Exam project in Optimization Modelling (42112)
DTU 2022

1 What you have to do

1. The assignment can be solved in groups of one or two students
2. The assignment consists of two parts:
   - Handball Referee Assignment
   - Zenith Chemicals Assignment
3. If you are in a group of two students, for each assignment one of you is responsible for the model (mathematically) and one for the text, and the roles should be different for the two parts of the assignment. Group members are jointly responsible for the Julia code. It should be clearly stated in the report who is responsible for the model and who is responsible for the text.
4. The assignment must be handed in online no later than **21/1, 11:59 pm** via DTU Learn.

Both assignments can, and should be modelled as MIP (linear!) models. The difficulty of the questions gradually increases within each of the two parts. For all questions, specify the model and you answer (and possibly any additional information that is requested). The description of the models (i.e. objective function and constraints) is important; however, you only need to introduce and describe the new components of the model when solving a particular question. It is not necessary to include the data tables in the report. You should just refer to these in the report as needed. Please remember that **all** parts of the model must be described. This includes Sets/indexes, data, variables, objective function(s), and constraints.

Handball Referee Assignment

Handball is the most popular sport in Denmark, after football, and was, in fact, invented in Denmark (Germans may disagree, but they are wrong!). This assignment focuses on optimizing the assignment of referees to handball matches. The data used in this assignment (more in the appendix) is based on a real case for some of the lower handball divisions in Denmark.

Two referees must be assigned to officiate (i.e. referee) any match. In the division, there are 31 teams (indexed from 1-31), and the season contains 198 matches (indexed from 1 to 198). These are played in 70 different arenas (indexed from 1 to 70). The matches are spread across 51 dates (indexed from 1 to 51). Each match is played in one of the 70 arenas. Which matches are played in which arenas has been predetermined. The referees that are selected to officiate a match must travel to the given arena and officiate the match between the home team and the visiting team. All tournaments in Danish handball are double round robin tournaments. This means that each team in the tournament meets every other team twice during a season, one match at each team's home arena.

There are some basic requirements that any feasible referee schedule must satisfy. These are listed below:

- Two referees must be assigned to each match.
- A referee can only officiate one match a day.
- The referees have jobs and can hence not officiate on all days. The days when referees are not available are given in the data.

Assignment H1

Formulate an initial mathematical model that can be used to find a referee schedule that minimizes the total distance travelled by the referees. The distances given in the data ($referee\_arena\_distances_a^r$) are round trip distances (i.e., these include the distance to and from the arena). Solve your model in Julia/JuMP

Answer H1: An optimal referee schedule that minimizes the distance traveled, the MIP model, and the Julia program used to obtain this result.

Assignment H2

After inspecting your solution to H1, you note that there is quite some variation in the number of matches each referee officiates. You have been asked to see if it is possible to come up with a referee schedule in which the referee with the most matches officiates at most 5 more matches than the referee with the least matches. Adjust your formulation so that adheres to this request. What is the total distance of the schedule (if such a schedule exists!)

Answer H2: An optimal referee schedule that minimizes the distance travelled, the modified MIP model, and the Julia program used to obtain this result.

Assignment H3

Ignore the schedule balance modification that you made in H2. The division organizers have received complaints by some referees regarding the distances they have to travel. You have been asked to investigate how much the total distance can be reduced by making modifications to the underlying match day allocation. If matches are allowed to be moved within a 14 day window of the currently allocated match day (i.e 7 days ahead of and 7 days later than the current day) but not more than 20 matches can be moved, what reduction in total is distance is possible? Note that teams can only play one match a day

Answer H3: An optimal referee and match schedule that minimizes the total distance travelled, the modified MIP model, and the Julia program used to obtain this result. State also the matches that get rescheduled.

Assignment H4

Ignore the problem extensions in parts H2 and H3. Since officiating a handball match is a joint effort, the referees are paired together into preferred teams. This is, however, not always possible due to the availability of each referee, sometimes two referees from two different pairs are assigned to officiate a game together.

