42112 Final Assignment Report

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Assignment Handball

0.1 Defining the Indexes

- M is a set of of matches, M = [1,198], $m \in M$
- R is a set of of referees, R = [1,52], $r \in R$
- A is a set of of arenas, A = [1,70], $a \in A$
- D is a set of of different dates when matches take place, D = [1,51], $d \in D$
- T is a set of of teams in the league, T = [1,31], $t \in T$
- $MatchArena_{m,a}$ is a set of of all of the games and in which area they take place in. True for the games that take place and false otherwise.
- $RefereeArenaDistances_{r,a}$ is a matrix of all the referees and their distance to each arena.
- $MatchTime_{m,d}$ is a set of of all matches and the date when the matches take place. True for dates when matches are played otherwise false.
- $RefPair_{r,r}$ represents the preferred referees pairs of two, True for referees that are pairs otherwise false.
- $RefNotAvailable_{r,d}$ is a set of of all the referees and the dates when they are not available for a game. True for the dates when they are not available and otherwise false.
- $TeamMatch_{t,m}$ is a set of of all the teams in the league and their matches. True for matches when they play otherwise false.

1 Assignment H1

Bjarki: Julia Code
Sverrir: Text

Minimizes the total distance travelled by the referees

1.1 Decision variables

- $x_{m,r} \in \{0,1\}$: $x_{m,r} = 1$ if the referee is assigned to the specific game otherwise 0.

1.2 Model

The model is:

$$
\begin{array}{lll}
\text{minimize} & \displaystyle\sum_{m=1}^{198}\sum_{r=1}^{52}\sum_{a=1}^{70} x_{m,r} \cdot MatchArena_{m,a} \cdot RefereeArenaDistance_{r,a} & \\
\text{subject to} & & \\
(1) & \sum_{r=1}^{52} x_{m,r} = 2 & \forall m \in \{1, M\} \\
(2) & \sum_{m=1}^{198} x_{m,r} \cdot MatchTime_{m,d} \leq 1 & \forall d \in \{1, D\}, r \in \{1, R\} \\
(3) & 1 - RefNotAvailable_{r,d} \geq MatchTime_{m,d} \cdot x_{m,r} & \forall d \in \{1, D\}, \\
 & r \in \{1, R\}, m \in \{1, M\} &
\end{array}
$$

Objective Value = $\underline{7{,}269}$

Constraints
(1) Two referees must be assigned to each match.
(2) A referee can only officiate one match a day.
(3) The referees have jobs and can not officiate on all days.

2 Assignment H2

Minimizes the total distance travelled by the referees

2.1 Decision variables

- $x_{m,r} \in \{0, 1\}$: $x_{m,r} = 1$ if the referee is assigned to the specific game otherwise 0.

2.2 Model

The model is:

$$
\begin{aligned}
&\text{minimize} && \sum_{m=1}^{198}\sum_{r=1}^{52}\sum_{a=1}^{70} x_{mr} \cdot MatchArena_{m,a} \cdot RefereeArenaDistance_{r,a} \\
&\text{subject to} && \\
&(1) && \sum_{r=1}^{52} x_{m,r} = 2 && \forall m \in \{1, M\} \\
&(2) && \sum_{m=1}^{198} x_{m,r} \cdot MatchTime_{m,d} \leq 1 && \forall d \in \{1, D\}, r \in \{1, R\} \\
&(3) && 1 - RefNotAvailable_{r,d} \geq MatchTime_{m,d} \cdot x_{m,r} && \forall d \in \{1, D\}, \\
& && r \in \{1, R\}, m \in \{1, M\} \\
&(4)\underline{(New)} && \sum_{m=1}^{198} x_{m,r} - 5 \leq \sum_{m=1}^{198} x_{m,r2} && \forall r \in \{1, R\}, r2 \in \{1, R\}
\end{aligned}
$$

Objective Value = <u>14,738</u>

New Constraint

(4) Referee with the most matches, can officates at most 5 more matches than the referee with the least matches

3 Assignment H3

We ran out of time.

4 Assignment H4

4.1 A

- The problem extensions in parts H2 and H3 are ignored.

Minimizes the total distance traveled and then maximizes the number of matches with preferred referee pairs

4.2 Decision variables

- $x_{m,r} \in \{0,1\} : x_{m,r} = 1$ if the referee is assigned to the specific game otherwise 0.

4.3 Model

From model in H1 we get:

$$
\begin{array}{lll}
\text{minimize} & \displaystyle\sum_{m=1}^{198}\sum_{r=1}^{52}\sum_{a=1}^{70} x_{m,r} \cdot MatchArena_{m,a} \cdot RefereeArenaDistance_{r,a} & \\
\text{subject to} & & \\
(1) & \sum_{r=1}^{52} x_{m,r} = 2 & \forall m \in \{1, M\} \\
(2) & \sum_{m=1}^{198} x_{m,r} \cdot MatchTime_{m,d} \leq 1 & \forall d \in \{1, D\}, r \in \{1, R\} \\
(3) & 1 - RefNotAvailable_{r,d} \geq MatchTime_{m,d} \cdot x_{m,r} & \forall d \in \{1, D\}, r \in \{1, R\}, \\
 & & m \in \{1, M\}
\end{array}
$$

Objective Value = $\underline{7{,}269}$ = stored as distance constrain

Constraints
(1) Two referees must be assigned to each match.
(2) A referee can only officiate one match a day.
(3) The referees have jobs and can not officiate on all days.

