Разпознаване на емоции в

сигнали от реч и ЕЕГ

• ... бе Словото

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- Експлицитен и имплицитен канал при общуване

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Попълването на вица е оставено за упражнение на читателя

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- Прозодия (ритъм, интонация, ударение)





Попълването на вица е оставено за упражнение на читателя

• Съчетаване на първичен (ЕЕГ) и вторичен (реч) канал

- ⊳ Нулева зона (какво е емоция)
- ⊳ Сигнал от реч

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- Съчетаване на двата сигнала
- ⊳ Резултати
- ⊳ Заключение

• Теорията на Дарвин

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  - "Принцип на полезните навици"

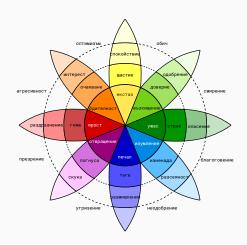
- Теорията на Дарвин
  - "Принцип на полезните навици"
  - "Принцип на противоположностите"

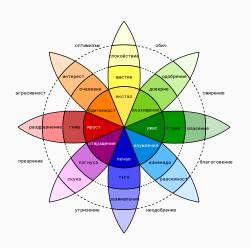
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- Продължението на Плутчик (1980)

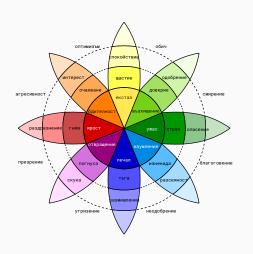
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- Продължението на Плутчик (1980)

Емоцията е сложна верига от събития, която започва с някакъв стимул. В следствие настъпва фаза на "изпитване на емоция" и фаза на физиологични промени. Те предизвикват целенасочено държание, което цели да премахне дразнението на стимула и да върне състоянието на еквилибриум.





+ Всяка емоция може да се изрази като комбинация на основните



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Осем е голямо число

• VAD модела - Алберт Мейерабиан и Джеймс Ръсел (1974)

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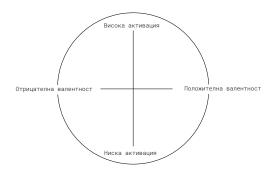
• В сигнала от реч се измерва по-лесно активацията

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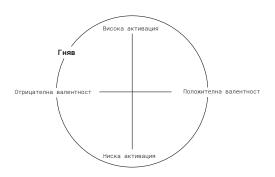
- В сигнала от реч се измерва по-лесно активацията
- В сигнала от ЕЕГ се измерва по-лесно валентността

## Избрани емоции:



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• Гняв



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• Щастие



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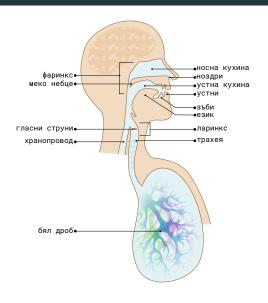
• Щастие

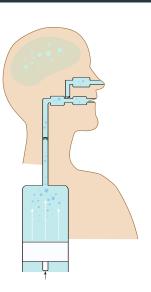
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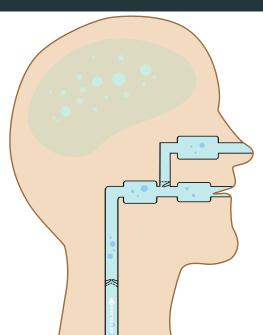
• Тъго



## Сигнал от реч







Видове звуци:

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 $Peч \rightarrow gymu$ 

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Реч ightarrow думи ightarrow фонеми

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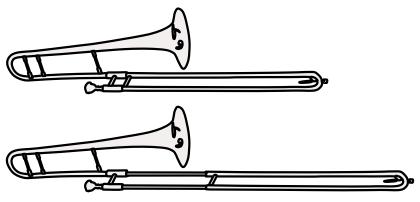
 $Peч \rightarrow gymu \rightarrow фонеми$ 

"Страхът стискаше гърлото, задушаваше гласа."

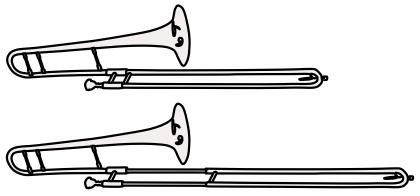
• Спектрални характеристики

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За да изследваме подлежащата емоция, трябва да изследваме спектралните свойства на статична конфигурация на вокалния тракт.

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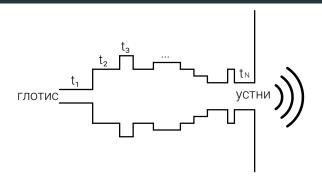
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Ако  $g(t) \overset{\mathcal{FS}}{\longleftrightarrow} \mathcal{G}(z), v(t) \overset{\mathcal{FS}}{\longleftrightarrow} \mathcal{V}(z), r(t) \overset{\mathcal{FS}}{\longleftrightarrow} \mathcal{R}(z)$ , а сигналът, който получаваме накрая,  $y(t) = g(t) * v(t) * r(t), y(t) \overset{\mathcal{FS}}{\longleftrightarrow} \mathcal{Y}(z)$ , е изпълнено, че

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•  $\mathcal{Y}(z) = \mathcal{G}(z)\mathcal{V}(z)\mathcal{R}(z)$ 



- $\cdot$  c скорост на звука в еластична среда
- ho плътност на въздуха в тръбите
- A лицето на напречното сечение в тръба (константа)
- $\mathit{u} = \mathit{u}(\mathit{x},\mathit{t})$  е обемната скорост на позиция  $\mathit{x}$  в момента  $\mathit{t}$
- p=p(x,t) е звуковото налягане

### Уравнения на Навие-Стокс:

$$\begin{split} -\frac{\partial \rho}{\partial x} &= \frac{\rho}{A} \frac{\partial u}{\partial t} \\ -\frac{\partial u}{\partial x} &= \frac{A}{\rho c^2} \frac{\partial \rho}{\partial t} \end{split}$$

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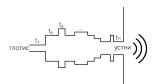
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С решения от вида:

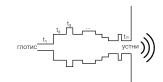
$$\begin{split} u(x,t) &= \left[ u^+ \left( t - \frac{x}{c} \right) - u^- \left( t + \frac{x}{c} \right) \right] \\ p(x,t) &= \frac{\rho c}{A} \left[ u^+ \left( t - \frac{x}{c} \right) + u^- \left( t + \frac{x}{c} \right) \right] \end{split}$$

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$$u_k(x,t) = \left[ u_k^+ \left( t - \frac{x}{c} \right) - u_k^- \left( t + \frac{x}{c} \right) \right]$$
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 U uspaseme  $u_{k+1}^+$ 

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$$u_{k+1}^{+}(t) = u_{k}^{+}(t - \tau_{k})\left[\frac{2A_{k+1}}{A_{k} + A_{k+1}}\right] + u_{k+1}^{-}(t)\left[\frac{A_{k+1} - A_{k}}{A_{k} + A_{k+1}}\right]$$

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$$t_k = \frac{2A_{k+1}}{A_k + A_{k+1}}$$

$$\begin{aligned} u_k(x,t) &= \left[ u_k^+ \left( t - \frac{x}{c} \right) - u_k^- \left( t + \frac{x}{c} \right) \right] & u_k(l_k,t) = u_{k+1}(0,t) \\ p_k(x,t) &= \frac{\rho c}{A_k} \left[ u_k^+ \left( t - \frac{x}{c} \right) + u_k^- \left( t + \frac{x}{c} \right) \right] & \\ u_k^+ \left( t - \tau_k \right) - u_k^- \left( t + \tau_k \right) &= u_{k+1}^+(t) - u_{k+1}^-(t) \\ &\frac{A_{k+1}}{A_k} \left[ u_k^+ \left( t - \tau_k \right) + u_k^- \left( t + \tau_k \right) = u_{k+1}^+(t) + u_{k+1}^-(t) \right] \\ u_k^- \left( t + \tau_k \right) &= u_k^+ \left( t - \tau_k \right) - u_{k+1}^+(t) + u_{k+1}^-(t) \\ u_{k+1}^+(t) &= u_k^+ \left( t - \tau_k \right) \left[ \frac{2A_{k+1}}{A_k + A_{k+1}} \right] + u_{k+1}^-(t) \left[ \frac{A_{k+1} - A_k}{A_k + A_{k+1}} \right] \end{aligned}$$

$$t_k = \frac{2A_{k+1}}{A_k + A_{k+1}} \qquad r_k = \frac{A_{k+1} - A_k}{A_k + A_{k+1}}$$

$$u_{k}(x,t) = \left[u_{k}^{+}\left(t - \frac{x}{c}\right) - u_{k}^{-}\left(t + \frac{x}{c}\right)\right] \qquad u_{k}(l_{k},t) = u_{k+1}(0,t)$$

$$p_{k}(x,t) = \frac{\rho c}{A_{k}} \left[u_{k}^{+}\left(t - \frac{x}{c}\right) + u_{k}^{-}\left(t + \frac{x}{c}\right)\right] \qquad u_{k}(l_{k},t) = p_{k+1}(0,t)$$

$$u_{k}^{+}\left(t - \tau_{k}\right) - u_{k}^{-}\left(t + \tau_{k}\right) = u_{k+1}^{+}\left(t\right) - u_{k+1}^{-}\left(t\right)$$

$$\frac{A_{k+1}}{A_{k}} \left[u_{k}^{+}\left(t - \tau_{k}\right) + u_{k}^{-}\left(t + \tau_{k}\right) = u_{k+1}^{+}\left(t\right) + u_{k+1}^{-}\left(t\right)\right]$$

$$u_{k}^{-}\left(t + \tau_{k}\right) = u_{k}^{+}\left(t - \tau_{k}\right) - u_{k+1}^{+}\left(t\right) + u_{k+1}^{-}\left(t\right)$$

$$u_{k+1}^{+}\left(t\right) = u_{k}^{+}\left(t - \tau_{k}\right) \left[\frac{2A_{k+1}}{A_{k} + A_{k+1}}\right] + u_{k+1}^{-}\left(t\right) \left[\frac{A_{k+1} - A_{k}}{A_{k} + A_{k+1}}\right]$$

$$t_{k} = \frac{2A_{k+1}}{A_{k+1}} \qquad r_{k} = \frac{A_{k+1} - A_{k}}{A_{k+1}}$$

26

 $r_k = \frac{A_{k+1} - A_k}{A_k + A_{k+1}}$ 

$$u_{k}(x,t) = \left[u_{k}^{+}\left(t - \frac{x}{c}\right) - u_{k}^{-}\left(t + \frac{x}{c}\right)\right] \qquad u_{k}(l_{k},t) = u_{k+1}(0,t)$$

$$p_{k}(x,t) = \frac{\rho c}{A_{k}}\left[u_{k}^{+}\left(t - \frac{x}{c}\right) + u_{k}^{-}\left(t + \frac{x}{c}\right)\right] \qquad p_{k}(l_{k},t) = p_{k+1}(0,t)$$

$$u_{k}^{+}\left(t - \tau_{k}\right) - u_{k}^{-}\left(t + \tau_{k}\right) = u_{k+1}^{+}(t) - u_{k+1}^{-}(t)$$

$$\frac{A_{k+1}}{A_{k}}\left[u_{k}^{+}\left(t - \tau_{k}\right) + u_{k}^{-}\left(t + \tau_{k}\right) = u_{k+1}^{+}(t) + u_{k+1}^{-}(t)\right]$$

$$u_{k}^{-}\left(t + \tau_{k}\right) = u_{k}^{+}\left(t - \tau_{k}\right) - u_{k+1}^{+}(t) + u_{k+1}^{-}(t)$$

$$u_{k+1}^{+}(t) = u_{k}^{+}\left(t - \tau_{k}\right)\left[\frac{2A_{k+1}}{A_{k} + A_{k+1}}\right] + u_{k+1}^{-}(t)\left[\frac{A_{k+1} - A_{k}}{A_{k} + A_{k+1}}\right]$$

 $r_k = \frac{A_{k+1} - A_k}{A_k + A_{k+1}}$ 

$$u_{k}(x,t) = \begin{bmatrix} u_{k}^{+} \left( t - \frac{x}{c} \right) - u_{k}^{-} \left( t + \frac{x}{c} \right) \end{bmatrix} \qquad u_{k}(l_{k},t) = u_{k+1}(0,t)$$

$$p_{k}(l_{k},t) = p_{k+1}(0,t) \qquad r_{k} = \frac{A_{k+1} - A_{k}}{A_{k} + A_{k+1}}$$

$$p_{k}(x,t) = \frac{\rho c}{A_{k}} \left[ u_{k}^{+} \left( t - \frac{x}{c} \right) + u_{k}^{-} \left( t + \frac{x}{c} \right) \right]$$

$$u_{k}^{+} (t - \tau_{k}) - u_{k}^{-} (t + \tau_{k}) = u_{k+1}^{+}(t) - u_{k+1}^{-}(t)$$

$$\frac{A_{k+1}}{A_{k}} \left[ u_{k}^{+} (t - \tau_{k}) + u_{k}^{-} (t + \tau_{k}) = u_{k+1}^{+}(t) + u_{k+1}^{-}(t) \right]$$

 $u_{k+1}^+(t) = u_k^+(t - \tau_k) \left| \frac{2A_{k+1}}{A_k + A_{k+1}} \right| + u_{k+1}^-(t) \left| \frac{A_{k+1} - A_k}{A_k + A_{k+1}} \right|$ 

$$\begin{split} u_k(x,t) &= \left[ u_k^+ \left( t - \frac{x}{c} \right) - u_k^- \left( t + \frac{x}{c} \right) \right] & u_k(l_k,t) = u_{k+1}(0,t) \\ p_k(x,t) &= \frac{\rho c}{A_k} \left[ u_k^+ \left( t - \frac{x}{c} \right) + u_k^- \left( t + \frac{x}{c} \right) \right] \\ u_k^+ \left( t - \tau_k \right) - u_k^- \left( t + \tau_k \right) &= u_{k+1}^+(t) - u_{k+1}^-(t) \\ \frac{A_{k+1}}{A_k} \left[ u_k^+ \left( t - \tau_k \right) + u_k^- \left( t + \tau_k \right) = u_{k+1}^+(t) + u_{k+1}^-(t) \right] \\ u_{k+1}^+(t) &= u_k^+ \left( t - \tau_k \right) \left[ \frac{2A_{k+1}}{A_k + A_{k+1}} \right] + u_{k+1}^-(t) \left[ \frac{A_{k+1} - A_k}{A_k + A_{k+1}} \right] \\ u_k^+(t - \tau_k) &= u_{k+1}^+(t) \left[ \frac{A_k + A_{k+1}}{2A_{k+1}} \right] + u_{k+1}^-(t) \left[ \frac{A_k - A_{k+1}}{2A_{k+1}} \right] \end{split}$$

