ASTRO 6530 FINAL PAPER
TECHNOSIGNATURE PROPAGATION FROM EXOPLANETS TO US

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Breakthrough Listen and other collaborations search for technosignatures—radio-frequency signals unlikely to have been produced by natural sources—to establish the existence of relatively advanced extraterrestrial civilizations. One way this is done is by scouring radio dynamic spectra, or 2D time-frequency plots generated by taking the Fourier transform of 1D voltage data in blocks, for signals with unusual frequency drift rates characteristic of the Doppler effect from the radial acceleration of a planet orbiting a star [4]. Signals of interest are then further analyzed via intensive comparisons with local sources of radio-frequency interference, where "local" can mean anything from "right next to the telescope" to "within the heliopause," and follow-up campaigns that attempt to reproduce the observation [13]. However, the diversity of the potential signals (e.g. in frequency, luminosity, polarization, modulation, etc.), under the "technosignature" umbrella makes the search for such signals a Herculean task [1]. One contributor to this diversity, apart from variations in drift rate, is the fact that the same signal produced at different source planets may look considerably different when it arrives at Earth due to differences in dispersive effects along the path it takes. In this project, I model the propagation of radio technosignatures on their journey from a putative source planet to a receiver on Earth.

The propagation occurs primarily in three media: the source planet's atmosphere, Earth's atmosphere, and the interstellar medium (ISM) along the line of sight. In each medium, under certain simplifying assumptions, the equation of radiative transfer applies,

$$\frac{dI_\nu}{ds} = j_\nu - \alpha_\nu I_\nu, \tag{1}$$

where I_ν is the intensity of the signal, ds is a differential length along the line of sight, and j_ν and α_ν are coefficients discussed below. Due to the complexity of exoplanet atmospheric spectra, a great deal of their interpretation is done using computer retrieval models [11]. The objective of this project is not to replicate those models, but to demonstrate how radio-frequency radiation changes during its path from the surface of one planet to the surface of another.

In the first part of this project, I will show a relatively straightforward solution to Equation (1) and how it can be applied using observable parameters of the media. (I will neglect the scattering term in the general radiative transfer equation for the sake of brevity, though Rybicki and Lightman present a derivation with scattering included [18].) For the second part of this project, I will apply the solution to a dataset of exoplanets for which the parameters are known.

The second term on the right-hand side contains the absorption coefficient, which can be derived from the composition of the medium,

$$\alpha_\nu = n\sigma_\nu, \tag{2}$$

where n is the number density of particles and σ_ν is the frequency-dependent cross-section of scatterers.

For the ISM, which is dominated by free electrons, σ_ν is simply the Thomson cross-section, $\sigma_T = \frac{8\pi}{3}r_e^2 \approx 0.665$ barn $= 6.65 \times 10^{-25}$ cm^2. For planetary atmospheres, σ_ν has a potentially complicated frequency dependence and generally must be inferred from absorption spectra. For simplicity, and since exoplanet spectra seldom extend to radio frequencies anyway, I will assume that the exoplanet's atmosphere absorbs at these frequencies in essentially the way Earth's atmosphere does.

The first term, the emission coefficient, can be inferred from the source function S_ν,

$$j_\nu = \alpha_\nu S_\nu, \tag{3}$$

where α_ν is the absorption coefficient previously discussed. In planetary atmospheres, where the rate of particle collisions is relatively high, local thermodynamic equilibrium (LTE) is a reasonable assumption [12]. In this case, from Kirchhoff's Law, the source function matches the Planck function,

$$S_\nu = B_\nu = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/k_B T} - 1}, \tag{4}$$

so the emission coefficient is

$$j_\nu = \frac{2h\nu^3}{c^2} \frac{n\sigma_\nu}{e^{h\nu/k_B T} - 1}. \tag{5}$$

In the ISM, meanwhile, the free electrons are sparsely distributed and LTE is not a reasonable assumption. We do have $\alpha_\nu = n\sigma_T$, so the equation of radiative transfer becomes

$$\frac{dI_\nu}{ds} = n\sigma_T(S_\nu - I_\nu), \tag{6}$$

and we will need to assume a form for the source function to make progress.

