

Time value of Money

Time Value of Money: borrowing/renting money has a cost for the borrower; lending money should create value for the lender/investor.

Interest Rate: cost to borrow/compensation to the lender

Effective Interest Rate: actual but not usually stated. (annual interest rate compounded annually) $i_e = r_{(y/y)}$

Nominal Interest Rate r : annual interest rate i_s : compounding interest rate every sub period

Per m , Compounded x (compounding period) $r_{(x/m)} =$ Nominal rates with common compounding period: $\frac{r_{x/m_1}}{m_1/x} = \frac{r_{x/m_2}}{m_2/x}$
 m/x : # of compounding periods x in m . e.g. $r_{(m/y)}/12 = r_{(m/s)}/6$

$$(1 + i_s)^m = (1 + \frac{r}{m})^m = 1 + i_e = e^{i_s} = e^{\frac{r}{m}}$$

i_e effective annual rate: for any period: $(1 + i_s)^{\frac{m}{f}} = (1 + \frac{r}{m})^{\frac{m}{f}} = 1 + i_e$

$$i_e = (1 + \frac{r_{n/m}}{m})^{\frac{m}{n}} - 1$$

Where $\frac{m}{n}$ is # of compounding periods n in time m

And $\frac{y}{n}$ is # of compounding periods n per year.

f : frequency of payments per year (1: annual 12: monthly)

$$X\% \text{ per } A \text{ based on } B \text{ compounding : } r_{\text{effective annual}} = \left(1 + \frac{X\%}{\text{how many } B \text{ in } A} \right)^{\text{how many } B \text{ per year}}$$

If B =continuous $r_{\text{effective annual}} = \exp(\text{how many } A \text{ per year} \times X\%)$

working/trading 252 days/year 52 weeks/year

Note: interest rate $X\%$ (this is annual nominal interest rate!!!) compounded monthly, then $i_s = X\%/12$

Categories of Cash Flows

First/Capital Cost: expense to build or to buy and install

Operation and Maintenance (O&M) Cost: annual expense, e.g., electricity, labour, repairs

Salvage Value(s): receipt at project termination for disposal of equipment (can be a cost)

Revenues: annual receipts due to sale of products or services

Overhauls: major capital expenditure that occurs part way through the life of an asset

Prepaid Expenses: annual expenses, such as leases and insurance payments that must be paid in advance

Cash Flow Analysis

Cash Flow Diagram:

Compounding Interest Factor for Discrete Compounding:

Single Payment (Receipt/ Disbursement)	Compound Amount Factor $(F/P, i, N) = (1+i)^N$	Present Worth Factor $(P/F, i, N) = \frac{1}{(1+i)^N}$
Annuity	Sinking Fund Factor $(A/F, i, N) = \frac{i}{(1+i)^N - 1}$ <small>how much should be put aside per term to get F in the future</small>	Uniform Series Compound Amount Factor $(F/A, i, N) = \frac{(1+i)^N - 1}{i}$
	Capital Recovery Factor $(A/P, i, N) = \frac{i(1+i)^N}{(1+i)^N - 1}$ <small>how much to collect per term to recover an investment P today</small>	Series Present Worth Factor $(P/A, i, N) = \frac{(1+i)^N - 1}{i(1+i)^N}$
Arithmetic Gradients & Geometric Gradients	Arithmetic Gradient to Annuity Conversion Factor $(A/G, i, N) = \frac{1}{i} - \frac{N}{(1+i)^N - 1}$ $A_{\text{total}} = A_{\text{base}} + G(A/G, i, N)$	Geometric Gradient to Present Worth Factor $(P/A, g, i, N) = \frac{(P/A, i^\circ, N)}{1+g} = \left(\frac{(1+i^\circ)^N - 1}{i^\circ(1+i^\circ)^N} \right) \frac{1}{(1+g)}$ $i^\circ = \frac{1+i}{1+g} - 1$
	Perpetuities $N \rightarrow \infty$ Capitalized Value Formula $P = A/i$	Deferred Annuity $P_0 = A(P/A, i, N-J)(P/F, i, J)$ <small>(by J periods, first A at J+1)</small>
Capital Recovery Formula: $A(\text{amount to pay every term for investment today}) = [P(\text{investment}) - S(\text{Salvage/Scrap Value})] \times (A/P, i, N) + Si$		

