EECS 106B/206B
Discussion 2

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Action Items

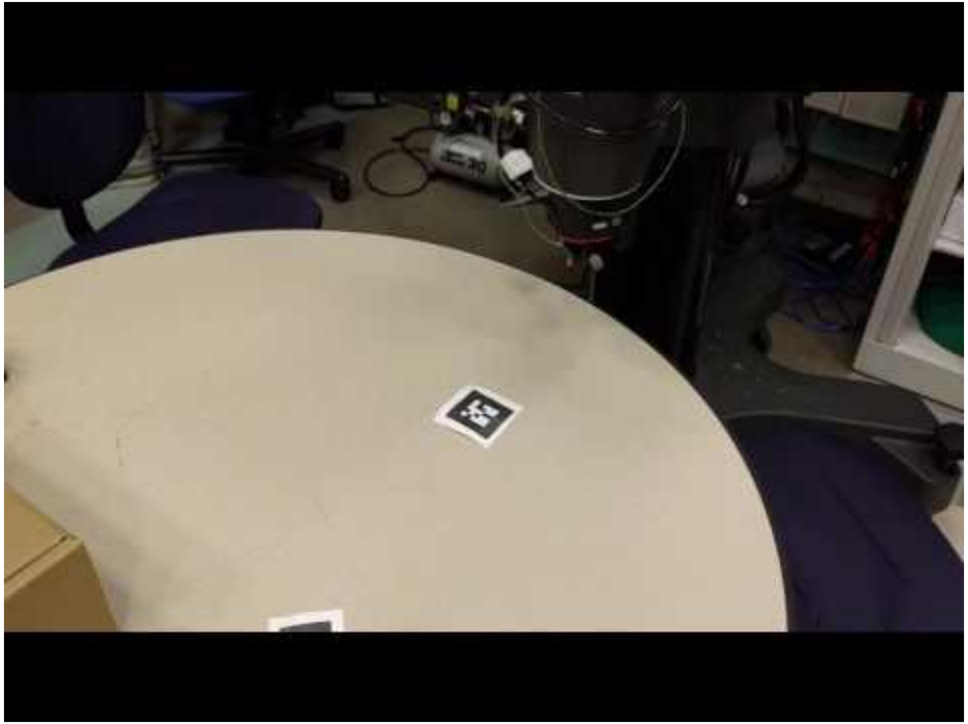
- BCourses/Piazza - is everyone on?
- Concurrent Enrollment students fill out both forms:
  - Student accounts
  - Lab access
- Does everyone else have Lab access?
- Remember, all students who did not take 106a must finish lab 0 with a GSI before you're allowed to use the robots
  - M 1-2, W 4-5, Th 2-4

Lab 1: Trajectory Tracking with Baxter / Sawyer

- Three Path Types
  - Linear Path
  - Circular Path
  - 3D Rectangular path
- Three controller types (and MoveIt)
  - Workspace Velocity
  - Jointspace Velocity
  - Jointspace Torque
- Visual Servoing

