HW 9

4-5

(1): $3^{-n+1}u[n-1]$

原式
$$=3^{-(n-1)}u[n-1]$$
 $3^{-n}u[n]<-->rac{1}{3-e^{-jw}}$ $3^{-(n-1)}u[n-1]<-->e^{-jw}rac{1}{3-e^{-jw}}$

(3): $2^n u[-n]$

$$2^n u[-n] = (rac{1}{2})^{-n} u[-n]$$
 $(rac{1}{2})^n u[n] < --> rac{1}{1 - rac{1}{2}e^{-jw}}$ $(rac{1}{2})^{-n} u[-n] < --> rac{1}{1 - rac{1}{2}e^{jw}}$

 $(5):\delta[n-2]+\delta[n+2]$

$$\delta[n] < --> 1$$

$$\delta[n-2] + \delta[n+2] < --> e^{-2jw} + e^{2jw}$$

(6):u[n-1]-u[n-5]

$$egin{align} u[n-1] - u[n-5] &= u[n+2-3] - u[n-2-3] \ &= u[n+2] - u[n-2] < --> rac{sin(2.5w)}{sin(0.5w)} \ &= u[n-1] - u[n-5] < --> e^{-3jw} rac{sin(2.5w)}{sin(0.5w)} \ &= u[n-1] - u[n-5] < --> e^{-3jw} rac{sin(2.5w)}{sin(0.5w)} \ &= u[n-1] - u[n-5] < --> e^{-3jw} rac{sin(2.5w)}{sin(0.5w)} \ &= u[n-1] - u[n-5] < --> e^{-3jw} rac{sin(2.5w)}{sin(0.5w)} \ &= u[n-1] - u[n-5] < --> e^{-3jw} rac{sin(2.5w)}{sin(0.5w)} \ &= u[n-1] - u[n-5] < --> e^{-3jw} rac{sin(2.5w)}{sin(0.5w)} \ &= u[n-1] - u[n-5] < --> e^{-3jw} rac{sin(2.5w)}{sin(0.5w)} \ &= u[n-1] - u[n-5] < --> e^{-3jw} rac{sin(2.5w)}{sin(0.5w)} \ &= u[n-1] - u[n-5] < --> e^{-3jw} rac{sin(2.5w)}{sin(0.5w)} \ &= u[n-1] - u[n-5] < --> e^{-3jw} rac{sin(2.5w)}{sin(0.5w)} \ &= u[n-1] - u[n-5] < --> e^{-3jw} rac{sin(2.5w)}{sin(0.5w)} \ &= u[n-1] - u[n-5] < --> e^{-3jw} rac{sin(2.5w)}{sin(0.5w)} \ &= u[n-1] - u[n-5] < --> e^{-3jw} rac{sin(2.5w)}{sin(0.5w)} \ &= u[n-1] - u[n-5] < --> e^{-3jw} rac{sin(2.5w)}{sin(0.5w)} \ &= u[n-1] - u[n-1]$$

 $\text{(9):} n2^{-n}u[n]$

$$egin{split} nx[n] < --> jrac{dX(jw)}{dw} \ 2^{-n}u[n] < --> rac{1}{1-rac{1}{2}e^{-jw}} \ n2^{-n}u[n] < --> -rac{1}{2}rac{1}{1-rac{1}{2}e^{-jw}} \end{split}$$

(11):

$$egin{align} x[n] &= u[n] - u[n-4] \ &= u[n+2-2] - u[n-2-2] \ u[n+2] - u[n-2] < -- > rac{sin(2.5w)}{sin(0.5w)} \ x[n] < -- > e^{-2jw} rac{sin(2.5w)}{sin(0.5w)} \ \end{array}$$

(12):

$$egin{split} X(e^{jw}) &= \sum x[n]e^{-jwn} \ &= rac{1}{2}e^{3jw} + e^{2jw} + rac{3}{2}e^{jw} + 2 + rac{1}{2}e^{-3jw} + e^{-2jw} + rac{3}{2}e^{-jw} \ &= 2 + 3cosw + 2cos2w + cos3w \end{split}$$

