HW 12

6-1

(1)

$$L\{e^{at}u(t)\} <--> \frac{1}{s-a}$$
$$\text{收敛域}: Re\{s\} > a$$
$$\text{零极点图}: X(s) = \frac{1}{s-a}$$

(3)

$$e^{-t}u(t) + e^{-2t}u(t) <--> \frac{1}{s+1} + \frac{1}{s+2}$$
$$\text{收敛域}: Re\{s\} > -1$$
$$\text{零极点图} X(s) = \frac{2s+3}{(s+1)(s+2)}$$

(5)

$$e^{3t}u(-t) + e^{5t}u(-t) <--> -(\frac{1}{s-3} + \frac{1}{s-5})$$
$$\text{收敛域}: Re\{s\} < 3$$
$$\text{零极点图} X(s) = -\frac{2s-8}{(s-3)(s-5)}$$

6-2

(1)

$$\frac{1}{s^2+4} <--> sin(2tu(t))/2$$

(3)

$$\frac{s}{s^2+25} <--> cos(5tu(t))$$

(5)

$$
\begin{gathered}
\frac{s+1}{s^2+5s+6}=-\frac{1}{s+2}+2\frac{1}{s+3} \\
=2e^{-3t}u(t)+e^{-2t}u(-t)
\end{gathered}
$$

6-3

(1)

$$
\begin{gathered}
x(t)=u(t)-u(t-5) \\
L\{x(t)\}=\frac{1}{s}-e^{-5s}\frac{1}{s}, Re\{s\}>0
\end{gathered}
$$

(3)

$$
\begin{gathered}
x(t)=4sin\pi(t-3)u(t-3) \\
L\{x(t)\}=e^{-4s}\frac{4\pi}{s^2+\pi^2}
\end{gathered}
$$

(5)

$$
\begin{gathered}
x(t)=\frac{du(t)}{dt} \\
L\{u(t)\}=\frac{1}{s} \\
L\{\frac{dx(t)}{dt}\}=sX(s) \\
L\{x(t)\}=1
\end{gathered}
$$

6-4

$$
\begin{gathered}
L\{e^{-5t}u(t)+e^{-\beta t}u(t)\}=\frac{1}{s+5}+\frac{1}{s+\beta} \\
ROC: Re\{s\}>-3, Re\{\beta\}=3, Im\{\beta\}\text{为任意值}
\end{gathered}
$$

6-5

(1)

$$
X(s)=\frac{12}{s(s+4)}=3(\frac{1}{s}-\frac{1}{s+4})
$$

$$= 3[u(t) - e^{-4t}u(t)]$$

(3)

$$X(s) = \frac{s}{s+3}$$
$$L\{e^{-3t}u(t)\} = \frac{1}{s+3}$$
$$L\{\frac{dx(t)}{dt}\} = sX(s)$$
$$L\{\frac{d(e^{-3t}u(t))}{dt}\} = s\frac{1}{s+3} = X(s)$$

6-7

$$G(jw) = X(j(w-2))$$
$$G(s) = X(s+2), ROC = R+2$$
如果$x(t)$是左边信号,则$G(s)$在小于-1的时候收敛，矛盾
如果$x(t)$是右边信号,则$G(s)$在大于1的时候收敛,矛盾
综上,$x(t)$是双边信号

6-8

(a)

$$x_1(t) = A - \frac{A}{\tau}t$$
$$= A - \frac{At}{\tau}[u(t-\tau) - u(t)]$$
$$X_1(s) = \frac{d[\frac{A}{\tau}[\frac{e^{-\tau}}{s} - \frac{1}{s}]]}{ds}$$

(b)

$$设x_1(t) = u(t-1) - u(t)$$
$$x_2(t) = x_1(t) * x_1(t)$$
$$X_2(s) = X_1(s)X_1(s)$$
$$X_1(s) = \frac{e^{-s}}{s} - \frac{1}{s}$$
$$X_2(s) = [\frac{e^{-s}-1}{s}]^2, Re\{s\} > 0$$

(c)

$$x_3(t) = u(t-3) - u(t) + u(t-2) - u(t-1)$$
$$X_3(s) = \frac{d^{-3s} + e^{-2s} - e^{-s} - 1}{s}$$

(d)

$$x_4(t) = u(t-1) - u(t) - u(t-2) + u(t-1)$$
$$= 2u(t-1) - u(t) - u(t-2)$$
$$X_4(s) = \frac{2e^{-s} - 1 - e^{-2s}}{s}$$

(e)

$$x_5(t) = -sint\cdot[u(t-\pi) - u(t)]$$
$$\text{由于有} sinw_0tu(t) <-^{S}-> \frac{w_0}{s^2+w_0^2}$$
$$x_5(t) = sin(t-\pi)u(t-\pi) + sintu(t)$$
$$X_5(s) = (e^{-\pi s}\frac{1}{s^2+1} + \frac{1}{s^2+1})$$

(f)

$$x_6(t) = 2[u(t-1) - u(t-4) + u(t-2) - u(t-3)]$$
$$X_6(s) = 2\frac{e^{-s} + e^{-2s} - e^{-3s} - e^{-4s}}{s}$$

6-9

$$y(t) = x(t) + Ax(-t), x(t) = Be^{-t}u(t)$$
$$Y(s) = \frac{s}{s^2-1}, Re\{s\} \in (-1, 1)$$
$$Y(s) = [\frac{1}{s+1} + \frac{1}{s-1}]/2$$
$$y(t) = Be^{-t}u(t) + ABe^{t}u(-t)$$
$$B = 2, A = 1$$