T-Bill and Bonds

T-Bill: a government bond that pays Face Value at time of maturity with no coupon.

$$B = I \times \left[\frac{1 - 1/(1+k)^n}{k} \right] + F \times \left[\frac{1}{(1+k)^n} \right]$$

Bond: (a form of debt)

B: bond purchase price, Market Value F: Face/Par Value I: Interest = F × CR (coupon rate) Yield/Interest rate: k (annually)

Yield to Maturity (return on investment): actual interest earned from a bond over holding period

Semi-annual compounded; coupon rate not dividable, only get paid at specific dates:

find bond price of its last coupon payment, use yield rate to calculate future value (of today)

Coupon payment period = Interest compound period

Mortgage: Loan(F/P, i, N) - Payment/term(F/A, i, N)

Principal owing How much you owe after N mortgage terms, in term of repaying on the initial amount of loan

Loan Type: \$1000, N=5 years, interest rate=10%

① Business: 0 payback until end of time horizon(N years), in the end, pay (interest rate+1)^N × money

② Bond with Coupon/Interest only payment: pay coupon rate × money every year and (coupon rate+1) × money in the end

③ Equal Payment: 按annuity计算, (Capital Recovery Factor)

④ Constant Principle: 每年还 P/N + rest × interest rate e.g 1st year 1000/5 + 1000 × (1+10%) = 300 2nd year 1000/5 + (1000-200) × (1+10%) = 280

Comparison Method

Project: investment opportunities

Classification of projects: independent, mutually exclusive, related but not mutually exclusive, contingent

Minimum Acceptable Rate of Return (MARR): an interest rate that must be earned for any project to be accepted. (a lower limit for investment acceptability)

Cost of Capital: minimum return required to induce investors to invest.

① Present Worth Method (PW): compare projects by present worth of all cash flows associated with projects

(Future Worth Method FW)

② Annual Worth/Cost Method (AW): convert all cash flows to a uniform series (annuity)

③ Comparison of unequal lives Alternatives

i) Repeated lives; ii) Study period with Salvage Value assumption.

④ Payback Period Method: how long it takes to pay back investments (when interest rate=0)

⑤ Internal Rate of Return (IRR):

$$\sum_{t=1}^T \frac{Receipt_t}{(1+i^*)^t} = \sum_{t=1}^T \frac{Disbursement_t}{(1+i^*)^t}$$

$$i^* = r_1 + \frac{r_2 - r_1}{(Factor_2) - (Factor_1)} [(Factor_{i^*}) - (Factor_1)]$$

Interpolation:

Risk, Reward, Arbitrage

Market Investment:	Risk Free Investment	Cash Flow Scenario
$Market \begin{matrix} \nearrow c\% & Market_{high} \\ \searrow d\% & Market_{low} \end{matrix}$	$RiskFree \begin{matrix} \nearrow & RiskFree \\ \searrow & RiskFree \end{matrix}$	$FairPrice \begin{matrix} \nearrow & FairPrice_{high} \\ \searrow & FairPrice_{low} \end{matrix}$

$$Price_{high} = a \times Market_{high} + b \times RiskFree$$

$$FairPrice_{low} = a \times Market_{low} + b \times RiskFree$$

$$Fair Price = a \times Market Price + b \times RiskFree Price$$

$$\beta = \frac{r - r_f}{r_{mp} - r_f} \quad \text{where} \quad r = \frac{(c\% \times FairPrice_{high} + d\% \times FairPrice_{low}) - FairPrice}{FairPrice}$$

Given c% high and d% low,

Foreign Exchange:

$$FX_{future} = FX_{current} \times \frac{P_{1,current}(1+i_1)}{P_{2,current}(1+i_2)}$$

External Return for Portfolio(CAMP): $E[R_c] = r_f + \beta[E(R_{mp}) - r_f]$ $\beta = (\sigma_{c,mp}) / (\sigma_{mp}^2)$ σ_{mp} : variance of MP
 R_c : company rate r_f : risk free rate r_{mp} : market rate $\sigma_{c,mp}$: covariance of company & market portfolio

