42106 Financial Risk Management

Project 2

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Introduction

In this Financial Risk Management project the team will act as credit risk analysts. The task is to calculate CVA charges for a portfolio of interest rate swaps.

1 The Portfolio

The portfolio consists of three off-market interest rate swaps (meaning that they have a value different from zero at time 0).

1. A 10-year interest swap on a notional of \$4 million, where we pay a fixed swap rate of 1% and receive the floating rate.

2. A 7-year interest swap on a notional of \$10 million, where we pay a fixed swap rate of 1.5% and receive the floating rate.

3. A 4-year interest swap on a notional of \$1 million dollars, where you pay the floating rate and receive a fixed rate of 2%.

All swap payments are made on a quarterly basis and we believe that the evolution of the interest rate can be described using a Vasicek model with the following parameters:

- Initial spot rate of $r_0 = 0.002$
- Mean reversion rate of $k = 0.2$
- Mean reversion level of $\theta = 0.01$
- Interest rate volatility of $\sigma = 0.01$

In the Vasicek-model, the short rate is assumed to satisfy the stochastic differential equation:

$$dr(t) = k(\theta - r(t))dt + \sigma dW(t))$$

where k, θ, σ are defined as above and W is a Brownian motion under the risk-neutral measure. The use of Brownian motion introduces a random pattern, that can be used to simulate many different evolutions of interest rates.

In the same Vasicek-model, the price of a zero-coupon bond with maturity T at time $t \in [0, T]$ is given by

$$P(t, T) = A(t, T)e^{-r(t)B(t,T)}$$

where

$$B(t, T) = \frac{1 - e^{-k(T-t})}{k}$$

and

$$A(t, T) = exp\left\{\left(\theta - \frac{\sigma^2}{2k^2}\right)(B(t, T) - T + t) - \frac{\sigma^2}{4k}B^2(t, T)\right\}$$

The prices of the zero-coupon bonds will be used as discount factors when the CVA values are calculated later on.

The interest rate swaps are entered with Morgan Stanley and their creditworthiness can be evaluated using the following CDS spreads along with an assumption that R=62%.

Tenor (in bps)	**Morgan Stanley**
6M	19.04
1Y	27.79
2Y	38.62
3Y	52.07
4Y	63.73
5Y	78.30
7Y	100.51
10Y	119.08
20Y	144.48
30Y	151.93

2 Calculation of exposure

Interest rate simulations were used to calculate the future values of each of the interest rate swap contracts. The simulation created 50.000 different interest rate evolutions and each contract was valued at each time step in each simulation. The simulated future values were then used to find the expected exposure, the peak exposure (97.5%), and the maximum peak exposure of each swap contract.

The value of the fixed leg of an interest rate swap, at time t_j, is the present value of all future payments

$$V_{fix}(t_i) = r_{fix} \sum_{j=i+1}^{n} P(t_i, t_j)\Delta t$$

where r_{fix} is the fixed rate of the interest rate swap, and $P(t_i, t_j)$ is as defined above. The value of the floating leg at time t_i is

$$V_{float}(t_j) = 1 - P(t_j, t_n)$$

The difference between the two legs is the value of the interest rate swap, at any given time. The direction of the legs depends on which position is held in the interest rate swap. In our case is the fixed rated paid on the 10 and 7-year swaps, and the floating rate paid on the 4-year swap.

Since these calculations are only relevant for the positive site, the exposure at each time, in each simulation can be defined as

$$Exposure = max(f_i^k, 0)$$

where f_i^k denotes the MtM-value of the swap at time i during simulation k.

The expected exposure at any given time point, overall the simulations, is then given by

$$EE(t_i) = \frac{1}{K} \sum_{k=1}^{K} max(f_i^k, 0)$$

where K is the total number of simulations.

The peak exposure of each contract can be found by extracting the 97.5% percentile from the simulations. This is done at each time point, by taking all the 50.000 simulated exposures and sorting them. Then the $50000 * (1 - 0.975) = 250$th highest value is reported as the peak exposure. The maximum peak exposure is the single highest exposure found across all the time steps and simulations.

