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Population dynamics with consideration of technology efficiency

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Abstract: In this paper, how technology efficiency is correlated with the classical population dynamics problem will be demonstrated. In addition, I will discuss technology efficiency in terms of having a direct impact to the birth rate or death rate.

1. Introduction

Most of the human population models we have studied in class involve carrying capacity which define the rate of population change as:

$$\frac{dN}{dt} = rN(1 - \frac{N}{k})$$

where k is the carrying capacity constant. I think this model only considered what can be limiting the rate of population increase and is lack of factors that can have a positive impact on the rate. I believe the change in technology efficiency can have an enormous influence on human population growth which can be supported by many historical events such as the population change after the first industrial revolution or the baby boomer which took place in the US from 1946 to 1964. To justify this idea, this paper introduce a function of technological efficiency into the calculation of $\frac{dN}{dt}$ and from here, the question of how does technology efficiency impact the rate of change of populationw will be answered by finding the fixed point and determining each stability in the model that will be discussed in the next section.

2. Simplification

To answer my question, two equations will be used from the article written by A.-S. Lafuite and M. Loreau[1] which are the following:

$$\frac{dH}{dt} = \mu_{max} H(1 - e^{y_1^{min} - \gamma_1 B^{\Omega} \frac{T}{T_m}})e^{-b_2 \gamma_2 \frac{T}{T_m}} \tag{1}$$

$$\frac{dT}{dt} = \sigma T(1 - \frac{T}{T_m}) \tag{2}$$

2.1. Constants, Variables, and aggregate parameters

Constants (numbers in the third column are default values from the reference 1):

[1]

η	Agents preference for agricultural goods	0.5
α_1	Labor intensity in the agricultural sector	0.3
α_2	Labor intensity in the industrial sector	0.3
B	Biodiversity	1
σ	Rate of technological change	0.5
κ	Land operating cost	0.35
μ_{max}	Maximum growth rate	1
y_1^{min}	Minimum per capita agricultural consumption	0.5
b_2	Sensitivity to industrial goods' consumption	0.2
Ω	Concavity of the BES relationship	1

Variables:

[1]	H	Human population
	T	Technology
	T_m	Maximum technological efficiency

Aggregate Parameters:

$$\gamma_1 = \eta T_m \alpha_1^{\alpha_1} (\frac{(1-\alpha_1)}{\kappa})^{1-\alpha_1} \tag{3}$$

$$\gamma_2 = (1-\eta) T_m \alpha_2^{\alpha_2} (\frac{(1-\alpha_2)}{\kappa})^{1-\alpha_2} \tag{4}$$

2.2. Explaining the equations and Assumption

2.2.1. Explanation

The exponential terms in $\frac{dH}{dt}$(1) separately represent the agricultural goods consumption and industrial goods consumption both with respect to the technology efficiency. This model is developed on this equation:

$$[2]\mu = \mu_{max}(1 - e^{y_1^{min} - y_1})e^{-b_2 y_2} \tag{5}$$

where this equation doesn't take into account technology efficiency as a part of the growth rate. From this equation, we can easily interpret that when y_1 is greater than y_1^{min}, we will have a higher agricultural goods consumption and a larger 1 minus exponential component comparing to when y_1^{min} is greater than y_1 will increase net human population growth rate. Conversely, as the consumption of the industrial goods y_2 increases, the net human growth rate tends to decline since as civilization becomes more industrialized, the whole population will shift from high birth rate and death rate to low birth rate and death rate which in the end will cause a steadily decreasing net growth rate. This trend is called the demographic transition and is captured by the b_2 term in the equation. With consideration of technology efficiency, both the consumption of agriculture and industrial goods will increase which precisely described the world we live in today.

2.2.2. Assumptions

- The original system of the equation from [1] includes three equations with the third equation representing biodiversity but since this paper will only be considering the effect of technology efficiency, every term related to biodiversity will be treated as a constant.
- In the later part of this paper that discuss whether technology can have a direct impact on the birth rate of the death rate, the μ_{max} in (1) will be replaced by the birth rate minus the death rate.
- The aggregate parameters will be calculated using the default value provided by[1].
- The function of technology efficiency is assumed to follow the trend of logistic growth.

