$tmvar$, x, y, f, m, n	variables
$covar$, c	coercion variables
$datacon$, K	
$const$, T	
$tyfam$, F	
$index$, i	indices

$\mathit{role},\ R$	$::=$			Role
	$\mid$	**Nom**		
	$\mid$	**Rep**		
	$\mid$	$R_1 \cap R_2$	S	
$\mathit{relflag},\ \rho$	$::=$			relevance flag
	$\mid$	$+$		
	$\mid$	$-$		
$\mathit{constraint},\ \phi$	$::=$			props
	$\mid$	$a \sim_{A/R} b$		
	$\mid$	(ϕ)	S	
	$\mid$	$\phi\{b/x\}$	S	
	$\mid$	$\lvert\phi\rvert$	S	
$\mathit{tm},\ a,\ b,\ v,\ w,\ A,\ B$	$::=$			types and kinds
	$\mid$	$\star$		
	$\mid$	x		
	$\mid$	$\lambda^{\rho} x\!:\!A/R.b$	bind x in b	
	$\mid$	$\lambda^{\rho} x.b$	bind x in b	
	$\mid$	$a\ b^{\rho}$		
	$\mid$	F		
	$\mid$	T		
	$\mid$	$\Pi^{\rho} x\!:\!A/R \to B$	bind x in B	
	$\mid$	$a \triangleright \gamma$		
	$\mid$	$\forall c\!:\!\phi.B$	bind c in B	
	$\mid$	$\Lambda c\!:\!\phi.b$	bind c in b	
	$\mid$	$\Lambda c.b$	bind c in b	
	$\mid$	$a[\gamma]$		
	$\mid$	$\Box$		
	$\mid$	K		
	$\mid$	**match** a **with** brs		
	$\mid$	$a\{b/x\}$	S	
	$\mid$	a	S	
	$\mid$	$a\{\gamma/c\}$	S	
	$\mid$	a	S	
	$\mid$	(a)	S	
	$\mid$	a	S	parsing precedence is hard
	$\mid$	$\lvert a\rvert$	S	
	$\mid$	**Int**	S	
	$\mid$	**Bool**	S	
	$\mid$	**Nat**	S	
	$\mid$	**Vec**	S	
	$\mid$	0	S	
	$\mid$	S	S	
	$\mid$	**True**	S	
	$\mid$	**Fix**	S	

	|	$a \to b$	S	
	|	$\phi \Rightarrow A$	S	
	|	$a\ b$	S	
	|	$\lambda x.a$	S	
	|	$\lambda x : A.a$	S	
	|	$\forall\, x : A/R \to B$	S	
brs	$::=$			case branches
	|	$\mathbf{none}$		
	|	$K \Rightarrow a; brs$		
	|	$brs\{a/x\}$	S	
	|	$brs\{\gamma/c\}$	S	
	|	(brs)	S	
$co,\ \gamma$	$::=$			explicit coercions
	|	$\bullet$		
	|	c		
	|	$\mathbf{red}\, a\, b$		
	|	$\mathbf{refl}\, a$		
	|	$(a \mid = \mid_{\gamma}\ b)$		
	|	$\mathbf{sym}\, \gamma$		
	|	$\gamma_1; \gamma_2$		
	|	$\Pi^{\rho} x : \gamma_1.\gamma_2$	bind x in γ_2	
	|	$\lambda^{\rho} x : \gamma_1.\gamma_2$	bind x in γ_2	
	|	$\gamma_1\ \gamma_2^{\rho}$		
	|	$\mathbf{piFst}\, \gamma$		
	|	$\mathbf{cpiFst}\, \gamma$		
	|	$\mathbf{isoSnd}\, \gamma$		
	|	$\gamma_1 @ \gamma_2$		
	|	$\forall c : \gamma_1.\gamma_3$	bind c in γ_3	
	|	$\lambda c : \gamma_1.\gamma_3 @ \gamma_4$	bind c in γ_3	
	|	$\gamma(\gamma_1, \gamma_2)$		
	|	$\gamma @ (\gamma_1 \sim \gamma_2)$		
	|	$\gamma_1 \triangleright \gamma_2$		
	|	$\gamma_1 \sim_{A/R} \gamma_2$		
	|	$\mathbf{conv}\ \phi_1 \sim_{\gamma} \phi_2$		
	|	$\mathbf{eta}\, a$		
	|	$\mathbf{left}\, \gamma\, \gamma'$		
	|	$\mathbf{right}\, \gamma\, \gamma'$		
	|	(γ)	S	
	|	γ	S	
	|	$\gamma\{a/x\}$	S	
sig_sort	$::=$			signature classifier
	|	$\mathbf{Cs}\, A$		
	|	$\mathbf{Ax}\, a\, A\, R$		

sort	::=			binding classifier
	|	$\mathbf{Tm}\,A\,R$		
	|	$\mathbf{Co}\,\phi$		
context, Γ	::=			contexts
	|	$\varnothing$		
	|	$\Gamma, x : A/R$		
	|	$\Gamma, c : \phi$		
	|	$\Gamma\{b/x\}$	M	
	|	$\Gamma\{\gamma/c\}$	M	
	|	Γ, Γ'	M	
	|	$\lvert\Gamma\rvert$	M	
	|	(Γ)	M	
	|	Γ	M	
available_props, Δ	::=			
	|	$\varnothing$		
	|	Δ, c		
	|	$\widetilde{\Gamma}$	M	
	|	(Δ)	M	
sig, Σ	::=			signatures
	|	$\varnothing$		
	|	$\Sigma \cup \{\, T : A/R\}$		
	|	$\Sigma \cup \{\, F \sim a : A/R\}$		
	|	Σ_0	M	
	|	Σ_1	M	
	|	$\lvert\Sigma\rvert$	M	
terminals	::=			
	|	$\leftrightarrow$		
	|	$\Leftrightarrow$		
	|	$\longrightarrow$		
	|	min		
	|	$\equiv$		
	|	$\forall$		
	|	$\in$		
	|	$\notin$		
	|	$\Leftarrow$		
	|	$\Rightarrow$		
	|	$\Rightarrow^*$		
	|	$\rightarrow$		
	|	Λ		
	|			
	|	$\Box$		
	|	$\vdash$		
	|	$\dashv$		

