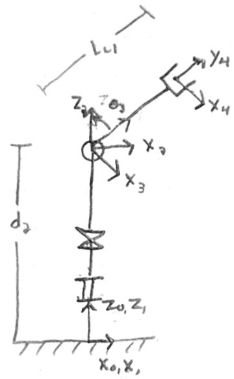
CS225A

Problem 1



a. $${}^0P_2 = \begin{bmatrix} 0 \\ 0 \\ d_2 \end{bmatrix}$$

$${}^2P_4 = \begin{bmatrix} -L_4 \sin\theta_3 \\ 0 \\ L_4 \cos\theta_3 \end{bmatrix}$$

$${}^0P_4 = {}^0R_2 \, {}^2P_4$$

$${}^0R_2 = {}^0R_1 \, {}^1R_2$$

$$= \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -L_4 \sin\theta_3 \\ 0 \\ L_4 \cos\theta_3 \\ 1 \end{bmatrix}$$

$$\boxed{{}^0P_4 = \begin{bmatrix} -L_4 c\theta_1 s\theta_3 \\ -L_4 s\theta_1 s\theta_3 \\ L_4 c\theta_3 + d_2 \end{bmatrix}}$$

b. $${}^0R_4 = {}^0R_2 \, {}^2R_4 = {}^0R_2 \, {}^2R_3 \, {}^3R_4$$

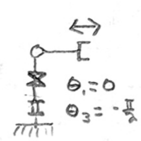
$$= \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 \\ s\theta_1 & c\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 \\ s\theta_3 & c\theta_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 \\ s\theta_1 & c\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 \\ 0 & 0 & -1 \\ -s\theta_3 & c\theta_3 & 0 \end{bmatrix} = \boxed{\begin{bmatrix} c\theta_1 c\theta_3 & -c\theta_1 s\theta_3 & s\theta_1 \\ s\theta_1 c\theta_3 & -s\theta_1 s\theta_3 & -c\theta_1 \\ -s\theta_3 & c\theta_3 & 0 \end{bmatrix}}$$

c. $$\boxed{{}^0J_v = \begin{bmatrix} L_4 s\theta_1 s\theta_3 & 0 & -L_4 c\theta_1 c\theta_3 \\ -L_4 c\theta_1 s\theta_3 & 0 & -L_4 s\theta_1 c\theta_3 \\ 0 & 1 & -L_4 s\theta_3 \end{bmatrix}}$$

d. $${}^0J_\omega = \begin{bmatrix} 0 & 0 & s\theta_1 \\ 0 & 0 & -c\theta_1 \\ 1 & 0 & 0 \end{bmatrix}$$

e.



$\theta_1 = 0$
$\theta_3 = -\frac{\pi}{2}$

$\theta_1 = 90°$
$\theta_3 = 0$

f. $$J = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \qquad J = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

Can't move in the x-direction

$\theta_1 = 0° : \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T$

$\theta_1 = 90° : \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}^T$

Can't move in the x-direction

$\theta_1 = 0° : \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}^T$

$\theta_1 = 90° : \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T$

g. $M(q) = \sum m_i J_{v_i}^T J_{v_i} + J_{\omega i} I_{ci} J_{\omega i}$

$= J_{v4}^T J_{v4}$ (since $m_{1,2,3} = 0$, $I_{ci} = 0$)

$$= \begin{bmatrix} L_4 s\theta_1 s\theta_3 & 0 & -L_4 c\theta_1 c\theta_3 \\ -L_4 c\theta_1 s\theta_3 & 0 & -L_4 s\theta_1 c\theta_3 \\ 0 & 1 & -L_4 s\theta_3 \end{bmatrix}^T \begin{bmatrix} L_4 s\theta_1 s\theta_3 & 0 & -L_4 c\theta_1 c\theta_3 \\ -L_4 c\theta_1 s\theta_3 & 0 & -L_4 s\theta_1 c\theta_3 \\ 0 & 1 & -L_4 s\theta_3 \end{bmatrix}$$

