HW 6

3-5

(1)

$$
\begin{aligned}
& e^{-2(t-2)}u(t-2) \\
& = e^{-2jw} \cdot \frac{1}{2+jw}
\end{aligned}
$$

(3)

$$
\begin{aligned}
& \delta(t+\pi) + \delta(t-\pi) \\
& = e^{jw\pi} + e^{-jw\pi}
\end{aligned}
$$

(5)

$$
x(t) = e^{-t}[u(t) - u(t-1)]
$$

$$
\text{设} x_1(t) = u(t) - u(t-1) = -[u(t - \frac{1}{2} - \frac{1}{2}) - u(t - \frac{1}{2} + \frac{1}{2})]
$$

$$
X_1(t) = -e^{-j\frac{1}{2}w} Sa(\frac{1}{2}w)
$$

$$
X(t) = X_1(j(w+1)) = -e^{\frac{1}{2}(w+1)} Sa(\frac{1}{2}j(w+1))
$$

3-6

(1)

$$
X(jw) = \pi[\delta(w+3\pi) + \delta(w-2\pi)]
$$

$$
x(t) = \frac{1}{2}[e^{-3t} + e^{2t}]
$$

3-7

(1)

$$
x_1(t) = x(-t+3)
$$

$$
X_1(jw) = X(-jw)e^{-3jw}
$$

(3)

$$
x_2(t) = x_1(3t)
$$

$$
X_2(jw) = \frac{1}{3} X(j\frac{1}{3}w)
$$

3-8

(2)

$$e^{-at}cos(w_0t)\cdot u(t)$$
$$= cos(w_0t)e^{-at}\cdot u(t)$$
$$x_1(t) = e^{-at}\cdot u(t)$$
$$X_1(jw) = \frac{1}{a+jw}$$
$$x(t) = x_1(t)cos(w_0t) = \frac{1}{2}[X_1(j(w-w_0)) + X_1(j(w+w_0))]$$
$$X(jw) = \frac{1}{2}[\frac{1}{a-w-1} + \frac{1}{a-w+1}]$$

(5)

$$\delta'(t) + 2\delta(3-2t)$$
$$\text{由于}\frac{dx(t)}{dt} --> jwX(jw)$$
$$\delta'(t) --> jw$$
$$\delta(-2t) --> \frac{1}{2}$$
$$\delta(3-2t) --> e^{3jw}\cdot\frac{1}{2}$$
$$\delta'(t) + 2\delta(3-2t) --> jw + \frac{1}{2}e^{3jw}$$

(7)

$$\frac{sin(\pi t)}{\pi t}\frac{sin(2\pi(t-1))}{\pi(t-1)}$$
$$\text{设}x_1(t) = \frac{sin(\pi t)}{\pi t}, X_1(jw) = u(w-\pi) - u(w+\pi)$$
$$\text{设}x_2(t) = \frac{sin(2\pi(t-1))}{\pi(t-1)}, X_2(jw) = e^{-jw}[u(w-2\pi) - u(w+2\pi)]$$
$$x_1(t)\cdot x_2(t) = \frac{1}{2\pi}X_1(jw) * X_2(jw)$$
$$= \frac{-j}{2\pi}(1+e^{-jw})[u(w+3\pi) - u(w+\pi) + u(w-3\pi) - u(w-\pi)]$$

(8)

$$x(t) = u(t) + u(t-2) - 2u(t-1)$$
$$u(t) --> \frac{1}{jw} + \pi\delta(w)$$
$$X(jw) = [\frac{1}{jw} + \pi\delta(w)][1 + e^{-2jw} - 2e^{-jw}]$$

(9)

$$x(t) = \frac{dx_1(t)}{dt}$$
$$x_1(t) = u(t+1) - u(t-1) - \delta(t+2) - \delta(t-2)$$
$$X_1(jw) = 2Sa(w) - (e^{2jw} + e^{-2jw})$$
$$X(t) = \frac{X_1(jw)}{jw} = \frac{2Sa(w) - (e^{2jw} + e^{-2jw})}{jw}$$

3-9

(1)

$$X(jw)=u(w)-u(w-2)$$
$$X_1(jw)=u(w+1)-u(w-1)$$
$$x_1(t)=\frac{sint}{\pi t}$$
$$X(jw)=X_1(j(w-1))$$
$$x(t)=e\cdot\frac{sint}{\pi t}$$

(2)

3-10

(1)

$$X(jw)=\frac{2sin3(w-\pi)}{w-\pi}$$
$$X_1(jw)=\frac{2sin3w}{w}$$
$$易知x_1(t)=u(t-3)-u(t+3)$$
$$x(t)=e^{jt\pi}x_1(t)=e^{jt\pi}[u(t-3)-u(t+3)]$$

(3)

$$X(jw)=cos(2w+\frac{\pi}{3})$$
$$=\frac{e^{j(2w+\frac{\pi}{3})}+e^{-j(2w+\frac{\pi}{3})}}{2}$$
$$=\frac{e^{j\frac{\pi}{3}}\delta(t+2)+e^{-j\frac{\pi}{3}}\delta(t-2)}{2}$$