4-6

(2):

$$egin{split} X(e^{jw}) &= 1 - e^{-jw} + 2e^{-j2w} - 3e^{-3jw} + 4e^{-4jw} \ & \delta[n-n_0] < -- > e^{-jwn_0} \ & X(e^{jw}) < -- > \delta[n] - \delta[n-1] + 2\delta[n-2] - 3\delta[n-3] + 4\delta[n-4] \end{split}$$

(3)

$$X(e^{jw})=e^{-jrac{w}{2}},w\in[-\pi,\pi]$$
 $rac{sinW_0n}{\pi n}-->X(e^{jw})=1(|w|< W_0),X(e^{jw})=0(|w|>W_0)$,周期为 2π $x[n-rac{1}{2}]<-->e^{-jrac{w}{2}}X(e^{jw})$ $x[n]=rac{sinW_0(n-rac{1}{2})}{\pi(n-rac{1}{2})}$

(4)

$$egin{align} X(e^{jw}) &= cos^2w + jsin3w \ &= rac{cos2w+1}{2} + jsin3w \ &= rac{e^{2jw} + e^{-2jw} + 2}{4} + rac{e^{3jw} - e^{-3jw}}{2} \ &= rac{\delta[n-2] + \delta[n+2] + 2\delta[n]}{4} + rac{\delta[n+3] - \delta[n-3]}{2} \ \end{array}$$

(6)

$$X(e^{jw}) = \frac{1 - e^{-jw}}{1 - \frac{5}{6}e^{-jw} + \frac{1}{6}e^{-2jw}}$$

$$X(e^{jw}) = \frac{1 - e^{-jw}}{(1 - \frac{1}{2}e^{-jw})(1 - \frac{1}{3}e^{-jw})}$$

$$= \frac{1}{1 - \frac{1}{3}e^{-jw}} - (\frac{1}{1 - \frac{1}{2}e^{-jw}} - \frac{1}{1 - \frac{1}{3}e^{-jw}}) \times 3$$

$$= 4\frac{1}{1 - \frac{1}{3}e^{-jw}} - 3\frac{1}{1 - \frac{1}{2}e^{-jw}}$$

$$x[n] = 4(\frac{1}{3})^n u[n] - 3(\frac{1}{2})^n u[n]$$

(7)

$$egin{align} X(e^{jw}) &= rac{1-(2e^{jw})^{-8}}{1-(2e^{jw})^{-1}} \ &= 1+\sum_{n=1}^7 (rac{1}{2}e^{-jw})^n \ &= \delta[n] + \sum_{m=1}^7 (rac{1}{2})^m \delta[n-m] \ \end{aligned}$$

4-8

(1)

$$X(e^{jw}) = \sum x[n]e^{-jwn} \ X(e^{j0}) = 6$$

(2)

(3)

$$egin{align} x[n] &= rac{1}{2\pi} \int_{2\pi} X(e^{jw}) e^{jw} dw \ x[0] &= rac{1}{2\pi} \int_{2\pi} X(e^{jw}) dw = 2 \ \int_{-\pi}^{\pi} X(e^{jw}) dw = 4\pi \ \end{cases}$$

(4)

$$X(e^{j\pi}) = \sum x[n]e^{-j\pi n}
onumber \ = \sum x[n](-1)^n = 2$$

(5)

$$Re\{X(e^{jw})\} < --> \frac{x[n] + x[-n]}{2}$$

$$n = 7, \frac{x[n] + x[-n]}{2} = \frac{-1}{2}$$

$$n = 6, \frac{x[n] + x[-n]}{2} = \frac{0}{2}$$

$$n = 5, \frac{x[n] + x[-n]}{2} = \frac{1}{2}$$

$$n = 4, \frac{x[n] + x[-n]}{2} = \frac{2}{2}$$

$$n = 3, \frac{x[n] + x[-n]}{2} = \frac{0}{2}$$

$$n = 2, \frac{x[n] + x[-n]}{2} = \frac{0}{2}$$

$$n = 1, \frac{x[n] + x[-n]}{2} = \frac{2}{2}$$

$$n = 0, \frac{x[n] + x[-n]}{2} = \frac{2}{2}$$

$$n = -1, \frac{x[n] + x[-n]}{2} = \frac{2}{2}$$

$$n = -2, \frac{x[n] + x[-n]}{2} = \frac{0}{2}$$

$$n = -3, \frac{x[n] + x[-n]}{2} = \frac{0}{2}$$

(6)

$$\int_{-\pi}^{\pi}|X(e^{jw})|^2dw \ \sum |x[n]|^2 = rac{1}{2\pi}\int_{-\pi}^{\pi}|X(e^{jw})|^2dw \ \int_{-\pi}^{\pi}|X(e^{jw})|^2dw = 2\pi\sum |x[n]|^2 = 28\pi$$

