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# Lecture Notes for **A Mathematical Introduction to Robotic Manipulation**

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# Chapter 5 Multifingered Hand

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4 Hand Kinematics

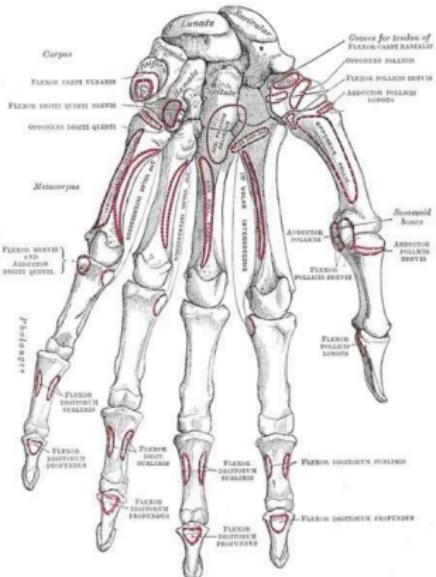
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## 5.1 Introduction

*Hand function:*

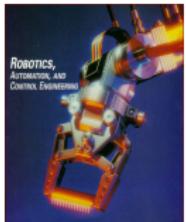
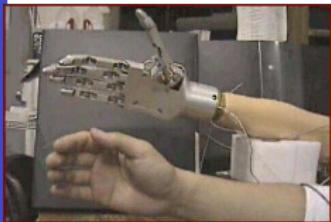


- Hand function:
    - Interface with external world
  - Hand operation:
    - Grasping
    - Dextrous manipulation
    - Fine manipulation
    - Exploration

# 5.1 Introduction

## □ *History of hand design:*

- Prosthetic devices (1509)
- Dextrous end-effectors
- Multiple manipulator/agents coordination
- Human hand study

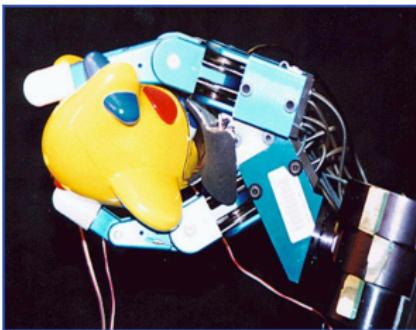


## 5.1 Introduction

### □ *Hand Design Issues:*

- Mechanical systems
- Sensor/actuators
- Control hardware

### □ *A Sample List of Hand Prototypes:*



The salisbury Hand (1982)

The Utah-MIT Hand (1987)

# 5.1 Introduction

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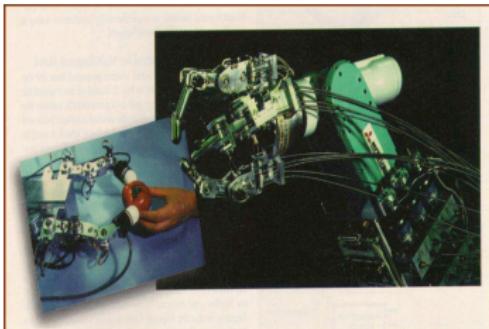
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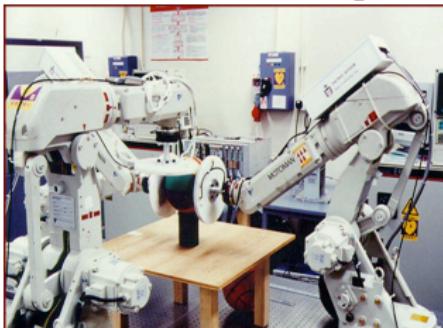
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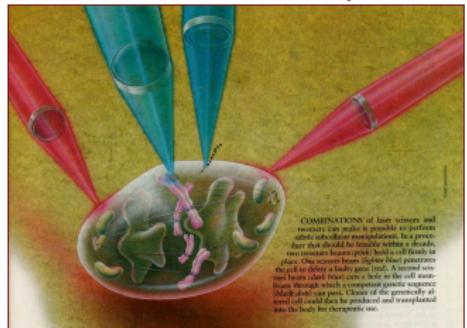
Toshiba Hand (Japan)



The HKUST Hand (1993)



DLR hand (Germany, 1993)



Micro/Nano Hand

# 5.1 Introduction

◊ Example: More Hands

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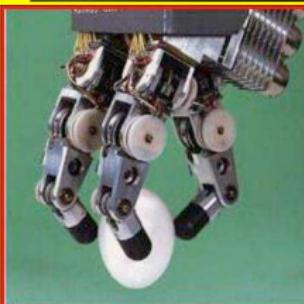
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# 5.1 Introduction

”The hand is indeed an instrument of creation par excellence.”

Rodin (1840–1917)

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Rodin Hand (1898)

# 5.1 Introduction

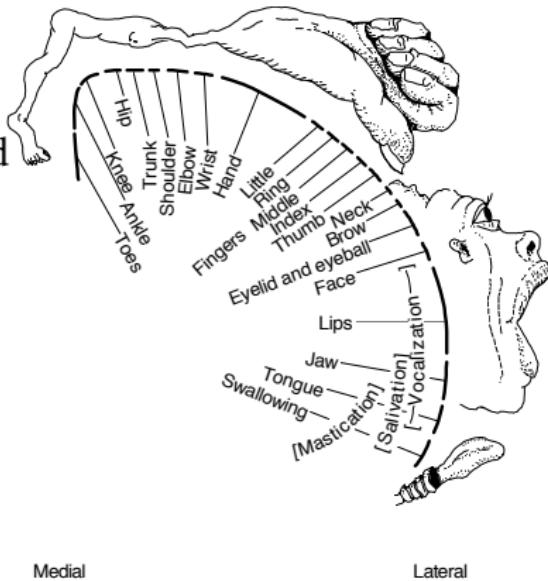
## □ *Lessons from Biological Systems:*

% of motor cortex for hand control:

Human 30% ~ 40%

Monkey 20% ~ 30%

Dog < 10%



Medial

Lateral

Goal: A manipulation theory for robotic hand based on physical laws and rigorous mathematical models!

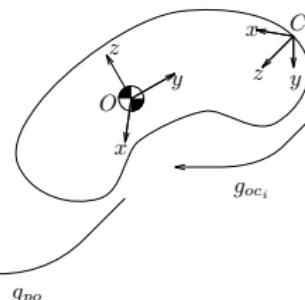
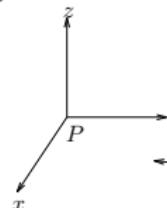
## 5.2 Grasp Statics

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### □ Contact Models:

#### 1 Frictionless Point Contact (FPC)

$$F_i = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} x_i, x_i \geq 0$$



#### 2 Point Contact with friction (PCWF)

$$F_i = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} x_i, x_i \in FC_i$$

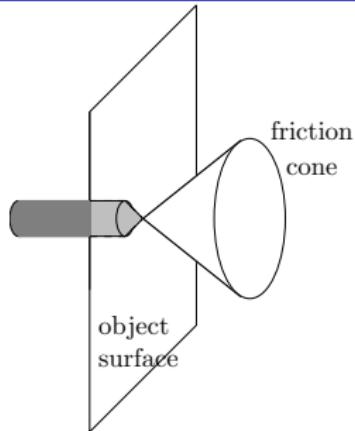
$$FC_i = \left\{ x_i \in \mathbb{R}^3 : \sqrt{x_{i,1}^2 + x_{i,2}^2} \leq \mu_i x_{i,3}, x_{i,3} \geq 0 \right\}$$

$\mu_i$ : Coulomb coefficient of friction

## 5.2 Grasp Statics

### 3 Soft finger contact (SFC)

$$F_i = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} x_i, x_i \in FC_i$$



### Elliptic Model:

$$FC_i = \{x_i \in \mathbb{R}^4 | x_{i,3} \geq 0, \sqrt{\frac{1}{\mu_i^2}(x_{i,1}^2 + x_{i,2}^2)} + \frac{1}{\mu_{it}} x_{i,4}^2 \leq x_{i,3}\}$$

### Linear Model:

$$FC_i = \{x_i \in \mathbb{R}^4 | x_{i,3} \geq 0, \frac{1}{\mu_i} \sqrt{x_{i,1}^2 + x_{i,2}^2} + \frac{1}{\mu_{it}} |x_{i,4}| \leq x_{i,3}\}$$

$\mu_{it}$ : Torsional coefficient of friction

## 5.2 Grasp Statics

$$F_i = B_i \cdot x_i, x_i \in FC_i, B_i \in \mathbb{R}^{6 \times m_i}; \text{wrench basis}$$

### Property 1:

- 1  $FC_i$  is a closed subset of  $\mathbb{R}^{m_i}$ , with nonempty interior.
- 2  $\forall x_1, x_2 \in FC_i, \alpha x_1 + \beta x_2 \in FC_i, \forall \alpha, \beta > 0$

### The Grasp Map:

#### 1 Single contact:

$$F_o = \text{Ad}_{g_{oc_i}^{-1}}^T F_i = \underbrace{\begin{bmatrix} R_{oc_i} & 0 \\ \hat{p}_{oc_i} R_{oc_i} & R_{oc_i} \end{bmatrix}}_{G_i} B_i x_i, x_i \in FC_i, G_i = \text{Ad}_{g_{oc_i}^{-1}}^T B_i$$

#### 2 Multifingered grasp:

$$F_o = \sum_{i=1}^k G_i x_i = [\text{Ad}_{g_{oc_1}^{-1}}^T B_1, \dots, \text{Ad}_{g_{oc_k}^{-1}}^T B_k] \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_k \end{bmatrix} \triangleq G \cdot x$$

$$x \in FC \triangleq FC_1 \times \dots \times FC_k \subset \mathbb{R}^m, m = \sum_{i=1}^k m_i$$

## 5.2 Grasp Statics

### Definition: Grasp

$(G, FC)$  is called a grasp.

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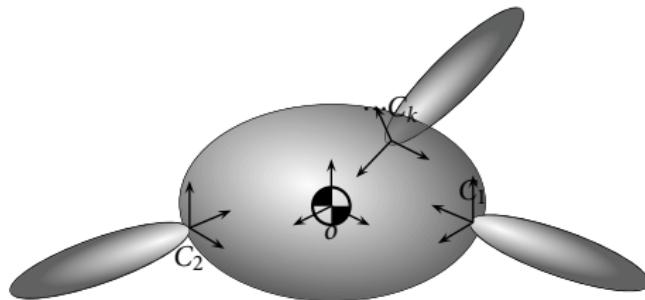
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## 5.2 Grasp Statics

