Problem 1

a.

$$||x||_\infty = max|x_i|$$
$$||x-\tilde{x}||_\infty = max|(x-\tilde{x})_i| = 0.5$$
$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 6 \end{bmatrix}$$
$$A\tilde{x} - b = (1, -1.3, 1.8)^t$$
$$||A\tilde{x} - b||_\infty = 1.8$$

b.

$$||x||_\infty = max|x_i|$$
$$||x-\tilde{x}||_\infty = max|(x-\tilde{x})_i| = 0.9$$
$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 6 \end{bmatrix}$$
$$A\tilde{x} - b = (1.27, -1.16, 2.21)^t$$
$$||A\tilde{x} - b||_\infty = 2.21$$

Problem 2

$$\text{由于} ||A||_2 = [\rho(A^tA)^{\frac{1}{2}}]$$
$$\text{若}A\text{是对称的，则} A^tA = A * A$$
$$||A||_2 = [\rho(A^tA)^{\frac{1}{2}}] = \rho(A)$$

Problem 3

> 本题采用三位精度

a.

$$s_1 = 58.9 \quad s_2 = 6.1$$

$$\frac{|a_{11}|}{s_1} = 0.000509337$$

$$\frac{|a_{21}|}{s_2} = 0.870491803$$

$$\frac{|a_{21}|}{s_2} > \frac{|a_{11}|}{s_1}$$

交换后的矩阵为：

$$\begin{bmatrix} 5.31 & -6.1 & 47 \\ 0.03 & 58.9 & 59.2 \end{bmatrix}$$

一次高斯消元以后得到的矩阵如下：

$$\begin{bmatrix} 5.31 & -6.1 & 47 \\ -1.5 * 10^{-6} & 58.934 & 58.934 \end{bmatrix}$$

$$=>$$

$$\begin{bmatrix} 5.31 & -6.1 & 47 \\ 0 & 58.934 & 58.934 \end{bmatrix}$$

解得：$x_2 = 1 \; x_1 = 10$

b.

$$s_1 = 12.1 \; s_2 = 21.1 \; s_3 = 21$$

$$|\frac{a_{11}}{s_1}| = 0.2504$$

$$|\frac{a_{21}}{s_2}| = 0.2504$$

$$|\frac{a_{31}}{s_3}| = 0.2909$$

交换后的矩阵为：

$$\begin{matrix} 6.11 & -14.2 & 21 & -139 \\ [-3.03 & 12.1 & -7 & 120\ ] \\ 3.03 & -12.1 & 14 & -119 \end{matrix}$$

一次高斯消元后得到：

$$\begin{matrix} 6.11 & -14.2 & 21 & -139 \\ [-5.55 * 10^{-3} & 5.071 & 3.395 & 51.195\ ] \\ 5.55 * 10^{-3} & -5.07 & 3.605 & -70.195 \end{matrix}$$

$$=>$$

$$\begin{matrix} 6.11 & -14.2 & 21 & -139 \\ [\ 0 & 5.071 & 3.395 & 51.195\ ] \\ 0 & -5.071 & 3.605 & -50.195 \end{matrix}$$

$E_3 + E_2$：

$$\begin{matrix} 6.11 & -14.2 & 21 & -139 \\ [\ 0 & 5.071 & 3.395 & 51.195] \\ 0 & 0 & 7 & 1 \end{matrix}$$

解得：$x_3 = 0.142 \;\; x_2 = 10.001 \;\; x_1 = 0.00527$

Problem4

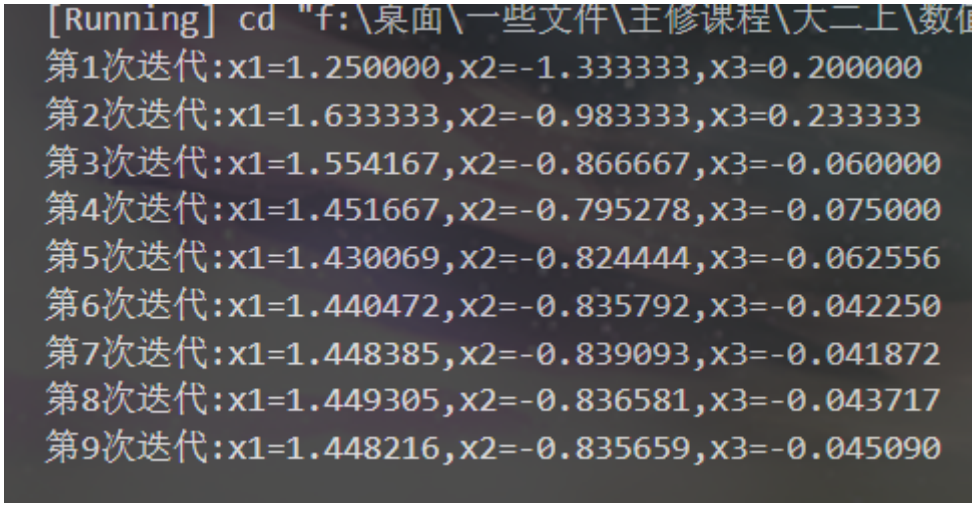
a.

