

# Counters

# Overview

- Ripple Counter
- Synchronous Binary Counters
  - Design with D Flip-Flops
  - Design with J-K Flip-Flops
- Serial Vs. Parallel Counters
- Up-down Binary Counter
- Binary Counter with Parallel Load
- BCD Counter, Arbitrary sequence Counters

# Counters

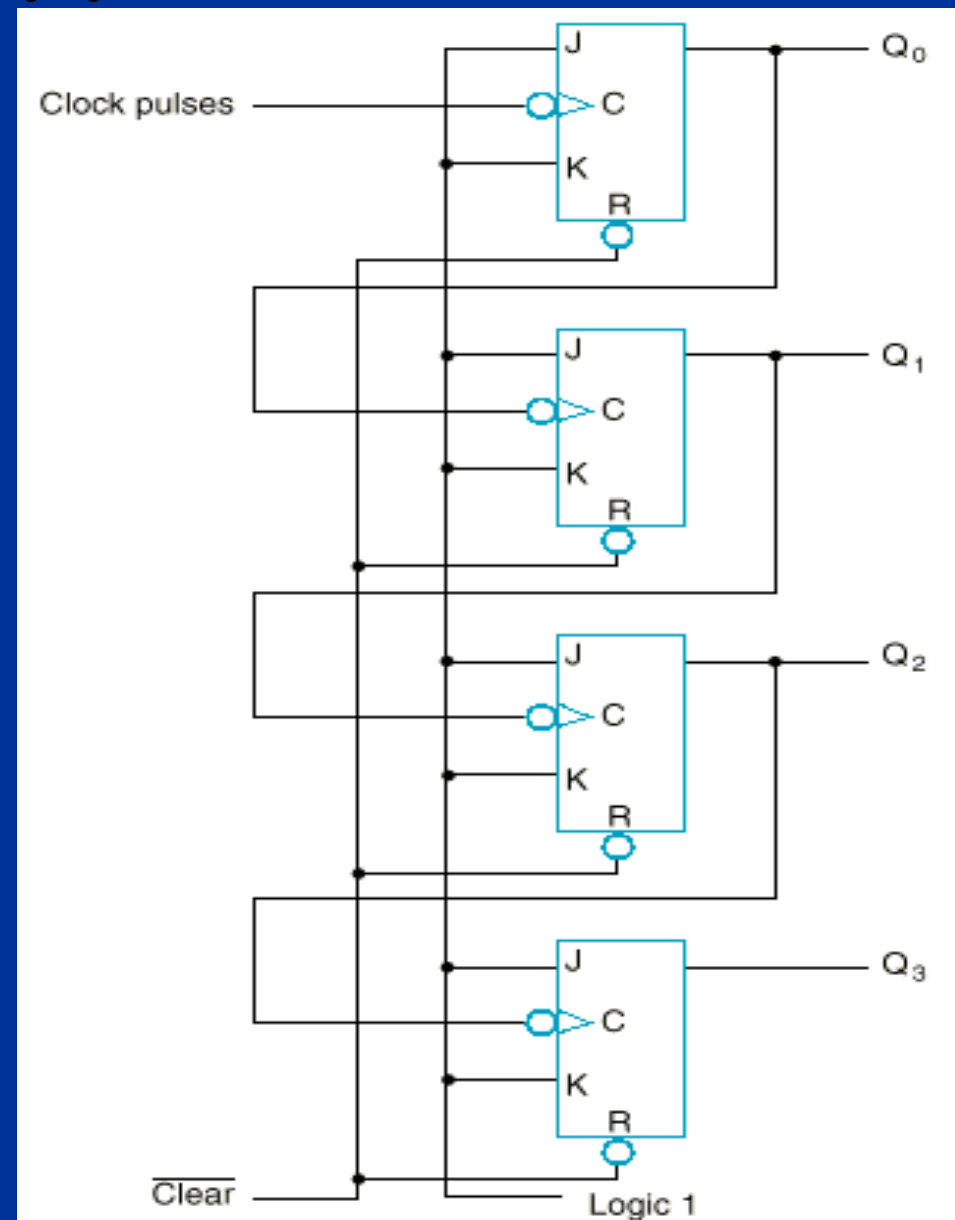
- A counter is a register that goes through a predetermined sequence of states upon the application of clock pulses.
- Counters are categorized as:
  - Ripple Counters:  
The FF output transition serves as a source for triggering other FFs. No common clock.
  - Synchronous Counter:  
All FFs receive the common clock pulse, and the change of state is determined from the present state.

# Example: A 4-bit Upward Counting Ripple Counter

Less Significant Bit output is Clock for Next Significant Bit!  
(Clock is negative edge)

Recall...

(a) JK Flip-Flop			
J	K	$Q(t+1)$	Operation
0	0	$Q(t)$	No change
0	1	0	Reset
1	0	1	Set
1	1	$\overline{Q}(t)$	Complement



# Example (cont.)

- The output of each FF is connected to the C input of the next FF in sequence.
- The FF holding the least significant bit receives the incoming clock pulses.
- The J and K inputs of all FFs are connected to a permanent logic 1.
- The bubble next to the C label indicates that the FFs respond to the negative-going transition of the input.

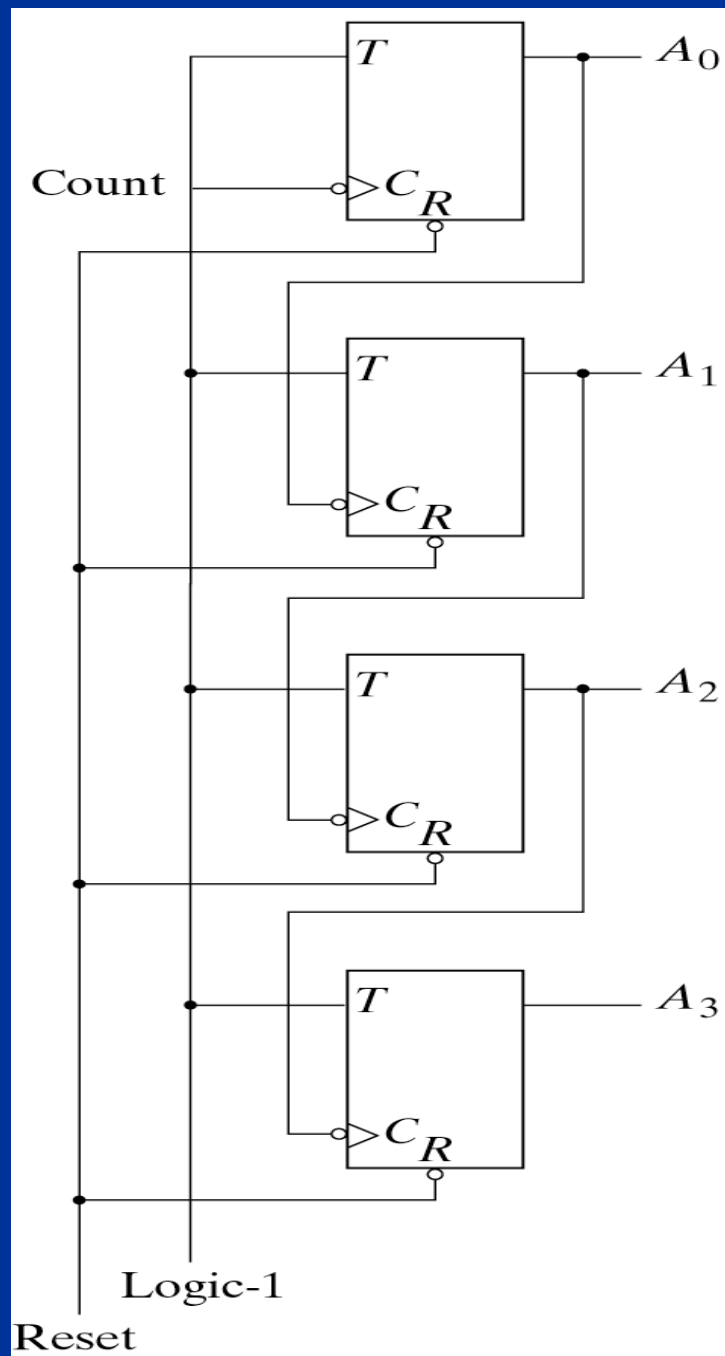
# Example (cont.)

