inertia frame (of reference): a frame of reference in which an object experiencing zero net force(free object) at rest/ constant velocity
An event is anything with a location in space and a time.
Einstein's Postulates:
1.The laws of physics are the same in all inertial frames.
2.The speed of light is same in all inertial frames.

①relative simultaneity:
Two events occurring at the same locations that are simultaneous in one frame of reference will not be simultaneous in a frame of reference moving relative to the first.

②time dilation(时间膨胀): $\Delta t' = \Delta t / \gamma$ t'=proper time
Two events occurring at the same location in one frame will be separated by a longer time interval in a frame moving relative to the first.

③length contraction(长度收缩) $L\gamma = L'$ L'=proper length
The length of an object in a frame through which the object moves is smaller than its length in the frame in which it is at rest.

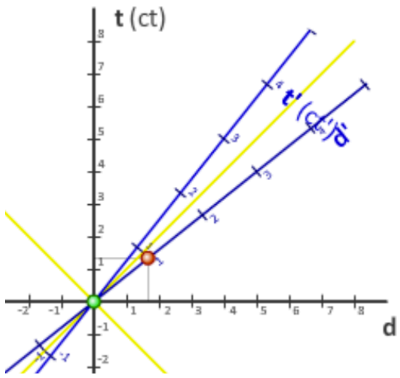
Relativistic Factor: $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ ≥1 =1 stationary
classic limit $v \ll c$
Note: binomial approximation: $(1-x)^a = 1-ax$ when $x \ll 1$

Lorentz Transformation:

S reference frame x t	S' reference frame x' t'
$t' = \gamma\left(-\frac{v}{c^2}x + t\right)$	$t = \gamma\left(\frac{v}{c^2}x' + t'\right)$
$x' = \gamma(x - vt)$	$x = \gamma(x' + vt')$

Note: if stand in S' frame, then v-> -v since sees in opposite direction.
v: S' moves at +v relative to S

Minkowski diagram (space-time diagram) in special relativity
In the Diagram Events has: x=1.67 ct=1.33 x'=1 t'=0 v=0.8c



Relativistic Doppler Effect:

$$f_{observer} = f_{source}\frac{\sqrt{1-\frac{v^2}{c^2}}}{1+\frac{v}{c}\cos\theta}$$

Toward: θ=180 Away: θ=0 circle: : θ=90

$$f = \frac{c}{\lambda}$$

$f_{observer}$ smaller   fo/fs>1

Relativistic Velocity Transformation:

$$u'_x = \frac{u_x - v}{1-\frac{u_x v}{c^2}} \qquad u'_y = \frac{u_y/\gamma}{1-\frac{u_x v}{c^2}}$$

u_x'=object observed in S' u_x=object observed in S   v=velocity S' to S

60° north of west = from west 60° to north (和north 30°夹角)

Energy and Momentum:

Momentum: $p = \gamma mu$

Energy: $E = \gamma mc^2$ c:universal constant

Total Energy = Kinetic Energy + Internal Energy

stationary object energy: $E_{internal}=mc^2$

object with zero mass $E_{total}=pc$

Kinetic Energy

=Energy of a Moving Object - Energy At rest(its internal energy)

$=\gamma mc^2-mc^2$

$=(\gamma-1)mc^2$ c:light speed at vacuum

$Kinetic\ Energy = \gamma mc^2 - mc^2 = (\gamma - 1)mc^2$ m: rest mass

true mass m ; mass at rest m0 $m = \gamma m_0$

$\gamma mc^2 = (1-u^2/c^2)^{½} mc^2 \approx (1+(-1/2)(-u^2/c^2)) mc^2 = mc^2 + ½mu^2$ $(u \ll c)$

Conservation of Total Energy:

mass of a proton=1.67×10^{-27}kg

potential difference V (accelerate to that speed) $KE=q\Delta V$ $(q=1.6\times10^{-19}C)$

Electron Volt(energy per charge) $eV=1.6\times10^{-19}J$

Momentum Conservation: $\gamma_1 m_1 u_1+\gamma_2 m_2 u_2=\gamma mu$

Energy Conservation: $\gamma_1 m_1+\gamma_2 m_2=\gamma m$

Change in Kinetic Energy= - Change in Potential Energy= $-\Delta mc^2$

Rest mass: rest mass never changes(invariant) in all inertia frames of reference.

