

42106 FINANCIAL RISK MANAGEMENT PROJECT 1

Authors:

Arnar Tjörvi Charlesson, s202024 Bjarki Thor Hilmarsson, s202034 Kristín Björk Lilliendahl, s192296 Sverrir Ólafur Torfason, s205636 Teacher: Nina Lange

22.03.2021

Contents

1	Dat	a Presentation	1
2	VaF	R models	5
	2.1	Conversion of foreign stocks to USD-values and using simple Historical Simulation	
		with N of your choosing	5
	2.2	Conversion of foreign stocks to USD-value and using Historical Simulation with	
		probability weighting	6
	2.3	Conversion of foreign stocks to USD-value and Model Based Approach using equal	
		weights for determining the covariance matrix	7
		2.3.1 Backtesting	9
	2.4	Conversion of foreign stocks to USD-value and Model Based Approach using	
		EWMA weights for determining the covariance matrix	10
		2.4.1 Backtesting	11
	2.5	Mapping of portfolio into stock returns and FX returns (total of $10+2$ risk factors)	
		and using Model Based Approach	12
		2.5.1 Backtesting	12
	2.6	Mapping of portfolio into stock returns and market index returns (total of $3+2$	
		risk factors) and using Model Based Approach	14
		2.6.1 Backtesting	15
3	Con	nclusions	16
	3.1	Advantages and disadvantages of each method	16
	3.2	Recommended Model	16

Introduction

In this Financial Risk Management project the team will perform a market risk analysis. A portfolio of 10 different stocks was chosen and different approaches to VaR (Value at Risk) and ES (Expected Shortfall) were calculated in order to recommend the best suitable model going forward. Additionally the potential benefits and shortcomings of the most trustworthy method will be discussed.

1 Data Presentation

Stock data was provided for a number of US stocks, UK stocks and German stocks from January 4, 2010 to December 31, 2020. The exchange rates into USD were also provided. The following analysis will all be performed in USD and the chosen portfolio must contain ten stocks with at least two from each currency. For further analysis the stock market indices for the three countries are also included in the dataset. The portfolio consists of \$10 million dollars, with \$1 million dollars invested in each stock.

The team chose two German stocks, two UK stocks and eight US stocks as seen in table 1.

Stock ticker	Company name	Market (Currency)
BMW.DE	Bayerische Motoren Werke Aktiengesellschaft	Germany (EUR)
DBK.DE	Deutsche Bank Aktiengesellschaft	Germany (EUR)
RSA.L	RSA Insurance Group plc	UK (GBP)
LRE.L	Lancashire Holdings Limited	UK (GBP)
COST	Costco Wholesale Corporation	US (USD)
FDX	FedEx Corporation	US (USD)
IBM	International Business Machines Corporation	US (USD)
XOM	Exxon Mobil Corporation	US (USD)
DIS	The Walt Disney Company	US (USD)
AAPL	Apple Inc.	US (USD)

Table 1: Chosen stocks and their industry groups.

The historical prices of the stocks for the time period January 4, 2010 until December 31, 2020 can be seen in figure 1. In order to be able to display all the stocks in a single graph, the prices of the stock were converted into USD and then scaled by the starting price of each stock. That way the relative prices are in a comparable format. This way the stock of each price starts as 1, and then develops based on the price changes each day.

Project 1 Page 1 / 16

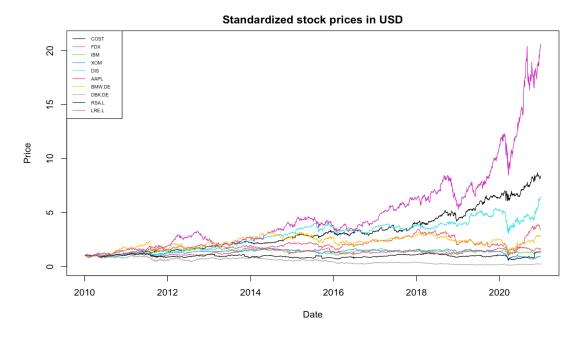


Figure 1: Standardized prices of the chosen stocks.

From figure 1 it can be seen that the price development of the 10 chosen stocks are drastically different. The majority of the stocks seem to have a similar price development, having a final standardized price below 5. However there are three stocks that show a very different trend. AAPL has a final standardized price of around 20, meaning that the price in 2020 is around 20 times higher than it was in 2010. The two other stocks with very high returns are COST and DIS. All three are US stocks so here the standardized price developments reflect only the difference in prices, since all the stocks are listed in USD. Therefore the differences in prices are not caused by any differences in the currency exchange rates.

If a similar strategy is applied only to the German stocks, along with the German Index, figure 2 shows that BMW is outperforming the index, while DBK is underperforming.

