

Problem 1

a.

$$\begin{aligned} \|x\|_{\infty} &= \max |x_i| \\ \|x - \tilde{x}\|_{\infty} &= \max |(x - \tilde{x})_i| = 0.5 \\ A &= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 6 \end{bmatrix} \\ A\tilde{x} - b &= (1, -1.3, 1.8)^t \\ \|A\tilde{x} - b\|_{\infty} &= 1.8 \end{aligned}$$

b.

$$\begin{aligned} \|x\|_{\infty} &= \max |x_i| \\ \|x - \tilde{x}\|_{\infty} &= \max |(x - \tilde{x})_i| = 0.9 \\ A &= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 6 \end{bmatrix} \\ A\tilde{x} - b &= (1.27, -1.16, 2.21)^t \\ \|A\tilde{x} - b\|_{\infty} &= 2.21 \end{aligned}$$

Problem 2

$$\begin{aligned} \text{由于 } \|A\|_2 &= [\rho(A^t A)^{\frac{1}{2}}] \\ \text{若 } A \text{ 是对称的, 则 } A^t A &= A * A \\ \|A\|_2 &= [\rho(A^t A)^{\frac{1}{2}}] = \rho(A) \end{aligned}$$

Problem 3

本题采用三位精度

a.

$$s_1 = 58.9 \quad s_2 = 6.1$$

$$\frac{|a_{11}|}{s_1} = 0.000509337$$

$$\frac{|a_{21}|}{s_2} = 0.870491803$$

$$\frac{|a_{21}|}{s_2} > \frac{|a_{11}|}{s_1}$$

交换后的矩阵为:

$$\begin{bmatrix} 5.31 & -6.1 & 47 \\ 0.03 & 58.9 & 59.2 \end{bmatrix}$$

一次高斯消元以后得到的矩阵如下:

$$\begin{bmatrix} 5.31 & -6.1 & 47 \\ -1.5 * 10^{-6} & 58.934 & 58.934 \end{bmatrix}$$

=>

$$\begin{bmatrix} 5.31 & -6.1 & 47 \\ 0 & 58.934 & 58.934 \end{bmatrix}$$

$$\text{解得: } x_2 = 1 \quad x_1 = 10$$

b.

$$s_1 = 12.1 \quad s_2 = 21.1 \quad s_3 = 21$$

$$\left| \frac{a_{11}}{s_1} \right| = 0.2504$$

$$\left| \frac{a_{21}}{s_2} \right| = 0.2504$$

$$\left| \frac{a_{31}}{s_3} \right| = 0.2909$$

交换后的矩阵为:

$$\begin{bmatrix} 6.11 & -14.2 & 21 & -139 \\ -3.03 & 12.1 & -7 & 120 \\ 3.03 & -12.1 & 14 & -119 \end{bmatrix}$$

一次高斯消元后得到:

$$\begin{bmatrix} 6.11 & -14.2 & 21 & -139 \\ -5.55 * 10^{-3} & 5.071 & 3.395 & 51.195 \\ 5.55 * 10^{-3} & -5.07 & 3.605 & -70.195 \end{bmatrix}$$

=>

$$\begin{bmatrix} 6.11 & -14.2 & 21 & -139 \\ 0 & 5.071 & 3.395 & 51.195 \\ 0 & -5.071 & 3.605 & -50.195 \end{bmatrix}$$

$E_3 + E_2$:

$$\begin{bmatrix} 6.11 & -14.2 & 21 & -139 \\ 0 & 5.071 & 3.395 & 51.195 \\ 0 & 0 & 7 & 1 \end{bmatrix}$$

$$\text{解得: } x_3 = 0.142 \quad x_2 = 10.001 \quad x_1 = 0.00527$$

Problem4

a.

