A Full Type System of SEDEL

$\boxed{\Gamma \vdash A}$ *(Well formedness)*

$$\frac{}{\Gamma \vdash \top}\;\textsc{WF-top} \qquad \frac{}{\Gamma \vdash \mathsf{Int}}\;\textsc{WF-int} \qquad \frac{\Gamma \vdash A \qquad \Gamma \vdash B}{\Gamma \vdash A \to B}\;\textsc{WF-arrow} \qquad \frac{\Gamma \vdash A}{\Gamma \vdash \{l : A\}}\;\textsc{WF-record} \qquad \frac{\alpha * A \in \Gamma}{\Gamma \vdash \alpha}\;\textsc{WF-var}$$

$$\frac{\Gamma \vdash A \qquad \Gamma \vdash B \qquad \Gamma \vdash A * B}{\Gamma \vdash A \,\&\, B}\;\textsc{WF-and} \qquad \frac{\Gamma \vdash A \qquad \Gamma, \alpha * A \vdash B}{\Gamma \vdash \forall (\alpha * A).\, B}\;\textsc{WF-forall} \qquad \frac{\Gamma \vdash A \qquad \Gamma \vdash B}{\Gamma \vdash \mathbf{Trait}\,[A, B]}\;\textsc{WF-trait}$$

$\boxed{A <: B}$ *(Subtyping)*

$$\frac{}{A <: \top}\;\textsc{Sub-top} \qquad \frac{A_1 <: A_2 \qquad A_1 <: A_3}{A_1 <: A_2 \,\&\, A_3}\;\textsc{Sub-andR} \qquad \frac{}{\mathsf{Int} <: \mathsf{Int}}\;\textsc{Sub-int} \qquad \frac{A_1 <: A_3 \qquad A_3\ \mathsf{ordinary}}{A_1 \,\&\, A_2 <: A_3}\;\textsc{Sub-andL1}$$

$$\frac{A_2 <: A_3 \qquad A_3\ \mathsf{ordinary}}{A_1 \,\&\, A_2 <: A_3}\;\textsc{Sub-andL2} \qquad \frac{A <: B}{\{l : A\} <: \{l : B\}}\;\textsc{Sub-rec} \qquad \frac{}{\alpha <: \alpha}\;\textsc{Sub-var} \qquad \frac{B_1 <: A_1 \qquad A_2 <: B_2}{A_1 \to A_2 <: B_1 \to B_2}\;\textsc{Sub-arr}$$

$$\frac{B_1 <: B_2 \qquad A_2 <: A_1}{\forall (\alpha * A_1).\, B_1 <: \forall (\alpha * A_2).\, B_2}\;\textsc{Sub-forall} \qquad \frac{B_1 <: A_1 \qquad A_2 <: B_2}{\mathbf{Trait}\,[A_1, A_2] <: \mathbf{Trait}\,[B_1, B_2]}\;\textsc{Sub-trait}$$

$\boxed{\Gamma \vdash A * B}$ *(Disjointness)*

$$\frac{}{\Gamma \vdash \top * A}\;\textsc{D-top} \qquad \frac{}{\Gamma \vdash A * \top}\;\textsc{D-topSym} \qquad \frac{\alpha * A \in \Gamma \qquad A <: B}{\Gamma \vdash \alpha * B}\;\textsc{D-var} \qquad \frac{\alpha * A \in \Gamma \qquad A <: B}{\Gamma \vdash B * \alpha}\;\textsc{D-varSym}$$

$$\frac{\Gamma, \alpha * A_1 \,\&\, A_2 \vdash B * C}{\Gamma \vdash \forall (\alpha * A_1).\, B * \forall (\alpha * A_2).\, C}\;\textsc{D-forall} \qquad \frac{\Gamma \vdash A * B}{\Gamma \vdash \{l : A\} * \{l : B\}}\;\textsc{D-rec} \qquad \frac{l_1 \neq l_2}{\Gamma \vdash \{l_1 : A\} * \{l_2 : B\}}\;\textsc{D-recn}$$

$$\frac{\Gamma \vdash A_2 * B_2}{\Gamma \vdash A_1 \to A_2 * B_1 \to B_2}\;\textsc{D-arrow} \qquad \frac{\Gamma \vdash A_1 * B \qquad \Gamma \vdash A_2 * B}{\Gamma \vdash A_1 \,\&\, A_2 * B}\;\textsc{D-andL} \qquad \frac{\Gamma \vdash A * B_1 \qquad \Gamma \vdash A * B_2}{\Gamma \vdash A * B_1 \,\&\, B_2}\;\textsc{D-andR}$$

$$\frac{\Gamma \vdash A_2 * B_2}{\Gamma \vdash \mathbf{Trait}\,[A_1, A_2] * \mathbf{Trait}\,[B_1, B_2]}\;\textsc{D-trait} \qquad \frac{\Gamma \vdash A_2 * B_2}{\Gamma \vdash \mathbf{Trait}\,[A_1, A_2] * B_1 \to B_2}\;\textsc{D-traitArr1}$$

$$\frac{\Gamma \vdash A_2 * B_2}{\Gamma \vdash A_1 \to A_2 * \mathbf{Trait}\,[B_1, B_2]}\;\textsc{D-traitArr2} \qquad \frac{A *_{ax} B}{\Gamma \vdash A * B}\;\textsc{D-ax}$$