使用如下代码迭代：

```
#include<stdio.h>

int main(){
    double x[100][100] = {
        {4,1,-1,5},
        {-1,3,1,-4},
        {2,2,5,1},
    };
    double x1,x2,x3;
    double xTmp1,xTmp2,xTmp3;
    double sumTmp = 0;
    x1 = x2 = x3 = 0;
    int i=0;
    while(++i<10){
        xTmp1 = (-(x[0][1]*x2+x[0][2]*x3)+x[0][3])/x[0][0];
        xTmp2 = (-(x[1][0]*x1+x[1][2]*x3)+x[1][3])/x[1][1];
        xTmp3 = (-(x[2][0]*x1+x[2][1]*x2)+x[2][3])/x[2][2];
        x1 = xTmp1;
        x2 = xTmp2;
        x3 = xTmp3;
        printf("第%d次迭代:x1=%f,x2=%f,x3=%f\n",i,x1,x2,x3);
    }
}
```

得到的结果为：



b.

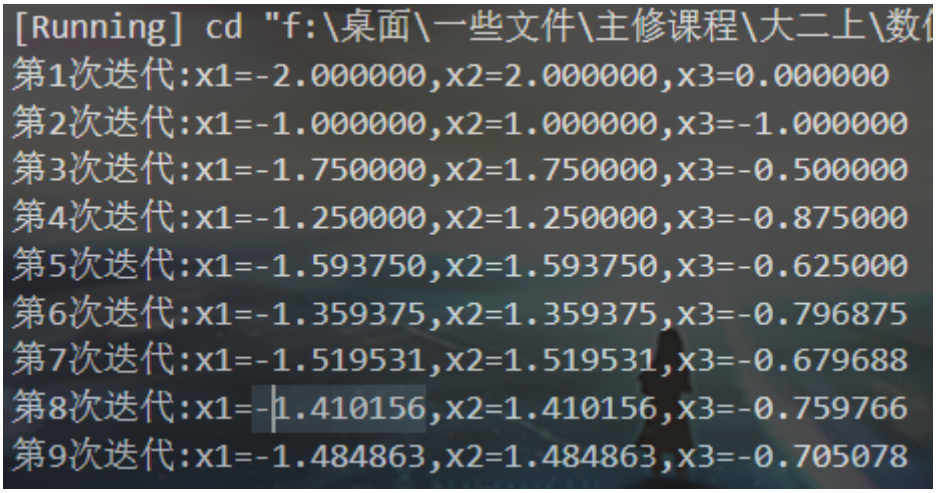
使用如下代码迭代：

```
#include<stdio.h>

int main(){
    double x[100][100] = {
        {-2,1,0.5,4},
```

```
        {1,-2,-0.5,-4},
        {0,1,2,0},
    };
    double x1,x2,x3;
    double xTmp1,xTmp2,xTmp3;
    double sumTmp = 0;
    x1 = x2 = x3 = 0;
    int i=0;
    while(++i<10){
        xTmp1 = (-(x[0][1]*x2+x[0][2]*x3)+x[0][3])/x[0][0];
        xTmp2 = (-(x[1][0]*x1+x[1][2]*x3)+x[1][3])/x[1][1];
        xTmp3 = (-(x[2][0]*x1+x[2][1]*x2)+x[2][3])/x[2][2];
        x1 = xTmp1;
        x2 = xTmp2;
        x3 = xTmp3;
        printf("第%d次迭代:x1=%f,x2=%f,x3=%f\n",i,x1,x2,x3);
    }
}
```

得到的结果如下：



Problem 5

a.