| $\models$

| $\models$

| $\neq$

| $\triangleright$

| ok

| $\_$

| $\leadsto$

| $\leadsto^{*}$

| $\leadsto$

| $\varnothing$

| $\circ$

| fv

| dom

| $\sim$

| $\asymp$

| $|$

| $\bullet$

| $\mathbf{fst}$

| $\mathbf{snd}$

| $| \Rightarrow |$

| $\vdash_{=}$

| $\mathbf{refl_2}$

| $++$

$\mathit{formula},\ \psi$ ::=

| $\mathit{judgement}$

| $x : A/R \ \in\ \Gamma$

| $c : \phi \ \in\ \Gamma$

| $T : A/R \ \in\ \Sigma$

| $F \sim a : A/R \ \in\ \Sigma$

| $K : T\,\Gamma \ \in\ \Sigma$

| $x \ \in\ \Delta$

| $c \ \in\ \Delta$

| $c\,\mathbf{not\ relevant} \ \in\ \gamma$

| $x \ \notin\ \mathsf{fv}\,a$

| $x \ \notin\ \mathsf{dom}\,\Gamma$

| $c \ \notin\ \mathsf{dom}\,\Gamma$

| $T \ \notin\ \mathsf{dom}\,\Sigma$

| $F \ \notin\ \mathsf{dom}\,\Sigma$

| $a = b$

| $\phi_1 = \phi_2$

| $\Gamma_1 = \Gamma_2$

| $\gamma_1 = \gamma_2$

| $\neg\psi$

| $\psi_1 \wedge \psi_2$

| $\psi_1 \vee \psi_2$

	$\mid$	$\psi_1 \Rightarrow \psi_2$	
	$\mid$	(ψ)	
	$\mid$	ψ	
	$\mid$	$c : (a : A \sim b : B) \in \Gamma$	
	$\mid$		suppress lc hypothesis generated by Ott
JSubRole	$::=$		
	$\mid$	$R_1 \leq R_2$	Subroling judgement
JValue	$::=$		
	$\mid$	$\textbf{CoercedValue}\, A$	Values with at most one coercion at the top
	$\mid$	$\mathsf{Value}\ A$	values
	$\mid$	$\textbf{ValueType}\, A$	Types with head forms (erased language)
Jconsistent	$::=$		
	$\mid$	$\textbf{consistent}\, a\, b$	(erased) types do not differ in their heads
Jerased	$::=$		
	$\mid$	$erased\_tm a$	
JChk	$::=$		
	$\mid$	$(\rho = +) \vee (x \notin \mathsf{fv}\ A)$	irrelevant argument check
Jpar	$::=$		
	$\mid$	$\vDash a \Rightarrow b$	parallel reduction (implicit language)
	$\mid$	$\vdash a \Rightarrow^* b$	multistep parallel reduction
	$\mid$	$\vdash a \Leftrightarrow b$	parallel reduction to a common term
Jbeta	$::=$		
	$\mid$	$\vDash a > b$	primitive reductions on erased terms
	$\mid$	$\vDash a \rightsquigarrow b$	single-step head reduction for implicit language
	$\mid$	$\vDash a \rightsquigarrow^* b$	multistep reduction
Jett	$::=$		
	$\mid$	$\Gamma \vDash \phi\ \mathsf{ok}$	Prop wellformedness
	$\mid$	$\Gamma \vDash a : A/R$	typing
	$\mid$	$\Gamma; \Delta \vDash \phi_1 \equiv \phi_2$	prop equality
	$\mid$	$\Gamma; \Delta \vDash a \equiv b : A/R$	definitional equality
	$\mid$	$\vDash \Gamma$	context wellformedness
Jsig	$::=$		
	$\mid$	$\vDash \Sigma$	signature wellformedness
Jann	$::=$		
	$\mid$	$\Gamma \vdash \phi\ \mathsf{ok}$	prop wellformedness
	$\mid$	$\Gamma \vdash a : A/R$	typing
	$\mid$	$\Gamma; \Delta \vdash \gamma : \phi_1 \sim \phi_2$	coercion between props

	$\mid$	$\Gamma; \Delta \vdash \gamma : A \sim_R B$	coercion between types
	$\mid$	$\vdash \Gamma$	context wellformedness
	$\mid$	$\vdash \Sigma$	signature wellformedness
Jred	::=		
	$\mid$	$\Gamma \vdash a \leadsto b$	single-step, weak head reduction to values for annotated langu
judgement	::=		
	$\mid$	*JSubRole*	
	$\mid$	*JValue*	
	$\mid$	*Jconsistent*	
	$\mid$	*Jerased*	
	$\mid$	*JChk*	
	$\mid$	*Jpar*	
	$\mid$	*Jbeta*	
	$\mid$	*Jett*	
	$\mid$	*Jsig*	
	$\mid$	*Jann*	
	$\mid$	*Jred*	
user_syntax	::=		
	$\mid$	*tmvar*	
	$\mid$	*covar*	
	$\mid$	*datacon*	
	$\mid$	*const*	
	$\mid$	*tyfam*	
	$\mid$	*index*	
	$\mid$	*role*	
	$\mid$	*relflag*	
	$\mid$	*constraint*	
	$\mid$	*tm*	
	$\mid$	*brs*	
	$\mid$	*co*	
	$\mid$	*sig_sort*	
	$\mid$	*sort*	
	$\mid$	*context*	
	$\mid$	*available_props*	
	$\mid$	*sig*	
	$\mid$	*terminals*	
	$\mid$	*formula*	

$\boxed{R_1 \leq R_2}$ Subroling judgement

$$\frac{}{\mathbf{Nom} \leq \mathbf{Rep}} \quad \textsc{NomRep}$$

$$\frac{}{R \leq R} \quad \textsc{Refl}$$

$$\frac{R_1 \leq R_2 \quad R_2 \leq R_3}{R_1 \leq R_3} \quad \textsc{Trans}$$

$\boxed{\textbf{CoercedValue}\ A}$ Values with at most one coercion at the top

$$\frac{\mathsf{Value}\ a}{\textbf{CoercedValue}\, a}\quad \text{CV}$$

$$\frac{\mathsf{Value}\ a}{\textbf{CoercedValue}\,(a \triangleright \gamma)}\quad \text{CC}$$

$\boxed{\mathsf{Value}\ A}$ values

$$\frac{}{\mathsf{Value}\ \star}\quad \textsc{Value\_Star}$$

$$\frac{}{\mathsf{Value}\ \Pi^{\rho} x\!:\!A/R \to B}\quad \textsc{Value\_Pi}$$

$$\frac{}{\mathsf{Value}\ \forall c\!:\!\phi.B}\quad \textsc{Value\_CPi}$$

$$\frac{}{\mathsf{Value}\ \lambda^{+} x\!:\!A/R.a}\quad \textsc{Value\_AbsRel}$$

$$\frac{}{\mathsf{Value}\ \lambda^{+} x.a}\quad \textsc{Value\_UAbsRel}$$

$$\frac{\mathsf{Value}\ a}{\mathsf{Value}\ \lambda^{-} x.a}\quad \textsc{Value\_UAbsIrrel}$$

$$\frac{\textbf{CoercedValue}\, a}{\mathsf{Value}\ \lambda^{-} x\!:\!A/R.a}\quad \textsc{Value\_AbsIrrel}$$

$$\frac{}{\mathsf{Value}\ \Lambda c\!:\!\phi.a}\quad \textsc{Value\_CAbs}$$

$$\frac{}{\mathsf{Value}\ \Lambda c.a}\quad \textsc{Value\_UCAbs}$$

$\boxed{\textbf{ValueType}\, A}$ Types with head forms (erased language)

$$\frac{}{\textbf{ValueType}\, \star}\quad \textsc{value\_type\_Star}$$

$$\frac{}{\textbf{ValueType}\, \Pi^{\rho} x\!:\!A/R \to B}\quad \textsc{value\_type\_Pi}$$

$$\frac{}{\textbf{ValueType}\, \forall c\!:\!\phi.B}\quad \textsc{value\_type\_CPi}$$

