Chapter 4

Planning Frictionless Enveloping Grasps

Anyone who has ever accidentally dropped a bar of soap in the shower or a soapy dish in the kitchen or lost his grip on an oily engine part or had a wet, shelled, hard-boiled egg slip away on the counter knows that slippery objects must be handled in a way different from how we hold most things. The reason for this is that the friction forces which are usually dominant become negligible, drastically reducing the set of forces which can be applied by our hands. After attempting a grasp which fails, we try again surrounding the object with our fingers, essentially building a cage around it. The grasp cannot be broken without bending one or more of our fingers out of the way of the object. A grasp so formed is an *enveloping* grasp; also called an *encompass* or*power* grasp [44]. It is the most secure type of grasp that we use. We also use an enveloping grasp when friction forces are significant to get a very good grip on something, *e.g.* a hammer or the handle of a suitcase. To improve the grasp further, the coefficient of friction can be enhanced, *e.g.* baseball players put pine tar or friction tape on the handle of a bat. Crossley and Umholtz, [13], Rovetta [67], and Skinner [72] realized the utility of this type of grasp and developed hands to execute it, but did not study its kinematic properties.

The main goal of this chapter is to establish a framework for planning finite manipulations. In the previous chapter, the basic equations for simulating grasping systems were developed. With those equations a simulator has been developed which is capable of verifying proposed manipulation plans. In our work on planning, we have chosen to concentrate on frictionless enveloping grasps for problems which can be reduced to the two-dimensional case. Most common objects are elongated and have an axis of symmetry or can be approximated by a generalized cylinder. The cross section of the object in a plane perpendicular to the axis of symmetry or to the spine of the generalized cylinder allows the analysis of the grasping problem in two dimensions. If the object is too complex to be approximated in one these ways, often there is a handle (as on a pitcher or cup) which can adequately be approximated and analyzed for grasping.

In the following discussions, the hand model is very simple, but can still be used to lift an object and gather it into the palm creating an enveloping grasp. Figure 4.1.1 shows the two-dimensional hand model. This hands falls into the category of grippers as outlined in Section 2.4.

The main difficulties in grasping slippery objects are as follows. First, one must determine where to place the equivalent of four contact points on the object (seven in three-dimensional problems) to obtain a stable grasp. Second, to manipulate the object, one must plan trajectories for three of the contact points relative to the fourth. In his recent thesis, Nguyen [50] developed a technique for placing fingers on an object to create stable grasps. However, he did not addressed the problem of moving those

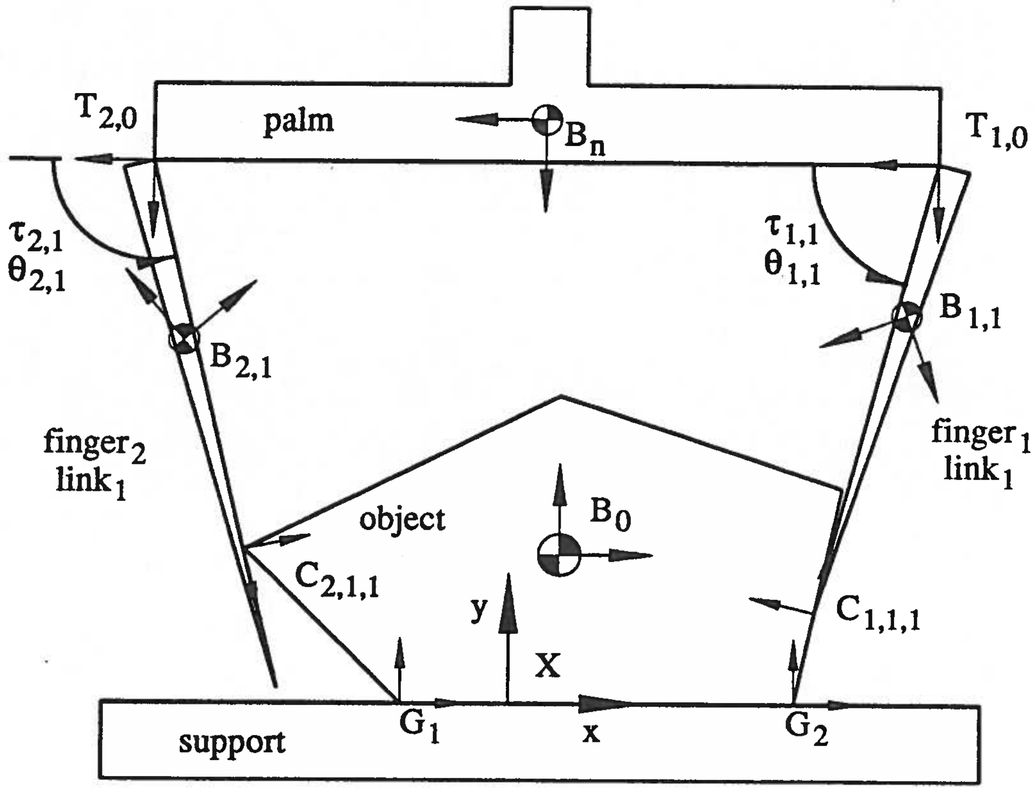

Figure 4.1.1: Initial Grasping Configuration

contacts to manipulate the object. Third, because the contacts slide on the object, an accurate description of the geometry of the surfaces of the contacting bodies is required over a larger region than would be required for bodies in rolling contact, because the relative positions of the object and hand can change more drastically than for rolling contacts.

Some aspects of grasping frictionless objects are simpler than the counterparts for perfectly rough objects. First, because there are no friction forces, the task of computing force directions [28] is obviated. Second, because friction forces are not used to stabilize the object, the contact forces applied to the object by the hand need not be controlled. This simplification is significant, for it allows frictionless grasping to proceed under position control with simple protective torque limits to prevent damaging the object and hand[9]. Third, the contacts always slide during manipulation. No tests for rolling versus slipping are needed. Fourth, since more contacts are required for a stable grasp, the handling load is spread over the object more uniformly. Fifth, the distribution of the contacts must be such that the hand surrounds the object to some extent, so it acts as a cage protecting the object from accidental collisions with obstacles in the environment.

In the remainder of this chapter, we formally state the problem and assumptions; Section 4.1. Then we develop a strategy which can be used to plan the lifting of slippery two-dimensional objects. The goal of the plan is to gain an enveloping grasp of the object. Under certain specified conditions, the resulting plan is guaranteed to succeed. The first phase of the strategy involves determining an initial grasp for which squeezing lifts the object; Section 4.2. The result of the analysis is the Initial Grasp Liftability Chart (IGLiC). Three geometric quantities of the initial grasp are needed to determine the liftability of the proposed grasp. In Section 4.3, we show how to partition the perimeter of any planar object into liftability regions. The liftability regions are dependent upon the geometry of the object and the position of one of the hand's contacts. The liftability regions are used to determine an initial grasp of the object from which the object can be lifted, instantaneously breaking all contact with the support. In Section 4.4 we show how to plan finite manipulation to bring the object into an enveloping grasp against the palm. Section 4.5 is concerned with adjusting the grasp without losing its enveloping character.

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9 Force control is more difficult than position control partly because small position errors tend to generate very large forces [64].

4.1. Problem Statement

The problem solved is that of picking up a frictionless object with an articulated mechanical hand and manipulating it into an enveloping grasp. It has been shown [36] that if a frictionless grasp is used to completely restrain an object, using only fingertips, a hand would need a minimum of four fingers (seven in the three-dimensional case). However, the necessary number of fingers may be reduced to two (three in three dimensions) if the hand's palmar surface is used. This includes the palm and those surfaces of the fingers which face the palm (see Figure 4.1.2).

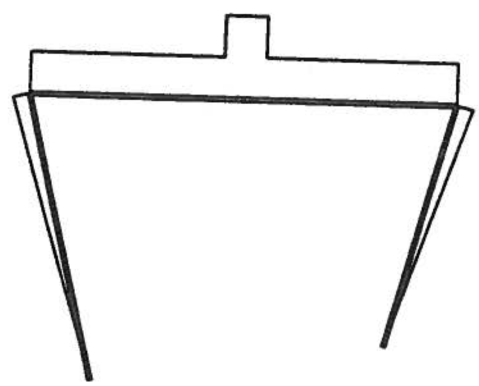

Figure 4.1.2: Palmar Edges are Drawn Bold

We solve the problem by computing a set of trajectories of the positions of the fingers and palm. The trajectories are found by executing a sequence of steps. First, we plan where to place the fingers so that the object can be lifted in the desired way. Then we plan the hand motion to lift the object into the palm creating an enveloping grasp. Finally we determine how to manipulate the object to improve the quality of the grasp without losing its enveloping character.

We make the following assumptions:

1) the fingers and hand move under position control,

2) all bodies (support, object, links of the hand, etc.) are rigid,

3) the mass and geometric properties of all of the bodies are known (possibly provided by an object recognition system and an internal data base of the hand),

4) the kinematic parameters of the hand are known,

5) the motion proceeds slowly enough to ignore dynamic effects,

6) there is no friction,

7) the initial position of the object is known (this information could be provided by a vision system).

4.2. Object Liftability

To achieve an enveloping grasp, the hand must be able to manipulate the object so as to break the contact between it and the support. In addition, the object must be gathered into the hand, creating contact between the palm and the object. In the most simple tasks, such as pick-and-place operations, we merely require a stable grasp. For more complex tasks, such as assembly operations, a stable grasp which facilitates subsequent stages of the task is needed.

Grasp planning is divided into three phases: the *pre-lift-off phase*, the *lifting phase*, and the *grip-adjustment phase*. Briefly, during the pre-lift-off phase, it is determined where to place the fingers on the object initially. The hand motion which creates an enveloping grasp of the object is determined in the lifting phase. The grip-adjustment phase is responsible for manipulating the enveloping grasp until the desired final grasp is achieved.

4.2.1. Pre-Lift-Off Phase

The goal of the pre-lift-off phase is to determine an appropriate initial grasp of the object. That is, one from which the object can be manipulated away from the support and into the hand. When the object is in contact with the support, there is an unknown force distribution along the contact interface (see Figure 4.2.1).

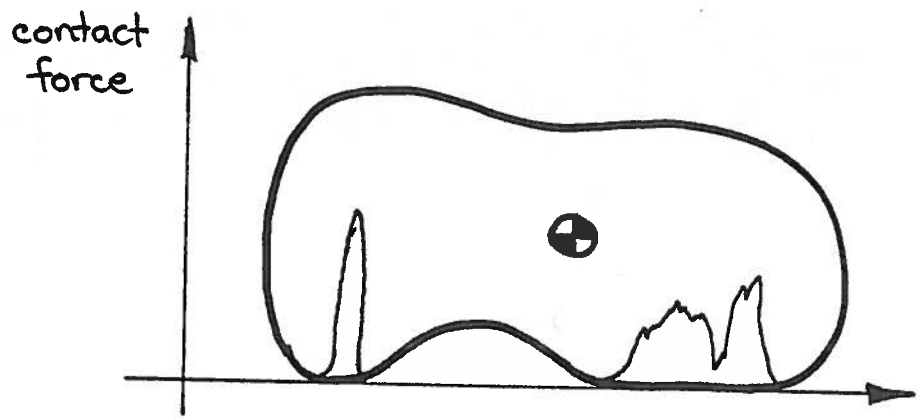

Figure 4.2.1: Hypothetical Contact Force Distribution

The exact force distribution is unimportant. What is important are the limiting cases of the force distribution which indicate impending motion. These cases are: 1) the force distribution becomes concentrated at one end of the contact interface or 2) the entire force distribution becomes zero. Therefore, we replace the actual distribution with point contact forces at the extreme ends of the contact interface (see Figure 4.2.2).

The hand shown in Figure 4.1.1 has two revolute joints. As the fingers begin to squeeze, four outcomes are possible: 1) the object tips leaving one point in contact with the support (first special force distribution), 2) the object loses all contact with the support (second special force distribution), 3) the object doesn't move; it is pressed against the support, or 4) the object slides along the support. The first and second outcomes cause the object to begin to rise, while the third and fourth do not. These observations

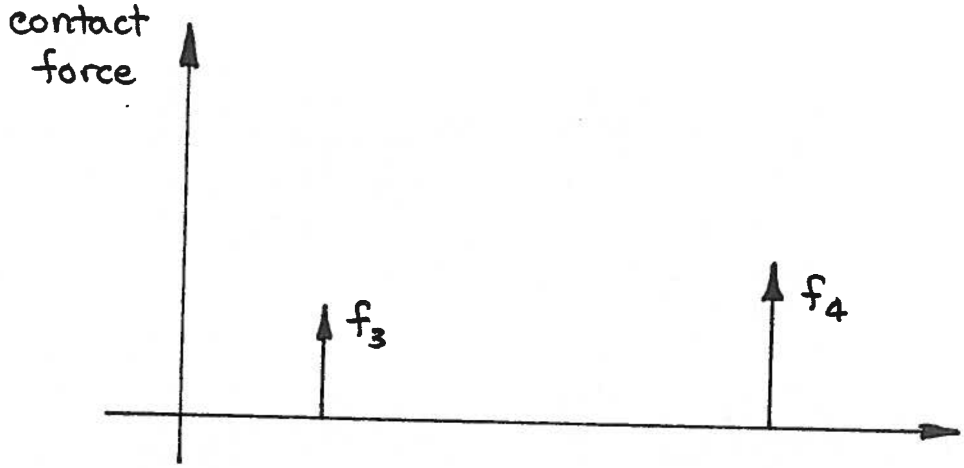

Figure 4.2.2: Modeled Contact Force Distribution

lead us to the definition of the *liftability* of an object.

Definition: An object is *liftable* if there exist finger contact positions on its perimeter (excluding the supporting edge) for which increasing those contact forces causes at least one of the supporting contact forces to become zero. Equivalently, an object is liftable if upon squeezing the fingers together, the center of gravity of the object rises.

Definition: The fingers are *squeezing* if the finger joint velocities satisfy the following inequality,

$$\dot{\theta}_{2,1} - \dot{\theta}_{1,1} \geq 0 . \tag{4.2.1}$$

If the equality is satisfied, then the joint torques must satisfy

$$\tau_{2,1} > 0 \qquad\qquad \tau_{1,1} < 0 \tag{4.2.2}$$

(see Figure 4.1.1). This definition allows one finger to rotate away from the palm as long as the other finger rotates toward the palm at a greater rate.

Definition: *Lifting* an object is synonymous with raising its center of gravity.

Definition: An initial grasp configuration is called *liftable* if it makes it possible to lift the grasped object.

It should be kept in mind that liftability is an instantaneous property depending on the geometry of the exposed perimeter and the location of the center of gravity of the object. To achieve an enveloping grasp, it is necessary but not sufficient that the object be liftable.

The liftability of an object may be determined by considering the equilibrium equations of initial grasps with one contact on each finger[10]. Referring to Figure 4.2.3 we write

$$\mathbf{W}_{obj}\,\mathbf{c}_{obj} = \begin{bmatrix} \hat{\mathbf{o}}_1 & \hat{\mathbf{o}}_2 & \hat{\mathbf{o}}_3 & \hat{\mathbf{o}}_4 \\ t_1 & t_2 & t_3 & t_4 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = -mg \begin{bmatrix} \hat{\mathbf{o}}_g \\ t_g \end{bmatrix} \tag{4.2.3}$$

$$\Phi_{obj}\,\mathbf{c}_{obj} \geq \mathbf{0} \tag{4.2.4}$$

where $\hat{\mathbf{o}}_i = \begin{bmatrix} \cos(\psi_i) \\ \sin(\psi_i) \end{bmatrix}$ is the i^{th} contact normal; ψ_i is the angle that the i^{th} contact normal makes with respect to the support; $\hat{\mathbf{o}}_g = [0\ -1]$ is the direction in which gravity acts; t_i and t_g are the moment arms of the i^{th} contact force and the gravity load, respectively, taken with respect to the summing point, $\mathbf{q}$; c_i is the magnitude of the i^{th} contact force; and Φ_{obj} is the identity matrix for frictionless contact points. Equation (4.2.3) can be solved analytically. First we choose $\mathbf{q}$ to be at the intersection of the lines of action of the first and fourth contact forces. Next, because $\mathbf{W}$ is three by four, we move its second column and c_2 to the right hand side. Now the equations may be solved in terms of c_2 by inverting the three by three matrix remaining on the left

[10] This is the simplest type of initial grasp which can result in quasistatic lifting.