(7)

$$\int_{-\pi}^{\pi} |rac{dX(e^{jw})}{dw}|^2 dw \ nx[n] < --> jrac{dX(e^{jw})}{dw} \ \int_{-\pi}^{\pi} |rac{dX(e^{jw})}{dw}|^2 dw = -2\pi \cdot rac{1}{2\pi} \int_{-\pi}^{\pi} |jrac{dX(e^{jw})}{dw}|^2 dw \ \sum |nx[n]|^2 < --> rac{1}{2\pi} \int_{-\pi}^{\pi} |jrac{dX(e^{jw})}{dw}|^2 dw \ \int_{-\pi}^{\pi} |rac{dX(e^{jw})}{dw}|^2 dw = 316\pi$$

(1):

$$\begin{split} \frac{\sin(\pi n/3)}{\pi n} & \frac{\sin(\pi n/4)}{\pi n} \\ & \frac{\sin(\pi n/3)}{\pi n} < --> X(e^{jw}) = \begin{cases} 1, |w| < \pi n/3 \\ 0, |w| > \pi n/3 \end{cases} \\ & \frac{\sin(\pi n/3)}{\pi n} < --> X(e^{jw}) = \begin{cases} 1, |w| < \pi n/4 \\ 0, |w| > \pi n/4 \end{cases} \\ & (0, |w| > \pi n/4) \\ & (0, |w| > \pi n/4) \end{cases} \\ x[n]y[n] < --> \frac{1}{2\pi} \int_{2\pi} X(e^{j\theta}) Y(e^{jw-\theta}) d\theta = \begin{cases} 1/4, |w| < \pi/12 \\ 7/24 - |w|/2\pi, |w| \in [\pi/12, 7\pi/12] \\ 0 \end{cases} \end{split}$$

(2)

$$(n+1)a^nu[n] \ (n+1)a^nu[n] = na^nu[n] + a^nu[n] \ nx[n] < --> jrac{dX(e^{jw})}{dw} \ a^nu[n] < --> rac{1}{1-ae^{-jw}} \ (n+1)a^nu[n] < --> rac{1-a}{1-ae^{-jw}}$$

(3)

$$egin{align} reve{m{y}} x_1[n] &= egin{cases} rac{\sqrt{E}}{N_1-1}, |n| < N_1 \ 0 \ &= x_1[n] * x_1[n] \ &= rac{E}{(N_1-1)^2} rac{sin^2(rac{N_1-1}{2}w)}{sin^2rac{w}{2}} \end{cases}$$

4-13

(1)

$$egin{aligned} x_1[n] &= x[1-n] + x[1-n] \ &x[-n] < -- > X(e^{-jw}) \ &x[n-1] < -- > e^{-jw}X(e^{jw}) \ &x[1-n] + x[1-n] < -- > 2e^{jw}X(e^{-jw}) \end{aligned}$$

(2)

$$egin{aligned} x_2[n] &= x[-n] cosw_0 n, 0 < w_0 < \pi \ &x[n] cosw_0 n < -- > rac{1}{2} (X(e^{j(w-w_0)}) + X(e^{j(w+w_0)})) \ &x[-n] < -- > X(e^{-jw}) \ &x[-n] cosw_0 n < -- > rac{1}{2} (X(e^{-j(w-w_0)}) + X(e^{-j(w+w_0)})) \end{aligned}$$

$$egin{align} x_3[n] &= rac{x^*[-n] + x[n]}{2} \ x^*[n] < -- > X^*(e^{-jw}) \ rac{x^*[-n] + x[n]}{2} < -- > [X^*(e^{jw}) + X(e^{jw})]/2 \ \end{cases}$$

