Birla Institute of Technology and Science, Pilani

II Semester 2012-2013, EEE F243 / INSTR F243, Signals & Systems

Comprehensive examination CLOSED BOOK

2/5/2013

MM: 110 Time: 3 hrs.

- Question paper is printed on BOTH sides (total 6 questions). Symbols have their usual notations.
- Answers should be CLEAR, CONCISE AND LEGIBLE. Specify your assumptions clearly.
- YOUR ANSWERS MUST BE JUSTIFIED. Simply writing yes/ no will not fetch you any marks
- All parts of a question must be answered together and in the same sequence as given in question paper, ELSE QUESTION SHALL NOT BE EVALUATED
- NO MARKS will be awarded for plots without proper labeling of axes.

Q1. Find the Z-transform & ROC of

(i) $x(n) = 2\left(\frac{5}{6}\right)^{n} u(-n-1) + 3\left(\frac{1}{2}\right)^{2n} u(n)$

Sketch the ROC and pole-zero location, also comment on causality & stability of this system. 8

(ii) Find the Z-transform of x (n), where x (n) is: 4

$x(n) = \{2, 1, \underset{\uparrow}{3}, 5, 0, 7\}$

(iii) Obtain T-equivalent model, of a Z - parameter matrix of a two port network at ω = 100 r/s given by:

$$[z] = \begin{bmatrix} 4 & 2+j1 \\ 2+j1 & -3j \end{bmatrix}$$

4

Q2 (i) Find the 8 point DFT using decimation in time FFT for the given signal. 10

x (t) = A cos (8πt). Assume A=2;

(ii) Find the sampling rate and sampling interval of the pulse signal as shown in the Figure 2. Assume that the pulse is generated from its first ten harmonics only. Assume T=0.2 sec.

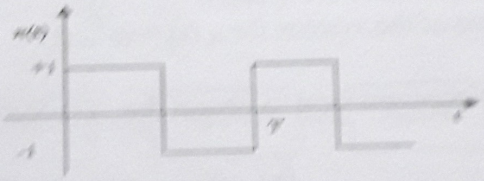

Figure 2. Pulse signal 10

Q3 (i) Find the inverse Fourier transform of the spectrum shown in Figure 3 (i)

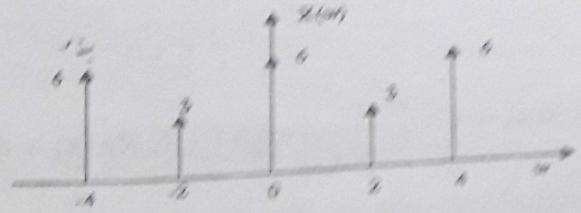

Figure 3(i) 10

(ii) For the signal x (t) shown in Figure 3(ii), evaluate the following quantities without explicit computing X(ω): $\int_{-\infty}^{\infty} X(\omega)d\omega$ (b) $\int_{-\infty}^{\infty} |X(\omega)|^2 d\omega$ (c) $\int_{-\infty}^{\infty} X(\omega)e^{j2\omega}d\omega$ (d) X(0) 8

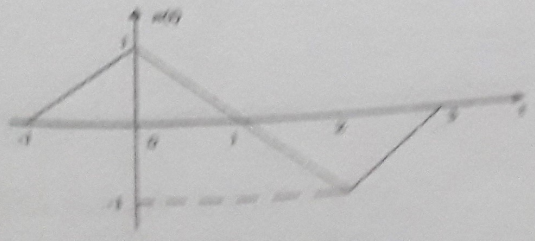

Figure 3(ii)

Q4. (i) Find the trigonometric Fourier series for the waveform shown in Figure 4(i).

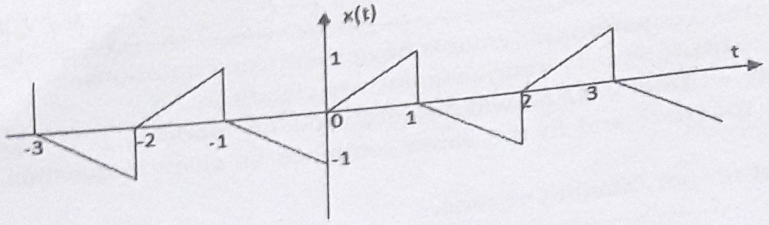

Figure 4(i) 10

(ii) Find the discrete-time Fourier series coefficients for x(n)

$$x(n) = \sin\left(\frac{\pi}{3}\right)n + \cos\left(\frac{\pi}{6}\right)n$$ 8

Q5. (i) Given the system having input x (n) and output y (n) and described by the relationship:

$$y(n) = \sum_{k=-\infty}^{n} x(k+2),$$

Determine if the system is:

(a) Memory-less, (b) stable, (c) casual, (d) linear and (e) time invariant. 5

(ii) Evaluate and sketch $y(t)=x(t)*h(t)$ for a continuous-time LTI system whose impulse respons $h(t)$ and the input $x(t)$ are given by:

$h(t)= u(t)-u(t-4)$ and $x(t)=2[\ u(t-4)-u(t-6)]$ [Use the graphical method only]. 6

(iii) The input & output of a causal LTI system are described by the differential equation:

$$\frac{d^2y(t)}{dt^2} + \frac{3dy(t)}{dt} + 2y(t) = x(t)$$

a) Find the frequency response of the system.
b) Find the impulse response of the system.
c) What is the response of the system for x (t) = $te^{-t}u(t)$? 8

Q6. (i) Use the properties of Laplace transform to obtain the X(s)of the following:

(a) $x(t) = \frac{d}{dt}\{ t e^{-t} u(t)\}$ (b) $x(t) = u(t-1) * e^{-2t} u(t-1)$ 4

(ii) Use the properties of Laplace Transforms to determine the time signals that correspond to the following bilateral Laplace transforms

(a) $X(s)= e^{5s}\frac{1}{s+2}$ with ROC Re (s) < - 2 (b) $X(s) = s^{-2}\frac{d}{ds}\left|\frac{e^{-3s}}{s}\right|$ with ROC Re (s) > 0

(iii)Suppose the following facts are given about the signal x(t) with Laplace transform X(s) :

(a) x(t) is real and even

(b) X(s) has four poles, no zeroes in the finite s-plane.

(c) X(s) has a pole at $s = (1/2)e^{j\pi/4}$

(d) $\int_{-\infty}^{\infty} x(t)dt = 4$

Determine X(s) and ROC.