BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI
FIRST SEMESTER 2016-2017
ME F212/MF F212: FLUID MECHANICS
COMPREHENSIVE EXAMINATION (Part-A) (CLOSED BOOK)

Date: 5th December, 2016 Max. Marks: 30 Duration: 60 Minutes

Q1.

$$\sum F_n = \oint_{cs} \upsilon_n (\rho \vec{\upsilon}.d\vec{A}) \quad \Rightarrow F_A = \rho V_{\text{jet}}^2 \sin\theta \left(\frac{\pi}{4} D_{\text{jet}}^2\right) = \underline{\mathbf{0.696\ N}} \qquad \textbf{[2M]}$$

$$\Rightarrow \dot{m}_{\text{jet}} = \dot{m}_2 + \dot{m}_3$$

$$\Rightarrow R_{\text{along the plate}} = \dot{m}_2 V_2 - \dot{m}_3 V_3 - \dot{m}_{\text{jet}} V_{\text{j}} \cos 30 = 0$$

$$\Rightarrow \frac{\dot{m}_2}{\dot{m}_3} = 13.93 \quad \textbf{[4M]}$$

$$\Rightarrow F_A = \rho \left(\frac{\pi}{4} D_{\text{jet}}^2\right) (V_{\text{jet}} - V)^2 \sin\theta = \underline{\mathbf{0.391\ N}} \qquad \textbf{[2M]}$$

Q2.

$$\bar{y} = 2 + \frac{1}{2}\sin 45 = 2.3535m \ \textbf{[1M]} \qquad y_{cp} = \bar{y} + \frac{I_{cg}\sin^2\alpha}{A\bar{y}} = 2.3712m \ \textbf{[2M]}$$

Hydrostatic force on gate: $F = \rho g\, \bar{y} A = 23.26\,\text{kN}$ **[2M]**

Moment about hinge O:

$$\sum M_0 = 0 \Rightarrow T\sin 60 = F \times \frac{y_{cp} - 2}{\sin 45} + W \times \frac{\cos 45}{2} \qquad \Rightarrow T = 15.12\,\text{kN} \ \textbf{[3M]}$$

Q3.

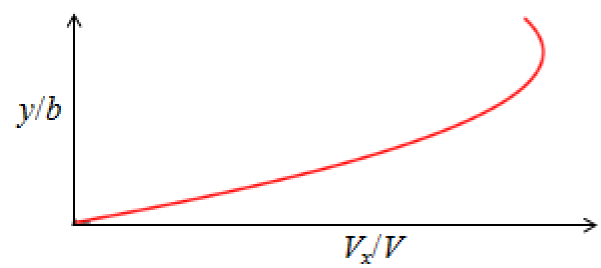
$$\Rightarrow V_x = \frac{Vy}{b} + \frac{1}{2\mu}\frac{dp}{dx}\left(y^2 - by\right) \qquad \text{set} \Rightarrow \frac{dV_x}{dy} = 0 \quad \Rightarrow y\big|_{V_{\max}} = \frac{b}{2} - \frac{\mu V}{b\frac{dp}{dx}} = \underline{\mathbf{2.08\ mm}} \quad \textbf{[2M]}$$

$$\Rightarrow V_{x\max} = \underline{\mathbf{0.6248\ m/sec}} \qquad \textbf{[2M]}$$

$$\Rightarrow \frac{Q}{width} = \int_0^b V_x dy = \frac{Vb}{2} - \frac{b^3}{12\mu}\frac{dp}{dx}$$

$= \underline{\mathbf{1.124\times10^{-3}\ m^3/s/m}}$ **[3M]**

Velocity distribution **[1M]**



Q4.

$$V_\theta = f(\omega, r, \tau, \rho, \mu) \quad \Rightarrow \frac{V_\theta}{\omega r} = \varphi\left(\frac{\mu}{\rho\omega r^2}, \omega\tau\right) \qquad \textbf{[2M]}$$

<u>Other solutions are also possible if we choose different repeating variables.</u>

From the above result π_2 containing the properties ρ, μ and π_3 containing the time τ. Combining them

$$\pi_2\pi_3 = \frac{\mu}{\rho\omega r^2}\omega\tau = \frac{\upsilon\tau}{r^2} \qquad \Rightarrow \upsilon_h\tau_h = \upsilon_w\tau_w$$

sine honey is more viscous then water

$\Rightarrow \tau_h < \tau_w$ so honey will attain steady motion faster. **[2M]**

<u>At steady state, solid body rotation exists. There are no relative motion and viscous forces and therefore, the Reynolds number would not be important.</u> **[2M]**

Part-B

Q1. $V = f(l, l_i, \rho, \mu, g, Q)$ $\Rightarrow \frac{Vl^2}{Q} = \varphi\left(\frac{l_i}{l}, \frac{Q^2}{l^5 g}, \frac{\rho Q}{l\mu}\right)$ **[3M]**

$$\Rightarrow \frac{Q_m}{Q_p} = \left(\frac{l_m}{l_p}\right)^{5/2} \qquad \text{- - - - (E1)}$$

From the last similarity requirement for same fluid

$$\Rightarrow \frac{Q_m}{Q_p} = \frac{l_m}{l_p} \qquad \text{- - - - (E2)}$$

Since these two requirements equation (1) and (2) are in conflict it follows that **<u>the similarity requirements cannot be satisfied</u>**. **[2M]**