$\boxed{A *_{ax} B}$ *(Disjointness axiom)*

$$\frac{B *_{ax} A}{A *_{ax} B}\;\textsc{Dax-sym} \qquad \frac{}{\mathsf{Int} *_{ax} A_1 \to A_2}\;\textsc{Dax-intArr} \qquad \frac{}{\mathsf{Int} *_{ax} \{l : A\}}\;\textsc{Dax-intRec} \qquad \frac{}{\mathsf{Int} *_{ax} \forall (\alpha * B_1).\, B_2}\;\textsc{Dax-intForall}$$

$$\frac{}{\mathsf{Int} *_{ax} \mathbf{Trait}\,[A_1, A_2]}\ \textsc{Dax-intTrait} \qquad \frac{}{A_1 \to A_2 *_{ax} \forall\,(\alpha * B_1).\,B_2}\ \textsc{Dax-arrForall} \qquad \frac{}{A_1 \to A_2 *_{ax} \{l : B\}}\ \textsc{Dax-arrRec}$$

$$\frac{}{\mathbf{Trait}\,[A_1, A_2] *_{ax} \forall\,(\alpha * B_1).\,B_2}\ \textsc{Dax-traitForall} \qquad \frac{}{\mathbf{Trait}\,[A_1, A_2] *_{ax} \{l : B\}}\ \textsc{Dax-traitRec} \qquad \frac{}{\forall\,(\alpha * A_1).\,A_2 *_{ax} \{l : B\}}\ \textsc{Dax-forallRec}$$

$\boxed{\Gamma \vdash E \Rightarrow A \rightsquigarrow e}$ *(Infer)*

$$\frac{}{\Gamma \vdash \top \Rightarrow \top \rightsquigarrow \top}\ \textsc{Inf-top} \qquad \frac{}{\Gamma \vdash i \Rightarrow \mathsf{Int} \rightsquigarrow i}\ \textsc{Inf-int} \qquad \frac{x : A \in \Gamma}{\Gamma \vdash x \Rightarrow A \rightsquigarrow x}\ \textsc{Inf-var} \qquad \frac{\Gamma \vdash E \Leftarrow A \rightsquigarrow e}{\Gamma \vdash E : A \Rightarrow A \rightsquigarrow e : |A|}\ \textsc{Inf-anno}$$

$$\frac{\Gamma \vdash E_1 \Rightarrow A_1 \to A_2 \rightsquigarrow e_1 \qquad \Gamma \vdash E_2 \Leftarrow A_1 \rightsquigarrow e_2}{\Gamma \vdash E_1\, E_2 \Rightarrow A_2 \rightsquigarrow e_1\, e_2}\ \textsc{Inf-app}$$

$$\frac{\Gamma \vdash E \Rightarrow \forall\,(\alpha * B).\,C \rightsquigarrow e \qquad \Gamma \vdash A \qquad \Gamma \vdash A * B}{\Gamma \vdash E\, A \Rightarrow [A/\alpha]C \rightsquigarrow e\,|A|}\ \textsc{Inf-tapp}$$

$$\frac{\Gamma \vdash E_1 \Rightarrow A \rightsquigarrow e_1 \qquad \Gamma \vdash E_2 \Rightarrow B \rightsquigarrow e_2 \qquad \Gamma \vdash A * B}{\Gamma \vdash E_1\,,,\,E_2 \Rightarrow A \,\&\, B \rightsquigarrow e_1\,,,\,e_2}\ \textsc{Inf-merge} \qquad \frac{\Gamma \vdash E \Rightarrow A \rightsquigarrow e}{\Gamma \vdash \{l = E\} \Rightarrow \{l : A\} \rightsquigarrow \{l = e\}}\ \textsc{Inf-rec}$$

$$\frac{\Gamma \vdash E \Rightarrow \{l : A\} \rightsquigarrow e}{\Gamma \vdash E.\,l \Rightarrow A \rightsquigarrow e.\,l}\ \textsc{Inf-proj} \qquad \frac{\Gamma \vdash A \qquad \Gamma, \alpha * A \vdash E \Rightarrow B \rightsquigarrow e}{\Gamma \vdash \Lambda\,(\alpha * A).\,E \Rightarrow \forall\,(\alpha * A).\,B \rightsquigarrow \Lambda\,(\alpha * |A|).\,e}\ \textsc{Inf-blam}$$

$$\frac{\Gamma, x : A \vdash E_1 \Leftarrow A \rightsquigarrow e_1 \qquad \Gamma, x : A \vdash E_2 \Rightarrow B \rightsquigarrow e_2}{\Gamma \vdash \mathbf{letrec}\, x : A = E_1 \,\mathbf{in}\, E_2 \Rightarrow B \rightsquigarrow \mathbf{letrec}\, x : |A| = e_1 \,\mathbf{in}\, e_2}\ \textsc{Inf-letE}$$

