1.

有
$$F(u,v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y)e^{-j(ux+vy)}dxdy$$

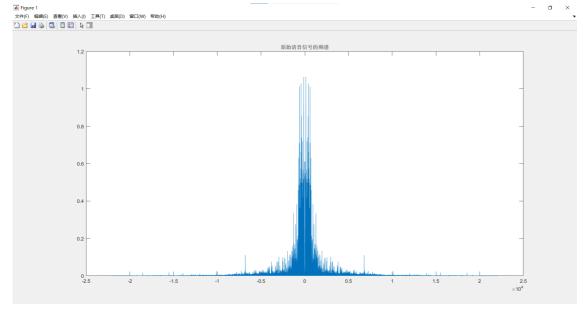
证明 $f(x,y) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(u,v)e^{j(ux+vy)}dudv$
设 $\overline{x} = (x,y), \overline{\epsilon} = (\epsilon_1,\epsilon_2)$
 $\overline{x} \cdot \overline{\epsilon} = x\epsilon_1 + y\epsilon_2$
 $e^{-2\pi j(\overline{x}\cdot\overline{\epsilon})} = e^{-2\pi j(x\epsilon_1+y\epsilon_2)}$
对于一维傅里叶逆变换,有
$$f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(u)e^{jux}du$$
$$f(\overline{x}) = \frac{1}{2\pi} \int_{R^2}^{\infty} e^{j(\overline{x}\cdot\overline{\epsilon})}F(\overline{\epsilon})d\overline{\epsilon}$$
$$= \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(u,v)e^{j(ux+vy)}dudv$$

编程题

1. 使用如下代码打印声音的频谱

```
[x1,fs] = audioread('voice3.mp3');
x1 = x1(:,1);
figure(1);
y1 = fft(x1);
y1 = fftshift(y1);
N = length(x1);
derta_fs = fs/N;
N1 = floor(fs/(2*derta_fs));
w = [-N1*derta_fs:derta_fs:(N-N1-1)*derta_fs];
plot(w,abs(y1)/fs);
title('原始语音信号的频谱');
```

得到的图像如下:



2. 使用如下代码进行四个信号的调制与解调; 首先读取音频文件

```
[x1,fs] = audioread('voice1.wma');

[x2,fs] = audioread('voice2.wma');

[x3,fs] = audioread('voice3.mp3');

[x4,fs] = audioread('voice4.mp3');

%只取左声道

x1 = x1(:,1);

x2 = x2(:,1);

x3 = x3(:,1);

x4 = x4(:,1);
```

接下来由于各个音频的长度不同,需要统一音频的长度

```
len1 = length(x1);
len2 = length(x2);
len3 = length(x3);
len4 = length(x4);
if len1>len2 && len1>len3 && len1>len4
    x2(len2+1:len1) = 0;
    x3(len3+1:len1) = 0;
    x4(len4+1:len1) = 0;
elseif len2>len1 && len2>len3 && len2>len4
    x1(len1+1:len2) = 0;
    x3(len3+1:len2) = 0;
    x4(len4+1:len2) = 0;
elseif len3>len1 && len3>len2 && len3>len4
    x1(len1+1:len3) = 0;
    x2(len2+1:len3) = 0;
    x4(len4+1:len3) = 0;
elseif len4>len1 && len4>len2 && len4>len3
    x1(len1+1:len4) = 0;
    x2(len2+1:len4) = 0;
    x3(len3+1:len4) = 0;
end
```

接下来进行滤波操作;设置截止频率为4000Hz

```
%低通滤波(FIR滤波器),在《数字信号处理》中要专门讲滤波器设计。
fp = 4000;
N1 = 2*pi*0.9/(0.1*pi);
wc1 = 2*pi*fp/fs;
if rem(N1,2)
   N1 = N1+1;
end
Window = blackman(N1+1);
b1 = fir1(N1,wc1/pi,Window);%低通滤波器,b1只有19个数,精度不算高。
figure(1);
freqz(b1,1,512);
title('低通滤波器的频率响应');
x1_low = filter(b1,1,x1);%将x1低通滤波
x2_low = filter(b1,1,x2);%将x2低通滤波
x3_low = filter(b1,1,x3);%将x3低通滤波
x4_low = filter(b1,1,x4);%将x4低通滤波
audiowrite('./res/voice1AfterLowpassFilter.wav', x1_low, fs);
audiowrite('./res/voice2AfterLowpassFilter.wav', x2_low, fs);%把低通滤波结果保
audiowrite('./res/voice3AfterLowpassFilter.wav', x3_low, fs);
audiowrite('./res/voice4AfterLowpassFilter.wav', x4_low, fs);
```