## Operation:

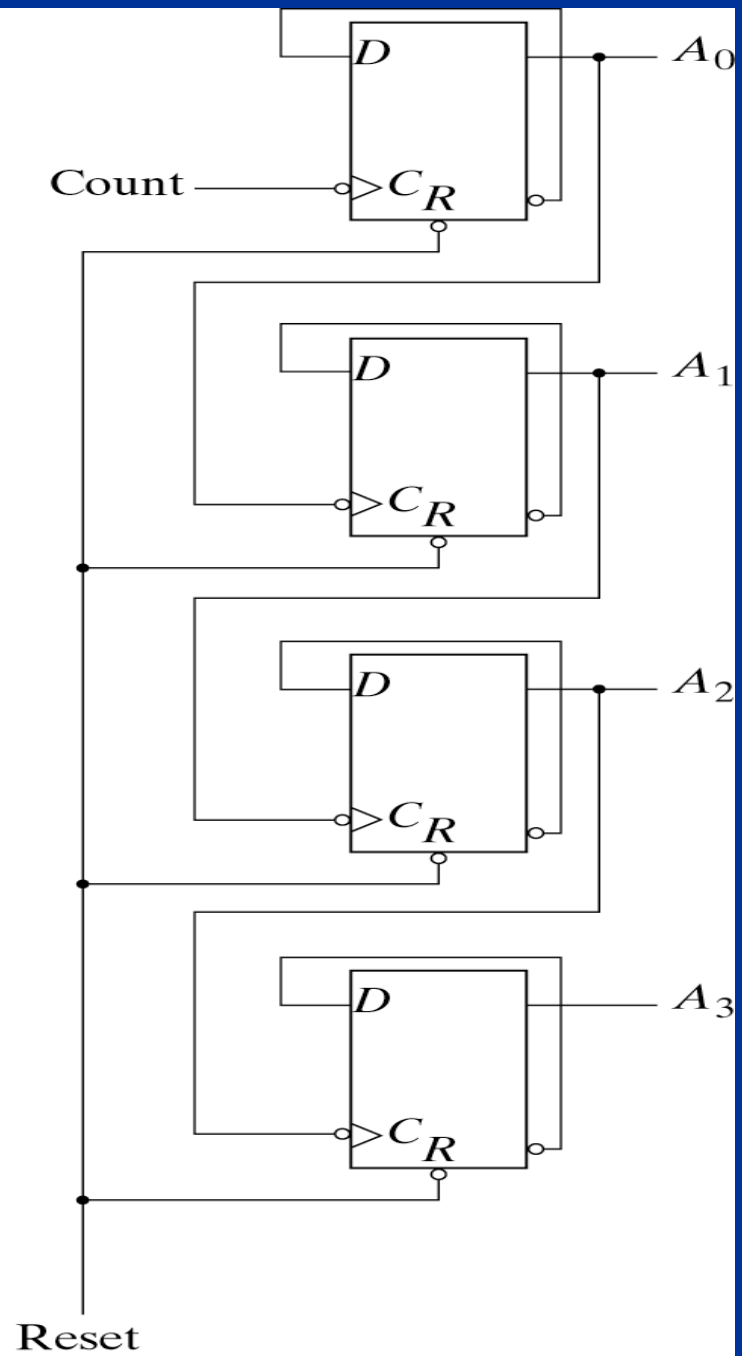
- The least significant bit ( $Q_0$ ) is complemented with each negative-edge clock pulse input.
- Every time that  $Q_0$  goes from 1 to 0,  $Q_1$  is complemented.
- Every time that  $Q_1$  goes from 1 to 0,  $Q_2$  is complemented.
- Every time that  $Q_2$  goes from 1 to 0,  $Q_3$  is complemented, and so on.

Upward Counting Sequence

$Q_3$	$Q_2$	$Q_1$	$Q_0$
0	0	0	0
0	0	0	1
0	0	1	0
0	0	1	1
0	1	0	0
0	1	0	1
0	1	1	0
0	1	1	1
1	0	0	0
1	0	0	1
1	0	1	0
1	0	1	1
1	1	0	0
1	1	0	1
1	1	1	0
1	1	1	1



(a) With T flip-flops



(b) With D flip-flops

Fig. 6-8 4-Bit Binary Ripple Counter

# A 4-bit Downward Counting Ripple Counter

- Use direct Set (S) signals instead of direct Reset (R), in order to start at 1111.
- Alternative designs:
  - Change edge-triggering to positive
  - Connect the complement output of each FF to the C input of the next FF in the sequence... (homework!)



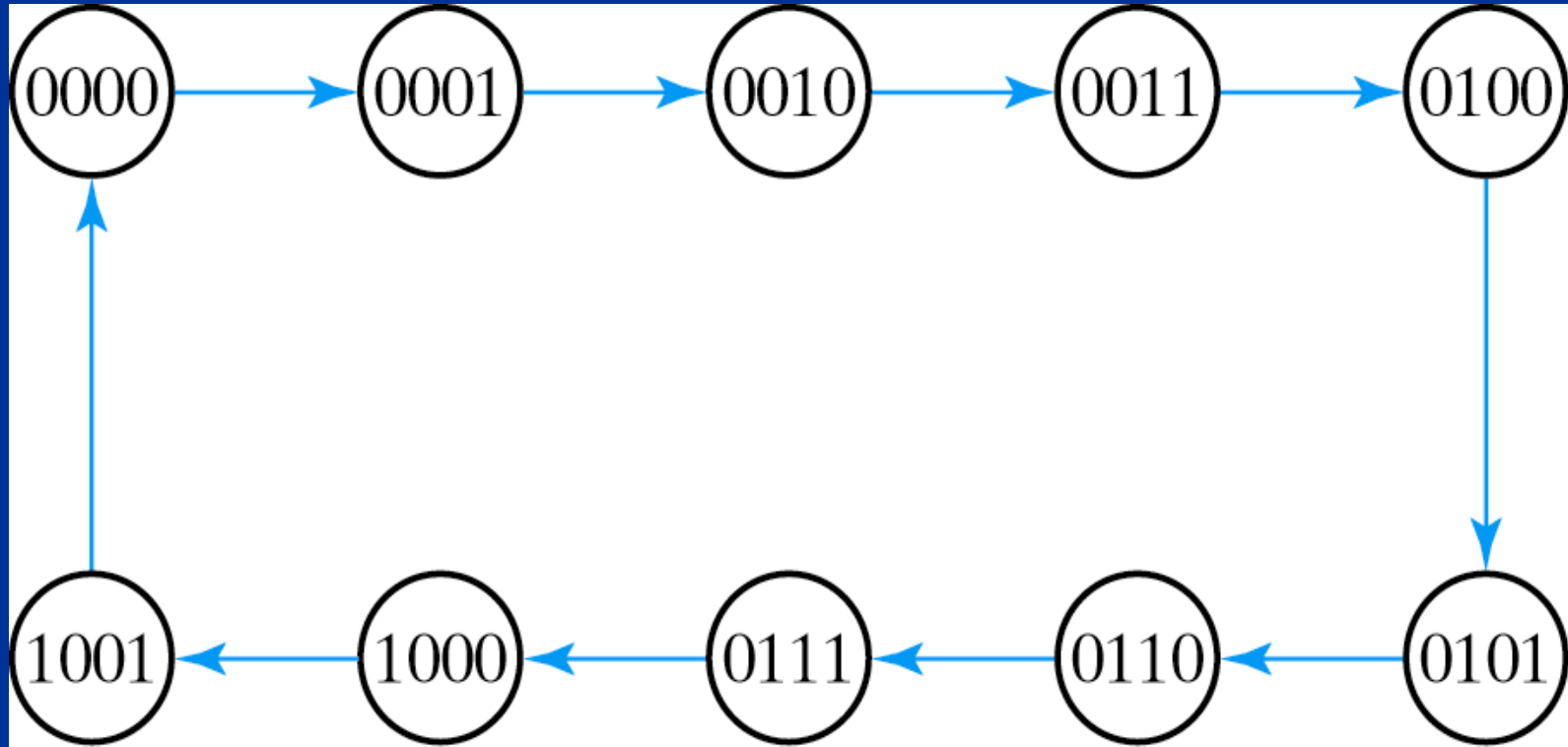
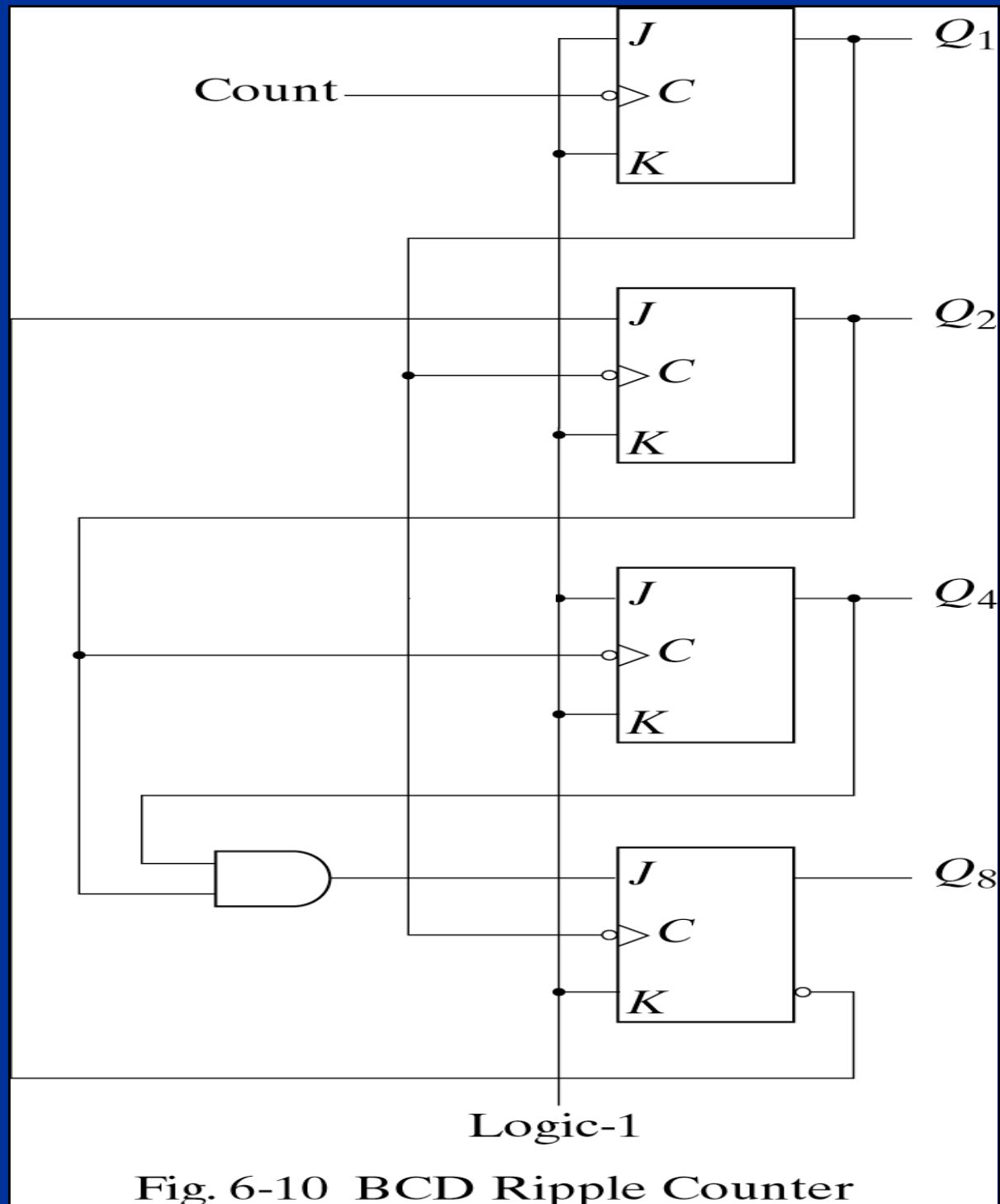


