HW 9

4-5

(1):$3^{-n+1}u[n-1]$

$$原式 = 3^{-(n-1)}u[n-1]$$
$$3^{-n}u[n] <--> \frac{1}{3-e^{-jw}}$$
$$3^{-(n-1)}u[n-1] <--> e^{-jw}\frac{1}{3-e^{-jw}}$$

(3):$2^n u[-n]$

$$2^n u[-n] = (\frac{1}{2})^{-n}u[-n]$$
$$(\frac{1}{2})^n u[n] <--> \frac{1}{1-\frac{1}{2}e^{-jw}}$$
$$(\frac{1}{2})^{-n}u[-n] <--> \frac{1}{1-\frac{1}{2}e^{jw}}$$

(5):$\delta[n-2]+\delta[n+2]$

$$\delta[n] <--> 1$$
$$\delta[n-2]+\delta[n+2] <--> e^{-2jw}+e^{2jw}$$

(6):$u[n-1]-u[n-5]$

$$u[n-1]-u[n-5] = u[n+2-3]-u[n-2-3]$$
$$u[n+2]-u[n-2] <--> \frac{sin(2.5w)}{sin(0.5w)}$$
$$u[n-1]-u[n-5] <--> e^{-3jw}\frac{sin(2.5w)}{sin(0.5w)}$$

(9):$n2^{-n}u[n]$

$$nx[n] <--> j\frac{dX(jw)}{dw}$$
$$2^{-n}u[n] <--> \frac{1}{1-\frac{1}{2}e^{-jw}}$$
$$n2^{-n}u[n] <--> -\frac{1}{2}\frac{1}{1-\frac{1}{2}e^{-jw}}$$

(11):

$$x[n] = u[n]-u[n-4]$$
$$= u[n+2-2]-u[n-2-2]$$
$$u[n+2]-u[n-2] <--> \frac{sin(2.5w)}{sin(0.5w)}$$
$$x[n] <--> e^{-2jw}\frac{sin(2.5w)}{sin(0.5w)}$$

(12):

$$X(e^{jw})=\sum x[n]e^{-jwn}$$
$$=\frac{1}{2}e^{3jw}+e^{2jw}+\frac{3}{2}e^{jw}+2+\frac{1}{2}e^{-3jw}+e^{-2jw}+\frac{3}{2}e^{-jw}$$
$$=2+3cosw+2cos2w+cos3w$$

4-6

(2):

$$X(e^{jw})=1-e^{-jw}+2e^{-j2w}-3e^{-3jw}+4e^{-4jw}$$
$$\delta[n-n_0]<-->e^{-jwn_0}$$
$$X(e^{jw})<-->\delta[n]-\delta[n-1]+2\delta[n-2]-3\delta[n-3]+4\delta[n-4]$$

(3)

$$X(e^{jw})=e^{-j\frac{w}{2}},w\in[-\pi,\pi]$$
$$\frac{sinW_0n}{\pi n}-->X(e^{jw})=1(|w|<W_0),X(e^{jw})=0(|w|>W_0),\text{周期为}2\pi$$
$$x[n-\frac{1}{2}]<-->e^{-j\frac{w}{2}}X(e^{jw})$$
$$x[n]=\frac{sinW_0(n-\frac{1}{2})}{\pi(n-\frac{1}{2})}$$

(4)

$$X(e^{jw})=cos^2w+jsin3w$$
$$=\frac{cos2w+1}{2}+jsin3w$$
$$=\frac{e^{2jw}+e^{-2jw}+2}{4}+\frac{e^{3jw}-e^{-3jw}}{2}$$
$$=\frac{\delta[n-2]+\delta[n+2]+2\delta[n]}{4}+\frac{\delta[n+3]-\delta[n-3]}{2}$$

(6)

$$X(e^{jw})=\frac{1-e^{-jw}}{1-\frac{5}{6}e^{-jw}+\frac{1}{6}e^{-2jw}}$$
$$X(e^{jw})=\frac{1-e^{-jw}}{(1-\frac{1}{2}e^{-jw})(1-\frac{1}{3}e^{-jw})}$$
$$=\frac{1}{1-\frac{1}{3}e^{-jw}}-(\frac{1}{1-\frac{1}{2}e^{-jw}}-\frac{1}{1-\frac{1}{3}e^{-jw}})\times 3$$
$$=4\frac{1}{1-\frac{1}{3}e^{-jw}}-3\frac{1}{1-\frac{1}{2}e^{-jw}}$$
$$x[n]=4(\frac{1}{3})^nu[n]-3(\frac{1}{2})^nu[n]$$