The simulated future values of the 10-year swap on a \$4 million notional, where we pay a fixed swap rate of 1% and receive the floating rate can be seen in figure 1. The maximum peak exposure is 0.3176406 million dollars, or \$317,641.

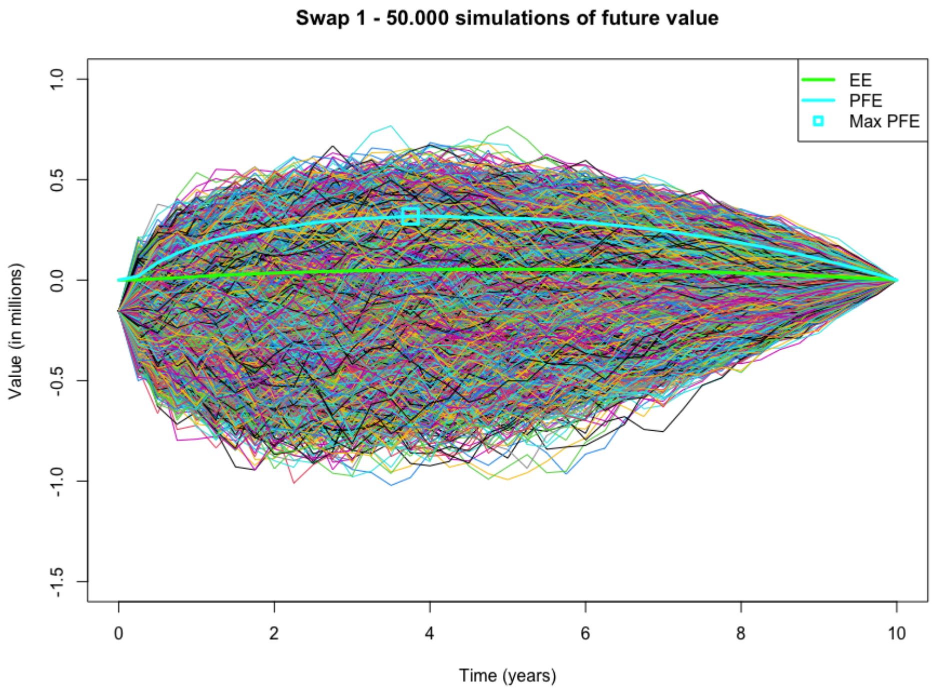

Figure 1: Simulated future values of swap 1, the expected exposure (EE), 97.5% peak exposure (PFE) and maximum peak exposure (Max PFE).

The simulated future values of the 7-year swap on a \$10 million notional, where we pay a fixed swap rate of 1.5% and receive the floating rate can be seen in figure 2. The maximum peak exposure is 0.3977598 million dollars, or \$397,760.