Arbitrage: practice of taking advantage of a price differential (mis-pricing) between two or more markets
 e.g. buy sth and sell it at \$12 one year later. At the same time, borrow money at 10%. (earn \$1 at year end)

Weighted Average Cost of Capital $r_{WACC} = \frac{E}{E+D}r_{equity} + \frac{D}{E+D}r_{debt}(1-tax)$

expected return of equity $r_c = r_k + \beta(r_m - r_k)$ CAPM(capital asset pricing model)

market premium $= r_m - r_k$

Present worth: for different timeline, duplicate with L.C.M. take into account interest and P/F factor

Annual worth: use A/P formulas

Payback period : First cost/ Annual savings, ignore time value of \$ and expected service lifetime

IRR: find interest rate of each project, must be larger than MACC to be acceptable.

Incremental IRR: To evaluate best alternative, take (challenger-defender) and find IRR, it's better if $IRR > MARR$.

MARR usually corresponds to cost of capital

ERR: for project that pays at the beginning.

Estimated ERR: Assume money cannot be reinvested in the project. Set inflow at MARR, outflow at ERR, future value = 0, then find ERR.

Actual ERR: Reinvest money into the project at ERR, get future value = 0. Same as IRR calculations

1.MARR

3.Present Worth/Cost

4.Annual Worth/Cost

5.Payback Period: First Cost – Annual Savings, ignore time value of money

6.IRR

incremental IRR: take (challenger – reference), If $\Delta IRR > MARR$, take challenger

7.ERR:

Estimated ERR:

assume money cannot be reinvested in the project, inflow@MARR outflow@ERR and future value=0

Actual ERR: reinvest money into project at ERR get future value=0, same as IRR calculation

1. Depreciation: loss in value of asset

2. Reason:

- ① Use-Related Physical loss: As something is used, parts wear out. measured by units of production.
- ② Time-Related Physical loss: due to environmental factor, endogenous physical factor (e.g iron rust) expressed in units of time.
- ③ Functional Loss: styles change, no longer fashionable, legislative change expressed in terms of particular unsatisfied function.

3. Value of Asset

- ① Market Value: actual value an asset can be sold for in an open market. (only way to determine actual MV is to sell it), so MV usually means an estimate of MV by a depreciation model (reasonably to captures true loss in value) etc.
- ② Book Value: depreciated value of an asset for accounting purposes, calculated with a depreciation model for some purposes. (maybe more or less than MV)
- ③ Scrap Value: actual value of an asset at the end of physical life (when it is broken up for material value of its parts); or an estimate of scrap value calculated using a depreciation model.
- ④ Salvage Value: actual value of an asset at the end of useful life (when it is sold); or an estimate of salvage value calculated using a depreciation model.

4. Depreciation Model/Method: estimate loss in value and remaining value of asset at any point in time

- ① **Straight-Line**: Book Value of an asset diminishes by an equal amount each year.

Depreciation Amount each year: $D = (P - S) \div N$ Book Value at Year n: $BV(n) = P - nD$

- ② **Declining-Balance**: Book Value of an asset diminishes by an equal proportion each year.

depreciation rate $d = 1 - (S/P)^{(1/n)}$

Book Value at Year n: $BV(n) = P(1 - d)^n = \$1172$ Depreciation Amount at year n: $D(n) = BV(n-1) - BV(n)$

- ③ **Sum-of-years'-Digits**: an accelerated method (like declining-balance), in which the depreciation rate is calculated as ratio of remaining years of life to the sum of the digits corresponding to the years of life.