 $r_k = \frac{A_{k+1} - A_k}{A_k + A_{k+1}}$ 

$$\begin{aligned} u_k(x,t) &= \left[ u_k^+ \left( t - \frac{x}{c} \right) - u_k^- \left( t + \frac{x}{c} \right) \right] & u_k(l_k,t) = u_{k+1}(0,t) \\ p_k(x,t) &= \frac{\rho c}{A_k} \left[ u_k^+ \left( t - \frac{x}{c} \right) + u_k^- \left( t + \frac{x}{c} \right) \right] \end{aligned} \qquad r_k = \frac{A_{k+1} - A_k}{A_k + A_{k+1}}$$

$$\begin{aligned} u_k^+ \left( t - \tau_k \right) - u_k^- \left( t + \tau_k \right) &= u_{k+1}^+(t) - u_{k+1}^-(t) \\ \frac{A_{k+1}}{A_k} \left[ u_k^+ \left( t - \tau_k \right) + u_k^- \left( t + \tau_k \right) = u_{k+1}^+(t) + u_{k+1}^-(t) \right] \end{aligned}$$

$$u_{k+1}^+(t) = u_k^+(t - \tau_k) \left[ \frac{2A_{k+1}}{A_k + A_{k+1}} \right] + u_{k+1}^-(t) \left[ \frac{A_{k+1} - A_k}{A_k + A_{k+1}} \right]$$

$$u_k^+(t - \tau_k) = u_{k+1}^+(t) \left[ \frac{A_k + A_{k+1}}{A_{k+1}} \right] + u_{k+1}^-(t) \left[ \frac{A_k - A_{k+1}}{A_{k+1}} \right]$$

$$u_{k}(x,t) = \begin{bmatrix} u_{k}^{+} \left( t - \frac{x}{c} \right) - u_{k}^{-} \left( t + \frac{x}{c} \right) \end{bmatrix} \qquad u_{k}(l_{k},t) = u_{k+1}(0,t)$$

$$p_{k}(l_{k},t) = p_{k+1}(0,t) \qquad r_{k} = \frac{A_{k+1} - A_{k}}{A_{k} + A_{k+1}}$$

$$v_{k}^{+} \left( t - \frac{x}{c} \right) + u_{k}^{-} \left( t + \frac{x}{c} \right) \end{bmatrix} \qquad r_{k} = \frac{A_{k+1} - A_{k}}{A_{k} + A_{k+1}}$$

$$u_{k}^{+} \left( t - \tau_{k} \right) - u_{k}^{-} \left( t + \tau_{k} \right) = u_{k+1}^{+}(t) - u_{k+1}^{-}(t)$$

$$\frac{A_{k+1}}{A_{k}} \left[ u_{k}^{+} \left( t - \tau_{k} \right) + u_{k}^{-} \left( t + \tau_{k} \right) = u_{k+1}^{+}(t) + u_{k+1}^{-}(t) \right]$$

$$u_{k+1}^{+} \left( t \right) = u_{k}^{+} \left( t - \tau_{k} \right) \left[ \frac{2A_{k+1}}{A_{k} + A_{k+1}} \right] + u_{k+1}^{-}(t) \left[ \frac{A_{k+1} - A_{k}}{A_{k} + A_{k+1}} \right]$$

$$u_{k}^{+} \left( t - \tau_{k} \right) = u_{k+1}^{+}(t) \left[ \frac{A_{k} + A_{k+1}}{2A_{k+1}} \right] + u_{k+1}^{-}(t) \left[ \frac{A_{k} - A_{k+1}}{2A_{k+1}} \right]$$

$$\begin{split} u_k(x,t) &= \left[ u_k^+ \left( t - \frac{x}{c} \right) - u_k^- \left( t + \frac{x}{c} \right) \right] & u_k(l_k,t) = u_{k+1}(0,t) \\ p_k(x,t) &= \frac{\rho c}{A_k} \left[ u_k^+ \left( t - \frac{x}{c} \right) + u_k^- \left( t + \frac{x}{c} \right) \right] \\ u_k^+ \left( t - \tau_k \right) - u_k^- \left( t + \tau_k \right) &= u_{k+1}^+(t) - u_{k+1}^-(t) \\ \frac{A_{k+1}}{A_k} \left[ u_k^+ \left( t - \tau_k \right) + u_k^- \left( t + \tau_k \right) = u_{k+1}^+(t) + u_{k+1}^-(t) \right] \\ u_{k+1}^+(t) &= u_k^+(t - \tau_k) \left[ \frac{2A_{k+1}}{A_k + A_{k+1}} \right] + u_{k+1}^-(t) \left[ \frac{A_{k+1} - A_k}{A_k + A_{k+1}} \right] \\ u_k^+(t - \tau_k) &= u_{k+1}^+(t) \left[ \frac{A_k + A_{k+1}}{2A_{k+1}} \right] + u_{k+1}^-(t) \left[ \frac{A_k - A_{k+1}}{2A_{k+1}} \right] \\ u_k^-(t + \tau_k) &= u_{k+1}^+(t) \left[ \frac{A_k - A_{k+1}}{2A_{k+1}} \right] + u_{k+1}^-(t) \left[ \frac{A_k + A_{k+1}}{2A_{k+1}} \right] \end{split}$$

$$\begin{split} u_k(x,t) &= \left[ u_k^+ \left( t - \frac{x}{c} \right) - u_k^- \left( t + \frac{x}{c} \right) \right] & u_k(l_k,t) = u_{k+1}(0,t) \\ p_k(x,t) &= \frac{\rho c}{A_k} \left[ u_k^+ \left( t - \frac{x}{c} \right) + u_k^- \left( t + \frac{x}{c} \right) \right] \\ u_k^+ \left( t - \tau_k \right) - u_k^- \left( t + \tau_k \right) &= u_{k+1}^+(t) - u_{k+1}^-(t) \\ \frac{A_{k+1}}{A_k} \left[ u_k^+ \left( t - \tau_k \right) + u_k^- \left( t + \tau_k \right) = u_{k+1}^+(t) + u_{k+1}^-(t) \right] \\ u_{k+1}^+(t) &= u_k^+(t - \tau_k) \left[ \frac{2A_{k+1}}{A_k + A_{k+1}} \right] + u_{k+1}^-(t) \left[ \frac{A_{k+1} - A_k}{A_k + A_{k+1}} \right] \\ u_k^+(t - \tau_k) &= u_{k+1}^+(t) \left[ \frac{A_k + A_{k+1}}{2A_{k+1}} \right] + u_{k+1}^-(t) \left[ \frac{A_k - A_{k+1}}{2A_{k+1}} \right] \\ u_k^-(t + \tau_k) &= u_{k+1}^+(t) \left[ \frac{A_k - A_{k+1}}{2A_{k+1}} \right] + u_{k+1}^-(t) \left[ \frac{A_k + A_{k+1}}{2A_{k+1}} \right] \end{split}$$

$$\begin{split} u_k(x,t) &= \left[ u_k^+ \left( t - \frac{x}{c} \right) - u_k^- \left( t + \frac{x}{c} \right) \right] & u_k(l_k,t) = u_{k+1}(0,t) \\ p_k(x,t) &= \frac{\rho c}{A_k} \left[ u_k^+ \left( t - \frac{x}{c} \right) + u_k^- \left( t + \frac{x}{c} \right) \right] \\ u_k^+ \left( t - \tau_k \right) - u_k^- \left( t + \tau_k \right) &= u_{k+1}^+(t) - u_{k+1}^-(t) \\ \frac{A_{k+1}}{A_k} \left[ u_k^+ \left( t - \tau_k \right) + u_k^- \left( t + \tau_k \right) = u_{k+1}^+(t) + u_{k+1}^-(t) \right] \\ u_{k+1}^+(t) &= u_k^+(t - \tau_k) \left[ \frac{2A_{k+1}}{A_k + A_{k+1}} \right] + u_{k+1}^-(t) \left[ \frac{A_{k+1} - A_k}{A_k + A_{k+1}} \right] \\ u_k^+(t - \tau_k) &= u_{k+1}^+(t) \left[ \frac{A_k + A_{k+1}}{2A_{k+1}} \right] + u_{k+1}^-(t) \left[ \frac{A_k - A_{k+1}}{2A_{k+1}} \right] \\ u_k^-(t + \tau_k) &= u_{k+1}^+(t) \left[ \frac{A_k - A_{k+1}}{2A_{k+1}} \right] + u_{k+1}^-(t) \left[ \frac{A_k + A_{k+1}}{2A_{k+1}} \right] \end{split}$$

$$u_{k}(x,t) = \begin{bmatrix} u_{k}^{+} \left( t - \frac{x}{c} \right) - u_{k}^{-} \left( t + \frac{x}{c} \right) \end{bmatrix} \qquad u_{k}(l_{k},t) = u_{k+1}(0,t)$$

$$p_{k}(l_{k},t) = p_{k+1}(0,t)$$

$$p_{k}(x,t) = \frac{\rho c}{A_{k}} \left[ u_{k}^{+} \left( t - \frac{x}{c} \right) + u_{k}^{-} \left( t + \frac{x}{c} \right) \right]$$

$$v_{k}^{+} (t - \tau_{k}) = u_{k+1}^{+}(t) \left[ \frac{A_{k} + A_{k+1}}{2A_{k+1}} \right] + u_{k+1}^{-}(t) \left[ \frac{A_{k} - A_{k+1}}{2A_{k+1}} \right]$$

$$u_{k}^{-} (t + \tau_{k}) = u_{k+1}^{+}(t) \left[ \frac{A_{k} - A_{k+1}}{2A_{k+1}} \right] + u_{k+1}^{-}(t) \left[ \frac{A_{k} + A_{k+1}}{2A_{k+1}} \right]$$

$$u_{k}(x,t) = \left[u_{k}^{+}\left(t - \frac{x}{c}\right) - u_{k}^{-}\left(t + \frac{x}{c}\right)\right] \qquad u_{k}(l_{k},t) = u_{k+1}(0,t)$$

$$p_{k}(x,t) = \frac{\rho c}{A_{k}}\left[u_{k}^{+}\left(t - \frac{x}{c}\right) + u_{k}^{-}\left(t + \frac{x}{c}\right)\right] \qquad u_{k}(l_{k},t) = p_{k+1}(0,t)$$

$$u_{k}^{+}(t - \tau_{k}) = u_{k+1}^{+}(t)\left[\frac{A_{k} + A_{k+1}}{2A_{k+1}}\right] + u_{k+1}^{-}(t)\left[\frac{A_{k} - A_{k+1}}{2A_{k+1}}\right]$$

$$u_{k}^{-}(t + \tau_{k}) = u_{k+1}^{+}(t)\left[\frac{A_{k} - A_{k+1}}{2A_{k+1}}\right] + u_{k+1}^{-}(t)\left[\frac{A_{k} + A_{k+1}}{2A_{k+1}}\right]$$

$$r_k = \frac{A_{k+1} - A_k}{A_k + A_{k+1}}$$

$$u_{k}(x,t) = \left[u_{k}^{+}\left(t - \frac{x}{c}\right) - u_{k}^{-}\left(t + \frac{x}{c}\right)\right] \qquad u_{k}(l_{k},t) = u_{k+1}(0,t)$$

$$p_{k}(x,t) = \frac{\rho c}{A_{k}}\left[u_{k}^{+}\left(t - \frac{x}{c}\right) + u_{k}^{-}\left(t + \frac{x}{c}\right)\right]$$

$$u_{k}^{+}(t - \tau_{k}) = \frac{1}{1 + r_{k}}u_{k+1}^{+}(t) - \frac{r_{k}}{1 + r_{k}}u_{k+1}^{-}(t)$$

$$u_{k}^{-}(t + \tau_{k}) = -\frac{r_{k}}{1 + r_{k}}u_{k+1}^{+}(t) + \frac{1}{1 + r_{k}}u_{k+1}^{-}(t)$$

$$u_{k}(x,t) = \begin{bmatrix} u_{k}^{+} \left( t - \frac{x}{c} \right) - u_{k}^{-} \left( t + \frac{x}{c} \right) \end{bmatrix} \qquad u_{k}(l_{k},t) = u_{k+1}(0,t)$$

$$p_{k}(l_{k},t) = p_{k+1}(0,t)$$

$$p_{k}(x,t) = \frac{\rho c}{A_{k}} \left[ u_{k}^{+} \left( t - \frac{x}{c} \right) + u_{k}^{-} \left( t + \frac{x}{c} \right) \right]$$

$$u_{k}^{+}(t-\tau_{k}) = \frac{1}{1+r_{k}} u_{k+1}^{+}(t) - \frac{r_{k}}{1+r_{k}} u_{k+1}^{-}(t)$$

$$u_{k}^{-}(t+\tau_{k}) = -\frac{r_{k}}{1+r_{k}} u_{k+1}^{+}(t) + \frac{1}{1+r_{k}} u_{k+1}^{-}(t)$$

$$\begin{split} u_k(x,t) &= \left[ u_k^+ \left( t - \frac{x}{c} \right) - u_k^- \left( t + \frac{x}{c} \right) \right] & u_k(l_k,t) = u_{k+1}(0,t) \\ p_k(x,t) &= \frac{\rho c}{A_k} \left[ u_k^+ \left( t - \frac{x}{c} \right) + u_k^- \left( t + \frac{x}{c} \right) \right] \end{split} \\ v_k^+(t-\tau_k) &= \frac{1}{1+r_k} u_{k+1}^+(t) - \frac{r_k}{1+r_k} u_{k+1}^-(t) \\ u_k^-(t+\tau_k) &= -\frac{r_k}{1+r_k} u_{k+1}^+(t) + \frac{1}{1+r_k} u_{k+1}^-(t) \\ \text{Heko } u_k(t) & \stackrel{\mathcal{FS}}{\longleftrightarrow} U_k(z) & z = e^{i\omega} \end{split}$$

$$\begin{split} u_k(x,t) &= \left[ u_k^+ \left( t - \frac{x}{c} \right) - u_k^- \left( t + \frac{x}{c} \right) \right] & u_k(l_k,t) = u_{k+1}(0,t) \\ p_k(x,t) &= \frac{\rho c}{A_k} \left[ u_k^+ \left( t - \frac{x}{c} \right) + u_k^- \left( t + \frac{x}{c} \right) \right] & \\ u_k^+(t-\tau_k) &= \frac{1}{1+r_k} u_{k+1}^+(t) - \frac{r_k}{1+r_k} u_{k+1}^-(t) \\ u_k^-(t+\tau_k) &= -\frac{r_k}{1+r_k} u_{k+1}^+(t) + \frac{1}{1+r_k} u_{k+1}^-(t) \\ & \text{Heko } u_k(t) \overset{\mathcal{FS}}{\longleftrightarrow} U_k(z) & z = e^{i\omega} \\ & \text{Tozaba} \ u_k(t-\tau_k) \overset{\mathcal{FS}}{\longleftrightarrow} z^{-\tau_k} U_k(z). \end{split}$$

$$\begin{split} u_k(x,t) &= \left[ u_k^+ \left( t - \frac{x}{c} \right) - u_k^- \left( t + \frac{x}{c} \right) \right] & u_k(l_k,t) = u_{k+1}(0,t) \\ p_k(x,t) &= \frac{\rho c}{A_k} \left[ u_k^+ \left( t - \frac{x}{c} \right) + u_k^- \left( t + \frac{x}{c} \right) \right] & u_k(l_k,t) = p_{k+1}(0,t) \\ u_k^+(t-\tau_k) &= \frac{1}{1+r_k} u_{k+1}^+(t) - \frac{r_k}{1+r_k} u_{k+1}^-(t) \\ u_k^-(t+\tau_k) &= -\frac{r_k}{1+r_k} u_{k+1}^+(t) + \frac{1}{1+r_k} u_{k+1}^-(t) \\ &\text{Heko } u_k(t) \overset{\mathcal{FS}}{\longleftrightarrow} U_k(z) & z = e^{i\omega} \\ &\text{Tozaba } u_k(t-\tau_k) \overset{\mathcal{FS}}{\longleftrightarrow} z^{-\tau_k} U_k(z). \\ U_k^+(z) &= \frac{z^{\tau_k}}{1+r_k} U_{k+1}^+(z) - \frac{r_k z^{\tau_k}}{1+r_k} U_{k+1}^-(z) \\ U_k^-(z) &= -\frac{r_k z^{-\tau_k}}{1+r_k} U_{k+1}^+(z) + \frac{z^{-\tau_k}}{1+r_k} U_{k+1}^-(z) \end{split}$$

