Introduction to Robotics (CS223A) Homework #2

(Winter 2017/2018) Due: **Wednesday, January 31, 4:30pm**

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Some tips for doing CS223A problem sets:

- Use abbreviations for trigonometric functions (e.g. $c\theta$ for $\cos(\theta)$, s_1 or $s\theta_1$ for $\sin(\theta_1)$) in situations where it would be tedious to repeatedly write sin,cos, etc.
- Unless instructed otherwise, leave square roots in symbolic form rather than writing out their decimal values.
- If you give a vector as an answer, make sure that you specify what frame it is given in (if it is not clear from context). The same rule applies to rotation and transformation matrices.

1. The vector ^AP is first rotated about $\hat{X}_A$ by θ degrees, next rotated about $\hat{Y}_A$ by ϕ degrees, and finally rotated about $\hat{Z}_A$ by ψ degrees. Find the four Euler parameters for the orientation of the rotated vector in frame $\{A\}$ when $\theta = 180$, $\phi = -60$, and $\psi = 60$.

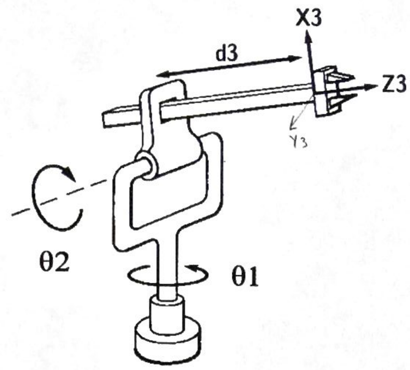
$$^AP' = R_z(\psi)R_y(\phi)R_x(\theta)$$

$$= \begin{bmatrix} c\psi & -s\psi & 0 \\ s\psi & c\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\phi & 0 & s\phi \\ 0 & 1 & 0 \\ -s\phi & 0 & c\phi \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\theta & -s\theta \\ 0 & s\theta & c\theta \end{bmatrix}$$

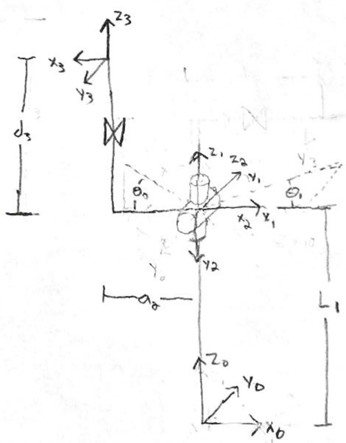
$$= \begin{bmatrix} 0.25 & 0.866 & 0.433 \\ 0.433 & -0.5 & 0.75 \\ 0.866 & 0 & -0.5 \end{bmatrix}$$

$$\Rightarrow (-0.75, -0.433, -0.433, 0.25)$$

2. Consider the following RRP manipulator.



(a) Draw a schematic of this manipulator, with the axes of frames {0} through {3} labeled. Note that frame {3} has been done for you, and your solution should be consistent with the given frame {3}. Also, include the parameters θ_1, θ_2, and d_3 on your schematic, as well as any link lengths you define. Assume that in this diagram, the slider bar is parallel to the ground and that this is the configuration where $\theta_1 = 0$ and $\theta_2 = 90°$



(b) Find the Denavit-Hartenberg parameters for this manipulator – that is, fill in the entries for the following table:

i	a_{i-1}	α_{i-1}	d_i	θ_i
1	0	0	L_1	θ_1
2	0	-90	0	θ_2
3	$-a_2$	90	d_3	180

if a_2 needs to be positive, add 180° offset to θ_2, row 3 becomes

a_2 -90 d_3 0

(c) In the code, implement the transformation between links based on DH parameters in *dh.py* (where it says HW2 Q2c). This part of the homework will be graded with Gradescope Autograder. Submit online.

(d) Find the matrix ${}^{0}_{3}T$ at the configuration from part (a). You may use your code from part (c) for this question, but in case your implementation is wrong, write down the product of matrices you computed for partial credit.

