EECS 106B/206B

Discussion 10

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Action Items

- Lab 2 released
- HW 3 due this weekend
- One set of lab 2 objects in the back cabinet
- OH scheduling form up

Control affine nonlinear systems

We're interested in controlling systems of this type

$$\dot{x} = g_0(x) + \sum_{i=1}^{m} g_i(x) u_i$$

How do these terms relate to the terms in a linear system?

$$\dot{x} = Ax + Bu$$

g0 is the "drift vector field". Why is it called this? How would you control a drifting system with PID?

Augmented control law as a control affine system

Manipulator Dynamics

$$M(\theta)\ddot{\theta} + C(\theta, \dot{\theta})\dot{\theta} + N(\theta, \dot{\theta}) = \tau$$

Differential Equation form:

$$\ddot{\theta} = -M(\theta)^{-1}C(\theta, \dot{\theta})\dot{\theta} - M(\theta)^{-1}N(\theta, \dot{\theta}) + M(\theta)^{-1}\tau$$

What’s g0? What are g1 to gn?

Unicycle Dynamics

Dynamics of a unicycle robot

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} V\cos(\theta) \\ V\sin(\theta) \\ \omega \end{bmatrix}$$

Is this driftless? What are the control inputs? What are g1 and g2?

Lie Bracket of the Unicycle Model

Component vector fields:

$$g_1 = \begin{bmatrix} \cos(\theta) \\ \sin(\theta) \\ 0 \end{bmatrix} \qquad g_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Lie Bracket:

$$g_3 = [g_1, g_2] = \frac{\partial g_2}{\partial q} g_1 - \frac{\partial g_1}{\partial q} g_2 = \begin{bmatrix} \sin(\theta) \\ -\cos(\theta) \\ 0 \end{bmatrix}$$

Lie Derivative

Derivative of a function along a vector field

$$L_f V = \frac{\partial V(q)}{\partial q} f(q)$$

We saw Lie derivatives when we covered Lyapunov stability

Lie Bracket

Lie derivative of a vector field along another vector field

$$L_f g = [f, g] = \frac{\partial g(q)}{\partial q} f(q) - \frac{\partial f(q)}{\partial q} g(q)$$

Lie brackets satisfy

1. Skew-symmetry:

$$[f, g] = -[g, f]$$

2. Jacobi identity:

$$[f, [g, h]] + [h, [f, g]] + [g, [h, f]] = 0$$

Vector Spaces and Distributions

A vector space is a set of vector fields (often represented by a basis of vector fields)

A distribution is the span of a vector space.

Lie Algebra

A Lie algebra is a vector space, where the Lie bracket of any two component vectors lies inside the vector space

$$f, g \in V \Rightarrow [f, g] \in V$$

The smallest representation of V is the *involutive closure* of V. You generally get this by taking successive Lie brackets of the basis vectors over and over until you get an orthogonal basis.

If your control input vectors don’t span R^n, but the involutive closure of your control input vectors does, then your system is nonholonomic

Lie groups and Lie algebras

All Lie groups have corresponding Lie algebras, and all Lie algebras have corresponding Lie groups. Thus, all the systems we look at are Lie groups, and we can use nonlinear control techniques on all Lie groups