$$\begin{split} u_k(x,t) &= \left[ u_k^+ \left( t - \frac{x}{c} \right) - u_k^- \left( t + \frac{x}{c} \right) \right] & u_k(l_k,t) = u_{k+1}(0,t) \\ p_k(x,t) &= \frac{\rho c}{A_k} \left[ u_k^+ \left( t - \frac{x}{c} \right) + u_k^- \left( t + \frac{x}{c} \right) \right] & u_k(l_k,t) = p_{k+1}(0,t) \\ u_k^+(t-\tau_k) &= \frac{1}{1+r_k} u_{k+1}^+(t) - \frac{r_k}{1+r_k} u_{k+1}^-(t) \\ u_k^-(t+\tau_k) &= -\frac{r_k}{1+r_k} u_{k+1}^+(t) + \frac{1}{1+r_k} u_{k+1}^-(t) \\ &\text{Heko } u_k(t) \overset{\mathcal{FS}}{\longleftrightarrow} U_k(z). \\ &\text{Tozobo} \ u_k(t-\tau_k) \overset{\mathcal{FS}}{\longleftrightarrow} z^{-\tau_k} U_k(z). \\ U_k^+(z) &= \frac{z^{\tau_k}}{1+r_k} U_{k+1}^+(z) - \frac{r_k z^{\tau_k}}{1+r_k} U_{k+1}^-(z) \\ U_k^-(z) &= -\frac{r_k z^{-\tau_k}}{1+r_k} U_{k+1}^+(z) + \frac{z^{-\tau_k}}{1+r_k} U_{k+1}^-(z) & \tau_k = 1/2 \end{split}$$

$$\begin{split} u_k(x,t) &= \left[ u_k^+ \left( t - \frac{x}{c} \right) - u_k^- \left( t + \frac{x}{c} \right) \right] & u_k(l_k,t) = u_{k+1}(0,t) \\ p_k(x,t) &= \frac{\rho c}{A_k} \left[ u_k^+ \left( t - \frac{x}{c} \right) + u_k^- \left( t + \frac{x}{c} \right) \right] \\ u_k^+(t-\tau_k) &= \frac{1}{1+r_k} u_{k+1}^+(t) - \frac{r_k}{1+r_k} u_{k+1}^-(t) \\ u_k^-(t+\tau_k) &= -\frac{r_k}{1+r_k} u_{k+1}^+(t) + \frac{1}{1+r_k} u_{k+1}^-(t) \\ \text{Hekg } u_k(t) & \stackrel{\mathcal{FS}}{\longleftrightarrow} U_k(z). \\ \text{Tozobo} \ u_k(t-\tau_k) & \stackrel{\mathcal{FS}}{\longleftrightarrow} z^{-\tau_k} U_k(z). \\ U_k^+(z) &= \frac{z^{1/2}}{1+r_k} U_{k+1}^+(z) - \frac{r_k z^{1/2}}{1+r_k} U_{k+1}^-(z) \\ U_k^-(z) &= -\frac{r_k z^{-1/2}}{1+r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1+r_k} U_{k+1}^-(z) \end{split}$$

$$u_k(x,t) = \left[ u_k^+ \left( t - \frac{x}{c} \right) - u_k^- \left( t + \frac{x}{c} \right) \right]$$

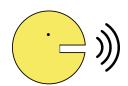
$$p_k(x,t) = \frac{\rho c}{A_k} \left[ u_k^+ \left( t - \frac{x}{c} \right) + u_k^- \left( t + \frac{x}{c} \right) \right]$$

$$U_k^+(z) = \frac{z^{1/2}}{1 + r_k} U_{k+1}^+(z) - \frac{r_k z^{1/2}}{1 + r_k} U_{k+1}^-(z)$$

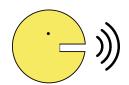
$$U_k^-(z) = -\frac{r_k z^{-1/2}}{1 + r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1 + r_k} U_{k+1}^-(z)$$

$$\begin{split} u_k(x,t) &= \left[ u_k^+ \left( t - \frac{x}{c} \right) - u_k^- \left( t + \frac{x}{c} \right) \right] \\ p_k(x,t) &= \frac{\rho c}{A_k} \left[ u_k^+ \left( t - \frac{x}{c} \right) + u_k^- \left( t + \frac{x}{c} \right) \right] \\ U_k^+(z) &= \frac{z^{1/2}}{1 + r_k} U_{k+1}^+(z) - \frac{r_k z^{1/2}}{1 + r_k} U_{k+1}^-(z) \\ U_k^-(z) &= -\frac{r_k z^{-1/2}}{1 + r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1 + r_k} U_{k+1}^-(z) \end{split}$$

$$\begin{split} u_k(x,t) &= \left[ u_k^+ \left( t - \frac{x}{c} \right) - u_k^- \left( t + \frac{x}{c} \right) \right] \\ p_k(x,t) &= \frac{\rho c}{A_k} \left[ u_k^+ \left( t - \frac{x}{c} \right) + u_k^- \left( t + \frac{x}{c} \right) \right] \\ U_k^+(z) &= \frac{z^{1/2}}{1 + r_k} U_{k+1}^+(z) - \frac{r_k z^{1/2}}{1 + r_k} U_{k+1}^-(z) \\ U_k^-(z) &= -\frac{r_k z^{-1/2}}{1 + r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1 + r_k} U_{k+1}^-(z) \end{split}$$

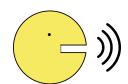


$$\begin{split} u_k(x,t) &= \left[ u_k^+ \left( t - \frac{x}{c} \right) - u_k^- \left( t + \frac{x}{c} \right) \right] \\ p_k(x,t) &= \frac{\rho c}{A_k} \left[ u_k^+ \left( t - \frac{x}{c} \right) + u_k^- \left( t + \frac{x}{c} \right) \right] \\ U_k^+(z) &= \frac{z^{1/2}}{1 + r_k} U_{k+1}^+(z) - \frac{r_k z^{1/2}}{1 + r_k} U_{k+1}^-(z) \\ U_k^-(z) &= -\frac{r_k z^{-1/2}}{1 + r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1 + r_k} U_{k+1}^-(z) \end{split}$$





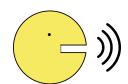
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$$\mathcal{P}_N(l_N,z) = Z_L(z)\mathcal{U}_N(l_N,z)$$

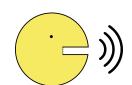
$$\begin{split} u_k(x,t) &= \left[ u_k^+ \left( t - \frac{x}{c} \right) - u_k^- \left( t + \frac{x}{c} \right) \right] \\ p_k(x,t) &= \frac{\rho c}{A_k} \left[ u_k^+ \left( t - \frac{x}{c} \right) + u_k^- \left( t + \frac{x}{c} \right) \right] \\ U_k^+(z) &= \frac{z^{1/2}}{1 + r_k} U_{k+1}^+(z) - \frac{r_k z^{1/2}}{1 + r_k} U_{k+1}^-(z) \\ U_k^-(z) &= -\frac{r_k z^{-1/2}}{1 + r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1 + r_k} U_{k+1}^-(z) \end{split}$$





$$\mathcal{P}_N(l_N,z) = Z_L \mathcal{U}_N(l_N,z)$$

$$\begin{split} u_k(x,t) &= \left[ u_k^+ \left( t - \frac{x}{c} \right) - u_k^- \left( t + \frac{x}{c} \right) \right] \\ p_k(x,t) &= \frac{\rho c}{A_k} \left[ u_k^+ \left( t - \frac{x}{c} \right) + u_k^- \left( t + \frac{x}{c} \right) \right] \\ U_k^+(z) &= \frac{z^{1/2}}{1 + r_k} U_{k+1}^+(z) - \frac{r_k z^{1/2}}{1 + r_k} U_{k+1}^-(z) \\ U_k^-(z) &= -\frac{r_k z^{-1/2}}{1 + r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1 + r_k} U_{k+1}^-(z) \end{split}$$

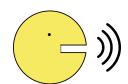




$$\mathcal{P}_N(l_N,z) = Z_L \mathcal{U}_N(l_N,z)$$

$$p(l_N,t) \stackrel{\mathcal{FS}}{\longleftrightarrow} \mathcal{P}_N(l_N,z), u_N(l_N,t) \stackrel{\mathcal{FS}}{\longleftrightarrow} \mathcal{U}_N(l_N,z)$$

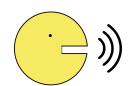
$$\begin{split} u_k(x,t) &= \left[ u_k^+ \left( t - \frac{x}{c} \right) - u_k^- \left( t + \frac{x}{c} \right) \right] \\ p_k(x,t) &= \frac{\rho c}{A_k} \left[ u_k^+ \left( t - \frac{x}{c} \right) + u_k^- \left( t + \frac{x}{c} \right) \right] \\ U_k^+(z) &= \frac{z^{1/2}}{1 + r_k} U_{k+1}^+(z) - \frac{r_k z^{1/2}}{1 + r_k} U_{k+1}^-(z) \\ U_k^-(z) &= -\frac{r_k z^{-1/2}}{1 + r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1 + r_k} U_{k+1}^-(z) \end{split}$$





$$p_N(l_N,t) = Z_L u_N(l_N,t)$$

$$\begin{aligned} u_k(x,t) &= \left[ u_k^+ \left( t - \frac{x}{c} \right) - u_k^- \left( t + \frac{x}{c} \right) \right] \\ p_k(x,t) &= \frac{\rho c}{A_k} \left[ u_k^+ \left( t - \frac{x}{c} \right) + u_k^- \left( t + \frac{x}{c} \right) \right] \\ U_k^+(z) &= \frac{z^{1/2}}{1 + r_k} U_{k+1}^+(z) - \frac{r_k z^{1/2}}{1 + r_k} U_{k+1}^-(z) \\ U_k^-(z) &= -\frac{r_k z^{-1/2}}{1 + r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1 + r_k} U_{k+1}^-(z) \end{aligned}$$





$$p_N(l_N, t) = Z_L u_N(l_N, t)$$

$$u_N^-(t + \tau_N) \frac{(\rho c + A_N Z_L)}{A_N} = u_N^+(t - \tau_N) \frac{(A_N Z_L - \rho c)}{A_N}$$

$$\begin{aligned} u_k(x,t) &= \left[ u_k^+ \left( t - \frac{x}{c} \right) - u_k^- \left( t + \frac{x}{c} \right) \right] \\ p_k(x,t) &= \frac{\rho c}{A_k} \left[ u_k^+ \left( t - \frac{x}{c} \right) + u_k^- \left( t + \frac{x}{c} \right) \right] \\ U_k^+(z) &= \frac{z^{1/2}}{1 + r_k} U_{k+1}^+(z) - \frac{r_k z^{1/2}}{1 + r_k} U_{k+1}^-(z) \\ U_k^-(z) &= -\frac{r_k z^{-1/2}}{1 + r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1 + r_k} U_{k+1}^-(z) \end{aligned}$$

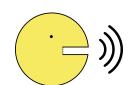




$$p_N(l_N, t) = Z_L u_N(l_N, t)$$

$$u_{N}^{-}(t+\tau_{N})\frac{(\rho c+A_{N}Z_{L})}{A_{N}}=u_{N}^{+}(t-\tau_{N})\frac{(A_{N}Z_{L}-\rho c)}{A_{N}} \qquad r_{L}=\frac{\frac{\rho c}{Z_{L}}-A_{N}}{\frac{\rho c}{Z_{L}}+A_{N}}$$

$$\begin{split} u_k(x,t) &= \left[ u_k^+ \left( t - \frac{x}{c} \right) - u_k^- \left( t + \frac{x}{c} \right) \right] \\ p_k(x,t) &= \frac{\rho c}{A_k} \left[ u_k^+ \left( t - \frac{x}{c} \right) + u_k^- \left( t + \frac{x}{c} \right) \right] \\ U_k^+(z) &= \frac{z^{1/2}}{1 + r_k} U_{k+1}^+(z) - \frac{r_k z^{1/2}}{1 + r_k} U_{k+1}^-(z) \\ U_k^-(z) &= -\frac{r_k z^{-1/2}}{1 + r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1 + r_k} U_{k+1}^-(z) \end{split}$$



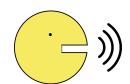


$$p_N(l_N, t) = Z_L u_N(l_N, t)$$

$$u_N^-(t+\tau_N) = -r_L u_N^+(t-\tau_N)$$
  $r_L = \frac{\frac{\rho c}{Z_L} - A_N}{\frac{\rho c}{Z_L} + A_N}$ 

$$\begin{aligned} u_k(x,t) &= \left[ u_k^+ \left( t - \frac{x}{c} \right) - u_k^- \left( t + \frac{x}{c} \right) \right] \\ p_k(x,t) &= \frac{\rho_c}{A_k} \left[ u_k^+ \left( t - \frac{x}{c} \right) + u_k^- \left( t + \frac{x}{c} \right) \right] \\ U_k^+(z) &= \frac{z^{1/2}}{1 + r_k} U_{k+1}^+(z) - \frac{r_k z^{1/2}}{1 + r_k} U_{k+1}^-(z) \\ U_k^-(z) &= -\frac{r_k z^{-1/2}}{1 + r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1 + r_k} U_{k+1}^-(z) \end{aligned}$$

$$r_L = \frac{\frac{\rho \, c}{Z_L} \, - \, A_N}{\frac{\rho \, c}{Z_L} \, + \, A_N} \label{eq:rL}$$





$$p_N(l_N,t) = Z_L u_N(l_N,t)$$

$$u_N^-(t+\tau_N) = -r_L u_N^+(t-\tau_N)$$
  $r_L = \frac{\frac{\rho c}{Z_L} - A_N}{\frac{\rho c}{Z_L} + A_N}$ 

$$\begin{split} u_k(x,t) &= \left[ u_k^+ \left( t - \frac{x}{c} \right) - u_k^- \left( t + \frac{x}{c} \right) \right] \\ p_k(x,t) &= \frac{\rho c}{A_k} \left[ u_k^+ \left( t - \frac{x}{c} \right) + u_k^- \left( t + \frac{x}{c} \right) \right] \\ U_k^+(z) &= \frac{z^{1/2}}{1 + r_k} U_{k+1}^+(z) - \frac{r_k z^{1/2}}{1 + r_k} U_{k+1}^-(z) \\ U_k^-(z) &= -\frac{r_k z^{-1/2}}{1 + r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1 + r_k} U_{k+1}^-(z) \end{split}$$

