

- Partial marks will be awarded only when all steps are shown clearly and units written.
- All final answers to be highlighted by enclosing in a box.
- Use $\rho_{\text{water}} = 1000 \text{ kg/m}^3$, $g = 9.81 \text{ m/s}^2$ and $P_{\text{atm}} = 101 \text{ kPa}$

Q1a. What do you mean by Dirichlet and Neumann boundary conditions ? [2M]

Q1b. A concrete wall (specific gravity = 2.7) of rectangular cross-section 0.8 m high and 0.2 m wide is designed to retain mud to a depth h (see FigQ1b). The friction coefficient between the ground and concrete wall is 0.3, and the density of the mud (ρ_m) is 1800 kg/m^3 . There is concern that the wall may slide or tip over the lower left edge (i.e. A) as the mud level rises. Determine the mud height (h) at which (i) the concrete wall will overcome friction and start sliding and (ii) the wall will tip over (iii) Which is more critical sliding or tipping? [5M]

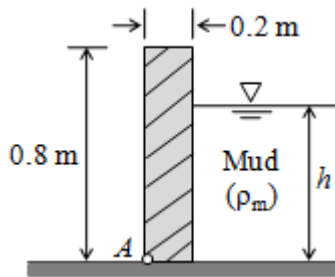


Fig Q1b

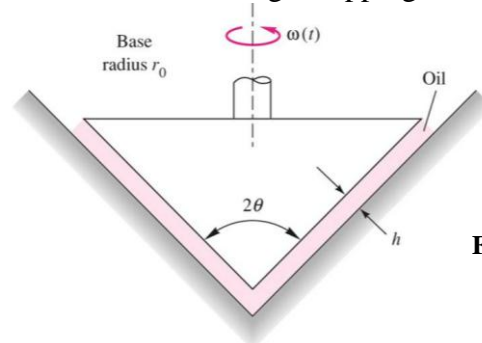


Fig Q2

Q2a. A solid cone of angle 2θ , base radius r_0 , and density ρ_c is rotating with initial angular velocity ω_0 inside a conical seat, as shown in Fig Q2. The clearance h is filled with oil of viscosity μ . Neglecting air drag and assuming the clearance is very small, derive an analytical expression for the cone's instantaneous angular velocity $\omega(t)$ if there is no applied torque. [6M]

Q2b. Show (i) Pseudoplastic fluid (ii) Dilatant fluid (iii) Bingham plastic fluid on shear stress-shear strain rate diagram.

Q3a. A pump is 2.5 m above the water level in the sump (water reservoir) (see Fig Q3a) and has a pressure of (-) 22 cm of mercury at the suction side (i.e. at pump inlet). The suction pipe is of 25 cm diameter and delivery pipe (pump outlet) is a 30 cm diameter pipe ending in a nozzle of 10 cm diameter. If the nozzle is directed vertically upwards at an elevation of 4 m above the sump water level (i.e. EL = 0 m). Neglecting all the losses, determine [8M]

1. The discharge (m^3/s)
2. The power input by pump (in kW)
3. Elevation (m) above the sump water level, to which the jet would reach.

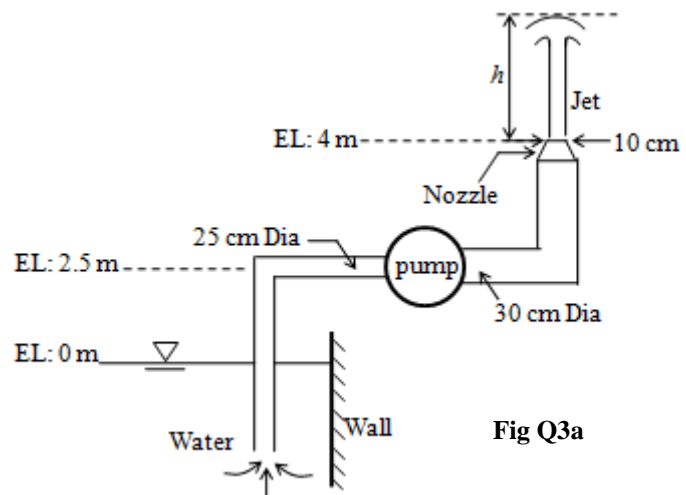


Fig Q3a

*EL = elevation

- Q3b. A simple flow system to be used for steady flow tests consists of a constant head tank connected to a length of 4 mm diameter tubing as shown in Fig Q3b. The liquid has a viscosity of 0.015 N-s/m^2 , a density of 1200 kg/m^3 , and discharges into the atmosphere with a mean velocity of 4 m/s . (i) The flow is fully developed in the last 3 m of the tube. What is the pressure at the pressure gage? (ii) What is the magnitude of the wall shearing stress, (τ) , in the fully developed region? [4M]