Model 2:

$$
\begin{aligned}
&\text{Maximize} && \sum_{m=1}^{198}\sum_{r1=1}^{52}\sum_{r2=1}^{52} x_{m,r1}\cdot RefPair_{r1,r2}\cdot x_{m,r2}\cdot 0.5 \\
&\text{subject to} && \\
&(1) && \sum_{r=1}^{52} x_{m,r} = 2 && \forall m \in \{1, M\} \\
&(2) && \sum_{m=1}^{198} x_{m,r}\cdot MatchTime_{m,d} \le 1 && \forall d \in \{1, D\}, r \in \{1, R\} \\
&(3) && 1 - RefNotAvailable_{r,d} \ge MatchTime_{m,d}\cdot x_{m,r} && \forall d \in \{1, D\}, r \in \{1, R\}, m \in \{1, M\} \\
&(4)\underline{\color{red}(New)} && \sum_{m=1}^{198}\sum_{r=1}^{52}\sum_{a=1}^{70} x_m\cdot MatchArena_{m,a}\cdot RefereeArenaDistance_{r,a} \le DistConst
\end{aligned}
$$

Objective Value = 16

New Constraint
(4) The total distance travelled by the referees can not be more than the distance solution from model above

4.4 B

- The problem extensions in parts H2 and H3 are ignored.

Maximizes the number of matches with preferred referee pairs and then minimizes distance travelled

4.5 Decision variables

- $x_{m,r} \in \{0, 1\}$: $x_{m,r} = 1$ if the referee is assigned to the specific game otherwise 0.

$$
\begin{aligned}
&\text{Maximize} && \sum_{m=1}^{198}\sum_{r1=1}^{52}\sum_{r2=1}^{52} x_{m,r1}\cdot RefPair_{r1,r2}\cdot x_{m,r2}\cdot 0.5 \\
&\text{subject to} && \\
&(1) && \sum_{r=1}^{52} x_{m,r} = 2 && \forall m \in \{1, M\} \\
&(2) && \sum_{m=1}^{198} x_{m,r}\cdot MatchTime_{m,d} \le 1 && \forall d \in \{1, D\}, r \in \{1, R\} \\
&(3) && 1 - RefNotAvailable_{r,d} \ge MatchTime_{m,d}\cdot x_{m,r} && \forall d \in \{1, D\}, r \in \{1, R\}, m \in \{1, M\}
\end{aligned}
$$

Objective Value = $\underline{163}$ = stored as pairs constraint

Constraints
(1) Two referees must be assigned to each match.
(2) A referee can only officiate one match a day.
(3) The referees have jobs and can not officiate on all days.

Model 2:

$$
\begin{array}{lll}
\text{minimize} & \displaystyle\sum_{m=1}^{198}\sum_{r=1}^{52}\sum_{a=1}^{70} x_{m,r} \cdot MatchArena_{m,a} \cdot RefereeArenaDistance_{r,a} & \\
\text{subject to} & & \\
(1) & \sum_{r=1}^{52} x_{m,r} = 2 & \forall m \in \{1, M\} \\
(2) & \sum_{m=1}^{198} x_{m,r} \cdot MatchTime_{m,d} \leq 1 & \\
 & \forall d \in \{1, D\}, r \in \{1, R\} & \\
(3) & 1 - RefNotAvailable_{r,d} \geq MatchTime_{m,d} \cdot x_{m,r} & \forall d \in \{1, D\}, \\
 & r \in \{1, R\}, m \in \{1, M\} & \\
(4)\underline{\color{red}{(New)}} & \sum_{m=1}^{198} \sum_{r1=1}^{52} \sum_{r2=1}^{52} x_{m,r1} \cdot RefPair_{r1,r2} \cdot x_{m,r2} \cdot 0.5 \geq PairsConstraint &
\end{array}
$$

Objective Value = $\underline{16{,}960}$

New Constraint
(4) The total pairs of referees can not be lower than the pairs solution from model above

5 Assignment H5

Minimize the total number of times the teams are refereed by the same referee more than 3 times and the number of times the same referee is used twice in row for the same team. A cost of 1 is given for each match above the requirements. Therefore minimize cost

5.1 Decision variables

- $x_{m,r} \in \{0, 1\}$: $x_{m,r} = 1$ if the referee is assigned to the specific game otherwise 0.
- $y_{t,r} \geq 0, \in Z$ (New) how many games in excess of three for team-referee pair occurs
- $z_{r,t,m} \geq 0, \in Z$ (New) how many games occur consecutively for team-referee pair