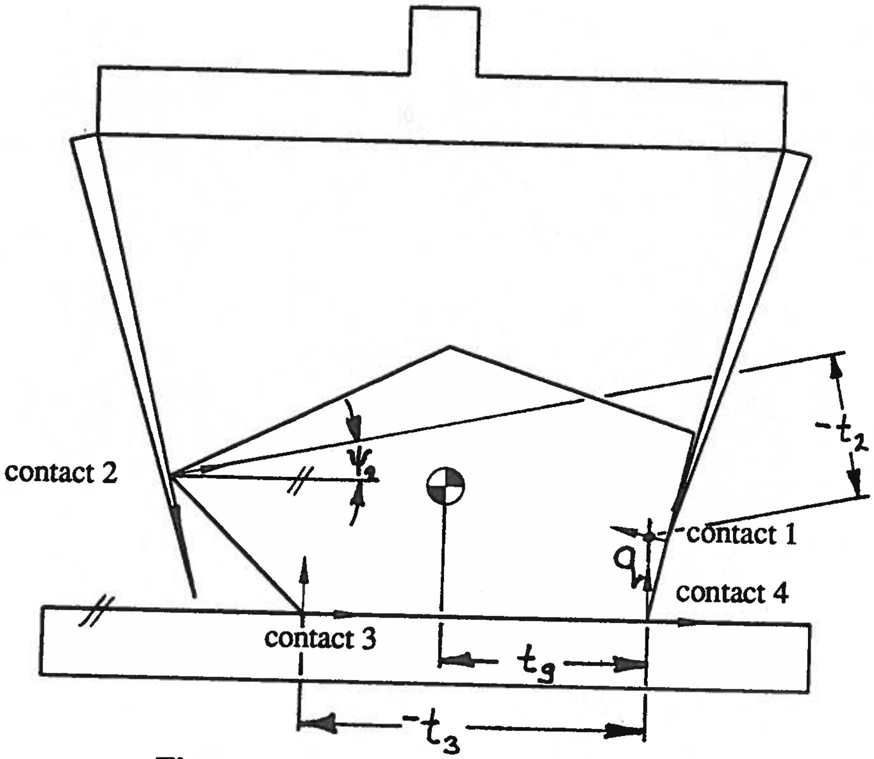

Figure 4.2.3: Two-Point Initial Grasp

$$\begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ c_{3,0} \\ c_{4,0} \end{bmatrix} + c_2 \begin{bmatrix} n_{21} \\ 1 \\ n_{23} \\ n_{24} \end{bmatrix} \tag{4.2.5}$$

where

$$c_{3,0} = mg\frac{t_g}{-t_3} \qquad c_{4,0} = mg\frac{t_3 + t_g}{t_3}, \tag{4.2.6}$$

$$n_{2,1} = -\frac{\cos(\psi_2)}{\cos(\psi_1)} \qquad n_{2,3} = -\frac{t_2}{t_3} \qquad n_{2,4} = \frac{t_2}{t_3} + \frac{\sin(\psi_1 - \psi_2)}{\cos(\psi_1)}. \tag{4.2.7}$$

The first term on the right hand side of equation (4.2.5) is the vector of contact force magnitudes before the hand contacts the object. For the object to be in stable equilibrium prior to being picked up, $c_{3,0}$ and $c_{4,0}$ must be strictly positive. This requirement induces restrictions on the values of t_g and t_3

$$t_3 < 0 \qquad t_g > 0 \qquad |t_3| > t_g \,. \tag{4.2.8}$$

Inequalities 4.2.8 imply that the line of action of the gravity force must pass between the two supporting contacts. In the limiting case as the supporting contacts move together, it is possible for an object to be in equilibrium on the support with only one contact (*e.g.* objects with smooth convex surfaces), however, stability depends on second order effects. Objects with only one supporting contact will be considered in Section 4.2.2.

The second term on the right hand side of equation (4.2.5) is the magnitude of the second contact force times the basis vector of the null space of the wrench matrix, **W**. It expresses how the contact force magnitudes must vary relative to one another to maintain the object's equilibrium. The null space vectors are commonly known as the internal grasp forces [14, 33, 47]. Plotting c_1, c_3, and c_4 as functions of c_2, we see that for an object to be liftable, at least one of $n_{2,3}$ and $n_{2,4}$ must be negative (see Figure 4.2.4). Before the hand contacts the object, c_2 is zero. After contact, the fingers begin to squeeze, increasing c_1 and c_2. Motion of the object begins when c_2 reaches the value c_2^*,

$$c_2^* = \min\{h_3, h_4\} \tag{4.2.9}$$

where

$$h_3 = -\frac{c_{3,0}}{n_{2,3}} \qquad h_4 = -\frac{c_{4,0}}{n_{2,4}} \,. \tag{4.2.10}$$

We now proceed with a closer examination of the null space vector. The first equation in expression (4.2.5)

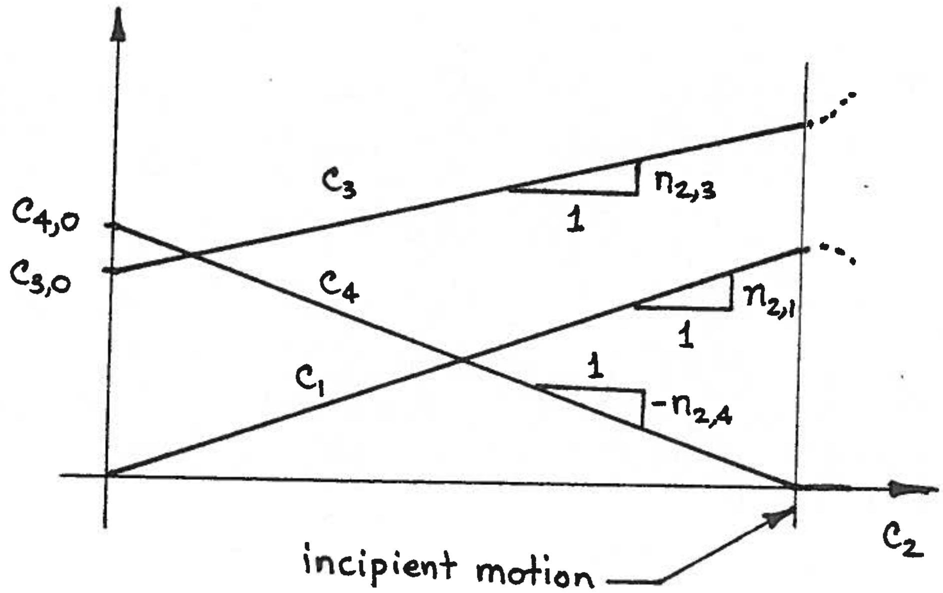

Figure 4.2.4: Contact Force Magnitudes

$$c_1 = -c_2 \frac{\cos(\psi_2)}{\cos(\psi_1)} \tag{4.2.11}$$

implies that the sum of the x-components of the two finger contact forces must be zero to maintain equilibrium. In addition, since c_1 must be positive, the signs of $\cos(\psi_1)$ and $\cos(\psi_2)$ must be opposite. This leads to a natural partitioning of the reachable perimeter of the object. We choose the valid contact region, I, for the first finger to be that part of the exposed perimeter for which the x-component of the surface normal is negative. The valid contact region, II, for the second finger must have a positive x-component (see Figure 4.2.5). Any point on the perimeter which has a vertical normal belongs to neither I nor II. Mathematically, the regions are defined as

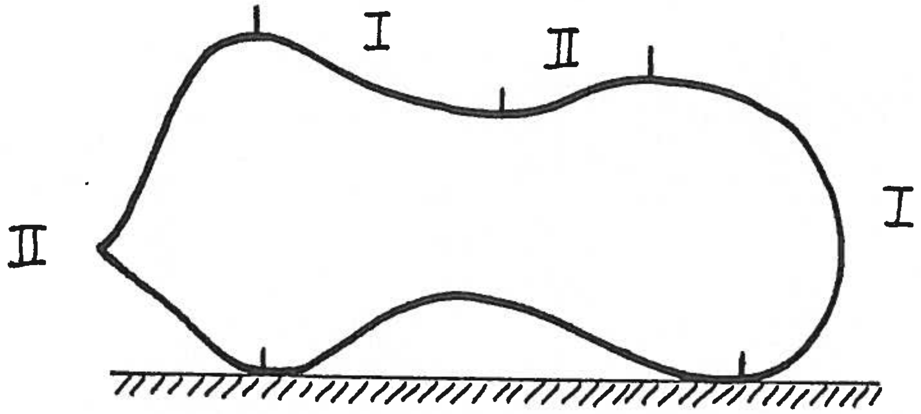

Figure 4.2.5: Regions *I* and *II*

$$\cos(\psi_1) < 0 \quad \text{or} \quad \frac{\pi}{2} < \psi_1 < \frac{3\pi}{2} \tag{4.2.12}$$

$$\cos(\psi_2) > 0 \quad \text{or} \quad \frac{-\pi}{2} < \psi_2 < \frac{\pi}{2} . \tag{4.2.13}$$

To break the third contact, $n_{2,3}$ must be less than zero

$$n_{2,3} = -\frac{t_2}{t_3} < 0 . \tag{4.2.14}$$

Since the moment of the third contact force, t_3, is fixed and negative, we see from equations (4.2.8) and (4.2.7) that t_2 must also be negative

$$t_2 < 0 . \tag{4.2.15}$$

Therefore, an object is liftable if finger contacts can be found for which the line of action of the second contact force passes above the summing point, $\mathbf{q}$. More precisely, the forces $\mathbf{f}_3$ and $\mathbf{f}_2$ must produce moments of the same sense about $\mathbf{q}$ which is true for the initial grasp shown in Figure 4.2.3.

To break the fourth contact, $n_{2,4}$ must be less than zero

$$n_{2,4} = \frac{t_2}{t_3} + \frac{\sin(\psi_1 - \psi_2)}{\cos(\psi_1)} < 0 . \tag{4.2.16}$$

Recalling that $\cos(\psi_1)$ is negative, inequality (4.2.16) can be written as

$$\sin(\psi_1 - \psi_2) > \frac{-t_2}{t_3} \cos(\psi_1) . \tag{4.2.17}$$

Figure 4.2.6 illustrates inequality (4.2.17).

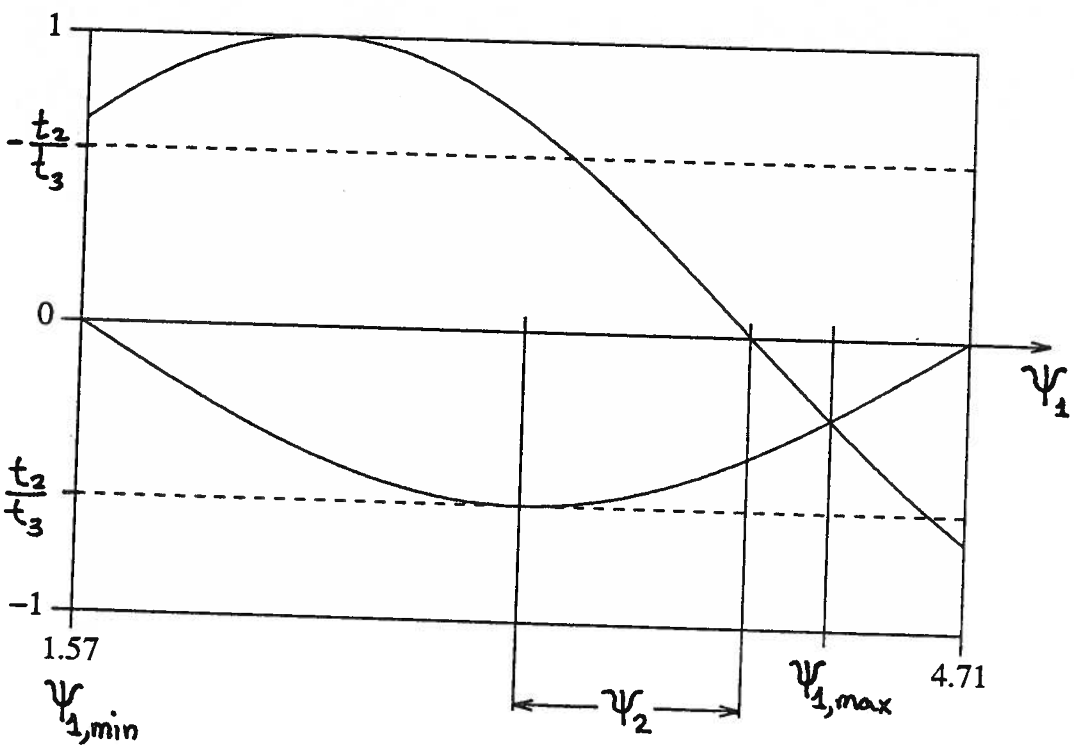

Figure 4.2.6: Graphical Interpretation of Inequality (4.2.17)

If the angle of the first contact lies within the open interval, $(\psi_{1,min}, \psi_{1,max})$, then the object can be lifted breaking the fourth contact.

The values of the ψ_1 and ψ_2 for which inequality (4.2.17) can be satisfied for a given moment arm ratio, $\frac{t_2}{t_3}$, can be plotted as shown in Figure 4.2.7. An initial grasp corresponding to a point below the curve may [11] cause the fourth contact to break, $B4$.

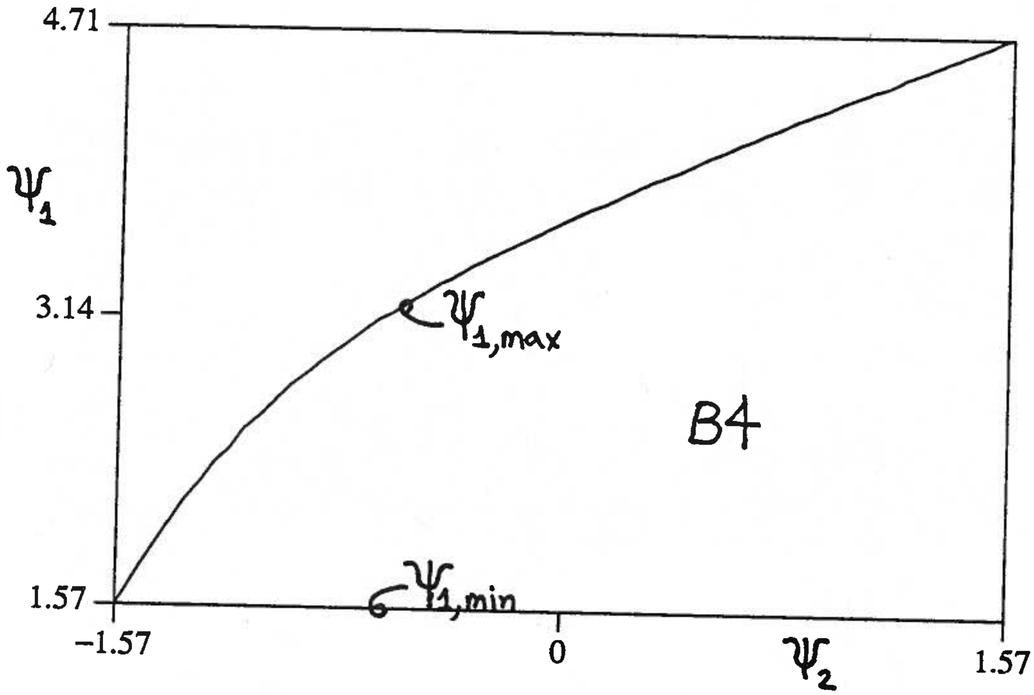

Figure 4.2.7: Region $B\,4$

Initial grasp configurations lying above or on the curve cannot. Letting the moment arm ratio take on all possible values, we can generate the Initial Grasp Liftability Chart, IGLiC. To determine the outcome of squeezing with any initial grasp, the following procedure may be followed.

1) Determine the value of the moment arm ratio.

2) Locate the curve on the IGLiC corresponding to that value. If the curve lies below the diagonal, the third contact may break.

3) Find the point corresponding to the finger contact angles.

If the point lies outside the IGLiC, then the object can only slide along the support, because the x-components of the finger contact forces are in the same direction. That is, both contacts are in the same region, *I* or *II*. If the point lies on the chart above the

[11] The word, may, is emphasized because it is possible that both inequalities (4.2.14) and (4.2.17) are satisfied, but the third contact will break.

