$tmvar$, x, y, f, m, n	variables
$covar$, c	coercion variables
$datacon$, K	
$const$, T	
$tyfam$, F	
$index$, i	indices

role, R	::=			Role
	|	**Nom**		
	|	**Rep**		
	|	$R_1 \cap R_2$	S	
relflag, ρ	::=			relevance flag
	|	$+$		
	|	$-$		
constraint, ϕ	::=			props
	|	$a \sim_{A/R} b$		
	|	(ϕ)	S	
	|	$\phi\{b/x\}$	S	
	|	$\lvert\phi\rvert$	S	
tm, a, b, v, w, A, B	::=			types and kinds
	|	$\star$		
	|	x		
	|	$\lambda^{\rho} x : A/R.b$	bind x in b	
	|	$\lambda^{R,\rho} x.b$	bind x in b	
	|	$a\ b^{R,\rho}$		
	|	F		
	|	T		
	|	$\Pi^{\rho} x : A/R \to B$	bind x in B	
	|	$a \triangleright \gamma$		
	|	$\forall c : \phi.B$	bind c in B	
	|	$\Lambda c : \phi.b$	bind c in b	
	|	$\Lambda c.b$	bind c in b	
	|	$a[\gamma]$		
	|	$\square$		
	|	K		
	|	**match** a **with** *brs*		
	|	$a\{b/x\}$	S	
	|	a	S	
	|	$a\{\gamma/c\}$	S	
	|	a	S	
	|	(a)	S	
	|	a	S	parsing precedence is hard
	|	$\lvert a\rvert$	S	
	|	**Int**	S	
	|	**Bool**	S	
	|	**Nat**	S	
	|	**Vec**	S	
	|	0	S	
	|	S	S	
	|	**True**	S	
	|	**Fix**	S	

	|	$a \to b$	S	
	|	$\phi \Rightarrow A$	S	
	|	$a\ b$	S	
	|	$\lambda^R x.a$	S	
	|	$\lambda x : A.a$	S	
	|	$\forall\, x : A/R \to B$	S	
brs	::=			case branches
	|	**none**		
	|	$K \Rightarrow a; brs$		
	|	$brs\{a/x\}$	S	
	|	$brs\{\gamma/c\}$	S	
	|	(brs)	S	
$co,\ \gamma$	::=			explicit coercions
	|	$\bullet$		
	|	c		
	|	**red** $a\ b$		
	|	**refl** a		
	|	$(a\ \mid=\mid_\gamma\ b)$		
	|	**sym** γ		
	|	$\gamma_1; \gamma_2$		
	|	$\Pi^\rho x : \gamma_1.\gamma_2$	bind x in γ_2	
	|	$\lambda^{R,\rho} x : \gamma_1.\gamma_2$	bind x in γ_2	
	|	$\gamma_1\ \gamma_2^{R,\rho}$		
	|	**piFst** γ		
	|	**cpiFst** γ		
	|	**isoSnd** γ		
	|	$\gamma_1 @ \gamma_2$		
	|	$\forall c : \gamma_1.\gamma_3$	bind c in γ_3	
	|	$\lambda c : \gamma_1.\gamma_3 @ \gamma_4$	bind c in γ_3	
	|	$\gamma(\gamma_1, \gamma_2)$		
	|	$\gamma @ (\gamma_1 \sim \gamma_2)$		
	|	$\gamma_1 \triangleright \gamma_2$		
	|	$\gamma_1 \sim_A \gamma_2$		
	|	**conv** $\phi_1 \sim_\gamma \phi_2$		
	|	**eta** a		
	|	**left** $\gamma\,\gamma'$		
	|	**right** $\gamma\,\gamma'$		
	|	(γ)	S	
	|	γ	S	
	|	$\gamma\{a/x\}$	S	
sig_sort	::=			signature classifier
	|	**Cs** A		
	|	**Ax** $a\ A\ R$		

sort	::=			binding classifier
	|	$\mathbf{Tm}\,A\,R$		
	|	$\mathbf{Co}\,\phi$		
context, Γ	::=			contexts
	|	$\varnothing$		
	|	$\Gamma, x : A/R$		
	|	$\Gamma, c : \phi$		
	|	$\Gamma\{b/x\}$	M	
	|	$\Gamma\{\gamma/c\}$	M	
	|	Γ, Γ'	M	
	|	$\lvert\Gamma\rvert$	M	
	|	(Γ)	M	
	|	Γ	M	
available_props, Δ	::=			
	|	$\varnothing$		
	|	Δ, c		
	|	$\widetilde{\Gamma}$	M	
	|	(Δ)	M	
sig, Σ	::=			signatures
	|	$\varnothing$		
	|	$\Sigma \cup \{\, T : A/R\}$		
	|	$\Sigma \cup \{\, F \sim a : A/R\}$		
	|	Σ_0	M	
	|	Σ_1	M	
	|	$\lvert\Sigma\rvert$	M	
terminals	::=			
	|	$\leftrightarrow$		
	|	$\Leftrightarrow$		
	|	$\longrightarrow$		
	|	min		
	|	$\equiv$		
	|	$\forall$		
	|	$\in$		
	|	$\notin$		
	|	$\Leftarrow$		
	|	$\Rightarrow$		
	|	$\Rightarrow^*$		
	|	$\rightarrow$		
	|	Λ		
	|			
	|	$\square$		
	|	$\vdash$		
	|	$\dashv$		

$\mid$	$\models$
$\mid$	$\models$
$\mid$	$\neq$
$\mid$	$\triangleright$
$\mid$	ok
$\mid$	$\_$
$\mid$	$\rightsquigarrow$
$\mid$	$\rightsquigarrow^*$
$\mid$	$\rightsquigarrow$
$\mid$	$\varnothing$
$\mid$	$\circ$
$\mid$	fv
$\mid$	dom
$\mid$	$\sim$
$\mid$	$\asymp$
$\mid$	$\mid$
$\mid$	$\bullet$
$\mid$	$\mathbf{fst}$
$\mid$	$\mathbf{snd}$
$\mid$	$\mid\Rightarrow\mid$
$\mid$	$\vdash_=$
$\mid$	$\mathbf{refl_2}$
$\mid$	$++$

$formula,\ \psi$	$::=$	
	$\mid$	$judgement$
	$\mid$	$x : A/R\ \in\ \Gamma$
	$\mid$	$c : \phi\ \in\ \Gamma$
	$\mid$	$T : A/R\ \in\ \Sigma$
	$\mid$	$F \sim a : A/R\ \in\ \Sigma$
	$\mid$	$K : T\,\Gamma\ \in\ \Sigma$
	$\mid$	$x\ \in\ \Delta$
	$\mid$	$c\ \in\ \Delta$
	$\mid$	$c\,\mathbf{not\,relevant}\ \in\ \gamma$
	$\mid$	$x\ \not\in\ \mathsf{fv}\,a$
	$\mid$	$x\ \not\in\ \mathsf{dom}\,\Gamma$
	$\mid$	$c\ \not\in\ \mathsf{dom}\,\Gamma$
	$\mid$	$T\ \not\in\ \mathsf{dom}\,\Sigma$
	$\mid$	$F\ \not\in\ \mathsf{dom}\,\Sigma$
	$\mid$	$a = b$
	$\mid$	$\phi_1 = \phi_2$
	$\mid$	$\Gamma_1 = \Gamma_2$
	$\mid$	$\gamma_1 = \gamma_2$
	$\mid$	$\neg\psi$
	$\mid$	$\psi_1 \wedge \psi_2$
	$\mid$	$\psi_1 \vee \psi_2$

