Please upload your assignment to Moodle by Wednesday, and also bring a paper copy to class on next day. This assignment will be graded anonymously, so please don't list your name, but only your MAC ID.

As noted by the syllabus as well as in class, the scope and content of assignments are set by lectures, instead of any specific textbook. Please beware that different textbooks may use different symbolism or definitions. Lemmon's as a very old textbook, for example, uses soundness and validity differently from the lectures.

Assignments are meant to be challenging! You are encouraged to discuss your answers with other students (but write up your own answers individually).

Assignments are meant to be challenging! It's okay if you don't know the answers right away. In that case, **first look at your class notes, notes posted in the shared folder, or textbooks**. Try different answers to see if anything works. You are encouraged to discuss your answers with other students (but write up your own answers individually).

1. (1.2 points) Find all the pairs of sentences in Assignment 3.2 that are tautologically equivalent. Fill the following table, by writing '+' in every square for which the row sentence is tautologically equivalent to the column sentence. You can use the truth tables that you constructed for the previous assignment.

	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

For each pair without a '+', show that there is a truth-value assignment to the sentential variables, under which the two sentences get different values.

Note: You can put '+' in the diagonal and you can also assume that the filled table is symmetric around the diagonal. (Can you see why?) This leaves fifteen pairs of sentential expressions for checking. Since equivalent sentences have always the same truth-values, they behave in the same way with respect to other sentences. Hence, the more equivalent pairs you discover at an early stage, the more you will economize in checking.

2. (1 point) Find all the tautologies and all the contradictions among the following sentences. For sentences that are not tautologies (or as contradictions), give a truth-value assignment to the sentential variables under which the sentence gets F (or gets T).

1. $\neg(A \vee B) \vee (A \vee B)$
2. $A \wedge (\neg(A \vee B) \vee (C \wedge \neg A))$
3. $(A \wedge B) \vee (\neg A \wedge \neg B)$
4. $(A \vee B) \wedge (\neg A \vee \neg B)$

3.(1.2 points) Express the following texts as sentential expressions using sentential variables and logical connectives. Also explain what each sentential variable represents; for example, A: Alex is happy. Then judge whether each argument is valid or not; you can get partial credit by identifying if Modus Ponens (the paw argument) or Modus Tollens (the tail argument) is involved .

(1) I'll go skiing only if I finish my math homework. But I won't finish that if I have to write a paper. I do have to write a paper. So I won't go skiing.

(2) Unless something has gone wrong, the battery still works. The car will start, provided the battery works. If the connection is bad, the car won't start. The connection is bad. So something has gone wrong. (Hint: you can symbolize "something has gone wrong" simply as S.)

(3) Al is telling the truth only if Bill is. Bill is lying, provided the car was locked. Al is not lying. So the car was unlocked.

4. (1.6 points) Each of the following is an instance of one of the basic equivalence laws. Find the law and the substitution that has been used to get the instance.

Note: Sometimes the two sides of the equivalence have been switched around.

1. $\neg(\neg A \wedge B \vee \neg A) \;\equiv\; \neg(\neg A \wedge B) \wedge \neg\neg A$
2. $\neg(A \vee \neg B) \wedge \neg(B \wedge C) \;\equiv\; \neg((A \vee \neg B) \vee (B \wedge C))$
3. $\neg(\neg(A \vee B) \wedge B) \;\equiv\; \neg\neg(A \vee B) \vee \neg B$
4. $\neg B \vee (C \wedge \neg A) \;\equiv\; (\neg B \vee C) \wedge (\neg B \vee \neg A)$
5. $(\neg(A \vee B) \wedge A) \vee (\neg(A \vee B) \wedge A) \;\equiv\; \neg(A \vee B) \wedge (A \vee A)$
6. $A \wedge (B \vee C) \vee A \wedge C \;\equiv\; A \wedge C \vee A \wedge (B \vee C)$
7. $(B \vee A) \wedge ((A \vee B) \wedge (B \vee A)) \;\equiv\; ((B \vee A) \wedge (A \vee B)) \wedge (B \vee A)$
8. $\neg(C \wedge (A \vee C)) \;\equiv\; \neg C \vee \neg(A \vee C)$

There's a handout given in class that gives a summary of important equivalence laws. Here're some of them:

Double Negation	$\neg\neg A \equiv A$
Commutativity	$A \wedge B \equiv B \wedge A$ $A \vee B \equiv B \vee A$
Associativity	$(A \wedge B) \wedge C \equiv A \wedge (B \wedge C)$ $(A \vee B) \vee C \equiv A \vee (B \vee C)$
Idempotence	$A \wedge A \equiv A$ $A \vee A \equiv A$
Distributivity	$A \wedge (B \vee C) \equiv (A \wedge B) \vee (A \wedge C)$ $A \vee (B \wedge C) \equiv (A \vee B) \wedge (A \vee C)$
De Morgan's Laws	$\neg(A \wedge B) \equiv \neg A \vee \neg B$ $\neg(A \vee B) \equiv \neg A \wedge \neg B$