HW 2: Dynamics and Control

Due Feb 17, 2019

We don't mind if you work with other students on your homework. However, each student must write up and turn in their own assignment (i.e. no copy & paste). If you worked with other students, please **acknowledge who you worked with** at the top of your homework.

1. **Inverted Pendulum Control**
   Consider an Inverted Pendulum. You have a cart that can move left and right along a track, and a stick that can rotate about it.

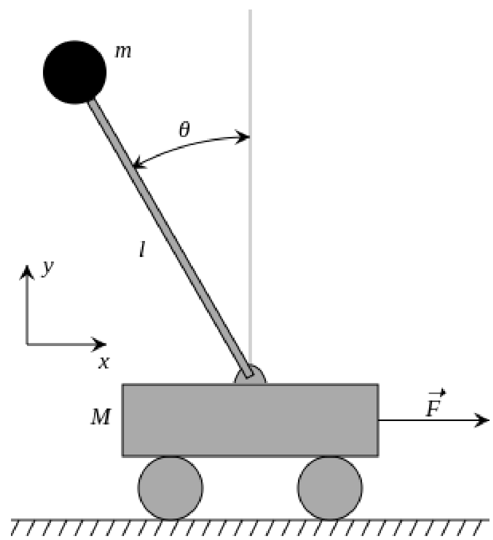

Figure 1: Inverted Pendulum

For this problem, feel free to use a symbolic math program (Matlab has a good one, but many others exist). However if you do, please write down results of your intermediate steps in your writeup and include your code. Note that all the lab computers and all the computers in Kresge have installations of Matlab. You can also request a license from the school.

(a) Find the dynamics of this system using Lagrangian Dynamics (4.2 from Homework 1). Your q should be $q = [\theta, x]^T$ The dynamics should be

$$\begin{bmatrix} L^2 m & -Lm\cos(\theta) \\ -Lm\cos(\theta) & M+m \end{bmatrix} \begin{bmatrix} \ddot{\theta} \\ \ddot{x} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 2Lm\dot{\theta}\sin(\theta) & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ \dot{x} \end{bmatrix} + \begin{bmatrix} -Lgm\sin(\theta) \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} F$$

(b) Construct a state-space representation of the system with state $z = [\theta, x, \dot{\theta}, \dot{x}]$, and linearize about $z = [0, 0, 0, 0]$. You should get

$$\dot{z} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{g}{L} + \frac{mg}{ML} & 0 & 0 & 0 \\ \frac{mg}{M} & 0 & 0 & 0 \end{bmatrix} z + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{LM} \\ \frac{1}{M} \end{bmatrix} F$$

(c) Use Lyapunov's Indirect Method to prove that this system is unstable about $\theta = 0$ (the upright position) with a stationary cart and zero force input. Note: If the linearized system isn't stable, there's still a possibility that the nonlinear system will be stable, and therefore Lyapunov's Indirect method cannot prove instabilty. However, for our purposes this is a good enough indication.

(d) Use state feedback control to design a controller for the linearized system, with cart force F as the input. Note that if you take the state of the system to be $q = [\theta, x]^T$, this is a PD controller.

(e) Use Lyapunov's Indirect Method to find the controller gains under which the nonlinear system becomes locally stable about $\theta = 0$. The Routh-Hurwitz Stability test or a symbolic math program might be helpful.

2. **Short Problems**

   (a) Why is closed loop control beneficial?

   (b) When would you use impedance control over standard position control?

3. **Intro to Grasping**
   Prove Theorem 5.6 in MLS: Show that a planar grasp composed of two point contacts with friction is in force closure if and only if the line connecting the contact points lies inside both friction cones. *Hint: Consider proving the contrapositive — if the line between the contacts is not in the cones, then the grasp cannot be in force closure*

4. **Research Comprehension**
   Read this paper and describe the Ferrari Canny metric with words (and an equation if you'd like).