This in fact leads to two different objectives functions: total distance travelled and the maximum number of games where established teams officiate. To get a feeling for what is better, you must now find the two **lexicographic** optimal solutions: First find the best solution according to one objective, then find the best solution to the second objective, but ensure that the first objective is **not** worsened. Complete the following:

a) Find the lexicographic optimal solution that first minimizes the total distance traveled and then maximizes the number of matches with preferred referee teams.

b) Find the lexicographic optimal solution that first maximizes the number of matches with preferred referee teams and then minimizes distance travelled

Answer H4: Both lexicographic optimal solutions, i.e. two solutions, one that minimizes the total distance travelled and one that maximizes the number of games with preferred referees, two MIP models for each question (it may be a good idea to write two models for each question although this is not strictly necessary), and four working Julia programs, two for each question.

Assignment H5

Beside the wishes to minimize distance and maximize number of preferred teams refereed games, there are also some extra requests, i.e. things which characterize a good plan. These are state below:

- Preferably, a referee must not officiate the same team more than 3 times during a season.
- Preferably, a referee must not officiate the same team in two consecutive rounds.

The idea with these limitations is to make the referees more impartial. This is difficult if they officiate the same team too many times during the season. If a team has the same referee two matches in a row, some of the players may be angered by (what they consider to be) wrong judgements by the referee in the previous match. It is assumed that this is only a problem for the teams and that the referees can officiate the same team in a row without a problem.

Minimize the total number of times the teams are refereed by the same referee more than 3 times and the number of times the same referee is used twice in row for the same team. As an example, if a referee officiates the same team five times during the season, a "cost" of 2 would be incurred in the first component of the objective.

Regarding the two objective functions from H4, add constraints that ensure that the distance travelled is at most 35% worse than the best solution to H4a and at least 70% of the number of matches given in the solution to H4b are refereed by referee pairs.

Answer H5: The optimal referee schedule, (i.e., one that minimizes the number of times where a referee officiates the same team more than 3 times during the season plus the number of times a team is refereed two times in a row by the same referee), the MIP model, and the Julia program used to obtain this result.

2 Appendix

Data for the handball-referee problem can be found in the file: "handball_data.jl". This file contains the definition of a number of Julia vectors:

- referee_arena_distances : Float64 two-dimensional vector giving the distance from referee (first index) (home) to sports arena (second index)
- match_arena : Bool two-dimensional vector, true if match (first index) takes place at arena (second index)
- match_time : Bool two-dimensional vector, true if match (first index) takes place at time (second index)
- ref_pair : Bool two-dimensional vector, true if two referees are part of a pair
- ref_not_available : Bool two-dimensional vector, true if referee (first index) not available at time (second index)
- team_match : Bool two-dimensional vector, true if team (first index) play in match (second index)

Zenith Chemicals

Zenith Chemicals produces a wide range of industrial chemicals at their various plants in Louisiana, Texas, Missouri and Delaware. Three major lines of chemical products are produced at their eighteen month old St. Louis facility. These chemical products, which share common raw materials as well as production line capacity, are sold in bulk on the wholesale market in the Midwest and South. Packaging of the products into retail consumer sizes is done by the individual wholesale companies under their own product labels. Zenith does not sell these products directly to the retail market.

The St. Louis facility has limited storage capacity for bulk finished product. The chemical products can be shipped to the wholesale customers immediately after production, or they can be stored in a bulk storage area for future shipments. All product shipments are made by rail at the expense of Zenith Chemicals. Due to current industry practices, Zenith charges the same price to all wholesale customers. As a consequence of this policy, Zenith must inflate their product selling prices to reflect the average cost of shipping.

Zenith Chemicals schedules their St. Louis production levels based on a four month estimation of demand levels, revenues and costs. Zenith believes that its demand estimates are relatively accurate, but certainly not entirely accurate, over the planning horizon.

The next planning horizon under consideration covers the period from May 1, to August 31 (i.e. 4 months).