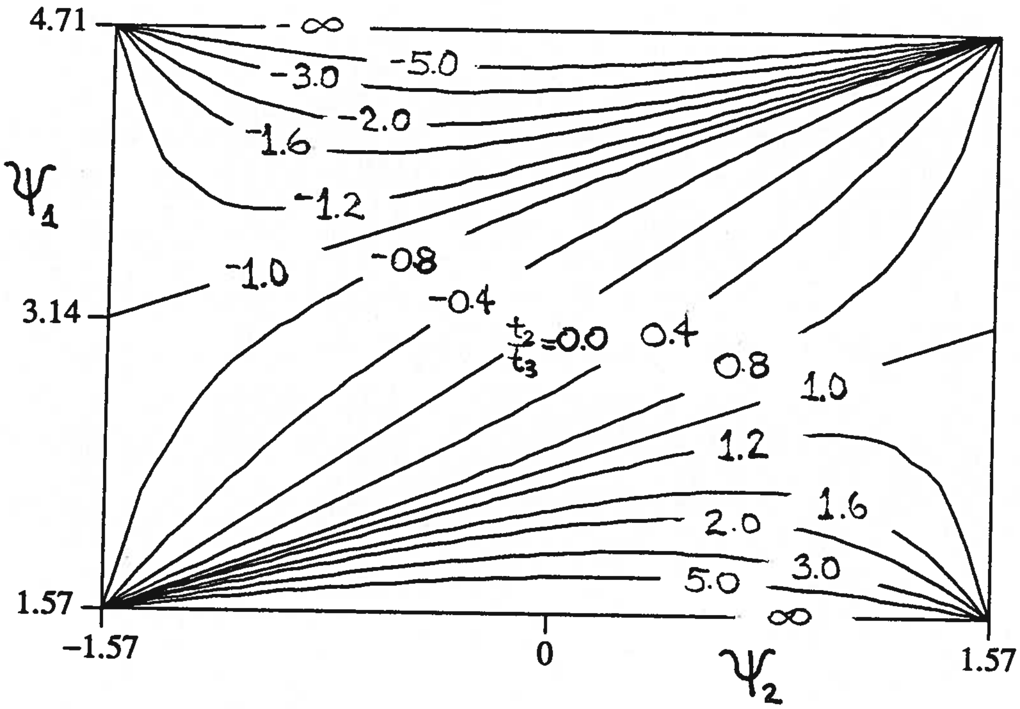

Figure 4.2.8: Initial Grasp Liftability Chart, IGLiC

curve, then the fourth contact cannot be broken. If the point lies below the curve, then the fourth contact may be broken. If the point lies below the curve which lies below the diagonal, then either the third or fourth contact may break. Which one will break can be determined by examining the values of h_3 and h_4

$$h_3 = \frac{mgt_g}{-t_2} \tag{4.2.18}$$

$$h_4 = \frac{-mg(t_3 + t_g)\cos(\psi_1)}{t_2 \cos(\psi_1) + t_3 \sin(\psi_1 - \psi_2)} . \tag{4.2.19}$$

The functions h_3 and h_4 are the values of the magnitude of the second contact force, c_2, for which the third and fourth contacts will break, respectively. The contact which will break is the one with the lower value of h.

Note that the IGLiC is valid for any grasping problem for which the relative motion of the object and the hand is planar, there are 2 or more contacts on a flat support, and friction is negligible.

Example 4.2.1

This example shows five possible initial grasps of and object. The contact angles point is shown for each initial grasp on the IGLiC shown below.

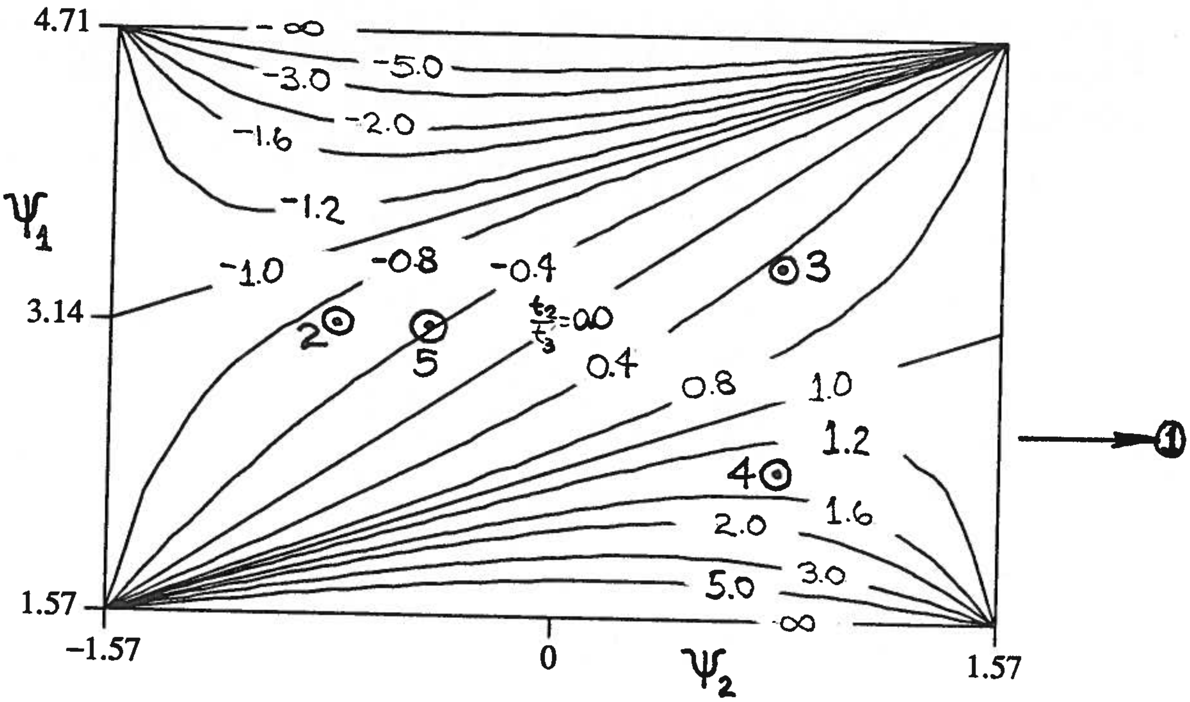

Figure 4.2.9: Initial Grasp Configurations

Figure 4.2.10 shows a possible initial grasp. The parameters of interest are

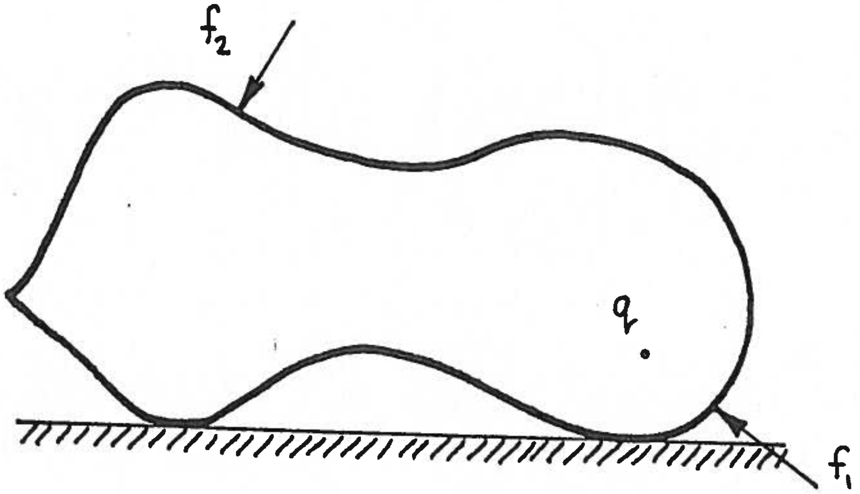

Figure 4.2.10: Possible Initial Grasp Points

$$\frac{t_2}{t_3} = *** \qquad \psi_1 = \frac{3\pi}{4} \qquad \psi_2 = \frac{4\pi}{3} .$$

The point corresponding to the contact angles lies outside of the IGLiC, so squeezing causes the object to slide along the support. Note that the value of the moment arm ratio is not relevant.

Figure 4.2.11 shows another possible initial grasp.

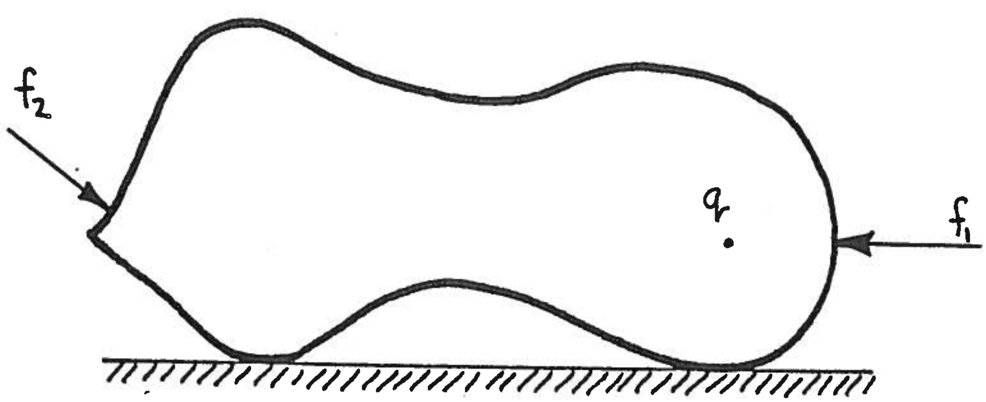

Figure 4.2.11: Possible Initial Grasp Points

The parameters of interest are

$$\frac{t_2}{t_3} = -0.75 \quad \psi_1 = \pi \quad \psi_2 = -\frac{\pi}{4}.$$

The contact angles lie on the IGLiC below the curve defined by $\frac{t_2}{t_3} = -0.75$ which lies above the diagonal, so the fourth contact will break.

Figure 4.2.12 shows another possible initial grasp.

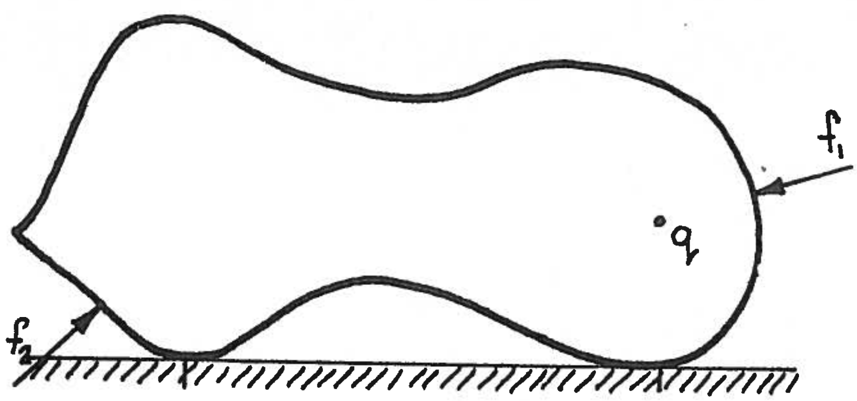

Figure 4.2.12: Possible Initial Grasp Points

The parameters of interest are

$$\frac{t_2}{t_3} = 0.7 \quad \psi_1 = \frac{7\pi}{6} \quad \psi_2 = \frac{\pi}{4}.$$

The contact angles lie on the IGLiC above the curve defined by $\frac{t_2}{t_3} = 0.7$ which lies below the diagonal, so the third contact will break.

Figure 4.2.13 shows another possible initial grasp. The parameters of interest are

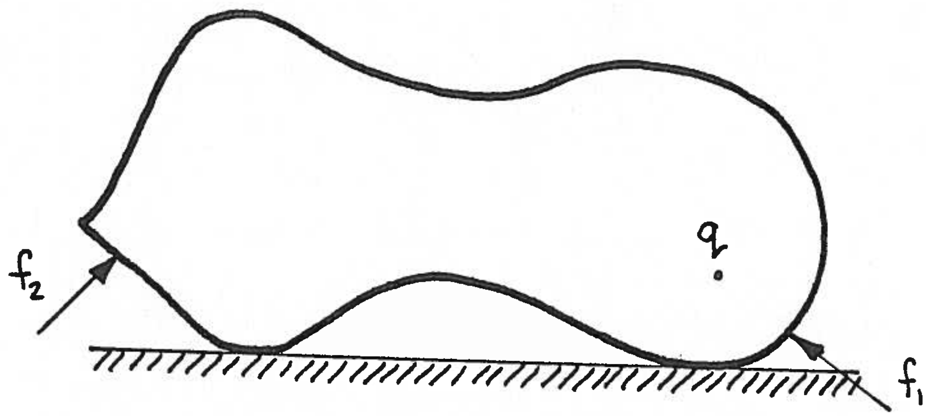

Figure 4.2.13: Possible Initial Grasp Points

$$\frac{t_2}{t_3} = 1.0 \quad \psi_1 = \frac{3\pi}{4} \quad \psi_2 = \frac{\pi}{4}.$$

The contact angles lie on the IGLiC below the curve defined by $\frac{t_2}{t_3} = 1.0$ which lies below the diagonal, so either the third or fourth contact will break.

Figure 4.2.14 shows another possible initial grasp.

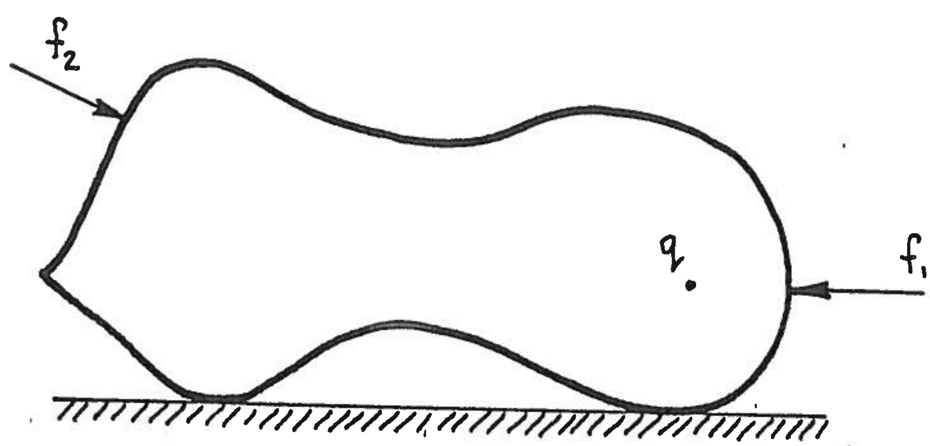

Figure 4.2.14: Possible Initial Grasp Points

The parameters of interest are

$$\frac{t_2}{t_3} = -0.25 \quad \psi_1 = \pi \quad \psi_2 = -\frac{\pi}{6} .$$

The contact angles lie on the IGLiC above the curve defined by $\frac{t_2}{t_3} = -0.25$ which lies above the diagonal, so neither the third nor fourth contact may break; the object will be pressed against the support.

4.3. Liftability Regions

In the previous section, we described a natural partition of the perimeter of any object. That partitioning only guaranteed that the x-components of the fingers' contact forces would cancel. It gave no information as to the motion resulting when squeezing the object.

In this section, we show how to partition the perimeter into four liftability regions, $B3$, $B4$, T, and J. The regions correspond to breaking the third contact, breaking the fourth contact (see Figure 4.2.3 for contact numbers) translating with an upward component of velocity, and a jamming region for which the object cannot be lifted, respectively.

Equations (4.2.18) and (4.2.19) give the magnitudes of the second contact force, c_2, necessary to break the third and fourth contacts, respectively

$$h_3 = \frac{mgt_g}{-t_2} = c_2 \tag{4.2.18}$$

$$h_4 = \frac{-mg(t_3 + t_g)\cos(\psi_1)}{t_2 \cos(\psi_1) + t_3 \sin(\psi_1 - \psi_2)} = c_2 . \tag{4.2.19}$$

Of the third and fourth contacts, the one requiring the smallest positive value of c_2 will break. For a given object, the variables which change as a function of the configuration of the grasp are t_2, ψ_1, and ψ_2. All of the other quantities are constant for a given an initial orientation of the object. In the following development of the liftability regions, the position of the contact of the first finger is held fixed at a point in region I making ψ_1 constant. The position of the second contact point may be anywhere in II, so the values of ψ_2 and t_2 vary. Computing the values of h_3 and h_4 at all points on the perimeter for which the second finger may be placed, the regions $B3$, $B4$, T, and J may be defined (see Figure 4.3.1). Using these regions, the outcome of squeezing is known prior to placing the finger on the object.

