Machine Learning for Improved Autotune Identification Method

A

MTP Report

Submitted in partial fulfillment of the requirement for the degree of

Master of Technology

Submitted by

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May, 2022

DECLARATION

I hereby declare that the work which is being presented in the thesis entitled, **Machine Learning for Improved Autotune Identification Method** *in partial fulfillment of the requirements for the award of the degree of Master of Technology in SYSTEMS, CONTROL, AND AUTOMATION at Indian Institute of Technology Guwahati is an authentic record of my work carried out under the supervision of* **Prof. Somanath Majhi**, *and refers to other researcher's work which is duly listed in the reference section. The contents of this thesis, in full or in parts, have not been submitted to any other Institute or University for the award of any degree or diploma.*

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CERTIFICATE

This is to certify that the work contained in the thesis entitled, **Machine Learning for Improved Autotune Identification Method** *is a bonafide work of* **Bittu Kumar (Roll No. 204102502)**, *which has been carried out in the Department of Electronics and Electrical Engineering, Indian Institute of Technology Guwahati under my supervision and this work has not been submitted elsewhere for a Degree.*

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ACKNOWLEDGEMENT

I am extremely grateful to my supervisor **Prof. Somanath Majhi** for this work. The research work that I have done is a little development of his idea. Without his valuable feedback and constant support, this development might not have been possible. The freedom, regular follow-up, and the relaxed environment that he gave created a conducive environment for research. Apart from research work, I am very much thankful for the profound life experience during this one year. It was an absolute pleasure to work under him.

I would also like to express gratitude towards my family for being the very basis of my life and for the constant love and support all through the journey of my life. I am obliged to all faculty members of the EEE Department for the knowledge they have provided. I am thankful to all my colleagues for the joyful environment. I would also like to thank the EEE Department staff members for their help during this period.

Sincerely
Bittu Kumar

ABSTRACT

A set of general explicit expressions is derived for the identification of a simple transfer function model based on describing function approach. Using these expressions, parameters of transfer function models are obtained. However, due to various dynamic errors, this approach leads to significant errors in determining transfer function model parameters. On the basis of these expressions, a large set of data is generated to train machine learning models for various transfer function models. This procedure is proposed to increase the accuracy of autotune identification method. Simulation examples show promising results by the proposed autotune identification method.

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List of Abbreviations

DF	Descrining Function
FOPDT	First order plus delay time
SOPDT	Second order plus delay time
SNR	Signal to noise ratio
SISO	Single input single output

List of Publications

Conference Publications

1. Bittu and Majhi S., "Machine Learning for Improved Autotune Identification Method", IEEE Conference of Inetrnational Federation of Automatic Control on Cyber-Physical Human system (CPHS 2022), Texas 2022.

Chapter 1

Introduction

Process identification is important for both analyzing process operation and designing model based controllers. It refers to the mathematical modeling of an unknown process in terms of transfer function [1], using measurements of input and output signals. On the basis of the Astrom-Hagglund [2] relay feedback system, Luyben [3] suggested an approach for detecting process transfer functions, also called original ATV (autotune variation) method.

There are two steps to the technique. To begin, a relay feedback experiment is carried out, with the eventual gain and frequency being recorded. Second, this data is fitted to a conventional process control transfer function, such as a first-, second-, or third-order plus delay time system.

This method identifies unknown process information around frequency called ultimate frequency using describing function (DF) approach. We can determine describing function using approximated gain of non-linear device, relay [1]. There are several methods to determine process information but relay feedback based identification is widely used in industrial process due to its advantages over other methods such as [8]:

i) It helps in identifying process inform ation around ultimate frequency at phase angle $-\Pi$.
ii) Feedback approach doesn't let the system drift away from nominal operating point.
iii)Less time consuming with processes having higher time constants.
iv) This method can be easily modified to use with disturbances.
v) A very simple method to identify process dynamics where operators can easily understand how it works.

Despite its apparent effectiveness in industrial applications, the ATV approach may be greatly enhanced in order to provide a more precise estimate of system characteristics. The reason for this is that nonlinear behavior of relay lead to inaccuracy when we try to back calculate parameters of the linear models. So, the describing function's projected ultimate gain N and estimated ultimate frequency are simply an approximation of information at critical frequency. When numerous higher harmonics are ignored in favour of merely accounting for fundamental harmonics, severe inaccuracies in the ultimate gain N and ultimate frequency for a typical transfer function result.

1.1 Problem Statement

Regardless of reliable sensors, measurement noise caused by control valves, measuring equipment or the process itself corrupts the process output in the real world. The ultimate time period and amplitude of the output signal are difficult to evaluate due to noisy output, resulting in erroneous process identification. As a result, it's critical to get rid of these shortcomings. Our main approach would be to rectify original output signal and determine parameters using describing function approach.

Relay based identification method depicts a simpler approach to determine process dynamics using relay, but its major limitation is lower accuracy with increase in the ratio of time constant and time delay. Our major strategy would be to use a machine learning algorithm to properly locate the process parameter in the presence of disturbances. This research aims to derive analytical formulas for the period of oscillation and amplitude ratio for relay feedback systems, as well as a machine learning technique for better process transfer function estimates.

1.1.1 Methodology

As we have seen in the problem statement that noisy output may lead to severe inaccuracy and malfunctioning of relay. In order to demonstrate the influence of noise in simulations, a white Gaussian noise with a fixed mean and variance is injected into the output signal, resulting in noisy oscillations. The process dynamics parameter is calculated in the presence of measurement noise with an SNR of 20dB to demonstrate the efficacy of the suggested ML model. To create this noise, the simulation employs a white Gaussian noise with a mean of zero and a variance of 1.35×10^{-4} . After induction of noise a bandwidth of noisy signal is calculated and a band pass filter is implemented to filter out noisy signal from output.

As describing function depicts a simpler approach to estimate process model parameters. A variety of process model is simulated, using describing function approach we have estimated parameters of numerous process models. These data are then further used to train machine learning model to estimate system parameter accurately. Various machine learning models are used to predict process model parameters such as Decision tree regressor model, Random forest regressor model, Linear regressor model and Artificial neural network regressor model.

1.2 Ideal Relay

Relay is an electromechanical device used a switch. It operates in cascade with process model to generate limit cycle output.

1.2.1 Describing Function Approach

Describing function approach is a method to estimate system parameters from generating limit cycle output. It is an effective and simple approach to estimate system parameters. It is widely used in industrial application. For stable process the limit cycle output is shown in the Figure 1.2. Here, the Fourier series analysis of relay output (U) can be done to calculate approximated relay gain which is used to calculate system parameters.

From Fourier series analysis of U(t), Approximated relay gain,

$$\mathrm{N} = \frac{4\mathrm{h}}{\pi \mathrm{A}} \tag{1.1}$$

where, h is the relay height and A is amplitude of the output signal

1.3 Identification Method

In relay based identification, describing function approach to determine unknown parameters of model transfer function is widely used due to ease of computation involved and simpler approach. There are two type of identification.

1.3.1 Offline Identification Method

This section describes offline identification of system parameters using describing function (DF) approach based on closed loop test. The conventional offline identification structure of non-minimum process model is shown in the Figure 1.1, which consists ideal relay with typical non-minimum phase stable transfer function with dead time is assumed as process model G(s).

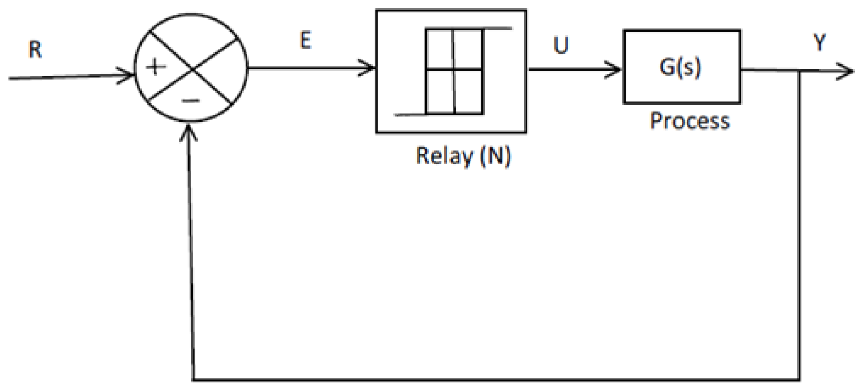

FIGURE 1.1: Offline autotune identification scheme

Where, R is reference input, U is input applied from relay and Y is output of the process model. This relay feedback test generates a limit cycle output. The ultimate frequency on the limit cycle output is, $\omega = 2\pi/\text{pu}$, where pu is the ultimate time period. Using Fourier series expansion, the amplitude (A) is determined from fundamental harmonics of limit cycle output. Now equivalent gain of non-linearity (relay) is approximated as $\text{N} = 4\text{h}/\pi\text{A}$, where h is relay height. In the identification process the reference input is made zero so that system can generate sustained oscillatory output or limit cycle output as shown in the Figure 1.2.

In industrial processes offline identification method is used to determine the parameters of process model by deriving an analytical expression (chapter 3) using amplitude and time period from sustained limit cycle output.

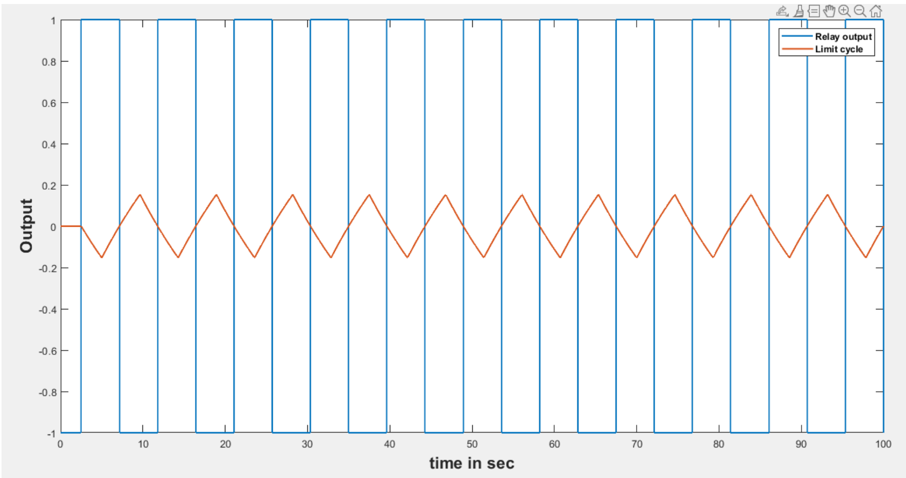

FIGURE 1.2: Sustained limit cycle output

1.3.2 Online Identification Method

This section describes online identification of system parameters using describing function (DF) approach based on closed loop test. The online identification is an advance approach of offline identification where, it is used to autotune the PID controller and estimate the process model simultaneously. A PID controller Gc(s) is connected in parallel with the relay in loop.The relay sees the controller and process model G(s) working in parallel with each other. The process gets stabilised from inner feedback loop of controller and relay. Online identification is a simultaneous process of parameter identification of process model and autotuning of PID controller. First, the relay is connected with PID controller and we can determine the parameter of process model from the relay and process loop. Now, we can disconnect the relay and tune the PID controller according to process model. The representational structure of online identification method is shown in the Figure 1.3.

