HW 5

Problem 1

(1)

$$\frac{df(x,y,t)}{dt}=k[\frac{d^2f(x,y,t)}{dx^2}+\frac{d^2f(x,y,t)}{dy^2}]$$
$$f(x,y,0)=f(x,y)$$
$$当f(x,y)=B_0时,$$
$$f(x,t)=B_0$$
$$当f(x,y)=Bcos(wx)sin(wy)时,$$
$$f(x,y,t)=Bcos(wx)sin(wy)e^{-2Bw^2t}$$
$$当f(x,y)=Ccos(wx)cos(wy)时$$
$$f(x,y,t)=Ccos(wx)cos(wy)e^{-2Cw^2t}$$
$$当f(x,y)=Dsin(wx)sin(wy)时$$
$$f(x,y,t)=Dsin(wx)sin(wy)e^{-2Dw^2t}$$
$$当f(x,y)=Esin(wx)cos(wy)时$$
$$f(x,y,t)=Esin(wx)sin(wy)e^{-2Ew^2t}$$
$$当f(x,y)是常数,sin(wx)sin(wy),sin(wx)cos(wy),cos(wx)cos(wy),cos(wx)sin(wy)$$
$$的线性组合时，f(x,y,t)是对应的线性组合$$

(2)

$$二维傅里叶级数：$$
$$f(u,v)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}f(x,y)e^{-j2\pi(ux+vy)}dxdy$$

Problem 2

$$x(t)=sin(\pi t),x\in[0,1]$$
$$x(t)=a_1\phi_1(t)+a_2\phi_2(t)+a_3\phi_3(t)$$
$$<x(t)\cdot\phi_1(t)>=a_1<\phi_1(t)\cdot\phi_1(t)>$$
$$\frac{2}{\pi}=a_1\cdot 1$$
$$<x(t)\cdot\phi_2(t)>=a_2<\phi_2(t)\cdot\phi_2(t)>$$
$$0=a_2\cdot 1,a_2=0$$
$$<x(t)\cdot\phi_3(t)>=a_3<\phi_3(t)\cdot\phi_3(t)>$$
$$0=a_3\cdot 1,a_3=0$$

Problem 3

$$当n=2时，有 <\beta_2\cdot\beta_1>=0,\beta_2,\beta_1正交$$
$$当n>=2时,设n<=m时\{\beta_1,\beta_2\ldots\beta_m\}是正交基$$
$$当n=m+1时：$$
$$\beta_{m+1}=a_{m+1}-\sum_{i=1}^{m}\frac{<a_{m+1}\cdot\beta_i>}{<\beta_i\cdot\beta_i>}\beta_i$$
$$<\beta_{m+1}\cdot\beta_j>=<a_{m+1}\cdot\beta_j>-\sum_{i=1}^{m}\frac{<a_{m+1}\cdot\beta_i>}{<\beta_i\cdot\beta_i>}<\beta_i,\beta_j>$$
$$=<a_{m+1}\cdot\beta_j>-\frac{<a_{m+1}\cdot\beta_j>}{<\beta_j\cdot\beta_j>}<\beta_j,\beta_j>=0$$
$$n=m+1时满足条件，证毕$$

Problem 4

(1)

$$a_1(t)=1,a_2(t)=t,a_3(t)=t^2,a_4(t)=t^3$$
$$\beta_1=a_1(t)=1$$
$$\beta_2=a_2(t)-\frac{<a_2(t)\cdot\beta_1>}{<\beta_1\cdot\beta_1>}\beta_1=t-\frac{1}{2}$$
$$\beta_3=t^2-t+\frac{1}{6}$$
$$\beta_4=t^3-\frac{3}{2}t^2+\frac{3}{5}t-\frac{1}{20}$$

(2)

