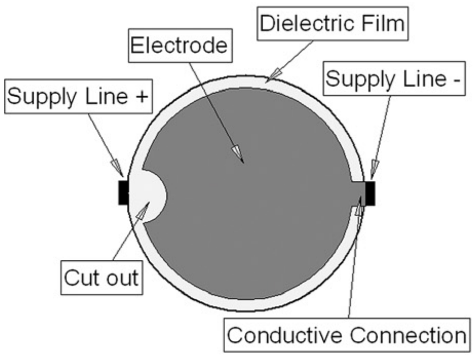
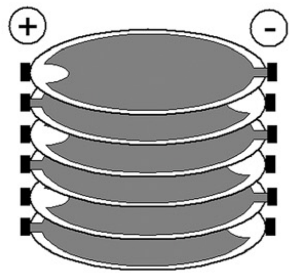
Stacked Dielectric Elastomer

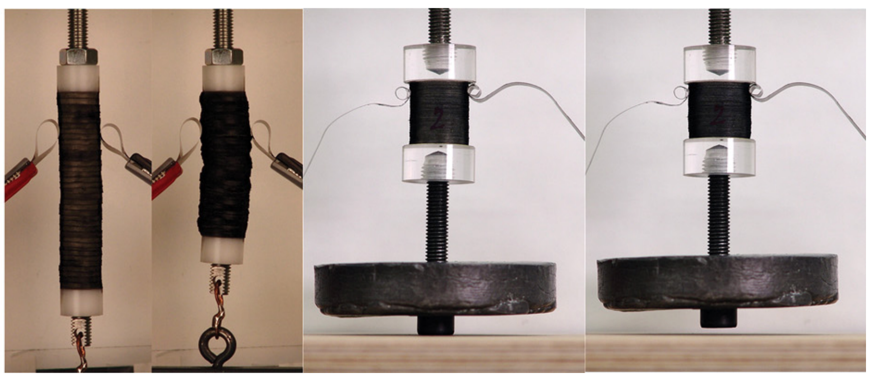
Stacked dielectric elastomer actuator for tensile force transmission

G. Kovacs*, L. Düring, S. Michel, G. Terrasi

Sensors and Actuators A 155 (2009) 299–307





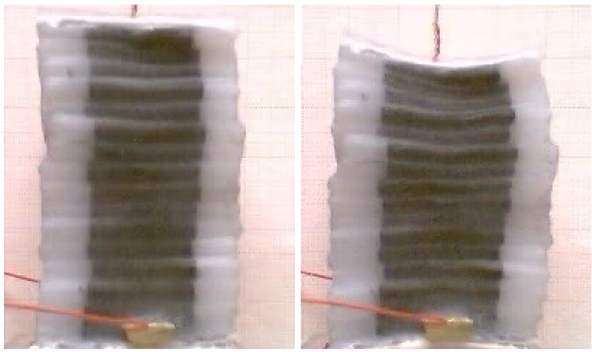


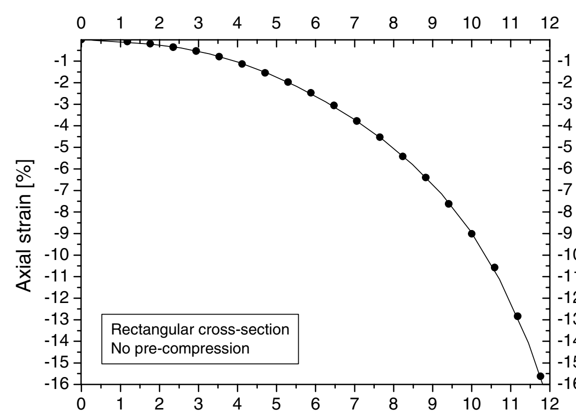
Dielectric: VHB 4910 Acrylic Elastomer infused w/ a penetrating network of 1,6-hexanediol diacrylate for enhanced dielectric breakdown strength
Conductor: Ketjenblack 600 (Akzo Nobel) Carbon Powder

Folded dielectric elastomer actuators

Federico Carpi, Claudio Salaris and Danilo De Rossi

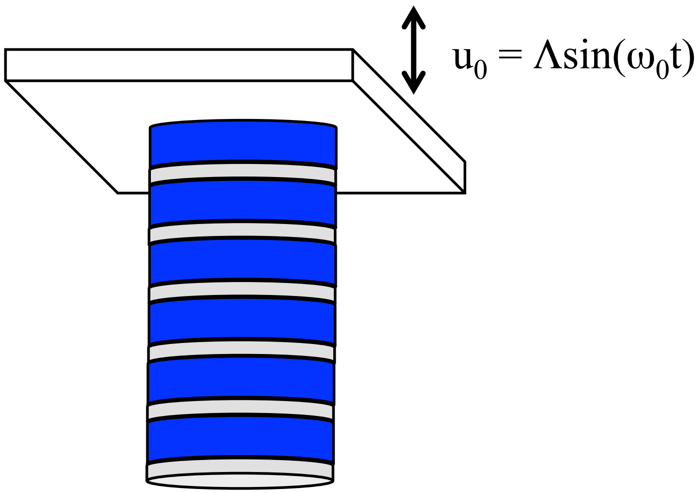
Smart Mater. Struct. **16** (2007) S300–S305

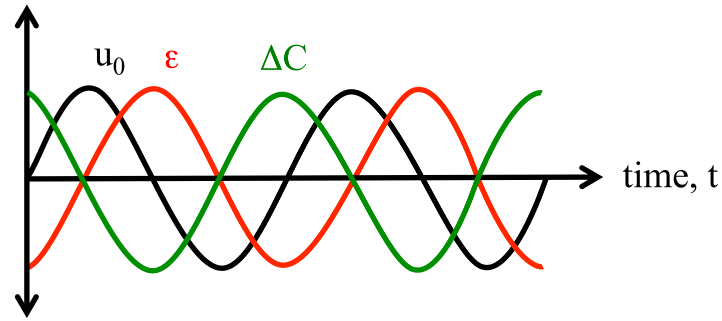


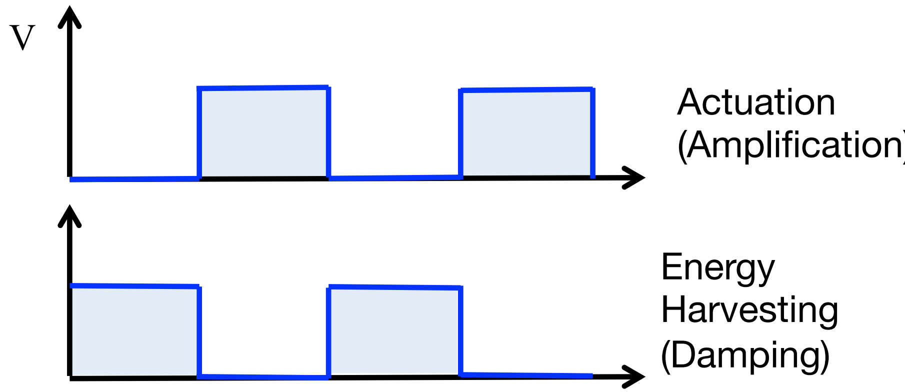


Dielectric: BJB TC-5005 Silicone/PDMS
Conductor: CAF 4 Rhodorsil Silicone + Vulcan XC R72 Carbon Powder

