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VIBRATION OF CIRCULAR MEMBRANE BACKED BY CYLINDRICAL CAVITY

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Abstract—The natural vibration of a circular membrane backed by a cylindrical air cavity is investigated using the multimodal approach. The cavity-backed membrane is modeled as a dynamical system composed of two subsystems, and their modal receptance or "inverse receptance" characteristics are used to study the system vibration. The natural frequencies of the system are determined for four typical cases and the results show that the resulting modes can be categorized into three groups depending on the strength of the sybsystem interaction. © 1998 Elsevier Science Ltd. All rights reserved

Keywords: vibration, circular membrane, cylindrical cavity.

NOTATION

$[\mathbf{A}_n^{(a)}]$	membrane receptance matrix
a	radius
c_a	speed of sound in air $(p_\mathrm{a}/\rho)^{1/2}$
c_s	wave speed in membrane $(T/\sigma)^{1/2}$
J_n	bessel function of order n
$k_{n,i,j}^{(a)}$	cavity rejectance coefficients
$[\mathbf{K}_n^{(a)}]$	cavity rejectance matrix
L	cavity depth
p	air pressure (gauge)
p_a	ambient pressure
$p^{(b)}$	back pressure
$p_{n,m}^{(s)}, p_{n,i}^{(a)}$	back pressure components
$q^{(e)}$	external pressure (gauge)
$q_{n,m}^{(s)}, q_{n,i}^{(a)}$	external pressure components
r	non-dimensional radial coordinate
t	time
T	tension
w	membrane displacement
$w_{n,m}^{(s)}, w_{n,i}^{(a)}$	displacement components
z	non-dimensional axial coordinate
$\alpha_{n,l,m}^{(s)}, \alpha_{n,i,j}^{(a)}$	membrane receptance coefficients
γ	specific heat ratio
$\zeta_{n,i}$	defined in Eqn (23)
θ	angular coordinate
$\lambda_{n,i,m}$	defined in Eqns (34) and (35)
$\mu_{n,m}$	mth zero of J_n
$v_{n,i}$	ith zero J_n'; $v_{0,0} = 0$
ρ	ambient air density
σ	membrane mass per unit area
$\phi_{n,i}(r)$	cavity modal functions, Eqn (18)
$\psi_{n,m}(r)$	membrane modal functions, Eqn (5)
ω	frequency
$\omega_{n,i,k}^{(a)}$	cavity frequencies
$\omega_{n,m}^{(s)}$	membrane frequencies
Ω	non-dimensional frequency, $\Omega = (a\omega/c_a)$

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INTRODUCTION

In several engineering applications, it is necessary to control the noise level in an enclosure and to keep it below a certain preassigned limit. Invariably, the source of the interior noise is an external disturbance transmitted through the structure. The principal parameters which influence the transmission of the exterior noise into the enclosure are the flexibility and inertia of the wall structure. In most cases, noise transmission through the wall may be controlled by altering the structure design parameters. However, in certain applications, where reduced weight of the wall structure is also a design criterion, active noise control methods may be utilized to achieve the desired noise level reduction [1].

Previous studies have used a rectangular parallelopiped enclosure with one flexible face as a model to study the noise transmission problem. The focus of these investigations is mainly on the forced-vibration analysis of the cavity-backed panel. Even though the underlying theory can be used to study the free vibration of the cavity-backed panel, accurate numerical results for the system natural frequencies are not readily available. A review of the published literature on the vibration of the cavity-backed panel is included in Ref. [2]. Contemporary methods can be classified into three groups, i.e. (i) the multimodal approach [3–5], (ii) the acoustoelasticity theory method [6] and (iii) the variational energy formulation [7]. Several investigations using the above methods have contributed to the physical understanding of sound transmission through flexible panels [2].

The cavity-backed rectangular panel can be represented by a closed-loop control system. When the plate deflects under the pressure difference, the cavity induces a back pressure to oppose the plate displacement. Such an approach which considers the cavity-backed panel as a dynamical system composed of two interacting subsystems, namely the isolated plate and the open-end cavity, was used in Ref. [8] to determine accurately the system frequencies. This investigation showed that the natural frequencies of the system are different from the subsystem frequencies. When the linear dimensions are in simple integer ratios, the subsystems had multiple frequencies associated with distinct modes.

Many engineering situations involving cavities bounded by flexible surfaces can be modeled as cylindrical structure–cavity systems. Also, a circular plate backed by a cylindrical cavity is a convenient model to understand the vibration characteristics of a cavity backed panel. Since the circular plate frequencies are far apart compared to the cylindrical cavity frequencies, a circular membrane is substituted for the plate in the system model for the present investigation. The vibration of a composite circular membrane backed by a cylindrical cavity was studied in Ref. [9] using a "Kettle drum" model [10] to represent the air cavity effect on the membrane and following the Rayleigh–Ritz method. A multimodal approach is employed to analyze the cylindrical membrane–cavity system in the present investigation. Results and discussion are presented for four typical cases.