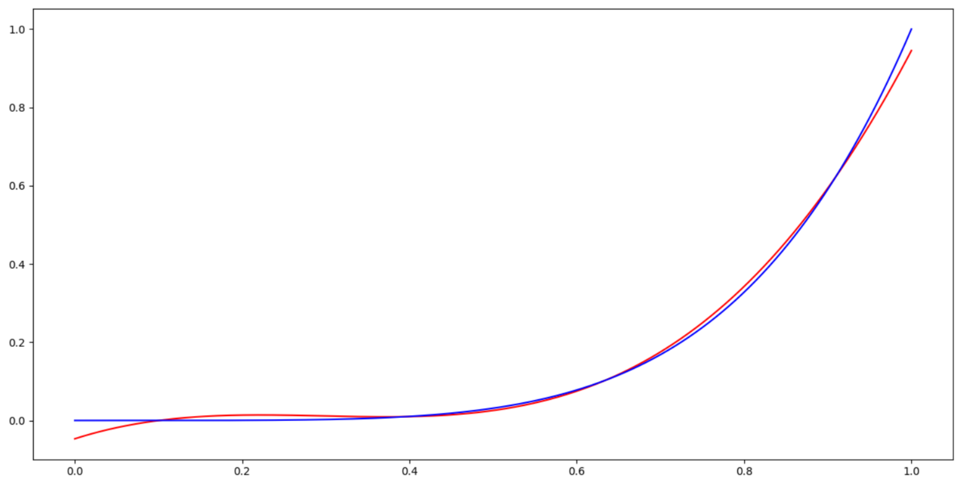
$$x(t)=b_1\beta_1(t)+b_2\beta_2(t)+b_3\beta_3(t)+b_4\beta_4(t)$$
$$<x(t)\cdot\beta_1(t)>=b_1<\beta_1(t)\cdot\beta_1(t)>$$
$$\frac{1}{6}=b_1\cdot 1,b_1=0.1666667$$
$$<x(t)\cdot\beta_2(t)>=b_2<\beta_2(t)\cdot\beta_2(t)>$$
$$\frac{5}{84}=b_2\cdot\frac{1}{12},b_2=\frac{5}{7}=0.71428$$
$$<x(t)\cdot\beta_3(t)>=b_3<\beta_3t)\cdot\beta_3(t)>$$
$$\frac{5}{504}=\frac{1}{180}b_3,b_3=\frac{100}{59}=1.694915254$$
$$<x(t)\cdot\beta_4(t)>=b_4<\beta_4(t)\cdot\beta_4(t)>$$
$$9.9206\times 10^{-4}=b_4\cdot\frac{1}{2800},b_4=2.7778$$
$$b_1=0.1666667,b_2=\frac{5}{7}=0.71428,b_3=\frac{100}{59}=1.694915254,b_4=2.7778$$

(3)

采用如下代码进行拟合，结果如下图

```
from re import X
from turtle import color
import numpy as np
import matplotlib.pyplot as plt

'''
```

```
beta_1 = 1
beta_2 = t-0.5
beta_3 = t^2-t+1/6
beta_4 = t^3-1.5t^2+0.6t-1/20
b_1 = 0.1666667
b_2 = 0.71428
b_3 = 1.694915245
b_4 = 2.777778
画出y = \sum (b_i*beta_i)
'''

b_1 = 0.1666667
b_2 = 0.71428
b_3 = 1.694915245
b_4 = 2.777778
x =  np.linspace(0,1,1000)
y = b_1*1 + b_2*(x-0.5) + b_3*(x**2-x+1/6) + b_4*(x**3-1.5*x**2+0.6*x-1/20)
y_1 = x**5
plt.plot(x,y,color="red")
plt.plot(x,y_1,color="blue")
plt.show()
```



Problem 5

$$先证明\{e^{j\frac{2\pi}{N}nk}\}是一组正交基：$$

$$< e^{j\frac{2\pi}{N}nk}\cdot e^{j\frac{2\pi}{N}nl} >= \sum_{n=0}^{N-1} e^{j\frac{2\pi}{N}n(k-l)}$$

$$当k=l时, \sum_{n=0}^{N-1} e^{j\frac{2\pi}{N}n(k-l)} = N$$

$$当k\neq l时, \sum_{n=0}^{N-1} e^{j\frac{2\pi}{N}n(k-l)} = \sum_{n=0}^{N-1} cos(\frac{n}{N}2\pi(k-l)) = 0$$

$$\{e^{j\frac{2\pi}{N}nk}\}是一组正交基$$

$$x[n] = \sum_{k=0}^{N-1} a_k e^{j\frac{2\pi}{N}nk}$$

$$< x[n]\cdot e^{j\frac{2\pi}{N}nk} >= a_k\cdot N$$

$$a_k = \frac{1}{N}\sum_{n=0}^{N-1} x[n]e^{-j\frac{2\pi}{N}nk}$$

Problem 6

(1)

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}[x_{cos(w)}cos(wt) + x_{sin(w)}sin(wt)]dw$$

$$x_{cos(w)} = \int_{-\infty}^{\infty} x(t)cos(wt)dt$$

$$x_{sin(w)} = \int_{-\infty}^{\infty} x(t)sin(wt)dt$$

$$证明：$$

$$由于是非周期信号，有：$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(jw)e^{jwt}dw$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} X(jw)(cos(wt) + isin(wt))dw$$

$$X(jw) = \int_{-\infty}^{\infty} x(t)e^{-jwt}dt$$

$$= \int_{-\infty}^{\infty} x(t)(cos(wt) - isin(wt))dt$$

$$代入：$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}[(\int_{-\infty}^{\infty} x(t)cos(wt)dt)cos(wt) + (\int_{-\infty}^{\infty} x(t)sin(wt)dt)sin(wt)]dw$$

$$=> x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}[x_{cos(w)}cos(wt) + x_{sin(w)}sin(wt)]dw$$

