Разпознаване на емоции в

сигнали от реч и ЕЕГ

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- Експлицитен и имплицитен канал при общуване

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• Съчетаване на първичен (ЕЕГ) и вторичен (реч) канал

- ⊳ Нулева зона (какво е емоция)

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- ⊳ Резултати
- ⊳ Заключение

• Теорията на Дарвин

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 - "Принцип на полезните навици"

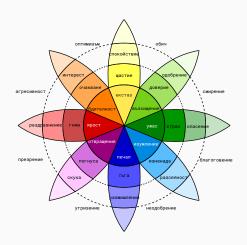
- Теорията на Дарвин
 - "Принцип на полезните навици"
 - "Принцип на противоположностите"

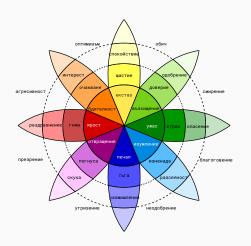
- Теорията на Дарвин
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- Продължението на Плутчик (1980)

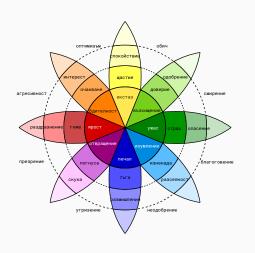
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Емоцията е сложна верига от събития, която започва с някакъв стимул. В следствие настъпва фаза на "изпитване на емоция" и фаза на физиологични промени. Те предизвикват целенасочено държание, което цели да премахне дразнението на стимула и да върне състоянието на еквилибриум.





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Осем е голямо число

• VAD модела - Алберт Мейерабиан и Джеймс Ръсел (1974)

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• В сигнала от реч се измерва по-лесно активацията

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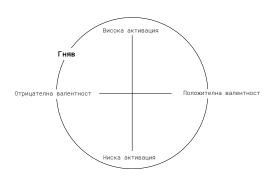
- В сигнала от реч се измерва по-лесно активацията
- В сигнала от ЕЕГ се измерва по-лесно валентността

Избрани емоции:



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• Гняв



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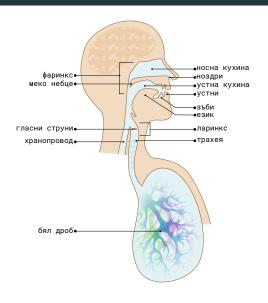
• Щастие

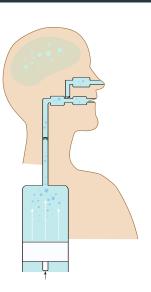
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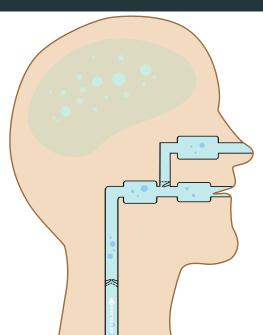
• Тъго



Сигнал от реч







Видове звуци:

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Реч ightarrow думи ightarrow фонеми

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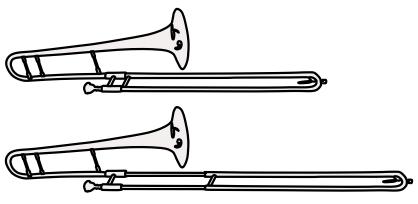
 $Peч \rightarrow gymu \rightarrow фонеми$

"Страхът стискаше гърлото, задушаваше гласа."

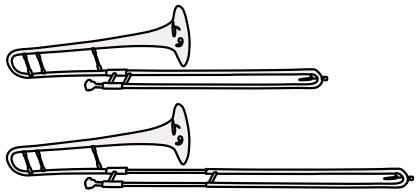
• Спектрални характеристики

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За да изследваме подлежащата емоция, трябва да изследваме спектралните свойства на статична конфигурация на вокалния тракт.

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- Глотисът д трепти, вокалният тракт v филтрира сигнала, и вълната евентуално излиза и допълнително се променя от устните r

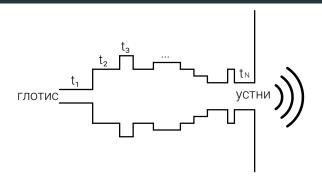
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, а новият сигнал е $y \in y(t) = g(t) * v(t) * r(t), y(t) \stackrel{\mathcal{FS}}{\longleftrightarrow} Y(z)$, е изпълнено, че

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$$\mathcal{Y}(z) = \mathcal{G}(z)\mathcal{V}(z)\mathcal{R}(z)$$



- \cdot c скорост на звука в еластична среда
- ho плътност на въздуха в тръбите
- A лицето на напречното сечение в тръба (константа)
- $\mathit{u} = \mathit{u}(\mathit{x},\mathit{t})$ е обемната скорост на позиция x в момента t
- p=p(x,t) е звуковото налягане

Уравнения на Навие-Стокс:

$$\begin{split} -\frac{\partial \rho}{\partial x} &= \frac{\rho}{A} \frac{\partial u}{\partial t} \\ -\frac{\partial u}{\partial x} &= \frac{A}{\rho c^2} \frac{\partial \rho}{\partial t} \end{split}$$

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С решения от вида:

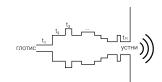
$$\begin{split} u(x,t) &= \left[u^+ \left(t - \frac{x}{c} \right) - u^- \left(t + \frac{x}{c} \right) \right] \\ p(x,t) &= \frac{\rho c}{A} \left[u^+ \left(t - \frac{x}{c} \right) + u^- \left(t + \frac{x}{c} \right) \right] \end{split}$$

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$$u_{k+1}^{+}(t) = u_{k}^{+}\left(t - \tau_{k}\right) \left[\frac{2A_{k+1}}{A_{k} + A_{k+1}}\right] + u_{k+1}^{-}(t) \left[\frac{A_{k+1} - A_{k}}{A_{k} + A_{k+1}}\right]$$

 $r_k = \frac{A_{k+1} - A_k}{A_{k+1} - A_k}$

$$u_{k}(x,t) = \left[u_{k}^{+}\left(t - \frac{x}{c}\right) - u_{k}^{-}\left(t + \frac{x}{c}\right)\right] \qquad u_{k}(l_{k},t) = u_{k+1}(0,t)$$

$$p_{k}(x,t) = \frac{\rho c}{A_{k}} \left[u_{k}^{+}\left(t - \frac{x}{c}\right) + u_{k}^{-}\left(t + \frac{x}{c}\right)\right] \qquad p_{k}(l_{k},t) = p_{k+1}(0,t)$$

$$u_{k}^{+}\left(t - \tau_{k}\right) - u_{k}^{-}\left(t + \tau_{k}\right) = u_{k+1}^{+}(t) - u_{k+1}^{-}(t)$$

$$\frac{A_{k+1}}{A_{k}} \left[u_{k}^{+}\left(t - \tau_{k}\right) + u_{k}^{-}\left(t + \tau_{k}\right) = u_{k+1}^{+}(t) + u_{k+1}^{-}(t)\right]$$

$$u_{k}^{-}\left(t + \tau_{k}\right) = u_{k}^{+}\left(t - \tau_{k}\right) - u_{k+1}^{+}(t) + u_{k+1}^{-}(t)$$

$$u_{k+1}^{+}(t) = u_{k}^{+}\left(t - \tau_{k}\right) \left[\frac{2A_{k+1}}{A_{k} + A_{k+1}}\right] + u_{k+1}^{-}(t) \left[\frac{A_{k+1} - A_{k}}{A_{k} + A_{k+1}}\right]$$

$$t_{k} = \frac{2A_{k+1}}{A_{k+1} + A_{k+1}} \qquad r_{k} = \frac{A_{k+1} - A_{k}}{A_{k+1} + A_{k+1}}$$