$$\frac{\overline{\Gamma, \mathsf{self} : B \vdash E_i \Rightarrow \mathbf{Trait}\,[B_i, C_i] \rightsquigarrow e_i}^{i \in 1..n} \qquad \Gamma, \mathsf{self} : B \vdash \{\,\overline{l_j = E'_j}^{j \in 1..m}\,\} \Rightarrow C \rightsquigarrow e \\ \overline{B <: B_i}^{i \in 1..n} \qquad \Gamma \vdash C_1 \,\&\, ..\, \&\, C_n \,\&\, C \qquad C_1 \,\&\, ..\, \&\, C_n \,\&\, C <: A}{\Gamma \vdash \mathbf{trait}\,[\mathsf{self} : B]\,\mathbf{inherits}\,\overline{E_i}^{i \in 1..n}\,\{\,\overline{l_j = E'_j}^{j \in 1..m}\,\} : A \Rightarrow \mathbf{Trait}\,[B, A] \rightsquigarrow \lambda(\mathsf{self} : |B|).\,(\overline{(e_i\,\mathsf{self})}^{i \in 1..n})\,,,\,e}\ \textsc{Inf-trait}$$

$$\frac{\overline{\Gamma, \mathsf{self} : B \vdash E_i \Rightarrow \mathbf{Trait}\,[B_i, C_i] \rightsquigarrow e_i}^{i \in 1..n} \\ \Gamma, \mathsf{self} : B, \mathsf{super} : C_1 \,\&\, ..\, \&\, C_n \vdash \{\,\overline{l_j = E'_j}^{j \in 1..m}\,\} \Rightarrow C \rightsquigarrow e \\ \overline{B <: B_i}^{i \in 1..n} \qquad \Gamma \vdash C_1 \,\&\, ..\, \&\, C_n \,\&\, C \qquad C_1 \,\&\, ..\, \&\, C_n \,\&\, C <: A}{\Gamma \vdash \mathbf{trait}\,[\mathsf{self} : B]\,\mathbf{inherits}\,\overline{E_i}^{i \in 1..n}\,\{\,\overline{l_j = E'_j}^{j \in 1..m}\,\} : A \Rightarrow \mathbf{Trait}\,[B, A] \rightsquigarrow \\ \lambda\,(\mathsf{self} : |B|).\,\mathbf{let}\,\mathsf{super} = \overline{(e_i\,\mathsf{self})}^{i \in 1..n}\,\mathbf{in}\,\mathsf{super}\,,,\,e}\ \textsc{Inf-traitSuper}$$

$$\frac{\overline{\Gamma \vdash E_i \Rightarrow \mathbf{Trait}\,[A_i, B_i] \rightsquigarrow e_i}^{i \in 1..n} \\ \overline{A <: A_i}^{i \in 1..n} \qquad \Gamma \vdash B_1 \,\&\, ..\, \&\, B_n \qquad B_1 \,\&\, ..\, \&\, B_n <: A}{\Gamma \vdash \mathbf{new}\,[A](\,\overline{E_i}^{i \in 1..n}\,) \Rightarrow A \rightsquigarrow \mathbf{letrec}\,\mathsf{self} : |A| = \overline{(e_i\,\mathsf{self})}^{i \in 1..n}\,\mathbf{in}\,\mathsf{self}}\ \textsc{Inf-new}$$

$$\frac{\Gamma \vdash E_1 \Rightarrow \mathbf{Trait}\,[A, B] \rightsquigarrow e_1 \qquad \Gamma \vdash E_2 \Leftarrow A \rightsquigarrow e_2}{\Gamma \vdash E_1\,\hat{}\,E_2 \Rightarrow B \rightsquigarrow e_1\, e_2}\ \textsc{Inf-forward}$$

$\boxed{\Gamma \vdash E \Leftarrow A \rightsquigarrow e}$ *(Check)*

$$\textsc{Chk-abs}\quad \frac{\Gamma \vdash A \qquad \Gamma, x : A \vdash E \Leftarrow B \rightsquigarrow e}{\Gamma \vdash \lambda x.\, E \Leftarrow A \to B \rightsquigarrow \lambda x.\, e}$$

$$\textsc{Chk-sub}\quad \frac{\Gamma \vdash E \Rightarrow A \rightsquigarrow e \qquad A <: B \qquad \Gamma \vdash B}{\Gamma \vdash E \Leftarrow B \rightsquigarrow e}$$

▶ **Definition 7** (Type translation)**.**

$$\begin{aligned}
|\alpha| &= \alpha \\
|\mathsf{Int}| &= \mathsf{Int} \\
|\top| &= \top \\
|A \to B| &= |A| \to |B| \\
|A \,\&\, B| &= |A| \,\&\, |B| \\
|\{l : A\}| &= \{l : |A|\} \\
|\forall (\alpha * A).\, B| &= \forall (\alpha * |A|).\, |B| \\
|\textbf{Trait}\,[A, B]| &= |A| \to |B|
\end{aligned}$$

B Metatheory

▶ **Lemma 8.** *If* $\Gamma \vdash A$ *then* $|\Gamma| \vdash |A|$.

Proof. By simple induction on the derivation of the judgement. ◀

▶ **Lemma 9.** *If* $A *_{ax} B$ *then* $|A| *_{ax} |B|$.