◇ Example: Grasp map offrictionless point contact

$$F_o = \begin{bmatrix} R_{c_i} & 0 \\ \hat{p}_{c_i} R_{c_i} & R_{c_i} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} x_i = \begin{bmatrix} p_{c_i} n_{c_i} \\ n_{c_i} \times n_{c_i} \end{bmatrix} x_i, x_i \geq 0$$

$$\Rightarrow F_o = \begin{bmatrix} p_{c_1} n_{c_1} & \cdots & p_{c_k} n_{c_k} \\ n_{c_1} \times n_{c_1} & \cdots & n_{c_k} \times n_{c_k} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_k \end{bmatrix} = Gx$$

$$F_o \in \mathbb{R}^6, x \geq 0.$$

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## 5.2 Grasp Statics

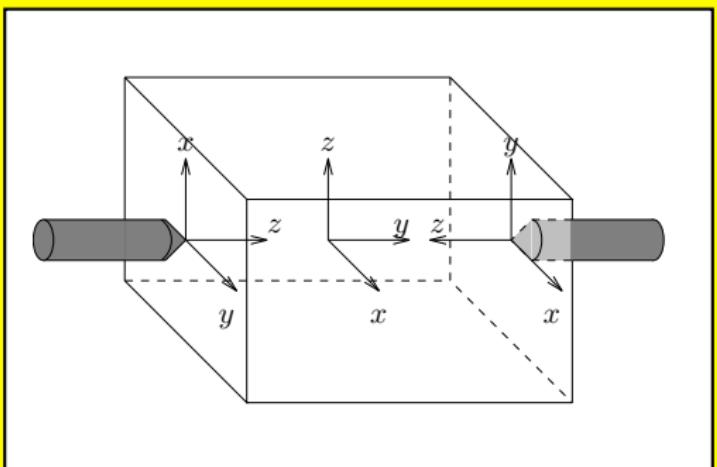
◇ Example: Soft finger grasp of a box

$$R_{c_1} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix},$$

$$p_{c_1} = \begin{bmatrix} 0 \\ -r \\ 0 \end{bmatrix},$$

$$R_{c_2} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix},$$

$$p_{c_2} = \begin{bmatrix} 0 \\ r \\ 0 \end{bmatrix},$$



$$G_i = \begin{bmatrix} R_{oc_i} & 0 \\ \hat{p}_{oc_i} R_{oc_i} & R_{oc_i} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

## 5.2 Grasp Statics

$$G = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & r & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & r & 0 & 0 & -r & 0 & 0 & 0 \end{bmatrix}$$

$$\boldsymbol{x} = [x_{1,1} \quad x_{1,2} \quad x_{1,3} \quad x_{1,4} \quad x_{2,1} \quad x_{2,2} \quad x_{2,3} \quad x_{2,4}] \in \mathbb{R}^8$$

$$FC = FC_1 \times FC_2$$

$$FC_i = \left\{ \boldsymbol{x}_i \in \mathbb{R}^4 \left| \sqrt{\frac{1}{\mu_i}(x_{i,1}^2 + x_{i,2}^2)} + \mu_{it}x_{i,4}^2 \leq x_{i,3}, x_{i,3} \geq 0 \right. \right\}, i = 1, 2$$

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## 5.2 Grasp Statics

### □ Friction Cone Representation:

FPC  $P_i(x_i) = [x_i]$

PCWF  $P_i(x_i) = \begin{bmatrix} \mu_i x_{i,3} + x_{i,1} & x_{i,2} \\ x_{i,2} & \mu_i x_{i,3} - x_{i,1} \end{bmatrix}$

SFC  $P_i(x_i) = \begin{bmatrix} x_{i,3} + x_{i,1}/\mu_i & x_{i,2}/\mu_i - j x_{i,4}/\mu_{it} \\ x_{i,2}/\mu_i + j x_{i,4}/\mu_{it} & x_{i,3} - x_{i,1}/\mu_i \end{bmatrix}, j^2 = -1$

$$P(x) \triangleq \text{Diag}(P_1(x_1), \dots, P_k(x_k)) \in \mathbb{R}^{\bar{m} \times \bar{m}} (\bar{m} = 2k)$$

$$x = (x_1, \dots, x_m)^T = (x_1^T, \dots, x_k^T)^T \in \mathbb{R}^m$$

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## 5.2 Grasp Statics

### Property 2:

1  $P_i = P_i^T, P = P^T$

2  $x_i \in FC_i$  (or  $x \in FC$ )  $\Leftrightarrow P_i(x_i) \geq 0$  (or  $P(x) \geq 0$ )

3  $x_i \in FC_i \Leftrightarrow P_i(x_i) = \underbrace{S_i^0}_{=0} + \sum_{j=1}^{m_i} S_i^j x_{i,j} \geq 0, S_i^{j^T} = S_i^j, j = 1, \dots, m_i$

$$x \in FC \Leftrightarrow P(x) = \underbrace{S_0}_{=0} + \sum_{i=1}^m S_i x_i \geq 0, S_i^T = S_i, i = 1, \dots, m$$

4 Let  $Q(x) = \underbrace{S_0}_{=0} + \sum_{i=1}^m x_i S_i, S_i^T = S_i$ , then

$A_Q = \{x \in \mathbb{R}^m | Q(x) \geq 0\}$  and  $B_Q = \{x \in \mathbb{R}^m | Q(x) > 0\}$  are both convex.

## 5.2 Grasp Statics

### Proof of Property 2:

For PCWF,

$$\sigma(P_i) = \mu_i x_{i,3} \pm \sqrt{x_{i,1}^2 + x_{i,2}^2}$$

Thus,

$$\begin{aligned} x_i \in FC_i &\Leftrightarrow x_{i,3} \geq 0, \sqrt{x_{i,1}^2 + x_{i,2}^2} \leq \mu_i x_{i,3} \\ &\Leftrightarrow \sigma(P_i) \geq 0 \\ &\Leftrightarrow P_i \geq 0 \end{aligned}$$

Furthermore,

$$P_i(x_i) = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} x_{i,1} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} x_{i,2} + \begin{bmatrix} \mu_i & 0 \\ 0 & \mu_i \end{bmatrix} x_{i,3} \quad \square$$

**Exercise:** Verify this for SFC.

## 5.2 Grasp Statics

### □ Force Closure:

#### Definition:

A grasp  $(G, FC)$  is force closure if  $\forall F_o \in \mathbb{R}^p$  ( $p = 3$  or  $6$ ),  $\exists x \in FC$ , s.t.  $Gx = F_o$

#### Problem 1: Force-closure Problem

Determine if a grasp  $(G, FC)$  is force-closure or not.

#### Problem 2: Force Feasibility Problem

Given  $F_o \in \mathbb{R}^p$ ,  $p = 3$  or  $6$ , determine if there exists  $x \in FC$  s.t.

$$Gx = F_o$$

#### Problem 3: Force Optimization Problem

Given  $F_o \in \mathbb{R}^p$ ,  $p = 3$  or  $6$ , find  $x \in FC$  s.t.  $Gx = F_o$  and  $x$  minimizes  $\Phi(x)$ .

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### Definition: internal force

$x_N \in FC$  is an internal force if  $Gx_N = 0$  or  $x_N \in (\ker G \cap FC)$ .

**Property 3:**  $(G, FC)$  is force closure iff  $G(FC) = \mathbb{R}^p$  and

$\exists x_N \in \ker G$  s.t.  $x_N \in \text{int}(FC)$ .

## 5.2 Grasp Statics

### Proof of Property 3:

#### Sufficiency:

For  $F_o \in \mathbb{R}^p$ , let  $x'$  be s.t.  $F_o = Gx'$ . Since  $\lim_{\alpha \rightarrow \infty} \frac{x' + \alpha x_N}{\alpha} = x_N \in \text{int}(FC)$ , there  $\exists \alpha'$ , sufficiently large, s.t.

$$\frac{x' + \alpha' x_N}{\alpha'} \in \text{int}(FC) \subset FC$$

$$\Rightarrow x = x' + \alpha' x_N \in \text{int}(FC)$$

$$\Rightarrow Gx = Gx' = F_o$$

#### Necessity:

Choose  $x_1 \in \text{int}(FC)$  s.t.  $F_o = Gx_1 \neq 0$ , and choose  $x_2 \in FC$  s.t.  $Gx_2 = -F_o$ . Define  $x_N = x_1 + x_2$ ,  $Gx_N = 0 \Rightarrow x_N \in \text{int}(FC)$  □

## 5.2 Grasp Statics

- *Solutions of the force-closure and the force-feasibility problems (P1&P2):*

◊ *Review:*

**Proposition: Linear matrix inequality (LMI) property**

Given  $Q(x) = S_0 + \sum_{l=1}^m x_l S_l$ , where  $S_l = S_l^T$ ,  $l = 0, \dots, m$ , the sets  $A_Q = \{x \in \mathbb{R}^m | Q(x) \geq 0\}$  and  $B_Q = \{x \in \mathbb{R}^m | Q(x) > 0\}$  are convex.

◊ *Review:*

**LMI Feasibility Problem:**

Determine if the set  $A_Q$  or  $B_Q$  is empty or not.

## 5.2 Grasp Statics

### ◊ Review:

Convex constraints on  $x \in \mathbb{R}^m$  (e.g., linear, (convex) quadratic, or matrix norm) can be transformed into LMI's.

- $x = \begin{bmatrix} x_1 \\ \vdots \\ x_m \end{bmatrix} \geq 0 \Leftrightarrow \text{diag}(x) \geq 0$

- For  $A \in \mathbb{R}^{n \times m}$ ,  $b \in \mathbb{R}^n$ ,  $d \in \mathbb{R}$ ,

$$\|Ax + b\| \leq C^T x + d \Leftrightarrow \begin{bmatrix} (C^T x + d)I & Ax + b \\ (Ax + b)^T & C^T x + d \end{bmatrix} \geq 0$$

### ◊ Review: Condition on force-closure

- 1 Rank( $G$ ) = 6

- 2  $x_N \in \mathbb{R}^m$ ,  $P(x_N) > 0$  and  $Gx_N = 0$

Let  $V = [v_1, \dots, v_k]$  whose columns are the basis of  $\ker G$

$$x_N = Vw, w \in \mathbb{R}^k$$

$$\tilde{P}(w) = P(Vw) = \sum_{l=1}^k w_l \tilde{S}_l, \tilde{S}_l = S_l^T, l = 1, \dots, k$$

## 5.2 Grasp Statics

◊ *Review:*

**Proposition:** Given  $Q(x) = S_0 + \sum_{l=1}^m x_l S_l$ , where  $S_l = S_l^T$ ,  $l = 0, \dots, m$ . Let  $x = Az + b$  where  $A \in \mathbb{R}^{m \times n}$ ,  $b \in \mathbb{R}^m$ , then  $\tilde{Q}(z) = Q(Az + b) = \tilde{S}_0 + \sum_{l=1}^n z_l \tilde{S}_l$ , where  $\tilde{S}_l = \tilde{S}_l^T$ ,  $l = 0, \dots, n$

**Theorem 1:**

$(G, FC)$  is force-closure iff  $B_{\bar{P}} = \{w \in \mathbb{R}^k \mid \sum_{l=1}^k w_l \tilde{S}_l > 0\}$  is non-empty.

- Force feasibility Problem

$$Gx = F_o \Rightarrow x_o = \underbrace{G^\dagger F_o}_{\text{generalized inverse}} \quad (\text{may not lie in } FC)$$

$$x = G^\dagger F_o + Vw, \text{span}(V) = \ker G,$$

$$\bar{P}(w) = P(G^\dagger F_o + Vw)$$

$$= \tilde{S}_0 + \sum_{l=1}^k \tilde{S}_l w_l, \text{where } \tilde{S}_l = \tilde{S}_l^T, l = 0, \dots, k$$

## 5.2 Grasp Statics

### Theorem 2:

The force-feasibility problem for a given  $F_o \in \mathbb{R}^6$  is solvable iff  $A_{\bar{P}}(w) = \{w \in \mathbb{R}^k | \bar{S}_0 + \sum_{l=1}^k \bar{S}_l w_l > 0\}$  is non-empty.

◊ *Review: LMI Feasibility Problem as Optimization Problem*

$$\begin{aligned} Q \geq 0 &\Leftrightarrow \exists t \leq 0 \text{ s.t. } Q + tI \geq 0 \\ &\Leftrightarrow t \geq -\lambda_{\min}(Q) \\ &\Leftrightarrow \lambda_{\min}(Q) \geq 0 \end{aligned}$$

**Problem 2':**  $\min t$

$$\text{subject to } Q(x) + tI \geq 0$$

If  $t^* \leq 0$ , then the LMI is feasible.