Connecting controller in parallel with relay has advantages such as it helps in stabilising the sustained oscillatory output in the presence of noise by making asymmetrical output symmetrical. If output is asymmetrical we can't estimate the amplitude and time period of limit cycle output correctly which can lead to significant error in the parameter estimation.

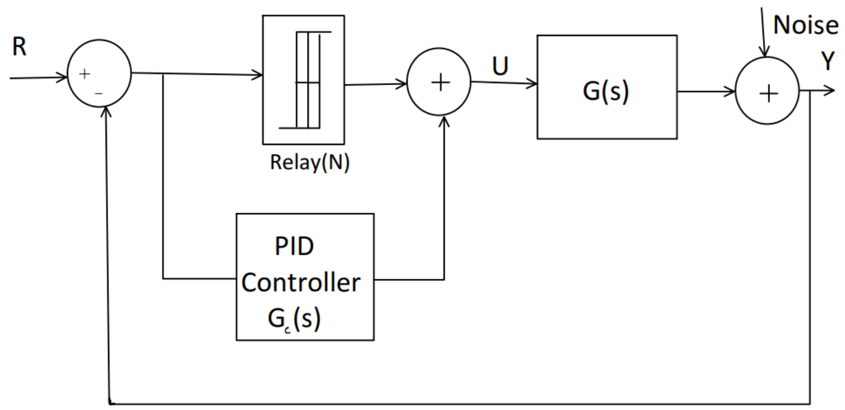

FIGURE 1.3: Online autotune identification scheme

1.4 Outline of this Thesis

The report is organised as follows.

1.4.1 Chapter 2

In this chapter literature survey is presented. This chapter includes various methods for identification of stable and unstable process model. The results of different methods have been compared after simulation. It can be concluded from result that parameter estimation of process model is still not very accurate.

1.4.2 Chapter 3

This chapter deals with the process identification of FOPDT, overdamped SOPDT and underdamped SOPDT process models. It includes expressions for parameter estimation using offline identification technique.

1.4.3 Chapter 4

This chapter outlines a machine learning approach to improve the accuracy of parameter estimation. It includes input and output data generation. Machine learning algorithm to train and test data set is also stated.

1.4.4 Chapter 5

In this chapter results and discussions are presented. Results are shown as R2 score, prediction error plot and residual plot. Few simulation examples are used to predict system parameters. From these results it is evident that we can analyse parameter of process model effectively using machine learning technique.

1.4.5 Chapter 6

Conclusions and some additional remarks are included in this chapter. Future direction of this works is also suggested.

Chapter 2

Literature Survey

IN this chapter, various methods are compared for parameter estimation of process model.

Original ATV method proposed by Luyben [3] involves one autotune test to determine ultimate frequency (ω) and amplitude (A) on the limit cycle to determine parameters of transfer function model. But, the effectiveness of this method is limited to a certain range of parameters.

Researchers devised many approaches to improve the accuracy and to autotune controllers utilizing relay-based identification.Bajrangbali et al [1] proposed a method to identify process model parameters using hysteresis relay in the presence of measurement noise. Kumar and Padhy [4] described analytical expressions to determine measurement sensitivity, which is the variation in relative error between the time constant (T) and the time delay (D) with regard to the limit cycle amplitude (A) and frequency (ω). Luyben extended this method to determine the transfer function of non-linear and complicated processes [3]. Sharma et al. [5] used neural networks to identify parameters of stable FOPDT (first order plus dead time) transfer function. Gerov estimated parameters using Lamber W function [6]. Majhi and Padhy presented a method to determine the system dynamics of first-order stable and unstable transfer functions [7]. Li et al. [12] used two relay tests to identify stable and unstable process model. But, use of two relay increases time and complexity. Chang et al. [11] provided a method to determine first order, second-order, and higher-order transfer functions using z-transform. Lavanya et al.[13] proposed a method to estimate SOPDT process model parameters, but this method is not very accurate. Shen et al. [15] employed dual input describing function approach to calculate two points on Nyquist curve to estimate stable process model parameter. Lee et. al [16] used describing function approach using hysteresis relay to estimate stable FOPDT process model parameter. Vivek and Chidambaram [17] suggested an Laplace transform approach to calculate stable process model parameters using symmetrical relay. Majhi and Atherton [9] used state space technique to determine various process model parameters. These algorithms are accuarte but requires computation of complex non-linear equations.Fedele [18] proposed FOPDT model for this process control system using step input method. Even though method suggested by Fedele has less estimation error but, online identification process is not considered.

2.1 Simulation studies of various process models

2.1.1 Analysis of FOPDT process model

We have done simulation for FOPDT process model to determine ultimate time period and amplitude. These parameters are further used to calculate system parameters.

Let the process transfer function is ,

$$G(s) = \frac{1e^{-2s}}{10s + 1} \tag{2.1}$$

Simulation diagram of FOPDT process model is shown in the Figure 2.1. Various methods are compared for estimating FOPDT process model parameters shown in Table 2.1.

From Table 2.1 we can estimate that Bajrangabali et al.[1] determines the parameters more accurately than any other method for FOPDT transfer function model.

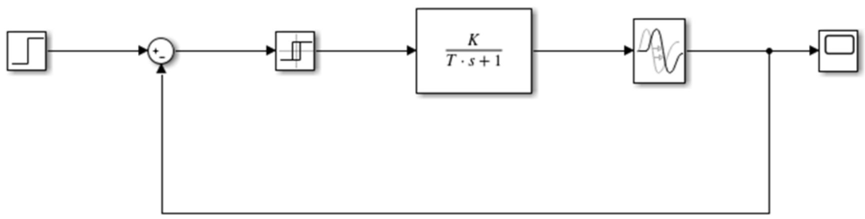

FIGURE 2.1: Simulation of FOPDT transfer function model

Method	Process Model	% Error in T	% Error in D
Offline	$\frac{e^{-1.99s}}{8.121s+1}$	18.79	0.5
Li. et al.[12]	$\frac{e^{-1.99s}}{8.02s+1}$	19.8	0
Kumar et al.(offline)[4]	$\frac{e^{-2.021s}}{8.179s+1}$	18.27	1.08
Kumar et al.(online)[4]	$\frac{e^{-1.995s}}{9.79s+1}$	2.09	0.48
Bajrngbali et al.[1]	$\frac{0.9994e^{-2s}}{9.9957s+1}$	0.043	0

Table 2.1: Comparison of FOPDT process mdoel parameters

2.1.2 Analysis of Overdamped SOPDT process model

We have done simulation for SOPDT process model to determine ultimate time period and amplitude. These parameters are further used to calculate system parameters.

Let the stable SOPDT process transfer function is ,

$$G(s) = \frac{1e^{-2s}}{(10s + 1)(s + 1)} \tag{2.2}$$

Simulation diagram of SOPDT process model is shown in the Figure 2.2. Various methods are compared for estimating SOPDT process model parameters shown in Table 2.2.

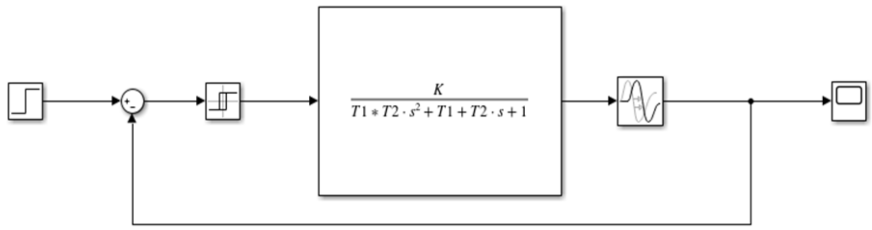

FIGURE 2.2: Simulation of SOPDT transfer function model

Method	Process Model	% Error in T_1	% Error in T_2
Offline	$\frac{e^{-2s}}{(9.87s+1)(0.9907s+1)}$	1.3	0.93
Li. et al.[12]	$\frac{e^{-2s}}{(7.416s+1)(1.15s+1)}$	19.8	0
Shen et al.[15]	$\frac{0.998e^{-2s}}{(9.14s+1)(1.044s+1)}$	8.6	4.4
Bajrngbali et al.[1]	$\frac{0.9923e^{-2s}}{(9.8997s+1)(1.0105s+1)}$	1.003	1.05

TABLE 2.2: Comparison of overdamped SOPDT process mdoel parameters

From Table 2.2 we can analyse that Bajaranbali et al. [1] is estimating process model parameters more accurately. This method depicts the use of hysteresis relay and state space estimation. But, state state estimation makes this method more computational and time consuming.

2.1.3 Analysis of Underdamped SOPDT process model

Similerly, we have simulated for stable underdamped SOPDT process model to determine ultimate time period and amplitude. These parameters are further used to calculate system parameters.

Now consider a underdamped SOPDT process model,

$$G(s) = \frac{1e^{-s}}{9s^2 + 2.4s + 1} \tag{2.3}$$

Simulation diagram of underdamped SOPDT process model is shown in the Figure 2.3. Various methods are compared for estimating SOPDT process model parameters shown in Table 2.3.

Offline identification is used to calculate the model transfer function. Analytical expressions are explained in chapter 3.

