

0.Problem Statement and Analysis:

Design a color organ, a device often used at concerts and similar venues to light up different colors of lights depending on the frequency content of the incoming music signal. For instance, one light may turn on for bass (low frequencies), and one for treble (high frequencies). Commercial versions often have multiple lights corresponding to different bands of frequencies.

—The two to be detected bands are bass and treble, with frequency in the range of human hearings.

Bass: 60Hz to 250 Hz

Treble: 2kHz to 16kHz

The range of human hearing is generally considered to be 20 Hz to 20 kHz, but it is far more sensitive to sounds between 1 kHz and 4 kHz.

—Lights can be treated as a ‘black box’ component with two states: $V_{in} < 3\text{Volts}$, light off; $V_{in} > 3\text{Volts}$, light on.

Lights are self-powered and draw no input current.

—Power supplies of any desired (finite) voltage and ideal op-amps are available.

—The incoming music signal (**input**) ranges from ~50-100 mV peak-to-peak at low volumes to ~500mV-1000mV peak-to-peak at high volume. You can assume an ideal music signal source (zero output resistance).

1.Engineering Specification:

1.Requirements of products:

- 1.The product should be able to detect two bands separately, bass and treble, with frequency in the range of human hearings.
- 2.Filter and/or amplifier sets are implemented to split off signals in specific frequency range, amplify and output them to the Light circuit.
- 3.All values picked for passive devices (including resistors, capacitors, and inductors) should be standardized, real-world decade values.
- 4.The budget and power budget shall be minimized if possible.
- 5.The order of filters are designed to be as less as possible, to reduce the total number of op-amps in circuit, meaning the complexity of the circuit.
- 6.The final gain of each filter & amplifier should be at least 60 (35dB), which corresponds to zero margin.

2.Assumptions:

- 1.Bass ranges from 60Hz to 250 Hz, while treble ranges from 2kHz to 16kHz.
- 2.Lights are treated as black box, self-powered and draw no input current; It turns on when the input voltage exceeds 3 Volts, otherwise, stays off.
- 3.Ideal music signal source is with zero output resistance and it ranges from ~50-100mV peak-to-peak to ~500mV-1000mV peak-to-peak.
- 4.Assume ideal op-amps (infinite voltage gain, infinite input impedance & zero output impedance, infinite bandwidth, zero input offset voltage)

2. Block diagram of Conceptual Design Solution:

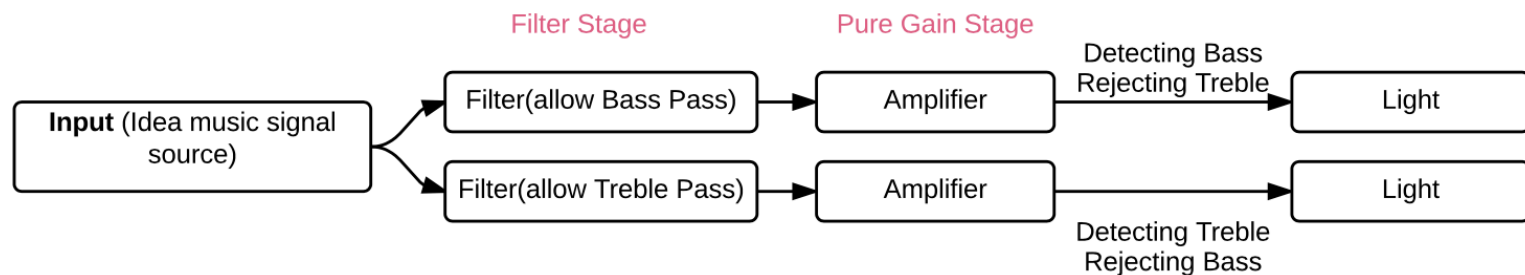


Figure · Conceptual Design Graph

- ①The input(music signal source) are voltage with zero output resistance ranges from $\sim 50\text{-}100\text{mV}$ peak-to-peak to $\sim 500\text{mV-}1000\text{mV}$ peak-to-peak.
- ②The filter allowing bass with amplifier is used to detect bass and reject treble, the output of this amplifier should be more than 3 Volts peak-to-peak, in order to illuminate the light successfully. Similarly, the filter allowing treble with amplifier is used to detect treble and reject bass.
- ③Keep in mind that if the output signal of the filter stage is already more than 3 Volts, then the pure gain stage is redundant and it can be removed from the circuit (its existence will be determined later).
- ④Lights are connected to the output of Filters & Amplifiers, which are the display devices of output.

Based on the research, the frequency of bass ranges from 60Hz to 250 Hz, while treble ranges from 2kHz to 16kHz. After converting them into unit in terms of rad/s ($\omega=2\pi f$, $6.28\text{rad/s}=1\text{Hz}$), the bass range is $[0.3\text{k}, 2\text{k}]$ rad/s, and the treble is around $[10\text{k}, 100\text{k}]$ rad/s. In this design, we can safely round those to numbers that are comfortable to work with. It is because the design purpose is to “separate” the input based on its frequency range and detect the existence of two types of input, categorized by range. If the design purpose is to detect a signal at a specific frequency, rounding would be prohibited or limited as much as possible.

3.Desired Bode Plot for each filtering element & Function of blocks:

1.Discussion of bode plot requirements:

Only the magnitude part of Bode Plot is necessary, since the input signal is supposed to be consistent and phase shift would not affect the functionality of detecting or rejecting. Designing based on merely the magnitude part provides more flexibility of poles/zeros localizations for transfer functions of the filters.