## 5.2 Grasp Statics

### □ Constructive force-closure for PCWF:

$$G = \begin{bmatrix} n_{c_1} \\ p_{c_1} \times n_{c_1} & \cdots & n_{c_k} \\ p_{c_k} \times n_{c_k} \end{bmatrix}, FC = \{x \in \mathbb{R}^k | x_i \geq 0\}$$

$G(FC) = \mathbb{R}^6 \Leftrightarrow$  Positive linear combination of column of  $G = \mathbb{R}^6$

#### Definition:

- $v_1, \dots, v_k, v_i \in \mathbb{R}^p$  is positively dependent if  $\exists \alpha_i > 0$  such that  $\sum \alpha_i v_i = 0$
- $v_1, \dots, v_k, v_i \in \mathbb{R}^n$  positively span  $\mathbb{R}^n$  if  $\forall x \in \mathbb{R}^n, \exists \alpha_i > 0$ , such that  $\sum \alpha_i v_i = x$

#### Definition:

- A set  $K$  is convex if  $\forall x, y \in K, \lambda x + (1 - \lambda)y \in K, \lambda \in [0, 1]$
- Given  $S = v_1, \dots, v_k, v_i \in \mathbb{R}^p$ , the convex hull of  $S$ :  

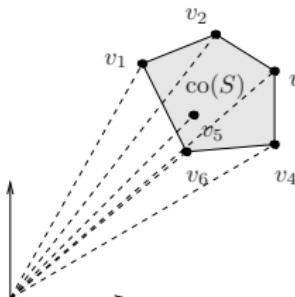
$$co(S) = \{v = \sum \alpha_i v_i, \sum \alpha_i = 1, \alpha_i \geq 0\}$$

## 5.2 Grasp Statics

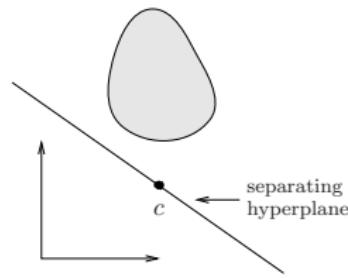
### Property 4:

Let  $G = \{G_1, \dots, G_k\}$ , the following are equivalent:

- 1  $(G, FC)$  is force-closure.
- 2 The columns of  $G$  positively span  $\mathbb{R}^p, p = 3, 6$
- 3 The convex hull of  $G_i$  contains a neighborhood of the origin.
- 4 There does not exist a vector  $v \in \mathbb{R}^p, v \neq 0$  s.t.  
 $\forall i = 1, \dots, k, v \cdot G_i \geq 0$



(a)

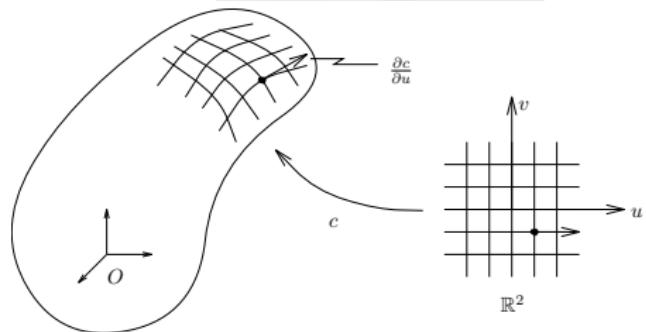
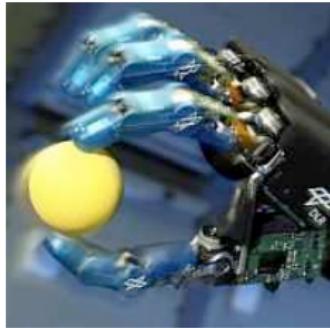


(b)

# 5.3 Kinematics of Contact

## □ Surface Model:

$$c : U \subset \mathbb{R}^2 \rightarrow \mathbb{R}^3, c(U) \subset S$$



$$c_u = \frac{\partial c}{\partial u} \in \mathbb{R}^3$$

$$c_v = \frac{\partial c}{\partial v} \in \mathbb{R}^3$$

First Fundamental form:  $I_p = \begin{bmatrix} c_u^T c_u & c_u^T c_v \\ c_v^T c_u & c_v^T c_v \end{bmatrix}$

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**Orthogonal Coordinates Chart:**  $c_u^T c_v = 0$  (assumption)

$$I_p = \begin{bmatrix} \|c_u\|^2 & 0 \\ 0 & \|c_v\|^2 \end{bmatrix} = M_p \cdot M_p$$

**Metric tensor:**

$$M_p = \begin{bmatrix} \|c_u\| & 0 \\ 0 & \|c_v\| \end{bmatrix}$$

**Gauss map:**

$$N : S \rightarrow \mathbb{S}^2 : N(u, v) = \frac{c_u \times c_v}{\|c_u \times c_v\|} := n$$

**2nd Fundamental form:**

$$II_p = \begin{bmatrix} c_u^T n_u & c_u^T n_v \\ c_v^T n_u & c_v^T n_v \end{bmatrix}, n_u = \frac{\partial n}{\partial u}, n_v = \frac{\partial n}{\partial v}$$

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**Curvature tensor:**

$$K_p = M_p^{-T} H_p M_p^{-1} = \begin{bmatrix} \frac{c_u^T n_u}{\|c_u\|^2} & \frac{c_u^T n_v}{\|c_u\| \|c_v\|} \\ \frac{c_v^T n_u}{\|c_u\| \|c_v\|} & \frac{c_v^T n_v}{\|c_v\|^2} \end{bmatrix}$$

**Gauss frame:**

$$[x, y, z] = \begin{bmatrix} \frac{c_u}{\|c_u\|} & \frac{c_v}{\|c_v\|} & n \end{bmatrix}, K_p = \begin{bmatrix} x^T \\ y^T \end{bmatrix} \begin{bmatrix} \frac{n_u}{\|c_u\|} & \frac{n_v}{\|c_v\|} \end{bmatrix}$$

**Torsion form:**

$$T_p = y^T \begin{bmatrix} \frac{x_u}{\|c_u\|} & \frac{x_v}{\|c_v\|} \end{bmatrix}$$

$(M_p, K_p, T_p)$ : Geometric parameter of the surface.

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◊ Example: Geometric parameters of a sphere in  $\mathbb{R}^3$

$$c(u, v) = \begin{bmatrix} \rho \cos u \cos v \\ \rho \cos u \sin v \\ \rho \sin u \end{bmatrix}$$

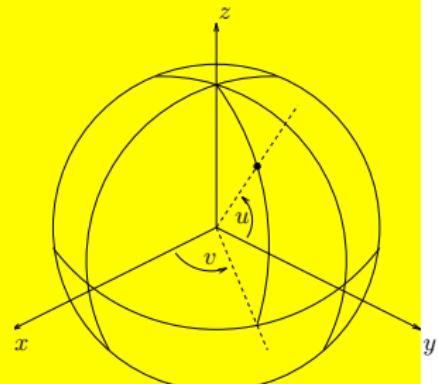
$$U = \{(u, v) | -\frac{\pi}{2} < u < \frac{\pi}{2}, -\pi < v < \pi\}$$

$$c_u = \begin{bmatrix} -\rho \sin u \cos v \\ -\rho \sin u \sin v \\ \rho \cos v \end{bmatrix}$$

$$c_v = \begin{bmatrix} -\rho \cos u \sin v \\ \rho \cos u \cos v \\ 0 \end{bmatrix}$$

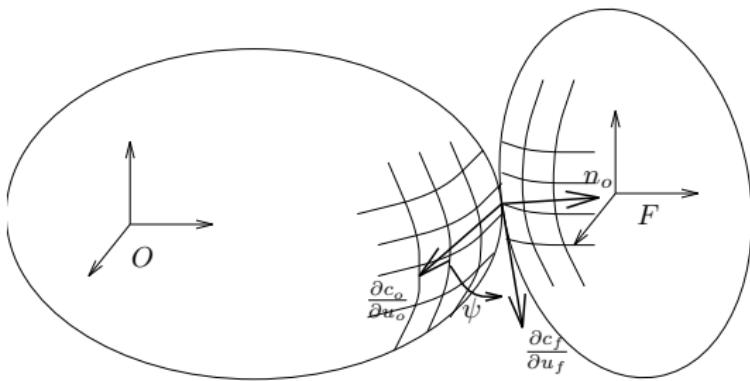
$$c_u^T c_v = 0$$

$$K = \begin{bmatrix} \frac{1}{\rho} & 0 \\ 0 & \frac{1}{\rho} \end{bmatrix}, M = \begin{bmatrix} \rho & 0 \\ 0 & \rho \cos u \end{bmatrix}, T = \begin{bmatrix} 0 & \frac{\tan v}{\rho} \end{bmatrix}$$



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## □ Gauss Frame:



$$g_{oc}(t) \in SE(3)$$

$$V_{oc}^b = ?$$

$$p_{oc}(t) = p(t) = c(\alpha(t)), R_{oc}(t) = [x(t), y(t), z(t)] = \begin{bmatrix} \frac{c_u}{\|c_u\|} & \frac{c_v}{\|c_v\|} & \frac{c_u \times c_v}{\|c_u \times c_v\|} \end{bmatrix}$$

$$v_{oc} = R_{oc}^T \dot{p}_{oc}(t) = \begin{bmatrix} x^T \\ y^T \\ z^T \end{bmatrix} \frac{\partial c}{\partial \alpha} \dot{\alpha} = \begin{bmatrix} x^T \\ y^T \\ z^T \end{bmatrix} \begin{bmatrix} c_u & c_v \end{bmatrix} \dot{\alpha} = \begin{bmatrix} \|c_u\| & 0 \\ 0 & \|c_v\| \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} M \dot{\alpha} \\ 0 \end{bmatrix}$$

$$\hat{\omega}_{oc} = R_{pc}^T \dot{R}_{oc} = \begin{bmatrix} x^T \\ y^T \\ z^T \end{bmatrix} \begin{bmatrix} \dot{x} & \dot{y} & \dot{z} \end{bmatrix} = \begin{bmatrix} 0 & x^T \dot{y} & x^T \dot{z} \\ y^T \dot{x} & 0 & y^T \dot{z} \\ z^T \dot{x} & z^T \dot{y} & 0 \end{bmatrix}$$

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$$y^T \dot{x} = y^T [x_u \ x_v] \dot{\alpha} = TM\dot{\alpha}$$

$$\begin{bmatrix} x^T \dot{z} \\ y^T \dot{z} \end{bmatrix} = \begin{bmatrix} x^T \\ y^T \end{bmatrix} \dot{z} = \begin{bmatrix} x^T \\ y^T \end{bmatrix} [n_u \ n_v] \dot{\alpha} = KM\dot{\alpha}$$

$$\hat{\omega}_{oc} = \left[ \begin{array}{cc|c} 0 & -TM\dot{\alpha} & KM\dot{\alpha} \\ TM\dot{\alpha} & 0 & \\ \hline & -(KM\dot{\alpha})^T & 0 \end{array} \right]$$