Fig. 6-9 State Diagram of a Decimal BCD-Counter



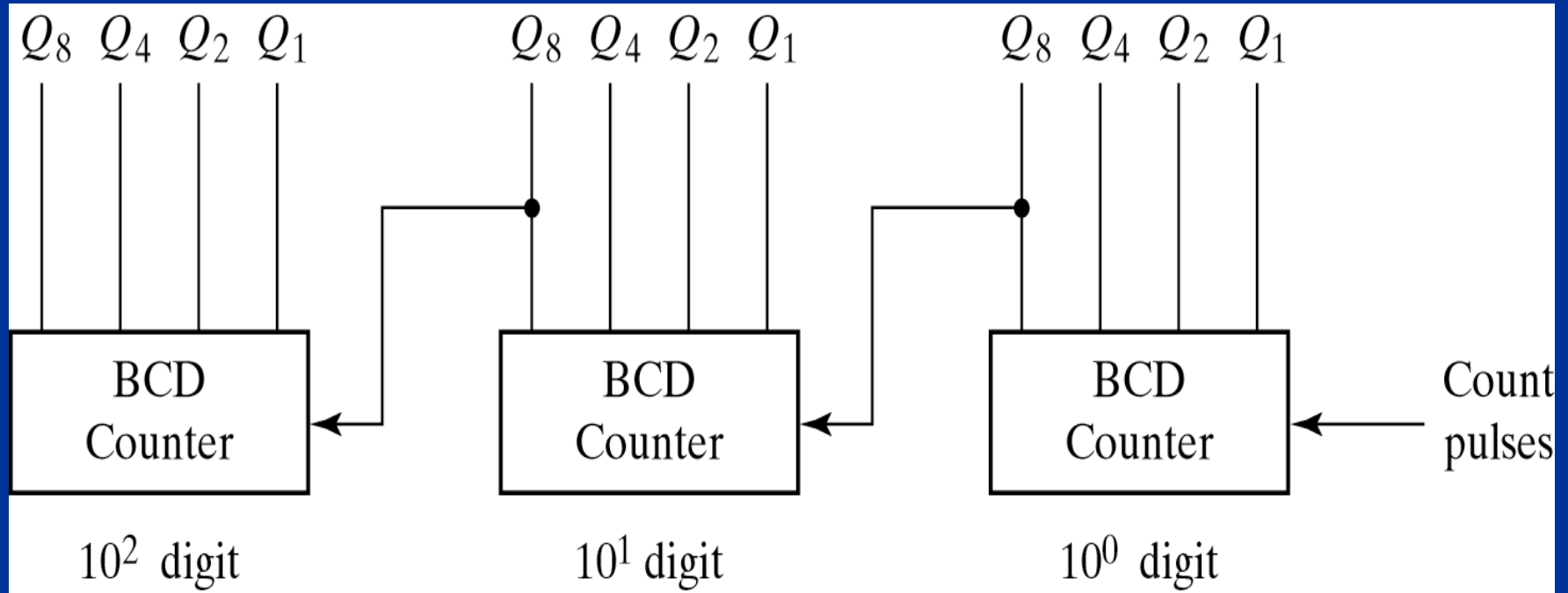


Fig. 6-11 Block Diagram of a Three-Decade Decimal BCD Counter

# Synchronous Binary Counters

- The design procedure for a binary counter is the same as any other synchronous sequential circuit.
- The primary inputs of the circuit are the CLK and any control signals (EN, Load, etc).
- The primary outputs are the FF outputs (present state).
- Most efficient implementations usually use T-FFs or JK-FFs. We will examine JK and D flip-flop designs.