(4)

$$egin{aligned} x_4[n] &= (n-1)^2 x[n] \ &= n^2 x[n] - 2n x[n] + x[n] \ &= -rac{dX(e^{jw})}{dw} - 2jrac{dX(e^{jw})}{dw} + X(e^{jw}) \end{aligned}$$

4-14

- (1): $X(e^{jw})$ 是虚信号,且既不是奇信号也不是偶信号,x[n]是虚信号,且既不是奇信号也不是偶信号
- (2):x[n]是实奇信号
- (3):x[n]是实信号,且既不是奇信号也不是偶信号

4-16

全部满足下列条件之一

4-17

$$egin{align} X(e^{jw}) &= rac{1}{1-e^{-jw}}(rac{sin1.5w}{sin0.5w}) + 3\pi\delta(w), w \in (-\pi,\pi] \ & x_1[n] &= egin{cases} 1, |n| <= 1 \ 0, |n| > 1 \ & X_1(e^{jw}) &= rac{sin1.5w}{sin0.5w} \ & \sum_{m=-\infty}^n x[m] - - > rac{X(e^{jw})}{1-e^{-jw}} + \pi X(e^{j0}) \sum \delta(w-2k\pi) \ & x[n] &= \sum_{k=-\infty}^n x_1[k] \end{cases}$$

4-18

$$egin{align} rac{x[n]-x[-n]}{2} < -- > jIm\{X(e^{jw}\} = j[sinw-sin2w] \ & X(e^{jw}) = A + e^{jw} + e^{-2jw} \ & 2\pi \sum |x[n]|^2 = \int_{-\pi}^{\pi} |X(e^{jw})|^2 dw = 6\pi \ & x[n] = \delta[n] + \delta[n+1] - \delta[n+2] \end{aligned}$$

4-19

(1)

$$egin{aligned} y[n] &= (rac{sinrac{\pi}{4}n}{\pi n})^2 * (rac{sinw_c n}{\pi n}) \ &|w_c| <= \pi \ &|w_c| \in [rac{\pi}{2},\pi] \end{aligned}$$

(2)

$$egin{aligned} y[n] &= (rac{sinrac{\pi n}{4}}{\pi n}cos(rac{\pi n}{2}))*(rac{sinw_c n}{\pi n})\ &|w_c| \in [rac{3}{4}\pi,\pi] \end{aligned}$$

4-20

(b)

$$X_1(e^{jw}) = [X(e^{j(w+\pi)}) + X(e^{j(w-\pi)})]/2 \ x_1[n] = e^{j\pi n}x[n]$$

(f)

$$egin{aligned} X_5(e^{jw}) &= 1 - X(e^{j(w-rac{\pi}{2})}) - X(e^{j(w+rac{\pi}{2})}) \ x_5[n] &= \delta[n] - e^{jrac{\pi}{2}n}x[n] - e^{jrac{\pi}{2}n}x[n] \end{aligned}$$

4-21

$$egin{aligned} x[n] < -- > A(w) + jB(w) \ Y(e^{jw}) &= B(w) + A(w)e^{-jw} \ &rac{x[n] + x[-n]}{2} < -- > A(w) \ &rac{x[n] - x[-n]}{2} < -- > jB(w) \ &y[n] &= rac{x[n-1] + x[-n+1]}{2} - jrac{x[n] - x[-n]}{2} \end{aligned}$$

4-23

4-25

(1)

$$\begin{split} y[n] + \frac{1}{6}y[n-1] - \frac{1}{6}y[n-2] &= x[n] - x[n-1] \\ Y(e^{jw}) + \frac{1}{6}e^{-jw}Y(e^{jw}) - \frac{1}{6}e^{-2jw}Y(e^{jw}) &= X(e^{jw}) - e^{-jw}X(e^{jw}) \\ H(e^{jw}) &= \frac{Y(e^{jw})}{X(e^{jw})} \\ &= \frac{1 - e^{-jw}}{1 + \frac{1}{6}e^{-jw} - \frac{1}{6}e^{-2jw}} \end{split}$$

