HW 1: Review Rigid Transformations, Kinematics, and Dynamics

Due Feb 3, 2019

We don't mind if you work with other students on your homework. However, each student must write up and turn in their own assignment (i.e. no copy & paste). If you worked with other students, please **acknowledge who you worked with** at the top of your homework.

1. **Adjoint Identities, MLS 2.14**
   Use homogeneous representations to prove that the following identities hold:

   (a) $(\mathrm{Ad}_g)^{-1} = \mathrm{Ad}_{g^{-1}}$ for all $g \in SE(3)$.

   (b) $\mathrm{Ad}_{g_1 g_2} = \mathrm{Ad}_{g_1}\mathrm{Ad}_{g_2}$ for all $g_1, g_2 \in SE(3)$.

2. **Proposition 2.15, MLS 2.15**
   Prove proposition 2.15: $V^b_{ac} = \mathrm{Ad}_{g^{-1}_{bc}} V^b_{ab} + V^b_{bc}$

3. **Kinematics and Jacobians**
   For each of the manipulators shown below:

   (a) Write a transformation matrix (G matrix) for each link.

   (b) Write G_{OE} from the origin to the end effector in terms of the matrices from part a.

   (c) Derive the Spatial and Body Jacobians in the current configuration. Assume that the elbow angle of the SCARA manipulator is 90°, and the angles of the pointer's joints are 30° and 60° respectively.

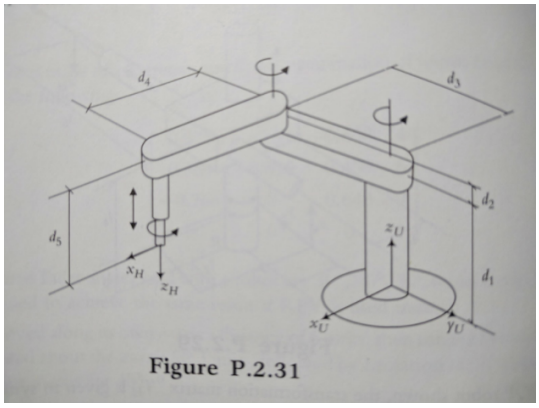

Figure 1: SCARA manipulator

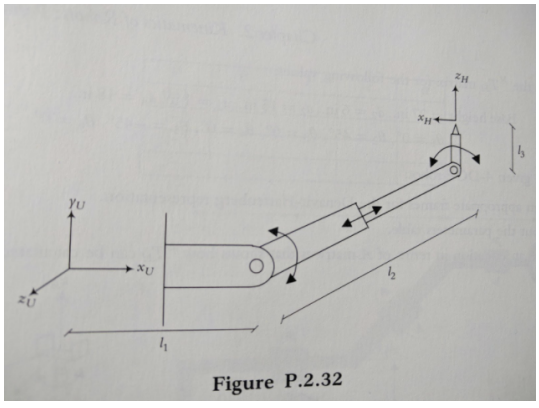

Figure 2: 2D Pointer

4. **Equations of Motion**
Derive the equations of motion for the systems below, using Lagrangian Mechanics.

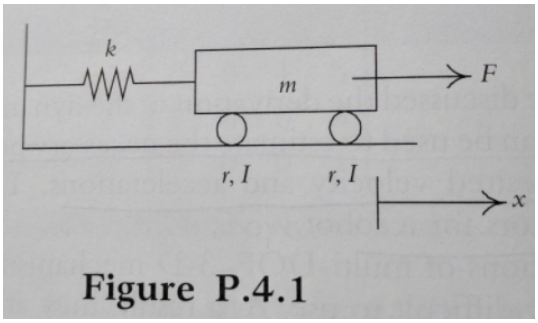

Figure 3: Mass-Spring with wheels

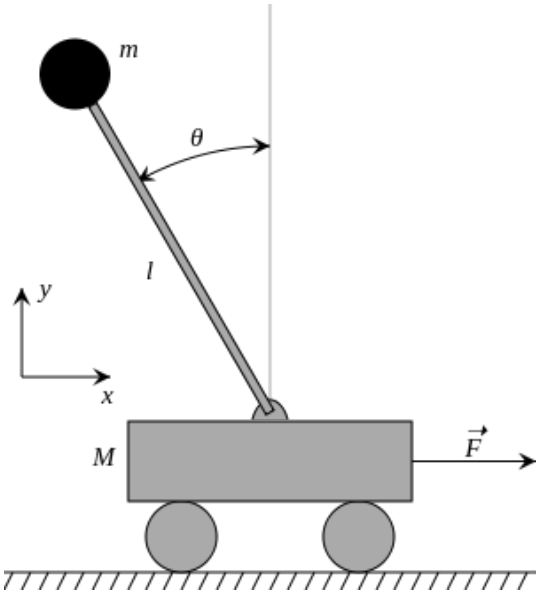

Figure 4: Inverted Pendulum

5. **Inverse Kinematics**
Read *Kinematic Control of Redundant Robot Manipulators*, by Bruno Siciliano, which is posted on BCourses, and answer the following questions:

   (a) Describe Task Space Augmentation in 2-3 paragraphs. Describe a case in which it would be useful, and describe what secondary task the algorithm would solve. Do not choose an example from the paper.

   (b) The pseudoinverse Jacobian-based gradient projection method is a very commonly used inverse kinematics algorithm. Using this algorithm, find the $\dot{q}$ that minimizes Yoshikawa's manipulability metric. Read Chapter 3, section 4.4 in the textbook for more information.

   (c) Sciavicco and Siciliano as well as Slotline and Yoerger independently proposed using the transpose of the Jacobian matrix rather than the pseudoinverse. Imagine that you're trying to find the inverse kinematics solution $\hat{q}$, where $FK(\hat{q}) = \hat{x}$. You seed your algorithm with some $q = q_0$, and update using the following equation:

$$\dot{q} = AJ^T(q)(\hat{x} - x) \tag{1}$$

   where A is a positive definite Hermitian matrix and $x = FK(q)$. Using the equation for Lyapunov stability of a linear function, prove that q converges to $\hat{q}$.