 $r_k = \frac{A_{k+1} - A_k}{A_k + A_{k+1}}$

$$u_{k}(x,t) = \left[u_{k}^{+}\left(t - \frac{x}{c}\right) - u_{k}^{-}\left(t + \frac{x}{c}\right)\right] \qquad u_{k}(l_{k},t) = u_{k+1}(0,t)$$

$$p_{k}(x,t) = \frac{\rho c}{A_{k}} \left[u_{k}^{+}\left(t - \frac{x}{c}\right) + u_{k}^{-}\left(t + \frac{x}{c}\right)\right] \qquad p_{k}(l_{k},t) = p_{k+1}(0,t)$$

$$u_{k}^{+}\left(t - \tau_{k}\right) - u_{k}^{-}\left(t + \tau_{k}\right) = u_{k+1}^{+}(t) - u_{k+1}^{-}(t)$$

$$\frac{A_{k+1}}{A_{k}} \left[u_{k}^{+}\left(t - \tau_{k}\right) + u_{k}^{-}\left(t + \tau_{k}\right) = u_{k+1}^{+}(t) + u_{k+1}^{-}(t)\right]$$

$$u_{k}^{-}\left(t + \tau_{k}\right) = u_{k}^{+}\left(t - \tau_{k}\right) - u_{k+1}^{+}(t) + u_{k+1}^{-}(t)$$

$$u_{k+1}^{+}(t) = u_{k}^{+}\left(t - \tau_{k}\right) \left[\frac{2A_{k+1}}{A_{k} + A_{k+1}}\right] + u_{k+1}^{-}(t) \left[\frac{A_{k+1} - A_{k}}{A_{k} + A_{k+1}}\right]$$

 $r_k = \frac{A_{k+1} - A_k}{A_k + A_{k+1}}$

$$u_{k}(x,t) = \left[u_{k}^{+}\left(t - \frac{x}{c}\right) - u_{k}^{-}\left(t + \frac{x}{c}\right)\right] \qquad u_{k}(l_{k},t) = u_{k+1}(0,t) \qquad r_{k} = \frac{A_{k+1} - A_{k}}{A_{k} + A_{k+1}}$$

$$p_{k}(x,t) = \frac{\rho c}{A_{k}} \left[u_{k}^{+}\left(t - \frac{x}{c}\right) + u_{k}^{-}\left(t + \frac{x}{c}\right)\right] \qquad r_{k} = \frac{A_{k+1} - A_{k}}{A_{k} + A_{k+1}}$$

$$u_{k}^{+}\left(t - \tau_{k}\right) - u_{k}^{-}\left(t + \tau_{k}\right) = u_{k+1}^{+}(t) - u_{k+1}^{-}(t)$$

$$\frac{A_{k+1}}{A_{k}} \left[u_{k}^{+}\left(t - \tau_{k}\right) + u_{k}^{-}\left(t + \tau_{k}\right) = u_{k+1}^{+}(t) + u_{k+1}^{-}(t)\right]$$

$$u_{k+1}^{+}(t) = u_{k}^{+}\left(t - \tau_{k}\right) \left[\frac{2A_{k+1}}{A_{k} + A_{k+1}}\right] + u_{k+1}^{-}(t) \left[\frac{A_{k+1} - A_{k}}{A_{k} + A_{k+1}}\right]$$

$$u_{k}(x,t) = \left[u_{k}^{+}\left(t - \frac{x}{c}\right) - u_{k}^{-}\left(t + \frac{x}{c}\right)\right] \qquad u_{k}(l_{k},t) = u_{k+1}(0,t)$$

$$p_{k}(x,t) = \frac{\rho c}{A_{k}}\left[u_{k}^{+}\left(t - \frac{x}{c}\right) + u_{k}^{-}\left(t + \frac{x}{c}\right)\right] \qquad p_{k}(l_{k},t) = p_{k+1}(0,t)$$

$$u_{k}^{+}\left(t - \tau_{k}\right) - u_{k}^{-}\left(t + \tau_{k}\right) = u_{k+1}^{+}(t) - u_{k+1}^{-}(t)$$

$$\frac{A_{k+1}}{A_{k}}\left[u_{k}^{+}\left(t - \tau_{k}\right) + u_{k}^{-}\left(t + \tau_{k}\right) = u_{k+1}^{+}(t) + u_{k+1}^{-}(t)\right]$$

$$u_{k+1}^{+}(t) = u_{k}^{+}\left(t - \tau_{k}\right)\left[\frac{2A_{k+1}}{A_{k} + A_{k+1}}\right] + u_{k+1}^{-}(t)\left[\frac{A_{k+1} - A_{k}}{A_{k} + A_{k+1}}\right]$$

$$u_{k}^{+}\left(t - \tau_{k}\right) = u_{k+1}^{+}(t)\left[\frac{A_{k} + A_{k+1}}{2A_{k+1}}\right] + u_{k+1}^{-}(t)\left[\frac{A_{k} - A_{k+1}}{2A_{k+1}}\right]$$

 $r_k = \frac{A_{k+1} - A_k}{A_k + A_{k+1}}$

$$u_{k}(x,t) = \left[u_{k}^{+}\left(t - \frac{x}{c}\right) - u_{k}^{-}\left(t + \frac{x}{c}\right)\right] \qquad u_{k}(l_{k},t) = u_{k+1}(0,t)$$

$$p_{k}(x,t) = \frac{\rho c}{A_{k}} \left[u_{k}^{+}\left(t - \frac{x}{c}\right) + u_{k}^{-}\left(t + \frac{x}{c}\right)\right] \qquad p_{k}(l_{k},t) = p_{k+1}(0,t)$$

$$u_{k}^{+}\left(t - \tau_{k}\right) - u_{k}^{-}\left(t + \tau_{k}\right) = u_{k+1}^{+}(t) - u_{k+1}^{-}(t)$$

$$\frac{A_{k+1}}{A_{k}} \left[u_{k}^{+}\left(t - \tau_{k}\right) + u_{k}^{-}\left(t + \tau_{k}\right) = u_{k+1}^{+}(t) + u_{k+1}^{-}(t)\right]$$

$$u_{k+1}^{+}(t) = u_{k}^{+}\left(t - \tau_{k}\right) \left[\frac{2A_{k+1}}{A_{k} + A_{k+1}}\right] + u_{k+1}^{-}(t) \left[\frac{A_{k+1} - A_{k}}{A_{k} + A_{k+1}}\right]$$

$$u_{k}^{+}\left(t - \tau_{k}\right) = u_{k+1}^{+}(t) \left[\frac{A_{k} + A_{k+1}}{2A_{k+1}}\right] + u_{k+1}^{-}(t) \left[\frac{A_{k} - A_{k+1}}{2A_{k+1}}\right]$$

$$u_{k}(x,t) = \left[u_{k}^{+}\left(t - \frac{x}{c}\right) - u_{k}^{-}\left(t + \frac{x}{c}\right)\right] \qquad u_{k}(l_{k},t) = u_{k+1}(0,t)$$

$$p_{k}(x,t) = \frac{\rho c}{A_{k}}\left[u_{k}^{+}\left(t - \frac{x}{c}\right) + u_{k}^{-}\left(t + \frac{x}{c}\right)\right] \qquad u_{k}(l_{k},t) = p_{k+1}(0,t)$$

$$u_{k}^{+}\left(t - \tau_{k}\right) - u_{k}^{-}\left(t + \tau_{k}\right) = u_{k+1}^{+}(t) - u_{k+1}^{-}(t)$$

$$\frac{A_{k+1}}{A_{k}}\left[u_{k}^{+}\left(t - \tau_{k}\right) + u_{k}^{-}\left(t + \tau_{k}\right) = u_{k+1}^{+}(t) + u_{k+1}^{-}(t)\right]$$

$$u_{k+1}^{+}(t) = u_{k}^{+}\left(t - \tau_{k}\right)\left[\frac{2A_{k+1}}{A_{k} + A_{k+1}}\right] + u_{k+1}^{-}(t)\left[\frac{A_{k+1} - A_{k}}{A_{k} + A_{k+1}}\right]$$