Line Task

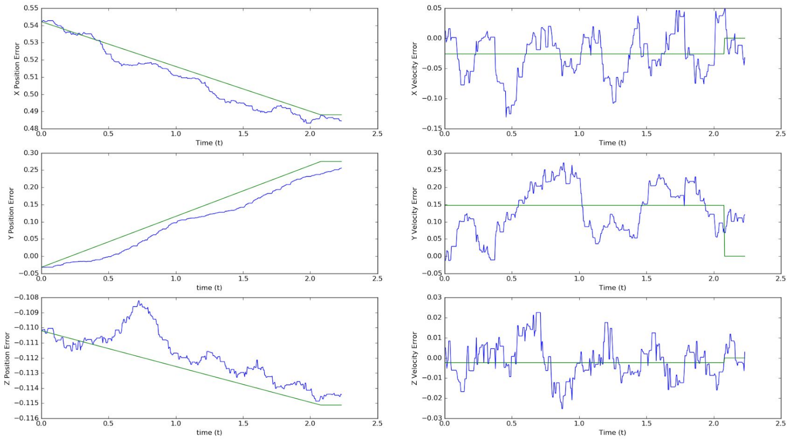
Circular Task

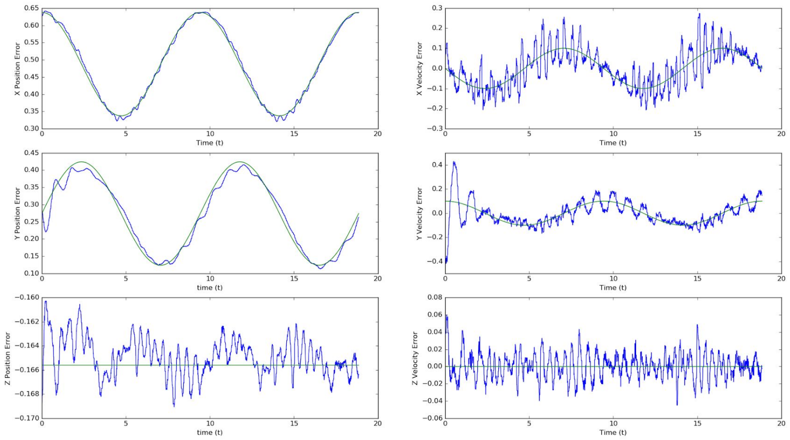


Multiple Paths

Visual Servoing

Example Trajectories





Dynamics

Kinematics: The study of motion

Dynamics: The study of the effect *forces* and *torques* have on motion

Fundamental Dynamics Laws

Newton’s Laws

Conservation of Mass

Conservation of Momentum

Conservation of Energy

Fundamental Dynamics Laws

Newton’s Laws:

$$\sum \boldsymbol{F} = 0 \Leftrightarrow \frac{d\boldsymbol{v}}{dt} = 0$$

$$\boldsymbol{F} = \frac{d\boldsymbol{p}}{dt} = m\frac{d\boldsymbol{v}}{dt} = m\boldsymbol{a}$$

$$F_{A \to B} = -F_{B \to A}$$

Conservation of Mass: $$\frac{dm}{dt} = 0$$

Conservation of Momentum: $$\frac{d}{dt}\sum p = 0$$

Conservation of Energy: $$\sum E = E_k + E_p$$

Newtonian vs Lagrangian Dynamics

Newtonian/Classical Mechanics: Focus on Forces

Lagrangian Dynamics: Focus on *Energy*

Lagrangian: Difference between kinetic and potential energy

$$L(q, \dot{q}) = T(q, \dot{q}) - V(q)$$

Lagrange's Equation

Change in momentum = external force + constraint forces

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} = \frac{\partial L}{\partial q_i} + \Upsilon_i$$

Kinetic Energy of a Rigid Body

Kinetic Energy has translational and rotational components

$$E_k = \frac{1}{2}(mv^2 + \mathcal{I}\omega^2)$$

$$T = \left[\begin{array}{cc} v & \omega \end{array} \right] \left[\begin{array}{cc} mI & 0 \\ 0 & \mathcal{I} \end{array} \right] \left[\begin{array}{c} v \\ \omega \end{array} \right]$$

$$\mathcal{M} = \left[\begin{array}{cc} mI & 0 \\ 0 & \mathcal{I} \end{array} \right]$$ Generalized Inertia Matrix

Kinetic Energy of an Open-Chain Manipulator

$$V_{sl_i}^b = J_{sl_i}^b(\theta)\dot{\theta}$$ Why do we use the body velocity?

$$T_i(\theta, \dot{\theta}) = \frac{1}{2}(V_{sl_i}^b)^T \mathcal{M}_i V_{sl_i}^b = \frac{1}{2}\dot{\theta}^T J_i^T(\theta)\mathcal{M}_i J_i(\theta)\dot{\theta}$$

$$T(\theta, \dot{\theta}) = \sum_{i=1}^{n} T_i(\theta, \dot{\theta}) =: \frac{1}{2}\dot{\theta}^T M(\theta)\dot{\theta}$$

$$M(\theta) = \sum_{i=1}^{n} J_i^T(\theta)\mathcal{M}_i J_i(\theta)$$ Manipulator Inertia Matrix

Lagrangian of a Manipulator

Potential Energy: only gravity

$$V(\theta) = \sum_{i=1}^{n} m_i g h_i(\theta)$$

Lagrangian:

$$L(\theta, \dot{\theta}) = \frac{1}{2}\dot{\theta}^T M(\theta)\dot{\theta} - V(\theta)$$

Dynamics of a Manipulator:

$$M(\theta)\ddot{\theta} + C(\theta, \dot{\theta})\dot{\theta} + N(\theta, \dot{\theta}) = \tau$$