THEORY

The cavity backed membrane, shown in Fig. 1, can be considered as a dynamical system composed of two interacting subsystems. The membrane and the open-end cavity are the subsystems and the pressure–displacement characteristics of these subsystems at their interface can be used to study the forced vibration of the system. The present analysis is confined to the determination of the system response for harmonic excitation on the external surface of the membrane.

In the theoretical model, the membrane is considered as a receptor, subjected to a pressure difference across its surface resulting it into membrane displacement. The air cavity is regarded as a "rejector" (inverse receptor) which opposes the membrane displacement with a back pressure. Thus, the cavity backed membrane can be represented by the closed-loop control system, whose block diagram is shown in Fig. 2.

Vibration of circular membrane

The equation of motion of the membrane can be expressed in non-dimensional polar coordinates (r, θ) as

$$\frac{T}{a^2}\left[\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}\right]w-\sigma\frac{\partial^2 w}{\partial t^2}=-q^{(e)}+p^{(b)}. \tag{1}$$

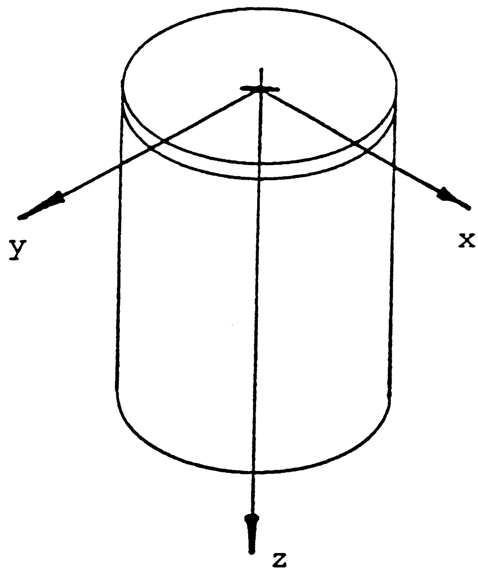

Fig. 1. Cavity-backed membrane.

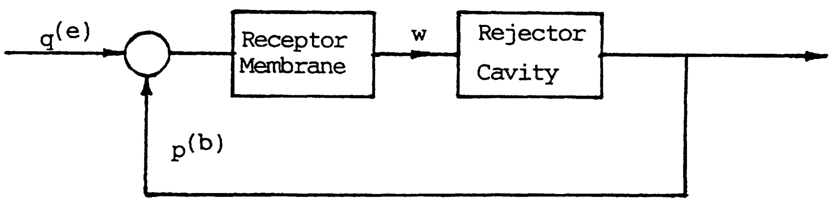

Fig. 2. Block diagram.

The modal receptance characteristics of the membrane can be used to obtain the relationship between the membrane displacement w and the pressure difference $(q^{(e)} - p^{(b)})$ for harmonic excitation at frequency ω. Thus, an investigation of the membrane natural vibration is necessary.

Natural vibration of circular membrane. For the natural vibration analysis, the homogeneous part of Eqn (1) can be written as

$$\left[\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}\right] w - \frac{a^2}{c_s^2}\frac{\partial^2 w}{\partial t^2} = 0, \tag{2}$$

where $c_s = (T/\sigma)^{1/2}$. The (n, m) modal displacement of the membrane can be expressed as

$$w = \psi_{n,m}(r) \cos n\theta \sin \omega_{n,m}^{(s)} t. \tag{3}$$

Substitution of Eqn (3) into Eqn (2) gives

$$\frac{\mathrm{d}^2}{\mathrm{d}r^2}\psi_{n,m} + \frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\psi_{n,m} + \left(\mu_{n,m}^2 - \frac{n^2}{r^2}\right)\psi_{n,m} = 0. \tag{4}$$

Using the boundary conditions (i) $\psi_{n,m}(0) =$ finite and (ii) $\psi_{n,m}(1) = 0$, the normalized solution for $\psi_{n,m}(r)$ in Eqn (4) can be obtained as

$$\psi_{n,m}(r) = \frac{\sqrt{2} J_n(\mu_{n,m} r)}{J_n(\mu_{n,m})}, \tag{5}$$

where

$$J_n(\mu_{n,m}) = 0. \tag{6}$$

In the standard notation for the zeroes of Bessel functions $\mu_{n,m} = j_{n,m}$ and consequently, the expression for the (n, m) membrane frequency, $\omega^{(s)}_{n,m}$ becomes $(j_{n,m}c_s/a)$. Further, the coefficient in Eqn (5) is chosen to normalize functions $\psi_{n,m}(r)$ using the conditions

$$\int_0^1 \psi_{n,m}(r)\psi_{n,l}(r)\, r\, \mathrm{d}r = \begin{cases} 1 & \text{for } l = m, \\ 0 & \text{for } l \neq m. \end{cases} \tag{7}$$