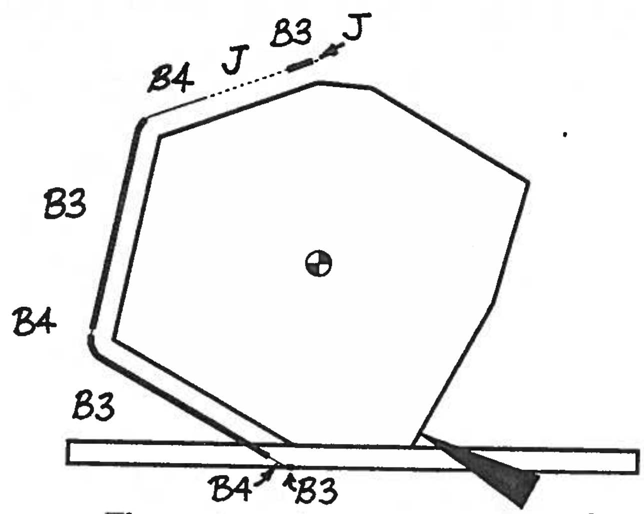

Figure 4.3.1: Liftability Regions [12]

4.3.1. Liftability Regions of Polygons

A polygon can be used to approximate any two dimensional object with arbitrary precision. The polygon also contains both extremes of the possible local radius of curvature of any object. Therefore we discuss the liftability regions of polygons in detail.

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[12] The arrowhead indicates the position of the contact point of the first finger. The dashed, thin, and bold curve offset to the left of the object encodes the information as to how the object will lift-off when squeezed. It is explained at the end of Section 4.3.4.

First, consider the edge of the polygon shown in Figure 4.3.2.

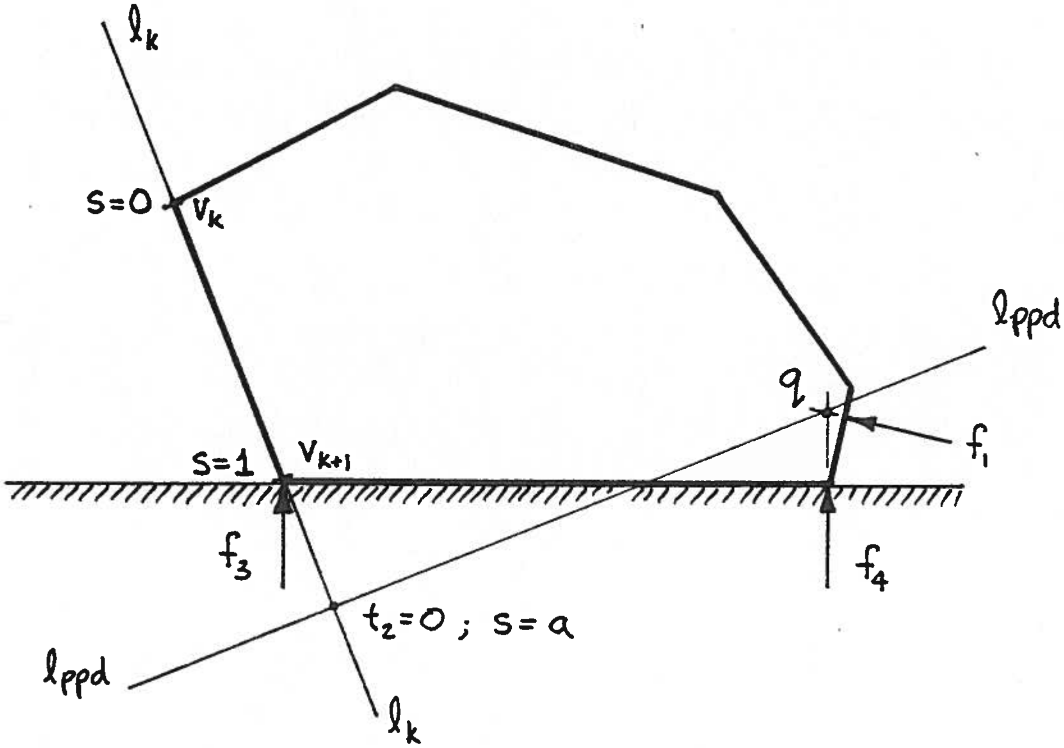

Figure 4.3.2: Quantities for Edge Liftability Regions

Between the vertices $\mathbf{v}_k$ and $\mathbf{v}_{k+1}$ on the line l_k, lies the k^{th} edge which belongs to region *II*. Because of its orientation, the entire edge belongs to *II* and thus every point on it is valid for contact by the second fingertip. All points, $\mathbf{p}$, lying on the line can be represented in parametric form as

$$(1-s)\,\mathbf{v}_k + s\,\mathbf{v}_{k+1} = \mathbf{p} \tag{4.3.1}$$

where s is the distance along the line from $\mathbf{v}_k$ to $\mathbf{v}_{k+1}$. Constraining s in the following way defines the k^{th} edge of the polygon

$$0 \le s < 1\,. \tag{4.3.2}$$

The line, l_{ppd}, is the unique line which contains the summing point, $\mathbf{q}$, and is perpendicular to l_k. The intersection is the point on the k^{th} edge where the moment arm of the second contact force, t_2, is zero. The variables s and t_2 are linearly related by

$$t_2 = s - a \tag{4.3.3}$$

where a is the value of s at the intersection of l_k and l_{ppd}. Substituting equation (4.3.3) into inequality (4.2.15), the region of the edge for which the third contact can break is the half-line given by

$$s < a . \tag{4.3.4}$$

Since t_2 varies linearly along the edge, h_3, defined by equation 4.2.18, describes a hyperbola along l_k (see Figure 4.3.3).

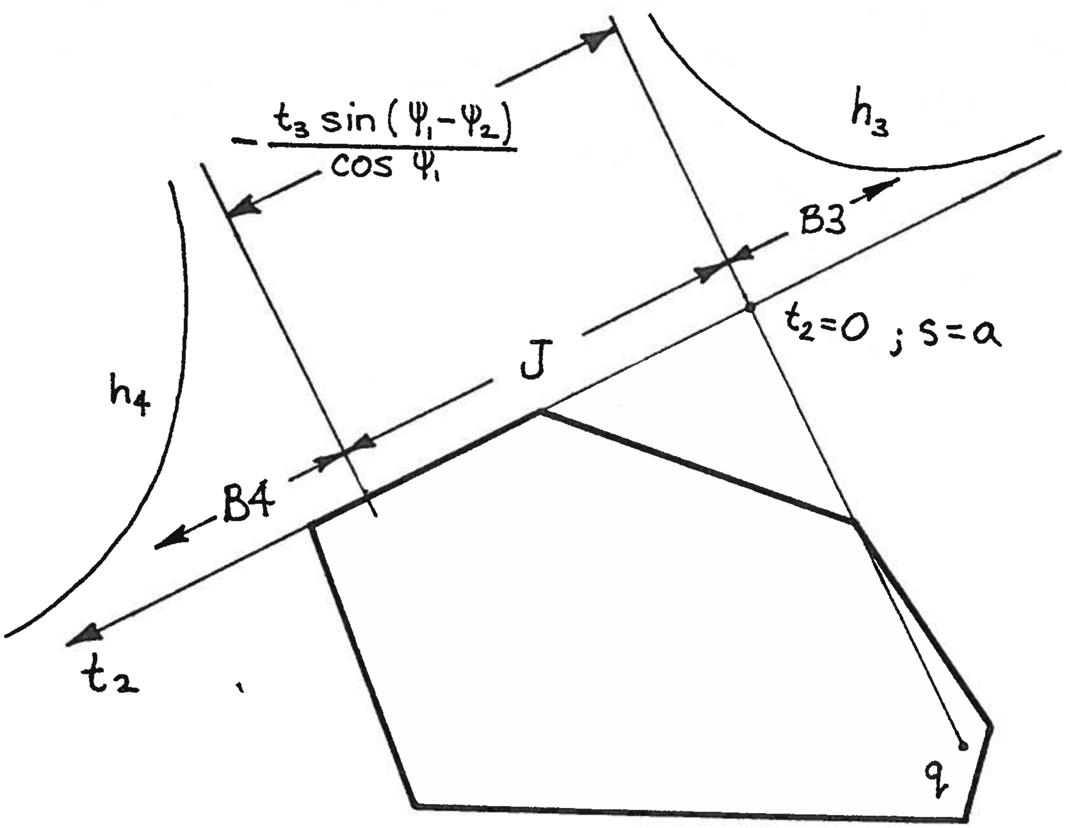

Figure 4.3.3: Liftability Regions, $sin(\psi_1 - \psi_2) < 0$

The function h_4 also defines a hyperbola, but on the half-line defined by (see Figure

4.3.3)

$$s > a - \frac{t_3 \sin(\psi_1 - \psi_2)}{\cos(\psi_1)} . \tag{4.3.5}$$

Note that the negative halves of these hyperbolas are unimportant since they imply tension rather than compression at the contact interface.

Figure 4.3.3 shows h_4 with its infinite asymptote at a positive value of t_2. This is the case defined by the following inequality

$$\sin(\psi_1 - \psi_2) < 0 . \tag{4.3.6}$$

When inequality (4.3.6) is satisfied, a jamming region, J, is that portion of the edge which lies between the two vertical asymptotes of the hyperbolas. The regions $B3$ and $B4$ are defined by inequalities (4.3.4) and (4.3.6) respectively, but of course do not extend beyond the ends of the edge,

$$-a \leq t_2 < 1 - a . \tag{4.3.7}$$

Since the values of h_3 and h_4 are not equal at any point on the edge, the translation region, T, is empty. The physical interpretation of inequality (4.3.6) is that the resultant of the finger contact forces is in the direction of the gravity force. Therefore, to avoid jamming and to cause tipping, one must push down on the edge in the correct place.

If the sense of inequality (4.3.6) is reversed,

$$\sin(\psi_1 - \psi_2) > 0 , \tag{4.3.8}$$

then the functions h_3 and h_4 over lap, eliminating the jamming region (see Figure 4.3.4). In this case, the liftability regions, $B3$ and $B4$, meet at the cross-over point defined by

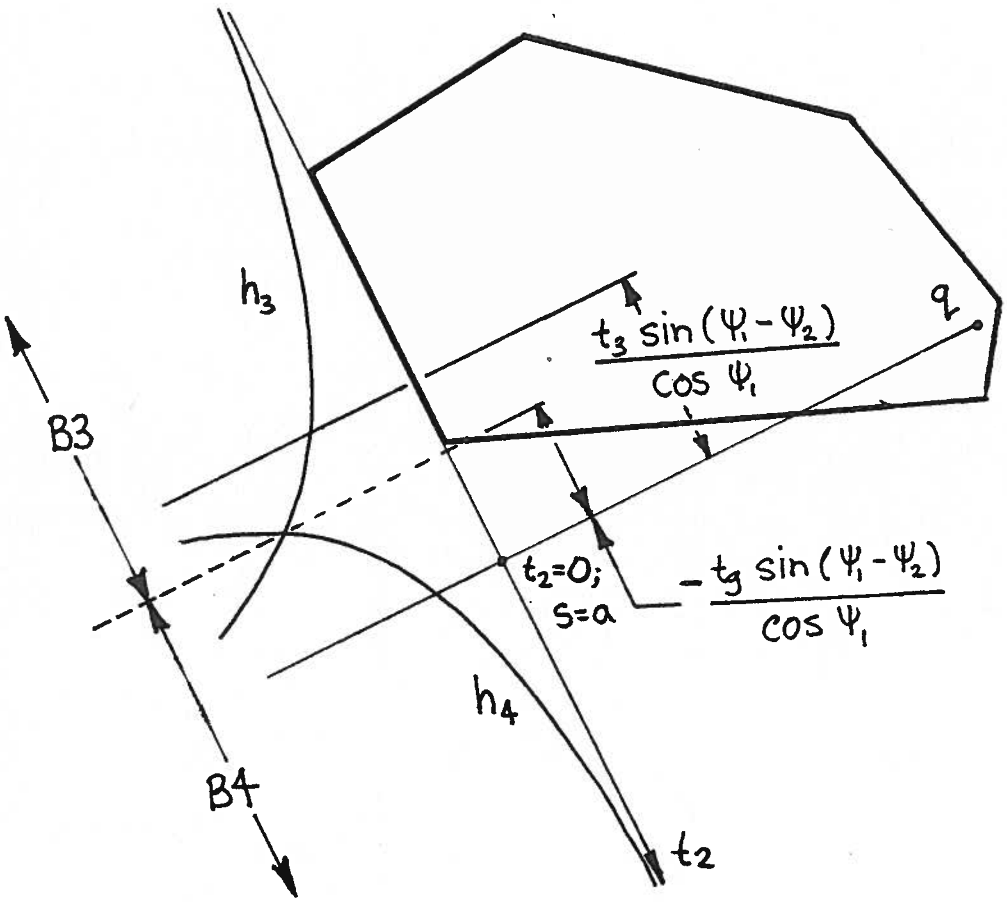

Figure 4.3.4: Liftability Regions, $sin(\psi_1 - \psi_2) > 0$

$$t_{2,crossover} = \frac{t_g \sin(\psi_1 - \psi_2)}{cos(\psi_1)} < 0 \tag{4.3.9}$$

which is the only point on the edge which is an element of the liftability region T. Physically, if inequality (4.3.8) is satisfied, the resultant of the finger contact forces is in the direction opposite to gravity. Therefore, as the hand squeezes more and more tightly, the weight of the object is overcome and it must rise.

The second contact point need not occur on an edge of the polygon. The contact may occur at the k^{th} vertex with the contact angle free to vary between the inward normals of edges k and $k-1$ (see Figure 4.3.5). The contact point is fixed, but the moment arm of the contact varies as a function of the contact angle according to

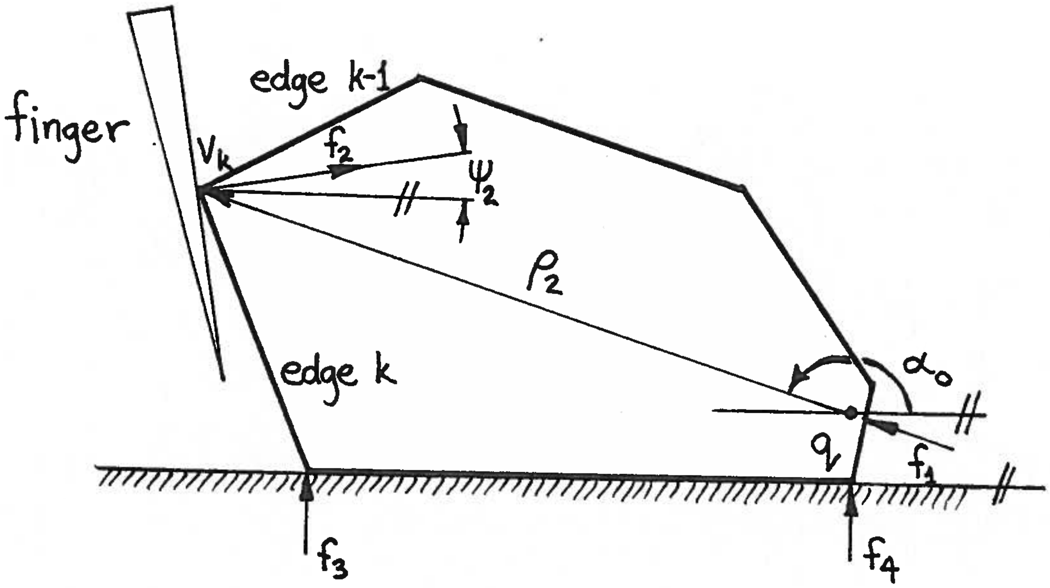

Figure 4.3.5: Quantities for Vertex Liftability Regions

$$t_2 = -|\rho_2| \sin(\psi_2 - \alpha_0)\,. \tag{4.3.10}$$

The liftability regions for a vertex are defined as partitions of the range of possible contact angles available at the k^{th} vertex. Substituting equation (4.3.10) into equations (4.2.18) and (4.2.19), the liftability regions can be determined as

$$h_3 = \frac{mgt_g}{|\rho_2| \sin(\psi_2 - \alpha_0)} \tag{4.3.11}$$

$$h_4 = \frac{-mg(t_3 - t_g)\, cos(\psi_1)}{-|\rho_2| \sin(\psi_2 - \alpha_0)\, cos(\psi_1) + t_3\, sin(\psi_1 - \psi_2)}\,. \tag{4.3.12}$$

Figure 4.3.6 shows h_3 and h_4 and the liftability regions of a typical vertex. The edge of the finger is shown against the vertex in the only orientation of T. Tilting the finger clockwise or counterclockwise changes the contact to region $B4$ or $B3$ respectively. Note that the regions are shown using the outward normal so that the display does not obscure the edges of the polygon.