# Synchronous Binary Counters:

## J-K Flip Flop Design of a 4-bit Binary Up Counter

TABLE 4-10  
Flip-Flop Excitation Tables

(a) JK Flip-Flop

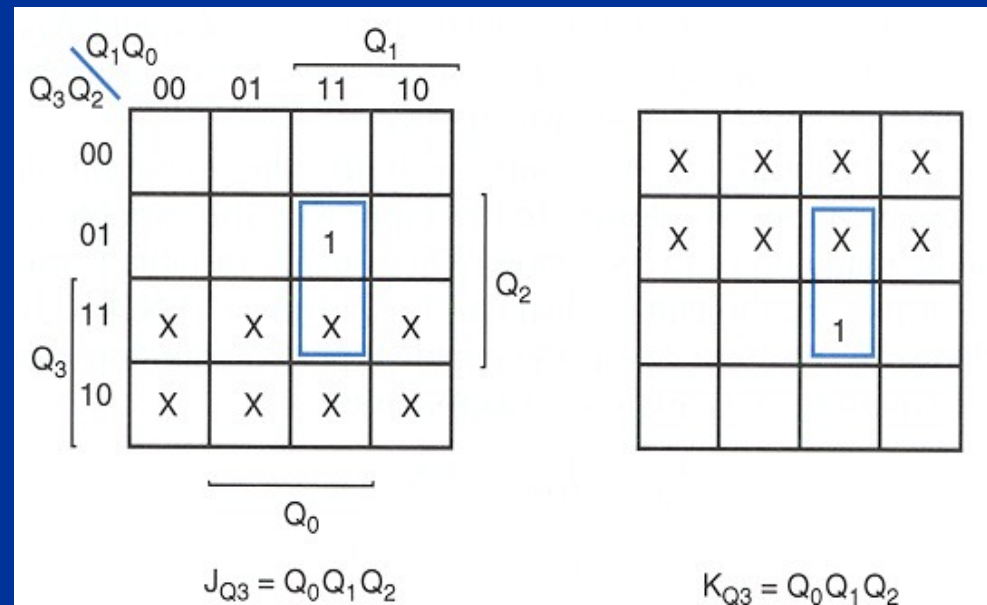
$Q(t)$	$Q(t+1)$	J	K
0	0	0	X
0	1	1	X
1	0	X	1
1	1	X	0

Present state				Next state				Flip-flop inputs							
$Q_3$	$Q_2$	$Q_1$	$Q_0$	$Q_3$	$Q_2$	$Q_1$	$Q_0$	$J_{Q3}$	$K_{Q3}$	$J_{Q2}$	$K_{Q2}$	$J_{Q1}$	$K_{Q1}$	$J_{Q0}$	$K_{Q0}$
0	0	0	0	0	0	0	1	0	X	0	X	0	X	1	X
0	0	0	1	0	0	1	0	0	X	0	X	1	X	X	1
0	0	1	0	0	0	1	1	0	X	0	X	X	0	1	X
0	0	1	1	0	1	0	0	0	X	1	X	X	1	X	1
0	1	0	0	0	1	0	1	0	X	X	0	0	X	1	X
0	1	0	1	0	1	1	0	0	X	X	0	1	X	X	1
0	1	1	0	0	1	1	1	0	X	X	0	X	0	1	X
0	1	1	1	1	0	0	0	1	X	X	1	X	1	X	1
1	0	0	0	1	0	0	1	X	0	0	X	0	X	1	X
1	0	0	1	1	0	1	0	X	0	0	X	1	X	X	1
1	0	1	0	1	0	1	1	X	0	0	X	X	0	1	X
1	0	1	1	1	1	0	0	X	0	1	X	X	1	X	1
1	1	0	0	1	1	0	1	X	0	X	0	0	X	1	X
1	1	0	1	1	1	1	0	X	0	X	0	1	X	X	1
1	1	1	0	1	1	1	1	X	0	X	0	X	0	1	X
1	1	1	1	0	0	0	0	X	1	X	1	X	1	X	1

# Synchronous Binary Counters:

## J-K Flip Flop Design of a Binary Up Counter (cont.)

Present state				Next state					
$Q_3$	$Q_2$	$Q_1$	$Q_0$	$Q_3$	$Q_2$	$Q_1$	$Q_0$	$J_{Q3}$	$K_{Q3}$
0	0	0	0	0	0	0	1	0	X
0	0	0	1	0	0	1	0	0	X
0	0	1	0	0	0	1	1	0	X
0	0	1	1	0	1	0	0	0	X
0	1	0	0	0	1	0	1	0	X
0	1	0	1	0	1	1	0	0	X
0	1	1	0	0	1	1	1	0	X
0	1	1	1	1	0	0	0	1	X
1	0	0	0	1	0	0	1	X	0
1	0	0	1	1	0	1	0	X	0
1	0	1	0	1	0	1	1	X	0
1	0	1	1	1	1	0	0	X	0
1	1	0	0	1	1	0	1	X	0
1	1	0	1	1	1	1	0	X	0
1	1	1	0	1	1	1	1	X	0
1	1	1	1	0	0	0	0	X	1



# Synchronous Binary Counters:

## J-K Flip Flop Design of a Binary Up Counter (cont.)

Present state				Next state				Flip-flop	
$Q_3$	$Q_2$	$Q_1$	$Q_0$	$Q_3$	$Q_2$	$Q_1$	$Q_0$	$J_{Q_2}$	$K_{Q_2}$
0	0	0	0	0	0	0	1	0	X
0	0	0	1	0	0	1	0	0	X
0	0	1	0	0	0	1	1	0	X
0	0	1	1	0	1	0	0	1	X
0	1	0	0	0	1	0	1	X	0
0	1	0	1	0	1	1	0	X	0
0	1	1	0	0	1	1	1	X	0
0	1	1	1	1	0	0	0	X	1
1	0	0	0	1	0	0	1	0	X
1	0	0	1	1	0	1	0	0	X
1	0	1	0	1	0	1	1	0	X
1	0	1	1	1	1	0	0	1	X
1	1	0	0	1	1	0	1	X	0
1	1	0	1	1	1	1	0	X	0
1	1	1	0	1	1	1	1	X	0
1	1	1	1	0	0	0	0	X	1

		1	
X	X	X	X
X	X	X	X
		1	

$$J_{Q_2} = Q_0 Q_1$$

X	X	X	X
		1	
		1	
X	X	X	X

$$K_{Q_2} = Q_0 Q_1$$

# Synchronous Binary Counters:

## J-K Flip Flop Design of a Binary Up Counter (cont.)

Present state				Next state				Flip inputs	
$Q_3$	$Q_2$	$Q_1$	$Q_0$	$Q_3$	$Q_2$	$Q_1$	$Q_0$	$J_{Q1}$	$K_{Q1}$
0	0	0	0	0	0	0	1	0	X
0	0	0	1	0	0	1	0	1	X
0	0	1	0	0	0	1	1	X	0
0	0	1	1	0	1	0	0	X	1
0	1	0	0	0	1	0	1	0	X
0	1	0	1	0	1	1	0	1	X
0	1	1	0	0	1	1	1	X	0
0	1	1	1	1	0	0	0	X	1
1	0	0	0	1	0	0	1	0	X
1	0	0	1	1	0	1	0	1	X
1	0	1	0	1	0	1	1	X	0
1	0	1	1	1	1	0	0	X	1
1	1	0	0	1	1	0	1	0	X
1	1	0	1	1	1	1	0	1	X
1	1	1	0	1	1	1	1	X	0
1	1	1	1	0	0	0	0	X	1