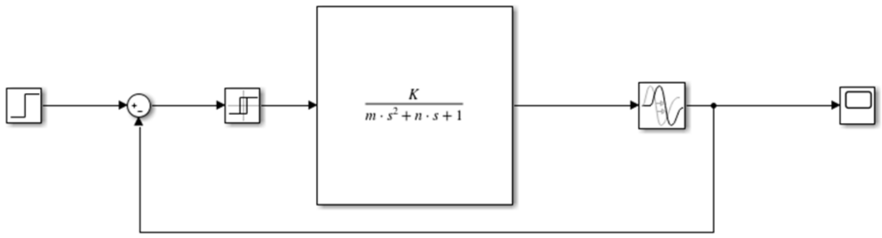

FIGURE 2.3: Simulation of underdamped SOPDT transfer function model

Method	Process Model	% Error
Offline	$\frac{1e^{-s}}{9.09s^2+2.285s+1}$	1.28
Chen et al.[19]	$\frac{1e^{-3.35s}}{(3.96s+1)}$	29.48
Bajrngbali et al.[1]	$\frac{1e^{-s}}{9.099s^2+2.289s+1}$	1.24

TABLE 2.3: Comparison of underdamped SOPDT process mdoel parameters

For this example, various methods are compared for stable underdamped process model.From Table 2.3 we can conclude that Bajrangbai et al.[1] presents accurate method where, absolute integral error is 0.0124, which is very less as compared to any other method.

Chapter 3

Process Identification

THIS chapter outlines the expressions and analysis of different process model parameters.

3.1 Introduction

From above chapters we can analyse that relay feedback based identification is less computational and time efficient method to determine process model parameters.But, the major limitation of this method is its inaccuracy of determining time constant (T) and time delay (D) for higher time constant to time delay ratio.

Although the relay-based identification technique is routinely used to derive transfer function parameters, the ATV methodology can offer a far more exact estimate of system characteristics. This is because the predicted ultimate gain N and anticipated ultimate frequency of the describing function are essentially approximations of information at critical frequency. There is a large variation between real and calculated parameters, especially for greater time constant to time delay ratio. We utilise a machine-learning technique to train varied data and identify the value of parameters using the same model to determine actual parameters and reduce error values. Therefore, with the help of machine learning techniques we can estimate the parameters accurately after training them with actual parameters.

3.2 Analytical Expressions for Identification of FOPDT Process Model parameter

Typical non minimum phase stable transfer function with dead time is assumed as process model. Autotune identification scheme is shown in the Figure 3.1 where, G(s) is the process transfer function model.

Transfer function:

$$\mathrm{G(s)} = \frac{\mathrm{Ke^{-Ds}}}{1 + \mathrm{Ts}} \tag{3.1}$$

Let relay height be denoted by h and hysteresis ϵ. When relay height, $\mathrm{h} \neq 0$, $\epsilon = 0$ (conventional relay), $\omega = 2\pi/\mathrm{pu}$

$$\mathrm{NG(j\omega)} = -1 \tag{3.2}$$

Where, N is the equivalent gain of the relay.

$$\mathrm{N} = \frac{4\mathrm{h}}{\pi \mathrm{A}} \tag{3.3}$$

From equation (3.1), (3.2) and (3.3)

$$\frac{4\mathrm{h}}{\pi \mathrm{A}} \frac{\mathrm{Ke^{-j\omega}}}{1 + \mathrm{j\omega T}} \tag{3.4}$$

From above we can time constant T and Time delay D as,

$$\mathrm{T} = \frac{\sqrt{(\frac{4\mathrm{hK}}{\pi\mathrm{A}})^2 - 1}}{\omega} \tag{3.5}$$

$$\mathrm{D} = \frac{\pi - \arctan(\omega\mathrm{T})}{\omega} \tag{3.6}$$

FOPDT system consists time constant (T), time delay (D) and gain (K) as unknown parameters. ATV method consists following steps:

i) The steady state gain (K) is estimated from steady state analysis of the system.
ii) Ultimate gain (N) and ultimate frequency (ω) is calculated from relay feedback Simulink analysis as shown in the Figure 2.1.
iii) First order transfer function is fitted to data to determine unknown time constant and time delay as given in (3.5) and (3.6).

3.3 Analytical Expressions for Identification of Over-damped SOPDT Process Model parameter

The transfer function model of a second order plus dead time process model is given by

$$\mathrm{G(s)} = \frac{\mathrm{Ke}^{-\mathrm{Ds}}}{(1+\mathrm{T}_1\mathrm{s})(1+\mathrm{T}_2\mathrm{s})} \tag{3.7}$$

Which in frequency domain becomes

$$\mathrm{G}(\mathrm{j}\omega) = \frac{\mathrm{Ke}^{-\mathrm{j}\omega\mathrm{D}}}{(1+\mathrm{j}\omega\mathrm{T}_1)(1+\mathrm{j}\omega\mathrm{T}_2)} \tag{3.8}$$

Where, T_1 and T_2 are the process time constants. We have to estimate T_1 and T_2 from offline identification. Substituting (3.3) and (3.8) in the condition (3.2) for offline identification method. We get,

$$\frac{4hKe^{-Dj\omega}}{\pi A(1+T_1j\omega)(1+T_2j\omega)} = -1 \tag{3.9}$$

Taking magnitude of the above equation,

$$\frac{4hK}{\pi A\sqrt{(1+(\omega T_1)^2)(1+(\omega T_2))^2}} = -1 \tag{3.10}$$

Further solving (3.10) we get,

$$T_1 + T_2 = \sqrt{\frac{1}{\omega^2}((\frac{4hK}{\pi A})^2 - 1) + 2T_1T_2 - (\omega T_1T_2)^2} \tag{3.11}$$

Taking phase of the (3.9)

$$\arctan(\omega T_1) + \arctan(\omega T_2) = \pi - \omega D \tag{3.12}$$

Using (4.5) and (4.6) and further solving we get the expression for stable SOPDT process

$$T_1 = \frac{2hK\sin(D\omega)}{\pi A\omega} + \frac{1}{\omega}\sqrt{(\frac{2hK\sin(D\omega)}{\pi A})^2 - \frac{4hK\cos(D\omega)}{\pi A} - 1} \tag{3.13}$$

$$T_2 = \frac{2hK\sin(D\omega)}{\pi A\omega} - \frac{1}{\omega}\sqrt{(\frac{2hK\sin(D\omega)}{\pi A})^2 - \frac{4hK\cos(D\omega)}{\pi A} - 1} \tag{3.14}$$

Hence, we can estimate process model parameter T_1 and T_2 by solving (3.13) and (3.14) for stable processes.

Overdamped SOPDT system consists time constants T_1 and T_2, time delay (D) and gain (K) as unknown parameters. ATV method consists following steps:

i) The steady state gain (K) is estimated from steady state analysis of the system.

ii) To estimate the process delay time (D), Majhi[14] proposed a method which involves estimation of t_0 and t_1 , where t_0 is the time instant at which relay switching takes place and t_1 is the time where discontinuity occurs for the first time in second derivative output of limit cycle. In our method we have used ideal relay so, relay switching takes place at $t_0 = 0$ always.

iii) First order transfer function is fitted to data to determine unknown time constant and time delay as given in (3.13) and (3.14).

3.4 Analytical Expressions for Identification of Underdamped SOPDT Process Model parameter

The transfer function model of a under damped second order plus dead time process model is given by

$$G(s) = \frac{Ke^{-Ds}}{ms^2 + ns + 1} \tag{3.15}$$

which in frequency domain becomes

$$G(j\omega) = \frac{Ke^{-j\omega D}}{m\omega^2 + j\omega n + 1} \tag{3.16}$$

Where $m = T_1\ T_2$ and $n = T_1 + T_2$ with the condition $n^2 < 4m$ for underdamped process. Rewriting (3.16) in the following form

$$G(j\omega) = \frac{Ke^{-j\omega D}\alpha_1\beta_1}{(j\omega - \alpha_1)(j\omega - \beta_1)} \tag{3.17}$$

Where,

$$\alpha_1 = \frac{-2}{n + j\sqrt{4m - n^2}} \text{and} \beta_1 = \frac{-2}{n - j\sqrt{4m - n^2}} \tag{3.18}$$

We have to estimate m and n from offline identification. Substituting (3.3) and (3.16) in the condition (3.2) for offline identification method. We get,

$$\frac{4hKe^{-j\omega D}\alpha_1\beta_1}{\pi A(j\omega - \alpha_1)(j\omega - \beta_1)} = -1 \tag{3.19}$$

Equating magnitude of (3.19) we get,

$$\alpha_1\beta_1 = \omega\pi\mathrm{A}\sqrt{\frac{\omega^2+\alpha_1^2+\beta_1^2}{4\mathrm{hK}^2-\pi\mathrm{A}^2}} \tag{3.20}$$

Equating phase of (3.19) we get,

$$\alpha_1+\beta_1 = (\frac{\alpha_1\beta_1-\omega^2}{\omega})\tan(\pi+\omega\mathrm{D}) \tag{3.21}$$

After solving (3.20) and (3.21) we get the following expressions,

$$\mathrm{m} = \frac{1}{\omega^2}[1+\cos(\omega\mathrm{D})\frac{4\mathrm{hK}}{\pi\mathrm{A}}] \tag{3.22}$$

$$\mathrm{n} = \sin(\omega\mathrm{D})\frac{4\mathrm{hK}}{\pi\mathrm{A}\omega} \tag{3.23}$$

Hence, the parameters m and n are calculated from (3.22) and (3.23), respectively for online identification method.

Underdamped SOPDT system consists time coefficient m and n, time delay (D) and gain (K) as unknown parameters. ATV method consists following steps:
i) The steady state gain (K) is estimated from steady state analysis of the system.
ii) To estimate the process delay time (D), Majhi[14] proposed a method which involves estimation of t_0 and t_1 , where t_0 is the time instant at which relay switching takes place and t_1 is the time where discontinuity occurs for the first time in second derivative output of limit cycle. In our method we have used ideal relay so, relay switching takes place at $t_0 = 0$ always.
iii) First order transfer function is fitted to data to determine unknown time constant and time delay as given in (3.22) and (3.23).

Chapter 4

Machine Learning to improve accuracy of estimating transfer function parameters

THIS chapter outlines the architecture of systolic array and how we use it to benefit the speeds of matrix computation in the overall design.