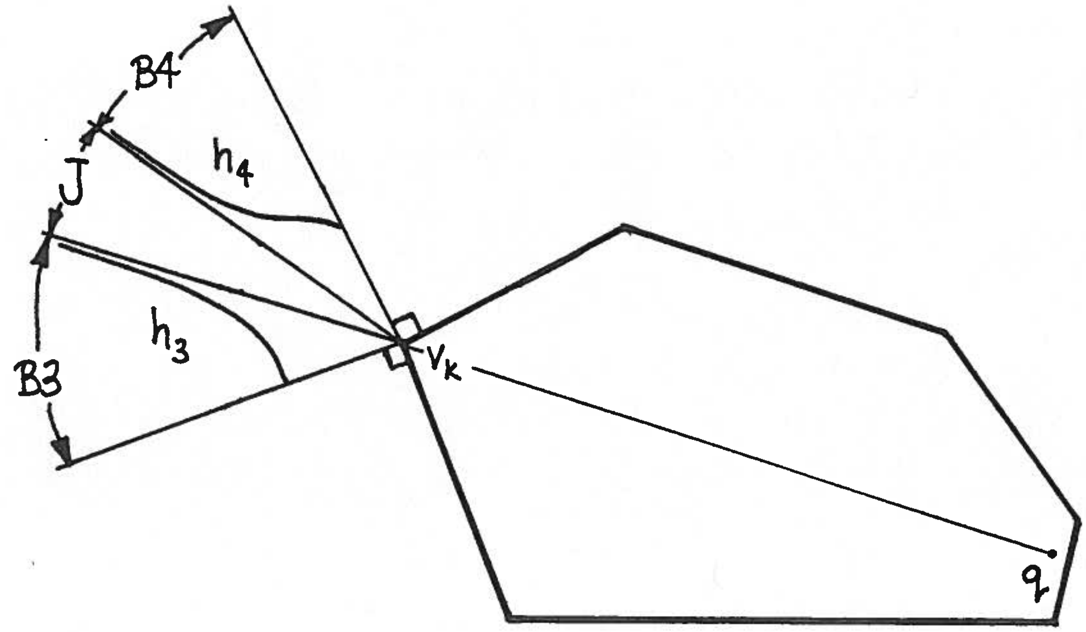

Figure 4.3.6: Vertex Liftability Regions

4.3.2. Lift-Off from One Point

Using the liftability regions, an initial grasp can be chosen which enables the hand to break only one of the supporting contact points[13]. While this constitutes progress toward achieving an enveloping grasp, there is no provision for breaking the remaining support contact. Since we want never to to lose control of the object, we require that the grasp have either force or form closure. Therefore there must be at least three contact points on the object. An initial grasp which utilizes the liftability regions has one contact point on each finger (this is called a two-point initial grasp) and two on the support. After squeezing the fingers a small amount, there are three contacts: one on the support and two on the fingers. In order to break the contact on the support, the hand

[13] Both contacts cannot be broken simultaneously because T consists of distinct points. The second finger cannot be made to contact a single point or to contact a vertex at an exact angle unless there is no error in the system.

must gain a third contact.

If a new contact (which we denote by 5) is achieved in the correct configuration, then squeezing the fingers can cause the object to be lifted away from the support (see Figure 4.3.7) and the resulting grasp is stable by force closure.

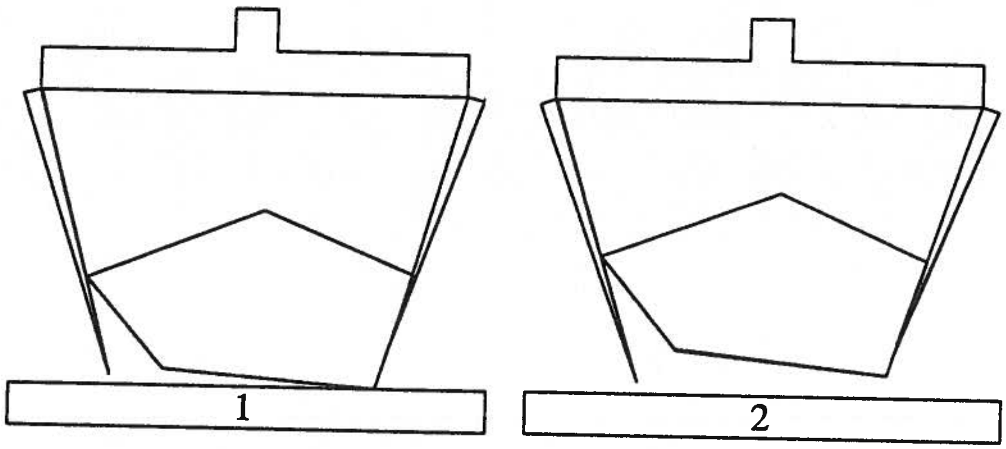

Figure 4.3.7: Lift-Off from One Point

The equation of equilibrium prior to lift-off is

$$\mathbf{W}_{obj}\,\mathbf{c}_{obj} = \begin{bmatrix} \hat{\mathbf{o}}_1 & \hat{\mathbf{o}}_2 & \hat{\mathbf{o}}_3 & \hat{\mathbf{o}}_5 \\ t_1 & t_2 & t_3 & t_5 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_5 \end{bmatrix} = -mg \begin{bmatrix} \hat{\mathbf{o}}_g \\ t_g \end{bmatrix} \tag{4.3.13}$$

which is the same as equation (4.2.3) except that the fourth contact has become the fifth. For the remaining support contact to break, a solution of equation (4.3.13) must exist which satisfies the following relationships

$$c_3 = 0 \quad c_1 > 0 \quad c_2 > 0 \quad c_5 > 0\,. \tag{4.3.14}$$

Removing the third column of the wrench matrix and the variable, c_3, the solution to equation (4.3.13) is

$$\begin{bmatrix} c_1 \\ c_2 \\ c_5 \end{bmatrix} = mg\frac{\begin{bmatrix} t_2 cos(\psi_5) - t_5 cos(\psi_2) - t_g \sin(\psi_5 - \psi_2) \\ t_5 cos(\psi_1) - t_1 cos(\psi_5) - t_g \sin(\psi_1 - \psi_5) \\ t_1 cos(\psi_2) - t_2 cos(\psi_1) - t_g \sin(\psi_2 - \psi_1) \end{bmatrix}}{t_2 sin(\psi_1 - \psi_5) + t_5 sin(\psi_2 - \psi_1) + t_1 sin(\psi_5 - \psi_2)} . \tag{4.3.15}$$

This equation is valid unless all three lines of the contact forces intersect at a point. If c_1, c_2, and c_5 are positive, the third contact will break. Next we choose the summing point, **q**, to be at the intersection of the gravity force and the first contact force so that t_1 and t_g become zero. Examining the solution (4.3.15), it can be shown that for the support contact to break, the moment arm of the second contact must have the same sign as the denomiriator of the solution and the moment arm of the fifth contact must have the opposite sign,

$$sign(t_2) = sign[t_2 sin(\psi_1 - \psi_5) + t_5 sin(\psi_2 - \psi_1) + t_1 sin(\psi_5 - \psi_2)] \tag{4.3.16}$$
$$sign(t_2) = -sign(t_5) . \tag{4.3.17}$$

The grasp in the first frame of Figure 4.3.7 satisfies these requirements, so squeezing the fingers together causes the support contact to break.

4.3.3. Translational Lift-Off

One strategy for breaking both support contacts is to use a two-point initial grasp to break one support contact, then manipulate the object to gain a new contact on the fingers which facilitates breaking the other support contact. The first part of the strategy can be accomplished using the tippability regions, but no satisfactory technique is available to gain the contact required to break the remaining support contact. In the interest of simplifying the above strategy, we seek an initial grasp with three contacts (called a three-point initial grasp) which allows the object to break both support contacts simultaneously. This strategy results in an initial grasp which is capable of breaking both support contacts instantaneously, whereas the original strategy requires planning finite hand motions to achieve the same end.

A desirable grasp will have one of the fingers lying against one of the edges of the object as shown in Figure 4.3.8.

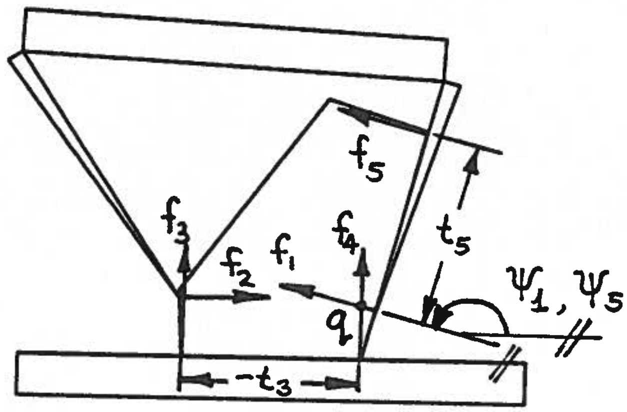

Figure 4.3.8: Three-Point Initial Grasp

The equilibrium equation are

$$\mathbf{W}_{obj}\,\mathbf{c}_{obj} = \begin{bmatrix} \hat{\mathbf{o}}_1 & \hat{\mathbf{o}}_2 & \hat{\mathbf{o}}_3 & \hat{\mathbf{o}}_4 & \hat{\mathbf{o}}_5 \\ t_1 & t_2 & t_3 & t_4 & t_5 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \end{bmatrix} = -mg \begin{bmatrix} \hat{\mathbf{o}}_g \\ t_g \end{bmatrix} . \tag{4.3.18}$$

The particular solution of equation (4.3.18) in which we are interested is the one for which

$$c_3 = 0 \quad c_4 = 0 \qquad c_1 > 0 \quad c_2 > 0 \quad c_5 > 0 . \tag{4.3.19}$$

Removing columns 3 and 4 from $\mathbf{W}_{obj}$ and solving gives equation (4.3.15). Next we choose $\mathbf{q}$ to be the intersection of the lines of action of the 1^{st} and 4^{th} contact forces. Noting that $\psi_1 = \psi_5$ and combining equations (4.3.15) and inequalities (4.3.19) yields the inequality constraints which must be satisfied for translational lift-off

$$\frac{-mg}{t_5 sin(\psi_1 - \psi_2)} \begin{bmatrix} t_2 cos(\psi_1) - t_5 cos(\psi_2) - t_g sin(\psi_1 - \psi_2) \\ t_5 cos(\psi_1) \\ -t_2 cos(\psi_1) + t_g sin(\psi_1 - \psi_2) \end{bmatrix} > \mathbf{0}\,. \qquad (4.3.20)$$

The inequalities may be rewritten as

$$sin(\psi_1 - \psi_2) > 0 \qquad (4.3.21)$$

$$t_5 > 0 \qquad (4.3.22)$$

$$\frac{t_g sin(\psi_1 - \psi_2)}{cos(\psi_1)} + \frac{t_5 \cos(\psi_2)}{cos(\psi_1)} < t_2 < \frac{t_g sin(\psi_1 - \psi_2)}{cos(\psi_1)} < 0\,. \qquad (4.3.23)$$

These inequalities define a portion of edge known as the translation region for which squeezing with the second finger causes the object to break both support contacts simultaneously. In Figure 4.3.9 the translation region is indicated by the double bold line segment.

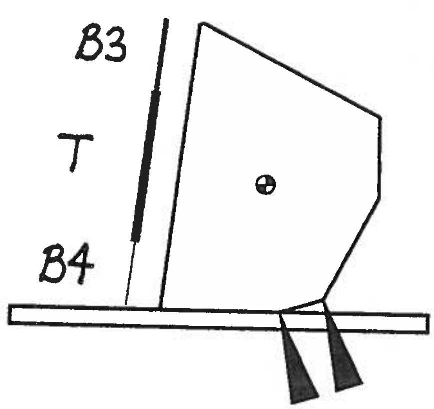

Figure 4.3.9: Region, *T*, for an Edge

The translation region for a vertex is determined by substituting equation (4.3.10) into inequality (4.3.23) yielding

$$sin(\psi_1 - \psi_2) > 0 \qquad (4.3.21)$$

$$t_5 > 0 \tag{4.3.22}$$

$$\frac{t_g sin\,(\psi_1 - \psi_2)}{cos\,(\psi_1)} + \frac{t_5 \cos(\psi_2)}{cos\,(\psi_1)} < -|\rho_2| \sin(\psi_2 - \alpha_0) < \frac{t_g sin\,(\psi_1 - \psi_2)}{cos\,(\psi_1)} < 0\,. \tag{4.3.24}$$

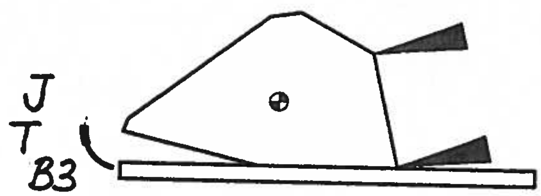

Figure 4.3.10: Region, *T*, of a Vertex

An interesting property of the translation region for a given edge is that the force to lift the object is independent of the position of the second contact within *T*. Also, as expected, the force is greater than the maximum force required to tip the object keeping one contact on the support.

4.3.4. Graphical Construction of Liftability Regions

The initial grasp liftability chart and the translation curves may be used to determine the incipient motion of an object given the grasp configuration. In planning, we would prefer to choose a point on the IGLiC which represents the desired initial grasp and then determine the configuration of the grasp. Finding the configuration from a point on the IGLiC is difficult, especially for smooth, concave objects.

The following graphical method provides a technique for determining the liftability regions of any object for two-point and three-point initial grasps. It is best to explain the method with the following example.

First, the perimeter of the object is partitioned into regions I and II. Next a contact point is chosen in I (see Figure 4.3.11) The region, I, is the curve segments for which all local contact normals have a component in the negative x direction. The region, II, is those segments whose normals have positive x-components. Therefore, the boundary between I and II are those points on the perimeter for which the normal is vertical.

$$I: \cos(\psi) < 0$$
$$II: \cos(\psi) > 0$$

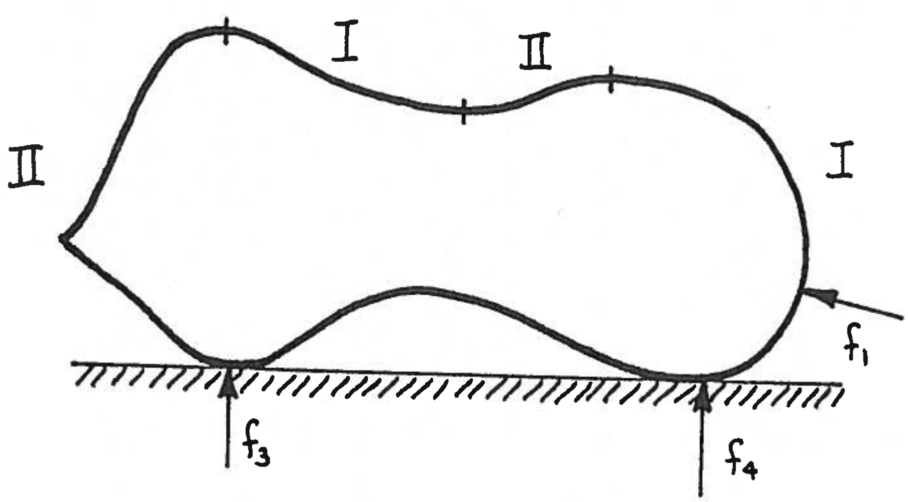

Figure 4.3.11: Regions *I* and *II*

Second, since the first finger is in I, the second finger must make contact with the object in the region II. We divide region II into regions of possible translation, PT, and possible jamming, PJ, based on the direction of the *translation limit angle* at $\mathbf{f}_1$. The translation limit angle, ψ_{tl} is defined by

$$\psi_{tl} = \psi_1 + \pi .$$

The partitions are shown in Figure 4.3.12

$$PT: \ sin(\psi_1 - \psi_2) > 0 \equiv \psi_2 > \psi_{tl}$$
$$PJ: \ sin(\psi_1 - \psi_2) < 0 \equiv \psi_2 < \psi_{tl} .$$

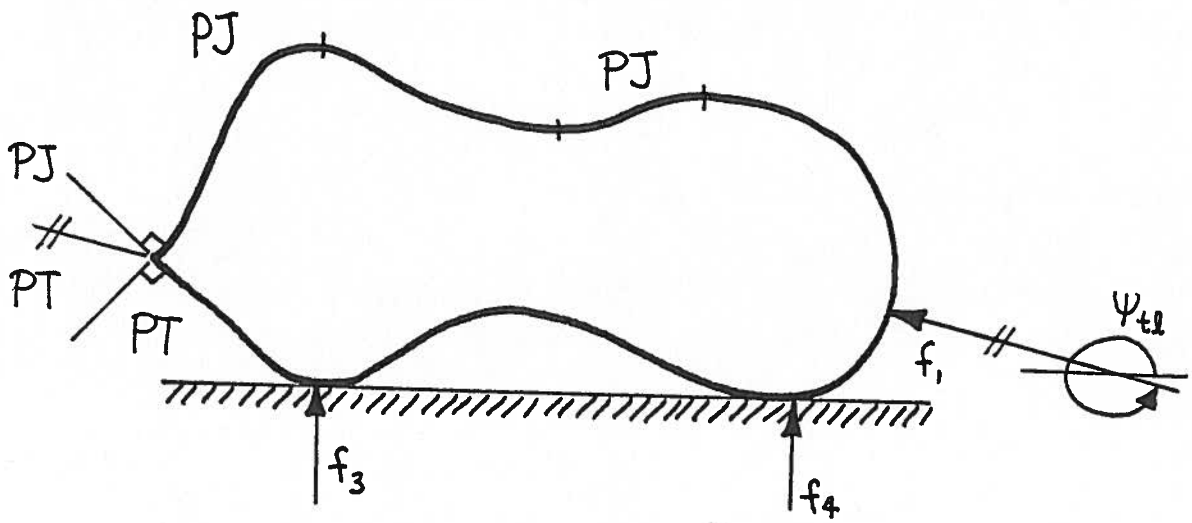

Figure 4.3.12: Regions *PT* and *PJ*

It should be noted that the translation limit angle is related to the pivotal quantity, $sin(\psi_1 - \psi_2)$, discussed in Section 4.3.1.

