# Problem 1

## a.

$$对于x = 1.1时，采用Three - Point \ \ Formulas$$
$$x_1 = x_0 + h \quad x_2 = x_0 + 2h$$
$$f'(x_0) = \frac{1}{h}[-1.5f(x_0) + 2f(x_1) - 0.5f(x_2)] + \frac{h^2}{3}f^{(3)}(\xi)$$
$$f'(1.1) = 17.769705$$
$$对于x > 1.1时，采用Three - Point \ \ Midpoint \ \ Formula$$
$$f'(x_0) = \frac{1}{2h}[f(x_0 + h) - f(x_0 - h)] - \frac{h^2}{6}f^{(3)}(\xi)$$
$$对于x = 1.4, 采用f'(x_0) = \frac{1}{2h}[f(x_0 - 2h) - 4f(x_0 - h) + 3f(x_0)]$$

计算结果如下表：

| x | f(x) | f'(x) |
|---|---|---|
| 1.1 | 9.025013 | 17.769705 |
| 1.2 | 11.02318 | 22.193635 |
| 1.3 | 13.46374 | 27.10735 |
| 1.4 | 16.44465 | 21.70365 |

## b.

如题a，当x=8.1时，用Three-Point Formulas，当x>8.1时，用Three-Point Midpoint Formula，计算结果如下表：

| x | f(x) | f'(x) |
|---|---|---|
| 8.1 | 16.94410 | 3.09205 |
| 8.3 | 17.56492 | 3.11615 |
| 8.5 | 18.19056 | 3.139975 |
| 8.7 | 18.82091 | 3.363525 |

# Problem 2

$$M = N(h) + K_1 h^2 + K_2 h^4 + K_3 h^6 \dots$$
$$M = N(\frac{h}{3}) + K_1 \frac{h^2}{3^2} + K_2 \frac{h^4}{3^4} + K_3 \frac{h^6}{3^6} \dots$$
$$M = N(\frac{h}{9}) + K_1 \frac{h^2}{9^2} + K_2 \frac{h^4}{9^4} + K_3 \frac{h^6}{9^6} \dots$$
$$\text{设}x, y\text{解方程以得到}O(h^6)$$
$$\frac{1}{3^2}x + \frac{1}{9^2}y = 1$$
$$\frac{1}{3^4}x + \frac{1}{9^4}y = 1$$
$$x = 90, y = -729$$
$$M_{O^6(h)} = M_h - xM_{\frac{h}{3}} - yM_{\frac{h}{9}} =$$

# Problem 3

## Trapezoidal rule

**a.**

$$\text{由于} \int_a^b f(x)dx = \frac{h}{2}[f(x_0) + f(x_1)] - \frac{h^3}{12} f''(\xi)$$
$$\int_{-0.25}^{0.25} (cosx)^2 dx = 0.25[f(-0.25) + f(0.25)] = 0.4999$$

**b.**

$$\int_{-0.5}^{0} xln(x+1)dx = 0.25[f(-0.5) + f(0)] = 0.0866433$$

**c.**

$$\int_{0.75}^{1.3} ((sinx)^2 - 2xsinx + 1)dx = \frac{1.3 - 0.75}{2}[f(0.75) + f(1.3)] = 0.5285677628$$

**d.**

$$\int_e^{e+1} \frac{1}{xlnx} = 0.5[f(e+1) + f(e)] = 0.28633417$$

## Simpson's rule

**a.**

$$\text{由于} \int_{x_0}^{x_2} f(x)dx = \frac{h}{3}[f(x_0) + 4f(x_1) + f(x_2)] - \frac{h^5}{90} f^{(4)}(\xi)$$
$$\int_{-0.25}^{0.25} (cosx)^2 dx = \frac{0.25}{3}[f(-0.25) + 4f(0) + f(0.25)] = 0.4999968269$$

**b.**

$$\int_{-0.5}^{0} xln(x+1)dx = \frac{0.25}{3}[f(-0.5) + 4f(-0.25) + f(0)] = 0.0528546$$

**c.**

$$\int_{0.75}^{1.3}((sinx)^2-2xsinx+1)dx=\frac{0.275}{3}[f(0.75)+4f(1.025)+f(1.3)]=0.5295269385$$

**d.**

$$\int_{e}^{e+1}\frac{1}{xlnx}=\frac{0.5}{3}[f(e)+4f(e+0.5)+f(e+1)]=0.2726704524$$