	$\mid$	$\psi_1 \Rightarrow \psi_2$	
	$\mid$	(ψ)	
	$\mid$	ψ	
	$\mid$	$c : (a : A \sim b : B) \in \Gamma$	
	$\mid$		suppress lc hypothesis generated by Ott
JSubRole	$::=$		
	$\mid$	$R_1 \leq R_2$	Subroling judgement
JValue	$::=$		
	$\mid$	$\textbf{CoercedValue}\, A$	Values with at most one coercion at the top
	$\mid$	$\mathsf{Value}\ A$	values
	$\mid$	$\textbf{ValueType}\, A$	Types with head forms (erased language)
Jconsistent	$::=$		
	$\mid$	$\textbf{consistent}\, a\, b$	(erased) types do not differ in their heads
Jerased	$::=$		
	$\mid$	$erased\_tm a$	
JChk	$::=$		
	$\mid$	$(\rho = +) \vee (x \notin \mathsf{fv}\ A)$	irrelevant argument check
Jpar	$::=$		
	$\mid$	$\vDash a \Rightarrow b$	parallel reduction (implicit language)
	$\mid$	$\vdash a \Rightarrow^* b$	multistep parallel reduction
	$\mid$	$\vdash a \Leftrightarrow b$	parallel reduction to a common term
Jbeta	$::=$		
	$\mid$	$\vDash a > b$	primitive reductions on erased terms
	$\mid$	$\vDash a \leadsto b$	single-step head reduction for implicit language
	$\mid$	$\vDash a \leadsto^* b$	multistep reduction
Jett	$::=$		
	$\mid$	$\Gamma \vDash \phi\ \ \mathsf{ok}$	Prop wellformedness
	$\mid$	$\Gamma \vDash a : A/R$	typing
	$\mid$	$\Gamma; \Delta \vDash \phi_1 \equiv \phi_2$	prop equality
	$\mid$	$\Gamma; \Delta \vDash a \equiv b : A/R$	definitional equality
	$\mid$	$\vDash \Gamma$	context wellformedness
Jsig	$::=$		
	$\mid$	$\vDash \Sigma$	signature wellformedness
Jann	$::=$		
	$\mid$	$\Gamma \vdash \phi\ \ \mathsf{ok}$	prop wellformedness
	$\mid$	$\Gamma \vdash a : A/R$	typing
	$\mid$	$\Gamma; \Delta \vdash \gamma : \phi_1 \sim \phi_2$	coercion between props

	|	$\Gamma;\Delta \vdash \gamma : A \sim_R B$	coercion between types
	|	$\vdash \Gamma$	context wellformedness
	|	$\vdash \Sigma$	signature wellformedness
Jred	::=		
	|	$\Gamma \vdash a \leadsto b$	single-step, weak head reduction to values for annotated langu
judgement	::=		
	|	*JSubRole*	
	|	*JValue*	
	|	*Jconsistent*	
	|	*Jerased*	
	|	*JChk*	
	|	*Jpar*	
	|	*Jbeta*	
	|	*Jett*	
	|	*Jsig*	
	|	*Jann*	
	|	*Jred*	
user_syntax	::=		
	|	*tmvar*	
	|	*covar*	
	|	*datacon*	
	|	*const*	
	|	*tyfam*	
	|	*index*	
	|	*role*	
	|	*relflag*	
	|	*constraint*	
	|	*tm*	
	|	*brs*	
	|	*co*	
	|	*sig_sort*	
	|	*sort*	
	|	*context*	
	|	*available_props*	
	|	*sig*	
	|	*terminals*	
	|	*formula*	

$\boxed{R_1 \leq R_2}$ Subroling judgement

$$\frac{}{\textbf{Nom} \leq \textbf{Rep}} \quad \text{NomRep}$$

$$\frac{}{R \leq R} \quad \text{Refl}$$

$$\frac{R_1 \leq R_2 \qquad R_2 \leq R_3}{R_1 \leq R_3} \quad \text{Trans}$$

$\boxed{\textbf{CoercedValue}\ A}$ Values with at most one coercion at the top

$$\frac{\mathsf{Value}\ a}{\mathbf{CoercedValue}\ a}\quad \text{CV}$$

$$\frac{\mathsf{Value}\ a}{\mathbf{CoercedValue}\ (a \rhd \gamma)}\quad \text{CC}$$

$\boxed{\mathsf{Value}\ A}$ values

$$\frac{}{\mathsf{Value}\ \star}\quad \text{VALUE\_STAR}$$

$$\frac{}{\mathsf{Value}\ \Pi^{\rho} x\!:\!A/R \to B}\quad \text{VALUE\_PI}$$

$$\frac{}{\mathsf{Value}\ \forall c\!:\!\phi.B}\quad \text{VALUE\_CPI}$$

$$\frac{}{\mathsf{Value}\ \lambda^{+} x\!:\!A/R.a}\quad \text{VALUE\_ABSREL}$$

$$\frac{}{\mathsf{Value}\ \lambda^{R,+} x.a}\quad \text{VALUE\_UABSREL}$$

$$\frac{\mathsf{Value}\ a}{\mathsf{Value}\ \lambda^{R,-} x.a}\quad \text{VALUE\_UABSIRREL}$$

$$\frac{\mathbf{CoercedValue}\ a}{\mathsf{Value}\ \lambda^{-} x\!:\!A/R.a}\quad \text{VALUE\_ABSIRREL}$$

$$\frac{}{\mathsf{Value}\ \Lambda c\!:\!\phi.a}\quad \text{VALUE\_CABS}$$

$$\frac{}{\mathsf{Value}\ \Lambda c.a}\quad \text{VALUE\_UCABS}$$

$\boxed{\mathbf{ValueType}\ A}$ Types with head forms (erased language)

$$\frac{}{\mathbf{ValueType}\ \star}\quad \text{VALUE\_TYPE\_STAR}$$

$$\frac{}{\mathbf{ValueType}\ \Pi^{\rho} x\!:\!A/R \to B}\quad \text{VALUE\_TYPE\_PI}$$

$$\frac{}{\mathbf{ValueType}\ \forall c\!:\!\phi.B}\quad \text{VALUE\_TYPE\_CPI}$$