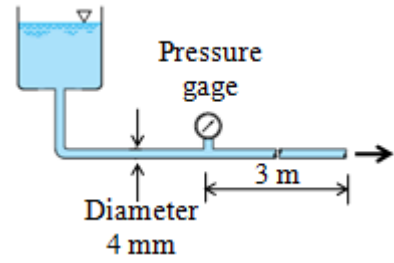


Fig Q3b

- Q4. Consider steady, incompressible, fully developed laminar flow of a Newtonian fluid in the annulus (See Fig Q4a) between two concentric pipes. Assume that the pressure is constant everywhere. The inner pipe is moving in axial direction (z -direction) with a steady velocity V_o to the right, and the outer pipe is stationary. [10M]
- State the boundary conditions and simplify z -component of Navier-Stokes equation.
 - Find out a general expression for the velocity profile $V_z(r)$ and the shear stress $\tau_{rz}(r)$.
 - A 10 cm diameter rod (Fig Q4b) is pulled steadily at 0.2 m/s through a 2.5 m long fixed cylinder whose clearance of 2.5 mm filled with a lubricating oil of density $\rho = 870 \text{ kg/m}^3$ and viscosity $\mu = 0.1 \text{ Pa-sec}$. Estimate the force required to pull the rod, using the velocity distribution deduced in part-(ii).
 - Calculate the shear stress ratio (i.e. $\tau_{rz} / \tau_{planar}$), where τ_{rz} is the shear stress obtained from (ii) and τ_{planar} is the shear stress at the surface of the inner cylinder that computed from a planar approximation obtained by assuming the annulus into a plane and a linear velocity profile in the gap.

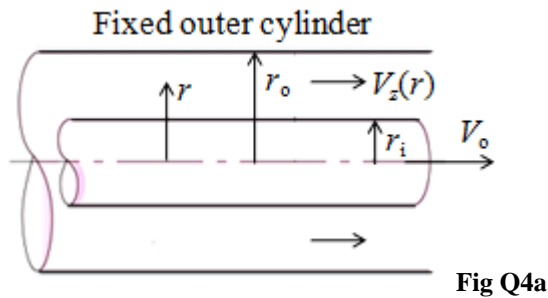


Fig Q4a

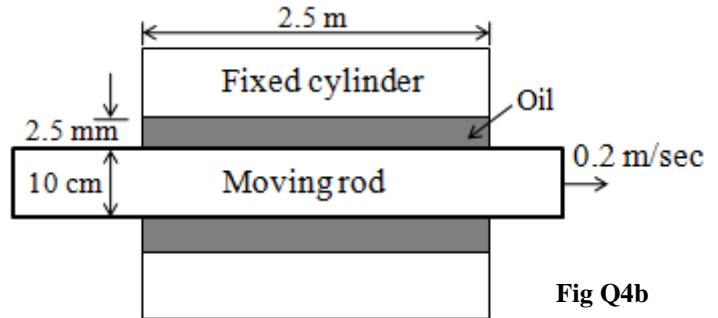


Fig Q4b

z- momentum equation:
$$\frac{\partial v_z}{\partial t} + (\bar{v} \cdot \bar{\nabla}) v_z = -\frac{1}{\rho} \frac{\partial p}{\partial z} + g_z + \frac{\mu}{\rho} \nabla^2 v_z$$

Laplacian operator:
$$\nabla^2 = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2}$$

Convective time derivative:
$$\bar{v} \cdot \bar{\nabla} = v_r \frac{\partial}{\partial r} + \frac{v_\theta}{r} \frac{\partial}{\partial \theta} + v_z \frac{\partial}{\partial z}$$

Continuity equation:
$$\frac{1}{r} \frac{\partial}{\partial r} (r v_r) + \frac{1}{r} \frac{\partial}{\partial \theta} (v_\theta) + \frac{\partial}{\partial z} (v_z) = 0$$

Shear stress
$$\tau_{rz} = \mu \left(\frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z} \right)$$