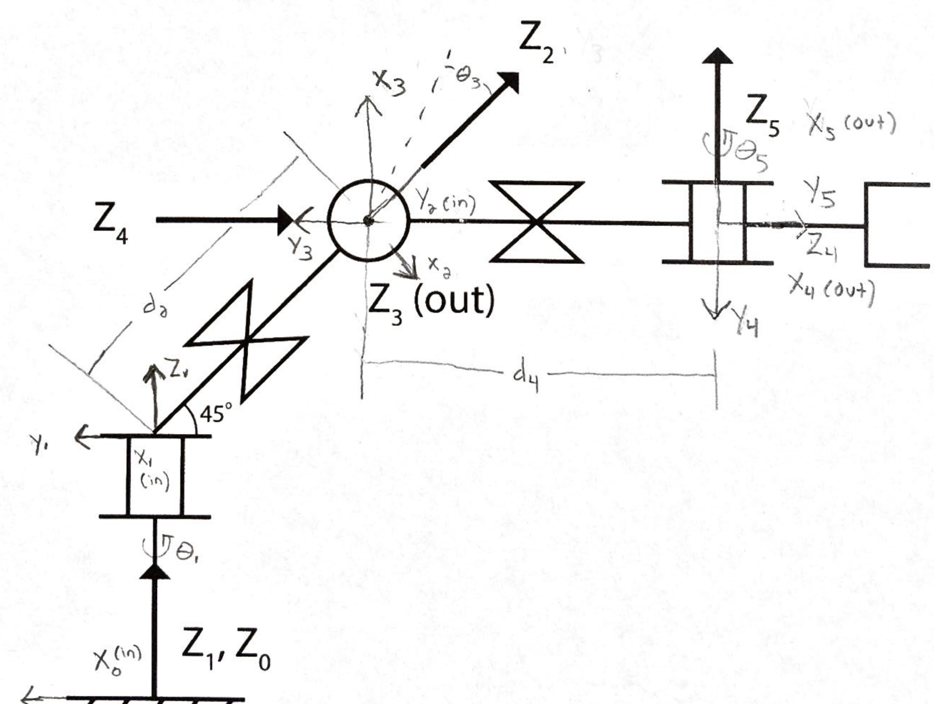
Assume $\theta_1 = 0$
$\theta_2 = 90°$

$${}^{i-1}_{i}T = R_x(\alpha)\, D_x(a)\, R_z(\theta)\, D_z(d)$$

$${}^{0}_{1}T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^{1}_{2}T = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^{2}_{3}T = \begin{bmatrix} -1 & 0 & 0 & -a_3 \\ 0 & 0 & -1 & -d_2 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^{0}_{2}T = \begin{bmatrix} 0 & 0 & 1 & d_3 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & L_1 + a_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3. (a) Assign and label the axes of frames {0} through {5}. The positive direction of the $\hat{Z}_i$ axes have been drawn for you, but they don't necessarily start at the frame origins. Include $\theta_1, d_2, \theta_3, d_4$, and θ_5 on your diagram. For the case when $\hat{Z}_i$ and $\hat{Z}_{i+1}$ axes are intersecting, take the perpendicular to both in the point of intersection and assign $\hat{X}_i$ along it in such a direction that the angle α_i from axis i to $i+1$ is measured in a positive sense.



$250

(b) Find the Denavit-Hartenberg parameters for this manipulator.

i	a_{i-1}	α_{i-1}	d_i	θ_i	
1	0	0	L_1	θ_1	
2	0	45°	d_2	-90°	
3	0	90°	0	θ_3	$+135^\circ$
4	0	90°	d_4	90°	
5	0	90°	0	θ_5	

(c) Find the link transformation matrices ${}^0_1T$, ${}^1_2T$, ${}^2_3T$, ${}^3_4T$, ${}^4_5T$.

$${}^0_1T = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

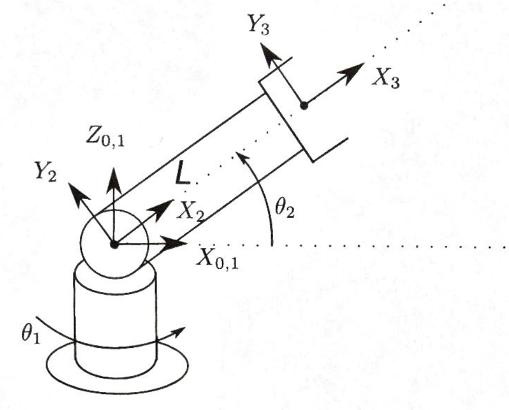
$${}^1_2T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{\sqrt{2}}{2} & 0 & -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2}d_2 \\ -\frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2}d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^2_3T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta_3^* & -s\theta_3^* & 0 & 0 \\ s\theta_3^* & c\theta_3^* & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c\theta_3^* & -s\theta_3^* & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s\theta_3^* & c\theta_3^* & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \theta_3^* = \theta_3 + 135^\circ$$

$${}^3_4T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & -d_4 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^4_5T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta_5 & -s\theta_5 & 0 & 0 \\ s\theta_5 & c\theta_5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c\theta_5 & -s\theta_5 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s\theta_5 & c\theta_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4. Let us consider the RR manipulator below. The constant length of the link is L. When $\theta_1 = 0$ the frames $\{0\}$ and $\{1\}$ are superposed.