$\boxed{\mathbf{consistent}\ a\ b}$ (erased) types do not differ in their heads

$$\frac{}{\mathbf{consistent}\ \star\ \star}\quad \text{CONSISTENT\_A\_STAR}$$

$$\frac{}{\mathbf{consistent}\ (\Pi^{\rho} x_1\!:\!A_1/R \to B_1)\ (\Pi^{\rho} x_2\!:\!A_2/R \to B_2)}\quad \text{CONSISTENT\_A\_PI}$$

$$\frac{}{\mathbf{consistent}\ (\forall c_1\!:\!\phi_1.A_1)\ (\forall c_2\!:\!\phi_2.A_2)}\quad \text{CONSISTENT\_A\_CPI}$$

$$\frac{\neg\mathbf{ValueType}\ b}{\mathbf{consistent}\ a\ b}\quad \text{CONSISTENT\_A\_STEP\_R}$$

$$\frac{\neg\mathbf{ValueType}\ a}{\mathbf{consistent}\ a\ b}\quad \text{CONSISTENT\_A\_STEP\_L}$$

$\boxed{erased\_tm\, a}$

$$\frac{}{erased\_tm\,\Box}\quad \text{ERASED\_A\_BULLET}$$

$$\frac{}{erased\_tm\star} \quad \textsc{erased\_a\_Star}$$

$$\frac{}{erased\_tm x} \quad \textsc{erased\_a\_Var}$$

$$\frac{erased\_tm a}{erased\_tm(\lambda^{R,\rho}x.a)} \quad \textsc{erased\_a\_Abs}$$

$$\frac{\begin{array}{c}erased\_tm a\\ erased\_tm b\end{array}}{erased\_tm(a\ b^{R,\rho})} \quad \textsc{erased\_a\_App}$$

$$\frac{\begin{array}{c}erased\_tm A\\ erased\_tm B\end{array}}{erased\_tm(\Pi^{\rho}x\!:\!A/R \to B)} \quad \textsc{erased\_a\_Pi}$$

$$\frac{\begin{array}{c}erased\_tm a\\ erased\_tm b\\ erased\_tm A\\ erased\_tm B\end{array}}{erased\_tm(\forall c\!:\!a \sim_{A/R} b.B)} \quad \textsc{erased\_a\_CPi}$$

$$\frac{erased\_tm b}{erased\_tm(\Lambda c.b)} \quad \textsc{erased\_a\_CAbs}$$

$$\frac{erased\_tm a}{erased\_tm(a[\bullet])} \quad \textsc{erased\_a\_CApp}$$

$$\frac{}{erased\_tm F} \quad \textsc{erased\_a\_Fam}$$

$$\frac{}{erased\_tm T} \quad \textsc{erased\_a\_Const}$$

$\boxed{(\rho = +) \vee (x \notin \mathsf{fv}\ A)}$ irrelevant argument check

$$\frac{}{(+ = +) \vee (x \notin \mathsf{fv}\ A)} \quad \textsc{Rho\_Rel}$$

$$\frac{x\ \notin\ \mathsf{fv} A}{(- = +) \vee (x \notin \mathsf{fv}\ A)} \quad \textsc{Rho\_IrrRel}$$

$\boxed{\vDash a \Rightarrow b}$ parallel reduction (implicit language)

$$\frac{}{\vDash a \Rightarrow a} \quad \textsc{Par\_Refl}$$

$$\frac{\begin{array}{c}\vDash a \Rightarrow (\lambda^{R,\rho}x.a')\\ \vDash b \Rightarrow b'\end{array}}{\vDash a\ b^{R,\rho} \Rightarrow a'\{b'/x\}} \quad \textsc{Par\_Beta}$$

$$\frac{\begin{array}{c}\vDash a \Rightarrow a'\\ \vDash b \Rightarrow b'\end{array}}{\vDash a\ b^{R,\rho} \Rightarrow a'\ b'^{R,\rho}} \quad \textsc{Par\_App}$$

$$\frac{\vDash a \Rightarrow (\Lambda c.a')}{\vDash a[\bullet] \Rightarrow a'\{\bullet/c\}} \quad \textsc{Par\_CBeta}$$

$$\frac{\vDash a \Rightarrow a'}{\vDash a[\bullet] \Rightarrow a'[\bullet]} \quad \textsc{Par\_CApp}$$

$$\frac{\vDash a \Rightarrow a'}{\vDash \lambda^{R,\rho} x.a \Rightarrow \lambda^{R,\rho} x.a'} \quad \textsc{Par\_Abs}$$

$$\frac{\vDash A \Rightarrow A' \quad \vDash B \Rightarrow B'}{\vDash \Pi^{\rho} x\!:\!A/R \to B \Rightarrow \Pi^{\rho} x\!:\!A'/R \to B'} \quad \textsc{Par\_Pi}$$

$$\frac{\vDash a \Rightarrow a'}{\vDash \Lambda c.a \Rightarrow \Lambda c.a'} \quad \textsc{Par\_CAbs}$$

$$\frac{\vDash A \Rightarrow A' \quad \vDash B \Rightarrow B' \quad \vDash a \Rightarrow a' \quad \vDash A_1 \Rightarrow A_1'}{\vDash \forall c\!:\!A \sim_{A_1/R} B.a \Rightarrow \forall c\!:\!A' \sim_{A_1'/R} B'.a'} \quad \textsc{Par\_CPi}$$

$$\frac{F \sim a : A/R \in \Sigma_0}{\vDash F \Rightarrow a} \quad \textsc{Par\_Axiom}$$

$\boxed{\vdash a \Rightarrow^* b}$ multistep parallel reduction

$$\frac{}{\vdash a \Rightarrow^* a} \quad \textsc{MP\_Refl}$$

$$\frac{\vDash a \Rightarrow b \quad \vdash b \Rightarrow^* a'}{\vdash a \Rightarrow^* a'} \quad \textsc{MP\_Step}$$

$\boxed{\vdash a \Leftrightarrow b}$ parallel reduction to a common term

$$\frac{\vdash a_1 \Rightarrow^* b \quad \vdash a_2 \Rightarrow^* b}{\vdash a_1 \Leftrightarrow a_2} \quad \textsc{join}$$

$\boxed{\vDash a > b}$ primitive reductions on erased terms

$$\frac{\mathsf{Value}\ (\lambda^{R,\rho} x.v)}{\vDash (\lambda^{R,\rho} x.v)\ b^{R,\rho} > v\{b/x\}} \quad \textsc{Beta\_AppAbs}$$

$$\frac{}{\vDash (\Lambda c.a')[\bullet] > a'\{\bullet/c\}} \quad \textsc{Beta\_CAppCAbs}$$

$$\frac{F \sim a : A/R \in \Sigma_0}{\vDash F > a} \quad \textsc{Beta\_Axiom}$$

$\boxed{\vDash a \leadsto b}$ single-step head reduction for implicit language

$$\frac{\vDash a \leadsto a'}{\vDash \lambda^{R,-} x.a \leadsto \lambda^{R,-} x.a'} \quad \textsc{E\_AbsTerm}$$

$$\frac{\vDash a \leadsto a'}{\vDash a\ b^{R,\rho} \leadsto a'\ b^{R,\rho}} \quad \textsc{E\_AppLeft}$$

$$\frac{\vDash a \leadsto a'}{\vDash a[\bullet] \leadsto a'[\bullet]} \quad \textsc{E\_CAppLeft}$$

$$\frac{\mathsf{Value}\ (\lambda^{R,\rho} x.v)}{\vDash (\lambda^{R,\rho} x.v)\ a^{R,\rho} \leadsto v\{a/x\}} \quad \textsc{E\_AppAbs}$$

$$\frac{}{\vDash (\Lambda c.b)[\bullet] \leadsto b\{\bullet/c\}} \quad \textsc{E\_CAppCAbs}$$

$$\frac{F \sim a : A/R \in \Sigma_0}{\vDash F \leadsto a} \quad \textsc{E\_Axiom}$$