3. Calculations

3.1. Fixed points

The fixed points of our equations can be found when $\frac{dH}{dt}$ (1) and $\frac{dT}{dt}$ (2) are set equal to zero. By plugging in default values, I get γ_1 and γ_2 both are equal to $0.566T_m$. Then equation 1 will become:

$$\frac{dH}{dt} = \mu_{max}H(1 - e^{y_1^{min} - 0.566B^{\Omega}T})e^{-b_2 0.566T} \tag{6}$$

The fixed point for this function will be H = 0. By setting the equation 2 to zero,

$$0 = \sigma T(1 - \frac{T}{T_m})$$

, it will give us two fixed points, T = 0 and T = T_m. So for the system of equation F(H,T), we have two sets of fixed points in total: (0, 0) and (0, T_m).

3.2. Stability

To find the stability of each set of fixed point, I will first find the partial derivative to F(H, T) and G(H, T) and matrix A should be

$$\begin{bmatrix} \frac{\partial F}{\partial H} & \frac{\partial F}{\partial T} \\ \frac{\partial G}{\partial H} & \frac{\partial G}{\partial T} \end{bmatrix}$$

And it is equal to:

$$\begin{bmatrix} (1 - e^{0.5 - 0.566T})e^{-0.1132T} & -0.1132He^{-0.1132T} + 0.6692He^{0.5 - 0.6692T} \\ 0 & \sigma - \frac{2\sigma T}{T_m} \end{bmatrix}$$

By plugging in the first set of fixed point (0, 0), we will get the following matrix:

$$\begin{bmatrix} 1 - e^{0.5} & 0 \\ 0 & \sigma \end{bmatrix}$$

At this point. we have λ_1 = 1 - $e^{0.5}$ and $\lambda_2 = \sigma$ so this should be a saddle.

Then we plug in the second set of the fixed point (0, T_m) and we will get the following matrix:

$$\begin{bmatrix} (1 - e^{0.5 - 0.566T_m})e^{-0.1132T_m} & 0 \\ 0 & -\sigma \end{bmatrix}$$

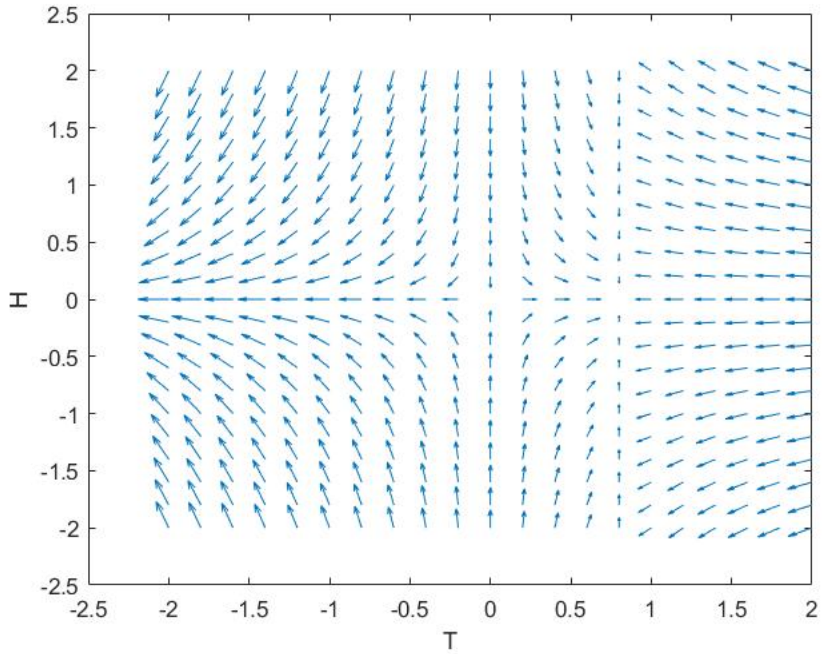
In this case, the λ_1 is equal to 1 - $e^{0.5-0.556T_m}e^{-0.1132T_m}$ and λ_2 is equal to -σ.

Therefore, when T_m is smaller than $\frac{0.5}{0.566}$, we will have a stable fixed point and otherwise, it is a saddle point.