Proof. Note that $|\textbf{Trait}\,[A, B]| = |A| \to |B|$, the rest are immediate. ◀

▶ **Lemma 10.** *If* $A <: B$ *then* $|A| <: |B|$.

Proof. Most of them are just F_i subtyping. We only show rule Sub-trait.

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$$\textsc{Sub-trait}\quad \frac{B_1 <: A_1 \qquad A_2 <: B_2}{\textbf{Trait}\,[A_1, A_2] <: \textbf{Trait}\,[B_1, B_2]}$$

$\lvert B_1\rvert <: \lvert A_1\rvert$	By i.h.
$\lvert A_2\rvert <: \lvert B_2\rvert$	By i.h.
$\lvert A_1\rvert \to \lvert A_2\rvert <: \lvert B_1\rvert \to \lvert B_2\rvert$	By S$\to$

◀

▶ **Lemma 11.** *If* $\Gamma \vdash A * B$ *then* $|\Gamma| \vdash |A| * |B|$.

Proof. By induction on the derivation of the judgement.

- Rules D-top, D-topSym, and D-recn are immediate.
-

$$\textsc{D-var}\quad \frac{\alpha * A \in \Gamma \qquad A <: B}{\Gamma \vdash \alpha * B}$$

$\lvert A\rvert <: \lvert B\rvert$	By Lemma 10
$\alpha * A \in \Gamma$	Given
$\alpha * \lvert A\rvert \in \lvert\Gamma\rvert$	Above
$\lvert\Gamma\rvert \vdash \alpha * \lvert B\rvert$	By Dα

■

$$\textsc{D-varSym}\quad \frac{\alpha * A \in \Gamma \qquad A <: B}{\Gamma \vdash B * \alpha}$$

$\lvert A\rvert <: \lvert B\rvert$	By Lemma 10
$\alpha * A \in \Gamma$	Given
$\alpha * \lvert A\rvert \in \lvert\Gamma\rvert$	Above
$\lvert\Gamma\rvert \vdash \lvert B\rvert * \alpha$	By DαSym

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$$\textsc{D-forall}\quad \frac{\Gamma, \alpha * A_1 \,\&\, A_2 \vdash B * C}{\Gamma \vdash \forall\,(\alpha * A_1).\, B * \forall\,(\alpha * A_2).\, C}$$

$\lvert\Gamma\rvert, \alpha * \lvert A_1\rvert \,\&\, \lvert A_2\rvert \vdash \lvert B\rvert * \lvert C\rvert$	By i.h.
$\lvert\Gamma\rvert \vdash \forall(\alpha * \lvert A_1\rvert).\, B * \forall(\alpha * \lvert A_2\rvert).\, C$	By D$\forall$

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$$\textsc{D-rec}\quad \frac{\Gamma \vdash A * B}{\Gamma \vdash \{l : A\} * \{l : B\}}$$

$\lvert\Gamma\rvert \vdash \lvert A\rvert * \lvert B\rvert$	By i.h.
$\lvert\Gamma\rvert \vdash \{l : \lvert A\rvert\} * \{l : \lvert B\rvert\}$	By DRec$_=$

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$$\textsc{D-arrow}\quad \frac{\Gamma \vdash A_2 * B_2}{\Gamma \vdash A_1 \to A_2 * B_1 \to B_2}$$

$\lvert\Gamma\rvert \vdash \lvert A_2\rvert * \lvert B_2\rvert$	By i.h.
$\lvert\Gamma\rvert \vdash \lvert A_1\rvert \to \lvert A_2\rvert * \lvert B_1\rvert \to \lvert B_2\rvert$	By D$\to$

■

$$\textsc{D-andL}\quad \frac{\Gamma \vdash A_1 * B \qquad \Gamma \vdash A_2 * B}{\Gamma \vdash A_1 \,\&\, A_2 * B}$$

$|\Gamma| \vdash |A_1| * |B|$ By i.h.
$|\Gamma| \vdash |A_2| * |B|$ By i.h.
$|\Gamma| \vdash |A_1| \,\&\, |A_2| * |B|$ By D&L

■

$$\textsc{D-andR}\quad \frac{\Gamma \vdash A * B_1 \qquad \Gamma \vdash A * B_2}{\Gamma \vdash A * B_1 \,\&\, B_2}$$

$|\Gamma| \vdash |A| * |B_1|$ By i.h.
$|\Gamma| \vdash |A| * |B_2|$ By i.h.
$|\Gamma| \vdash |A| * |B_1| \,\&\, |B_2|$ By D&R

■

$$\textsc{D-trait}\quad \frac{\Gamma \vdash A_2 * B_2}{\Gamma \vdash \mathbf{Trait}\,[A_1, A_2] * \mathbf{Trait}\,[B_1, B_2]}$$

$|\Gamma| \vdash |A_2| * |B_2|$ By i.h.
$|\Gamma| \vdash |A_1| \to |A_2| * |B_1| \to |B_2|$ By D$\to$

■

$$\textsc{D-traitArr1}\quad \frac{\Gamma \vdash A_2 * B_2}{\Gamma \vdash \mathbf{Trait}\,[A_1, A_2] * B_1 \to B_2}$$

