HW1

Problem 1

(a):

$$E_1 = zcos(wt - kz)$$
$$\nabla^2 E_1 - u_0\varepsilon_0 \frac{d^2}{dt^2} E_1 = \frac{d^2}{dz^2} E_1 - u_0\varepsilon_0 \frac{d^2}{dt^2} E_1 = 0$$
$$\text{易知，} E_1 \text{是上述方程的一个解}'$$
$$=> k^2 = w^2 u_0 \epsilon_0$$
$$E_2 = (x+z)cos(wt + \frac{k|x-z|}{2^{0.5}})$$
$$\nabla^2 E_2 - u_0\varepsilon_0 \frac{d^2}{dt^2} E_2 = \frac{d^2}{dz^2} E_2 - u_0\varepsilon_0 \frac{d^2}{dt^2} E_2 \neq 0$$
$$E_3 = (x+z)cos(wt + ky)$$
$$\nabla^2 E_3 - u_0\varepsilon_0 \frac{d^2}{dt^2} E_3 = u_0\epsilon_0 w^2 sin(wt + ky) = 0$$
$$\text{综上，} E_1, E_3 \text{满足} \nabla^2 E - u_0\varepsilon_0 \frac{d^2}{dt^2} E = 0\text{，} E_2 \text{不满足}$$
$$k^2 = w^2 u_0 \epsilon_0$$

(b):

$$D = \epsilon_0 E$$
$$\nabla \times H = \frac{d}{dt} D + J$$
$$\text{由于} J = 0, \text{得到：}$$
$$\nabla \times H = \epsilon \frac{d}{dt} E$$
$$B = \frac{E}{c} = uH$$
$$H = \frac{E}{c \cdot u_0}$$
$$H_1 = z\frac{cos(wt - kz)}{c \cdot u_0}$$
$$H_2 = (x+z)\frac{cos(wt + \frac{k|x-z|}{2^{0.5}})}{c \cdot u_0}$$
$$H_3 = (x+z)\frac{cos(wt + ky)}{c \cdot u_0}$$

(c):

E_1, E_3属于电磁波，E_2违反了$\nabla \cdot B = 0$

对于E_1, E_2,有:

$$\nabla \times E = -\frac{d}{dt}B$$

$$\nabla \cdot B = 0$$

由叉乘的定义可知，B, E的传播方向相互垂直

Problem 2

(a):

$\lambda = \frac{c}{f}$

- f=60hz, $\lambda = 5000000m$
- f=535-1605khz, $\lambda = 186 - 560m$
- f=88-108Mhz, $\lambda = 2.7 - 3.4m$
- f=4-6Ghz, $\lambda = 0.05 - 0.075m$
- f=~10^{14}hz, $\lambda = \ 3 * 10^{-6}m$
- f=~10^{18}hz, $\lambda = \ 3 * 10^{-10}m$

(b):

$f = \frac{c}{\lambda}$

- $\lambda = 1000m$, f = $3 * 10^5$
- $\lambda = 1m$, f = $3 * 10^8$
- $\lambda = 0.01m$, f = $3 * 10^{10}$
- $\lambda = 0.0000001$, f = $3 * 10^{14}$
- $\lambda = 10^{-10}m$, f = $3 * 10^{18}$

(c):

$k = \frac{1}{\lambda}K_0$

- $\lambda = 1000m, k = 10^{-3}K_0$
- $\lambda = 1m, k = 1K_0$
- $\lambda = 0.01m, k = 10^2K_0$
- $\lambda = 10^{-6}m, k = 10^6K_0$
- $\lambda = 10^{-10}m, k = 10^{10}K_0$

Problem 3

(1):

$$\nabla \times (\nabla \times E) = \nabla \times [\begin{matrix} x & y & z \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ E_x & E_y & E_z \end{matrix}]$$

$$= [\begin{matrix} x & y & z \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ \frac{d}{dy}E_z - \frac{d}{dz}E_y & \frac{d}{dz}E_x - \frac{d}{dx}E_z & \frac{d}{dy}E_x - \frac{d}{dx}E_y \end{matrix}]$$

$$= \sum \frac{d}{dy}(\frac{d}{dy}E_x - \frac{d}{dx}E_y) - \frac{d}{dz}(\frac{d}{dz}E_x - \frac{d}{dx}E_z)x$$

$$= \nabla(\nabla \cdot E) - \nabla^2 E$$

(2):

$$\nabla\cdot(E\times H)=\nabla\cdot\begin{matrix} x & y & z \\ [E_x & E_y & E_z] \\ H_x & H_y & H_z \end{matrix}$$

$$=\sum\frac{d}{dx}(E_yH_z-E_zH_y)$$

$$\sum H_x(\frac{d}{dy}E_z-\frac{d}{dz}E_y)-\sum E_x(\frac{d}{dy}H_z-\frac{d}{dz}H_y)$$

$$=H\cdot(\nabla\times E)-E\cdot(\nabla\times H)$$

(3):

$$\nabla\cdot(\nabla\times A)=\nabla\cdot\begin{matrix} x & y & z \\ [\frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz}] \\ A_x & A_y & A_z \end{matrix}=0$$