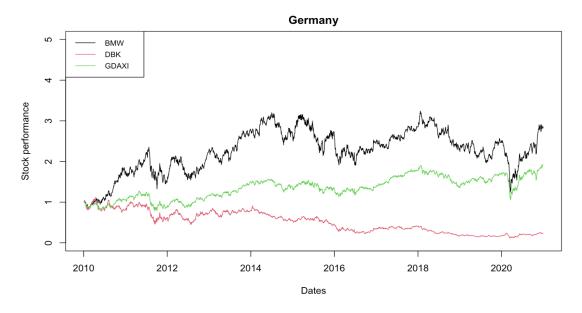


Figure 2: Standardized prices of the chosen German stocks along with the German index.

Project 1 Page 2 / 16

The UK stocks seem to follow the UK index slightly better, especially RSA, which mirrors it closely from 2017-2020. However at the end of 2020, both stocks are over performing the index, after a sharp increase in RSA at the end of the year.

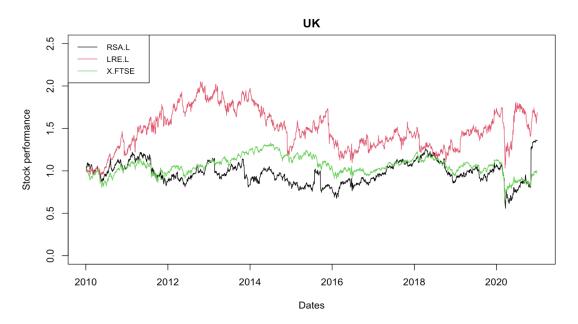


Figure 3: Standardized prices of the chosen UK stocks along with the UK index.

If the currency exchange rates are standardized using a similar method some interesting things become clear as figure 4 shows. From 2010 until 2020, both the EURUSD and GBPUSD currency exchange rates have dropped by close to 20%. This means that for a US investor each EUR or GBP has become cheaper to purchase, So the returns on the foreign stocks are higher because of the differences in exchange rates.

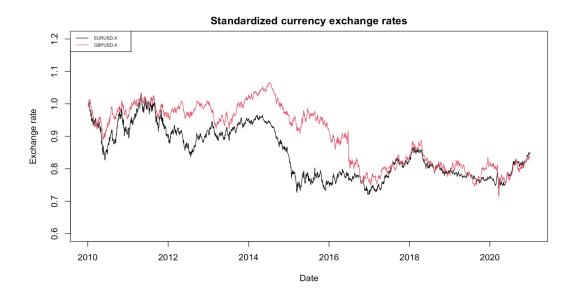


Figure 4: Standardized currency exchange rates.

In table 2 the annualized average historical return and the annualized standard deviation of return is presented for each stock. AAPL has the highest average return, or 38.29% and DBK.DE

Project 1 Page 3 / 16

has the lowest average return or -4.87%. The standard deviations of returns range from 19.57% to 41.26%.

Table 2: Chosen stocks and their annualized returns and standard deviations.

Stock ticker	Average return	Average standard deviation of return
COST	24.47%	19.57%
FDX	17.13%	29.18%
IBM	5.54%	22.68%
XOM	1.87%	23.97%
DIS	23.01%	25.22%
AAPL	38.29%	28.71%
BMW.DE	15.80%	31.36%
DBK.DE	-4.87%	41.26%
RSA.L	7.83%	31.06%
LRE.L	9.24%	27.96%

The correlation between each of the stocks was computed, after the conversion into US dollars, and presented in Figure 5. Many stocks are highly correlated and the highest correlation remains between AAPL and COST with 94.83% correlation. Most of the stocks have a positive correlation which means that the prices of the stocks tend to move in the same direction but some of them do have a negative correlation which means the exact opposite. What might come shockingly is that the two German stocks, BMW and DBK are not that highly correlated but only around negative 36% so their prices tend to move in opposite directions. Same goes for the UK stocks, RSA and LRE but they are very low negative correlated, only around -0.5%.

