		

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	Column I	Column II	Column III
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group 1	1 0001	0,2 00-0	0,2,8,10 -0-0
	2 0010	0,8 -000	0,8,1,9 -00-
	8 1000	1,5 0-01	
		1,9 -001	
group 2	5 0101	2,6 0-10	
	6 0110	2,10 -010	
	9 1001	8,9 100-	
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	Column I	Column II	Column III
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	2 0010	0,8 -000	0,8,1,9 -00-
	8 1000	1,5 0-01	
group 2	5 0101	1,9 -001	
	6 0110	2,6 0-10	
	9 1001	2,10 -010	
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		6,14 -110	
		10,14 1-10	

Quine-McCluskey Method
An Example

	Column I	Column II	Column III
group 0	0 0000	0,1 000-	0,1,8,9 -00-
group 1	1 0001	0,2 00-0	0,2,8,10 -0-0
	2 0010	0,8 -000	0,8,1,9 -00-
	8 1000	1,5 0-01	0,8,2,10 -0-0
group 2	5 0101	1,9 -001	
	6 0110	2,6 0-10	
	9 1001	2,10 -010	
	10 1010	8,9 100-	
group 3	7 0111	8,10 10-0	
	14 1110	5,7 01-1	
		6,7 011-	
		6,14 -110	
		10,14 1-10	

Quine-McCluskey Method
An Example

	Column I	Column II	Column III
group 0	0 0000	0,1 000-	0,1,8,9 -00-
group 1	1 0001	0,2 00-0	0,2,8,10 -0-0
	2 0010	0,8 -000	0,8,1,9 -00-
	8 1000	1,5 0-01	0,8,2,10 -0-0
group 2	5 0101	1,9 -001	
	6 0110	2,6 0-10	
	9 1001	2,10 -010	
	10 1010	8,9 100-	
group 3	7 0111	8,10 10-0	
	14 1110	5,7 01-1	
		6,7 011-	
		6,14 -110	
		10,14 1-10	

Quine-McCluskey Method
An Example

	Column I	Column II	Column III
group 0	0 0000	0,1 000-	0,1,8,9 -00-
group 1	1 0001	0,2 00-0	0,2,8,10 -0-0
	2 0010	0,8 -000	0,8,1,9 -00-
	8 1000	1,5 0-01	0,8,2,10 -0-0
group 2	5 0101	1,9 -001	
	6 0110	2,6 0-10	
	9 1001	2,10 -010	
	10 1010	8,9 100-	
group 3	7 0111	8,10 10-0	
	14 1110	5,7 01-1	
		6,7 011-	
		6,14 -110	
		10,14 1-10	

Quine-McCluskey Method
An Example

	Column I	Column II	Column III
group 0	0 0000	0,1 000-	0,1,8,9 -00-
group 1	1 0001	0,2 00-0	0,2,8,10 -0-0
	2 0010	0,8 -000	0,8,1,9 -00-
	8 1000	1,5 0-01	0,8,2,10 -0-0
group 2	5 0101	1,9 -001	
	6 0110	2,6 0-10	
	9 1001	2,10 -010	
	10 1010	8,9 100-	
group 3	7 0111	8,10 10-0	
	14 1110	5,7 01-1	
		6,7 011-	
		6,14 -110	
		10,14 1-10	

Quine-McCluskey Method
An Example

	Column I	Column II	Column III
group 0	0 0000	0,1 000-	0,1,8,9 -00-
group 1	1 0001	0,2 00-0	0,2,8,10 -0-0
	2 0010	0,8 -000	0,8,1,9 -00-
	8 1000	1,5 0-01	0,8,2,10 -0-0
group 2	5 0101	1,9 -001	
	6 0110	2,6 0-10	
	9 1001	2,10 -010	
	10 1010	8,9 100-	
group 3	7 0111	8,10 10-0	
	14 1110	5,7 01-1	
		6,7 011-	
		6,14 -110	
		10,14 1-10	

Quine-McCluskey Method
An Example

	Column I	Column II	Column III
group 0	0 0000	0,1 000-	0,1,8,9 -00-
group 1	1 0001	0,2 00-0	0,2,8,10 -0-0
	2 0010	0,8 -000	0,8,1,9 -00-
	8 1000	1,5 0-01	0,8,2,10 -0-0
group 2	5 0101	1,9 -001	2,6,10,14 --10
	6 0110	2,6 0-10	
	9 1001	2,10 -010	
	10 1010	8,9 100-	
group 3	7 0111	8,10 10-0	
	14 1110	5,7 01-1	
		6,7 011-	
		6,14 -110	
		10,14 1-10	