Useful Life $N = 10$ years, $\text{Sum} = \frac{1}{2}N(N+1) = 55$

Total Depreciation amount	$D(2) = (P - S) \times (10 + 9) / 55$	$D(3) = (P - S) \times (10 + 9 + 8) / 55$
Book Value	$BV(2) = P - (P - S) \times (10 + 9) / 55$	$BV(3) = P - (P - S) \times (10 + 9 + 8) / 55$
Depreciation Amount	$D(3) = (P - S) \times 8 / 55 = 334.54$	

- ④ **Double-Declining**: a declining-balance method in which the depreciation rate is calculated as $2/N$ for an asset with a service life of N years. depreciation rate $d = 2/N = 0.2$ Book Value $BV(n) = P(1 - d)^n$

- ⑤ **150%-Declining**: a declining-balance method in which the depreciation rate is calculated as $1.5/N$ for an asset with a service life of N years. depreciation rate $d = 1.5/N$ Book Value $BV(n) = P(1 - d)^n$

- ⑥ **Units-of-Production**: depreciation rate is calculated per unit of production as the ratio of units produced in a particular year to the total estimated units produced over the asset's lifetime.

Units Depreciation $= (P - S) / \text{total units}$ Depreciation Amount: $D(1) = \text{units depreciation} \times \text{units of production year 1}$

1.

Income Statement 2010			
Revenue			
	Sales		780000
	Management fees earned		N/A
Expense			
	Cost of Sales	Material: Raw Materials Costs	-115000
		Labor: Manufacturing Labor Costs	-180000
	Gross Margin = Sales-Cost of Sales or COGS		780000-115000-180000=485000
	Selling, General, Administrative Expenses (SG&A)	Selling Expenses	-95670
		Administrative Expenses	-60000
		Miscellaneous Expenses 杂项费用	-18700
		Indirect Cost	-26000
	EBITDA		485000- (all) =284630
	Depreciation		-41000
	EBIT		284630-41000=243630
	Interest Expense		-5600
	Other loss/gain (Loss from investment) 投资别家股票		4000
Profit (EBT)	243630-5600+4000=242030		
Taxes	(taxes for the year) provision for taxes		-109200
Net Earning/Profit	132830 (After TAX income)		
	Extraordinary Gains/Losses		N/A
Profit after taxes	Net Earning-Extraordinary Gains/Losses=....		
	(Account in Balance Sheet) Retained Earning = Net income - Dividends (Addition to Equity)		

Note: Cost of Sales=material+labor (manufacturing company) is equivalent to Cost of Goods Sold (merchandising company)

2.

Balance Sheet/Position Statement 2010

Asset			Liability		
Current Asset	215+55.8+92+12+55=429.8		Current Liability	40+22+18+109.2=189.2K	
	Cash	215000		Account Payable	40000
	Account Receivables	55800		Bond Payable	22000
	Inventories	92000		Payroll 工资单Taxes Payable	18000
	Prepaid Expenses (Paid Utility for 2011)	12000		Taxes for the year	109200
	Investments	55000		Wage Payable	N/A
				Short Term Debt	N/A
Non-Current Asset	land + build&equip, Net =85+120+655=860K			Accrued Liabilities	N/A
	Land (无 depreciation)	85000	Non-Current Liability	45K	
	Newly Purchased Land	120000		Long term Notes Payable/Debt	45000
	Plant and Equip. at Cost	①	Total Liability	234.2K	
	Less: Depreciation	②	Equity		
	Building and Equipment NET of Depreciation	①-②=655000		Retaining Earning 2010	110000
Total Asset	1289.8K			Retaining Earning 2011 (Net Income-Dividends)	124830
				Common Stock Paid in Capital, Primary	620970
				Paid in Capital, Others (Beginning of 2010)	199800
			Total Equity	1055.6K	
			Total L+E	1289.8K	

Balance Sheet

Asset:

resources owned by a business that will provide future economic benefit

Liabilities: claims of creditors, obligations of the business resulting from past transactions

SE: claims of owners=common stock+retained earnings + paid-in capital,others

① **Current Assets** (assets that a company expects to convert to cash or use up within one year or its operating cycle, whichever is longer/resources whose benefits will be realized within one year)

Listed order: liquidity (the order in which they are expected to be converted into cash)

cash; short-term investment; (note/accounts/interest) receivables; inventories; (insurance, supplies) prepaid expenses

Non-Current Assets: ② **Long-term investments**

investments in stocks/equity securities & bonds/debt of other corporations that are held/not sold within more than one year; short-term debt (t-bills)

long-term assets (land/buildings that not currently using in operating activities)