For detecting purpose, the circuit should function on the incoming signal with lowest voltage. Function here means being able to amplify the input signal to a detectable voltage level, namely 3 Volts in this case. The minimum voltage of input is around 50mV peak-to-peak. As an output exceeding 3 Volts can succeed in powering up the light, the magnitude of amplification is mandatory to be more than $3V/50mV=60$ (35dB). However, amplification rate is set to be 100, allowing a safety factor to be 66.67%. It could be redundant, but picking 100 gives convenience for conversion into Decibel base. Also, there is a 3dB difference at the corner of frequency for single pole/zero, adding a safety margin would avoid problems generated by the 3dB drop. As far as we know, $1dB = 20*\log_{10}(\text{Magnitude})$, so $100=40dB$.

The maximum voltage of input is 1000mV. Simply speaking, the output is potentially amplified to 100Volts, which is hazardous to Light circuit without doubt. However, the output of op-amps is always restricted by its power supply. It is feasible to choose op-amps with desirable saturated voltage to protect the Light circuit.

For rejecting purpose, the circuit should function on the incoming signal with highest voltage. Function here means being able to restrict the output corresponding to this input signal lower than a detectable voltage level, namely 3 Volts in this case. The maximum voltage of input is 1000mV. As an output less than 3 Volts, the magnitude of amplification is mandatory to be less than $3V/1000mV=3$ (9.5dB, round down to 9dB).

2.Requirements of filter allowing bass:

To amplifying the bass signal, the magnitude of bode plot on the range [0.3k, 2k] rad/s should be more than 40dB.

To rejecting the treble signal, the magnitude of bode plot on the range [10k, 100k] rad/s should be less than 9dB.

3.Requirements of filter allowing treble:

To amplifying the treble signal, the magnitude of bode plot on the range [10k, 100k] rad/s should be more than 40dB.

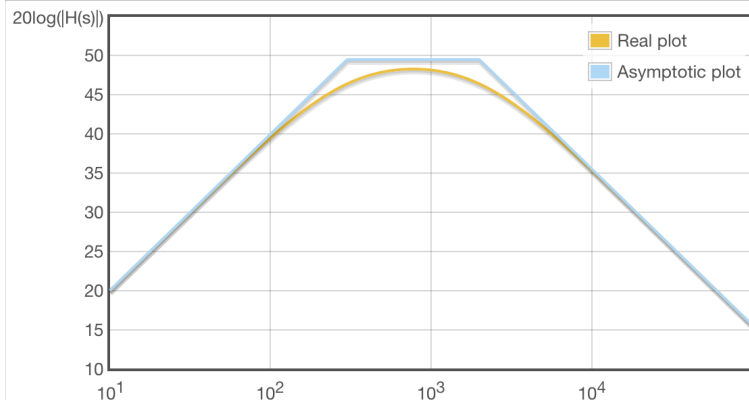
To rejecting the bass signal, the magnitude of bode plot on the range [0.3k, 2k] rad/s should be less than 9dB.

4.First Try: Filter Stage to be fully functional & no Pure Gain Stage:

The first discussion is on omitting the pure gain stage and achieve all this functions by merely the filter stage (each of the two filters may consist of multiple operational amplifiers, though). This would be the most preferable design as long as we can select real-value components for all passive components. As adding the pure gain stages, two more operational amplifier is required. This would potentially add up not only the budgeting, but also the complexity of the circuit.

For Bass, the amplifying range is [0.3k, 2k] rad/s with desired final gain to be 40dB. Simply pick:

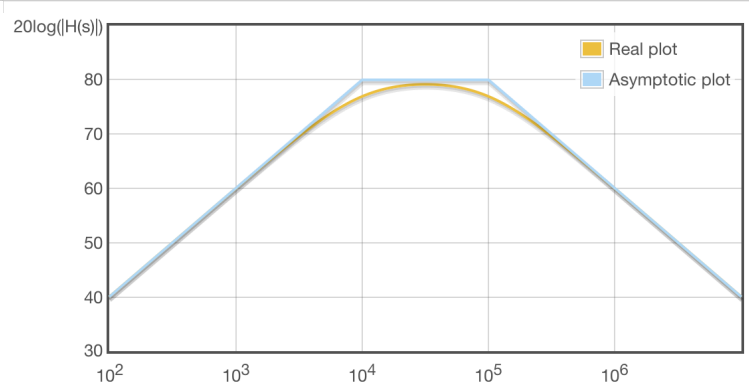
$$H_{bass} = K_1 \frac{s}{(s+0.3k)(s+2k)} = K_1 \frac{s}{s^2 + 2.3 \times 10^3 s + 6 \times 10^6}$$



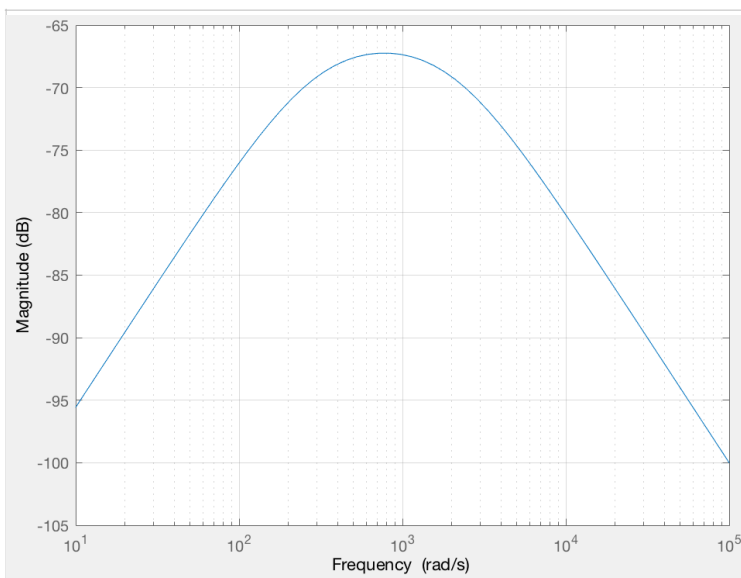
Bode Plot for H_{bass}

For Treble, the amplifying range is [10k, 100k] rad/s with desired final gain to be 40dB. Simply pick:

$$H_{treble} = K_2 \frac{s}{(s+10k)(s+100k)} = K_2 \frac{s}{s^2 + 1.1 \times 10^5 s + 10^9}$$



Bode Plot for H_{treble}

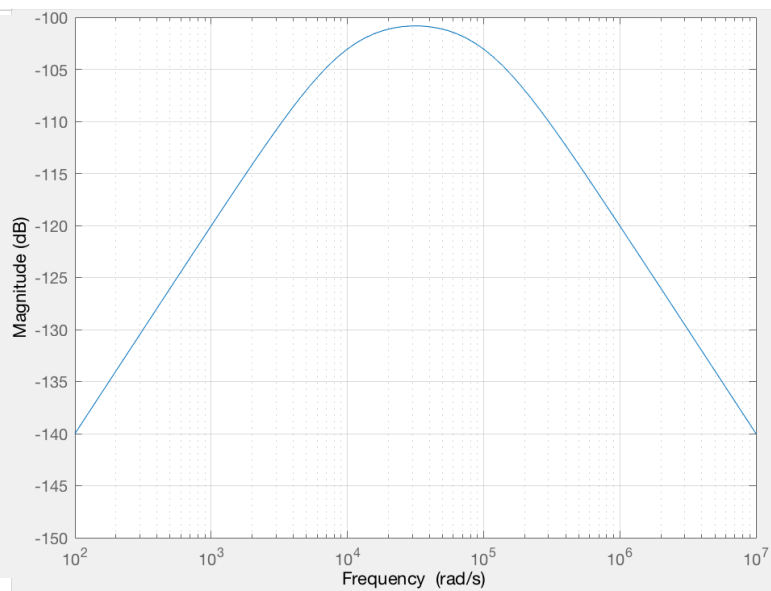


Bode Plot for H_{bass} (with $K1=1$)

Constraint of K1: $|H_{bass}(\omega = 0.3k)| \geq 40dB$ with $s=-j\omega$

Constraint of K1: $|H_{bass}(\omega = 10k)| < 9dB$ with $s=-j\omega$

$$K1 = 378600$$



Bode Plot for H_{treble} (with $K2=1$)

Constraint of K2: $|H_{treble}(\omega = 10k)| \geq 40dB$

Constraint of K2: $|H_{treble}(\omega = 2k)| < 9dB$ with $s=-j\omega$

$$K2 = 10000000$$

Note: The calculations of K1 and K2 are in appendix

5.Further Investigation: (TBD)

It will be proceeded to step 4~6, after step 6, if not all real-world passive components can be selected to achieve a desirable behavior. Going back to this step and focusing on other design options are required, as design is an iterative process. Even if a real-world circuit could be found through first trial, more investigations are sometimes also expected to plan for back-up solutions, or conduct cost/complexity analysis to optimize design.

4&5.Circuits to implement each block and Ideal Design of Filter Circuits:

Rule of Thumb states that Resistor-Capacitor circuit is universally used for low frequencies, while Resistor-Inductor is more suitable for high frequencies.

Known that, $H_{bass} = K_1 \frac{s}{(s+0.3k)(s+2k)}$ with $K_1=378600$ and $H_{treble} = K_2 \frac{s}{(s+10k)(s+100k)}$ with $K_2=10000000$

With an inverting amplifier, for every OpAmp we use, we can realize up to two distinct Poles or Zeros, and (as a side-effect) some fixed Gain. [reference: lecture notes]

1.For the bass part:

Group poles and zeros as pair 1 {zero@0, pole@0.3k} and pair 2 {no zero, pole@2k}

now we can see that the transfer function is divided as one PZG block & one first-order pole block.

①Pair 1 {zero@0, pole@0.3k} in terms of $H_{pair1} = K_{pair1} \frac{s+z}{s+p}$

Suppose we pick resistor-capacitor series combination for pair 1, then $K = -\frac{R_2}{R_1}$ $p = \frac{1}{R_1 C_1} = 300$ $z = \frac{1}{R_2 C_2} = 0$, which

is not realistic since we can never achieve a zero at origin using finite component values.

Then we try resistor-capacitor parallel combination, which is unrealistic as well.

Afterwards, move to resistor-inductor series combination, then $K = -\frac{L_2}{L_1}$ $p = \frac{R_1}{L_1} = 300$ $z = \frac{R_2}{L_2} = 0$

From rule of thumb, $L_1=400mH=400 \times 10^{-3}H$, then $R_1=400 \times 10^{-3}H \times 300=120\Omega$. $R_2=0$ and L_2 will be selected later to benefit combined circuit (pair1 + pair2) final gain.