Third, we define the points $\mathbf{q}_{1,3}$, $\mathbf{q}_{1,4}$, and $\mathbf{q}_{1,g}$. They are at the intersections of third, fourth, and gravity forces, respectively, with the line of action of the first contact force (see Figure 4.3.13).

Fourth, the region, *PT*, is broken into $B3_T$, $B4_T$, and T. The point, $\mathbf{q}_{1,g}$ is called the *translation window*. Points in *PT* whose contact normals pass through the translation window belong to T. Other points belong to $B4_T$ or $B3_T$ if their contact normals produce positive or negative moments, respectively, with respect to $\mathbf{q}_{1,g}$ (see Figure 4.3.13 again).

Fifth, *PJ* is divided into $B3_J$, $B4_J$, and J. The line segment defined by $\mathbf{q}_{1,3}$ and $\mathbf{q}_{1,4}$ is the *jamming window*. Points in *PJ* whose contact normals intersect the jamming window are elements of J. The points whose normals do not intersect the jamming

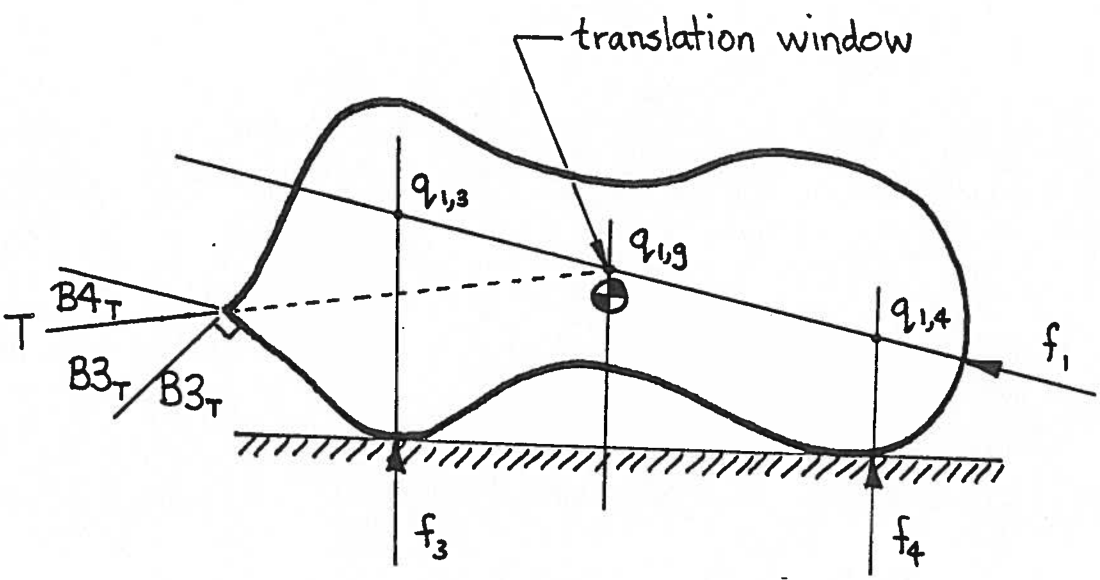

Figure 4.3.13: Regions $B\,3_T$, $B\,4_T$, and T

window and generate a positive or negative moment with respect to $\mathbf{q}_{1,g}$ belong to region $B\,4_J$ or $B\,3_J$, respectively (see Figure 4.3.14).

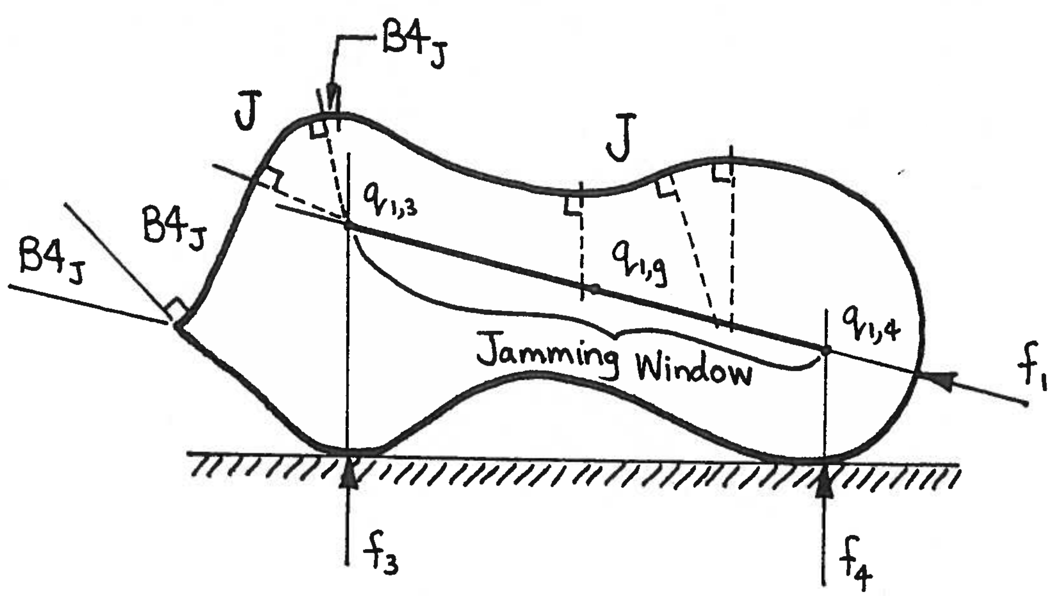

Figure 4.3.14: Regions $B\,3_J$, $B\,4_J$, and J

Finally, the liftability regions, J and T, are complete. However, the region, $B\,3$, is formed by the union of the individual $B\,3's$ found in steps 4 and 5. Similarly, $B\,4$ is found by union (see Figure 4.3.15).

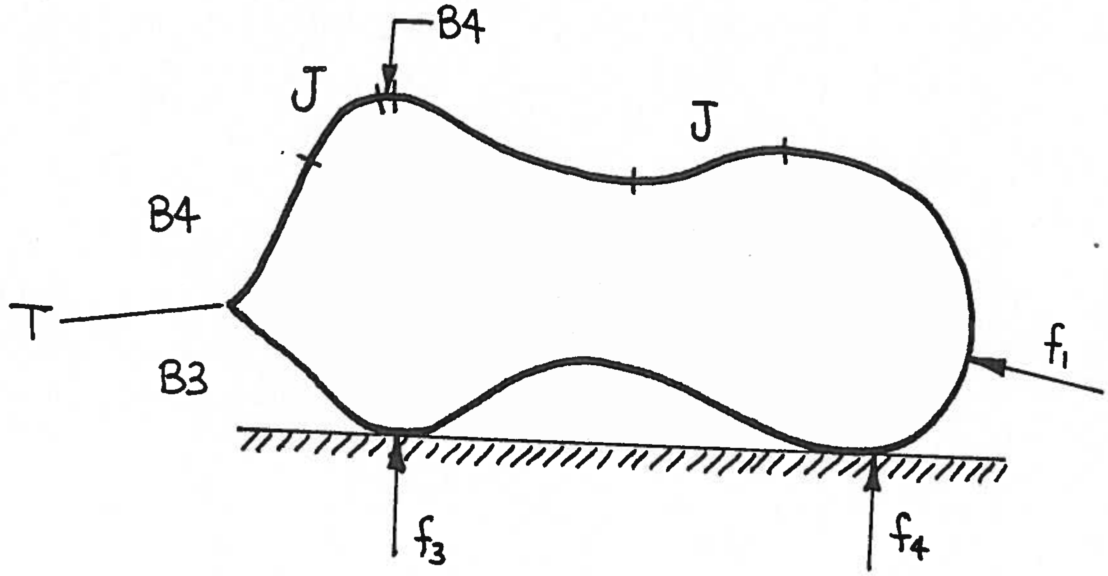

Figure 4.3.15: Liftability Regions for Two-Point Grasp

The translation region for this example contains only one point. For two-point grasps of general objects, the translation region has no length, making it physically impossible to achieve a grasp which will result in breaking both of the support contacts simultaneously when squeezing. Since it is desirable to break both support contacts at the instant of lift-off we consider a three-point initial grasp (see Figure 4.3.16). The liftability regions are determined in the same way as described above, only the translation window, the jamming window, and the translation limit angle must be redefined. The translation window is the line segment defined by the points $\mathbf{q}_{1,g}$ and $\mathbf{q}_{5,g}$. The jamming window is the line segment between the points $\mathbf{q}_3$ and $\mathbf{q}_4$, where $\mathbf{q}_3$ is the point $\mathbf{q}_{i,3}$, $i = \{1,5\}$ with the smaller z component (in this case, $\mathbf{q}_{5,3}$) and $\mathbf{q}_4$ is the point $\mathbf{q}_{i,4}$, $i = \{1,5\}$ with the higher z component (in this case, $\mathbf{q}_{5,4}$). The translation limit angle is the angle of the jamming window measured from $\mathbf{q}_3$ to $\mathbf{q}_4$. The liftability regions with the third hand contact are shown in Figure 4.3.16

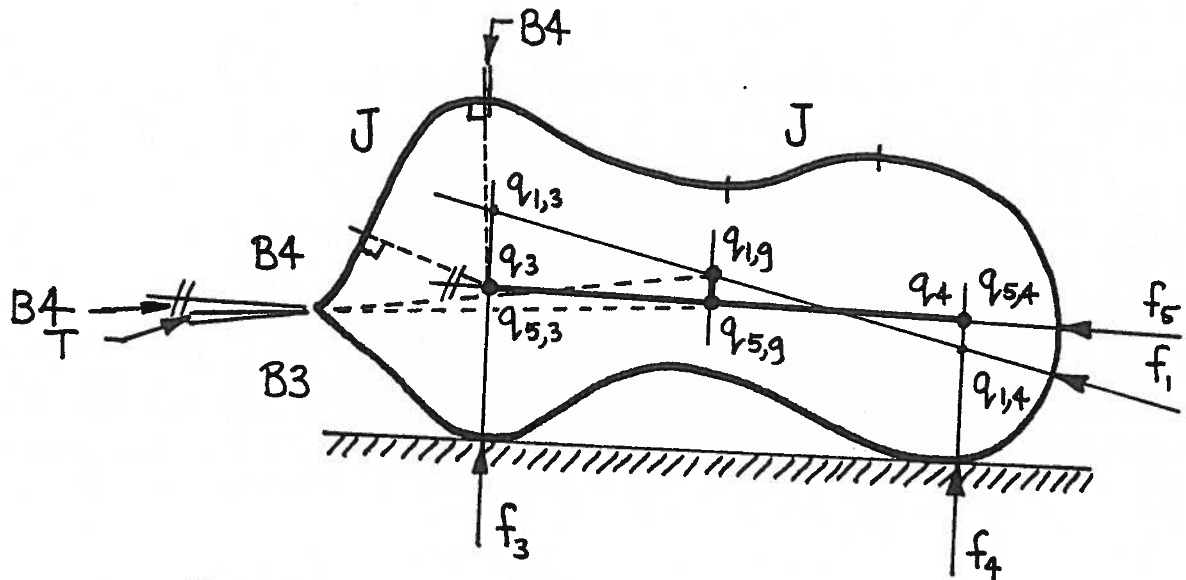

Figure 4.3.16: Liftability Regions for Three-Point Grasp

The effect of the extra contact is that now the region, T, is a portion of the perimeter with finite length, rather than a set of discrete points. If the kinematics of the hand linkage allow the second finger to contact the object in T while maintaining contacts 1 and 5, then squeezing the fingers can cause the object to break both support contacts. Importantly, this results in a force closure grasp with all of the contacts on the hand. It is also important to note that the length of the translation window has a direct effect on the length of the translation region. Therefore an attempt should be made to make the translation window as large as possible. In the case of two contacts on a single edge of a polygon, the length of the translation window is directly proportional to the distance between the contacts. Therefore, if possible, the contact should be made to occur at the ends of the edge. Figure 4.3.17 shows some examples of the liftability regions of several polygons. The perimeter of each object is grown and coded to illustrate the regions. Placing the second finger tip against the object where the offset perimeter is dashed indicates that squeezing the fingers will cause jamming, J. Placing the second

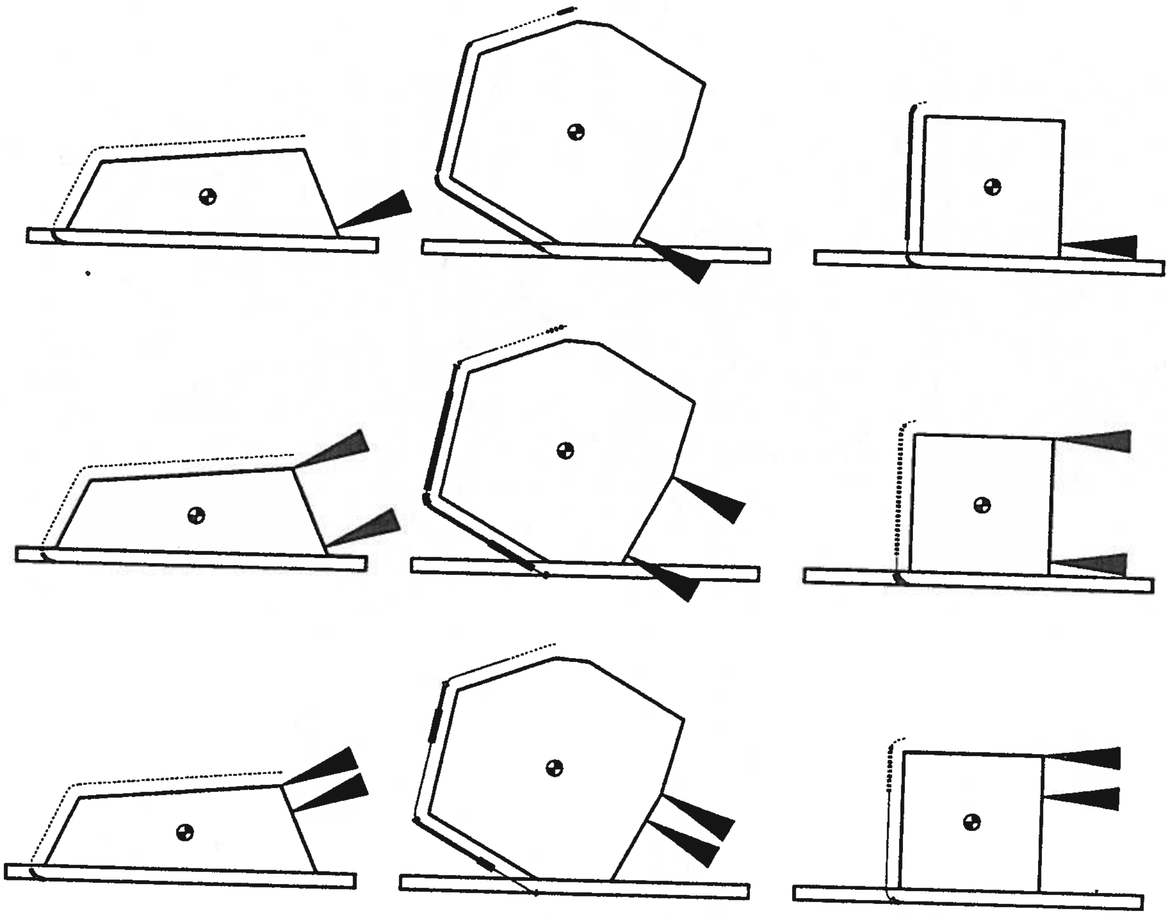

Figure 4.3.17: Liftability Regions

finger against the object beside the thin solid line, the bold solid line, and the double bold solid line indicate that squeezing will cause the right support contact to break, $B\,4$, the left support contact to break, $B\,3$, or both contacts to break, T, respectively. The first row of the figure shows the liftability regions for two-point initial grasps; no translation regions are visible. The second row shows the regions for three point grasps. Note that T has grown, but so has the unusable region, J. The third row shows that the translation regions shrink as the distance between the contacts on the edges of the polygons is reduced.

