This assignment will be graded anonymously, so please don't list your name, but only your MAC ID.

As noted by the syllabus as well as in class, the scope and content of assignments are set by lectures, instead of any specific textbook. Please beware that different textbooks may use different symbolism or definitions. Lemmon's as a very old textbook, for example, uses soundness and validity differently from the lectures.

Assignments are meant to be challenging! It's okay if you don't know the answers right away. In that case, **first look at your class notes, notes posted in the shared folder, or textbooks**. Try different answers to see if anything works. You are encouraged to discuss your answers with other students (but write up your own answers individually).

1. (0.6 points) Please review your notes (including earlier ones on logical connectives) and summarize the inference rules that we have learned in this class (for example, conjunction introduction or *modus ponens*). You can state these inference rules either in the format of an argument or as tautological implications. (Hint: there should be at least 6 inference rules that we have learned.)

2. (2 points) Complete a formal proof/derivation for each of the following arguments, showing that it is valid by using the inference rules summarized above.

(1) Assumption 1: **(P→Q)**; Assumption 2: **¬¬P**. Show: **¬¬Q**.

(2) Assumption 1: **Q**; Assumption 2: **(¬P→¬Q)**. Show: **P**.

(3) Assumption 1: **¬S**; Assumption 2: **((P^Q) → S)**; Assumption 3: **((P^Q) v R)**. Show: **R**.

(4) Assumption 1: **(S→¬Q)**; Assumption 2: **(P→S)**; Assumption 3: **¬¬P**. Show: **¬Q**.

(5) Assumption 1: **(P v S)**; Assumption 2: **(T → ¬S)**; Assumption 3: **T**. Show: **((P v Q) v R)**.

(6) Assumption 1: **((R→S)→Q)**; Assumption 2: **¬Q**; Assumption 3: **(¬(R→S)→V)**. Show: **V**.

(7) Assumption 1: **(P→(Q→R))**; Assumption 2: **¬(Q→R)**. Show: **¬P**.

(8) Assumption 1: **(¬(Q→R)→P)**; Assumption 2: **¬P**; Assumption 3: **Q**. Show: **R**.

(9) Assumption 1: **(P→(Q→R))**; Assumption 2: **P**; Assumption 3: **((Q→R)→¬S)), ((T→V)→S)**. Show: **¬(T→V)**.

(10) Assumption 1: **(PvQ)**; Assumption 2: **(Q → S)**; Assumption 3: **(¬S^T)**. Show: **(T^P)**.

3.(1.2 points) Make your own key to translate into sentential logic the portions of the following argument that are in bold. Using a forma proof/derivation, prove that the resulting argument is valid.

Inspector Tarski told his assistant, Mr. Carroll, "**If Wittgenstein had mud on his boots, then he was in the field**. Furthermore, **if Wittgenstein was in the field, then he is the prime suspect for the murder of Dodgson**. **Wittgenstein did have mud on his boots.** We conclude, **Wittgenstein is the prime suspect for the murder of Dodgson**."

4. (1 point) A logical implication does NOT hold if it is *possible* to assign the sentential variables truth-values under which the left-hand side of **|=** gets **T** and the right-hand side gets **F**. Thus, we just need to identify such an assignment in order to show an implication does not hold. For example, in the case of $A \to B \models \neg A$,

- Assume that B is true. From the truth table of conditionals, we know that $A \to B$ gets T, regardless whether A is true or false. If A gets T, then $\neg A$ gets F.

  Therefore, the implication does not hold, because you find a counterexample in which the premise is T but the conclusion is false.

A logical implication holds if it is *impossible* to assign the sentential variables truth-values under which the left-hand side of **|=** gets **T** and the right-hand side gets **F**. Thus, to show that an implication holds, we need check all possible assignments and see in every case in which the left-hand side of **|=** gets **T**, the right-hand side also gets **T**. For example, in the case of $\neg A \models A \to B$,

(i) Assume that B is false. If $\neg A$ gets T, then A must get F. In that case, from the truth table of conditionals, we know that $A \to B$ gets T;

(ii) Assume that B is true. If $\neg A$ gets T, then A must get F. In that case, from the truth table of conditionals, we know that $A \to B$ gets T.

Therefore, there is no assignment (to the sentential variables) in which $\neg A$ gets T and $A \to B$ doesn't.

Find, for each of (1) $A \vee B \models A$, (2) $B \models A \to B$, whether the implication holds for all B, in each of the following cases:

(I) A is true. (II) A is false.

Use the same reasoning as above to show that the implication holds, or does not hold (by giving a suitable counterexample).

5. (0.2 points) Is ‘A|=B’ the same as ‘A→B’? If not, what are some of the differences? How are they related?

6. (Optional) Using the “distributive laws” of Assignment 5, push → all the way in, in the following sentences:

(1) $(A \vee B) \rightarrow (C \vee D)$

(2) $(A \vee B) \rightarrow (C \wedge D)$

(3) $(A \wedge B) \rightarrow (C \vee D)$

(4) $(A \wedge B) \rightarrow (C \wedge D)$

Note: Using conditional, we can form the tautology $A \rightarrow A$, which is perhaps simpler than our previous standard tautology $A \vee \neg A$.