HW 3

2-3

$x_1(t) = u(t+1) - u(t-1), x_2(t) = \delta(t+5) + \delta(t-5), x_3(t) = \delta(t+\frac{1}{2}) + \delta(t-\frac{1}{2})$

(1):$x_1(t) * x_2(t)$

$$\text{由于有} x(t+t_0) * h(t-t_0) = x(t) * h(t), x(t) * \delta(t) = x(t)$$
$$[u(t+1) - u(t-1)] * [\delta(t+5) + \delta(t-5)]$$
$$= \delta(t+5) * x_1(t) + \delta(t-5) * x_1(t)$$
$$= x_1(t-5) + x_1(t+5)$$

(2):$x_1(t) * x_2(t) * x_3(t)$

$$\text{由第一小题，} x_1(t) * x_2(t) * x_3(t) = [x_1(t+5) + x_1(t-5)] * x_3(t)$$
$$= x_1(t+5) * x_3(t) + x_1(t-5) * x_3(t)$$
$$x_1(t+5) * x_3(t) = x_1(t+5) * \delta(t+\frac{1}{2}) + x_1(t+5) * \delta(t-\frac{1}{2})$$
$$= x_1(t+\frac{1}{2}) + x_1(t-\frac{1}{2})$$
$$\text{同理，} x_1(t-5) * x_3(t) = x_1(t+\frac{1}{2}) + x_1(t-\frac{1}{2})$$
$$x_1(t) * x_2(t) * x_3(t) = 2[x_1(t+\frac{1}{2}) + x_1(t-\frac{1}{2})]$$

(3):$x_1(t) * x_3(t)$

$$x_1(t) * x_3(t) = x_1(t) * \delta(t+\frac{1}{2}) + x_1(t) * \delta(t-\frac{1}{2})$$
$$= x_1(t+\frac{1}{2}) + x_1(t-\frac{1}{2})$$