5.2 Model

$$\text{minimize} \quad \sum_{t=1}^{31} \sum_{r=1}^{52} y_{t,r} + \sum_{r=1}^{52} \sum_{t=1}^{31} \sum_{m=1}^{198} z_{r,t,m}$$

subject to

$$(1) \quad \sum_{r=1}^{52} x_{m,r} = 2 \qquad \forall m \in \{1, M\}$$

$$(2) \quad \sum_{m=1}^{198} x_{m,r} \cdot MatchTime_{m,d} \leq 1 \qquad \forall d \in \{1, D\}, r \in \{1, R\}$$

$$(3) \quad 1 - RefNotAvailable_{r,d} \geq MatchTime_{m,d} \cdot x_{m,r} \qquad \forall d \in \{1, D\}, r \in \{1, R\}, m \in \{1, M\}$$

$$(4)(New) \quad \sum_{m=1}^{198} \sum_{r1=1}^{52} \sum_{r2=1}^{52} x_{m,r1} \cdot RefPair_{r1,r2} \cdot x_{m,r2} \geq Pairs_{H4B} \cdot 0.7$$

$$(5)(New) \quad \sum_{m=1}^{198} \sum_{r=1}^{52} \sum_{a=1}^{70} x_{m,r} \cdot MatchArena_{m,a} \cdot RefereeArenaDistance_{r,a} \leq Dist_{H4A} \cdot 1.35$$

$$(6)(New) \quad \sum_{m=1}^{198} x_{m,r} \cdot TeamMatch_{t,m} \leq y_{t,r} + 3 \qquad \forall r \in \{1, R\}, t \in \{1, T\}$$

$$(7)(New) \quad x_{r,Tset[m]} + x_{r,Tset[m-1]} \leq 1 + z_{r,t,Tset[m]} \qquad \forall r \in \{1, R\}, t \in \{1, T\}, m \in \{2, Tset[m-1]\}$$

Objective Value = 0, meaning we found a feasible solution yielding zero penalty points.

New Constraints

(4) At least 70% of the number of matches given in the solution to H4b are refereed by referee pairs.

(5) Distance travelled is at most 35% worse than the best solution to H4a

(6) Count times games excess 3 for referee-team pair

(7) Count times referee has consecutive games with a team

Assignment Zenith Chemicals

5.3 Defining the indexes

- MaxStorageTotal: number that represent the max products that can be stored each month.
- TSPperMonth: number of total TSP raw material received each month.
- HCMDperMonth: number of total HCMD raw material received each month.
- BlendingMachines: Total amount of available blending machines.
- GratingMachines: Total amount of available grading machines.
- M is a set of of months, M = [1,4], $m \in M$
- R is a set of of regions, R = [1,4], $r \in R$
- C is a set of of products, C = [1,3], $c \in C$
- B is a set of of machines, B = [1,2], $b \in B$
- Y is a set of of raw material, Y = [1,2], $y \in Y$
- $TimeMonth_{1,m}$ is the available hours for every machine for each month. The value in the vector has already taking it into account that one day each month is off due to cleaning.
- $StorageCost_{1,c}$ is the cost for storing each product per month.
- $UseRawMat_{c,y}$ represents the total amount of TSP and HMCD needed for producing one product of Alpha, Beta and Delta.
- $ProductTime_{c,b}$ represents the time in hours that is needed for producing one product of Alpha, Beta and Delta. The time is given for both types of machines, blending and grating.
- $ProductionCost_{c,m}$ is the production cost for each product in each month.
- $ProductionPrice_{c,m}$ is the price of each product in each month.
- $DemandAlpha_{r,m}$ is the demand for Alpha in given regions for each month.
- $DemandBeta_{r,m}$ is the demand for Beta in given regions for each month.
- $DemandDelta_{r,m}$ is the demand for Delta in given regions for each month.
- $TransportationCost_{c,m}$ is the transportation cost for every product for each month.

6 Assignment Z1

Bjarki: Text
Sverrir: Julia Code

Maximize Zenith Chemicals profits

6.1 Decision variables

- $x_{c,m} \geq 0$ Variable for products manufactured each month.
- $y_{c,m,r} \geq 0$ Variable for sold products in every month and in every region.
- $StorageLevel_{c,m} \geq 0, m \in \{1, M+1\}$ Variable for stored products at end of each month starting with the inherited storage level from end of month april.
- $machines_{b,c,m} \geq 0$ How many machines of type b are operating, what product c they are producing in what month m.

6.2 Model

The model is:

$$\begin{aligned}
\text{Maximize} \quad & \sum_{c=1}^{3}\sum_{m=1}^{4}\sum_{r=1}^{4} y_{c,m,r} \cdot ProdPrice_{c,m} \\
& - \sum_{c=1}^{3}\sum_{m=1}^{4}\sum_{r=1}^{4} TransCost_{c,m} \cdot y_{c,m,r} \\
& - \sum_{c=1}^{3}\sum_{m=1}^{4} ProdCost_{c,m} \cdot x_{c,m} \\
& - \sum_{c=1}^{3}\sum_{m=1}^{4} StorageLevel_{c,m} \cdot StorCost_{c}
\end{aligned}$$