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$$U_1(0, z) = U_G(z) - \frac{P_1(0, z)}{Z_G(z)}$$

$$\begin{split} u_k(x,t) &= \left[ u_k^+ \left( t - \frac{x}{c} \right) - u_k^- \left( t + \frac{x}{c} \right) \right] \\ p_k(x,t) &= \frac{\rho c}{A_k} \left[ u_k^+ \left( t - \frac{x}{c} \right) + u_k^- \left( t + \frac{x}{c} \right) \right] \\ U_k^+(z) &= \frac{z^{1/2}}{1 + r_k} U_{k+1}^+(z) - \frac{r_k z^{1/2}}{1 + r_k} U_{k+1}^-(z) \\ U_k^-(z) &= -\frac{r_k z^{-1/2}}{1 + r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1 + r_k} U_{k+1}^-(z) \end{split}$$

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$$U_1(0,z) = U_G(z) - \frac{P_1(0,z)}{Z_G}$$

$$u_1(0,t) \stackrel{\mathcal{FS}}{\longleftrightarrow} U_1(0,z), u_G(t) \stackrel{\mathcal{FS}}{\longleftrightarrow} U_G(z), p_1(0,t) \stackrel{\mathcal{FS}}{\longleftrightarrow} P_1(0,z)$$

$$\begin{split} u_k(x,t) &= \left[ u_k^+ \left( t - \frac{x}{c} \right) - u_k^- \left( t + \frac{x}{c} \right) \right] \\ p_k(x,t) &= \frac{\rho c}{A_k} \left[ u_k^+ \left( t - \frac{x}{c} \right) + u_k^- \left( t + \frac{x}{c} \right) \right] \\ U_k^+(z) &= \frac{z^{1/2}}{1 + r_k} U_{k+1}^+(z) - \frac{r_k z^{1/2}}{1 + r_k} U_{k+1}^-(z) \\ U_k^-(z) &= -\frac{r_k z^{-1/2}}{1 + r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1 + r_k} U_{k+1}^-(z) \end{split}$$

$$U_1(0,z) = U_G(z) - \frac{P_1(0,z)}{Z_G}$$

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$$u_1(0,t) \stackrel{\mathcal{FS}}{\longleftrightarrow} U_1(0,z), u_G(t) \stackrel{\mathcal{FS}}{\longleftrightarrow} U_G(z), p_1(0,t) \stackrel{\mathcal{FS}}{\longleftrightarrow} P_1(0,z)$$

$$u_1(0,t) = u_G(t) - \frac{p_1(0,t)}{Z_G}$$

$$\begin{split} u_k(x,t) &= \left[ u_k^+ \left( t - \frac{x}{c} \right) - u_k^- \left( t + \frac{x}{c} \right) \right] \\ p_k(x,t) &= \frac{\rho c}{A_k} \left[ u_k^+ \left( t - \frac{x}{c} \right) + u_k^- \left( t + \frac{x}{c} \right) \right] \\ U_k^+(z) &= \frac{z^{1/2}}{1 + r_k} U_{k+1}^+(z) - \frac{r_k z^{1/2}}{1 + r_k} U_{k+1}^-(z) \\ U_k^-(z) &= -\frac{r_k z^{-1/2}}{1 + r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1 + r_k} U_{k+1}^-(z) \end{split}$$

$$\begin{split} &U_{1}(0,z)=U_{G}(z)-\frac{P_{1}(0,z)}{Z_{G}}\\ &u_{1}(0,t) \xleftarrow{\mathcal{FS}} U_{1}(0,z), u_{G}(t) \xleftarrow{\mathcal{FS}} U_{G}(z), p_{1}(0,t) \xleftarrow{\mathcal{FS}} P_{1}(0,z)\\ &u_{1}(0,t)=u_{G}(t)-\frac{p_{1}(0,t)}{Z_{G}}\\ &u_{1}^{+}(t)=u_{G}(t)\left[\frac{A_{1}Z_{G}}{A_{1}Z_{G}+\rho c}\right]+u_{1}^{-}(t)\left[\frac{A_{1}Z_{G}-\rho c}{A_{1}Z_{G}+\rho c}\right] \end{split}$$

$$\begin{split} u_k(x,t) &= \left[ u_k^+ \left( t - \frac{x}{c} \right) - u_k^- \left( t + \frac{x}{c} \right) \right] \\ p_k(x,t) &= \frac{\rho c}{A_k} \left[ u_k^+ \left( t - \frac{x}{c} \right) + u_k^- \left( t + \frac{x}{c} \right) \right] \\ U_k^+(z) &= \frac{z^{1/2}}{1 + r_k} U_{k+1}^+(z) - \frac{r_k z^{1/2}}{1 + r_k} U_{k+1}^-(z) \\ U_k^-(z) &= -\frac{r_k z^{-1/2}}{1 + r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1 + r_k} U_{k+1}^-(z) \end{split}$$

$$U_{1}(0,z) = U_{G}(z) - \frac{P_{1}(0,z)}{Z_{G}}$$

$$u_{1}(0,t) \stackrel{\mathcal{FS}}{\longleftrightarrow} U_{1}(0,z), u_{G}(t) \stackrel{\mathcal{FS}}{\longleftrightarrow} U_{G}(z), p_{1}(0,t) \stackrel{\mathcal{FS}}{\longleftrightarrow} P_{1}(0,z)$$

$$u_{1}(0,t) = u_{G}(t) - \frac{p_{1}(0,t)}{Z_{G}}$$

$$u_{1}^{+}(t) = u_{G}(t) \left[ \frac{A_{1}Z_{G}}{A_{1}Z_{G} + \rho c} \right] + u_{1}^{-}(t) \left[ \frac{A_{1}Z_{G} - \rho c}{A_{1}Z_{G} + \rho c} \right] \qquad r_{G} = \left( \frac{A_{1}Z_{G} - \rho c}{A_{1}Z_{G} + \rho c} \right)$$

$$\begin{split} u_k(x,t) &= \left[ u_k^+ \left( t - \frac{x}{c} \right) - u_k^- \left( t + \frac{x}{c} \right) \right] \\ p_k(x,t) &= \frac{\rho c}{A_k} \left[ u_k^+ \left( t - \frac{x}{c} \right) + u_k^- \left( t + \frac{x}{c} \right) \right] \\ U_k^+(z) &= \frac{z^{1/2}}{1 + r_k} U_{k+1}^+(z) - \frac{r_k z^{1/2}}{1 + r_k} U_{k+1}^-(z) \\ U_k^-(z) &= -\frac{r_k z^{-1/2}}{1 + r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1 + r_k} U_{k+1}^-(z) \end{split}$$

$$\begin{split} &U_{1}(0,z)=U_{G}(z)-\frac{P_{1}(0,z)}{Z_{G}}\\ &u_{1}(0,t) \xleftarrow{\mathcal{FS}} U_{1}(0,z), u_{G}(t) \xleftarrow{\mathcal{FS}} U_{G}(z), p_{1}(0,t) \xleftarrow{\mathcal{FS}} P_{1}(0,z)\\ &u_{1}(0,t)=u_{G}(t)-\frac{p_{1}(0,t)}{Z_{G}}\\ &u_{1}^{+}(t)=u_{G}(t)\left[\frac{1+r_{G}}{2}\right]+r_{G}u_{1}^{-}(t) \qquad r_{G}=\left(\frac{A_{1}Z_{G}-\rho c}{A_{1}Z_{G}+\rho c}\right) \end{split}$$

$$\begin{split} u_k(x,t) &= \left[ u_k^+ \left( t - \frac{x}{c} \right) - u_k^- \left( t + \frac{x}{c} \right) \right] \\ p_k(x,t) &= \frac{\rho c}{A_k} \left[ u_k^+ \left( t - \frac{x}{c} \right) + u_k^- \left( t + \frac{x}{c} \right) \right] \\ U_k^+(z) &= \frac{z^{1/2}}{1 + r_k} U_{k+1}^+(z) - \frac{r_k z^{1/2}}{1 + r_k} U_{k+1}^-(z) \\ U_k^-(z) &= -\frac{r_k z^{-1/2}}{1 + r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1 + r_k} U_{k+1}^-(z) \end{split}$$

#### Огрнаничения при глотиса

$$U_{1}(0,z) = U_{G}(z) - \frac{P_{1}(0,z)}{Z_{G}}$$

$$u_{1}(0,t) \stackrel{\mathcal{FS}}{\longleftrightarrow} U_{1}(0,z), u_{G}(t) \stackrel{\mathcal{FS}}{\longleftrightarrow} U_{G}(z), p_{1}(0,t) \stackrel{\mathcal{FS}}{\longleftrightarrow} P_{1}(0,z)$$

$$u_{1}(0,t) = u_{G}(t) - \frac{p_{1}(0,t)}{Z_{G}}$$

$$u_{1}^{+}(t) = u_{G}(t) \left[ \frac{1+r_{G}}{2} \right] + r_{G}u_{1}^{-}(t) \qquad r_{G} = \left( \frac{A_{1}Z_{G} - \rho c}{A_{1}Z_{G} + \rho c} \right)$$

$$U_{1}^{+}(z) = U_{G}(z) \left[ \frac{1+r_{G}}{2} \right] + r_{G}U_{1}^{-}(z)$$

$$\begin{split} u_k(x,t) &= \left[ u_k^+ \left( t - \frac{x}{c} \right) - u_k^- \left( t + \frac{x}{c} \right) \right] \\ p_k(x,t) &= \frac{\rho c}{A_k} \left[ u_k^+ \left( t - \frac{x}{c} \right) + u_k^- \left( t + \frac{x}{c} \right) \right] \\ U_k^+(z) &= \frac{z^{1/2}}{1 + r_k} U_{k+1}^+(z) - \frac{r_k z^{1/2}}{1 + r_k} U_{k+1}^-(z) \\ U_k^-(z) &= -\frac{r_k z^{-1/2}}{1 + r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1 + r_k} U_{k+1}^-(z) \end{split}$$

#### Огрнаничения при глотиса

$$U_{1}(0,z) = U_{G}(z) - \frac{P_{1}(0,z)}{Z_{G}}$$

$$u_{1}(0,t) \stackrel{\mathcal{FS}}{\longleftrightarrow} U_{1}(0,z), u_{G}(t) \stackrel{\mathcal{FS}}{\longleftrightarrow} U_{G}(z), p_{1}(0,t) \stackrel{\mathcal{FS}}{\longleftrightarrow} P_{1}(0,z)$$

$$u_{1}(0,t) = u_{G}(t) - \frac{p_{1}(0,t)}{Z_{G}}$$

$$u_{1}^{+}(t) = u_{G}(t) \left[ \frac{1+r_{G}}{2} \right] + r_{G}u_{1}^{-}(t) \qquad r_{G} = \left( \frac{A_{1}Z_{G} - \rho c}{A_{1}Z_{G} + \rho c} \right)$$

$$U_{1}^{+}(z) = U_{G}(z) \left[ \frac{1+r_{G}}{2} \right] + r_{G}U_{1}^{-}(z)$$

$$\begin{split} u_k(x,t) &= \left[ u_k^+ \left( t - \frac{x}{c} \right) - u_k^- \left( t + \frac{x}{c} \right) \right] \\ p_k(x,t) &= \frac{\rho c}{A_k} \left[ u_k^+ \left( t - \frac{x}{c} \right) + u_k^- \left( t + \frac{x}{c} \right) \right] \\ U_k^+(z) &= \frac{z^{1/2}}{1 + r_k} U_{k+1}^+(z) - \frac{r_k z^{1/2}}{1 + r_k} U_{k+1}^-(z) \\ U_k^-(z) &= -\frac{r_k z^{-1/2}}{1 + r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1 + r_k} U_{k+1}^-(z) \end{split}$$

$$\begin{split} r_L &= \frac{\frac{\rho \, c}{Z_L} - A_N}{\frac{\rho \, c}{Z_L} + A_N} \qquad \qquad r_G = \left(\frac{A_1 \, Z_G - \rho \, c}{A_1 \, Z_G + \rho \, c}\right) \\ &\qquad \qquad U_1^+(z) = \, U_G(z) \left[\frac{1 + r_G}{2}\right] + r_G \, U_1^-(z) \end{split}$$

$$\begin{split} U_k^+(z) &= \frac{z^{1/2}}{1 + r_k} U_{k+1}^+(z) - \frac{r_k z^{1/2}}{1 + r_k} U_{k+1}^-(z) \\ U_k^-(z) &= -\frac{r_k z^{-1/2}}{1 + r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1 + r_k} U_{k+1}^-(z) \end{split} \qquad r_L &= \frac{\frac{\rho c}{Z_L} - A_N}{\frac{\rho c}{Z_L} + A_N} \\ U_L^-(z) &= -\frac{r_k z^{-1/2}}{1 + r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1 + r_k} U_{k+1}^-(z) \\ U_L^+(z) &= U_G(z) \left[ \frac{1 + r_G}{2} \right] + r_G U_1^-(z) \end{split}$$

$$\begin{split} U_k^+(z) &= \frac{z^{1/2}}{1 + r_k} U_{k+1}^+(z) - \frac{r_k z^{1/2}}{1 + r_k} U_{k+1}^-(z) \\ U_k^-(z) &= -\frac{r_k z^{-1/2}}{1 + r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1 + r_k} U_{k+1}^-(z) \end{split} \qquad r_L &= \frac{\frac{\rho c}{Z_L} - A_N}{\frac{\rho c}{Z_L} + A_N} \\ U_L^-(z) &= -\frac{r_k z^{-1/2}}{1 + r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1 + r_k} U_{k+1}^-(z) \\ U_L^+(z) &= U_G(z) \left[ \frac{1 + r_G}{2} \right] + r_G U_1^-(z) \end{split}$$

$$\begin{split} U_k^+(z) &= \frac{z^{1/2}}{1 + r_k} U_{k+1}^+(z) - \frac{r_k z^{1/2}}{1 + r_k} U_{k+1}^-(z) \\ U_k^-(z) &= -\frac{r_k z^{-1/2}}{1 + r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1 + r_k} U_{k+1}^-(z) \\ \end{split} \qquad \qquad \begin{split} r_L &= \frac{\frac{\rho c}{Z_L} - A_N}{\frac{\rho c}{Z_L} + A_N} \\ U_L^+(z) &= \frac{r_K z^{-1/2}}{1 + r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1 + r_k} U_{k+1}^-(z) \\ U_L^+(z) &= U_G(z) \left[ \frac{1 + r_G}{2} \right] + r_G U_1^-(z) \end{split}$$

$$U_{N+1}^+(z) = U_L(z)$$

$$\begin{split} U_k^+(z) &= \frac{z^{1/2}}{1+r_k} U_{k+1}^+(z) - \frac{r_k z^{1/2}}{1+r_k} U_{k+1}^-(z) \\ U_k^-(z) &= -\frac{r_k z^{-1/2}}{1+r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1+r_k} U_{k+1}^-(z) \end{split} \qquad r_L &= \frac{\frac{\rho c}{Z_L} - A_N}{\frac{\rho c}{Z_L} + A_N} \qquad r_G = \left(\frac{A_1 Z_G - \rho c}{A_1 Z_G + \rho c}\right) \\ U_1^+(z) &= U_G(z) \left[\frac{1+r_G}{2}\right] + r_G U_1^-(z) \end{split}$$

$$U_{N+1}^{+}(z) = U_{L}(z)$$
  
 $U_{N+1}^{-}(z) = 0$ 

$$\begin{split} U_k^+(z) &= \frac{z^{1/2}}{1+r_k} U_{k+1}^+(z) - \frac{r_k z^{1/2}}{1+r_k} U_{k+1}^-(z) \\ U_k^-(z) &= -\frac{r_k z^{-1/2}}{1+r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1+r_k} U_{k+1}^-(z) \end{split} \qquad r_L &= \frac{\frac{\rho c}{Z_L} - A_N}{\frac{\rho c}{Z_L} + A_N} \qquad r_G = \left(\frac{A_1 Z_G - \rho c}{A_1 Z_G + \rho c}\right) \\ U_1^+(z) &= U_G(z) \left[\frac{1+r_G}{2}\right] + r_G U_1^-(z) \end{split}$$

$$U_{N+1}^{+}(z) = U_{L}(z)$$
  
 $U_{N+1}^{-}(z) = 0$   $r_{N} = r_{L}$ 

$$\begin{split} U_k^+(z) &= \frac{z^{1/2}}{1+r_k} U_{k+1}^+(z) - \frac{r_k z^{1/2}}{1+r_k} U_{k+1}^-(z) \\ U_k^-(z) &= -\frac{r_k z^{-1/2}}{1+r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1+r_k} U_{k+1}^-(z) \\ \end{split} \qquad \begin{aligned} r_L &= \frac{\frac{\rho c}{Z_L} - A_N}{\frac{\rho c}{Z_L} + A_N} \\ U_L^+(z) &= \frac{r_k z^{-1/2}}{\frac{\rho c}{Z_L} + A_N} U_L^+(z) \\ U_L^+(z) &= U_G(z) \left[ \frac{1+r_G}{2} \right] + r_G U_L^-(z) \end{aligned}$$