接下来进行调制操作,4个声音的载波频率分别是fc1=9000Hz,fc2=18000Hz,fc3=27000Hz,fc4=36000Hz

```
x5 = zeros(1,len1);

fc1 = 9000;

fc2 = 18000;

fc3 = 27000;

fc4 = 36000;

for i =1:length(x3)

    x5(i) = x1_low(i)*cos(2*pi*fc1*(i-1)/fs)+x2_low(i)*cos(2*pi*fc2*(i-1)/fs)+x3_low(i)*cos(2*pi*fc3*(i-1)/fs)+x4_low(i)*cos(2*pi*fc4*(i-1)/fs);%两个加起来

end

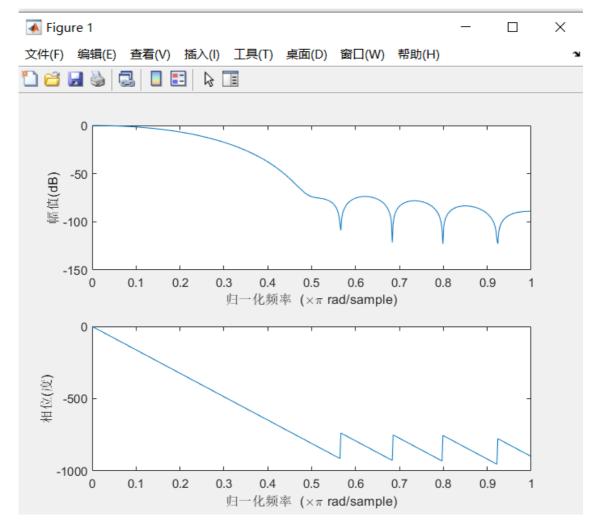
audiowrite('./res/voice1and2andVoice3andVoice4AfterModulation.wav', x5, fs);
```

解调操作

```
x1_afterModulation = zeros(1,len1);
x2_afterModulation = zeros(1,len2);
x3_afterModulation = zeros(1,len3);
x4_afterModulation = zeros(1,len4);
for i =1:length(x3)
   x1_afterModulation(i) = x3(i)*cos(2*pi*fc1*(i-1)/fs);
   x2_afterModulation(i) = x3(i)*cos(2*pi*fc2*(i-1)/fs);%两个信号各自乘以相应的
载波
   x3_afterModulation(i) = x3(i)*cos(2*pi*fc3*(i-1)/fs);
   x4\_afterModulation(i) = x3(i)*cos(2*pi*fc4*(i-1)/fs);
end
x1_afterModulation = filter(b1,1,x1_afterModulation);
x2_afterModulation = filter(b1,1,x2_afterModulation);%低通滤波。
x3_afterModulation = filter(b1,1,x3_afterModulation);
x4_afterModulation = filter(b1,1,x4_afterModulation);
audiowrite('./res/voice1AfterDemodulation.wav', x1_afterModulation, fs);
```

```
audiowrite('./res/voice2AfterDemodulation.wav', x2_afterModulation, fs);
audiowrite('./res/voice3AfterDemodulation.wav', x3_afterModulation, fs);
audiowrite('./res/voice4AfterDemodulation.wav', x4_afterModulation, fs);
```

运行代码得到的结果如下



当调低载波频率时,声音信号间的相互干扰增大

附录

第一题的完整代码如下

```
[x1,fs] = audioread('voice3.mp3');
x1 = x1(:,1);
figure(1);
y1 = fft(x1);
y1 = fftshift(y1);
N = length(x1);
derta_fs = fs/N;

N1 = floor(fs/(2*derta_fs));
w = [-N1*derta_fs:derta_fs:(N-N1-1)*derta_fs];
plot(w,abs(y1)/fs);
title('原始语音信号的频谱');
```