,

$$v_{oc} = \begin{bmatrix} M\dot{\alpha} \\ 0 \end{bmatrix}$$

# 5.3 Kinematics of Contact

## □ Contact Kinematics:

$$p_t \in S_0 \mapsto p_f(t) \in S_f$$

Local coordinate:

$$c_0 : U_0 \subset \mathbb{R}^2 \rightarrow S_0$$

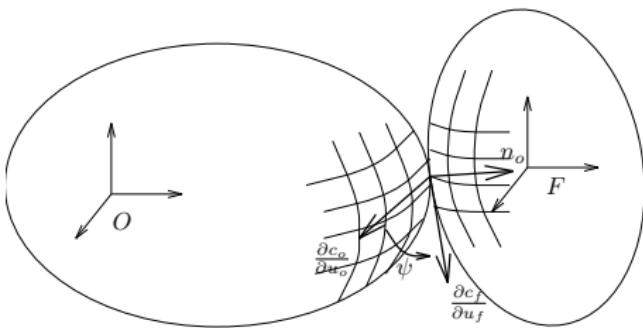
$$c_f : U_f \subset \mathbb{R}^2 \rightarrow S_f$$

$$\alpha_0(t) = c_0^{-1}(p_0(t))$$

$$\alpha_f(t) = c_f^{-1}(p_f(t))$$

Angle of contact:  $\phi$

Contact coordinates:  $\eta = (\alpha_f, \alpha_0, \phi)$



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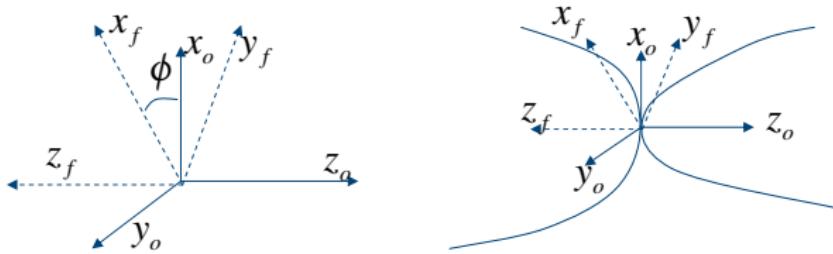
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Rotation about the  $z$ -axis of  $C_o$  by  $-\phi$  aligns the  $x$  axis of  $C_f$  with that of  $C_o$

$$\Rightarrow R_{c_o c_f} = \begin{bmatrix} \cos \phi & -\sin \phi & 0 \\ -\sin \phi & -\cos \phi & 0 \\ 0 & 0 & -1 \end{bmatrix}, p_{c_o c_f} = 0 \in \mathbb{R}^3$$



## 5.3 Kinematics of Contact

Define  $L_0(\tau)$ :

At  $\tau = t$ ,  $L_0(\tau)$  coincide with the Gauss frame at  $p_0(t)$ .

$L_f(\tau)$  : coincide with  $C_f(t)$  at  $\tau = t$

$$v_{l_0} l_f = (v_x, v_y, v_z),$$

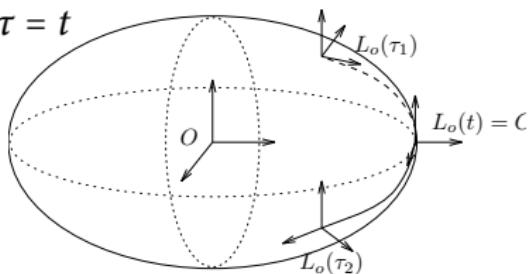
$$\omega_{l_0} l_f = (\omega_x, \omega_y, \omega_z),$$

$\begin{bmatrix} \omega_x \\ \omega_y \end{bmatrix}$ : Rolling velocities

$\begin{bmatrix} v_x \\ v_y \end{bmatrix}$ : Sliding velocities

$v_z$  : Linear velocity in the normal direction

$V_{l_0} l_f = Ad_{g_{f_l f}} V_{o f}$  : Velocity of the finger relative to the object



# 5.3 Kinematics of Contact

Define:  $\tilde{K}_0 = R_\phi K_0 R_\phi$  :Curvature of O relative to  $C_f$   
 $K_f + \tilde{K}_0$  : Relative Curvature.

## Theorem 3: Montana Equations of contact

$$\begin{cases} \dot{\alpha}_f = M_f^{-1}(K_f + \tilde{K}_o)^{-1} \left( \begin{bmatrix} -\omega_y \\ \omega_x \end{bmatrix} - \tilde{K}_o \begin{bmatrix} v_x \\ v_y \end{bmatrix} \right) \\ \dot{\alpha}_o = M_o^{-1}R(K_f + \tilde{K}_o)^{-1} \left( \begin{bmatrix} -\omega_y \\ \omega_x \end{bmatrix} + \tilde{K}_f \begin{bmatrix} v_x \\ v_y \end{bmatrix} \right)_\psi \\ \dot{\psi} = \omega_z + T_f M_f \dot{\alpha}_f + T_o M_o \dot{\alpha}_o \\ v_z = 0 \end{cases}$$

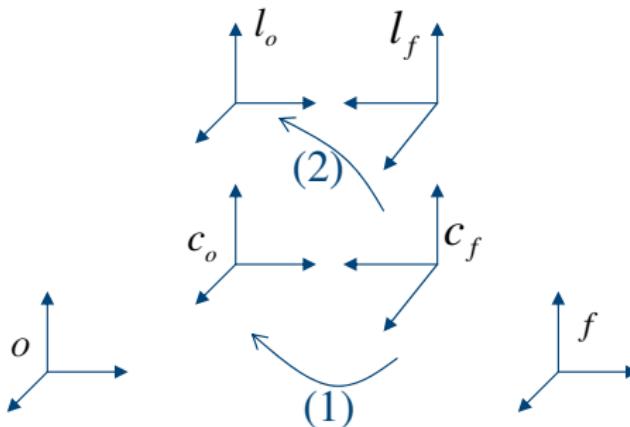
# 5.3 Kinematics of Contact

## Proof of Theorem 3:

$$V_{flf} = 0$$

$$V_{fc_f} = Ad_{g_{l_f c_f}^{-1}} V_{fl_f} + V_{l_f c_f} = V_{l_f c_f}$$

$$V_{oc_o} = Ad_{g_{l_o c_o}^{-1}} V_{ol_o} + V_{l_o c_o} = V_{l_o c_o}$$



## 5.3 Kinematics of Contact

(1). At time t,  $P_{l_f c_f} = 0$  ,  $R_{l_f c_f} = I \Rightarrow V_{l_o c_f} = V_{l_o l_f} + V_{l_f c_f}$

$$(2) p_{c_o c_f} = 0 \Rightarrow V_{l_o c_f} = \begin{bmatrix} R_{c_o c_f}^T & 0 \\ 0 & R_{c_o c_f}^T \end{bmatrix} V_{l_o c_o} + V_{c_o c_f}$$

$$\Rightarrow V_{l_o l_f} + V_{f c_f} = \begin{bmatrix} R_{c_o c_f}^T & 0 \\ 0 & R_{c_o c_f}^T \end{bmatrix} V_{o c_o} + V_{c_o c_f}$$

$$p_{c_o c_f} = 0 \Rightarrow V_{c_o c_f} = 0 \quad R_{c_o c_f} = \begin{bmatrix} R_\psi & 0 \\ 0 & -1 \end{bmatrix} \Rightarrow \omega_{c_o c_f} = \begin{bmatrix} 0 \\ 0 \\ \dot{\psi} \end{bmatrix}$$

$$V_{l_o l_f} = \begin{bmatrix} v_x \\ v_y \\ v_z \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}, V_{f c_f} = \begin{bmatrix} M_f \dot{\alpha}_f \\ 0 \end{bmatrix}$$

$$\hat{\omega}_{f c_f} = \left[ \begin{array}{cc|c} 0 & -T_f M_f \dot{\alpha}_f & K_f M_f \dot{\alpha}_f \\ T_f M_f \dot{\alpha}_f & 0 & \\ \hline -(K_f M_f \dot{\alpha}_f)^T & & 0 \end{array} \right]$$

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$$\nu_{oc_o} = \begin{bmatrix} M_o \dot{\alpha}_o \\ 0 \end{bmatrix}, \quad \hat{\omega}_{oc_o} = \begin{bmatrix} 0 & -T_o M_o \dot{\alpha}_o & K_o M_o \dot{\alpha}_o \\ T_o M_o \dot{\alpha}_o & 0 & 0 \\ -(K_o M_o \dot{\alpha}_o)^T & & 0 \end{bmatrix}$$

$$\text{Linear component: } \begin{bmatrix} M_f \dot{\alpha}_f \\ 0 \end{bmatrix} + \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = \begin{bmatrix} R_\psi M_o \dot{\alpha}_o \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} K_f M_f \dot{\alpha}_f \\ T_f M_f \dot{\alpha}_f \end{bmatrix} + \begin{bmatrix} \omega_y \\ -\omega_x \\ \omega_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \dot{\psi} \end{bmatrix} - \begin{bmatrix} R_\psi K_o M_o \dot{\alpha}_o \\ T_o M_o \dot{\alpha}_o \end{bmatrix}$$

$\Rightarrow$  Theorem result



**Corollary:** Rolling contact motion.

$$\dot{\alpha}_f = M_f^{-1} (K_f + \tilde{K}_o)^{-1} \begin{bmatrix} -\omega_y \\ \omega_x \end{bmatrix} \dot{\alpha}_o = M_o^{-1} R_\psi$$

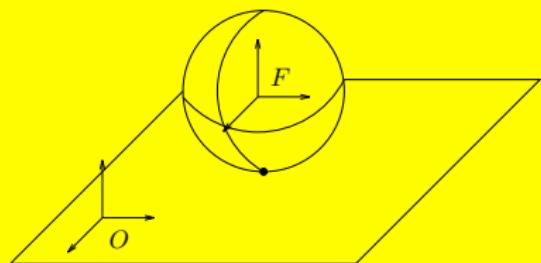
$$(K_f + \tilde{K}_o)^{-1} \begin{bmatrix} -\omega_y \\ \omega_x \end{bmatrix} \dot{\psi} = T_f M_f \dot{\alpha}_f + T_o M_o \dot{\alpha}_o$$

## 5.3 Kinematics of Contact

◇ Example: A sphere rolling on a plane

$$c_f(u, v) = \begin{bmatrix} \rho \cos u_f \cos v_f \\ \rho \cos u_f \sin v_f \\ \rho \sin u_f \end{bmatrix}$$

$$c_o(u, v) = \begin{bmatrix} u_o \\ v_o \\ 0 \end{bmatrix}$$



$$K_o = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, K_f = \begin{bmatrix} \frac{1}{\rho} & 0 \\ 0 & \frac{1}{\rho} \end{bmatrix}$$

$$M_o = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, M_f = \begin{bmatrix} \rho & 0 \\ 0 & \rho \end{bmatrix},$$

$$T_o = [0 \ 0], T_f = [0 \ -\frac{1}{\rho} \tan u_f]$$

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$$\begin{bmatrix} \dot{u}_f \\ \dot{v}_f \\ \dot{u}_o \\ \dot{v}_o \\ \psi \end{bmatrix} = \begin{bmatrix} 0 \\ \sec u_f \\ -\rho \sin \psi \\ -\rho \cos \psi \\ -\tan u_f \end{bmatrix} \omega_x + \begin{bmatrix} -1 \\ 0 \\ -\rho \cos \psi \\ \rho \sin \psi \\ 0 \end{bmatrix} \omega_y$$

$$\dot{\eta} = g_1(\eta) \underbrace{\omega_x}_{u_1(t)} + g_2(\eta) \underbrace{\omega_y}_{u_2(t)} \quad (*)$$

Q: Given  $\eta_0, \eta_f$ , how to find a path  $u: [0, T] \rightarrow \mathbb{R}^2$   
so that solution of (\*) links  $\eta_0$  to  $\eta_f$ ?