#### Общ вид на $\mathcal V$

$$U_{N+1}^-(z) = 0$$
  $r_N = r_L \rightarrow A_{N+1} = \frac{\rho c}{Z_L}$ 

 $U_{N+1}^{+}(z) = U_{L}(z)$ 

$$U_{k}^{+}(z) = \frac{z^{1/2}}{1 + r_{k}} U_{k+1}^{+}(z) - \frac{r_{k}z^{1/2}}{1 + r_{k}} U_{k+1}^{-}(z) \qquad r_{L} = \frac{\frac{\rho c}{2L} - A_{N}}{\frac{\rho c}{2L} + A_{N}} \qquad r_{G} = \left(\frac{A_{1}Z_{G} - \rho c}{A_{1}Z_{G} + \rho c}\right)$$

$$U_{k}^{-}(z) = -\frac{r_{k}z^{-1/2}}{1 + r_{k}} U_{k+1}^{+}(z) + \frac{z^{-1/2}}{1 + r_{k}} U_{k+1}^{-}(z) \qquad U_{1}^{+}(z) = U_{G}(z) \left[\frac{1 + r_{G}}{2}\right] + r_{G} U_{1}^{-}(z)$$

#### Общ вид на ${\cal V}$

$$U_{N+1}^{-}(z) = 0$$
  $r_N = r_L \rightarrow A_{N+1} = \frac{\rho c}{Z_L}$ 

$$U_k = Q_k U_{k+1}$$

 $U_{N+1}^{+}(z) = U_{L}(z)$ 

$$\begin{split} U_k^+(z) &= \frac{z^{1/2}}{1 + r_k} U_{k+1}^+(z) - \frac{r_k z^{1/2}}{1 + r_k} U_{k+1}^-(z) \\ U_k^-(z) &= -\frac{r_k z^{-1/2}}{1 + r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1 + r_k} U_{k+1}^-(z) \\ \end{split} \qquad \begin{aligned} r_L &= \frac{\frac{\rho c}{Z_L} - A_N}{\frac{\rho c}{Z_L} + A_N} \\ U_L^+(z) &= \frac{r_G}{2} \left( \frac{A_1 Z_G - \rho c}{A_1 Z_G + \rho c} \right) \\ U_L^+(z) &= U_G(z) \left[ \frac{1 + r_G}{2} \right] + r_G U_1^-(z) \end{aligned}$$

$$U_{N+1}^{+}(z) = U_{L}(z)$$

$$U_{N+1}^{-}(z) = 0 r_{N} = r_{L} \to A_{N+1} = \frac{\rho c}{Z_{L}}$$

$$U_{k} = Q_{k}U_{k+1}$$

$$U_{k} = \begin{bmatrix} U_{k}^{+}(z) \\ U_{T}^{-}(z) \end{bmatrix}$$

$$Q_{k} = \begin{bmatrix} \frac{z^{1/2}}{1+r_{k}} & \frac{-r_{k}z^{1/2}}{1+r_{k}} \\ -r_{k}z^{-1/2} & \frac{z^{-1/2}}{1+r_{k}} \end{bmatrix}$$

$$\begin{split} U_k^+(z) &= \frac{z^{1/2}}{1 + r_k} U_{k+1}^+(z) - \frac{r_k z^{1/2}}{1 + r_k} U_{k+1}^-(z) \\ U_k^-(z) &= -\frac{r_k z^{-1/2}}{1 + r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1 + r_k} U_{k+1}^-(z) \\ \end{split} \qquad \begin{aligned} r_L &= \frac{\frac{\rho c}{Z_L} - A_N}{\frac{\rho c}{Z_L} + A_N} \\ U_L^+(z) &= \frac{r_G}{A_1 Z_G - \rho c} \\ U_L^+(z) &= U_G(z) \left[ \frac{1 + r_G}{2} \right] + r_G U_1^-(z) \end{aligned}$$

$$U_{N+1}^{+}(z) = U_{L}(z)$$

$$U_{N+1}^{-}(z) = 0 r_{N} = r_{L} \to A_{N+1} = \frac{\rho c}{Z_{L}}$$

$$U_{k} = Q_{k}U_{k+1}$$

$$U_{k} = \begin{bmatrix} U_{k}^{+}(z) \\ U_{k}^{-}(z) \end{bmatrix}$$

$$U_{1} = \begin{bmatrix} U_{k}^{+}(z) \\ U_{k}^{-}(z) \end{bmatrix}$$

$$\begin{split} U_k^+(z) &= \frac{z^{1/2}}{1 + r_k} U_{k+1}^+(z) - \frac{r_k z^{1/2}}{1 + r_k} U_{k+1}^-(z) \\ U_k^-(z) &= -\frac{r_k z^{-1/2}}{1 + r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1 + r_k} U_{k+1}^-(z) \\ \end{split} \qquad \begin{aligned} r_L &= \frac{\frac{\rho c}{Z_L} - A_N}{\frac{\rho c}{Z_L} + A_N} \\ U_L^+(z) &= \frac{r_K z^{-1/2}}{2} U_{k+1}^+(z) + \frac{z^{-1/2}}{1 + r_k} U_{k+1}^-(z) \\ U_L^+(z) &= U_G(z) \left[ \frac{1 + r_G}{2} \right] + r_G U_1^-(z) \end{aligned}$$

$$U_{N+1}^{+}(z) = U_{L}(z)$$

$$U_{N+1}^{-}(z) = 0 r_{N} = r_{L} \to A_{N+1} = \frac{\rho c}{Z_{L}}$$

$$U_{k} = Q_{k} U_{k+1}$$

$$U_{k} = \begin{bmatrix} U_{k}^{+}(z) \\ U_{k}^{-}(z) \end{bmatrix}$$

$$U_{1} = Q_{1} U_{2}$$

$$Q_{k} = \begin{bmatrix} \frac{z^{1/2}}{1 + r_{k}} & \frac{-r_{k} z^{1/2}}{1 + r_{k}} \\ \frac{-r_{k} z^{-1/2}}{1 + r_{k}} & \frac{z^{-1/2}}{1 + r_{k}} \end{bmatrix}$$

$$U_{1} = Q_{1} U_{2}$$

 $U_1 = Q_1 U_2 = Q_1 Q_2 U_3$ 

$$\begin{split} U_k^+(z) &= \frac{z^{1/2}}{1 + r_k} U_{k+1}^+(z) - \frac{r_k z^{1/2}}{1 + r_k} U_{k+1}^-(z) \\ U_k^-(z) &= -\frac{r_k z^{-1/2}}{1 + r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1 + r_k} U_{k+1}^-(z) \\ \end{split} \qquad \begin{aligned} r_L &= \frac{\frac{\rho c}{Z_L} - A_N}{\frac{\rho c}{Z_L} + A_N} \\ U_L^+(z) &= \frac{r_K z^{-1/2}}{2} U_{k+1}^+(z) + \frac{z^{-1/2}}{1 + r_k} U_{k+1}^-(z) \\ U_L^+(z) &= U_G(z) \left[ \frac{1 + r_G}{2} \right] + r_G U_1^-(z) \end{aligned}$$

$$U_{N+1}^{+}(z) = U_{L}(z)$$

$$U_{N+1}^{-}(z) = 0 r_{N} = r_{L} \to A_{N+1} = \frac{\rho c}{Z_{L}}$$

$$U_{k} = Q_{k} U_{k+1}$$

$$U_{k} = \begin{bmatrix} U_{k}^{+}(z) \\ U_{k}^{-}(z) \end{bmatrix}$$

$$Q_{k} = \begin{bmatrix} \frac{z^{1/2}}{1 + r_{k}} & \frac{-r_{k} z^{1/2}}{1 + r_{k}} \\ \frac{-r_{k} z^{-1/2}}{1 + r_{k}} & \frac{z^{-1/2}}{1 + r_{k}} \end{bmatrix}$$

$$\begin{split} U_k^+(z) &= \frac{z^{1/2}}{1 + r_k} U_{k+1}^+(z) - \frac{r_k z^{1/2}}{1 + r_k} U_{k+1}^-(z) \\ U_k^-(z) &= -\frac{r_k z^{-1/2}}{1 + r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1 + r_k} U_{k+1}^-(z) \end{split} \qquad r_L &= \frac{\frac{\rho c}{Z_L} - A_N}{\frac{\rho c}{Z_L} + A_N} \\ v_L &= \frac{\frac{\rho c}{Z_L} - A_N}{\frac{\rho c}{Z_L} + A_N} \\ v_L &= \frac{\rho c}{Z_L} - A_N \\ \frac{\rho c}{Z_L} &= \frac{\rho c}{A_1 Z_G - \rho c} \\ U_1^+(z) &= U_G(z) \left[ \frac{1 + r_G}{2} \right] + r_G U_1^-(z) \end{split}$$

$$U_{N+1}^{+}(z) = U_{L}(z)$$

$$U_{N+1}^{-}(z) = 0 r_{N} = r_{L} \to A_{N+1} = \frac{\rho c}{Z_{L}}$$

$$Q_{k} = \begin{bmatrix} \frac{z^{1/2}}{1 + r_{k}} & \frac{-r_{k}z^{1/2}}{1 + r_{k}} \\ -r_{k}z^{-1/2} & \frac{z^{-1/2}}{1 + r_{k}} \end{bmatrix}$$

$$U_{k} = \begin{bmatrix} U_{k}^{+}(z) \\ U_{k}^{-}(z) \end{bmatrix}$$

$$U_{1} = Q_{1}U_{2} = Q_{1}Q_{2}U_{3} = \dots = \begin{bmatrix} \prod_{k=1}^{N} Q_{k} \end{bmatrix} U_{N+1}$$

$$\begin{split} U_k^+\left(z\right) &= \frac{z^{1/2}}{1+r_k} U_{k+1}^+\left(z\right) - \frac{r_k z^{1/2}}{1+r_k} U_{k+1}^-\left(z\right) & r_L &= \frac{\frac{\rho c}{Z_L} - A_N}{\frac{\rho c}{Z_L} + A_N} & r_G &= \left(\frac{A_1 Z_G - \rho c}{A_1 Z_G + \rho c}\right) \\ U_k^-\left(z\right) &= -\frac{r_k z^{-1/2}}{1+r_k} U_{k+1}^+\left(z\right) + \frac{z^{-1/2}}{1+r_k} U_{k+1}^-\left(z\right) & U_1^+\left(z\right) &= U_G(z) \left[\frac{1+r_G}{2}\right] + r_G U_1^-\left(z\right) \end{split}$$

$$U_{N+1}^{+}(z) = U_{L}(z)$$

$$U_{N+1}^{-}(z) = 0 r_{N} = r_{L} \to A_{N+1} = \frac{\rho c}{Z_{L}}$$

$$U_{k} = Q_{k}U_{k+1}$$

$$U_{k} = \begin{bmatrix} U_{k}^{+}(z) \\ U_{k}^{-}(z) \end{bmatrix}$$

$$U_{1} = Q_{1}U_{2} = Q_{1}Q_{2}U_{3} = \dots = \begin{bmatrix} \prod_{i=1}^{N} Q_{i} \end{bmatrix} U_{N+1}$$

$$Q_{k} = \begin{bmatrix} \frac{z^{1/2}}{1+r_{k}} & \frac{-r_{k}z^{1/2}}{1+r_{k}} \\ \frac{-r_{k}z^{-1/2}}{1+r_{k}} & \frac{z^{-1/2}}{1+r_{k}} \end{bmatrix}$$

$$U_{1} = Q_{1}U_{2} = Q_{1}Q_{2}U_{3} = \dots = \begin{bmatrix} \prod_{i=1}^{N} Q_{i} \end{bmatrix} U_{N+1}$$

$$= \begin{bmatrix} \prod_{i=1}^{N} Q_{i} \end{bmatrix} \begin{bmatrix} U_{N+1}^{+}(z) \\ U_{N+1}^{-}(z) \end{bmatrix}$$

$$\begin{split} U_k^+\left(z\right) &= \frac{z^{1/2}}{1+r_k} U_{k+1}^+\left(z\right) - \frac{r_k z^{1/2}}{1+r_k} U_{k+1}^-\left(z\right) & r_L &= \frac{\frac{\rho c}{Z_L} - A_N}{\frac{\rho c}{Z_L} + A_N} & r_G &= \left(\frac{A_1 Z_G - \rho c}{A_1 Z_G + \rho c}\right) \\ U_k^-\left(z\right) &= -\frac{r_k z^{-1/2}}{1+r_k} U_{k+1}^+\left(z\right) + \frac{z^{-1/2}}{1+r_k} U_{k+1}^-\left(z\right) & U_1^+\left(z\right) &= U_G(z) \left[\frac{1+r_G}{2}\right] + r_G U_1^-\left(z\right) \end{split}$$