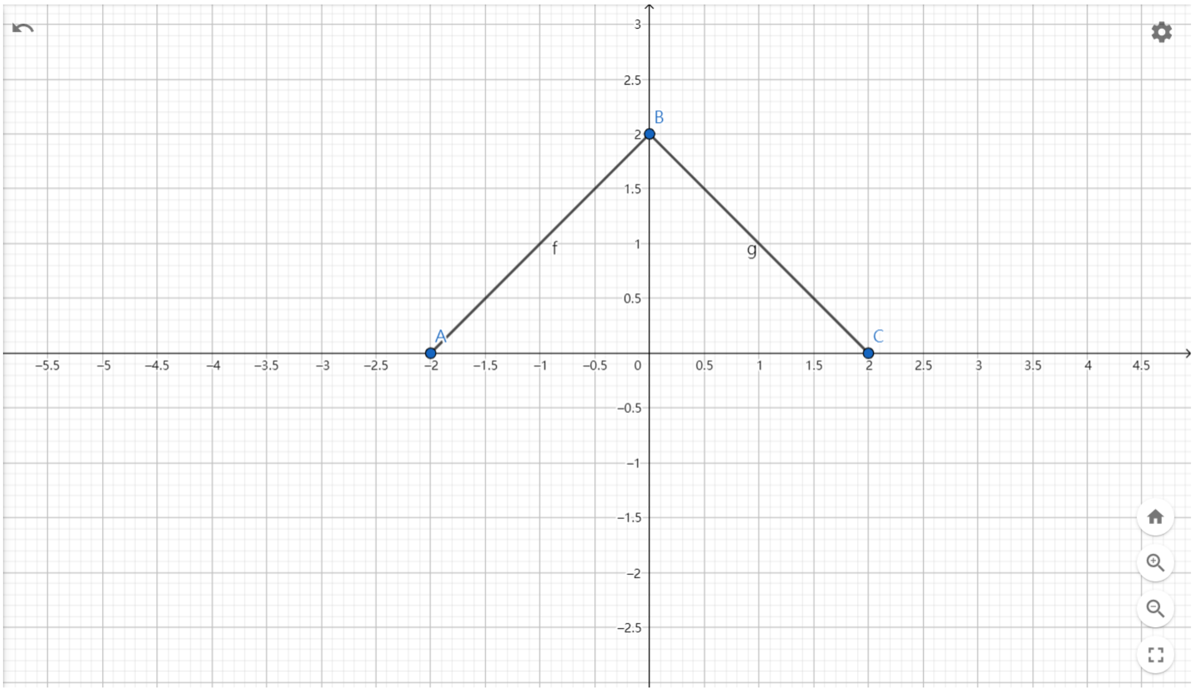
2-4

$y(t) = e^{-t}u(t) * \sum \delta(t-3k)$

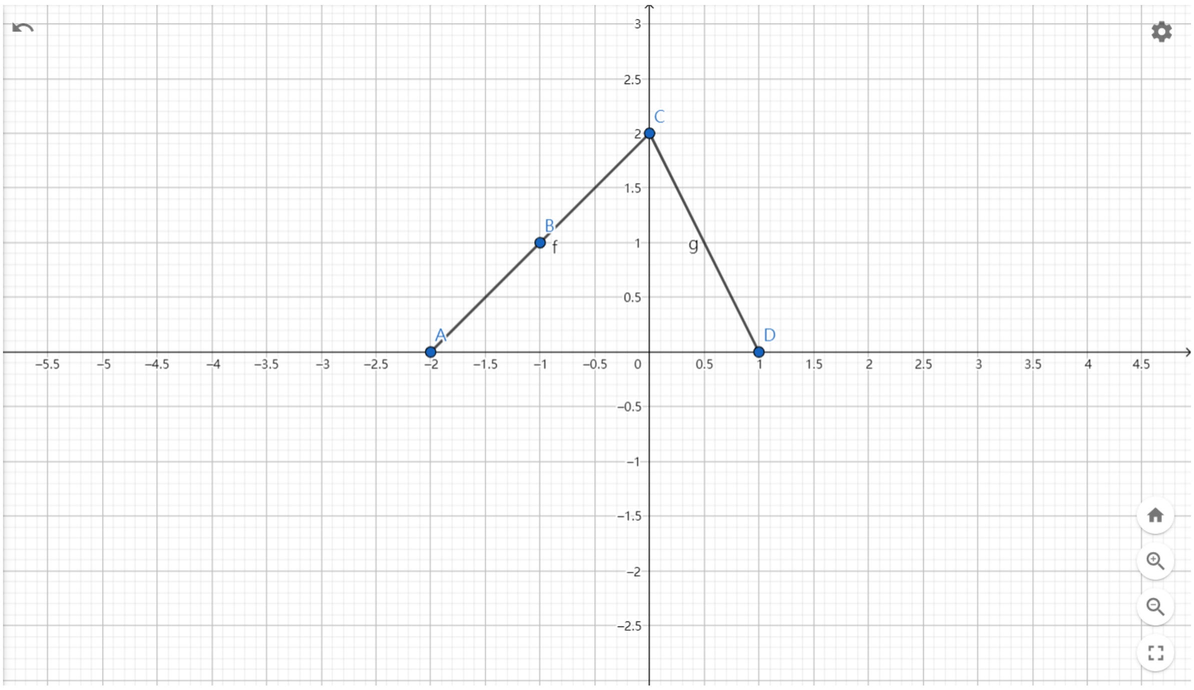
$$\text{证明：}$$
$$y(t) = \sum e^{-t}u(t) * \delta(t-3k)$$
$$= \sum e^{-(t-k)}u(t-3k) \quad \ldots 1$$
$$\text{由于} 0 < t \leq 3$$
$$1\text{式等价于} \sum_{k=-\infty}^{0} e^{-(t-3k)}$$
$$= \frac{e^{-t}}{1-e^{-3t}}$$
$$A = \frac{1}{1-e^{-3t}}$$