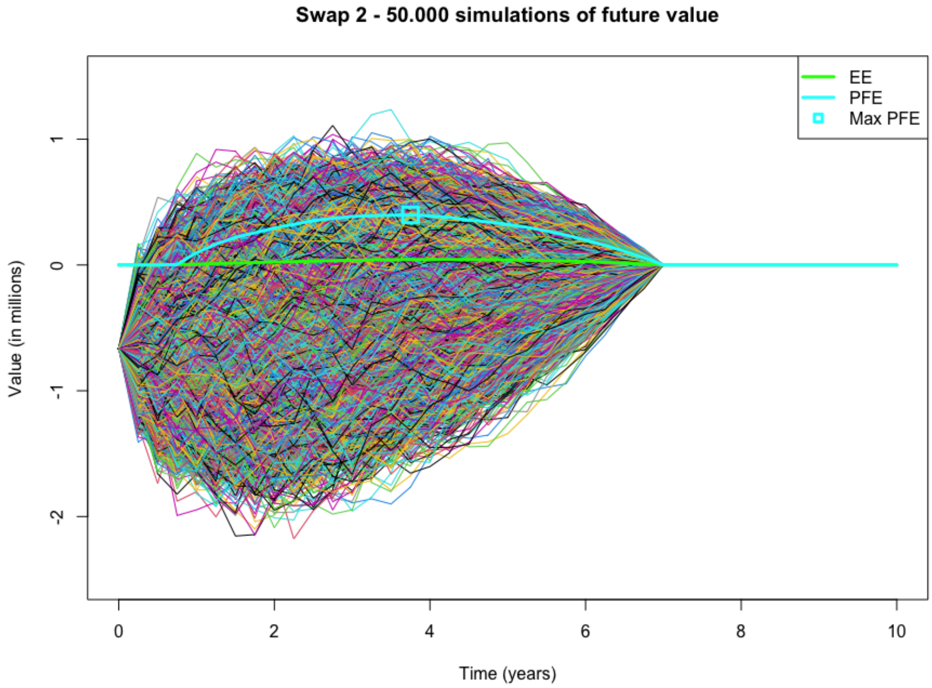

Figure 2: Simulated future values of swap 2, the expected exposure (EE), 97.5% peak exposure (PFE) and maximum peak exposure (Max PFE).

The simulated future values of the 4-year swap on a $1 million notional, where we pay the floating rate and receive a fixed rate of 2% can be seen in figure 3. The maximum peak exposure is 0.08884552 million dollars, or $88,846.

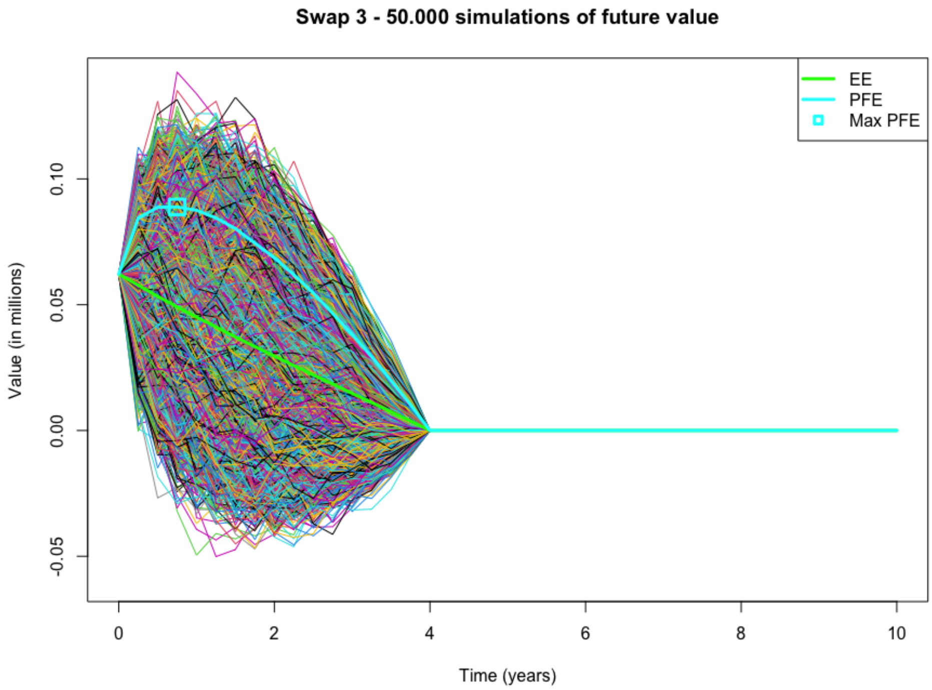

Figure 3: Simulated future values of swap 3, the expected exposure (EE), 97.5% peak exposure (PFE) and maximum peak exposure (Max PFE).

From figures 1, 2 and 3 it can clearly be seen that the swaps are of-market swaps, as their values are different from zero at time 0. Swaps 1 and 2 have a negative starting value, as the initial spot rate is higher than the fixed rate of both swaps. Swap 3 has a positive value at time zero since the opposite applies there, and we receive the fixed rate.

The expected exposures, 97.5% peak exposures, and the maximum peak exposures of all the swaps were plotted together in figure 4 for easier comparison.