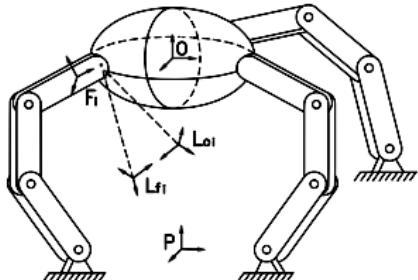
**A question of nonholonomic motion planning!**

# 5.4 Hand Kinematics

$$g_{po} = g_{pfi}(\theta_i) g_{fil_{f_i}} g_{l_{f_i}l_{o_i}} g_{l_{o_i}}$$

$$V_{po} = \text{Ad}_{g_{l_{o_i}o}^{-1}} V_{pfi} + \text{Ad}_{g_{l_{o_i}o}^{-1}} V_{l_{f_i}l_{o_i}}$$

$$\text{Ad}_{g_{l_{o_i}o}} V_{po} = \text{Ad}_{g_{f_il_{o_i}}^{-1}} V_{pfi} + V_{l_{f_i}l_{o_i}}$$



**PCWF:**  $V_z = 0 \Rightarrow [0 \ 0 \ 1 \ 0 \ 0 \ 0] V_{l_{f_i}l_{o_i}} = B_i^T V_{l_{f_i}l_{o_i}} = 0$

$$B_i^T \text{Ad}_{g_{l_{o_i}o}} V_{po} = \underbrace{\text{Ad}_{g_{f_il_{o_i}}} V_{pfi}}_{J_i(\theta_i)\dot{\theta}_i}$$



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## 5.4 Hand Kinematics

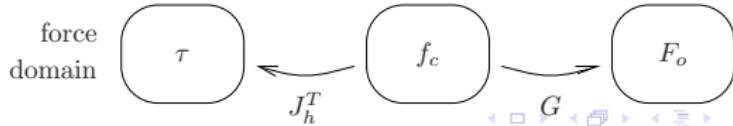
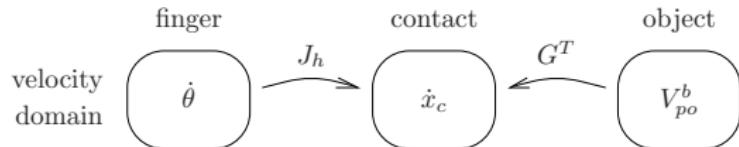
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$$\begin{bmatrix} B_1^T \text{Ad}_{g_{l_{o_i}} o} \\ \vdots \\ B_k^T \text{Ad}_{g_{l_{o_k}} o} \end{bmatrix} V_{po} = \begin{bmatrix} \text{Ad}_{g_{f_l l_{o_1}}^{-1}} J_1(\theta_1) \\ \ddots \\ \text{Ad}_{g_{f_k l_{o_k}}^{-1}} J_k(\theta_k) \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \vdots \\ \dot{\theta}_k \end{bmatrix}$$

$$G^T(\eta) V_{po} = J_h(\theta, x_0, \eta) \dot{\theta}$$

$\theta = (\theta_1, \dots, \theta_k) \in \mathbb{R}^n, n = \sum_{i=1}^k n_i, J_h \in \mathbb{R}^{m \times n}$ : Hand Jacobian

**Definition:**  $\Omega = (G, FC, J_h)$  is called a multifingered grasp.



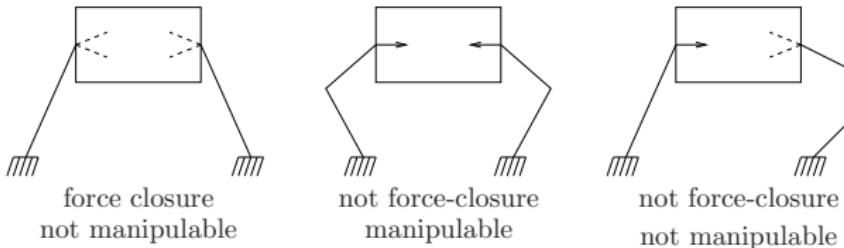
# 5.4 Hand Kinematics

## Definition:

A multifingered grasp  $\Omega = (G, FC, J_h)$  is manipulable at a configuration  $(\theta, x_0)$  if for any object motion,  $V_{po} \in \mathbb{R}^6$ ,  $\exists \dot{\theta} \in \mathbb{R}^n$  s.t.  $G^T V_{po} = J_h \dot{\theta}$ .

## Proposition:

$\Gamma$  is manipulable at  $(\theta, x^\circ)$  iff  $\text{Im}(G^T) \subset \text{Im}(J_h(\theta, x_0))$



# 5.4 Hand Kinematics

Table 5.4: Grasp properties.

Property	Definition	Description
Force-closure	Can resist any applied wrench	$G(FC) = \mathbb{R}^p$
Manipulable	Can accommodate any object motion	$\mathcal{R}(G^T) \subset \mathcal{R}(J_h)$
Internal forces	Contact forces $f_N$ which cause no net object wrench	$f_N \in \mathcal{N}(G) \cap \text{int}(FC)$
Internal motions	Finger motions $\dot{\theta}_N$ which cause no object motion	$\dot{\theta}_N \in \mathcal{N}(J_h)$
Structural forces	Object wrench $F_I$ which causes no net joint torques	$G^+ F_I \in \mathcal{N}(J_h^T)$

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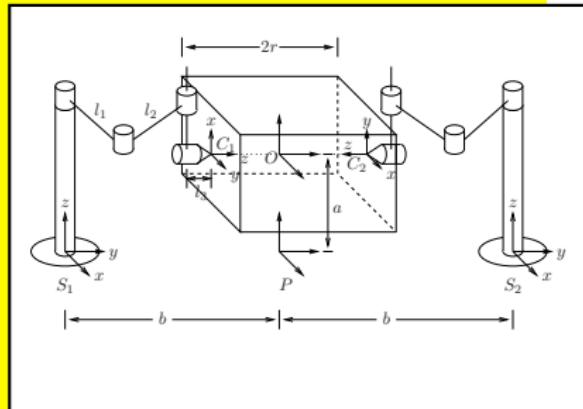
◇ Example: Two SCARA fingers grasping a box

## Soft finger

$$G = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ -r & 0 & 0 & 0 & 0 & 0 & r & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & +r & 0 & 0 & -r & 0 & 0 & 0 \end{bmatrix}$$

$$B_{ci} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_{po} = I, p_{po} = \begin{bmatrix} 0 \\ 0 \\ a \end{bmatrix}$$



# 5.4 Hand Kinematics

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$$J_h = \begin{bmatrix} J_{11} & 0 \\ 0 & J_{22} \end{bmatrix}$$

$$J_{11} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ -b+r & -b+r+l_1c_1 & -b+r+l_1c_1+l_2c_{12} & 0 \\ 0 & l_1s_1 & l_1s_1+l_2s_{12} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$J_{22} = \begin{bmatrix} b-r & b-r+l_3c_3 & b-r+l_3c_3+l_4c_{34} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -l_3s_3 & -l_3s_3-l_4s_{34} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The grasp is not manipulable, as

$$G^T \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ -1 \end{bmatrix} \in \text{Im}(J_h), \dot{\theta}_{N_1} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \dot{\theta}_{N_2} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

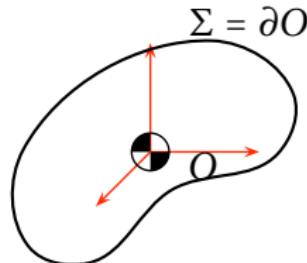
## 5.5 Grasp Planning

### □ *Bounds on number of required contacts:*

Consider PCWF, let

$$\Lambda(\Sigma) = \left\{ \left[ \begin{array}{c} p_{c_i} \\ n_{c_i} \end{array} \right] \middle| c_i \in \Sigma \right\}$$

be the set of all wrenches, where  $n_{c_i}$  is inward normal.



#### **Definition: Exceptional surface**

The convex hull of  $\Lambda(\Sigma)$  does not contain a neighborhood of  $o$  in  $\mathbb{R}^p$ .

E.g. a Sphere or a circle.

#### **Theorem 4: Caratheodory**

If a set  $X = (v_1, \dots, v_k)$  positively spans  $\mathbb{R}^p$ , then  $k \geq p + 1$

# 5.5 Grasp Planning

⇒ Lower bound on the number of fingers.

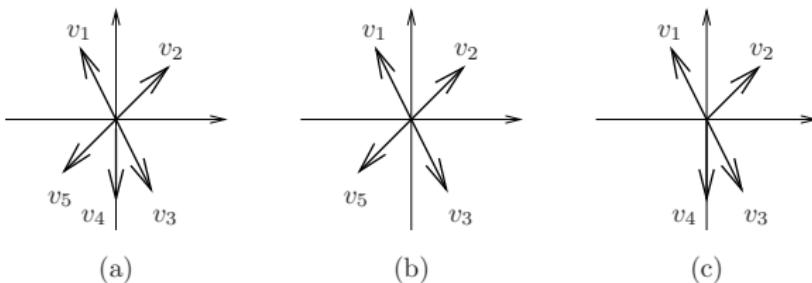


Figure 5.11: Sets of vectors which positively span  $\mathbb{R}^2$ .

## Theorem 5: Steinitz

If  $S \subset \mathbb{R}^p$  and  $q \in \text{int}(co(S))$ , then there exists  $X = (v_1, \dots, v_k) \subset S$  such that  $q \in \text{int}(co(X))$  and  $k \leq 2p$ .

⇒ upper bound on the number of minimal fingers.

# 5.5 Grasp Planning

Table 5.3: Lower bounds on the number of fingers required to grasp an object.

Space	Object type	Lower	Upper	FPC	PCWF	SF
Planar ( $p = 3$ )	Exceptional	4	6	n/a	3	3
	Non-exceptional			4	3	3
Spatial ( $p = 6$ )	Exceptional	7	12	n/a	4	4
	Non-exceptional			12	4	4
	Polyhedral			7	4	4

## □ Constructing force-closure grasps:

### Theorem 1: Planar antipodal grasp

A planar grasp with two point contacts with friction is force-closure iff the line connecting the contact point lies inside both friction cones.

# 5.5 Grasp Planning

## Theorem 2: Spatial antipodal grasps

A spatial grasp with two soft-finger contacts is force-closure iff the line connecting the contact points lies inside both friction cones.

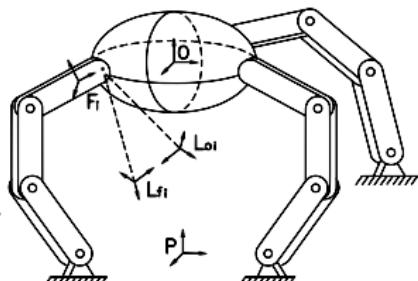
## Problem 4: Optimal Grasp Synthesis

Plan a set of contact points on the object so that  $(G, FC)$  is force closure and optimal in some sense.

**Idea: construct a quality function:**

$$\psi : \alpha = \begin{bmatrix} \alpha_1 \\ \vdots \\ \alpha_k \end{bmatrix} \in \mathbb{R}^{2k} \rightarrow \mathbb{R}$$

with computable gradient such that the optimal solution of  $\psi$  is also force closure.