COST	FDX	IBM	XOM	DIS	AAPL	BMW.DE	DBK.DE	RSA.L	LRE.L
1.000000	0.70791	0.042056	-0.097266	0.890890	0.949835	0.34421	-0.84651	-0.0780271	-0.0299604
0.707907	1.00000	0.129304	0.279356	0.811573	0.682863	0.69372	-0.71000	0.1682934	-0.2151844
0.042056	0.12930	1.000000	0.607679	0.138307	0.015787	0.50459	-0.11761	0.0621373	0.6570299
-0.097266	0.27936	0.607679	1.000000	0.243330	-0.258882	0.66290	-0.16849	0.0363532	0.1584685
0.890890	0.81157	0.138307	0.243330	1.000000	0.785411	0.59242	-0.85404	-0.1166395	-0.0805470
0.949835	0.68286	0.015787	-0.258882	0.785411	1.000000	0.29140	-0.73320	-0.0205681	0.0377892
0.344214	0.69372	0.504595	0.662905	0.592419	0.291399	1.00000	-0.35944	0.0982798	0.2263258
-0.846508	-0.71000	-0.117608	-0.168492	-0.854041	-0.733196	-0.35944	1.00000	0.2175790	0.1678084
-0.078027	0.16829	0.062137	0.036353	-0.116640	-0.020568	0.09828	0.21758	1.0000000	-0.0049508
-0.029960	-0.21518	0.657030	0.158468	-0.080547	0.037789	0.22633	0.16781	-0.0049508	1.0000000
	1.000000 0.707907 0.042056 -0.097266 0.890890 0.949835 0.344214 -0.846508 -0.078027	1.000000 0.70791 0.707907 1.00000 0.042056 0.12930 -0.097266 0.27936 0.890890 0.81157 0.949835 0.68286 0.344214 0.69372 -0.846508 -0.71000	1.000000 0.70791 0.042056 0.707907 1.00000 0.129304 0.042056 0.12930 1.00000 -0.097266 0.27936 0.607679 0.890890 0.81157 0.138307 0.949835 0.68286 0.015787 0.344214 0.69372 0.504595 -0.846508 -0.71000 -0.117608 -0.078027 0.16829 0.062137	1.000000 0.70791 0.042056 -0.097266 0.707907 1.00000 0.129304 0.279356 0.042056 0.12930 1.000000 0.607679 -0.097266 0.27936 0.607679 1.000000 0.890890 0.81157 0.138307 0.243330 0.949835 0.68286 0.015787 -0.258882 0.344214 0.69372 0.504595 0.662905 -0.846508 -0.71000 -0.117608 -0.168492 -0.078027 0.16829 0.062137 0.036353	1.000000 0.70791 0.042056 -0.097266 0.890890 0.707907 1.00000 0.129304 0.279356 0.811573 0.042056 0.12930 1.000000 0.607679 0.138307 -0.097266 0.27936 0.607679 1.000000 0.243330 0.890890 0.81157 0.138307 0.243330 1.000000 0.949835 0.68286 0.015787 -0.258882 0.785411 0.344214 0.69372 0.504595 0.662905 0.592419 -0.846508 -0.71000 -0.117608 -0.168492 -0.854041 -0.078027 0.16829 0.062137 0.036353 -0.116640	1.000000 0.70791 0.042056 -0.097266 0.890890 0.949835 0.707907 1.00000 0.129304 0.279356 0.811573 0.682863 0.042056 0.12930 1.000000 0.607679 0.138307 0.015787 -0.097266 0.27936 0.607679 1.000000 0.243330 -0.258882 0.890890 0.81157 0.138307 0.243330 1.000000 0.785411 0.949835 0.68286 0.015787 -0.258882 0.785411 1.000000 0.344214 0.69372 0.504595 0.662905 0.592419 0.291399 -0.846508 -0.71000 -0.117608 -0.168492 -0.854041 -0.733196 -0.078027 0.16829 0.062137 0.036353 -0.116640 -0.020568	1.000000 0.70791 0.042056 -0.097266 0.890890 0.949835 0.34421 0.707907 1.00000 0.129304 0.279356 0.811573 0.682863 0.69372 0.042056 0.12930 1.000000 0.607679 0.138307 0.015787 0.50459 -0.097266 0.27936 0.607679 1.000000 0.243330 -0.258882 0.66290 0.890890 0.81157 0.138307 0.243330 1.000000 0.785411 0.59242 0.949835 0.68286 0.015787 -0.258882 0.785411 1.000000 0.29140 0.344214 0.69372 0.504595 0.662905 0.592419 0.291399 1.00000 -0.846508 -0.71000 -0.117608 -0.168492 -0.854041 -0.733196 -0.35944 -0.078027 0.16829 0.062137 0.036353 -0.116640 -0.020568 0.09828	1.000000 0.70791 0.042056 -0.097266 0.890890 0.949835 0.34421 -0.84651 0.707907 1.00000 0.129304 0.279356 0.811573 0.682863 0.69372 -0.71000 0.042056 0.12930 1.000000 0.607679 0.138307 0.015787 0.50459 -0.11761 -0.097266 0.27936 0.607679 1.000000 0.243330 -0.258882 0.66290 -0.16849 0.890890 0.81157 0.138307 0.243330 1.000000 0.785411 0.59242 -0.85404 0.949835 0.68286 0.015787 -0.258882 0.785411 1.000000 0.29140 -0.73320 0.344214 0.69372 0.504595 0.662905 0.592419 0.291399 1.00000 -0.35944 -0.846508 -0.71000 -0.117608 -0.168492 -0.854041 -0.733196 -0.35944 1.00000 -0.078027 0.16829 0.062137 0.036353 -0.116640 -0.020568 0.09828 0.21758	1.000000 0.70791 0.042056 -0.097266 0.890890 0.949835 0.34421 -0.84651 -0.0780271 0.707907 1.00000 0.129304 0.279356 0.811573 0.682863 0.69372 -0.71000 0.1682934 0.042056 0.12930 1.000000 0.607679 0.138307 0.015787 0.50459 -0.11761 0.0621373 -0.097266 0.27936 0.607679 1.000000 0.243330 -0.258882 0.66290 -0.16849 0.0363532 0.890890 0.81157 0.138307 0.243330 1.000000 0.785411 0.59242 -0.85404 -0.1166395 0.949835 0.68286 0.015787 -0.258882 0.785411 1.000000 0.29140 -0.73320 -0.0205681 0.344214 0.69372 0.504595 0.662905 0.592419 0.291399 1.00000 -0.35944 0.0982798 -0.846508 -0.71000 -0.117608 -0.168492 -0.854041 -0.733196 -0.35944 1.00000 0.2175790 -0.078027 0.16829 0.062137 0.036353 -0.116640 -0.020568

Figure 5: Correlation between Stocks when converted into US dollars.