	1	X	X
	1	X	X
	1	X	X
	1	X	X

$$J_{Q1} = Q_0$$

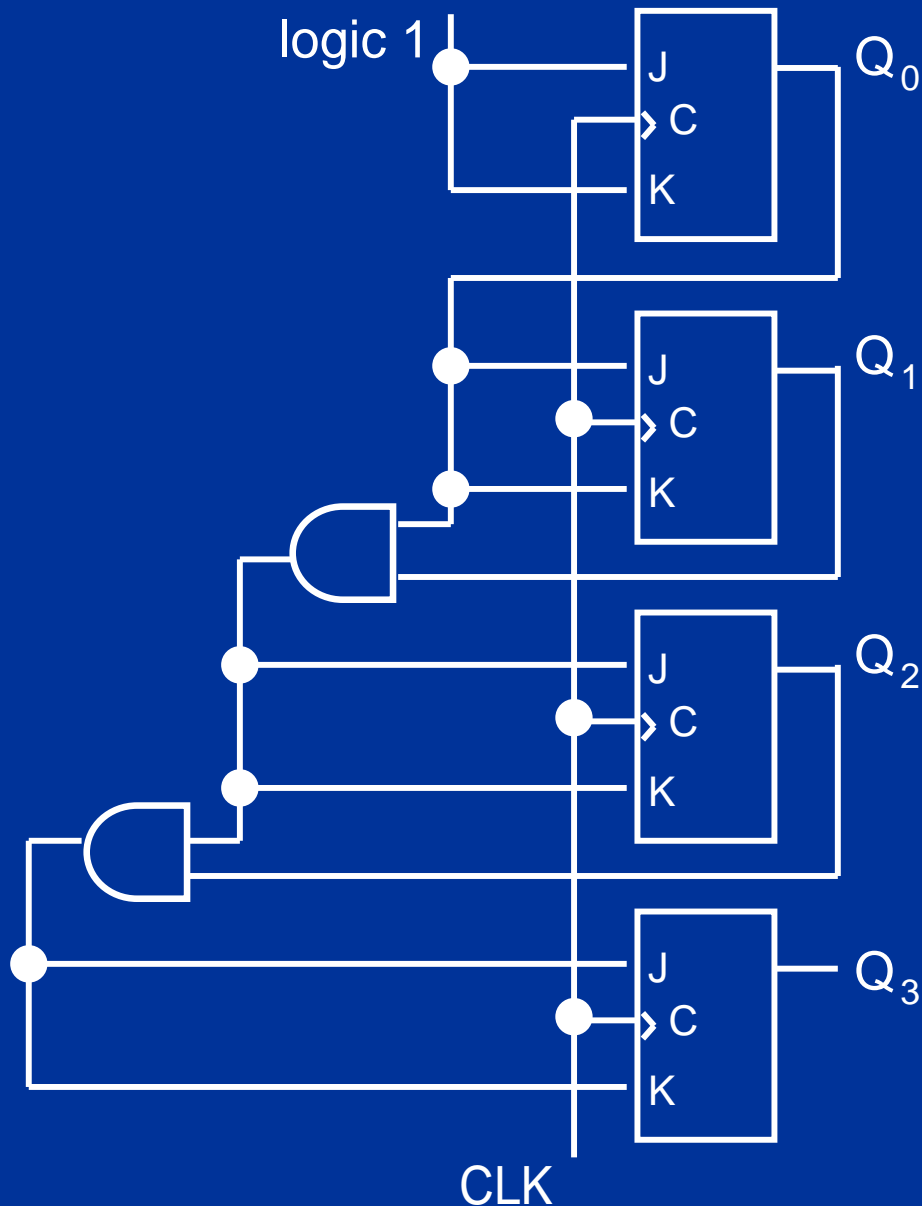
X	X	1	
X	X	1	
X	X	1	
X	X	1	

$$K_{Q1} = Q_0$$



# Synchronous Binary Counters:

## J-K Flip Flop Design of a Binary Up Counter (cont.)



$$J_{Q_0} = 1$$
$$K_{Q_0} = 1$$

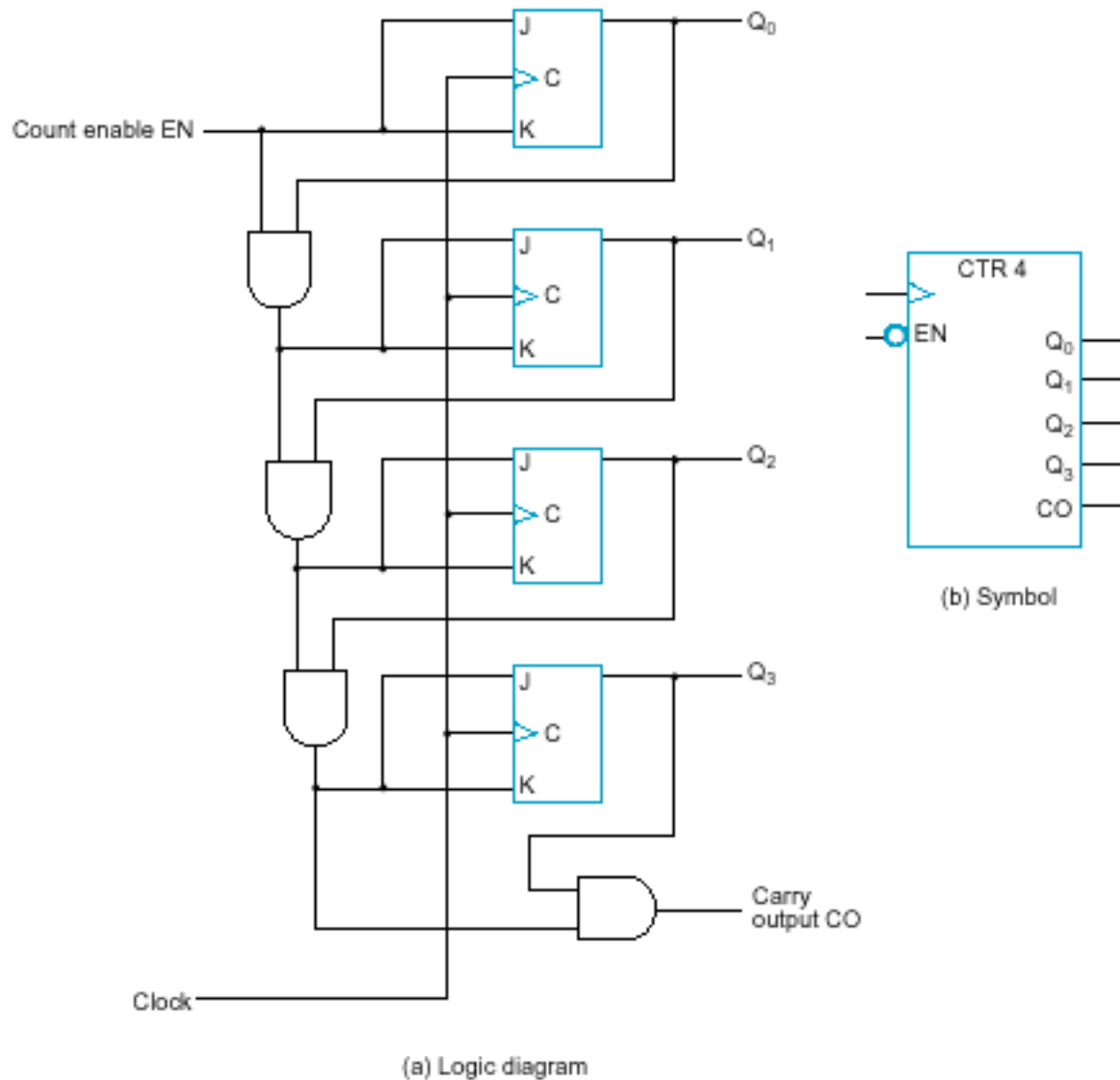
$$J_{Q_1} = Q_0$$
$$K_{Q_1} = Q_0$$

$$J_{Q_2} = Q_0 Q_1$$
$$K_{Q_2} = Q_0 Q_1$$

$$J_{Q_3} = Q_0 Q_1 Q_2$$
$$K_{Q_3} = Q_0 Q_1 Q_2$$

# Synchronous Binary Counters:

## J-K Flip Flop Design of a Binary Up Counter with EN and CO



EN = enable control signal, when 0 counter remains in the same state, when 1 it counts