2-5

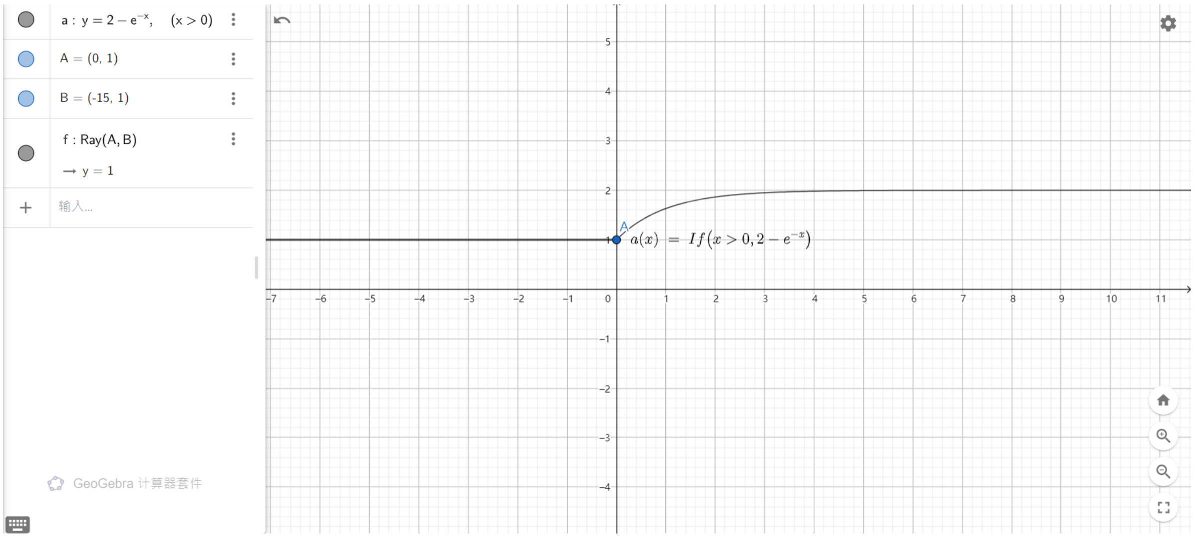
(a)



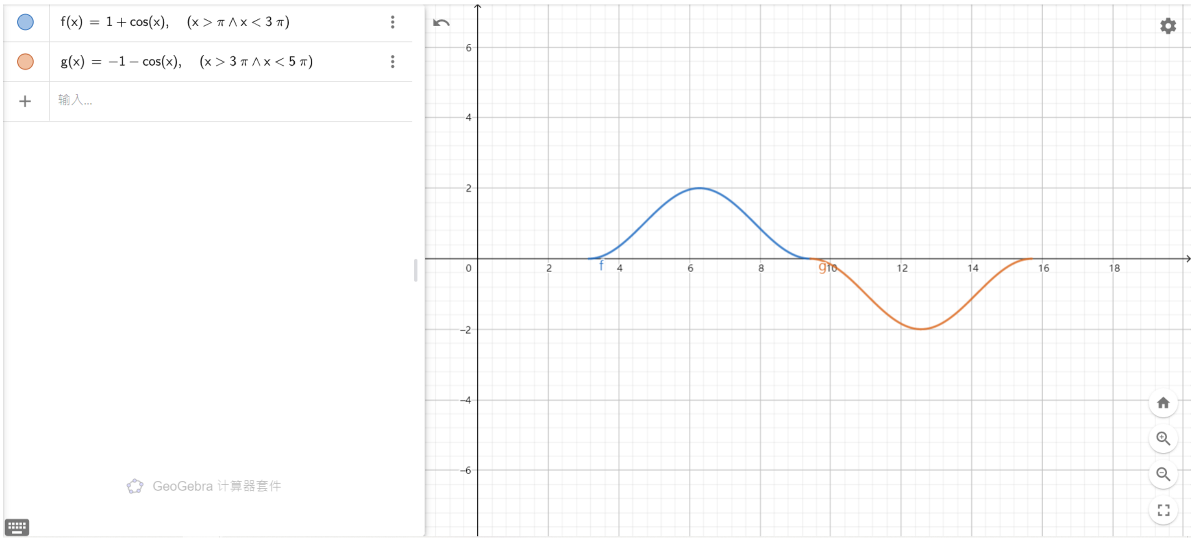
(b)