subject to

$$
\begin{array}{lll}
(1) & \sum_{c=1}^{3} UseRawMat_{c,1} \cdot x_{c,m} \leq TSPperMonth & \forall m \in \{1, M\} \\
(2) & \sum_{c=1}^{3} UseRawMat_{c,2} \cdot x_{c,m} \leq HCMDperMonth & \forall m \in \{1, M\} \\
(3) & ProdTime_{c,b} \cdot x_{c,m} \leq TimeMonth_{m} \cdot 8 \cdot machines_{b,c,m} & \\
 & \forall m \in \{1, M\}, c \in \{1, C\}, b \in \{1, B\} & \\
(4) & \sum_{c=1}^{3} machines_{1,c,m} \leq BlendingMachines & \forall m \in \{1, M\} \\
(5) & \sum_{c=1}^{3} machines_{2,c,m} \leq GratingMachines & \forall m \in \{1, M\} \\
(6) & StorageLevel_{c,1} = 100 & \forall c \in \{1, C\} \\
(7) & StorageLevel_{c,m} + x_{c,m} - \sum_{r=1}^{4} y_{c,m,r} = StorageLevel_{c,m+1} & \\
 & \forall c \in \{1, C\}, m \in \{1, M\} & \\
(8) & \sum_{c=1}^{3} StorageLevel_{c,m} \leq 2500 & \forall m \in \{1, M\} \\
(9) & y_{1,m,r} \leq DemandAlpha_{r,m} & \forall r \in \{1, R\}, m \in \{1, M\} \\
(10) & y_{2,m,r} \leq DemandBeta_{r,m} & \forall r \in \{1, R\}, m \in \{1, M\} \\
(11) & y_{3,m,r} \leq DemandDelta_{r,m} & \forall r \in \{1, R\}, m \in \{1, M\} \\
(12) & StorageLevel_{c,m} + x_{c,m} \geq \sum_{r=1}^{3} y_{c,m,r} & \forall c \in \{1, C\}, m \in \{1, M\} \\
(13) & StorageLevel_{c,5} \geq 100 & \forall c \in \{1, C\}
\end{array}
$$

Objective Value = <u>446,988.77</u>

Constraints

(1) Recieved monthly TSP raw material must be greater or equal to the amount used
(2) Recieved monthly HCMD raw material must be greater or eaqual to the amount used
(3) Bleending/grading time constraint for all machines
(4) allocated machines every month over all products
(5) allocated machines every month over all products
(6) Storage in May has 100 products of Alpha, Beta and Delta
(7) Storage level for the next month is equal to the storage from the previous month plus the net flow of production and sales
(8) Storage capacity has a capacity limit of 2500
(9) The sales possibilities for Alpha
(10) The sales possibilities for Beta
(11) The sales possibilities for Delta
(12) The total number of products in storage + the total number of production each month always has to be higher than sold products for each month
(13) At the end of August at least 100 products has to be in storage for each products

Beta	Month			
Machine	**1**	**2**	**3**	**4**
Blending	0	0	0	0
Grating	0	0	0	1

Alpha	Month			
Machine	**1**	**2**	**3**	**4**
Blending	6	11	10	0
Grating	5	9	8	3

Delta	Month			
Machine	**1**	**2**	**3**	**4**
Blending	30	25	26	36
Grating	15	11	12	16

7 Assignment Z2

Maximize Zenith Chemicals profits with 4 workers

(New) W is the number for workers w = 4

7.1 Decision variables

- $x_{c,m} \geq 0$ Variable for products manufactured each month.
- $y_{c,m,r} \geq 0$ Variable for sold products in every month and in every region.
- $StorageLevel_{c,m} \geq 0, m \in \{1, M+1\}$ Variable for stored products at end of each month.
- $machines_{b,c,m} \geq 0$ Variable for products produce in each month for each of the machine.
- (New) $workers_{b,m} \geq 0$: workers $\in Z^{+}$ How many operators are operating machine type b in month b

7.2 Model

The model is:

$$
\begin{aligned}
\text{Maximize} \quad & \sum_{c=1}^{3}\sum_{m=1}^{4}\sum_{r=1}^{4} y_{c,m,r} \cdot ProdPrice_{c,m} \\
& - \sum_{c=1}^{3}\sum_{m=1}^{4}\sum_{r=1}^{4} TransCost_{c,m} \cdot y_{c,m,r} \\
& - \sum_{c=1}^{3}\sum_{m=1}^{4} ProdCost_{c,m} \cdot x_{c,m} \\
& - \sum_{c=1}^{3}\sum_{m=1}^{4} StorageLevel_{c,m} \cdot StorCost_{c}
\end{aligned}
$$

subject to

$$
\begin{array}{lll}
(1) & \sum_{c=1}^{3} UseRawMat_{c,1} \cdot x_{c,m} \le TSPperMonth & \forall m \in \{1, M\} \\
(2) & \sum_{c=1}^{3} UseRawMat_{c,2} \cdot x_{c,m} \le HCMDperMonth & \forall m \in \{1, M\} \\
(3) & ProdTime_{c,b} \cdot x_{c,m} \le TimeMonth_{m} \cdot 8 \cdot machines_{b,c,m} & \\
 & \forall m \in \{1, M\}, c \in \{1, C\}, b \in \{1, B\} & \\
(4) & \sum_{c=1}^{3} machines_{1,c,m} \le BlendingMachines & \forall m \in \{1, M\} \\
(5) & \sum_{c=1}^{3} machines_{2,c,m} \le GratingMachines & \forall m \in \{1, M\} \\
(6) & StorageLevel_{c,1} = 100 & \forall c \in \{1, C\} \\
(7) & StorageLevel_{c,m} + x_{c,m} - \sum_{r=1}^{4} y_{c,m,r} = StorageLevel_{c,m+1} & \\
 & \forall c \in \{1, C\}, m \in \{1, M\} & \\
(8) & \sum_{c=1}^{3} StorageLevel_{c,m} \le 2500 & \forall m \in \{1, M\} \\
(9) & y_{1,m,r} \le DemandAlpha_{r,m} & \forall r \in \{1, R\}, m \in \{1, M\} \\
(10) & y_{2,m,r} \le DemandBeta_{r,m} & \forall r \in \{1, R\}, m \in \{1, M\} \\
(11) & y_{3,m,r} \le DemandDelta_{r,m} & \forall r \in \{1, R\}, m \in \{1, M\} \\
(12) & StorageLevel_{c,m} + x_{c,m} \ge \sum_{r=1}^{3} y_{c,m,r} & \forall c \in \{1, C\}, m \in \{1, M\} \\
(13) & StorageLevel_{c,5} \ge 100 & \forall c \in \{1, C\} \\
(14)\underline{(New)} & \sum_{b=1}^{2} workers_{b,m} = 4 & \forall m \in \{1, M\} \\
(15)\underline{(New)} & workers_{1,m} \cdot 3 = \sum_{c=1}^{3} machines_{1,c,m} & \forall m \in \{1, M\} \\
(16)\underline{(New)} & workers_{2,m} \cdot 2 = \sum_{c=1}^{3} machines_{2,c,m} & \forall m \in \{1, M\}
\end{array}
$$