$\boxed{\textbf{consistent}\ a\ b}$ (erased) types do not differ in their heads

$$\frac{}{\textbf{consistent}\ \star\ \star}\quad \textsc{consistent\_a\_Star}$$

$$\frac{}{\textbf{consistent}\ (\Pi^{\rho} x_1\!:\!A_1/R \to B_1)\ (\Pi^{\rho} x_2\!:\!A_2/R \to B_2)}\quad \textsc{consistent\_a\_Pi}$$

$$\frac{}{\textbf{consistent}\ (\forall c_1\!:\!\phi_1.A_1)\ (\forall c_2\!:\!\phi_2.A_2)}\quad \textsc{consistent\_a\_CPi}$$

$$\frac{\neg\textbf{ValueType}\, b}{\textbf{consistent}\ a\ b}\quad \textsc{consistent\_a\_Step\_R}$$

$$\frac{\neg\textbf{ValueType}\, a}{\textbf{consistent}\ a\ b}\quad \textsc{consistent\_a\_Step\_L}$$

$\boxed{\mathit{erased\_tm}\, a}$

$$\frac{}{\mathit{erased\_tm}\, \Box}\quad \textsc{erased\_a\_Bullet}$$

$$\frac{}{erased\_tm \star} \quad \text{ERASED\_A\_STAR}$$

$$\frac{}{erased\_tm x} \quad \text{ERASED\_A\_VAR}$$

$$\frac{erased\_tm a}{erased\_tm(\lambda^{\rho} x.a)} \quad \text{ERASED\_A\_ABS}$$

$$\frac{\begin{array}{c} erased\_tm a \\ erased\_tm b \end{array}}{erased\_tm(a\ b^{\rho})} \quad \text{ERASED\_A\_APP}$$

$$\frac{\begin{array}{c} erased\_tm A \\ erased\_tm B \end{array}}{erased\_tm(\Pi^{\rho} x\!:\!A/R \to B)} \quad \text{ERASED\_A\_PI}$$

$$\frac{\begin{array}{c} erased\_tm a \\ erased\_tm b \\ erased\_tm A \\ erased\_tm B \end{array}}{erased\_tm(\forall c\!:\!a \sim_{A/R} b.B)} \quad \text{ERASED\_A\_CPI}$$

$$\frac{erased\_tm b}{erased\_tm(\Lambda c.b)} \quad \text{ERASED\_A\_CABS}$$

$$\frac{erased\_tm a}{erased\_tm(a[\bullet])} \quad \text{ERASED\_A\_CAPP}$$

$$\frac{}{erased\_tm F} \quad \text{ERASED\_A\_FAM}$$

$$\frac{}{erased\_tm T} \quad \text{ERASED\_A\_CONST}$$

$\boxed{(\rho = +) \vee (x \notin \mathsf{fv}\ A)}$ irrelevant argument check

$$\frac{}{(+ = +) \vee (x \notin \mathsf{fv}\ A)} \quad \text{RHO\_REL}$$

$$\frac{x \notin \mathsf{fv} A}{(- = +) \vee (x \notin \mathsf{fv}\ A)} \quad \text{RHO\_IRRREL}$$

$\boxed{\vDash a \Rightarrow b}$ parallel reduction (implicit language)

$$\frac{}{\vDash a \Rightarrow a} \quad \text{PAR\_REFL}$$

$$\frac{\begin{array}{c} \vDash a \Rightarrow (\lambda^{\rho} x.a') \\ \vDash b \Rightarrow b' \end{array}}{\vDash a\ b^{\rho} \Rightarrow a'\{b'/x\}} \quad \text{PAR\_BETA}$$

$$\frac{\begin{array}{c} \vDash a \Rightarrow a' \\ \vDash b \Rightarrow b' \end{array}}{\vDash a\ b^{\rho} \Rightarrow a'\ b'^{\rho}} \quad \text{PAR\_APP}$$

$$\frac{\vDash a \Rightarrow (\Lambda c.a')}{\vDash a[\bullet] \Rightarrow a'\{\bullet/c\}} \quad \text{PAR\_CBETA}$$

$$\frac{\vDash a \Rightarrow a'}{\vDash a[\bullet] \Rightarrow a'[\bullet]} \quad \text{PAR\_CAPP}$$

$$\frac{\vDash a \Rightarrow a'}{\vDash \lambda^\rho x.a \Rightarrow \lambda^\rho x.a'} \quad \textsc{Par\_Abs}$$

$$\frac{\begin{array}{c}\vDash A \Rightarrow A' \\ \vDash B \Rightarrow B'\end{array}}{\vDash \Pi^\rho x\!:\!A/R \to B \Rightarrow \Pi^\rho x\!:\!A'/R \to B'} \quad \textsc{Par\_Pi}$$

$$\frac{\vDash a \Rightarrow a'}{\vDash \Lambda c.a \Rightarrow \Lambda c.a'} \quad \textsc{Par\_CAbs}$$

$$\frac{\begin{array}{c}\vDash A \Rightarrow A' \\ \vDash B \Rightarrow B' \\ \vDash a \Rightarrow a' \\ \vDash A_1 \Rightarrow A_1'\end{array}}{\vDash \forall c\!:\!A \sim_{A_1/R} B.a \Rightarrow \forall c\!:\!A' \sim_{A_1'/R} B'.a'} \quad \textsc{Par\_CPi}$$

$$\frac{F \sim a : A/R \in \Sigma_0}{\vDash F \Rightarrow a} \quad \textsc{Par\_Axiom}$$

$\boxed{\vdash a \Rightarrow^* b}$ multistep parallel reduction

$$\frac{}{\vdash a \Rightarrow^* a} \quad \textsc{MP\_Refl}$$

$$\frac{\begin{array}{c}\vDash a \Rightarrow b \\ \vdash b \Rightarrow^* a'\end{array}}{\vdash a \Rightarrow^* a'} \quad \textsc{MP\_Step}$$

$\boxed{\vdash a \Leftrightarrow b}$ parallel reduction to a common term

$$\frac{\begin{array}{c}\vdash a_1 \Rightarrow^* b \\ \vdash a_2 \Rightarrow^* b\end{array}}{\vdash a_1 \Leftrightarrow a_2} \quad \textsc{join}$$