Project 1 Page 4 / 16

2 VaR models

The ultimate goal of this analysis is to make VaR and ES predictions at a 95% and a 99% level on the final day of the dataset. In order to be able to recommend a method many different methods had to be proposed an carefully backtested over a year (using the most recent data). Six different methods were tried and each will be addressed in a specific subsection.

2.1 Conversion of foreign stocks to USD-values and using simple Historical Simulation with N of your choosing

The most popular approach for calculating value at risk and expected shortfall for market risk is Historical simulation. The approach involves using historical data as guidance to help make predictions of what might happen in the future. Historical day-to-day changes in values of market variables are used in a direct way to estimate the probability distribution of the change in the value of the portfolio between today and tomorrow.

The first step is to identify the variables that affect the portfolio. These variables typically include exchange rates, interest rates, stock indices, volatilities and they are often referred to as risk factors. The data is then collected on movements in the market variables over the past N days which will essentially provide N-1 alternative scenarios for what can happen between today and tomorrow. For this part of the project, N equals 2686 with corresponding 2685 scenarios.

The first day of the data-set is denoted as Day 0 and the second day is denoted as Day 1 and so on. Scenario 1 is then defined as the percentage change in the values of all variables are the same as they were between Day 0 and Day 1 and then Scenarios 2 is where they are the same between Day 1 and Day 2.

The corresponding currency change in the value of the portfolio between today and tomorrow is calculated which defines the probability distribution for the daily loss in the value of the portfolio and gains are counted as negative losses.

The 99 percentile of the distribution can be estimated as the 1-day VaR where the equation is $N \times (1-99)th$ lowest value and 1-day expected shortfall is the average of the $N \times (1-99)th$ lowest values.

The estimate of VaR is the loss when we are at this 99 percentile point and it can be concluded that with 99% certainty that the portfolio will not suffer a loss greater than the VaR estimate if the percentage changes in market variables in the past N days are representative of what will happen between today and tomorrow. The Expected Shortfall is then defined as the average loss conditional that we are in the 1% tail of the loss distribution, that is, the average of the losses that are worse than VaR. The losses for the N different scenarios are then ranked in order and from there the VaR and ES values can be found from the 99 percentile.

The formula for the Value under ith scenario is the following, where V_i is the value of a market variable on day i and n stands for the day today. The historical simulation approach assumes that the value of the market variable tomorrow will be:

Value under *i*th Scenario =
$$\frac{V_n \times V_i}{V_{i-1}}$$
 (1)

Project 1 Page 5 / 16

For some variables such as interest rates, credit spreads, and volatilities, actual rather than percentage changes in market variables are considered. It is then the equation becomes:

Value under *i*th Scenario =
$$V_n + V_i - V_{i-1}$$
 (2)

The following table shows the results of the one-day Value at Risk (VaR) and Expected Shortfall (ES) using the method.

Since 2686 data points were used then we had to interpoleate on values 26 and 27 for the 99% VaR and for 95% Var we had to interpoleate on values 134 and 135. 2585 Scenarios were created.

Level	Value At Risk (VaR)	Expected Shortfall (ES)
95%	165,562.5	271,105.4
99%	318,520.5	461,040.7

Table 3: The one-day risk calculations.

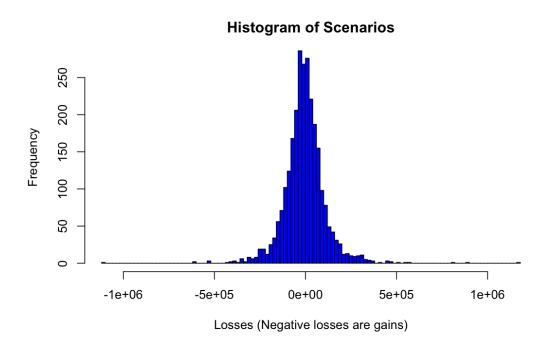


Figure 6: Scenario Losses

2.2 Conversion of foreign stocks to USD-value and using Historical Simulation with probability weighting

The basic historical simulation approach assumes that each day in the past is given equal weight. It has been suggested by several experts that more recent observations should be given more weight as they reflect more accurately the current conditions of the market, for example, macroeconomics and volatilities.