“Stacked” Soft-Matter Capacitor

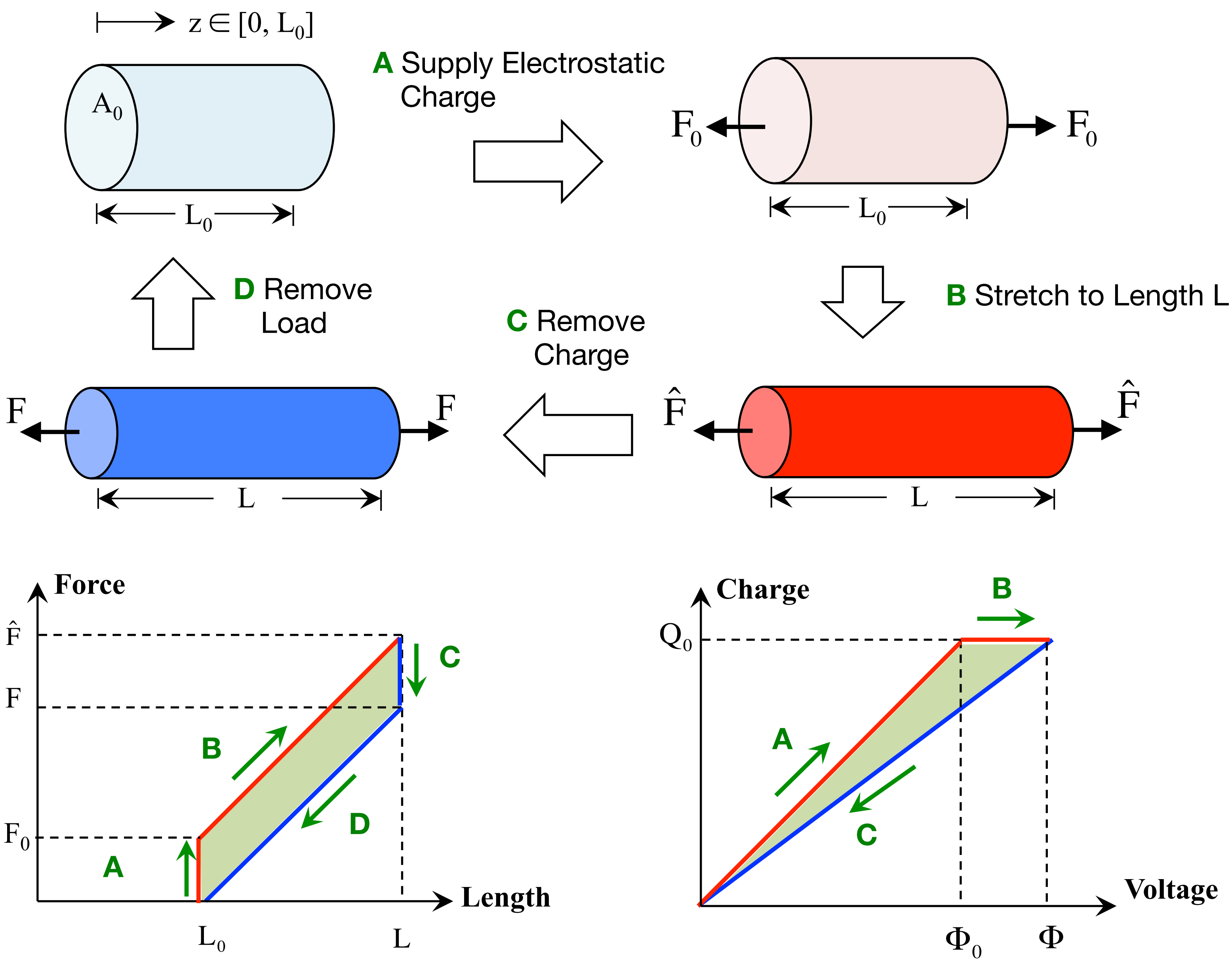






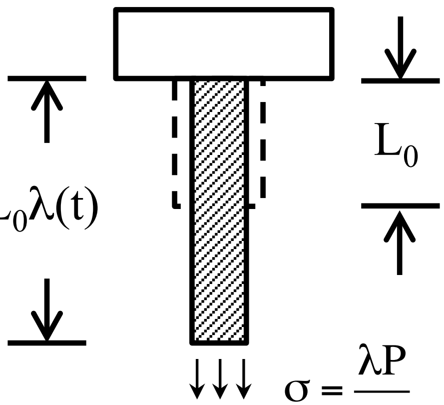
- Electromechanical coupling: $C/C_0 \sim \lambda^2$
- Harmonic excitation at base
- Stready state ΔC and λ controlled by Λ, $|\omega_n - \omega_0|$, rubber viscosity η, and electrostatic damping from *Maxwell Stress*
- Phase of electrostatic “loading” matters
  - w/ contraction $\Rightarrow$ amplification
  - w/ stretch $\Rightarrow$ damping

Energy Harvesting



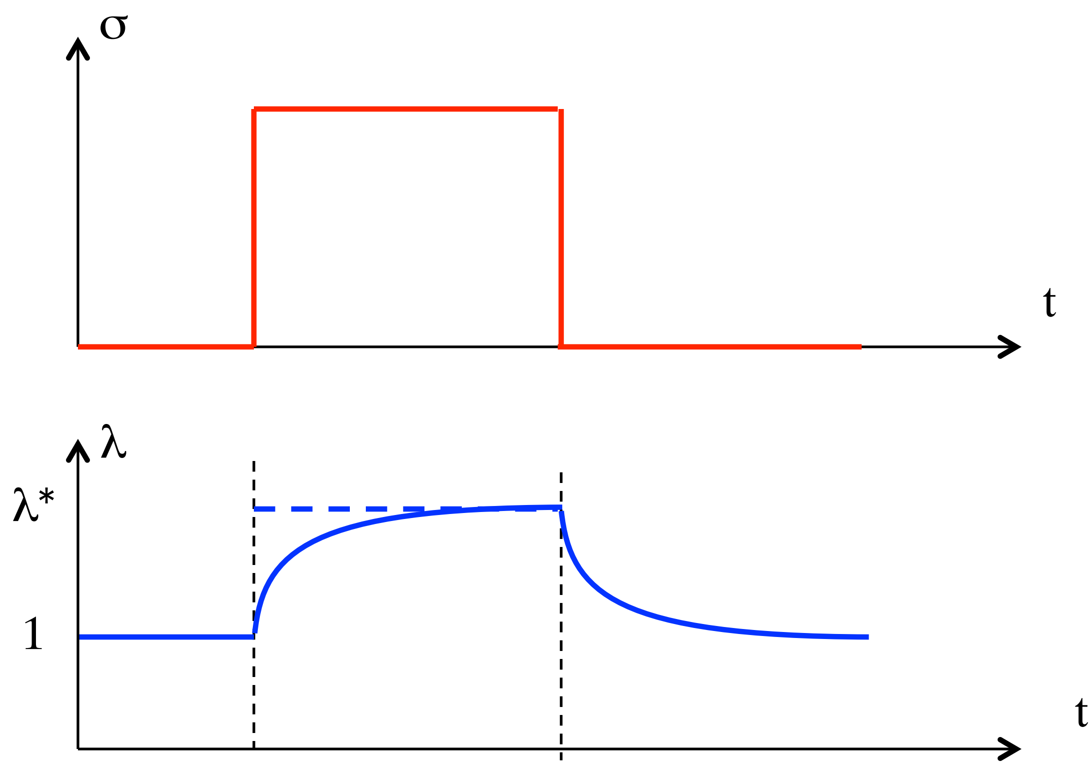
Viscoelasticity

When loaded with a dead weight P, most elastomers stretch over time:



$$\sigma = \frac{\lambda P}{A_0}$$

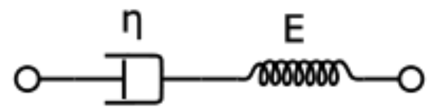
$$\lambda^* = \left\{ \lambda : \sigma = C_1\left(\lambda^2 - \frac{1}{\lambda}\right) \right\}$$

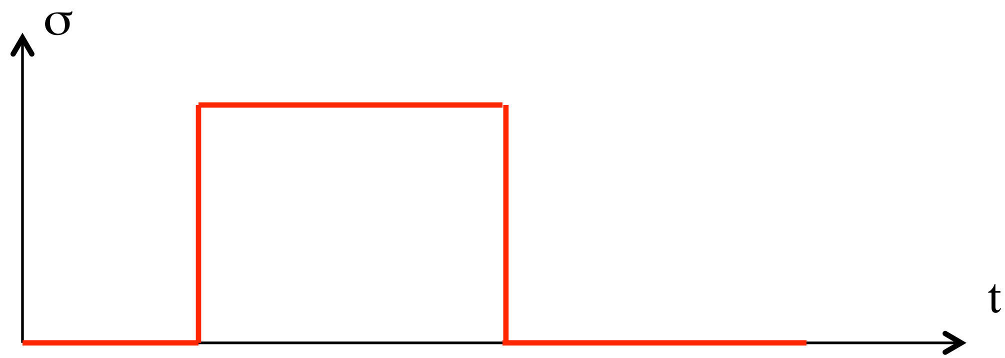


“Creep”

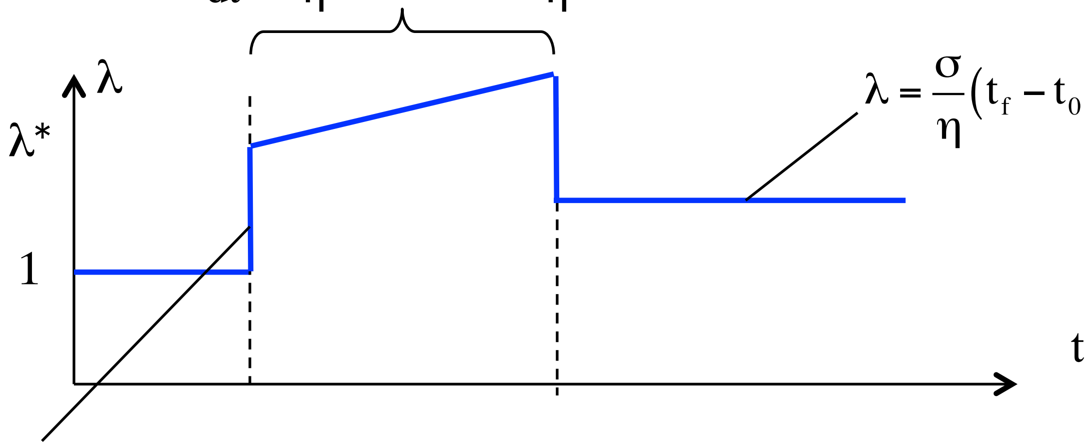
Viscoelasticity

Maxwell Model: $$\frac{d\lambda}{dt} = \frac{1}{E}\frac{d\sigma}{dt} + \frac{\sigma}{\eta}$$





$$\frac{d\lambda}{dt} \approx \frac{\sigma}{\eta} \Rightarrow \lambda = \lambda^{*} + \frac{\sigma}{\eta}\left(t - t_0\right)$$

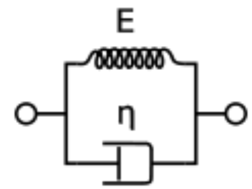


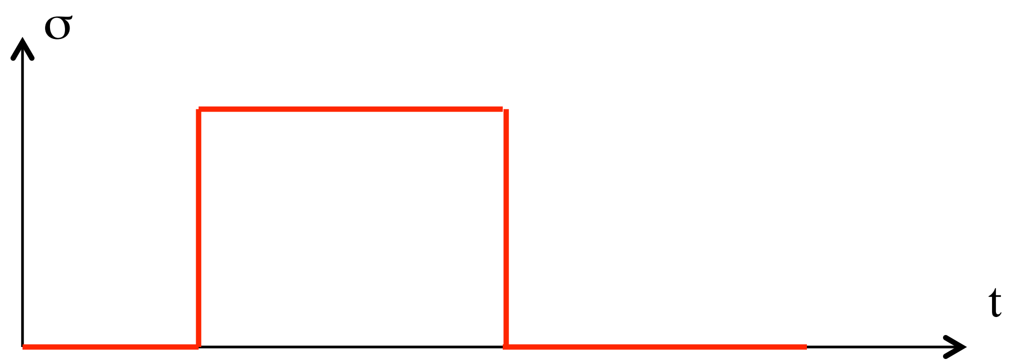
$$\lambda = \frac{\sigma}{\eta}\left(t_f - t_0\right)$$

$$\frac{d\lambda}{dt} \approx \frac{1}{E}\frac{d\sigma}{dt} \Rightarrow \sigma = E\left(\lambda - 1\right)$$

