JDMD: Extended Dynamic Mode Decomposition with Jacobian Residual Penalization for Learning Bilinear, Control-affine Koopman Models

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Abstract: We present a novel extension to previous data-driven approaches based on Koopman operator theory that combines many of the benefits of model-based and data-driven approaches. By including derivative information from an approximate analytical model, we show that we can quickly learn an efficient bilinear representation of the system dynamics, that, when combined with a novel linear MPC method that projects the dynamics into the original state space, offers performance improvements over nominal linear MPC with just a few training trajectories. We demonstrate increased sampling efficiency, increased generalization, and robustness to training hyperparameters compared to previous approaches

1 Introduction

Controlling complex, underactuated, and highly nonlinear autonomous systems remains an active area of research in robotics, despite decades of previous work exploring effective algorithms and the development of substantial theoretical analysis. Classical approaches typically rely on local linear approximations of the nonlinear system, which are then used in any of a multitude of linear control techniques, such as PID, pole placement, Bode analysis, H-infinity, LQR, or linear MPC. These approaches only work well if the states of the system always remain close to the equilibrium point or reference trajectory about which the system was linearized. The region for which these linearizations remain valid can be extremely small for highly nonlinear systems. Alternatively, model-based methods for nonlinear optimal control have shown great success, as long as the model is well known and an accurate estimate of the global state can be provided. These model-based techniques leverage decades of insight into dynamical systems and have demonstrated incredible performance on complicated autonomous systems [1, 2, 3, 4] . On the other hand, data-driven techniques such as reinforcement learning have received tremendous attention over the last decade and have begun to demonstrate impressive performance and robustness for complicated robotic systems in unstructured environments [5, 6, 7]. While these approaches are attractive since they require little to no previous knowledge about the system, they often require large amounts of data and fail to generalize outside of the domain or task on which they were "trained."

In this work we propose a novel method that combines the benefits of model-based and data-driven methods, based on recent work applying Koopman Operator Theory to controlled dynamical systems [8, 9, 10, 11, 12]. By leveraging data collected from an unknown dynamical system along with derivative information from an approximate analytical model, we can efficiently learn a bilinear representation of the system dynamics that performs well when used in traditional model-based control techniques such as linear MPC. By leveraging information from an analytical model, we can dramatically reduce the number of samples required to learn a good approximation of the true nonlinear dynamics. We also show the effectiveness of linear MPC on these systems when the learned bilinear system is linearized and projected back into the original state space. The result is a fast, robust, and sample-efficient pipeline for quickly learning a model that beats previous Koopman-

Submitted to the 6th Conference on Robot Learning (CoRL 2022). Do not distribute.

based linear MPC approaches as well as purely model-based linear MPC controllers that do not leverage data collected from the actual system. To learn these bilinear representations, we also propose a numerical technique that allows for large systems to be trained while limiting the peak memory required to solve the least-squares problem.

The current work is most closely related to recent work out of Caltech [11, 13, 14] which learn a bilinear model and apply nonlinear model-predictive control directly on the learned bilinear dynamics. Similarly, control Lyapunov functions were also recently used directly on the learned bilinear dynamics [15]. These works build on a foundational paper applying MPC to nonlinear systems using Koopman operators to learn a linear model directly [10].

In summary, our contributions are:

- A novel extension to extended dynamic mode decomposition (eDMD) that incorporates gradient information from an approximate analytic model;
- a simple linear MPC control technique for learned bilinear control systems that is computationally efficient online, which, when combined with the proposed extension to eDMD, requires extremely little training data to get a good control policy; and
- a recursive, batch QR algorithm solving the least-squares problems that arise when learning bilinear dynamical systems using eDMD.

The paper is organized as follows: in Section 2 we give some background on the application of Koopman operator theory to controlled dynamical systems and review some related works. Section 3 describes the proposed algorithm for combining data-driven and model-based approaches, while 4 proposes an efficient technique for controlling these learned models online. In Section 5 we outline a memory-efficient technique for solving the large, sparse linear least-squares problems that arise when applying the proposed adaptation to eDMD. Lastly, Section 6 provides numerical analysis of the proposed algorithm, applied to a simulated cartpole, quadrotor model, and a small foam airplane, all subject to significant model mismatch. In Section 7 we discuss the limitations of the method, and finish with some concluding thoughts in Section 8.

