HW 11

5-1

$$T=\frac{2\pi}{w}=10^{-3}s$$
$$w_s>2w_c$$
$$(1),(3),(4)\text{可以恢复}$$

5-2

(1)

$$x(t)=2+cos(1000\pi t)+sin(3000\pi t)$$
$$\text{奈奎斯特频率：}6000\pi$$

(3)

$$x(t)=(\frac{sinw_ct}{\pi t})^2=\frac{1-cos2w_ct}{2(\pi t)^2}$$
$$\text{奈奎斯特频率：}4w_c$$

(5)

$$x(t)=(\frac{sin1000\pi t}{\pi t})(\frac{sin2000\pi t}{\pi t})$$
$$\text{奈奎斯特频率：}6000\pi$$

5-3

$$x(t)=\sum_{k=0}^{5}(\frac{1}{2})^k sin(k\pi t)$$

(1)

$$w_s=10\pi=2w_M$$
$$\text{会发生混叠}$$

(2)

$$\text{由于截止频率为}5\pi$$
$$x_r(t)=sum_{k=0}^{4}(\frac{1}{2})^k sin(k\pi t)$$

5-4

$$x_r(t)=\sum x(nT)h(t-nT), x(t)=cos2\pi t, T=0.2$$
$$x_r(t)=x_p(t)*h(t)=[\sum x(nT)\delta(t-nT)]*h(t)$$
$$=\sum x(nT)h(t-nT)$$
$$X(jw)=\pi[\delta(w+2\pi)+\delta(w-2\pi)]$$
$$X_p(jw)=\frac{1}{T}\sum_{k=-\infty}^{\infty}X(j(w-kw_s)), w_s=\frac{2\pi}{T}$$
$$X_p(jw)=5\pi\sum[\delta(w+10k\pi+2\pi)+\delta(w+10k\pi-2\pi)]$$
$$H(jw)=0.2Sa^2(0.1w)$$
$$X_r(jw)=X_p(jw)\cdot H(jw)=\pi Sa^2(0.1w)\sum[\delta(w+10k\pi+2\pi)+\delta(w+10k\pi-2\pi)]$$

5-5

$$x_r(t)=\sum x(nT)\frac{Tw_c}{\pi}Sa(w_c(t-nT)), w_c=\frac{w_s}{2}$$
$$\text{证明}x_r(kT)=x(kT)$$
$$\text{证明：}$$
$$w_s=\frac{2\pi}{T}$$
$$w_c=\frac{\pi}{T}$$
$$x_r(kT)=\sum_{n=-\infty}^{\infty}x(nT)Sa(k\pi-n\pi)$$
$$\text{当}n=k\text{时}, Sa(k\pi-n\pi)=1$$
$$=>x_r(kT)=x(kT)$$

5-6

(1):$f(t)=x_1(t)+x_2(t)$

T = 1/600

(3):$f(t)=x_2(2t)$

T = 1/1200

(5):$f(t) = x_1(t) \cdot x_2(t/3)$

T = 1/200

5-8

$$A = T, w_b = w_2, T = \frac{2\pi}{w_2}, w_a \in (w_2 - w_1, w_1)$$

5-12

$$x_c(n \cdot 10^{-3}) <--> 1000X(1000jw)$$
$$x_d[n] <--> X(e^{jw})$$
$$X_c(jw) = 1000X_d(e^{1000jw})$$

(1):$X_c(jw)$为实函数

(2):

$$X_c(jw) = 1000X_d(e^{jw1000}) < 1$$
$$X_c(jw) < 1$$

(3):

$$X_c(jw) = 1000X_d(e^{jw1000}) = 0, |w| \in [\frac{3}{4}\pi, \pi]$$
$$X_c(jw) = 0, |w >= 750\pi|$$

(4):

$$X_c(jw) = 1000X_d(e^{jw1000}) = 1000X_d(e^{j(w-\pi)1000}) = X_c(j(w - 1000\pi))$$
$$X_c(jw) = X_c(j(w - 1000\pi))$$

5-16

$$X(e^{jw}) = \frac{1}{T} \sum X(j(w - 2k\pi)/T)$$
$$= 20 \cdot 10^3 \sum X(j(w - 2k\pi) \cdot 20 \cdot 10^3)$$
$$Y(e^{jw}) = X(e^{jw}) \cdot H(e^{jw})$$

$$Y_c(jw) = TY(e^{jwT}), T = \frac{1}{20kHz}$$

5-17

5-18

$$Y(e^{jw}) - \frac{1}{3}e^{-jw}Y(e^{jw}) = X(e^{jw})$$
$$H(e^{jw}) = \frac{1}{1-\frac{1}{3}e^{-jw}}$$
$$H(jw) = TH(e^{jwT}) = \frac{T}{1-\frac{1}{3}e^{-jwT}}$$

5-19

$$\frac{sin300\pi t - sin100\pi t}{2}$$

5-21

$$设x_1(t) = [x(t)`cos5wt] * h_1(t)$$
$$X_1(jw) = \frac{X(j(w-5w)) + X(j(w+5w))}{2} \cdot H_1(jw)$$
$$设x_2(t) = [x_1(t) \cdot cos3wt] * h_2(t)$$
$$X_2(jw) = \frac{H_1(j(w-3w)) + X_1(j(w+3w))}{2} \cdot H_2(jw)$$

5-26

(1)

$$R(w) = cosw$$
$$h(t) = \frac{\delta(t-1) + \delta(t+1)}{2}$$

(2)

$$R(w) = \frac{1}{1+w^2}$$
$$h(t) = e^{-|t|}/2$$
$$I(w) = R(w) * (-\frac{1}{\pi w})$$

(3)

$$
\begin{gathered}
\text{证明} H(jw) = \frac{1}{j\pi}\int \frac{H(j\lambda)}{w-\lambda}d\lambda \\
H(jw) = R(w) + jI(W) \\
R(w) = \frac{1}{\pi}\int \frac{I(\lambda)}{w-\lambda}d\lambda = I(w) * \frac{1}{\pi w} \\
I(w) = -\frac{1}{\pi}\int \frac{R(\lambda)}{w-\lambda}d\lambda = R(w) * (-\frac{1}{\pi w}) \\
H(jw) = \frac{1}{\pi}\int \frac{I(\lambda)}{w-\lambda}d\lambda - j\frac{1}{\pi}\int \frac{R(\lambda)}{w-\lambda}d\lambda \\
= \frac{1}{j\pi}\int \frac{H(j\lambda)}{w-\lambda}d\lambda
\end{gathered}
$$

5-27

(1)

$$
\begin{gathered}
x(t) = a(t)cosw_0t + b(t)sinw_0t \\
\widehat{x}(t) = x(t) * \frac{1}{\pi t} = \frac{1}{\pi}\int \frac{x(\tau)}{t-\tau}d\tau \\
\widehat{x}(t) = H[x(t)]
\end{gathered}
$$

(2)

$$
\begin{gathered}
\widehat{x}(t) = H[x(t)] \\
\text{瞬时包络}: |a(t)| = \sqrt{x(t)^2 + \widehat{x}(t)^2} = \sqrt{x(t)^2 + H[x(t)]^2} \\
\text{瞬时相位}: \phi(t) = arctan\frac{\widehat{x}(t)}{x(t)} = arctan\frac{H[x(t)]}{x(t)} \\
\text{瞬时频率}: w(t) = \frac{d\phi(t)}{dt} = \frac{d(arctan\frac{H[x(t)]}{x(t)})}{dt}
\end{gathered}
$$