

This assignment will be graded anonymously, so please don't list your name, but only your MAC ID.

As noted by the syllabus as well as in class, the scope and content of assignments are set by lectures, instead of any specific textbook. Please beware that different textbooks may use different symbolism or definitions. Lemmon's as a very old textbook, for example, uses soundness and validity differently from the lectures.

Assignments are meant to be challenging! It's okay if you don't know the answers right away. In that case, **first look at your class notes, notes posted in the shared folder, or textbooks**. Try different answers to see if anything works. You are encouraged to discuss your answers with other students (but write up your own answers individually).

1. (2 points) Using the two implication laws (Conditional Proof and Disjoining), prove, via top-down derivations, the following five implications. The goal should be reduced in the end to an obvious implication in which the conclusion is one of the premises. You can use substitution-of-equivalents based on simple equivalences laws as we did in class.

$$(1) \quad | = (A \rightarrow (B \rightarrow C)) \rightarrow (B \rightarrow (A \rightarrow C))$$

$$(2) \quad P \rightarrow Q, R \rightarrow S, Q \rightarrow R, P | = S$$

$$(3) \quad | = (A \rightarrow \neg A) \rightarrow \neg A$$

$$(4) \quad | = (A \rightarrow B) \rightarrow (\neg B \rightarrow \neg A)$$

$$(5) \quad \neg A \rightarrow B, B \rightarrow C \models \neg C \rightarrow A$$

2. (1 point) To give you a sense how adding more implication laws enables richer top-down derivations, consider an implication law that we introduced in class but didn't use for top-down derivations: $\Gamma, A \wedge B | = C$ iff $\Gamma, A, B | = C$. Call it *Conjunction Premise*. Try to prove the following implication via a top-down derivation:

$$(A \wedge B) \rightarrow C, B | = A \rightarrow C$$

3. (1.2 points) Translate the following sentences, using predicates, individual constants and connectives. Please also explain what each letter represents; for example, a: Alan.

- (1) The universe is immense.
- (2) My computer turns off only if its battery runs out.
- (3) Bob is not a male cat.

- (4) Pat and Bob are female.
- (5) Bob is older than Pat.
- (6) Pat is not older than both Sandi and Bob.

4. (0.8 points) Identify the predicate of the following sentences, and identify whether it's 1-place predicate or 2-place predicate, or 3-place predicate, etc.

- (1) Aletheia and Lorena are tall.
- (2) Margarita is younger than Aletheia, Lorena, and Stefano.
- (3) Margarita and Stefano live in Rome and Aletheia and Lorena live in Milan.
- (4) Lorena stands between Stefano and Aletheia.