2 Background and Related Work

2.1 Koopman Operator Theory

The theoretical underpinnings of the Koopman operator and its application to dynamical systems has been extensively studied, especially within the last decade [16, 17, 9, 18]. Rather than describe the theory in detail, we highlight the key concepts employed by the current work, and defer the motivated reader to the existing literature on Koopman theory.

We start by assuming we have some discrete approximation of a controlled nonlinear, time-dynamical system whose underlying continuous dynamics are Lipchitz continuous:

$$x^+ = f(x, u) \tag{1}$$

where $x \in \mathcal{X} \subseteq \mathbb{R}^{N_x}$ is the state vector, $u \in \mathbb{R}^{N_u}$ is the control vector, and x^+ is the state at the next time step. This discrete approximation can be obtained for any continuous-time, smooth dynamical system in many ways, including implicit and explicit Runge-Kutta methods, or by solving the Discrete Euler-Lagrange equations [19, 20, 21].

The key idea behind the Koopman operator is that the nonlinear finite-dimensional discrete dynamics (1) can be represented by an infinite-dimensional *bilinear* system:

$$y^+ = Ay + Bu + \sum_{i=1}^{m} u_i C_i y = g(y, u) \tag{2}$$

where $y = \phi(x)$ is a nonlinear mapping from the finite-dimensional state space $\mathcal{X}$ to the (possibly) infinite-dimensional Hilbert space of *observables* $y \in \mathcal{Y}$. We also assume the inverse map is approximately linear: $x = Gy$. In practice, we approximate (1) by choosing ϕ to be some arbitrary finite set of nonlinear functions of the state variables, which in general include the states themselves such that the linear mapping $G \in \mathbb{R}^{N_x \times N_y}$ is exact. Intuitively, ϕ "lifts" our states into a higher dimensional space where the dynamics are approximately (bi)linear, effectively trading dimensionality for (bi)linearity. This idea should be both unsurprising and familiar to most roboticists, since similar techniques have already been employed in other forms, such as maximal-coordinate representations of rigid body dynamics [22, 19, 21], the "kernel trick" for state-vector machines [23], or the observation that solving large, sparse nonlinear optimization problems is often more effective than solving small, tightly-coupled dense problems.

2.2 Extended Dynamic Mode Decomposition

The lifted bilinear system (2) can be easily learned from P samples of the system dynamics (x_j^+, x_j, u_j) using extended Dynamic Mode Decomposition (eDMD) [18, 13]. We can learn the matrix of model coefficients

$$E = [A \quad B \quad C_1 \quad \dots \quad C_m] \in \mathbb{R}^{N_y \times N_z}, \tag{3}$$

by solving the following linear least-squares problem:

$$\underset{E}{\text{minimize}} \; \left\| E Z_{1:P} - Y^2_{1:P} \right\|_2^2 \tag{4}$$

where $Z_{1:P} \in \mathbb{R}^{N_z \times P}$ and $Y^+_{1:P} \in \mathbb{R}^{N_y \times P}$ are the data matrices

$$Z_{1:P} = \begin{bmatrix} y_1 & y_2 & \dots & y_P \\ u_1 & u_2 & \dots & u_P \\ u_{1,1}y_1 & u_{2,1}y_2 & \dots & u_{P,1}y_P \\ \vdots & \vdots & \ddots & \vdots \\ u_{1,m}y_1 & u_{2,m}y_2 & \dots & u_{P,m}y_P \end{bmatrix}, \quad Y^+_{1:P} = \begin{bmatrix} y_1^+ & y_2^+ & \dots & y_P^+ \end{bmatrix}, \tag{5}$$

and $N_z = N_y + N_u + N_y \cdot N_u$.

3 EDMD with Jacobian Residual-Penalization

We derive our proposed algorithm, Jacobian-penalized Dynamic Mode Decomposition (jDMD) through a straightforward adaptation of the original eDMD algorithm described in Section 2.2.