Objective Value = 74,455

New Constraints

(14) We only have 4 operators each month across both types of equipment

(15) Each blending operator can only operate 3 blending machines each month
(16) Each grating operator can only operate 2 grating machines each month

	Month			
# Operators	4	4	4	4

8 Assignment Z3

Maximize Zenith Chemicals profits starting with 4 operators, and investigate if it pays off to hire more. Each operator has a monthly cost of 3000 and cannot be sacked if hired

8.1 Decision variables

- $x_{c,m} \geq 0$ Variable for products manufactured each month.
- $y_{c,m,r} \geq 0$ Variable for sold products in every month and in every region.
- $StorageLevel_{c,m} \geq 0, m \in \{1, M+1\}$ Variable for stored products at end of each month.
- $machines_{b,c,m} \geq 0$ Variable for products produce in each month for each of the machine.
- $workers_{b,m} \geq 0$: workers $\in Z^{+}$

8.2 Model

The model is:

$$
\begin{aligned}
\text{Maximize} \quad & \sum_{c=1}^{3}\sum_{m=1}^{4}\sum_{r=1}^{4} y_{c,m,r} \cdot ProdPrice_{c,m} \\
& -\sum_{c=1}^{3}\sum_{m=1}^{4}\sum_{r=1}^{4} TransCost_{c,m} \cdot y_{c,m,r} \\
& -\sum_{c=1}^{3}\sum_{m=1}^{4} ProdCost_{c,m} \cdot x_{c,m} \\
& -\sum_{c=1}^{3}\sum_{m=1}^{4} StorageLevel_{c,m} \cdot StorCost_{c} \\
& -\sum_{b=1}^{2}\sum_{m=1}^{4} workers_{b,m} \cdot 3000
\end{aligned}
$$

subject to

(1) $\sum_{c=1}^{3} UseRawMat_{c,1} \cdot x_{c,m} \leq TSPperMonth$ $\quad \forall m \in \{1, M\}$

(2) $\sum_{c=1}^{3} UseRawMat_{c,2} \cdot x_{c,m} \leq HCMDperMonth$ $\quad \forall m \in \{1, M\}$

(3) $ProdTime_{c,b} \cdot x_{c,m} \leq TimeMonth_{m} \cdot 8 \cdot machines_{b,c,m}$
$\forall m \in \{1, M\}, c \in \{1, C\}, b \in \{1, B\}$

(4) $\sum_{c=1}^{3} machines_{1,c,m} \leq BlendingMachines$ $\quad \forall m \in \{1, M\}$

(5) $\sum_{c=1}^{3} machines_{2,c,m} \leq GratingMachines$ $\quad \forall m \in \{1, M\}$

(6) $StorageLevel_{c,1} = 100$ $\quad \forall c \in \{1, C\}$

(7) $StorageLevel_{c,m} + x_{c,m} - \sum_{r=1}^{4} y_{c,m,r} = StorageLevel_{c,m+1}$
$\forall c \in \{1, C\}, m \in \{1, M\}$

(8) $\sum_{c=1}^{3} StorageLevel_{c,m} \leq 2500$ $\quad \forall m \in \{1, M\}$

(9) $y_{1,m,r} \leq DemandAlpha_{r,m}$ $\quad \forall r \in \{1, R\}, m \in \{1, M\}$

(10) $y_{2,m,r} \leq DemandBeta_{r,m}$ $\quad \forall r \in \{1, R\}, m \in \{1, M\}$

(11) $y_{3,m,r} \leq DemandDelta_{r,m}$ $\quad \forall r \in \{1, R\}, m \in \{1, M\}$

(12)*(Changes)* $StorageLevel_{c,m} \geq \sum_{r=1}^{3} y_{c,m,r}$ $\quad \forall c \in \{1, C\}, m \in \{1, M\}$

(13) $StorageLevel_{c,5} \geq 100$ $\quad \forall c \in \{1, C\}$

(14)*(Changes)* $\sum_{b=1}^{2} workers_{b,1} = 4$

(15) $workers_{1,m} \cdot 3 = \sum_{c=1}^{3} machinces_{1,c,m}$ $\quad \forall m \in \{1, M\}$

(16) $workers_{2,m} \cdot 2 = \sum_{c=1}^{3} machinces_{2,c,m}$ $\quad \forall m \in \{1, M\}$

(17)*(New)* $\sum_{b=1}^{2} workers_{b,m-1} \leq \sum_{b=1}^{2} workers_{b,m}$ $\quad \forall m \in \{2, M\}$