$$u_{k}^{+}\left(t - \tau_{k}\right) = u_{k+1}^{+}(t)\left[\frac{A_{k} + A_{k+1}}{2A_{k+1}}\right] + u_{k+1}^{-}(t)\left[\frac{A_{k} - A_{k+1}}{2A_{k+1}}\right]$$

$$\begin{aligned} u_k(x,t) &= \left[u_k^+ \left(t - \frac{x}{c} \right) - u_k^- \left(t + \frac{x}{c} \right) \right] & u_k(l_k,t) = u_{k+1}(0,t) \\ p_k(x,t) &= \frac{\rho c}{A_k} \left[u_k^+ \left(t - \frac{x}{c} \right) + u_k^- \left(t + \frac{x}{c} \right) \right] & u_k(l_k,t) = p_{k+1}(0,t) \end{aligned}$$

$$\begin{aligned} u_k^+ \left(t - \tau_k \right) &= u_k^- \left(t + \tau_k \right) = u_{k+1}^+(t) - u_{k+1}^-(t) \\ \frac{A_{k+1}}{A_k} \left[u_k^+ \left(t - \tau_k \right) + u_k^- \left(t + \tau_k \right) = u_{k+1}^+(t) + u_{k+1}^-(t) \right] \end{aligned}$$

$$\begin{aligned} u_k^+ \left(t - \tau_k \right) &= u_k^+ \left(t - \tau_k \right) \left[\frac{2A_{k+1}}{A_k + A_{k+1}} \right] + u_{k+1}^-(t) \left[\frac{A_{k+1} - A_k}{A_k + A_{k+1}} \right] \end{aligned}$$

$$u_k^+ \left(t - \tau_k \right) &= u_{k+1}^+(t) \left[\frac{A_k + A_{k+1}}{2A_{k+1}} \right] + u_{k+1}^-(t) \left[\frac{A_k - A_{k+1}}{2A_{k+1}} \right] \end{aligned}$$

$$u_k^- \left(t + \tau_k \right) &= u_{k+1}^+(t) \left[\frac{A_k - A_{k+1}}{2A_{k+1}} \right] + u_{k+1}^-(t) \left[\frac{A_k + A_{k+1}}{2A_{k+1}} \right] \end{aligned}$$

$$u_{k}(x,t) = \left[u_{k}^{+}\left(t - \frac{x}{c}\right) - u_{k}^{-}\left(t + \frac{x}{c}\right)\right] \qquad u_{k}(l_{k},t) = u_{k+1}(0,t)$$

$$p_{k}(x,t) = \frac{\rho c}{A_{k}}\left[u_{k}^{+}\left(t - \frac{x}{c}\right) + u_{k}^{-}\left(t + \frac{x}{c}\right)\right] \qquad u_{k}(l_{k},t) = p_{k+1}(0,t)$$

$$u_{k}^{+}\left(t - \tau_{k}\right) - u_{k}^{-}\left(t + \tau_{k}\right) = u_{k+1}^{+}(t) - u_{k+1}^{-}(t)$$

$$\frac{A_{k+1}}{A_{k}}\left[u_{k}^{+}\left(t - \tau_{k}\right) + u_{k}^{-}\left(t + \tau_{k}\right) = u_{k+1}^{+}(t) + u_{k+1}^{-}(t)\right]$$

$$u_{k+1}^{+}(t) = u_{k}^{+}\left(t - \tau_{k}\right)\left[\frac{2A_{k+1}}{A_{k} + A_{k+1}}\right] + u_{k+1}^{-}(t)\left[\frac{A_{k+1} - A_{k}}{A_{k} + A_{k+1}}\right]$$

$$u_{k}^{+}\left(t - \tau_{k}\right) = u_{k+1}^{+}(t)\left[\frac{A_{k} + A_{k+1}}{2A_{k+1}}\right] + u_{k+1}^{-}(t)\left[\frac{A_{k} - A_{k+1}}{2A_{k+1}}\right]$$

$$u_{k}^{-}\left(t + \tau_{k}\right) = u_{k+1}^{+}(t)\left[\frac{A_{k} - A_{k+1}}{2A_{k+1}}\right] + u_{k+1}^{-}(t)\left[\frac{A_{k} + A_{k+1}}{2A_{k+1}}\right]$$

$$\begin{split} u_k(x,t) &= \left[u_k^+ \left(t - \frac{x}{c} \right) - u_k^- \left(t + \frac{x}{c} \right) \right] & u_k(l_k,t) = u_{k+1}(0,t) \\ p_k(x,t) &= \frac{\rho c}{A_k} \left[u_k^+ \left(t - \frac{x}{c} \right) + u_k^- \left(t + \frac{x}{c} \right) \right] & u_k(l_k,t) = p_{k+1}(0,t) \\ u_k^+ \left(t - \tau_k \right) - u_k^- \left(t + \tau_k \right) &= u_{k+1}^+ (t) - u_{k+1}^- (t) \\ \frac{A_{k+1}}{A_k} \left[u_k^+ \left(t - \tau_k \right) + u_k^- \left(t + \tau_k \right) = u_{k+1}^+ (t) + u_{k+1}^- (t) \right] \\ u_{k+1}^+ (t) &= u_k^+ (t - \tau_k) \left[\frac{2A_{k+1}}{A_k + A_{k+1}} \right] + u_{k+1}^- (t) \left[\frac{A_{k+1} - A_k}{A_k + A_{k+1}} \right] \\ u_k^+ (t - \tau_k) &= u_{k+1}^+ (t) \left[\frac{A_k + A_{k+1}}{2A_{k+1}} \right] + u_{k+1}^- (t) \left[\frac{A_k - A_{k+1}}{2A_{k+1}} \right] \\ u_k^- (t + \tau_k) &= u_{k+1}^+ (t) \left[\frac{A_k - A_{k+1}}{2A_{k+1}} \right] + u_{k+1}^- (t) \left[\frac{A_k + A_{k+1}}{2A_{k+1}} \right] \end{split}$$

$$u_{k}(x,t) = \left[u_{k}^{+}\left(t - \frac{x}{c}\right) - u_{k}^{-}\left(t + \frac{x}{c}\right)\right] \qquad u_{k}(l_{k},t) = u_{k+1}(0,t)$$

$$p_{k}(x,t) = \frac{\rho c}{A_{k}} \left[u_{k}^{+}\left(t - \frac{x}{c}\right) + u_{k}^{-}\left(t + \frac{x}{c}\right)\right] \qquad p_{k}(l_{k},t) = p_{k+1}(0,t)$$

$$u_{k}^{+}(t - \tau_{k}) = u_{k+1}^{+}(t) \left[\frac{A_{k} + A_{k+1}}{2A_{k+1}}\right] + u_{k+1}^{-}(t) \left[\frac{A_{k} - A_{k+1}}{2A_{k+1}}\right]$$

$$u_{k}^{-}(t + \tau_{k}) = u_{k+1}^{+}(t) \left[\frac{A_{k} - A_{k+1}}{2A_{k+1}}\right] + u_{k+1}^{-}(t) \left[\frac{A_{k} + A_{k+1}}{2A_{k+1}}\right]$$

$$r_k = \frac{A_{k+1} - A_k}{A_k + A_{k+1}}$$

$$u_{k}(x,t) = \left[u_{k}^{+}\left(t - \frac{x}{c}\right) - u_{k}^{-}\left(t + \frac{x}{c}\right)\right] \qquad u_{k}(l_{k},t) = u_{k+1}(0,t)$$

$$p_{k}(x,t) = \frac{\rho c}{A_{k}} \left[u_{k}^{+}\left(t - \frac{x}{c}\right) + u_{k}^{-}\left(t + \frac{x}{c}\right)\right] \qquad p_{k}(l_{k},t) = p_{k+1}(0,t)$$

