Numerical Analysis 23 — Sheet 2

Linear Systems, Bisection Method

Numerical Analysis 23: Please hand in questions 1, 2, and 3 by 11am on Thursday 14 February.

1. Find an LU decomposition for the matrix

$$A = \begin{bmatrix} -1 & 1 & 1 \\ 2 & -1 & 1 \\ 1 & 1 & 2 \end{bmatrix}.$$

   By using your LU decomposition find the inverse of the matrix A. (Solve $A\mathbf{v}_j = \mathbf{e}_j$, $j \in \{1, 2, 3\}$, where $\mathbf{v}_j$ is the j-th column of A^{-1} and $\mathbf{e}_j$ is the j-th column of the identity matrix I).

2. Find a decomposition of the form

$$PA = LU$$

   for the matrix

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 4 & 2 & 3 \\ 2 & -1 & 4 \end{bmatrix}.$$

   Here P is a permutation matrix, L is lower triangular and U is upper triangular.

3. (a) Show that the function $f(x) = x^3 - x - 1/4$ has 3 roots, exactly one of which lies in the interval $[1, 2]$. (Hint: start by searching for maxima and minima of the function.)

   (b) Determine the number of interations of the Bisection method which would assure of finding this root to an accuracy of 10^{-4}.

   (c) Give 2 intervals $[a, b]$ which contain exactly one of the other 2 roots.

4. Consider $f(x) = x^2 - 2$ and perform five iterations of the Bisection method to find an approximation for $\sqrt{2}$. How many digits of your final approximation are guaranteed to be correct?