(2)

$$X(jw) = \int_{-\infty}^{\infty} x(t)e^{-jwt}dt$$

$$= \int_{-\infty}^{\infty} x(t)(cos(wt) - isin(wt))dt$$

$$X(jw) = x_{cos(w)} - ix_{sin(w)}$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(jw)e^{jwt}dw$$

$$x_{cos(w)} = \int_{-\infty}^{\infty} x(t)cos(wt)dt = \frac{1}{2\pi}\int_{-\infty}^{\infty}[(\int_{-\infty}^{\infty} X(jw)e^{jwt}dw)cos(wt)dt]$$

$$x_{sin(w)} = \int_{-\infty}^{\infty} x(t)sin(wt)dt = \frac{1}{2\pi}\int_{-\infty}^{\infty}[(\int_{-\infty}^{\infty} X(jw)e^{jwt}dw)sin(wt)dt]$$

Problem 7

(1)

$$a_k = \frac{1}{T_0}\int_{T_0} x(t)e^{-jkw_0t}dt$$

$$a_k = \frac{1}{4}\cdot[\frac{2}{jk\pi}(e^{jk\frac{\pi}{2}} - e^{-jk\frac{\pi}{2}})]$$

$$= \frac{sin(k\frac{\pi}{2})}{k\pi}$$

(2)

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jkw_0t}$$

$$= \sum_{k=-\infty}^{\infty} \frac{sin(k\frac{\pi}{2})}{k\pi}e^{jkw_0t}$$

$$= lim_{k->0}\frac{sin(k\frac{\pi}{2})}{k\pi}e^{jkw_0t} + \sum_{k=1}^{\infty}\frac{sin(k\frac{\pi}{2})}{k\pi}e^{jkw_0t}$$

$$= \frac{1}{2} + \sum_{k=1}^{\infty}\frac{sin(k\frac{\pi}{2})}{k\pi}e^{jkw_0t}$$