In the planetary atmosphere case, we can use the frequency-dependent optical depth $d\tau_\nu = \alpha_\nu ds$ to rewrite (1) as

$$\frac{dI_\nu}{d\tau_\nu} = B_\nu - I_\nu \tag{7}$$

and solve using an integrating factor. Since the equation follows the form $\frac{dy}{dx} = Q - Py$, the integrating factor is $e^{\int P dx} = e^{\tau_\nu}$. We multiply (7) by e^{τ_ν} to obtain

$$e^{\tau_\nu}\left(\frac{dI_\nu}{d\tau_\nu} + I_\nu\right) = e^{\tau_\nu} B_\nu. \tag{8}$$

Then, since $\frac{d}{d\tau_\nu}\left(e^{\tau_\nu} I_\nu\right) = e^{\tau_\nu}\frac{dI_\nu}{d\tau_\nu} + e^{\tau_\nu} I_\nu$,

$$\frac{d}{d\tau_\nu}\left(e^{\tau_\nu} I_\nu\right) = e^{\tau_\nu} B_\nu. \tag{9}$$

Integrating both sides with respect to τ_ν,

$$e^{\tau_\nu} I_\nu = \int_0^{\tau_\nu} e^{\tau'_\nu} B_\nu(\tau'_\nu) d\tau'_\nu + C, \tag{10}$$

where C is an integration constant that we see is $I_\nu(0)$ from the $\tau_\nu = 0$ boundary condition. I will denote this as $I_\nu^{(0)}$, the initial frequency-dependent intensity of the beam, hereafter. Dividing both sides by e^{τ_ν}, we arrive at the solution

$$I_\nu(\tau_\nu) = I_\nu^{(0)} e^{-\tau_\nu} + \int_0^{\tau_\nu} e^{-(\tau_\nu - \tau'_\nu)} B_\nu(\tau'_\nu) d\tau'_\nu. \tag{11}$$

If we let the I_ν of (7) be $I_\nu^{(\mathrm{ex})}$, the intensity of the light after propagating through the exoplanet atmosphere, and substitute in the Planck spectrum for B_ν, then we have

$$I_\nu^{(\mathrm{ex})}(\tau_\nu) = I_\nu^{(0)} e^{-\tau_\nu} + \int_0^{\tau_\nu} e^{-(\tau_\nu - \tau'_\nu)} \left(\frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/k_B T(\tau'_\nu)} - 1} \right) d\tau'_\nu. \tag{12}$$

Note that the dependence of temperature on optical depth, $T(\tau_\nu)$, remains to be derived. We first put back in the units of distance via $\tau_\nu = \alpha_\nu z$, where I now refer to distance in the atmosphere with z rather than s by analogy with other problems dealing with atmospheric height. With this substitution, (12) becomes

$$I_\nu^{(\mathrm{ex})}(z) = I_\nu^{(0)} e^{-\alpha_\nu z} + \int_0^{z} e^{-\alpha_\nu (z - z')} \left(\frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/k_B T(z')} - 1} \right) \alpha_\nu dz'. \tag{13}$$

$T(z)$ is still not straightforward to derive analytically. Ground can be made by assuming the atmosphere is everywhere an ideal gas, but observations of Solar System bodies with non-tenuous atmospheres like Earth, Venus, Titan, and the outer planets show significant deviations from this model [14]. Since exoplanet temperature profiles are not trivially obtained [15], I will assume an isothermal atmosphere here for simplicity. Thus, we obtain the following as a form for $I_\nu^{(\mathrm{ex})}$:

$$I_\nu^{(\mathrm{ex})}(z) = I_\nu^{(0)} e^{-\alpha_\nu z} + \alpha_\nu \int_0^{z} e^{-\alpha_\nu (z - z')} \left(\frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/k_B T} - 1} \right) dz' \tag{14}$$

$$I_\nu^{(\mathrm{ex})}(z) = I_\nu^{(0)} e^{-\alpha_\nu z} + \frac{2h\nu^3}{c^2}\left(\frac{1-e^{-\alpha_\nu z}}{e^{h\nu/k_B T}-1}\right). \tag{15}$$

The observables are the frequency-dependent absorption coefficient α_ν, obtained from absorption spectra; the total height of the atmosphere z, estimated from mass and radius measurements using a procedure like in Kipping et al. [16]; and the temperature of the atmosphere T, estimated from the host star's temperature and planet's orbital radius.