第二题的完整代码如下

```
[x1,fs] = audioread('voice1.wma');
```

```
[x2,fs] = audioread('voice2.wma');
[x3,fs] = audioread('voice3.mp3');
[x4,fs] = audioread('voice4.mp3');
%只取左声道
x1 = x1(:,1);
x2 = x2(:,1);
x3 = x3(:,1);
x4 = x4(:,1);
%统一两个信号的长度。
len1 = length(x1);
len2 = length(x2);
len3 = length(x3);
len4 = length(x4);
if len1>len2 && len1>len3 && len1>len4
   x2(len2+1:len1) = 0;
    x3(len3+1:len1) = 0;
    x4(len4+1:len1) = 0;
elseif len2>len1 && len2>len3 && len2>len4
    x1(len1+1:len2) = 0;
    x3(len3+1:len2) = 0;
    x4(len4+1:len2) = 0;
elseif len3>len1 && len3>len2 && len3>len4
    x1(len1+1:len3) = 0;
    x2(len2+1:len3) = 0;
    x4(len4+1:len3) = 0;
elseif len4>len1 && len4>len2 && len4>len3
   x1(len1+1:len4) = 0:
    x2(len2+1:len4) = 0;
    x3(len3+1:len4) = 0;
end
derta_fs = fs/length(x1);
%低通滤波(FIR滤波器),在《数字信号处理》中要专门讲滤波器设计。
fp = 4000;
N1 = 2*pi*0.9/(0.1*pi);
wc1 = 2*pi*fp/fs;
if rem(N1,2)
   N1 = N1+1;
end
Window = blackman(N1+1);
b1 = fir1(N1,wc1/pi,Window);%低通滤波器,b1只有19个数,精度不算高。
figure(1);
freqz(b1,1,512);
title('低通滤波器的频率响应');
x1_low = filter(b1,1,x1);%将x1低通滤波
x2_low = filter(b1,1,x2);%将x2低通滤波
x3_low = filter(b1,1,x3);%将x3低通滤波
x4_low = filter(b1,1,x4);%将x4低通滤波
audiowrite('./res/voice1AfterLowpassFilter.wav', x1_low, fs);
audiowrite('./res/voice2AfterLowpassFilter.wav', x2_low, fs);%把低通滤波结果保存
audiowrite('./res/voice3AfterLowpassFilter.wav', x3_low, fs);
audiowrite('./res/voice4AfterLowpassFilter.wav', x4_low, fs);
%调制
```

```
x5 = zeros(1,len1);
fc1 = 9000;
fc2 = 18000;
fc3 = 27000;
fc4 = 36000;
for i =1:length(x3)
          x5(i) = x1_low(i)*cos(2*pi*fc1*(i-1)/fs)+x2_low(i)*cos(2*pi*fc2*(i-1)/fs)+x2_low(i)*cos(2*pi*fc2*(i-1)/fs)+x2_low(i)*cos(2*pi*fc2*(i-1)/fs)+x2_low(i)*cos(2*pi*fc2*(i-1)/fs)+x2_low(i)*cos(2*pi*fc2*(i-1)/fs)+x2_low(i)*cos(2*pi*fc2*(i-1)/fs)+x2_low(i)*cos(2*pi*fc2*(i-1)/fs)+x2_low(i)*cos(2*pi*fc2*(i-1)/fs)+x2_low(i)*cos(2*pi*fc2*(i-1)/fs)+x2_low(i)*cos(2*pi*fc2*(i-1)/fs)+x2_low(i)*cos(2*pi*fc2*(i-1)/fs)+x2_low(i)*cos(2*pi*fc2*(i-1)/fs)+x2_low(i)*cos(2*pi*fc2*(i-1)/fs)+x2_low(i)*cos(2*pi*fc2*(i-1)/fs)+x2_low(i)*cos(2*pi*fc2*(i-1)/fs)+x2_low(i)*cos(2*pi*fc2*(i-1)/fs)+x2_low(i)*cos(2*pi*fc2*(i-1)/fs)+x2_low(i)*cos(2*pi*fc2*(i-1)/fs)+x2_low(i)*cos(2*pi*fc2*(i-1)/fs)+x2_low(i)*cos(2*pi*fc2*(i-1)/fs)+x2_low(i)*cos(2*pi*fc2*(i-1)/fs)+x2_low(i)*cos(2*pi*fc2*(i-1)/fs)+x2_low(i)*cos(2*pi*fc2*(i-1)/fs)+