Problem 1

牛顿方法如下：

$$p_n = p_{n-1} - \frac{f(p_{n-1})}{f'(p_{n-1})}$$

当p_0 = -1时，p_2 = -0.999949236,不能取p_0=0,因为f'(0)=0

Problem2

（Ⅰ）

$$X_k = 2X_{k-1} - bX_{k-1}^2$$
$$X_{k-1} = \frac{\xi_{k-1}+1}{b}$$
$$=> X_k = \frac{1-\xi_{k-1}^2}{b}$$
$$\xi_k = \frac{\frac{1}{b} - x_k}{\frac{1}{b}}$$
$$\xi_k = \xi_{k-1}^2$$
$$\text{也就是}|\xi_{k+1}| = \xi_k^2$$

（Ⅱ）

$$X_k = 2X_{k-1} - bX_{k-1}^2$$
$$x_0 \in (0, \frac{2}{b})$$
$$=> x_1 \in (0, \frac{1}{b})$$
$$\text{又有}\frac{1}{b} = 2\frac{1}{b} - b(\frac{1}{b})^2$$
$$\frac{1}{b}\text{是}f(x) = 2x - bx^2\text{的不动点}$$
$$g'(x) = (2x - bx^2 - x)' = 1 - 2bx$$
$$\text{当}x \in (0, \frac{1}{b})\text{时，}|g'(x)| < 1, \text{由}fixed - point\ theorem\text{可知}$$
$$f(x)\text{收敛于}\frac{1}{b}$$

problem3

a.

$$F(x_1,x_2,x_3)=(f_1(x_1,x_2,x_3),f_2(x_1,x_2,x_3),f_3(x_1,x_2,x_3))$$
$$f_1(x_1,x_2,x_3)=3x_1-cos(x_2x_3)-\frac{1}{2}$$
$$f_2(x_1,x_2,x_3)=4x_1^2-625x_2^2+2x_2-1$$
$$f_3(x_1,x_2,x_3)=e^{-x_1x_2}+20x_3+\frac{10\pi-3}{3}$$
$$Jacobian\ \ matrix:$$
$$J(x_1,x_2,x_3)=[\begin{matrix}3 & x_3sin(x_2x_3) & x_2sin(x_2x_3)\\ 8x_1 & -1250x_2+2 & 0\\ -x_2e^{-x_1x_2} & -x_1e^{-x_1x_2} & 20\end{matrix}]$$
$$x^{(0)}=0\quad F(x^{(0)})=(-\frac{3}{2},-1,\frac{10\pi}{3})^t$$
$$J(x^{(0)})=[\begin{matrix}3 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 20\end{matrix}]$$
$$J^{x(0)}y^{(0)}=-F(x^{(0)})$$
$$y^{(0)}=(\frac{1}{2},\frac{1}{2},\frac{-1\pi}{6})^t$$
$$x^{(1)}=x^{(0)}+y^{(0)}=(\frac{1}{2},\frac{1}{2},\frac{-1\pi}{6})^t$$
类似迭代
$$解得y^{(1)}=(0.00546566,0.24923252,-0.00610098)$$
$$x^{(2)}=y^{(1)}+x^{(1)}$$
$$x^{(2)}=(0.50546566,0.74923252,-0.52969976)$$

b.

$$F(x_1,x_2,x_3)=(f_1(x_1,x_2,x_3),f_2(x_1,x_2,x_3),f_3(x_1,x_2,x_3))$$
$$f_1(x_1,x_2,x_3)=x_1^2+x_2-37$$
$$f_2(x_1,x_2,x_3)=x_1-x_2^2-5$$
$$f_3(x_1,x_2,x_3)=x_1+x_2+x_3-3$$
$$\begin{matrix} & 2x_1 & 1 & 0 \\ J(x_1,x_2,x_3)=[& 1 & 2x_2 & 0] \\ & 1 & 1 & 1 \end{matrix}$$
$$x^{(0)}=0 \quad F(x^{(0)})=(-37,-5,-3)$$
$$\begin{matrix} & 0 & 1 & 0 \\ J(x^{(0)})=[& 1 & 0 & 0] \\ & 1 & 1 & 1 \end{matrix}$$
$$J^{x(0)}y^{(0)}=-F(x^{(0)})$$
$$y^{(0)}=(5,37,-39)^t$$
$$x^{(1)}=x^{(0)}+y^{(0)}$$
$$x^{(1)}=(5,37,-39)^t$$
同理，解得$y^{(1)}=(1.126835,-18.26835781,17.14152203)$
$$x^{(2)}=y^{(1)}+x^{(1)}$$
$$x^{(2)}=(6.126835,18.73164219,14.14152203)$$

Problem 4

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a.