Given P samples of the dynamics (x_i^+, x_i, u_i), and an approximate discrete dynamics model

$$x^+ = \tilde{f}(x, u) \tag{6}$$

we can evaluate the Jacobians of our approximate model $\tilde{f}$ at each of the sample points: $\tilde{A}_i = \frac{\partial \tilde{f}}{\partial x}, \tilde{B}_i = \frac{\partial \tilde{f}}{\partial u}$. After choosing a nonlinear mapping $\phi : \mathbb{R}^{N_x} \mapsto \mathbb{R}^{N_y}$ our goal is to find a bilinear dynamics model (2) that matches the Jacobians of our approximate model, while also matching our dynamics samples. We accomplish this by penalizing differences between the Jacobians of our learned bilinear model with respect to the original states x and controls u, and the Jacobians we expect from our analytical model. These *projected Jacobians* are calculated by differentiating through the *projected dynamics*:

$$x^+ = G\left(A\phi(x) + Bu + \sum_{i=1}^{m} u_i C_i \phi(x)\right) = \bar{f}(x, u). \tag{7}$$

Differentiating (7) with respect to x and u gives us

$$\bar{A}_j = \frac{\partial \hat{f}}{\partial x}(x_j, u_j) = G\left(A + \sum_{i=1}^{m} u_{j,i} C_i\right)\Phi(x_j) = GE\hat{A}(x_j, u_j) = GE\hat{A}_j \tag{8a}$$

$$\bar{B}_j = \frac{\partial \hat{f}}{\partial u}(x_j, u_j) = G\Big(B + [C_1 x_j \quad \dots \quad C_m x_j]\Big) = GE\hat{B}(x_j, u_j) = GE\hat{B}_j \tag{8b}$$

where $\Phi(x) = \partial\phi/\partial x$ is the Jacobian of the nonlinear map ϕ, and

$$\hat{A}(x,u) = \begin{bmatrix} I_{N_y} \\ 0 \\ u_1 I_{N_y} \\ u_2 I_{N_y} \\ \vdots \\ u_m I_{N_y} \end{bmatrix} \Phi(x) \in \mathbb{R}^{N_z \times N_x}, \quad \hat{B}(x,u) = \begin{bmatrix} 0 \\ I_{N_u} \\ [x \; 0 \; \dots \; 0] \\ [0 \; x \; \dots \; 0] \\ \vdots \\ [0 \; 0 \; \dots \; x] \end{bmatrix} \in \mathbb{R}^{N_z \times N_u}. \tag{9}$$

We then solve the following linear least-squares problem:

$$\underset{E}{\text{minimize}} \;\; (1-\alpha)\left\| E Z_{1:P} - Y_{1:P}^{+} \right\|_2^2 + \alpha \sum_{j=1}^{P} \left(\left\| G E \hat{A}_j - \tilde{A}_j \right\|_2^2 + \left\| G E \hat{B}_j - \tilde{B}_j \right\|_2^2 \right) \tag{10}$$

The resulting linear least-squares problem has $(N_y + N_x^2 + N_x \cdot N_u) \cdot P$ rows and $N_y \cdot N_z$ columns. Given that the number of rows in this problem grows quadratically with the state dimension, solving this problem can be challenging from a computational perspective. In the Section 5, we propose an algorithm for solving these problems without needing to move to a distributed-memory setup in order to solve these large linear systems. The proposed method also provides a straightforward way to approach incremental updates to the bilinear system, where the coefficients could be efficiently learned "live" while the robot gathers data by moving through its environment.

4 Projected Bilinear MPC

Here we propose a simple approach to model-predictive control for the bilinear systems learned using either classic eDMD or the proposed jDMD approach. The key idea is to use the projected Jacobians $\bar{A}$ and $\bar{B}$ in (8), effectively reducing the problem back to the original state dimension instead of the larger, lifted one. When tracking a reference trajectory as we do in the all of the following examples, this can be done by solving the following convex Quadratic Program (QP):

$$\begin{aligned} \underset{x_{1:N},\, u_{1:N-1}}{\text{minimize}} \quad & \frac{1}{2} x_N^T Q_N x_N + \frac{1}{2} \sum_{k=1}^{N-1} x_k^T Q_k x_k + u_k^T R_k u_k \\ \text{subject to} \quad & x_{k+1} = \bar{A}_k x_k + \bar{B}_k u_k + d_k, \\ & x_1 = x_{\text{init}} \end{aligned} \tag{11}$$

where here we define x and u to be the "delta" from the reference trajectory $\bar{x}_{1:N}, \bar{u}_{1:N-1}$. The affine dynamics term $d_k = f(\bar{x}_k, \bar{u}_k) - \bar{x}_{k+1}$ allows for dynamically infeasible reference trajectories. The projected Jacobians can be efficiently calculated from the bilinear dynamics either offline or online, and since the problem dimension is the same size as the linear MPC problem for the original dynamics, it is no more expensive to compute. This formulation also makes it trivial to enforce additional control or path constraints, and avoids the need to regularize or otherwise constrain the lifted states.