CO = carry output signal, used to extend the counter to more stages

$$J_{Q_0} = 1 \cdot EN$$

$$K_{Q_0} = 1 \cdot EN$$

$$J_{Q_1} = Q_0 \cdot EN$$

$$K_{Q_1} = Q_0 \cdot EN$$

$$J_{Q_2} = Q_0 Q_1 \cdot EN$$

$$K_{Q_2} = Q_0 Q_1 \cdot EN$$

$$J_{Q_3} = Q_0 Q_1 Q_2 \cdot EN$$

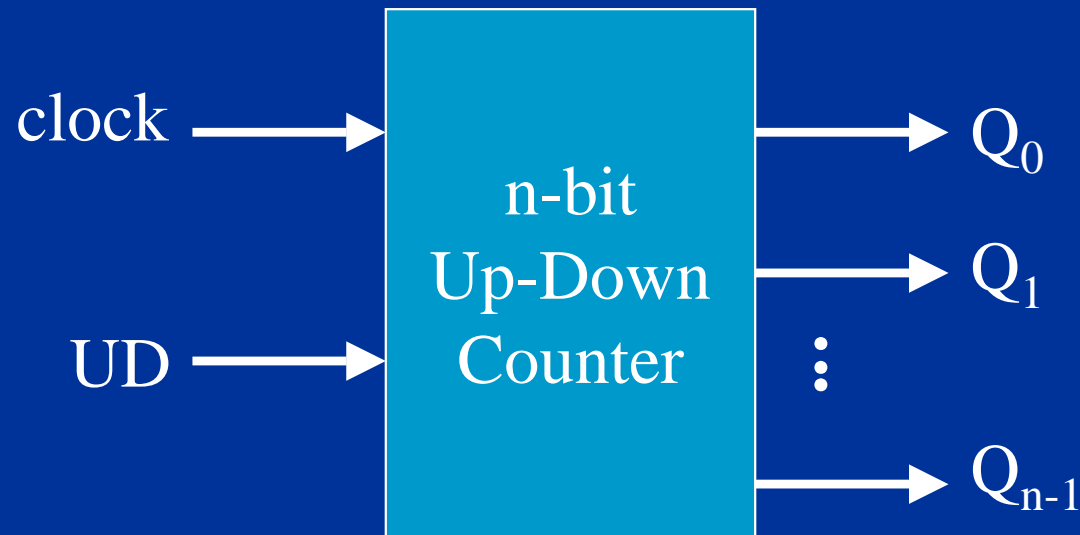
$$K_{Q_3} = Q_0 Q_1 Q_2 \cdot EN$$

$$CO = Q_0 Q_1 Q_2 Q_3 \cdot EN$$

# Serial Vs Parallel Counters

- If **serial gating** (chain of gates, info ripples through) is used  
→ serial counter
- If serial gating is replaced with **parallel gating** (this is analogous with ripple-logic replaced with carry-look ahead logic in our adder designs)  
→ parallel counter
- Advantage of parallel over serial counter:  
faster in certain occasions (1111 → 0000)

# Up-Down Binary Counter




UD = 0: count up


UD = 1: count down

# Up-Down Binary Counter (cont.)

UD	Q2	Q1	Q0	Q2.D	Q1.D	Q0.D	UD	Q2	Q1	Q0	Q2.D	Q1.D	Q0.D
0	0	0	0	0	0	1	1	0	0	0	1	1	1
0	0	0	1	0	1	0	1	0	0	1	0	0	0
0	0	1	0	0	1	1	1	0	1	0	0	0	1
0	0	1	1	1	0	0	1	0	1	1	0	1	0
0	1	0	0	1	0	1	1	1	0	0	0	1	1
0	1	0	1	1	1	0	1	1	0	1	1	0	0
0	1	1	0	1	1	1	1	1	1	0	1	0	1
0	1	1	1	0	0	0	1	1	1	1	1	1	0



Up-Counter



Down-Counter

# Up-Down Binary Counter (cont.)

		Q1 Q0			
		00	01	11	10
UD Q2	00				
	01				
	11				
	10				

Fill-in the Karnaugh maps for  $Q2.D$ ,  $Q1.D$ , and  $Q0.D$ , simplify, and derive the logic diagram using  
(a) D-FFs and (b) T-FFs