$\boxed{\vDash a \leadsto^* b}$ multistep reduction

$$\frac{}{\vDash a \leadsto^* a} \quad \textsc{Equal}$$

$$\frac{\begin{array}{l}\vDash a \leadsto b \\ \vDash b \leadsto^* a'\end{array}}{\vDash a \leadsto^* a'} \quad \textsc{Step}$$

$\boxed{\Gamma \vDash \phi \ \mathsf{ok}}$ Prop wellformedness

$$\frac{\begin{array}{l}\Gamma \vDash a : A/R \\ \Gamma \vDash b : A/R \\ \Gamma \vDash A : \star/R\end{array}}{\Gamma \vDash a \sim_{A/R} b \ \ \mathsf{ok}} \quad \textsc{E\_Wff}$$

$\boxed{\Gamma \vDash a : A/R}$ typing

$$\frac{\begin{array}{l}R_1 \leq R_2 \\ \Gamma \vDash a : A/R_1\end{array}}{\Gamma \vDash a : A/R_2} \quad \textsc{E\_SubRole}$$

$$\frac{\vDash \Gamma}{\Gamma \vDash \star : \star/R} \quad \textsc{E\_Star}$$

$$\frac{\begin{array}{l}\vDash \Gamma \\ x : A/R \in \Gamma\end{array}}{\Gamma \vDash x : A/R} \quad \textsc{E\_Var}$$

$$\frac{\begin{array}{l}\Gamma, x : A/R \vDash B : \star/R' \\ \Gamma \vDash A : \star/R \\ R \leq R'\end{array}}{\Gamma \vDash \Pi^\rho x\!:\!A/R \to B : \star/R'} \quad \textsc{E\_Pi}$$

$$\frac{\begin{array}{l}\Gamma, x : A/R \vDash a : B/R' \\ \Gamma \vDash A : \star/R \\ (\rho = +) \lor (x \notin \mathsf{fv}\ a) \\ R \leq R'\end{array}}{\Gamma \vDash \lambda^{R,\rho} x.a : (\Pi^\rho x\!:\!A/R \to B)/R'} \quad \textsc{E\_Abs}$$

$$\frac{\begin{array}{l}\Gamma \vDash b : \Pi^+ x\!:\!A/R \to B/R' \\ \Gamma \vDash a : A/R\end{array}}{\Gamma \vDash b\ \ a^{R,+} : B\{a/x\}/R'} \quad \textsc{E\_App}$$

$$\frac{\begin{array}{l}\Gamma \vDash b : \Pi^- x\!:\!A/R \to B/R' \\ \Gamma \vDash a : A/R\end{array}}{\Gamma \vDash b\ \Box^{R,-} : B\{a/x\}/R'} \quad \textsc{E\_IApp}$$

$$\frac{\begin{array}{l}\Gamma \vDash a : A/R \\ \Gamma; \widetilde{\Gamma} \vDash A \equiv B : \star/R \\ \Gamma \vDash B : \star/R\end{array}}{\Gamma \vDash a : B/R} \quad \textsc{E\_Conv}$$

$$\frac{\begin{array}{l}\Gamma, c : \phi \vDash B : \star/R \\ \Gamma \vDash \phi \ \mathsf{ok}\end{array}}{\Gamma \vDash \forall c{:}\phi.B : \star/R} \quad \text{E\_CPi}$$

$$\frac{\begin{array}{l}\Gamma, c : \phi \vDash a : B/R \\ \Gamma \vDash \phi \ \mathsf{ok}\end{array}}{\Gamma \vDash \Lambda c.a : \forall c{:}\phi.B/R} \quad \text{E\_CAbs}$$

$$\frac{\begin{array}{l}\Gamma \vDash a_1 : \forall c{:}(a \sim_{A/R} b).B_1/R' \\ \Gamma; \widetilde{\Gamma} \vDash a \equiv b : A/R\end{array}}{\Gamma \vDash a_1[\bullet] : B_1\{\bullet/c\}/R'} \quad \text{E\_CApp}$$

$$\frac{\begin{array}{l}\vDash \Gamma \\ F \sim a : A/R \ \in \ \Sigma_0 \\ \varnothing \vDash A : \star/R\end{array}}{\Gamma \vDash F : A/R} \quad \text{E\_Fam}$$

$\boxed{\Gamma; \Delta \vDash \phi_1 \equiv \phi_2}$ prop equality

$$\frac{\begin{array}{c}\Gamma; \Delta \vDash A_1 \equiv A_2 : A/R \\ \Gamma; \Delta \vDash B_1 \equiv B_2 : A/R\end{array}}{\Gamma; \Delta \vDash A_1 \sim_{A/R} B_1 \equiv A_2 \sim_{A/R} B_2} \quad \text{E\_PropCong}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash A \equiv B : \star/R \\ \Gamma \vDash A_1 \sim_{A/R} A_2 \ \mathsf{ok} \\ \Gamma \vDash A_1 \sim_{B/R} A_2 \ \mathsf{ok}\end{array}}{\Gamma; \Delta \vDash A_1 \sim_{A/R} A_2 \equiv A_1 \sim_{B/R} A_2} \quad \text{E\_IsoConv}$$

$$\frac{\Gamma; \Delta \vDash \forall c{:}\phi_1.B_1 \equiv \forall c{:}\phi_2.B_2 : \star/R}{\Gamma; \Delta \vDash \phi_1 \equiv \phi_2} \quad \text{E\_CPiFst}$$

$\boxed{\Gamma; \Delta \vDash a \equiv b : A/R}$ definitional equality

$$\frac{\begin{array}{l}\vDash \Gamma \\ c : (a \sim_{A/R} b) \ \in \ \Gamma \\ c \ \in \ \Delta\end{array}}{\Gamma; \Delta \vDash a \equiv b : A/R} \quad \text{E\_Assn}$$

$$\frac{\Gamma \vDash a : A/R}{\Gamma; \Delta \vDash a \equiv a : A/R} \quad \text{E\_Refl}$$

$$\frac{\Gamma; \Delta \vDash b \equiv a : A/R}{\Gamma; \Delta \vDash a \equiv b : A/R} \quad \text{E\_Sym}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a \equiv a_1 : A/R \\ \Gamma; \Delta \vDash a_1 \equiv b : A/R\end{array}}{\Gamma; \Delta \vDash a \equiv b : A/R} \quad \text{E\_Trans}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a \equiv b : A/R_1 \\ R_1 \leq R_2\end{array}}{\Gamma; \Delta \vDash a \equiv b : A/R_2} \quad \text{E\_Sub}$$