5 Efficient Recursive Least Squares

In its canonical formulation, a linear least squares problem can be represented as the following unconstrained optimization problem:

$$\min_x \|Fx - d\|_2^2. \tag{12}$$

We assume F is a large, sparse matrix and that solving it directly using a QR or Cholesky decomposition requires too much memory for a single computer. While solving (12) using an iterative method such as LSMR [24] or LSQR [25] is possible, we found these methods do not work well in practice for solving (10) since the matrices tend to be ill-conditioned. Recursive methods for solving

Figure 1: Airplane perching trajectory, a high angle-of-attack maneuver that minimizes velocity at the goal position

these problems are able to process the rows of the matrices sequentially to build QR decomposition of the full matrix, but also tend to be ill-conditioned [26, 27, 28]. We propose an alternative recursive method based off of numerical techniques that, to our knowledge, aren't well known to the learning community. We solve (12) by dividing up the rows of F into batches:

$$F^T F = F_1^T F_1 + F_2^T F_2 + \ldots + F_N^T F_N. \tag{13}$$

The main idea of this recursive method is to maintain an upper-triangular "square root" factor U_i of the first i terms of the sum (13). Given the factorization U_i, we can calculate U_{i+1} using the QR decomposition, as shown in [29]:

$$U_{i+1} = \sqrt{U_i^T U_i + F_{i+1}^T F_{i+1}} = \mathrm{QR_R}\left(\begin{bmatrix} U_i \\ F_{i+1} \end{bmatrix} \right), \tag{14}$$

where $\mathrm{QR_R}$ returns the upper triangular matrix R from the QR decomposition. For a maximally-efficient implementation, this function should be a "Q-less" QR decomposition, since the Q matrix is never needed.

We handle regularization of the normal equations, equivalent to adding L2 regularization to the original least squares problem, during the base case of our recursion. If we want to add an L2 regularization with weight λ, we calculate U_1 as:

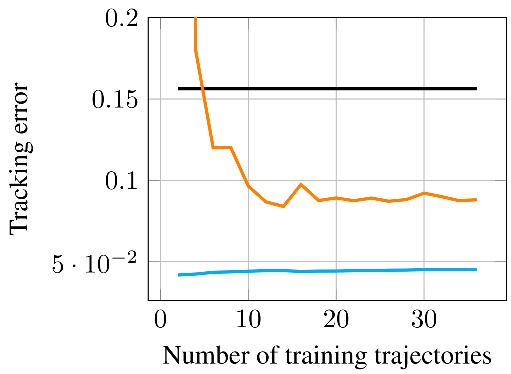
$$U_1 = \mathrm{QR_R}\left(\begin{bmatrix} F_1 \\ \sqrt{\lambda} I \end{bmatrix} . \right). \tag{15}$$

6 Results

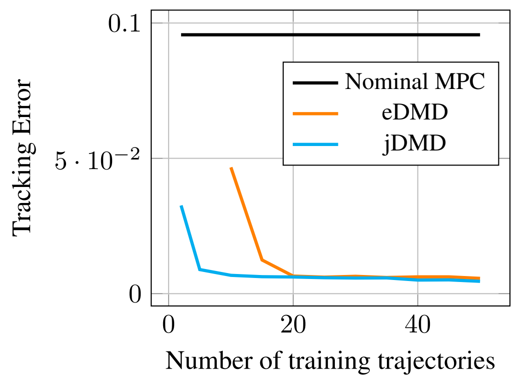
The following sections provide various numerical analyses of the proposed algorithm. In lieu of an actual hardware experiment (left for future work), for each simulated system we specify two models: a *nominal* model which is a simplified model approximating the *true* model, which contains both parametric and non-parametric model error from the nominal model. This true model is used exclusively for simulating the system.