使用如下代码迭代:

```
#include<stdio.h>

int main(){
    double x[100][100] = {
        {4,1,-1,5},
        {-1,3,1,-4},
        {2,2,5,1},
    };
    double x1,x2,x3;
    double xTmp1,xTmp2,xTmp3;
    double sumTmp = 0;
    x1 = x2 = x3 = 0;
    int i=0;
    while(++i<10){
        xTmp1 = (-(x[0][1]*x2+x[0][2]*x3)+x[0][3])/x[0][0];
        xTmp2 = (-(x[1][0]*x1+x[1][2]*x3)+x[1][3])/x[1][1];
        xTmp3 = (-(x[2][0]*x1+x[2][1]*x2)+x[2][3])/x[2][2];
        x1 = xTmp1;
        x2 = xTmp2;
        x3 = xTmp3;
        printf("第%d次迭代:x1=%f,x2=%f,x3=%f\n",i,x1,x2,x3);
    }
}
```

得到的结果为:

```
[Running] cd "f:\桌面\一些文件\主修课程\大二上\数值
第1次迭代:x1=1.250000,x2=-1.333333,x3=0.200000
第2次迭代:x1=1.633333,x2=-0.983333,x3=0.233333
第3次迭代:x1=1.554167,x2=-0.866667,x3=-0.060000
第4次迭代:x1=1.451667,x2=-0.795278,x3=-0.075000
第5次迭代:x1=1.430069,x2=-0.824444,x3=-0.062556
第6次迭代:x1=1.440472,x2=-0.835792,x3=-0.042250
第7次迭代:x1=1.448385,x2=-0.839093,x3=-0.041872
第8次迭代:x1=1.449305,x2=-0.836581,x3=-0.043717
第9次迭代:x1=1.448216,x2=-0.835659,x3=-0.045090
```

b.

使用如下代码迭代:

```
#include<stdio.h>

int main(){
    double x[100][100] = {
        {-2,1,0.5,4},
    };
}
```

```

        {1,-2,-0.5,-4},
        {0,1,2,0},
    };
    double x1,x2,x3;
    double xTmp1,xTmp2,xTmp3;
    double sumTmp = 0;
    x1 = x2 = x3 = 0;
    int i=0;
    while(++i<10){
        xTmp1 = (-(x[0][1]*x2+x[0][2]*x3)+x[0][3])/x[0][0];
        xTmp2 = (-(x[1][0]*x1+x[1][2]*x3)+x[1][3])/x[1][1];
        xTmp3 = (-(x[2][0]*x1+x[2][1]*x2)+x[2][3])/x[2][2];
        x1 = xTmp1;
        x2 = xTmp2;
        x3 = xTmp3;
        printf("第%d次迭代:x1=%f,x2=%f,x3=%f\n",i,x1,x2,x3);
    }
}

```

得到的结果如下：

```

[Running] cd "f:\桌面\一些文件\主修课程\大二上\数值
第1次迭代:x1=-2.000000,x2=2.000000,x3=0.000000
第2次迭代:x1=-1.000000,x2=1.000000,x3=-1.000000
第3次迭代:x1=-1.750000,x2=1.750000,x3=-0.500000
第4次迭代:x1=-1.250000,x2=1.250000,x3=-0.875000
第5次迭代:x1=-1.593750,x2=1.593750,x3=-0.625000
第6次迭代:x1=-1.359375,x2=1.359375,x3=-0.796875
第7次迭代:x1=-1.519531,x2=1.519531,x3=-0.679688
第8次迭代:x1=-1.410156,x2=1.410156,x3=-0.759766
第9次迭代:x1=-1.484863,x2=1.484863,x3=-0.705078

```

Problem 5

a.

