EECS 106B/206B

Discussion 12

GSI: Valmik Prabhu

Action Items

- Lab 2 due this weekend
- HW 4 released
- Potential office hour schedule change:
  - Valmik’s OH Monday 4-5, Weds 12-1
  - Chris’ OH Thursday 1:30-3:30
- Project Proposals due 3/22. More info forthcoming
  - Very broad. Must involve at least one of: sensing, actuation, or control
  - Must involve hardware, either IRL or in simulation (discouraged)
  - Must be research-based. You’ll need to find papers backing up your proposal

Holonomic Constraints

Defined:

$$h_i(q) = 0, \qquad i = 1, \ldots, k.$$

Jacobian

$$\frac{\partial h}{\partial q} = \begin{bmatrix} \frac{\partial h_1}{\partial q_1} & \cdots & \frac{\partial h_1}{\partial q_n} \\ & \ddots & \\ \frac{\partial h_k}{\partial q_1} & \cdots & \frac{\partial h_k}{\partial q_n} \end{bmatrix}$$

Rows Linearly Independent

They restrict motion to some subspace of Q

They shrink the state space of the system

h maps q to R

Holonomic constraints can always be removed by a coordinate parameterization

Pfaffian Constraints

A is some function of q

$$A(q) \cdot \delta q = 0$$

Pfaffian constraints restrict a *direction* of motion

They restrict the *action space*

Constraints are forces

Each constraint is a force normal to the hyperplane in Q

$$\Gamma = \frac{\partial h}{\partial q}^T \lambda,$$

Force

Coordinate basis
for each cons

Relative Magnitudes
between constraints
(you find this through
dynamics)

Examples:

- Cartesian
- Pendulum
- Welding Robot

Are Pfaffian constraints holonomic?

If they can be integrated, they're holonomic. There must exist h such that

$$A(q)\dot{q} = 0 \qquad \Longleftrightarrow \qquad \frac{\partial h}{\partial q}\dot{q} = 0.$$

Inserting a constraint into the dynamics

We replace h with A. Why?

$$\Gamma = A^T(q)\lambda,$$

Then put it into the Euler-Lagrange equation

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} + A^T(q)\lambda - \Upsilon = 0,$$

And find the Lagrange multipliers (constraint forces)

$$\lambda = (AM^{-1}A^T)^{-1}\left(AM^{-1}(F - C\dot{q} - N) + \dot{A}\dot{q}\right)$$

d'Alembert's Principle

Constraints do *no work*!

$$(A^T(q)\lambda) \cdot \delta q = 0$$

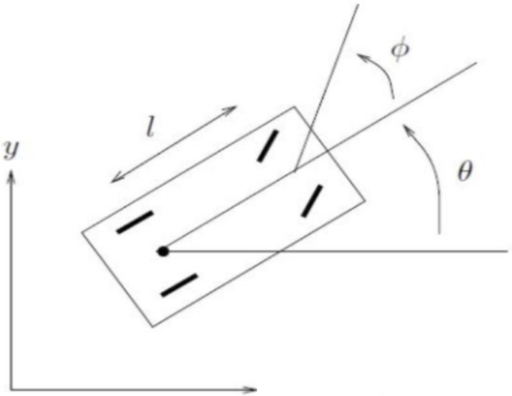
To test this, we find the nullspace of A, and project the equations of motion onto it

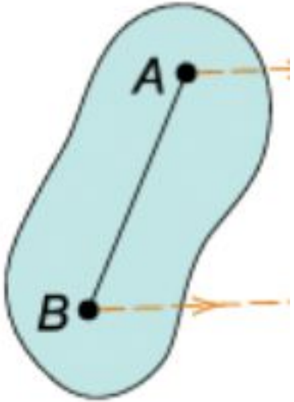
$$\left(\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} - \Upsilon\right) \cdot \delta q = 0,$$

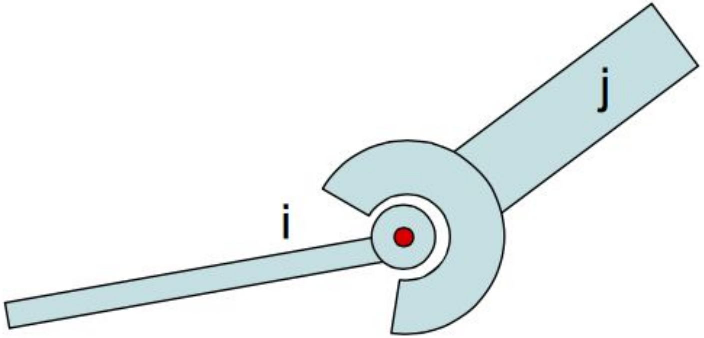
where $\delta q \in \mathbb{R}^n$ satisfies

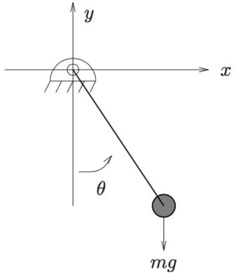
$$A(q)\delta q = 0.$$

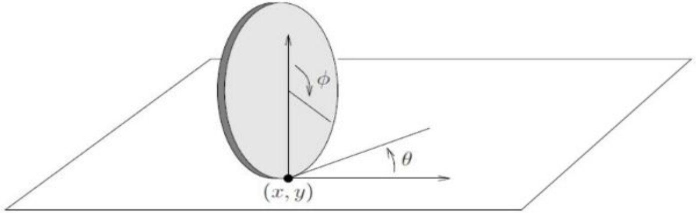
Holonomic or Nonholonomic?











Which is this?