Deadline: **Nov. 18 (can be extended to 22)**, 5pm. Please upload your assignment to Moodle by this time, and also bring a paper copy to class on next day. This assignment will be graded anonymously, so please don't list your name, but only your MAC ID.

As noted by the syllabus as well as in class, the scope, content, and convention of assignments are set by lectures, instead of any specific textbook. Please beware that different textbooks may use different symbolism or definitions.

Assignments are meant to be challenging! You are encouraged to discuss your answers with other students (but write up your own answers individually).

Please make sure your handwriting is legible.

1. Using the proof-by-contradiction method, check which of the following implications is valid. Give in each case either a top-down derivation or a counterexample.

   (1) $A \to (B \vee C), \neg(B \to A), \neg(C \to A) \models \bot$

   (2) $A \wedge (B \to C), B \models A \wedge C$

Optional:

   (3) $A \leftrightarrow B \models (A \vee B) \leftrightarrow A$

   (4) $\models ((A \to B) \wedge (A \to C)) \to ((A \vee B) \to C)$

   (5) $(A \vee B) \to C, C \models A \vee B$

2. Translate the following sentences, using predicates, **individual constants**, and connectives. The translation is to be based on the assumption that the sentences are interpreted in a universe consisting of six people: Jack, David, and Harry are American; Claire, Ann, and Edith are Canadian. (Please do **not use quantifiers** yet.)

   When you find a sentence ambiguous, give its possible translations and prove their non-equivalence by showing the existence of interpretations under which they get different truth-values.

   (1) Everyone is happy.

   (2) Every American is happy and every Canadian is happy.

   (3) Every American is happy or every Canadian is happy.

   (4) Someone is happy.

(5) Some American is happy and some Canadian is happy.

(6) Some American is happy or some Canadian is happy.

(7) Some Canadian is happy and some is not.

(8) Exactly one of Ann, Edith and Claire is happy.

(9) If Jack is not happy, none of the Canadians is.

(10) All Canadians are not happy, but all Americans are.

(11) If Americans are happy so are Canadians.

(12) Every Canadian is liked by some American.

(13) Some American likes every Canadian.

(14) Some Canadian likes herself, and some American does not.

(15) Nobody is happy who does not like himself/herself.

Optional:

(16) Some Canadians, who do not like themselves, like David.

(17) Claire likes an American who likes Edith.

(18) Unless liked by a Canadian, no American is happy.

3. Translate the following sentences, using predicates, universal or existential quantifier, and connectives.

(1) Some bunnies are fluffy, and all cats are cute.

(2) Every werewolf is afraid of the Moon.

(3) Everyone wants to be happy, but some people want to be rich.

4. (Optional) Give, for each of the following implication claims, a top-down derivation or a counterexample, using the implication laws we have learned. You can also use substitutions of equivalents. Please label each step (as well as number each step, unless you're using the tree structure).

(1) $\models [A \to (B \to C)] \to [(A \to B) \to (A \to C)]$

(2) $A \to (B \vee C), \neg B \vee \neg C \models \neg A$

(3) $A \vee (B \wedge C), B \vee (A \wedge C) \models (A \vee B) \wedge C$