Töluleg Greining

Upphitunar Verkefni Viku 3

Hópur 34

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1)

$$y''(x) + y(x) = 0 \qquad x \in [0, \tfrac{\pi}{2}]$$

$$f''(x) \approx \frac{f(x-h) - 2f(x) + f(x+h)}{h^2}$$

Við táknum i-th'ta nálgaða y gildið u_i

$$u_{i-1} - 2u_i + u_{i+1} + h^2 u_i = 0 \quad , \quad i = 2,3,4,\dots,n-1$$

$$\Rightarrow u_{i-1} + (-2+h^2)u_i + u_{i+1} = 0$$

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2) Við eigum $n-2$ jöfnur úr Dæmi 1

Við þurfum að nýta jaðar gildin okkar sem auka jöfnur til að fá full skilgreint jöfnuhneppi.

$$\begin{cases} y(0)=1 \\ y(\frac{\pi}{2}) = -5 \end{cases} \Rightarrow \begin{cases} u_1 = 1 \\ u_n = -5 \end{cases}$$

Við viljum svo skrifa heildar jöfnuhneppið á fylkja form, $A\bar{u}=\bar{b}$

$$\bar{u} = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix} \quad \bar{b} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \\ -5 \end{bmatrix}, \quad \alpha = -2+h^2$$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 1 & \alpha & 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & \alpha & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \alpha & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & & \vdots \\ 0 & \cdots & 0 & 0 & 1 & \alpha & 1 \\ 0 & \cdots & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

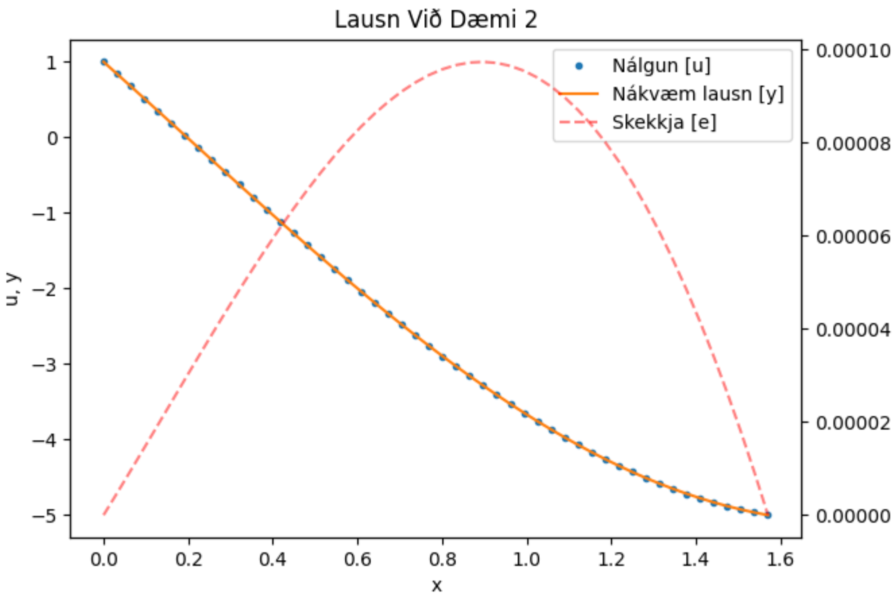
Uppsettning:

```
a = 0
b = np.pi/2
n = 50
h = (b-a)/(n-1)

b = np.concatenate([[1], np.zeros(n-2), [-5]])

alpha = -2+h**2
A = np.concatenate([
    [np.concatenate([[1], np.zeros(n-1)])],
    [np.concatenate([np.zeros(i),[1,alpha,1], np.zeros(n-i-3)]) for i in range(n-2)],
    [np.concatenate([np.zeros(n-1), [1]])]
])

u = np.linalg.solve(A, b)
```



3)

fyrir upphafs punktin getum við bara notað gildi u_{1+a}, $a \geq 0$

þannig við nálgum $y'(0) = 1$ sem:

$-3u_1 + 4u_2 - u_3 = 2h$

í endan getum við ekki notað gildi nærri en u_n þannig við nálgum $y'(\pi/2) = 3$ sem:

$-3u_n + 4u_{n-1} - u_{n-2} = -2h \cdot 3$

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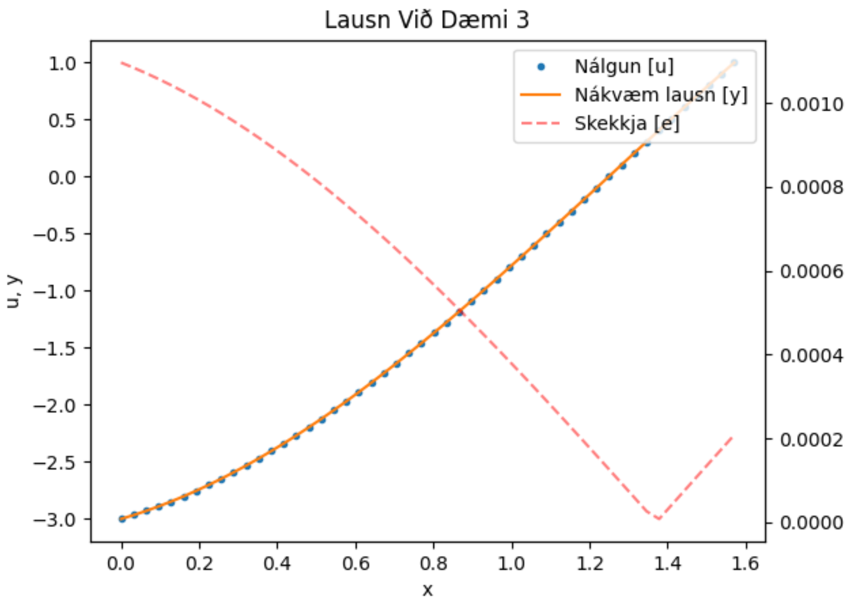
fylkja form s jöfnuhneppið er þá

$$b = \begin{bmatrix} 2h \\ 0 \\ 0 \\ \vdots \\ 0 \\ -2h\cdot 3 \end{bmatrix}, \qquad \alpha = -2 + h^2$$

$$A = \begin{bmatrix} -3 & 4 & -1 & 0 & 0 & \cdots & 0 \\ 1 & \alpha & 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & \alpha & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \alpha & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & & \vdots \\ 0 & \cdots & 0 & 0 & 1 & \alpha & 1 \\ 0 & \cdots & 0 & 0 & -1 & 4 & -3 \end{bmatrix}$$

```python
b = np.concatenate([[2*h], np.zeros(n-2), [-2*h*3]])

alpha = -2+h**2
A = np.concatenate([
    [np.concatenate([[-3,4,-1], np.zeros(n-3)])],
    [np.concatenate([np.zeros(i),[1,alpha,1], np.zeros(n-i-3)]) for i in range(n-2)],
    [np.concatenate([np.zeros(n-3), [-1,4,-3]])]
])
```



4)

Notum svipaða innsetningu fyrir y' og í dæmi 4.

fyrra: $-3u_1+4u_2-u_3-2hu_1=0 \Rightarrow (-3-2h)u_1+4u_2-u_3=0$

seinna: $-3u_n+4u_{n-1}-u_{n-2}-2h\cdot 2u_n=-2h \Rightarrow -u_{n-2}+4u_{n-1}+(-3-4h)u_n=-2h$