$|\Gamma| \vdash |A_2| * |B_2|$ By i.h.
$|\Gamma| \vdash |A_1| \to |A_2| * |B_1| \to |B_2|$ By D$\to$

■

$$\textsc{D-traitArr2}\quad \frac{\Gamma \vdash A_2 * B_2}{\Gamma \vdash A_1 \to A_2 * \mathbf{Trait}\,[B_1, B_2]}$$

$|\Gamma| \vdash |A_2| * |B_2|$ By i.h.
$|\Gamma| \vdash |A_1| \to |A_2| * |B_1| \to |B_2|$ By D$\to$

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$$\textsc{D-ax}\quad \frac{A *_{ax} B}{\Gamma \vdash A * B}$$

$|A| *_{ax} |B|$ By Lemma 9
$|\Gamma| \vdash |A| * |B|$ By DAx

◀

▶ **Theorem 12** (Type-safe translation)**.** *We have that:*

- *If* $\Gamma \vdash E \Rightarrow A \rightsquigarrow e$ *then* $|\Gamma| \vdash e \Rightarrow |A|$.
- *If* $\Gamma \vdash E \Leftarrow A \rightsquigarrow e$ *then* $|\Gamma| \vdash e \Leftarrow |A|$.

Proof. By induction on the typing judgement.

- Rules Inf-top, Inf-int, and Inf-var are immediate.
-
$$\textsc{Inf-anno}\quad \frac{\Gamma \vdash E \Leftarrow A \rightsquigarrow e}{\Gamma \vdash E : A \Rightarrow A \rightsquigarrow e : |A|}$$

$\lvert\Gamma\rvert \vdash e \Leftarrow \lvert A\rvert$	By i.h.
$\lvert\Gamma\rvert \vdash e : \lvert A\rvert \Rightarrow \lvert A\rvert$	By TI-anno

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$$\textsc{Inf-app}\quad \frac{\Gamma \vdash E_1 \Rightarrow A_1 \to A_2 \rightsquigarrow e_1 \qquad \Gamma \vdash E_2 \Leftarrow A_1 \rightsquigarrow e_2}{\Gamma \vdash E_1\, E_2 \Rightarrow A_2 \rightsquigarrow e_1\, e_2}$$

$\lvert\Gamma\rvert \vdash e_1 \Rightarrow \lvert A_1\rvert \to \lvert A_2\rvert$	By i.h.
$\lvert\Gamma\rvert \vdash e_2 \Leftarrow \lvert A_2\rvert$	By i.h.
$\lvert\Gamma\rvert \vdash e_1\, e_2 \Rightarrow \lvert A_2\rvert$	By TI-app

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$$\textsc{Inf-tapp}\quad \frac{\Gamma \vdash E \Rightarrow \forall (\alpha * B).\, C \rightsquigarrow e \qquad \Gamma \vdash A \qquad \Gamma \vdash A * B}{\Gamma \vdash E\, A \Rightarrow [A/\alpha]C \rightsquigarrow e\, |A|}$$

$\lvert\Gamma\rvert \vdash e \Rightarrow \forall(\alpha * \lvert B\rvert).\, \lvert C\rvert$	By i.h.
$\lvert\Gamma\rvert \vdash \lvert A\rvert$	By Lemma 8
$\lvert\Gamma\rvert \vdash \lvert A\rvert * \lvert B\rvert$	By Lemma 11
$\lvert\Gamma\rvert \vdash e\, \lvert A\rvert \Rightarrow [\alpha \mapsto \lvert A\rvert]\lvert C\rvert$	By TI-tapp

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$$\textsc{Inf-merge}\quad \frac{\Gamma \vdash E_1 \Rightarrow A \rightsquigarrow e_1 \qquad \Gamma \vdash E_2 \Rightarrow B \rightsquigarrow e_2 \qquad \Gamma \vdash A * B}{\Gamma \vdash E_1 \,,,\, E_2 \Rightarrow A \,\&\, B \rightsquigarrow e_1 \,,,\, e_2}$$

$\lvert\Gamma\rvert \vdash e_1 \Rightarrow \lvert A\rvert$	By i.h.
$\lvert\Gamma\rvert \vdash e_2 \Rightarrow \lvert B\rvert$	By i.h.
$\lvert\Gamma\rvert \vdash \lvert A\rvert * \lvert B\rvert$	By Lemma 11
$\lvert\Gamma\rvert \vdash e_1 ,, e_2 \Rightarrow \lvert A\rvert \,\&\, \lvert B\rvert$	By TI-merge

- $$\textsc{Inf-rec}\quad \frac{\Gamma \vdash E \Rightarrow A \rightsquigarrow e}{\Gamma \vdash \{l = E\} \Rightarrow \{l : A\} \rightsquigarrow \{l = e\}}$$

$\lvert\Gamma\rvert \vdash e \Rightarrow \lvert A\rvert$	By i.h.
$\lvert\Gamma\rvert \vdash \{l = e\} \Rightarrow \{l : \lvert A\rvert\}$	By TI-rec

- $$\textsc{Inf-proj}\quad \frac{\Gamma \vdash E \Rightarrow \{l : A\} \rightsquigarrow e}{\Gamma \vdash E.\,l \Rightarrow A \rightsquigarrow e.\,l}$$