$$u_{k}^{+}(t - \tau_{k}) = u_{k+1}^{+}(t) \left[\frac{A_{k} + A_{k+1}}{2A_{k+1}}\right] + u_{k+1}^{-}(t) \left[\frac{A_{k} - A_{k+1}}{2A_{k+1}}\right]$$

$$u_{k}^{-}(t + \tau_{k}) = u_{k+1}^{+}(t) \left[\frac{A_{k} - A_{k+1}}{2A_{k+1}}\right] + u_{k+1}^{-}(t) \left[\frac{A_{k} + A_{k+1}}{2A_{k+1}}\right]$$

$$r_k = \frac{A_{k+1} - A_k}{A_k + A_{k+1}}$$

$$u_{k}(x,t) = \left[u_{k}^{+}\left(t - \frac{x}{c}\right) - u_{k}^{-}\left(t + \frac{x}{c}\right)\right] \qquad u_{k}(l_{k},t) = u_{k+1}(0,t)$$

$$p_{k}(x,t) = \frac{\rho c}{A_{k}}\left[u_{k}^{+}\left(t - \frac{x}{c}\right) + u_{k}^{-}\left(t + \frac{x}{c}\right)\right] \qquad p_{k}(l_{k},t) = p_{k+1}(0,t)$$

$$u_{k}^{+}(t - \tau_{k}) = \frac{1}{1 + r_{k}}u_{k+1}^{+}(t) - \frac{r_{k}}{1 + r_{k}}u_{k+1}^{-}(t)$$

$$u_{k}^{-}(t + \tau_{k}) = -\frac{r_{k}}{1 + r_{k}}u_{k+1}^{+}(t) + \frac{1}{1 + r_{k}}u_{k+1}^{-}(t)$$

$$u_{k}(x,t) = \left[u_{k}^{+}\left(t - \frac{x}{c}\right) - u_{k}^{-}\left(t + \frac{x}{c}\right)\right] \qquad u_{k}(l_{k},t) = u_{k+1}(0,t)$$

$$p_{k}(x,t) = \frac{\rho c}{A_{k}} \left[u_{k}^{+}\left(t - \frac{x}{c}\right) + u_{k}^{-}\left(t + \frac{x}{c}\right)\right]$$

$$v_{k}^{+}(t - \tau_{k}) = \frac{1}{1 + r_{k}} u_{k+1}^{+}(t) - \frac{r_{k}}{1 + r_{k}} u_{k+1}^{-}(t)$$

$$u_{k}^{-}(t + \tau_{k}) = -\frac{r_{k}}{1 + r_{k}} u_{k+1}^{+}(t) + \frac{1}{1 + r_{k}} u_{k+1}^{-}(t)$$

$$\begin{split} u_k(x,t) &= \left[u_k^+ \left(t - \frac{x}{c} \right) - u_k^- \left(t + \frac{x}{c} \right) \right] & u_k(l_k,t) = u_{k+1}(0,t) \\ p_k(x,t) &= \frac{\rho c}{A_k} \left[u_k^+ \left(t - \frac{x}{c} \right) + u_k^- \left(t + \frac{x}{c} \right) \right] & r_k = \frac{A_{k+1} - A_k}{A_k + A_{k+1}} \\ u_k^+(t - \tau_k) &= \frac{1}{1 + r_k} u_{k+1}^+(t) - \frac{r_k}{1 + r_k} u_{k+1}^-(t) \\ u_k^-(t + \tau_k) &= -\frac{r_k}{1 + r_k} u_{k+1}^+(t) + \frac{1}{1 + r_k} u_{k+1}^-(t) \\ & \text{Heko } u_k(t) \overset{\mathcal{FS}}{\longleftrightarrow} U_k(z). \end{split}$$

$$\begin{split} u_k(x,t) &= \left[u_k^+ \left(t - \frac{x}{c} \right) - u_k^- \left(t + \frac{x}{c} \right) \right] & u_k(l_k,t) = u_{k+1}(0,t) \\ p_k(x,t) &= \frac{\rho c}{A_k} \left[u_k^+ \left(t - \frac{x}{c} \right) + u_k^- \left(t + \frac{x}{c} \right) \right] \end{split} \qquad r_k = \frac{A_{k+1} - A_k}{A_k + A_{k+1}} \\ u_k^+(t - \tau_k) &= \frac{1}{1 + r_k} u_{k+1}^+(t) - \frac{r_k}{1 + r_k} u_{k+1}^-(t) \\ u_k^-(t + \tau_k) &= -\frac{r_k}{1 + r_k} u_{k+1}^+(t) + \frac{1}{1 + r_k} u_{k+1}^-(t) \\ \text{Heko } u_k(t) & \stackrel{\mathcal{FS}}{\longleftrightarrow} U_k(z). \end{split}$$

$$\begin{aligned} u_k(x,t) &= \left[u_k^+ \left(t - \frac{x}{c} \right) - u_k^- \left(t + \frac{x}{c} \right) \right] & u_k(l_k,t) = u_{k+1}(0,t) \\ p_k(x,t) &= \frac{\rho c}{A_k} \left[u_k^+ \left(t - \frac{x}{c} \right) + u_k^- \left(t + \frac{x}{c} \right) \right] & p_k(l_k,t) = p_{k+1}(0,t) \\ u_k^+ (t - \tau_k) &= \frac{1}{1 + r_k} u_{k+1}^+ (t) - \frac{r_k}{1 + r_k} u_{k+1}^- (t) \\ u_k^- (t + \tau_k) &= -\frac{r_k}{1 + r_k} u_{k+1}^+ (t) + \frac{1}{1 + r_k} u_{k+1}^- (t) \\ \text{Hekg } u_k(t) &\iff U_k(z). \\ \text{Tozabo} & u_k(t - \tau_k) &\iff z^{-\tau_k} U_k(z). \end{aligned}$$

 $U_k^-(z) = -\frac{r_k z^{-\tau_k}}{1 + z} U_{k+1}^+(z) + \frac{z^{-\tau_k}}{1 + z} U_{k+1}^-(z)$

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$$u_k(x,t) = \left[u_k^+ \left(t - \frac{x}{c}\right) - u_k^- \left(t + \frac{x}{c}\right)\right] \qquad u_k(l_k,t) = u_{k+1}(0,t)$$

$$p_k(x,t) = \frac{\rho c}{A_k} \left[u_k^+ \left(t - \frac{x}{c}\right) + u_k^- \left(t + \frac{x}{c}\right)\right] \qquad p_k(l_k,t) = p_{k+1}(0,t)$$

$$u_k^+ (t - \tau_k) = \frac{1}{1 + r_k} u_{k+1}^+ (t) - \frac{r_k}{1 + r_k} u_{k+1}^- (t)$$

$$u_k^- (t + \tau_k) = -\frac{r_k}{1 + r_k} u_{k+1}^+ (t) + \frac{1}{1 + r_k} u_{k+1}^- (t)$$

$$\text{Hekg } u_k(t) \overset{\mathcal{FS}}{\longleftrightarrow} U_k(z).$$

$$\text{Tocabo } u_k(t - \tau_k) \overset{\mathcal{FS}}{\longleftrightarrow} z^{-\tau_k} U_k(z).$$

$$U_k^+ (z) = \frac{z^{\tau_k}}{1 + r_k} U_{k+1}^+ (z) - \frac{r_k z^{\tau_k}}{1 + r_k} U_{k+1}^- (z)$$