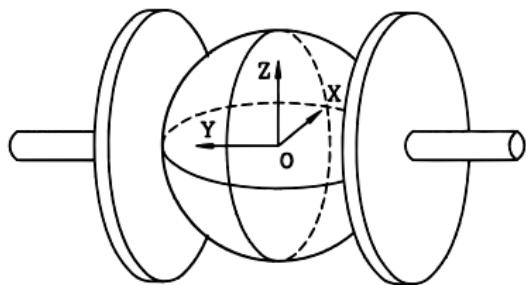
# 5.5 Grasp Planning

## □ *Grasp quality functions:*

- Two-finger grasps (Hong et al 90 & Chen and Burdick 93)

$$E = \frac{1}{2} \|X(\alpha_{o1}) - X(\alpha_{o2})\|^2$$

Solution: antipodal grasp



# 5.5 Grasp Planning

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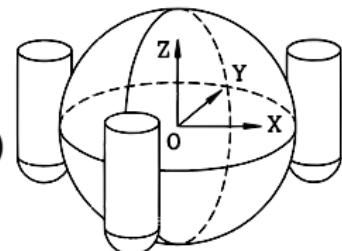
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$$E = \frac{1}{4} (\|X(\alpha_{o3}) - X(\alpha_{o1})\|^2 \|X(\alpha_{o3}) - X(\alpha_{o2})\|^2 - ((X(\alpha_{o3}) - X(\alpha_{o1})) \cdot (X(\alpha_{o3}) - X(\alpha_{o2})))^2)$$



Solution: symmetric grasp

Problem: not general w.r.t. no. of fingers and object geometry

# 5.5 Grasp Planning

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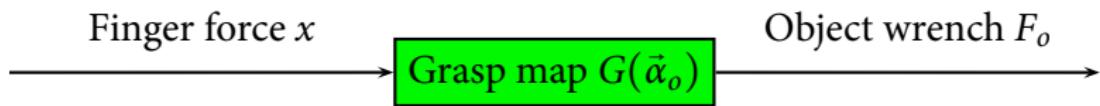
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## 3 Max-transfer problem (Ferrari and Canny 92)

$$\vec{\alpha}_o = (\alpha_{o1}^T, \dots, \alpha_{ok}^T)^T$$

$$g_0(\vec{\alpha}_o) = \min_{\|F_o\|=1} \max_{\substack{Gx = F_o, \|x\| \\ P(x) \geq 0}} \frac{1}{\|x\|}$$



**Problem:** Computational difficulties.

# 5.5 Grasp Planning

## ④ Min-analytic-center problem:

Analytic center  $x^*$ :  
(Boyd et. al. 1996)

$$\begin{aligned} & \min_x \log \det P(x)^{-1} \\ & \text{s.t. } Gx = F \end{aligned}$$

$$P(x) > 0$$

Interpretation: the grasping force  $x$  which is farthest from the boundary of the friction cone.

$$g(\vec{\alpha}_o) = \max_{F_o^T A F_o = 1} \min_{\substack{Gx = F_o, \\ P(x) > 0}} \log \det P(x)^{-1}$$

A: Task requirement

**Interpretation:** optimize worst case analytic center.

## 5.5 Grasp Planning

### □ Simplification for real-time optimization:

Center of FC:  $L = \{\xi t \mid \xi = (0, 0, 1, \dots, 0, 0, 1)^T, t > 0\}$  (PCWF)

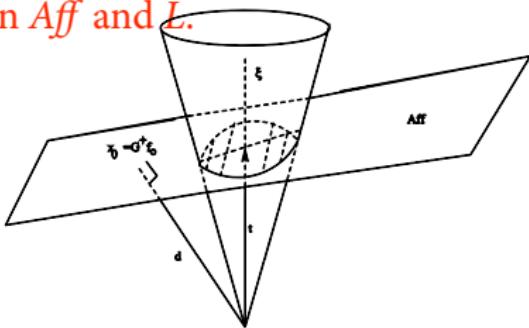
$$L = \{\xi t \mid \xi = (0, 0, 1, 0, \dots, 0, 0, 1, 0)^T, t > 0\} \quad (\text{SFCE})$$

Solution set:  $Aff = \{x \mid Gx = f_o\}$

$x^* \approx \text{the intersection point between } Aff \text{ and } L.$

$$x^* \approx \frac{\xi}{\sqrt{\xi^T G^T A G \xi}},$$

$$\psi(\vec{\alpha}_o) \approx \log \frac{(\xi^T G^T A G \xi)^k}{\prod_{i=1}^k \mu_i^2}$$



### Problem 4: (simplified)

Find  $\vec{\alpha}_o$  s.t. it minimizes  $\psi_1(\vec{\alpha}_o) = \xi^T G^T A G \xi$

Note: Optimization of  $\psi_1$  leads to antipodal and symmetric grasps, respectively.

# 5.5 Grasp Planning

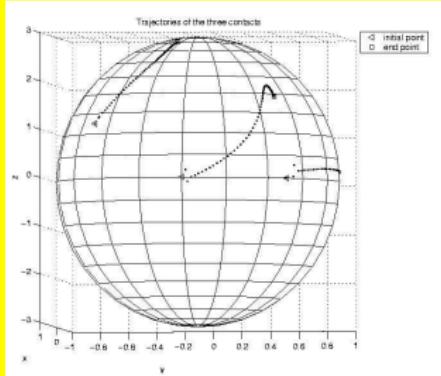
## □ *Simulation results:*

◊ *Example: A 3-fingered hand manipulating an ellipsoid*

■ Minimize  $\psi(\vec{\alpha}_o)$

$$C(\alpha_{oi}) = \begin{bmatrix} a \cos u_{oi} \cos v_{oi} \\ b \cos u_{oi} \sin v_{oi} \\ c \sin u_{oi} \end{bmatrix}$$

$$c = 3a = 3b = 3$$



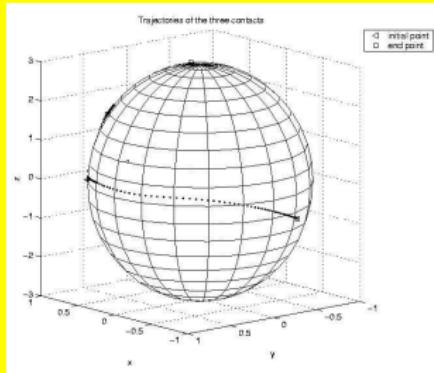
# 5.5 Grasp Planning

## ■ Initial contacts (not force closure)

$$\alpha_{o1} = (0, 0)^T$$

$$\alpha_{o2} = (0, \pi/4)^T$$

$$\alpha_{o3} = (\pi/8, -\pi/4)^T$$



## Advantages of the quality function approach:

- Objects with arbitrary geometry
- Arbitrary number of fingers
- Ability for real-time contact points servoing

# 5.6 Grasp Force Optimization

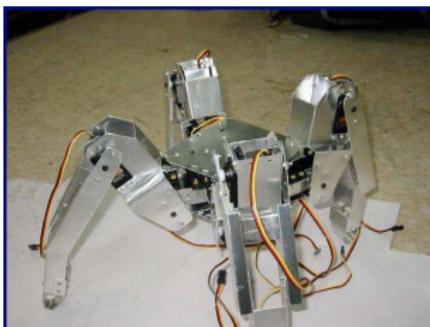
## □ *Grasping Force Optimization:*

### Problem 5:

Given  $F_o \in \mathbb{R}^p$ , find  $x \in FC$  s.t.  $Gx = F_o$  and  $x$  minimizes some suitable cost function.

### Other Applications

- 1 Optimal force distribution for multilegged robots;
- 2 Force control for cable-driven parallel robots



Legged robot



parallel robot

# 5.6 Grasp Force Optimization

## □ Wrench balance constraint:

$$Gx = F_o = (f_{o_1}, \dots, f_{o_6})^T \Leftrightarrow \text{Tr}(B_i P(z)) = f_{oi}, i = 1, \dots, 6 \quad (*)$$

$\Omega_P$  is convex (intersection of a convex cone with a convex hyperplane)

### Sketch of Proof for (\*)

$$\begin{aligned} \mathbb{R}^{2k \times 2k} &\mapsto P(x) = \begin{bmatrix} S_1^1 & \ddots & & \\ & \ddots & & \\ & & 0 & \\ & & & \end{bmatrix} x_1 + \begin{bmatrix} S_1^2 & \ddots & & \\ & \ddots & & \\ & & 0 & \\ & & & \end{bmatrix} x_2 \\ &\quad + \begin{bmatrix} S_1^3 & \ddots & & \\ & \ddots & & \\ & & 0 & \\ & & & \end{bmatrix} x_3 + \dots + \begin{bmatrix} 0 & 0 & & \\ 0 & \ddots & & \\ & & S_k^s & \\ & & & \end{bmatrix} x_m, m = 3k \end{aligned}$$

$$S_1^1 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, S_1^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, S_1^3 = \begin{bmatrix} \mu_1 & 0 \\ 0 & \mu_1 \end{bmatrix}$$

$$Gx = F_0 \Rightarrow \begin{bmatrix} G_{11} & \cdots & G_{1m} \\ \vdots & \ddots & \vdots \\ G_{61} & \cdots & G_{6m} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_m \end{bmatrix} = \begin{bmatrix} f_{o1} \\ \vdots \\ f_{o6} \end{bmatrix}$$

# 5.6 Grasp Force Optimization

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$$\begin{cases} \text{Tr}(B_1 P(x)) = f_{o1} \\ \vdots \\ \text{Tr}(B_6 P(x)) = f_{o6} \end{cases}, B_1 = \begin{bmatrix} B_1^1 & \ddots & 0 \\ 0 & \ddots & B_1^k \end{bmatrix} \in \mathbb{R}^{2k \times 2k}$$

$$\begin{cases} \text{Tr}(B_1^1 S_1^1) = G_{11} \\ \text{Tr}(B_1^1 S_1^2) = G_{12} \\ \text{Tr}(B_1^1 S_1^3) = G_{13} \end{cases}, B_1^1 = \begin{bmatrix} b_{11} & b_{12} \\ b_{12} & b_{22} \end{bmatrix} \Rightarrow \begin{cases} b_{11} - b_{22} = G_{11} \\ 2b_{12} = G_{12} \\ b_{11} + b_{22} = \frac{G_{13}}{\mu_1} \end{cases}$$

$$\Rightarrow B_1^1 = \begin{bmatrix} \frac{1}{2}(G_{11} + \frac{G_{12}}{\mu_1}) & \frac{G_{12}}{2} \\ \frac{G_{12}}{2} & \frac{1}{2}(\frac{G_{13}}{\mu_1} - G_{11}) \end{bmatrix}$$

The rest of  $B_1^i, i = 2, \dots, k$  and thus  $B_j^i, j = 2, \dots, 6$  can be figured out in a similar manner.

# 5.6 Grasp Force Optimization

## □ Wrench balance constraint (continued):

$$\Omega_P = \{x \in \mathbb{R}^n | P(x) > 0, \text{Tr}(B_i P) = f_{oi}, i = 1, \dots, 6\}$$

$\Omega_P$  is convex (intersection of a convex cone with a convex hyperplane)

### Problem 3 (a): Max-det Problem

$$\min \Phi(P) = \text{Tr}(CP) + \log \det P^{-1}$$

$$\text{subject to } \text{Tr}(B_i P) = f_{oi}, i = 1, \dots, 6$$

$$P > 0$$

or

$$\min \Phi(z) = C^T z + \log \det P^{-1}(z)$$

$$\text{subject to } Gx = F_o$$

$$P(x) = S_0 + \sum_{i=1}^m x_i S_i > 0, i = 1, \dots, m$$

$$c_i = \text{Tr}(CS_i)$$

## 5.6 Grasp Force Optimization

Configuration space  $S_{++}^n = \{P \in \mathbb{R}^{n \times n} | P^T = P, P > 0\}$ : Riemannian manifold of dimension  $\frac{n(n+1)}{2}$

$T_p S_{++}^n = \{\xi \in \mathbb{R}^{n \times n} | \xi^T = \xi\} = S^n$ :  $n \times n$  symmetric matrices.