Quine-McCluskey Method
An Example

	Column I	Column II	Column III
group 0	0 0000	0,1 000-	0,1,8,9 -00-
		0,2 00-0	0,2,8,10 -0-0
group 1	1 0001	0,8 -000	0,8,1,9 -00-
	2 0010	1,5 0-01	0,8,2,10 -0-0
	8 1000	1,9 -001	2,6,10,14 --10
		2,6 0-10	2,10,6,14 --10
group 2	5 0101	2,10 -010	
	6 0110	8,9 100-	
	9 1001	8,10 10-0	
	10 1010	5,7 01-1	
group 3	7 0111	6,7 011-	
	14 1110	6,14 -110	
		10,14 1-10	

Quine-McCluskey Method
An Example

	Column I	Column II	Column III
group 0	0 0000	0,1 000-	0,1,8,9 -00-
		0,2 00-0	0,2,8,10 -0-0
group 1	1 0001	0,8 -000	0,8,1,9 -00-
	2 0010	1,5 0-01	0,8,2,10 -0-0
	8 1000	1,9 -001	2,6,10,14 --10
		2,6 0-10	2,10,6,14 --10
group 2	5 0101	2,10 -010	
	6 0110	8,9 100-	
	9 1001	8,10 10-0	
	10 1010	5,7 01-1	
group 3	7 0111	6,7 011-	
	14 1110	6,14 -110	
		10,14 1-10	

Quine-McCluskey Method
An Example

	Column I	Column II	Column III
group 0	0 0000	0,1 000-	0,1,8,9 -00-
		0,2 00-0	0,2,8,10 -0-0
		0,8 -000	0,8,1,9 -00-
group 1	1 0001	1,5 0-01	0,8,2,10 -0-0
	2 0010	1,9 -001	2,6,10,14 --10
	8 1000	2,6 0-10	2,10,6,14 --10
		2,10 -010	
group 2	5 0101	8,9 100-	
	6 0110	8,10 10-0	
	9 1001	5,7 01-1	
	10 1010	6,7 011-	
group 3	7 0111	6,14 -110	
	14 1110	10,14 1-10	

Quine-McCluskey Method
An Example

	Column I	Column II	Column III
group 0	0 0000	0,1 000-	0,1,8,9 -00-
group 1	1 0001	0,2 00-0	0,2,8,10 -0-0
	2 0010	0,8 -000	0,8,1,9 -00-
	8 1000	1,5 0-01	0,8,2,10 -0-0
group 2	5 0101	1,9 -001	2,6,10,14 --10
	6 0110	2,6 0-10	2,10,6,14 --10
	9 1001	2,10 -010	
	10 1010	8,9 100-	
group 3	7 0111	8,10 10-0	
	14 1110	5,7 01-1	
		6,7 011-	
		6,14 -110	
		10,14 1-10	

No more combinations are possible, thus we stop here.

Quine-McCluskey Method
An Example

	Column I	Column II	Column III
group 0	0 0000	0,1 000-	0,1,8,9 -00-
group 1	1 0001	0,2 00-0	0,2,8,10 -0-0
	2 0010	0,8 -000	~~0,8,1,9 -00-~~
	8 1000	1,5 0-01	~~0,8,2,10 -0-0~~
group 2	5 0101	1,9 -001	2,6,10,14 --10
	6 0110	2,6 0-10	~~2,10,6,14 --10~~
	9 1001	2,10 -010	
	10 1010	8,9 100-	
group 3	7 0111	8,10 10-0	
	14 1110	5,7 01-1	
		6,7 011-	
		6,14 -110	
		10,14 1-10	

We can eliminate repeated combinations

Quine-McCluskey Method
An Example

Column I		Column II		Column III	
group 0	0 0000	0,1	000-	0,1,8,9	-00-
group 1	1 0001	0,2	00-0	0,2,8,10	-0-0
	2 0010	0,8	-000	2,6,10,14	--10
	8 1000	1,5	0-01		
group 2	5 0101	1,9	-001		
	6 0110	2,6	0-10		
	9 1001	2,10	-010		
	10 1010	8,9	100-		
group 3	7 0111	8,10	10-0		
	14 1110	5,7	01-1		
		6,7	011-		
		6,14	-110		
		10,14	1-10		