(7)

$$
\begin{aligned}
X(e^{jw}) &= \frac{1-(2e^{jw})^{-8}}{1-(2e^{jw})^{-1}} \\
&= 1+\sum_{n=1}^{7}(\frac{1}{2}e^{-jw})^n \\
&= \delta[n]+\sum_{m=1}^{7}(\frac{1}{2})^m\delta[n-m]
\end{aligned}
$$

4-8

(1)

$$
X(e^{jw}) = \sum x[n]e^{-jwn}
$$
$$
X(e^{j0}) = 6
$$

(2)

(3)

$$
x[n] = \frac{1}{2\pi}\int_{2\pi} X(e^{jw})e^{jw}dw
$$
$$
x[0] = \frac{1}{2\pi}\int_{2\pi} X(e^{jw})dw = 2
$$
$$
\int_{-\pi}^{\pi} X(e^{jw})dw = 4\pi
$$

(4)

$$
\begin{aligned}
X(e^{j\pi}) &= \sum x[n]e^{-j\pi n} \\
&= \sum x[n](-1)^n = 2
\end{aligned}
$$

(5)

$$Re\{X(e^{jw})\} <--> \frac{x[n]+x[-n]}{2}$$

$$n=7, \frac{x[n]+x[-n]}{2} = \frac{-1}{2}$$

$$n=6, \frac{x[n]+x[-n]}{2} = \frac{0}{2}$$

$$n=5, \frac{x[n]+x[-n]}{2} = \frac{1}{2}$$

$$n=4, \frac{x[n]+x[-n]}{2} = \frac{2}{2}$$

$$n=3, \frac{x[n]+x[-n]}{2} = \frac{0}{2}$$

$$n=2, \frac{x[n]+x[-n]}{2} = \frac{0}{2}$$

$$n=1, \frac{x[n]+x[-n]}{2} = \frac{2}{2}$$

$$n=0, \frac{x[n]+x[-n]}{2} = \frac{4}{2}$$

$$n=-1, \frac{x[n]+x[-n]}{2} = \frac{2}{2}$$

$$n=-2, \frac{x[n]+x[-n]}{2} = \frac{0}{2}$$

$$n=-3, \frac{x[n]+x[-n]}{2} = \frac{0}{2}$$

(6)

$$\int_{-\pi}^{\pi} |X(e^{jw})|^2 dw$$

$$\sum |x[n]|^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} |X(e^{jw})|^2 dw$$

$$\int_{-\pi}^{\pi} |X(e^{jw})|^2 dw = 2\pi \sum |x[n]|^2 = 28\pi$$

(7)

$$\int_{-\pi}^{\pi} |\frac{dX(e^{jw})}{dw}|^2 dw$$

$$nx[n] <--> j\frac{dX(e^{jw})}{dw}$$

$$\int_{-\pi}^{\pi} |\frac{dX(e^{jw})}{dw}|^2 dw = -2\pi \cdot \frac{1}{2\pi} \int_{-\pi}^{\pi} |j\frac{dX(e^{jw})}{dw}|^2 dw$$

$$\sum |nx[n]|^2 <--> \frac{1}{2\pi} \int_{-\pi}^{\pi} |j\frac{dX(e^{jw})}{dw}|^2 dw$$

$$\int_{-\pi}^{\pi} |\frac{dX(e^{jw})}{dw}|^2 dw = 316\pi$$

4-10

(1):

$$\frac{sin(\pi n/3)}{\pi n}\frac{sin(\pi n/4)}{\pi n}$$

$$\frac{sin(\pi n/3)}{\pi n} <--> X(e^{jw}) = \begin{cases} 1, |w| < \pi n/3 \\ 0, |w| > \pi n/3 \end{cases}$$

$$\frac{sin(\pi n/3)}{\pi n} <--> X(e^{jw}) = \begin{cases} 1, |w| < \pi n/4 \\ 0, |w| > \pi n/4 \end{cases}$$

$$x[n]y[n] <--> \frac{1}{2\pi}\int_{2\pi} X(e^{j\theta})Y(e^{jw-\theta})d\theta = \begin{cases} 1/4, |w| < \pi/12 \\ 7/24 - |w|/2\pi, |w| \in [\pi/12, 7\pi/12] \\ 0 \end{cases}$$

(2)

$$(n+1)a^n u[n]$$

$$(n+1)a^n u[n] = na^n u[n] + a^n u[n]$$

$$nx[n] <--> j\frac{dX(e^{jw})}{dw}$$

$$a^n u[n] <--> \frac{1}{1-ae^{-jw}}$$

$$(n+1)a^n u[n] <--> \frac{1-a}{1-ae^{-jw}}$$

(3)

$$设x_1[n] = \begin{cases} \frac{\sqrt{E}}{N_1-1}, |n| < N_1 \\ 0 \end{cases}$$

$$x[n] = x_1[n] * x_1[n]$$

$$X(e^{jw}) = X_1(e^{jw})^2 = \frac{E}{(N_1-1)^2}\frac{sin^2(\frac{N_1-1}{2}w)}{sin^2\frac{w}{2}}$$