### Общ вид на $\mathcal{V}$

$$U_{N+1}^-(z) = 0$$
  $r_N = r_L \rightarrow A_{N+1} = \frac{\rho c}{Z_L}$ 

$$U_k = Q_k U_{k+1}$$

 $U_{N+1}^{+}(z) = U_{L}(z)$ 

$$U_k = \begin{bmatrix} U_k^+(z) \\ U_k^-(z) \end{bmatrix}$$

$$U_1 = Q_1 U_2 = Q_1 Q_2 U_3 = \dots = \prod_{i=1}^{n} \prod_{j=1}^{n} q_j U_{ij} = \dots = \prod_{j=1}^{n} q_j U_{ij} = \dots$$

$$Q_k = \begin{bmatrix} \frac{z^{1/2}}{1+r_k} & \frac{-r_k z^{1/2}}{1+r_k} \\ \\ \frac{-r_k z^{-1/2}}{1+r_k} & \frac{z^{-1/2}}{1+r_k} \end{bmatrix}$$

$$U_{1} = Q_{1} U_{2} = Q_{1} Q_{2} U_{3} = \dots = \begin{bmatrix} \prod_{i=1}^{N} Q_{i} \end{bmatrix} U_{N+1} = \begin{bmatrix} \prod_{i=1}^{N} Q_{i} \end{bmatrix} \begin{bmatrix} U_{N+1}^{+}(z) \\ U_{N+1}^{-}(z) \end{bmatrix} = \begin{bmatrix} \prod_{i=1}^{N} Q_{i} \end{bmatrix} \begin{bmatrix} U_{L}(z) \\ 0 \end{bmatrix}$$

$$\begin{split} U_k^+(z) &= \frac{z^{1/2}}{1+r_k} U_{k+1}^+(z) - \frac{r_k z^{1/2}}{1+r_k} U_{k+1}^-(z) \\ U_k^-(z) &= -\frac{r_k z^{-1/2}}{1+r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1+r_k} U_{k+1}^-(z) \\ \end{split} \qquad \begin{aligned} r_L &= \frac{\frac{\rho c}{2L} - A_N}{\frac{\rho c}{Z_L} + A_N} \\ U_L^+(z) &= \frac{r_k z^{-1/2}}{\frac{\rho c}{Z_L} + A_N} U_1^+(z) &= U_G(z) \left[ \frac{1+r_G}{2L} \right] + r_G U_1^-(z) \end{aligned}$$

$$Q_k = \begin{bmatrix} \frac{z^{1/2}}{1+r_k} & \frac{-r_k z^{1/2}}{1+r_k} \\ \frac{-r_k z^{-1/2}}{1+r_k} & \frac{z^{-1/2}}{1+r_k} \end{bmatrix} \quad U_1 = \begin{bmatrix} \prod_{i=1}^N Q_i \end{bmatrix} \begin{bmatrix} U_L(z) \\ 0 \end{bmatrix}$$

$$U_k^+(z) = \frac{z^{1/2}}{1+r_k} U_{k+1}^+(z) - \frac{r_k z^{1/2}}{1+r_k} U_{k+1}^-(z) \qquad \qquad \\ r_L = \frac{\frac{\rho c}{Z_L} - A_N}{\frac{\rho c}{Z_L} + A_N} \qquad \qquad \\ r_G = \left(\frac{A_1 Z_G - \rho c}{A_1 Z_G + \rho c}\right) \\ U_k^-(z) = -\frac{r_k z^{-1/2}}{1+r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1+r_k} U_{k+1}^-(z) \qquad \qquad \\ U_1^+(z) = U_G(z) \left[\frac{1+r_G}{2}\right] + r_G U_1^-(z)$$

$$Q_k = \begin{bmatrix} \frac{z^{1/2}}{1+r_k} & \frac{-r_k z^{1/2}}{1+r_k} \\ \frac{-r_k z^{-1/2}}{1+r_k} & \frac{z^{-1/2}}{1+r_k} \end{bmatrix} \quad U_1 = \begin{bmatrix} \prod_{i=1}^N Q_i \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} U_L(z)$$

$$\begin{split} U_k^+(z) &= \frac{z^{1/2}}{1+r_k} U_{k+1}^+(z) - \frac{r_k z^{1/2}}{1+r_k} U_{k+1}^-(z) \\ U_k^-(z) &= -\frac{r_k z^{-1/2}}{1+r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1+r_k} U_{k+1}^-(z) \\ \end{split} \qquad \begin{aligned} r_L &= \frac{\frac{\rho c}{Z_L} - A_N}{\frac{\rho c}{Z_L} + A_N} \\ U_L^+(z) &= \frac{\rho c}{A_1 Z_G - \rho c} \\ U_L^+(z) &= U_G(z) \left[ \frac{1+r_G}{2} \right] + r_G U_1^-(z) \end{aligned}$$

$$Q_k = \begin{bmatrix} \frac{z^{1/2}}{1+r_k} & \frac{-r_k z^{1/2}}{1+r_k} \\ \frac{-r_k z^{-1/2}}{1+r_k} & \frac{z^{-1/2}}{1+r_k} \end{bmatrix} \quad U_1 = \begin{bmatrix} \prod_{i=1}^N Q_i \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} U_L(z)$$

$$U_G(z) =$$

$$\begin{split} U_k^+(z) &= \frac{z^{1/2}}{1+r_k} U_{k+1}^+(z) - \frac{r_k z^{1/2}}{1+r_k} U_{k+1}^-(z) \\ U_k^-(z) &= -\frac{r_k z^{-1/2}}{1+r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1+r_k} U_{k+1}^-(z) \\ \end{split} \qquad \begin{aligned} r_L &= \frac{\frac{\rho_C}{2L} - A_N}{\frac{\rho_C}{2L} + A_N} \\ U_L^-(z) &= \frac{r_k z^{-1/2}}{1+r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1+r_k} U_{k+1}^-(z) \\ \end{aligned} \qquad \qquad \begin{aligned} r_L &= \frac{\frac{\rho_C}{2L} - A_N}{\frac{\rho_C}{2L} + A_N} \\ U_1^+(z) &= U_G(z) \left[ \frac{1+r_G}{2L} \right] + r_G U_1^-(z) \end{aligned}$$

$$Q_k = \begin{bmatrix} \frac{z^{1/2}}{1+r_k} & \frac{-r_k z^{1/2}}{1+r_k} \\ \\ \frac{-r_k z^{-1/2}}{1+r_k} & \frac{z^{-1/2}}{1+r_k} \end{bmatrix} \quad U_1 = \begin{bmatrix} \prod_{i=1}^N Q_i \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} U_L(z)$$

$$U_G(z) = \begin{bmatrix} \frac{2}{1+r_G}, -\frac{2r_G}{1+r_G} \end{bmatrix} U_1$$

$$\begin{split} U_k^+(z) &= \frac{z^{1/2}}{1 + r_k} U_{k+1}^+(z) - \frac{r_k z^{1/2}}{1 + r_k} U_{k+1}^-(z) \\ U_k^-(z) &= -\frac{r_k z^{-1/2}}{1 + r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1 + r_k} U_{k+1}^-(z) \\ \end{split} \qquad \begin{aligned} r_L &= \frac{\frac{\rho c}{Z_L} - A_N}{\frac{\rho c}{Z_L} + A_N} \\ U_L^+(z) &= \frac{r_G}{2} \left( \frac{A_1 Z_G - \rho c}{A_1 Z_G + \rho c} \right) \\ U_L^+(z) &= U_G(z) \left[ \frac{1 + r_G}{2} \right] + r_G U_1^-(z) \end{aligned}$$

$$Q_k = \begin{bmatrix} \frac{z^{1/2}}{1+r_k} & \frac{-r_k z^{1/2}}{1+r_k} \\ \frac{-r_k z^{-1/2}}{1+r_k} & \frac{z^{-1/2}}{1+r_k} \end{bmatrix} \quad U_1 = \begin{bmatrix} \prod_{i=1}^N Q_i \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} U_L(z)$$

$$U_G(z) = \left[\frac{2}{1 + r_G}, -\frac{2r_G}{1 + r_G}\right] U_1 = \left[\frac{2}{1 + r_G}, -\frac{2r_G}{1 + r_G}\right] \left[\prod_{i=1}^{N} Q_i\right] \begin{bmatrix} 1\\ 0 \end{bmatrix} U_L(z)$$

$$\begin{split} U_k^+(z) &= \frac{z^{1/2}}{1+r_k} U_{k+1}^+(z) - \frac{r_k z^{1/2}}{1+r_k} U_{k+1}^-(z) \\ U_k^-(z) &= -\frac{r_k z^{-1/2}}{1+r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1+r_k} U_{k+1}^-(z) \end{split} \qquad r_L &= \frac{\frac{\rho c}{Z_L} - A_N}{\frac{\rho c}{Z_L} + A_N} \\ U_L^-(z) &= -\frac{r_k z^{-1/2}}{1+r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1+r_k} U_{k+1}^-(z) \\ U_L^+(z) &= U_G(z) \left[ \frac{1+r_G}{2} \right] + r_G U_1^-(z) \end{split}$$

$$Q_k = \begin{bmatrix} rac{z^{1/2}}{1+r_k} & rac{-r_k z^{1/2}}{1+r_k} \\ rac{-r_k z^{-1/2}}{1+r_c} & rac{z^{-1/2}}{1+r_c} \end{bmatrix} \qquad U_G(z) = \left[ rac{2}{1+r_G}, -rac{2r_G}{1+r_G} 
ight] \left[ \prod_{i=1}^N Q_i 
ight] \left[ rac{1}{0} 
ight] U_L(z)$$

$$\begin{split} U_k^+(z) &= \frac{z}{1+r_k} U_{k+1}^+(z) - \frac{r_k z}{1+r_k} U_{k+1}^-(z) \\ U_k^-(z) &= -\frac{r_k z^{-1/2}}{1+r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1+r_k} U_{k+1}^-(z) \end{split}$$

$$\begin{split} U_k^+(z) &= \frac{z^{1/2}}{1+r_k} U_{k+1}^+(z) - \frac{r_k z^{1/2}}{1+r_k} U_{k+1}^-(z) \\ U_k^-(z) &= -\frac{r_k z^{-1/2}}{1+r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1+r_k} U_{k+1}^-(z) \\ \end{split} \qquad \begin{aligned} r_L &= \frac{\frac{\rho c}{Z_L} - A_N}{\frac{\rho c}{Z_L} + A_N} \end{aligned} \qquad r_G = \left(\frac{A_1 Z_G - \rho c}{A_1 Z_G + \rho c}\right) \\ U_k^+(z) &= U_k^{-1/2} U_{k+1}^+(z) + \frac{z^{-1/2}}{1+r_k} U_{k+1}^-(z) \end{aligned} \qquad \qquad U_1^+(z) = U_G(z) \left[\frac{1+r_G}{2}\right] + r_G U_1^-(z) \end{aligned}$$

$$Q_k = \begin{bmatrix} -r_k z^{-1/2} & z^{-1/2} \\ \hline 1 + r_k & \overline{1 + r_k} \end{bmatrix}$$

$$Q_k = \begin{bmatrix} z^{1/2} & -r_k z^{1/2} \\ \hline 1 + r_k & \overline{1 + r_k} \\ \hline -r_k z^{-1/2} & z^{-1/2} \\ \hline 1 + r_k & \overline{1 + r_k} \end{bmatrix}$$

$$Q_k = \begin{bmatrix} \dfrac{z^{1/2}}{1+r_k} & \dfrac{-r_k z^{1/2}}{1+r_k} \\ \dfrac{-r_k z^{-1/2}}{1+r_l} & \dfrac{z^{-1/2}}{1+r_l} \end{bmatrix} \qquad U_G(z) = \begin{bmatrix} \dfrac{2}{1+r_G}, -\dfrac{2r_G}{1+r_G} \end{bmatrix} \begin{bmatrix} \prod\limits_{i=1}^N Q_i \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} U_L(z)$$

$$\begin{split} U_k^+(z) &= \frac{z^{1/2}}{1 + r_k} U_{k+1}^+(z) - \frac{r_k z^{1/2}}{1 + r_k} U_{k+1}^-(z) \\ U_k^-(z) &= -\frac{r_k z^{-1/2}}{1 + r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1 + r_k} U_{k+1}^-(z) \end{split} \qquad r_L &= \frac{\frac{\rho c}{Z_L} - A_N}{\frac{\rho c}{Z_L} + A_N} \\ V_L^-(z) &= -\frac{r_k z^{-1/2}}{1 + r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1 + r_k} U_{k+1}^-(z) \\ U_L^+(z) &= U_G(z) \left[ \frac{1 + r_G}{2} \right] + r_G U_L^-(z) \end{split}$$

$$Q_{k} = \begin{bmatrix} \frac{z^{1/2}}{1+r_{k}} & \frac{-r_{k}z^{1/2}}{1+r_{k}} \\ \frac{-r_{k}z^{-1/2}}{1+r_{k}} & \frac{z^{-1/2}}{1+r_{k}} \end{bmatrix} \qquad U_{G}(z) = \begin{bmatrix} \frac{2}{1+r_{G}}, -\frac{2r_{G}}{1+r_{G}} \end{bmatrix} \begin{bmatrix} \prod_{i=1}^{N} Q_{i} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} U_{L}(z)$$
$$\begin{bmatrix} \frac{z^{1/2}}{1+r_{G}} & \frac{-r_{k}z^{1/2}}{1+r_{G}} \end{bmatrix} \begin{bmatrix} \frac{1}{1+r_{G}}, -\frac{r_{k}}{1+r_{G}} \end{bmatrix}$$

$$Q_k = \begin{bmatrix} \frac{z^{1/2}}{1+r_k} & \frac{-r_k z^{1/2}}{1+r_k} \\ \frac{-r_k z^{-1/2}}{1+r_k} & \frac{z^{-1/2}}{1+r_k} \end{bmatrix} = z^{1/2} \begin{bmatrix} \frac{1}{1+r_k} & \frac{-r_k}{1+r_k} \\ \frac{-r_k z^{-1}}{1+r_k} & \frac{z^{-1}}{1+r_k} \end{bmatrix} = z^{1/2} \widehat{Q}_k$$

$$\begin{split} U_k^+(z) &= \frac{z^{1/2}}{1 + r_k} U_{k+1}^+(z) - \frac{r_k z^{1/2}}{1 + r_k} U_{k+1}^-(z) \\ U_k^-(z) &= -\frac{r_k z^{-1/2}}{1 + r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1 + r_k} U_{k+1}^-(z) \\ \end{split} \qquad \qquad \begin{split} r_L &= \frac{\frac{\rho_C}{2L} - A_N}{\frac{\rho_C}{2L} + A_N} \\ U_L^+(z) &= \frac{r_K z^{-1/2}}{2L} U_{k+1}^+(z) + \frac{z^{-1/2}}{1 + r_k} U_{k+1}^-(z) \\ U_L^+(z) &= U_G(z) \left[ \frac{1 + r_G}{2L} \right] + r_G U_1^-(z) \end{split}$$

$$Q_k = z^{1/2} \widehat{Q}_k \qquad U_G(z) = \left[ \frac{2}{1 + r_G}, -\frac{2r_G}{1 + r_G} \right] \left[ \prod_{i=1}^N Q_i \right] \begin{bmatrix} 1 \\ 0 \end{bmatrix} U_L(z)$$

$$\begin{split} U_k^+(z) &= \frac{z^{1/2}}{1+r_k} U_{k+1}^+(z) - \frac{r_k z^{1/2}}{1+r_k} U_{k+1}^-(z) \\ U_k^-(z) &= -\frac{r_k z^{-1/2}}{1+r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1+r_k} U_{k+1}^-(z) \\ \end{split} \qquad \begin{aligned} r_L &= \frac{\frac{\rho_C}{2L} - A_N}{\frac{\rho_C}{2L} + A_N} \\ U_L^-(z) &= \frac{r_k z^{-1/2}}{1+r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1+r_k} U_{k+1}^-(z) \\ \end{aligned} \qquad \qquad \\ U_L^+(z) &= U_G(z) \left[ \frac{1+r_G}{2L} \right] + r_G U_1^-(z) \end{split}$$