Modal receptance of the membrane. A particular solution to Eqn (1) can be obtained in the form,

$$w = w^{(s)}_{n,m}\psi_{n,m}(r)\cos n\theta \sin \omega t, \tag{8}$$

$$p^{(b)} = p^{(s)}_{n,m}\psi_{n,m}(r)\cos n\theta \sin \omega t, \tag{9}$$

$$q^{(e)} = q^{(s)}_{n,m}\psi_{n,m}(r)\cos n\theta \sin \omega t. \tag{10}$$

Substitution of Eqns (8)–(10) into Eqn (1) and simplifying using Eqn (4) gives

$$\{w^{(s)}_{n,m}/(q^{(s)}_{n,m} - p^{(s)}_{n,m})\} = 1/\{\sigma(\omega^{(s)^2}_{n,m} - \omega^2)\}.$$

The quotient on the LHS of the above equation is the direct modal receptance of the membrane. Thus, the cross-receptance between the (n, l) and (n, m) membrane modes can be expressed as

$$\alpha^{(s)}_{n,l,m} = \begin{cases} 1/\{\sigma(\omega^{(s)^2}_{n,m} - \omega^2)\} & \text{for } l = m, \\ 0 & \text{for } l \neq m, \end{cases} \tag{11}$$

Consequently, the membrane displacement can be related to the modal pressure difference by

$$w^{(s)}_{n,m} = \alpha^{(s)}_{n,m,m}(q^{(s)}_{n,m} - p^{(s)}_{n,m}). \tag{12}$$

The actual membrane displacement during the forced vibration of the system can be recreated as a superposition of the modal displacements.

Vibration of the open-ended cavity

The pressure variation in the air cavity is governed by the wave equation

$$\frac{1}{a^2}\left[\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}\right]p + \frac{1}{L^2}\frac{\partial^2}{\partial z^2}p = \frac{1}{c_a^2}\frac{\partial^2}{\partial t^2}p, \tag{13}$$

where $c_a = (\gamma p_a/\rho)^{1/2}$. The boundary conditions for the solution of Eqn (13) are

(i) $p|_{r=0} = \text{finite},$ (14)

(ii) $\left.\dfrac{\partial p}{\partial r}\right|_{r=1} = 0,$ (15)

(iii) $\left.\dfrac{\partial p}{\partial z}\right|_{z=1} = 0,$ (16)

(iv) $\left.\dfrac{\partial p}{\partial z}\right|_{z=0} = -\rho L\dfrac{\partial^2 w}{\partial t^2},$ (17)

where, Eqn (16) relates the pressure in the cavity to the particle displacement at the open end. In the above formulation, the mathematical problem reduces to the solution of a homogeneous partial differential equation subjected to inhomogeneous boundary conditions. The separation of variables method is not convenient to obtain the required solution corresponding to an arbitrarily specified particle displacement [11]. However, it is possible to study the above forced-vibration problem of the open-end cavity by considering its modal vibration characteristics. Natural vibration analysis of the cavity is a prerequisite to obtain the expression for the cavity modal interface displacement.

Natural vibration of the air cavity. The governing Eqn (13) and the boundary conditions (14)–(16) remain valid for the natural vibration analysis. But the boundary condition (17) at the membrane–cavity interface has to be changed to $p|_{z=0} = 0$. Using the separation of variables method, the modal solutions for the open-end cavity can be simplified to

$$p = \phi_{n,i}(r) \sin(k - \tfrac{1}{2})\, \pi z \cos n\theta \sin \omega^{(a)}_{n,i,k} t \tag{18}$$

where

$$\phi_{n,i}(r) = \begin{cases} \dfrac{\sqrt{2}\nu_{n,i}}{\sqrt{\nu^2_{n,i} - n^2}} \cdot \dfrac{J_n(\nu_{n,i} r)}{J_n(\nu_{n,i})} & \text{for } n \geqslant 0 \text{ and } i \geqslant 1, \\ \sqrt{2} & \text{for } n = 0 \text{ and } i = 0, \end{cases} \tag{19}$$

$$J'_n(\nu_{n,i}) = 0, \tag{20}$$

$$\omega^{(a)}_{n,i,k} = \left[\left\{ \frac{\nu_{n,i} c_a}{a} \right\}^2 + \left\{ \left(k - \frac{1}{2} \right) \frac{\pi c_a}{L} \right\}^2 \right]^{1/2}. \tag{21}$$

The natural frequencies $\omega^{(a)}_{n,i,k}$ given by Eqn (21) consist of two components. The component given within the second set of brackets on the RHS of Eqn (21) represents the frequencies of vibration in the axial direction. As $L \to \infty$, the natural frequency approaches $\{\nu_{n,i} c_a / a\}$, the frequency of vibration perpendicular to the z-axis. For the open-end cylindrical cavity, the natural vibration is a mixture of the in-plane and the axial vibrations.