$$f_1(x_1,x_2,x_3) = 15x_1 + x_2^2 - 4x_3 - 13$$
$$f_2(x_1,x_2,x_3) = x_1^2 + 10x_2 - x_3 - 11$$
$$f_3(x_1,x_2,x_3) = x_2^3 - 25x_3 + 22$$
$$x^{(0)} = 0$$
$$F(x^{(0)}) = (-13,-11,22)^t$$
$$J(x^{(0)}) = \begin{bmatrix} 15 & 0 & -4 \\ 0 & 10 & -1 \\ 0 & 0 & -25 \end{bmatrix}$$
$$g(x) = f_1(x_1,x_2,x_3)^2 + f_2(x_1,x_2,x_3)^2 + f_3(x_1,x_2,x_3)^2$$
$$g(x^{(0)}) = 774$$
$$\nabla g(x) = 2J(x)^t F(x)$$
$$\nabla g(^{(0)}) = \begin{bmatrix} -566 \\ -264 \\ -1100 \end{bmatrix}$$
$$z_0 = ||\nabla g(x^{(0)})||_2 = 1264.931619$$
$$z = \frac{1}{z_0}\nabla g(x^0) = (-0.4474550177, -0.2087069341, -0.8696122253)$$
$$取\alpha_1 = 0$$
$$g_1 = g(x^{(0)} - \alpha_1 z) = g(x^{(0)}) = 774$$
$$取\alpha_3 = 1$$
$$g_3 = g(x^{(0)} - \alpha_3 z) = 186.431202$$
$$取\alpha_2 = \alpha_3/2$$
$$g_2 = g(x^{(0)} - \alpha_2 z) = 360.171324$$
$$Set\ P(\alpha) = g_1 + h_1\alpha + h_3\alpha(\alpha - \alpha_2)$$
$$h_1 = (g_2 - g_1)/\alpha_2$$
$$h_2 = (g_3 - g_2)/(\alpha_3 - \alpha_2)$$
$$h_3 = (h_2 - h_1)/\alpha_3$$
$$由于要找到\alpha使得P最小，求导$$
$$P'(\alpha) = 0$$
$$=> \alpha = 1.077364$$
$$x^{(1)} = x^0 - \alpha z$$
$$由于g(x^{(1)}) - g(x^{(0)}) < TOL$$
$$答案是x^{(1)} = (0.48207, 0.22485, 0.93689)$$

b.

$$f_1(x_1,x_2,x_3)=10x_1-2x_2^2+x_2-2x_3-5$$
$$f_2(x_1,x_2,x_3)=8x_2^2+x_3^2-9$$
$$f_3(x_1,x_2,x_3)=8x_2x_3+4$$
$$x^{(0)}=0$$
$$F(x^{(0)})=(-5,-9,4)^t$$
$$J(x^{(0)})=\begin{bmatrix}10 & 1 & -2\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$
$$g(x)=f_1(x_1,x_2,x_3)^2+f_2(x_1,x_2,x_3)^2+f_3(x_1,x_2,x_3)^2$$
$$g(x^{(0)})=122$$
$$\nabla g(x)=2J(x)^tF(x)$$
$$\nabla g(x^{(0)})=\begin{bmatrix}-100\\ -18\\ -16\end{bmatrix}$$
$$z_0=||\nabla g(x^{(0)})||_2=102.859127$$
$$z=\frac{1}{z_0}\nabla g(x^0)=(-0.9722034681,-0.1749966243,-0.1555525549)^t$$
$$\text{取}\alpha_1=0$$
$$g_1=g(x^{(0)}-\alpha_1 z)=g(x^{(0)})=122$$
$$\text{取}\alpha_3=1$$
$$g_3=g(x^{(0)}-\alpha_3 z)=114.48938766804345$$
$$\text{取}\alpha_2=\alpha_3/2$$
$$g_2=g(x^{(0)}-\alpha_2 z)=96.2811284177945$$
$$Set\ P(\alpha)=g_1+h_1\alpha+h_3\alpha(\alpha-\alpha_2)$$
$$h_1=(g_2-g_1)/\alpha_2$$
$$h_2=(g_3-g_2)/(\alpha_3-\alpha_2)$$
$$h_3=(h_2-h_1)/\alpha_3$$
$$\text{由于要找到}\alpha\text{使得}P\text{最小，求导}$$
$$P'(\alpha)=0$$
$$=>\alpha=0.3955950485$$
$$x^{(1)}=x^0-\alpha z$$
$$\text{由于}g(x^{(1)})-g(x^{(0)})<TOL$$
$$\text{答案是}x^{(1)}=(0.3845988781148877,0.06922779807729477,0.06153582049996441)$$