Introduction to Robotics (CS223A) Homework #4

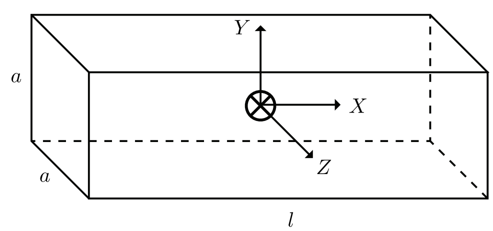
(Winter 2017/2018) Due: **Wednesday, February 28**

Name ____________________ **SUNet ID** ____________________

Some tips for doing CS223A problem sets:

- Use abbreviations for trigonometric functions (e.g. $c\theta$ for $\cos(\theta)$, s_1 or $s\theta_1$ for $\sin(\theta_1)$) in situations where it would be tedious to repeatedly write sin,cos, etc.
- Unless instructed otherwise, leave square roots in symbolic form rather than writing out their decimal values.
- Use common sense for decimals – – if the question states a = 1.34, then don't give answers like 2*a = 2.680001245735.
- If you give a vector as an answer, make sure that you specify what frame it is given in (if it is not clear from context). The same rule applies to rotation and transformation matrices.

1. (a) You are given a solid box with side length l along the X axis, and a along the Y and Z axes. Assume this box has a total mass m and constant density.

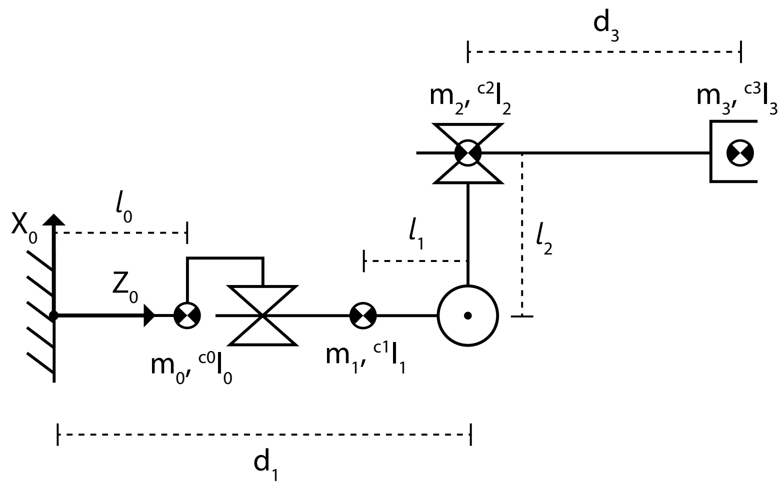


Integrate over the box's volume to find the inertia tensor I_{box} at the center of mass in terms of a, l, and m. You may use symmetry to avoid repeating integrals.

(b) Now calculate the inertia tensor with respect to a frame which has an offset of $(l/2, a/2, a/2)$ from the center of mass.

Hint: You can use parallel axis theorem instead of computing the integrals again.

2. You are given a PRP manipulator. Gravity is along $-X_0$ direction. Let m_0, m_1, m_2, and m_3 be the masses of links 0, 1, 2, and 3, respectively (m_2 doesn't move with d_3). Let ${}^{C_i}I_i$ be the the inertia tensor of link i expressed at the link's center of mass C_i with the same orientation as frame $\{i\}$. Assume the inertia tensors have nonzero off-diagonal entries, as shown below.



$$
{}^{C_i}I_i = \begin{bmatrix} I_{xx_i} & I_{xy_i} & I_{xz_i} \\ I_{xy_i} & I_{yy_1} & I_{yz_i} \\ I_{xz_i} & I_{yz_i} & I_{zz_1} \end{bmatrix}
$$

(a) Calculate the mass matrix $M(q)$ for this manipulator.

Hint: First derive the Jacobians, and then calculate the mass matrix. You will reuse the Jacobians in a later part.

Extra space for part (a):

(b) Draw a configuration of the robot where the inertial coupling between joints 1 and 3 is zero. Intuitively, why does this happen?

(c) Calculate the vector of centrifugal forces $C(q)[\dot{q}^2]$ for this manipulator.

(d) What do you notice about the first and third columns of the $C(q)$ matrix? How can you explain this?

(e) Suppose the robot is in zero configuration and $\dot{q}_2$ and $\dot{q}_3$ are positive. Draw the direction of the generalized torque at joint 3 given by $C(q)[\dot{q}^2]$. What is the physical interpretation of this torque?

(f) Calculate the vector of Coriolis forces $B(q)[\dot{q}\dot{q}]$ for this manipulator.

(g) Calculate the gravity vector $g(q)$ for this manipulator.

(h) Draw the robot in a configuration where it feels zero gravity (ignoring joint limits).

3. Implement `mass_matrix()` and `gravity_vector()` in `dynamics.py` for a general serial manipulator. This part will be evaluated by the autograder.

   Hint: Compare your implementation with your answer from Q2 for the given PRP manipulator. An example RP manipulator is given in the code.

4. Equations of Motion - Modeling the dynamics of a certain RRP manipulator, we arrive at the following equations of motion:

$$\begin{aligned}
\tau_1 &= (m_1 l^2/4 + m_2 l^2 + m_3(l^2 + d_3^2 s_2^2) + I_{ZZ_1})\ddot{\theta}_1 + m_3 l d_3 c_2 \ddot{\theta}_2 + m_3 l s_2 \ddot{d}_3 + 2m_3 d_3^2 s_2 c_2 \dot{\theta}_1 \dot{\theta}_2 \\
&\quad + 2m_3 d_3 s_2^2 \dot{\theta}_1 \dot{d}_3 + 2m_3 l c_2 \dot{\theta}_2 \dot{d}_3 - m_3 l d_3 s_2 \dot{\theta}_2^2 + b_1 \dot{\theta}_1 \\
\tau_2 &= ??? + m_3 d_3^2 \ddot{\theta}_2 + 0\ddot{d}_3 + 2m_3 d_3 \dot{\theta}_2 \dot{d}_3 - m_3 d_3^2 s_2 c_2 \dot{\theta}_1^2 - m_3 d_3 s_2 g + b_2 \dot{\theta}_2 \\
\tau_3 &= m_3 l s_2 \ddot{\theta}_1 + 0\ddot{\theta}_2 + m_3(c_2 + 1)\ddot{d}_3 + 2m_3 d_3 \dot{\theta}_2^2 \dot{d}_3 - m_3 d_3 s_2^2 \dot{\theta}_1^2 - m_3 d_3 \dot{\theta}_2^2 + m_3 c_2 g + b_3 \dot{d}_3.
\end{aligned}$$

A term, indicated by "???", has been left out.

(a) Give a Coriolis term acting at joint 2.

(b) Give a centrifugal term acting at joint 2.

(c) Give a gravity term acting at joint 2.

(d) Give a friction term acting at joint 2.

(e) Find the missing term "???" and explain how you found it. (Hint: write the mass matrix, $M(q)$.)

(f) Find the two incorrect terms acting at joint 3 and explain how you found them.

(g) **[Extra Credit]** Draw a schematic of the RRP manipulator. Label the link lengths, frames 0 to 3, the joint coordinates θ_1, θ_2, and d_3, the position of the center of mass of the links, the masses at these center of mass and the link inertias.