$$Q_k = z^{1/2} \widehat{Q}_k \qquad \qquad U_G(z) = \left[ \frac{2}{1 + r_G}, -\frac{2r_G}{1 + r_G} \right] \left[ \prod_{i=1}^N Q_i \right] \begin{bmatrix} 1\\0 \end{bmatrix} U_L(z)$$

$$U_G(z) = \left[ \frac{2}{1 + r_G}, -\frac{2r_G}{1 + r_G} \right] \left[ \prod_{i=1}^N Q_i \right] \begin{bmatrix} 1\\0 \end{bmatrix} U_L(z)$$

$$\begin{split} U_k^+(z) &= \frac{z^{1/2}}{1+r_k} U_{k+1}^+(z) - \frac{r_k z^{1/2}}{1+r_k} U_{k+1}^-(z) \\ U_k^-(z) &= -\frac{r_k z^{-1/2}}{1+r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1+r_k} U_{k+1}^-(z) \\ \end{split} \qquad \begin{aligned} r_L &= \frac{\frac{\rho c}{Z_L} - A_N}{\frac{\rho c}{Z_L} + A_N} \\ U_L^-(z) &= \frac{r_k z^{-1/2}}{1+r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1+r_k} U_{k+1}^-(z) \\ \end{aligned} \qquad \begin{aligned} r_L &= \frac{\frac{\rho c}{Z_L} - A_N}{\frac{\rho c}{Z_L} + A_N} \\ U_1^+(z) &= U_G(z) \left[ \frac{1+r_G}{2} \right] + r_G U_1^-(z) \end{aligned}$$

$$Q_{k} = z^{1/2} \widehat{Q}_{k} \qquad U_{G}(z) = \left[ \frac{2}{1+r_{G}}, -\frac{2r_{G}}{1+r_{G}} \right] \left[ \prod_{i=1}^{N} Q_{i} \right] \begin{bmatrix} 1\\0 \end{bmatrix} U_{L}(z)$$

$$U_{G}(z) = \left[ \frac{2}{1+r_{G}}, -\frac{2r_{G}}{1+r_{G}} \right] \left[ \prod_{i=1}^{N} Q_{i} \right] \begin{bmatrix} 1\\0 \end{bmatrix} U_{L}(z) =$$

$$= \left[ \frac{2}{1+r_{G}}, -\frac{2r_{G}}{1+r_{G}} \right] \left[ \prod_{i=1}^{N} z^{1/2} \widehat{Q}_{i} \right] \begin{bmatrix} 1\\0 \end{bmatrix} U_{L}(z)$$

$$\begin{split} U_k^+(z) &= \frac{z^{1/2}}{1+r_k} U_{k+1}^+(z) - \frac{r_k z^{1/2}}{1+r_k} U_{k+1}^-(z) \\ U_k^-(z) &= -\frac{r_k z^{-1/2}}{1+r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1+r_k} U_{k+1}^-(z) \\ \end{split} \qquad \begin{aligned} r_L &= \frac{\frac{\rho c}{Z_L} - A_N}{\frac{Z_L}{2} + A_N} \\ U_L^+(z) &= \frac{r_k z^{-1/2}}{2} U_{k+1}^+(z) + \frac{z^{-1/2}}{1+r_k} U_{k+1}^-(z) \\ U_L^+(z) &= U_G(z) \left[ \frac{1+r_G}{2} \right] + r_G U_1^-(z) \end{aligned}$$

$$Q_{k} = z^{1/2} \widehat{Q}_{k} \qquad U_{G}(z) = \left[ \frac{2}{1+r_{G}}, -\frac{2r_{G}}{1+r_{G}} \right] \left[ \prod_{i=1}^{N} Q_{i} \right] \left[ \frac{1}{0} \right] U_{L}(z)$$

$$U_{G}(z) = \left[ \frac{2}{1+r_{G}}, -\frac{2r_{G}}{1+r_{G}} \right] \left[ \prod_{i=1}^{N} Q_{i} \right] \left[ \frac{1}{0} \right] U_{L}(z) =$$

$$= \left[ \frac{2}{1+r_{G}}, -\frac{2r_{G}}{1+r_{G}} \right] \left[ \prod_{i=1}^{N} z^{1/2} \widehat{Q}_{i} \right] \left[ \frac{1}{0} \right] U_{L}(z)$$

$$= z^{N/2} \left[ \frac{2}{1+r_{G}}, -\frac{2r_{G}}{1+r_{G}} \right] \left[ \prod_{i=1}^{N} \widehat{Q}_{i} \right] \left[ \frac{1}{0} \right] U_{L}(z)$$

$$\begin{split} U_k^+(z) &= \frac{z^{1/2}}{1+r_k} U_{k+1}^+(z) - \frac{r_k z^{1/2}}{1+r_k} U_{k+1}^-(z) \\ U_k^-(z) &= -\frac{r_k z^{-1/2}}{1+r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1+r_k} U_{k+1}^-(z) \\ \end{split} \qquad \begin{aligned} r_L &= \frac{\frac{\rho_C}{L} - A_N}{\frac{\rho_C}{Z_L} + A_N} \\ U_L^-(z) &= \frac{r_k z^{-1/2}}{2L} U_{k+1}^+(z) + \frac{z^{-1/2}}{1+r_k} U_{k+1}^-(z) \\ U_L^+(z) &= U_G(z) \left[ \frac{1+r_G}{2} \right] + r_G U_1^-(z) \end{aligned}$$

$$Q_k = z^{1/2} \widehat{Q}_k$$
  $U_G(z) = z^{N/2} \left[ \frac{2}{1 + r_G}, -\frac{2r_G}{1 + r_G} \right] \left[ \prod_{i=1}^N \widehat{Q}_i \right] \begin{bmatrix} 1 \\ 0 \end{bmatrix} U_L(z)$ 

$$\begin{split} U_k^+(z) &= \frac{z^{1/2}}{1 + r_k} U_{k+1}^+(z) - \frac{r_k z^{1/2}}{1 + r_k} U_{k+1}^-(z) \\ U_k^-(z) &= -\frac{r_k z^{-1/2}}{1 + r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1 + r_k} U_{k+1}^-(z) \\ \end{split} \qquad \qquad \begin{split} r_L &= \frac{\frac{\rho_C}{2L} - A_N}{\frac{\rho_C}{2L} + A_N} \\ U_L^+(z) &= \frac{r_k z^{-1/2}}{2L} U_{k+1}^+(z) + \frac{z^{-1/2}}{1 + r_k} U_{k+1}^-(z) \\ U_L^+(z) &= U_G(z) \left[ \frac{1 + r_G}{2L} \right] + r_G U_1^-(z) \end{split}$$

$$Q_k = z^{1/2} \widehat{Q}_k \qquad U_G(z) = z^{N/2} \left[ \frac{2}{1 + r_G}, -\frac{2r_G}{1 + r_G} \right] \left[ \prod_{i=1}^N \widehat{Q}_i \right] \begin{bmatrix} 1\\0 \end{bmatrix} U_L(z)$$

$$U_L(z) = U_G(z) \mathcal{V}(z)$$

$$\begin{split} U_k^+(z) &= \frac{z^{1/2}}{1+r_k} U_{k+1}^+(z) - \frac{r_k z^{1/2}}{1+r_k} U_{k+1}^-(z) \\ U_k^-(z) &= -\frac{r_k z^{-1/2}}{1+r_k} U_{k+1}^+(z) + \frac{z^{-1/2}}{1+r_k} U_{k+1}^-(z) \\ \end{split} \qquad \begin{aligned} r_L &= \frac{\frac{\rho c}{Z_L} - A_N}{\frac{\rho c}{Z_L} + A_N} \\ U_L^+(z) &= \frac{r_k z^{-1/2}}{2} U_{k+1}^+(z) + \frac{z^{-1/2}}{1+r_k} U_{k+1}^-(z) \\ U_L^+(z) &= U_G(z) \left[ \frac{1+r_G}{2} \right] + r_G U_1^-(z) \end{aligned}$$

$$Q_k = z^{1/2} \widehat{Q}_k \qquad \qquad U_G(z) = z^{N/2} \left[ \frac{2}{1 + r_G}, -\frac{2r_G}{1 + r_G} \right] \left[ \prod_{i=1}^N \widehat{Q}_i \right] \begin{bmatrix} 1\\0 \end{bmatrix} U_L(z)$$

$$U_L(z) = U_G(z) \mathcal{V}(z) \longleftrightarrow$$

$$\frac{1}{\mathcal{V}(z)} = \frac{U_G(z)}{U_L(z)}$$

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$$Q_k = z^{1/2} \widehat{Q}_k \qquad U_G(z) = z^{N/2} \left[ \frac{2}{1+r_G}, -\frac{2r_G}{1+r_G} \right] \left[ \prod_{i=1}^N \widehat{Q}_i \right] \begin{bmatrix} 1\\0 \end{bmatrix} U_L(z)$$

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$$\mathcal{V}(z) = \frac{0.5(1 + r_G) \prod_{i=1}^N (1 + r_i) z^{-N/2}}{1 - \sum_{i=1}^N \alpha_i z^{-i}}$$

$$\mathcal{V}(z) = \frac{0.5(1 + r_G) \prod\limits_{i=1}^{N} (1 + r_i)z^{-N/2}}{1 - \sum\limits_{i=1}^{N} \alpha_i z^{-i}}$$

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$$\mathcal{G}(z)$$
?

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$$G(z)$$
?

$$\mathcal{R}(z)$$
?

$$\mathcal{V}(z) = \frac{0.5(1+r_G)\prod\limits_{i=1}^{N}{(1+r_i)z^{-N/2}}}{1-\sum\limits_{i=1}^{N}{\alpha_iz^{-i}}}$$

Общ вид на  ${\mathcal R}$ 

$$\mathcal{V}(z) = \frac{0.5(1+r_G)\prod\limits_{i=1}^{N}{(1+r_i)z^{-N/2}}}{1-\sum\limits_{i=1}^{N}{\alpha_iz^{-i}}}$$

#### Общ вид на $\mathcal R$

• Моделира се трудно

$$\mathcal{V}(z) = \frac{0.5(1 + r_G) \prod\limits_{i=1}^{N} (1 + r_i)z^{-N/2}}{1 - \sum\limits_{i=1}^{N} \alpha_i z^{-i}}$$

#### Общ вид на $\mathcal R$

- Моделира се трудно
- Обикновено се ползва някакво много опростено представяне:

$$\mathcal{R}(z) = 1 - \gamma z^{-1}, \gamma < 1$$

$$\mathcal{V}(z) = \frac{0.5(1 + r_G) \prod\limits_{i=1}^{N} (1 + r_i)z^{-N/2}}{1 - \sum\limits_{i=1}^{N} \alpha_i z^{-i}}$$

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•  $\gamma \approx 0.97$ 

$$\mathcal{V}(z) = \frac{0.5(1 + r_G) \prod_{i=1}^{N} (1 + r_i)z^{-N/2}}{1 - \sum_{i=1}^{N} \alpha_i z^{-i}}$$

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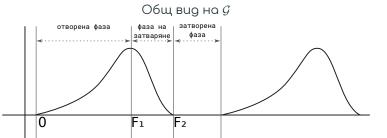
$$\mathcal{R}(z) = 1 - \gamma z^{-1}$$

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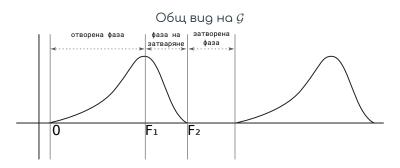
$$\mathcal{R}(z) = 1 - \gamma z^{-1}$$

Общ вид на  $\mathcal G$ 

$$\mathcal{V}(z) = \frac{0.5(1+r_G) \prod\limits_{i=1}^{N} (1+r_i)z^{-N/2}}{1-\sum\limits_{i=1}^{N} \alpha_i z^{-i}}$$
 
$$\mathcal{R}(z) = 1-\gamma z^{-1}$$



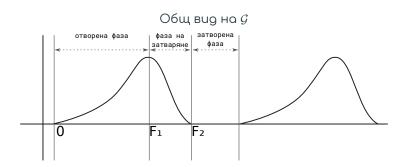
$$\mathcal{V}(z) = \frac{0.5(1+r_G) \prod_{i=1}^{N} (1+r_i)z^{-N/2}}{1-\sum\limits_{i=1}^{N} \alpha_i z^{-i}}$$
 
$$\mathcal{R}(z) = 1-\gamma z^{-1}$$



$$g(t) = \begin{cases} \frac{1}{2}(1 - \cos(\pi n/F1)), & 0 \le t \le F_1 \\ \cos(\pi (n - F_1)/2(F_2 - F_1)), & F_1 \le t \le F_2 \\ 0, & \text{uhave} \end{cases}$$

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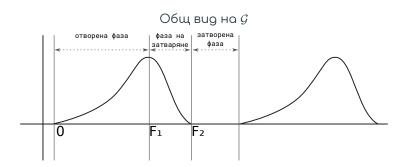
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$$\mathcal{V}(z) = \frac{0.5(1 + r_G) \prod_{i=1}^{N} (1 + r_i)z^{-N/2}}{1 - \sum_{i=1}^{N} \alpha_i z^{-i}}$$

$$\mathcal{R}(z) = 1 - \gamma z^{-1}$$

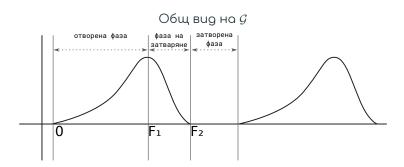


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$$\mathcal{V}(z) = \frac{0.5(1 + r_G) \prod_{i=1}^{N} (1 + r_i)z^{-N/2}}{1 - \sum_{i=1}^{N} \alpha_i z^{-i}}$$

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Общ вид на  ${\cal Y}$ 

$$\mathcal{V}(z) = \frac{0.5(1+r_G)\prod\limits_{i=1}^{N}(1+r_i)z^{-N/2}}{1-\sum\limits_{i=1}^{N}\alpha_iz^{-i}} \\ \mathcal{R}(z) = 1-\gamma z^{-1} \\ \mathcal{G}(z) = \frac{\prod\limits_{i=0}^{K}(1-\beta_iz^{-1})}{(1-\beta z)^2}$$

Общ вид на У

$$\mathcal{Y}(z) = \mathcal{G}(z)\mathcal{V}(z)\mathcal{R}(z)$$

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Общ вид на  $\mathcal{Y}$ 

$$\mathcal{Y}(z) = \mathcal{G}(z)\mathcal{V}(z)\mathcal{R}(z)$$

$$= \left[ \frac{\prod\limits_{i=0}^{K} (1 - \beta_i z^{-1})}{(1 - \beta z)^2} \right] \left[ \frac{0.5(1 + r_G) \prod\limits_{i=1}^{N} (1 + r_i) z^{-N/2}}{1 - \sum\limits_{i=1}^{N} \alpha_i z^{-i}} \right] \left[ \frac{\prod\limits_{i=0}^{K} (1 - \beta_i z^{-1})}{(1 - \beta z)^2} \right]$$

$$\mathcal{V}(z) = \frac{0.5(1+r_G) \prod\limits_{i=1}^{N} (1+r_i)z^{-N/2}}{1-\sum\limits_{i=1}^{N} \alpha_i z^{-i}} \\ \mathcal{R}(z) = 1-\gamma z^{-1} \\ \mathcal{G}(z) = \frac{\prod\limits_{i=0}^{K} (1-\beta_i z^{-1})}{(1-\beta z)^2}$$

