Please upload your assignment to Moodle by Wednesday, and also bring a paper copy to class on next day. This assignment will be graded anonymously, so please don't list your name, but only your MAC ID.

As noted by the syllabus as well as in class, the scope and content of assignments are set by lectures, instead of any specific textbook. Please beware that different textbooks may use different symbolism or definitions. Lemmon's as a very old textbook, for example, uses soundness and validity differently from the lectures.

Assignments are meant to be challenging! You are encouraged to discuss your answers with other students (but write up your own answers individually).

Assignments are meant to be challenging! It's okay if you don't know the answers right away. In that case, **first look at your class notes, notes posted in the shared folder, or textbooks**. Try different answers to see if anything works. You are encouraged to discuss your answers with other students (but write up your own answers individually).

1. (0.9 points) Let G be the sentence: $\neg[(A \vee \neg B) \wedge \neg C] \wedge \neg[\neg C \vee (\neg D \wedge E)]$

Construct the following sentences:

G_1: Obtained from G by pushing-in negations all the way.

G_2: Obtained from G_1 by pushing-in conjunctions all the way.

G_3: Obtained from G_1 by pushing-in disjunctions all the way.

If possible, simplify G_2 and G_3 by using the Idempotence laws.

Hints: We had done a couple of examples of pushing-in negations in class. Conjunctions are pushed-in by distributing repeatedly conjunction over disjunction: we do it by going left-to-right in the distributive laws for conjunction:

$$A \wedge (B \vee C) \equiv (A \wedge B) \vee (A \wedge C)$$

$$(B \vee C) \wedge A \equiv (B \wedge A) \vee (C \wedge A)$$

It increases the size (both 'A' and '$\wedge$' occur once on the left, but twice on the right), but decreases the scope of the conjunction (instead of $B \vee C$ we have the scopes B and C). It also increases the scope of the disjunction. Here is an example of pushing-in conjunctions:

$$A \wedge [(\neg B \vee C) \wedge \neg D] \equiv A \wedge [(\neg B \wedge \neg D) \vee (C \wedge \neg D)] \equiv (A \wedge \neg B \wedge \neg D) \vee (A \wedge C \wedge \neg D)$$

If we keep pushing-in conjunctions, we must eventually arrive at a point where no further pushing-in is possible; at that stage, there are no components either of the form $A \wedge (B \vee C)$ or of the form $(B \vee C) \wedge A$.

Pushing-in disjunctions, which is carried out via the distributive laws for disjunction, is very similar. It reduces the scopes of ∨, but enlarges those of ∧. Repeated pushing-in of disjunction terminates at a stage where there are no components either of the form A ∨ (B ∧ C) or of the form (B ∧ C) ∨ A.

2. (1.2 points) Let H = ¬G, where G is as above. Construct:

H1: obtained from H by pushing-in negations all the way.

H_2 : obtained from H_1 by pushing-in conjunctions all the way.

H_3: obtained from H_1 by pushing-in disjunctions all the way.

WATCH IT: It is inadvisable to mix pushing-in conjunctions and pushing-in disjunctions. The first reduces the scopes of ∧ and enlarges those of ∨. The second has the opposite effect. And both increase sentence size. If you interlace them you will get longer and longer sentences. For example:

$$A \wedge (B \vee C) \equiv (A \wedge B) \vee (A \wedge C) \equiv [(A \wedge B) \vee A] \wedge [(A \wedge B) \vee C] \equiv$$

$$\{[(A \wedge B) \vee A] \wedge (A \wedge B)\} \vee \{[(A \wedge B) \vee A] \wedge C\} \equiv \ldots$$

Occasionally you may want to apply the two kinds of pushing-in to separate components, or to combine them with other operations in between.

3. (1.2 points) Simplify the following sentential expressions. Try to get simplification that are as simple, or as short as possible. Check two things to see if the expression can be further simplified: (i) what is the scope of negation? (ii) Can the parenthesis be removed?

Please number the steps and indicate which equivalence law(s) are used.

(1) ¬B∨ (B∧A)

(2) (A∧B) ∨ (¬A∧B)

(3) (A∨B) ∧ (A∨¬B)

(4) (A∨ (B∧C)) ∧ ¬(B∧C)

(5) (A∨B) ∧ (A∨¬B) ∧ (¬A∨B) ∧ (¬A∨¬B)

(6) (A → B) ↔ (B → A)

(Hint: $A \leftrightarrow B \equiv (A \rightarrow B) \wedge (B \rightarrow A)$; $A \leftrightarrow B \equiv (A \wedge B) \vee (\neg A \wedge \neg B)$.)

or Simplify ¬[¬A ∨ (¬B ∨ C)] ∨ [¬(¬A ∨ B) ∨ (¬A ∨ C)] to C ∨¬C.

Optional:

(7) $\neg((A\wedge B) \vee (A\wedge C) \vee (B\wedge C))$

(8) $(A\vee\neg B) \wedge \neg(\neg B\vee A)$

(9) $(\neg B\vee (A\wedge B)) \wedge (A\vee (\neg A\wedge B))$

(10) $\neg[(A\vee B\vee C)\wedge(\neg A\vee\neg B\vee\neg C)]$

4. (0.5 points) Prove one of the following (your choice). Please number the steps and indicate which equivalence law(s) are used.

(1) $A\wedge(A\vee B)\equiv A$ (if you're not sure how to start, check the hints given in class)

(2) $((A\vee B)\wedge\neg B)\vee((A\wedge C)\vee\neg(A\rightarrow C))\equiv A$

5. (0.9 points) In certain cases conditionals can be distributed over conjunctions and disjunctions. For example, $A\rightarrow(B\wedge C) \equiv(A\rightarrow B)\wedge(A\rightarrow C)$. But sometimes the "distributing" involves a change in the other connective (the conjunction or the disjunction over which conditional is distributed). Find the "distributive laws" for the following cases. (Hint: they look similar to the example.)

(1) $A\rightarrow(B\vee C)$

(2) $(A\vee B)\rightarrow C$

(3) $(A\wedge B)\rightarrow C$

6. (Optional) Consider the expression: $A\rightarrow B\rightarrow C\rightarrow D$. By inserting parentheses, we can obtain different sentential expressions from it; for example, $A\rightarrow((B\rightarrow C)) \rightarrow D)$ and $(A\rightarrow B)\rightarrow(C\rightarrow D)$. Please first list at least 5 possible expressions. Then find whether any two of sentential expressions obtained in this way are logically equivalent.