Decision: $L_1=500mH=500 \times 10^{-3}H$, $R_1=150\Omega$, $R_2=0$, L_2 (TBD)

②Pair 2 {no zero, pole@2k} in terms of $H_{pair2} = K_{pair2} \frac{1}{s+p}$

Pick Z_4 be a resistor $Z_4=R_4$ ($L_4=0$, no inductor) and Z_3 be a resistor in series with an inductor, $Z_3=R_3+sL_3$.

$$\text{Now } K = -\frac{R_4}{L_3} \quad p = \frac{R_3}{L_3} = 2000$$

From rule of thumb, $L_3=50\text{mH}$, then $R_3=50 \times 10^{-3}\text{H} \times 2000 = 100\Omega$.

R_2 will, be select later to benefit combined circuit final gain.

Decision: $L_3=50\text{mH}$, $R_3=100\Omega$, R_4 (TBD)

③Selection of Gain:

$$\text{The total gain combined with pair 1 and pair 2 are } K = \left(-\frac{L_2}{L_1} \right)_{pair1} \left(-\frac{R_4}{L_3} \right)_{pair2} = \frac{L_2}{L_1} \frac{R_4}{L_3}$$

$$K=378600 \text{ requires } \frac{L_2}{L_1} \frac{R_4}{L_3} = 378600 \text{ with } (L_1)_{pair1}=500\text{mH and } (L_3)_{pair2}=50\text{mH, which results in } L_2 \cdot R_4=9465$$

We can safely select $(L_2)_{pair1}=500\text{mH}$ and $R_4=18930\Omega$

Decision: $(L_2)_{pair1}=500\text{mH}$ and $R_4=18930\Omega$

$$H_{bass} = \left(-\frac{sL_2}{sL_1 + R_1} \right) \left(-\frac{R_4}{sL_3 + R_3} \right) = \left(\frac{sL_2}{sL_1 + R_1} \right) \left(\frac{R_4}{sL_3 + R_3} \right)$$

$$\begin{aligned} \text{④Ideal Transfer Function:} \quad &= (9465 \cdot s) / ((s/20 + 100) \cdot (s/2 + 150)) \\ &= 378600 \frac{s}{(s + 300)(s + 2000)} \end{aligned}$$

2.For the treble part:

first group poles and zeros as pair 1 {zero@0, pole@10k} and pair 2 {no zero, pole@100k}

now we can see that the transfer function is divided as one PZG block & one first-order pole block.

①Pair 1 {zero@0, pole@10k} in terms of $H_{pair1} = K_{pair1} \frac{s+z}{s+p}$

$$\text{Try resistor-inductor series combination, then } K = -\frac{L_6}{L_5} \quad p = \frac{R_5}{L_5} = 10k \quad z = \frac{R_6}{L_6} = 0$$

From rule of thumb, $L_5=10\text{mH}=10 \times 10^{-3}\text{H}$, then $R_5=10 \times 10^{-3}\text{H} \times 10k=100\Omega$. $R_6=0$ and L_6 will be selected later to benefit combined circuit final gain.

Decision: $L_5=10\text{mH}=10 \times 10^{-3}\text{H}$, $R_5=100\Omega$, $R_6=0$, L_6 (TBD)

②Pair 2 {no zero, pole@100k} in terms of $H_{pair2} = K_{pair2} \frac{1}{s+p}$

Pick Z_2 be a resistor $Z_2=R_2$ and Z_1 be a resistor in series with an inductor, $Z_1=R_1+sL_1$.

$$\text{Now } K = -\frac{R_8}{L_7} \quad p = \frac{R_7}{L_7} = 100k$$

From rule of thumb, $L_7=50\text{mH}$, then $R_7=50 \times 10^{-3}\text{H} \times 100k=5k\Omega$. R_8 will be select later to benefit combined circuit final gain.

Decision: $L_7=50\text{mH}$, $R_7=5k\Omega$, R_8 (TBD)

③Selection of Gain:

$$\text{The total gain combined with pair 1 and pair 2 are } K = \left(-\frac{L_6}{L_5} \right)_{pair1} \left(-\frac{R_8}{L_7} \right)_{pair2} = \left(\frac{L_6}{L_5} \right)_{pair1} \left(\frac{R_8}{L_7} \right)_{pair2}$$

$$K_2 = 10000000 \text{ requires } \left(\frac{L_6}{L_5} \right)_{pair1} \left(\frac{R_8}{L_7} \right)_{pair2} = 10000000 \text{ with } L_5=10\text{mH and } L_7=50\text{mH, which results in } L_6 \cdot R_8=5000$$

We can safely select $L_6=500\text{mH}$ and $R_8=10000\Omega$.

Decision: $L_6=500\text{mH}$ and $R_8=10000\Omega$

$$H_{treble} = \left(-\frac{sL_6}{sL_5 + R_5} \right) \left(-\frac{R_8}{sL_7 + R_7} \right) = \left(\frac{sL_6}{sL_5 + R_5} \right) \left(\frac{R_8}{sL_7 + R_7} \right)$$

④Ideal Transfer Function:

$$= (5000*s)/((s/100 + 100)*(s/20 + 5000))$$
$$=10000000 \frac{s}{(s + 10k)(s + 100k)}$$

6.Schematics:

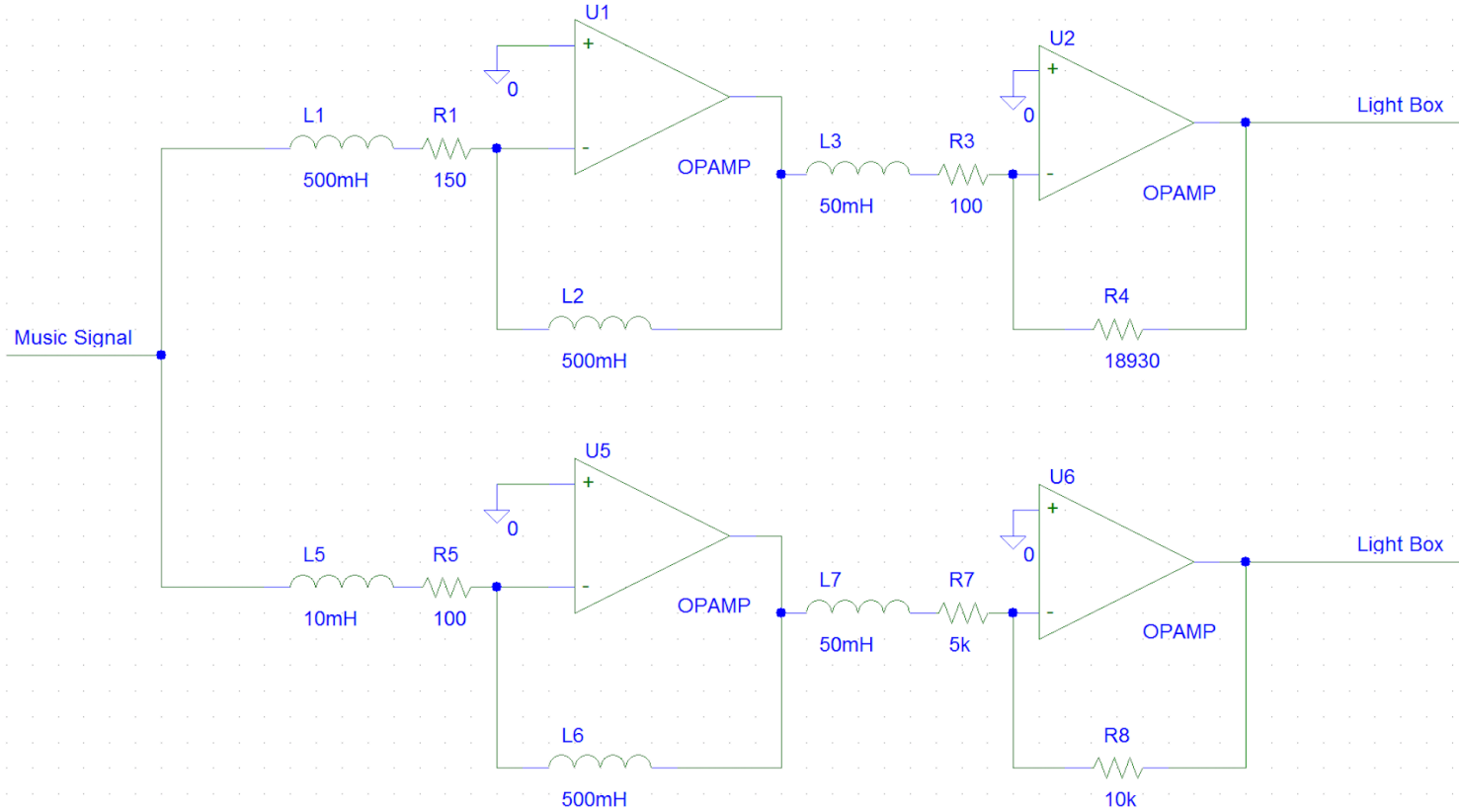


Figure · Schematics of Ideal Circuit

7.Standardized, real-world design:

See Appendix for the table of real-world passive components values. It is feasible to reach different passive components value by connecting multiple parts in series or parallel. However, selection of single passive component to achieve goals is prioritized.

Passive Component requires for idea design:

Component	Ideal	Real-World Value	Real-World
L1	500mH	500mH	http://catalog.triadmagnetics.com/Asset/C-36X.pdf
R1	150Ω	150Ω	https://www.seielect.com/Catalog/SEI-CF_CFM.pdf
L2	500mH	500mH	http://catalog.triadmagnetics.com/Asset/C-36X.pdf
L3	50mH	50mH	http://www.bourns.com/docs/Product-Datasheets/70f_series.pdf
R3	100Ω	100Ω	https://www.seielect.com/Catalog/SEI-CF_CFM.pdf
R4	18930Ω	1.8kΩ	https://www.seielect.com/Catalog/SEI-CF_CFM.pdf
L5	10mH	10mH	https://en.tdk.eu/inf/30/db/ind_2008/b82144a.pdf
R5	100Ω	100Ω	https://www.seielect.com/Catalog/SEI-CF_CFM.pdf
L6	500mH	500mH	http://catalog.triadmagnetics.com/Asset/C-36X.pdf
L7	50mH	50mH	http://www.bourns.com/docs/Product-Datasheets/70f_series.pdf

R ₇	5kΩ	5kΩ	http://www.ohmite.com/cat/res_50.pdf
R ₈	10kΩ	10kΩ	https://www.seielect.com/Catalog/SEI-CF_CFM.pdf

Note that there is resistor with 18900Ω and available as well, though not standard-value:
https://www.seielect.com/Catalog/SEI-RNF_RNMF.pdf
https://www.seielect.com/Catalog/SEI-CF_CFM.pdf

Op-Amp	LM324	http://www.st.com/content/ccc/resource/technical/document/datasheet/58/13/52/7e/39/8c/4c/ba/CD00000457.pdf/files/CD00000457.pdf/jcr:content/translations/en.CD00000457.pdf
Voltage Supply	Computer Power Supplies will be selected, which can provide 15V to Op-Amp	

One LM324 electronic contains 4 Op-Amp, which is enough for proposed solution. Jumper wires and bread board are suggested to use for the completion of the circuit.