③ **Property, Plant, Equipment** (assets with relatively long useful lives that are currently used in operating the business)

Listed order: Permanency

land; buildings; equipment; delivery vehicles; furniture

④ **Intangible Assets** (no physical substance, represent a privilege/right)

goodwill; patent; copyright; trademark; trade name (exclusive right of use for a specified period of time)

1. Liabilities

① **Current Liabilities** (obligations that the company is to pay within the coming year or operating cycle, whichever is longer) due within one year

(accounts/wages/notes & bank loans/interest/income taxes) payable; bank advances; current maturities/portions of long-term obligations (payments to be made within the next year on long-term debt)

② **Non-Current/Long-term Liabilities** (obligations that company expects to pay after one year)

(bonds/mortgages/long-term notes) payable; liabilities (lease/pension); derivative instrument

2. Stockholders Equity

① **Share Capital/Common Stock**: (investments of assets into the business by the stockholders in exchange for common/preferred shares; primary ownership interest in a corporation)
(Paid in Capital-Primary)

② **Retained Earnings** (income retained for use in the business/accumulated earnings of corporation that have not distributed to shareholders)

Revenue - Expenses - Dividends Paid

③ **Paid in Capital-Others**: brought in partners

3.

T-account																
D/C Rules	Assets		Liabilities		Stockholders' Equity											
					Common Stock		Retained Earnings =		Revenues		-Expenses		-Dividends		Paid-in Capital	
(Normal Balance)	Dr.	Cr.	Dr.	Cr.	Dr.	Cr.	Dr.	Cr.	Dr.	Cr.	Dr.	Cr.	Dr.	Cr.	Dr.	Cr.
	+	-	-	+	-	+	-	+	-	+	+	-	+	-	-	+

Transactions v.s Account			
	Assets	Liabilities	Stockholders' Equity
cash investment by stockholders	cash +		common stock +
note issued in exchange for cash	cash +	notes payment +	
purchase of equipment for cash	cash - equipment +		
cash receipt in advance from customer	cash +	unearned service revenue +	no record revenue until it has performed work
Paid rent for next month	cash - prepaid expenses +		
Sold goods for cash	cash +		RE State revenue + (revenue increases SE)
services provided for cash	cash +		RE State revenue + (revenue increases SE)
services provided on account	account receivable +		RE State revenue +
inventories purchased on account	inventory +	account payable +	
inventory sold	Inventory - Cash +		RE State Cost of Sales + Revenue +
Depreciation	Plant & Equipment -		Deprecation expense +
payment of rent	cash -		RE State expense + (expenses decrease SE)
expenses paid in cash	cash -		SGA +

payment of insurance policy for cash	cash - prepaid insurance +		
purchase of supplies on account	supplies +	account payable +	
hire new employees	accounting transaction not occurred		
payment of Dividend	cash -		RE State dividend + (dividends are reduction of SE but not an expense)
payment of cash of employee salaries	cash -		RE State SGA expense + (expenses decrease SE)
Collection of account receivable	cash + account receivable -		
Paid account payable	cash -	account payable -	
Tax (to be paid later)		tax payable +	RE State Tax expense +
Buy Stocks	cash - investment +		
bring in a partner for	cash +		Paid-in Capital other + (contributed surplus)

Income Statement will be close to "Retaining Earning" on Balance Sheet (**RE State**)

4. Financial Ratio Analysis/Performance Measures:

Comparison for the same company from previous year (trend analysis) or with industry standard ratios.

① Liquidity Ratio:

short-term ability of a business to meet its current liability/cash obligations, to weather unforeseen fluctuations in cash flows.