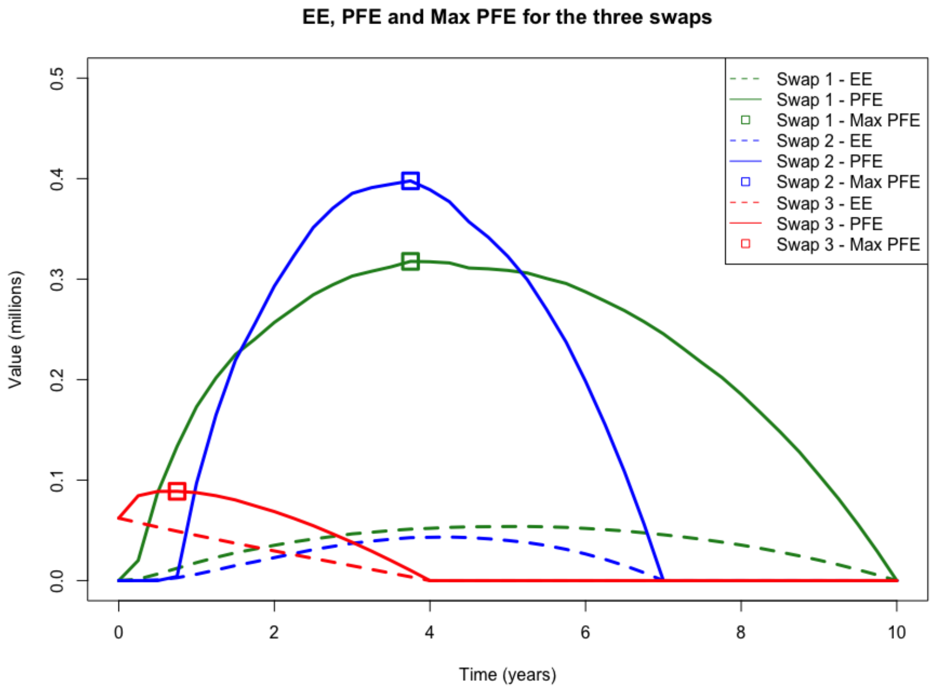

Figure 4: The expected exposure (EE), 97.5% peak exposure (PFE) and maximum peak exposure (Max PFE) of all three swaps.

Figure 4 shows that swaps 1 and 2, have expected exposures and peak exposures that start at 0 at time 0. This is due to the fact that both swaps have a negative value in the beginning and the expected exposures only care about the positive values of the swaps. The expected exposures and peak exposures then increase until around year 4 and then fall back to zero at their respective maturities. Both swaps have a maximum peak exposure around year 4, with swap 2 having the highest value of max peak exposure. This is not surprising as that swap has the highest notional, $10 million. Swap 3 has a different curve, that starts with a positive expected exposure and peak exposure value at time 0. The expected exposure value is then decreasing linearly until the maturity of the swap in year 4. The maximum peak exposure is reached before year 1, and the values are much lower than the other swaps, partly since the notional on swap 3 is lower, at $1 million.

3 Calculation of CVA charge

The credit value adjustment (CVA) is the amount by which a dealer must reduce the value of transactions because of counterparty default risk. The CVA is the present value of the expected loss from the default of the counterparty. Under the simplest assumptions, the CVA can be calculated as

$$CVA \approx (1-R)\sum_{i=1}^{n} DF(t_i)EE(t_i)PD(t_{i-1},t_i)$$

where R is the estimated recovery rate, $DF(t_i)$ are the discount factors at each time t_i, $EE(t_i)$ are the expected exposures at each time t_i and $PD(t_{i-1},t_i)$ are the default probabilities during the time interval $[t_{i-1},t_i]$. The assumption is made that the default probabilities (PD) and the expected exposures (EE) are independent. This is usually not the case, but makes for a very simple CVA model as there is no need to estimate parameters for a multivariate model.

