LIVEMATH NEW NOTEBOOK LITE

Clamped Cubic Spline (slopes at each end fixed at u, v)

$y = A_i + B_i(x - x_i) + C_i(x - x_i)^2 + D_i(x - x_i)^3$

At point i, $\Delta y = A_i$ Substitute Substitute Substitute

derivatives

implies

at point 0 (i=0)

at point 1, use both 0 and 1 eqns

at point 2, use both 1 and 2 eqns

at point 3, use both 2 and 3 eqns

at point 4, use eqn for 3 plus slope

Collect all equations (12 eqn, 12 unk)

Substitute for x, y diffs

Solve forwards 0, 1, 2 etc. eliminate D 's

Solve backwards to eliminate C 's

Test case

x and y values (knots)

$x = (1,3,4,6,7.5)$

$x_0 = 1$

$x_1 = 3$ $\Delta x_1 = 2$ Substitute Isolate

$x_2 = 4$ $\Delta x_2 = 1$ Substitute Isolate

$x_3 = 6$ $\Delta x_3 = 2$ Substitute Isolate

$x_4 = 7.5$ $\Delta x_4 = 1.5$ Substitute Isolate

$y = (2,6,5,7,1.5)$

$y_0 = 2$

$y_1 = 6$ $\Delta y_1 = 4$ Substitute Isolate

$y_2 = 5$ $\Delta y_2 = -1$ Substitute Isolate

$y_3 = 7$ $\Delta y_3 = 2$ Substitute Isolate

$y_4 = 1.5$ $\Delta y_4 = -5.5$ Substitute Isolate

Calculate slopes at ends

$u = \frac{y_1 - y_0}{x_1 - x_0}$ $u = 2$ Substitute

$v = \frac{y_4 - y_3}{x_4 - x_3}$ $v = -3.6667$ Substitute

At point i

Solve for coefficients

$A_0 = 2$ Substitute

$A_1 = 6$ Substitute

$A_2 = 5$ Substitute

$A_3 = 7$ Substitute

$B_0 = 2$ Substitute Isolate

$B_1 = -0.27597$ Substitute Substitute Substitute Substitute ...

$B_2 = -0.17209$ Substitute Substitute Substitute Substitute ...

$B_3 = -1.4155$ Substitute Substitute Isolate

$C_0 = 1.138$ Substitute Isolate

$C_1 = -2.276$ Substitute Substitute Substitute

$C_2 = 2.3798$ Substitute Substitute

$C_3 = -3.0016$ Substitute Substitute

$D_0 = -0.56899$ Substitute Substitute Isolate

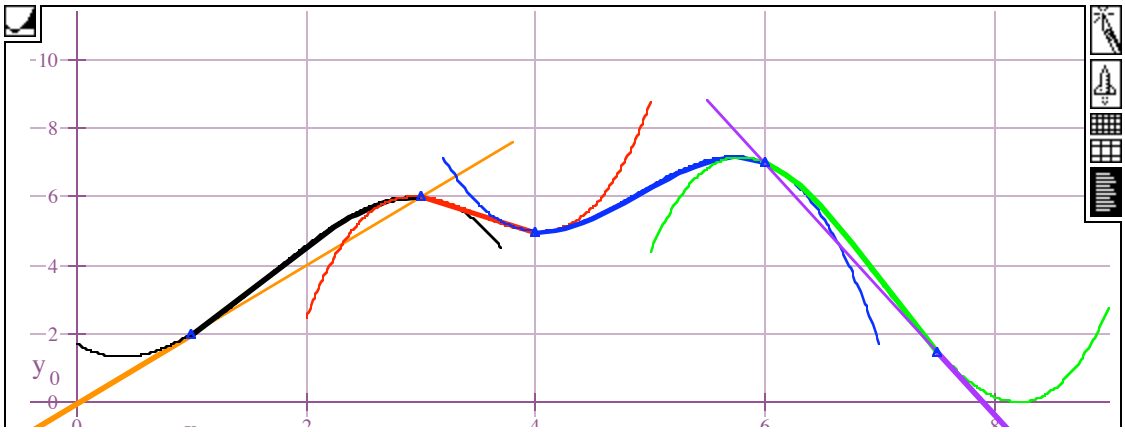
$D_1 = 1.5519$ Substitute Isolate

$D_2 = -0.8969$ Substitute Isolate

$D_3 = 1.0005$ Substitute Substitute Isolate

yields equations for splines

Add straight line interpolations at ends



cut

page 3a

– 0.4 ... 9 = left...right Stretch to Fit

– 1.8 ... 11.4 = bottom...top cropping Tightly

Graph Building Blocks

Curve at (x, z_1) where x = left ... 3.8 with a normal line, colored Orange.

Curve at (x, z_1) where x = left ... x_0 with a heavy line, colored Orange.

Curve at (x, y_0) where $x = 0 \ldots 3.7$ with a normal line, colored Black.

Curve at (x, y_0) where $x = x_0 \ldots x_1$ with a heavy line, colored Black.

Curve at (x, y_1) where $x = 2 \ldots 5$ with a normal line, colored Red.

Curve at (x, y_1) where $x = x_1 \ldots x_2$ with a heavy line, colored Red.

Curve at (x, y_2) where $x = 3.2 \ldots 7$ with a normal line, colored Blue.

Curve at (x, y_2) where $x = x_2 \ldots x_3$ with a heavy line, colored Blue.

Curve at (x, y_3) where $x = 5 \ldots 9$ with a normal line, colored Green.

Curve at (x, y_3) where $x = x_3 \ldots x_4$ with a heavy line, colored Green.

Curve at (x, z_2) where x = 5.5 ... right with a normal line, colored Purple.

Curve at (x, z_2) where $x = x_4 \ldots$ right with a heavy line, colored Purple.

Scatter plot of (x_{k_1}, y_{k_1}) where $k_1 = 1 \ldots 5$ using 5 point triangles colored Blue.