All models were trained by simulating the true system with an arbitrary controller to collect data in the region of the state space relevant to the task. A set of fixed-length trajectories were collected, each at a sample rate of 20-25 Hz. The bilinear eDMD model was trained using the same approach introduced by Folkestad and Burdick [13]. All continuous dynamics were discretized with an explicit fourth-order Runge Kutta integrator. We organize the following results section by topic, and briefly describe the three simulated models used to produce the results in the paragraphs below.

The true cartpole model included a $\tanh$ model of Coulomb friction between the cart and the floor, viscous damping at both joints, and a control dead band that was not included in the nominal model. Additionally, the mass of the cart and pole model were altered by 20% and 25% with respect to the nominal model, respectively. The following nonlinear mapping was used when learning the bilinear models: $\phi(x) = [1, x, \sin(x), \cos(x), \sin(2x), \sin(4x), T_2(x), T_3(x), T_4(x)] \in \mathbb{R}^{33}$, where $T_i(x)$ is a Chebyshev polynomial of the first kind of order i. All reference trajectories for the swing up task were generated using ALTRO [29, 30].



(a) Cartpole



(b) Airplane

Figure 2: MPC tracking error vs training trajectories for both the cartpole (left) and airplane (right). Tracking error is defined as the average L2 error over all the test trajectories between the reference and simulated trajectories.

We also include 2 quadrotor systems: a 3-DOF, planar model and a 6-DOF quadrotor model with experimentally-determined force and torque curves for the motors. For both systems, the true model includes aerodynamic drag terms not included in the nominal model, as well as parametric error of about 5% on the system properties (e.g. mass, rotor arm length, etc.). The planar model was trained using a nonlinear mapping of $\phi(x) = [1,\, x,\, \sin(x),\, \cos(x),\, \sin(2x),\, T_2(x)] \in \mathbb{R}^{25}$ while the full quadrotor model was train using a nonlinear mapping of $\phi(x) = [1,\, x,\, T_2(x),\, \sin(p),\, \cos(p),\, R^T v,\, v^T R R^T v,\, p \times v,\, p \times \omega,\, \omega \times \omega] \in \mathbb{R}^{44}$, where p is the quadrotor's position, v and ω are the translational and angular velocities respectively, and R is the Rotation matrix. Reference trajectories were produced by nominal MPC tracking controllers attempting to track infeasible, point-to-point, linear trajectories with various initial conditions.

Our most high-fidelity model is an airplane model with lift and drag coefficients fit from rigorous wind-tunnel testing [31]. We produce a high-angle-of-attack (up to $40°$) manuever by solving for "perching" trajectories with ALTRO that encourage minimal velocity subject to a constraint on the final position (see Figure 1). These trajectories are tracked using a QP-based linear MPC solver. The nominal model used a simple flat-plate wing model with linear lift and quadratic drag coefficient curves. To train the bilinear models, we used a 68-dimensional nonlinear mapping composed of dynamics terms such as the rotation matrix (expressed in terms of a Modified Rodriguez Parameter), powers of the angle of attack and side slip angle, the body frame velocity, various cross products with the angular velocity, and some 3rd and 4th order Chebyshev polynomials.

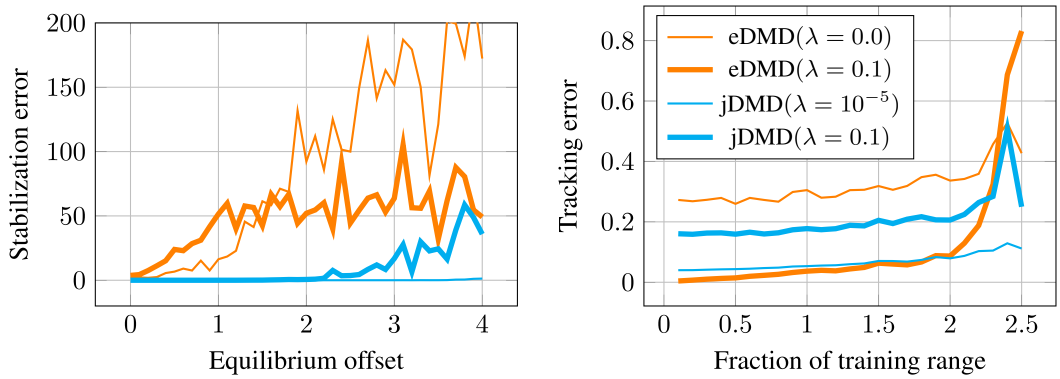
The code for the experiments is located at TODO: include after review.

