Iteration method

Friday, 28 January 2022 9:14 AM

$T(n) = T(n/2^1) + c$ iter 1

$T(n) = T(n/2^2) + 2c$ iter 2

$= T(n/2^3) + 3c$ iter 3

$= T(n/2^4) + 4c$ iter 4

$= T(1) + \dots$

Let $T(1)$ is reached at k^{th} iteration

$T(n) = T(n/2^k) + kc$

$= T(1) + kc$ (1)

ie, $\frac{n}{2^k} = 1$

$n = 2^k$

$k = \log_2 n = \lg n$

Sub 'k' in (1)

$T(n) = \lg n \times c + T(1)$

$= \cancel{c} \lg n + \cancel{T(1)}$

$= O(\lg n)$

$T(n/2) = 2T(n/4) + n/2$

(2) $T(n) = 2^1 T(n/2^1) + n$ iter 1

$= 2\left(2T(n/4) + n/2\right) + n$

$= 2^2 T(n/2^2) + n + n$ iter 2

$= 2^3 T(n/2^3) + 3n$ iter 3

This continue until $T(1)$

Let $T(1)$ is reached at k^{th} iter

At k^{th} level

$T(n) = 2^k T\left(\frac{n}{2^k}\right) + kn$ (1)

ie, $\frac{n}{2^k} = 1$

$n = 2^k$

$k = \lg n$

Sub: ing k in (1)

$T(n) = 2^{\log_2 n} T(1) + n \lg n$

$= n^{\log_2 2} T(1) + n \lg n$

$= n \cancel{T(1)} + n \lg n$

$= O(n \lg n)$