$Working\ Capital = Current\ Assets - Current\ Liabilities$	(+)-short-term debt paying ability
$Working\ Capital\ Ratio = Current\ Ratio = \frac{Current\ Assets}{Current\ Liabilities}$	(+): improved ability to meet maturing short-term obligations) short-term liabilities paying ability, >1 not good financial health, 2 is considered adequate Note: no consider the composition of the current assets
$Acid - Test\ Ratio = Quick\ Ratio = \frac{Quick\ Assets}{Current\ Liabilities}$	Quick Asset (highly liquid) = Cash + Account/Note Receivable + temporary Investments i marketable securities 1 is considered adequate Note: inventory and prepaid expenses more difficult to turn into cash

② Leverage Ratio/Debt-Management Ratio: how a firm relies on debt for its operation

$Equity\ Ratio = \frac{Total\ Equity}{Total\ Assets}$	(+): The smaller the ratio, the more dependent the firm is on debt for its operations and the higher are the risks the company faces.
$Debt\ to\ Total\ Assets = \frac{Total\ Liabilities}{Total\ Assets}$	

③ Efficiency Ratio/Asset-Management Ratio: how efficiently a firm is using its assets

$Inventory\ Turnover = \frac{Sales}{Inventories} = ? \text{ turns/year}$	$Days\ in\ Inventory = \frac{365\ days}{Inventory\ Turnover} = ? \text{ days/turn}$
how efficiently a firm is using its resources to manage its inventories (number of times that its inventories are replaced)	(-)-number of days inventory is held
$Inventory\ Turnover = \frac{Cost\ of\ Goods\ Sold}{Average\ Inventory}$	
(+): take the company shorter to sell inventory and less chance of inventory obsolescence) liquidity of inventory	
$Net\ Profit\ Ratio = \frac{Net\ Income}{Total\ Assets}$	

④ Profitability Ratio: how productively assets have been employed in producing a profit

$Return\ on\ Assets = Net\ Profit\ Ratio = \frac{Net\ Income}{Total\ Assets}$	Net income before extraordinary items!!! (+)
ROA	
$Return\ on\ Equity = \frac{Net\ Income}{Total\ Equity}$	(+) Measure of Investment Performance. how much profit a company has earned in comparison to the amount of capital that the owners have tied up in the company
ROE	

⑤ Market Ratio:

$Earnings\ Per\ Share = \frac{Net\ Income - Preferred\ Stock\ Dividends}{Average\ Common\ Shares\ Outstanding}$	
$Price - Earnings\ Ratio = \frac{Stock / Market\ Price\ Per\ Share}{Earnings\ Per\ Share}$	

Liquidity ratios *Acid-test ratio: (Cash & equivalents + AR)/ CL (excl. inv and prepaid exp)

*Interval measure (days): (cash+securities+AR)/(COGS+SGA)

Debt-management ratio

*Leverage ratio: Asset/Equity *Receivables in Days: AR/ daily sales *Cash flow coverage ratio: EBITDA/(Int+LT Debt repayment)

*Payables in Days: AP/ daily purchases (purchases = COGS + End Inv. - Start Inv.)

✱Times interest earned: EBIT/ Interest exp.

✱Cash flow/debt: cash from operation/ total debt

Asset management ratio

✱Inv.-turnover ratio: COGS (or Sales)/ Avg. Inv.

✱Inv. In days: 365/Inv.-turnover ratio

✱Working capital turnover: Sales/WC

Profitability ratio

✱Gross margin %: GM/sales

✱Profit margin: Net income/Sales

✱Operating profit margin: (EBIT-taxes)/Sales

✱EBIT margin: EBIT/Sales

✱EPS: (net income-pref div)/#common shares

Dividend policy dividend per common share=common stock dividends/number of common shares

✱Dividend yield: dividend per common share/ market price of common shares

✱Dividend Payout: Dividend on common share / (net income-preferred dividend)

Overall Performance Measure

✱ROA (net profit ratio): Net income/ total assets

✱ROA* = (NI+I(1-tax))/ total assets

✱ROPC (permanent cap.): (NI+I(1-tax))/(LT liab. + equities)

✱ROIC (invested cap.): (NI+I(1-tax))/(LT liab. + Current portion of LT debt + equities)

✱ROE: net income/ total equity

Return on Sales=Net Profit/Sales

Receivables in Days=Account Receivables/Daily Sales

Inventory in Days=Inventory/Daily COGS

Payment Period in Days=Account Payables/Daily Purchases

Profitability Index/profit investment ratio/value investment ratio=PV of future CF/Initial investment (>1)

is the ratio of payoff to investment of a proposed project. It is a useful tool for ranking projects because it allows you to quantify the amount of value created per unit of investment.

