EECS 106B/206B
Discussion 5

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Action Items

- Concurrent Enrollment students fill out both forms:
  - Student accounts
  - Lab access
- Concurrent Enrollment students should be accepted
- Homework 2 released
- Lab Due in 11 days!!!!

Trajectory Control of Manipulators

$$M(\theta)\ddot{\theta} + C(\theta, \dot{\theta})\dot{\theta} + N(\theta, \dot{\theta}) = \tau$$

M: Inertial Term

C: Coriolis Term

N: External Forces (gravity and friction)

Tau: Joint Torques

Open-loop model

$$\tau = M(\theta_d)\ddot{\theta}_d + C(\theta_d, \dot{\theta}_d)\dot{\theta}_d + N(\theta_d, \dot{\theta}_d).$$

Not robust to initial conditions

Not robust to model parameters

No trajectory tracking

No disturbance rejection

Computed torque

Now we set a desired torque to eliminate manipulator inertia:

$$\tau = M(\theta)\ddot{\theta}_d + C(\theta, \dot{\theta})\dot{\theta} + N(\theta, \dot{\theta}).$$

We set this equation equal to the dynamics and get

$$\ddot{\theta} = \ddot{\theta}_d.$$

If initial position and velocity match desired, this will track

Add feedback control

Add a PD controller on position $e = \theta - \theta_d$

$$\tau = M(\theta)\left(\ddot{\theta}_d - K_v\dot{e} - K_p e\right) + C(\theta,\dot{\theta})\dot{\theta} + N(\theta,\dot{\theta})$$

$$\ddot{e} + K_v\dot{e} + K_p e = 0.$$

$$\tau = \underbrace{M(\theta)\ddot{\theta}_d + C\dot{\theta} + N}_{\tau_{\mathrm{ff}}} + \underbrace{M(\theta)\left(-K_v\dot{e} - K_p e\right)}_{\tau_{\mathrm{fb}}}.$$

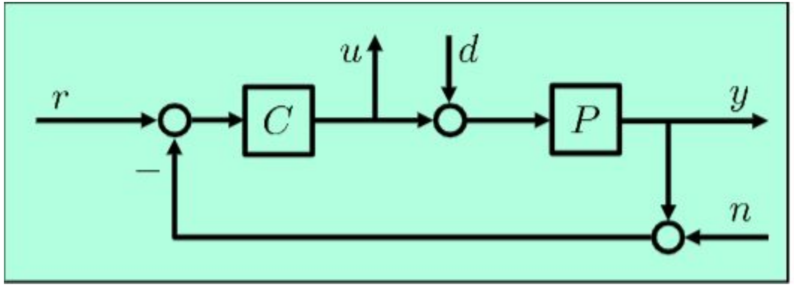
Augmented Control

Let’s add the Coriolis matrix to the feedback term (which brings desired velocity to the feed-forward term)

$$\tau = M(\theta)\ddot{\theta}_d + C(\theta,\dot{\theta})\dot{\theta}_d + N(\theta,\dot{\theta}) - K_v\dot{e} - K_p e$$

Controller Tuning: Trade-off between Disturbance Rejection and Noise

$$\begin{bmatrix} Y \\ U \end{bmatrix} = \begin{bmatrix} \frac{PC}{1+PC} & \frac{P}{1+PC} & \frac{-PC}{1+PC} \\ \frac{C}{1+PC} & \frac{-PC}{1+PC} & \frac{-C}{1+PC} \end{bmatrix} \begin{bmatrix} R \\ D \\ N \end{bmatrix}$$



Tuning Tips

- Your feed-forward term matters a lot
- First do static tuning
  - Tune the P term first until you get reasonably stable motion with a fast rise time
  - Tune the D term to bring it down
- Ziegler-Nichols method - somewhat problematic
  - Other similar heuristic methods
- Tune in different regions of the arm