For $n \geqslant 0$ and $i \geqslant 1$, the root $\nu_{n,i}$ of Eqn (20) can be expressed in standard notation as $j'_{n,i}$. In this notation, $j'_{n,i}$ denotes the ith positive zero of the Bessel function derivative J'_n. When $n \geqslant 1$, the function set $\{\phi_{n,i}(r) | i = 1, 2, \dots\}$ can be used to express the pressure and displacement variations in the radial direction. However, when $n = 0$, an additional function $\phi_{0,0}(r)$ must also be included to have the complete base function set $\{\phi_{0,i}(r) | i = 0, 1, \dots\}$. It can be observed that the expression for $\phi_{0,0}(r)$ in Eqn (19) can be obtained from the general expression $\phi_{0,i}(r)$ for $i \geqslant 1$ as a limiting case corresponding to $\nu_{0,0} \to 0$. Thus, the notation $\nu_{n,i}$ is extended to include the case ($n = 0$, $i = 0$) by defining $\nu_{0,0}$ as zero. The coefficients in Eqn (19) for the orthogonal function set $\{\phi_{n,i}\}$ are chosen to normalize the set using the conditions

$$\int_0^1 \phi_{n,i}(r) \phi_{n,j}(r) r \, \mathrm{d}r = \begin{cases} 1 & \text{for } j = i, \\ 0 & \text{for } j \neq i. \end{cases} \tag{22}$$

The particle displacement at the open end corresponding to the modal pressure variation, given in Eqn (18), can be evaluated from Eqn (17) as

$$w = \left[\left\{ \left(k - \frac{1}{2} \right) \frac{\pi}{L} \right\} \Big/ \{\rho \omega^{(a)\,2}_{n,i,k}\} \right] \phi_{n,i}(r) \cos n\theta \sin \omega^{(a)}_{n,i,k} t.$$

Modal rejectance of the air cavity. The air cavity is considered as a rejector which opposes the interface displacement with a back pressure. For the forced vibration analysis of the air cavity the modal interface displacement can be expressed as

$$w = w^{(a)}_{n,i} \phi_{n,i}(r) \cos n\theta \sin \omega t \tag{23}$$

and solution of Eqn (13) subjected to the boundary conditions (14)–(17) becomes

$$p = p^{(a)}_{n,i} \phi_{n,i}(r) \zeta_{n,i}(z) \cos n\theta \sin \omega t. \tag{24}$$

When $\zeta_{n,i}(0) = 1$, the quotient $p^{(a)}_{n,i} / w^{(a)}_{n,i}$ represents the "modal rejectance" of the air cavity. Here the term modal rejectance is used to denote the inverse of the modal receptance of the air cavity. The expression for the modal rejectance can be simplified from Eqns (17), (23) and (24) as $\rho L \omega^2 / \zeta'_{n,i}(0)$.

By substituting Eqn (24) into Eqn (13) the differential equation for $\zeta_{n,i}$ can be obtained as

$$\zeta''_{n,i} + \frac{L^2}{a^2}\left(\frac{\omega^2 a^2}{c_a^2} - v_{n,i}^2\right)\zeta_{n,i} = 0.$$

From the solution of the above equation, subjected to the boundary conditions (i) $\zeta_{n,i}(0) = 1$, and (ii) $\zeta'_{n,i}(1) = 0$, the expression for rejectance between (n, i) and (n, j) cavity modes can be obtained as

$$k^{(a)}_{n,i,j} = \begin{cases} -\left(\frac{\gamma p_a}{a}\right)\frac{\Omega^2}{\sqrt{v_{n,i}^2 - \Omega^2}}\coth\frac{l}{a}\sqrt{v_{n,i}^2 - \Omega^2} & \text{for } j = i \text{ and } \Omega < v_{n,i}, \\ \infty & \text{for } j = i \text{ and } \Omega = v_{n,i}, \\ \left(\frac{\gamma p_a}{a}\right)\frac{\Omega^2}{\sqrt{\Omega^2 - v_{n,i}^2}}\cot\frac{l}{a}\sqrt{\Omega^2 - v_{n,i}^2} & \text{for } j = i \text{ and } \Omega > v_{n,i}, \\ 0 & \text{for } j \neq i, \end{cases} \tag{25}$$

where $\Omega = \omega a/c_a$. The cavity rejectance analysis relates the back pressure for the modal interface displacement as

$$p^{(a)}_{n,i} = k^{(a)}_{n,i,i} w^{(a)}_{n,i}. \tag{26}$$

The actual interface displacement during the forced vibration of the system can be expressed as a superposition of the modal cavity displacements at the interface.

Vibration of cavity backed membrane

Using the modal receptance analysis of the membrane, the interface displacement and the internal and external pressure acting on the membrane can be expressed in terms of the membrane modal components as

$$w = \sum_m w^{(s)}_{n,m}\psi_{n,m}(r)\cos n\theta \sin \omega t, \tag{27}$$

$$p^{(b)} = \sum_m p^{(s)}_{n,m}\psi_{n,m}(r)\cos n\theta \sin \omega t, \tag{28}$$

$$q^{(e)} = \sum_m q^{(s)}_{n,m}\psi_{n,m}(r)\cos n\theta \sin \omega t. \tag{29}$$

The superscript (s) in the modal coefficients $w^{(s)}_{n,m}$, $p^{(s)}_{n,m}$ and $q^{(s)}_{n,m}$ indicates the presence of the radial functions $\psi_{n,m}(r)$ in the series expression. These modal coefficients satisfy Eqns (11) and (12).