An alternative way to determine the liftability regions of an initial grasp with three contacts is by determining the regions for two two-point grasps. The first grasp is the one which includes the first through fourth contacts. The second one includes the second through fifth contacts. The two grasps compete to dictate which contact will break. If the first two-point grasp would cause the object to be lifted breaking the fourth contact and other two-point grasp would cause the third contact to break, then the three-point grasp will cause both of the support contacts to break simultaneously (*i.e.* translation). Using similar reasoning, we note that if both two-point grasps cause the same result upon squeezing, then the three-point grasp will also cause that result. Given the liftability regions for the first two-point grasp, $B3_5$, $B4_5$, T_5, J_5 and the second two-point grasp, $B3_1$, $B4_1$, T_1, J_1, the liftability regions for the three-point grasp can be computed as

$$B3 = B3_1 \cap B3_5 \tag{4.3.25}$$
$$B4 = B4_1 \cap B4_5 \tag{4.3.26}$$
$$T = (B3_1 \cap B4_5) \cup (B3_5 \cap B4_1) \tag{4.3.27}$$
$$J = (B3 \cup B4 \cup T)' \tag{4.3.28}$$

where the apostrophe indicates the set complement operation.

4.4. Lifting Phase

The lifting phase begins when the object no longer contacts the support. For this to occur, the object must be in a force closure grasp in the hand. All contacts must be on the palm and fingers. The goal of the lifting phase is to manipulate the object into an enveloping grasp. In doing so, the object may either be stable through force closure or unstable. Instability is undesirable, because it results in the object's falling. Even though the object will eventually come to rest in a stable configuration, it could be contacting the support or, worse yet, completely lose contact with the hand. Therefore, it is imperative that an enveloping grasp be gained without ever losing force closure.

Assume that the initial grasp has been chosen in a translation region (defined in Section 4.3.3). As lifting begins, the object contacts the hand at two points on one finger and at one point on the other. Since force closure requires three contacts, all of the contacts must be maintained until a fourth contact is achieved. Then if the object is enveloped, *i.e.* the grasp has form closure, the lifting phase ends and the grip adjustment phase begins. If not, one of the contacts must break as manipulation continues. Thus it is apparent that during the lifting phase, the object must translate relative to one of the fingers (assuming flat fingers) until the object contacts the palm. Once the palm has been contacted, translation is possible only if one finger loses contact with the object. In an enveloping grasp both fingers and the palm must contact the object. Therefore, we prefer to manipulate the object maintaining contact with both fingers. However, we also analyze a planning strategy which requires contact to be lost with one finger as the object slides on the other.

A force closure grasp is one for which the negative of the gravitational force acting on the object is within the convex cone, C^+, defined by

$$\mathbf{W}_{obj}\,\mathbf{c}_{obj} = -\mathbf{g}_{ext|o} \tag{3.1.12}$$

$$\mathbf{c}_{obj} \geq \mathbf{0}\,. \tag{3.3.2}$$

Figure 4.4.1 shows a convex cone and an external force for a typical force closure grasp. Writing the equilibrium equations with respect to the inertial frame, X, the i^{th} column of $\mathbf{W}_{obj}$ is given by

$$\mathbf{y}_i = \begin{bmatrix} \cos\psi_i \\ \sin\psi_i \\ t_i \end{bmatrix} \tag{4.4.1}$$

where ψ_i is the angle of the i^{th} contact normal measured counterclockwise from the x

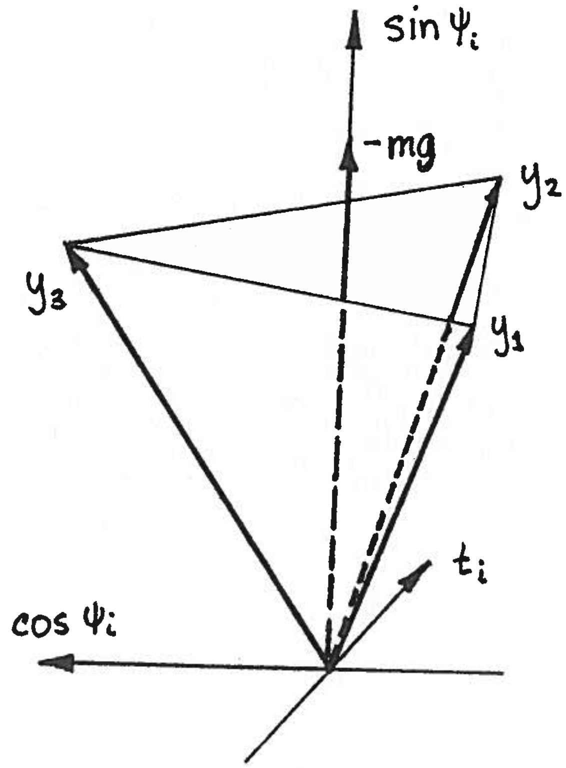

Figure 4.4.1: Convex Cone and External Force

axis of **X** and t_i is the moment of the contact normal with respect to the summing point, **q**. Choosing **q** to be the center of gravity of the object, the gravity force is constant during manipulation

$$\mathbf{g}_{ext|o} = \begin{bmatrix} 0 \\ -mg \\ 0 \end{bmatrix} . \tag{4.4.2}$$

The convex cone can be projected onto the plane formed by $\cos\psi_i$ and t_i, called the Lifting Phase Plane, (LPP). In this plane, the cone becomes the LPP triangle, the gravity force maps to the origin, and the two contacts on a common finger map to points on a vertical line separated by the same distance that separates the contacts on the finger (see Figure 4.4.2). The necessary and sufficient conditions for a grasp to have force

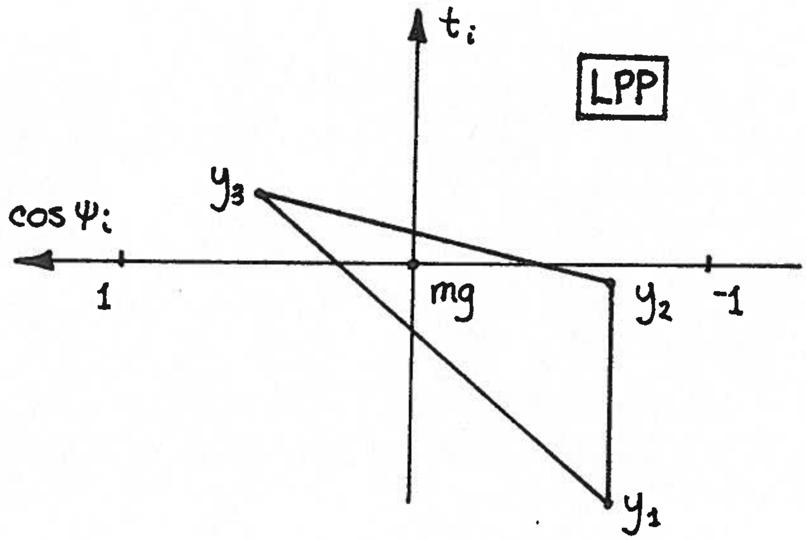

Figure 4.4.2: Convex Cone Mapped onto the LPP

closure are that the LPP triangle enclose the origin and the *sine* of the difference of the contact angles on the two fingers be greater than zero

$$\sin(\psi_1 - \psi_2) > 0 \,. \tag{4.4.3}$$

What we would like to do is squeeze the object until it contacts the palm. However, while squeezing, we must make sure that the LPP triangle always contains the origin and inequality (4.4.3) is never violated. Initially both of the conditions are satisfied, because the grasp has force closure. Restricting manipulation to squeezing, the quantity, $\psi_1 - \psi_2$, may only decrease (if the singly-contacted finger contacts a vertex of the object) or remain constant (if the singly-contacted finger contacts an edge of the object) and because the palm eventually prevents squeezing from continuing, the quantity is bounded from below by zero. At the start of manipulation the angular difference between the contact normals, $\psi_1 - \psi_2$, begins in the interval bounded by zero and π. During squeezing, the difference reduces, but remains in the same interval. Because the *sine* function is positive in that interval, the second necessary and sufficient condition is guaranteed to be satisfied throughout the entire manipulation.

The condition that the LPP triangle contain the origin at all times can only be checked by considering the trajectories of its vertices whose positions are affected by three variables, the two joint angles, $\theta_{1,1}$ and $\theta_{2,1}$, and the angle of the palm, θ_p, (if the hand is rotated far enough). Consider the hypothetical trajectory shown in Figure 4.4.3.

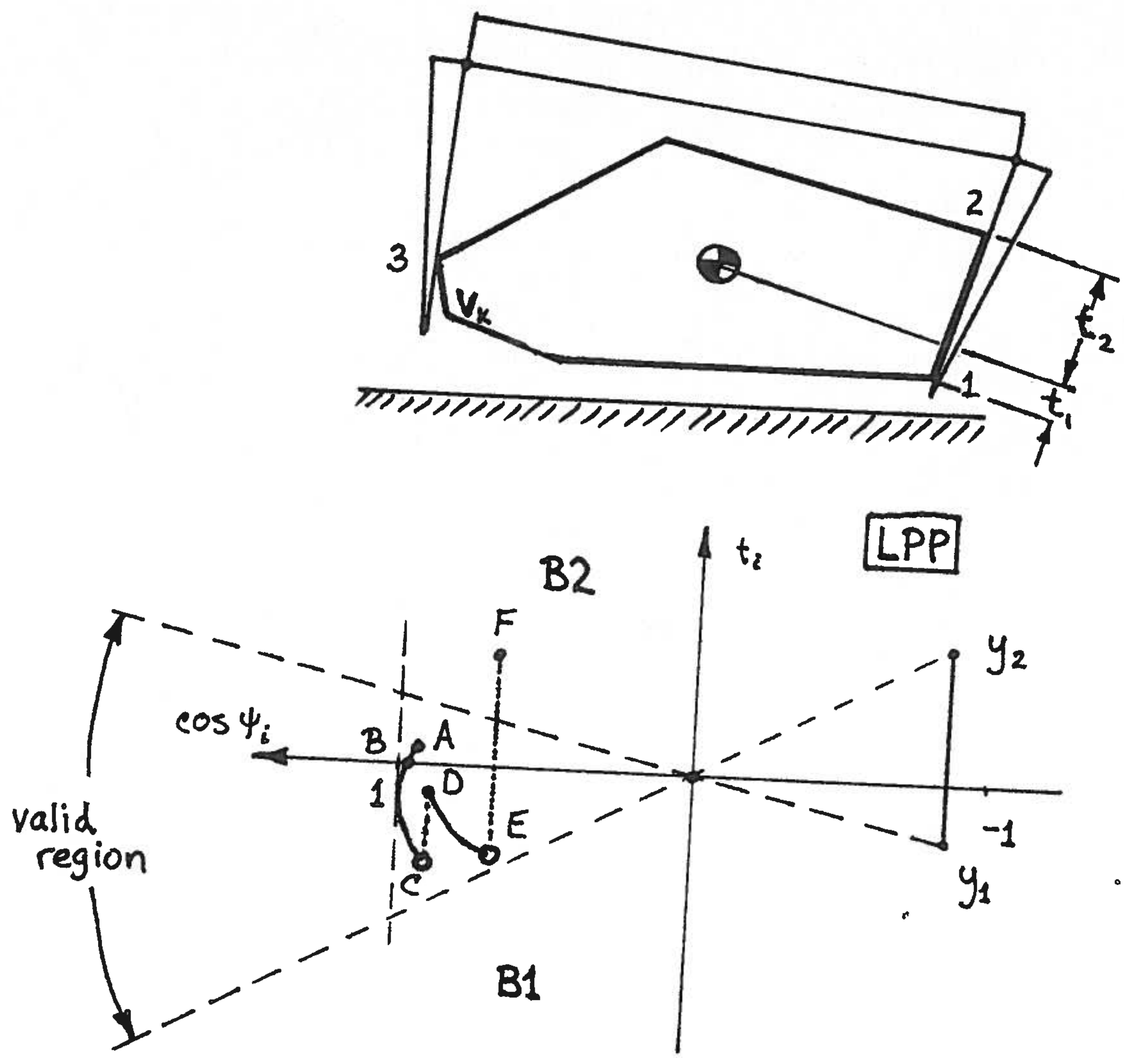

Figure 4.4.3: Trajectory of the Pusher

Because the object will translate up finger 1, finger 2, which we call the *pusher*, is rotated clockwise while finger 1 is held stationary. As the pusher rotates, vertices y_1 and y_2 remain fixed as vertex y_3 follows a path qualitatively like the one shown beginning at point A. At the point C, the vertex jumps to D. The discontinuity is caused by the edge of the second finger contacting the k^{th} vertex of the object. At the instant of contact, there are four contacts, but only three can be maintained as the fingers continues to squeeze. Since D is within the valid region for the third vertex, the new contact remains and the previous contact on that finger breaks. At the point E, the trajectory jumps outside of the valid region to the point, F. If the new contact were to remain (as it did at D) the interior of the LPP triangle would exclude the origin and the object would become unstable. However, the trajectory jumped into the region labeled $B2$. This means that the second contact point (which is on finger 1) will break. The new LPP triangle has vertices labeled y_1, y_3, and y_4.

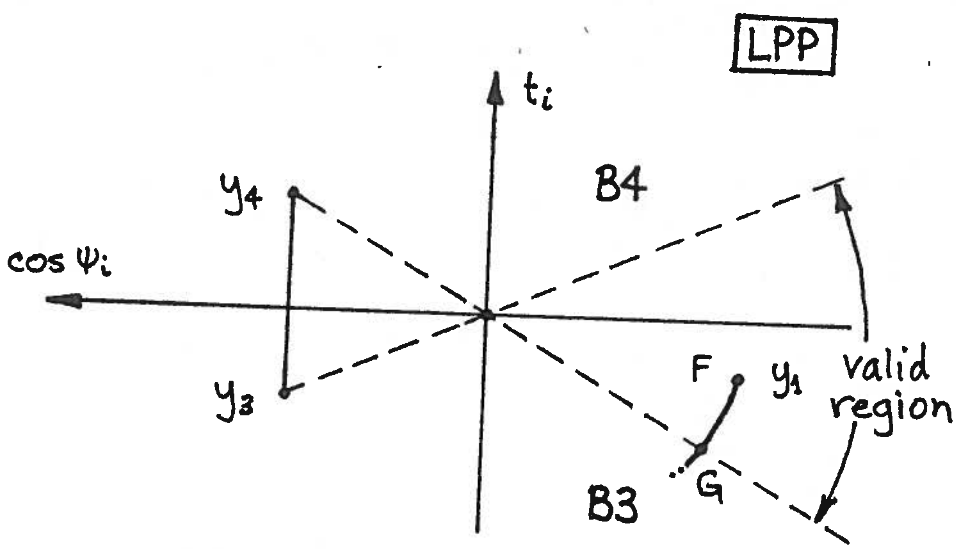

Figure 4.4.4: Trajectory after Switching Pushers

It contains the origin, so the grasp is still stable, but now the object will slide up the second finger rather than the first. To continue squeezing, the first finger now acts as the pusher, rotating counterclockwise and the second finger is held fixed. This strategy of using on finger as a pusher and holding the other finger fixed is called the *pusher strategy*.

The figure shows the trajectory of vertex y_1 crossing the boundary of the new valid region into the region $B3$. When this happens, the third contact breaks, leaving only the first contact on finger 1 and the fourth contact on finger 2. The grasp loses force closure, becoming unstable and the object falls. If an enveloping grasp is achieved before the object becomes unstable, then the lifting strategy can be used successfully. If not a different strategy must be considered.