4-13

(1)

$$x_1[n] = x[1-n] + x[1-n]$$

$$x[-n] <--> X(e^{-jw})$$

$$x[n-1] <--> e^{-jw}X(e^{jw})$$

$$x[1-n] + x[1-n] <--> 2e^{jw}X(e^{-jw})$$

(2)

$$x_2[n] = x[-n]cosw_0 n, 0 < w_0 < \pi$$

$$x[n]cosw_0 n <--> \frac{1}{2}(X(e^{j(w-w_0)}) + X(e^{j(w+w_0)}))$$

$$x[-n] <--> X(e^{-jw})$$

$$x[-n]cosw_0 n <--> \frac{1}{2}(X(e^{-j(w-w_0)}) + X(e^{-j(w+w_0)}))$$

(3)

$$x_3[n] = \frac{x^*[-n] + x[n]}{2}$$
$$x^*[n] <-- > X^*(e^{-jw})$$
$$\frac{x^*[-n] + x[n]}{2} <-- > [X^*(e^{jw}) + X(e^{jw})]/2$$

(4)

$$x_4[n] = (n-1)^2 x[n]$$
$$= n^2 x[n] - 2nx[n] + x[n]$$
$$= -\frac{dX(e^{jw})}{dw} - 2j\frac{dX(e^{jw})}{dw} + X(e^{jw})$$

4-14

(1):$X(e^{jw})$是虚信号，且既不是奇信号也不是偶信号，x[n]是虚信号，且既不是奇信号也不是偶信号

(2):x[n]是实奇信号

(3):x[n]是实信号，且既不是奇信号也不是偶信号

4-16

全部满足下列条件之一

4-17

$$X(e^{jw}) = \frac{1}{1-e^{-jw}}(\frac{sin1.5w}{sin0.5w}) + 3\pi\delta(w), w \in (-\pi, \pi]$$
$$x_1[n] = \begin{cases} 1, |n| <= 1 \\ 0, |n| > 1 \end{cases}$$
$$X_1(e^{jw}) = \frac{sin1.5w}{sin0.5w}$$
$$\sum_{m=-\infty}^{n} x[m] -- > \frac{X(e^{jw})}{1-e^{-jw}} + \pi X(e^{j0}) \sum \delta(w - 2k\pi)$$
$$x[n] = \sum_{k=-\infty}^{n} x_1[k]$$

4-18

$$\frac{x[n] - x[-n]}{2} <-- > jIm\{X(e^{jw}\} = j[sinw - sin2w]$$
$$X(e^{jw}) = A + e^{jw} + e^{-2jw}$$
$$2\pi \sum |x[n]|^2 = \int_{-\pi}^{\pi} |X(e^{jw})|^2 dw = 6\pi$$
$$x[n] = \delta[n] + \delta[n+1] - \delta[n+2]$$

4-19

(1)

$$y[n] = (\frac{sin\frac{\pi}{4}n}{\pi n})^2 * (\frac{sinw_c n}{\pi n})$$
$$|w_c| <= \pi$$
$$|w_c| \in [\frac{\pi}{2}, \pi]$$

(2)

$$y[n] = (\frac{sin\frac{\pi n}{4}}{\pi n} cos(\frac{\pi n}{2})) * (\frac{sinw_c n}{\pi n})$$
$$|w_c| \in [\frac{3}{4}\pi, \pi]$$

4-20

(b)

$$X_1(e^{jw}) = [X(e^{j(w+\pi)}) + X(e^{j(w-\pi)})]/2$$
$$x_1[n] = e^{j\pi n}x[n]$$

(f)

$$X_5(e^{jw}) = 1 - X(e^{j(w-\frac{\pi}{2})}) - X(e^{j(w+\frac{\pi}{2})})$$
$$x_5[n] = \delta[n] - e^{j\frac{\pi}{2}n}x[n] - e^{j\frac{-\pi}{2}n}x[n]$$

4-21

$$x[n] < -- > A(w) + jB(w)$$
$$Y(e^{jw}) = B(w) + A(w)e^{-jw}$$
$$\frac{x[n] + x[-n]}{2} < -- > A(w)$$
$$\frac{x[n] - x[-n]}{2} < -- > jB(w)$$
$$y[n] = \frac{x[n-1] + x[-n+1]}{2} - j\frac{x[n] - x[-n]}{2}$$