To implement this idealogy to the VaR model the natural weighting scheme is to use weights that decline exponentially. The weight assigned to Scenario 1 is λ times that assigned to Scenario

Project 1 Page 6 / 16

2. This in turn is λ times that given Scenario 3 and so on. The cumulative sum of the weights should add up to 1.

The formula for the implementation of the weights can be seen below, where Scenario is i and n is the total number of scenarios.

$$\frac{\lambda^{(n-i)}(1-\lambda)}{1-\lambda^n} \tag{3}$$

VaR is calculated by ranking the observations from the worst outcome to the best. The weights of the corresponding worst VaR are then summed up until the cumulative sum reaches the required percentile of the distribution. For example, to calculate VaR with a 99% confidence level, the cumulative sum of the weights should be or just exceed 0.01.

The parameter λ can be chosen by using different values and back-test with the value to see which value for λ is best.

One disadvantage of the exponential weighting approach is that the data-set for the portfolio is reduced effectively. This can be compensated for by using a larger data-set.

The following table shows the results of the one-day Value at Risk (VaR) and Expected Shortfall (ES) using the method.

Scenario used for 99% VaR was 10 and for 95% VaR was scenario 62. The 99% ES was then calculated as the average losses from scenario 1 to 10 and for 95% it was average losses from scenario 1 to 62. 2685 Scenarios were created.

 Level
 Value At Risk (VaR)
 Expected Shortfall (ES)

 95%
 245,415
 356,629.2

 99%
 445,869.3
 632,223.6

Table 4: The one-day risk calculations.

2.3 Conversion of foreign stocks to USD-value and Model Based Approach using equal weights for determining the covariance matrix

The model-building approach, also referred to as the variance-covariance approach is an alternative approach to the historical simulations approach for calculating value at risk and expected shortfall. The method involves assuming a model for the joint distribution of changes in market variables and using historical data to estimate the parameters of the model.

The method is tailored to situations where the change in the value of a portfolio is linearly dependent on changes in the values of the underlying market variables, for example, stock prices, exchange rates, indices, interest rates and more. Furthermore, if the daily changes in the underlying market variables are assumed to be multivariate normal then the model is much faster than the historical simulation approach. The approach is fundamentally an extension of the portfolio theory by Harry Markowitz.

When the change in the value of the portfolio is linearly dependent on the changes in the underlying market variables then the probability distribution of the change in the value is normal.

Project 1 Page 7 / 16

Therefore, the mean and the standard deviation of the change in the value of he portfolio can be calculated from the mean and standard deviation of the changes in the market variables as well as the correlations between those changes.

By supposing that the portfolio \mathbf{P} is dependent on \mathbf{n} market variables then it can be supposed that the change in the value of the portfolio is linearly related to the proportional changes in the market variables so that:

$$\Delta P = \sum_{i=1}^{n} \delta_i \Delta x_i \tag{4}$$

 ΔP is the dollar change in the value of the portfolio in one day and Δx_i is the proportional change in the *i*th market variable in one day. The parameter, δ_i is a variation on delta risk measure. The delta of the position w.r.t. a market variable is normally defined as

$$\delta_i = \frac{\Delta P}{\Delta S} \tag{5}$$

where ΔS is a small change in the value of the market variable when all other market variables remain the same and ΔP is the resultant change in the value of the portfolio.

If Δx_i is multivariate normal and ΔP is normally distributed then only mean and standard deviation of Δ P is needed to calculate value at risk. If the expected value of each Δx_i is zero then it implies that the mean of ΔP is zero. σ_i is the standard deviation of Δx_i and $\rho_{i,j}$ is correlation coefficient between Δx_i and Δx_j . Since only one day is considered at then σ_i is the volatility per day. The standard deviation of ΔP is then given by the following formula:

$$\sigma_P = \sqrt{\sum_{i=1}^n \sum_{j=1}^n \rho_{ij} \delta_i \delta_j \sigma_i \sigma_j}$$
 (6)

Another way of writing the formula is:

$$\sigma_P^2 = \sum_{i=1}^n \sum_{j=1}^n cov_{ij} \delta_i \delta_j \tag{7}$$

Where cov_{ij} is the covariance between Δx_i and Δx_j and by using matrix notation the formula above becomes:

$$\sigma_P^2 = \delta^T C \delta \tag{8}$$

Where δ is the vector whose ith element is δ_i , C is the variance-covariance matrix and δ^T is the transpose δ . The formula to calculate VaR with X% confidence level is:

$$VaR_{X\%} = N^{-1}(X)\sigma_P \tag{9}$$

and the formula for expected shortfall with X% confidence level is:

$$ES_{X\%} = \sigma_P \frac{e^{-Y^2/2}}{\sqrt{2\pi(1-X)}} \tag{10}$$

where Y is $N^{-1}(X)$ and stands for the inverse cumulative distribution. To calculate standard deviation, VaR or ES for a T days horizon then simply multiply them with \sqrt{T} .

Project 1 Page 8 / 16

Table 5: The one-day risk calculations using the model building approach with equal weights.