$$\frac{\begin{array}{l}\Gamma \vDash a_1 : B/R \\ \Gamma \vDash a_2 : B/R \\ \vDash a_1 > a_2\end{array}}{\Gamma; \Delta \vDash a_1 \equiv a_2 : B/R} \quad \text{E\_Beta}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash A_1 \equiv A_2 : \star / R \\ \Gamma, x : A_1/R; \Delta \vDash B_1 \equiv B_2 : \star / R' \\ \Gamma \vDash A_1 : \star / R \\ \Gamma \vDash \Pi^\rho x\!:\!A_1/R \to B_1 : \star / R' \\ \Gamma \vDash \Pi^\rho x\!:\!A_2/R \to B_2 : \star / R' \\ R \leq R'\end{array}}{\Gamma; \Delta \vDash (\Pi^\rho x\!:\!A_1/R \to B_1) \equiv (\Pi^\rho x\!:\!A_2/R \to B_2) : \star / R'} \quad \text{E\_PiCong}$$

$$\frac{\begin{array}{l}\Gamma, x : A_1/R; \Delta \vDash b_1 \equiv b_2 : B/R' \\ \Gamma \vDash A_1 : \star / R \\ R \leq R' \\ (\rho = +) \vee (x \notin \mathsf{fv}\ b_1) \\ (\rho = +) \vee (x \notin \mathsf{fv}\ b_2)\end{array}}{\Gamma; \Delta \vDash (\lambda^{R,\rho} x.b_1) \equiv (\lambda^{R,\rho} x.b_2) : (\Pi^\rho x\!:\!A_1/R \to B)/R'} \quad \text{E\_AbsCong}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a_1 \equiv b_1 : (\Pi^+ x\!:\!A/R \to B)/R' \\ \Gamma; \Delta \vDash a_2 \equiv b_2 : A/R\end{array}}{\Gamma; \Delta \vDash a_1\ \ {a_2}^{R,+} \equiv b_1\ \ {b_2}^{R,+} : (B\{a_2/x\})/R'} \quad \text{E\_AppCong}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a_1 \equiv b_1 : (\Pi^- x\!:\!A/R \to B)/R' \\ \Gamma \vDash a : A/R\end{array}}{\Gamma; \Delta \vDash a_1\ \square^{R,-} \equiv b_1\ \square^{R,-} : (B\{a/x\})/R'} \quad \text{E\_IAppCong}$$

$$\frac{\Gamma; \Delta \vDash \Pi^\rho x\!:\!A_1/R \to B_1 \equiv \Pi^\rho x\!:\!A_2/R \to B_2 : \star / R'}{\Gamma; \Delta \vDash A_1 \equiv A_2 : \star / R} \quad \text{E\_PiFst}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash \Pi^\rho x\!:\!A_1/R \to B_1 \equiv \Pi^\rho x\!:\!A_2/R \to B_2 : \star / R' \\ \Gamma; \Delta \vDash a_1 \equiv a_2 : A_1/R\end{array}}{\Gamma; \Delta \vDash B_1\{a_1/x\} \equiv B_2\{a_2/x\} : \star / R'} \quad \text{E\_PiSnd}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a_1 \sim_{A_1/R} b_1 \equiv a_2 \sim_{A_2/R} b_2 \\ \Gamma, c : a_1 \sim_{A_1/R} b_1; \Delta \vDash A \equiv B : \star / R' \\ \Gamma \vDash a_1 \sim_{A_1/R} b_1\ \ \mathsf{ok} \\ \Gamma \vDash \forall c\!:\!a_1 \sim_{A_1/R} b_1.A : \star / R' \\ \Gamma \vDash \forall c\!:\!a_2 \sim_{A_2/R} b_2.B : \star / R'\end{array}}{\Gamma; \Delta \vDash \forall c\!:\!a_1 \sim_{A_1/R} b_1.A \equiv \forall c\!:\!a_2 \sim_{A_2/R} b_2.B : \star / R'} \quad \text{E\_CPiCong}$$

$$\frac{\begin{array}{l}\Gamma, c : \phi_1; \Delta \vDash a \equiv b : B/R \\ \Gamma \vDash \phi_1\ \ \mathsf{ok}\end{array}}{\Gamma; \Delta \vDash (\Lambda c.a) \equiv (\Lambda c.b) : \forall c\!:\!\phi_1.B/R} \quad \text{E\_CAbsCong}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a_1 \equiv b_1 : (\forall c\!:\!(a \sim_{A/R} b).B)/R' \\ \Gamma; \widetilde{\Gamma} \vDash a \equiv b : A/R\end{array}}{\Gamma; \Delta \vDash a_1[\bullet] \equiv b_1[\bullet] : (B\{\bullet/c\})/R'} \quad \text{E\_CAppCong}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash \forall c\!:\!(a_1 \sim_{A/R} a_2).B_1 \equiv \forall c\!:\!(a_1' \sim_{A'/R'} a_2').B_2 : \star / R_0 \\ \Gamma; \widetilde{\Gamma} \vDash a_1 \equiv a_2 : A/R \\ \Gamma; \widetilde{\Gamma} \vDash a_1' \equiv a_2' : A'/R'\end{array}}{\Gamma; \Delta \vDash B_1\{\bullet/c\} \equiv B_2\{\bullet/c\} : \star / R_0} \quad \text{E\_CPiSnd}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a \equiv b : A/R \\ \Gamma; \Delta \vDash a \sim_{A/R} b \equiv a' \sim_{A'/R'} b'\end{array}}{\Gamma; \Delta \vDash a' \equiv b' : A'/R'} \quad \text{E\_Cast}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a \equiv b : A/R_1 \\ \Gamma; \widetilde{\Gamma} \vDash A \equiv B : \star/R_2 \\ R_1 \leq R_2\end{array}}{\Gamma; \Delta \vDash a \equiv b : B/R_2} \quad \text{E\_EqConv}$$

$$\frac{\Gamma; \Delta \vDash a \sim_{A/R} b \equiv a' \sim_{A'/R} b'}{\Gamma; \Delta \vDash A \equiv A' : \star/R} \quad \text{E\_IsoSnd}$$

$\boxed{\vDash \Gamma}$ context wellformedness

$$\frac{}{\vDash \varnothing} \quad \text{E\_Empty}$$

$$\frac{\begin{array}{l}\vDash \Gamma \\ \Gamma \vDash A : \star/R \\ x \notin \mathsf{dom}\,\Gamma\end{array}}{\vDash \Gamma, x : A/R} \quad \text{E\_ConsTm}$$

$$\frac{\begin{array}{l}\vDash \Gamma \\ \Gamma \vDash \phi \ \mathsf{ok} \\ c \notin \mathsf{dom}\,\Gamma\end{array}}{\vDash \Gamma, c : \phi} \quad \text{E\_ConsCo}$$

$\boxed{\vDash \Sigma}$ signature wellformedness

$$\frac{}{\vDash \varnothing} \quad \text{Sig\_Empty}$$

$$\frac{\begin{array}{l}\vDash \Sigma \\ \varnothing \vDash A : \star/R \\ \varnothing \vDash a : A/R' \\ F \notin \mathsf{dom}\,\Sigma \\ R' \leq R\end{array}}{\vDash \Sigma \cup \{F \sim a : A/R'\}} \quad \text{Sig\_ConsAx}$$