Objective Value = 33,844

New constraints/Changed constraints
(12) We can now only sell from the previous mont's storage, thus we remove the option to sell the volume produced during the month.
(14) We assume that we can not hire in the first month and therefore fix the amount of workers to 4 during m_1 and not all months like in problem Z2
(17) Cannot have fewer workers in the next month than previous month

	Month			
Operators hired	0	3	0	0

9 Assignment Z4

Maximize Zenith Chemicals profits from the 8 estimated scenarios from the demand

(New) S is the number for scenarios S = 8, $s \in S$

9.1 Decision variables

- $x_{s,c,m} \geq 0$ Variable for products manufactured each month.(Changes)
- $y_{s,c,m,r} \geq 0$ Variable for sold products in every month and in every region.(Changes)
- $StorageLevel_{s,c,m} \geq 0, m \in \{1, M+1\}$ Variable for stored products at end of each month.(Changes)
- $machines_{s,b,c,m} \geq 0$ Variable for products produce in each month for each of the machine.(Changes)
- $workers_{s,b,m,w} \geq 0$: workers $\in Z^{+}$ (Changes)

9.2 Model

The model is:

$$
\begin{aligned}
\text{Maximize} \quad & \tfrac{1}{8}\Big(\sum_{s=1}^{8}\sum_{c=1}^{3}\sum_{m=1}^{4}\sum_{r=1}^{4} y_{s,c,m,r} \cdot ProdPrice_{c,m} \\
& - \sum_{s=1}^{8}\sum_{c=1}^{3}\sum_{m=1}^{4}\sum_{r=1}^{4} TransCost_{c,m} \cdot y_{s,c,m,r} \\
& - \sum_{s=1}^{8}\sum_{c=1}^{3}\sum_{m=1}^{4} ProdCost_{c,m} \cdot x_{s,c,m} \\
& - \sum_{s=1}^{8}\sum_{c=1}^{3}\sum_{m=1}^{4} StorageLevel_{s,c,m} \cdot StorCost_{c} \\
& - \sum_{s=1}^{8}\sum_{b=1}^{2}\sum_{m=1}^{4} workers_{s,b,m} \cdot 3000\Big)
\end{aligned}
$$

subject to

(1)	$\sum_{c=1}^{3} UseRawMat_{c,1} \cdot x_{s,c,m} \leq TSPperMonth$	$\forall s \in \{1, S\}, m \in \{1, M\}$
(2)	$\sum_{c=1}^{3} UseRawMat_{c,2} \cdot x_{s,c,m} \leq HCMDperMonth$	$\forall m \in \{1, M\}, s \in \{1, S\}$
(3)	$ProdTime_{c,b} \cdot x_{s,c,m} \leq TimeMonth_m \cdot 8 \cdot machines_{b,c,m}$	$\forall s \in \{1, S\}, m \in \{1, M\},$ $c \in \{1, C\}, b \in \{1, B\}$
(4)	$\sum_{c=1}^{3} machines_{s,1,c,m} \leq BlendingMachines$	$\forall s \in \{1, S\}, m \in \{1, M\}$
(5)	$\sum_{c=1}^{3} machines_{s,2,c,m} \leq GratingMachines$	$\forall s \in \{1, S\}, m \in \{1, M\}$
(6)	$StorageLevel_{s,c,1} = 100$	$\forall s \in \{1, S\}, c \in \{1, C\}$
(7)	$StorageLevel_{s,c,m} + x_{c,m} - \sum_{r=1}^{4} y_{s,c,m,r} = StorageLevel_{s,c,m+1}$ $\forall s \in \{1, S\}, c \in \{1, C\}, m \in \{1, M\}$	
(8)	$\sum_{c=1}^{3} StorageLevel_{s,c,m} \leq 2500$	$\forall s \in \{1, S\}, m \in \{1, M\}$
(9)	$y_{s,1,m,r} \leq DemandAlpha_{s,r,m} \cdot scenarios_{s,m}$	$\forall s \in \{1, S\}, r \in \{1, R\},$ $m \in \{1, M\}$
(10)	$y_{s,2,m,r} \leq DemandBeta_{s,r,m} \cdot scenarios_{s,m}$	$\forall s \in \{1, S\}, r \in \{1, R\},$ $m \in \{1, M\}$
(11)	$y_{s,3,m,r} \leq DemandDelta_{s,r,m} \cdot scenarios_{s,m}$	$\forall s \in \{1, S\}, r \in \{1, R\},$ $m \in \{1, M\}$
(12)	$StorageLevel_{s,c,m} \geq \sum_{r=1}^{3} y_{s,c,m,r}$	$\forall s \in \{1, S\}, c \in \{1, C\},$ $m \in \{1, M\}$
(13)	$StorageLevel_{s,c,5} \geq 100$	$\forall s \in \{1, S\}, c \in \{1, C\}$
(14)	$\sum_{b=1}^{2} workers_{s,b,1} = 4$	$\forall s \in \{1, S\}$
(15)	$\sum_{b=1}^{2} workers_{s,b,m-1} \leq \sum_{b=1}^{2} workers_{s,b,m}$	$\forall s \in \{1, S\}, m \in \{2, M\}$
(16)	$workers_{s,1,m} \cdot 3 = \sum_{c=1}^{3} machinces_{s,1,c,m}$	$\forall s \in \{1, S\}, m \in \{1, M\}$
(17)	$workers_{s,2,m} \cdot 2 = \sum_{c=1}^{3} machinces_{s,2,c,m}$	$\forall s \in \{1, S\}, m \in \{1, M\}$
(18)	$x_{s-1,c,1} = x_{s,c,m}$	$\forall s \in \{2, S\}, c \in \{1, C\},$ $m \in \{1, M\}$
(19)	$x_{s-1,c,2} = x_{s,c,m}$	$\forall s \in \{2, 4\}, c \in \{1, C\},$ $m \in \{1, M\}$
(20)	$x_{s-1,c,2} = x_{s,c,m}$	$\forall s \in \{6, 8\}, c \in \{1, C\},$ $m \in \{1, M\}$
(21)	$x_{1,c,3} = x_{2,c,m}$	$\forall c \in \{1, C\}$
(22)	$x_{3,c,3} = x_{4,c,m}$	$\forall c \in \{1, C\}$
(23)	$x_{5,c,3} = x_{6,c,m}$	$\forall c \in \{1, C\}$
(24)	$x_{7,c,3} = x_{8,c,m}$	$\forall c \in \{1, C\}$