Level	Value At Risk (VaR)	Expected Shortfall (ES)
95%	187,824.7	235,539.8
99%	265,644.1	304,339

2.3.1 Backtesting

The backtesting was done by calculating a daily VaR value over a year. We want to see how often our model was violated

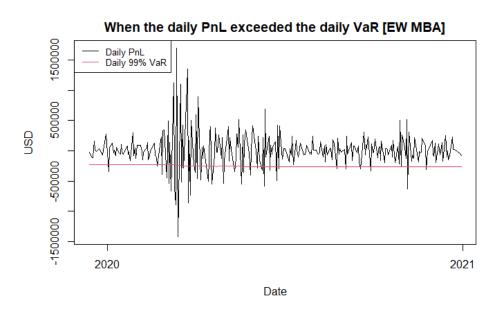


Figure 7: PnL vs 99% VaR over a year

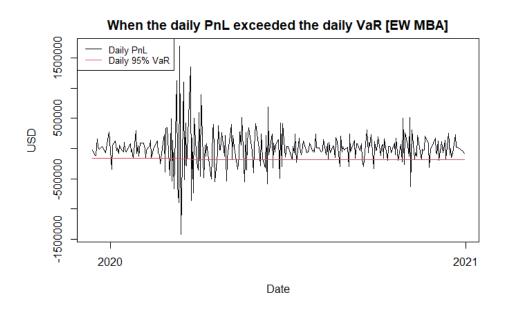


Figure 8: PnL vs 95% VaR over a year

Project 1 Page 9 / 16

It is obvious from observing the figures above that the exceedances are coming in clusters and therefore a preliminary observation would lead us to believe that the model is not accurate but we will to a statistical analysis as well.

In order to analyze our backtesting then we used the asymptomatic distribution of the LR_{UC} test statistic.

$$LR_{UC} = \frac{\pi_{exp}^{n_1} (1 - \pi_{exp})^{n_0}}{\pi_{obs}^{n_1} (1 - \pi_{obs})^{n_0}}$$
(11)

Where π_{exp} is the expected proportion of exceedences, π_{obs} is the observed proportion of exceedences, n_1 is the number of exceedences and $n_0 = n - n_1$

And then we check $-2ln(LR_{UC})$. For 99% VaR it should be lower than 2.71 and lower than 3.84 for 95% VaR.

We see from those figures that this year was obviously a very turbulent time in the market and that the VaR levels were exceeded many times. The 99% VaR was violated 36 times and the 95% VaR was violated 46 times. That gives us a LR_{UC} value of 129.21 for the 99% VaR and 57.22 for the 95% VaR. Which means we can confidently reject.

2.4 Conversion of foreign stocks to USD-value and Model Based Approach using EWMA weights for determining the covariance matrix

Here we will do an extension of the MBA model. In the former model there was equal probability of events for every observation. Now we will put more weight on more recent observations. By this extension the co-variance matrix we use for the model becomes:

$$\Sigma_n = \lambda \Sigma_{n-1} + (1 - \lambda) \mathbf{r}'_{n-1} \mathbf{r}_{n-1}$$
(12)

The following table shows the results of the one-day Value at Risk (VaR) and Expected Shortfall (ES) using the method where $\lambda=0.94$.

Table 6: The one-day risk calculations.

Level	Value At Risk (VaR)	Expected Shortfall (ES)
95%	190,386.9	238,752.9
99%	269,267.9	308,490.6

Project 1 Page 10 / 16

2.4.1 Backtesting

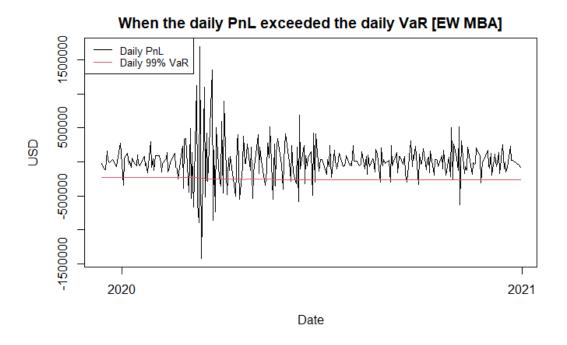


Figure 9: PnL vs 99% VaR over a year

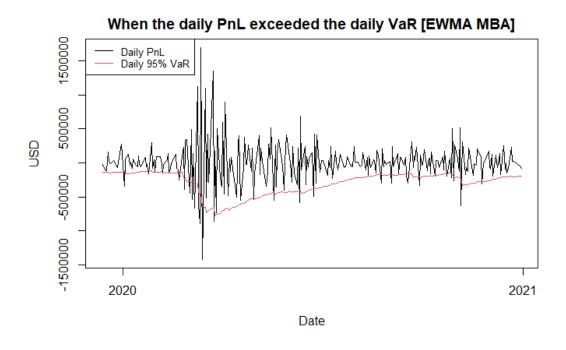


Figure 10: PnL vs 99% VaR over a year

We get here that the PnL breaks out of the VaR levels 14 times for the 99% VaR and 28 times for the 95% VaR which leads to LR_{UC} 5.87 at 99% and 5.2 at 95%. The exponential weights seem to be having a great impact on the model since it's quicker to react to the dramatic changes in stock prices during the start of the pandemic. But it's still too high and we must reject.

