Mechanics: study of motion

Translational Motion: all points of an object experience the same displacement

Motion in one-Dimension

Rectilinear Kinematics

choose axis(arbitrary: direction, zero)

※Position: (**r**) location of a system relative to a reference point

※Displacement: (Δ**r**) change in position $\Delta\vec{x} = x_f - x_i$

※Distance: length of path

※Velocity: (**v**) rate of change of position with respect to time magnitude+direction(vector)

average velocity: $\vec{v}_{avg} = \frac{\Delta x}{\Delta t}$ instantaneous velocity: $\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$

※Speed: (v) length of distance travelled per unit of time magnitude(scalar)

average speed: $v_{avg} = \frac{d}{\Delta t}$

※Acceleration: (**a**) rate of change of velocity with respect to time

average acceleration: $\vec{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$ slope of v-t graph

instantaneous acceleration: $\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$

the force on an object is proportional to the acceleration of the object. F∝a

Kinematic Equations: ① $x_f = x_i + vt$ (v constant)

②constant acceleration:

$$v_f = v_i + at \qquad v_{avg} = \frac{1}{2}(v_i + v_f) \qquad x_f = x_i + \frac{1}{2}(v_i + v_f)t = x_i + v_i t + \frac{1}{2}at^2 \qquad v_f^2 = v_i^2 + 2a(x_f - x_i)$$

Relative Motion and Galilean transformation equations

Motion in two-Dimension

Position Vector: vector from origin of coordinate system to location of particle $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

average velocity $\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t}$ instantaneous velocity: $\vec{v} = \lim_{t \to 0} \frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$

average acceleration $\vec{a}_{avg} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$ instantaneous acceleration $\vec{a} = \lim_{t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$

velocity vector as a function of time: $\vec{v}_f = \vec{v}_i + \vec{a}t$

position vector as a function of time: $\vec{r}_f = \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a}t^2$

1.**Projectile Motion**:

free-fall acceleration is constant and directed downwards; effect of air resistance is negligible

horizontal: $\vec{v}_{x,i} = \vec{v}_i \cos\theta$ constant velocity

vertical: $\vec{v}_{y,i} = \vec{v}_i \sin\theta$ constant acceleration

<u>maximum height</u>: time when projectile reaches peak $t = \frac{v_i \sin\theta_i}{g}$ height $h = \frac{v_i^2 \sin^2\theta_i}{2g}$

<u>horizontal range</u>: time when projectile reaches max range $t = \frac{2v_i \sin\theta_i}{g}$ range $R = \frac{2v_i^2 \sin\theta_i \cos\theta_i}{g} = \frac{v_i^2 \sin 2\theta_i}{g}$

R is maximum when θ=45°

2.Uniform **Circular/Curvilinear Motion**: (velocity constant)

Period: time interval required for one complete revolution of a particle $T = \frac{2\pi r}{v}$

Centripetal Acceleration a_c: $a_c = \frac{v^2}{r}$ r: radius

Total Acceleration: $\vec{a} = \vec{a}_r + \vec{a}_t$

①Radial Acceleration: orthogonal to the velocity of an object(only changes direction of velocity)

$a_r = -a_c = -\frac{v^2}{r}$

②Tangential Acceleration: parallel to the velocity of an object(changes magnitude and possibly sign of velocity, but not the linear path of an object) $a_t = \left|\frac{dv}{dt}\right|$

Laws of Motion(Dynamics)

Newton Laws of Motion (Dynamics): (hold for inertial/non-accelerating frames of reference)

Note: for non-inertial(accelerating) frames of reference, an "inertial force" $\mathbf{F}_I = -m\mathbf{a}_{frame}$ must be added for Newton Laws of motion to hold.

①Newton First Law:(law of inertia) When no force acts on an object/net external force acting on a system is zero, the acceleration of object is zero.