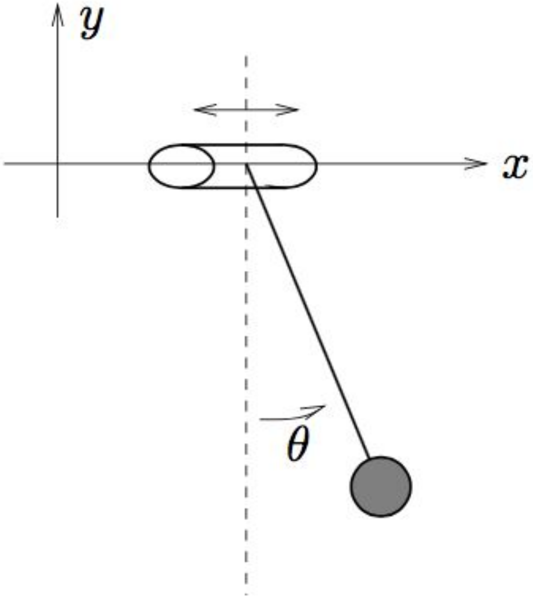
M: Inertial Term

C: Coriolis Term

N: External Forces (gravity and friction)

Tau: Joint Torques

Problem 1: Pendulum on a Cart



(a)

Note:

There are some errors in the following solutions. Typing out dynamics takes a while, but they'll hopefully be fixed over the weekend.

Problem 1: Solution pt 1

$$v_{cart} = \begin{bmatrix} \dot{x} \\ 0 \end{bmatrix} \qquad v_{pendulum} = \begin{bmatrix} \dot{x} + \dot{\theta} l \cos(\theta) \\ \dot{\theta} l \sin(\theta) \end{bmatrix} \qquad q = \begin{bmatrix} x \\ \theta \end{bmatrix}$$

$$V(q) = m_{pendulum} g(l - l\cos(\theta))$$

Problem 1: Solution pt 2

$$T(q,\dot{q})_{cart} = \frac{1}{2}m_{cart}\|v_{cart}\|^2 = \frac{1}{2}m_{cart}\dot{x}^2$$

$$\begin{aligned}
T(q,\dot{q})_{pendulum} &= \frac{1}{2}m_{pendulum}\|v_{pendulum}\|^2 = \frac{1}{2}m_{pendulum}(v^2_{x_{pendulum}} + v^2_{y_{pendulum}}) \\
&= \frac{1}{2}m_{pendulum}\left((\dot{x} + \dot{\theta}l\cos(\theta))^2 + (\dot{\theta}l\sin(\theta))^2\right) \\
&= \frac{1}{2}m_{pendulum}\left(\dot{x}^2 + 2\dot{\theta}\dot{x}l\cos\theta + \dot{\theta}^2l^2\cos^2(\theta) + \dot{\theta}^2l^2\sin^2(\theta)\right) \\
&= \frac{1}{2}m_{pendulum}\left(\dot{x}^2 + 2\dot{\theta}\dot{x}l\cos\theta + \dot{\theta}^2l^2\right)
\end{aligned}$$

$$L(q,\dot{q}) = \frac{1}{2}m_{cart}\dot{x}^2 + \frac{1}{2}m_{pendulum}\left(\dot{x}^2 + 2\dot{\theta}\dot{x}l\cos\theta + \dot{\theta}^2l^2\right) - (l - l\cos(\theta))$$

Problem 1: Solution pt 3

$$\Upsilon_1 = \frac{d}{dt}\frac{\partial L(q,\dot{q})}{\partial \dot{x}} - \frac{\partial L(q,\dot{q})}{\partial x} = \frac{d}{dt}\Big(m_{cart}\dot{x} + m_{pendulum}(\dot{x} + \dot{\theta} l \cos\theta)\Big) - 0$$

$$\Upsilon_2 = \frac{d}{dt}\frac{\partial L(q,\dot{q})}{\partial \dot{\theta}} - \frac{\partial L(q,\dot{q})}{\partial \theta} = \frac{d}{dt}\Big(m_{pendulum}(\dot{x} l \cos(\theta) + l^2\dot{\theta})\Big) - \Big(m_{pendulum}(-\dot{\theta}\dot{x} l \sin(\theta)) - m_{pendulum} g(0 + l\sin(\theta))\Big)$$

$$\Upsilon = \begin{bmatrix} m_{cart}\ddot{x} + m_{pendulum}(\ddot{x} + \ddot{\theta} l \cos(\theta) - \dot{\theta}^2 l \sin(\theta)) \\ m_{pendulum}(\ddot{x} l \cos(\theta) - \dot{x}\dot{\theta} l \sin(\theta) + l^2\ddot{\theta}) - \Big(m_{pendulum}(-\dot{\theta}\dot{x} l \sin(\theta)) - m_{pendulum} g(0 + l\sin(\theta))\Big) \end{bmatrix}$$