HW 10

4-1

(1):

$$T = 6$$
$$a_k = \frac{1}{N}\sum_{n=<N>} x[n]e^{-jk(\frac{2\pi}{N})n}$$
$$a_k = \frac{1}{6}(\sum_{k=0}^{4} e^{-jk(\frac{2\pi}{6})n})$$
$$a_k = \frac{1}{6}(cos\frac{\pi k}{3} + cos\frac{2\pi k}{3} - j(sin\frac{\pi k}{3} + sin\frac{2\pi k}{3}))$$
$$|a_k| = \frac{1}{3}|cos\frac{k}{2}\pi + cos\frac{k}{6}\pi|$$
$$\theta = -\frac{\pi}{2}$$

(3):$x[n] = \sum_{m=-\infty}^{\infty}\{2\delta[n-4m] + 4\delta[n-1-4m]\}$

$$T = 4$$
$$a_k = \frac{1}{N}\sum_{n=<N>} x[n]e^{-jk(\frac{2\pi}{N})n}$$
$$= \frac{1}{4}(2 + 4e^{-jk(\frac{2\pi}{N})})$$
$$a_k = \frac{1}{4}(2 + 4e^{-jk(\frac{\pi}{2})}) = \frac{1}{4}(2 + 4cosk\frac{\pi}{2} - 4jsink\frac{\pi}{2})$$
$$|a_k| = \sqrt{(\frac{1}{2} + cosk\frac{\pi}{2})^2 + (sink\frac{\pi}{2})^2}$$
$$相位: \theta = arctan(\frac{-sink\frac{\pi}{2}}{\frac{1}{2} + cosk\frac{\pi}{2}})$$

(5)$x[n] = 1 - sin(\pi n/4), n \in [0,3], x[n]$以4为周期

$$T = 4, a_k = \frac{1}{N}\sum_{n=<N>} x[n]e^{-jk(\frac{2\pi}{N})n}$$
$$a_k = \frac{1}{4}(1 + (1 - \frac{1}{\sqrt{2}})e^{-j\frac{\pi}{2}} + (1 + \frac{1}{\sqrt{2}})e^{-j\frac{3\pi}{2}})$$
$$= \frac{1}{4}(1 + (2 - \sqrt{2})cos(\frac{k\pi}{2}))$$
$$|a_k| = |\frac{1}{4}(1 + (2 - \sqrt{2})cos(\frac{k\pi}{2}))|$$
$$\theta = 0$$

4-2

(1)

$$a_k = cos(\frac{k}{4}\pi) + sin(\frac{3k\pi}{4}), T = 8$$
$$a_k = \frac{1}{N}\sum_{n=<N>} x[n]e^{-jk(\frac{2\pi}{N})n}$$
$$a_k = \frac{1}{8}\sum x[n](cos(\frac{\pi}{4}kn) - jsin(\frac{\pi}{4}kn))$$
$$x[-3] = 4j, x[-1] = 4, x[1] = 4, x[3] = -4j, \text{其余} x[n] = 0$$

(3)

$$\text{当} n \neq -2, 2 \text{时}, a_k = 1; n = -2 \text{或} 2 \text{时}, a_k = 0$$
$$x[n] = \sum_{k=-3}^{4} a_k e^{jn(\frac{2\pi}{N})k}$$
$$= 2 + 2cos\frac{n\pi}{2} + 2cos\frac{3}{2}n\pi$$

(5)

$$a_k = -a_{k-4}, x[2n+1] = (-1)^n$$
$$x[2n] = \sum_{k=<8>} a_k e^{jk\frac{\pi}{4}2n} = 0$$
$$x[2n] = 0, x[2n+1] = (-1)^n$$

4-3

(1)

$$x[n] = -x[n + \frac{N}{2}], \text{证明} a_{2k} = 0$$
$$a_{2k} = \frac{1}{N}\sum_{n=<N>} x[n]e^{-j2k(\frac{2\pi}{N})n}$$
$$\text{由于} x[n] = -x[n + \frac{N}{2}]$$
$$e^{-j2k(\frac{2\pi}{N})n} = e^{-j2k(\frac{2\pi}{N})(n+\frac{N}{2})}$$
$$a_{2k} = 0$$

(2)

$$N\%4 = 0, x[n] = -x[n + \frac{N}{4}]$$
$$a_{4k} = \frac{1}{N}\sum_{n=<N>} x[n]e^{-j4k(\frac{2\pi}{N})n}$$
$$\text{由于} x[n] = -x[n + \frac{N}{4}]$$
$$e^{-j4k(\frac{2\pi}{N})n} = e^{-j4k(\frac{2\pi}{N})(n+\frac{N}{4})}$$
$$a_{4k} = 0$$

4-4

(1)

$$a_k = b_k + jc_k$$
$$\text{证明} a_{-k} = a_k^*$$
$$x[n] < -^{FS} - > a_k$$
$$x^*[n] < -^{FS} - > a_{-k}^*$$
$$a_{-k}^* = a_k$$
$$a_{-k} = a_k^*$$
$$b_{-k} + jc_{-k} = b_k - jc_k$$
$$b_k = b_{-k}, c_{-k} = -c_k$$

(2)

$$N\text{为偶数}, \text{证明} c_{N/2} = 0, a_{N/2}\text{是实数}$$
$$a_{\frac{N}{2}} = \frac{1}{N} \sum_{n=<N>} x[n] e^{-j\frac{N}{2}(\frac{2\pi}{N})n}$$
$$= \frac{1}{N} \sum x[n] e^{-j\pi n}$$
$$= \frac{1}{N} \sum (-1)^n x[n]$$
$$\text{由于} x[n]\text{是实序列，} a_{N/2}\text{是实数，} c_{N/2} = 0$$