 $\tau_k = 1/2$

 $U_k^-(z) = -\frac{r_k z^{-\tau_k}}{1 + z} U_{k+1}^+(z) + \frac{z^{-\tau_k}}{1 + z} U_{k+1}^-(z)$

$$\begin{aligned} u_k(x,t) &= \left[u_k^+ \left(t - \frac{x}{c} \right) - u_k^- \left(t + \frac{x}{c} \right) \right] & u_k(l_k,t) = u_{k+1}(0,t) \\ p_k(x,t) &= \frac{\rho c}{A_k} \left[u_k^+ \left(t - \frac{x}{c} \right) + u_k^- \left(t + \frac{x}{c} \right) \right] & p_k(l_k,t) = p_{k+1}(0,t) \\ u_k^+(t-\tau_k) &= \frac{1}{1+r_k} u_{k+1}^+(t) - \frac{r_k}{1+r_k} u_{k+1}^-(t) \\ u_k^-(t+\tau_k) &= -\frac{r_k}{1+r_k} u_{k+1}^+(t) + \frac{1}{1+r_k} u_{k+1}^-(t) \\ &\text{Heko } u_k(t) \overset{\mathcal{FS}}{\longleftrightarrow} U_k(z). \\ &\text{Tozaba } u_k(t-\tau_k) \overset{\mathcal{FS}}{\longleftrightarrow} z^{-\tau_k} U_k(z). \end{aligned}$$

 $U_{k}^{-}(z) = -\frac{r_{k}z^{-1/2}}{1 + r_{k}}U_{k+1}^{+}(z) + \frac{z^{-1/2}}{1 + r_{k}}U_{k+1}^{-}(z)$

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$$u_k(x,t) = \left[u_k^+ \left(t - \frac{x}{c} \right) - u_k^- \left(t + \frac{x}{c} \right) \right]$$

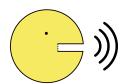
$$p_k(x,t) = \frac{\rho c}{A_k} \left[u_k^+ \left(t - \frac{x}{c} \right) + u_k^- \left(t + \frac{x}{c} \right) \right]$$

$$U_k^+(z) = \frac{z^{1/2}}{1 + r_k} U_{k+1}^+(z) - \frac{r_k z^{1/2}}{1 + r_k} U_{k+1}^-(z)$$

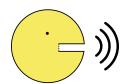
$$U_k^-(z) = -\frac{r_k z^{-1/2}}{1 + r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1 + r_k} U_{k+1}^-(z)$$

$$\begin{split} u_k(x,t) &= \left[u_k^+ \left(t - \frac{x}{c} \right) - u_k^- \left(t + \frac{x}{c} \right) \right] \\ p_k(x,t) &= \frac{\rho c}{A_k} \left[u_k^+ \left(t - \frac{x}{c} \right) + u_k^- \left(t + \frac{x}{c} \right) \right] \\ U_k^+(z) &= \frac{z^{1/2}}{1 + r_k} U_{k+1}^+(z) - \frac{r_k z^{1/2}}{1 + r_k} U_{k+1}^-(z) \\ U_k^-(z) &= -\frac{r_k z^{-1/2}}{1 + r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1 + r_k} U_{k+1}^-(z) \end{split}$$

$$\begin{split} u_k(x,t) &= \left[u_k^+ \left(t - \frac{x}{c} \right) - u_k^- \left(t + \frac{x}{c} \right) \right] \\ p_k(x,t) &= \frac{\rho c}{A_k} \left[u_k^+ \left(t - \frac{x}{c} \right) + u_k^- \left(t + \frac{x}{c} \right) \right] \\ U_k^+(z) &= \frac{z^{1/2}}{1 + r_k} U_{k+1}^+(z) - \frac{r_k z^{1/2}}{1 + r_k} U_{k+1}^-(z) \\ U_k^-(z) &= -\frac{r_k z^{-1/2}}{1 + r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1 + r_k} U_{k+1}^-(z) \end{split}$$

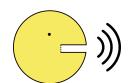


$$\begin{split} u_k(x,t) &= \left[u_k^+ \left(t - \frac{x}{c} \right) - u_k^- \left(t + \frac{x}{c} \right) \right] \\ p_k(x,t) &= \frac{\rho c}{A_k} \left[u_k^+ \left(t - \frac{x}{c} \right) + u_k^- \left(t + \frac{x}{c} \right) \right] \\ U_k^+(z) &= \frac{z^{1/2}}{1 + r_k} U_{k+1}^+(z) - \frac{r_k z^{1/2}}{1 + r_k} U_{k+1}^-(z) \\ U_k^-(z) &= -\frac{r_k z^{-1/2}}{1 + r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1 + r_k} U_{k+1}^-(z) \end{split}$$





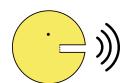
$$\begin{split} u_k(x,t) &= \left[u_k^+ \left(t - \frac{x}{c} \right) - u_k^- \left(t + \frac{x}{c} \right) \right] \\ p_k(x,t) &= \frac{\rho c}{A_k} \left[u_k^+ \left(t - \frac{x}{c} \right) + u_k^- \left(t + \frac{x}{c} \right) \right] \\ U_k^+(z) &= \frac{z^{1/2}}{1 + r_k} U_{k+1}^+(z) - \frac{r_k z^{1/2}}{1 + r_k} U_{k+1}^-(z) \\ U_k^-(z) &= -\frac{r_k z^{-1/2}}{1 + r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1 + r_k} U_{k+1}^-(z) \end{split}$$





$$\mathcal{P}_N(l_N,z) = Z_L(z)\mathcal{U}_N(l_N,z)$$

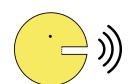
$$\begin{split} u_k(x,t) &= \left[u_k^+ \left(t - \frac{x}{c} \right) - u_k^- \left(t + \frac{x}{c} \right) \right] \\ p_k(x,t) &= \frac{\rho c}{A_k} \left[u_k^+ \left(t - \frac{x}{c} \right) + u_k^- \left(t + \frac{x}{c} \right) \right] \\ U_k^+(z) &= \frac{z^{1/2}}{1 + r_k} U_{k+1}^+(z) - \frac{r_k z^{1/2}}{1 + r_k} U_{k+1}^-(z) \\ U_k^-(z) &= -\frac{r_k z^{-1/2}}{1 + r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1 + r_k} U_{k+1}^-(z) \end{split}$$





$$\mathcal{P}_N(l_N, z) = Z_L \mathcal{U}_N(l_N, z)$$

$$\begin{split} u_k(x,t) &= \left[u_k^+ \left(t - \frac{x}{c} \right) - u_k^- \left(t + \frac{x}{c} \right) \right] \\ p_k(x,t) &= \frac{\rho c}{A_k} \left[u_k^+ \left(t - \frac{x}{c} \right) + u_k^- \left(t + \frac{x}{c} \right) \right] \\ U_k^+(z) &= \frac{z^{1/2}}{1 + r_k} U_{k+1}^+(z) - \frac{r_k z^{1/2}}{1 + r_k} U_{k+1}^-(z) \\ U_k^-(z) &= -\frac{r_k z^{-1/2}}{1 + r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1 + r_k} U_{k+1}^-(z) \end{split}$$

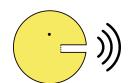




$$\mathcal{P}_N(l_N,z) = Z_L \mathcal{U}_N(l_N,z)$$

$$p(l_N, t) \stackrel{\mathcal{FS}}{\longleftrightarrow} \mathcal{P}_N(l_N, z), u_N(l_N, t) \stackrel{\mathcal{FS}}{\longleftrightarrow} \mathcal{U}_N(l_N, z)$$

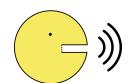
$$\begin{split} u_k(x,t) &= \left[u_k^+ \left(t - \frac{x}{c} \right) - u_k^- \left(t + \frac{x}{c} \right) \right] \\ p_k(x,t) &= \frac{\rho c}{A_k} \left[u_k^+ \left(t - \frac{x}{c} \right) + u_k^- \left(t + \frac{x}{c} \right) \right] \\ U_k^+(z) &= \frac{z^{1/2}}{1 + r_k} U_{k+1}^+(z) - \frac{r_k z^{1/2}}{1 + r_k} U_{k+1}^-(z) \\ U_k^-(z) &= -\frac{r_k z^{-1/2}}{1 + r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1 + r_k} U_{k+1}^-(z) \end{split}$$





$$p_N(l_N,t) = Z_L u_N(l_N,t)$$

$$\begin{aligned} u_k(x,t) &= \left[u_k^+ \left(t - \frac{x}{c} \right) - u_k^- \left(t + \frac{x}{c} \right) \right] \\ p_k(x,t) &= \frac{\rho c}{A_k} \left[u_k^+ \left(t - \frac{x}{c} \right) + u_k^- \left(t + \frac{x}{c} \right) \right] \\ U_k^+(z) &= \frac{z^{1/2}}{1 + r_k} U_{k+1}^+(z) - \frac{r_k z^{1/2}}{1 + r_k} U_{k+1}^-(z) \\ U_k^-(z) &= -\frac{r_k z^{-1/2}}{1 + r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1 + r_k} U_{k+1}^-(z) \end{aligned}$$