Viscoelasticity

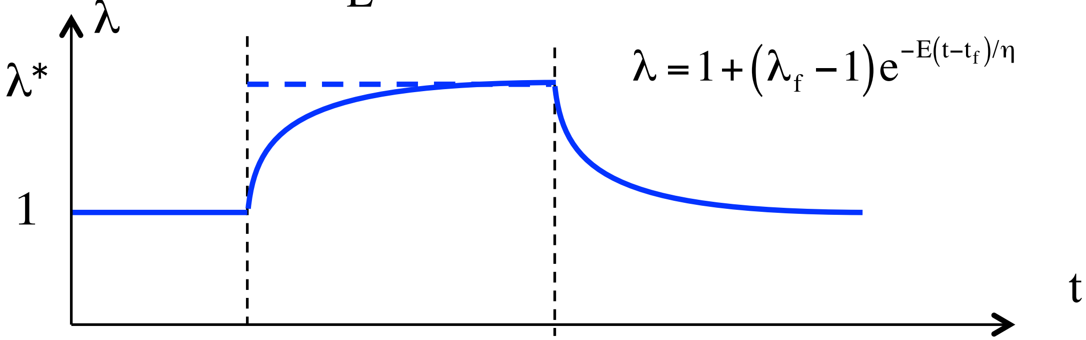
Kelvin-Voigt Model: $\sigma = E(\lambda - 1) + \eta \dfrac{d\lambda}{dt}$





$$\lambda = 1 + \frac{\sigma_0}{E}\left\{1 - e^{-E(t-t_0)/\eta}\right\}$$

$$\lambda = 1 + (\lambda_f - 1)e^{-E(t-t_f)/\eta}$$



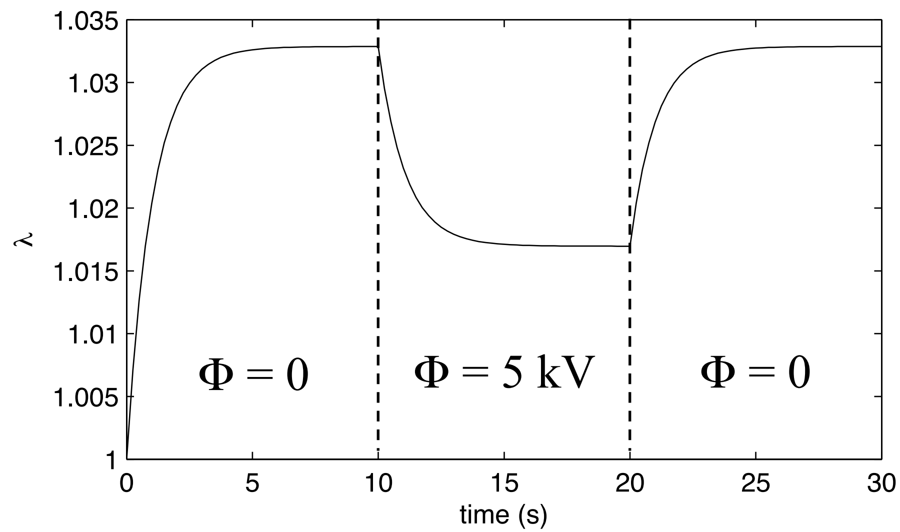
Viscoelasticity

For a Neo-Hookean Solid subject to unixaxial loading and Maxwell Stress,

$$\sigma_3 - \sigma_M = C_1\left(\lambda^2 - \frac{1}{\lambda}\right) \qquad \sigma_M = \varepsilon E^2 = \frac{\varepsilon\Phi^2}{\lambda^2 t_0^2}$$

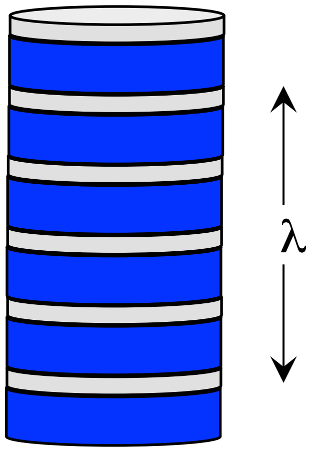
Including the viscoelasticity term, this becomes

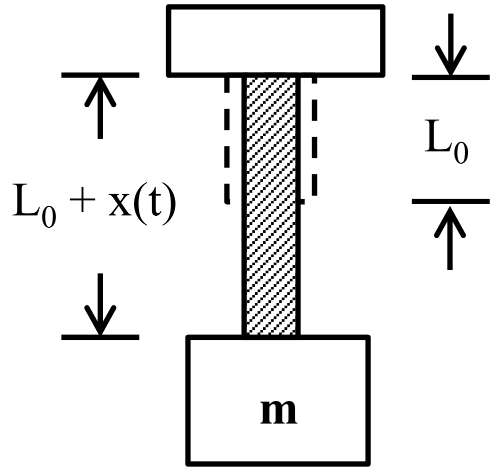
$$\sigma = C_1\left(\lambda^2 - \frac{1}{\lambda}\right) + \frac{\varepsilon\Phi^2}{\lambda^2 h_0^2} + \eta\frac{d\lambda}{dt}$$

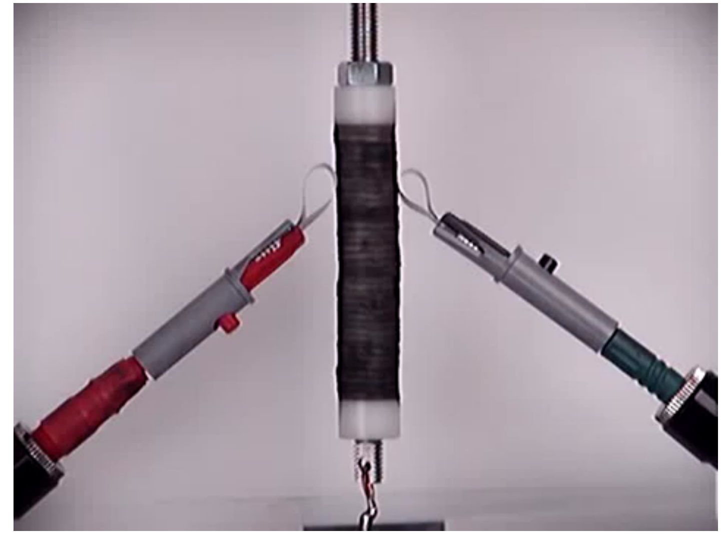


HW 3

Stacked Capacitor







Kovacs et al. (EMPA Switzerland)

4 Stacked DEA. As in lecture, consider a stacked DEA with an initial radius R_0 = 1cm length L_0 = 10 cm, and dielectric gap h_0 = 0.1 mm, The DEA contains $N = L_0/h_0 = 1000$ capacitive elements and the thickness and rigidity of the capacitive electrodes will be ignored.

Let the dielectric be treated as a Neo-Hookean solid with an added Maxwell stress and viscosity term

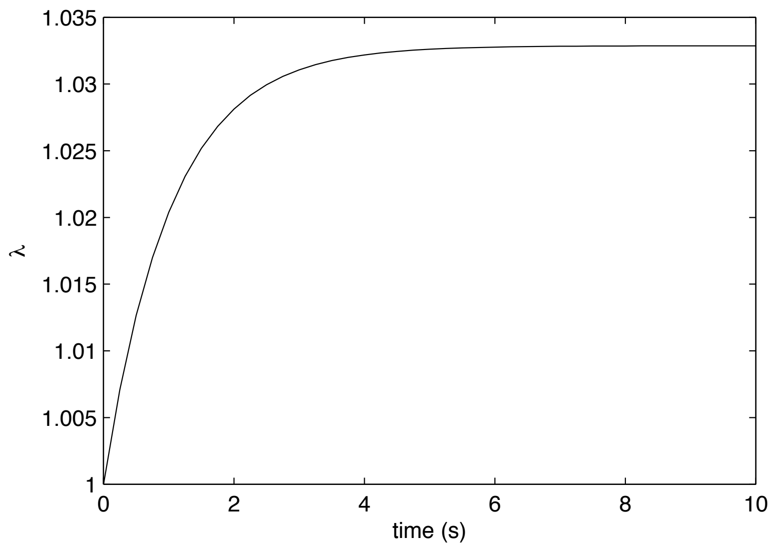
$$\sigma = 2C_1\left(\lambda^2 - \frac{1}{\lambda}\right) + \varepsilon\left(\frac{\Phi}{\lambda h_0}\right)^2 + \eta\frac{d\lambda}{dt}$$

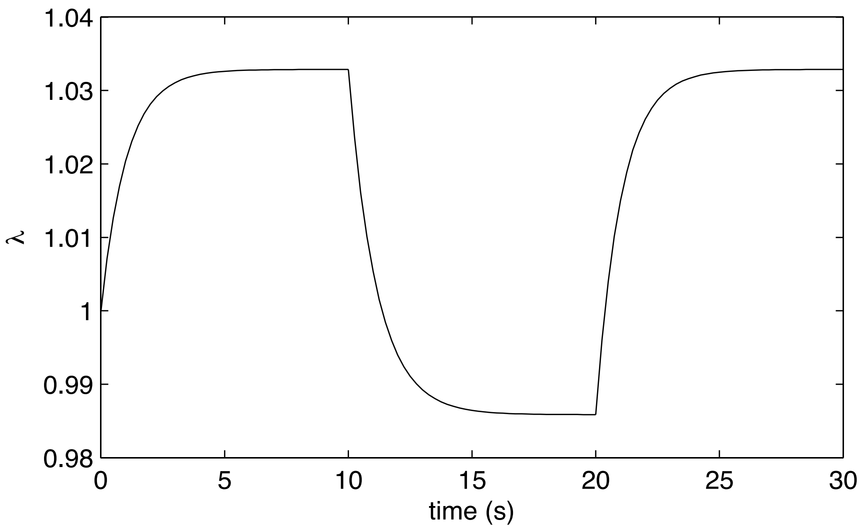
with modulus E = 1 MPa, dielectric constant $\varepsilon_r = 2$, and viscosity η = 1 MPa-s. [**4 points**]