The same interface displacement w and the back pressure $p^{(b)}$ can also be expanded in terms of the cavity modal components as

$$w = \sum_i w^{(a)}_{n,i}\phi_{n,i}(r)\cos n\theta \sin \omega t, \tag{30}$$

$$p^{(b)} = \sum_i p^{(a)}_{n,i}\phi_{n,i}(r)\cos n\theta \sin \omega t, \tag{31}$$

where, the functions $\phi_{n,i}(r)$ are used in the series expansion and the modal coefficients $w^{(a)}_{n,i}$ and $p^{(a)}_{n,i}$ with the superscript (a) satisfy Eqns (25) and (26).

Equations (27) and (30) represent two different series expansions of the same variable w. In matrix notation, the coefficient vector $\{w^{(s)}_{n,m}\}$ and $\{w^{(a)}_{n,i}\}$ are two different expressions of the same interface displacement vector with respect to two distinct base function sets $\{\psi_{n,m}\}$ and $\{\phi_{n,i}\}$. For a chosen n, these base function sets are two distinct orthogonal basis of the same function space. The relations between these base function sets can be expressed as

$$\phi_{n,i}(r) = \sum_m \lambda_{n,i,m} \psi_{n,m}(r), \tag{32}$$

$$\psi_{n,m}(r) = \sum_i \lambda_{n,i,m} \phi_{n,i}(r), \tag{33}$$

where

$$\lambda_{n,i,m} = \int_0^1 \phi_{n,i}(r)\psi_{n,m}(r) r \, \mathrm{d}r. \tag{34}$$

Substitution of the expressions for $\phi_{n,i}(r)$ and $\psi_{n,m}(r)$ from Eqns (5) and (18) into Eqn (34) gives

$$\lambda_{n,i,m} = \begin{cases} \dfrac{v_{n,i}}{\sqrt{v_{n,i}^2 - n^2}} \dfrac{2\mu_{n,m}}{(v_{n,i}^2 - \mu_{n,m}^2)} & \text{for } n \geqslant 1 \text{ and } i \geqslant 1, \\ \dfrac{2\mu_{n,m}}{(v_{n,i}^2 - \mu_{n,m}^2)} & \text{for } n = 0 \text{ and } i \geqslant 0, \end{cases} \tag{35}$$

where, the validity of Eqn (35) is extended to the case $n = 0$ and $i = 0$ using the definition $v_{0,0} = 0$ introduced earlier. Since both base function sets are orthonormal, $\lambda_{n,i,m}$ satisfies the orthogonality conditions

$$\sum_m \lambda_{n,i,m} \lambda_{n,j,m} = \begin{cases} 1 & \text{for } j = i, \\ 0 & \text{for } j \neq i, \end{cases}$$

$$\sum_i \lambda_{n,i,l} \lambda_{n,i,m} = \begin{cases} 1 & \text{for } l = m, \\ 0 & \text{for } l \neq m. \end{cases}$$

For a chosen n, any one of the function sets $\{\phi_{n,i}\}$ or $\{\psi_{n,m}\}$ can be used to express a physical variable as a column vector. The function set $\{\phi_{n,i}\}$ associated with the cavity mode is selected as the basis for this investigation. Since the other function set $\{\psi_{n,m}\}$ is used to express the membrane receptance characteristics, it is necessary to transform the membrane receptance matrix with respect to the chosen base function set. Using Eqns (32) and (33), the transformed receptance matrix can be simplified to the form

$$\alpha_{n,i,j}^{(a)} = \frac{1}{(\gamma p_a/a)(\sigma/a\rho)} \sum_m \frac{\lambda_{n,i,m}\lambda_{n,j,m}}{\{(c_s/c_a)^2 \mu_{n,m}^2 - \Omega^2\}}. \tag{36}$$

For a chosen n, Eqns (25) and (36) show the elements of the transformed membrane receptance matrix $[\mathbf{A}_n^{(a)}]$ and the cavity rejectance matrix $[\mathbf{K}_n^{(a)}]$. The equations governing the harmonic vibration of the system can be expressed in matrix notation as

$$\{\mathbf{A}_n^{(a)}\} = [\mathbf{A}_n^{(a)}]\{\mathbf{q}_n^{(a)} - \mathbf{p}_n^{(a)}\}, \tag{37}$$

$$\{\mathbf{p}_n^{(a)}\} = [\mathbf{K}_n^{(a)}]\{\mathbf{w}_n^{(a)}\}. \tag{38}$$

Elimination of $\{\mathbf{w}_n^{(a)}\}$ from Eqns (37) and (38) gives the harmonic response of the system as

$$[\mathbf{I} + \mathbf{K}_n^{(a)}\mathbf{A}_n^{(a)}]\{\mathbf{p}_n^{(a)}\} = [\mathbf{K}_n^{(a)}\mathbf{A}_n^{(a)}]\{\mathbf{q}_n^{(a)}\}. \tag{39}$$

The frequency equation of the system can be expressed in the form of an infinite determinantal equation as

$$\| \mathbf{I} + \mathbf{K}_n^{(a)}\mathbf{A}_n^{(a)} \| = 0. \tag{40}$$

The element of the matrix $[\mathbf{I} + \mathbf{K}_n^{(a)}\mathbf{A}_n^{(a)}]$ decreases progressively as its row and column order increases beyond a certain number corresponding to the assumed frequency and consequently a truncated form of this determinantal equation can be used in numerical computation. Since the elements of the matrices $[\mathbf{K}_n^{(a)}]$ and $[\mathbf{A}_n^{(a)}]$ are frequency dependent, this form of the frequency equation cannot be related to a standard matrix eigenvalue problem. In spite of the complicated form of the frequency equation, it can be used efficiently to determine the natural frequencies in a low frequency range.

