# **HW 6**

## 3-5

(1)

$$e^{-2(t-2)}u(t-2) \ = e^{-2jw} \cdot rac{1}{2+jw}$$

(3)

$$\delta(t+\pi) + \delta(t-\pi)$$
$$= e^{jw\pi} + e^{-jw\pi}$$

(5)

$$x(t)=e^{-t}[u(t)-u(t-1)]$$
  
设 $x_1(t)=u(t)-u(t-1)=-[u(t-rac{1}{2}-rac{1}{2})-u(t-rac{1}{2}+rac{1}{2})]$   $X_1(t)=-e^{-jrac{1}{2}w}Sa(rac{1}{2}w)$   $X(t)=X_1(j(w+1))=-e^{rac{1}{2}(w+1)}Sa(rac{1}{2}j(w+1))$ 

# 3-6

(1)

$$X(jw)=\pi[\delta(w+3\pi)+\delta(w-2\pi)] \ x(t)=rac{1}{2}[e^{-3t}+e^{2t}]$$

# 3-7

(1)

$$x_1(t)=x(-t+3) \ X_1(jw)=X(-jw)e^{-3jw}$$

(3)

$$x_2(t)=x_1(3t) \ X_2(jw)=rac{1}{3}X(jrac{1}{3}w)$$

# 3-8

(2)

$$egin{aligned} e^{-at}cos(w_0t)\cdot u(t)\ &=cos(w_0t)e^{-at}\cdot u(t)\ &x_1(t)=e^{-at}\cdot u(t)\ &X_1(jw)=rac{1}{a+jw}\ &x(t)=x_1(t)cos(w_0t)=rac{1}{2}[X_1(j(w-w_0))+X_1(j(w+w_0))]\ &X(jw)=rac{1}{2}[rac{1}{a-w-1}+rac{1}{a-w+1}] \end{aligned}$$

(5)

$$egin{aligned} \delta'(t) + 2\delta(3-2t) \ & \pm rac{dx(t)}{dt} - - > jwX(jw) \ & \delta'(t) - - > jw \ & \delta(-2t) - - > rac{1}{2} \ & \delta(3-2t) - - > e^{3jw} \cdot rac{1}{2} \ & \delta'(t) + 2\delta(3-2t) - - > jw + rac{1}{2}e^{3jw} \end{aligned}$$

(7)

$$rac{sin(\pi t)}{\pi t} rac{sin(2\pi(t-1))}{\pi(t-1)}$$
 
设 $x_1(t) = rac{sin(\pi t)}{\pi t}, X_1(jw) = u(w-\pi) - u(w+\pi)$  
设 $x_2(t) = rac{sin(2\pi(t-1))}{\pi(t-1)}, X_2(jw) = e^{-jw}[u(w-2\pi) - u(w+2\pi)]$ 
 $x_1(t) \cdot x_2(t) = rac{1}{2\pi} X_1(jw) * X_2(jw)$ 
 $= rac{-j}{2\pi} (1 + e^{-jw})[u(w+3\pi) - u(w+\pi) + u(w-3\pi) - u(w-\pi)]$ 

(8)

$$egin{split} x(t) &= u(t) + u(t-2) - 2u(t-1) \ &= u(t) - - > rac{1}{jw} + \pi \delta(w) \ &= [rac{1}{jw} + \pi \delta(w)][1 + e^{-2jw} - 2e^{-jw}] \end{split}$$

(9)

$$egin{aligned} x(t) &= rac{dx_1(t)}{dt} \ x_1(t) &= u(t+1) - u(t-1) - \delta(t+2) - \delta(t-2) \ X_1(jw) &= 2Sa(w) - (e^{2jw} + e^{-2jw}) \ X(t) &= rac{X_1(jw)}{jw} = rac{2Sa(w) - (e^{2jw} + e^{-2jw})}{jw} \end{aligned}$$

$$egin{aligned} X(jw) &= u(w) - u(w-2) \ X_1(jw) &= u(w+1) - u(w-1) \ x_1(t) &= rac{sint}{\pi t} \ X(jw) &= X_1(j(w-1)) \ x(t) &= e \cdot rac{sint}{\pi t} \end{aligned}$$

(2)

# 3-10

(1)

$$egin{align} X(jw) &= rac{2 sin 3(w-\pi)}{w-\pi} \ X_1(jw) &= rac{2 sin 3w}{w} \ \mathbb{B} \mathfrak{M} x_1(t) &= u(t-3) - u(t+3) \ x(t) &= e^{jt\pi} x_1(t) &= e^{jt\pi} [u(t-3) - u(t+3)] \end{aligned}$$

(3)

$$egin{split} X(jw) &= cos(2w+rac{\pi}{3}) \ &= rac{e^{j(2w+rac{\pi}{3})}+e^{-j(2w+rac{\pi}{3})}}{2} \ &= rac{e^{jrac{\pi}{3}}\delta(t+2)+e^{-jrac{\pi}{3}}\delta(t-2)}{2} \end{split}$$