Objective Value = 24,887

All New Constraints

(1) - (17) All the same, but now is scenarios included
(18) First month all the same x in all scenarios
(19) Second month all the same x in scenario 1:4
(20) Second month all the same x in scenario 5:8
(21) Third month all the same x in scenario 1 & 2
(22) Third month all the same x in scenario 3 & 4
(23) Third month all the same x in scenario 5 & 6
(24) Third month all the same x in scenario 7 & 8

Scenario	Profit
1	2,933.90
2	9,742.20
3	22,577.55
4	23,262.35
5	33,999.60
6	34,599.60
7	35,991.60
8	35,991.60

10 Assignment Z5

Maximize Zenith Chemicals profits

(New) K is a set of numbers K = {0.5, 0.6, 0.7, 0.8, 0.9, 1}, $k \in K$.

10.1 Decision variables

- $x_{s,c,m} \geq 0$ Variable for products manufactured each month.
- $y_{s,c,m,r} \geq 0$ Variable for sold products in every month and in every region.
- $StorageLevel_{s,c,m} \geq 0, m \in \{1, M+1\}$ Variable for stored products at end of each month.
- $machines_{s,b,c,m} \geq 0$ Variable for products produce in each month for each of the machine.
- $workers_{s,b,m,w} \geq 0$: workers $\in Z^+$
- $ExpectedProfit \in R^+$ Variable that will equal the objective value to make code more clear in a constraint. (New)

10.2 Model

The model is:

$$\begin{aligned}
\text{Maximize} \quad & \tfrac{1}{8}\Big(\sum_{s=1}^{8}\sum_{c=1}^{3}\sum_{m=1}^{4}\sum_{r=1}^{4} y_{s,c,m,r} \cdot ProdPrice_{c,m} \\
& - \sum_{s=1}^{8}\sum_{c=1}^{3}\sum_{m=1}^{4}\sum_{r=1}^{4} TransCost_{c,m} \cdot y_{s,c,m,r} \\
& - \sum_{s=1}^{8}\sum_{c=1}^{3}\sum_{m=1}^{4} ProdCost_{c,m} \cdot x_{s,c,m} \\
& - \sum_{s=1}^{8}\sum_{c=1}^{3}\sum_{m=1}^{4} StorageLevel_{s,c,m} \cdot StorCost_{c} \\
& - \sum_{s=1}^{8}\sum_{b=1}^{2}\sum_{m=1}^{4} workers_{s,b,m} \cdot 3000\Big)
\end{aligned}$$

subject to(1) $\sum_{c=1}^{3} UseRawMat_{c,1} \cdot x_{s,c,m} \leq TSPperMonth \quad \forall s \in \{1, S\}, m \in \{1, M\}$

(2) $\sum_{c=1}^{3} UseRawMat_{c,2} \cdot x_{s,c,m} \leq HCMDperMonth \quad \forall m \in \{1, M\}, s \in \{1, S\}$

(3) $ProdTime_{c,b} \cdot x_{s,c,m} \leq TimeMonth_m \cdot 8 \cdot machines_{b,c,m} \quad \forall s \in \{1, S\}, m \in \{1, M\}, c \in \{1, C\}, b \in \{1, B\}$

(4) $\sum_{c=1}^{3} machines_{s,1,c,m} \leq BlendingMachines \quad \forall s \in \{1, S\}, m \in \{1, M\}$

(5) $\sum_{c=1}^{3} machines_{s,2,c,m} \leq GratingMachines \quad \forall s \in \{1, S\}, m \in \{1, M\}$

(6) $StorageLevel_{s,c,1} = 100 \quad \forall s \in \{1, S\}, c \in \{1, C\}$

(7) $StorageLevel_{s,c,m} + x_{c,m} - \sum_{r=1}^{4} y_{s,c,m,r} = StorageLevel_{s,c,m+1}$
$\forall s \in \{1, S\}, c \in \{1, C\}, m \in \{1, M\}$

(8) $\sum_{c=1}^{3} StorageLevel_{s,c,m} \leq 2500 \quad \forall s \in \{1, S\}, m \in \{1, M\}$

(9) $y_{s,1,m,r} \leq DemandAlpha_{s,r,m} \cdot scenarios_{s,m} \quad \forall s \in \{1, S\}, r \in \{1, R\}, m \in \{1, M\}$