$\boxed{\vDash a > b}$ primitive reductions on erased terms

$$\frac{\mathsf{Value}\ (\lambda^\rho x.v)}{\vDash (\lambda^\rho x.v)\ \ b^\rho > v\{b/x\}} \quad \textsc{Beta\_AppAbs}$$

$$\frac{}{\vDash (\Lambda c.a')[\bullet] > a'\{\bullet/c\}} \quad \textsc{Beta\_CAppCAbs}$$

$$\frac{F \sim a : A/R \in \Sigma_0}{\vDash F > a} \quad \textsc{Beta\_Axiom}$$

$\boxed{\vDash a \leadsto b}$ single-step head reduction for implicit language

$$\frac{\vDash a \leadsto a'}{\vDash \lambda^- x.a \leadsto \lambda^- x.a'} \quad \textsc{E\_AbsTerm}$$

$$\frac{\vDash a \leadsto a'}{\vDash a\ \ b^\rho \leadsto a'\ \ b^\rho} \quad \textsc{E\_AppLeft}$$

$$\frac{\vDash a \leadsto a'}{\vDash a[\bullet] \leadsto a'[\bullet]} \quad \textsc{E\_CAppLeft}$$

$$\frac{\mathsf{Value}\ (\lambda^\rho x.v)}{\vDash (\lambda^\rho x.v)\ \ a^\rho \leadsto v\{a/x\}} \quad \textsc{E\_AppAbs}$$

$$\frac{}{\vDash (\Lambda c.b)[\bullet] \leadsto b\{\bullet/c\}} \quad \textsc{E\_CAppCAbs}$$

$$\frac{F \sim a : A/R \in \Sigma_0}{\vDash F \rightsquigarrow a} \quad \text{E\_Axiom}$$

$\boxed{\vDash a \rightsquigarrow^* b}$ multistep reduction

$$\frac{}{\vDash a \rightsquigarrow^* a} \quad \text{Equal}$$

$$\frac{\begin{array}{c}\vDash a \rightsquigarrow b \\ \vDash b \rightsquigarrow^* a'\end{array}}{\vDash a \rightsquigarrow^* a'} \quad \text{Step}$$

$\boxed{\Gamma \vDash \phi \ \mathsf{ok}}$ Prop wellformedness

$$\frac{\begin{array}{c}\Gamma \vDash a : A/R \\ \Gamma \vDash b : A/R \\ \Gamma \vDash A : \star/R\end{array}}{\Gamma \vDash a \sim_{A/R} b \ \mathsf{ok}} \quad \text{E\_Wff}$$

$\boxed{\Gamma \vDash a : A/R}$ typing

$$\frac{\begin{array}{l}R_1 \leq R_2 \\ \Gamma \vDash a : A/R_1\end{array}}{\Gamma \vDash a : A/R_2} \quad \text{E\_SubRole}$$

$$\frac{\vDash \Gamma}{\Gamma \vDash \star : \star/R} \quad \text{E\_Star}$$

$$\frac{\begin{array}{l}\vDash \Gamma \\ x : A/R \in \Gamma\end{array}}{\Gamma \vDash x : A/R} \quad \text{E\_Var}$$

$$\frac{\begin{array}{l}\Gamma, x : A/R \vDash B : \star/R \\ \Gamma \vDash A : \star/R\end{array}}{\Gamma \vDash \Pi^\rho x\!:\!A/R \to B : \star/R} \quad \text{E\_Pi}$$

$$\frac{\begin{array}{l}\Gamma, x : A/R \vDash a : B/R' \\ \Gamma \vDash A : \star/R \\ (\rho = +) \vee (x \notin \mathsf{fv}\ a) \\ R \leq R'\end{array}}{\Gamma \vDash \lambda^\rho x.a : (\Pi^\rho x\!:\!A/R \to B)/R'} \quad \text{E\_Abs}$$

$$\frac{\begin{array}{l}\Gamma \vDash b : \Pi^+ x\!:\!A/R \to B/R' \\ \Gamma \vDash a : A/R\end{array}}{\Gamma \vDash b\ a^+ : B\{a/x\}/R'} \quad \text{E\_App}$$

$$\frac{\begin{array}{l}\Gamma \vDash b : \Pi^- x\!:\!A/R \to B/R' \\ \Gamma \vDash a : A/R\end{array}}{\Gamma \vDash b\ \square^- : B\{a/x\}/R'} \quad \text{E\_IApp}$$

$$\frac{\begin{array}{l}\Gamma \vDash a : A/R \\ \Gamma; \widetilde{\Gamma} \vDash A \equiv B : \star/R \\ \Gamma \vDash B : \star/R\end{array}}{\Gamma \vDash a : B/R} \quad \text{E\_Conv}$$

$$\frac{\begin{array}{l}\Gamma, c : \phi \vDash B : \star/R \\ \Gamma \vDash \phi \ \mathsf{ok}\end{array}}{\Gamma \vDash \forall c\!:\!\phi.B : \star/R} \quad \text{E\_CPi}$$

$$\frac{\begin{array}{l}\Gamma, c:\phi \vDash a : B/R \\ \Gamma \vDash \phi \ \mathsf{ok}\end{array}}{\Gamma \vDash \Lambda c.a : \forall c{:}\phi.B/R} \quad \text{E\_CAbs}$$

$$\frac{\begin{array}{l}\Gamma \vDash a_1 : \forall c{:}(a \sim_{A/R} b).B_1/R' \\ \Gamma; \widetilde{\Gamma} \vDash a \equiv b : A/R\end{array}}{\Gamma \vDash a_1[\bullet] : B_1\{\bullet/c\}/R'} \quad \text{E\_CApp}$$

$$\frac{\begin{array}{l}\vDash \Gamma \\ F \sim a : A/R \in \Sigma_0 \\ \varnothing \vDash A : \star/R\end{array}}{\Gamma \vDash F : A/R} \quad \text{E\_Fam}$$

$\boxed{\Gamma; \Delta \vDash \phi_1 \equiv \phi_2}$ prop equality

$$\frac{\begin{array}{c}\Gamma; \Delta \vDash A_1 \equiv A_2 : A/R \\ \Gamma; \Delta \vDash B_1 \equiv B_2 : A/R\end{array}}{\Gamma; \Delta \vDash A_1 \sim_{A/R} B_1 \equiv A_2 \sim_{A/R} B_2} \quad \text{E\_PropCong}$$

$$\frac{\begin{array}{c}\Gamma; \Delta \vDash A \equiv B : \star/R \\ \Gamma \vDash A_1 \sim_{A/R} A_2 \ \mathsf{ok} \\ \Gamma \vDash A_1 \sim_{B/R} A_2 \ \mathsf{ok}\end{array}}{\Gamma; \Delta \vDash A_1 \sim_{A/R} A_2 \equiv A_1 \sim_{B/R} A_2} \quad \text{E\_IsoConv}$$

$$\frac{\Gamma; \Delta \vDash \forall c{:}\phi_1.B_1 \equiv \forall c{:}\phi_2.B_2 : \star/R}{\Gamma; \Delta \vDash \phi_1 \equiv \phi_2} \quad \text{E\_CPiFst}$$