Taxation

CCA(Capital Cost Allowance) rate $d=15\%$ inflation 2.88% risk-free rate r_k market portfolio return r_m
 straight-line depreciation no debt beta $\beta=0.5$
 tax rate $t=25\%$ current interest $r_c=r_k+\beta(r_m-r_k) = 7\%$ real interest $r_{real}=(1+r_c)/(1+f)-1 = 4\%$
 after tax interest rate $i=r_{real}$

$$MARR_{after\ tax} \cong MARR_{before\ tax}(1-tax) \quad IRR_{after\ tax} \cong IRR_{before\ tax}(1-tax)$$

$$Tax\ Benefit\ Factor = \frac{td}{i+d}$$

$$CCTF_{new} = \text{Capital Tax Factor} = 1 - \frac{td(1 + \frac{i}{2})}{(i+d)(1+i)}$$

$$CCTF_{old} = \text{Capital Salvage Factor} = 1 - \frac{td}{(i+d)}$$

$$NPV = -FC \times CTF + (Annual\ Revenue - Expenses) \times (1-t) \times \text{discounting factor} + S \times CSF \times \text{discounting factor}$$

Depreciation with CCA: if specific depreciation method not mentioned, assume declining balance with CCA provided. IF mentioned, either finding appropriate CCA formula or include the effects of tax savings in cashflows directly

CCTF:

Use the formulae to calculate these from tax rate, CCA (depreciation rate), and discount rate. Always use an after-tax actual discount rate. You use the $CCTF_{old}$ for salvage value no matter what. If the purchase is before 1981, also use $CCTF_{old}$ for the first cost; otherwise, use $CCTF_{new}$.

Discounting: If you have actual cash flows, use the after-tax actual discount rate.

If you have real cash flows, calculate the after-tax real discount rate and use that (only for discounting, not for CCTF stuff!).

If you have a combination of cashflows, pick one and convert the others to that.

Finding the after-tax actual discount rate:

• If the project is equity funded, just use r_e (recall equity not taxable).

• If there is both debt and equity, use $WACC_{AT} = \frac{E}{E+D}r_e + \frac{D}{E+D}r_d(1-t)$ E and D are market values of equity and debt.
 – r_d , if not given, can be estimated from the yield rate of an outstanding bond the company already has out

– r_e , if not given, can be found from CAPM: $\beta = \frac{r_e - r_f}{r_m - r_f}$

Inflation

Inflation: increase over time in average prices of goods and services; decrease in purchasing power of money over time. (most developed countries)

Deflation: decrease over time in average prices of goods and services; increase in purchasing power of money over time.

How inflation influence MARR, IRR, PW of a project?

Real Money in the future = Actual Money received in the future $\times (P/F, f, N)$

Money \rightarrow Money $\times (1+i)$ without inflation (consider time value of money)

Money \rightarrow Money $\times (1+i)/(1+f)$ with inflation

$$Real_n = \frac{Actual_n}{(1+f)^n} \quad \text{real} \quad 1+i_{real} = \frac{1+i_{Actual}}{1+f} \quad 1+MARR_{real} = \frac{1+MARR_{Actual}}{1+f} \quad 1+IRR_{real} = \frac{1+IRR_{Actual}}{1+f}$$

Note: use actual values with actual interest, Or real with reals, do not mix!

$$PW_firstcost = -FC + [0.5FC * TBF + 0.5FC * TBF * (P/F, i, 1)] \quad \text{or} \quad -FC * CCTF_{new}$$

$$AW_firstcost = PW_firstcost \times (A/P, i, N)$$

$$AW_savings = saving(1 - tax)$$

$$AW_salvage = salvage(1 - TBF)(A/F, i, N)$$

Replacement Decisions

$$EAC = (P - S)(A/P, i, N) + Si \quad \text{Capital cost} = (\text{First cost} - (\text{Trade-in offer price} - \text{market price}))$$

Same competitor and defender: if defender > economic life = replace

New competitor: calculator $EAC_{defender}$ to move fwd 1 year, compare to $EAC_{competitor}$ at optimum economy life