a) Find the position of the end effector (origin of frame $\{3\}$) expressed in frame $\{0\}$, and the transformation matrix between frames $\{0\}$ and $\{3\}$. For this simple RR manipulator, derive your answer without using DH parameters.

$${}^0_1T = R_z(\theta_1)$$

$${}^1_2T = R_x(90)R_z(\theta_2)$$

$${}^2_3T = D_x(L)$$

x, z, z

$${}^0_3T = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & 0 \\ s\theta_2 & c\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & L \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c\theta_1 & 0 & s\theta_1 & 0 \\ s\theta_1 & 0 & -c\theta_1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & c\theta_2 L \\ s\theta_2 & c\theta_2 & 0 & s\theta_2 L \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c\theta_1 c\theta_2 & -c\theta_1 s\theta_2 & s\theta_1 & c\theta_1 c\theta_2 L \\ s\theta_1 c\theta_2 & -s\theta_1 s\theta_2 & -c\theta_1 & s\theta_1 c\theta_2 L \\ s\theta_2 & c\theta_2 & 0 & s\theta_2 L \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

b) Let $\theta_1 = \alpha t$ and $\theta_2 = \beta t$ with α, β strictly positive numbers. Find the end effector velocity vector in function of time t (in frame $\{0\}$).

$$\bar{X} = \begin{bmatrix} c\theta_1 c\theta_2 L \\ s\theta_1 c\theta_2 L \\ s\theta_2 L \end{bmatrix} = \begin{bmatrix} \cos(\alpha t)\cos(\beta t)L \\ \sin(\alpha t)\cos(\beta t)L \\ \sin(\beta t)L \end{bmatrix}$$

$$\dot{\bar{X}} = \begin{bmatrix} -s\theta_1 c\theta_2 L & -c\theta_1 s\theta_2 L \\ c\theta_1 c\theta_2 L & -s\theta_1 s\theta_2 L \\ 0 & c\theta_2 L \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} = \begin{bmatrix} -\alpha\sin(\alpha t)\cos(\beta t) & -\beta\cos(\alpha t)\sin(\beta t) \\ \alpha\cos(\alpha t)\cos(\beta t) & -\beta\sin(\alpha t)\sin(\beta t) \\ 0 & \beta\cos(\beta t) \end{bmatrix}$$

$$= \begin{bmatrix} -\sin(\alpha t)\cos(\beta t) & -\cos(\alpha t)\sin(\beta t) \\ \cos(\alpha t)\cos(\beta t) & -\sin(\alpha t)\sin(\beta t) \\ 0 & \cos(\beta t) \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} L$$

3×2 2×1

c) let $\alpha = \beta$. What is the maximal velocity achieved by the end effector (L and α should appear in your answer), and in which configuration is it achieved. Do you have an intuitive explanation?

$$|V|^2 = [\alpha^2(-\sin\alpha\cos\alpha - \cos\alpha\sin\alpha)^2 +$$

$$\alpha^2(\cos\alpha\cos\alpha - \sin\alpha\sin\alpha)^2 +$$

$$\alpha^2 \cos^2\alpha]L$$

$$= \alpha^2[(-2\sin\alpha\cos\alpha)^2 + (\cos^2\alpha - \sin^2\alpha)^2 + \cos^2\alpha)]L$$

$$= \alpha^2[4\sin^2\alpha\cos^2\alpha + \cos^4\alpha - 2\sin^2\alpha\cos^2\alpha + \cos^4\alpha + \cos^2\alpha]L$$

$$= \alpha^2[\cos^4\alpha + 2\sin^2\alpha\cos^2\alpha + \cos^4\alpha + \cos^2\alpha]L$$