3-11

$$x(t)=cosw_0t\cdot(1-\frac{2}{\tau_1}|t|),|t|<\frac{\tau_1}{2}$$
$$设x_1(t)=cosw_0t,X_1(jw)=\delta(t+w_0)$$
$$设x_2(t)=1-\frac{2}{\tau_1}|t|$$
$$x_3(t)=u(t+\frac{\tau_1}{4})-u(t-\frac{\tau_1}{4})$$
$$x_2(t)=x_3(t)*x_3(t)$$
$$X_2(jw)=X_3(jw)^2=(\frac{\tau_1}{2}Sa(\frac{\tau_1}{4}w))^2$$
$$X(jw)=\frac{1}{2\pi}\cdot X_1(jw)*X_2(jw)$$
$$=\frac{1}{2\pi}\cdot[\frac{\tau_1}{2}Sa(\frac{\tau_1}{4}(w-w_0))]^2$$

3-13

$$x_2(t)=x(1-t)+x(1+t)$$
$$x(t)=\sum a_k e^{jkw_0t}$$
$$x_2(t)=\sum a_k e^{jkw_0(1-t)}+\sum a_k e^{jkw_0(1+t)}$$

3-14

(1)

由于实数信号的傅里叶变换的实部是偶函数，虚部是奇函数，$x_2(t),x_3(t)$是实值函数

(2)

当x(t)是实值偶函数时，其频谱也是实值偶函数，$x_2(t)$是实值偶函数

3-15

$$\text{当}|k|>1\text{时，}a_k=0$$
$$x(t)=a_0+a_1e^{-jw_0t}+a_{-1}e^{jw_0t}$$
$$\text{由于}x(t)\text{是实值偶函数},a_k=a_{-k}$$
$$x(t)=a_0+a_1(e^{jw_0t}+e^{-jw_0t})=a_0+2a_1cos\pi t$$
$$\frac{1}{2}\int_0^2|x(t)|^2dt=\frac{1}{2}\int_0^2|a_0^2+4a_1^2cos^2\pi t+4a_0a_1cos\pi t|dt=1$$
$$a_0^2+2a_1^2=1$$
$$x(t)=1\text{或}x(t)=\sqrt{2}cos(\pi t)$$

3-16

$$(2+jw)X(jw)<-^F->2x(t)+\frac{dx(t)}{dt}=Ae^{-t}u(t)\ldots(1)$$
$$\frac{1}{2\pi}\int_{-\infty}^{\infty}|X(jw)|^2dw=\int_{-\infty}^{\infty}|x(t)|^2dt=1\ldots(2)$$
$$\text{由}(1)\text{式可以解得：}$$
$$x(t)=A(e^{-t}-e^{-2t})u(t)$$
$$\text{代入}(2)\text{式},\text{解得}A=2\sqrt{3}$$
$$x(t)=2\sqrt{3}(e^{-t}-e^{-2t})u(t)$$

3-17

$$x(t)是实值信号,则X(jw)的实部是偶函数$$
$$\frac{1}{2\pi}\int_{-\infty}^{\infty}Re\{X(jw)\}e^{jwt}dw=e^{-|t|}$$
$$\frac{1}{2\pi}\int_{-\infty}^{\infty}Re\{X(jw)\}e^{jwt}dw=\frac{1}{2\pi}\int_{-\infty}^{\infty}Re\{X(jw)\}cos(wt)dw=e^{-|t|}$$
$$\int_{0}^{\infty}Re\{X(jw)\}cos(wt)dw=\pi e^{-|t|}$$
$$Re\{X(jw)\}=\frac{x(t)+x(-t)}{2}$$
$$=>\int_{0}^{\infty}x(t)cos(wt)dw=2\pi e^{-|t|}$$
$$x(t)=2e^{-t}u(t)$$

3-18

(1)

$$x(t)=E[u(t+\frac{\tau}{2}-\frac{\tau}{2})-u(t-\frac{\tau}{2}-\frac{\tau}{2})]$$
$$设x_1(t)=E[u(t+\frac{\tau}{2})-u(t-\frac{\tau}{2})]$$
$$X_1(jw)=\tau ESa(\frac{\tau}{2}w)$$
$$X(jw)=e^{-jw\frac{\tau}{2}}\tau ESa(\frac{\tau}{2}w)$$
$$\theta(w)=\frac{-\tau w}{2}$$

(2)

$$X(0)=\tau ESa(0)=\tau E$$

(3)

$$x(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}X(jw)e^{jwt}dw$$
$$\int_{-\infty}^{\infty}X(jw)dw=2\pi x(0)=2\pi E$$

(4)

$$\int_{-\infty}^{\infty}X(jw)\frac{2sinw}{w}e^{i2w}dw...(1)$$
$$设X_1(jw)=\frac{2sinw}{w}e^{i2w}$$
$$X_2(jw)=X_1(jw)X(jw)$$
$$\int_{-\infty}^{\infty}X(jw)\frac{2sinw}{w}e^{i2w}dw=2\pi x_2(t)$$
$$x_2(t)=x(t)*x_1(t)$$
$$当\tau<1时，(1)式=0$$
$$当\tau\in[1,3]时，(1)式=2\pi E(\tau-1)$$
$$当\tau\geq 3时，(1)式=4\pi E$$

(5)

$$\int |X(jw)|^2 dw = 2\pi \int |x(t)|^2 dt = 2\pi\tau E^2$$

(6)

$$Re\{X(jw)\} <-^{F}-> \frac{x(t)+x(-t)}{2}$$

$$= \frac{E}{2}[x(t+\tau) - x(t-\tau)]$$