(4):

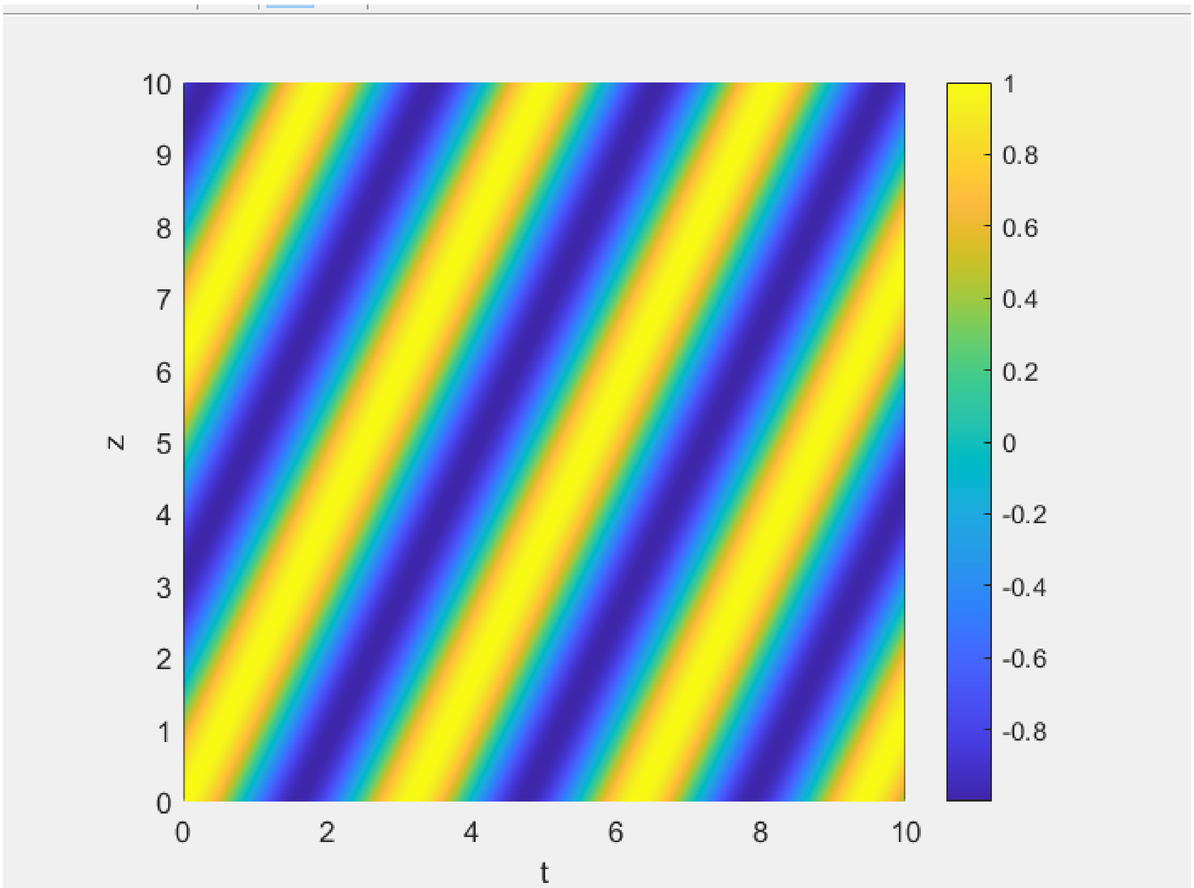
$$\nabla\times(\nabla\phi)=\begin{matrix} x & y & z \\ [\frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz}] \\ \frac{d\phi}{dx} & \frac{d\phi}{dy} & \frac{d\phi}{dz} \end{matrix}$$