The recovery rate is assumed to be 62% for Morgan Stanley as previously stated. The discount factors (DF) are not dependent on the simulations and were calculated from the Vasicek model. The expected exposures (EE) were found using the simulations as explained in the previous section. The default probabilities (PD) were estimated from Morgan Stanley's CDS spreads. The default intensity or hazard rate, λ, is the 'instantaneous' probability of default conditional on no earlier default. The average hazard rate function estimated from the CDS can be defined as

$$\bar{\lambda}(T_i) = \frac{s_i}{1-R}$$

where s_i is the spread of the CDS's yield over the risk-free rate at time 0 to T_i and R is the recovery rate. The piecewise linear average hazard rate was extracted from this in order to get the hazard rate for each time period. The time grid for this function is very sparse, as it is generated from the available CDS spreads of Morgan Stanley. The choice of a time grid will greatly affect the CVA calculations. Therefore the piece-wise linear average hazard rate function was converted into probability default rates with an equidistant grid, where each timestep was 3 months. The results of the CVA calculations can be seen in table 1.

Table 1: CVA of the three different interest rate swaps (IRS).

IRS	**CVA**
Swap 1	\$3,941.7
Swap 2	\$1,884.5
Swap 3	\$616.2

From table 1 it can be seen that swap 1 has the highest CVA value, more than twice as high as swap 1. Swap 3 has by far the lowest CVA values. These results are in agreement with what can be observed from figure 4. By looking at the expected exposure (EE) curves, it can be seen that swap 1 is always higher than swap 2, and has a longer time until maturity.

4 Impact of netting and collateral

In order to investigate the impact of netting and collateral, the expected exposure (EE), the 97.5% peak exposure (PFE), the maximum peak exposure (Max PFE) and CVA charge were calculated for the following three cases.

1. Netting is allowed and no collateral posted.
2. No netting, but Morgan Stanley sets a collateral of \$100.000 per swap contract.
3. Netting is allowed and Morgan Stanley sets a collateral of \$100.000 in total.

The netting that typically relates to counterparty risk is called closeout netting. It allows the termination of all contracts between the defaulting party and the institution, together with the offsetting of all transaction values. This means that when the counterparty defaults, they have to default on all contracts, also those that have a positive value for them.

Without netting the exposure of a portfolio containing three swaps can be defined as

$$Expoure = \mathbb{E}[max(f_i^1, 0)] + \mathbb{E}[max(f_i^2, 0)] + \mathbb{E}[max(f_i^3, 0)]$$

where f_i^k is the MtM-value of swap k at time i.

However when netting is used, the exposure of the same portfolio becomes:

$$Expoure = \mathbb{E}[max(f_i^1 + f_i^2 + f_i^3, 0)]$$

Netting reduces the counterparty risk, by reducing the expected exposure, and the CVA is therefore lower (or the same) when netting is used.

Collateral can be posted for each contract or by a counterparty in general. When collateral is posted it reduces the exposures, as some money (collateral) has been set aside. Posting collateral removes some credit risk from the transactions, and therefore lowers the counterparty risk and the CVA calculations.

When the portfolio of the three interest rate swaps is evaluated using no netting and no collateral, the results are as indicated in figure 5.

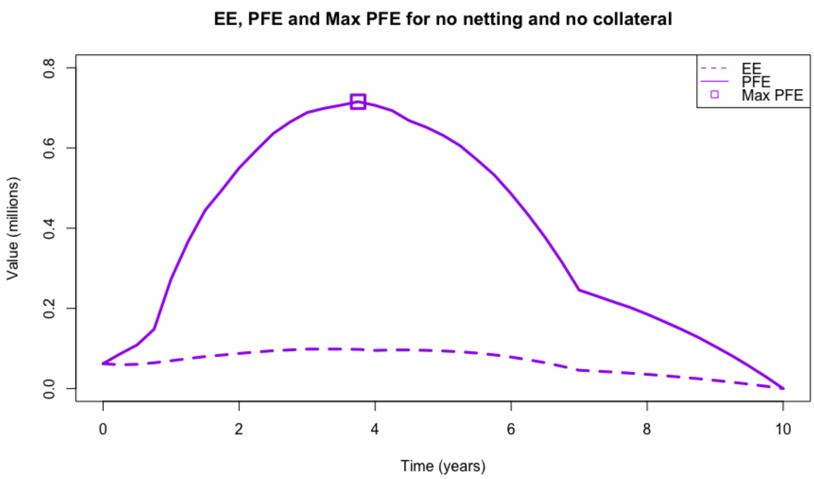

Figure 5: The expected exposure (EE), 97.5% peak exposure (PFE) and maximum peak exposure (Max PFE) for a portfolio using no netting and no collateral.