The following three non-dimensional parameters are necessary for analysis of the cavity-backed membrane vibration:

(i) slenderness ratio $= L/a$,
(ii) mass ratio $= \sigma/a\rho$,
(iii) speed ratio $= (c_s/c_a)$.

It is useful to compare the natural frequencies of the system with those of its subsystems. For this purpose, the non-dimensional natural frequencies of the membrane and the open-end cavity can be obtained from Eqns (21) and (36) as $(c_s/c_a)\mu_{n,m}$ and $[\nu_{n,i}^2 + \{(k-\frac{1}{2})\pi a/L\}^2]^{1/2}$ respectively. Using an analysis similar to that given earlier, the non-dimensional frequencies of the closed-end cavity can be obtained as $[\nu_{n,i}^2 + \{k\pi a/L\}^2]^{1/2}$.

It is desirable to compare the natural frequencies of the system with those of the membrane, the open-end cavity, and the closed-end cavity. Since the membrane and the open-end cavity do not share a common vibration mode, the back pressure cannot be absent during the natural vibration of the system. Thus, the subsystems interact during the natural vibration of the system and consequently the natural frequencies of the system must differ from those of the subsystems. Because of the presence of back pressure during the natural vibration of a closed-end cavity, the frequency of vibration of the system cannot also be the same as that of the closed-end cavity. However, in the extreme cases of a very deep cavity, a very slack light membrane, or a very taut, heavy membrane, a few lower natural frequencies of the system can be in the vicinities of the frequencies of the membrane, the open-end cavity, or the closed-end cavity.

RESULTS AND DISCUSSION

Computations have been undertaken for the four typical cases of the cavity-backed membrane given in Table 1. In system A, the fundamental cavity frequency is lower than the membrane frequencies and consequently this parameter combination represents a membrane backed by a deep cavity. The parameters of system B correspond to a membrane backed by a shallow cavity, where the fundamental cavity frequency is greater than the first membrane frequency. For system C, the fundamental frequencies of the membrane and the open-end cavity are equal. System D is another special case where the membrane fundamental frequency is equal to the first natural frequency of the closed-end cavity.

The frequency Eqn (40) is an infinite determinantal equation in which the infinite determinant is truncated to a finite order for numerical computation. For the four chosen cases, the values of the truncated determinant are evaluated at the non-dimensional frequencies in the range from 0 to 10 in 0.001 steps. The frequencies at which the determinant vanishes are then interpolated. The convergence of the numerical scheme is validated for system B using determinants of order 5, 10, 15, 20, 25 and 30, and the result of the convergence test is shown in Table 2. It shows that numerical convergence can be achieved in a low frequency range by utilizing a sufficiently large order of the determinant. Determinants of order 30 are used for computation of the natural frequencies of the four chosen systems, and the results are shown in Table 3. In order to illustrate the convergence, the results obtained by using determinants of order 25 are also included in Table 3.

Table 1. Data for computation

System	Slenderness ratio, L/a	Mass ratio, $\sigma/(a\rho)$	Speed ratio, c_s/c_a
A	5.0	3.3	0.6
B	1.5	3.3	0.3
C	2.0	3.3	0.326592
D	2.0	3.3	0.653185

Table 2. Convergence test results for system B

$N = 5$	$N = 10$	$N = 15$	$N = 20$	$N = 25$	$N = 30$
0.7607	0.7605	0.7605	0.7605	0.7605	0.7605
1.5933	1.5905	1.5902	1.5902	1.5902	1.5901
2.1766	2.1760	2.1760	2.1760	2.1760	2.1760
2.5706	2.5573	2.5563	2.5561	2.5560	2.5560
3.5299	3.4816	3.4788	3.4781	3.4779	3.4778
3.8577	3.8576	3.8576	3.8576	3.8576	3.8576
4.2307	4.2275	4.2273	4.2272	4.2272	4.2274
4.4060	4.3834	4.3814	4.3810	4.3808	4.3807
—	4.4763	4.4724	4.4715	4.4713	4.4712
5.3084	5.3837	5.3714	5.3689	5.3681	5.3678
5.7378	5.7155	5.7155	5.7155	5.7155	5.7155
6.3241	6.2955	6.2869	6.2849	6.2843	6.2840
—	6.3623	6.3475	6.3453	6.3446	6.3444
7.0311	7.0285	7.0273	7.0273	7.0272	7.0272
7.3545	7.2913	7.2541	7.2480	7.2461	7.2453
—	7.3604	7.3571	7.3568	7.3567	7.3566
7.3907	7.3965	7.3933	7.3929	7.3928	7.3928
8.1998	8.1919	8.1790	8.1749	8.1738	8.1728
—	8.3061	8.2307	8.2244	8.2226	8.2220
8.4038	8.4058	8.4039	8.4039	8.4038	8.4038
—	—	9.1590	9.1434	9.1390	9.1372
9.2360	9.2325	9.2410	9.2396	9.2389	9.2388
9.4419	9.4378	9.4415	9.4415	9.4414	9.4415