4.1 Introduction

In order to improve accuracy, effort is made to determine the true value of ultimate gain and frequency. But due to non-linear behavior of relay it is very difficult to analyse ultimate gain and frequency accurately. Chang et al. [11] derived a method to analyse the close value of ultimate gain and frequency using z-transform method but this method involves calculation of complex nonlinear equations for higher order systems. The method is more time consuming and solution may not converge to a specific value sometimes. Therefore, we use the analytical expressions described in chapter 3 to determine various parameters. In addition, we determine the number of parameters for various models in order to train the machine learning model mentioned in the following section 4.3.

4.2 Input and output data analysis

Input data is generated using python random generator with wide variety of system parameters, such as gain ranging from 1-15, time delay ranging from 1-12 with resolution of 1 and time constants ranging from 1-20 with resolution of 0.5. The input data is sorted in a random order.

4.2.1 Input and output data analysis for FOPDT process model without noise

For the FOPDT transfer function model, Machine learning model is trained as output actual T and actual D using relay height, gain, amplitude, time period, estimated T (3.5) and estimated D (3.6) as input as given in Table 3.1 to properly predict system parameters T and D. To acquire the highest accuracy for the test data, we have explored a variety of models in which decision tree regressor model for predicting T and linear regressor model for predicting D fit well.

S.No.	Relay height	Gain	Amplitude	Time period	Estimated T	Estimated D
0	1	1	0.8540	5.248	0.9237	1.9260
1	1	1	0.7362	5.656	0.1.2703	1.9680
2	1	1	0.6316	5.960	1.6604	1.9823
3	1	1	0.5484	6.194	2.0666	1.9872
4	1	1	0.4866	6.379	2.4550	1.9928
...	...	...	...	...	...	....
580	1	9	3.227	27.038	14.663	7.9877
581	1	9	3.16	27.133	15.053	7.9895
582	1	9	3.089	27.225	15.479	7.9887
583	1	9	3.027	27.313	15.873	7.9901
584	1	9	2.966	27.396	16.274	7.9909

Table 4.1: Input data for training FOPDT transfer function model

4.2.2 Input and output data analysis for FOPDT process model in the presence of measurement noise

Regardless of reliable sensors, measurement noise caused by control valves, measuring equipment or the process itself corrupts the process output in the real world. The ultimate time period and amplitude of the output signal are difficult to evaluate due to noisy output, resulting in erroneous process identification. As a result, it's critical to get rid of these shortcomings.

In order to demonstrate the influence of noise in simulations, a white Gaussian noise with a fixed mean and variance is injected into the output signal, resulting in noisy oscillations. The process dynamics parameter is calculated in the presence of measurement noise with an SNR of 20dB to demonstrate the efficacy of the suggested ML model. To create this noise, the simulation employs a white

Gaussian noise with a mean of zero and a variance of 1.3526×10^{-4} as shown in Figure 4.1. The filtered signal shown in Figure 4.2 is approximated using a band pass filter.

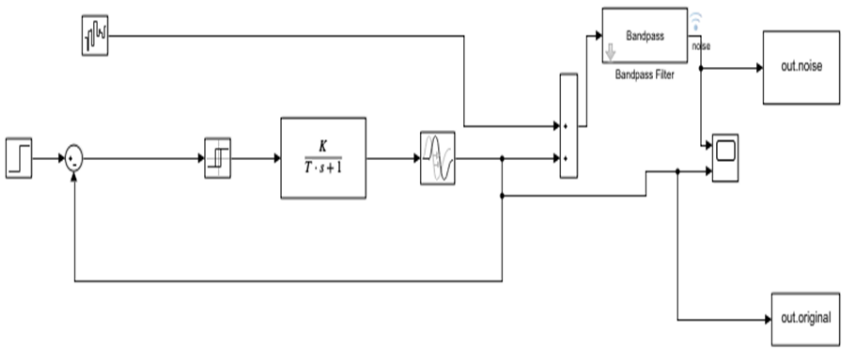

FIGURE 4.1: Simulation of FOPDT transfer function model in the presence of measurement noise

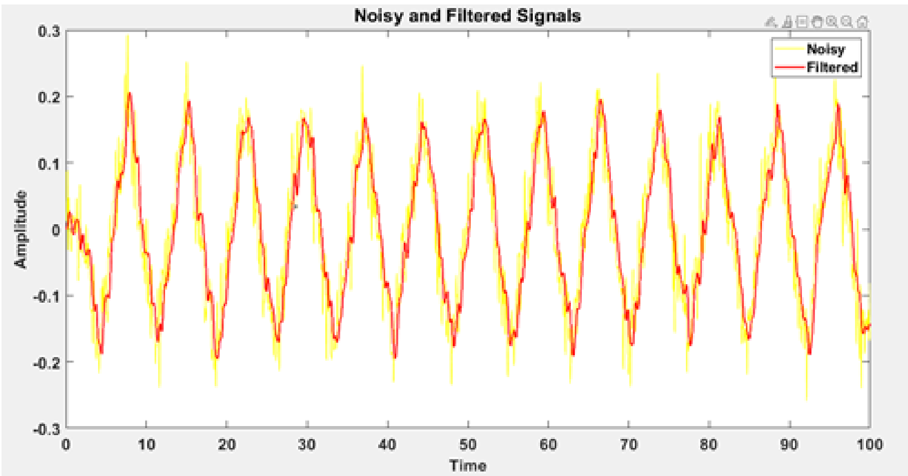

FIGURE 4.2: Noisy (yellow) and denoised (red) output signal of FOPDT transfer function model

Input data for FOPDT in the presence of measurement noise is given in Table 4.2. In which decision tree regressor model for predicting T and linear regressor model for predicting D fit well.

S.No.	Relay height	Gain	Amplitude	Time period	Estimated T(N)	Estimated D(N)
0	1	1	0.8540	5.248	0.9042	1.9350
1	1	1	0.7298	5.656	1.2870	1.9534
2	1	1	0.6310	5.960	1.6625	1.9818
3	1	1	0.5460	6.194	2.0769	1.9854
4	1	1	0.4788	6.379	2.5018	1.9961
...	...	...	...	...	...	....
580	1	9	3.1930	27.038	14.833	7.9744
581	1	9	3.1230	27.133	15.246	7.9750
582	1	9	3.0620	27.225	15.627	7.9781
583	1	9	3.0050	27.313	15.997	7.9815
584	1	9	2.9300	27.396	16.488	7.9768

TABLE 4.2: Input data for training FOPDT transfer function model in the presence of disturbance

4.2.3 Input and output data analysis for Overdamped SOPDT process model

For the overdamped SOPDT process model, unknown parameters are T_1 and T_2 as outputs and relay height, gain, time delay, amplitude, time period, estimated T_1 (3.13), estimated T_2 (3.14), as inputs (see Table 4.3) to train the model. To acquire the highest accuracy for the test data, we explored a variety of models in which decision tree regressor models fits well with the test data.

S.No.	Relay height	Gain	Time Delay	Amplitude	Time period	Estimated T_1	Estimated T_2
0	1	1	2	0.5490	8.209	2.161	0.803
1	1	1	2	0.4484	8.877	3.009	0.955
2	1	1	2	0.3723	9.319	3.978	0.9701
3	1	1	2	0.3148	9.605	5.016	10.9547
4	1	1	2	0.2729	9.90	6.039	0.9805
...	...	...	...	...	...	...	...
1430	1	10	4	2.6120	25.030	11.306	5.076
1431	1	3	7	1.7030	26.350	7.542	1.818
1432	1	6	6	3.619	23.668	5.481	2.4683
1433	1	11	5	5.3710	20.851	6.724	1.9113
1434	1	8	4	1.987	26.32	11.458	6.070

TABLE 4.3: Input data for training overdamped SOPDT transfer function model

4.2.4 Input and output data analysis for underdamped SOPDT process model

For the underdamped SOPDT process model, unknown parameters are m and n as outputs and relay height, gain, time delay, amplitude, time period, estimated m (4.22) and estimated n (4.23) as inputs (see Table 4.4) to train the model. To obtain the maximum accuracy for the test data, we explored a number of models, finding that the decision tree regressor model matches the data well..

S.No.	Relay height	Gain	Time Delay	Amplitude	Time period	Estimated m	Estimated n
0	1	8	4	9.385	8.209	8.8909	2.9958
1	1	3	2	1.383	8.877	7.1633	4.4513
2	1	8	2	3.448	9.319	6.2828	4.6488
3	1	6	5	7.421	9.605	9.4989	3.2182
4	1	5	8	16.840	9.90	10.307	1.1136
...	...	...	...	...	...	...	...
727	1	1	3	2.152	25.030	0.9786	0.5508
728	1	1	3	2.746	26.350	1.4860	0.5508
729	1	1	3	2.800	23.668	0.9956	0.3649
730	1	1	3	2.536	20.851	0.9869	0.4252
731	1	1	1	0.759	26.32	7.627	1.5900

TABLE 4.4: Input data for training underdamped SOPDT transfer function model

4.3 Methodology and Algorithms

4.3.1 Methodology

We have employed various machine learning regressor models to train and test input data shown in Table 4.1-4.4. For estimation of time constants, the Decision Tree regressor model predicts the output more precisely, whereas the Linear regressor model predicts the output more accurately for the estimation of time delay. Decision tree is a simple and most commonly used regressor model. It works like a tree structured classifier where initial node is root node which represents the entire sample which divides further in two sub nodes. Interior nodes represent data set, branches represent decision rules and finally leaf nodes represent the outcome. For a particular data, it runs through complete tree to the leaf node. Final value is the average of the particular data present in the leaf node. After numerous iterations, tree is able to predict the projected data. Linear regression model predicts the linear relationship between input data and output data. The model seeks to predict output value in such a way that the error difference between the predicted and real value is as small as possible by reaching the best-fit regression line.