6.1 Sample Efficiency

We highlight the sample efficiency of the proposed algorithm in Figure 2. For both the cartpole swing up and the airplane perch trajectory tracking tasks, the proposed method achieves better tracking than the nominal MPC controller with just two sample trajectories, and performs better than eDMD on both trajectory tracking tasks. To achieve comparable performance on the perching task, eDMD requires about 4x the number of samples (20 vs 5) compared to the proposed approach.

6.2 Generalization

We demonstrate the generalizability of the proposed method to tasks outside of its training domain in Figure 4. In both the planar quadrotor stabilization (Figure 3a) and trajectory tracking (Figure 3b) tasks, we trained the models by sampling uniformly from a given window of offsets, centered about the origin. To test the generalizability of the methods we increased the relative size of the window



(a) LQR stabilization error over increasing equilibrium offset

(b) Tracking error for the quadrotor MPC reference trajectory tracking task.

Figure 3: Generalizability with respect to initial conditions sampled outside of the training domain. The initial conditions are sampled from a uniform distribution, whose limits are determined by a scaling of the limits used for the training distribution. A training range fraction greater than 1 indicates the distribution range is beyond that used to generate the training trajectories. The thick lines represent the algorithm with a heavy regularization parameter.

from which the test data was sampled, e.g. if the initial lateral position was trained on data in the interval $[-1.5, +1.5]$, we sampled the test initial condition from the window $[-\gamma 1.5, +\gamma 1.5]$. As shown in the results, while the performance of the proposed algorithm remains relatively constant even when $\gamma = 2.5$, whereas the classic eDMD approach looses performance and fails to generalize at all for the stabilization task using and LQR controller (like due to poor derivative information), and up to $\gamma = 2$ for the tracking task using a linear MPC controller.

In Figure 3a we show the effect of changing the equilibrium position for the planar quadrotor, but keeping the delta initial conditions within the training window. As shown, eDMD doesn't generalize to other equilibrium points, despite the fact that the underlying dynamics are invariant to the equilibrium position. Our proposed approach, however, easily learns this from the derivative information provided by the nominal model.

	Nominal MPC	**eDMD**	**jDMD**
Mean Tracking Err.	0.30	0.63	**0.11**
Success Rate	**82%**	18%	80%

Table 1: Performance summary of MPC tracking of 6-DOF quadrotor

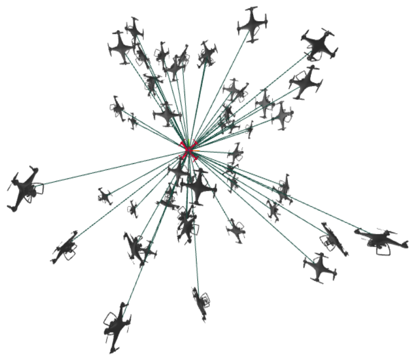
To demonstrate that this generalizability extends to more complex dynamics, we additionally show the tracking capabilities of infeasible, point-to-point trajectories for the 6-DOF quadrotor with full attitude dynamics. As seen in Figure 4a, the generated reference trajectories have a wide scope in sampling despite being sparse in nature. As shown in Table 1, jDMD has the best tracking performance and closely matches nominal MPC's ability to successfully reach the equilibrium. Meanwhile, eDMD has significantly worse performance in tracking and succeeding in stabilizing about the goal state. Despite the aggressive starting state, jDMD is still able to successfully account for the attitude dynamics, reach the goal state, and stabilize, as seen in Figure 4b.

6.3 Lifted versus Projected MPC

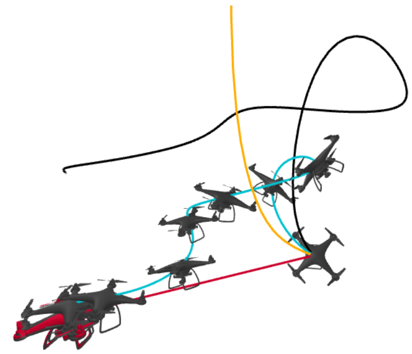
We performed a simple experiment to highlight the value of the proposed "projected" MPC, outlined in Section 4. We trained eDMD and jDMD models with an increasing number of training trajectories, and recorded the first sample size at which the "lifted" and "projected" MPC controllers consis-

MPC	**eDMD**	**jDMD**
Lifted	17	15
Projected	18	**2**

Table 2: Training trajectories required to beat nominal MPC



(a) Generated point-to-point trajectories and initial conditions for testing tracking MPC of 6-DOF quadrotor.