The CVA of the portfolio, is the sum of the CVAs of the three individual swaps, \$6,442.3.

4.1 Netting and no collateral

In the case of netting but no collateral the exposure is as previously shown

$$Expoure = \mathbb{E}[max(f_i^1 + f_i^2 + f_i^3, 0)]$$

The results of the expected exposure, peak exposure and max peak exposure using netting can be seen in figure 6 along with a comparison with a portfolio not using netting. Case 0 shows the portfolio without netting, while case 1 uses netting. The effect of netting can clearly be seen at the start, where the expected exposure (EE) is lower when netting is used. This is due to swap 3 having a positive MtM-value in the beginning, while swaps 1 and 2 have a negative value. This difference is offset in the netting, resulting in a lower expected exposure for the portfolio using netting. The expected exposures converge in year 4, when swap 3 has reached its maturity. The peak exposure and max peak exposures are nearly identical, apart from the peak exposure being lower in the first year when netting is applied.

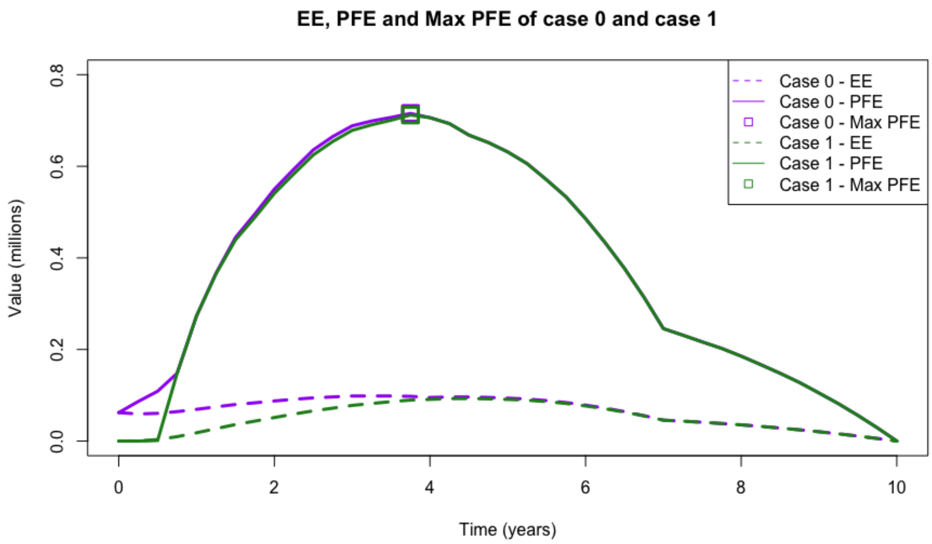

Figure 6: The expected exposure (EE), 97.5% peak exposure (PFE) and maximum peak exposure (Max PFE) for a portfolio using no netting and no collateral (case 0) and with netting (case 1).

The resulting CVA value of the portfolio with netting is \$5,601.6, slightly lower than the original CVA value with no netting of \$6,442.3.

4.2 No netting, collateral on each swap contract

Without netting, but with collateral C on each swap contract, the exposure of a portfolio containing three swaps can be defined as

$$Expoure = \mathbb{E}[max(f_i^1 - C, 0)] + \mathbb{E}[max(f_i^2 - C, 0)] + \mathbb{E}[max(f_i^3 - C, 0)]$$

In this case the value of the collateral is $C = \$100.000$ for each swap contract. The results of the expected exposure, peak exposure and max peak exposure using collateral can be seen in figure

7 along with a comparison with a portfolio without collateral and netting. Case 0 shows the base case of a portfolio that uses no netting and no collateral. Case 2 shows the same portfolio where collateral has been posted on each swap. The expected exposure curve is shifted downwards, with the difference being biggest at the start, and slowly converging until the maturity of the last swap.