From theoretical considerations, the natural frequencies of the cavity-backed membrane are different from those of the membrane, the open-end cavity, or the closed-end cavity. However, it is desirable to compare the system frequencies with those of the chosen degenerated cases of the system. The results of the comparisons for the four chosen systems are shown graphically in Figs. 3–6.

The parameters of system A are chosen to keep the first two cavity frequencies lower than the membrane fundamental frequency. Figure 3 shows that the first two system frequencies are close to those of the closed-end cavity. The third system frequency is near the fundamental membrane frequency. Further, the first nine system frequencies are in the vicinities of the membrane and the closed-end cavity frequencies. For the system B parameters, the fundamental membrane frequency is lower than the open-end cavity frerquency. Figure 4 shows that the first two system frequencies are close to the membrane frequencies. Here again, the first five system frequencies are near those of the membrane and the closed-end cavity frequencies. The tenth frequency of system A and the sixth system B frequency are close to the open-end cavity frequency. Thus, the higher modes of the cavity-backed membrane need not necessarily have pronounced vibration either in the membrane alone or in the cavity.

The Indian musical drum studied in Ref. [9], has a harmonic relation between its natural frequencies. Previous researchers cited in Ref. [9] attempted to explain this relation by analyzing the vibration of the membrane, which has a composite construction. Reference [9] describes the influence of the air cavity on the natural frequencies. Table 3 shows that the natural frequencies of the membrane–cavity system have an almost harmonic relation among themselves. It may be possible to explain the harmonic relation between natural frequencies of the musical drum if the present analysis is extended to include the composite nature of the membrane.

For system C, the fundamental frequencies of the membrane and the open-end cavity are equal. However, the fundamental vibration modes of the subsystems at their interface are distinct. Thus, the fundamental system frequency cannot be precisely the same as that of its subsystems. Figure 5 shows that the fundamental system frequency is close to the coincident subsystem frequencies. Further, the first six system frequencies lie in the vicinities of the membrane and the closed-end cavity frequencies. System D is a special case of a membrane backed by a deep cavity, where the fundamental frequencies of the membrane and the closed-end cavity are equal. Figure 6 shows that

Table 3. Natural frequencies of systems, A, B, C and D

System A		System B		System C		System D	
$N = 25$	$N = 30$	$N = 25$	$N = 30$	$N = 25$	$N = 30$	$N = 25$	$N = 30$
0.6127	0.6127	0.7605	0.7605	0.7776	0.7776	1.3493	1.3493
1.1828	1.1828	1.5902	1.5901	1.5875	1.5875	1.7837	1.7837
1.4768	1.4768	2.1760	2.1760	1.8096	1.8096	3.1457	3.1457
1.9275	1.9275	2.5560	2.5560	2.7598	2.7597	3.4679	3.4679
2.5299	2.5299	3.4779	3.4778	3.1915	3.1915	3.8704	3.8704
3.1017	3.1017	3.8576	3.8576	3.7765	3.7764	4.2046	4.2046
3.2341	3.2341	4.2272	4.2274	3.8615	3.8615	4.7401	4.7401
3.7888	3.7888	4.3808	4.3807	4.1819	4.1819	4.9814	4.9814
3.8392	3.8392	4.4713	4.4712	4.7282	4.7282	5.5273	5.5272
3.9052	3.9052	5.3681	5.3678	4.8292	4.8290	6.1092	6.1092
4.0540	4.0540	5.7155	5.7155	4.9949	4.9949	6.3090	6.3090
4.2872	4.2872	6.2843	6.2840	5.8420	5.8417	7.0260	7.0260
4.4098	4.4098	6.3446	6.3444	6.1024	6.1024	7.2058	7.2058
4.5935	4.5935	7.0272	7.0272	6.3075	6.3075	7.3689	7.3689
4.9462	4.9462	7.2461	7.2453	6.8690	6.8684	7.5617	7.5616
5.0227	5.0227	7.3567	7.3566	7.0270	7.0270	7.7446	7.7446
5.1161	5.1161	7.3928	7.3928	7.7129	7.7129	7.8803	7.8803
5.3955	5.3955	8.1738	8.1728	7.3809	7.3809	8.4773	8.4773
5.6670	5.6670	8.2226	8.2220	7.7051	7.7051	8.7565	8.7565
5.8463	5.8463	8.4038	8.4038	7.8612	7.8610	9.4195	9.4195
6.2916	6.2916	9.1390	9.1372	7.9159	7.9152	9.6311	9.6307
6.3293	6.3293	9.2389	9.2388	8.4704	8.4704		
6.8283	6.8283	9.4414	9.4415	8.7541	8.7541		
6.9004	6.9003			8.9296	8.9282		
6.9542	6.9541			9.4350	9.4350		
7.0218	7.0217			9.9522	9.9502		
7.0649	7.0649						
7.1481	7.1481						
7.2814	7.2814						
7.3706	7.3706						
7.4663	7.4663						
7.5489	7.5489						
7.6988	7.6988						
7.9746	7.9746						
8.1749	8.1749						
8.2888	8.2888						
8.4634	8.4634						
8.6349	8.6349						
8.7925	8.7925						
8.8521	8.8518						
9.0227	9.0227						
9.0340	9.0340						
9.4268	9.4268						
9.8558	9.8558						