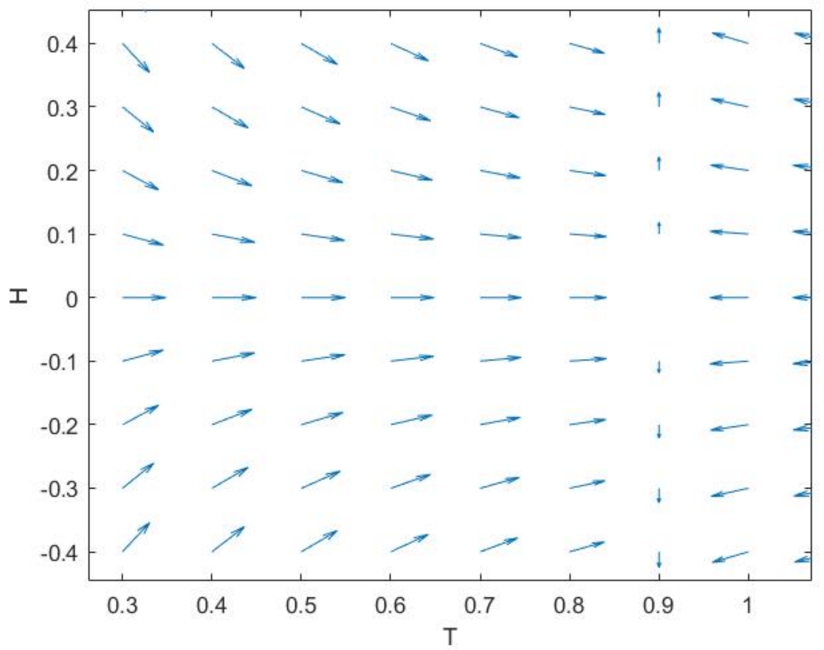
3.3. Plot and explanation

3.3.1. Plot

To support the above analysis of stability of each fixed point, I will set T_m to 0.8 and plot the direction field with y-axis representing the H and x-axis representing the T and this is the corresponding plot:



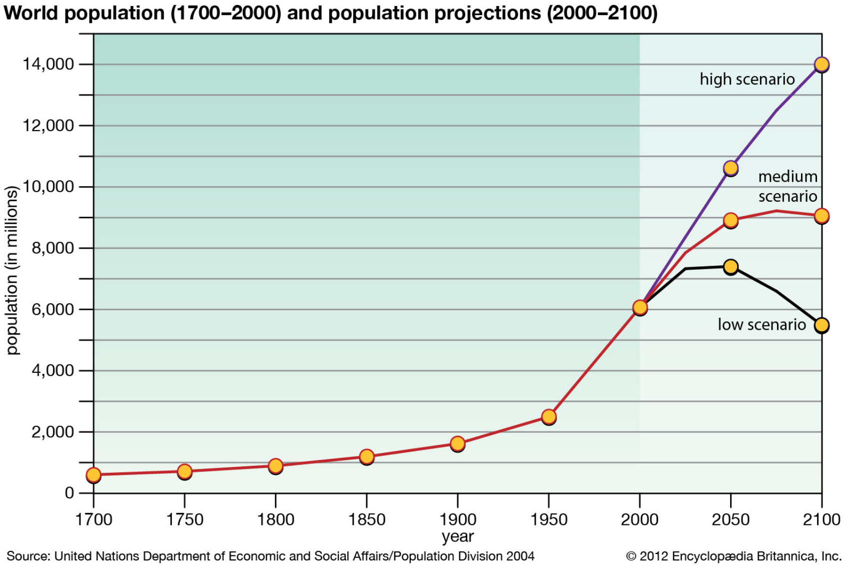
From this plot, we can easily tell that we have a saddle point (0, 0) and a stable fixed point at (0, 0.8). If we choose T_m is equal to 0.9 which is greater than $\frac{0.5}{0.566}$, then we will yield the following plot:



From this plot, we can see that we have a saddle point at (0, 0.9) which supports our prediction.