If an object does not interact with other objects, it is possible to identify a reference frame(inertial frame of reference) in which the object has zero acceleration.
Inertia is a property of a material object to resist attempts to change its velocity.
(mass is a measure of inertia)

②Newton Second Law: force is the cause of changes in motion $\sum \vec{F}_{external} = \vec{F}_{net} = \frac{d}{dt}\vec{p} = \frac{d}{dt}(m\vec{v}) = m\vec{a}$

Gravitational Force and Weight:

③Newton Third Law: For every force(action force) there is a force(reaction force) equal in magnitude that acts the opposite direction.

normal force, tension, friction

Friction:
Static Friction Force f_s: force to keep object from moving
Static Friction Force f_k: force for an object in motion
$f_s \leq \mu_s n$ n: normal force μ: coefficient of static friction
$f_k = \mu_k n$ μ: coefficient of kinetic friction

Mass on a spring (restoring force) Hooke's law spring constant
simple harmonic motion (amplitude frequency period phase)
simple pendulum

Energy

※System: a collection of physical entities in the universe chosen for physical analysis
※Environment: non-system physical entities (surroundings)

isolated: no matter/energy transfers between system and environment
closed: no matter transfers between system and environment, but energy may move across system 's boundaries.
open: both matter & energy may enter/leave system

※Energy: ability/potential to do work

①kinetic energy: translational $K = \frac{1}{2}mv^2$ rotational $K = \frac{1}{2}I\omega^2$

②potential energy: gravitational $U = -\frac{GMm}{r}$ $\Delta U \approx mgh$ elastic $U = \frac{1}{2}kx^2$

※Force: a physical attempt to accelerate mass

①conservative force: internal force that conserves total mechanical energy (K+U)

②nonconservative force: internal force that does not conserve total mechanical energy; converts some energy to internal(thermal) energy.

※equilibrium(stable/unstable/neutral)

※Mechanical Work (W): integral of dot product of force and infinitesimal distances travelled by the point of application of force $W = \int_{r_i}^{r_f} \vec{F}\, d\vec{r}$

Energy Conservation: ①Total energy of an isolated system is constant.

②Energy cannot be created or destroyed, only transferred and transformed.

Energy (isolated system) $W_{external} = 0$ $\Delta K + \Delta U + \Delta E_{internal} = 0$

①conservative force: ΔE=0 ②non-conservative force $\Delta E = \mu F_N \Delta d$ (kinetic friction)

(non-isolated system) W≠0 ΔE≠0

Power:

Hook Spring System: $F_{spring} = -kx$ $F_{damping} = -bv$ $U_{spring} = \frac{1}{2}kx^2$

Linear Momentum and Collision

Linear Momentum: $\vec{p} = m\vec{v}$ [kg·m/s]

time rate of change of linear momentum of a particle is equal to net force acting on the particle $\sum \vec{F} = \frac{d\vec{p}}{dt}$

Momentum Conservation: total momentum of an isolated system is constant. dp/dt=0 $p_{1i}+p_{2i}=p_{1f}+p_{2f}$

Impulse: $\vec{I} = \int_{t_i}^{t_f} \sum \vec{F}\, dt$

Impulse-Momentum Theorem: change in momentum of a particle is equal to impulse of the net force acting on the particle $\Delta\vec{p} = \vec{I}$

Collision: two particles come close to each other and interact by means of forces

momentum of system must be conserved in any collision, but total kinetic energy may or may not be conserved.

①elastic: total energy conserved

(1-D) conservation energy $\frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_2v_{2i}^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2$ momentum $m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$

solution: $v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right)v_{2i}$ $v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right)v_{1i} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)v_{2i}$

special case: i) $m_1=m_2$, $v_{1f}=v_{2i}$ $v_{2f}=v_{1i}$ ii) $v_{2i}=0$ iii) $m_1 \gg m_2$ & $v_{2i}=0$, $v_{1f} \approx v_{1i}$ $v_{2f} \approx 2v_{1i}$ iv) $m_2 \gg m_1$ & $v_{2i}=0$, $v_{1f} \approx -v_{1i}$ $v_{2f} \approx 0$