$= [$

h. $P(q) = \sum m_i g^T r_{ci}$

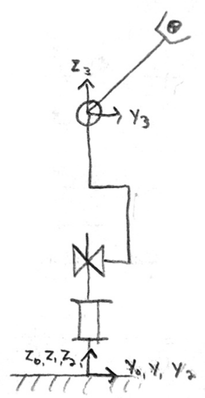
$G(q) = \frac{\partial P}{\partial q} = \sum m_i g^T \frac{\partial r_{ci}}{\partial q}$

$= g^T J_{v4}$

$$= \begin{bmatrix} L_4 s\theta_1 s\theta_3 & 0 & -L_4 c\theta_1 c\theta_3 \\ -L_4 c\theta_1 s\theta_3 & 0 & -L_4 s\theta_1 c\theta_3 \\ 0 & 1 & -L_4 s\theta_3 \end{bmatrix}$$

$$= g \begin{bmatrix} 0 \\ 1 \\ -L_4 s\theta_3 \end{bmatrix}$$

Problem 2



a. See right

b. ${}^3P_4 = [0 \;\; 0 \;\; L_4]^T$

c. Output: $[0 \;\; 1 \;\; 1]^T$
Expected: $[1 \;\; 0 \;\; 1]^T$

Output: $[-1 \;\; 0 \;\; 1]^T$
Expected: $[0 \;\; 1 \;\; 1]^T$

They're different because the frames are different; in q' the third joint was along $-Y$, now it's along $+X$ at $\theta_1 = 0$.

d. Output: $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ Expected: $\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

Different for the same reason as before, the base frame is different.

Output: $\begin{bmatrix} 0 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ Expected: $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

In [1]:
```
using Plots
using LinearAlgebra
using DelimitedFiles
```

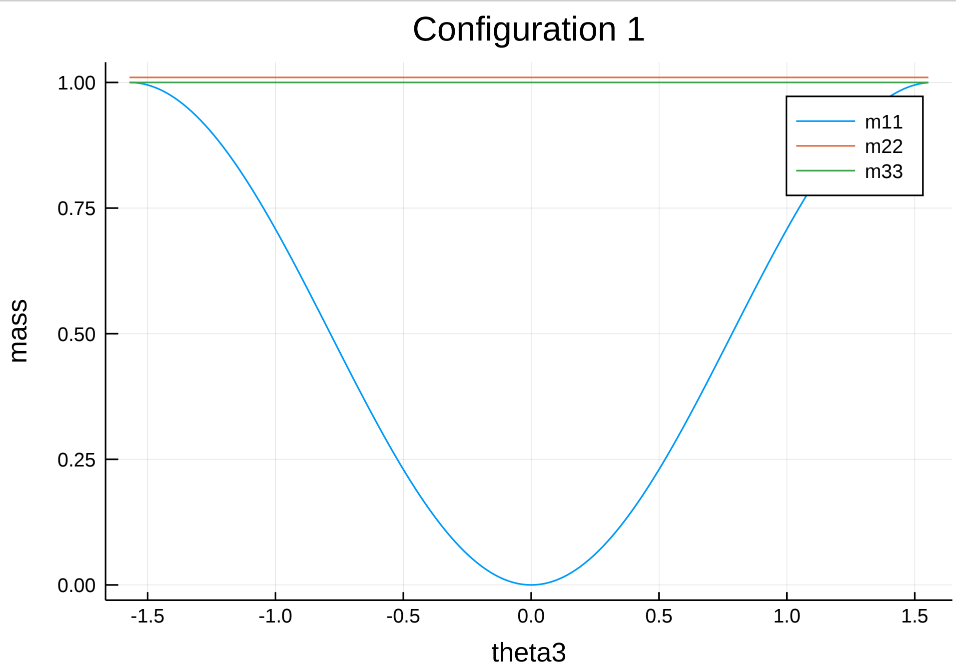
In [2]:
```
# Forward Kinematics
L4 = 1;
Jv(q) = [L4*sin(q[1])*sin(q[3]) 0 -L4*cos(q[1])*cos(q[3]);
        -L4*cos(q[1])*sin(q[3]) 0 -L4*sin(q[1])*cos(q[3]);
         0                      1 -L4*sin(q[3])];

ee(q) = [-L4*cos(q[1])*sin(q[3]), -L4*sin(q[1])*sin(q[3]), L4*cos(q[3])+q[2]]
```