# Problem 4

## a.

$$R_{3,3}=R_{3,2}+\frac{1}{15}(R_{3,2}-R_{2,2})$$
$$R_{3,2}=R_{3,1}+\frac{1}{3}(R_{3,1}-R_{2,1})$$
$$R_{2,2}=R_{2,1}+\frac{1}{3}(R_{2,1}-R_{1,1})$$
$$h=a-b$$
$$\int_{a}^{b}f(x)dx=\frac{h}{2^n}[f(a)+f(b)+2\sum_{j=1}^{n-1}f(x_j)]$$
$$R_{3,3}=1.452814$$

```
#include<stdio.h>
#include<math.h>

double fn(double x){
    return pow(cos(x),2);
}

//romberg intergration to count R_{3,3}
int main(){
    double R33,R32,R31,R22,R21,R11;
    double h;
    double a,b;
    a=-1,b=1;
    //count R11
    h=b-a;
    R11=(h/2)*(fn(a)+fn(b));
    //count R21
    R21=(h/4)*(fn(a)+2*fn(a+h/2)+fn(b));
    //count R31
    R31=(h/8)*(fn(a)+2*(fn(a+h/4)+fn(a+h/2)+fn(a+3*h/4))+fn(b));
    //count R32
    R32=R31+(R31-R21)/3;
    //count R22
    R22=R21+(R21-R11)/3;
    //count R33
    R33=R32+(R32-R22)/15;
    printf("R_{33}=%f\n",R33);
}
```

## b.

$$\text{类似}a\text{，}\int_{0.75}^{0.75} xln(x+1)dx = 0.327959$$

```
#include<stdio.h>
#include<math.h>

double fn(double x){
    return x*log(x+1);
}

//romberg intergration to count R_{3,3}
int main(){
    double R33,R32,R31,R22,R21,R11;
    double h;
    double a,b;
    a=-0.75,b=0.75;
    //count R11
    h=b-a;
    R11=(h/2)*(fn(a)+fn(b));
    //count R21
    R21=(h/4)*(fn(a)+2*fn(a+h/2)+fn(b));
    //count R31
    R31=(h/8)*(fn(a)+2*(fn(a+h/4)+fn(a+h/2)+fn(a+3*h/4))+fn(b));
    //count R32
    R32=R31+(R31-R21)/3;
    //count R22
    R22=R21+(R21-R11)/3;
    //count R33
    R33=R32+(R32-R22)/15;
    printf("R_{33}=%f\n",R33);
}
```

## c.

$$\text{类似的，}\int_{1}^{4} ((sinx)^2 - 2xsin(x) + 1)dx = 1.387063$$

```
#include<stdio.h>
#include<math.h>

double fn(double x){
    return pow(sin(x),2)-2*x*sin(x)+1;
}

//romberg intergration to count R_{3,3}
int main(){
    double R33,R32,R31,R22,R21,R11;
    double h;
    double a,b;
    a=1,b=4;
    //count R11
    h=b-a;
    R11=(h/2)*(fn(a)+fn(b));
    //count R21
```

```
    R21=(h/4)*(fn(a)+2*fn(a+h/2)+fn(b));
    //count R31
    R31=(h/8)*(fn(a)+2*(fn(a+h/4)+fn(a+h/2)+fn(a+3*h/4))+fn(b));
    //count R32
    R32=R31+(R31-R21)/3;
    //count R22
    R22=R21+(R21-R11)/3;
    //count R33
    R33=R32+(R32-R22)/15;
    printf("R_{33}=%f\n",R33);
}
```

## d.

$$\int_{e}^{e+1} \frac{1}{xlnx} dx = 0.272515$$

```
#include<stdio.h>
#include<math.h>

double fn(double x){
    return 1/(log(x)*x);
}

//romberg intergration to count R_{3,3}
int main(){
    double R33,R32,R31,R22,R21,R11;
    double h;
    double a,b;
    a=exp(1),b=exp(1)+1;
    //count R11
    h=b-a;
    R11=(h/2)*(fn(a)+fn(b));
    //count R21
    R21=(h/4)*(fn(a)+2*fn(a+h/2)+fn(b));
    //count R31
    R31=(h/8)*(fn(a)+2*(fn(a+h/4)+fn(a+h/2)+fn(a+3*h/4))+fn(b));
    //count R32
    R32=R31+(R31-R21)/3;
    //count R22
    R22=R21+(R21-R11)/3;
    //count R33
    R33=R32+(R32-R22)/15;
    printf("R_{33}=%f\n",R33);
}
```

# Problem 5

## a.

$$由于w_0 = \alpha$$
$$w_{i+1} = w_i + hf(t_i, w_i)$$
$$n = \frac{2-1}{0.1} = 10$$
$$w_0 = y(1) = 1$$
$$t_{i+1} = t_i + h$$
$$t_0 = 1$$
$$计算得到：y(2) = 1.170652$$

```
#include<stdio.h>
#include<math.h>

//use Eluer's method to approximate the solution for each of the following
initial conditions value

double fn(double t,double w){
    return w/t-pow(w/t,2);
}

int main(){
    double w=1;
    double n=10;
    double h=0.1;
    double t=1;
    for(int i=0;i<n;i++){
        w=w+h*fn(t,w);
        t=t+h;
    }
    printf("w=%f\n",w);
}
```

