HW 7

3-19

$$H(jw) = \frac{sin3w}{w}cosw$$
$$设H_1(jw) = \frac{sin3w}{w}$$
$$h_1(t) = \frac{1}{2}[u(t+3) - u(t-3)]$$
$$H_2(jw) = cosw \ = \frac{e^{jw} + e^{-jw}}{2}$$
$$h_2(t) = \frac{\delta(t+1) + \delta(t-1)}{2}$$
$$h(t) = h_1(t) * h_2(t) = \frac{1}{4}[u(t+4) + u(t+2) - u(t-2) - u(t-4)]$$

3-20

$$H(jw) = \frac{1}{jw+3}$$
$$对于x(t), y(t) = e^{-3t}u(t) - e^{-4t}u(t)$$
$$y(t) = x(t) * h(t)$$
$$Y(jw) = X(jw) \cdot H(jw) = \frac{1}{3+jw} - \frac{1}{4+jw}$$
$$X(jw) \cdot \frac{1}{3+jw} = \frac{1}{3+jw} - \frac{1}{4+jw}$$
$$X(jw) = \frac{1}{4+jw}$$
$$x(t) = e^{-4t}u(t)$$

3-23

(1)

$$x(t) = e^{-t}u(t), h(t) = e^{-3t}u(t)$$
$$y(t) = x(t) * h(t)$$
$$Y(jw) = X(jw) \cdot H(jw)$$
$$Y(jw) = \frac{1}{(1+jw)(3+jw)}$$
$$= \frac{1}{2}[\frac{1}{1+jw} - \frac{1}{3+jw}]$$
$$y(t) = \frac{e^{-t}u(t) - e^{-3t}u(t)}{2}$$

(2)

$$x(t) = te^{-t}u(t), h(t) = e^{-3t}u(t)$$
$$y(t) = x(t) * h(t)$$
$$Y(jw) = X(jw) \cdot H(jw)$$
$$Y(jw) = \frac{1}{3+jw} \cdot \frac{1}{(1+jw)^2}$$
$$y(t) = \frac{1}{4}(2te^{-t} - e^{-t} + e^{-3t})$$

3-24*

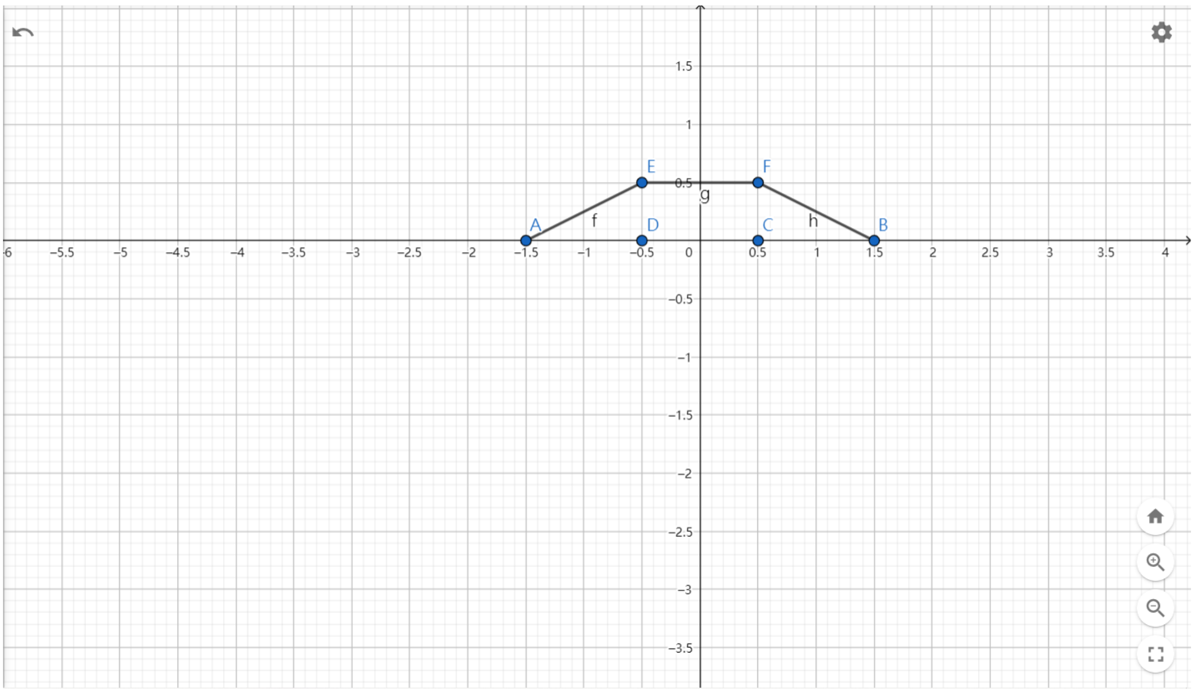
(1)

$$y(t) = x(t)p(t) = \sum a_k x(t) e^{jw_0 kt}$$
$$Y(jw) = \sum a_k X(j(w - w_0 k))$$