In the ISM, the emission and absorption processes result in a modification from $I_\nu^{(\mathrm{ex})}$ to $I_\nu^{(\mathrm{ISM})}$. This modification is like the modification from $I_\nu^{(0)}$ to $I_\nu^{(\mathrm{ex})}$, but with an arbitrary source function instead of the Planck function. Performing the same integration of the equation of radiative transfer, we obtain

$$I_\nu^{(\mathrm{ISM})} = I_\nu^{(\mathrm{ex})} e^{-\alpha_{\nu,(\mathrm{ISM})} L} + \int_0^L e^{-\alpha_{\nu,(\mathrm{ISM})}(L-\ell)} S_\nu(\ell) d\ell, \tag{16}$$

where the integral is along the line of sight, with differential distance $d\ell$ and total distance L, and $\alpha_{\nu,(\mathrm{ISM})}$ is the absorption coefficient for the interstellar medium.

Here, for simplicity, we assume a source function that's invariant with respect to position along the line of sight—i.e., $S_\nu(\ell) = S_\nu$. This is obviously not a great approximation, since the ISM is full of local variations in electron density and the source function will change significantly as the beam passes through, for example, HII regions or giant molecular clouds. More than 20 years ago, Cordes and Lazio presented the NE2001 model for the local ISM, which takes into account several of these variations and offers an excellent way to model the propagation of radiation from pulsars and other radio sources. NE2001 is still a useful standard in the literature, seeing a native Python implementation by Ocker and Cordes this year [6], but it is also itself still being refined by Ocker, Cordes, and others in work that is yet unpublished. An interesting extension to this calculation, outside the scope of this paper, would be to define $S_\nu(\ell)$ numerically in terms of the NE2001 model, using the exoplanet system's right ascension and declination as inputs to calculate how the source function varies along the corresponding line of sight.

By dimensional analysis, we see that $S_\nu = Ah\nu^3/c^2$, where A is a dimensionless constant of proportionality. Using this form, we see that

$$I_\nu^{(\mathrm{ISM})} = I_\nu^{(\mathrm{ex})} e^{-\alpha_{\nu,(\mathrm{ISM})}L} + \frac{Ah\nu^3}{c^2}\left(1 - e^{-\alpha_{\nu,(\mathrm{ISM})}L}\right), \tag{17}$$

or, expanding $I_\nu^{(\mathrm{ex})}$ into the form found in (15),

$$I_\nu^{(\mathrm{ISM})} = \left[I_\nu^{(0)} e^{-\alpha_\nu z} + \frac{2h\nu^3}{c^2}\left(\frac{1 - e^{-\alpha_\nu z}}{e^{h\nu/k_B T} - 1}\right)\right] e^{-\alpha_{\nu,(\mathrm{ISM})}L} + \frac{Ah\nu^3}{c^2}\left(1 - e^{-\alpha_{\nu,(\mathrm{ISM})}L}\right). \tag{18}$$

We can simplify this a little using Wien's approximation (i.e. that $\nu \ll 1$), since we're in the radio regime. If we do, the solution for $I_\nu^{(\mathrm{ISM})}$ looks like

$$I_\nu^{(\mathrm{ISM})} = \left[I_\nu^{(0)} e^{-\alpha_\nu z} + \frac{2\nu^2 k_B T}{c^2}\left(1 - e^{-\alpha_\nu z}\right)\right] e^{-\alpha_{\nu,(\mathrm{ISM})}L} + \frac{h\nu^3}{c^2}\left(1 - e^{-\alpha_{\nu,(\mathrm{ISM})}L}\right), \tag{19}$$

or, distributing the exponential into the brackets and slightly restructuring our notation,

$$\boxed{I_\nu^{(\mathrm{ISM})} = I_\nu^{(0)} e^{-(\alpha_\nu z + \beta_\nu L)} + \frac{2\nu^2 k_B T}{c^2}\left(\frac{1 - e^{-\alpha_\nu z}}{e^{\beta_\nu L}}\right) + \frac{h\nu^3}{c^2}\left(1 - e^{-\beta_\nu L}\right)}. \tag{20}$$

Here I'm redefining the ISM absorption coefficient $\alpha_{\nu,(\mathrm{ISM})} \to \beta_\nu$ for brevity, and also setting $A = 1$ to assume the form $h\nu^3/c^2$ for the ISM source function.