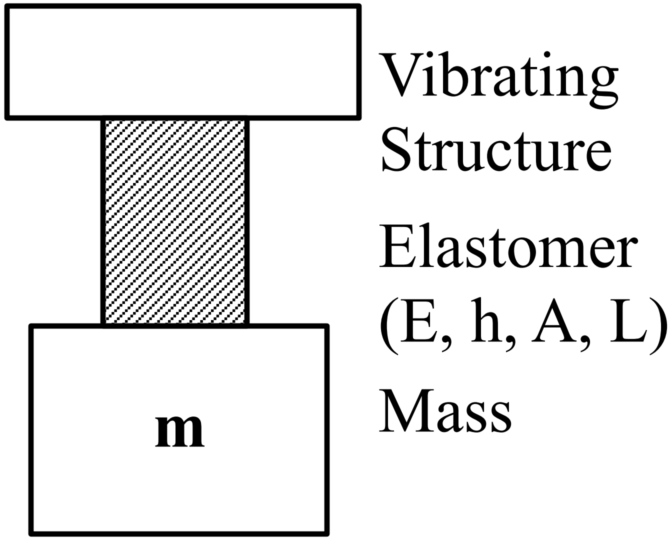
HW 3

a) Suppose that at time t = 0s, the DEA is loaded with a tensile deadweight P = 10 N. Plot λ vs. t over the domain $0s \le t \le 10s$.

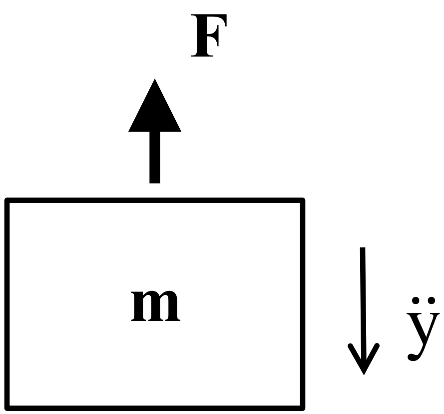
b) Now suppose that starting at time t = 10s, a voltage Φ = 5 kV is applied for 10s. Reconstruct the plot below for λ vs. time t over the domain $0s \le t \le 30s$.







$$\boxed{-F \equiv m\ddot{y}}$$

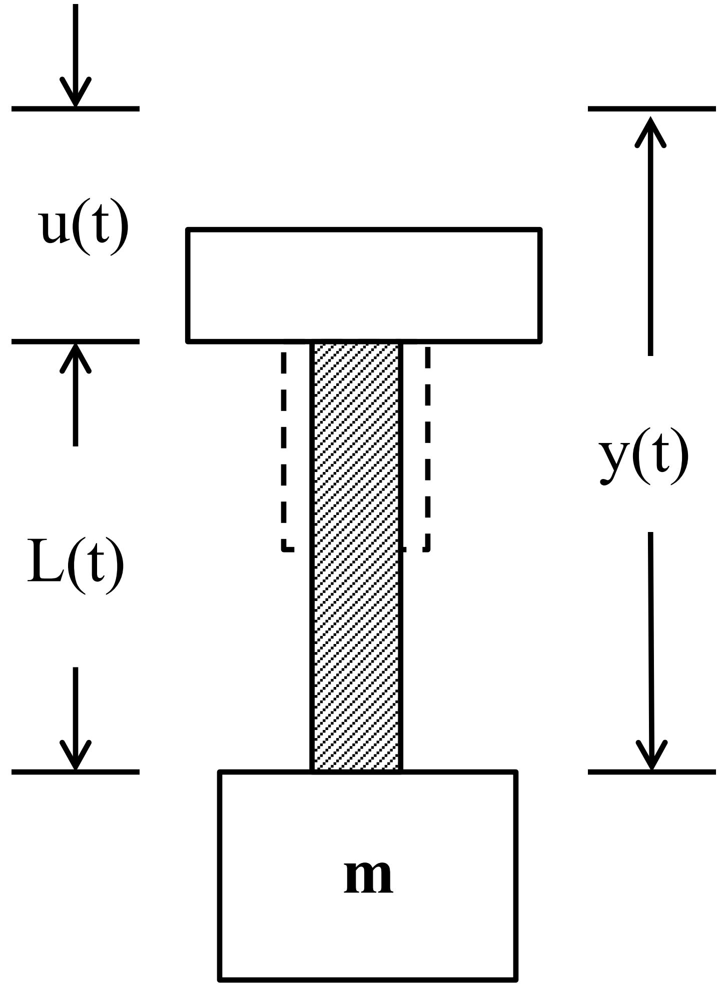


$$y = u + L$$

$$u = u_0 \sin(\omega t)$$

$$y(0) = L_0$$

$$\dot{y}(0) = 0$$



$$L = L_0 + x = \lambda L_0$$

$$\Rightarrow y = L_0 + u + x$$

Linearized Model

Kelvin-Voigt Solid: $\sigma = E\varepsilon + \eta\dot{\varepsilon}$ $\qquad \varepsilon = \dfrac{x}{L_0} \qquad \dot{\varepsilon} = \dfrac{\dot{x}}{L_0}$

$$F \approx \sigma A_0 = \underbrace{\frac{EA_0}{L_0}}_{k} x + \underbrace{\frac{\eta A_0}{L_0}}_{c} \dot{x}$$

$$-F \equiv m\ddot{y} \Rightarrow \ddot{x} + \frac{F}{m} = -\ddot{u}$$

$$\ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m}x = u_0\omega^2 \sin(\omega t)$$

$$\begin{aligned} y(0) = L_0 &\Rightarrow \boxed{\begin{aligned} x(0) &= 0 \\ \dot{x}(0) &= -u_0\omega \end{aligned}} \\ \dot{y}(0) = 0 &\Rightarrow \end{aligned}$$

It is convenient to define the following:

Natural Frequency: $\omega_n = \sqrt{\dfrac{k}{m}}$

Damping Ratio: $\zeta = \dfrac{c}{2\sqrt{mk}}$

$$\boxed{\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = u_0\omega^2 \sin(\omega t)}$$

$$x(0) = 0$$
$$\dot{x}(0) = -u_0\omega$$

Solution: $x = x_c + x_p$

"complementary solution" "particular solution"

Natural Frequency

Recall that for a spring mass system, the natural frequency is

$$\omega_n = \sqrt{\frac{k}{m}}$$

For a Hookean solid, $k = EA_0/L_0$

$$\left.\begin{array}{ll} L_0 = 10\ \text{cm} & E = 1\ \text{MPa} \\ R_0 = 1\ \text{cm} & \eta = 100\ \text{Pa-s} \\ h_0 = 0.1\ \text{mm} & m = 10\ \text{mg} \\ u_0 = 1\ \text{mm} & \varepsilon_r = 2 \end{array}\right\} \omega_n = 560.5\ \text{Hz}$$

Solution Summary

$$x = A_1 e^{\omega_n(-\zeta + i\omega_d)t} + A_2 e^{\omega_n(-\zeta - i\omega_d)t} + X_1 \sin(\omega t) + X_2 \cos(\omega t)$$

$$\omega_d = \sqrt{1-\zeta^2} \qquad \lambda_{1,2} = \omega_n\left(-\zeta \pm \sqrt{\zeta^2 - 1}\right)$$

$$A_1 = \frac{\lambda_2 X_2 - u_0\omega - \omega X_1}{\lambda_1 - \lambda_2} \qquad A_2 = \frac{\lambda_1 X_2 - u_0\omega - \omega X_1}{\lambda_2 - \lambda_1}$$

$$X_1 = \frac{u_0\omega^2\left(\omega_n^2 - \omega^2\right)}{\left(\omega_n^2 - \omega^2\right)^2 + \left(2\zeta\omega_n\omega\right)^2} \qquad X_2 = \frac{-u_0\omega^2\left(2\zeta\omega_n\omega\right)}{\left(\omega_n^2 - \omega^2\right)^2 + \left(2\zeta\omega_n\omega\right)^2}$$

At steady state, this converges to

$$x = \frac{u_0\omega^2 \sin(\omega t + \phi)}{\sqrt{\left(\omega_n^2 - \omega^2\right)^2 + \left(2\zeta\omega_n\omega\right)^2}} \quad \text{where} \quad \phi = n\pi + \tan^{-1}\left(\frac{X_2}{X_1}\right)$$

and $X = \dfrac{u_0\omega^2}{\sqrt{\left(\omega_n^2 - \omega^2\right)^2 + \left(2\zeta\omega_n\omega\right)^2}}$ is the steady state amplitude

Complementary Solution

$$\ddot{x}_c + 2\zeta\omega_n\dot{x}_c + \omega_n^2 x_c = 0$$

$$x_c = \sum A_i e^{\lambda_i t}$$

$$\Rightarrow \sum A_i e^{\lambda_i t}\left\{\lambda_i^2 + 2\zeta\omega_n\lambda_i + \omega_n^2\right\} = 0$$

For this to be satisfied for arbitrary A_i and t,

$$\lambda_i^2 + 2\zeta\omega_n\lambda_i + \omega_n^2 = 0$$

This only has two linearily independent solutions.
Therefore in general,

$$x_c = A_1 e^{\lambda_1 t} + A_2 e^{\lambda_2 t} \quad \text{where} \quad \lambda_{1,2} = \omega_n\left(-\zeta \pm \sqrt{\zeta^2 - 1}\right)$$

Particular Solution

We postulate that x_p has the form

$$x_p = X_1 \sin(\omega t) + X_2 \cos(\omega t)$$

Substituting $x_c + x_p$ into the governing ODE and noting

$$\ddot{x}_c + 2\zeta\omega_n \dot{x}_c + \omega_n^2 x_c = 0,$$

it follows that $\ddot{x}_p + 2\zeta\omega_n \dot{x}_p + \omega_n^2 x_p = u_0 \omega^2 \sin(\omega t)$