Realistic Circuit Transfer Function:

$$H_{bass} = \left(-\frac{sL_2}{sL_1 + R_1}\right)\left(-\frac{R_4}{sL_3 + R_3}\right) = \left(\frac{sL_2}{sL_1 + R_1}\right)\left(\frac{R_4}{sL_3 + R_3}\right)$$

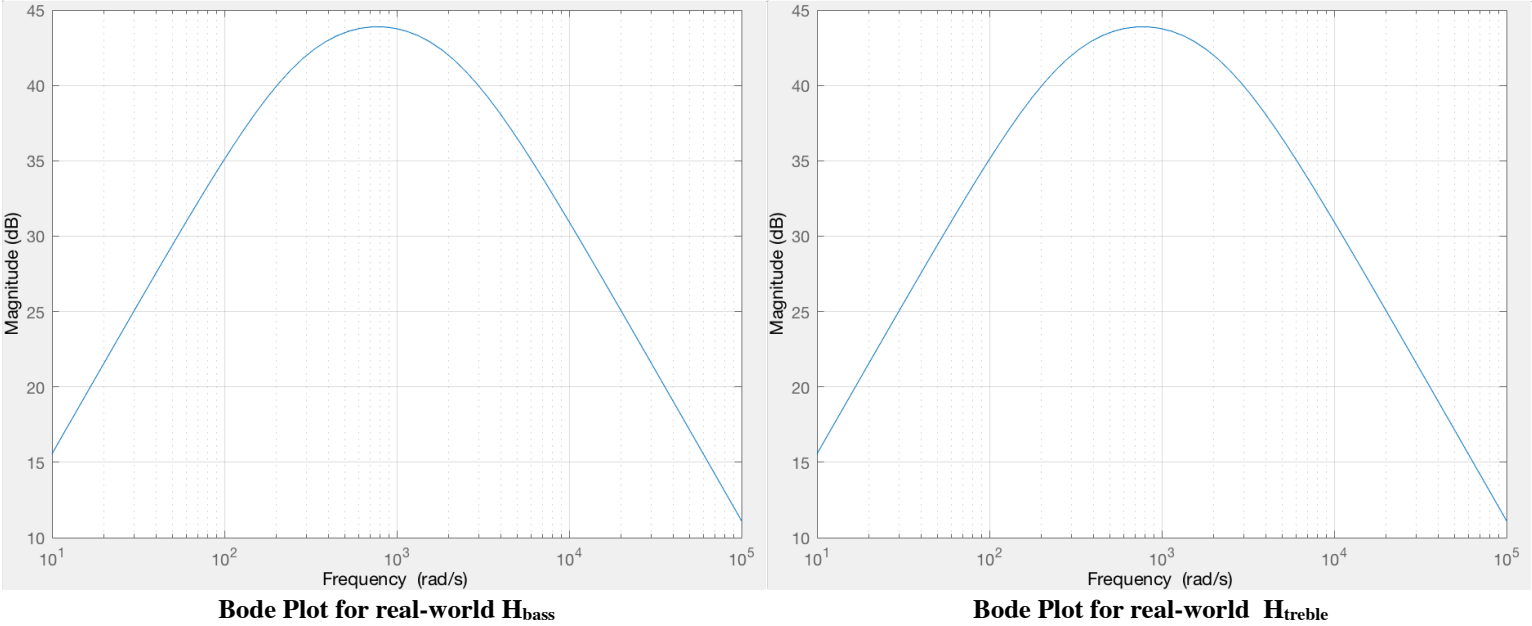
For Bass Part:

$$= (9000*s)/((s/20 + 100)*(s/2 + 150))$$

$$= 360000 \frac{s}{(s + 300)(s + 2000)}$$

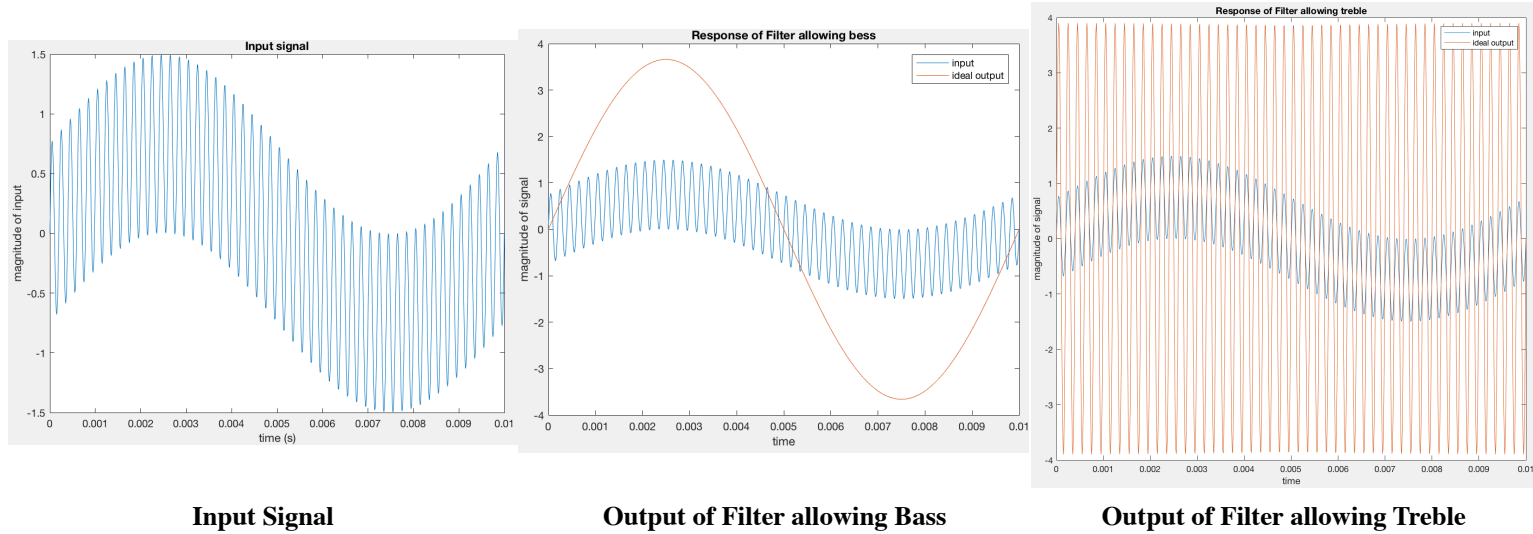
with all components with ideal value except R₄ is rounded to 1.8kΩ