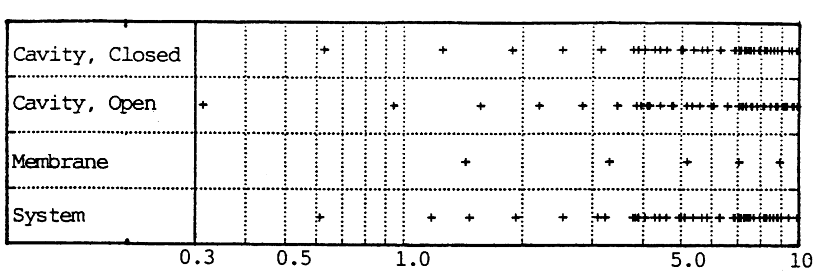

Fig. 3. Comparison of frequencies for system A.

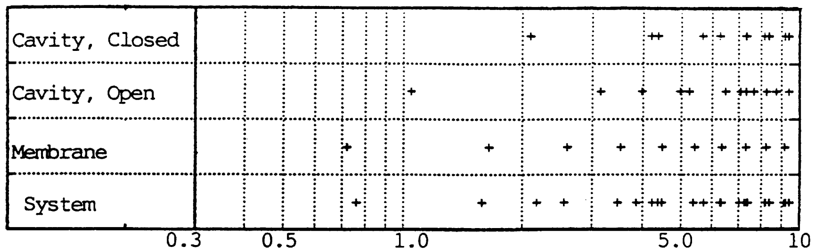

Fig. 4. Comparison of frequencies for system B.

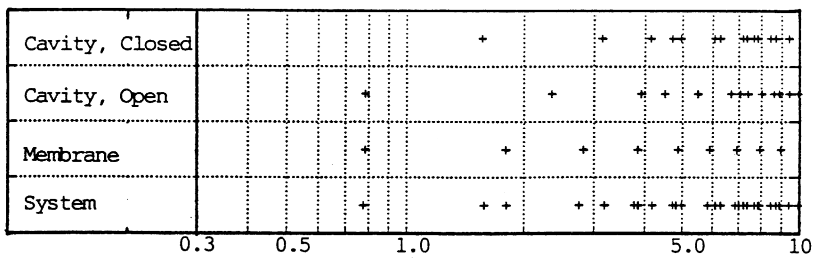

Fig. 5. Comparison of frequencies for system C.

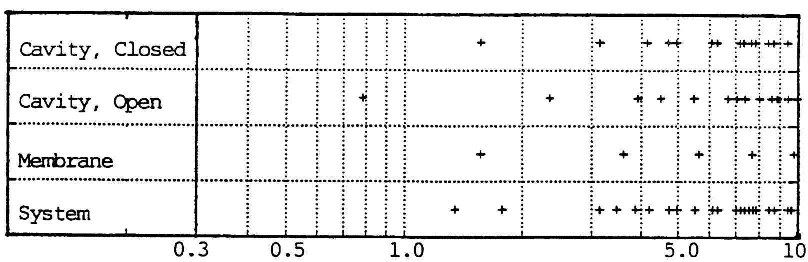

Fig. 6. Comparison of frequencies for system D.

the first two system frequencies are different from the coincident fundamental frequencies of the membrane and the closed-end cavity. This indicates a strong interaction between the subsystem modes even at the first two natural frequencies of the system. Further, the opposing influence of the membrane and closed-cavity modes becomes weaker at frequencies far away from the coincident frequency.

CONCLUSIONS

The natural vibration of a cavity-backed membrane is investigated using a receptor–rejector system model. Numerical results pertaining to the axisymmetrical vibration of four typical systems are presented. The natural frequencies of the system are compared with those of the isolated membrane, the open-end cavity, and the closed-end cavity. None of the system frequencies is found to coincide with the frequencies of the three degenerated cases chosen for comparison. The vibrational modes of the system can be categorized into three groups:

(i) The modes for which the vibration is predominantly in the membrane, and consequently the corresponding system frequency is close to that of the isolated membrane.
(ii) The modes where the vibration is pronounced in the cavity and the corresponding system frequency is near that of the closed-end cavity.
(iii) The modes in which the vibration is significant in the membrane as well as the cavity. In this case the system frequency is neither close to the membrane nor to the closed-end cavity frequencies.

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