4.3.2 Algorithms

Algorithm to estimate time constants and time coefficients:

Defining output and input:

y1 = data['Time Constant']

y2 = data['Delay']

X = data

Splitting train and test data in 80:20 ratios:

from sklearn.model_selection import train_test_split

X_train, X_test, y_train, y_test = train_test_split(X, y1, test_size = 0.2, random$_s$tate = 1)

Importing Regressor Model:

from sklearn.tree import DecisionTreeRegressor

regressor1 = DecisionTreeRegressor(random$_s$tate = 2)

regressor1.fit(X_train, y_train)

Predicted output:

y_pred = regressor1.predict(X_test)

Importing r2 score:

from sklearn.metrics import r2_score

r2_score(y_test, y_pred)

Algorithm to estimate time delay:

Defining output and input:

y1 = data['Time Constant']

y2 = data['Delay']

X = data

Splitting train and test data in 80:20 ratios:

from sklearn.model_selection import train_test_split

X_train, X_test, y_train, y_test = train_test_split(X, y2, test_size = 0.2, random_state = 1)

Importing Regressor Model:

from sklearn.linear$_{\mathrm{m}}$odelimportLinearRegression

regressor3 = LinearRegression(random$_{\mathrm{s}}$tate = 2)

regressor3.fit(X_train, y_train)

Predicted output:

y_pred = regressor3.predict(X_test)

Importing r2 score:

from sklearn.metrics import r2_score

r2_score(y_test, y_pred)

Chapter 5

Results and Discussions

5.1 R-Squared (R2) score

R-squared is a statistical measure that represents the goodness of fit of a regression model. The ideal value for r-square is 1. The closer the value of r-square to 1, the better is the model fitted. R-square is a comparison of residual sum of squares (SSres) with total sum of squares (SStot). Total sum of squares is calculated by summation of squares of perpendicular distance between data points and the average line. Residual sum of squares in calculated by the summation of squares of perpendicular distance between data points and the best fitted line.

$$\mathrm{R}^2 = 1 - \frac{\mathrm{SS}_{\mathrm{res}}}{\mathrm{SS}_{\mathrm{total}}} \tag{5.1}$$

R-squared score of models are shown in table 5.1.

Models	Parameters	R2 score
FOPDT	Time constant(T)	0.991
FOPDT	Time delay(D)	0.997
FOPDT(Noise)	Time constant(T)	0.941
FOPDT(Noise)	Time delay(D)	0.966
SOPDT(Overdamped)	Time constant(T_1)	0.949
SOPDT(Overdamped)	Time constant(T_2)	0.949
SOPDT(Uerdamped)	Time coefficient(m)	0.993
SOPDT(Uerdamped)	Time coefficient(m)	0.998

Table 5.1: R2 score of different models

From above table we can see that machine learning models have given high R2 score for the test data prediction. Which concludes that these models can give accurate result for random input data.

5.2 Prediction error plot

The actual data from the data set is compared to the anticipated values generated by our model in a prediction error plot. This enables us to see how much variation the model has. By comparing regression models to the 45-degree line, where the forecast completely fits the model, we may diagnose regression models. Prediction plots for various process models are given below.

Algorithm:

print(prediction_error(regressor1, X_train, y_train, X_test, y_test))

For FOPDT process model, prediction error for time constant (T) is shown in Figure 5.1.

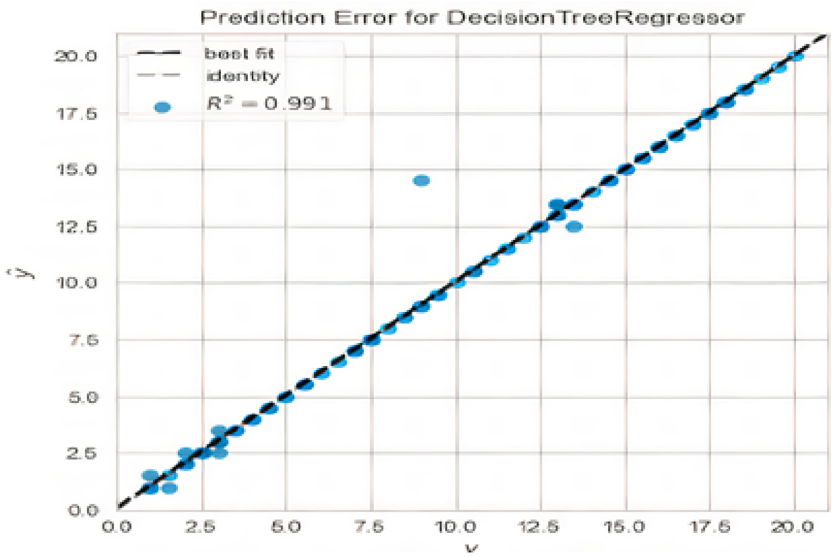

FIGURE 5.1: Prediction error plot between y and y_pred

For FOPDT process model, prediction error for time delay (D) is shown in Figure 5.2.

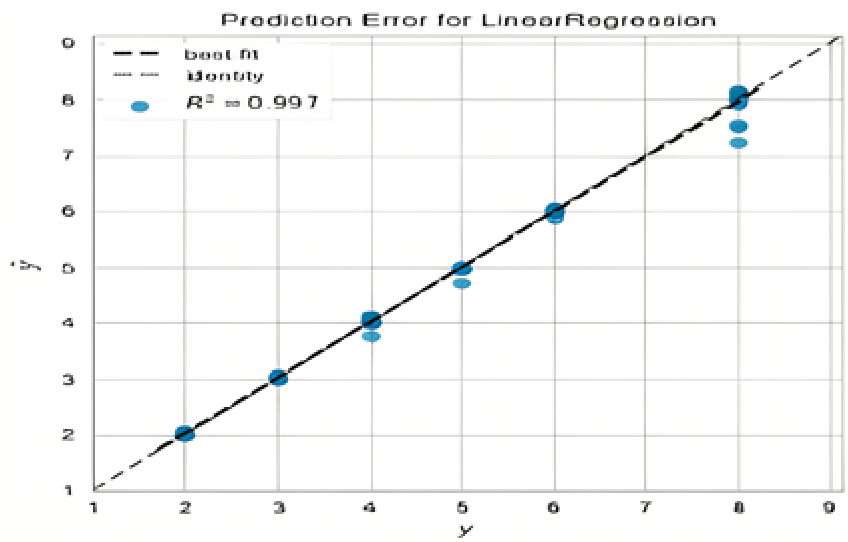

FIGURE 5.2: Prediction error plot between y and y_pred

For FOPDT process model with measurement noise, prediction error for time constant (T) is shown in Figure 5.3.

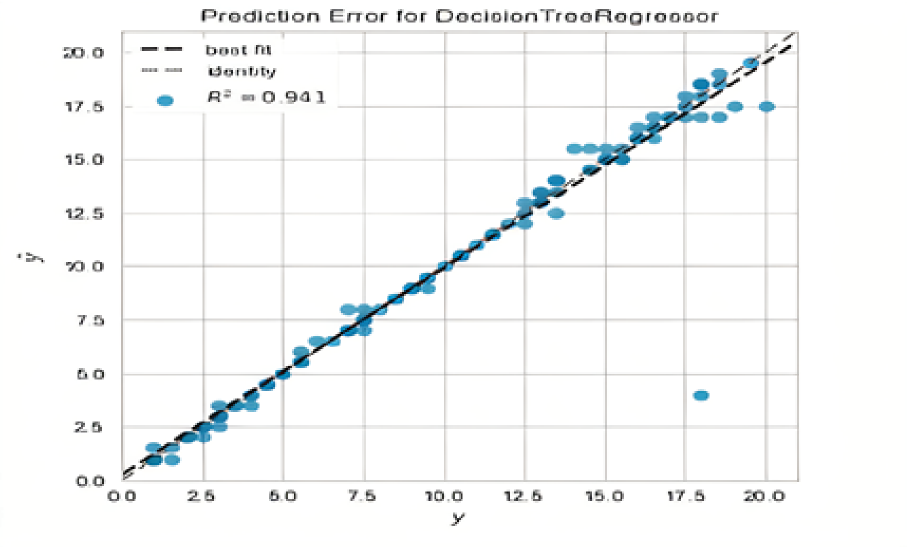

FIGURE 5.3: Prediction error plot between y and y_pred

For FOPDT process model with measurement noise, prediction error for time delay (D) is shown in Figure 5.4.

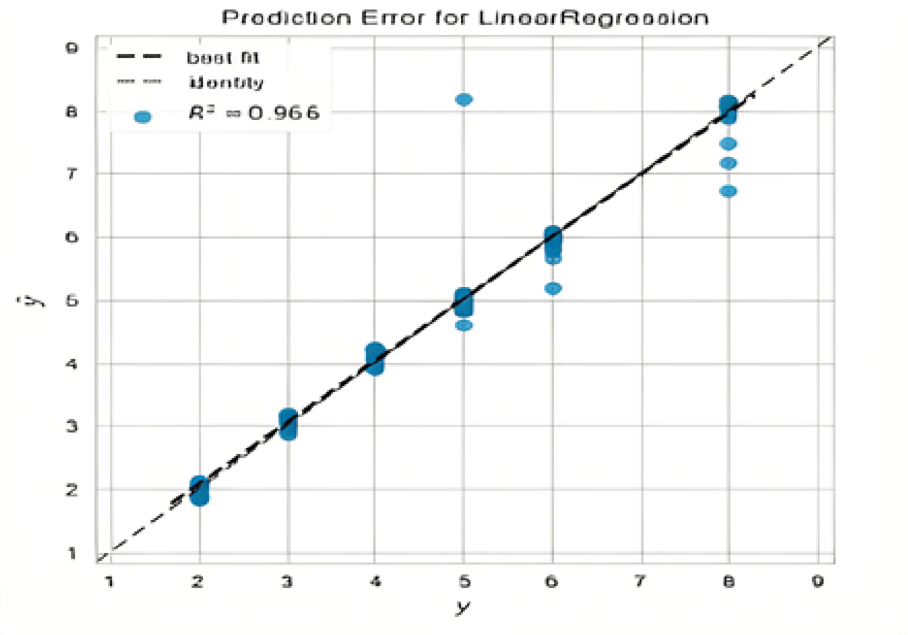

FIGURE 5.4: Prediction error plot between y and y_pred

For overdamped SOPDT process mode, prediction error for time constant (T_1) is shown in Figure 5.5.

