HW 4: Nonholonomic Motion and Constrained Manipulation

Due March 17, 2018

We don't mind if you work with other students on your homework. However, each student must write up and turn in their own assignment (i.e. no copy & paste). If you worked with other students, please **acknowledge who you worked with** at the top of your homework.

1. **Nonholonomic Kinematics**

   (a) Suppose that we have the following model of a turtlebot:

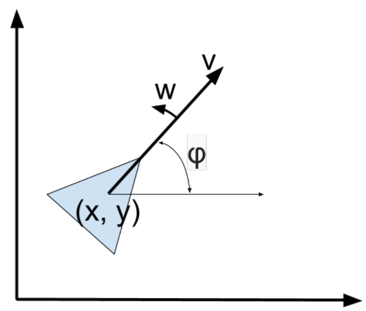

Figure 1: Unicycle Model of a Car

The dynamics of the system are given by:

$$\dot{x} = v\cos(\theta) \tag{1}$$
$$\dot{y} = v\sin(\theta) \tag{2}$$
$$\dot{\theta} = \omega \tag{3}$$

Suppose we can control the speed of the car v and the yaw rate ω.

i. Find the Pfaffian constraints $w(q, \dot{q})\dot{q} = 0$ and rewrite the dynamics in the form of

$$\dot{q} = g_1(q)u_1 + g_2(q)u_2$$

ii. Use Lie brackets to find the Lie algebra of the system. What's the highest order of the Lie bracket you need to use? Is this system nonholonomic?

iii. Are any of the Pfaffian constraint(s) integrable? If so, what are the integrated constraints?

iv. The presence of a holonomic constraint indicates that the system can be represented by a smaller set of state variables (thus eliminating the constraint). If this system is holonomic, what's the minimum number of state variables needed? What are the dynamics of the system in those state variables?

(b) Suppose we have the following model of a cart:

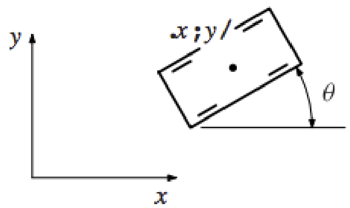

Figure 2: Model of a Cart

The dynamics of the system are given by:

$$\begin{aligned}\dot{x} &= v\cos(\theta)\\ \dot{y} &= v\sin(\theta)\\ \dot{\theta} &= 0\end{aligned}$$

Suppose we can control the speed of the cart v but not the angle θ.

i. Find the Pfaffian constraints $w(q,\dot{q})\dot{q} = 0$ and rewrite the dynamics in the form of

$$\dot{q} = g_1(q)u_1 + g_2(q)u_2$$

ii. Use Lie brackets to find the Lie algebra of the system. What's the highest order of the Lie bracket you need to use? Is this system nonholonomic?

iii. Are any of the Pfaffian constraint(s) integrable? If so, what are the integrated constraints?

iv. The presence of a holonomic constraint indicates that the system can be represented by a smaller set of state variables (thus eliminating the constraint). If this system is holonomic, what's the minimum number of state variables needed? What are the dynamics of the system in those state variables?

(c) Suppose we have the following model of a car:

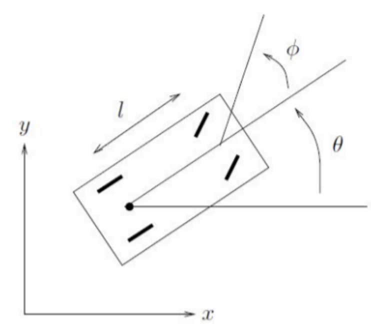

Figure 3: Bicycle Model of a Car

The dynamics of the system are given by:

$$\begin{aligned}\dot{x} &= v\cos(\theta)\\ \dot{y} &= v\sin(\theta)\\ \dot{\theta} &= \frac{v}{l}\tan\phi\end{aligned}$$

where l is the length of the car, and ϕ is the steering angle of with respect to the car body. Suppose we can control the speed of the car v, and the steering velocity $\dot{\phi}$.

i. Find the Pfaffian constraints $w(q,\dot{q})\dot{q} = 0$ and rewrite the dynamics in the form of

$$\dot{q} = g_1(q)u_1 + g_2(q)u_2$$

ii. Use Lie brackets to find the Lie algebra of the system. What's the highest order of the Lie bracket you need to use? Is this system nonholonomic?

iii. Are any of the Pfaffian constraint(s) integrable? If so, what are the integrated constraints?

iv. The presence of a holonomic constraint indicates that the system can be represented by a smaller set of state variables (thus eliminating the constraint). If this system is holonomic, what's the minimum number of state variables needed? What are the dynamics of the system in those state variables?

2. **Constrained Control**

Consider the two-link planar manipulator below:

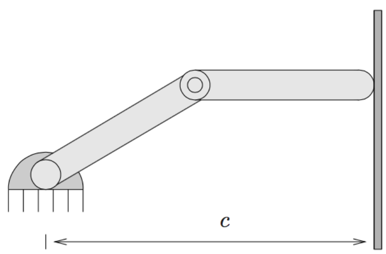

Figure 4: 2-link manipulator

(a) Calculate the kinetic constraint on the system, in terms of the system state (θ_1, θ_2).

(b) Calculate the dynamics of the manipulator assuming that the end-effector remains in contact with the wall at $x = c$. Model the inertia of each link as a point mass concentrated at the middle of the link.

(c) Calculate the internal forces due to the manipulator dynamics.

3. **Short Problems**

(a) What is the Lie algebra of the following system (or rather, what are the Lie brackets you'd need to calculate to find it)?

$$\dot{q} = g_0 + g_1(q)u_1 + g_2(q)u_2$$

(b) What's the difference between force and stress? Why do we use stress and strain rather than force and elongation?

4. **Research Comprehension**

Read one of the following papers:

- Grasp Planning
- Dynamic Regrasping
- Personalized kinematics for human-robot collaborative manipulation

Provide a summary with the following

(a) A brief summary of the work.

(b) What did you find most interesting about the paper?

(c) How might you apply the ideas presented here?

(d) If you were implementing this, what would you add or do differently?

(e) Look at the paper's references. Which 2-3 seem to be the most important or foundational to understand the paper's methodology and why?