$\boxed{\Gamma; \Delta \vDash a \equiv b : A/R}$ definitional equality

$$\frac{\begin{array}{l}\vDash \Gamma \\ c : (a \sim_{A/R} b) \in \Gamma \\ c \in \Delta\end{array}}{\Gamma; \Delta \vDash a \equiv b : A/R} \quad \text{E\_Assn}$$

$$\frac{\Gamma \vDash a : A/R}{\Gamma; \Delta \vDash a \equiv a : A/R} \quad \text{E\_Refl}$$

$$\frac{\Gamma; \Delta \vDash b \equiv a : A/R}{\Gamma; \Delta \vDash a \equiv b : A/R} \quad \text{E\_Sym}$$

$$\frac{\begin{array}{c}\Gamma; \Delta \vDash a \equiv a_1 : A/R \\ \Gamma; \Delta \vDash a_1 \equiv b : A/R\end{array}}{\Gamma; \Delta \vDash a \equiv b : A/R} \quad \text{E\_Trans}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a \equiv b : A/R_1 \\ R_1 \leq R_2\end{array}}{\Gamma; \Delta \vDash a \equiv b : A/R_2} \quad \text{E\_Sub}$$

$$\frac{\begin{array}{l}\Gamma \vDash a_1 : B/R \\ \Gamma \vDash a_2 : B/R \\ \vDash a_1 > a_2\end{array}}{\Gamma; \Delta \vDash a_1 \equiv a_2 : B/R} \quad \text{E\_Beta}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vDash A_1 \equiv A_2 : \star/R \\ \Gamma, x : A_1/R;\Delta \vDash B_1 \equiv B_2 : \star/R \\ \Gamma \vDash A_1 : \star/R \\ \Gamma \vDash \Pi^\rho x\!:\!A_1/R \to B_1 : \star/R \\ \Gamma \vDash \Pi^\rho x\!:\!A_2/R \to B_2 : \star/R\end{array}}{\Gamma;\Delta \vDash (\Pi^\rho x\!:\!A_1/R \to B_1) \equiv (\Pi^\rho x\!:\!A_2/R \to B_2) : \star/R} \quad \textsc{E\_PiCong}$$

$$\frac{\begin{array}{l}\Gamma, x : A_1/R;\Delta \vDash b_1 \equiv b_2 : B/R' \\ \Gamma \vDash A_1 : \star/R \\ R \leq R' \\ (\rho = +) \vee (x \notin \mathsf{fv}\ b_1) \\ (\rho = +) \vee (x \notin \mathsf{fv}\ b_2)\end{array}}{\Gamma;\Delta \vDash (\lambda^\rho x.b_1) \equiv (\lambda^\rho x.b_2) : (\Pi^\rho x\!:\!A_1/R \to B)/R'} \quad \textsc{E\_AbsCong}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vDash a_1 \equiv b_1 : (\Pi^+ x\!:\!A/R \to B)/R' \\ \Gamma;\Delta \vDash a_2 \equiv b_2 : A/R\end{array}}{\Gamma;\Delta \vDash a_1\ \ a_2{}^+ \equiv b_1\ \ b_2{}^+ : (B\{a_2/x\})/R'} \quad \textsc{E\_AppCong}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vDash a_1 \equiv b_1 : (\Pi^- x\!:\!A/R \to B)/R' \\ \Gamma \vDash a : A/R\end{array}}{\Gamma;\Delta \vDash a_1\ \Box^- \equiv b_1\ \Box^- : (B\{a/x\})/R'} \quad \textsc{E\_IAppCong}$$

$$\frac{\Gamma;\Delta \vDash \Pi^\rho x\!:\!A_1/R_1 \to B_1 \equiv \Pi^\rho x\!:\!A_2/R_2 \to B_2 : \star/R'}{\Gamma;\Delta \vDash A_1 \equiv A_2 : \star/R_1 \cap R_2} \quad \textsc{E\_PiFst}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vDash \Pi^\rho x\!:\!A_1/R_1 \to B_1 \equiv \Pi^\rho x\!:\!A_2/R_2 \to B_2 : \star/R' \\ \Gamma;\Delta \vDash a_1 \equiv a_2 : A_1/R_1 \cap R_2\end{array}}{\Gamma;\Delta \vDash B_1\{a_1/x\} \equiv B_2\{a_2/x\} : \star/R'} \quad \textsc{E\_PiSnd}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vDash \phi_1 \equiv \phi_2 \\ \Gamma, c : \phi_1;\Delta \vDash A \equiv B : \star/R \\ \Gamma \vDash \phi_1\ \ \mathsf{ok} \\ \Gamma \vDash \forall c\!:\!\phi_1.A : \star/R \\ \Gamma \vDash \forall c\!:\!\phi_2.B : \star/R\end{array}}{\Gamma;\Delta \vDash \forall c\!:\!\phi_1.A \equiv \forall c\!:\!\phi_2.B : \star/R} \quad \textsc{E\_CPiCong}$$

$$\frac{\begin{array}{l}\Gamma, c : \phi_1;\Delta \vDash a \equiv b : B/R \\ \Gamma \vDash \phi_1\ \ \mathsf{ok}\end{array}}{\Gamma;\Delta \vDash (\Lambda c.a) \equiv (\Lambda c.b) : \forall c\!:\!\phi_1.B/R} \quad \textsc{E\_CAbsCong}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vDash a_1 \equiv b_1 : (\forall c\!:\!(a \sim_{A/R} b).B)/R' \\ \Gamma;\widetilde{\Gamma} \vDash a \equiv b : A/R\end{array}}{\Gamma;\Delta \vDash a_1[\bullet] \equiv b_1[\bullet] : (B\{\bullet/c\})/R'} \quad \textsc{E\_CAppCong}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vDash \forall c\!:\!(a_1 \sim_{A/R} a_2).B_1 \equiv \forall c\!:\!(a_1' \sim_{A'/R'} a_2').B_2 : \star/R_0 \\ \Gamma;\widetilde{\Gamma} \vDash a_1 \equiv a_2 : A/R \\ \Gamma;\widetilde{\Gamma} \vDash a_1' \equiv a_2' : A'/R'\end{array}}{\Gamma;\Delta \vDash B_1\{\bullet/c\} \equiv B_2\{\bullet/c\} : \star/R_0} \quad \textsc{E\_CPiSnd}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vDash a \equiv b : A/R \\ \Gamma;\Delta \vDash a \sim_{A/R} b \equiv a' \sim_{A'/R'} b'\end{array}}{\Gamma;\Delta \vDash a' \equiv b' : A'/R'} \quad \textsc{E\_Cast}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vDash a \equiv b : A/R_1 \\ \Gamma;\widetilde{\Gamma} \vDash A \equiv B : \star/R_2 \\ R_1 \leq R_2\end{array}}{\Gamma;\Delta \vDash a \equiv b : B/R_2} \quad \textsc{E\_EqConv}$$

$$\frac{\Gamma; \Delta \vDash a \sim_{A/R} b \equiv a' \sim_{A'/R'} b'}{\Gamma; \Delta \vDash A \equiv A' : \star/R \cap R'} \quad \textsc{E\_IsoSnd}$$

$\boxed{\vDash \Gamma}$ context wellformedness

$$\frac{}{\vDash \varnothing} \quad \textsc{E\_Empty}$$

$$\frac{\begin{array}{l} \vDash \Gamma \\ \Gamma \vDash A : \star/R \\ x \notin \mathsf{dom}\,\Gamma \end{array}}{\vDash \Gamma, x : A/R} \quad \textsc{E\_ConsTm}$$

$$\frac{\begin{array}{l} \vDash \Gamma \\ \Gamma \vDash \phi \ \mathsf{ok} \\ c \notin \mathsf{dom}\,\Gamma \end{array}}{\vDash \Gamma, c : \phi} \quad \textsc{E\_ConsCo}$$