An interesting property of the LPP trajectory is that if the pusher's vertex moves out of the valid region in a continuous manner (as at G), the object becomes unstable, because a contact point is lost without gaining a new one. However, if it jumps outside of the valid region (as at F), the object remains stable and the finger on which the object translates switches. Any motion of the trajectory within the valid region represents stable translation of the object without switching fingers.

If the pusher strategy fails, one could try the *roll strategy* during which the finger angles are fixed as the palm is rotated. If the hand can be rotated far enough without the object losing force closure, the object will slide down the finger until it touches the palm. Afterward, the fingers may be closed around the object creating and enveloping grasp. Figure 4.4.5 shows the trajectories of the vertices of the LPP triangle corresponding to rotating the hand shown in Figure 4.4.3 clockwise. As the hand rolls, the object does not move relative to it and therefore, the moments, t_i, of the contacts do not change. The result is that the corners of the triangle can only move horizontally and since the normals of contacts 1 and 2 have the same direction, y_1 and y_2 move at a common rate. At B the right finger becomes horizontal. After slightly more rotation, the third contact breaks and the object slides toward the palm. Closing the fingers

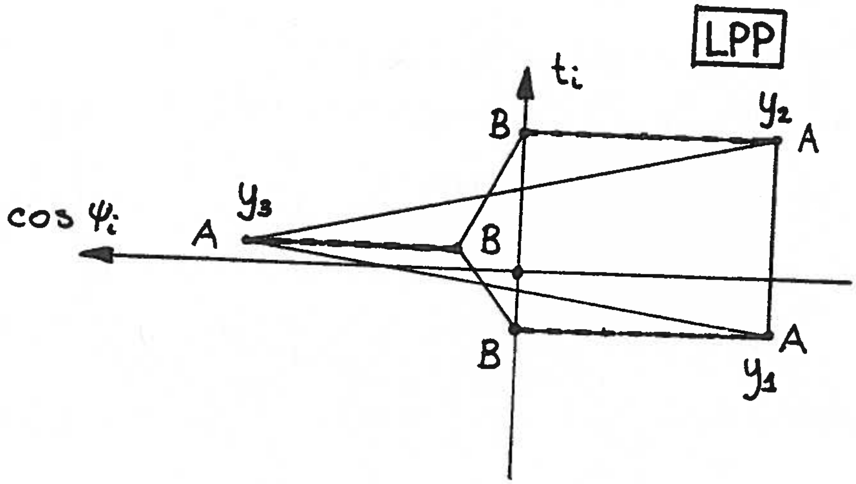

Figure 4.4.5: Hand Rolling Strategy

around the object completes the lifting phase.

Next we would like to know what conditions guarantee the success of the rotation strategy. A condition of necessity is that t_1 and t_2 have opposite signs. If they have the same sign, the object could not be stable when sliding down the finger without tipping because the gravity force would not pass between the two supporting contacts. Given that necessity is met, a sufficient condition is that t_3 equal zero. The validity of this condition can be argued for as follows. Because the grasp satisfies $\sin(\psi_1 - \psi_2) > 0$, y_3 is always on the left side of the lifting phase plane. As the hand rotates clockwise, y_1 and y_2 move toward the left. Until the right finger becomes horizontal, y_1 and y_2 are on the right side of the LPP. Therefore the LPP triangle always contains the origin. As the right finger passes through horizontal, y_1 and y_2 cross the t_i axis causing the third contact to break as the object slides toward the palm.

4.5. Grip Adjustment Phase

The lifting phase ends and the grip adjustment phase begins when a form closure grasp is achieved. Form closure is signaled by the infeasibility of the velocity linear program, equations (3.5.4 - 3.5.6) for a squeezing motion of the fingers. Since the goal is to improve the form closure grasp, the object is manipulated in a way which maintains form closure. Thus there must always be four contact points, yielding four constraints on the variables effecting the grasp. Because manipulation is done to move the object with respect to the hand, the three variables defining the position of the palm are unimportant. Thus there are five variables of interest, three defining the position of the object and the two joint angles. The four constraints, leave only one independent variable with which to manipulate the object and improve the grasp.

The following sequence of frames, shows the adjustment of a form closure grasp. The objective was to minimize the sum of the contact force magnitudes acting on the object while holding the palm still.

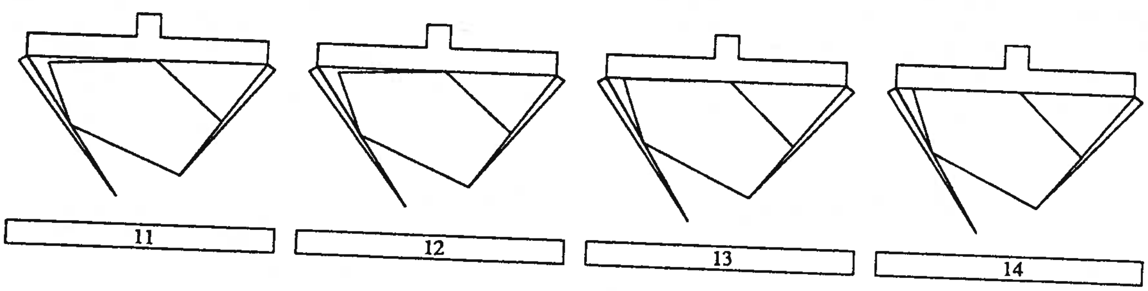

Figure 4.5.1: Adjusting an Enveloping Grasp

The final grasp, shown in frame 14, has five contact points which cause a discontinuous branch point in the objective function. The branches occur because only four contact points can be maintained while manipulating the grasp, but 5 possible sets of four contacts exist. Two of the five sets do not maintain form closure, so they are not considered for further grip adjustments. Figure 4.5.2 shows the grip adjustment objective function in the neighborhood of frame 14. The square on the far right corresponds to

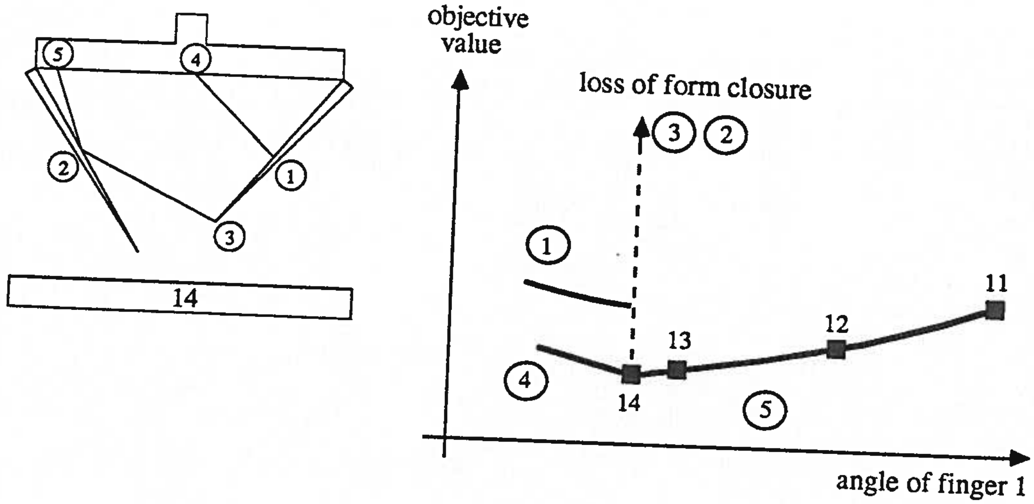

Figure 4.5.2: Grip Adjustment Objective Function [14]

frame 11. Moving to the left, the objective value decreases until the object gains the fifth contact point, frame 14. To continue manipulating the grip, one of the contacts must be chosen as the one to break, and the finger motions must be computed to be consistent with maintaining the other four contacts. If either contact 2 or 3 is broken, the grasp loses form closure. This is fairly obvious for the 2^{nd} contact, since it corresponds to removing the second finger. In the case of the 3^{rd} contact, form closure is lost since the object may rotate counterclockwise about the intersection of the 1^{st} and 4^{th} contact normals, losing contact at points 2 and 5. If contact 1 is broken, the objective value jumps because one of the palm contact forces (which was previously zero) becomes nonzero, effectively adding to the gravity load which must be absorbed by the two remaining contacts on the fingers. Breaking the 5^{th} contact, the object moves back to a previous grasping configuration. Thus removing the 4^{th} contact is the only remaining

[14] The grasping objective as a function of the angle of finger 1 and the choice of the contact to break. The frame numbers are indicated by squares on the right branch. Each branch is caused by manipulating the object to break one contact and maintain the other four. The circled number beside each branch indicates the lost contact leading to that branch.

possibility for improving the grasp. However, it is seen that the grip objective increases when breaking that contact, so frame 14 is the (locally) optimal solution.

4.6. Three-Phase Planning

The three planning phases, pre-lift-off, lifting, and grip adjustment, must be invoked serially.

1) Pre-Lift-Off - First, the liftability regions of the object are computed assuming that the right hand finger is laid along the clockwise-most edge of the object.

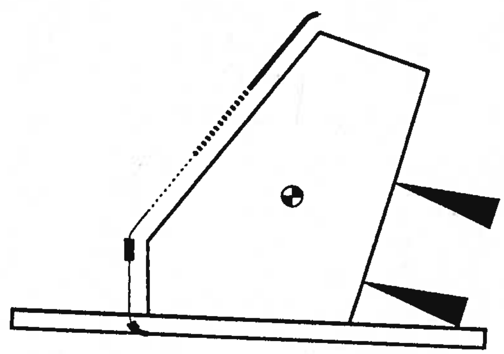

Figure 4.6.1: Liftability Regions

Then, given this finger position, the joint angles are determined which place the left fingertip in the part of the translation region (show by the double bold solid line) closest to the support. That part of the translation region is most desirable because the hand surrounds the object more than if another part of the translation region is contacted. In the case that the fingertip is placed in the translation region and on the support, as in Figure 4.6.2, the robot is required to have two special features. First the fingertip must be sharp with respect to the contacted vertex so that the finger's edge may contact the vertex (*i.e.* the fingertip must be able to be positioned slightly under the object as when we use a fingernail to pick up a coin resting on a flat surface). Second, the robot must be compliantly controlled so that the fingernail may slide along the support and under

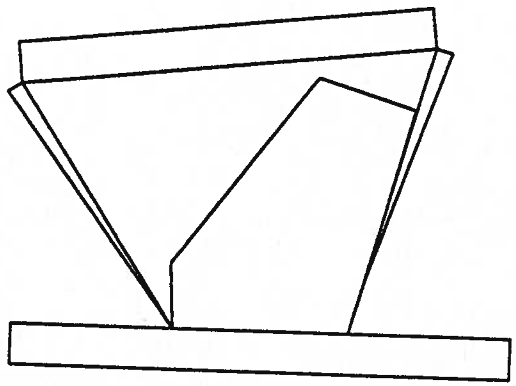

Figure 4.6.2: Placement of Sharp Fingertip

the supporting vertex of the object. If the left finger does not have a nail, then its tip should be placed on the object in a part of the translation region away from the supporting vertex. If the robot cannot be compliantly controlled, the procedure must be changed to avoid contact between the robot and the support. The liftability regions are still computed assuming that the right finger is laid against the clockwise-most edge of the object, but its tip is assumed to be placed far enough up the edge to guarantee that the position error of the robot will not cause the tip to contact the support. Also, the left fingertip is restricted to that part of the translation region with at least as much clearance as given to the right fingertip. The following figure shows the liftability regions for a square for the right finger laid against the right side of the square. Note that the square cannot be translated up the finger without a fingernail and compliant control, because the translation region exists at only the left supporting vertex. However, the square can be tipped with only one point of contact per finger.

Sometimes it is impossible to place the left tip in the translation region due to interference between the object and the hand. If this occurs, the planning procedure may be repeated, laying a finger against each edge of the object and attempting to place the other fingertip in the translation region. If an initial grasp in the translation region

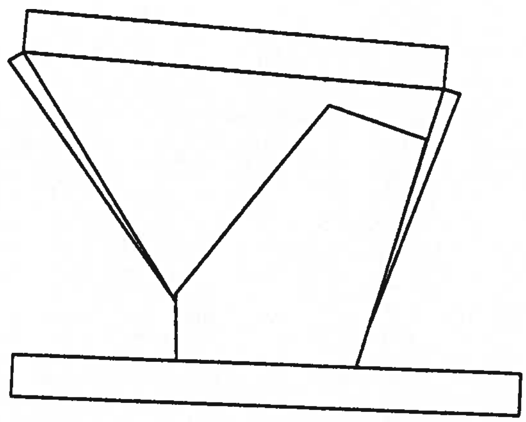

Figure 4.6.3: Placement of Blunt Fingertip

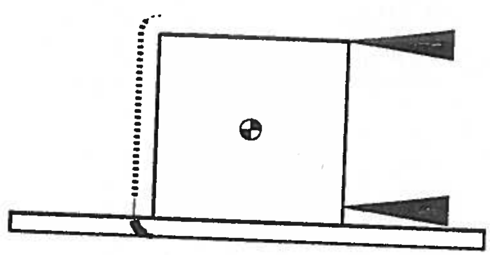

Figure 4.6.4: Liftability Regions of Square

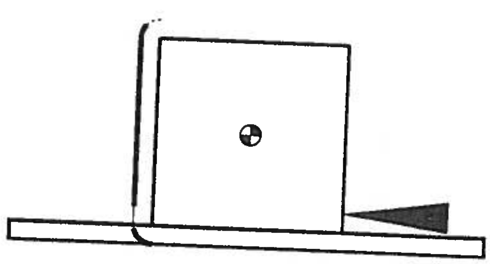

Figure 4.6.5: Liftability Regions of Square

can still not be found, then one would have to settle for an initial grasp which only tips the object and hope that squeezing eventually results in breaking the other contact on the support.

2) Lifting Phase - At the beginning of the lifting phase, there are two contacts on one finger and one on the other. The finger with one contact is designated the pusher. It moves to push the object toward the palm while the palm and other finger are held fixed. The trajectory of the LPP triangle (discussed in Section 4.4) is computed as the object is pushed. If an enveloping grasp is achieved before the object becomes unstable, then the lifting phase will succeed.

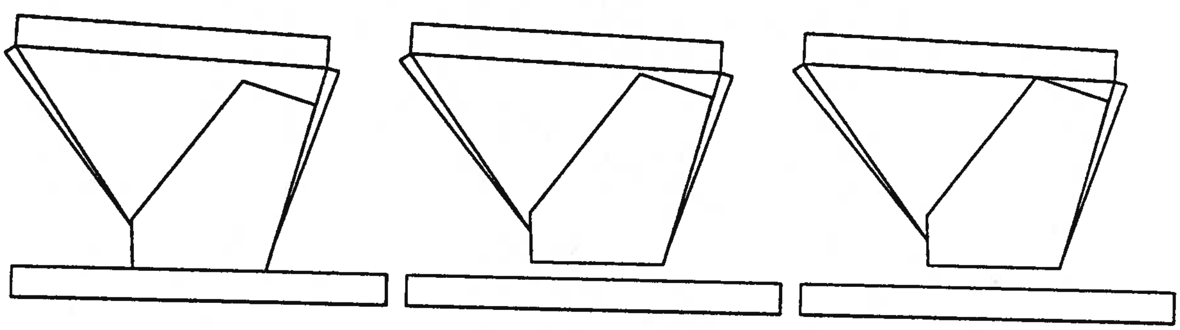

Figure 4.6.6: Successful Pushing Strategy

If not, the roll strategy may be used. Figures 4.4.3 and 4.4.4 show a pusher plan which fails at the point G. Before failing, however, the LPP triangle gains a configuration (at point B) which guarantees that the roll strategy will succeed. Therefore, the pushing strategy should be executed until B, followed by the hand rolling strategy which cannot fail.

3) Grip Adjustment Phase - The grasp is now enveloping, *i.e.* the grasp has form closure. In this phase, the grasp may be adjusted with one degree of freedom (for the two-dimensional hand under consideration) to optimize an objective function. To do so, the increments of motion of the object and one finger are computed given the incre-

ment of motion of the other finger. The resulting change in the objective function dictates the direction in which to manipulate the object. When the objective function reaches a local optimum, the grip adjustment phase ends and the plan is complete.

Note that the various frames of motion shown throughout this chapter were generated by an object motion simulator. The simulator is coupled with a planner which is capable of computing the liftability regions, determining the initial grasp, and invoking the lifting strategy.