$$= \alpha^2[(\sin^2\alpha + \cos^2\alpha)^2 + \cos^2\alpha]L$$

$$= \alpha^2 L(1 + \cos^2\alpha)$$

d) **[extra credit]** What is the trajectory of the end effector

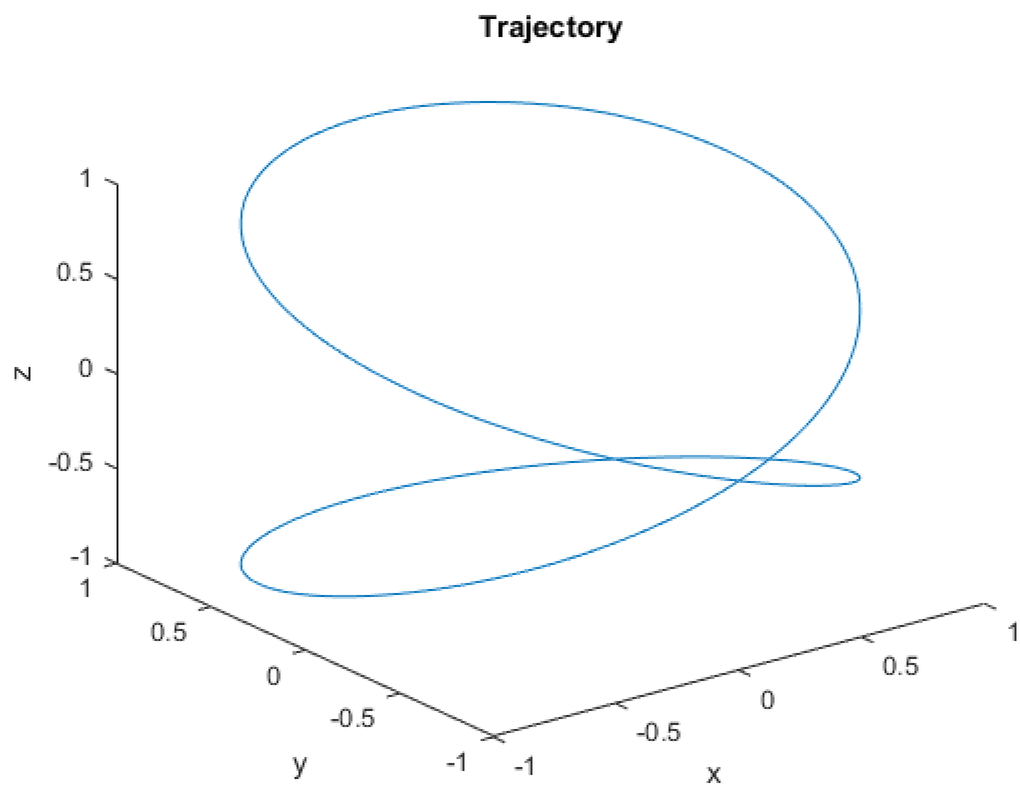
c) let $\alpha = \beta$. What is the maximal velocity achieved by the end effector (L and α should appear in your answer), and in which configuration is it achieved. Do you have an intuitive explanation?

$$|V|^2 = [\alpha^2 (-\sin\alpha\cos\alpha - \cos\alpha\sin\alpha)^2 +$$
$$\alpha^2 (\cos\alpha\cos\alpha - \sin\alpha\sin\alpha)^2 +$$
$$\alpha^2 \cos^2\alpha] L^2$$
$$= \alpha^2[(-2\sin\alpha\cos\alpha)^2 + (\cos^2\alpha - \sin^2\alpha)^2 + \cos^2\alpha)] L^2$$
$$= \alpha^2[4\sin^2\alpha\cos^2\alpha + \cos^4\alpha - 2\sin^2\alpha\cos^2\alpha + \cos^4\alpha + \cos^2\alpha] L^2$$
$$= \alpha^2[\cos^4\alpha + 2\sin^2\alpha\cos^2\alpha + \cos^4\alpha + \cos^2\alpha] L^2$$
$$= \alpha^2[(\sin^2\alpha + \cos^2\alpha)^2 + \cos^2\alpha] L^2$$
$$= \alpha^2 L^2 (1 + \cos^2\alpha)$$
$$|V| = \alpha L \sqrt{1 + \cos^2\alpha}$$

Max when $\cos^2\alpha t$ is maximum, or when $\theta_2 = 0$, which makes sense because it will have the largest radial distance from the center of rotation

d) **[extra credit]** What is the trajectory of the end effector

```
T = linspace(0,10,10000);
for i = 1:length(T)
t = T(i);
x(1:3,i) = [-sin(t).*cos(t) -cos(t).*sin(t); cos(t).*cos(t) -
sin(t).*sin(t); 0 cos(t)]*[1;1];
end
figure
plot3(x(1,:),x(2,:),x(3,:))
xlabel('x')
ylabel('y')
zlabel('z')
title('Trajectory')
```



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