$\boxed{\vDash \Sigma}$ signature wellformedness

$$\frac{}{\vDash \varnothing} \quad \textsc{Sig\_Empty}$$

$$\frac{\begin{array}{l} \vDash \Sigma \\ \varnothing \vDash A : \star/R \\ \varnothing \vDash a : A/R' \\ F \notin \mathsf{dom}\,\Sigma \\ R' \leq R \end{array}}{\vDash \Sigma \cup \{F \sim a : A/R'\}} \quad \textsc{Sig\_ConsAx}$$

$\boxed{\Gamma \vdash \phi \ \mathsf{ok}}$ prop wellformedness

$$\frac{\begin{array}{l} \Gamma \vdash a : A/R \\ \Gamma \vdash b : B/R \\ |A| = |B| \end{array}}{\Gamma \vdash a \sim_{A/R} b \ \mathsf{ok}} \quad \textsc{An\_Wff}$$

$\boxed{\Gamma \vdash a : A/R}$ typing

$$\frac{\vdash \Gamma}{\Gamma \vdash \star : \star/R} \quad \textsc{An\_Star}$$

$$\frac{\begin{array}{l} \vdash \Gamma \\ x : A/R \in \Gamma \end{array}}{\Gamma \vdash x : A/R} \quad \textsc{An\_Var}$$

$$\frac{\begin{array}{l} \Gamma, x : A/R \vdash B : \star/R' \\ \Gamma \vdash A : \star/R \end{array}}{\Gamma \vdash \Pi^\rho x{:}A/R \to B : \star/R'} \quad \textsc{An\_Pi}$$

$$\frac{\begin{array}{l} \Gamma \vdash A : \star/R \\ \Gamma, x : A/R \vdash a : B/R' \\ (\rho = +) \vee (x \notin \mathsf{fv}\ |a|) \\ R \leq R' \end{array}}{\Gamma \vdash \lambda^\rho x{:}A/R.a : (\Pi^\rho x{:}A/R \to B)/R'} \quad \textsc{An\_Abs}$$

$$\frac{\Gamma \vdash b : (\Pi^\rho x\!:\!A/R \to B)/R' \qquad \Gamma \vdash a : A/R}{\Gamma \vdash b\ a^\rho : (B\{a/x\})/R'} \quad \textsc{An\_App}$$

$$\frac{\Gamma \vdash a : A/R \qquad \Gamma; \widetilde{\Gamma} \vdash \gamma : A \sim_R B \qquad \Gamma \vdash B : \star/R}{\Gamma \vdash a \triangleright \gamma : B/R} \quad \textsc{An\_Conv}$$

$$\frac{\Gamma \vdash \phi \ \mathsf{ok} \qquad \Gamma, c : \phi \vdash B : \star/R}{\Gamma \vdash \forall c\!:\!\phi.B : \star/R} \quad \textsc{An\_CPi}$$

$$\frac{\Gamma \vdash \phi \ \mathsf{ok} \qquad \Gamma, c : \phi \vdash a : B/R}{\Gamma \vdash \Lambda c\!:\!\phi.a : (\forall c\!:\!\phi.B)/R} \quad \textsc{An\_CAbs}$$

$$\frac{\Gamma \vdash a_1 : (\forall c\!:\!a \sim_{A_1/R} b.B)/R' \qquad \Gamma; \widetilde{\Gamma} \vdash \gamma : a \sim_R b}{\Gamma \vdash a_1[\gamma] : B\{\gamma/c\}/R'} \quad \textsc{An\_CApp}$$

$$\frac{\vdash \Gamma \qquad F \sim a : A/R \in \Sigma_1 \qquad \varnothing \vdash A : \star/R}{\Gamma \vdash F : A/R} \quad \textsc{An\_Fam}$$

$\boxed{\Gamma; \Delta \vdash \gamma : \phi_1 \sim \phi_2}$ coercion between props

$$\frac{\Gamma; \Delta \vdash \gamma_1 : A_1 \sim_R A_2 \qquad \Gamma; \Delta \vdash \gamma_2 : B_1 \sim_R B_2 \qquad \Gamma \vdash A_1 \sim_{A/R} B_1 \ \mathsf{ok} \qquad \Gamma \vdash A_2 \sim_{A/R} B_2 \ \mathsf{ok}}{\Gamma; \Delta \vdash (\gamma_1 \sim_{A/R} \gamma_2) : (A_1 \sim_{A/R} B_1) \sim (A_2 \sim_{A/R} B_2)} \quad \textsc{An\_PropCong}$$

$$\frac{\Gamma; \Delta \vdash \gamma : \forall c\!:\!\phi_1.A_2 \sim_R \forall c\!:\!\phi_2.B_2}{\Gamma; \Delta \vdash \mathbf{cpiFst}\,\gamma : \phi_1 \sim \phi_2} \quad \textsc{An\_CPiFst}$$

$$\frac{\Gamma; \Delta \vdash \gamma : \phi_1 \sim \phi_2}{\Gamma; \Delta \vdash \mathbf{sym}\,\gamma : \phi_2 \sim \phi_1} \quad \textsc{An\_IsoSym}$$

$$\frac{\Gamma; \Delta \vdash \gamma : A \sim_R B \qquad \Gamma \vdash a_1 \sim_{A/R} a_2 \ \mathsf{ok} \qquad \Gamma \vdash a_1' \sim_{B/R} a_2' \ \mathsf{ok} \qquad |a_1| = |a_1'| \qquad |a_2| = |a_2'|}{\Gamma; \Delta \vdash \mathbf{conv}\ (a_1 \sim_{A/R} a_2) \sim_\gamma (a_1' \sim_{B/R} a_2') : (a_1 \sim_{A/R} a_2) \sim (a_1' \sim_{B/R} a_2')} \quad \textsc{An\_IsoConv}$$

$\boxed{\Gamma; \Delta \vdash \gamma : A \sim_R B}$ coercion between types

$$\frac{\vdash \Gamma \qquad c : a \sim_{A/R} b \in \Gamma \qquad c \in \Delta}{\Gamma; \Delta \vdash c : a \sim_R b} \quad \textsc{An\_Assn}$$

$$\frac{\Gamma \vdash a : A/R}{\Gamma; \Delta \vdash \mathbf{refl}\,a : a \sim_R a} \quad \textsc{An\_Refl}$$

$$\frac{\begin{array}{l}\Gamma \vdash a : A/R \\ \Gamma \vdash b : B/R \\ |a| = |b| \\ \Gamma; \widetilde{\Gamma} \vdash \gamma : A \sim_R B\end{array}}{\Gamma; \Delta \vdash (a \mathrel{|{=}|_\gamma} b) : a \sim_R b} \quad \textsc{An\_EraseEq}$$