Now we form f with the terms not checked

f = a'c'd

Quine-McCluskey Method
An Example

	Column I	Column II	Column III
group 0	0 0000	0,1 000-	0,1,8,9 -00-
group 1	1 0001	0,2 00-0	0,2,8,10 -0-0
	2 0010	0,8 -000	2,6,10,14 --10
	8 1000	1,5 0-01	
group 2	5 0101	1,9 -001	
	6 0110	2,6 0-10	
	9 1001	2,10 -010	
	10 1010	8,9 100-	
group 3	7 0111	8,10 10-0	
	14 1110	5,7 01-1 ←	
		6,7 011-	
		6,14 -110	
		10,14 1-10	

f = a'c'd + a'bd

Quine-McCluskey Method
An Example

Column I		Column II		Column III	
group 0	0 0000	0,1	000-	0,1,8,9	-00-
group 1	1 0001	0,2	00-0	0,2,8,10	-0-0
	2 0010	0,8	-000	2,6,10,14	--10
	8 1000	1,5	0-01		
group 2	5 0101	1,9	-001		
	6 0110	2,6	0-10		
	9 1001	2,10	-010		
	10 1010	8,9	100-		
group 3	7 0111	8,10	10-0		
	14 1110	5,7	01-1		
		6,7	011-		
		6,14	-110		
		10,14	1-10		

f = a'c'd + a'bd + a'bc

Quine-McCluskey Method
An Example

Column I		Column II		Column III	
group 0	0 0000	0,1	000-	0,1,8,9	-00-
group 1	1 0001	0,2	00-0	0,2,8,10	-0-0
	2 0010	0,8	-000	2,6,10,14	--10
	8 1000	1,5	0-01		
group 2	5 0101	1,9	-001		
	6 0110	2,6	0-10		
	9 1001	2,10	-010		
	10 1010	8,9	100-		
group 3	7 0111	8,10	10-0		
	14 1110	5,7	01-1		
		6,7	011-		
		6,14	-110		
		10,14	1-10		

f = a'c'd + a'bd + a'bc + b'c'

Quine-McCluskey Method
An Example

	Column I	Column II	Column III
group 0	0 0000	0,1 000-	0,1,8,9 -00-
group 1	1 0001	0,2 00-0	0,2,8,10 -0-0 ←
	2 0010	0,8 -000	2,6,10,14 --10
	8 1000	1,5 0-01	
group 2	5 0101	1,9 -001	
	6 0110	2,6 0-10	
	9 1001	2,10 -010	
	10 1010	8,9 100-	
group 3	7 0111	8,10 10-0	
	14 1110	5,7 01-1	
		6,7 011-	
		6,14 -110	
		10,14 1-10	

f = a'c'd + a'bd + a'bc + b'c'
+ b'd'

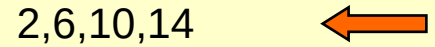
Quine-McCluskey Method
An Example

Column I		Column II		Column III	
group 0	0 0000	0,1	000-	0,1,8,9	-00-
group 1	1 0001	0,2	00-0	0,2,8,10	-0-0
	2 0010	0,8	-000	2,6,10,14	--10
	8 1000	1,5	0-01		
group 2	5 0101	1,9	-001		
	6 0110	2,6	0-10		
	9 1001	2,10	-010		
	10 1010	8,9	100-		
group 3	7 0111	8,10	10-0		
	14 1110	5,7	01-1		
		6,7	011-		
		6,14	-110		
		10,14	1-10		

f = a'c'd + a'bd + a'bc + b'c'
+ b'd' + cd'

Quine-McCluskey Method
An Example

Column I		Column II	Column III
group 0	0	0,1	0,1,8,9
group 1	1	0,2	0,2,8,10
	2	0,8	2,6,10,14
	8	1,5	
group 2	5	1,9	
	6	2,6	
	9	2,10	
	10	8,9	
group 3	7	8,10	
	14	5,7	
		6,7	
		6,14	
		10,14	

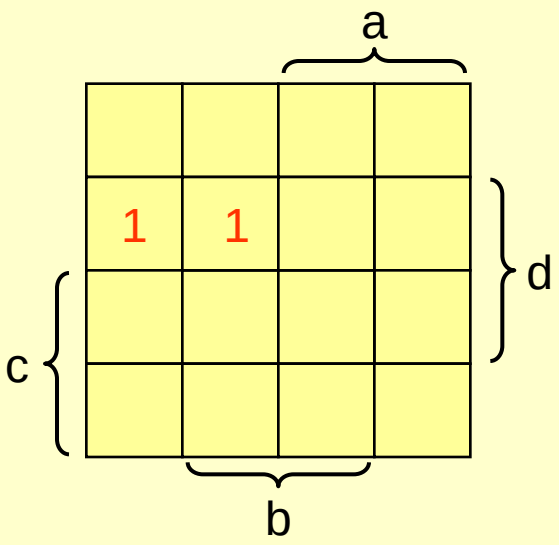


f = a'c'd + a'bd + a'bc + b'c'
+ b'd' + cd'