$\boxed{\Gamma \vdash \phi \ \mathsf{ok}}$ prop wellformedness

$$\frac{\begin{array}{l}\Gamma \vdash a : A/R \\ \Gamma \vdash b : B/R \\ |A| = |B|\end{array}}{\Gamma \vdash a \sim_{A/R} b \ \mathsf{ok}} \quad \text{An\_Wff}$$

$\boxed{\Gamma \vdash a : A/R}$ typing

$$\frac{\vdash \Gamma}{\Gamma \vdash \star : \star/R} \quad \text{An\_Star}$$

$$\frac{\begin{array}{l}\vdash \Gamma \\ x : A/R \in \Gamma\end{array}}{\Gamma \vdash x : A/R} \quad \text{An\_Var}$$

$$\frac{\begin{array}{l}\Gamma, x : A/R \vdash B : \star/R' \\ \Gamma \vdash A : \star/R\end{array}}{\Gamma \vdash \Pi^{\rho} x\!:\!A/R \to B : \star/R'} \quad \text{An\_Pi}$$

$$\frac{\begin{array}{l}\Gamma \vdash A : \star/R \\ \Gamma, x : A/R \vdash a : B/R' \\ (\rho = +) \vee (x \notin \mathsf{fv}\ |a|) \\ R \leq R'\end{array}}{\Gamma \vdash \lambda^{\rho} x\!:\!A/R.a : (\Pi^{\rho} x\!:\!A/R \to B)/R'} \quad \textsc{An\_Abs}$$

$$\frac{\begin{array}{l}\Gamma \vdash b : (\Pi^{\rho} x\!:\!A/R \to B)/R' \\ \Gamma \vdash a : A/R\end{array}}{\Gamma \vdash b\ \ a^{R,\rho} : (B\{a/x\})/R'} \quad \textsc{An\_App}$$

$$\frac{\begin{array}{l}\Gamma \vdash a : A/R \\ \Gamma; \widetilde{\Gamma} \vdash \gamma : A \sim_R B \\ \Gamma \vdash B : \star/R\end{array}}{\Gamma \vdash a \triangleright \gamma : B/R} \quad \textsc{An\_Conv}$$

$$\frac{\begin{array}{l}\Gamma \vdash \phi\ \ \mathsf{ok} \\ \Gamma, c : \phi \vdash B : \star/R\end{array}}{\Gamma \vdash \forall c\!:\!\phi.B : \star/R} \quad \textsc{An\_CPi}$$

$$\frac{\begin{array}{l}\Gamma \vdash \phi\ \ \mathsf{ok} \\ \Gamma, c : \phi \vdash a : B/R\end{array}}{\Gamma \vdash \Lambda c\!:\!\phi.a : (\forall c\!:\!\phi.B)/R} \quad \textsc{An\_CAbs}$$

$$\frac{\begin{array}{l}\Gamma \vdash a_1 : (\forall c\!:\!a \sim_{A_1/R} b.B)/R' \\ \Gamma; \widetilde{\Gamma} \vdash \gamma : a \sim_R b\end{array}}{\Gamma \vdash a_1[\gamma] : B\{\gamma/c\}/R'} \quad \textsc{An\_CApp}$$

$$\frac{\begin{array}{l}\vdash \Gamma \\ F \sim a : A/R\ \in\ \Sigma_1 \\ \varnothing \vdash A : \star/R\end{array}}{\Gamma \vdash F : A/R} \quad \textsc{An\_Fam}$$

$\boxed{\Gamma; \Delta \vdash \gamma : \phi_1 \sim \phi_2}$ coercion between props

$$\frac{\begin{array}{l}\Gamma; \Delta \vdash \gamma_1 : A_1 \sim_R A_2 \\ \Gamma; \Delta \vdash \gamma_2 : B_1 \sim_R B_2 \\ \Gamma \vdash A_1 \sim_{A/R} B_1\ \ \mathsf{ok} \\ \Gamma \vdash A_2 \sim_{A/R} B_2\ \ \mathsf{ok}\end{array}}{\Gamma; \Delta \vdash (\gamma_1 \sim_A \gamma_2) : (A_1 \sim_{A/R} B_1) \sim (A_2 \sim_{A/R} B_2)} \quad \textsc{An\_PropCong}$$

$$\frac{\Gamma; \Delta \vdash \gamma : \forall c\!:\!\phi_1.A_2 \sim_R \forall c\!:\!\phi_2.B_2}{\Gamma; \Delta \vdash \mathbf{cpiFst}\,\gamma : \phi_1 \sim \phi_2} \quad \textsc{An\_CPiFst}$$

$$\frac{\Gamma; \Delta \vdash \gamma : \phi_1 \sim \phi_2}{\Gamma; \Delta \vdash \mathbf{sym}\,\gamma : \phi_2 \sim \phi_1} \quad \textsc{An\_IsoSym}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vdash \gamma : A \sim_R B \\ \Gamma \vdash a_1 \sim_{A/R} a_2\ \ \mathsf{ok} \\ \Gamma \vdash a_1' \sim_{B/R} a_2'\ \ \mathsf{ok} \\ |a_1| = |a_1'| \\ |a_2| = |a_2'|\end{array}}{\Gamma; \Delta \vdash \mathbf{conv}\ (a_1 \sim_{A/R} a_2) \sim_{\gamma} (a_1' \sim_{B/R} a_2') : (a_1 \sim_{A/R} a_2) \sim (a_1' \sim_{B/R} a_2')} \quad \textsc{An\_IsoConv}$$

$\boxed{\Gamma; \Delta \vdash \gamma : A \sim_R B}$ coercion between types

$$\frac{\begin{array}{l}\vdash \Gamma \\ c : a \sim_{A/R} b \in \Gamma \\ c \in \Delta\end{array}}{\Gamma; \Delta \vdash c : a \sim_R b} \quad \textsc{An\_Assn}$$

$$\frac{\Gamma \vdash a : A/R}{\Gamma; \Delta \vdash \mathbf{refl}\, a : a \sim_R a} \quad \textsc{An\_Refl}$$

$$\frac{\begin{array}{l}\Gamma \vdash a : A/R \\ \Gamma \vdash b : B/R \\ |a| = |b| \\ \Gamma; \widetilde{\Gamma} \vdash \gamma : A \sim_R B\end{array}}{\Gamma; \Delta \vdash (a \mid=\mid_\gamma b) : a \sim_R b} \quad \textsc{An\_EraseEq}$$