Substituting the expression for x_p into the above ODE,

$$\left\{-X_1\omega^2 \sin(\omega t) - X_2\omega^2 \cos(\omega t)\right\}$$

$$+2\zeta\omega_n \left\{X_1\omega \cos(\omega t) - X_2\omega \sin(\omega t)\right\}$$

$$+\omega_n^2 \left\{X_1 \sin(\omega t) + X_2 \cos(\omega t)\right\} = u_0\omega^2 \sin(\omega t)$$

Rearranging terms,

$$\begin{aligned}&\left\{-X_1\omega^2 - 2\zeta\omega_n\omega X_2 + \omega_n^2 X_1\right\}\sin(\omega t)\\ &+\left\{-X_2\omega^2 + 2\zeta\omega_n\omega X_1 + \omega_n^2 X_2\right\}\cos(\omega t)\end{aligned} = u_0\omega^2\sin(\omega t)$$

$$\Rightarrow\begin{cases}\left(\omega_n^2-\omega^2\right)X_1 - 2\zeta\omega_n\omega X_2 = u_0\omega^2\\ 2\zeta\omega_n\omega X_1 + \left(\omega_n^2-\omega^2\right)X_2 = 0\end{cases}$$

$$\boxed{X_1 = \frac{u_0\omega^2\left(\omega_n^2-\omega^2\right)}{\left(\omega_n^2-\omega^2\right)^2+\left(2\zeta\omega_n\omega\right)^2}\qquad X_2 = \frac{-u_0\omega^2\left(2\zeta\omega_n\omega\right)}{\left(\omega_n^2-\omega^2\right)^2+\left(2\zeta\omega_n\omega\right)^2}}$$

Noting that sin(A+B) = sin(A)cos(B) + cos(A)sin(B), it follows that x_p can also be expressed as

$$x_p = X\sin\left(\omega t + \phi\right),$$

where $X = \sqrt{X_1^2 + X_2^2} = \dfrac{u_0\omega^2}{\sqrt{\left(\omega_n^2 - \omega^2\right)^2 + \left(2\zeta\omega_n\omega\right)^2}}$

is the amplitude, and

$\phi = n\pi + \tan^{-1}\left(\dfrac{X_2}{X_1}\right)$ is the phase shift.

Boundary Conditions

Lastly, A_1 and A_2 are determined by applying the boundary conditions $x(0)=0 \quad \dot{x}(0)=-u_0\omega$

$$x = A_1 e^{\lambda_1 t} + A_2 e^{\lambda_2 t} + X_1 \sin(\omega t) + X_2 \cos(\omega t)$$

$$\dot{x} = \lambda_1 A_1 e^{\lambda_1 t} + \lambda_2 A_2 e^{\lambda_2 t} + \omega X_1 \cos(\omega t) - \omega X_2 \sin(\omega t)$$

$$x(0) = 0 \Rightarrow A_1 + A_2 + X_2 = 0 \Rightarrow \boxed{A_1 = -(A_2 + X_2)}$$

$$\dot{x}(0) = -u_0\omega \Rightarrow -\lambda_1(A_2 + X_2) + \lambda_2 A_2 + \omega X_1 = -u_0\omega$$

$$\Rightarrow \boxed{A_2 = \frac{\lambda_1 X_2 - u_0\omega - \omega X_1}{\lambda_2 - \lambda_1}}$$

Solution

The coefficients, A_1, A_2, X_1, and X_2 are real. However, for an underdamped system ($\zeta < 1$) λ_1 and λ_2 will be imaginary.

$$x = A_1 e^{\omega_n(-\zeta + i\omega_d)t} + A_2 e^{\omega_n(-\zeta - i\omega_d)t} + X_1 \sin(\omega t) + X_2 \cos(\omega t)$$

$$= e^{-\zeta\omega_n t}\left\{A_1 e^{i\omega_n\omega_d t} + A_2 e^{-i\omega_n\omega_d t}\right\} + X_1 \sin(\omega t) + X_2 \cos(\omega t)$$

where $\omega_d = \sqrt{1-\zeta^2}$.

The first term vanishes as t approaches ∞, and so it is known as the "transient solution." The solution converges to the last two terms (particular solution), which is known as the "steady state solution."

The smaller the damping, the longer it takes to reach steady state and the larger the steady state amplitude X.

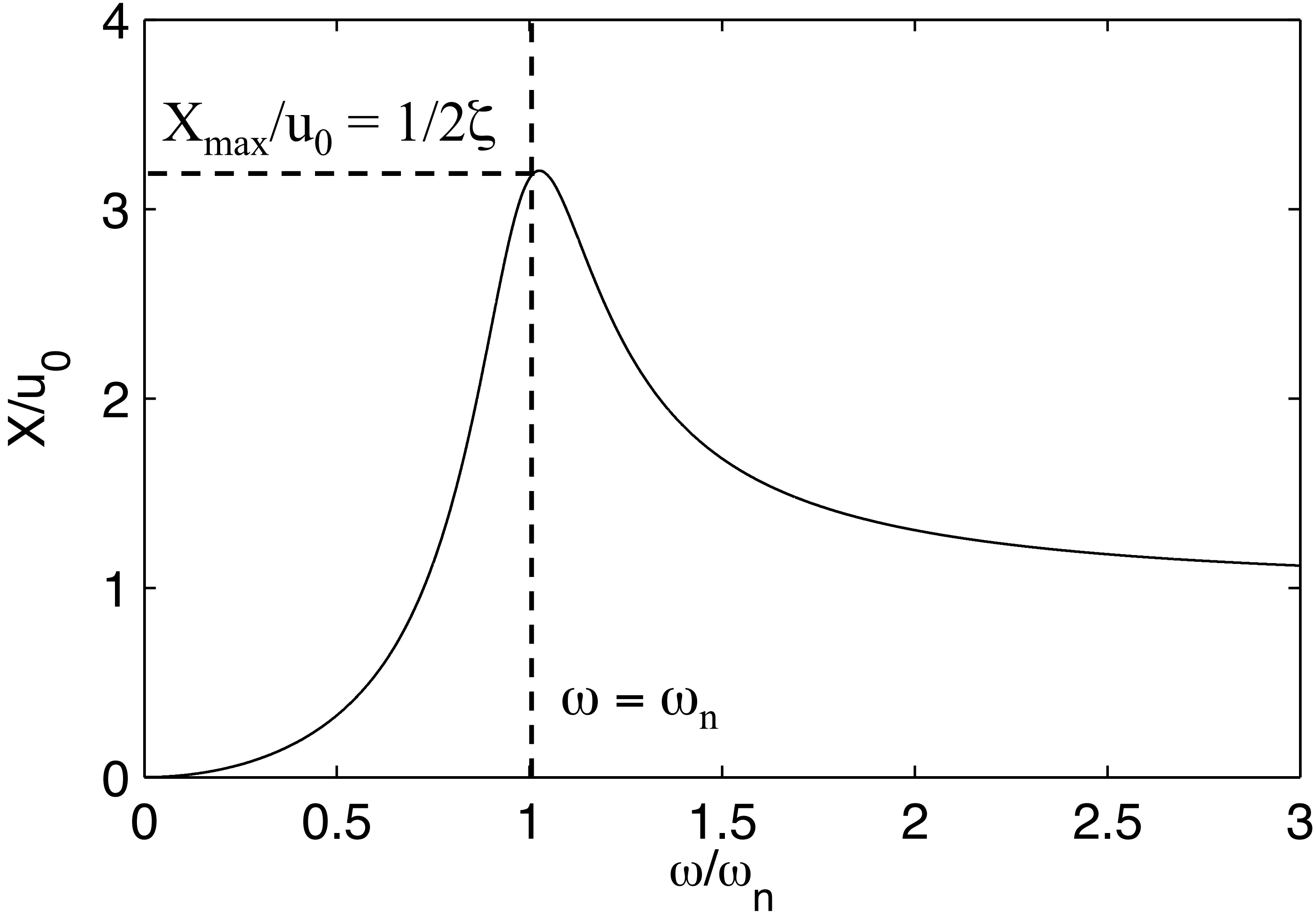
Once the steady state solution is reached, the length of the viscoelastic solid will change with amplitude

$$X = \frac{u_0\omega^2}{\sqrt{\left(\omega_n^2 - \omega^2\right)^2 + \left(2\zeta\omega_n\omega\right)^2}}$$

Clearly, this is maximized when the excitation frequency ω of the external (base) vibration matches the natural frequency ω_n of the system: $X_{max} = u_0/2\zeta$.

Interestingly, X approaches zero when ω is small. This means that the elastomer doesn't deform and the mass vibrates with the base.

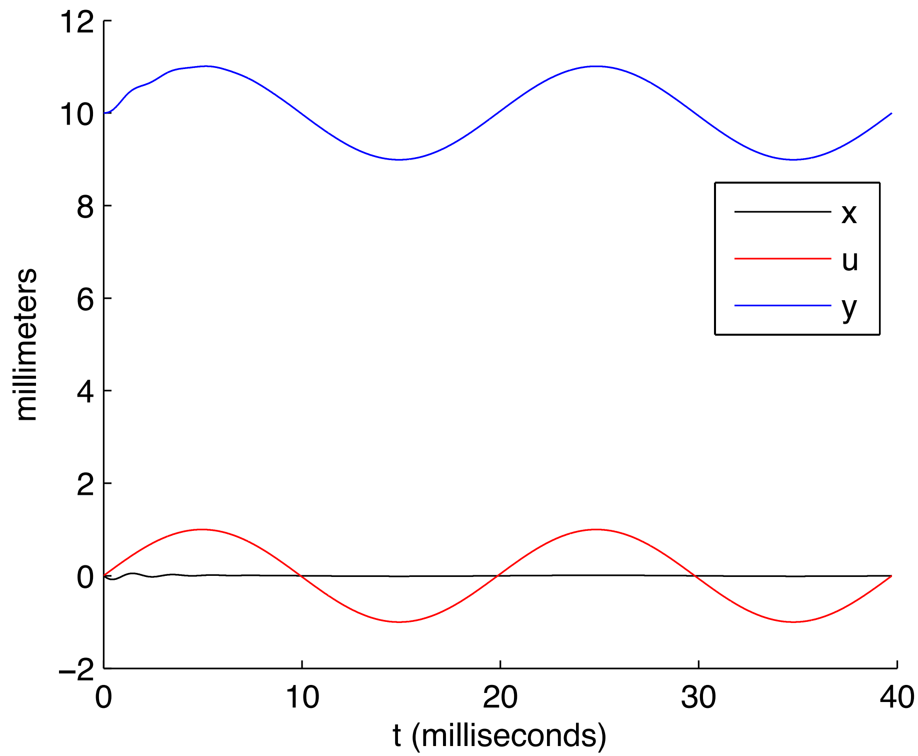
When ω gets large, X approaches u_0. This is not intuitive and should be examined in more detail.