(2)

$$egin{aligned} h[n] < --> H(e^{jw}) &= rac{1 - e^{-jw}}{1 + rac{1}{6}e^{-jw} - rac{1}{6}e^{-2jw}} \ &= rac{1 - e^{-jw}}{(1 + rac{1}{2}e^{-jw})(1 - rac{1}{3}e^{-jw})} \ h[n] &= rac{9}{5}(rac{-1}{2})^n u[n] - rac{4}{5}(rac{1}{3})^n u[n] \end{aligned}$$

(3)

$$\begin{split} x[n] &= 4^{-n}u[n] \\ Y(e^{jw}) &= \frac{1 - e^{-jw}}{1 + \frac{1}{6}e^{-jw} - \frac{1}{6}e^{-2jw}} X(e^{jw}) \\ &= \frac{1 - e^{-jw}}{1 + \frac{1}{6}e^{-jw} - \frac{1}{6}e^{-2jw}} \cdot \frac{1}{1 - \frac{1}{4}e^{-jw}} \\ y[n] &= \frac{6}{5}(\frac{-1}{2})^n u[n] + 3(\frac{1}{4})^n u[n] - \frac{16}{5}(\frac{1}{3})^n u[n] \end{split}$$

4-26

$$(rac{2}{3})^n u[n] < --> n (rac{2}{3})^n u[n]$$

(1)

$$egin{align} y[n] &= x[n] * h[n] \ Y(e^{jw}) &= X(e^{jw}) \cdot H(e^{jw}) \ X(e^{jw}) &= rac{1}{1 - rac{2}{3}e^{-jw}} \ Y(e^{jw}) &= jrac{dX(e^{jw})}{dw} = -rac{2}{3}e^{-jw}rac{1}{(1 - rac{2}{3}e^{-jw})^2} \ H(e^{jw}) &= rac{Y(e^{jw})}{X(e^{jw})} = rac{2}{3}e^{-jw}rac{1}{(1 - rac{2}{2}e^{-jw})} \ \end{array}$$

(2)

$$egin{align} H(e^{jw}) &= rac{\sum_{k=0}^M b_k e^{-jkw}}{\sum_{k=0}^N a_k e^{-jkw}} = rac{2}{3} e^{-jw} rac{1}{(1-rac{2}{3}e^{-jw})} \ & \ b_0 &= 0, b_1 = rac{-2}{3} \ & \ a_0 &= 1, a_1 = rac{-2}{3} \ & \ y[n] - rac{2}{3}y[n-1] = rac{2}{3}x[n-1] \ \end{array}$$

4-28

(1):
$$x[n] = (-1)^n$$

$$x[n] = cos\pi n < --> \pi \sum [\delta(w-\pi) + \delta(w+\pi)] \ y[n] = x[n] * h[n] \ Y(e^{jw}) = X(e^{jw}) \cdot H(e^{jw}) = 0$$
 (2): $x[n] = 1 + sin(rac{3\pi}{8}n + rac{\pi}{4}) + rac{1}{2}cos(rac{\pi}{2}n + rac{\pi}{6}) + rac{1}{4}sin(rac{2\pi}{3}n + rac{\pi}{4}) \ y[n] = sin(rac{3\pi}{8} + rac{\pi}{4})$

4-32

$$egin{align*} h_1[n] &= \delta[n] - rac{sin(\pi n/2)}{\pi n} \ y[n] &= (-x[n]*h_1[n] + x[n])*h_3[n] + x[n]*h_1[n]*h_2[n] \ Y(e^{jw}) &= H_3[e^{jw}](-X[e^{jw}] \cdot H_1[e^{jw}] + X[e^{jw}]) + X[e^{jw}] \cdot H_1[e^{jw}] \cdot H_2[e^{jw}] \ &= X[e^{jw}](-H_3(e^{jw}) \cdot H_1(e^{jw}) + H_3(e^{jw}) + H_1[e^{jw}] \cdot H_2[e^{jw}] \ H(e^{jw}) &= -H_3(e^{jw}) \cdot H_1(e^{jw}) + H_3(e^{jw}) + H_1[e^{jw}] \cdot H_2[e^{jw}] \ y[n] &= 16(rac{sinn\pi}{4})cosrac{\pi}{2}n \end{gathered}$$