Out[2]:
```
ee (generic function with 1 method)
```

Part (e): Mass Matrix

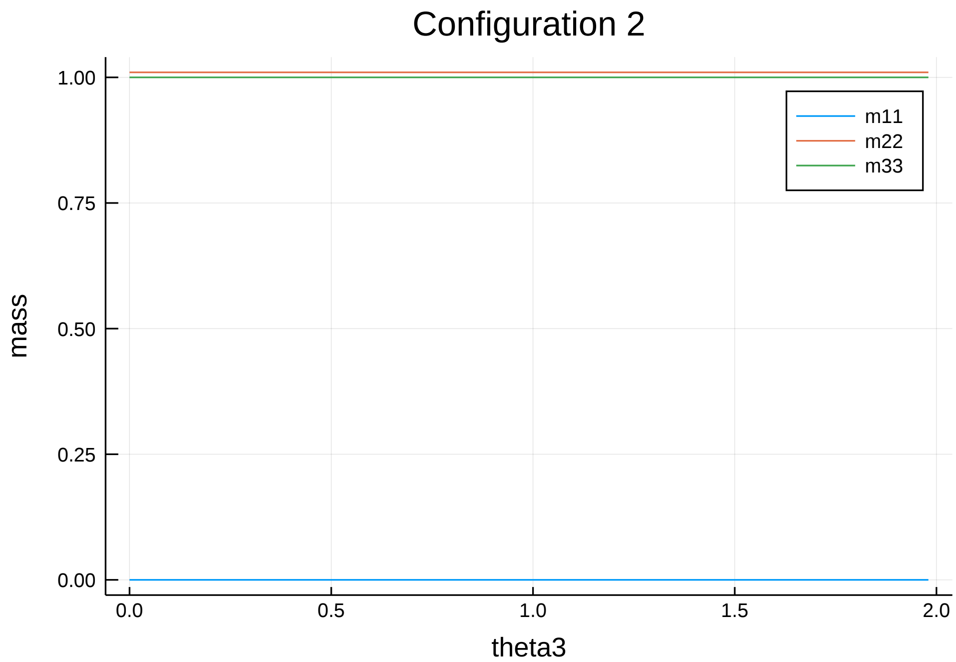
In [5]:
```
# Read in the data file for configuration 1
M1 = readdlm("../bin/hw0/mass_matrix1.txt", ',', Float64, '\n');
plot(M1[:,1],M1[:,2:4],title="Configuration 1",ylabel="mass",xlabel="theta3",la
bel=["m11" "m22" "m33"])
```

Out[5]:



In [6]:
```
# Read in the data file for configuration 2
M2 = readdlm("../bin/hw0/mass_matrix2.txt", ',', Float64, '\n');
plot(M2[:,1],M2[:,2:4],title="Configuration 2",ylabel="mass",xlabel="theta3",la
bel=["m11" "m22" "m33"])
```

Out[6]:



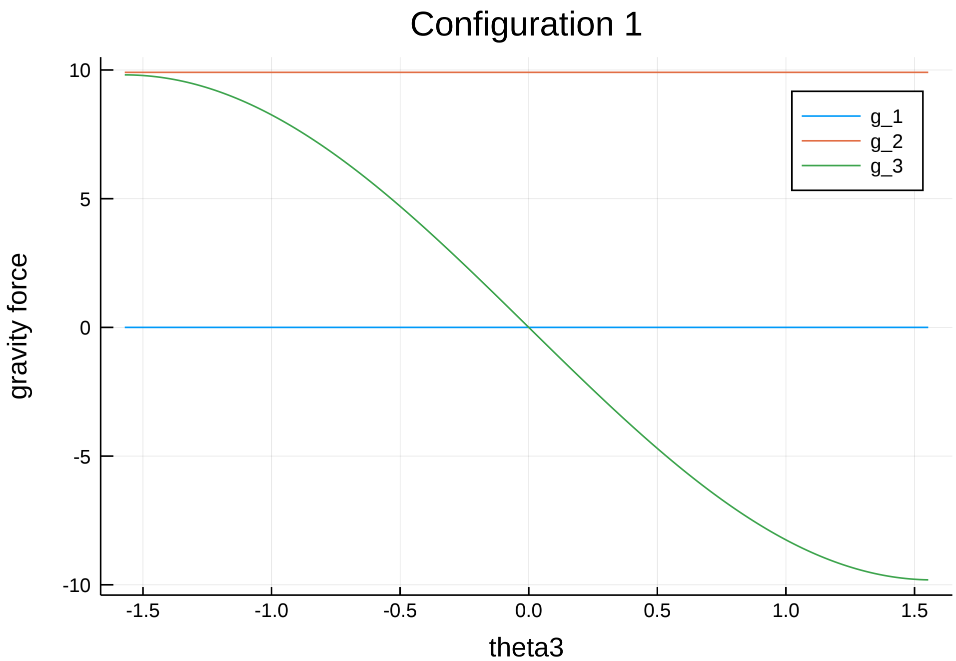
The mass felt by the first joint will decrease to zero when the arm is straight up. The prismatic joint will always just feel the weight of the end effector (so is constant) and the third joint only has the end effector left so will also be constant.