(c)

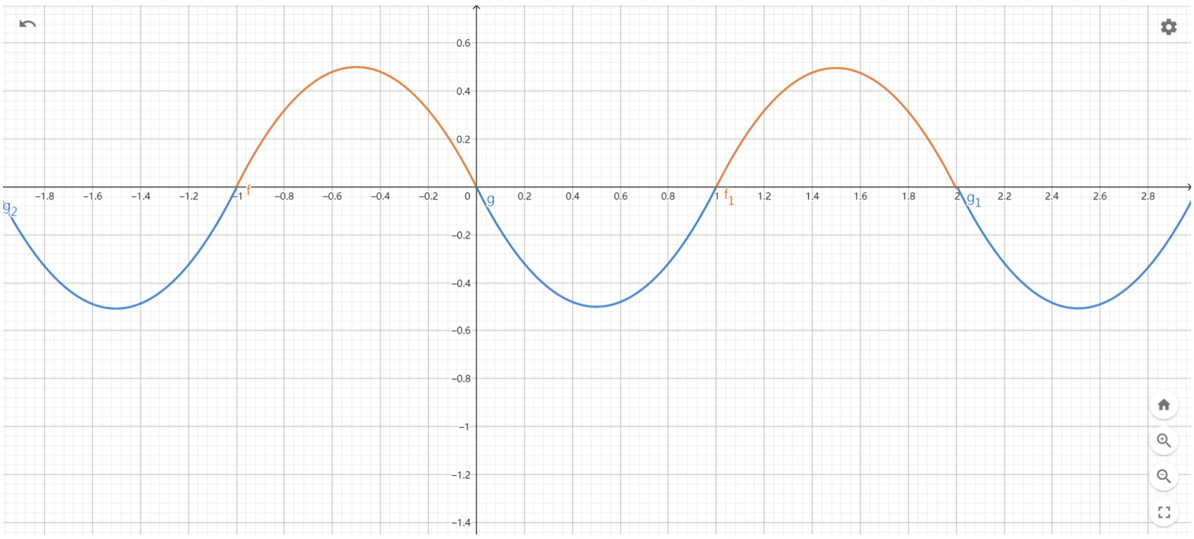


(d)



(e)

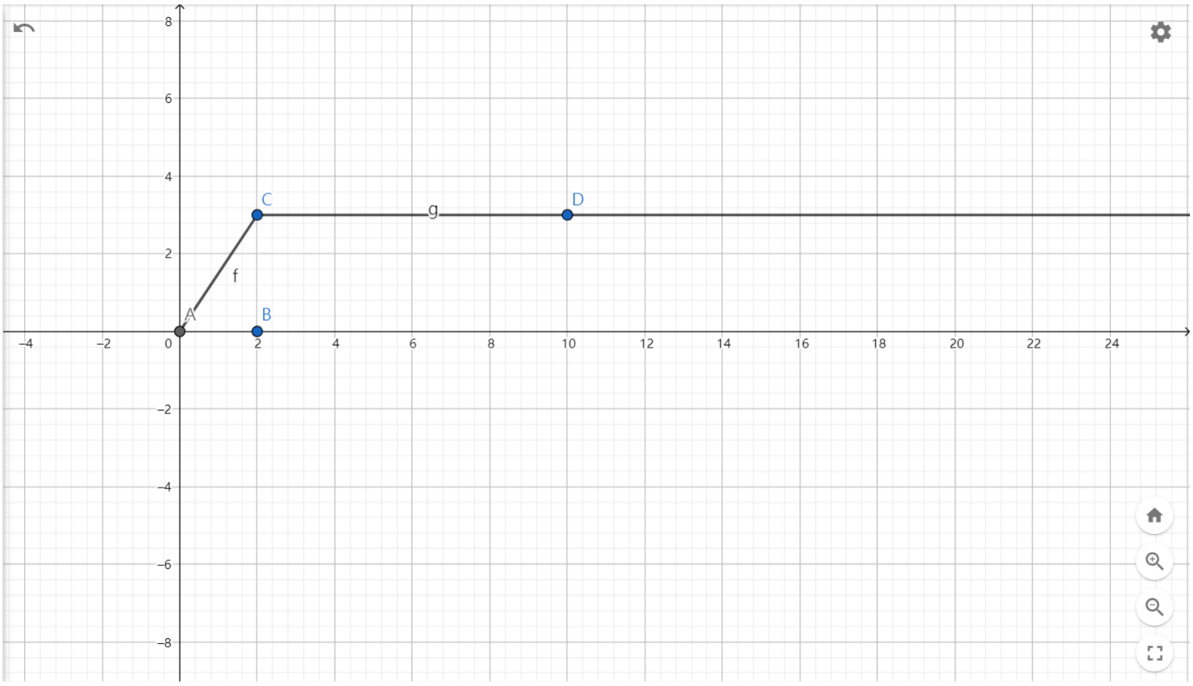
$$y(t) = -2t^2 - 2t, t \in [-1, 0]$$
$$y(t) = 2t^2 - 2t, t \in [0, 1]$$
$$\text{且} y(t) \text{以} 2 \text{为周期}$$



