EECS 106B/206B
Discussion 1

GSI: Valmik Prabhu

About Us

Valmik, MS MechE ‘19

Board Game Fanatic

Chris, MS EECS ‘19

‘Niners Fanatic

Nandita, BA CS ‘19

Shy Programmer

Action Items

- BCourses - talk to me if you aren’t on it
- Lab access?
- Piazza
- Gradescope
- Book - it’s free online, but a physical copy can be nice
- Discussions - Monday AND Wednesday

Agenda

Discussions and Logistics

Labs, Lab Safety

Brief Course Overview

Review and Practice Problems

Discussions

- Discussions are MONDAYS AND WEDNESDAYS, same times
- They will usually cover material from the previous lecture, supplementary material, and/or lab introductions. See syllabus
- Email me questions by the 6 pm the day before to guarantee I’ll add them
- How much question time?

Labs

- Labs are essentially self-paced:
  - Introduction (in discussion). 2-3 weeks to do the lab
  - Need Python + ROS; A 106A member would be good for each group
- Lab 0: ROS review (self-paced)
- Lab 1: Trajectory tracking on Baxter, 3 weeks
- Lab 2: Grasp Metrics, 3 weeks
- Lab 3: Nonholonomic Control on Turtlebots, 2 weeks (new)
- Lab 4: Hyper-redundant and Soft Robots, 2 weeks (new)

Lab Rules

Do NOT work alone

Do not leave the room while the robot is running

ALWAYS have the E-stop handy

Terminate all your processes when you log off

    pkill -u *username*

Tell course staff if you break something

Lab Tips

- If something seems broken, power cycle
- NEVER use ctrl-z to kill a process
  - Use ctrl-c (terminate) or ctrl-\ (hard-kill)
  - kill -9 %*process-number*
- Always run: source devel/setup.bash
- Organize your workspace properly
- ROS is open-source. For each package:
  - Check documentation
  - Check source code
- Check your bashrc
  - ROS_MASTER_URI, ros version, etc
- Connect to the robot in *all terminals*

Semester Overview:

- Review: MLS 2, 3, 4.1-4.3
    - Rigid body motion; screws, twists, velocities, wrenches
    - forward and inverse kinematics, the manipulator jacobian, and singularities
    - Newtonian and Lagrangian dynamics
- Dynamics and Control: MLS 4.3-4.5
    - System stability, kinematic and dynamic control of robot manipulators
- Hand Kinematics and Grasping: MLS 5.1-5.5
    - What is a grasp? How do you define one? How to we manipulate held objects?
- Nonholonomic Systems and Hand Dynamics: 4.6, 6.1-6.3
    - How do dynamics work with constraints? How do we design controllers that work for them?
- Hyper-redundant and Soft Robotics: not in book
- Path Planning: not in book
- Special Topics

Notation

R_{AB} transforms q_B to q_A

$$q_A = R_{AB} q_B$$
$$q_B = R_{BA} q_A$$

Capital Letters are matrices

Lowercase letters are (generally) vectors

What's Rigid Body Motion?

Rotation + Translation

$$q_A = R_{AB} q_A + t_{AB}$$

$$q_B = R_{BA} q_B + t_{BA}$$

Transformation Matrix:

$$\bar{g}_{AB} = \begin{bmatrix} R_{AB} & t_{AB} \\ \mathbf{0} & 1 \end{bmatrix}$$

$$\bar{q}_A = \begin{bmatrix} q_A \\ 1 \end{bmatrix} \quad \bar{q}_B = \begin{bmatrix} q_B \\ 1 \end{bmatrix}$$

$$\bar{q}_A = \bar{g}_{AB} \bar{q}_B$$

$$\bar{q}_A - \bar{q}_B = \begin{bmatrix} q_A - q_B \\ 0 \end{bmatrix}$$

Two Rigid Body Transformations?