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$$= \mathcal{G}(z) \frac{\sum_{m=0}^{M} b_m z^{-m}}{\sum_{k=0}^{K} a_k z^{-k}},$$

$$\mathcal{Y}(z) = \mathcal{G}(z)\mathcal{V}(z)\mathcal{R}(z) = \mathcal{G}(z)\frac{\sum\limits_{m=0}^{M}{}^{b_{m}z^{-m}}}{\sum\limits_{k=0}^{K}{}^{a_{k}z^{-k}}}$$

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• Система (дефиниция)

$$\mathcal{Y}(z) = \mathcal{G}(z)\mathcal{V}(z)\mathcal{R}(z) = \mathcal{G}(z)\frac{\sum\limits_{m=0}^{M}b_{m}z^{-m}}{\sum\limits_{k=0}^{K}a_{k}z^{-k}}$$

 Система (дефиниция) - Механизъм, който манипулира един или повече сигнали с някаква цел до получаване на нов сигнал, се нарича система.

$$\mathcal{Y}(z) = \mathcal{G}(z)\mathcal{V}(z)\mathcal{R}(z) = \mathcal{G}(z)\frac{\sum\limits_{m=0}^{M}b_{m}z^{-m}}{\sum\limits_{k=0}^{K}a_{k}z^{-k}}$$

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- Филтър

$$\mathcal{Y}(z) = \mathcal{G}(z)\mathcal{V}(z)\mathcal{R}(z) = \mathcal{G}(z)\frac{\sum\limits_{m=0}^{M}b_{m}z^{-m}}{\sum\limits_{k=0}^{K}a_{k}z^{-k}}$$

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- Филтър
- $g[n] \mapsto y[n]$

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- Линейна система (Дефиниция)

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- Филтър
- $g[n] \mapsto y[n]$
- Линейна система (Дефиниция) Ако  $x_1[n]\mapsto y_1[n]$  и  $x_2[n]\mapsto y_2[n]$ , то системата е линейна  $\longleftrightarrow$   $\forall a,b\in\mathbb{R}\ (ax_1[n]+bx_2[n]\mapsto ay_1[n]+by_2[n])$

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- $g[n] \mapsto y[n]$
- Линейна система (Дефиниция) Ако  $x_1[n]\mapsto y_1[n]$  и  $x_2[n]\mapsto y_2[n]$ , то системата е линейна  $\longleftrightarrow$   $\forall a,b\in\mathbb{R}\ (ax_1[n]+bx_2[n]\mapsto ay_1[n]+by_2[n])$
- Времево-инвариантна система (Дефиниция)

$$\mathcal{Y}(z) = \mathcal{G}(z)\mathcal{V}(z)\mathcal{R}(z) = \mathcal{G}(z)\frac{\sum\limits_{m=0}^{M}b_{m}z^{-m}}{\sum\limits_{k=0}^{K}a_{k}z^{-k}}$$

- Система (дефиниция) Механизъм, който манипулира един или повече сигнали с някаква цел до получаване на нов сигнал, се нарича система.
- Филтър
- $g[n] \mapsto y[n]$
- Линейна система (Дефиниция) Ако  $x_1[n] \mapsto y_1[n]$  и  $x_2[n] \mapsto y_2[n]$ , то системата е линейна  $\longleftrightarrow$   $\forall a,b \in \mathbb{R} \ (ax_1[n] + bx_2[n] \mapsto ay_1[n] + by_2[n])$
- Времево-инвариантна система (Дефиниция) Нека  $x[n]\mapsto y[n]$ . Тогава, ако за всяко  $n_0:x[n-n_0]\mapsto y[n-n_0]$ , то системата е времево-инвариантна.

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$$y[n]=(g*h)[n]$$
  $y[n] \stackrel{\mathcal{FS}}{\longleftrightarrow} Y(z), g[n] \stackrel{\mathcal{FS}}{\longleftrightarrow} G(z), h[n] \stackrel{\mathcal{FS}}{\longleftrightarrow} H(z), z=e^{iw}$   $Y(z)=G(z)H(z)$   $H(z)=rac{Y(z)}{G(z)}$  - предавателна функция

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$$\begin{split} y[n] &= (g*h)[n] \qquad \qquad y[n] \stackrel{\mathcal{FS}}{\longleftrightarrow} Y(z), g[n] \stackrel{\mathcal{FS}}{\longleftrightarrow} G(z), h[n] \stackrel{\mathcal{FS}}{\longleftrightarrow} H(z), z = e^{iw} \\ Y(z) &= G(z)H(z) \\ H(z) &= \frac{Y(z)}{G(z)} \text{- предавателна функция} \\ \sum_{}^{N} a_k y[n-k] &= \sum_{}^{M} b_m g[n-m] \end{split}$$

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$$\mathcal{Y}(z) = \mathcal{G}(z)\mathcal{V}(z)\mathcal{R}(z) = \mathcal{G}(z)\frac{\sum\limits_{m=0}^{M}b_{m}z^{-m}}{\sum\limits_{k=0}^{K}a_{k}z^{-k}} \qquad \qquad \sum\limits_{k=0}^{N}a_{k}y[n-k] = \sum\limits_{m=0}^{M}b_{m}x[n-m]$$
 
$$y[n] = (g*h)[n] \qquad y[n] \stackrel{\mathcal{FS}}{\longleftrightarrow} Y(z), g[n] \stackrel{\mathcal{FS}}{\longleftrightarrow} G(z), h[n] \stackrel{\mathcal{FS}}{\longleftrightarrow} H(z), z = e^{iw}$$
 
$$Y(z) = G(z)H(z)$$
 
$$H(z) = \frac{Y(z)}{G(z)} - \text{предовомелно функция}$$
 
$$\sum\limits_{k=0}^{N}a_{k}y[n-k] = \sum\limits_{m=0}^{M}b_{m}g[n-m] \qquad y[n] \stackrel{\mathcal{FS}}{\longleftrightarrow} Y(z), g[n] \stackrel{\mathcal{FS}}{\longleftrightarrow} G(z)$$
 
$$\left[\sum\limits_{k=0}^{N}a_{k}z^{-k}\right]\mathcal{Y}(z) = \left[\sum\limits_{m=0}^{M}b_{m}z^{-m}\right]\mathcal{G}(z)$$
 
$$\frac{\mathcal{Y}(z)}{\mathcal{G}(z)} = \frac{\sum\limits_{m=0}^{M}b_{m}z^{-m}}{\sum\limits_{N}^{N}a_{k}z^{-k}}$$

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y[n] = (q \* h)[n]  $y[n] \stackrel{\mathcal{FS}}{\longleftrightarrow} Y(z), q[n] \stackrel{\mathcal{FS}}{\longleftrightarrow} G(z), h[n] \stackrel{\mathcal{FS}}{\longleftrightarrow} H(z), z = e^{iw}$ 

$$\begin{split} Y(z) &= G(z)H(z) \\ H(z) &= \frac{Y(z)}{G(z)} \text{- предавателна функция} \\ \sum_{k=0}^{N} a_k y[n-k] &= \sum_{m=0}^{M} b_m g[n-m] \qquad y[n] \overset{\mathcal{FS}}{\longleftrightarrow} Y(z), g[n] \overset{\mathcal{FS}}{\longleftrightarrow} G(z) \\ \left[\sum_{k=0}^{N} a_k z^{-k}\right] \mathcal{Y}(z) &= \left[\sum_{m=0}^{M} b_m z^{-m}\right] \mathcal{G}(z) \\ \frac{\mathcal{Y}(z)}{\mathcal{G}(z)} &= H(z) &= \frac{\sum_{m=0}^{M} b_m z^{-m}}{\sum_{k=0}^{N} a_k z^{-k}} \end{split}$$

$$\mathcal{Y}(z) = \mathcal{G}(z)\mathcal{V}(z)\mathcal{R}(z) = \mathcal{G}(z)\frac{\sum\limits_{m=0}^{M}b_{m}z^{-m}}{\sum\limits_{k=0}^{K}a_{k}z^{-k}} \qquad \qquad \frac{\mathcal{Y}(z)}{\mathcal{G}(z)} = H(z) = \frac{\sum\limits_{m=0}^{M}b_{m}z^{-m}}{\sum\limits_{k=0}^{N}a_{k}z^{-k}}$$

$$y[n]=(g*h)[n] \qquad \qquad y[n] \stackrel{\mathcal{FS}}{\longleftrightarrow} Y(z), \\ g[n] \stackrel{\mathcal{FS}}{\longleftrightarrow} G(z), \\ h[n] \stackrel{\mathcal{FS}}{\longleftrightarrow} H(z), \\ z=e^{iw}$$
 
$$Y(z)=G(z)H(z)$$
 
$$H(z)=\frac{Y(z)}{G(z)} \text{- предавателна функция}$$

$$\sum_{k=0}^{N} a_k y[n-k] = \sum_{m=0}^{M} b_m g[n-m] \qquad y[n] \stackrel{\mathcal{FS}}{\longleftrightarrow} Y(z), g[n] \stackrel{\mathcal{FS}}{\longleftrightarrow} G(z)$$

$$\left[\sum_{k=0}^{N} a_k z^{-k}\right] \mathcal{Y}(z) = \left[\sum_{m=0}^{M} b_m z^{-m}\right] \mathcal{G}(z)$$

$$\frac{\mathcal{Y}(z)}{\mathcal{G}(z)} = H(z) = \frac{\sum_{m=0}^{M} b_m z^{-m}}{\sum_{k=0}^{N} a_k z^{-k}}$$

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$$\mathcal{Y}(z) = \mathcal{G}(z)\mathcal{V}(z)\mathcal{R}(z) \cup \mathcal{Y}(z) = \mathcal{G}(z)\mathcal{H}(z)$$

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- $\mathcal{Y}(z) = \mathcal{G}(z)\mathcal{V}(z)\mathcal{R}(z)$  u  $\mathcal{Y}(z) = \mathcal{G}(z)\mathcal{H}(z)$
- Производството на реч се описва от система  $g[n] \mapsto y[n]$

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- Производството на реч се описва от система  $g[n]\mapsto y[n]$
- $\mathcal{Y}(z)=\mathcal{G}(z)\mathcal{H}(z)$ , трансферната функция  $\mathcal{H}=\mathcal{V}(z)\mathcal{R}(z)$  съдържа информация за вокалния тракт

$$\mathcal{Y}(z) = \mathcal{G}(z)\mathcal{V}(z)\mathcal{R}(z) = \mathcal{G}(z)\frac{\sum\limits_{m=0}^{M}b_{m}z^{-m}}{\sum\limits_{k=0}^{K}a_{k}z^{-k}} \qquad \qquad \frac{\mathcal{Y}(z)}{\mathcal{G}(z)} = H(z) = \frac{\sum\limits_{m=0}^{M}b_{m}z^{-m}}{\sum\limits_{k=0}^{N}a_{k}z^{-k}}$$

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- Характеристиките, които ще ползваме, трябва да отделят информацията за  ${\mathcal G}$  от тази за  ${\mathcal H}$

Обща идея

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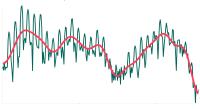


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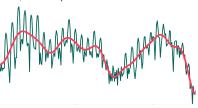


• Логаритъмът подчертава периодичността

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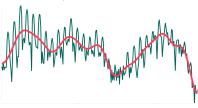


- Логаритъмът подчертава периодичността
- Kencmър

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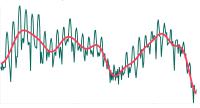


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- $c_h[n]$  ще са в ниските

## Извличане

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## Извличане

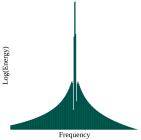
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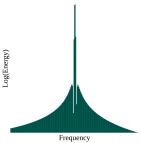


#### Извличане

• Трябва да направим сигнала във всеки фрейм периодичен

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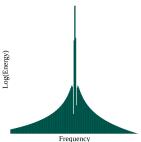


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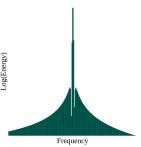


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$$a_{k,t} = \frac{1}{L} \sum_{t=1}^{L-1} x_t[n] e^{-\frac{2\pi i k n}{L}}$$

### Извличане

• Няколко особености на слуха

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- Мел скала

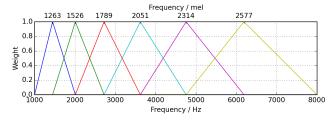
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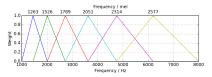
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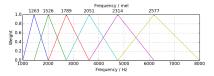
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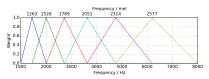


## Извличане



Взимаме логаритъм от енергиите в критичните области (М = 23):





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$$c_{m,t} = \log\left(\sum_{k=0}^{L-1} |a_{k,t}|^2 H_m[k, f[m-1], f[m], f[m+1]]\right), m = 0, 1, \dots, M-1, M$$

$$H_m[k, start, center, end] = \begin{cases} \frac{k - start}{center - start}, & start \le k \le center \\ \frac{end - k}{end - center}, & center < k \le end \end{cases}$$

$$f[m] = \frac{L}{F_s} melToHerz\left(\frac{m \times maxMel}{M+1}\right), maxMel = herzToMel\left(\frac{F_s}{2}\right)$$

## Извличане

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# Сигнал от реч - характеристики

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- За всеки фрейм получаваме 39 коефициента

# Сигнал от реч - характеристики

- Започваме от дискретен сигнал: s[n], n = 0, 1, ..., N
- Разделяме сигнала s[n] на феймове:
- $s_t[i] = s[tS + i],$  Прилагаме прозоречна функция:
  - $x_t[n] = s_t[n] w_{hamming}[n]$ , където
- Намираме Фурие коефициентите:

$$a_{k,t} = \frac{1}{L} \sum_{n=0}^{L-1} x_t[n] e^{-\frac{2\pi i k n}{L}}$$

 Взимаме логаритъм от енергиите в критичните области:

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• Гаусови смески

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 За всяка емоция намираме Гаусовата смеска, която максимизира логаритъм от правдоподобието на съответните вектори

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• Правдоподобието на всички вектори X с етикет e спрямо  $(\hat{\pi}^e, \hat{\mu}^e, \hat{\Sigma}^e)$ :

$$p(X|(\hat{\pi}^e, \hat{\mu}^e, \hat{\Sigma}^e)) = \prod_{i=1}^n \sum_{k=1}^K \pi_k^e \mathcal{N}(x_i; \mu_k^e, \Sigma_k^e).$$

 За всяка емоция намираме Гаусовата смеска, която максимизира логаритъм от правдоподобието на съответните вектори - EM метод

- Гаусови смески всяко непрекъснато разпределение върху  $\mathbb{R}^n$  може да се приближи с линейна комбинация на достатъчно на брой гаусиани
- Имаме по една смеска за всяка емоция  $(\hat{\pi}^e, \hat{\mu}^e, \hat{\Sigma}^e)$
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- За първоначално разбиване на векторите се ползва К-теапs++