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EXTENDS *Integers*, *FiniteSets*

CONSTANT *N*

ASSUME  $N \in \text{Nat} \setminus \{0, 1\}$

$\text{Procs} \triangleq 1 \dots (N - 1)$  Or  $0 \dots N$  should *J* be equal to procs or not?

$\text{Numberize}(S) \triangleq \text{IF } S = \text{TRUE} \text{ THEN } 1 \text{ ELSE } 0$

Dijkstra's 4 state

```
--algorithm 4state{
  variable up = [i ∈ 0 .. N ↦ Numberize(i = 0)], c = [i ∈ 0 .. N ↦ 0];
  variable up = [i ∈ 0 .. N ↦ Numberize((i%2) = 0)], c = [i ∈ 0 .. N ↦ (i%2)];
  fair process ( i ∈ {0} )
  {
    I0: up[self] := 1;
    I1: while ( TRUE )
      { await ((c[0] = c[1]) ∧ up[1] = 0);
        c[0] := (c[0] + 1)%2;
      }
  }
  fair process ( n ∈ {N} )
  {
    N0: up[self] := 0;
    N1: while ( TRUE )
      { await (c[(N - 1)] ≠ c[N]);
        c[N] := c[(N - 1)];
      }
  }
  fair process ( j ∈ Procs )
  {
    J1: while ( TRUE )
    {
      either
      { await (c[self] ≠ c[(self - 1)]);
        c[self] := c[(self - 1)];
        up[self] := 1;
      }
      or
      { await ((c[(self)] = c[(self + 1)]) ∧ up[(self + 1)] = 0 ∧ up[self] = 1);
        up[self] := 0;
      }
    }
  }
}
```

```

    }
  }
}

```

BEGIN TRANSLATION

VARIABLES  $up, c, pc$

$vars \triangleq \langle up, c, pc \rangle$

$ProcSet \triangleq (\{0\}) \cup (\{N\}) \cup (Procs)$

$Init \triangleq$  Global variables  
 $\wedge up = [i \in 0 \dots N \mapsto Numberize(i = 0)]$   
 $\wedge c = [i \in 0 \dots N \mapsto 0]$   
 $\wedge pc = [self \in ProcSet \mapsto \text{CASE } self \in \{0\} \rightarrow \text{"I0"}$   
 $\quad \square \quad self \in \{N\} \rightarrow \text{"N0"}$   
 $\quad \square \quad self \in Procs \rightarrow \text{"J1"}]$

$I0(self) \triangleq$   $\wedge pc[self] = \text{"I0"}$   
 $\wedge up' = [up \text{ EXCEPT } ![self] = 1]$   
 $\wedge pc' = [pc \text{ EXCEPT } ![self] = \text{"I1"}]$   
 $\wedge c' = c$

$I1(self) \triangleq$   $\wedge pc[self] = \text{"I1"}$   
 $\wedge ((c[0] = c[1]) \wedge up[1] = 0)$   
 $\wedge c' = [c \text{ EXCEPT } ![0] = (c[0] + 1) \% 2]$   
 $\wedge pc' = [pc \text{ EXCEPT } ![self] = \text{"I1"}]$   
 $\wedge up' = up$

$i(self) \triangleq I0(self) \vee I1(self)$

$N0(self) \triangleq$   $\wedge pc[self] = \text{"N0"}$   
 $\wedge up' = [up \text{ EXCEPT } ![self] = 0]$   
 $\wedge pc' = [pc \text{ EXCEPT } ![self] = \text{"N1"}]$   
 $\wedge c' = c$

$N1(self) \triangleq$   $\wedge pc[self] = \text{"N1"}$   
 $\wedge (c[(N - 1)] \neq c[N])$   
 $\wedge c' = [c \text{ EXCEPT } ![N] = c[(N - 1)]]$   
 $\wedge pc' = [pc \text{ EXCEPT } ![self] = \text{"N1"}]$   
 $\wedge up' = up$

$n(self) \triangleq N0(self) \vee N1(self)$

$J1(self) \triangleq$   $\wedge pc[self] = \text{"J1"}$   
 $\wedge \vee \wedge (c[self] \neq c[(self - 1)])$   
 $\quad \wedge c' = [c \text{ EXCEPT } ![self] = c[(self - 1)]]$   
 $\quad \wedge up' = [up \text{ EXCEPT } ![self] = 1]$   
 $\vee \wedge ((c[(self)] = c[(self + 1)]) \wedge up[(self + 1)] = 0 \wedge up[self] = 1)$

$$\begin{aligned}
& \wedge up' = [up \text{ EXCEPT } ![self] = 0] \\
& \wedge c' = c \\
& \wedge pc' = [pc \text{ EXCEPT } ![self] = \text{"J1"}]
\end{aligned}$$

$$j(self) \triangleq J1(self)$$

$$\begin{aligned}
Next \triangleq & (\exists self \in \{0\} : i(self)) \\
& \vee (\exists self \in \{N\} : n(self)) \\
& \vee (\exists self \in Procs : j(self))
\end{aligned}$$

$$\begin{aligned}
Spec \triangleq & \wedge Init \wedge \Box [Next]_{vars} \\
& \wedge \forall self \in \{0\} : \text{WF}_{vars}(i(self)) \\
& \wedge \forall self \in \{N\} : \text{WF}_{vars}(n(self)) \\
& \wedge \forall self \in Procs : \text{WF}_{vars}(j(self))
\end{aligned}$$

END TRANSLATION

$$T\_0 \triangleq \text{IF } ((c[0] = c[1]) \wedge (up[1] = 0)) \text{ THEN } 1 \text{ ELSE } 0$$

$$T\_N \triangleq \text{IF } c[N] \neq c[N-1] \text{ THEN } 1 \text{ ELSE } 0$$

$$T \triangleq \text{Cardinality}(\{k \in Procs : (c[k] \neq c[k-1]) \vee ((c[k] = c[k+1]) \wedge (up[k] = 1) \wedge (up[k+1] = 0))\}) + T\_0 -$$

$$Invariant \triangleq T = 1$$

$$Stabilization \triangleq \Box \Diamond Invariant$$

$$LowerBound \triangleq T \geq 1$$

$$DoesNotMoveAway \triangleq \Box [T' \leq T]_{vars}$$

$$MovesTowards \triangleq \forall M \in 1 \dots N+1 : \Box \Diamond (T \leq M)$$

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\ \* Modification History  
\ \* Last modified Sun Dec 13 01:56:07 EST 2015 by chartoin  
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