Two transformations are a single transformation:

$$q_A = R_{AB}(R_{BC}q_C + t_{BC}) + t_{AB}$$

$$\bar{q}_A = \bar{g}_{AB}\bar{g}_{BC}\bar{q}_C$$

$$\bar{q}_A = \bar{g}_{AC}\bar{q}_C$$

Twists

Twists represent a *subset* of rigid body motions called *screw motions*

$$\xi = \begin{bmatrix} \nu \\ \omega \end{bmatrix} \qquad\qquad g(\theta) = e^{\hat{\xi}\theta} g(0)$$

Rotation and Translation are along the same axis, and are *proportional*

$$h = \frac{d}{\theta} \qquad\qquad \xi = \begin{bmatrix} -\omega \times q + h\omega \\ \omega \end{bmatrix}$$

$$g = \begin{bmatrix} e^{\hat{\omega}\theta} & (I - e^{\hat{\omega}\theta})q + h\theta\omega \\ 0 & 1 \end{bmatrix}$$

Most Joints are Twists

Revolute Joint: $h = 0 \quad \xi = \begin{bmatrix} -\omega \times q \\ \omega \end{bmatrix}$

$$g = \begin{bmatrix} e^{\hat{\omega}\theta} & (I - e^{\hat{\omega}\theta})q \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} R(\theta) & (I - R(\theta))q \\ 0 & 1 \end{bmatrix}$$

Prismatic Joint: $\omega = 0$

$$\xi = \begin{bmatrix} v \\ 0 \end{bmatrix} \qquad g = \begin{bmatrix} I & v\theta \\ 0 & 1 \end{bmatrix}$$

Forward Kinematics

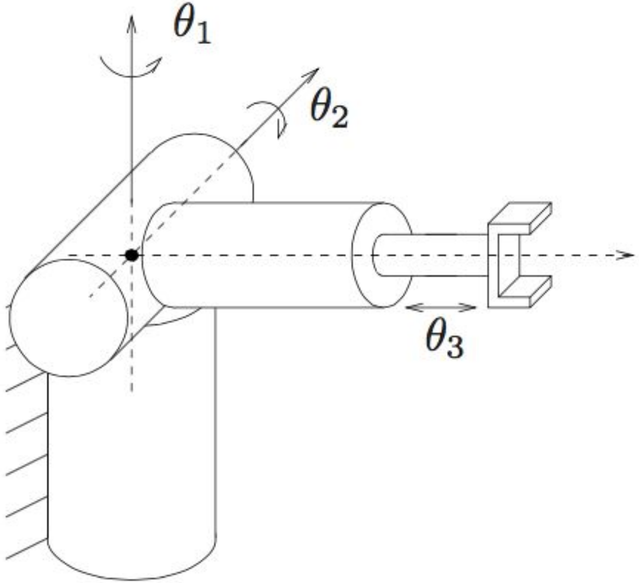
We need an initial position:

$$g(\theta) = e^{\hat{\xi}\theta} g(0)$$

$$g_{st}(0) = \begin{bmatrix} R_{st} & q_{st} \\ 0 & 1 \end{bmatrix}$$

Problem 1: Forward Kinematics

Find the Forward Kinematics Map:



Problem 1: Solution pt 1

$$q_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad \omega_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad \xi_1 = \begin{bmatrix} -\omega \times q \\ \omega \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$R_1(\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$q_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad \omega_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad \xi_2 = \begin{bmatrix} -\omega \times q \\ \omega \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$R_2(\theta) = \begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{bmatrix}$$

Problem 1: Solution pt 2

$$v_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \omega_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad \xi_3 = \begin{bmatrix} v \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad R_3(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$g(0) = \begin{bmatrix} 1 & 0 & 0 & l_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad g(\theta_1) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 & 0 \\ \sin(\theta) & \cos(\theta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$g(\theta_2) = \begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad g(\theta_3) = \begin{bmatrix} 1 & 0 & 0 & \theta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Problem 3: Solution pt 3

$$g(\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 & 0 \\ \sin(\theta) & \cos(\theta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & \theta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & l_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Inverse Kinematics