## 3-11

$$egin{aligned} x(t) &= cosw_0t \cdot (1 - rac{2}{ au_1}|t|), |t| < rac{ au_1}{2} \ orall x_1(t) &= cosw_0t, X_1(jw) = \delta(t+w_0) \ orall x_2(t) &= 1 - rac{2}{ au_1}|t| \ x_3(t) &= u(t + rac{ au_1}{4}) - u(t - rac{ au_1}{4}) \ x_2(t) &= x_3(t) * x_3(t) \ X_2(jw) &= X_3(jw)^2 = (rac{ au_1}{2}Sa(rac{ au_1}{4}w))^2 \ X(jw) &= rac{1}{2\pi} \cdot X_1(jw) * X_2(jw) \ &= rac{1}{2\pi} \cdot [rac{ au_1}{2}Sa(rac{ au_1}{4}(w-w_0))]^2 \end{aligned}$$

3-13

$$x_2(t) = x(1-t) + x(1+t) \ x(t) = \sum a_k e^{jkw_0t} \ x_2(t) = \sum a_k e^{jkw_0(1-t)} + \sum a_k e^{jkw_0(1+t)}$$

#### 3-14

(1)

由于实数信号的傅里叶变换的实部是偶函数,虚部是奇函数, $x_2(t),x_3(t)$ 是实值函数

(2)

当 $\mathbf{x}(t)$ 是实值偶函数时,其频谱也是实值偶函数, $x_2(t)$ 是实值偶函数

### 3-15

当
$$|k|>1$$
时, $a_k=0$   $x(t)=a_0+a_1e^{-jw_0t}+a_{-1}e^{jw_0t}$  由于 $x(t)$ 是实值偶函数, $a_k=a_{-k}$   $x(t)=a_0+a_1(e^{jw_0t}+e^{-jw_0t})=a_0+2a_1cos\pi t$   $rac{1}{2}\int_0^2|x(t)|^2dt=rac{1}{2}\int_0^2|a_0^2+4a_1^2cos^2\pi t+4a_0a_1cos\pi t|dt=1$   $a_0^2+2a_1^2=1$   $x(t)=1$ 或 $x(t)=1$ 或 $x(t)=\sqrt{2}cos(\pi t)$ 

#### 3-16

$$(2+jw)X(jw) < -^F - > 2x(t) + rac{dx(t)}{dt} = Ae^{-t}u(t)\dots(1)$$
  $rac{1}{2\pi}\int_{-\infty}^{\infty}|X(jw)|^2dw = \int_{-\infty}^{\infty}|x(t)|^2dt = 1\dots(2)$  由(1)式可以解得:  $x(t) = A(e^{-t} - e^{-2t})u(t)$  代入(2)式,解得 $A = 2\sqrt{3}$   $x(t) = 2\sqrt{3}(e^{-t} - e^{-2t})u(t)$ 

#### 3-17

x(t)是实值信号,则X(jw)的实部是偶函数

$$egin{aligned} rac{1}{2\pi} \int_{-\infty}^{\infty} Re\{X(jw)\}e^{jwt}dw &= e^{-|t|} \ rac{1}{2\pi} \int_{-\infty}^{\infty} Re\{X(jw)\}e^{jwt}dw &= rac{1}{2\pi} \int_{-\infty}^{\infty} Re\{X(jw)\}cos(wt)dw &= e^{-|t|} \ \int_{0}^{\infty} Re\{X(jw)\}cos(wt)dw &= \pi e^{-|t|} \ Re\{X(jw)\} &= rac{x(t) + x(-t)}{2} \ &=> \int_{0}^{\infty} x(t)cos(wt)dw &= 2\pi e^{-|t|} \ x(t) &= 2e^{-t}u(t) \end{aligned}$$

### 3-18

(1)

$$egin{aligned} x(t) &= E[u(t+rac{ au}{2}-rac{ au}{2}) - u(t-rac{ au}{2}-rac{ au}{2})] \ orall x_1(t) &= E[u(t+rac{ au}{2}) - u(t-rac{ au}{2})] \ X_1(jw) &= au ESa(rac{ au}{2}w) \ X(jw) &= e^{-jwrac{ au}{2}} au ESa(rac{ au}{2}w) \ heta(w) &= rac{- au w}{2} \end{aligned}$$

(2)

$$X(0) = \tau E Sa(0) = \tau E$$

(3)

$$egin{aligned} x(t) &= rac{1}{2\pi} \int_{-\infty}^{\infty} X(jw) e^{jwt} dw \ \int_{-\infty}^{\infty} X(jw) dw &= 2\pi x(0) = 2\pi E \end{aligned}$$

(4)

$$\int_{-\infty}^{\infty} X(jw) rac{2sinw}{w} e^{i2w} dw \dots (1)$$
  $orall X_1(jw) = rac{2sinw}{w} e^{i2w}$   $X_2(jw) = X_1(jw)X(jw)$   $\int_{-\infty}^{\infty} X(jw) rac{2sinw}{w} e^{i2w} dw = 2\pi x_2(t)$   $x_2(t) = x(t) * x_1(t)$  当 $au < 1$ 时, $(1)$ 式  $= 0$  当 $au \in [1,3]$ 时, $(1)$ 式  $= 2\pi E( au - 1)$  当 $au \geq 3$ 时, $(1)$ 式  $= 4\pi E$ 

$$\int |X(jw)|^2 dw = 2\pi \int |x(t)|^2 dt = 2\pi au E^2$$

(6)

$$egin{split} Re\{X(jw)\} < -^F - > rac{x(t) + x(-t)}{2} \ &= rac{E}{2}[x(t+ au) - x(t- au)] \end{split}$$