$$\frac{\begin{array}{l}\Gamma \vdash b : B/R \\ \Gamma \vdash a : A/R \\ \Gamma; \widetilde{\Gamma} \vdash \gamma_1 : B \sim_R A \\ \Gamma; \Delta \vdash \gamma : b \sim_R a\end{array}}{\Gamma; \Delta \vdash \mathbf{sym}\, \gamma : a \sim_R b} \quad \textsc{An\_Sym}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vdash \gamma_1 : a \sim_R a_1 \\ \Gamma; \Delta \vdash \gamma_2 : a_1 \sim_R b \\ \Gamma \vdash a : A/R \\ \Gamma \vdash a_1 : A_1/R \\ \Gamma; \widetilde{\Gamma} \vdash \gamma_3 : A \sim_R A_1\end{array}}{\Gamma; \Delta \vdash (\gamma_1; \gamma_2) : a \sim_R b} \quad \textsc{An\_Trans}$$

$$\frac{\begin{array}{l}\Gamma \vdash a_1 : B_0/R \\ \Gamma \vdash a_2 : B_1/R \\ |B_0| = |B_1| \\ \vDash |a_1| > |a_2|\end{array}}{\Gamma; \Delta \vdash \mathbf{red}\, a_1\, a_2 : a_1 \sim_R a_2} \quad \textsc{An\_Beta}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vdash \gamma_1 : A_1 \sim_R A_2 \\ \Gamma, x : A_1/R; \Delta \vdash \gamma_2 : B_1 \sim_R B_2 \\ B_3 = B_2\{x \triangleright \mathbf{sym}\, \gamma_1/x\} \\ \Gamma \vdash \Pi^\rho x\!:\!A_1/R \to B_1 : \star/R \\ \Gamma \vdash \Pi^\rho x\!:\!A_1/R \to B_2 : \star/R \\ \Gamma \vdash \Pi^\rho x\!:\!A_2/R \to B_3 : \star/R\end{array}}{\Gamma; \Delta \vdash \Pi^\rho x\!:\!\gamma_1.\gamma_2 : (\Pi^\rho x\!:\!A_1/R \to B_1) \sim_R (\Pi^\rho x\!:\!A_2/R \to B_3)} \quad \textsc{An\_PiCong}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vdash \gamma_1 : A_1 \sim_R A_2 \\ \Gamma, x : A_1/R; \Delta \vdash \gamma_2 : b_1 \sim_{R'} b_2 \\ b_3 = b_2\{x \triangleright \mathbf{sym}\, \gamma_1/x\} \\ \Gamma \vdash A_1 : \star/R \\ \Gamma \vdash A_2 : \star/R \\ (\rho = +) \vee (x \notin \mathsf{fv}\, |b_1|) \\ (\rho = +) \vee (x \notin \mathsf{fv}\, |b_3|) \\ \Gamma \vdash (\lambda^\rho x\!:\!A_1/R.b_2) : B/R' \\ R \leq R'\end{array}}{\Gamma; \Delta \vdash (\lambda^\rho x\!:\!\gamma_1.\gamma_2) : (\lambda^\rho x\!:\!A_1/R.b_1) \sim_{R'} (\lambda^\rho x\!:\!A_2/R.b_3)} \quad \textsc{An\_AbsCong}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vdash \gamma_1 : a_1 \sim_{R'} b_1 \\ \Gamma; \Delta \vdash \gamma_2 : a_2 \sim_R b_2 \\ \Gamma \vdash a_1\ {a_2}^\rho : A/R' \\ \Gamma \vdash b_1\ {b_2}^\rho : B/R' \\ \Gamma; \widetilde{\Gamma} \vdash \gamma_3 : A \sim_{R'} B \\ R \leq R'\end{array}}{\Gamma; \Delta \vdash \gamma_1\ \gamma_2^\rho : a_1\ {a_2}^\rho \sim_{R'} b_1\ {b_2}^\rho} \quad \textsc{An\_AppCong}$$

$$\frac{\begin{array}{l}\Gamma;\Delta\vdash\gamma:\Pi^{\rho}x\!:\!A_1/R_1\to B_1\sim_R\Pi^{\rho}x\!:\!A_2/R_2\to B_2\\ R_3\leq R_1\\ R_3\leq R_2\end{array}}{\Gamma;\Delta\vdash\mathbf{piFst}\,\gamma:A_1\sim_{R_3}A_2}\ \textsc{An\_PiFst}$$

$$\frac{\begin{array}{l}\Gamma;\Delta\vdash\gamma_1:\Pi^{\rho}x\!:\!A_1/R_1\to B_1\sim_{R'}\Pi^{\rho}x\!:\!A_2/R_2\to B_2\\ \Gamma;\Delta\vdash\gamma_2:a_1\sim_R a_2\\ \Gamma\vdash a_1:A_1/R\\ \Gamma\vdash a_2:A_2/R\\ R\leq R_1\\ R\leq R_2\end{array}}{\Gamma;\Delta\vdash\gamma_1@\gamma_2:B_1\{a_1/x\}\sim_{R'}B_2\{a_2/x\}}\ \textsc{An\_PiSnd}$$

$$\frac{\begin{array}{l}\Gamma;\Delta\vdash\gamma_1:\phi_1\sim\phi_2\\ \Gamma,c:\phi_1;\Delta\vdash\gamma_3:B_1\sim_R B_2\\ B_3=B_2\{c\triangleright\mathbf{sym}\,\gamma_1/c\}\\ \Gamma\vdash\forall c\!:\!\phi_1.B_1:\star/R\\ \Gamma\vdash\forall c\!:\!\phi_2.B_3:\star/R\\ \Gamma\vdash\forall c\!:\!\phi_1.B_2:\star/R\end{array}}{\Gamma;\Delta\vdash(\forall c\!:\!\gamma_1.\gamma_3):(\forall c\!:\!\phi_1.B_1)\sim_R(\forall c\!:\!\phi_2.B_3)}\ \textsc{An\_CPiCong}$$

$$\frac{\begin{array}{l}\Gamma;\Delta\vdash\gamma_1:\phi_1\sim\phi_2\\ \Gamma,c:\phi_1;\Delta\vdash\gamma_3:a_1\sim_R a_2\\ a_3=a_2\{c\triangleright\mathbf{sym}\,\gamma_1/c\}\\ \Gamma\vdash(\Lambda c\!:\!\phi_1.a_1):\forall c\!:\!\phi_1.B_1/R\\ \Gamma\vdash(\Lambda c\!:\!\phi_1.a_2):B/R\\ \Gamma\vdash(\Lambda c\!:\!\phi_2.a_3):\forall c\!:\!\phi_2.B_2/R\\ \Gamma;\widetilde{\Gamma}\vdash\gamma_4:\forall c\!:\!\phi_1.B_1\sim_R\forall c\!:\!\phi_2.B_2\end{array}}{\Gamma;\Delta\vdash(\lambda c\!:\!\gamma_1.\gamma_3@\gamma_4):(\Lambda c\!:\!\phi_1.a_1)\sim_R(\Lambda c\!:\!\phi_2.a_3)}\ \textsc{An\_CAbsCong}$$