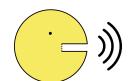




$$p_N(l_N, t) = Z_L u_N(l_N, t)$$

$$u_N^-(t + \tau_N) \frac{(\rho c + A_N Z_L)}{A_N} = u_N^+(t - \tau_N) \frac{(A_N Z_L - \rho c)}{A_N}$$

$$\begin{aligned} u_k(x,t) &= \left[u_k^+ \left(t - \frac{x}{c} \right) - u_k^- \left(t + \frac{x}{c} \right) \right] \\ p_k(x,t) &= \frac{\rho c}{A_k} \left[u_k^+ \left(t - \frac{x}{c} \right) + u_k^- \left(t + \frac{x}{c} \right) \right] \\ U_k^+(z) &= \frac{z^{1/2}}{1 + r_k} U_{k+1}^+(z) - \frac{r_k z^{1/2}}{1 + r_k} U_{k+1}^-(z) \\ U_k^-(z) &= -\frac{r_k z^{-1/2}}{1 + r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1 + r_k} U_{k+1}^-(z) \end{aligned}$$



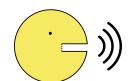


$$p_N(l_N,t) = Z_L u_N(l_N,t)$$

$$u_{N}^{-}(t+\tau_{N})\frac{(\rho c+A_{N}Z_{L})}{A_{N}}=u_{N}^{+}(t-\tau_{N})\frac{(A_{N}Z_{L}-\rho c)}{A_{N}} \qquad r_{L}=\frac{\frac{\rho c}{Z_{L}}-A_{N}}{\frac{\rho c}{Z_{L}}+A_{N}}$$

$$r_L = \frac{\frac{\rho c}{Z_L} - A_L}{\frac{\rho c}{Z_L} + A_L}$$

$$\begin{split} u_k(x,t) &= \left[u_k^+ \left(t - \frac{x}{c} \right) - u_k^- \left(t + \frac{x}{c} \right) \right] \\ p_k(x,t) &= \frac{\rho c}{A_k} \left[u_k^+ \left(t - \frac{x}{c} \right) + u_k^- \left(t + \frac{x}{c} \right) \right] \\ U_k^+(z) &= \frac{z^{1/2}}{1 + r_k} U_{k+1}^+(z) - \frac{r_k z^{1/2}}{1 + r_k} U_{k+1}^-(z) \\ U_k^-(z) &= -\frac{r_k z^{-1/2}}{1 + r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1 + r_k} U_{k+1}^-(z) \end{split}$$



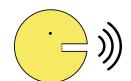


$$p_N(l_N,t)=Z_Lu_N(l_N,t)$$

$$u_N^-(t+\tau_N) = -r_L u_N^+(t-\tau_N)$$
 $r_L = \frac{\frac{\rho c}{Z_L} - A_N}{\frac{\rho c}{Z_L} + A_N}$

$$\begin{split} u_k(x,t) &= \left[u_k^+ \left(t - \frac{x}{c} \right) - u_k^- \left(t + \frac{x}{c} \right) \right] \\ p_k(x,t) &= \frac{\rho c}{A_k} \left[u_k^+ \left(t - \frac{x}{c} \right) + u_k^- \left(t + \frac{x}{c} \right) \right] \\ U_k^+(z) &= \frac{z^{1/2}}{1 + r_k} U_{k+1}^+(z) - \frac{r_k z^{1/2}}{1 + r_k} U_{k+1}^-(z) \\ U_k^-(z) &= -\frac{r_k z^{-1/2}}{1 + r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1 + r_k} U_{k+1}^-(z) \end{split}$$

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$$p_N(l_N,t)=Z_Lu_N(l_N,t)$$

$$u_N^-(t+\tau_N) = -r_L u_N^+(t-\tau_N)$$
 $r_L = \frac{\frac{\rho_C}{Z_L} - A_N}{\frac{\rho_C}{Z_L} + A_N}$

$$\begin{split} u_k(x,t) &= \left[u_k^+ \left(t - \frac{x}{c} \right) - u_k^- \left(t + \frac{x}{c} \right) \right] \\ p_k(x,t) &= \frac{\rho c}{A_k} \left[u_k^+ \left(t - \frac{x}{c} \right) + u_k^- \left(t + \frac{x}{c} \right) \right] \\ U_k^+(z) &= \frac{z^{1/2}}{1 + r_k} U_{k+1}^+(z) - \frac{r_k z^{1/2}}{1 + r_k} U_{k+1}^-(z) \\ U_k^-(z) &= -\frac{r_k z^{-1/2}}{1 + r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1 + r_k} U_{k+1}^-(z) \end{split}$$

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$$U_1(0,z) = U_G(z) - \frac{P_1(0,z)}{Z_G(z)}$$

$$\begin{split} u_k(x,t) &= \left[u_k^+ \left(t - \frac{x}{c} \right) - u_k^- \left(t + \frac{x}{c} \right) \right] \\ p_k(x,t) &= \frac{\rho c}{A_k} \left[u_k^+ \left(t - \frac{x}{c} \right) + u_k^- \left(t + \frac{x}{c} \right) \right] \\ U_k^+(z) &= \frac{z^{1/2}}{1 + r_k} U_{k+1}^+(z) - \frac{r_k z^{1/2}}{1 + r_k} U_{k+1}^-(z) \\ U_k^-(z) &= -\frac{r_k z^{-1/2}}{1 + r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1 + r_k} U_{k+1}^-(z) \end{split}$$

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$$\begin{split} U_1(0,z) &= U_G(z) - \frac{P_1(0,z)}{Z_G} \\ u_1(0,t) & \stackrel{\mathcal{FS}}{\longleftrightarrow} U_1(0,z), u_G(t) \stackrel{\mathcal{FS}}{\longleftrightarrow} U_G(z), p_1(0,t) \stackrel{\mathcal{FS}}{\longleftrightarrow} P_1(0,z) \end{split}$$

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$$U_1(0,z) = U_G(z) - \frac{P_1(0,z)}{Z_G}$$

$$u_1(0,t) \stackrel{\mathcal{FS}}{\longleftrightarrow} U_1(0,z), u_G(t) \stackrel{\mathcal{FS}}{\longleftrightarrow} U_G(z), p_1(0,t) \stackrel{\mathcal{FS}}{\longleftrightarrow} P_1(0,z)$$

