Assignment for Lecture 1

Order of Magnitude, Estimation and Divide-and-Conquer

Lecture Date: 3/4/2026

"C" denotes for "computational" problems.

please write in pdf format and submit to `bjcai@fudan.edu.cn` ***before or on 3/10/2026***

1. One of the root of the equation $ax^2 + bx + c = 0$ with $abc \neq 0$ and $b > 0$ is given by

$$x^* = \frac{-b + \sqrt{b^2 - 4ac}}{2a} = \frac{b}{2a}\left(\sqrt{1 - \frac{4ac}{b^2}} - 1\right). \tag{1-1}$$

   Assume that a is small in the sense $k = 4ac/b^2 \ll 1$, try to obtain the approximation for x^* from (1-1) by expanding the square root to order k^2 using the formula $\sqrt{1+w} \approx 1 + w/2 - w^2/8$ for small w. The same result could also be obtained via firstly solving the linear equation $bx + c = 0$ and then adding some perturbation p to the solution $-c/b$ as $x = -(c/b)(1+p)$. Determine the expression for p to first order.

2. Consider the equation generalized from (11), namely $x^n(t) = \Omega + t x^m(t)/\Lambda$ with $m < n$, develop its approximated solution, for both-small t and large-t limits.

3. Is $2^{n+1} = \mathcal{O}(2^n)$? Is $2^{2n} = \mathcal{O}(2^n)$?

4. Show that the nth Fibonacci number satisfies the equality,

$$F_n = \frac{\phi^n - \widehat{\phi}^n}{\sqrt{5}} = \left\lfloor \frac{1}{2} + \frac{\phi^n}{\sqrt{5}} \right\rfloor, \ \phi = \frac{1+\sqrt{5}}{2}, \ \widehat{\phi} = \frac{1-\sqrt{5}}{2}, \tag{1-2}$$

   which is to say that the nth Fibonacci number is equal to $\phi^n/\sqrt{5}$ rounded to the nearest integer. Thus, Fibonacci numbers grow exponentially. Here $\lfloor x \rfloor$ is the greatest integer less than or equal to x, and similarly $\lceil x \rceil$ is the least integer greater than or equal to x.

5. [C] Write a program to solve the equation $x^8(t) = 1 + t x(t)/2$ both using approximated algorithms and numerical methods.