$\lvert\Gamma\rvert \vdash e \Rightarrow \{l : \lvert A\rvert\}$	By i.h.
$\lvert\Gamma\rvert \vdash e.l \Rightarrow \lvert A\rvert$	By TI-proj

- $$\textsc{Inf-blam}\quad \frac{\Gamma \vdash A \qquad \Gamma, \alpha * A \vdash E \Rightarrow B \rightsquigarrow e}{\Gamma \vdash \Lambda\,(\alpha * A).\,E \Rightarrow \forall\,(\alpha * A).\,B \rightsquigarrow \Lambda\,(\alpha * \lvert A\rvert).\,e}$$

$\lvert\Gamma\rvert \vdash \lvert A\rvert$	By Lemma 8
$\lvert\Gamma\rvert, \alpha * \lvert A\rvert \vdash e \Rightarrow \lvert B\rvert$	By i.h.
$\lvert\Gamma\rvert \vdash \Lambda(\alpha * \lvert A\rvert).e \Rightarrow \forall(\alpha * \lvert A\rvert).\,\lvert B\rvert$	By TI-blam

- $$\textsc{Inf-lete}\quad \frac{\Gamma, x : A \vdash E_1 \Leftarrow A \rightsquigarrow e_1 \qquad \Gamma, x : A \vdash E_2 \Rightarrow B \rightsquigarrow e_2}{\Gamma \vdash \mathbf{letrec}\; x : A = E_1 \;\mathbf{in}\; E_2 \Rightarrow B \rightsquigarrow \mathbf{letrec}\; x : \lvert A\rvert = e_1 \;\mathbf{in}\; e_2}$$

$\lvert\Gamma\rvert, x : \lvert A\rvert \vdash e_1 \Leftarrow \lvert A\rvert$	By i.h.
$\lvert\Gamma\rvert, x : \lvert A\rvert \vdash e_2 \Rightarrow \lvert B\rvert$	By i.h.
$\lvert\Gamma\rvert \vdash \mathbf{letrec}\; x : \lvert A\rvert = e_1 \;\mathbf{in}\; e_2 \Rightarrow \lvert B\rvert$	By TI-lete

- $$\textsc{Inf-new}\quad \frac{\overline{A <: A_i}^{\,i\in 1..n} \qquad \overline{\Gamma \vdash E_i \Rightarrow \mathbf{Trait}\,[A_i, B_i] \rightsquigarrow e_i}^{\,i\in 1..n} \quad \Gamma \vdash B_1 \,\&\, ..\, \&\, B_n \qquad B_1 \,\&\, ..\, \&\, B_n <: A}{\Gamma \vdash \mathbf{new}\,[A](\overline{E_i}^{\,i\in 1..n}) \Rightarrow A \rightsquigarrow \mathbf{letrec}\,\mathsf{self} : \lvert A\rvert = \overline{(e_i\,\mathsf{self})}^{\,i\in 1..n} \;\mathbf{in}\,\mathsf{self}}$$

$\lvert\Gamma\rvert \vdash e_i \Rightarrow \lvert A_i\rvert \to \lvert B_i\rvert$	By i.h.
$\lvert A\rvert <: \lvert A_i\rvert$	By Lemma 10
$\lvert\Gamma\rvert \vdash \lvert B_1\rvert \,\&\, ..\, \&\, \lvert B_n\rvert$	By Lemma 11

$\lvert B_1\rvert \,\&\, .. \,\&\, \lvert B_n\rvert <: \lvert A\rvert$	By Lemma 10
$\lvert\Gamma\rvert, \mathsf{self} : \lvert A\rvert \vdash \mathsf{self} \Rightarrow \lvert A\rvert$	By TI-VAR
$\lvert\Gamma\rvert, \mathsf{self} : \lvert A\rvert \vdash \mathsf{self} \Leftarrow \lvert A_i\rvert$	By TC-SUB
$\lvert\Gamma\rvert, \mathsf{self} : \lvert A\rvert \vdash e_i\, \mathsf{self} \Rightarrow \lvert B_i\rvert$	By TI-APP
$\lvert\Gamma\rvert, \mathsf{self} : \lvert A\rvert \vdash (e_1\, \mathsf{self}) \,,,\, .. \,,,\, (e_n\, \mathsf{self}) \Rightarrow \lvert B_1\rvert \,\&\, .. \,\&\, \lvert B_n\rvert$	By TI-MERGE and above
$\lvert\Gamma\rvert, \mathsf{self} : \lvert A\rvert \vdash (e_1\, \mathsf{self}) \,,,\, .. \,,,\, (e_n\, \mathsf{self}) \Leftarrow \lvert A\rvert$	By TC-SUB
$\lvert\Gamma\rvert \vdash \mathbf{letrec}\, \mathsf{self} : \lvert A\rvert = (e_1\, \mathsf{self}) \,,,\, .. \,,,\, (e_n\, \mathsf{self})\, \mathbf{in}\, \mathsf{self} \Rightarrow \lvert A\rvert$	By TI-LETE