3.3.2. Discussion

Despite all the calculations, we all know that the purpose of technology development is to increase our survivability in this ecosystem thus means that a higher technology level allows our population to be stabilized. This situation can be frequently discovered in the population dynamics model of developed countries where the overall trend is a logistic growth and the current location on that graph is approaching the stabilized point where the population is limited by the carrying capacity. If a specific population's technology level is declining then we might see the population go through something like the "low scenario" in this image:



[3] All of above analysis are made more based on intuition than the actual data because it is hard to see it directly from all the fixed-point analysis and the plots in the previous part so then let's move on to discussion of how the fixed points and plots are related to the central question of this paper which is how technology efficiency affect the rate of change of population.

The definition of technology efficiency is "the effectiveness with which a given set of inputs is used to produce an output.[4]" From our fixed points, we know that we have a stable fixed point when T is equal to T_m which means that our current technology efficiency has reached maximum efficiency. When the technology efficiency has reached or close to a relatively high level, then from equation 1 we know that the consumption of industrial goods will rise thus leads to a declining rate of change of population. As the rate of change continues to decline, the population will become stabilized and reach a carrying capacity which is similar to my intuitive judgment made from previous part and is why I think figure [3] have an accurate prediction for the future trends of population where low scenario is when technology efficiency decrease, medium scenario is when efficiency reached or close to maximum efficiency. A reasonable explanation for the high scenario is when the maximum efficiency increased due to a significant technological breakthrough(ex: recycle wastewater to become usable fuel) which created a gap between our current technology efficiency and the maximum efficiency. Then the rate of change of population might increase as the current efficiency catch up to the maximum efficiency.

Recall that when we are determining the stability of the fixed point (0, T_m), it turns out that we only get a stable fixed point when T_m is smaller than $\frac{0.5}{0.566}$ and otherwise it is a saddle point. This can be explained by the Jevons paradox which it stated: "when technological progress or government policy increases the efficiency with which a resource is used (reducing the amount necessary for any one use), but the rate of consumption of that resource rises due to increasing demand.[5]" Following this statement, the carrying capacity will eventually decrease since the number of resources is a major factor in the carrying capacity. With the drop in the carrying capacity, the human population will start to decline as it adjusts to the new carrying capacity which explains the saddle point(unstable) fixed point at (0, T_m).

4. Technology efficiency direct impact on birth and death rate

4.1. Setup

All of the previous discussions are based on an analysis of how technology efficiency makes a difference in the rate of change of population concerning the maximum growth rate μ_{max}. In this section, μ_{max} will replaced by the birth rate minus the death rate to see if there is a difference in the result. If there is a difference then it means that technology efficiency does indeed have a direct impact on the birth rate or the death rate. With the replacement, our $\frac{dH}{dt}$ function will become:

$$\frac{dH}{dt} = (Birth - Death)H(1 - e^{y_1^{min} - \gamma_1 B^{\Omega} \frac{T}{T_m}})e^{-b_2\gamma_2 \frac{T}{T_m}} \tag{7}$$

I will make the following assumptions for this section:

- The $(1 - e^{y_1^{min} - \gamma_1 B^{\Omega} \frac{T}{T_m}})e^{-b_2\gamma_2 \frac{T}{T_m}}$ term should be directly proportional to the birth rate.
- The above item should be inversely proportional the death rate.
- Birth rate and death rate are both positive variables.

With the addition of the assumptions, I will adjust the function into:

$$\frac{dH}{dt} = (Birth(1 - e^{y_1^{min} - \gamma_1 B^{\Omega} \frac{T}{T_m}})e^{-b_2\gamma_2 \frac{T}{T_m}} - Death)H \tag{8}$$

$$\frac{dH}{dt} = (Birth - \frac{Death}{(1 - e^{y_1^{min} - \gamma_1 B^{\Omega} \frac{T}{T_m}})e^{-b_2\gamma_2 \frac{T}{T_m}}})H \tag{9}$$

where the first equation (8) is relating birth rate with technology efficiency and second equation (9) is relating death rate with technology efficiency. For this section, "Birth" represent the birth rate of a population and "Death" represent the death rate of that population

4.2. Calculations

For this section, I will continue to use the default value for all of the constants and find all fixed points for each equation and their stability.