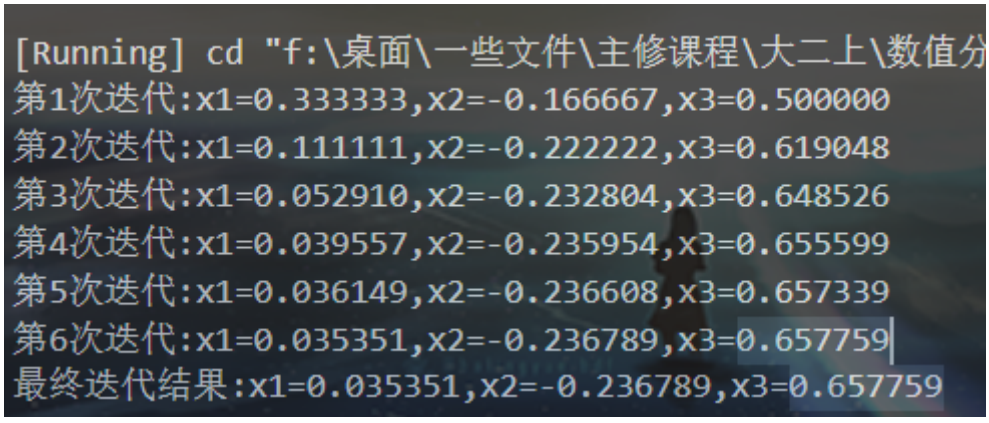
采用如下代码进行迭代：

```
#include<stdio.h>
#include<math.h>
#define TOL 0.001

int main(){
    double x[100][100] = {
        {3,-1,1,1},
        {3,6,2,0},
        {3,3,7,4},
    };
    double x1,x2,x3;
    double xTmp1,xTmp2,xTmp3;
    double sumTmp = 0;
    x1 = x2 = x3 = 0;
    int i=0;
```

```
    double maxNorms = 10;
    while(maxNorms > TOL){
        xTmp1 = (-(x[0][1]*x2+x[0][2]*x3)+x[0][3])/x[0][0];
        xTmp2 = (-(x[1][0]*xTmp1+x[1][2]*x3)+x[1][3])/x[1][1];
        xTmp3 = (-(x[2][0]*xTmp1+x[2][1]*xTmp2)+x[2][3])/x[2][2];
        maxNorms = fabs(xTmp1-x1);
        if(fabs(xTmp2-x2)>maxNorms)
            maxNorms = fabs(xTmp2-x2);
        if(fabs(xTmp3-x3)>maxNorms)
            maxNorms = fabs(xTmp3-x3);
        //printf("maxNorms: %f\n",maxNorms);
        x1 = xTmp1;
        x2 = xTmp2;
        x3 = xTmp3;
        i++;
        printf("第%d次迭代:x1=%f,x2=%f,x3=%f\n",i,x1,x2,x3);
    }
    printf("最终迭代结果:x1=%f,x2=%f,x3=%f\n",x1,x2,x3);
}
```

得到的迭代结果为：x1=0.035351,x2=-0.236789,x3=0.657759



b.

采用如下代码进行迭代：

```
#include<stdio.h>
#include<math.h>
#define TOL 0.001

int main(){
    double x[100][100] = {
        {10,-1,0,9},
        {-1,10,-2,7},
        {0,-2,10,6},
    };
    double x1,x2,x3;
    double xTmp1,xTmp2,xTmp3;
    double sumTmp = 0;
    x1 = x2 = x3 = 0;
    int i=0;
    double maxNorms = 10;
    while(maxNorms > TOL){
        xTmp1 = (-(x[0][1]*x2+x[0][2]*x3)+x[0][3])/x[0][0];
        xTmp2 = (-(x[1][0]*xTmp1+x[1][2]*x3)+x[1][3])/x[1][1];
```

```
        xTmp3 = (-(x[2][0]*xTmp1+x[2][1]*xTmp2)+x[2][3])/x[2][2];
        maxNorms = fabs(xTmp1-x1);
        if(fabs(xTmp2-x2)>maxNorms)
            maxNorms = fabs(xTmp2-x2);
        if(fabs(xTmp3-x3)>maxNorms)
            maxNorms = fabs(xTmp3-x3);
        //printf("maxNorms: %f\n",maxNorms);
        x1 = xTmp1;
        x2 = xTmp2;
        x3 = xTmp3;
        i++;
        printf("第%d次迭代:x1=%f,x2=%f,x3=%f\n",i,x1,x2,x3);
    }
    printf("最终迭代结果:x1=%f,x2=%f,x3=%f\n",x1,x2,x3);
}
```

得到的迭代结果为：x1=0.995748,x2=0.957874,x3=0.791575