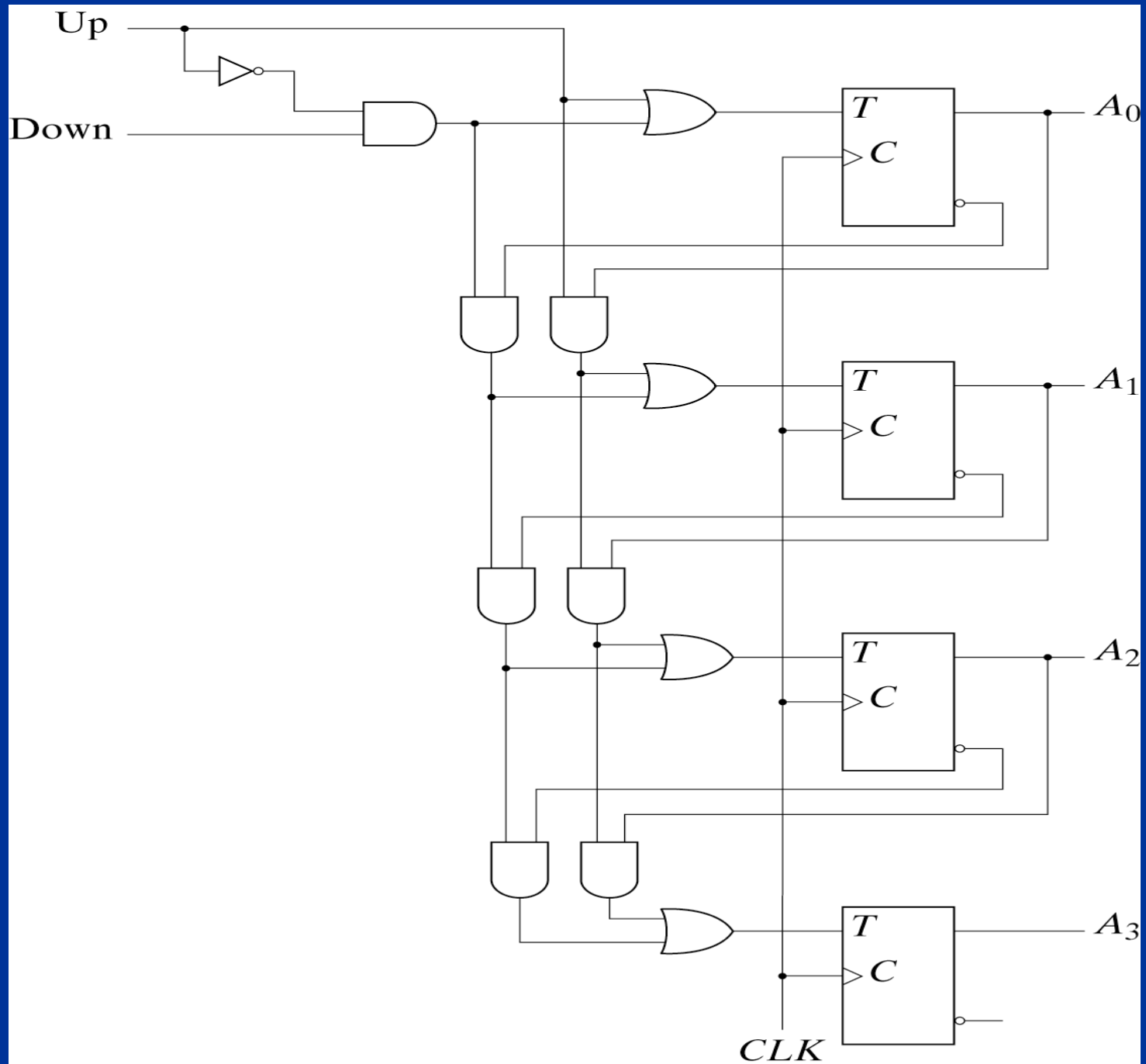


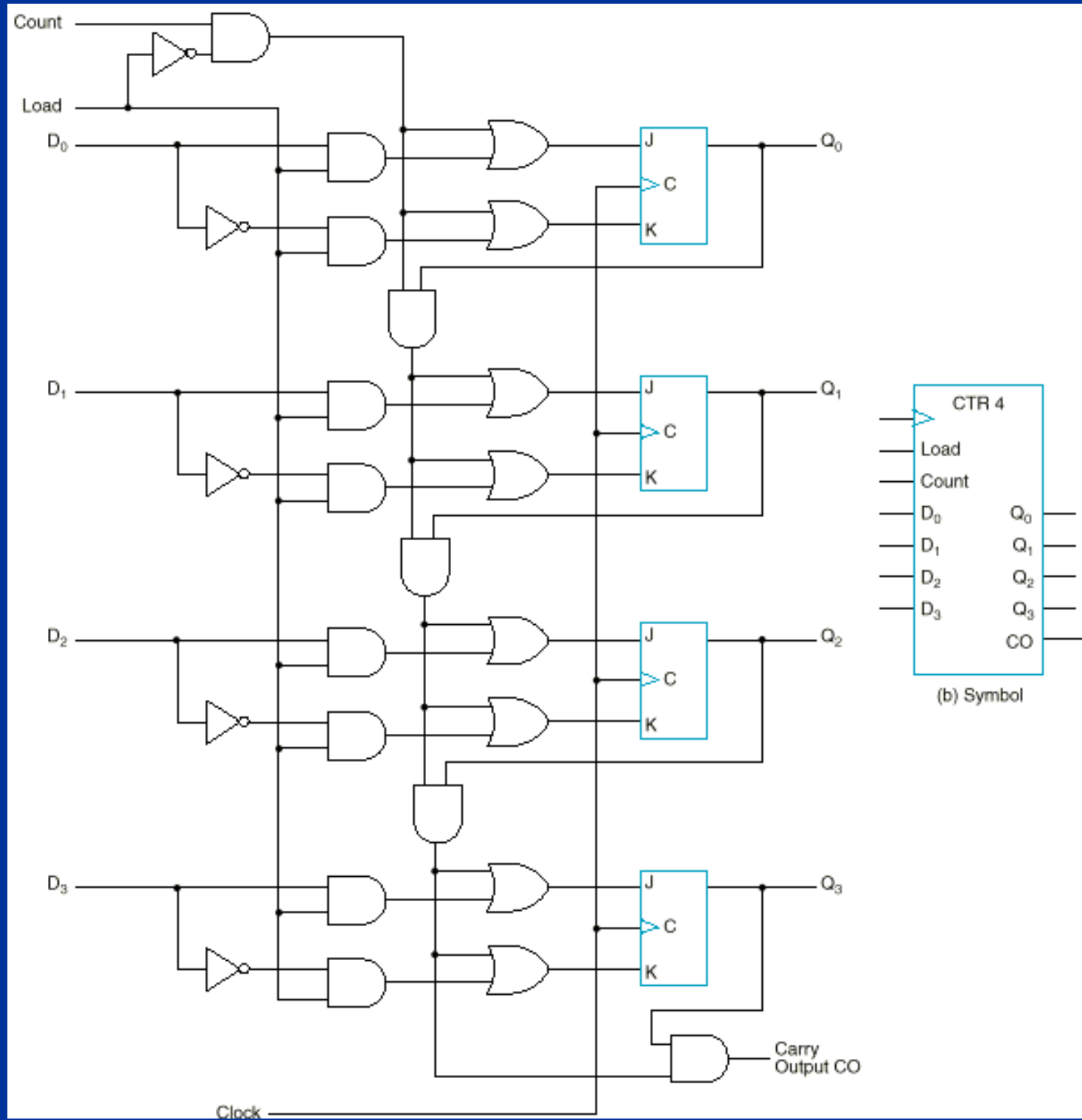
Fig. 6-13 4-Bit Up-Down Binary Counter

# Binary Counter with Parallel Load

- (Next slide) gives the logic diagram and symbol of a 4-bit synchronous binary counter with parallel load capability. The function table for this binary counter is

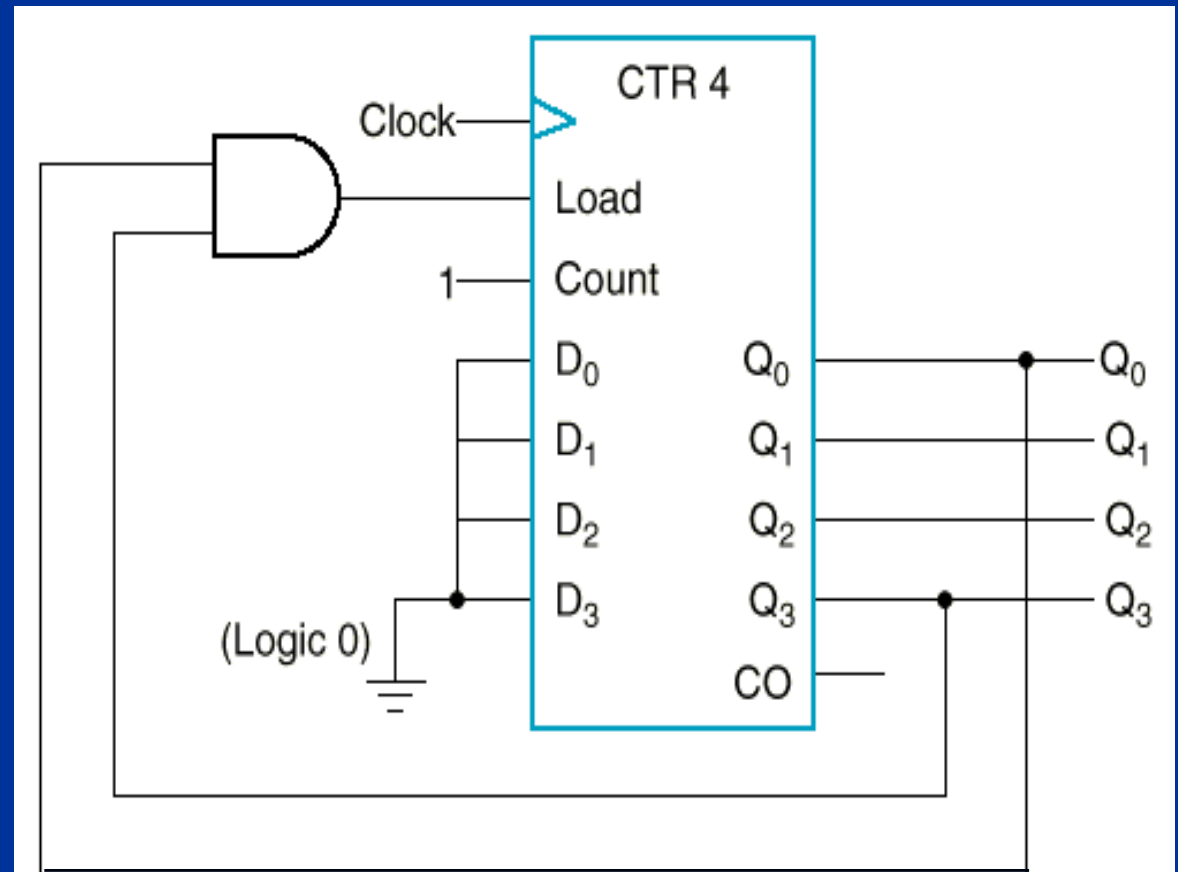
Load	Count	Operation
0	0	Nothing
0	1	Count
1	x	Load





# BCD counter

- The binary counter with parallel load can be converted into a synchronous BCD counter by connecting an external AND gate to it.



# BCD counter (cont.)

- The counter starts with an all-zero output.
- As long as the output of the AND gate is 0, each positive clock pulse transition increments the counter by one.
- When the output reaches the count of 1001, both  $Q_0$  and  $Q_3$  become 1, making the output of the AND gate equal to 1. This condition makes Load active, so on the next clock transition, the counter does not count, but is loaded from its four inputs.
- The value loaded then is 0000.

# Arbitrary Sequence Counter

- Given an arbitrary sequence, design a counter that will generate this sequence.
- Procedure:
  - Derive state table/ diagram based on given sequence
  - Simplify (using K-maps, etc)
  - Draw logic diagram
- Example: Use JK-FFs to draw the logic diagram for sequence generator (counter) for:  
 $0 \rightarrow 1 \rightarrow 2 \rightarrow 4 \rightarrow 5 \rightarrow 6 \rightarrow 0$