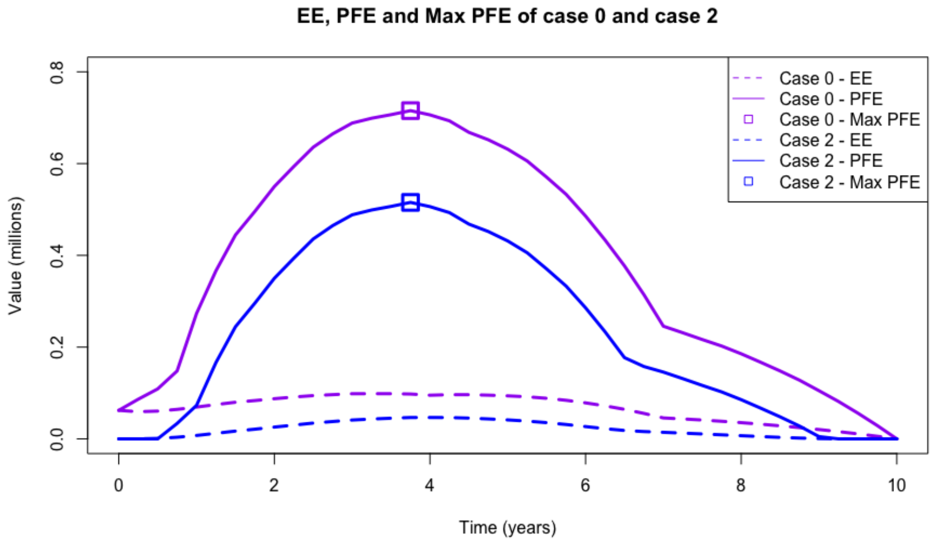

Figure 7: The expected exposure (EE), 97.5% peak exposure (PFE) and maximum peak exposure (Max PFE) for a portfolio using no netting and no collateral (case 0) and with collateral (case 2).

The resulting CVA value of the portfolio with collateral posted on each contract is \$2,239.3, which is much lower than the original CVA value of \$6,442.3, using no netting and no collateral.

4.3 Netting and collateral

With netting and collateral C posted in total, the exposure of a portfolio containing three swaps can be defined as

$$Expoure = \mathbb{E}[max(f_i^1 + f_i^2 + f_i^3 - C, 0)]$$

In this case the netting is first applied, and then a collateral of $C = \$100.000$ is extracted. The results of the expected exposure, peak exposure and max peak exposure using netting and collateral can be seen in figure 8 along with a comparison with a portfolio without collateral and netting. Case 0 shows the base case of a portfolio that uses no netting and no collateral. Case 3 shows the same portfolio where netting has been applied and collateral posted. The expected exposure curve and peak exposure curves have been shifted downwards. The shape of the expected exposure curve changes, and the curve starts at zero, and increases until around year 5.