2-8

(1)

$$x(t) = 3x_0(t), h(t) = h_0(t)$$
$$y(t) = 3x_0(t) * h_0(t) = 3y_0(t)$$

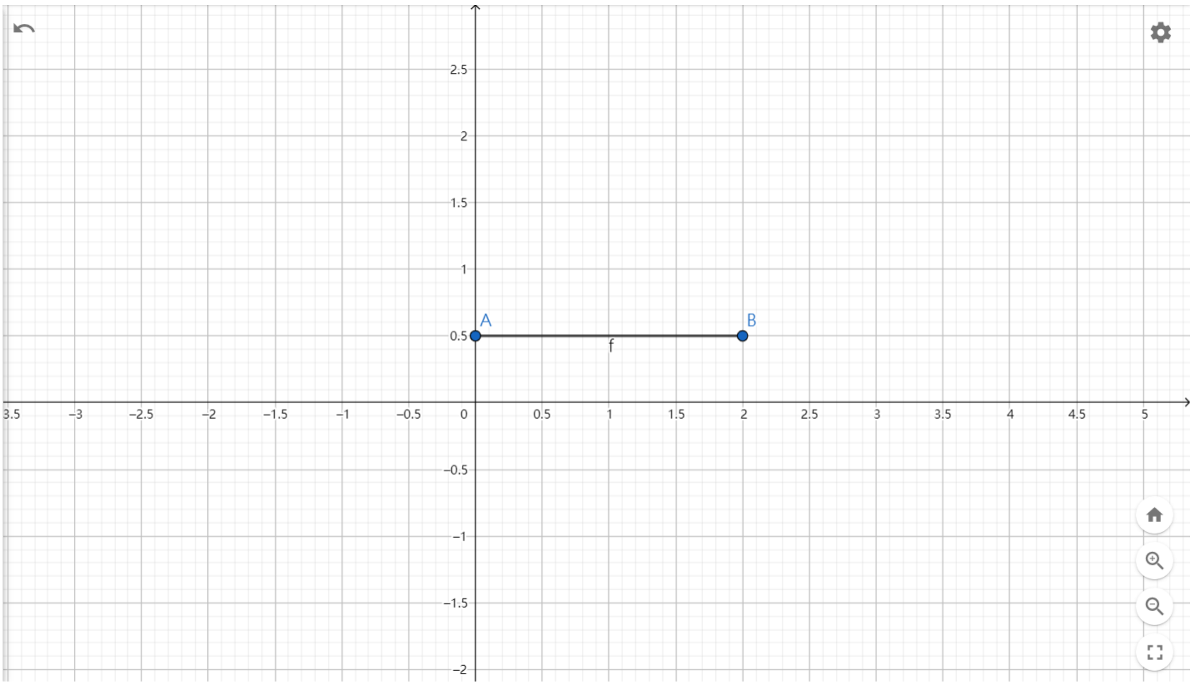


(3)

