Birla Institute of Technology & Science, Pilani

Comprehensive Exam **(Closed Book)** **Part A**

Course Name : Mathematical Methods (MATH F241) **Date: 05-05-16**

Max. Time: 105 Minutes **Max. Marks: 75**

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1. Solve the following BVP with the help of Green's function

   $y'' + y = 1, y(0) = 1,\ y(1) = 0.$ [8]

2. Prove that the eigenvalues of real symmetric kernel $K(x,\xi)$ are real. [8]

3. Find the characteristic values of λ for the integral equation

$$y(x) = \lambda \int_0^{2\pi} \sin(x+\xi)\, y(\xi)\, d\xi$$ [10]

4. Using the calculus of variations, find the shortest distance between the line $y = x$ and the curve $y = \sqrt{x-1}$. [11]

5. Using the Rayleigh-Ritz method, find the first approximate solution of the non linear differential equation $2y'' - 3y^2 = 0$, that satisfies the conditions $y(0) = 4, y(1) = 1$. [12]

6. Consider a simple plane pendulum consisting of a mass m attached to a string of length l. After the pendulum is set into motion, the length of the string is shortened at a constant rate i. e. $\frac{dl}{dt} = -\alpha$ (α is a constant). The suspension point remains fixed. Compute the Lagrangian L and hence derive the equation of motion. [10]

7. Obtain the Euler's equation and the associated natural boundary conditions for the problem

$$I = \delta\left[\int_a^b F(x, y, y')dx - \beta y(b) + \alpha y(a)\right] = 0$$ where α, β are given constants and $y(a)$ and $y(b)$ are not preassigned. [8]

8. Using parametric equation of the curve, determine a closed curve C of given (fixed) length (perimeter) which encloses maximum area by Calculus of Variation. [8]

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Comprehensive Exam **(Open Book)** **Part B**

Course Name : Mathematical Methods (MATH F241) **Date: 05-05-16**

Max. Time: 75 Minutes **Max. Marks: 60**

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1. Solve the integral $\int_0^\infty f(x)\cos(\xi x)\,dx = \begin{cases} 1-\xi, & 0 \le \xi \le 1 \\ 0, & \xi > 1 \end{cases}$. Hence deduce that $\int_0^\infty \frac{\sin^2 t}{t^2}dt = \frac{\pi}{2}$. [10]

2. Find the Fourier sine transform of $f(x) = \begin{cases} x, & 0 < x < 1 \\ 3-x, & 1 < x < 3. \\ 0, & x > 3 \end{cases}$, [8]

3. Find the solution of partial differential equation $\frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial x^2}$ if $U(0,t) = 0$, $U(x,0) = \begin{cases} 2, & 0 < x < 2 \\ 0, & x \ge 2 \end{cases}$ and $U(x,t)$ is bounded where $x > 0, t > 0$. [10]

4. Find inverse Fourier transform of $\frac{1}{(4+\xi^2)(9+\xi^2)}$ using convolution theorem. [8]

5. Using the assumption $y(x) \approx c_1 + c_2 x + c_3 x^2$ find the approximate solution of the equation $y(x) = x + \int_0^1 K(x,\xi)\,y(\xi)\,d\xi$, where $K(x,\xi) = \begin{cases} x(1-\xi), & x < \xi \\ \xi(1-x), & x > \xi \end{cases}$ using the method of collocation at the points $x = 0,\ 0.4\, and\, 1.0$. [12]

6. Find the Fourier Transform of $f(x) = \begin{cases} a^2 - x^2, & |x| < a \\ 0, & |x| > a \end{cases}$. Hence, deduce $\int_0^\infty \frac{\sin x - x\cos x}{x^3}dx = \frac{\pi}{4}$ [12]