## b.

$$类似a题目$$
$$w_0 = 0$$
$$t_0 = 1$$
$$n = 10$$
$$h = 0.2$$
$$计算得：y(3) = 4.514277$$

```
#include<stdio.h>
#include<math.h>

//use Eluer's method to approximate the solution for each of the following
initial conditions value

double fn(double t,double w){
    return 1+w/t+pow(w/t,2);
}

int main(){
    double w=0;
    double n=10;
```

```
    double h=0.2;
    double t=1;
    for(int i=0;i<n;i++){
        w=w+h*fn(t,w);
        t=t+h;
    }
    printf("w=%f\n",w);
}
```

# Problem 6

$$[6,6]_{sinx} = \frac{(x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!})(1+q_1x+q_2x^2+q_3x^3+q_4x^4+q_5x^5+q_6x^6)}{p_0+p_1x+p_2x^2+p_3x^3+p_4x^4+p_5x^5+p_6x^6}$$

$$\sum_{i=0}^{k} a_i q_{k-i} = p_k$$

使用如下代码进行计算：

```
#include<stdio.h>
#include<math.h>
//use pade rational approximation to count sin(x)[6,6],m=6,n=6

double factorial(int n){
    int i;
    double fact=1;
    for(i=1;i<=n;i++)
        fact*=i;
    return fact;
}
int main(){
    int N = 12;
    int n = 6;
    int m = 6;
    double a[12] =
{0,1,0,-1/factorial(3),0,1/factorial(5),0,-1/factorial(7),0,1/factorial(9),0,-1/
factorial(11)};
    double b[100][100];
    double q[12] = {0};
    double p[12] = {0};
    q[0] = 1;
    p[0] = a[0];
    for(int i=1;i<=N;i++){
        for(int j=1;j<i;j++){
            b[i][j] = 0;
        }
        if(i <= 6)
            b[i][i] = 1;
        for(int j=i+1;j<=N;j++)
            b[i][j] = 0;
        for(int j=1;j<=i;j++){
            if(j <= 6)
                b[i][6+j] = -a[i-j];
        }
        for(int j=6+i+1;j<=N;j++)
            b[i][j] = 0;
```

```
        b[i][N+1] = a[i];
    }
    for(int i=n+1;i<=N-1;i++){
        //let k be the integer with |b[k][i]| = max|b[j][i]|,i<=j<=N
        double max = 0;
        int k;
        for(int j=n+1;j<=N;j++){
            if(fabs(b[j][i]) > max){
                max = fabs(b[j][i]);
                k = j;
            }
        }
        for(int j=i;j<=N+1;j++){
            double temp = b[k][j];
            b[k][j] = b[i][j];
            b[i][j] = temp;
        }
        for(int j=i+1;j<=N;j++){
            b[j][i] /= b[i][i];
            for(int l=i+1;l<=N+1;l++){
                b[j][l] -= b[j][i]*b[i][l];
            }
        }
    }
    for(int i=1;i<=m;i++)
        q[i] = b[N][N+1]/b[N][N];
    for(int i=N-1;i>=n+1;i--){
        q[i] = b[i][N+1];
        for(int j=n+1;j<=N;j++){
            q[i] -= b[i][j]*q[j-n];
        }
        q[i] /= b[i][i];
    }
    for(int i=n;i>=1;i--){
        p[i] = b[i][N+1];
        for(int j=n+1;j<=N;j++){
            p[i] -= b[i][j]*p[j-n];
        }
    }
    //output
    for(int i=1;i<=m;i++){
        printf("q[%d] = %f \n",i,q[i]);
    }
    for(int i=1;i<=n;i++){
        printf("p[%d] = %f \n",i,p[i]);
    }
}
```

没有得到正确结果

# Problem 7

由于:

| i | x | y |
| --- | --- | --- |
| 1 | 0 | 6 |
| 2 | 2 | 8 |
| 3 | 4 | 14 |
| 4 | 5 | 20 |

$$a_0 = \frac{\sum x_i^2 \sum y_i - \sum x_i y_i \sum x_i}{m(\sum x_i^2) - (\sum x_i)^2}$$

$$a_1 = \frac{m \sum x_i y_i - \sum x_i \sum y_i}{m(\sum x_i^2) - (\sum x_i)^2}$$

$$P(x_i) = a_1 x_i + a_0$$

$$E = \sum [y_i - P(x_i)]^2$$

计算得到：

$$P(x_i) = 2.711864407x + 4.542372881$$

$$E = 11.525424$$