$$=0$$

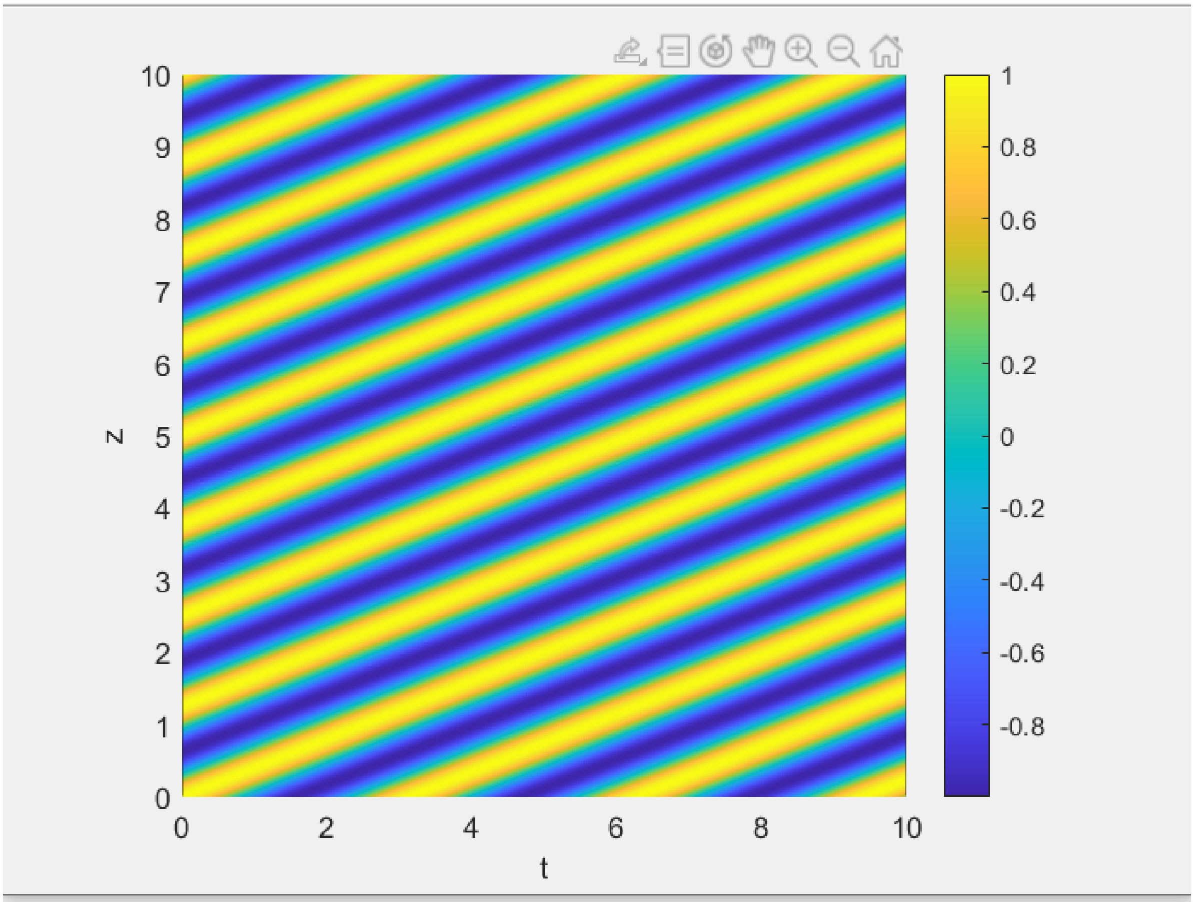
Problem 4

不能用对z的导数来表示波速，波速应该是有关时间t的导数

k=1时，运行代码得到如下图像：



当k=5时，得到如下图像：



当k增加时，线的斜率降低，波长减小

Problem 5

$$\nabla(A\cdot A)=2(A\cdot\nabla)A+2A\times(\nabla\times A)$$

证明：

$$2(A\cdot\nabla)A+2A\times(\nabla\times A)=$$

$$(2\sum\frac{d}{dx}A_x)(A_x+A_y+A_z)+2A\times\left[\begin{matrix}x&y&z\\ \frac{d}{dx}&\frac{d}{dy}&\frac{d}{dz}\\ A_x&A_y&A_z\end{matrix}\right]$$

$$=(2\sum\frac{d}{dx}A_x)(A_x+A_y+A_z)+2\left[\begin{matrix}x&y&z\\ A_x&A_y&A_z\\ \frac{d}{dy}Az-\frac{d}{dz}A_y&\frac{d}{dz}A_x-\frac{d}{dx}A_z&\frac{d}{dx}A_y-\frac{d}{dy}A_x\end{matrix}\right]$$

$$=(2\sum\frac{d}{dx}Ax)(A_x+A_y+A_z)+2\sum[(\frac{d}{dx}A_y-\frac{d}{dy}A_x)A_y-(\frac{d}{dz}A_x-\frac{d}{dx}A_z)A_z]$$

$$=\nabla(A\cdot A)$$