4.2.1. Birth rate function

In this case, the sets of the fixed point are still (0, 0) and (0, T_m) but the matrix that is used to determine the stability should have some changes.

By performing a partial derivative of F(H, T) and G(H, T) just like before, the matrix is the following:

$$\begin{bmatrix} Birth(1 - e^{0.5-0.566T})e^{-0.1132T} - Death & Birth * H(-0.1132e^{-0.1132T} + 0.6692e^{0.5-0.6692T}) \\ 0 & \sigma - \frac{2\sigma T}{T_m} \end{bmatrix}$$

By plugging in the first fixed point, we will get the following matrix:

$$\begin{bmatrix} Birth(1 - e^{0.5}) - Death & 0 \\ 0 & \sigma \end{bmatrix}$$

Since this is a diagonal matrix, $\lambda 1$ is [Birth(1 - $e^{0.5}$) - Death] and $\lambda 2$ is σ. For this case, since both birth rate and death rate are always positive variable, this fixed point will be a saddle point.

By plugging in the second fixed point, we will get the following result:

$$\begin{bmatrix} Birth(1 - e^{0.5-0.566T_m})e^{-0.1132T_m} - Death & 0 \\ 0 & -\sigma \end{bmatrix}$$

To determine the stability of this fixed point, we will use T_m is equal to 0.8 from the previous graphing section. The $\lambda 1$ for this matrix is equal to Birth(1 - $e^{0.5-0.566T_m}$)$e^{-0.1132T_m}$ - Death) and $\lambda 2$ is equal to $-\sigma$. Let's make a hypothesis that this T_m value will result in a stable fixed point just like before so by setting $\lambda 1$ is less than zero, we will get:

$$-0.04833Birth(0.8989) - Death < 0 \tag{10}$$

$$Death > -0.0434Birth \tag{11}$$

$$-22.65Death < Birth \tag{12}$$

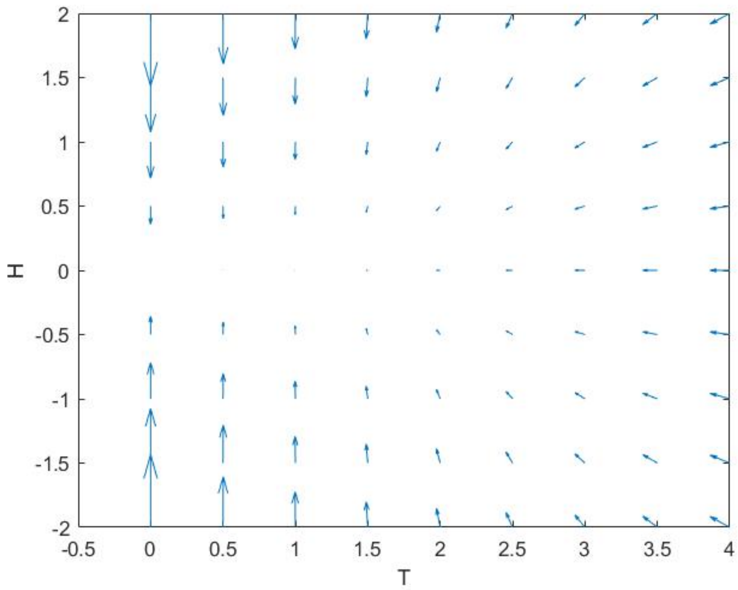
This result makes sense since both of our Birth rates and Death rates are positive so the negative product of death rate should always be less than the birth rate.

This proves that our hypothesis is correct since if we had use greater than zero(saddle point) then we would get -23*Death > Birth which can't be right.

4.2.2. Plot for the birth function

For this section, I will be using the United States 2019 birth rate and death rate which are 11.979 and 8.782 [6].