$$x(t) = x_0(t-2), h(t) = h_0(t+1)$$
$$y(t) = x_0(t-2) * h_0(t+1) = y_0(t-1)$$

(5)

$$x(t) = \frac{dx_0(t)}{dt}, h(t) = h_0(t)$$
$$y(t) = \frac{dy_0(t)}{dt}$$



2-9

$$h_1[n] = a^n u[n] + \beta^n u[n]$$
$$h_2[n] = (-\frac{1}{2})^n u[n]$$
$$x[n] = \delta[n] + \frac{1}{2}\delta[n-1]$$
$$y[n] = h_1[n] * h_2[n] * x[n]$$
$$y[n] = [(a^n u[n] + \beta^n u[n]) * ((-\frac{1}{2})^n u[n]) * (\delta[n] + \frac{1}{2}\delta[n-1])]$$
$$= [\frac{1-(-\frac{1}{2}a)^{n+1}}{1+\frac{1}{2}a}u[n] + \frac{1-(-\frac{1}{2}\beta)^{n+1}}{1+\frac{1}{2}\beta}u[n]] * (\delta[n] + \frac{1}{2}\delta[n-1]))$$
$$= \frac{1-(-\frac{1}{2}a)^{n+1}}{1+\frac{1}{2}a}u[n] + \frac{1-(-\frac{1}{2}\beta)^{n+1}}{1+\frac{1}{2}\beta}u[n] + \frac{1}{2}u[n-1][\frac{1-(-\frac{1}{2}a)^{n}}{1+\frac{1}{2}a} + \frac{1-(-\frac{1}{2}\beta)^{n}}{1+\frac{1}{2}\beta}]$$

2-10

$$y[n] = x_1[n] * x_1[n] * x_3[n]$$
$$x_1[n] = (0.5)^n u[n], x_2[n] = u[n+3], x_3[n] = \delta[n] - \delta[n-1]$$

(1)求$x_1[n] * x_2[n]$

$$x_1[n] * x_2[n] = (0.5)^n u[n] * u[n+3]$$
$$= \sum_{k=-\infty}^{\infty} (0.5)^n u[k] \times u[n-k+3]$$
$$= (\sum_{k=0}^{n+3} (0.5)^n) u[n+3]$$
$$= (2 - 0.5^{n+3}) u[n+3]$$

(2)求$x_1[n] * x_2[n] * x_3[n]$

$$x_1[n] * x_2[n] * x_3[n]$$
$$= (2 - 0.5^{n+3}) u[n+3] * (\delta[n] - \delta[n-1])$$
$$= (2 - 0.5^{n+3}) u[n+3] - (2 - 0.5^{n+2}) u[n+2]$$

(3)求$x_2[n] * x_3[n]$

$$x_2[n] * x_3[n] = u[n+3] * (\delta[n] - \delta[n-1])$$
$$= u[n+3] - u[n+2] = \delta[n+3]$$

