

HW 7

Problem 1

自由空间中的平面波以波导半空间 θ 角入射；对于足够大的 $\sigma/w\epsilon_0$ ，通过证明 $\theta_t = \tan^{-1}k_x/k_{zR}$ 来证明透射波垂直于边界

$$\begin{aligned}\frac{\sin\theta_i}{\sin\theta_t} &= \frac{k_t}{k_0} \\ \tan\theta_t &= \frac{k_0 \sin\theta_0}{\sqrt{k_t^2 - k_0^2 \sin^2\theta_0}} = \frac{k_x}{k_z} = \frac{k_x}{k_{zR}} \\ \text{由于 } \frac{\sigma}{w\epsilon_0} &\gg 1 \\ k_x &= \sqrt{\frac{w\epsilon}{2\sigma}} k(i-1) \\ k_x^2 + k_z^2 &= k^2 \\ \tan\theta_t &= \frac{k_x}{\sqrt{k^2 - k_x^2}} = \frac{i-1}{\sqrt{2}\sqrt{\frac{\sigma}{w\epsilon} + i}} = 0\end{aligned}$$

Problem 2

$$\text{对于 } TE \text{ 波, } R_{TE} = \frac{1 - p_{0t}}{1 + p_{0t}}$$

$$T_{TE} = \frac{2}{1 + p_{0t}}$$

$$p_{0t} = \frac{u_0 k_{tz}}{u_t k_z}$$

$$\text{对于 } TM \text{ 波, } R_{TM} = \frac{1 - p_{0t}}{1 + p_{0t}}$$

$$T_{TM} = \frac{2}{1 + p_{0t}}$$

$$p_{0t} = \frac{\epsilon k_{tz}}{\epsilon_t k_x}$$

由于 TE 、 TM 波入射的是相同介质，得到的 TEM 波反射率、透射率相同

Problem 3

$$\begin{aligned}
E_i &= \left(\frac{1}{\sqrt{2}}x + \frac{1}{\sqrt{2}}z + iy \right) e^{i\frac{1}{\sqrt{2}}k_0x - i\frac{1}{\sqrt{2}}k_0z} \\
&= \left(\frac{1}{\sqrt{2}}x + \frac{1}{\sqrt{2}}z + iy \right) \left[\cos\left(\frac{1}{\sqrt{2}}k_0(x-z)\right) + i\sin\left(\frac{1}{\sqrt{2}}k_0(x-z)\right) \right] \\
E_x &= \frac{1}{\sqrt{2}} \left[\cos\left(\frac{1}{\sqrt{2}}k_0(x-z)\right) + i\sin\left(\frac{1}{\sqrt{2}}k_0(x-z)\right) \right] x \\
E_y &= \frac{1}{\sqrt{2}} \left[\cos\left(\frac{1}{\sqrt{2}}k_0(x-z)\right) + i\sin\left(\frac{1}{\sqrt{2}}k_0(x-z)\right) \right] iy
\end{aligned}$$

由于入射方向是 z 方向，电场方向在 x, z 平面内，入射波是 TM 波，

$$\begin{aligned}
&\text{对于 } TM \text{ 波, } E_r = R^{TM} E_i \\
R_{TM} &= \frac{1 - p_{0t}}{1 + p_{0t}}, T_{TM} = \frac{2}{1 + p_{0t}}, p_{0t} = \frac{\epsilon k_{tz}}{\epsilon_t k_x}
\end{aligned}$$

由于是从 $+z$ 入射到 $-z$ 方向，入射波是左旋的

$$\begin{aligned}
&\text{又由于 } \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 = 1, \text{ 入射波是圆极化波} \\
&\Rightarrow \text{反射波是右旋圆极化波}
\end{aligned}$$