■

INF-TRAIT

$$\frac{\overline{\Gamma, \mathsf{self} : B \vdash E_i \Rightarrow \mathbf{Trait}\,[B_i, C_i] \rightsquigarrow e_i}^{i\in 1..n} \quad \Gamma, \mathsf{self} : B \vdash \{\, \overline{l_j = E'_j}^{j\in 1..m} \,\} \Rightarrow C \rightsquigarrow e \quad \overline{B <: B_i}^{i\in 1..n} \quad \Gamma \vdash C_1 \,\&\, .. \,\&\, C_n \,\&\, C \quad C_1 \,\&\, .. \,\&\, C_n \,\&\, C <: A}{\Gamma \vdash \mathbf{trait}\,[\mathsf{self} : B]\, \mathbf{inherits}\, \overline{E_i}^{i\in 1..n} \,\{\, \overline{l_j = E'_j}^{j\in 1..m} \,\} : A \Rightarrow \mathbf{Trait}\,[B, A] \rightsquigarrow \lambda(\mathsf{self} : \lvert B\rvert).\, (\overline{(e_i\, \mathsf{self})}^{i\in 1..n}) \,,,\, e}$$

$\lvert\Gamma\rvert, \mathsf{self} : \lvert B\rvert \vdash e_i \Rightarrow \lvert B_i\rvert \to \lvert C_i\rvert$	By i.h.
$\lvert\Gamma\rvert, \mathsf{self} : \lvert B\rvert \vdash e \Rightarrow \lvert C\rvert$	By i.h.
$\lvert B\rvert <: \lvert B_i\rvert$	By Lemma 10
$\lvert\Gamma\rvert \vdash \lvert C_1\rvert \,\&\, .. \,\&\, \lvert C_n\rvert \,\&\, \lvert C\rvert$	By Lemma 8
$\lvert C_1\rvert \,\&\, .. \,\&\, \lvert C_n\rvert \,\&\, \lvert C\rvert <: \lvert A\rvert$	By Lemma 10
$\lvert\Gamma\rvert, \mathsf{self} : \lvert B\rvert \vdash \mathsf{self} \Rightarrow \lvert B\rvert$	By TI-VAR
$\lvert\Gamma\rvert, \mathsf{self} : \lvert B\rvert \vdash \mathsf{self} \Leftarrow \lvert B_i\rvert$	By TC-SUB
$\lvert\Gamma\rvert, \mathsf{self} : \lvert B\rvert \vdash e_i\, \mathsf{self} \Rightarrow \lvert C_i\rvert$	By TI-APP
$\lvert\Gamma\rvert, \mathsf{self} : \lvert B\rvert \vdash (e_1\, \mathsf{self}) \,,,\, .. \,,,\, (e_n\, \mathsf{self}) \,,,\, e \Rightarrow \lvert C_1\rvert \,\&\, .. \,\&\, \lvert C_n\rvert \,\&\, \lvert C\rvert$	By TI-MERGE
$\lvert\Gamma\rvert, \mathsf{self} : \lvert B\rvert \vdash (e_1\, \mathsf{self}) \,,,\, .. \,,,\, (e_n\, \mathsf{self}) \,,,\, e \Leftarrow \lvert A\rvert$	By TC-SUB
$\lvert\Gamma\rvert \vdash \lambda(\mathsf{self} : \lvert B\rvert).\, (e_1\, \mathsf{self}) \,,,\, .. \,,,\, (e_n\, \mathsf{self}) \,,,\, e \Rightarrow \lvert B\rvert \to \lvert A\rvert$	By TI-ABS (annotated lambda typing)

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INF-TRAITSUPER

$$\frac{\overline{\Gamma, \mathsf{self} : B \vdash E_i \Rightarrow \mathbf{Trait}\,[B_i, C_i] \rightsquigarrow e_i}^{i\in 1..n} \quad \Gamma, \mathsf{self} : B, \mathsf{super} : C_1 \,\&\, .. \,\&\, C_n \vdash \{\, \overline{l_j = E'_j}^{j\in 1..m} \,\} \Rightarrow C \rightsquigarrow e \quad \overline{B <: B_i}^{i\in 1..n} \quad \Gamma \vdash C_1 \,\&\, .. \,\&\, C_n \,\&\, C \quad C_1 \,\&\, .. \,\&\, C_n \,\&\, C <: A}{\Gamma \vdash \mathbf{trait}\,[\mathsf{self} : B]\, \mathbf{inherits}\, \overline{E_i}^{i\in 1..n} \,\{\, \overline{l_j = E'_j}^{j\in 1..m} \,\} : A \Rightarrow \mathbf{Trait}\,[B, A] \rightsquigarrow \lambda\,(\mathsf{self} : \lvert B\rvert).\, \mathbf{let}\, \mathsf{super} = \overline{(e_i\, \mathsf{self})}^{i\in 1..n}\, \mathbf{in}\, \mathsf{super} \,,,\, e}$$