By using quiver in MATLAB, we will get the following graph:



4.2.3. Death rate function

For this section, we will be using the equation (9) as our $\frac{dH}{dt}$ function to find the fixed point and stability of this system of equation. The sets of fixed point are still (0, 0) and (0, T_m) when you set both function to zero. Similar to the previous part, the partial derivative of function F(H, T) and G(H, T) will give the following result:

$$\begin{bmatrix} Birth - \frac{Death}{(1-e^{0.5-0.566T})e^{-0.1132T}} & \frac{-Death*H(-0.1132e^{-0.1132T}+0.6692e^{0.5-0.6692T})}{(e^{-0.1132T}-e^{0.5-0..6692T})^2} \\ 0 & \sigma - \frac{2\sigma T}{T_m} \end{bmatrix}$$

The first set of fixed point (0, 0) will result in:

$$\begin{bmatrix} Birth - \frac{Death}{1-e^{0.5}} & 0 \\ 0 & \sigma \end{bmatrix}$$

With both positive birth and death rate, λ 1 and 2 will be both positive which result in a unstable fixed point at (0, 0).

For the second fixed point, we will get the following matrix:

$$\begin{bmatrix} Birth - \frac{Death}{(1-e^{0.5-0.566T_m})e^{-0.1132T_m}} & 0 \\ 0 & -\sigma \end{bmatrix}$$

Similarly, we will use $T_m = 0.8$ and assume this fixed point is stable therefore,

$$Birth - \frac{Death}{(1 - e^{0.5-0.566T_m})e^{-0.1132T_m}} < 0 \tag{13}$$

must be less than zero. By plugging T_m, we get:

$$Birth - \frac{Death}{-0.04414} < 0 \tag{14}$$

$$Birth < -22 * Death \tag{15}$$

With birth rate and death rate both positive, the above equation can't be satisfied therefore, this fixed point can't be stable. If we set the equation such that:

$$Birth - \frac{Death}{(1 - e^{0.5-0.566T_m})e^{-0.1132T_m}} > 0 \tag{16}$$

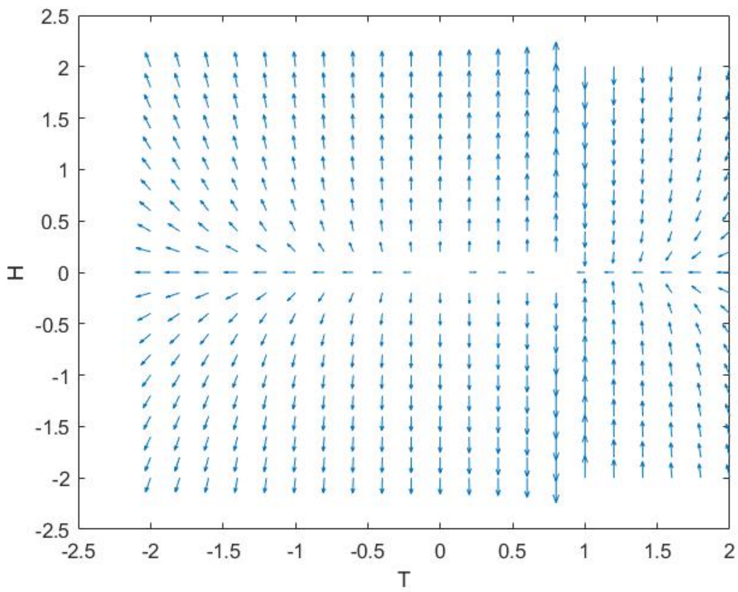
we would get:

$$Birth > -22 * Death \tag{17}$$

which positive birth and death rate will always satisfy the above equation so this is a saddle point.

4.2.4. Plot for death function

The following direction field is created using MATLAB command quiver:



4.3. Discussion of the results

The first part of this section focused on founding the fixed points of the system of equations (2) and (8) and determining their stability. It was clear that the fixed points of this system of equation and the original system of equations share the same characteristic where (0, 0) is saddle point and (0, T_m) is a stable fixed point. This makes sense since both systems, the technology efficiency factor is directly proportional to the intrinsic growth rate of the population and birth rate and μ_{max} are just mathematical symbols so no matter which one we use, it should always give us a similar result. The MATLAB plot continued to support the above statement. Even though the arrows are less tilted but the general trend of pointing toward [0, 0.8(which is T_m)] doesn't change so the plot does indeed tell us that there is a saddle point at (0, 0) and a stable fixed point at (0, T_m).

