Numerical Analysis 23 **Sheet 1**

Linear Systems

Please hand in questions 1, 2 and 3 by 11am on Thursday 7 February.

1. In the lectures the linear system $U\mathbf{x} = \mathbf{b}$ was solved for a general $n \times n$ upper triangular matrix U by *backward substitution*. Solve the linear system $L\mathbf{x} = \mathbf{b}$ for a general $n \times n$ lower triangular matrix L by *forward substitution* , i.e. specify the equations that iteratively determine the unknowns x_i for $i = 1, \ldots, n$.

2. Solve the linear system:

$$\begin{aligned} -x_1 + x_2 + x_3 + x_4 &= -2 \\ x_1 - x_2 + x_3 + x_4 &= -2 \\ x_1 + x_2 - x_3 + x_4 &= 2 \\ x_1 + x_2 + x_3 - x_4 &= 2 \end{aligned}$$

   by Gaussian elimination. Demonstrate that your solution works.

3. Consider the matrix

$$A = \begin{bmatrix} 1 & -1 & \lambda \\ -2 & 1 & -2\lambda \\ \lambda & -2 & 1 \end{bmatrix}$$

   where λ is some constant. By using Gaussian elimination solve the linear system

$$A\mathbf{x} = \begin{bmatrix} -2 \\ 3 \\ -1 \end{bmatrix}, \qquad (1)$$

   assuming that it has a unique solution.

   Calculate the determinant of the matrix A and hence find the values of λ for which the linear system (1) has no unique solution. In addition, specify the value of λ for which it has no solution, and the value of λ for which it has an infinite number of solution.