Euclidean metric  $\langle , \rangle : T_p S_{++}^n \times T_p S_{++}^n \mapsto \mathbb{R}^n, (\xi, \eta) \mapsto \text{Tr}(\xi \eta)$

◇ Example:  $S_{++}^2 = \{P \in \mathbb{R}^{2 \times 2} | P = P^T, P > 0\}$

$$\left\{ P = \begin{bmatrix} P_{11} & P_{12} \\ P_{12} & P_{22} \end{bmatrix} \middle| P > 0 \right\} \Leftrightarrow P_{11} > 0, P_{11}P_{22} - P_{12}^2 > 0$$

$$\Rightarrow \{P | P > 0\} \cong \left\{ \begin{bmatrix} P_{11} \\ P_{12} \\ P_{22} \end{bmatrix} \in \mathbb{R}^3 \middle| P_{11} > 0, P_{11}P_{22} - P_{12}^2 > 0 \right\}$$

in  $\mathbb{R}^3$

$T_P S_{++}^2 = \{B \in \mathbb{R}^{2 \times 2} | B = B^T\}$ : vector space of dimension 3.

$$\ll B, C \gg = \text{Tr}(BC) = b_{11}c_{11} + b_{12}c_{12} + b_{12}c_{12} + b_{22}c_{22}$$

$$\text{Dimension of } S^n: \frac{n^2 - n}{2} + n = \frac{n^2 + n}{2}$$

# 5.6 Grasp Force Optimization

◇ Example:  $S^n = T \oplus T^\perp$

Assumption:  $\{B_i\}, i = 1, \dots, 6$  are linearly independent. By Gram-Schmidt process, orthonormalize the  $B_i$ 's if necessary.

$$\text{Thus } \text{Tr}(B_i B_j) = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases}$$

Let  $T = \{\eta \in S^n | \text{Tr}(B_i \eta) = 0, i = 1, \dots, 6\}$  be the subspace of constrained velocities, with  $\dim(T) = \frac{1}{2}n(n+1) - 6$  ( $\dim T_q Q$ )  
 $T^\perp = \text{span}\{B_1, \dots, B_6\}$  ( $T_q Q^\perp$ )

## Property 6:

- $\Phi(P)$  is a convex function
- $\Omega_z$  is a convex set

## 5.6 Grasp Force Optimization

Let  $Q \in S_{++}^n$  be s.t.  $P = Q^2$ ,  $P(t) = Q e^{Q^{-1}\xi t Q^{-1}}$   $Q$  satisfies:

$$P(0) = P, \dot{P}(0) = \xi$$

$$\Rightarrow D\Phi(p)(\xi) = \left. \frac{d}{dt} \right|_{t=0} \Phi(P(t)) = \text{Tr}(C\xi) - \text{Tr}(P^{-1}\xi)$$

where the second term follows from:

$$\left. \frac{d}{dt} \right|_{t=0} \log \det P^{-1}(t) = - \left. \frac{d}{dt} \right|_{t=0} \log \det P(t)$$

$$= - \left. \frac{d}{dt} \right|_{t=0} (\log \det Q + \log \det e^{Q^{-1}\xi t Q^{-1}})$$

$$= - \left. \frac{d}{dt} \right|_{t=0} \log e^{\text{Tr}(Q^{-1}\xi t Q^{-1})}$$

$$= - \left. \frac{d}{dt} \right|_{t=0} \text{Tr}(Q^{-1}\xi t Q^{-1}) = -\text{Tr}(Q^{-1}\xi Q^{-1}) = -\text{Tr}(P^{-1}\xi)$$

## 5.6 Grasp Force Optimization

$\Rightarrow \nabla\Phi(P) \in S^n$  is defined by

$$\text{Tr}(\nabla\Phi(P)\xi) = D\Phi(P)(\xi), \forall \xi \in S^n$$

$$\Rightarrow \nabla\Phi(P) = C - P^{-1}$$

$$\Pi : S^n \mapsto T : \nabla\Phi(P) \mapsto \nabla_T\Phi(P)$$

$$\nabla_T\Phi(P) = C - P^{-1} - \sum_{i=1}^6 \gamma_i B_i,$$

$$\gamma_i = \text{Tr}(B_i(C - P^{-1})), i = 1, \dots, 6$$

# 5.6 Grasp Force Optimization

Constraint subspace:  $T = \{\eta \in S^n | \text{Tr}(B_i \eta) = 0, j = 1, \dots, 6\}$

Euclidean gradient:  $\nabla_T \Phi(P) = C - P^{-1} - \sum_{i=1}^6 \gamma_i B_i$   
 $\gamma_i = \text{Tr}(B_i(C - P^{-1}))$

## □ Computation of $D^2\phi(P)$ :

Consider the curve  $P(t) = Qe^{Q^{-1}\eta t Q^{-1}}$ ,  $P(0) = Q^2 = \Gamma$ ,  $\dot{P}(0) = \eta$ .  
 Then,

$$D^2\phi(\Gamma)(\xi, \eta) = \left. \frac{d}{dt} \right|_{t=0} D\phi(P(t))(\xi) = \text{Tr}(\Gamma^{-1}\xi\Gamma^{-1}\eta), \forall \xi, \eta \in S^n$$

and  $D^2\phi(P)(\xi, \xi) = \text{Tr}(\Gamma^{-1}\xi\Gamma^{-1}\xi) > 0, \forall \xi \neq 0$

$\Rightarrow \phi(\cdot)$  is a convex function. Define  $\ll \xi, \eta \gg_g = \text{Tr}(\Gamma^{-1}\xi\Gamma^{-1}\eta)$

## 5.6 Grasp Force Optimization

### Algorithm 1: Dikin-type Euclidean algorithm (BFM98)

$$P_{k+1} = P_k - \alpha_k \frac{\nabla_T \Phi(P_k)}{\|\nabla_T \Phi(P_k)\|_M},$$

$$\|\nabla_T \Phi(P_k)\|_g = \sqrt{\text{Tr}(P_k^{-1} \nabla_T \Phi(P_k) P_k^{-1} \nabla_T \Phi(P_k))}$$

$$P_k > 0, \alpha_k \in [0, 1) \Rightarrow P_{k+1} > 0$$

Using line-search method for the optimal  $\alpha_k^*$

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# 5.6 Grasp Force Optimization

**Algorithm 2: Newton algorithm with estimated step size  
(HHM02)**

$$P_{k+1} = P_k - \alpha_k (D^2 \Phi(P_k))^{-1} \nabla_T \Phi(P_k) = P_k - \alpha_k \nabla_T \Phi(P_k)$$

$$\alpha_k = \frac{1 + 2\lambda(P_k) - \sqrt{1 + 4\lambda(P_k)}}{2(\lambda(P_k))^2}, \lambda(P_k) = \sqrt{\text{Tr}(\nabla_T \Phi \nabla T \Phi)}$$

## □ LMI Model:

$$P = P_0 + P_1 x_1 + \cdots + P_m x_m \geq 0$$

Elimination of linear constraints  $G \cdot x = F_o$

$$x = G^\dagger F_o + V y, y \in \mathbb{R}^{m-6}, GV = 0$$

$$P = \tilde{P}(y) = \tilde{P}_0 + \tilde{P}_1 y_1 + \cdots + \tilde{P}_{m-6} y_{m-6} \geq 0$$

# 5.6 Grasp Force Optimization

## Algorithm 3: Interior point algorithm (HTL00)

$$\min \Psi(y) = C^T y + \log \det \tilde{P}(y)^{-1}$$

subject to  $\tilde{P}(y) \in \mathbb{R}^{n \times n} = \tilde{P}_0 + \tilde{P}_1 y_1 + \cdots + \tilde{P}_{m-6} y_{m-6} \geq 0$

$$F(y) = F_0 + F_1 y_1 + \cdots + F_{m-6} y_{m-6} > 0$$

$$c_j = \text{Tr}(C\tilde{P}_j), j = 1, \dots, m-6$$

Choose  $F_i$  s.t.  $\text{diag}(\tilde{P}_i, F_i)$ 's are linearly independent for  $i = 1, \dots, m-6$  (e.g.  $F_0 = 1, F_i = 0, i \geq 1$ )

- 1 Solved efficiently using Interior Point Algorithm
- 2 Polynomial-type algorithms w.r.t. the problem dimension (i.e.  $m-6, n$ )

## 5.6 Grasp Force Optimization

### □ *Initial Point Computation:*

#### Problem 5:

Find an initial point  $x$  or  $y$  such that  $P(x) > 0$  or  $\tilde{P}(y) > 0$

[HTL00]

$$\min e^T z = z_{m-6+1} \quad (e = [0 \cdots 0 \ 1]^T)$$

subject to  $\tilde{P}(z) = 1 \geq 0$

$$F(z) = \tilde{P}_0 + \tilde{P}_1 z_1 + \cdots + \tilde{P}_{m-6} z_{m-6} + I z_{m-6+1} \geq 0$$

Solved using the **Interior Point Algorithm** with initial point  $z = [0, \dots, -\lambda_{\min}(\tilde{P}_0) + \beta]^T, \beta > 0$ .

## 5.6 Grasp Force Optimization

### □ *Algorithm analysis & evaluation:*

**Property 8:** Quadratic convergence property

$$d(P_{k+1}, P^*) \leq d^2(P_k, P^*)$$

platform algorithms	No. of iteration	SUN Ultra 60, UNIX
Algorithm 1 (BFM 98)	5	2s/1000 times
Algorithm 2 (HHM 02)	6	3s/1000 times
Algorithm 3 (HTL 00)	2	2s/1000 times

# 5.7 Coordinated Control

## □ *Coordinated Motion Generation:*

### **Problem 6: Coordinated finger motion generation**

Given desired object velocity  $V_{po} \in \mathbb{R}^6$ , find fingertip velocity  $V_{pf_i} \in \mathbb{R}^6$ , that satisfies the non-slippage and the force closure constraints.

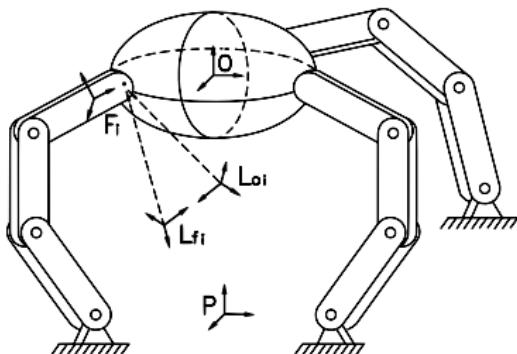
## Kinematics

$$g_{po} = g_{pf_i} \cdot g_{f_i l f_i} \cdot g_{l f_i l_{o_i}} \cdot g_{l_{o_i} o}$$

$$V_{po} = \text{Ad}_{g_{f_i o}}^{-1} V_{pf_i} + \text{Ad}_{g_{l_{o_i} o}}^{-1} V_{l f_i l_{o_i}}$$

$$\tilde{V}_{pf_i} = \text{Ad}_{g_{l_{o_i} o}} V_{po} - V_{l f_i l_{o_i}} \quad (*)$$

$$V_{pf_i} = \text{Ad}_{g_{f_i l_{o_i}}} \tilde{V}_{pf_i}$$



# 5.7 Coordinated Control

## ■ Grasp optimization

$$\tilde{V}_{pf_i} = \text{Ad}_{gl_{o_i} o} V_{po} - B_{c_i}^c \begin{bmatrix} \omega_x^i \\ \omega_y^i \end{bmatrix}$$

constraints

$$B_{c_i}^c = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}^T$$

## ■ Contact equation

$$\begin{bmatrix} \omega_x^i \\ \omega_y^i \end{bmatrix} = R_{\psi_i} (K_{o_i} + \tilde{K}_{f_i}) M_{o_i} \dot{\alpha}_{o_i}$$

## ■ Grasp quality measure

$$g : [ \alpha_{o_1} \quad \cdots \quad \alpha_{o_k} ]^T \in \mathbb{R}^{2k} \rightarrow \mathbb{R}$$

