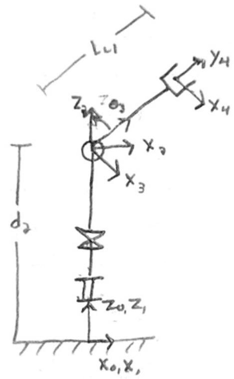
CS225A

Problem 1



a. ${}^0P_2 = \begin{bmatrix} 0 \\ 0 \\ d_2 \end{bmatrix}$

${}^2P_4 = \begin{bmatrix} -L_4 \sin\theta_3 \\ 0 \\ L_4 \cos\theta_3 \end{bmatrix}$

${}^0P_4 = {}^0R_2\,{}^2P_4$

${}^0R_2 = {}^0R_1\,{}^1R_2$

$$= \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -L_4 \sin\theta_3 \\ 0 \\ L_4 \cos\theta_3 \\ 1 \end{bmatrix}$$

$$\boxed{{}^0P_4 = \begin{bmatrix} -L_4 c\theta_1 s\theta_3 \\ -L_4 s\theta_1 s\theta_3 \\ L_4 c\theta_3 + d_2 \end{bmatrix}}$$

b. ${}^0R_4 = {}^0R_2\,{}^2R_4 = {}^0R_2\,{}^2R_3\,{}^3R_4$

$$= \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 \\ s\theta_1 & c\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 \\ s\theta_3 & c\theta_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 \\ s\theta_1 & c\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 \\ 0 & 0 & -1 \\ -s\theta_3 & c\theta_3 & 0 \end{bmatrix} = \boxed{\begin{bmatrix} c\theta_1 c\theta_3 & -c\theta_1 s\theta_3 & s\theta_1 \\ s\theta_1 c\theta_3 & -s\theta_1 s\theta_3 & -c\theta_1 \\ -s\theta_3 & c\theta_3 & 0 \end{bmatrix}}$$

c. $$\boxed{{}^0J_v = \begin{bmatrix} L_4 s\theta_1 s\theta_3 & 0 & -L_4 c\theta_1 c\theta_3 \\ -L_4 c\theta_1 s\theta_3 & 0 & -L_4 s\theta_1 c\theta_3 \\ 0 & 1 & -L_4 s\theta_3 \end{bmatrix}}$$

d. $${}^0J_\omega = \begin{bmatrix} 0 & 0 & s\theta_1 \\ 0 & 0 & -c\theta_1 \\ 1 & 0 & 0 \end{bmatrix}$$

e.

$\theta_1 = 0$
$\theta_3 = -\frac{\pi}{2}$

$\theta_1 = 90°$
$\theta_3 = 0$

f. $J = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

Can't move in the x-direction

$\theta_1 = 0° : \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T$

$\theta_1 = 90° : \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}^T$

$J = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

Can't move in the x-direction

$\theta_1 = 0° : \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}^T$

$\theta_1 = 90° : \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T$

g. $M(q) = \sum m_i J_{v_i}^T J_{v_i} + J_{\omega i} I_{c_i} J_{\omega i}$

$= J_{v4}^T J_{v4}$ (since $m_{1,2,3} = 0$, $I_{ci} = 0$)

$$= \begin{bmatrix} L_4 s\theta_1 s\theta_3 & 0 & -L_4 c\theta_1 c\theta_3 \\ -L_4 c\theta_1 s\theta_3 & 0 & -L_4 s\theta_1 c\theta_3 \\ 0 & 1 & -L_4 s\theta_3 \end{bmatrix}^T \begin{bmatrix} L_4 s\theta_1 s\theta_3 & 0 & -L_4 c\theta_1 c\theta_3 \\ -L_4 c\theta_1 s\theta_3 & 0 & -L_4 s\theta_1 c\theta_3 \\ 0 & 1 & -L_4 s\theta_3 \end{bmatrix}$$

$= [$

h. $P(q) = \sum m_i g^T r_{ci}$

$G(q) = \frac{\partial P}{\partial q} = \sum m_i g^T \frac{\partial r_{ci}}{\partial q}$

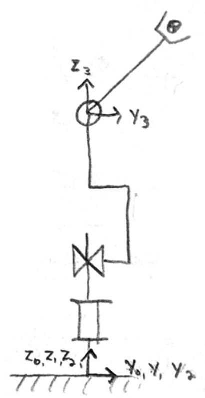
$$= g^T J_{v4}$$

$$= \begin{bmatrix} L_4 s\theta_1 s\theta_3 & 0 & -L_4 c\theta_1 c\theta_3 \\ -L_4 c\theta_1 s\theta_3 & 0 & -L_4 s\theta_1 c\theta_3 \\ 0 & 1 & -L_4 s\theta_3 \end{bmatrix}$$

$$= g \begin{bmatrix} 0 \\ 1 \\ -L_4 s\theta_3 \end{bmatrix}$$

Problem 2

a. See right



b. ${}^3P_4 = [0 \ 0 \ L_4]^T$

c. Output: $[0 \ 1 \ 1]^T$
Expected: $[1 \ 0 \ 1]^T$

Output: $[-1 \ 0 \ 1]^T$
Expected: $[0 \ 1 \ 1]^T$

They're different because the frames are different; in q! the third joint was along -Y, now its along +X at $\theta_1 = 0$.

d. Output: $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ Expected: $\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

Output: $\begin{bmatrix} 0 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ Expected: $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

Different for the same reason as before, the base frame is different.