$$b=\begin{bmatrix}0\\0\\0\\0\\\vdots\\0\\-2h\end{bmatrix}$$

$\alpha=-2+h^2$

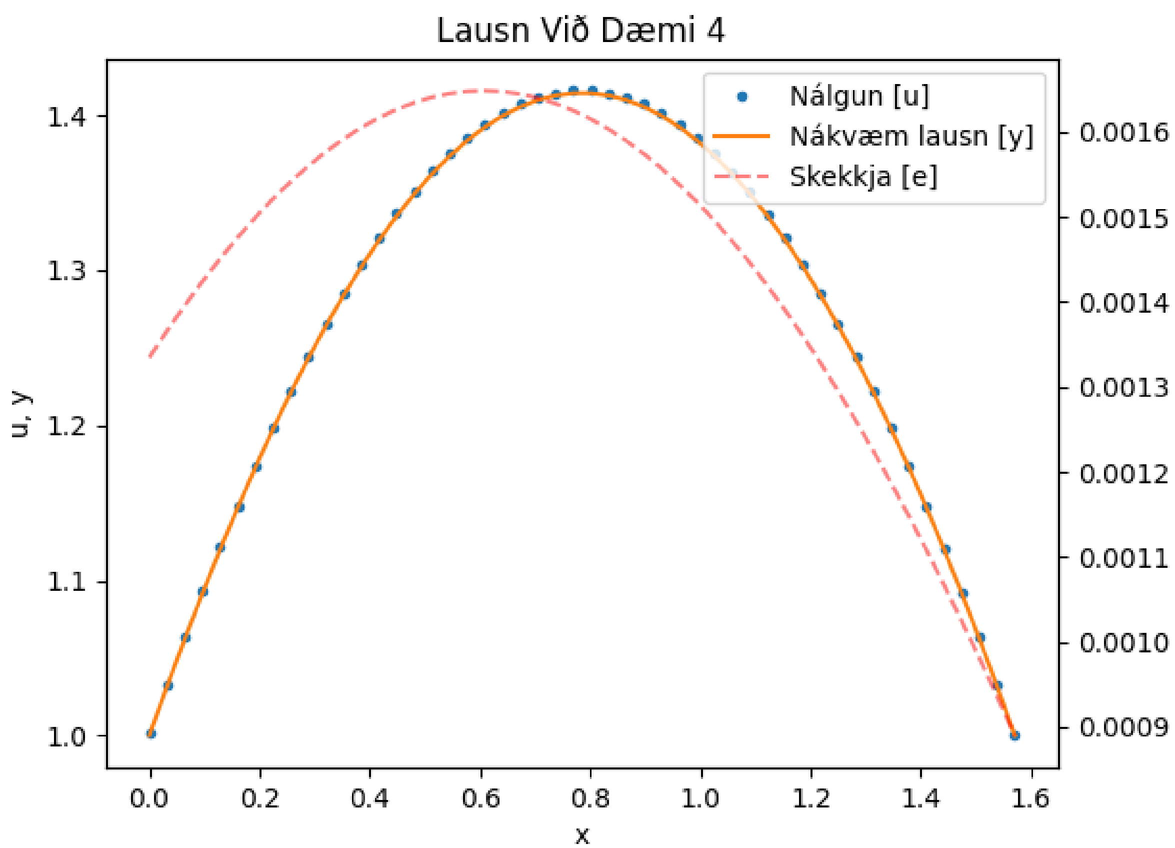
$C=-3-2h$

$d=-3-4h$

$$A=\begin{bmatrix}C & 4 & -1 & 0 & 0 & \cdots & 0\\ 1 & \alpha & 1 & 0 & 0 & \cdots & 0\\ 0 & 1 & \alpha & 1 & 0 & \cdots & 0\\ 0 & 0 & 1 & \alpha & 1 & \cdots & 0\\ \vdots & \vdots & \ddots & \ddots & \ddots & & \vdots\\ 0 & \cdots & 0 & 0 & 1 & \alpha & 1\\ 0 & \cdots & 0 & 0 & -1 & 4 & d\end{bmatrix}$$

```
b = np.concatenate([[0], np.zeros(n-2), [-2*h]])

alpha = -2+h**2
c = -3-2*h
d = -3-4*h
A = np.concatenate([
    [np.concatenate([[c,4,-1], np.zeros(n-3)])],
    [np.concatenate([np.zeros(i),[1,alpha,1], np.zeros(n-i-3)]) for i in range(n-2)],
    [np.concatenate([np.zeros(n-3), [-1,4,d]])]
])
```



Við Sjáum hér smá sýnilega skekkju, einstaklega í kringum hápunkt ferilsins.