Quine-McCluskey Method
An Example

But, the form below is not minimized, using a Karnaugh map we can obtain:

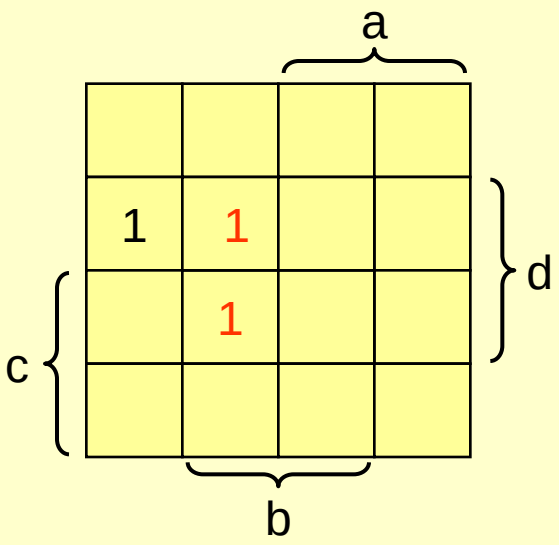
f = a'c'd + a'bd + a'bc + b'c' + b'd' + cd'



Quine-McCluskey Method
An Example

But, the form below is not minimized, using a Karnaugh map we can obtain:

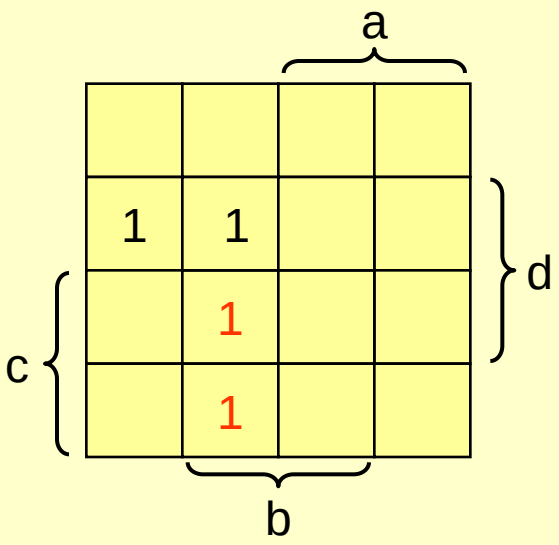
f = a'c'd + a'bd + a'bc + b'c' + b'd' + cd'



Quine-McCluskey Method
An Example

But, the form below is not minimized, using a Karnaugh map we can obtain:

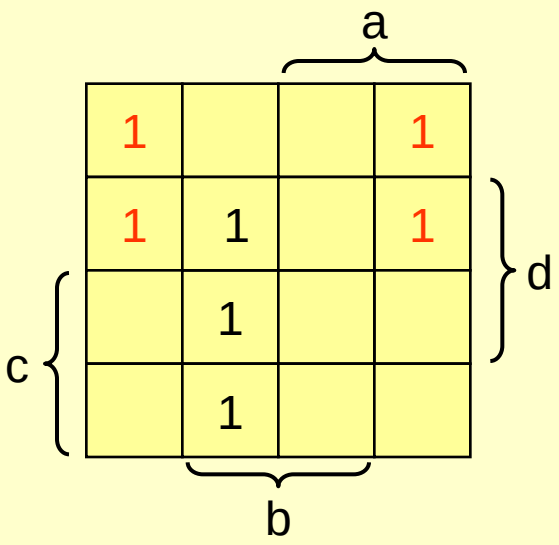
f = a'c'd + a'bd + a'bc + b'c' + b'd' + cd'



Quine-McCluskey Method
An Example

But, the form below is not minimized, using a Karnaugh map we can obtain:

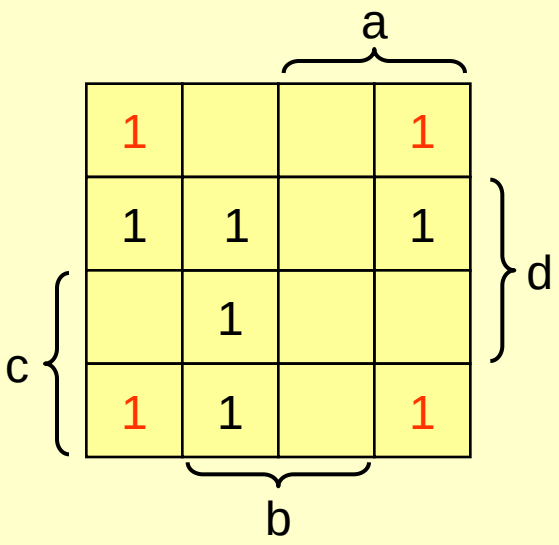
f = a'c'd + a'bd + a'bc + b'c' + b'd' + cd'