Moving the prismatic joint will not change how much mass is felt by the other two joints.

Part (f): Gravity Vector

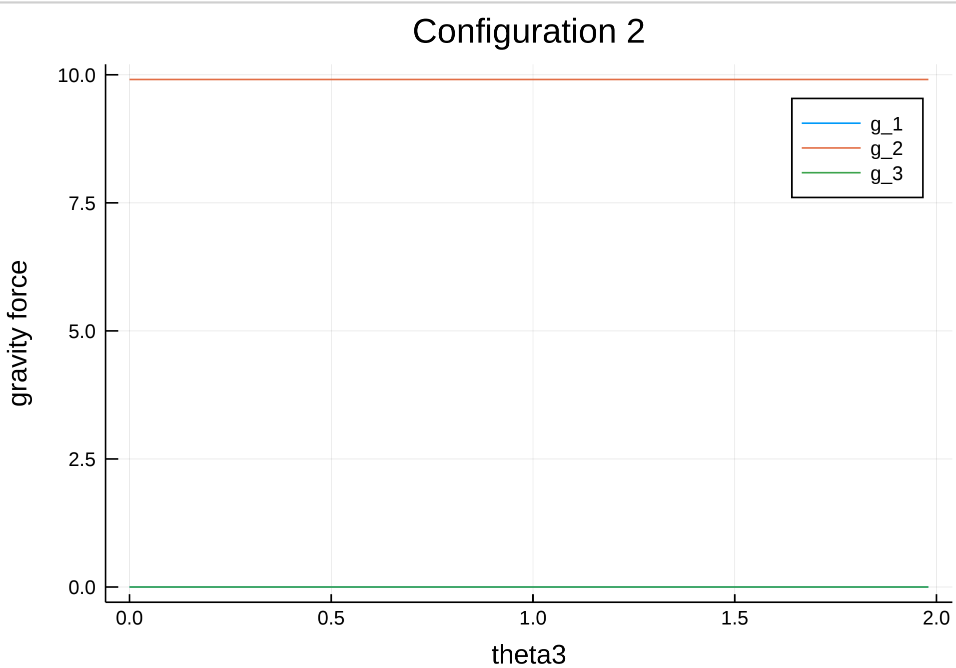
In [7]:
```
# Configuration 1
G1 = readdlm("../bin/hw0/G_vals.txt", ',', Float64, '\n');
plot(G1[:,1],G1[:,2:4],title="Configuration 1",ylabel="gravity force",xlabel="t
heta3",label=["g_1" "g_2" "g_3"])
```

Out[7]:



In [8]:
```
# Configuration 2
G2 = readdlm("../bin/hw0/G_vals2.txt", ',', Float64, '\n');
plot(G2[:,1], G2[:,2:4],title="Configuration 2",ylabel="gravity force",xlabel="
theta3",label=["g_1" "g_2" "g_3"])
```

Out[8]:



As the arm moves to vertical the third joint will not have to support as much of the weight so will require less torque/force to combat gravity. The prismatic joint, since it is aligned with gravity, will always have to compensate for the weight of the effector, while the first joint will always see zero force due to gravity since it's axis is aligned with gravity. Moving the joint up and down will not change any of this.