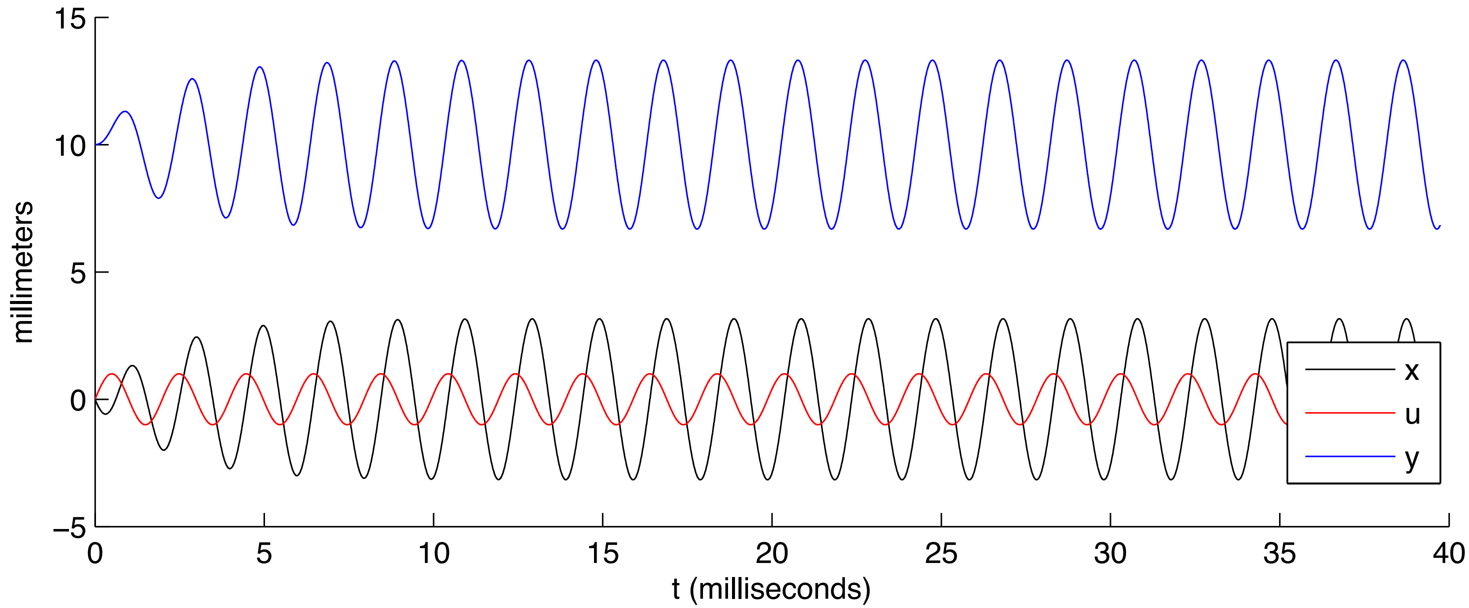
$X_{max}/u_0 = 1/2\zeta$
$\omega = \omega_n$
X/u_0
ω/ω_n
0
1
2
3
4
0
0.5
1
1.5
2
2.5
3

$$\omega = 0.1\omega_n$$

$$\boxed{\begin{aligned} &x \approx 0 \quad \forall t \\ &y \approx L_0 + u_0 \sin(\omega t) \end{aligned}}$$



$$\omega = \omega_n$$



$$x \to \frac{u_0}{2\zeta}\sin\left(\omega_n t - \frac{\pi}{2}\right)$$

$$y \to L_0 + u_0 \sin(\omega_n t) + \frac{u_0}{2\zeta}\sin\left(\omega_n t - \frac{\pi}{2}\right)$$

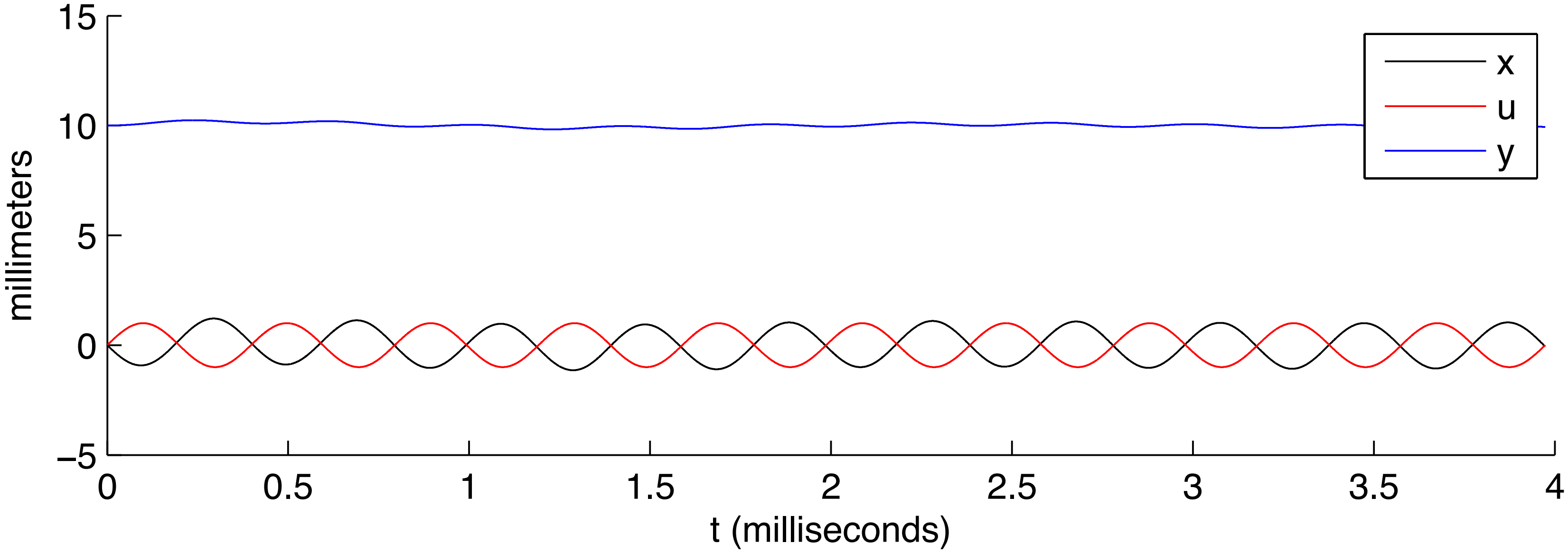
$$= L_0 + u_0 \sin(\omega_n t) - \frac{u_0}{2\zeta}\cos(\omega_n t)$$

$$\boxed{\therefore y \to L_0 + Y\sin(\omega_n t - \phi)}$$

$$Y = u_0\sqrt{1 + (2\zeta)^{-2}}$$

$$\phi = \tan^{-1}(1/2\zeta)$$

$\omega = 5\omega_n$



$$X_1 \approx -u_0 \quad X_2 \approx 0 \quad \Rightarrow \quad x \approx -u_0 \sin(\omega_n t)$$

$$\boxed{y \approx L_0}$$

Displacements are equal-and-opposite → mass remains stationary

Nonlinear Model

Kelvin-Voigt Solid: $\sigma = 2C_1\left(\lambda^2 - \dfrac{1}{\lambda}\right) + \eta\dfrac{d\lambda}{dt} + \varepsilon\left(\dfrac{\Phi}{\lambda h_0}\right)^2$

$$\lambda = \frac{L}{L_0} \qquad \frac{d\lambda}{dt} = \frac{1}{L_0}\frac{dL}{dt}$$

$$F = \frac{\sigma A_0}{\lambda} = 2C_1A_0\left(\lambda - \frac{1}{\lambda^2}\right) + \frac{\eta A_0}{\lambda}\frac{d\lambda}{dt} + \frac{\varepsilon A_0}{\lambda^3 h_0^2}\Phi^2$$

$$-F \equiv m\ddot{y} = m\frac{d}{dt}\left\{\lambda L_0 + u\right\}$$

$$\Rightarrow mL_0\ddot{\lambda} - mu_0\omega^2\sin\left(\omega t\right) + 2C_1A_0\left(\lambda - \frac{1}{\lambda^2}\right) + \frac{\eta A_0}{\lambda}\dot{\lambda} + \frac{\varepsilon A_0}{h_0^2\lambda^3}\Phi^2 = 0$$

ODE: $$\ddot{\lambda} = \frac{u_0\omega^2}{L_0}\sin\left(\omega t\right) - \frac{\varepsilon A_0}{mL_0h_0^2\lambda^3}\Phi^2 - \frac{2C_1A_0}{mL_0}\left(\lambda - \frac{1}{\lambda^2}\right) - \frac{\eta A_0}{mL_0\lambda}\dot{\lambda}$$

BCs: $$y\left(0\right) = L_0 \Rightarrow \lambda\left(0\right) = 1$$

$$\dot{y}\left(0\right) = 0 \Rightarrow \dot{\lambda}\left(0\right) = 1 - \frac{u_0\omega}{L_0}$$