## ■ Optimize $F(\cdot)$

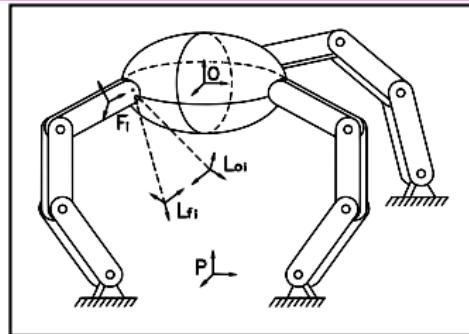
$$\dot{\alpha}_o = -\lambda \nabla g(\alpha_o) = -\lambda \nabla \xi G^T A G \xi, \lambda \in (0, 1)$$

# 5.7 Coordinated Control

## □ *Control System Architecture:*

### Problem 7: Formulation of control objectives

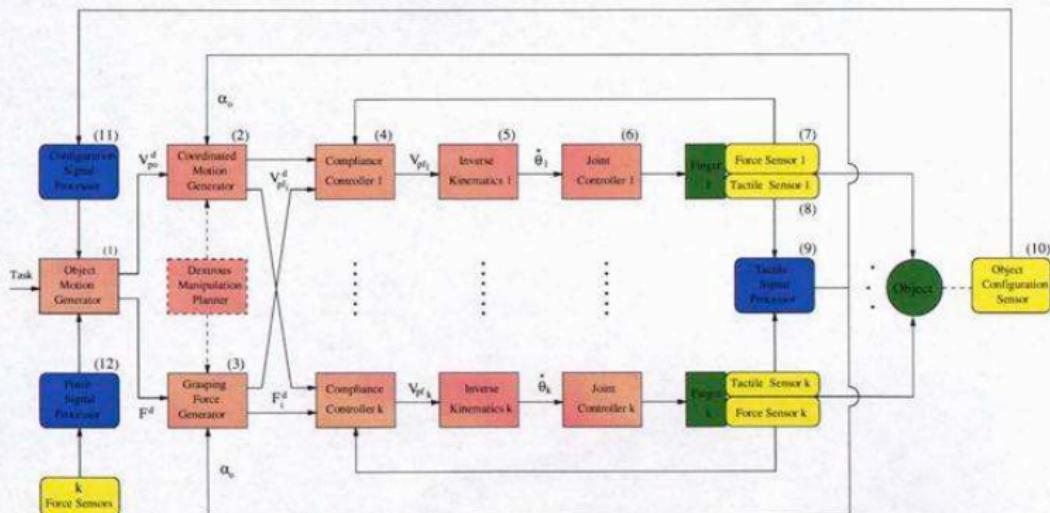
- 1 Desired object velocity  $V_{po}^d$
- 2 Desired object force  $f_o^d$
- 3 Suitable grasp quality  $\alpha_o^d$



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# 5.7 Coordinated Control



CoSAM2 – A unified Control System  
Architecture for Multifingered Manipulation

# 5.7 Coordinated Control

## □ *Coordinated Motion Generation:*

**Input** Desired object velocity  $V_{po}^d \in \mathbb{R}^6$

**Sensors** Tactile sensors

**Output** Fingertip velocity  $V_{pf_i} \in \mathbb{R}^6$

### Constraints

- Rolling/finger gaiting (non-slippage)
- Force closure

# 5.7 Coordinated Control

## □ *Grasping force generation:*

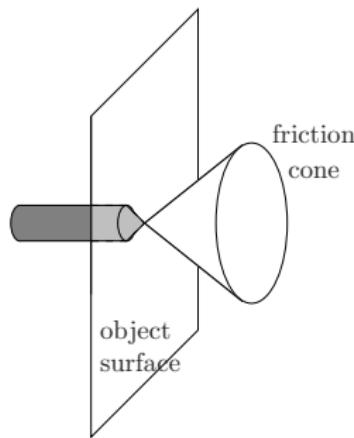
**Input** Desired object force  $f_o^d \in \mathbb{R}^6$

**Sensors** Tactile and contact force sensors

**Output** Fingertip force  $x \in \mathbb{R}^m$

**Constraints**  $-Gx = F^d$

$$-x \in FC$$

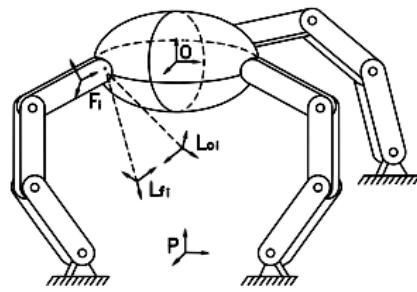


# 5.7 Coordinated Control

## □ Compliance Motion Control Module:

- **Input** Fingertip velocity  $V_{pf_i}^d \in \mathbb{R}^6$  from the CMG module and desired fingertip force  $F_i^d$  from the GFG module
- **Sensors** Contact force sensors
- **Output** total finger velocity  $V_{pf_i} \in \mathbb{R}^6$

$$V_{pf_i} = V_{pf_i}^d + K_{ci}(F_i^d - F_i^m)$$



$K_{ci} \in \mathbb{R}^{6 \times 6}$ : Finger compliance matrix;  $F_i^m$ : Measured force.

$$V_{pf_i}^d = \underbrace{\text{Ad}_{g_{f_i o}}(\eta_i)V_{po}^d}_{\text{Object motion}} + \underbrace{\text{Ad}_{g_{f_i l f_i}}(\eta_i)V_{l_{oi} l_{fi}}(\eta_i, \dot{\eta}_i^d)}_{\text{Grasp quality}} + \underbrace{K_{ci}(F_i^d - F_i^m)}_{\text{Object force}}$$

# 5.7 Coordinated Control

## □ Inverse Kinematics Module:

- Input Desired Fingertip velocity  $V_{pf_i}^d \in \mathbb{R}^6$
- Sensors Finger joint encoders
- Output Finger joint velocity  $\dot{\theta} \in \mathbb{R}^{n_i}$
- Constraints

$$V_{pf_i} = J_{f_i}(\theta_i)\dot{\theta}_i, n_i \leq 6$$

–Collision constraints

$$\min_{\dot{\theta}_i \in \mathbb{R}^{n_i}} \langle J_{f_i}\dot{\theta}_i - V_{pf_i}, Q(J_{f_i}\dot{\theta}_i - V_{pf_i}) \rangle$$

subject to  $\langle \dot{\theta}_i, M\dot{\theta}_i \rangle \leq \alpha^2$

$$\psi(\theta_i) + D\psi(\theta_i)\dot{\theta}_i \leq 0$$



# 5.7 Coordinated Control

## □ *Implementation:*



The three-fingered HKUST hand

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Control

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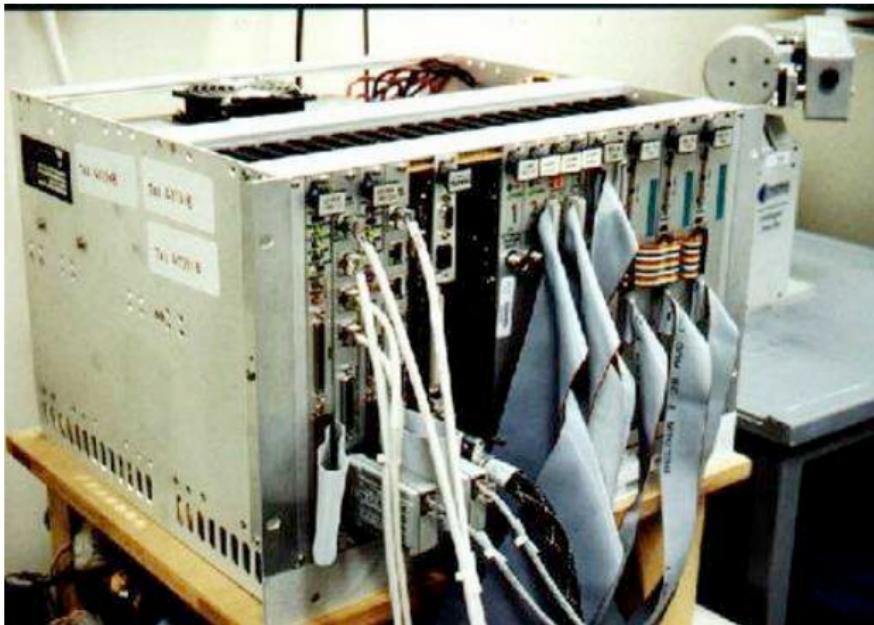
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Microprocessor control system for HKUST hand

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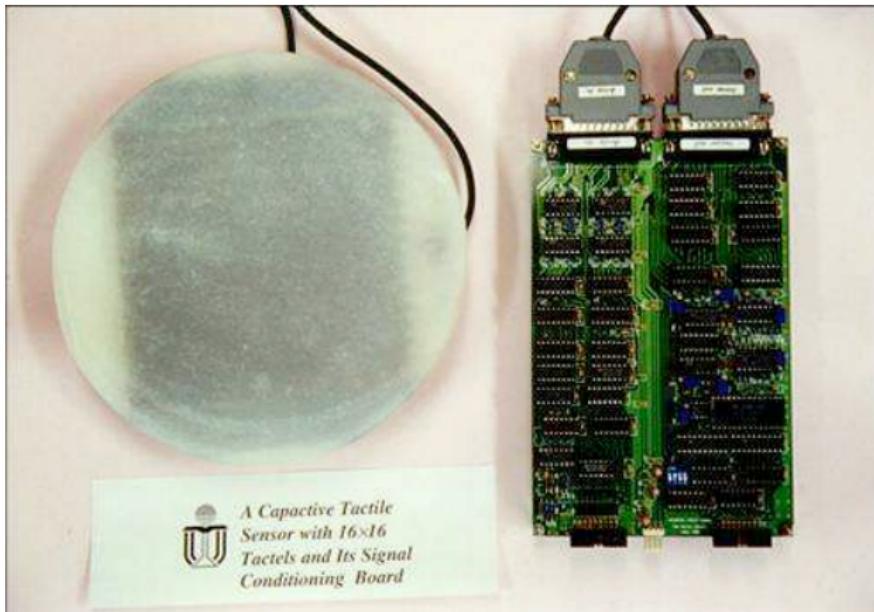
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Tactile sensor and signal conditioning unit for HKUST hand

# 5.7 Coordinated Control

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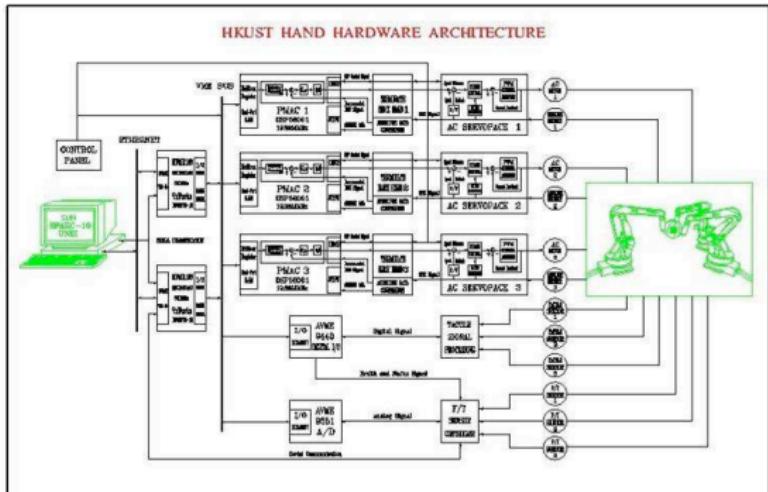
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HKUST hand hardware architecture

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HKUST-Hand system

Sensors

Manipulation: single  
finger and two fingers

Experiment 2  
3-Figured Manipulation