

Assignment for Lecture 1

ORDER OF MAGNITUDE, ESTIMATION AND DIVIDE-AND-CONQUER

Lecture Date: 3/4/2026

“C” denotes for “computational” problems.

please write in pdf format and submit to bjcai@fudan.edu.cn before or on 3/10/2026

1. One of the root of the equation $ax^2 + bx + c = 0$ with $abc \neq 0$ and $b > 0$ is given by

$$x^* = \frac{-b + \sqrt{b^2 - 4ac}}{2a} = \frac{b}{2a} \left(\sqrt{1 - \frac{4ac}{b^2}} - 1 \right). \quad (1-1)$$

Assume that a is small in the sense $k = 4ac/b^2 \ll 1$, try to obtain the approximation for x^* from (1-1) by expanding the square root to order k^2 using the formula $\sqrt{1+w} \approx 1 + w/2 - w^2/8$ for small w . The same result could also be obtained via firstly solving the linear equation $bx + c = 0$ and then adding some perturbation p to the solution $-c/b$ as $x = -(c/b)(1 + p)$. Determine the expression for p to first order.

2. Consider the equation generalized from (11), namely $x^n(t) = \Omega + tx^m(t)/\Lambda$ with $m < n$, develop its approximated solution, for both small t and large t limits.
3. Show that the n th Fibonacci number satisfies the equality,

$$F_n = \frac{\phi^n - \hat{\phi}^n}{\sqrt{5}} = \left\lfloor \frac{1}{2} + \frac{\phi^n}{\sqrt{5}} \right\rfloor, \quad \phi = \frac{1 + \sqrt{5}}{2}, \quad \hat{\phi} = \frac{1 - \sqrt{5}}{2}, \quad (1-2)$$

which is to say that the n th Fibonacci number is equal to $\phi^n / \sqrt{5}$ rounded to the nearest integer. Thus, Fibonacci numbers grow exponentially. Here $\lfloor x \rfloor$ is the greatest integer less than or equal to x , and similarly $\lceil x \rceil$ is the least integer greater than or equal to x .

4. [C] Write a program to solve the equation $x^8(t) = 1 + tx(t)/2$ both using approximated algorithms and numerical methods.
5. [C] Compute the area formed by the exact curve, the large- t and the small- t curves to first order and second order shown in FIG. 3 of the lecture note, respectively. What is the value for this quantity at third order?