For Treble Part: since all ideal passive components value can be selected from standard-value components library, the ideal and realistic transfer function is exactly the same. $H_{treble} = 10000000 \frac{s}{(s + 10k)(s + 100k)}$



8.Simulation: response to a given input:

Given a high-volume input of two pure sine waves, $v_{in}(t)=A\sin(\omega_1t)+B\sin(\omega_2t)$, where $A=B=750\text{mV}$, $\omega_1=100\text{Hz}=2\pi\cdot 100\text{ rad/s}$, $\omega_2=5000\text{Hz}=2\pi\cdot 5000\text{ rad/s}$. The input is $v_{in}(t)=0.75\sin(200\pi t)+0.75\sin(10000\pi t)$



9.Imperfection of Design:

1.As the frequency content is too high, light blinking cannot be properly perceived by human eyes. Human's persistence of vision is considered to be 0.1s, so frequency content more than $1/0.1\text{s} = 10\text{Hz}$ is rarely perceivable by human. As the treble contains signal from 2kHz to 16kHz, and bass from 60Hz to 250 Hz, some equipments are needed to modify the signal frequency content, which will be fed into light box.

One possible solution would be add an AC-DC converter between the output of circuit and the Light box, converting the AC signal to a direct current signal.

2.The imperfections (meaning not fully rejection of enemy signals) of current solution as I mentioned in Appendix. The solution would be, consider double poles at $\omega=2\text{k rad/s}$ for bass, double zeros at origin and double poles at $\omega=10\text{k rad/s}$.

Then the transfer function of filter allowing bass is $H_{bass} = K_1 \frac{s}{(s+0.3k)(s+2k)^2}$

and the transfer function of filter allowing treble is $H_{treble} = K_2 \frac{s^2}{(s+10k)^2(s+100k)}$

Discussion on the value of K_1 & K_2 :

At the rate of change (slope of bode plot) is $\pm 20\text{dB/decade}$ with single pole or zero. So the range $[2\text{k}, 10\text{k}] \text{ rad/s}$ enjoys a magnitude drop at -40dB/decade . To achieve less than 9dB at $\omega=10\text{k rad/s}$ requires the magnitude at $\omega=2\text{k rad/s}$ no more than $40\text{dB} \cdot [10\text{k}-2\text{k}]/10\text{k} + 9\text{dB} = 41\text{dB}$. Unfortunately, double poles at $\omega=2\text{k rad/s}$ will lead to a 6dB magnitude drop at the corner of frequency. But thank to the safety margin, the magnitude at $\omega=2\text{k rad/s}$ after 6dB drop is around 35dB, which is still able to amplify the input signal with lowest voltage to a detectable level.

So magnitude of H at $\omega=2\text{k rad/s}$ as well as $\omega=0.3\text{k rad/s}$ is selected to be 41dB.

$K_{\text{start}} = 41\text{dB} - 20\text{dB} - 20\text{dB} - 20\text{dB} \cdot (2\text{k}-1\text{k})/10\text{k} = -5\text{dB} = 10^{(-5/20)} = 0.5623$

$K_1 = 0.5623 \cdot 0.3\text{k} \cdot 2\text{k} = 337380$

Same principle can be applied on the calculation of K_2 .

Reference

Frequency of Bass: <http://www.teachmeaudio.com/mixing/techniques/audio-spectrum>

Frequency of Treble: <http://greenboy.us/fEARful/frequencytables.htm>

Human Hearing Frequency: <http://hypertextbook.com/facts/2003/ChrisDAmbrose.shtml>

Bode Plot Online Generator: <http://www.onmyphd.com/?p=bode.plot.online.generator>

SchemeIt: <http://www.digikey.ca/schemeit/project/>

Resistive decade value: <http://www.digikey.com/en/pdf/v/vishay-dale/standard-electronic-decade-table>

Capacitor decade value: <http://www.ece.uidaho.edu/ee/classes/ECE311S09/docs/>

[resistor & capacitor standard values.pdf](#)

Inductor decade value: <http://www.ietlabs.com/pdf/Datasheets/1492.pdf>

Appendix

Real Value of Passive Component: [Reference: MIE346 Lab2 Manual]

However, real components cannot take on an infinite range of values. Instead, they are available in common decade increments only. In the lab, you will have available the following values of resistors, capacitors, and inductors only:

Resistors				
100 Ω	220 Ω	470 Ω	680 Ω	1 K Ω
2 K Ω	3.9 K Ω	8.2 K Ω	15 K Ω	18 K Ω
36 K Ω	39 K Ω	47 K Ω	82 K Ω	100 K Ω
Capacitors				
100 nF	220 nF	680 nF	820 nF	
1 μ F	2.2 μ F	4.7 μ F	10 μ F	
Inductors				
12 μ H	120 μ H	1.2 mH		

Calculate the minimum and maximum pure gain for each transfer function MATLAB:

①For K1

```
s = - sqrt(-1) * 300;  
H1 = s / ((s + 300) * (s+2000));  
minK = 40 / abs(20* log10(H1))
```

```
s = - sqrt(-1) * 10000;  
H1 = s / ((s + 300) * (s+2000));  
gain = abs(20* log10(H1))  
maxK = 35 / gain
```

②For K2

```
s = - sqrt(-1) * 10000;  
H1 = s / ((s + 10000) * (s+100000));  
gain = abs(20* log10(H1))  
minK = 40 / gain
```

Transfer Function of the Filters & Amplifiers can be defined as: $H(s) = K \frac{\prod_{i=1}^m (s - z_i)}{\prod_{i=1}^n (s - p_i)}$, where K is the gain, m is the

number of zeros and n is the number of poles.

①**For bass**, the transfer function is $H_{bass} = K_1 \frac{s}{(s + 0.3k)(s + 2k)}$

|H_{bass}| is supposed to be more than be 40dB at $\omega=0.3k$ rad/s and less than 9dB at $\omega=10k$ rad/s.

The starting point of magnitude Bode Plot can be estimated by $K_{start} \approx K(z_1) \dots (z_m) \left(\frac{1}{p_1}\right) \dots \left(\frac{1}{p_n}\right)$

$K_{start,dB} \approx 20 \log(K_{start})$, where the starting point refers to the approximated magnitude in Decibel corresponding to frequency $\omega=1$ rad/s. The value of gain K1 can be approximated by the |H($\omega=1$ rad/s)|. At the rate of change (slope of bode plot) is $\pm 20\text{dB/decade}$ with single pole or zero.

In order to make the magnitude at $\omega=10k$ less than 9dB, it is required that magnitude of H at $\omega=2k$ rad/s no more than 9dB + 20dB*[(10000-2000)/10000] = 25 dB \leq 40dB (unrealistic with single pole). Note that the value is calculated based on linear approximation.

With |H_{bass}($\omega=0.3k$ rad/s)| = 40dB, it is impossible to reject all treble signals, a small branches of treble signal with lower frequencies and high volume will be passed as well. (treated as imperfections of circuit or noises, since we do not want to make the circuit more complex, case of conjugate complex poles will not be studied in this document)

>>> simply chose |H_{bass}($\omega=0.3k$ rad/s)| = 40dB

$K_{start} = 40\text{dB} - 20\text{dB} - 20\text{dB} - 20\text{dB} * (300-100)/1000 = -4\text{dB} = 10^{(-4/20)} = 0.6310$

$K = 0.6310 * 0.3k * 2k = 378600$

②**For treble**, the transfer function is $H_{treble} = K_2 \frac{s}{(s + 10k)(s + 100k)}$

|H_{bass}| is supposed to be more than be 40dB at $\omega=10k$ rad/s and less than 9dB at $\omega=2k$ rad/s.

Similarly, this is unrealistic with single pole as well. But just simply chose |H_{bass}($\omega=10k$ rad/s)| = 40dB,

$K_{start} = 40\text{dB} - 20\text{dB} * 4 = -40\text{dB} = 10^{(-40/20)} = 0.0100$

$K = 0.0100 * 10k * 100k = 10000000$

MATLAB Code to generate bode plot

```
clear all
close all
clc

poles = [10000 100000];          % poles at negative poles(i)
zeros = [0];                    % zeros at negative zeros(i)
K = 1    % gain

syms s
% s=-jw
TF = 1;
for i = 1: length(poles)
    TF = TF/(s - poles(i));
end
for i = 1: length(zeros)
    TF = TF * (s - zeros(i));
end

[nf, df] = numden(K * TF);

TF = tf(sym2poly(nf),sym2poly(df))

opts = bodeoptions('cstprefs');
% opts.FreqUnits = 'Hz';
opts.PhaseVisible = 'off';
% Bode Plot
figure
hold on
bode(tf(sym2poly(nf),sym2poly(df)), opts), grid
```

MATLAB Code to plot circuit response

```
clear all
close all
clc

pi = 3.14159;

poles = [300 2000];             % poles at negative poles(i)
zeros = [0];                    % zeros at negative zeros(i)
K = -360000;    % gain

syms s
% s=-jw
TF = 1;
for i = 1: length(poles)
    TF = TF/(s - poles(i));
end
for i = 1: length(zeros)
    TF = TF * (s - zeros(i));
end

[nf, df] = numden(K * TF);

% opts = bodeoptions('cstprefs');
% % opts.FreqUnits = 'Hz';
% opts.PhaseVisible = 'off';
% % Bode Plot
% figure (1)
% bode(tf(sym2poly(nf),sym2poly(df)), opts), grid

% input graph
% figure (1)
% plot(t, v_in)
% title('Input signal')
% xlabel('time (s)')
% ylabel('magnitude of input')

sys = tf(sym2poly(nf),sym2poly(df))
clf
t = linspace(0, 0.01, 1000);    % total time = 15s
v_in = 0.75 * sin(200*pi*t) + 0.75 * sin(10000 * pi *t);
lsim(sys,v_in,t)    % u,t define the input signal
```