$$\frac{\begin{array}{l}\Gamma \vdash b : B/R \\ \Gamma \vdash a : A/R \\ \Gamma; \widetilde{\Gamma} \vdash \gamma_1 : B \sim_R A \\ \Gamma; \Delta \vdash \gamma : b \sim_R a\end{array}}{\Gamma; \Delta \vdash \mathbf{sym}\, \gamma : a \sim_R b} \quad \textsc{An\_Sym}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vdash \gamma_1 : a \sim_R a_1 \\ \Gamma; \Delta \vdash \gamma_2 : a_1 \sim_R b \\ \Gamma \vdash a : A/R \\ \Gamma \vdash a_1 : A_1/R \\ \Gamma; \widetilde{\Gamma} \vdash \gamma_3 : A \sim_R A_1\end{array}}{\Gamma; \Delta \vdash (\gamma_1; \gamma_2) : a \sim_R b} \quad \textsc{An\_Trans}$$

$$\frac{\begin{array}{l}\Gamma \vdash a_1 : B_0/R \\ \Gamma \vdash a_2 : B_1/R \\ |B_0| = |B_1| \\ \vDash |a_1| > |a_2|\end{array}}{\Gamma; \Delta \vdash \mathbf{red}\, a_1\, a_2 : a_1 \sim_R a_2} \quad \textsc{An\_Beta}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vdash \gamma_1 : A_1 \sim_{R'} A_2 \\ \Gamma, x : A_1/R; \Delta \vdash \gamma_2 : B_1 \sim_{R'} B_2 \\ B_3 = B_2\{x \triangleright \mathbf{sym}\, \gamma_1/x\} \\ \Gamma \vdash \Pi^\rho x\!:\!A_1/R \to B_1 : \star/R' \\ \Gamma \vdash \Pi^\rho x\!:\!A_1/R \to B_2 : \star/R' \\ \Gamma \vdash \Pi^\rho x\!:\!A_2/R \to B_3 : \star/R' \\ R \leq R'\end{array}}{\Gamma; \Delta \vdash \Pi^\rho x\!:\!\gamma_1.\gamma_2 : (\Pi^\rho x\!:\!A_1/R \to B_1) \sim_{R'} (\Pi^\rho x\!:\!A_2/R \to B_3)} \quad \textsc{An\_PiCong}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vdash \gamma_1 : A_1 \sim_R A_2 \\ \Gamma, x : A_1/R; \Delta \vdash \gamma_2 : b_1 \sim_{R'} b_2 \\ b_3 = b_2\{x \triangleright \mathbf{sym}\, \gamma_1/x\} \\ \Gamma \vdash A_1 : \star/R \\ \Gamma \vdash A_2 : \star/R \\ (\rho = +) \vee (x \notin \mathsf{fv}\, |b_1|) \\ (\rho = +) \vee (x \notin \mathsf{fv}\, |b_3|) \\ \Gamma \vdash (\lambda^\rho x\!:\!A_1/R.b_2) : B/R' \\ R \leq R'\end{array}}{\Gamma; \Delta \vdash (\lambda^{R,\rho} x\!:\!\gamma_1.\gamma_2) : (\lambda^\rho x\!:\!A_1/R.b_1) \sim_{R'} (\lambda^\rho x\!:\!A_2/R.b_3)} \quad \textsc{An\_AbsCong}$$

$$\frac{\begin{array}{c}\Gamma;\Delta \vdash \gamma_1 : a_1 \sim_{R'} b_1 \\ \Gamma;\Delta \vdash \gamma_2 : a_2 \sim_R b_2 \\ \Gamma \vdash a_1\ \ a_2{}^{R,\rho} : A/R' \\ \Gamma \vdash b_1\ \ b_2{}^{R,\rho} : B/R' \\ \Gamma;\widetilde{\Gamma} \vdash \gamma_3 : A \sim_{R'} B\end{array}}{\Gamma;\Delta \vdash \gamma_1\ \ \gamma_2^{R,\rho} : a_1\ \ a_2{}^{R,\rho} \sim_{R'} b_1\ \ b_2{}^{R,\rho}} \quad \textsc{An\_AppCong}$$

$$\frac{\Gamma;\Delta \vdash \gamma : \Pi^\rho x\!:\!A_1/R \to B_1 \sim_{R'} \Pi^\rho x\!:\!A_2/R \to B_2}{\Gamma;\Delta \vdash \mathbf{piFst}\,\gamma : A_1 \sim_R A_2} \quad \textsc{An\_PiFst}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vdash \gamma_1 : \Pi^\rho x\!:\!A_1/R \to B_1 \sim_{R'} \Pi^\rho x\!:\!A_2/R \to B_2 \\ \Gamma;\Delta \vdash \gamma_2 : a_1 \sim_R a_2 \\ \Gamma \vdash a_1 : A_1/R \\ \Gamma \vdash a_2 : A_2/R\end{array}}{\Gamma;\Delta \vdash \gamma_1@\gamma_2 : B_1\{a_1/x\} \sim_{R'} B_2\{a_2/x\}} \quad \textsc{An\_PiSnd}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vdash \gamma_1 : a_1 \sim_{A_1/R} b_1 \sim a_2 \sim_{A_2/R} b_2 \\ \Gamma, c : a_1 \sim_{A_1/R} b_1;\Delta \vdash \gamma_3 : B_1 \sim_{R'} B_2 \\ B_3 = B_2\{c \triangleright \mathbf{sym}\,\gamma_1/c\} \\ \Gamma \vdash \forall c\!:\!a_1 \sim_{A_1/R} b_1.B_1 : \star/R' \\ \Gamma \vdash \forall c\!:\!a_2 \sim_{A_2/R} b_2.B_3 : \star/R' \\ \Gamma \vdash \forall c\!:\!a_1 \sim_{A_1/R} b_1.B_2 : \star/R'\end{array}}{\Gamma;\Delta \vdash (\forall c\!:\!\gamma_1.\gamma_3) : (\forall c\!:\!a_1 \sim_{A_1/R} b_1.B_1) \sim_R (\forall c\!:\!a_2 \sim_{A_2/R} b_2.B_3)} \quad \textsc{An\_CPiCong}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vdash \gamma_1 : b_0 \sim_{A_1/R} b_1 \sim b_2 \sim_{A_2/R} b_3 \\ \Gamma, c : b_0 \sim_{A_1/R} b_1;\Delta \vdash \gamma_3 : a_1 \sim_{R'} a_2 \\ a_3 = a_2\{c \triangleright \mathbf{sym}\,\gamma_1/c\} \\ \Gamma \vdash (\Lambda c\!:\!b_0 \sim_{A_1/R} b_1.a_1) : \forall c\!:\!b_0 \sim_{A_1/R} b_1.B_1/R' \\ \Gamma \vdash (\Lambda c\!:\!b_0 \sim_{A_1/R} b_1.a_2) : B/R' \\ \Gamma \vdash (\Lambda c\!:\!b_2 \sim_{A_2/R} b_3.a_3) : \forall c\!:\!b_2 \sim_{A_2/R} b_3.B_2/R' \\ \Gamma;\widetilde{\Gamma} \vdash \gamma_4 : \forall c\!:\!b_0 \sim_{A_1/R} b_1.B_1 \sim_{R'} \forall c\!:\!\phi_2.B_2\end{array}}{\Gamma;\Delta \vdash (\lambda c\!:\!\gamma_1.\gamma_3@\gamma_4) : (\Lambda c\!:\!b_0 \sim_{A_1/R} b_1.a_1) \sim_{R'} (\Lambda c\!:\!b_2 \sim_{A_2/R} b_3.a_3)} \quad \textsc{An\_CAbsCong}$$