4-23

4-25

(1)

$$y[n] + \frac{1}{6}y[n-1] - \frac{1}{6}y[n-2] = x[n] - x[n-1]$$
$$Y(e^{jw}) + \frac{1}{6}e^{-jw}Y(e^{jw}) - \frac{1}{6}e^{-2jw}Y(e^{jw}) = X(e^{jw}) - e^{-jw}X(e^{jw})$$
$$H(e^{jw}) = \frac{Y(e^{jw})}{X(e^{jw})}$$
$$= \frac{1 - e^{-jw}}{1 + \frac{1}{6}e^{-jw} - \frac{1}{6}e^{-2jw}}$$

(2)

$$h[n] <--> H(e^{jw}) = \frac{1-e^{-jw}}{1+\frac{1}{6}e^{-jw}-\frac{1}{6}e^{-2jw}}$$
$$= \frac{1-e^{-jw}}{(1+\frac{1}{2}e^{-jw})(1-\frac{1}{3}e^{-jw})}$$
$$h[n] = \frac{9}{5}(\frac{-1}{2})^n u[n] - \frac{4}{5}(\frac{1}{3})^n u[n]$$

(3)

$$x[n] = 4^{-n}u[n]$$
$$Y(e^{jw}) = \frac{1-e^{-jw}}{1+\frac{1}{6}e^{-jw}-\frac{1}{6}e^{-2jw}}X(e^{jw})$$
$$= \frac{1-e^{-jw}}{1+\frac{1}{6}e^{-jw}-\frac{1}{6}e^{-2jw}} \cdot \frac{1}{1-\frac{1}{4}e^{-jw}}$$
$$y[n] = \frac{6}{5}(\frac{-1}{2})^n u[n] + 3(\frac{1}{4})^n u[n] - \frac{16}{5}(\frac{1}{3})^n u[n]$$

4-26

$$(\frac{2}{3})^n u[n] <--> n(\frac{2}{3})^n u[n]$$

(1)

$$y[n] = x[n] * h[n]$$
$$Y(e^{jw}) = X(e^{jw}) \cdot H(e^{jw})$$
$$X(e^{jw}) = \frac{1}{1-\frac{2}{3}e^{-jw}}$$
$$Y(e^{jw}) = j\frac{dX(e^{jw})}{dw} = -\frac{2}{3}e^{-jw}\frac{1}{(1-\frac{2}{3}e^{-jw})^2}$$
$$H(e^{jw}) = \frac{Y(e^{jw})}{X(e^{jw})} = \frac{2}{3}e^{-jw}\frac{1}{(1-\frac{2}{3}e^{-jw})}$$

(2)

$$H(e^{jw}) = \frac{\sum_{k=0}^{M} b_k e^{-jkw}}{\sum_{k=0}^{N} a_k e^{-jkw}} = \frac{2}{3}e^{-jw}\frac{1}{(1-\frac{2}{3}e^{-jw})}$$

$$b_0 = 0, b_1 = \frac{-2}{3}$$
$$a_0 = 1, a_1 = \frac{-2}{3}$$
$$y[n] - \frac{2}{3}y[n-1] = \frac{2}{3}x[n-1]$$

4-28

(1):$x[n] = (-1)^n$

$$x[n] = cos\pi n <--> \pi \sum[\delta(w-\pi)+\delta(w+\pi)]$$

$$y[n] = x[n] * h[n]$$

$$Y(e^{jw}) = X(e^{jw}) \cdot H(e^{jw}) = 0$$

(2):$x[n] = 1 + sin(\frac{3\pi}{8}n + \frac{\pi}{4}) + \frac{1}{2}cos(\frac{\pi}{2}n + \frac{\pi}{6}) + \frac{1}{4}sin(\frac{2\pi}{3}n + \frac{\pi}{4})$

$$y[n] = sin(\frac{3\pi}{8} + \frac{\pi}{4})$$

4-32

$$h_1[n] = \delta[n] - \frac{sin(\pi n/2)}{\pi n}$$

$$y[n] = (-x[n] * h_1[n] + x[n]) * h_3[n] + x[n] * h_1[n] * h_2[n]$$

$$Y(e^{jw}) = H_3[e^{jw}](-X[e^{jw}] \cdot H_1[e^{jw}] + X[e^{jw}]) + X[e^{jw}] \cdot H_1[e^{jw}] \cdot H_2[e^{jw}]$$

$$= X[e^{jw}](-H_3(e^{jw}) \cdot H_1(e^{jw}) + H_3(e^{jw}) + H_1[e^{jw}] \cdot H_2[e^{jw})$$

$$H(e^{jw}) = -H_3(e^{jw}) \cdot H_1(e^{jw}) + H_3(e^{jw}) + H_1[e^{jw}] \cdot H_2[e^{jw}]$$

$$y[n] = 16(\frac{\frac{sinn\pi}{4}}{\pi n})cos\frac{\pi}{2}n$$