For complete similarity from equation (2)

$$\Rightarrow \left(\frac{l_m}{l_p}\right)^{3/2} = \left(\frac{\upsilon_m}{\upsilon_p}\right) \Rightarrow \upsilon_m = \frac{10^{-6}}{(13)^{3/2}} = 4.8 \times 10^{-8}\ \mathrm{m^2/s}$$ **[3M]**

Q2. **<u>Option 1</u>**:

Given p_1 = 200 kPa, l = 23 m, D = 0.019 m, $\frac{\varepsilon}{D} = 0$

SFEE $\Rightarrow V = \sqrt{\frac{2D(200 - g \times 15)}{fl}}$ **[1M]**

For smooth pipe make a guess $\Rightarrow f_1 = \frac{0.008 + 0.042}{2} = 0.025$ **[1M]**

Calculate V_1 using SFEE =1.868 m/s $\Rightarrow \mathrm{Re}_1 = \frac{\rho V_1 D}{\mu}$

Colebrook equation $\Rightarrow \frac{1}{\sqrt{f_2}} = -2\log\left(\frac{2.51}{\mathrm{Re}_1 \sqrt{f_2}}\right)$ $\Rightarrow f_2 = 0.0222$ **[1M]**

$\Rightarrow V_2$ = 1.98 m/s **[1M]**

Then flow rate $\Rightarrow Q = 5.61 \times 10^{-4}\ \mathrm{m^3/s}$ **[2M]**

<u>Option 2</u>:

Given p_1' = 300 kPa, l' = 16 m, D = 0.0127 m, $\frac{\varepsilon}{D} = 0.05$

$$\Rightarrow \frac{300\,kPa}{\rho g} = 15 + \frac{f l' V^2}{2gD}$$ **[2M]**

For pipe with e/D = 0.5 make a guess $\Rightarrow f_1 = 0.07$

Calculate V_1 using SFEE $\Rightarrow V_1$ = 1.87 m/s $\Rightarrow \mathrm{Re}_1 = \frac{\rho V_1 D}{\mu}$

Colebrook equation $\Rightarrow f_2 = 0.0725$ **[1M]**

$\Rightarrow V_2$ =1.83 m/s **[1M]**

Then flow rate $\Rightarrow Q = \frac{\pi}{4}(0.0127)^2 \times 1.83 = 2.32 \times 10^{-4}\ \mathrm{m^3/s}$ **[2M]**

<u>Option 1 is 2.42 times more effective</u>. **[2M]**

Q3. $\Rightarrow p_{s\,gage} = \frac{\rho_{air} V_\infty^2}{2}(1 - 4\sin^2\theta)$ **[4M]**

$\Rightarrow F_y = \frac{5}{3}\rho_{air} V_\infty^2 (aL)$ **[4M]**

Q4. **Boundary conditions [2M]**

(1) At $y = -h$ $V_{x1} = 0$ (2) At $y = 0$ $V_{x1} = V_{x2}$

(3) At $y = h$ $V_{x2} = 0$ (4) At $y = 0$ $\mu_1 \frac{dV_{x1}}{dy} = \mu_2 \frac{dV_{x2}}{dy}$

Obtaining these constants

$C_1 = -750; C_2 = 2.5; C_3 = -187.5; C_4 = 2.5$ **[4M]**

$\Rightarrow V_{\text{interface}} = 2.5$ m/s **[2M]** $\Rightarrow y_{\max} = -1.5 mm$ **[1M]**

$\Rightarrow V_{x\max} = 3.06$ m/s **[2M]** **Velocity variation [1M]**

Q5. $\mathrm{Re}_x = \frac{V_\infty x}{\upsilon} = 5\times10^5$ $\Rightarrow x = 0.25m$ **[1M]** $\delta = \frac{4.64x}{\sqrt{\mathrm{Re}_x}} = 1.64\times10^{-3}m$ **[1M]**

$\delta_{\text{turb}} = \frac{0.37x}{(\mathrm{Re}_x)^{1/5}} \Rightarrow 6.16\times10^{-2}m$ **[1M]** $\Rightarrow \tau_{\mathrm{w}} = \frac{0.059}{(\mathrm{Re}_l)^{1/5}}\frac{\rho \mathrm{V}_\infty^2}{2} \Rightarrow$ **<u>4.912 Pa</u> [1M]**

Drag force due to laminar boundary layer $\Rightarrow F_{lam} = \frac{1.328}{\sqrt{\mathrm{Re}_{x=0.25m}}}\frac{\rho \mathrm{V}_\infty^2}{2}(bx)$ = **<u>1.4085 N</u> [1M]**

<u>Total drag force on the plate with transition</u> at Re$_x$ = 5×10^5 $\mathrm{Re}_l = \frac{V_\infty l}{\upsilon} = 8\times10^6$

$\Rightarrow F_{total} = F_{tutb}$ over full length $- F_{tutb}$ over length $x_{cr} + F_{lam}$ over length x_{cr}

$\Rightarrow F_{total}$ = **<u>35.118 N</u>** **[1M]** $\Rightarrow F_{turb}$ = **<u>33.71 N</u>** **[1M]** $\Rightarrow \frac{F_{turb}}{F_{lam}} = 23.94$ **[1M]**