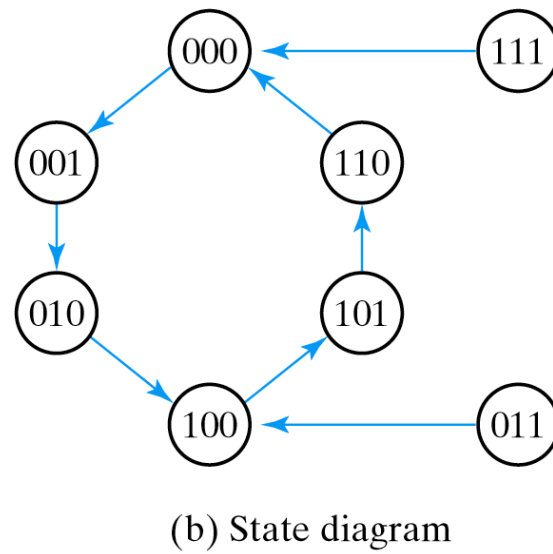
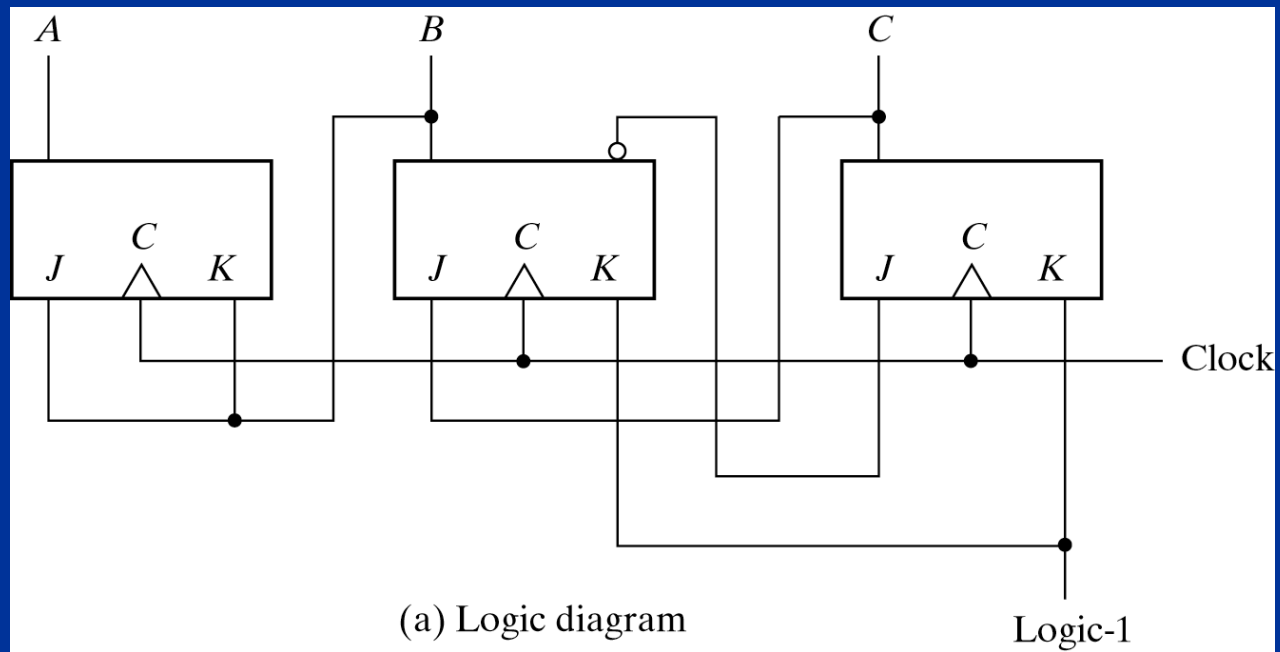
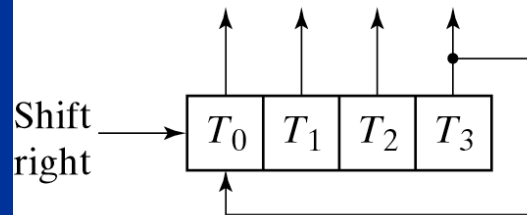
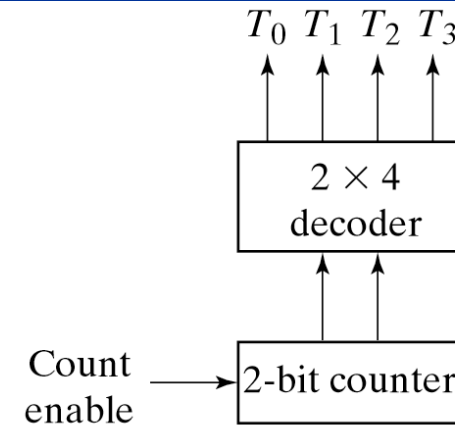


Fig. 6-16 Counter with Unused States

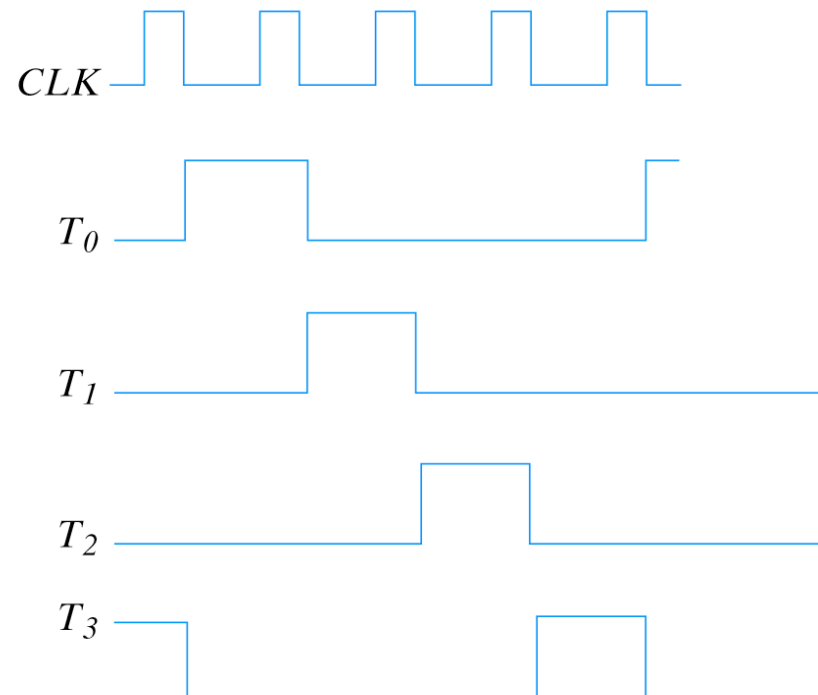
# RING COUNTER



(a) Ring-counter (initial value = 1000)



(b) Counter and decoder

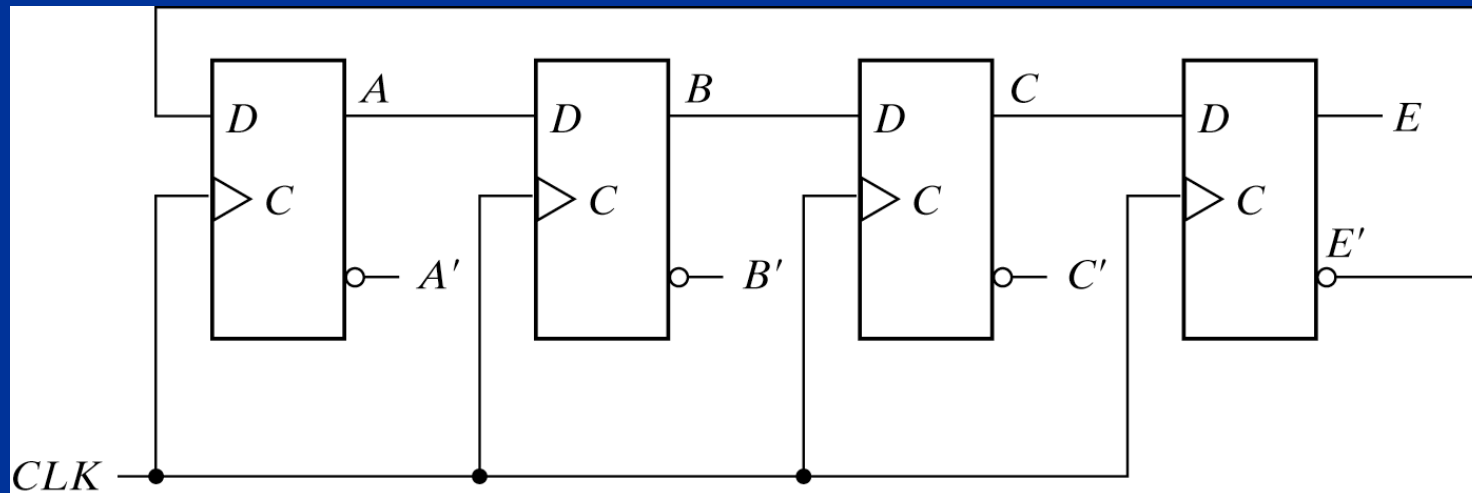


(c) Sequence of four timing signals

Fig. 6-17 Generation of Timing Signals

JOHNSON COUNTER/  
SWITCH TAIL COUNTER/  
TWISTED RING COUNTER





(a) Four-stage switch-tail ring counter

Sequence number	Flip-flop outputs				AND gate required for output
	A	B	C	E	
1	0	0	0	0	$A'E'$
2	1	0	0	0	$AB'$
3	1	1	0	0	$BC'$
4	1	1	1	0	$CE'$
5	1	1	1	1	$AE$
6	0	1	1	1	$A'B$
7	0	0	1	1	$B'C$
8	0	0	0	1	$C'E$

(b) Count sequence and required decoding

Fig. 6-18 Construction of a Johnson Counter