Once the light reaches the edge of Earth's atmosphere, it will undergo the same process as in the exoplanet atmosphere. Since we're assuming an Earthlike atmosphere for the exoplanet, the modification from $I_\nu^{(\mathrm{ex})}$ to $I_\nu^{\oplus}$, the shape of the beam at the receiver, uses all the same variables as (15):

$$I_\nu^{\oplus}(\tau_\nu) = I_\nu^{(\mathrm{ISM})} e^{-\alpha_\nu z} + \frac{2\nu^2 k_B T}{c^2}\left(1 - e^{-\alpha_\nu z}\right). \tag{21}$$

Here, again, I have employed the Wien approximation to simplify $(e^{h\nu/k_BT}-1) \approx h\nu/k_BT$. Substituting (20) for $I_\nu^{(\mathrm{ISM})}$, we obtain

(22)

$$I_\nu^{\oplus} = I_\nu^{(0)} e^{-(2\alpha_\nu z+\beta_\nu L)} + \frac{2\nu^2 k_B T}{c^2}\left(\frac{1-e^{-\alpha_\nu z}}{e^{\alpha_\nu z}e^{\beta_\nu L}}\right) + \frac{h\nu^3}{c^2}\left(\frac{1-e^{-\beta_\nu L}}{e^{\alpha_\nu z}}\right) + \frac{2\nu^2 k_B T}{c^2}\left(1-e^{-\alpha_\nu z}\right)$$

or, combining the second and fourth terms,

(23)

$$\boxed{I_\nu^{\oplus} = I_\nu^{(0)} e^{-(2\alpha_\nu z+\beta_\nu L)} + \frac{2\nu^2 k_B T}{c^2}\left(1-e^{-\alpha_\nu z}\right)\left(1+e^{-(\alpha_\nu z+\beta_\nu L)}\right) + \frac{h\nu^3}{c^2}\left(\frac{1-e^{-\beta_\nu L}}{e^{\alpha_\nu z}}\right)}.$$

This is the shape in frequency space of the beam $I_\nu^{(0)}$ after it propagates through three media: two planetary atmospheres, assumed identical for simplicity; and the ISM. It is in terms of five observables:

- the absorption coefficient of the atmosphere, α_ν;
- the absorption coefficient of the ISM, β_ν;
- the height of the atmosphere, z;
- the line-of-sight distance, L; and
- the temperature of the atmosphere, T.

Three of these (α_ν, z, and T) are easily obtained from measurements of Earth's own atmosphere, L is typically well known for star systems that are well-studied enough to have confirmed exoplanets, and, for the ISM, $\beta_\nu = n\sigma_T$, where n is estimated from pulsar dispersion measures in the Galactic plane to be ~0.01–0.1 electrons per cm^3 on average [17]. Using these observables, one could calculate what a given beam shape would look like after the propagation. This is a potential area of expansion for this project.

It must be emphasized that I have taken several liberties with my assumptions for this setup. Real planetary atmospheres are not isothermal and rarely Earthlike, the real ISM source function is not spatially invariant and certainly not exactly $h\nu^3/c^2$, and one cannot realistically neglect scattering in the equation of radiative transfer. The latter effect is discussed in detail in Rybicki and Lightman—scattering of photons off of particles in a medium can deflect radiation both into and out of the range of the receiver, and the resulting beam is typically broadened in both real and frequency space—and they present a numerical approximation for solving the integrodifferential equation that results from incorporating these effects into the equation of radiative transfer [18]. Such a solution is beyond the scope of this paper. However, I find the assumptions I've made reasonable, and would be interested in applying the results. How do radio-frequency beams change as they cross the cosmos? Given the depression of their amplitudes by radiative transfer effects, what are the limits of our current detectors as they try to pick these signals out from noise? Which currently known exoplanets could we in principle detect these signals from, what does it mean that we haven't yet, and how many yet-to-be-discovered exoplanets might there still be within our cosmic "earshot"? These are interesting questions that are relevant not only for humanity, but for any extraterrestrial civilizations capable of detecting Earth's own radio transmissions, and the equation of radiative transfer offers a way to answer them.

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