②inelastic: perfect inelastic: objects stick together after collision $\vec{v}_f = \frac{m_1\vec{v}_{1i} + m_2\vec{v}_{2i}}{m_1 + m_2}$

Center Of Mass

Center of Mass: average position $x_{cm} = \frac{\sum_{i=1}^{n} m_i x_i}{M} = \frac{1}{M}\int x\,dm$ where $M = \sum_{i=1}^{n} m_i$

$\vec{r}_{cm} = \frac{1}{M}\int \vec{r}\,dm$

center of gravity: when g constant

System of many particles:

①velocity of CM of SMP: $\vec{v}_{cm} = \frac{\sum_{i=1}^{n} m_i \vec{v}_i}{M}$ ②total momentum of SMP: $\vec{p}_{total} = M\vec{v}_{cm}$

③acceleration of CM of SMP: $\vec{a}_{cm} = \frac{\sum_{i=1}^{n} m_i \vec{a}_i}{M}$ ④total force of SMP: $\vec{F}_{net} = M\vec{a}_{cm}$

⑤Newton 2 Law: $\sum \vec{F}_{external} = M\vec{a}_{cm}$ ⑥Impulse-momentum theorem: $\Delta\vec{p}_{total} = \vec{I}$

Deformable System

Rotational Motion:

Rotation of Rigid Object about a fixed axis

※Angular Position: θ angular location of a system relative to a reference point $\theta = \frac{s}{r}$ [radian, rad] arc of length s, radius r

※Angular Velocity: ω rate of change of angular position with respect to time

instantaneous angular velocity: $\omega = \lim_{t\to 0}\frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$ [rad/s] v=ωr (r:radius of rotation)

※Angular Acceleration: α rate of change of angular velocity with respect to time

average angular acceleration: $\alpha_{avg} = \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{\Delta\omega}{\Delta t}$ instantaneous: $\alpha = \lim_{t\to 0}\frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$

$a_t = \alpha r$ (r:radius of rotation)

Kinematic Equations:

constant angular acceleration:

$\omega_f = \omega_i + \alpha t$ $\theta_f = \theta_i + \frac{1}{2}(\omega_i + \omega_f)t = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$ $\omega_f^2 = \omega_i^2 + 2\alpha(\theta_f - \theta_i)$

Translation Particle (a constant)	Rotation Rigid Body (α constant)

Relationship between translation & rotation:

$v = r\omega$ $a_{tangential} = r\alpha$ $a_{centripetal} = v^2 / r = r\omega^2$ $a = \sqrt{a_t^2 + a_c^2} = r\sqrt{\alpha^2 + \omega^4}$

Rotational kinetic energy: $K_r = \frac{1}{2}\sum_i m_i v_i^2 = \frac{1}{2}\sum_i m_i r_i^2 \omega_i^2 = \frac{1}{2}I\omega^2$

Moment of Inertia: $I = \sum_i m_i r_i^2$

measure resistance of an object to changes in its rotational motion (like mass, measure of tendency of an object to resist changes in translational motion)

$$I=\lim_{\Delta m_i\to 0}\sum_i r_i^2\Delta m_i=\int r^2\,dm=\int \rho r^2\,dV$$

volumetric mass density $\rho=m/V$ (mass per unit volume)
surface mass density $\sigma=\rho t$ (t:thickness)
linear mass density $\lambda=M/L=\rho A$ (A: cross section area)

Hoop/Thin Cylindrical Shell $I_{cm}=MR^2$ Hollow Cylinder $I_{cm}=½M(R_1^2+R_2^2)$ Solid Cylinder/Disk $I_{cm}=½MR^2$
rectangular plate: $I_{cm}=(1/12)M(a^2+b^2)$
Long, thin rod with rotation axis through center $I_{cm}=½ML^2$
Long, thin rod with rotation axis through end $I_{cm}=(1/3)ML^2$
Solid Sphere $I_{cm}=(2/5)MR^2$ Thin spherical shell $I_{cm}=(2/3)MR^2$