Production

The three chemical final products will be referred to in this document simply as Alpha, Beta and Delta. For our purposes, the products will be measured in in units of equal volume. These units will be denoted as DCM's (i.e., ten cubic meters.) In reality, Zenith must measure the products in terms of their weight as well as their volume.

Two major raw materials are used in the production of the three chemical products at the St. Louis plant. These raw materials, known as TSP and HCMD, will also be measured in DCM units. TSP, which is used in a wide range of applications, is produced by Zenith Chemicals at three of their chemical plants in New Orleans. TSP is delivered to the St. Louis facility by barge at a rate of 183,000 DCM/month.

The second major raw material, HCMD, is purchased from a variety of industrial chemical producers. Current HCMD deliveries are made at a rate of 212,000 DCM per month. Zenith Chemicals does not have the capability for long term storage of either raw material. Instead, the material remains in the delivery vehicle (i.e., barge or hopper (rail) car) until consumed. Although it is beyond the scope of this project, Zenith is currently examining a network optimization model to aid them in their scheduling of raw material deliveries to St. Louis. The usage of raw materials is specified in Table 1 below.

	TSP	HCMD
alpha	15.0	31.5
beta	27.0	18.0
delta	22.0	23.5

Table 1: Raw material needed (DCM)

The chemical products are processed on two types of machines, blending and grating. There are 36 identical blending machines and 20 grating machines at the St. Louis plant. We will assume that the machines run 8 hours all days of the month, except the first day, which is used for cleaning. Since it takes time to clean the machines, when shifting from production of one type of final product to another type. Hence it is decided that each month, a machine can only produce one type of product. The first day of the month will then be used to clean the machines, the remaining days for production. The amount of production time needed for each DCM of final product on each machine is given below in Table 2.

Product	Blending	Grating
Alpha	1.25	1.00
Beta	2.50	3.50
Delta	1.75	0.75

Table 2: Required process time for each dcm of final product per final product (hour/DCM)

The combined production costs of producing the products are given in Table 3. The monthly cost of storage in the facility is \$0.70 per DCM of Alpha, \$1.25 per DCM of Beta and \$1.85 per DCM of Delta. It is projected that there will be 100 DCM of each product in storage on May 1. Likewise, it is desirable to have at least 100 DCM of each product in storage at the end of the four month planning horizon. At most 2500 DCM of final products can be stored, i.e. the total storage capacity for all products is 2500 DCM. We use ultimo storage in this assignment, i.e. the storage use is measured in the end of a month.

	May	June	July	August
ALPHA	10.00	11.00	12.00	13.00
BETA	17.00	17.00	19.50	19.50
DELTA	15.50	18.00	18.00	15.50

Table 3: Production costs for finished products ($ per dcm)

Product Pricing

The demand for Zenith's three major industrial chemicals is dependent on a number of interrelated factors. Foremost among these are the selling prices charged by Zenith and the selling prices charged by Zenith's competitors for similar products. At the present time, Zenith Chemicals charges $28 per DCM for the Alpha product, $52.50 per DCM for Beta and $42.75 per DCM for Delta. For planning purposes, Zenith has assured its wholesale clients that these selling prices will remain in effect through the months of May and June. Historically in the chemical industry, product price increases are announced in either January or July. Although Zenith Chemicals has not yet notified its customers, the current plans call for selling prices to increase on July 1 to $41.00 per DCM for Alpha, $57.75 per DCM for Beta, and $47.00 per DCM for Delta. These scheduled increases reflect Zenith's best estimation of the market and the competition.

Demand Forecasts

By their very nature, demand estimations are subject to error. The following tables provide Zenith's best estimation of demand for its chemical products over the four months under consideration. These demand estimates are based on the current and planned selling prices presented in the previous section. Management is relatively confident that these figures are accurate.

Although Zenith sells its industrial chemical products to over fifty major wholesale clients in the Midwest and South, the data given here will be aggregated into four principal demand zones to simplify our analysis. The demand for the Alpha product is given in Table 4, for the Beta product Table 5 and the Delta product Table 6.