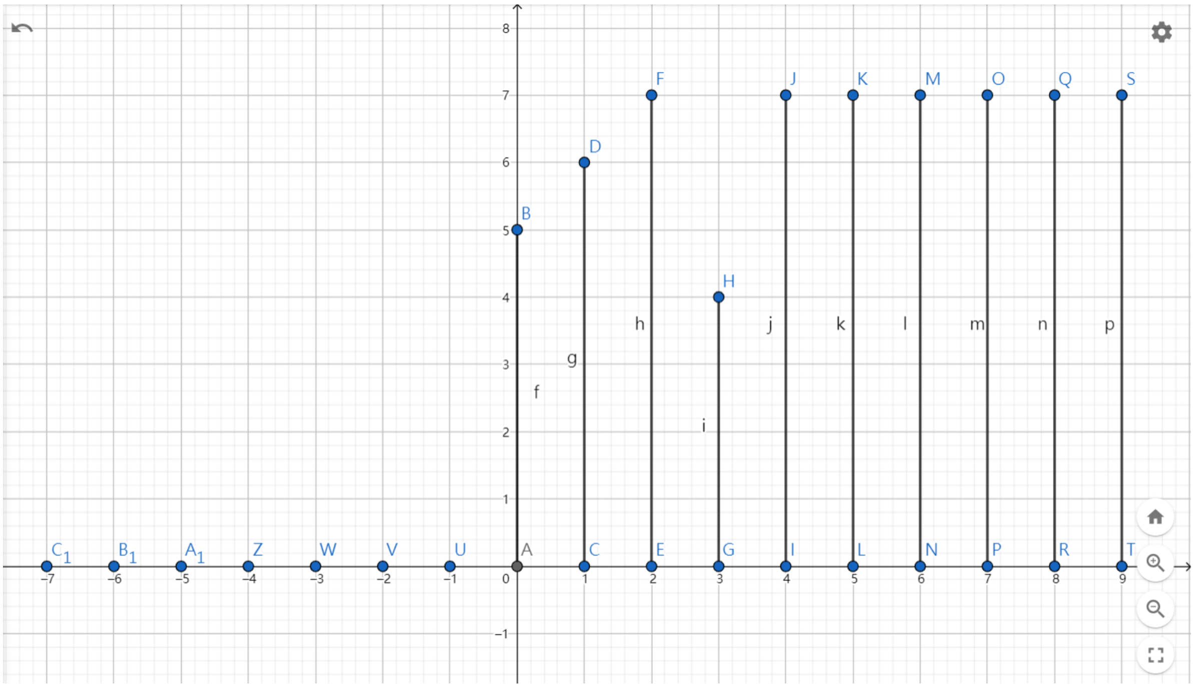
2-11

(1)

$$h[n] = h_1[n] * h_2[n] - h_1[n] * h_3[n] * h_4[n] + h_5[n]$$

(2)

$$
\begin{aligned}
h_1[n] &= 4(\frac{1}{2})^n(u[n]-u[n-3]) \\
h_2[n] &= h_3[n] = (n+1)u[n] \\
h_4[n] &= \delta[n-1] \\
h_5[n] &= \delta[n] - 4\delta[n-3] \\
h[n] &= h_1[n]*h_2[n] - h_1[n]*h_3[n]*h_4[n] + h_5[n] \\
&= h_1[n]*(h_2[n]-h_2[n-1]) + h_5[n] \\
&= h_1[n]*(u[n]) + h_5[n] \\
&= [(8-(\frac{1}{2})^{n-2})u[n]] - [(8-(\frac{1}{2})^{n+1})u[n+3]] + h_5[n]
\end{aligned}
$$



(3)

2-12

$$
y(t) = \int_{-\infty}^{t} e^{-(t-\tau)} x(\tau-2)d\tau
$$

(1)

单位冲激响应：

$$
\begin{aligned}
y(t) &= \int_{-\infty}^{t} e^{-(t-\tau)} x(\tau-2)d\tau \\
&= \int_{-\infty}^{t} e^{-(t-\tau)} \delta(\tau-2)d\tau \\
h(t) &= e^{-t+2}u(t-2)
\end{aligned}
$$

(2)

$$x(t) = -u(t-2) + u(t+1)$$
$$y(t) = \int_{-\infty}^{t} e^{-(t-\tau)} x(\tau-2) d\tau$$
$$= \int_{-\infty}^{t} e^{-(t-\tau)} [-u(\tau-4) + u(\tau-1)] d\tau$$
$$= -(1-e^{4-t})u[t-4] + (1-e^{1-t})u[t-1]$$

2-13

(1)

$$h(t) = h_1(t) + h_2(t) * h_3(t) * h_1(t)$$
$$= u(t) + u(t) * \delta(t-1) * -\delta(t)$$
$$= u(t) - u(t-1)$$

(2)

$$x(t) = u(t+1) - u(t-2)$$
$$y(t) = x(t) * h(t) = [u(t+1) - u(t-2)] * [u(t) - u(t-1)]$$
$$= (t+1)u(t+1) - tu(t) - (t-2)u(t-2) + (t-3)u(t-3)$$