$$\frac{\begin{array}{l}\Gamma;\Delta\vdash\gamma_1:a_1\sim_R b_1\\ \Gamma;\widetilde{\Gamma}\vdash\gamma_2:a_2\sim_{R'}b_2\\ \Gamma;\widetilde{\Gamma}\vdash\gamma_3:a_3\sim_{R'}b_3\\ \Gamma\vdash a_1[\gamma_2]:A/R\\ \Gamma\vdash b_1[\gamma_3]:B/R\\ \Gamma;\widetilde{\Gamma}\vdash\gamma_4:A\sim_R B\end{array}}{\Gamma;\Delta\vdash\gamma_1(\gamma_2,\gamma_3):a_1[\gamma_2]\sim_R b_1[\gamma_3]}\ \textsc{An\_CAppCong}$$

$$\frac{\begin{array}{l}\Gamma;\Delta\vdash\gamma_1:(\forall c_1\!:\!a\sim_{A/R}a'.B_1)\sim_{R_0}(\forall c_2\!:\!b\sim_{B/R'}b'.B_2)\\ \Gamma;\widetilde{\Gamma}\vdash\gamma_2:a\sim_R a'\\ \Gamma;\widetilde{\Gamma}\vdash\gamma_3:b\sim_{R'}b'\end{array}}{\Gamma;\Delta\vdash\gamma_1@(\gamma_2\sim\gamma_3):B_1\{\gamma_2/c_1\}\sim_{R_0}B_2\{\gamma_3/c_2\}}\ \textsc{An\_CPiSnd}$$

$$\frac{\begin{array}{l}\Gamma;\Delta\vdash\gamma_1:a\sim_{R_1}a'\\ \Gamma;\Delta\vdash\gamma_2:a\sim_{A/R_1}a'\sim b\sim_{B/R_2}b'\end{array}}{\Gamma;\Delta\vdash\gamma_1\triangleright\gamma_2:b\sim_{R_2}b'}\ \textsc{An\_Cast}$$

$$\frac{\begin{array}{l}\Gamma;\Delta\vdash\gamma:(a\sim_{A/R_1}a')\sim(b\sim_{B/R_2}b')\\ R_1\leq R_0\\ R_2\leq R_0\end{array}}{\Gamma;\Delta\vdash\mathbf{isoSnd}\,\gamma:A\sim_{R_0}B}\ \textsc{An\_IsoSnd}$$

$\boxed{\vdash\Gamma}$ context wellformedness

$$\frac{}{\vdash \varnothing} \quad \textsc{An\_Empty}$$

$$\frac{\begin{array}{l}\vdash \Gamma \\ \Gamma \vdash A : \star / R \\ x \notin \mathsf{dom}\,\Gamma\end{array}}{\vdash \Gamma, x : A/R} \quad \textsc{An\_ConsTm}$$

$$\frac{\begin{array}{l}\vdash \Gamma \\ \Gamma \vdash \phi \ \mathsf{ok} \\ c \notin \mathsf{dom}\,\Gamma\end{array}}{\vdash \Gamma, c : \phi} \quad \textsc{An\_ConsCo}$$

$\boxed{\vdash \Sigma}$ signature wellformedness

$$\frac{}{\vdash \varnothing} \quad \textsc{An\_Sig\_Empty}$$

$$\frac{\begin{array}{l}\vdash \Sigma \\ \varnothing \vdash A : \star / R \\ \varnothing \vdash a : A/R \\ F \notin \mathsf{dom}\,\Sigma\end{array}}{\vdash \Sigma \cup \{F \sim a : A/R\}} \quad \textsc{An\_Sig\_ConsAx}$$

$\boxed{\Gamma \vdash a \rightsquigarrow b}$ single-step, weak head reduction to values for annotated language

$$\frac{\Gamma \vdash a \rightsquigarrow a'}{\Gamma \vdash a\ b^{\rho} \rightsquigarrow a'\ b^{\rho}} \quad \textsc{An\_AppLeft}$$

$$\frac{\mathsf{Value}\ (\lambda^{\rho} x\!:\!A/R.w)}{\Gamma \vdash (\lambda^{\rho} x\!:\!A/R.w)\ a^{\rho} \rightsquigarrow w\{a/x\}} \quad \textsc{An\_AppAbs}$$

$$\frac{\Gamma \vdash a \rightsquigarrow a'}{\Gamma \vdash a[\gamma] \rightsquigarrow a'[\gamma]} \quad \textsc{An\_CAppLeft}$$

$$\frac{}{\Gamma \vdash (\Lambda c\!:\!\phi.b)[\gamma] \rightsquigarrow b\{\gamma/c\}} \quad \textsc{An\_CAppCAbs}$$

$$\frac{\begin{array}{c}\Gamma \vdash A : \star / R \\ \Gamma, x : A/R \vdash b \rightsquigarrow b'\end{array}}{\Gamma \vdash (\lambda^{-} x\!:\!A/R.b) \rightsquigarrow (\lambda^{-} x\!:\!A/R.b')} \quad \textsc{An\_AbsTerm}$$

$$\frac{F \sim a : A/R \in \Sigma_1}{\Gamma \vdash F \rightsquigarrow a} \quad \textsc{An\_Axiom}$$

$$\frac{\Gamma \vdash a \rightsquigarrow a'}{\Gamma \vdash a \triangleright \gamma \rightsquigarrow a' \triangleright \gamma} \quad \textsc{An\_ConvTerm}$$

$$\frac{\mathsf{Value}\ v}{\Gamma \vdash (v \triangleright \gamma_1) \triangleright \gamma_2 \rightsquigarrow v \triangleright (\gamma_1; \gamma_2)} \quad \textsc{An\_Combine}$$

$$\frac{\begin{array}{l}\mathsf{Value}\ v \\ \Gamma; \widetilde{\Gamma} \vdash \gamma : \Pi^{\rho} x_1\!:\!A_1/R_1 \to B_1 \sim_R \Pi^{\rho} x_2\!:\!A_2/R_2 \to B_2 \\ b' = b \triangleright \mathbf{sym}\,(\mathbf{piFst}\,\gamma) \\ \gamma' = \gamma@(b' \mid=\mid_{(\mathbf{piFst}\,\gamma)} b)\end{array}}{\Gamma \vdash (v \triangleright \gamma)\ b^{\rho} \rightsquigarrow (v\ b'^{\rho}) \triangleright \gamma'} \quad \textsc{An\_Push}$$

$$\frac{\begin{array}{l}\mathsf{Value}\ v\\ \Gamma;\widetilde{\Gamma}\vdash\gamma:\forall c_1\!:\!\phi_1.A_1\sim_R\forall c_2\!:\!\phi_2.A_2\\ \gamma_1'=\gamma_1\triangleright\mathbf{sym}\,(\mathbf{cpiFst}\,\gamma)\\ \gamma'=\gamma@(\gamma_1'\sim\gamma_1)\end{array}}{\Gamma\vdash(v\triangleright\gamma)[\gamma_1]\rightsquigarrow(v[\gamma_1'])\triangleright\gamma'}\quad\textsc{An\_CPush}$$

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Definition rules:        146 good    0 bad
Definition rule clauses: 430 good    0 bad
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