Given $g_d(\theta)$ find θ

- Not injective
- ~~Paden Kahan~~
- Pseudoinverse-Jacobian-based gradient descent (usually)

Instantaneous Kinematics: Manipulator Jacobian

Describes the relationship between:

- Joint velocities
- End effector velocity (in either the body or the spatial frame)

Its transpose describes the relationship between:

- End effector wrench (in either the body or the spatial frame)
- Joint torques

In Calculus

A matrix of partial derivatives

$$x = \begin{bmatrix} x_1 & x_2 & \dots & x_n \end{bmatrix}^T$$
$$f(x) = \begin{bmatrix} f_1 & f_2 & \dots & f_m \end{bmatrix}^T$$

$$\mathbf{J} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_m}{\partial x_1} & \cdots & \frac{\partial f_m}{\partial x_n} \end{bmatrix}$$

In Robotics

For a robot with n joints:

$$J^s(\theta) = \begin{bmatrix} \xi_1 & \xi_2' & \cdots & \xi_n' \end{bmatrix}$$

$$\xi_i' = Ad_{e^{\hat{\xi}_1 \theta_1} \dots e^{\hat{\xi}_{i-1} \theta_{i-1}}} \xi_i$$

Where each twist is expressed in the current robot configuration

This is the same thing!

~~Analytic vs geometric jacobian~~

Matrix Adjoint

The matrix adjoint is used to transform a twist between frames.

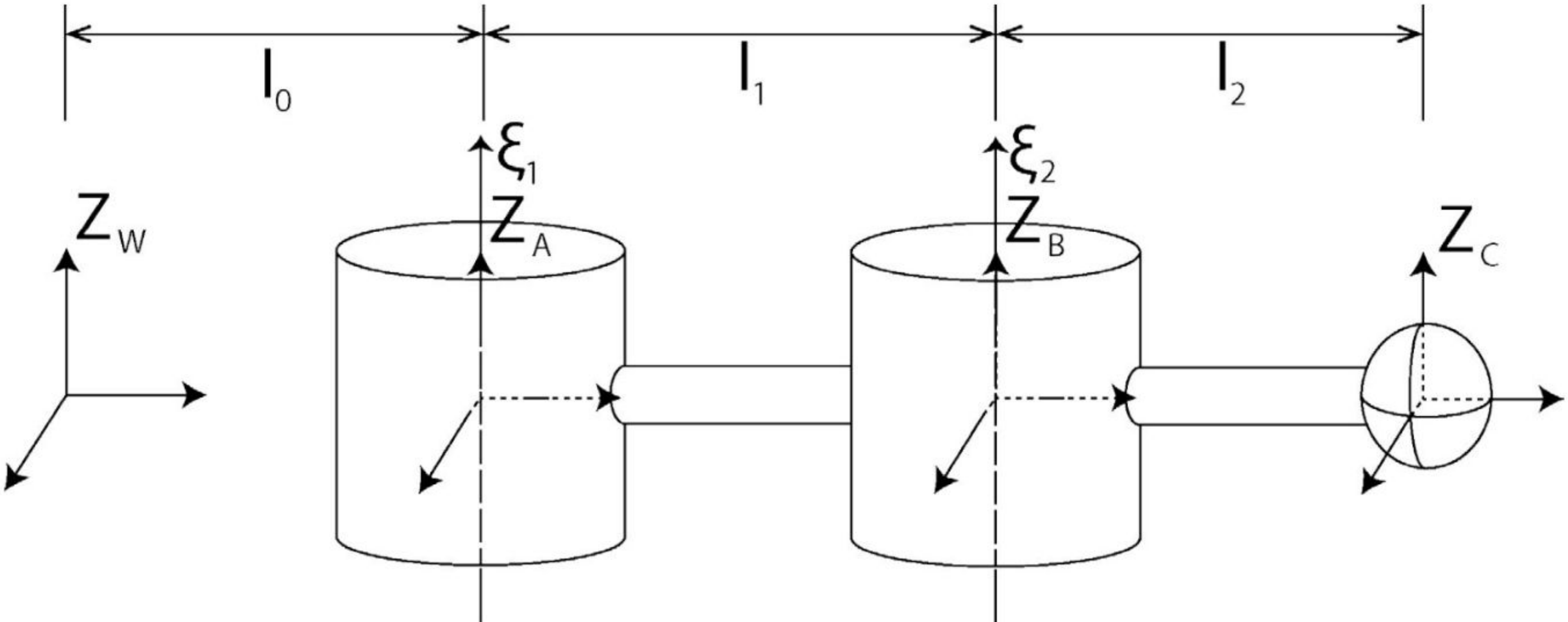
$$V_{ab}^{s} = \begin{bmatrix} v_{ab}^{s} \\ \omega_{ab}^{s} \end{bmatrix} = \begin{bmatrix} R_{ab} & \widehat{p}_{ab} R_{ab} \\ 0 & R_{ab} \end{bmatrix} \begin{bmatrix} v_{ab}^{b} \\ \omega_{ab}^{b} \end{bmatrix} \qquad \mathrm{Ad}_g = \begin{bmatrix} R & \widehat{p} R \\ 0 & R \end{bmatrix}$$