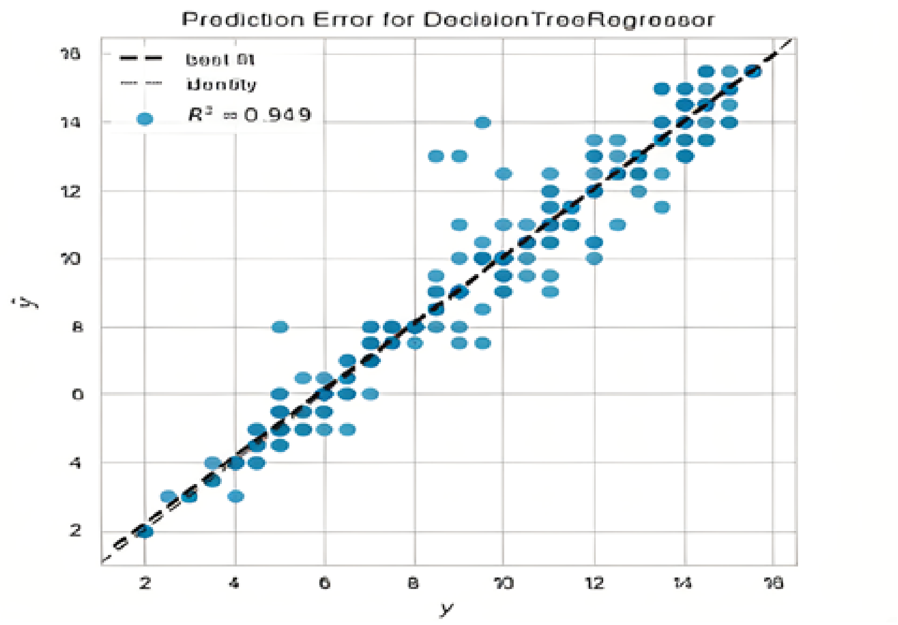

FIGURE 5.5: Prediction error plot between y and y_pred

For overdamped SOPDT process mode, prediction error for time constant (T_2) is shown in Figure 5.6.

FIGURE 5.6: Prediction error plot between y and y_pred

For underdamped SOPDT process mode, prediction error for time coefficient (m) is shown in Figure 5.7.

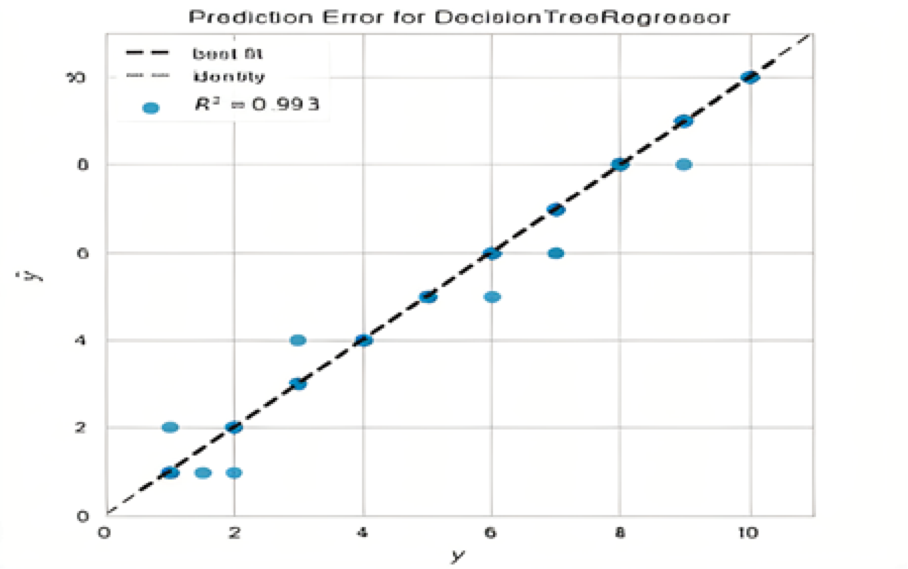

FIGURE 5.7: Prediction error plot between y and y_pred

For underdamped SOPDT process mode, prediction error for time coefficient (n) is shown in Figure 5.8.

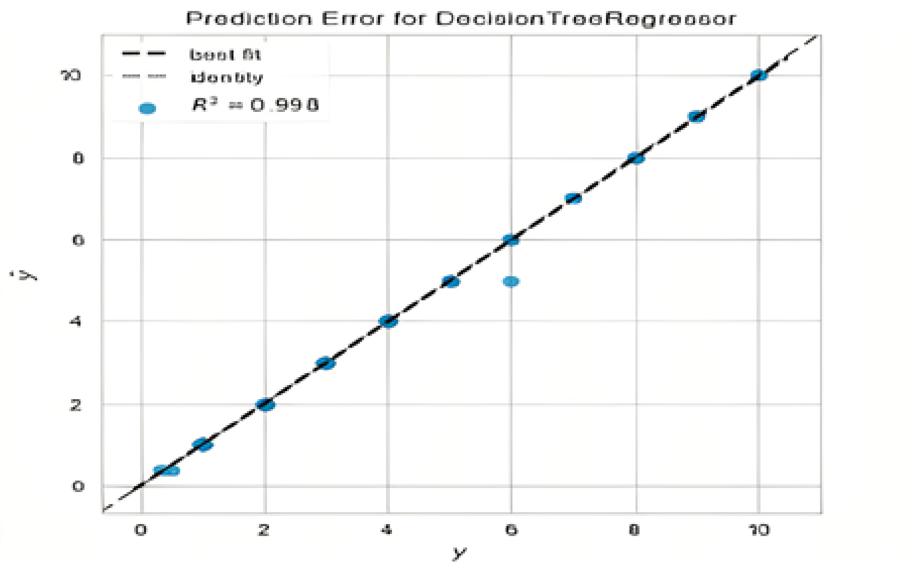

FIGURE 5.8: Prediction error plot between y and y_pred

5.3 Residual Plot

The disparities between actual and anticipated data values are known as residuals in a statistical or machine learning model. They're a diagnostic tool for evaluating the quality of a model. Errors are another name for them. When determining the quality of a model, residuals are crucial. You may look at residuals to see how big they are and if they create a pattern. The model predicts exactly when all of the residuals are zero. The model becomes less accurate as the residuals get further away from zero. In linear regression, the smaller the R-squared statistic is the bigger the sum of squared residuals, everything else being equal.

Algorithm:

print(Residuals Plot)

print(residuals_plot(regressor1, X_train, y_train, X_test, y_test))

For FOPDT process model, residual plot for time constant (T) is shown in Figure 5.9.

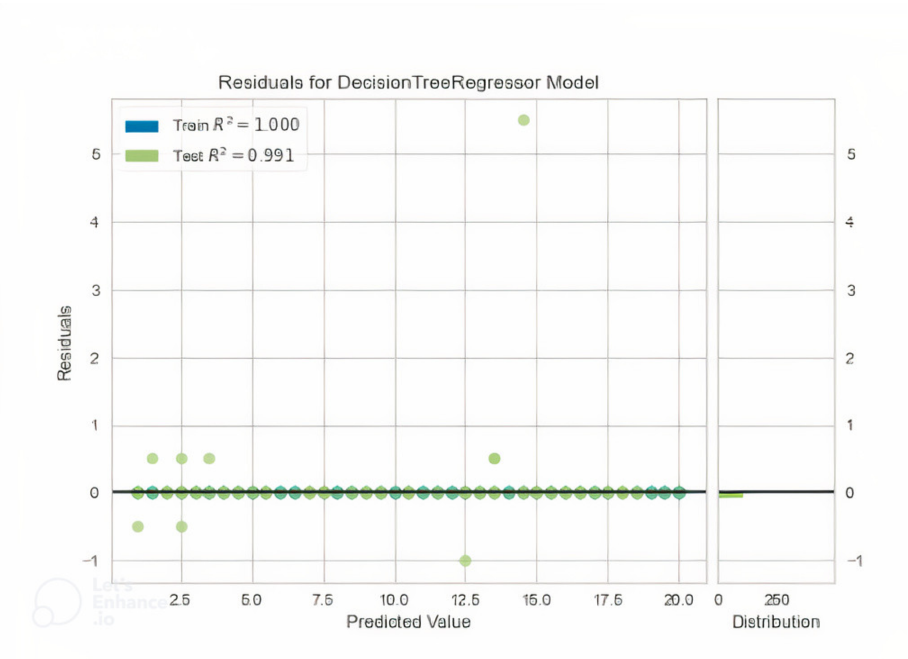

FIGURE 5.9: Residual plot between y and y_pred

For FOPDT process model, residual plot for time delay (D) is shown in Figure 5.10.

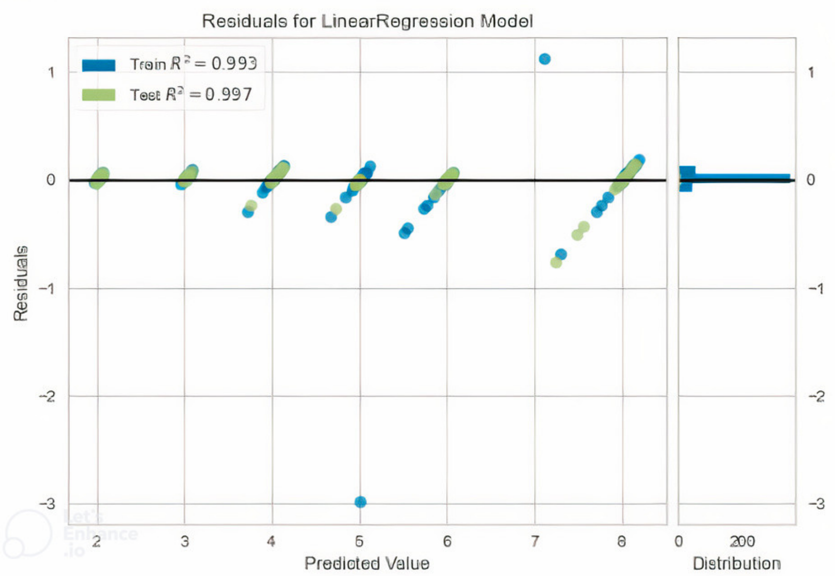

FIGURE 5.10: Residual plot between y and y_pred

For FOPDT process model with measurement noise, residual plot for time constant (T) is shown in Figure 5.11.