Quine-McCluskey Method
An Example

But, the form below is not minimized, using a Karnaugh map we can obtain:

f = a'c'd + a'bd + a'bc + b'c' + b'd' + cd'

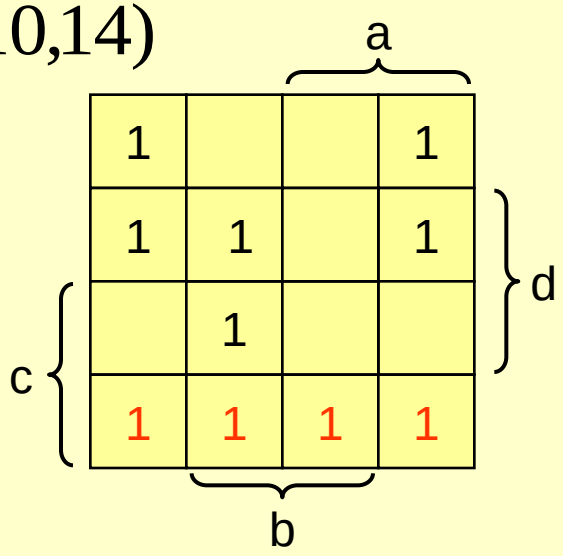


Quine-McCluskey Method
An Example

But, the form below is not minimized. Using a Karnaugh map we can obtain:

f = a'c'd + a'bd + a'bc + b'c' + b'd' + cd'

$$f(a,b,c,d) = \sum m(0,1,2,5,6,7,8,9,10,14)$$

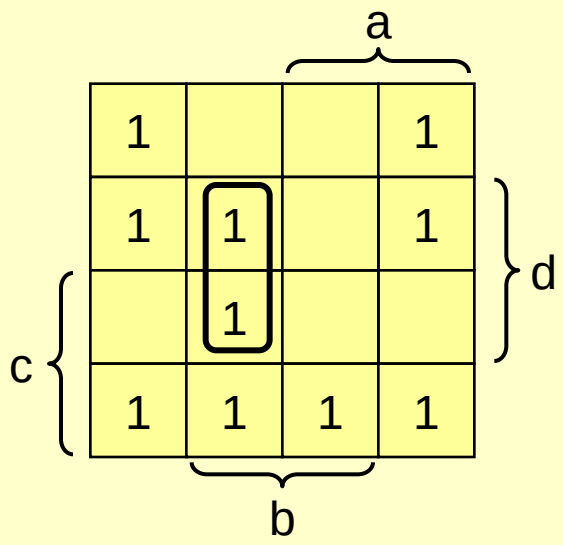


Quine-McCluskey Method
An Example

But, the form below is not minimized, using a Karnaugh map we can obtain:

f = a'c'd + a'bd + a'bc + b'c' + b'd' + cd'

F = a'bd

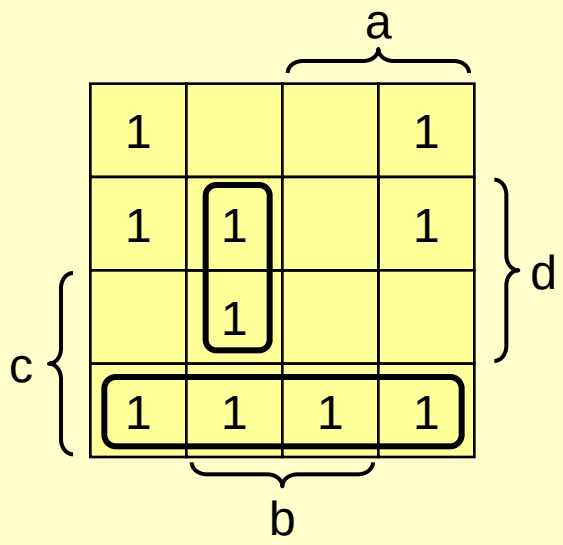


Quine-McCluskey Method An Example

But, the form below is not minimized, using a Karnaugh map we can obtain:

f = a'c'd + a'bd + a'bc + b'c' + b'd' + cd'

F = a'bd + cd'