Code

```
function stacked_DEA

global L0 A0 h0 u0 C1 eta m eps omega V0 psi

L0 = 0.1;
R0 = 0.01;
A0 = pi*R0^2;
h0 = 0.1e-3;
u0 = 1e-3;

E = 1e6;
C1 = E/6;

eta = 100;
m = 10e-3;
er = 2;
eps = er*8.85e-12;

k = E*A0/L0;
omega_n = sqrt(k/m);
omega = omega_n;
T = 2*pi/omega;

V0 = 0;
psi = 0;
```

```
%%%% Determine Lambda %%%%

tf = 5*T;
lambda0 = 1;
lambdadot0 = 1 - u0*omega/L0;

[t,x] = ode45(@get_lambda,[0 tf],[lambda0 lambdadot0]);

lambda = x(:,1);
lambdadot = x(:,2);
u = u0*sin(omega*t);
y = u + lambda*L0;
```

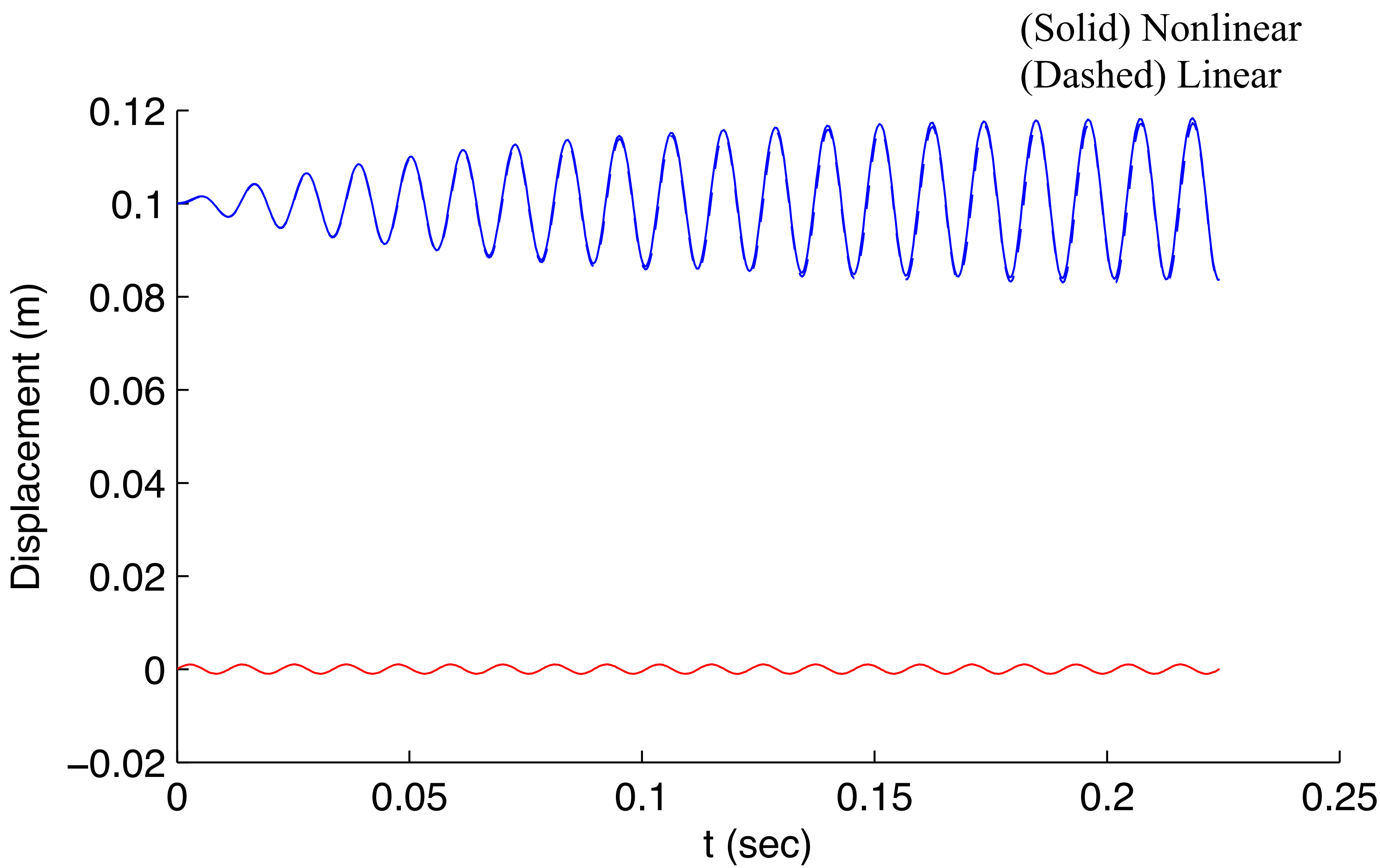
```
function xdot = get_lambda(t,x)

global L0 A0 h0 u0 C1 eta m eps omega V0 psi

V = V0*(1 + sin(omega*t - psi))/2;

xdot(1,1) = x(2);
xdot(2,1) = u0*omega^2*sin(omega*t)/L0 - eps*A0*V^
```

Comparison: No Voltage, $\omega = \omega_n$



Assume a sinusoidal applied voltage with amplitude Φ_0 , frequency ω, and phase ψ:

$$\Phi = \frac{1}{2}\Phi_0\left\{1+\sin\left(\omega t-\psi\right)\right\}$$

Power in: $P_{in} = n\Phi\dot{q} = \left(\frac{L}{h_0}\right)\Phi\frac{d}{dt}\left(\frac{\varepsilon A}{h}\Phi\right)$

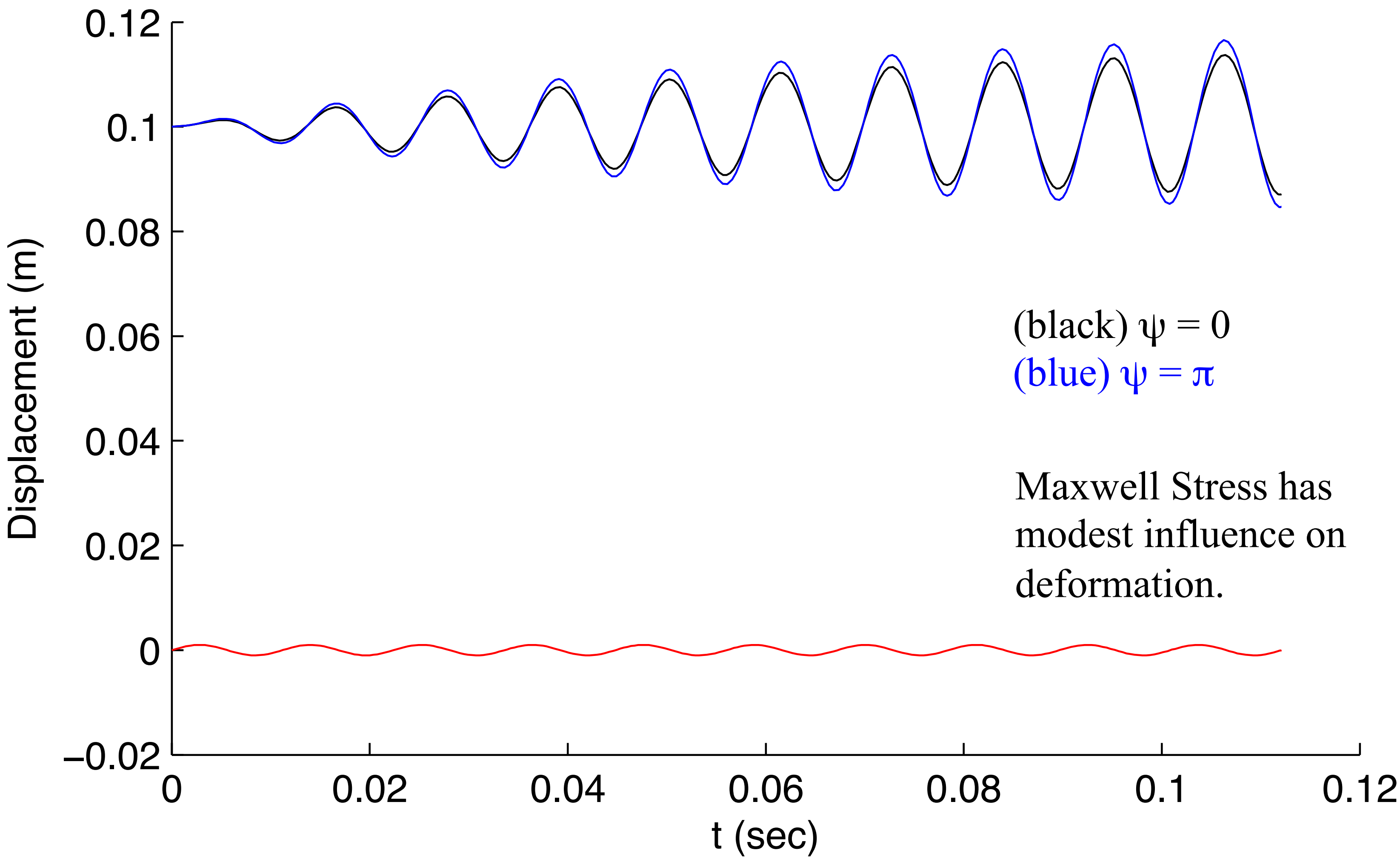
$$= \left(\frac{L}{h_0}\right)\Phi\frac{d}{dt}\left\{\frac{\varepsilon A_0}{\lambda^2 h_0}\Phi\right\} = \frac{\varepsilon A_0 L_0}{h_0^2\lambda^2}\Phi\left\{\dot{\Phi}-\frac{2\dot{\lambda}\Phi}{\lambda}\right\}$$