$$\frac{\begin{array}{c}\Gamma;\Delta \vdash \gamma_1 : a_1 \sim_R b_1 \\ \Gamma;\widetilde{\Gamma} \vdash \gamma_2 : a_2 \sim_{R'} b_2 \\ \Gamma;\widetilde{\Gamma} \vdash \gamma_3 : a_3 \sim_{R'} b_3 \\ \Gamma \vdash a_1[\gamma_2] : A/R \\ \Gamma \vdash b_1[\gamma_3] : B/R \\ \Gamma;\widetilde{\Gamma} \vdash \gamma_4 : A \sim_R B\end{array}}{\Gamma;\Delta \vdash \gamma_1(\gamma_2,\gamma_3) : a_1[\gamma_2] \sim_R b_1[\gamma_3]} \quad \textsc{An\_CAppCong}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vdash \gamma_1 : (\forall c_1\!:\!a \sim_{A/R} a'.B_1) \sim_{R_0} (\forall c_2\!:\!b \sim_{B/R'} b'.B_2) \\ \Gamma;\widetilde{\Gamma} \vdash \gamma_2 : a \sim_R a' \\ \Gamma;\widetilde{\Gamma} \vdash \gamma_3 : b \sim_{R'} b'\end{array}}{\Gamma;\Delta \vdash \gamma_1@(\gamma_2 \sim \gamma_3) : B_1\{\gamma_2/c_1\} \sim_{R_0} B_2\{\gamma_3/c_2\}} \quad \textsc{An\_CPiSnd}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vdash \gamma_1 : a \sim_{R_1} a' \\ \Gamma;\Delta \vdash \gamma_2 : a \sim_{A/R_1} a' \sim b \sim_{B/R_2} b'\end{array}}{\Gamma;\Delta \vdash \gamma_1 \triangleright \gamma_2 : b \sim_{R_2} b'} \quad \textsc{An\_Cast}$$

$$\frac{\Gamma;\Delta \vdash \gamma : (a \sim_{A/R} a') \sim (b \sim_{B/R} b')}{\Gamma;\Delta \vdash \mathbf{isoSnd}\,\gamma : A \sim_R B} \quad \textsc{An\_IsoSnd}$$

$\boxed{\vdash \Gamma}$ context wellformedness

$$\frac{}{\vdash \varnothing} \quad \text{AN\_EMPTY}$$

$$\frac{\begin{array}{l}\vdash \Gamma \\ \Gamma \vdash A : \star/R \\ x \notin \mathsf{dom}\,\Gamma\end{array}}{\vdash \Gamma, x : A/R} \quad \text{AN\_CONSTM}$$

$$\frac{\begin{array}{l}\vdash \Gamma \\ \Gamma \vdash \phi \;\; \mathsf{ok} \\ c \notin \mathsf{dom}\,\Gamma\end{array}}{\vdash \Gamma, c : \phi} \quad \text{AN\_CONSCO}$$

$\boxed{\vdash \Sigma}$ signature wellformedness

$$\frac{}{\vdash \varnothing} \quad \text{AN\_SIG\_EMPTY}$$

$$\frac{\begin{array}{l}\vdash \Sigma \\ \varnothing \vdash A : \star/R \\ \varnothing \vdash a : A/R \\ F \notin \mathsf{dom}\,\Sigma\end{array}}{\vdash \Sigma \cup \{F \sim a : A/R\}} \quad \text{AN\_SIG\_CONSAX}$$

$\boxed{\Gamma \vdash a \leadsto b}$ single-step, weak head reduction to values for annotated language

$$\frac{\Gamma \vdash a \leadsto a'}{\Gamma \vdash a\ b^{R,\rho} \leadsto a'\ b^{R,\rho}} \quad \text{AN\_APPLEFT}$$

$$\frac{\mathsf{Value}\ (\lambda^{\rho} x{:}A/R.w)}{\Gamma \vdash (\lambda^{\rho} x{:}A/R.w)\ a^{R,\rho} \leadsto w\{a/x\}} \quad \text{AN\_APPABS}$$

$$\frac{\Gamma \vdash a \leadsto a'}{\Gamma \vdash a[\gamma] \leadsto a'[\gamma]} \quad \text{AN\_CAPPLEFT}$$

$$\frac{}{\Gamma \vdash (\Lambda c{:}\phi.b)[\gamma] \leadsto b\{\gamma/c\}} \quad \text{AN\_CAPPCABS}$$

$$\frac{\begin{array}{c}\Gamma \vdash A : \star/R \\ \Gamma, x : A/R \vdash b \leadsto b'\end{array}}{\Gamma \vdash (\lambda^{-} x{:}A/R.b) \leadsto (\lambda^{-} x{:}A/R.b')} \quad \text{AN\_ABSTERM}$$

$$\frac{F \sim a : A/R \in \Sigma_1}{\Gamma \vdash F \leadsto a} \quad \text{AN\_AXIOM}$$

$$\frac{\Gamma \vdash a \leadsto a'}{\Gamma \vdash a \rhd \gamma \leadsto a' \rhd \gamma} \quad \text{AN\_CONVTERM}$$

$$\frac{\mathsf{Value}\ v}{\Gamma \vdash (v \rhd \gamma_1) \rhd \gamma_2 \leadsto v \rhd (\gamma_1; \gamma_2)} \quad \text{AN\_COMBINE}$$

$$\frac{\begin{array}{l}\mathsf{Value}\ v \\ \Gamma; \widetilde{\Gamma} \vdash \gamma : \Pi^{\rho} x_1{:}A_1/R \to B_1 \sim_{R'} \Pi^{\rho} x_2{:}A_2/R \to B_2 \\ b' = b \rhd \mathbf{sym}\,(\mathbf{piFst}\,\gamma) \\ \gamma' = \gamma @ (b' \mid=\mid_{(\mathbf{piFst}\,\gamma)}\ b)\end{array}}{\Gamma \vdash (v \rhd \gamma)\ b^{R,\rho} \leadsto (v\ b'^{R,\rho}) \rhd \gamma'} \quad \text{AN\_PUSH}$$

$$\frac{\begin{array}{l}\mathsf{Value}\ v \\ \Gamma; \widetilde{\Gamma} \vdash \gamma : \forall c_1 \!:\! a_1 \sim_{B_1/R} b_1 . A_1 \sim_{R'} \forall c_2 \!:\! a_2 \sim_{B_2/R} b_2 . A_2 \\ \gamma_1' = \gamma_1 \triangleright \mathbf{sym}\,(\mathbf{cpiFst}\,\gamma) \\ \gamma' = \gamma @ (\gamma_1' \sim \gamma_1)\end{array}}{\Gamma \vdash (v \triangleright \gamma)[\gamma_1] \leadsto (v[\gamma_1']) \triangleright \gamma'} \quad \textsc{An\_CPush}$$

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Definition rules:         146 good    0 bad
Definition rule clauses: 426 good    0 bad
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