2-1

$$p(t) = cos\frac{t}{2} = \frac{e^{jt/2} + e^{-jt/2}}{2}$$
$$y(t) = p(t)x(t)$$
$$Y(jw) = \frac{1}{2}[X(j(w - \frac{1}{2})) + X(j(w + \frac{1}{2}))]$$

图像如下：



2-4

$$p(t) = \sum \delta(t - \pi k)$$
$$= \frac{1}{2\pi} \sum e^{j2kt}$$
$$y(t) = x(t)p(t)$$
$$= \frac{1}{2\pi} \sum x(t) e^{j2kt}$$
$$Y(jw) = \frac{1}{2\pi} \sum X(j(w - 2k))$$

2-6

$$
\begin{aligned}
p(t) &= \sum \delta(t-4\pi k) \\
&= \frac{1}{4\pi}\sum e^{j\frac{k}{2}t} \\
y(t) &= x(t)p(t) \\
&= \frac{1}{4\pi}\sum x(t)e^{j\frac{k}{2}t} \\
Y(jw) &= \frac{1}{4\pi}\sum X(j(w-\frac{k}{2}))
\end{aligned}
$$

3-26

> 证明LTI系统对周期信号的响应仍然是周期信号且不会产生新的谐波分量或新的频率分量

$$
\begin{gathered}
\text{设输入为} x(t) \\
\text{由于} x(t) \text{是周期函数，} x(t) = \sum a_k e^{jkw_0t} \\
y(t) = x(t) * h(t) \\
Y(jw) = X(jw)\cdot H(jw) \\
= \sum 2\pi a_k \delta(w-kw_0)H(jw) \\
y(t)\text{可以写为}\sum a_{k1}e^{jkw_0t}\text{, 仍然是周期信号且不会产生新的谐波分量或新的频率分量}
\end{gathered}
$$

3-27

(1)

$$
\begin{gathered}
h(t) = \frac{sin5(t-1)}{\pi(t-1)} \\
x(t) = cos(7t+\frac{\pi}{3}) \\
y(t) = x(t) * h(t) \\
Y(jw) = X(jw)H(jw) \\
H(jw) = e^{-5jw}[u(w+5)-u(w-5)] \\
x(t) = cos(7t+\frac{\pi}{3}) = \frac{1}{2}cos7t - \frac{\sqrt{3}}{2}sin7t \\
X(jw) = \frac{1}{2}\pi[\delta(w+7)+\delta(w-7)] - \frac{\sqrt{3}}{2}\frac{\pi}{j}[\delta(w+7)-\delta(w-7)] \\
Y(jw) = 0 \\
y(t) = 0
\end{gathered}
$$

(3)

$$h(t) = \frac{sin5(t-1)}{\pi(t-1)}$$
$$x(t) = \frac{sin5(t+1)}{\pi(t+1)}$$
$$Y(jw) = H(jw)X(jw)$$
$$H(jw) = e^{-5jw}[u(w+5) - u(w-5)]$$
$$X(jw) = e^{5jw}[u(w+5) - u(w-5)]$$
$$Y(jw) = u(w+5) - u(w-5)$$
$$y(t) = \frac{sin5t}{\pi t}$$

3-33

$H(jw) = [u(w+(w_0+w_c)) - u(w-(w_0+w_c))] - [u(w+(w_0-w_c)) - u(w-(w_0-w_c))]$

$$h(t) <-^F-> H(jw)$$
$$h(t) = \frac{sin((w_0+w_c)t)}{\pi t} - \frac{sin((w_0-w_c)t)}{\pi t}$$

由于$h(t)$是非因果的，该滤波器在时域上不可行

3-34

$$y(t) = x(t) * h(t)$$
$$Y(jw) = X(jw)H(jw)$$
$$x(t) = 2E[u(t) - 2u(t-T) + 2u(t-2T) - u(t-3T)]$$
$$H(jw) = e^{-jwt_0}$$
$$h(t) = \frac{w_c}{\pi}\frac{sin[w_c(t-t_0)]}{w_c(t-t_0)}$$

低通滤波器的阶跃响应为$s(t) = \frac{1}{2} + \frac{1}{\pi}Si[w_c(t-t_0)]$

$$Si(y) = \int_0^y \frac{sinx}{x}dx$$
$$y(t) = x(t) * h(t)$$

$= \frac{2E}{\pi}[Si[w_c(t-t_0)] - 2Si[w_c(t-t_0-T)] + 2Si[w_c(t-t_0-2T)] - Si[w_c(t-t_0-3T)]]$