$$\mathrm{Ad}_g^{-1} = \begin{bmatrix} R^T & -(R^T p)^\wedge R^T \\ 0 & R^T \end{bmatrix} = \begin{bmatrix} R^T & -R^T \widehat{p} \\ 0 & R^T \end{bmatrix} = \mathrm{Ad}_{g^{-1}}$$

(look up the Adjoint Representation of a Lie group for more info)

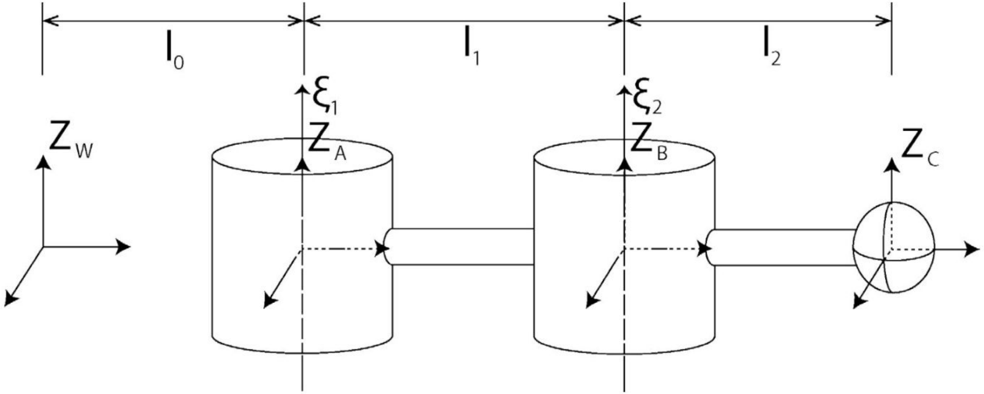
Problem 2: Jacobians

Find the spatial and body Jacobian for the manipulator in this configuration.



Problem 2: Solution pt 1

$$J_{WT}^{s}(\boldsymbol{\theta}) = \begin{bmatrix} \begin{bmatrix} \begin{bmatrix} l_0 \\ 0 \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \end{bmatrix} & \begin{bmatrix} \begin{bmatrix} l_0 + l_1 c_1 \\ l_1 s_1 \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \end{bmatrix} \end{bmatrix}$$



Problem 2: Solution pt 2

$$J_{WT}^{b}(\boldsymbol{\theta}) = \begin{bmatrix} \begin{bmatrix} \begin{bmatrix} -l_1 c_2 - l_2 \\ l_1 s_2 \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \end{bmatrix} & \begin{bmatrix} \begin{bmatrix} -l_2 \\ 0 \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \end{bmatrix} \end{bmatrix}$$