x2_low(i)*cos(2*pi*fc2*(i-1)/fs)+x2_low(i)*cos(2*pi*fc2*(i-1)/fs)+x2_low(i)*cos(2*pi*fc2*(i-1)/fs)+x2_low(i)*cos(2*pi*fc2*(i-1)/fs)+x2_low(i)*cos(2*pi*fc2*(i-1)/fs)+x2_low(i)*cos(2*pi*fc2*(i-1)/fs)+x2_low(i)*cos(2*pi*fc2*(i-1)/fs)+x2_low(i)*cos(2*pi*fc2*(i-1)/fs)+x2_low(i)*cos(2*pi*fc2*(i-1)/fs)+x2_low(i)*cos(2*pi*fc2*(i-1)/fs)+x2_low(i)*cos(2*pi*fc2*(i-1)/fs)+x2_low(i)*cos(2*pi*fc2*(i-1)/fs)+x2_low(i)*cos(2*pi*fc2*(i-1)/fs)+x2_low(i)*cos(2*pi*fc2*(i-1)/fs)+x2_low(i)*cos(2*pi*fc2*(i-1)/fs)+x2_low(i)*cos(2*pi*fc2*(i-1)/fs)+x2_low(i)*cos(2*pi*fc2*(i-1)/fs)+x2_low(i)*cos(2*pi*fc2*(i-1)/fs)+x2_low(i)*cos(2*pi*fc2*(i-1)/fs)+x2_low(i)*cos(2*pi*fc2*(i-1)/fs)+x2_low(i)*cos(2*pi*fc2*(i-1)/fs)+x2_low(i)*cos(2*pi*fc2*(i-1)/fs)+x2_low(i)*cos(2*pi*fc2*(i-1)/fs)+x2_low(i)*cos(2*pi*fc2*(i-1)/fs)+x2_low(i)*cos(2*pi*fc2*(i-1)/fs)+x2_low(i)*cos(2*pi*fc2*(i-1)/fs)+x2_low(i)*cos(2*pi*fc2*(i-1)/fs)+x2_low(i)*cos(2*pi*fc2*(i-1)/fs)+x2_low(i)*cos(2*pi*fc2*(i-1)/fs)+x2_low(i)*cos(2*pi*fc2*(i-1)/fs)+x2_low(i)*cos(2*pi*fc2*(i-1)/fs)+x2_low(i)*cos(2*pi*fc2*(i-1)/fs)+x2_low(i)*cos(2*pi*fc2*(i-1)/fs)+x2_low(i)*cos(2*pi*fc2*(i-1)/fs)+x2_low(i)*cos(2*pi*fc2*(i-1)/fs)+x2_low(i)*cos(2*pi*fc2*(i-1)/fs)+x2_low(i)*cos(2*pi*fc2*(i-1)/fs)+x2_low(i)*cos(2*pi*fc2*
1)/fs)+x3_low(i)*cos(2*pi*fc3*(i-1)/fs)+x4_low(i)*cos(2*pi*fc4*(i-1)/fs);%两个加起
来
audiowrite('./res/voice1and2andVoice3andVoice4AfterModulation.wav', x5, fs);
%解调
x1_afterModulation = zeros(1,len1);
x2_afterModulation = zeros(1,len1);
x3_afterModulation = zeros(1,len1);
x4_afterModulation = zeros(1,len1);
for i =1:length(x3)
          x1_afterModulation(i) = x3(i)*cos(2*pi*fc1*(i-1)/fs);
          x2_afterModulation(i) = x3(i)*cos(2*pi*fc2*(i-1)/fs);%两个信号各自乘以相应的载波
          x3_afterModulation(i) = x3(i)*cos(2*pi*fc3*(i-1)/fs);
          x4\_afterModulation(i) = x3(i)*cos(2*pi*fc4*(i-1)/fs);
end
x1_afterModulation = filter(b1,1,x1_afterModulation);
x2_afterModulation = filter(b1,1,x2_afterModulation);%低通滤波。
x3_afterModulation = filter(b1,1,x3_afterModulation);
x4_afterModulation = filter(b1,1,x4_afterModulation);
audiowrite('./res/voice1AfterDemodulation.wav', x1_afterModulation, fs);
audiowrite('./res/voice2AfterDemodulation.wav', x2_afterModulation, fs);
audiowrite('./res/voice3AfterDemodulation.wav', x3_afterModulation, fs);
audiowrite('./res/voice4AfterDemodulation.wav', x4_afterModulation, fs);
```