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EXTENDS *Integers*, *FiniteSets*

CONSTANT *N*

ASSUME  $N \in \text{Nat} \setminus \{0, 1\}$

$\text{Procs} \triangleq 1 \dots (N - 1)$

$\text{Numberize}(S) \triangleq \text{IF } S = \text{TRUE} \text{ THEN } 1 \text{ ELSE } 0$

Dijkstra's 3 state

```
--algorithm 3state{
  variable  $c = [i \in 0 \dots N \mapsto \text{Numberize}(i = 0)]$ ;
  variable  $c = [i \in 0 \dots N \mapsto ((i + 1)\%3)]$ ;
  fair process (  $i \in \{0\}$  )
  {
    I0:  $c[0] := 1$ ;
    I1: while ( TRUE )
    {
      await  $((c[0] + 1)\%3 = c[1])$ ;
       $c[0] := (c[1] + 1)\%3$ ;
    }
  }
  fair process (  $n \in \{N\}$  )
  {
    N1: while ( TRUE )
    {
      await  $((c[(N - 1)] = c[0]) \wedge (c[N] \neq ((c[N - 1] + 1)\%3)))$ ;
       $c[N] := (c[(N - 1)] + 1)\%3$ ;
    }
  }
  fair process (  $j \in \text{Procs}$  )
  {
    J1: while ( TRUE )
    {
      either
      {
        await  $((c[\text{self}] + 1)\%3 = c[(\text{self} - 1)])$ ;
         $c[\text{self}] := c[(\text{self} - 1)]$ ;
      }
      or
      {
        await  $((c[\text{self}] + 1)\%3 = c[(\text{self} + 1)])$ ;
         $c[\text{self}] := c[(\text{self} + 1)]$ ;
      }
    }
  }
}
```

```

    }
  }
}

```

**BEGIN TRANSLATION**

VARIABLES  $c, pc$

$vars \triangleq \langle c, pc \rangle$

$ProcSet \triangleq (\{0\}) \cup (\{N\}) \cup (Procs)$

$Init \triangleq$  **Global variables**  
 $\wedge c = [i \in 0 \dots N \mapsto Numberize(i = 0)]$   
 $\wedge pc = [self \in ProcSet \mapsto \text{CASE } self \in \{0\} \rightarrow \text{"I0"}$   
 $\quad \quad \quad \square \quad self \in \{N\} \rightarrow \text{"N1"}$   
 $\quad \quad \quad \square \quad self \in Procs \rightarrow \text{"J1"}]$

$I0(self) \triangleq$   $\wedge pc[self] = \text{"I0"}$   
 $\wedge c' = [c \text{ EXCEPT } ![0] = 1]$   
 $\wedge pc' = [pc \text{ EXCEPT } ![self] = \text{"I1"}]$

$I1(self) \triangleq$   $\wedge pc[self] = \text{"I1"}$   
 $\wedge (((c[0] + 1) \% 3) = c[1])$   
 $\wedge c' = [c \text{ EXCEPT } ![0] = (c[1] + 1) \% 3]$   
 $\wedge pc' = [pc \text{ EXCEPT } ![self] = \text{"I1"}]$

$i(self) \triangleq I0(self) \vee I1(self)$

$N1(self) \triangleq$   $\wedge pc[self] = \text{"N1"}$   
 $\wedge ((c[(N - 1)] = c[0]) \wedge (c[N] \neq ((c[N - 1] + 1) \% 3)))$   
 $\wedge c' = [c \text{ EXCEPT } ![N] = (c[(N - 1)] + 1) \% 3]$   
 $\wedge pc' = [pc \text{ EXCEPT } ![self] = \text{"N1"}]$

$n(self) \triangleq N1(self)$

$J1(self) \triangleq$   $\wedge pc[self] = \text{"J1"}$   
 $\wedge \vee \wedge (((c[self] + 1) \% 3) = c[(self - 1)])$   
 $\quad \wedge c' = [c \text{ EXCEPT } ![self] = c[(self - 1)]]$   
 $\vee \wedge (((c[self] + 1) \% 3) = c[(self + 1)])$   
 $\quad \wedge c' = [c \text{ EXCEPT } ![self] = c[(self + 1)]]$   
 $\wedge pc' = [pc \text{ EXCEPT } ![self] = \text{"J1"}]$

$j(self) \triangleq J1(self)$

$Next \triangleq$   $(\exists self \in \{0\} : i(self))$   
 $\vee (\exists self \in \{N\} : n(self))$   
 $\vee (\exists self \in Procs : j(self))$

$$\begin{aligned}
Spec &\triangleq \wedge Init \wedge \Box[Next]_{vars} \\
&\wedge \forall self \in \{0\} : \text{WF}_{vars}(i(self)) \\
&\wedge \forall self \in \{N\} : \text{WF}_{vars}(n(self)) \\
&\wedge \forall self \in Procs : \text{WF}_{vars}(j(self))
\end{aligned}$$

END TRANSLATION

$$\begin{aligned}
T\_0 &\triangleq \text{IF } (((c[0] + 1) \% 3) = c[1]) \text{ THEN } 1 \text{ ELSE } 0 \\
T\_N &\triangleq \text{IF } ((c[(N - 1)] = c[0]) \wedge (c[N] \neq ((c[N - 1] + 1) \% 3))) \text{ THEN } 1 \text{ ELSE } 0 \\
T &\triangleq \text{Cardinality}(\{k \in Procs : (((c[k] + 1) \% 3) = c[k - 1]) \vee (((c[k] + 1) \% 3) = c[k + 1])\}) + T\_0 + T\_N \\
Invariant &\triangleq T = 1 \\
Stabilization &\triangleq \Box \Diamond Invariant \\
LowerBound &\triangleq T \geq 1 \\
DoesNotMoveAway &\triangleq \Box[T' \leq T]_{vars} \\
MovesTowards &\triangleq \forall M \in 1 .. N + 1 : \Box \Diamond (T \leq M)
\end{aligned}$$

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