(10) $y_{s,2,m,r} \leq DemandBeta_{s,r,m} \cdot scenarios_{s,m} \quad \forall s \in \{1, S\}, r \in \{1, R\}, m \in \{1, M\}$

(11) $y_{s,3,m,r} \leq DemandDelta_{s,r,m} \cdot scenarios_{s,m} \quad \forall s \in \{1, S\}, r \in \{1, R\}, m \in \{1, M\}$

(12) $StorageLevel_{s,c,m} \geq \sum_{r=1}^{3} y_{s,c,m,r} \quad \forall s \in \{1, S\}, c \in \{1, C\}, m \in \{1, M\}$

(13) $StorageLevel_{s,c,5} \geq 100 \quad \forall s \in \{1, S\}, c \in \{1, C\}$

(14) $\sum_{b=1}^{2} workers_{s,b,1} = 4 \quad \forall s \in \{1, S\}$

(15) $\sum_{b=1}^{2} workers_{s,b,m-1} \leq \sum_{b=1}^{2} workers_{s,b,m} \quad \forall s \in \{1, S\}, m \in \{2, M\}$

(16) $workers_{s,1,m} \cdot 3 = \sum_{c=1}^{3} machinces_{s,1,c,m} \quad \forall s \in \{1, S\}, m \in \{1, M\}$

(17) $workers_{s,2,m} \cdot 2 = \sum_{c=1}^{3} machinces_{s,2,c,m} \quad \forall s \in \{1, S\}, m \in \{1, M\}$

(18) $x_{s-1,c,1} = x_{s,c,m} \quad \forall s \in \{2, S\}, c \in \{1, C\}, m \in \{1, M\}$

(19) $x_{s-1,c,2} = x_{s,c,m} \quad \forall s \in \{2, 4\}, c \in \{1, C\}, m \in \{1, M\}$

(20) $x_{s-1,c,2} = x_{s,c,m} \quad \forall s \in \{6, 8\}, c \in \{1, C\}, m \in \{1, M\}$

(21) $x_{1,c,3} = x_{2,c,m} \quad \forall c \in \{1, C\}$

(22) $x_{3,c,3} = x_{4,c,m} \quad \forall c \in \{1, C\}$

(23) $x_{5,c,3} = x_{6,c,m} \quad \forall c \in \{1, C\}$

(24) $x_{7,c,3} = x_{8,c,m} \quad \forall c \in \{1, C\}$

$$(25) \quad ExpectedProfit = \frac{1}{8}(\sum_{s=1}^{8}\sum_{c=1}^{3}\sum_{m=1}^{4}\sum_{r=1}^{4} y_{s,c,m,r} \cdot ProdPrice_{c,m}$$
$$-\sum_{s=1}^{8}\sum_{c=1}^{3}\sum_{m=1}^{4}\sum_{r=1}^{4} TransCost_{c,m} \cdot y_{s,c,m,r}$$
$$-\sum_{s=1}^{8}\sum_{c=1}^{3}\sum_{m=1}^{4} ProdCost_{c,m} \cdot x_{s,c,m}$$
$$-\sum_{s=1}^{8}\sum_{c=1}^{3}\sum_{m=1}^{4} StorageLevel_{s,c,m} \cdot StorCost_{c}$$
$$-\sum_{s=1}^{8}\sum_{b=1}^{2}\sum_{m=1}^{4} workers_{s,b,m} \cdot 3000)$$

$$(26) \quad ExpectedProfit \cdot k \leq \sum_{s=1}^{8}\sum_{c=1}^{3}\sum_{m=1}^{4}\sum_{r=1}^{4} y_{s,c,m,r} \cdot ProdPrice_{c,m}$$
$$-\sum_{s=1}^{8}\sum_{c=1}^{3}\sum_{m=1}^{4}\sum_{r=1}^{4} TransCost_{c,m} \cdot y_{s,c,m,r}$$
$$-\sum_{s=1}^{8}\sum_{c=1}^{3}\sum_{m=1}^{4} ProdCost_{c,m} \cdot x_{s,c,m}$$
$$-\sum_{s=1}^{8}\sum_{c=1}^{3}\sum_{m=1}^{4} StorageLevel_{s,c,m} \cdot StorCost_{c}$$
$$-\sum_{s=1}^{8}\sum_{b=1}^{2}\sum_{m=1}^{4} workers_{s,b,m} \cdot 3000 \qquad \forall s \in \{1, S\}, k \in \ K$$

Objective Value = See table

New Constraints
(25) The variable $ExpectedProfit$ takes the value of the objective function
(26) The profit in every scenario s must be greater or equal to the expected profit from the objective function times a factor k.

	K					
Scenario	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1**
1	4,954.70	4,954.70	4,954.70	4,954.70	4,954.70	4,954.65
2	5,008.10	6,362.85	5,027.15	4,958.60	4,980.15	4,954.65
3	10,860.55	10,648.10	4,960.85	4,980.25	6,186.40	4,954.65
4	11,367.20	9,087.30	4,954.80	5,000.30	5,868.70	4,954.65
5	5,582.70	10,665.55	4,996.00	7,405.50	4,955.85	4,954.65
6	5,800.40	10,583.00	9,949.5	4,954.80	5,268.78	4,954.65
7	17,429.30	8,772.80	12,753.30	9,649.30	6,844.80	4,954.65
8	18,269.15	4,987.85	9,027.15	7,643.55	4,982.40	4,954.65