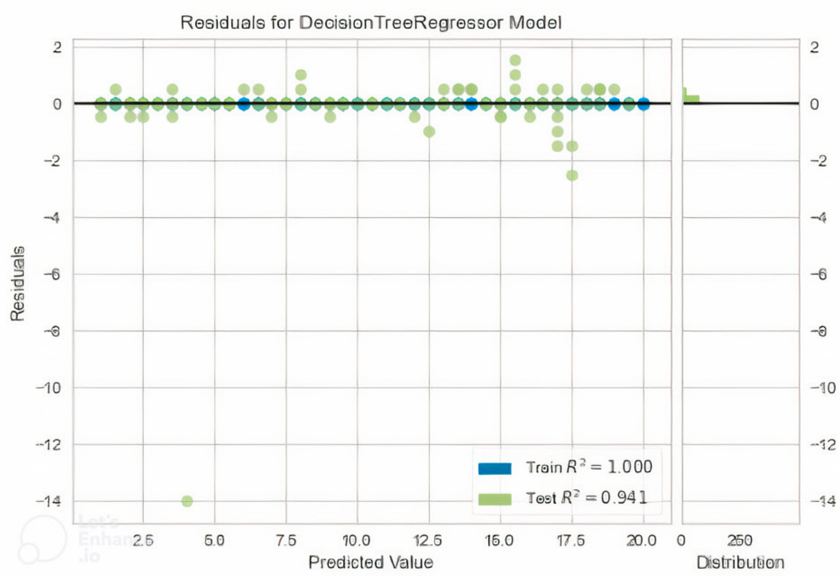

FIGURE 5.11: Residual plot between y and y_pred

For FOPDT process model with measurement noise, residual plot for time delay (D) is shown in Figure 5.12.

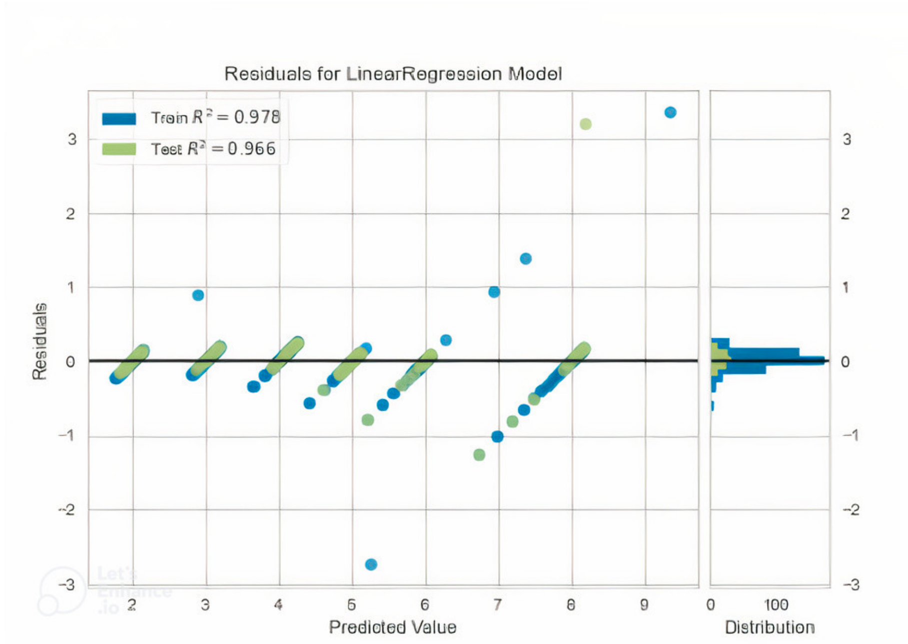

FIGURE 5.12: Residual plot between y and y_pred

For overdamped SOPDT process mode, residual plot for time constant (T_1) is shown in Figure 5.13.

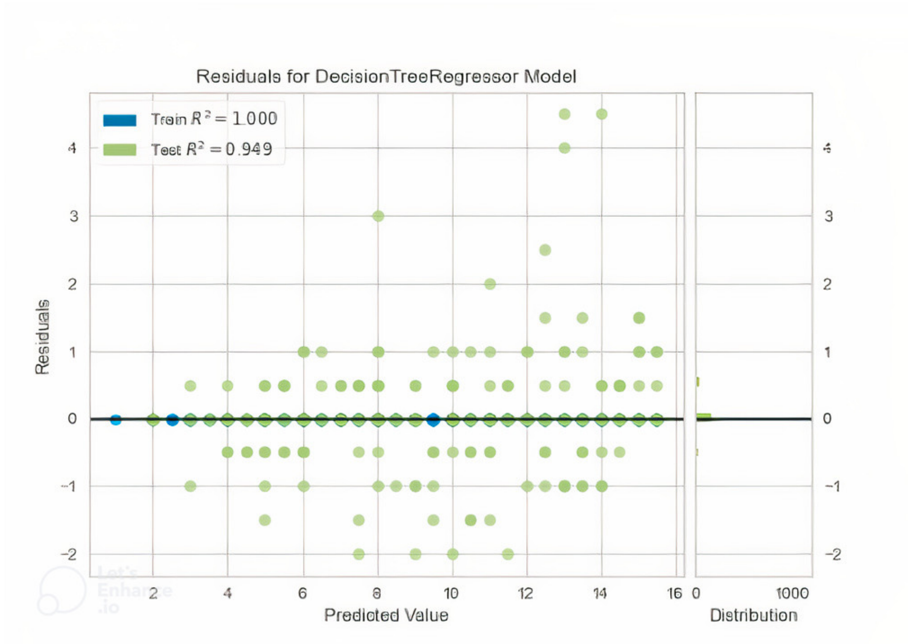

FIGURE 5.13: Residual plot between y and y_pred

For overdamped SOPDT process mode, residual plot for time constant (T_2) is shown in Figure 5.14.

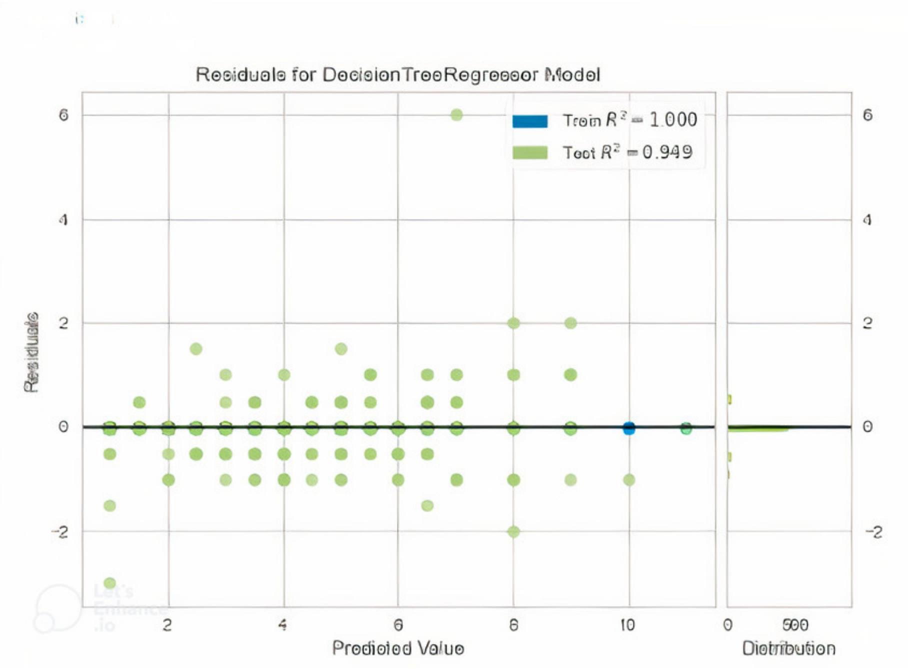

FIGURE 5.14: Residual plot between y and y_pred

For underdamped SOPDT process mode, residual plot for time coefficient (m) is shown in Figure 5.15.

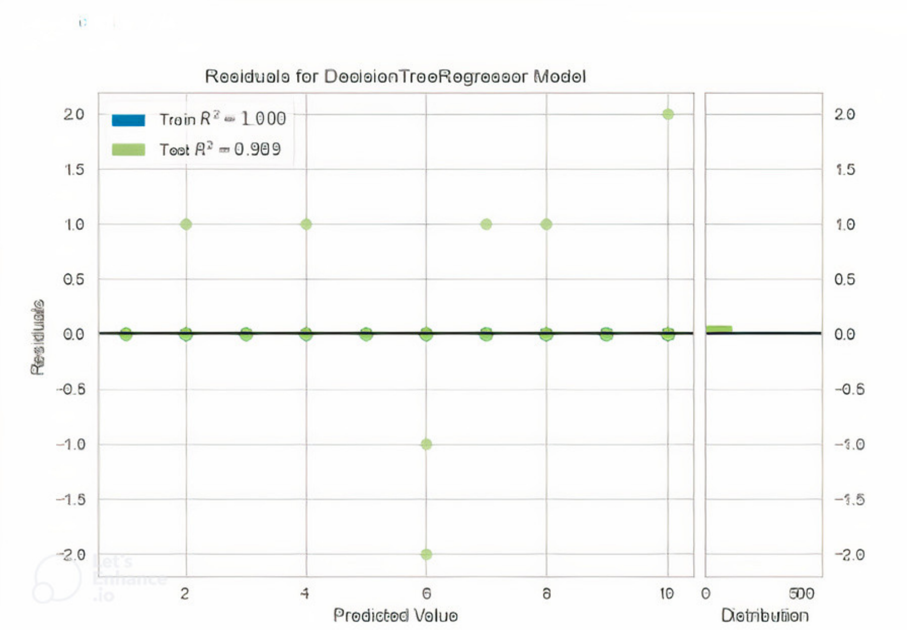

FIGURE 5.15: Residual plot between y and y_pred

For underdamped SOPDT process mode, residual plot for time coefficient (n) is shown in Figure 5.16.