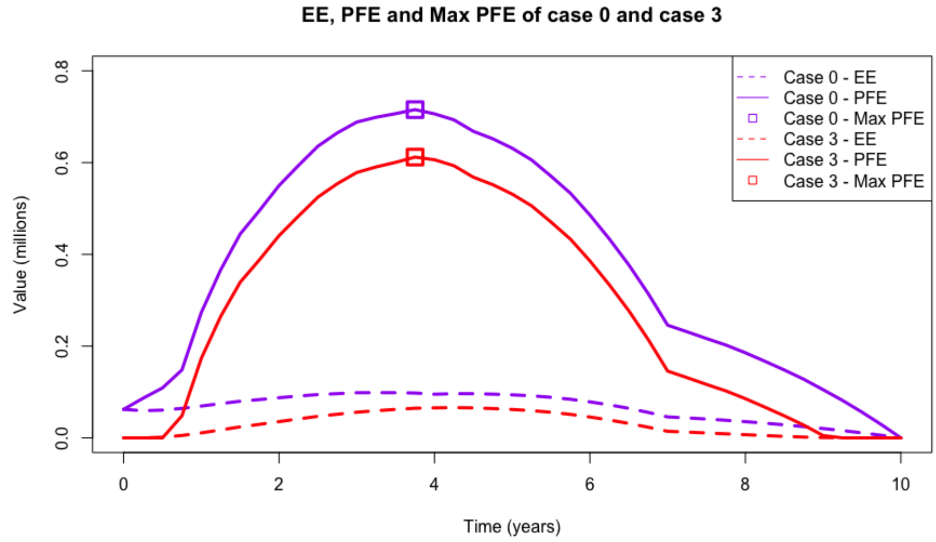

Figure 8: The expected exposure (EE), 97.5% peak exposure (PFE) and maximum peak exposure (Max PFE) for a portfolio using no netting and no collateral (case 0) and with netting and collateral (case 3).

The resulting CVA value of the portfolio with netting and collateral posted on each contract is $3,206.1, close to half of the original CVA value of $6,442.3, using no netting and no collateral.

4.4 Comparison

By observing table 2, it can be seen that the CVA differs with a different implementation of methods to the portfolio. By allowing netting or posting collateral, the CVA for the portfolio gets lower in all cases. This can be a useful way to hedge risk and therefore lowers the exposure the party is facing when entering into multiple contracts.

When netting is allowed but no collateral is posted the CVA is $5,601.6 but when no netting is allowed but collateral is posted for each swap the CVA lowers to $2,239.3. However, it might not be an attractive investment for Morgan Stanley to post collateral of a total of $300,000 as that money would be locked for the sole purpose of being collateral and therefore, Morgan Stanley would not be able to use it for other investing purposes. By lowering the collateral to $100,00 and by allowing netting the CVA is calculated to be $3,206.1 which differs not that much from $2,239.3, when netting is not allowed but the total collateral is 3 times more. It might be more attractive for both parties to have netting allowed and lowering the total collateral. By observing the CVA charges it can be seen that allowing netting and posting collateral changes the CVA significantly and it can be estimated that entering three contracts at once and treating them as a portfolio can be deemed as a far less risky investment than entering them as an individual investment.

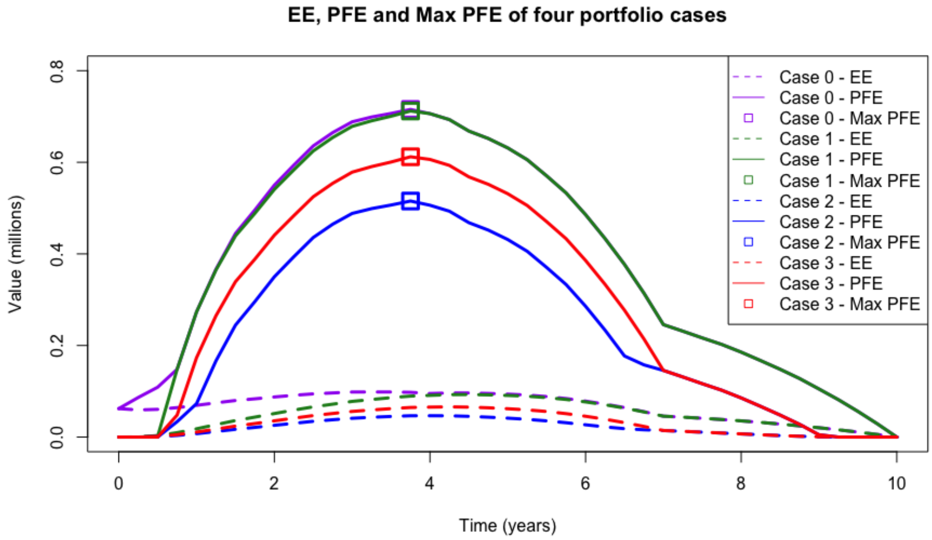

Figure 9: The expected exposure (EE), 97.5% peak exposure (PFE) and maximum peak exposure (Max PFE) for all portfolio cases.

Table 2: CVA of the four different portfolio cases.

Portfolio	**CVA**
Case 0 - No netting, no collateral	\$6,442.3
Case 1 - Netting, no collateral	\$5,601.6
Case 2 - No netting, collateral on each swap	\$2,239.3
Case 3 - Netting, collateral in total	\$3,206.1