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$$\begin{split} &U_{1}(0,z) = U_{G}(z) - \frac{P_{1}(0,z)}{Z_{G}} \\ &u_{1}(0,t) \overset{\mathcal{FS}}{\longleftrightarrow} U_{1}(0,z), u_{G}(t) \overset{\mathcal{FS}}{\longleftrightarrow} U_{G}(z), p_{1}(0,t) \overset{\mathcal{FS}}{\longleftrightarrow} P_{1}(0,z) \\ &u_{1}(0,t) = u_{G}(t) - \frac{p_{1}(0,t)}{Z_{G}} \\ &u_{1}^{+}(t) = u_{G}(t) \left[\frac{A_{1}Z_{G}}{A_{1}Z_{G} + \rho c} \right] + u_{1}^{-}(t) \left[\frac{A_{1}Z_{G} - \rho c}{A_{1}Z_{G} + \rho c} \right] \end{split}$$

$$\begin{split} u_k(x,t) &= \left[u_k^+ \left(t - \frac{x}{c} \right) - u_k^- \left(t + \frac{x}{c} \right) \right] \\ p_k(x,t) &= \frac{\rho c}{A_k} \left[u_k^+ \left(t - \frac{x}{c} \right) + u_k^- \left(t + \frac{x}{c} \right) \right] \\ U_k^+(z) &= \frac{z^{1/2}}{1 + r_k} U_{k+1}^+(z) - \frac{r_k z^{1/2}}{1 + r_k} U_{k+1}^-(z) \\ U_k^-(z) &= -\frac{r_k z^{-1/2}}{1 + r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1 + r_k} U_{k+1}^-(z) \end{split}$$

$$\begin{split} &U_{1}(0,z) = U_{G}(z) - \frac{P_{1}(0,z)}{Z_{G}} \\ &u_{1}(0,t) \overset{\mathcal{FS}}{\longleftrightarrow} U_{1}(0,z), u_{G}(t) \overset{\mathcal{FS}}{\longleftrightarrow} U_{G}(z), p_{1}(0,t) \overset{\mathcal{FS}}{\longleftrightarrow} P_{1}(0,z) \\ &u_{1}(0,t) = u_{G}(t) - \frac{p_{1}(0,t)}{Z_{G}} \\ &u_{1}^{+}(t) = u_{G}(t) \left[\frac{A_{1}Z_{G}}{A_{1}Z_{G} + \rho c} \right] + u_{1}^{-}(t) \left[\frac{A_{1}Z_{G} - \rho c}{A_{1}Z_{G} + \rho c} \right] \qquad r_{G} = \left(\frac{A_{1}Z_{G} - \rho c}{A_{1}Z_{G} + \rho c} \right) \end{split}$$

$$\begin{split} u_k(x,t) &= \left[u_k^+ \left(t - \frac{x}{c} \right) - u_k^- \left(t + \frac{x}{c} \right) \right] \\ p_k(x,t) &= \frac{\rho c}{A_k} \left[u_k^+ \left(t - \frac{x}{c} \right) + u_k^- \left(t + \frac{x}{c} \right) \right] \\ U_k^+(z) &= \frac{z^{1/2}}{1 + r_k} U_{k+1}^+(z) - \frac{r_k z^{1/2}}{1 + r_k} U_{k+1}^-(z) \\ U_k^-(z) &= -\frac{r_k z^{-1/2}}{1 + r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1 + r_k} U_{k+1}^-(z) \end{split}$$

$$\begin{split} &U_{1}(0,z)=U_{G}(z)-\frac{P_{1}(0,z)}{Z_{G}}\\ &u_{1}(0,t) \xleftarrow{\mathcal{FS}} U_{1}(0,z), u_{G}(t) \xleftarrow{\mathcal{FS}} U_{G}(z), p_{1}(0,t) \xleftarrow{\mathcal{FS}} P_{1}(0,z)\\ &u_{1}(0,t)=u_{G}(t)-\frac{p_{1}(0,t)}{Z_{G}}\\ &u_{1}^{+}(t)=u_{G}(t)\left[\frac{1+r_{G}}{2}\right]+r_{G}u_{1}^{-}(t) \qquad r_{G}=\left(\frac{A_{1}Z_{G}-\rho c}{A_{1}Z_{G}+\rho c}\right) \end{split}$$

$$\begin{split} u_k(x,t) &= \left[u_k^+ \left(t - \frac{x}{c} \right) - u_k^- \left(t + \frac{x}{c} \right) \right] \\ p_k(x,t) &= \frac{\rho c}{A_k} \left[u_k^+ \left(t - \frac{x}{c} \right) + u_k^- \left(t + \frac{x}{c} \right) \right] \\ U_k^+(z) &= \frac{z^{1/2}}{1 + r_k} U_{k+1}^+(z) - \frac{r_k z^{1/2}}{1 + r_k} U_{k+1}^-(z) \\ U_k^-(z) &= -\frac{r_k z^{-1/2}}{1 + r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1 + r_k} U_{k+1}^-(z) \end{split}$$

$$U_{1}(0,z) = U_{G}(z) - \frac{P_{1}(0,z)}{Z_{G}}$$

$$u_{1}(0,t) \stackrel{\mathcal{FS}}{\longleftrightarrow} U_{1}(0,z), u_{G}(t) \stackrel{\mathcal{FS}}{\longleftrightarrow} U_{G}(z), p_{1}(0,t) \stackrel{\mathcal{FS}}{\longleftrightarrow} P_{1}(0,z)$$

$$u_{1}(0,t) = u_{G}(t) - \frac{p_{1}(0,t)}{Z_{G}}$$

$$u_{1}^{+}(t) = u_{G}(t) \left[\frac{1+r_{G}}{2}\right] + r_{G}u_{1}^{-}(t) \qquad r_{G} = \left(\frac{A_{1}Z_{G} - \rho c}{A_{1}Z_{G} + \rho c}\right)$$

$$U_{1}^{+}(z) = U_{G}(z) \left[\frac{1+r_{G}}{2}\right] + r_{G}U_{1}^{-}(z)$$

$$\begin{split} u_k(x,t) &= \left[u_k^+ \left(t - \frac{x}{c} \right) - u_k^- \left(t + \frac{x}{c} \right) \right] \\ p_k(x,t) &= \frac{\rho c}{A_k} \left[u_k^+ \left(t - \frac{x}{c} \right) + u_k^- \left(t + \frac{x}{c} \right) \right] \\ U_k^+(z) &= \frac{z^{1/2}}{1 + r_k} U_{k+1}^+(z) - \frac{r_k z^{1/2}}{1 + r_k} U_{k+1}^-(z) \\ U_k^-(z) &= -\frac{r_k z^{-1/2}}{1 + r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1 + r_k} U_{k+1}^-(z) \end{split}$$

$$r_{L} = \frac{\frac{\rho c}{Z_{L}} - A_{N}}{\frac{\rho c}{Z_{L}} + A_{N}} \qquad r_{G} = \left(\frac{A_{1} Z_{G} - \rho c}{A_{1} Z_{G} + \rho c}\right)$$
$$U_{1}^{+}(z) = U_{G}(z) \left[\frac{1 + r_{G}}{2}\right] + r_{G} U_{1}^{-}(z)$$

$$U_{1}(0,z) = U_{G}(z) - \frac{P_{1}(0,z)}{Z_{G}}$$

$$u_{1}(0,t) \stackrel{\mathcal{FS}}{\longleftrightarrow} U_{1}(0,z), u_{G}(t) \stackrel{\mathcal{FS}}{\longleftrightarrow} U_{G}(z), p_{1}(0,t) \stackrel{\mathcal{FS}}{\longleftrightarrow} P_{1}(0,z)$$

$$u_{1}(0,t) = u_{G}(t) - \frac{p_{1}(0,t)}{Z_{G}}$$

$$u_{1}^{+}(t) = u_{G}(t) \left[\frac{1+r_{G}}{2} \right] + r_{G}u_{1}^{-}(t) \qquad r_{G} = \left(\frac{A_{1}Z_{G} - \rho c}{A_{1}Z_{G} + \rho c} \right)$$

$$U_{1}^{+}(z) = U_{G}(z) \left[\frac{1+r_{G}}{2} \right] + r_{G}U_{1}^{-}(z)$$

$$\begin{split} u_k(x,t) &= \left[u_k^+ \left(t - \frac{x}{c} \right) - u_k^- \left(t + \frac{x}{c} \right) \right] \\ p_k(x,t) &= \frac{\rho c}{A_k} \left[u_k^+ \left(t - \frac{x}{c} \right) + u_k^- \left(t + \frac{x}{c} \right) \right] \\ U_k^+(z) &= \frac{z^{1/2}}{1 + r_k} U_{k+1}^+(z) - \frac{r_k z^{1/2}}{1 + r_k} U_{k+1}^-(z) \\ U_k^-(z) &= -\frac{r_k z^{-1/2}}{1 + r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1 + r_k} U_{k+1}^-(z) \end{split}$$