采用如下代码进行迭代：

```

#include<stdio.h>
#include<math.h>
#define TOL 0.001

int main(){
    double x[100][100] = {
        {3,-1,1,1},
        {3,6,2,0},
        {3,3,7,4},
    };
    double x1,x2,x3;
    double xTmp1,xTmp2,xTmp3;
    double sumTmp = 0;
    x1 = x2 = x3 = 0;
    int i=0;
}

```

```

double maxNorms = 10;
while(maxNorms > TOL){
    xTmp1 = (-(x[0][1]*x2+x[0][2]*x3)+x[0][3])/x[0][0];
    xTmp2 = (-(x[1][0]*xTmp1+x[1][2]*x3)+x[1][3])/x[1][1];
    xTmp3 = (-(x[2][0]*xTmp1+x[2][1]*xTmp2)+x[2][3])/x[2][2];
    maxNorms = fabs(xTmp1-x1);
    if(fabs(xTmp2-x2)>maxNorms)
        maxNorms = fabs(xTmp2-x2);
    if(fabs(xTmp3-x3)>maxNorms)
        maxNorms = fabs(xTmp3-x3);
    //printf("maxNorms: %f\n",maxNorms);
    x1 = xTmp1;
    x2 = xTmp2;
    x3 = xTmp3;
    i++;
    printf("第%d次迭代:x1=%f,x2=%f,x3=%f\n",i,x1,x2,x3);
}
printf("最终迭代结果:x1=%f,x2=%f,x3=%f\n",x1,x2,x3);
}

```

得到的迭代结果为: x1=0.035351,x2=-0.236789,x3=0.657759

```

[Running] cd "f:\桌面\一些文件\主修课程\大二上\数值分
第1次迭代:x1=0.333333,x2=-0.166667,x3=0.500000
第2次迭代:x1=0.111111,x2=-0.222222,x3=0.619048
第3次迭代:x1=0.052910,x2=-0.232804,x3=0.648526
第4次迭代:x1=0.039557,x2=-0.235954,x3=0.655599
第5次迭代:x1=0.036149,x2=-0.236608,x3=0.657339
第6次迭代:x1=0.035351,x2=-0.236789,x3=0.657759
最终迭代结果:x1=0.035351,x2=-0.236789,x3=0.657759

```

b.

采用如下代码进行迭代:

```

#include<stdio.h>
#include<math.h>
#define TOL 0.001

int main(){
    double x[100][100] = {
        {10,-1,0,9},
        {-1,10,-2,7},
        {0,-2,10,6},
    };
    double x1,x2,x3;
    double xTmp1,xTmp2,xTmp3;
    double sumTmp = 0;
    x1 = x2 = x3 = 0;
    int i=0;
    double maxNorms = 10;
    while(maxNorms > TOL){
        xTmp1 = (-(x[0][1]*x2+x[0][2]*x3)+x[0][3])/x[0][0];
        xTmp2 = (-(x[1][0]*xTmp1+x[1][2]*x3)+x[1][3])/x[1][1];

```

```

xTmp3 = (-(x[2][0]*xTmp1+x[2][1]*xTmp2)+x[2][3])/x[2][2];
maxNorms = fabs(xTmp1-x1);
if(fabs(xTmp2-x2)>maxNorms)
    maxNorms = fabs(xTmp2-x2);
if(fabs(xTmp3-x3)>maxNorms)
    maxNorms = fabs(xTmp3-x3);
//printf("maxNorms: %f\n",maxNorms);
x1 = xTmp1;
x2 = xTmp2;
x3 = xTmp3;
i++;
printf("第%d次迭代:x1=%f,x2=%f,x3=%f\n",i,x1,x2,x3);
}
printf("最终迭代结果:x1=%f,x2=%f,x3=%f\n",x1,x2,x3);
}

```

得到的迭代结果为: x1=0.995748,x2=0.957874,x3=0.791575

```

[Running] cd "f:\桌面\一些文件\主修课程\大二上\数值分
第1次迭代:x1=0.900000,x2=0.790000,x3=0.758000
第2次迭代:x1=0.979000,x2=0.949500,x3=0.789900
第3次迭代:x1=0.994950,x2=0.957475,x3=0.791495|
第4次迭代:x1=0.995748,x2=0.957874,x3=0.791575
最终迭代结果:x1=0.995748,x2=0.957874,x3=0.791575

```