AE/ME8112 Problem Set 4 **Due Wednesday July 8th at 4:00PM**

Instructions:
For each problem, state all your assumptions, explain your approach and solution method, include a good copy of any preparation work that needs to be done by hand (derivations, discretizations… etc.) in your report.

If the problem requires writing a computer program, include your code and its output in your report.

In addition, email your codes to Prof. Dworkin by the due date and time. Your codes may be run to verify output, or put through a plagiarism checker. All codes that you email should be as separate attachments, titled PSnum_Qnum_Lastname_Firstname_Studentnum.dat (or .f90 or .f95 or .c etc.)
Example: PS4_Q2_Dworkin_Seth_0000001.f90. Make sure that they can be opened and read in a text editor such as Notepad or equivalent.

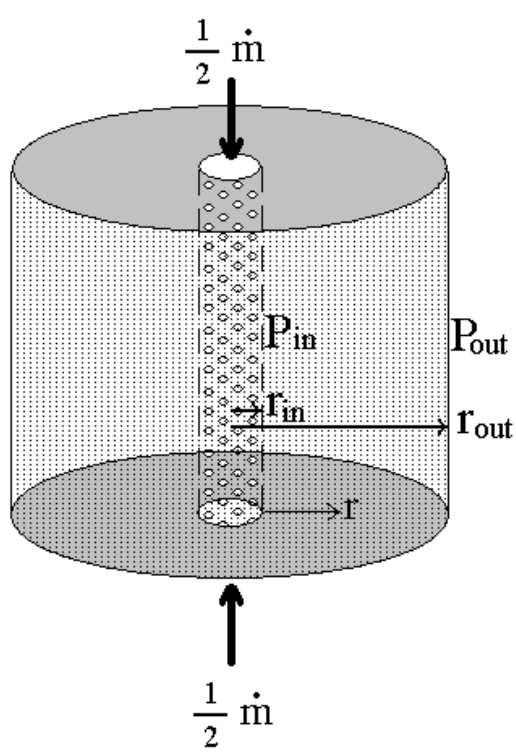
Email your codes and report in the same email. Your email should be timestamped by Dr. Dworkin's email provider by 3:59:59 PM on the due date or else your assignment will be considered late. All of your report must be contained in a single PDF file. (I.e., your email should have one PDF report which contains your codes, and in addition it should have each code that you wrote as a separate attachment.)

For all programs that you write, include a header with your full name as registered at the university, student number, date, assignment number, question number, description of the program, and programming language used.

Before you hand in your assignment, refer to the document on assignment preparation instructions posted on D2L under Content – Problem Sets.

Questions
1. Consider two concentric porous cylinders as depicted in the picture below.

$$\dot{m} = 10^{-5} * \left(e^{2t} - 1\right)\ m^3/s$$
$$\rho = 1.2\ kg/m^3$$
$$r_{in} = 0.01\ m$$
$$r_{out} = 0.11\ m$$
$$P_{out} = 101325\ Pa$$
$$\mu = 1.84e-5\ kg/m*s$$



A gas flows into the central cylinder from both ends at a total rate of $\dot{m}$. The gas then flows through the pores of the inner cylinder, radially outward, and out the pores of the outer cylinder.

a) Derive a finite volume formulation for the 1D Navier-Stokes equations on a staggered grid. State all of your assumptions and detail the solution procedure that you plan to use. You should not need to solve any linear systems in your program. During your discretization, discretize the pressure terms last. Discretize all other terms centred at i+1/2, the centre of your shifted velocity cell. That way, when you get to your pressure term, it will end up being a function of $(P_{i+1} - P_i)$. Show that the units of all of your discretized terms are consistent.

b) Write a program to solve the 1D Navier-Stokes equations for fluid velocity and pressure as functions of radial location and time (since $\dot{m}$ is a function of time) on a staggered grid. You may neglect the effects of the upper and lower surfaces, which are 0.5 m away from each other. Use 100 equally spaced grid points in the radial direction and a timestep of 0.01 s.

Your algorithm should proceed by first calculating the velocity field from continuity, then the pressure field from the momentum equation, then by incrementing the timestep and repeating. When solving the momentum equation for pressure, if you solve it backwards, from P_{out} to P_{in}, you avoid the need for solving a linear system.

Run the simulation until t = 1 s. Plot your solution at 0.1 s intervals to see how the velocity and pressure profiles develop.

Note: The pressure changes will be small, likely much smaller than 1 Pa. Therefore, to capture the pressure changes without them getting caught in the noise, you may want to set $P_{out} = 0$ in the simulation and then add 101,325 Pa to all of the computed values afterwards.

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2. Return to your code from question 1. Design and perform an analysis on its performance, accuracy, or efficiency. You may set the parameters of your own analysis and it is expected that it will require modification and rerunning of your code. Your report should proceed in a logical manner. You should state the problem, procedure, show the steps, observations, and then draw conclusions.

For example:
I wish to test the grid independence of my solution.
I will continually double my grid, starting from 8 points, until my solution is no longer changing based on some tolerance I have set. That tolerance is ...
Based on the calculated P_{in} at t = 1 s, I see from this plot that my solution is no longer changing after *x* gridpoints.
Below is the table of the error as a function of grid points defined as....
The optimal grid for this problem is...

In addition to the above example, you may want to look at things such as timestep size dependence, the effect of neglecting viscosity, solution variation as a function of gas properties, effectiveness and optimization of the code for a steady problem, implementation in parallel, or any other problem you are interested in tackling. It is your choice which of these to study.

You should consider tackling at least one *challenging* question such as: use of a non-equispaced grid, use of finite differences instead of finite volumes, solving a different physical problem (such as adaptation to cylindrical or Cartesian coordinates via use of flags)...etc, or a problem of your design.

The ideal report will tackle some of the more basic efficiency and accuracy question, and will show independent and creative thought, design and execution of one or more case studies of your choice, and a clear and logical report of the process and results.

I encourage you to consult with your peers and check codes and solutions for **question 1**, but do question 2 independently. I would like to see originality in your work.

In total, I expect you to devote 20 to 24 hours to this problem set with your solution to problem 1 taking somewhere around a third to a half of that time. Try to have some fun with it.