Demand	May	June	July	August
Region A	500	600	700	400
Region B	800	950	1000	750
Region C	250	300	250	200
Region D	500	600	500	400
Total	2050	2450	2450	1750

Table 4: Estimated Alpha demand (dcm)

Zone	May	June	July	August
Region A	250	250	200	200
Region B	400	400	400	400
Region C	150	125	100	75
Region D	275	250	225	225
Total	1075	1025	925	900

Table 5: Estimated Beta demand (dcm)

Transport costs

The costs of transporting finished products to the regions is given in Table 7

Zone	May	June	July	August
Region A	1000	1200	1500	1900
Region B	1600	1750	2100	2450
Region C	650	800	1000	1600
Region D	1000	1000	1500	2000
Total	4250	4750	6100	7950

Table 6: Estimated Delta demand (dcm)

	May	June	July	August
ALPHA	4.50	3.75	5.75	4.25
BETA	12.00	10.00	13.00	12.00
DELTA	7.40	7.00	7.75	7.25

Table 7: Transport costs for finished products ($ per dcm)

Assignment Z1

Formulate a MIP model that can be used to maximize Zenith Chemical's profits.

Answer: The MIP model, in mathematical notation. Also report the total profit and tables for each final product showing which machines are used for the production in each month. And the julia program which implements the MIP model.

Assignment Z2

It takes time to transport the final products to the regions (by train or barge), hence it is decided that products of the current month cannot be sold, only what is on storage from the previous month can be sold this month. Furthermore, each of the 36 blending machines and 20 grating machines needs to be operated by a human operator. There are 4 machine operators employed at Zenith. The operators can handle both types of machines, but one operator can at most handle 3 blending machines and at most 2 grating machines, and only one type of machine at a time. Extend the model from Z1 to the new requirements. Make a table of how many machines are operated in each month and how many operators are operating which machines in each month.

Answer: The modified MIP model, in mathematical notation. Also report the total profit and how many machine operators are employed

for each month. And the julia program which implements the MIP model.

Assignment Z3

Zenith is considering to hire more operators. Starting with four machine operators in May, Zenith wants to investigate if it pays off to hire more. Each operator has a monthly cost of \$3000. Due to local union agreements, once a person is hired, this person cannot be sacked (at least not in this 4 month period), hence once hired in one month, that person has to stay for the entire time-period.

Answer: The modified MIP model, in mathematical notation. Also report the total profit and the number of operators to hire and when. And the julia program which implements the MIP model.

Assignment Z4

To get a more precise production plan, and hiring plan, the market research unit has made 8 scenario estimates of how the demand is going to develop. We will assume that all scenarios are equally likely. The data is described below in Table 8.

	May	June	July	August
1	1	0.343	0.125	0.064
2	1	0.343	0.125	0.216
3	1	0.343	0.729	0.512
4	1	0.343	0.729	1
5	1	2.197	1.331	1
6	1	2.197	1.331	1.728
7	1	2.197	3.375	2.744
8	1	2.197	3.375	4.096

Table 8: Demand scenario factor, i.e. the factor change of the demand pr. scenario pr. month.

Create a MIP model to maximize the expected (multi-stage stochastic) profit

Answer: The modified MIP model, in mathematical notation. Also report the profit in each scenario. And the julia program which implements the MIP model.

Assignment Z5

The management look at the profit, which is satisfactory, but they are less satisfied with the variation in the results for the 8 scenarios. Hence, they want to look for production plans which may be less profitable, but where the variation is smaller, that is, where the scenarios with low profit are better. To do this, they want to see a list of production solutions where the worst scenario is only a factor $0.5 \leq K \leq 1.0$ worse than the average over all the scenarios (the objective). They want to see the result for the K values: $0.5, 0.6, 0.7, 0.8, 0.9, 1.0$.

Answer: he modified MIP model, in mathematical notation. For each K value, report the optimal objective value for each scenario. And the julia program which implements the MIP model.