Project 1 Page 11 / 16

2.5 Mapping of portfolio into stock returns and FX returns (total of 10+2 risk factors) and using Model Based Approach

Mapping is an extension to the Model Based Approach, since that can become tricky, when the portfolio is complex and the covariance matrix becomes large and difficult to estimate. By mapping the portfolio into fewer risk factors, it is easier to estimate and additionally it can be easier to see where the risk in the portfolio is coming from, as the risk factors can correspond to the components of the risk of the portfolio.

The method used here involves mapping the portfolio into 12 different risk components, the stocks themselves and then the currency or FX factors. Since the stocks are 10, the stock risk factors will be 10 as well, one for each stock. There are two currency exchange risk factors, GBP-USD and EUR-USD, so there is a total of two currency risk factors.

If a group of assets is closely related to the same factor, then the portfolio value change can be described in terms of chosen risk factors and the exposure to each risk factor in the following way:

$$\Delta P = \sum_{i=1}^{n} w_i r^i \tag{13}$$

where w_i describes the exposure to the risk factor i and r^i the return on risk factor i.

Here r^1 to r^{10} will be the risk factors indicated by the returns of the ten stocks, r^{11} will be the return on EURUSD and r^{12} the return on GBPUSD. The weights for each stock risk factor are the same \$1.000.000, while the weights of the FX risk factors will both be \$2.000.000 since the portfolio is constructed of two German stocks, two British stocks and six American stocks.

The weights in the risk factors and the returns can then be combined and the portfolio variance can be found as in formula 8. Then the VaR and ES can be calculated as in formulas 9 and 10.

Table 7: The one-day risk calculations using the mapping method.

Level	Value At Risk (VaR)	Expected Shortfall (ES)
95%	187,851	235,572
99%	265,681	304,381

2.5.1 Backtesting

For the backtesting of the method, a one-day VaR was computed daily for the last year of the dataset. The results can be seen in figure 11. The VaR starts at around -165,000 but with increased volatility in markets around Covid, it decreases to around -185,000.

Project 1 Page 12 / 16

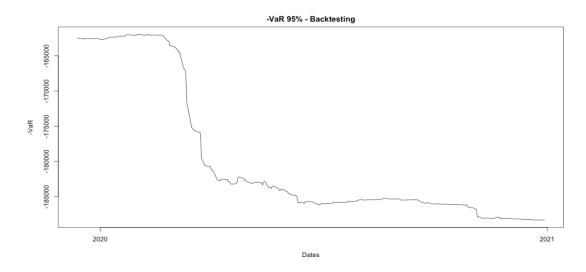


Figure 11: Computing one-day 95% VaR for the last year.

Then the daily Profit-and-Loss (PnL) of the portfolio for the same year was computed and compared to the negative VaR values. Whenever the portfolio loss is greater than the VaR for the same day, the lines will cross.

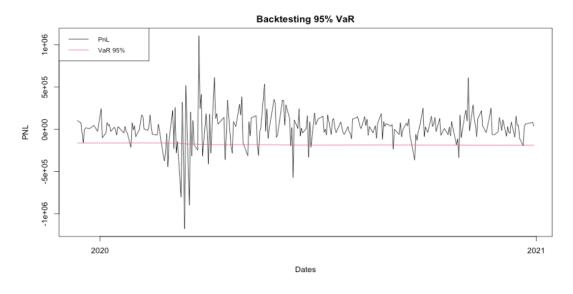


Figure 12: Backtesting 95% VaR for the last year.

From figure 12 it can be seen that the PnL values are of such a dramatically different scale than the VaR values, that the VaR line seems to be a straight line. Even when figure 11 showed that not to be the case.

The expected number of exceedances, based on the sample size should be around 13 times for the 95% VaR, around 2-3 times for the 99% VaR.

For the 95% VaR this method exceeds the VaR limit 29 times, which leads to a LR_uc value of 16.7. This means that we can with reject at a very low level, and conclude that the number of exceedances is very high.

Similarly for the 99% VaR, figure 13 shows that the number of exceedances are very high.

Project 1 Page 13 / 16

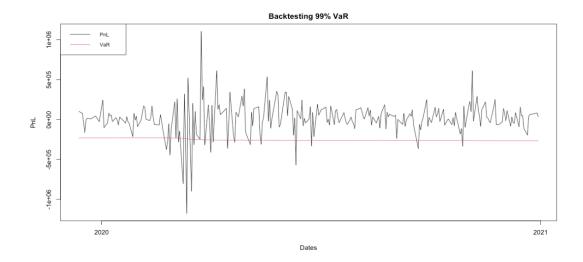


Figure 13: Backtesting 99% VaR for the last year.

For the 99% VaR, this method exceeds the VaR limit 20 times, which leads to a LC_uc value of 49.2. This means that we can confidently reject that this is a good VaR model, since the number of exceedances is way to high. For a 99% VaR limit, it should only exceed the risk limit around 2-3 times a year.