(b) Generated MPC trajectories for nominal MPC (black), eDMD (orange), and jDMD (cyan) for tracking infeasible, point-to-point trajectory (red).

Figure 4: Generalizability with respect to initial conditions sampled outside of the training domain. The initial conditions are sampled from a uniform distribution, whose limits are determined by a scaling of the limits used for the training distribution. A training range fraction greater than 1 indicates the distribution range is beyond that used to generate the training trajectories. The thick lines represent the algorithm with a heavy regularization parameter.

Friction (μ)	0.0	0.1	0.2	0.3	0.4	0.5	0.6
Nominal	✓	✓	✗	✗	✗	✗	✗
eDMD	3	19	6	14	✗	✗	✗
jDMD	2	2	2	2	3	7	12

Table 3: Training trajectories required to stabilize the cartpole with the given friction coefficient

tently stabilized the system (i.e. stabilized 95% of the test initial conditions for the cartpole system for that sample size and subsequent ones). The results are summarized in Table 2. The results quantitatively show what we qualitatively observed while training and testing these various examples: the projected MPC approach usually required far fewer samples to "train" and usually had better performance than its lifted counterpart that used the bilinear lifted dynamics. This was especially pronounced when combined with the proposed jDMD approach, which makes sense given that the approach explicitly encourages these Jacobians to match the analytical ones, so quickly converges to reasonable values with just a few training examples.

6.4 Sensitivity to Model Mismatch

While we've introduced a significant mount of model mismatch in all of the examples so far, a natural argument against model-based methods is that they're only as good as your model is at capturing the salient dynamics of the system. We investigated the effect of increasing model mismatch by incrementally increasing the Coulomb friction coefficient between the cart and the floor for the cartpole stabilization task (recall the nominal model assumed zero friction). The results are shown in Table 3. As expected, the number of training trajectories required to find a good stabilizing controller increases for the proposed approach. We achieved the results above by setting $\alpha = 0.01$, corresponding to a decreased confidence in our model, thereby placing greater weight on the experimental data. The standard eDMD approach always required more samples, and was unable to find a good enough model above friction values of 0.4. While this could likely be remedied by adjusting the nonlinear mapping ϕ, the proposed approach works well with the given bases. Note that the nominal MPC controller failed to stabilize the system above friction values of 0.1, so again, we demonstrate that we can improve MPC performance substantially with just a few training samples by combining analytical gradient information and data sampled from the true dynamics.

7 Limitations

As with most data-driven techniques, it is hard to definitively declare that the proposed method will increase performance in all cases. It is possible that having an extremely poor analytical model may hurt rather than help the training process. However, we found that even when the α parameter is extremely small (placing little weight on the Jacobians during the learning process), it still dramatically improves the sample efficiency over standard eDMD, . It is also quite possible that the performance gaps between eDMD and jDMD shown here can be reduced through better selection of basis functions and better training data sets; however, given that the proposed approach converges to eDMD as $\alpha \to 0$, we see no reason to not adopt the proposed methodology as simply tune α based on the confidence of the model and the quantity (and quality) of training data.

8 Conclusion and Future Work

We have presented a simple but powerful extension to eDMD, a model-based method for learning a bilinear representation of arbitrary dynamical systems, that incorporates derivative information from an analytical model. When combined with a simple linear MPC policy that projects the learned dynamics back into the original state space, we have shown that the resulting pipeline can dramatically increase sample efficiency, often improving over a nominal MPC policy with just a few sample trajectories. Substantial areas for future work remain: most notably testing the proposed pipeline on hardware. Additional directions include lifelong learning or adaptive control applications, combining simulated and real data through the use of modern differentiable physics engines [21, 32], residual dynamics learning, as well as the development of specialized numerical methods for solving nonlinear optimal control problems using the learned bilinear dynamics.

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