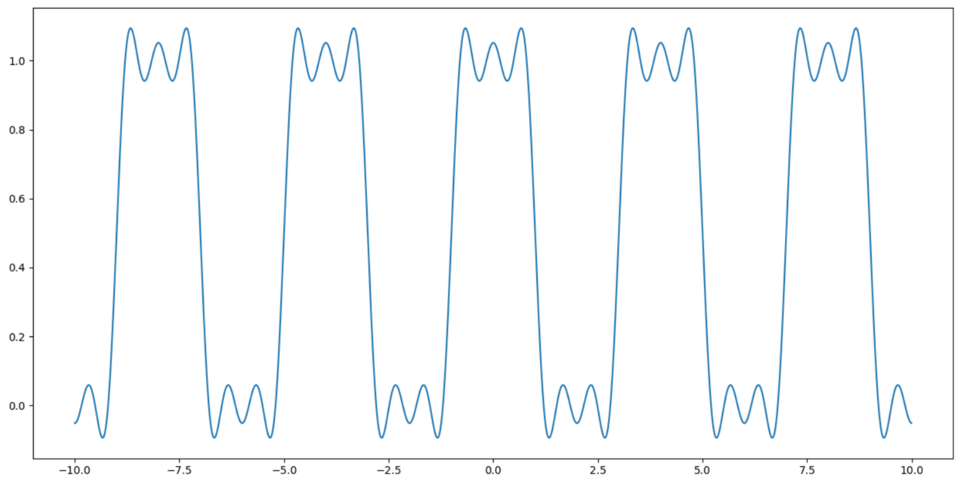
吉布斯现象

使用如下代码进行画图

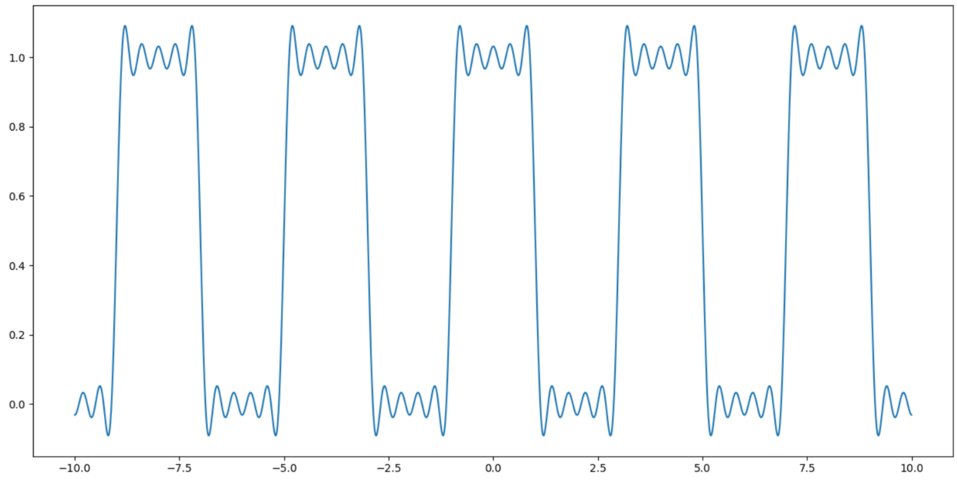
```
from cProfile import label
import math
from turtle import color
import numpy as np
import matplotlib.pyplot as plt


fig = plt.figure()
n = np.array([5,10,100,1000])
x = np.arange(-10,10,0.01)
for i in n:
    y = []
    for j in np.arange(-10,10,0.01):
        res = 0.5
```

```
        for k in np.arange(1,i+1,1):
            tmp = (2*math.sin(k*math.pi/2))/(k*math.pi)*
(math.cos(k*math.pi/2*j))
            res += tmp
        y.append(res)
    plt.plot(x,y)
plt.show()
```

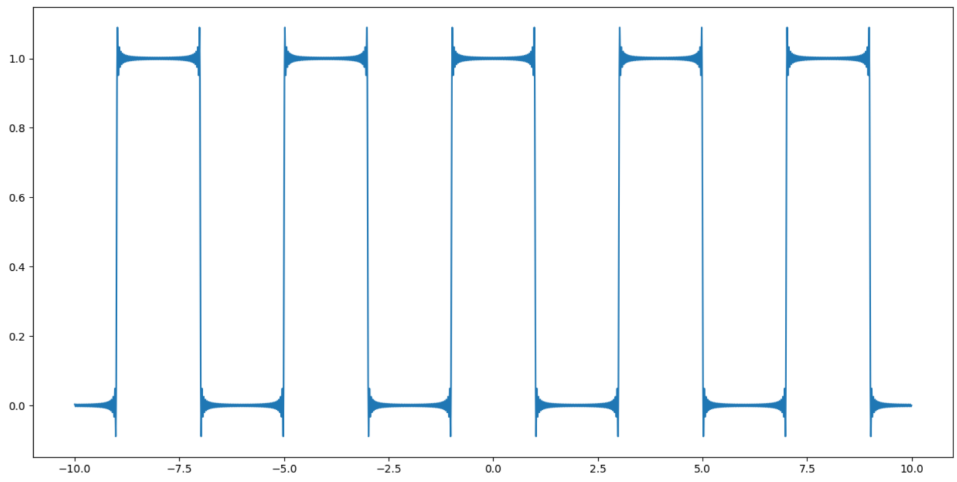
N = 5时，图像如下



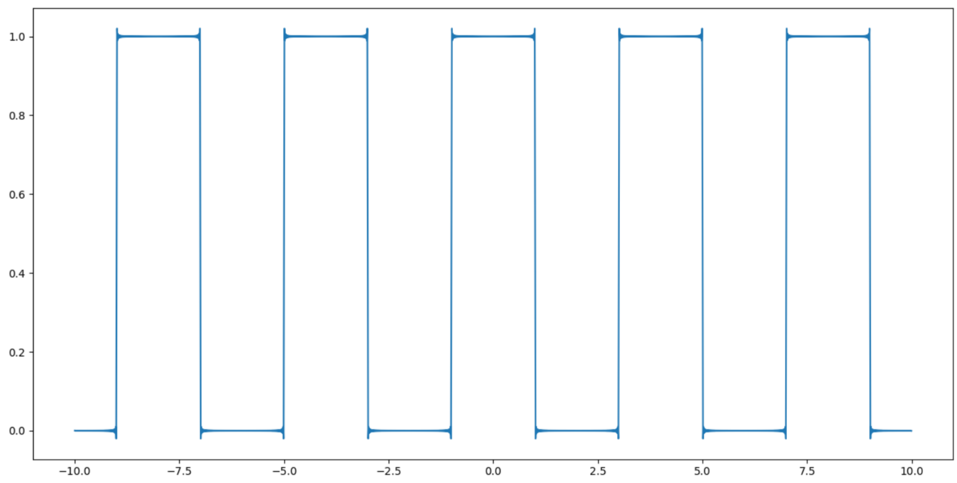
N = 10时



N = 100时


1.0
0.8
0.6
0.4
0.2
0.0
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0

N = 1000时


1.0
0.8
0.6
0.4
0.2
0.0
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0