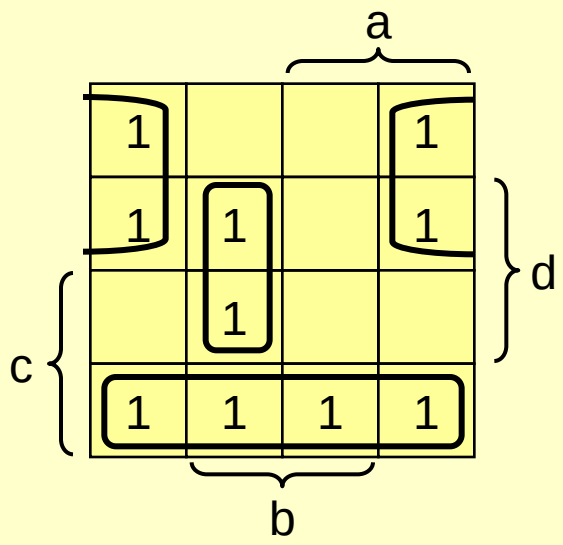


Quine-McCluskey Method
An Example

But, the form below is not minimized, using a Karnaugh map we can obtain:

f = a'c'd + a'bd + a'bc + b'c' + b'd' + cd'

F = a'bd + cd' + b'c'



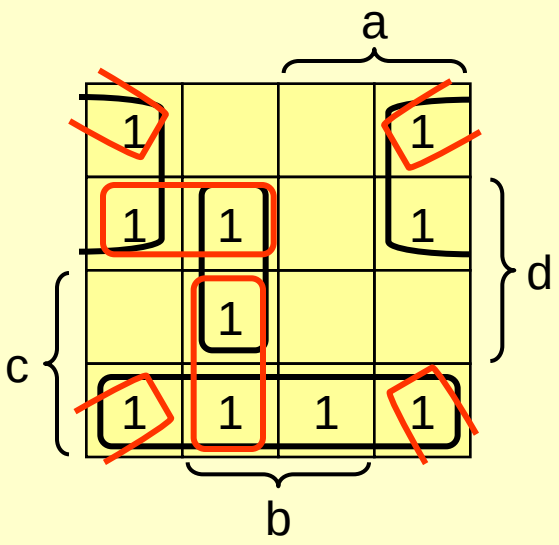
Quine-McCluskey Method
An Example

What are the extra terms in the solution obtained with the Quine-McCluskey method?

f = a'c'd + a'bd + a'bc + b'c' + b'd' + cd'

F = a'bd + cd' + b'c'

Thus, we need a method to eliminate this redundant terms from the Quine-McCluskey solution.



The Prime Implicant Chart

The prime implicant chart is the second part of the Quine-McCluskey procedure.

It is used to select a minimum set of prime implicants.

Similar to the Karnaugh map, we first select the essential prime implicants, and then we select enough prime implicants to cover all the minterms of the function.

Prime Implicant Chart (Example)

Prime Implicants		minterms									
		0	1	2	5	6	7	8	9	10	14
(0,1,8,9)	b'c'	X	X					X	X		
(0,2,8,10)	b'd'	X		X				X		X	
(2,6,10,14)	cd'			X		X				X	X
(1,5)	a'c'd		X		X						
(5,7)	a'bd				X		X				
(6,7)	a'bc					X	X				

Question: Given the prime implicant chart above, how can we identify the essential prime implicants of the function?

Prime Implicant Chart (Example)

Prime Implicants		minterms 0	1	2	5	6	7	8	9	10	14
(0,1,8,9)	b'c'	X	X					X	X		
(0,2,8,10)	b'd'	X		X				X		X	
(2,6,10,14)	cd'			X		X				X	X
(1,5)	a'c'd		X		X						
(5,7)	a'bd				X		X				
(6,7)	a'bc					X	X				

Similar to the Karnaugh map, all we have to do is to look for minterms that are covered by a single term.

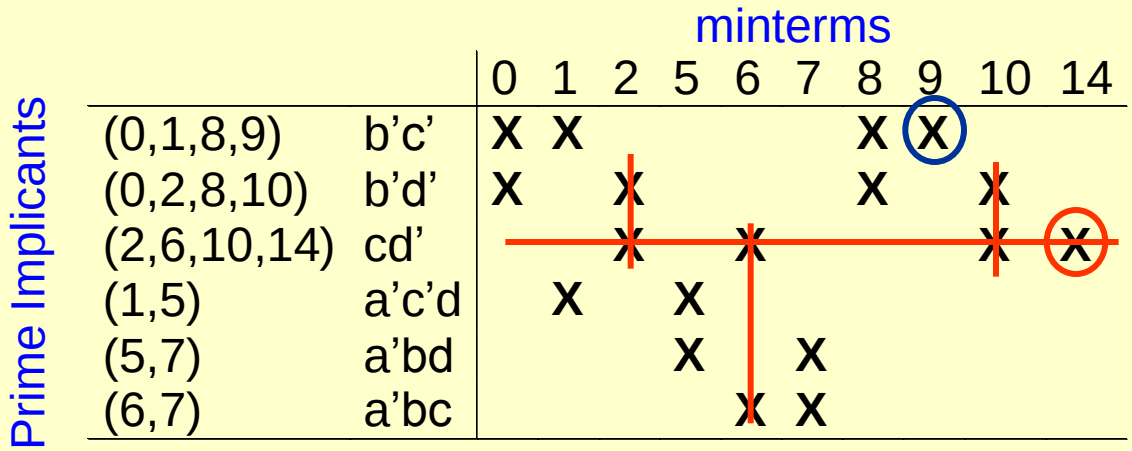
Prime Implicant Chart (Example)