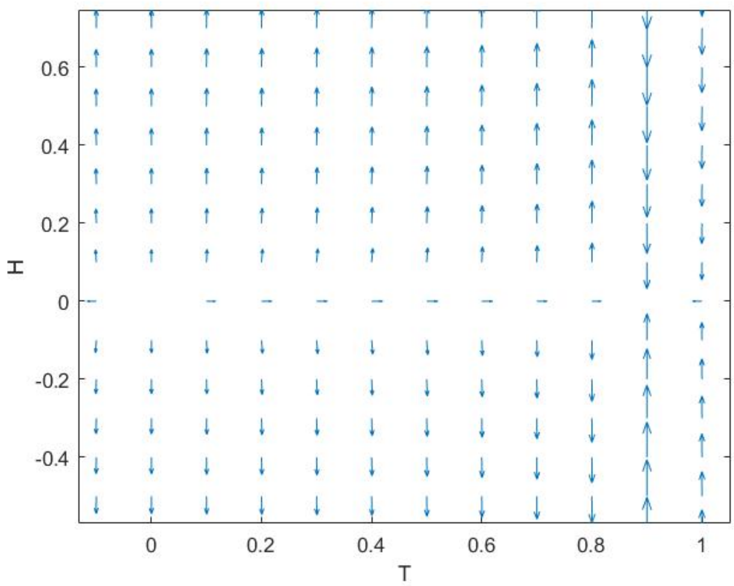
In contrast, fixed points of the system of the equation of (2) and (9) show a different behavior comparing the fixed points of the original system of equations. In this case, we get a unstable fixed point at (0, 0) and a saddle point at (0, T_m). Since a saddle point is also considered to be an unstable fixed point, therefore, I will not discuss the difference of a saddle point in the original function and an unstable fixed point in this function at (0, 0). The plot of this system supports the fact that there is a saddle point at (0, T_m) but why does this fixed point doesn't follow the trends of the previous two functions? One possible explanation is that let's assume the technology efficiency has successfully decreased the death rate of our current population without a change in the birth rate. If this is the case, our population will continue to increase dramatically which means we will have more individuals that are going to consume the limited resource of our ecosystem. Similar to the case of Jevons Paradox of the original system, with the increasing consumption of resources, the carrying capacity will decrease thus cause this system to be unstable. Another possible reasoning focus on the child mortality rate part of the death rate. As technology efficiency increases, there is a possibility that the child mortality rate decrease which means that families are less likely to have more children comparing to when there is a high child mortality rate. With decrease desire of some families to have more children, we might see the rate of change of the population start to decrease which causes this system to become unstable. This phenomenon is called the Child Survival Hypothesis and even though it is disapproved by several papers such as [7] but this could still be a possible explanation for this situation.

In order to get a stable fixed point at (0, T_m) for this system of equations, let change the value of T_m to be some number greater than $\frac{0.5}{0.566}$ so that the denominator of the death term in equation 13 will be positive. so if T_m is equal to 0.9 then the equation will become:

$$Birth - \frac{Death}{0.008} < 0 \tag{18}$$

$$Birth < 106.88 * Death \tag{19}$$

The value of birth rate death rate that was used before would satisfy the above equation and the direction field will become:



From this graph, we can see that we have a stable fixed point at (0, 0.9) so T_m equals 0.9 will give us a stable fixed point at (0, T_m).

5. Improvement

One improvement that can contribute to yield a better result is to use the original three equation system from the reference [1] because by giving up one equation and changing that function to become a constant, the system lost many of its correlations which could cost an inaccuracy of the result. If I were to use the three equation system, the calculation of this paper would be much difficult since it will involve more aggregated parameters and this system would require a 3 by 3 matrix and 3 λ to determine the stability of each fixed point. Even if I find all of the lambda correctly, I still don't know how to determine its stability using three lambdas which is why solving this three equations system could be difficult and challenging.

6. Conclusion

In a nutshell, we can conclude that when the technology efficiency level of our population reaches the maximum technology efficiency, our population will become stabilized which can be supported by the fixed point analysis in section 3.2 and the plot in section 3.3. In the latter part, we see that it doesn't matter what kind of mathematical notation we use, as long as the relation of the notation with the rest of the function stays the same, we will yield a similar result.

References

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