$E^2 = p^2c^2+m^2c^4$ m: rest mass

http://www.trell.org/div/minkowski.html

※ intensity(energy per unit time per unit area)∝(amplitude of electric field oscillation E_0)^2

like rate of energy arrives.

Photon: $E = hf = h\frac{c}{\lambda}$ h=6.626×10^{-34}J·s

black body radiation curve

※ **Photoelectric Effect**: (光电效应)

Electrons are emitted from solids, liquids, gases when they absorb energy from light. Light produce a flow of electricity.

A light beam directed at the surface of a metal could liberate electrons.

※ **Working Function ϕ**: minimum energy required to free an electron/ minimum energy to eject an electron(溢出功) $\varphi = hf_0$

light: intensity (P/A) frequency(c/λ)

Stopping Potential: potential difference that barely stops the flow

※ **Threshold Wavelength**: maximum wavelength for which electrons are freed corresponding minimum frequency (threshold frequency)

electron: KE, time lag between light and ejection

Electron Volt (eV): Energy that one electron gets by travelling through 1 V of electric potential

1eV=1.6 x 10^{-19} J Mass (eV / c2)

Electron's mass is 511 keV / c2 Proton's mass is 938 MeV /c2

Production of X-Ray (10^{-2}nm~10nm)

cutoff wavelength: termination of the spectrum.

※ **Compton-Debye effect/scattering**: (康普顿效应/散射)

scattering of electromagnetic radiation from free stationary electrons

>> object with zero mass $E = pc$

momentum *density*: energy/volume=momentum/volume ×c

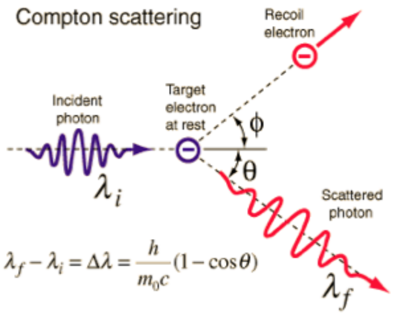
momentum of proton: $p = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda}$

当X射线或伽马射线的光子跟物质相互作用，因失去能量而导致波长变长的现象

波长变化的幅度被称为康普顿偏移

$$\lambda - \lambda_0 = \Delta\lambda = \frac{h}{mc}(1 - \cos\theta)$$

Effect is between 0 and 0.00485 nm (2 h / m_ec)

Derivation:	
 	Photon momentum: $p=E/c=h/\lambda$ Electron momentum: $p=\gamma m_e v$ Momentum Conserved: x-component $\frac{h}{\lambda} = \frac{h}{\lambda'}\cos\theta + \gamma_u m_e u \cos\varphi$ y-component $0 = \frac{h}{\lambda'}\sin\theta - \gamma_u m_e u \sin\varphi$ Energy Conserved: $h\frac{c}{\lambda} + m_e c^2 = h\frac{c}{\lambda'} + \gamma_u m_e c^2$

※ **Wave Function Ψ**: probability amplitude Ψ^2 >> probability

when a phenomenon is detected as particles, we cannot predict with certainty where a given particle will be found. The most we can determine is a probability of finding it in a given region, which is proportional to the square of the amplitude of the associated wave in that region.

$$\left(\begin{array}{c}\text{probability of finding}\\ \textit{particle}\text{ in a region}\end{array}\right) \propto \left(\begin{array}{c}\text{amplitude of }\textit{wave}\\ \text{in that region}\end{array}\right)^2$$

※ **Matter Wave**:

double slip experiment-- constructive/destructive interference

Properties:

① de Broglie wavelength: $\lambda = \frac{h}{p}$

② frequency: $f = \frac{E}{h}$

wave number k $k = \frac{2\pi}{\lambda}$ & angular frequency ω $\omega = \frac{2\pi}{T}$:

let $\hbar = \frac{h}{2\pi} = 1.055\times10^{-34}\, J\cdot s$ so $p = \frac{h}{\lambda} = \hbar k$ $E = hf = \hbar\omega$

③ velocity: $v_{wave} = f\lambda = \frac{E}{h}\frac{h}{p} = \frac{E}{p}$

young's Double Slit Experiment

※ **Bohr's Model**:

Angular momentum of electron is quantized: $L = \frac{nh}{2\pi}$

energy of an electron： $E_n = -\frac{13.6eV}{n^2}$ 电子定态跃迁 $\Delta E = h\nu$

n: principal quantum number

Balmer series

Lyman series
Paschen series

Bragg Diffraction

Heisenberg Uncertainty Principle: $\Delta p \Delta x \geq \frac{\hbar}{2}$
Gaussian wave form $\psi(x,0)=Ce^{-(x/2\varepsilon)^2}$, $\Delta p \Delta x=\hbar/2$
well-defined property: no uncertainty $\Delta=0$
undefined property: Δ=infinite

Energy-Time Uncertainty Principle: $\Delta E \Delta t \geq \frac{\hbar}{2}$

Schrödinger Equation

Wave Function Ψ(probability amplitude) quantum state of a particle and how its behaves

probability density $|\Psi^2|$

Free-Particle Schrödinger Equation: (no external force/interaction)

wave function obeys wave equation

> wave on string: > electromagnetic waves:

> matter waves:

$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2\psi(x,t)}{\partial x^2}\right)=i\hbar\left(\frac{\partial\psi(x,t)}{\partial t}\right)$$

Plane Wave: general solution $\psi(x,t)=Ae^{i(kx-\omega t)}$ and $|\psi(x,t)|=A$

Bound State:particle's motion is restricted by an external force to a finite region of space

Time-Dependent Schrödinger Equation

$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2\psi(x,t)}{\partial x^2}\right)+U(x)\psi(x,t)=i\hbar\left(\frac{\partial\psi(x,t)}{\partial t}\right)$$

(external force~potential energy associated with the force (even nonconservative forces))

$(KE+U(x))\psi(x,t)=E\psi(x,t)$

Stationary States: energy is well defined E=ℏω(ΔE=0) and probability density independent of time.

<u>Separation of Variables</u>: $\psi(x,t)=\varphi(x)\phi(t)$ spatial part×temporal part

so $$-\frac{\hbar^2}{2m}\frac{1}{\varphi(x)}\left(\frac{\partial^2\varphi(x)}{\partial x^2}\right)+U(x)=i\hbar\frac{1}{\phi(t)}\left(\frac{\partial\phi(t)}{\partial t}\right)=C$$ (separation constant=energy)

Temporal part $\phi(t)$: $\psi(x,t)=\varphi(x)e^{-i(E/\hbar)t}$ Spatial part $\varphi(x)$:

Time-Independent Schrödinger Equation

$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2\varphi(x)}{\partial x^2}\right)+U(x)\varphi(x)=E\varphi(x)$$

Well-Behaved Function:

①Normalization: $$\int_{all\ space}|\psi(x,t)|^2\,dx=1$$

②Smoothness:

Continuity of wavefunction(finite momentum) & its derivative(finite kinetic energy)

Case 1: Particle in a Box / Infinite Wall

$$U(x)=\begin{cases} 0 & 0<x<L \\ \infty & x<0, x>L \end{cases}$$

$$\psi_n(x)=\begin{cases} \sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L} & 0<x<L \\ 0 & x<0, x>L \end{cases}$$

$$E_n=\frac{n^2\pi^2\hbar^2}{2mL^2}$$

Ground State: n=1

Case 2: Finite Wall

$$U(x)=\begin{cases} 0 & 0<x<L \\ U_0 & x<0, x>L \end{cases}$$

$E>U_0$: particle given enough energy to escape, so restrict $E<U_0$

Case 3: Simple Harmonic Oscillator

$$U(x)=\frac{1}{2}kx^2$$

$$E=(n+\frac{1}{2})\hbar\omega_0 \qquad \omega_0=\sqrt{\frac{k}{m}}$$

unstationary states:

Expectation Value: average value of *position*

$$\bar{x}=\int_{allspace} x\,|\varphi(x)|^2\,dx$$

Uncertainties: $\Delta x=\sqrt{\overline{x^2}-\bar{x}^2}$

Derivation:

$$\Delta x=\sqrt{\int_{allspace}(x-\bar{x})^2\,|\varphi(x)|^2\,dx}$$

EV of square of the position

$$\overline{x^2}=\int_{allspace} x^2\,|\varphi(x)|^2\,dx \quad \& \quad \int_{allspace}|\varphi(x)|^2\,dx=1$$

Observable Q: property that can be measured

$$\bar{Q}=\int_{allspace}\psi^*(x,t)\hat{Q}\psi(x,t)\,dx \qquad \Delta Q=\sqrt{\overline{Q^2}-\bar{Q}^2}$$