Prime Implicants		minterms 0	1	2	5	6	7	8	9	10	14
(0,1,8,9)	b'c'	X	X					X	X		
(0,2,8,10)	b'd'	X		X				X		X	
(2,6,10,14)	cd'			X		X				X	X
(1,5)	a'c'd		X		X						
(5,7)	a'bd				X		X				
(6,7)	a'bc					X	X				

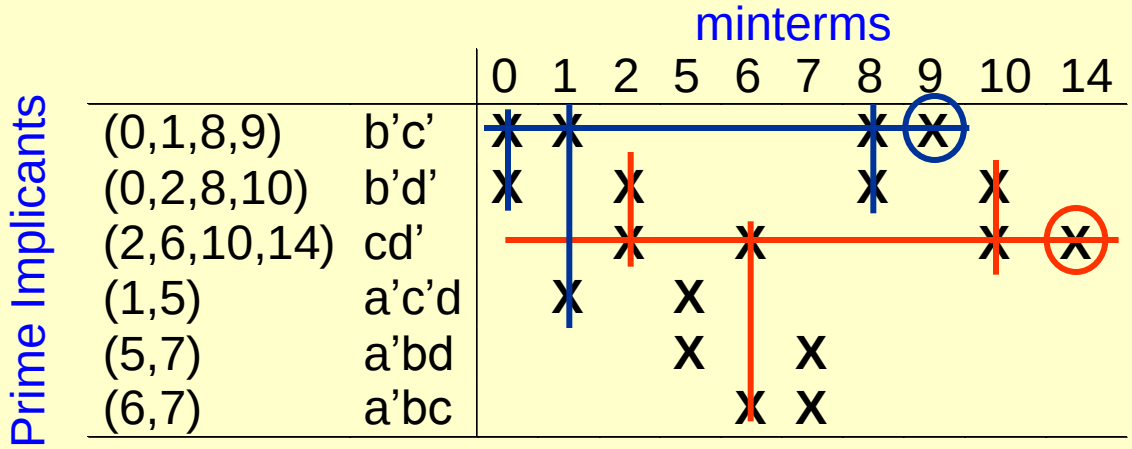
Once a term is included in the solution, all the minterms covered by that term are covered.

Therefore we may now mark the covered minterms and find terms that are no longer useful.

Prime Implicant Chart (Example)

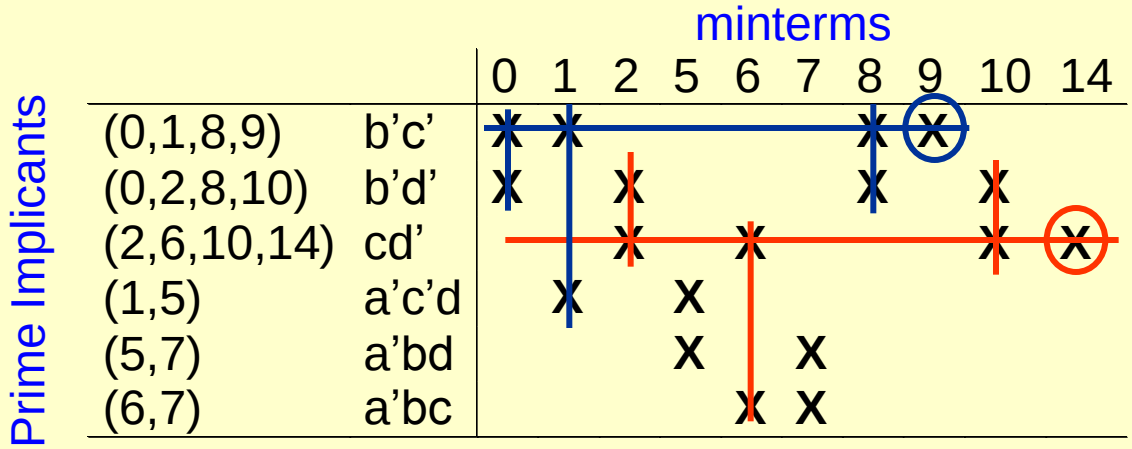


Prime Implicant Chart (Example)