$$\begin{split} r_L &= \frac{\frac{\rho c}{Z_L} - A_N}{\frac{\rho c}{Z_L} + A_N} \\ &\qquad r_G = \left(\frac{A_1 Z_G - \rho c}{A_1 Z_G + \rho c}\right) \\ &\qquad U_1^+(z) = U_G(z) \left[\frac{1 + r_G}{2}\right] + r_G U_1^-(z) \end{split}$$

$$\begin{split} U_k^+(z) &= \frac{z^{1/2}}{1 + r_k} U_{k+1}^+(z) - \frac{r_k z^{1/2}}{1 + r_k} U_{k+1}^-(z) \\ U_k^-(z) &= -\frac{r_k z^{-1/2}}{1 + r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1 + r_k} U_{k+1}^-(z) \end{split} \qquad r_L &= \frac{\frac{\rho c}{Z_L} - A_N}{\frac{\rho c}{Z_L} + A_N} \qquad r_G = \left(\frac{A_1 Z_G - \rho c}{A_1 Z_G + \rho c}\right) \\ U_1^+(z) &= U_G(z) \left[\frac{1 + r_G}{2}\right] + r_G U_1^-(z) \end{split}$$

$$\begin{split} U_k^+(z) &= \frac{z^{1/2}}{1+r_k} U_{k+1}^+(z) - \frac{r_k z^{1/2}}{1+r_k} U_{k+1}^-(z) \\ U_k^-(z) &= -\frac{r_k z^{-1/2}}{1+r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1+r_k} U_{k+1}^-(z) \\ \end{split} \qquad \begin{aligned} r_L &= \frac{\frac{\rho_C}{2L} - A_N}{\frac{\rho_C}{2L} + A_N} \\ U_L^-(z) &= \frac{r_k z^{-1/2}}{2L} U_{k+1}^+(z) + \frac{z^{-1/2}}{1+r_k} U_{k+1}^-(z) \\ U_L^+(z) &= U_G(z) \left[\frac{1+r_G}{2L} \right] + r_G U_1^-(z) \end{aligned}$$

$$\begin{split} U_k^+(z) &= \frac{z^{1/2}}{1+r_k} U_{k+1}^+(z) - \frac{r_k z^{1/2}}{1+r_k} U_{k+1}^-(z) \\ U_k^-(z) &= -\frac{r_k z^{-1/2}}{1+r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1+r_k} U_{k+1}^-(z) \end{split} \qquad r_L &= \frac{\frac{\rho c}{Z_L} - A_N}{\frac{\rho c}{Z_L} + A_N} \qquad r_G = \left(\frac{A_1 Z_G - \rho c}{A_1 Z_G + \rho c}\right) \\ U_1^+(z) &= U_G(z) \left[\frac{1+r_G}{2}\right] + r_G U_1^-(z) \end{split}$$

$$U_{N+1}^+(z) = U_L(z)$$

$$\begin{split} U_k^+(z) &= \frac{z^{1/2}}{1+r_k} U_{k+1}^+(z) - \frac{r_k z^{1/2}}{1+r_k} U_{k+1}^-(z) \\ U_k^-(z) &= -\frac{r_k z^{-1/2}}{1+r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1+r_k} U_{k+1}^-(z) \\ \end{split} \qquad \begin{aligned} r_L &= \frac{\frac{\rho c}{Z_L} - A_N}{\frac{\rho c}{Z_L} + A_N} \\ v_L &= \frac{\frac{\rho c}{Z_L} - A_N}{\frac{\rho c}{Z_L} + A_N} \end{aligned} \qquad \qquad \\ r_G &= \left(\frac{A_1 Z_G - \rho c}{A_1 Z_G + \rho c}\right) \\ U_1^+(z) &= U_G(z) \left[\frac{1+r_G}{2}\right] + r_G U_1^-(z) \end{aligned}$$

$$U_{N+1}^{+}(z) = U_{L}(z)$$

 $U_{N+1}^{-}(z) = 0$

$$U_{k}^{+}(z) = \frac{z^{1/2}}{1 + r_{k}} U_{k+1}^{+}(z) - \frac{r_{k}z^{1/2}}{1 + r_{k}} U_{k+1}^{-}(z) \qquad r_{L} = \frac{\frac{\rho c}{Z_{L}} - A_{N}}{\frac{\rho c}{Z_{L}} + A_{N}} \qquad r_{G} = \left(\frac{A_{1}Z_{G} - \rho c}{A_{1}Z_{G} + \rho c}\right)$$

$$U_{k}^{-}(z) = -\frac{r_{k}z^{-1/2}}{1 + r_{k}} U_{k+1}^{+}(z) + \frac{z^{-1/2}}{1 + r_{k}} U_{k+1}^{-}(z) \qquad U_{1}^{+}(z) = U_{G}(z) \left[\frac{1 + r_{k}}{Z_{L}} \right]$$

Общ вид

$$U_{N+1}^{+}(z) = U_L(z)$$

$$U_{N+1}^{-}(z) = 0$$

$$r_N = r_L$$

 $U_1^+(z) = U_G(z) \left[\frac{1 + r_G}{2} \right] + r_G U_1^-(z)$

$$\begin{split} U_k^+(z) &= \frac{z^{1/2}}{1+r_k} U_{k+1}^+(z) - \frac{r_k z^{1/2}}{1+r_k} U_{k+1}^-(z) \\ U_k^-(z) &= -\frac{r_k z^{-1/2}}{1+r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1+r_k} U_{k+1}^-(z) \end{split}$$

$$\begin{split} r_L &= \frac{\frac{\rho c}{Z_L} - A_N}{\frac{\rho c}{Z_L} + A_N} \\ &\qquad \qquad r_G = \left(\frac{A_1 Z_G - \rho c}{A_1 Z_G + \rho c}\right) \\ &\qquad \qquad U_1^+(z) = U_G(z) \left\lceil \frac{1 + r_G}{2} \right\rceil + r_G U_1^-(z) \end{split}$$

Общ вид

$$U_{N+1}^{-}(z) = 0$$

$$r_N = r_L \rightarrow A_{N+1} = \frac{\rho c}{Z_L}$$

 $U_{N+1}^{+}(z) = U_{L}(z)$

$$\begin{split} U_k^+(z) &= \frac{z^{1/2}}{1+r_k} U_{k+1}^+(z) - \frac{r_k z^{1/2}}{1+r_k} U_{k+1}^-(z) \\ U_k^-(z) &= -\frac{r_k z^{-1/2}}{1+r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1+r_k} U_{k+1}^-(z) \end{split}$$

$$r_L = \frac{\frac{\rho c}{Z_L} - A_N}{\frac{\rho c}{Z_L} + A_N} \qquad r_G = \left(\frac{A_1 Z_G - \rho c}{A_1 Z_G + \rho c}\right)$$
$$U_1^+(z) = U_G(z) \left[\frac{1 + r_G}{2}\right] + r_G U_1^-(z)$$

$$U_{N+1}^+(z) = U_L(z)$$

$$U_{N+1}^-(z) = 0$$

$$r_N = r_L \to A_{N+1} = \frac{\rho c}{Z_L}$$

$$U_k = Q_k U_{k+1}$$
 зо

$$U_{k} = \begin{bmatrix} U_{k}^{+}(z) \\ U_{k}^{-}(z) \end{bmatrix}$$

$$Q_k = egin{bmatrix} rac{z^{1/2}}{1+r_k} & rac{-r_k z^{1/2}}{1+r_k} \ rac{-r_k z^{-1/2}}{1+r_k} \ rac{-r_k z^{-1/2}}{1+r_k} \end{bmatrix}$$