Deadline: October 14, 5pm. Please upload your assignment to Moodle by this time, and also bring a paper copy to class on next day. This assignment will be graded anonymously, so please don't list your name, but only your MAC ID.

As noted by the syllabus as well as in class, the scope, content, and convention of assignments are set by lectures, instead of any specific textbook. Please beware that different textbooks may use different symbolism or definitions.

Assignments are meant to be challenging! You are encouraged to discuss your answers with other students (but write up your own answers individually).

1. Let G be the sentence: $\neg[(A \vee \neg B) \wedge \neg C] \wedge \neg[\neg C \vee (\neg D \wedge E)]$

Construct the following sentences:

G_1: Obtained from G by pushing-in negations all the way.

G_2: Obtained from G_1 by pushing-in conjunctions all the way.

G_3: Obtained from G_1 by pushing-in disjunctions all the way.

If possible, simplify G_2 and G_3 by using the Idempotence laws.

If you're not sure what all these mean, please check the readings first.

2. Let H = ¬G, where G is as above. Construct

H_1: obtained from H by pushing-in negations all the way

H_2 : obtained from H_1 by pushing-in conjunctions all the way, and

H_3: obtained from H_1 by pushing-in disjunctions all the way.

WATCH IT: It is inadvisable to mix pushing-in conjunctions and pushing-in disjunctions. The first reduces the scopes of $\wedge$ and enlarges those of $\vee$. The second has the opposite effect. And both increase sentence size. If you interlace them you will get longer and longer sentences. For example:

$$A \wedge (B \vee C) \equiv (A \wedge B) \vee (A \wedge C) \equiv [(A \wedge B) \vee A] \wedge [(A \wedge B) \vee C] \equiv$$

$$\{[(A \wedge B) \vee A] \wedge (A \wedge B)\} \vee \{[(A \wedge B) \vee A] \wedge C\} \equiv \ldots$$

Occasionally you may want to apply the two kinds of pushing-in to separate components, or to combine them with other operations in between.

3. Simplify the following sentential expressions. Try to get simplification that are as simple, or as short as possible. Check two things to see if the expression can be further simplified: (i) what is the scope of negation? (ii) Can the parenthesis be removed?

   Please number the steps and indicate which equivalence law(s) are used.

(1) ¬B∨ (B∧A)

(2) (A∧B) ∨ (¬A∧B)

(3) (A∨B) ∧ (A∨¬B)

(4) (A∨ (B∧C)) ∧ ¬(B∧C)

(5) (A∨B) ∧ (A∨¬B) ∧ (¬A∨B) ∧ (¬A∨¬B)

(6) Simplify $\neg[\neg A \vee (\neg B \vee C)] \vee [\neg(\neg A \vee B) \vee (\neg A \vee C)]$ to C ∨¬C.

Optional:

(7) ¬((A∧B) ∨ (A∧C) ∨ (B∧C))

(8) (A∨¬B) ∧ ¬(¬B∨A)

(9) (¬B∨ (A∧B)) ∧ (A∨ (¬A∧B))

(10) ¬[(A∨B∨C)∧(¬A∨¬B∨¬C)]

4. Show: $(A \wedge B) \vee C \equiv (A \vee C) \wedge (B \vee C)$. Please number the steps and indicate which equivalence law(s) are used.

5. In certain cases conditionals can be distributed over conjunctions and disjunctions. For example, A→(B∧C) ≡(A→B)∧(A→C). But sometimes the “distributing” involves a change in the other connective (the conjunction or the disjunction over which conditional is distributed). Find the “distributive laws” for the following cases.

   - A→(B∨C)
   - (A∨B)→C
   - (A∧B)→C

6. (Optional) Consider the expression: A→B→C→D
   By inserting parentheses, we can obtain different sentential expressions from it; for example, A→((B→C)) →D) and (A→B)→(C→D). Please first list at least 5 possible

expressions. Then find whether any two of sentential expressions obtained in this way are logically equivalent.

7. (Optional) Prove the following tautological equivalences:

$$((A \vee B) \wedge \neg B) \vee ((A \wedge C) \vee \neg(A \rightarrow C)) \equiv A$$