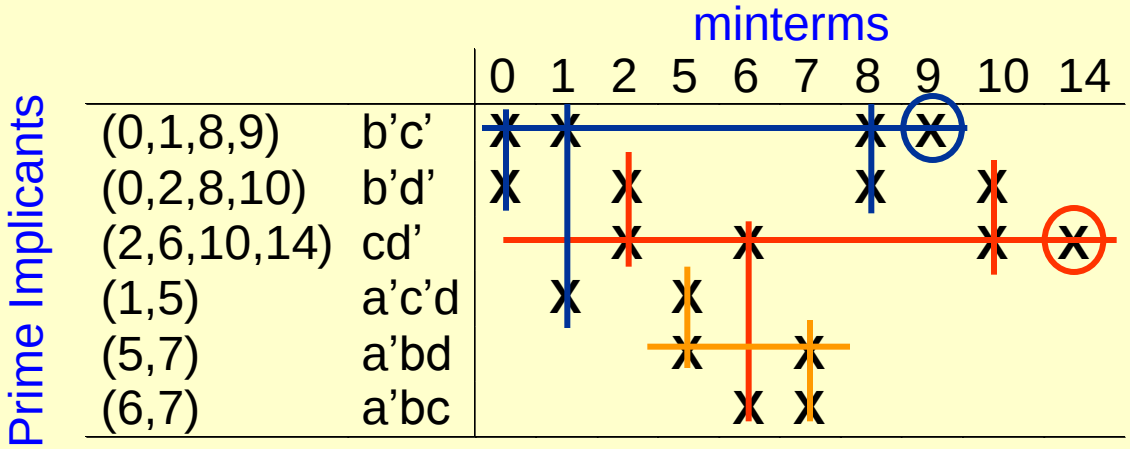
As we have not covered all the minterms with essential prime implicants, we must choose enough non-essential prime implicants to cover the remaining minterms.

Prime Implicant Chart (Example)



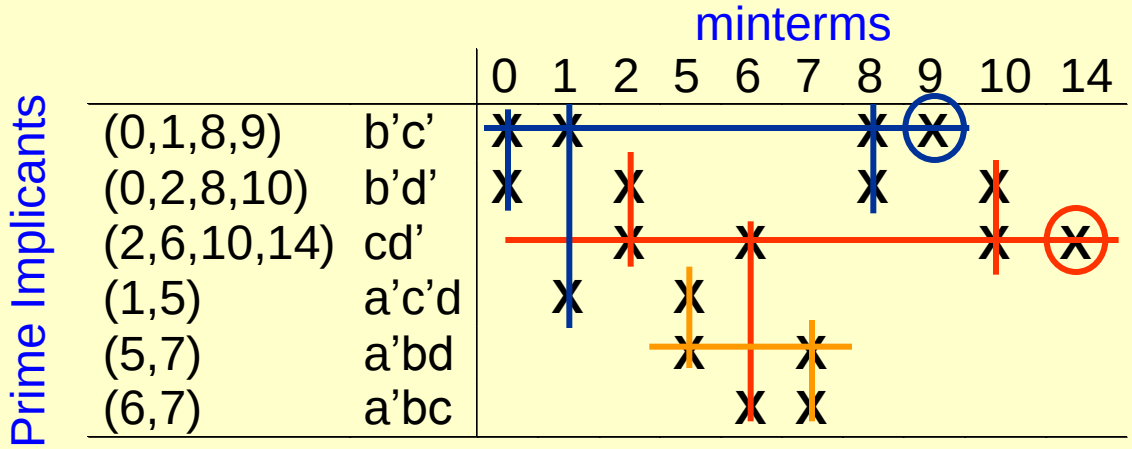
What strategy should we use to find a minimum cover for the remaining minterms?

Prime Implicant Chart (Example)



We choose first prime implicants that cover the most minterms.

Prime Implicant Chart (Example)



Therefore our minimum solution is:

f(a,b,c,d) = b'c' + cd' + a'bd

To Simplify in POS Form:

- **Take 0's as initial list of minterms**
- **Simplify to get the compliment of the function in SOP form**
- **Compliment to get the function in POS form**

Function with Don’t care inputs

- Don’t cares included while computing Prime implicants

-In the Selection of Essential Prime implicants don’t cares not used.

Simplify Using QM Method

F(A,B,C,D) = Σ(6,7,14)
d(A,B,C,D) = Σ(0,8,15)