6-13

$$y'' + 2y' + 5y = 2x' + 3x$$

(1)

$$s^2Y(s) + 2sY(s) + 5Y(s) = 2sX(s) + 3X(s)$$
$$\frac{Y(s)}{X(s)} = \frac{2s+3}{s^2+2s+5}$$
$$x(t) = u(t), X(s) = \frac{1}{s}$$
$$Y(s) = \frac{1}{s}\frac{2s+3}{s^2+2s+5}$$
$$y_{zs}(t) = (\frac{5}{4}e^t - \frac{1}{3}e^{-3t} - 1)u(t)$$

(2)

$$x(t) = e^{-t}u(t), X(s) = \frac{1}{s+1}$$
$$y_{zs}(t) = (\frac{5}{8}e^t - \frac{3}{8}e^{-3t} - \frac{1}{4}e^{-t})u(t)$$

6-14

$$H(s) = \frac{s}{s^2+4}, y(0^-) = 0, y'(0^-) = 1$$
$$y'' + 4y = x'$$
$$s^2Y(s) - sy(0) - y'(0) + 4Y(s) = sX(s) - x(0)$$
$$Y(s) = \frac{sX(s) - x(0) + sy(0) + y'(0)}{s^2+4}$$
$$\text{零输入响应}: Y_{zi}(s) = \frac{1}{s^2+4}$$

6-15

$$y'' + 3y' + y = x' + 4x(t)$$
$$x(t) = e^{-t}u(t)$$
$$s^2Y(s) + 3sY(s) + Y(s) = sX(s) + 4X(s)$$
$$Y_{zs}(s) = \frac{(s+4)X(s)}{s^2+3s+1} = \frac{s+4}{(s+1)(s^2+3s+1)}$$
$$\text{由初值定理}, x(0^+) = lim_{s->\infty}sX(s)$$
$$\text{由终值定理}, lim_{t->\infty}x(t) = lim_{s->0}sX(s)$$
$$y_{zs}(0^+) = 0, y_{zs}(\infty) = 0$$

6-16

$$\frac{dh(t)}{dt}+2h(t)=e^{-4t}u(t)+bu(t)$$
$$sH(s)+2H(s)=\frac{1}{s+4}+\frac{b}{s}$$
$$H(s)=\frac{bs+4b+s}{s(s+2)(s+4)}$$
$$s=2\text{时},H(s)=\frac{1}{6}$$
$$=>b=1,H(s)=\frac{2}{s(s+4)}$$

6-17

$$y(t)=-\frac{2}{3}e^{2t}u(-t)+\frac{1}{3}e^{-t}u(t)$$
$$x(t)=0,X(s)=\frac{s+2}{s-2}$$
$$Y(s)=\frac{2}{3}\frac{1}{s-2}+\frac{1}{3}\frac{1}{s+1}=H(s)\cdot X(s)$$
$$H(s)=\frac{2}{3}\frac{1}{s+2}+\frac{1}{3}\frac{s-2}{(s+1)(s+2)}$$
$$h(t)=$$

6-18

$$y''+3y'+2y=x(t),y(0^-)=3,y'(0^-)=-5$$
$$x(t)=2u(t)$$
$$s^2Y(s)-sy(0)-y'(0)+3[sY(s)-y(0)]+2Y(s)=X(s)=\frac{2}{s}$$
$$Y(s)=\frac{3s+4+\frac{2}{s}}{s^2+3s+2}$$
$$Y_{zs}(s)=\frac{3s+4}{s^2+3s+2}$$
$$Y_{zi}(s)=\frac{\frac{2}{s}}{s^2+3s+2}$$

6-21

(1)

$$H_1(s)=\frac{1}{(s+1)(s+3)},\text{是低通系统}$$

(2)

$$H_2(s) = \frac{s^2}{s^2+2s+1}, \text{是高通系统}$$

6-23

(a)

ROC:Re{x} > -1 : 稳定且因果

ROC：-2<Re{x}<-1 : 不稳定且不因果

ROC：Re{x} <-2 : 不稳定且不因果

(b)

ROC:Re{x} > 1 : 不稳定稳定且因果

ROC：-1<Re{x}<1 : 稳定且不因果

ROC：-2<Re{x} <-1 : 不稳定且不因果

ROC:Re{x} <-2 : 不稳定且不因果

6-25

$$s(t) = (1-e^{-2t})u(t), y(t) = (-e^{-2t}+e^{-t})u(t)$$
$$x(t) = e^{-t}u(t)$$

6-27

(1)

$$x(t) = e^{-2t}u(t+1)$$
$$e^{-2t}u(t) <--> \frac{1}{s+2}$$
$$e^{-2t}u(t+1) <--> e^{-t}\frac{1}{s+2}, ROC: Re\{x\} > -2$$

(3)

$$x(t) = e^{-2t}u(t) + e^{-4t}u(t)$$
$$X(s) = \frac{1}{s+2} + \frac{1}{s+4}, ROC: Re\{x\} > -2$$

6-28

(1)

$$V_2(s) = \frac{Ks}{s^2+4s+4}[V_1(s)+V_2(s)]$$
$$H(s) = \frac{V_2(s)}{V_1(s)} = \frac{Ks}{s^2+(4-K)s+4}$$

(2)

K<=4

(3)

$$K=4, H(s) = \frac{4s}{s^2+4}$$
$$h(t) = 4cos2tu(t)$$