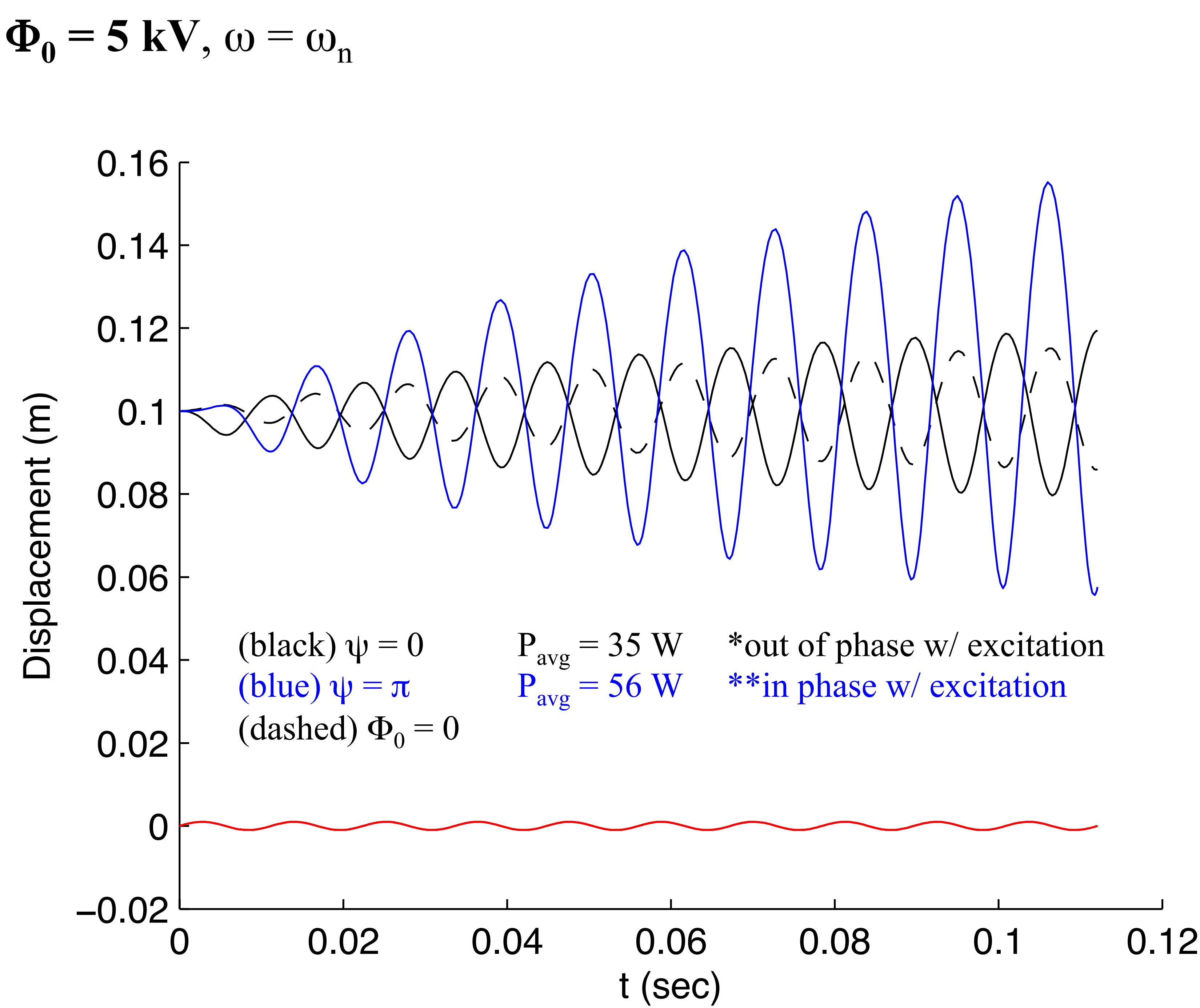
$$\therefore P_{in} = \frac{\varepsilon A_0 L_0}{h_0^2\lambda^2}\Phi\left\{\frac{1}{2}\omega\cos\left(\omega t-\psi\right)-\frac{2\dot{\lambda}\Phi}{\lambda}\right\}$$

Average Power: $P_{avg} = \int_{t_0}^{t_0+T}\frac{\varepsilon A_0 L_0}{h_0^2 T\lambda^2}\Phi\left\{\dot{\Phi}-\frac{2\dot{\lambda}\Phi}{\lambda}\right\}dt$

Period of Oscillation: $T = 2\pi/\omega_n$

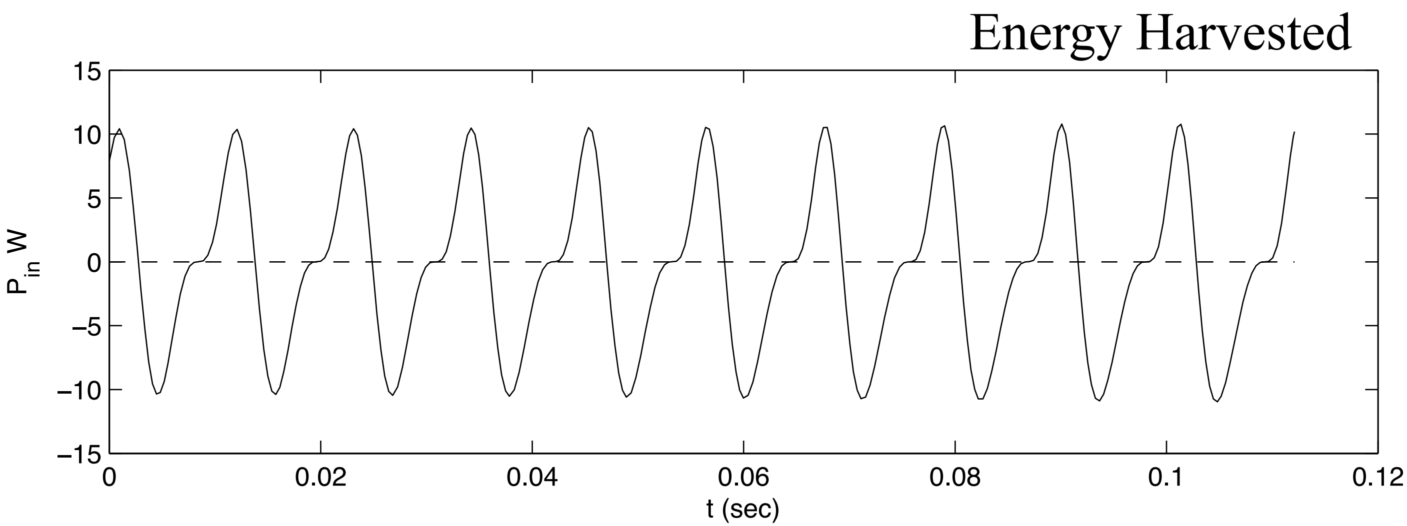
Φ_0 = 1 kV, $\omega = \omega_n$





Φ_0 = 5 kV, $\omega = \omega_n$
Displacement (m)
t (sec)
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
−0.02
0
0.02
0.04
0.06
0.08
0.1
0.12
(black) $\psi = 0$
(blue) $\psi = \pi$
(dashed) $\Phi_0 = 0$
P_{avg} = 35 W
P_{avg} = 56 W
*out of phase w/ excitation
**in phase w/ excitation

Φ_0 = 1 kV, $\psi = 0$, $\omega = \omega_n$



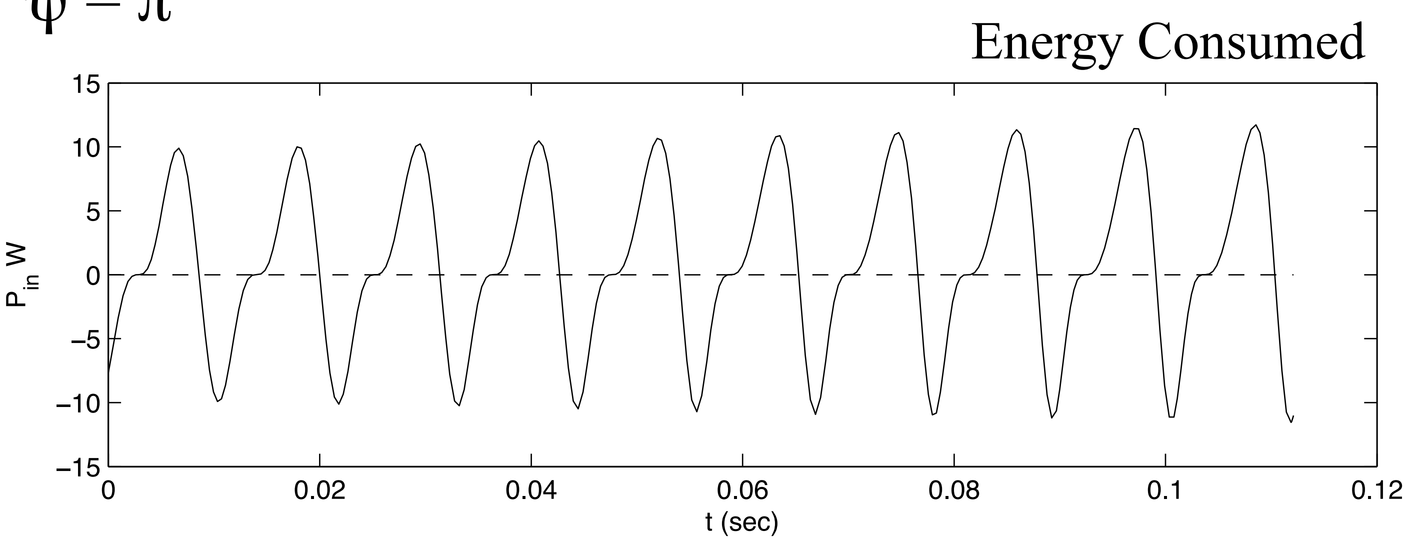
Supplied

Harvested

$P_{avg} = -1$ W

$\psi = \pi$

~10 μW/mm³



$P_{avg} = 1$ W