$\lvert\Gamma\rvert, \mathsf{self} : \lvert B\rvert \vdash e_i \Rightarrow \lvert B_i\rvert \to \lvert C_i\rvert$	By i.h.
$\lvert\Gamma\rvert, \mathsf{self} : \lvert B\rvert, \mathsf{super} : \lvert C_1\rvert \,\&\, .. \,\&\, \lvert C_n\rvert \vdash e \Rightarrow \lvert C\rvert$	By i.h.
$\lvert B\rvert <: \lvert B_i\rvert$	By Lemma 10
$\lvert\Gamma\rvert \vdash \lvert C_1\rvert \,\&\, .. \,\&\, \lvert C_n\rvert \,\&\, \lvert C\rvert$	By Lemma 8
$\lvert C_1\rvert \,\&\, .. \,\&\, \lvert C_n\rvert \,\&\, \lvert C\rvert <: \lvert A\rvert$	By Lemma 10
$\lvert\Gamma\rvert, \mathsf{self} : \lvert B\rvert \vdash \mathsf{self} \Rightarrow \lvert B\rvert$	By TI-VAR
$\lvert\Gamma\rvert, \mathsf{self} : \lvert B\rvert \vdash \mathsf{self} \Leftarrow \lvert B_i\rvert$	By TC-SUB
$\lvert\Gamma\rvert, \mathsf{self} : \lvert B\rvert \vdash e_i\, \mathsf{self} \Rightarrow \lvert C_i\rvert$	By TI-APP
$\lvert\Gamma\rvert, \mathsf{self} : \lvert B\rvert \vdash (e_1\, \mathsf{self}) \,,,\, .. \,,,\, (e_n\, \mathsf{self}) \Rightarrow \lvert C_1\rvert \,\&\, .. \,\&\, \lvert C_n\rvert$	By TI-MERGE
$\lvert\Gamma\rvert, \mathsf{self} : \lvert B\rvert, \mathsf{super} : \lvert C_1\rvert \,\&\, .. \,\&\, \lvert C_n\rvert \vdash \mathsf{super} \,,,\, e \Rightarrow \lvert C_1\rvert \,\&\, .. \,\&\, \lvert C_n\rvert \,\&\, \lvert C\rvert$	By TI-MERGE

$\lvert\Gamma\rvert, \mathsf{self} : \lvert B\rvert \vdash \mathbf{let}\ \mathsf{super} = \overline{(e_i\ \mathsf{self})}^{i\in 1..n}\ \mathbf{in}\ \mathsf{super}\ ,,\ e \Rightarrow \lvert C_1\rvert \,\&\, ..\, \&\, \lvert C_n\rvert \,\&\, \lvert C\rvert$	By TI-LET (non-recursive let)
$\lvert\Gamma\rvert, \mathsf{self} : \lvert B\rvert \vdash \mathbf{let}\ \mathsf{super} = \overline{(e_i\ \mathsf{self})}^{i\in 1..n}\ \mathbf{in}\ \mathsf{super}\ ,,\ e \Leftarrow \lvert A\rvert$	By TC-SUB
$\lvert\Gamma\rvert \vdash \lambda(\mathsf{self} : \lvert B\rvert).\ \mathbf{let}\ \mathsf{super} = \overline{(e_i\ \mathsf{self})}^{i\in 1..n}\ \mathbf{in}\ \mathsf{super}\ ,,\ e \Rightarrow \lvert B\rvert \to \lvert A\rvert$	By TI-ABS (annotated lambda typing)

▬

INF-FORWARD

$$\frac{\Gamma \vdash E_1 \Rightarrow \mathbf{Trait}\,[A, B] \rightsquigarrow e_1 \qquad \Gamma \vdash E_2 \Leftarrow A \rightsquigarrow e_2}{\Gamma \vdash E_1\,\hat{}\,E_2 \Rightarrow B \rightsquigarrow e_1\ e_2}$$

$\lvert\Gamma\rvert \vdash e_1 \Rightarrow \lvert A\rvert \to \lvert B\rvert$	By i.h.
$\lvert\Gamma\rvert \vdash e_2 \Leftarrow \lvert A\rvert$	By i.h.
$\lvert\Gamma\rvert \vdash e_1\ e_2 \Rightarrow \lvert B\rvert$	By TI-APP

CHK-ABS

$$\frac{\Gamma \vdash A \qquad \Gamma, x : A \vdash E \Leftarrow B \rightsquigarrow e}{\Gamma \vdash \lambda x.\ E \Leftarrow A \to B \rightsquigarrow \lambda x.\ e}$$

▬

$\lvert\Gamma\rvert \vdash \lvert A\rvert$	By Lemma 8
$\lvert\Gamma\rvert, x : \lvert A\rvert \vdash e \Leftarrow \lvert B\rvert$	By i.h.
$\lvert\Gamma\rvert \vdash \lambda x.\ e \Leftarrow \lvert A\rvert \to \lvert B\rvert$	By TC-ABS

CHK-SUB

$$\frac{\Gamma \vdash E \Rightarrow A \rightsquigarrow e \qquad A <: B \qquad \Gamma \vdash B}{\Gamma \vdash E \Leftarrow B \rightsquigarrow e}$$

▬

$\lvert\Gamma\rvert \vdash e \Rightarrow \lvert A\rvert$	By i.h.
$\lvert A\rvert <: \lvert B\rvert$	By Lemma 10
$\lvert\Gamma\rvert \vdash \lvert B\rvert$	By Lemma 8
$\lvert\Gamma\rvert \vdash e \Leftarrow \lvert B\rvert$	By TC-SUB

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