2.6 Mapping of portfolio into stock returns and market index returns (total of 3+2 risk factors) and using Model Based Approach

Similar to the mapping method in the previous section, the mapping will now be done with fewer risk factor. Here the stocks will all be mapped to the appropriate indices along with their currencies. So the American stocks will be mapped to the American Index, DJI, the German stocks to the German index, GDAXI, and the British stocks to the British Index, FTSE.

The method used for mapping the stocks to the corresponding indices is similar as before but now the historical returns were computed and regressed on the chosen risk factors to obtain a suitable description of the portfolio.

From the linear regression the weights are computed and used in a similar way to the previous section. Since there is now a total of 5 risk factors, the covariance matrix is a 5x5 matrix, instead of the 12x12 covariance matrix before.

The following table shows the results of the one-day Value at Risk (VaR) and Expected Shortfall (ES) using the method.

Table 8: The one-day risk calculations.

Level	Value At Risk (VaR)	Expected Shortfall (ES)
95%	167,875	210,522
99%	237,428	272,013

Project 1 Page 14 / 16

2.6.1 Backtesting

The backtesting was done in a similar way as in the previous section. The VaR values for the last year for the 95% VaR varied between 142,417 and 169,162 while the 99% VaR varied between 201,423 and 239,249.

The daily Profit-and-Loss (PnL) of the portfolio for the year was computed and compared to the negative VaR values. Whenever the portfolio loss is greater than the VaR for the same day, the lines will cross.

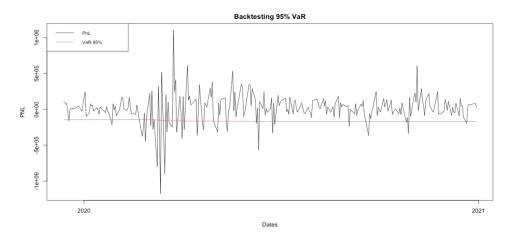


Figure 14: Backtesting 95% VaR for the last year.

from figure 14 it can be seen that the 95% VaR is exceeded multiple times. The expected number of exceedances, based on the sample size should be around 13 times. However, in reality the number of exceedances for the last year was 32 times. Based on the results of the LR_uc statistical test, 22.5, we can therefore confidently reject that the model has a 5% probability of exceedance.

The same method was used for backtesting the 99% VaR model. Here the expected number of exceedances is around 2-3 times, given the one year sample size used. The actual number of exceedance is 24 times, which leads to a LR_uc value of 67.1. If the value is above 3.84, we can reject at a 5% level.

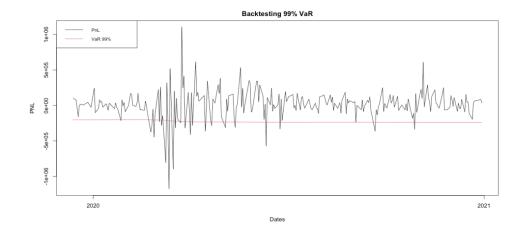


Figure 15: Backtesting 99% VaR for the last year.

Project 1 Page 15 / 16

3 Conclusions

COVID-19 happened in late 2019 and started progressing in early 2020 which had major impacts on all businesses and therefore, the stock markets. Many companies declined since there was now a lot of uncertainty in the whole world society and many governments were starting to put on lockdowns and restrictions in the society to counter the further spread of the virus. That meant many businesses could not operate as usual and after some time with these restrictions then many companies ended up going bankrupt. As can be seen by all of the value at risk backtesting graphs, then there is a major violation during early 2020 which can be traced directly to the COVID-19 effect on the market.

3.1 Advantages and disadvantages of each method

With Model-building approach, results can be produced very quickly and can be easily used in conjunction with volatility and correlation updating. But the quick results can only be produced when the change in the portfolio is linearly related to the proportional or actual changes in the risk factors are assumed to have multivariate normal distribution. On other hand Daily changes in risk factors often have distribution that are different from normal. This model is also known as variance-covariance approach. The advantage to the Model-Based-Approach is that its a lot faster than Historical Simulation and by using the EWMA extension then we are taking putting heavier weight on more recent data which reflects more the current market conditions.

However with the Historical simulation, volatility changes can be incorporated into this method but rather in more artificial way. This approach is most frequently used by organisations. As the name suggests, we consider daily changes in historical values to compute the likelihood of the variations in values of current portfolio between given time frame. The other advanced version of this model places more emphasis on recent observations. The key assumption in historical simulation is that the set of possible future outcomes is fully represented by what occurred in a definite historical time frame. One disadvantage to the Historical Simulation approach is that it is time-consuming.

The mapping method for model-based-approach is convenient when you have a large data-set as the covariance-variance matrix would be really big and by using mapping then you're reducing the size of that matrix significantly, dimension redundancy.

3.2 Recommended Model

Based on the generated backtesting over the year of 2020 then we can see the importance of modelling with more focus on recent data. Since volatility tends to cluster. Our results lead us to recommend the exponentially weighted moving average version of the model building approach. That model captured the stockmarket crash at the start of covid quickly and that model would then inform management more accurately about their risk exposure than a model that would put the same weight on data that's many years old and irrelevant to what's going on in the markets today.

Project 1 Page 16 / 16