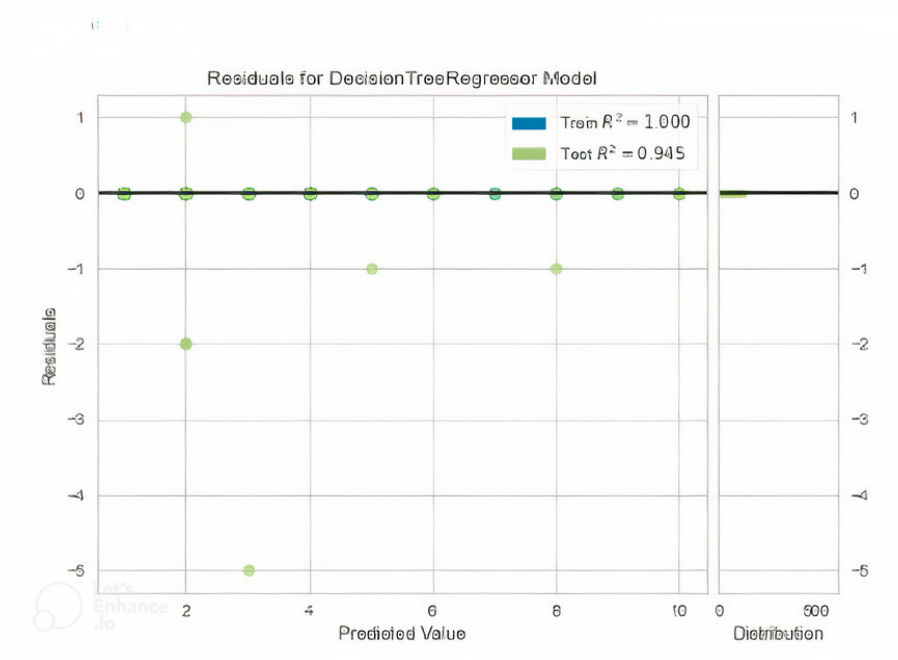

FIGURE 5.16: Residual plot between y and y_pred

Above prediction plots show that predicted data fits well with the actual data in test data analysis. From this we can analyse that prediction of parameter can be done accurately using these models.

5.4 Simulation Examples

5.4.1 Example 1

Let the process transfer function is ,

$$G(s) = \frac{1.65e^{-10s}}{20s + 1} \tag{5.2}$$

For offline identification of the FOPDT process model, a relay feedback test is performed. To create limit cycle output with the parameter A = 0.6491 and time period $p_u = 33.272$, use relay height, h = 1. From the steady-state analysis, the steady-state gain K = 1.65 is calculated. The remaining process model parameters such as time constant, T = 16.301, and time delay, D = 9.981 are calculated after

Method	Process Model	% Error in T	% Error in D
Offline	$\frac{1.65e^{-9.981s}}{16.301s+1}$	18.49	0.19
Chang et al.[11]	$\frac{1.65e^{-10s}}{19.97s+1}$	0.15	0
Proposed method	$\frac{1.65e^{-9.92s}}{20s+1}$	20	0.8

Table 5.2: Comparison of FOPDT process mdoel parameters of Example 1

solving (3.5) and (3.6). We get T = 20 and D = 9.92 correspondingly after feeding these parameters into the machine learning model outlined in chapter 4.

Chang et al. [11] found an accurate method, however it requires a significant number of complex nonlinear equation computations. Higher-order process models are more complicated to calculate, and they may not necessarily converge to a specific solution. In comparison, our recommended method yields an accurate output with lower processing time.

5.4.2 Example 2

Let the process transfer function is ,

$$G(s) = \frac{8.1e^{-5.2s}}{9.4s + 1} \tag{5.3}$$

For offline identification of the FOPDT process model, a relay feedback test is performed. To create limit cycle output with the parameter A = 3.441 and time period $p_u = 17.057$, use relay height, h = 1. From the steady-state analysis, the

Method	Process Model	% Error in T	% Error in D
Offline	$\frac{8.1e^{-5.187s}}{7.67s+1}$	23.3	0.25
Proposed method	$\frac{8.1e^{-5.247s}}{9.5s+1}$	1.05	0.9

TABLE 5.3: Comparison of FOPDT process mdoel parameters of Example 2

steady-state gain K = 8.1 is calculated. The remaining process model parameters such as time constant, T = 7.67, and time delay, D = 5.187 are calculated after solving (3.5) and (3.6). We get T = 9.5 and D = 5.247 correspondingly after feeding these parameters into the machine learning model outlined in chapter 4.

From above table we can say that proposed method estimated the parameter more accurately in comparison to offline identification method.

5.4.3 Example 3

Let the SOPDT process transfer function is ,

$$G(s) = \frac{e^{-4s}}{(10s + 1)(2s + 1)} \tag{5.4}$$

During offline identification of the SOPDT process model, a relay feedback test is performed. To create limit cycle output with the parameter A = 0.3129 and time period p_u = 19.117, use relay height, h = 1. From the steady-state analysis, the steady-state gain K = 1 is calculated. The remaining process model parameters such as time constants, T_1 = 10.1189, and T_1 = 1.8578 are calculated after solving

Method	Process Model	% Error
Offline	$\frac{e^{-2s}}{(10.1198s+1)(1.8578s+1)}$	1.56
Fedele	$\frac{1.0015e^{-5.647s}}{(10.4495s+1)}$	0.0143
Proposed method	$\frac{1e^{-2s}}{(10s+1)(2s+1)}$	0

TABLE 5.4: Comparison of SOPDT process mdoel parameters of example 3

(4.13) and (4.14). Process control parameter, time delay is estimated from Majhi et al. [14].

We get T1 = 10 and T2 = 2 correspondingly after feeding these parameters into the machine learning model outlined in chapter 4.

Fedele [10] proposed FOPDT model for this process control system using step input method. Even though method suggested by Fedele has less estimation error but, proposed method estimates the parameter more accurately.

5.4.4 Example 4

Let the SOPDT process transfer function is ,

$$G(s) = \frac{1.5e^{-5s}}{(7.5s + 1)(4.2s + 1)} \tag{5.5}$$

During offline identification of the SOPDT process model, a relay feedback test is performed. To create limit cycle output with the parameter A = 0.6194 and time

Method	Process Model	% Error in T_1	% Error in T_2
Offline	$\frac{1.5e^{-5s}}{(7.8231s+1)(3.7532s+1)}$	4.3	9.92
Proposed method	$\frac{1.5e^{-5s}}{(7.5s+1)(4s+1)}$	0	4.76

Table 5.5: Comparison of SOPDT process mdoel parameters of example 4

period p_u = 24.674, use relay height, h = 1. From the steady-state analysis, the steady-state gain K = 1.5 is calculated. The remaining process model parameters such as time constants, $T_1 = 7.8231$, and $T_1 = 3.7532$ are calculated after solving (4.13) and (4.14). Process control parameter, time delay is estimated from Majhi et al. [14].

We get T1 = 7.5 and T2 = 4 correspondingly after feeding these parameters into the machine learning model outlined in chapter 4.Percentage error comparisons of parameters estimated with different methods are shown in the Table 5.5.

5.4.5 Example 5

Consider following stable underdamped SOPDT process model [13].

$$G(s) = \frac{1e^{-3s}}{s^2 + 0.4s + 1} \tag{5.6}$$

For identification of the underdamped SOPDT process model, a relay feedback test is performed. Limit cycle output is generated with the parameter A = 2.536 and time period $p_u = 7.861$ sec when subjected to a relay height h = 1. Steady state analysis is used to calculate gain K=1. The remaining process model parameters

Method	Process Model	% Error in m	% Error in n
Offline	$\frac{1e^{-3s}}{1.32s^2+0.68s+1}$	32.6	70
Lavanya et al.[13]	$\frac{1e^{-3s}}{0.9869s^2+0.4252s+1}$	2.45	1.1
Proposed method	$\frac{1e^{-3s}}{s^2+0.391s+1}$	0	2.25

TABLE 5.6: Comparison of underdamped SOPDT process mdoel parameters of example 5

such as time constants, m = 1.32, and n = 0.68 are calculated after solving (4.22) and (4.23). Percentage error comparisons of parameters estimated with different methods are shown in the Table 5.6.

From above comparison table we can analyse that proposed methods estimate parameters effectively for random process model.

5.4.6 Example 6

Consider following stable underdamped SOPDT process model .

$$G(s) = \frac{3.5e^{-7s}}{9.4s^2 + 3.7s + 1} \tag{5.7}$$

During offline identification of the underdamped SOPDT process model, a relay feedback test is performed. To create limit cycle output with the parameter A = 4.956 and time period $p_u = 23.076$, use relay height, h = 1. From the steady-state analysis, the steady-state gain K = 3.5 is calculated. The remaining process model parameters such as time constants, m = 8.625, and n = 3.801 are calculated after

Method	Process Model	% Error in m	% Error in n
Offline	$\frac{3.5e^{-7s}}{8.652s^2+3.801s+1}$	7.9	2.73
Proposed method	$\frac{3.5e^{-7s}}{9s^2+3.61s+1}$	4.25	2.43

TABLE 5.7: Comparison of underdamped SOPDT process mdoel parameters of example 6

solving (4.22) and (4.23). Process control parameter, time delay is estimated from Majhi et al. [14] method.

From Table 5.7, we can analyse that proposed method estimate parameters accurately for random process model.

Chapter 6

Conclusions and Future Scope

6.1 Conclusions

This thesis has carried out various identification method for stable processes based on relay feedback approach at sustained limit cycle output.We have concluded the major contribution of this thesis and then examined the many prospective additions in the following final remarks.

- Relay based identification method is widely used in process control. However, this method can lead to significant error if the system parameters are estimated from the approximate ultimate gain and ultimate frequency. The main motivation of this thesis leads to estimation of the process model parameter accurately.

- To estimate parameter of process model accurately, describing function approach is used for initial estimation.Large set of data is generated after simulation and offline identification of process model

- The various machine learning models are proposed to predict the parameters accurately in the face of measurement noise, process variation, etc. In which Decision tree regressor model and Linear regressor model fit well for the test data.

- The results suggest that using the proposed technique, the process model parameters can be estimated more accurately. If we train the model with additional robust data, the suggested method's accuracy can improve even significantly.

6.2 Future Scope

- In this thesis, parameter estimation is done only for stable FOPDT and SOPDT process model. This work can be further extended for parameter estimation of unstable and higher order process model.

- In this work, offline identification is used to calculate initial parameter, now to achieve further accuracy online identification can also be used.

- We have considered approximately 600-1600 data for parameter estimation, by increasing the range of data such as for larger time constant and time delay, machine learning model can predict process model parameter more accurately.

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