2-14

(1)$h(t) = e^{-4t}u(t-2)$

因果性：当t<0时，有h(t)=0;因此h(t)是因果的

稳定性：$\sum |h(t)| = \sum_{t=2}^{\infty} e^{-4t}$，有界，h(t)是稳定的

(3)$h(t) = e^{-2t}u(t+50)$

因果性：当t<0时，存在t使得h(t)>0;h(t)是非因果的

稳定性：$\sum |h(t)| = \sum_{t=-50}^{\infty} e^{-2t}$，有界，h(t)是稳定的

(5)$h(t) = e^{-6|t|}$

因果性：当t<0时，有h(t)>0;因此h(t)是非因果的

稳定性：$\sum |h(t)| = 2\sum_{t=0}^{\infty} e^{-6t}$，有界，h(t)是稳定的

2-15

(1)$h[n] = 0.2^n u[n]$

因果性：当n<0时，有h[n]=0;h[n]是因果的

稳定性：$\sum |h[n]| = \sum_{n=0}^{\infty} 0.2^n$，有界，h[n]是稳定的

(3)$h[n] = (-0.5)^n u[-n]$

因果性：当n<0时，h[n]≠0;h[n]是非因果的

稳定性：$\sum |h[n]| = \sum_{n=-\infty}^{0} 0.5^n$，无界，h[n]是不稳定的

(5)$(-\frac{1}{2})^n u[n] + (1.01)^n u[n-1]$

因果性：当n<0时，h[n]=0;h[n]是因果的

稳定性：$\sum |h[n]| = \sum_{n=0}^{\infty} 0.5^n + \sum_{n=1}^{\infty} 1.01^n$，无界，h[n]是不稳定的

(7)$h[n]=n(\frac{1}{3})^n u[n-1]$

因果性：当n<0时，h[n]=0;h[n]是因果的

稳定性：$\sum|h[n]|=\sum_{n=1}^{\infty}\frac{n}{3^n}$，有界，h[n]是稳定的

2-16

(1)

正确，若$lim_{t->\infty}|h(t)|=c,$则有$\sum|h(t)|->\infty;$与系统稳定矛盾

(2)

正确，
$\sum|h(t)|>\sum|h(t)|;$设一个周期内的$\sum|h(t)|=c>0;$则有$\sum|h(t)|=\sum c->\infty;h(t)$不稳定

(3)

错误

$\delta(t+1)*\delta(t-1)=\delta(t),\delta(t-1)$因果，$\delta(t+1)$非因果

(4)

错，$\sum|h[n|\leq\sum k->\infty;$无法证明收敛, 以$h[n]$为单位脉冲响应的系统不一定稳定

(5)

正确，由于h[n]的长度有限且有界，不妨设该长度为从0到k$\sum h|n|<\sum_0^k|h[n|$，有界，h[n]稳定

(6)

错误，系统因果无法推出系统稳定

(7)

错误

$$h(t)=h_1(t)*h_2(t),\text{当}t<0\text{时},h(t)=\int_{-\infty}^{\infty}h_2(\tau)\times h_1(t-\tau)d\tau$$
$$\text{设}h_2(t)\text{是因果的}$$
$$h(t)=\int_0^{\infty}h_2(\tau)\times h_1(t-\tau)d\tau$$
$$\text{当}h_2(t)=0\text{时，}h(t)=0,\text{此时}h(t)\text{因果}$$

(8)

错误，是充分不必要条件

(9)

正确·，是定义

2-18

$$x_1(t) = u(t) - u(t-2)$$

$$y_1(t) = h(t) * x_1(t)$$

$$\text{易知}, h(t) = u(t) - u(t-1)$$

$$y_2(t) = [u(t) - u(t-1)] * sin\pi t[u(t) - u(t-1)]$$

$$y_2(t) = \frac{1 - cos\pi t}{\pi}[u(t) - u(t-2)]$$