Parallel-Axis Theorem: moment of inertia of an object about an arbitrary axis

$I=I_{cm}+MD^2$ D:distance from rotation axis to axis through center of mass

derivation:

Torque: τ tendency of a force to rotate an object about some axis **τ=r×F** τ=Frsinθ

$\tau=Fd$ d: moment/level arm (perpendicular distance from rotation axis to line of action of force)

	Translation of Particle	**Rotation of Rigid Body**
Position, Velocity, Acceleration	x, v=dx/dt, a=dv/dt	θ, ω=dθ/dt, α=dω/dt
Kinematic Equations (a/α constant)	$v_f=v_i+at$ $x_f=x_i+\frac{1}{2}(v_i+v_f)t=x_i+v_it+\frac{1}{2}at^2$ $v_f^2=v_i^2+2a(x_f-x_i)$	$\omega_f=\omega_i+\alpha t$ $\theta_f=\theta_i+\frac{1}{2}(\omega_i+\omega_f)t=\theta_i+\omega_it+\frac{1}{2}\alpha t^2$ $\omega_f^2=\omega_i^2+2\alpha(\theta_f-\theta_i)$
Resistance of an object to changes in motion	mass m	moment of inertia I
cause a change in motion	force F (a physical attempt to accelerate mass)	torque τ (a physical attempt to cause angular acceleration)
Net Force/Torque & Acceleration	net force causes an acceleration of object, and acceleration is *proportional* to net force $\sum F=ma$	angular acceleration of a rigid body rotating about a fixed axis is *proportional* to the net torque acting about the axis. $\sum\tau=I\alpha$
Kinetic Energy	$K=\frac{1}{2}\sum_i m_iv_i^2$	$K_r=\frac{1}{2}\sum_i m_iv_i^2=\frac{1}{2}\sum_i m_ir_i^2\omega_i^2=\frac{1}{2}I\omega^2$
Work **(work-kinetic energy theorem)**	$W=\int F\,dx$ $W=\frac{1}{2}mv_f^2-\frac{1}{2}mv_i^2$	$W=\int\tau\,d\theta$ $W=\frac{1}{2}I\omega_f^2-\frac{1}{2}I\omega_i^2$
Conservation of Energy		
Power	$P=Fv$	$P=\tau\omega$
Momentum	Linear Momentum $\vec{p}=m\vec{v}$ $\sum\vec{F}=\frac{d\vec{p}}{dt}$	Angular Momentum L $\vec{L}=\vec{r}\times\vec{p}$ $L_z=I\omega$ $\sum\vec{\tau}=\frac{d\vec{L}}{dt}$
Conservation of Momentum (isolated system: net external force/torque on the system is zero)	$\sum F_{external}=\frac{d\vec{p}}{dt}=0$ $\vec{p}_i=\vec{p}_f=m_i\vec{v}_i=m_f\vec{v}_f$	$\sum\tau_{external}=\frac{d\vec{L}_{total}}{dt}=0$ $\vec{L}_{total}=I_i\omega_i=I_f\omega_f=constant$
Impulse	Linear $\vec{I}=\int_{t_i}^{t_f}\sum\vec{F}\,dt$	

$$\sum\tau_{external}=\int\alpha r^2\,dm=I\alpha$$

Note: radial components produces zero torque about the axis.

Work-Kinetic Energy Theorem: when work is done on a system and the only change in the system is in its speed, the net work done on the system equals the change in kinetic energy of the system.

instantaneous angular momentum L of a particle relative to an axis through the origin O is cross product of the particle instantaneous position vector r and instantaneous linear momentum p $\vec{L} = \vec{r} \times \vec{p}$

L=Iω=mvrsinθ

Kepler Laws of Planetary Motion:

①All planets follow elliptical orbits, with the Sun at one focus.

②The radius vector drawn from the Sun to a planet sweeps out equal areas of space in equal time intervals

③The square of the orbital period of a planet is directly proportional to the cube of the semi-major axis of the elliptical orbit.