WH #8

1) a) $P(62 \le W \le 78) = \int_{62}^{78} f(x)\,dx =$

$W \sim N(\mu = 63.7975,\ \sigma = 9.6149)$

$Z = \frac{(63.7975)-(63.7975)}{\sigma} = \frac{0}{\sigma} = 0$

$Z = \frac{78-63.7975}{9.6149} = \frac{14.2025}{9.6149} = 1.477$

$Z = \frac{62-63.7975}{9.6149} = -.1869$

$\Phi(1.477) - \Phi(-.1869) = .5043 = 50.43\%$

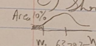
b) $M \sim N(\mu = 71.2499,\ \sigma = 14.8530)$

$Z = \frac{(78-71.2499)}{14.8530} = .4545$

$Z = \frac{(62-71.2499)}{14.8530} = -.6228$

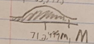
$\Phi(.4545) - \Phi(-.6228) = .4085 = 40.85\%$

c) Shortest 10% of women: area $= .1$, $\sigma = 9.6149$, $\mu = 63.7975$



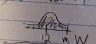
using calculator: inverse normal $= \boxed{51.4755}$ to

using calculator: inverse normal $= \boxed{90.2848}$



area $= .9$, $\sigma = 14.8530$, $\mu = 71.2499$

2) $\mu = 12$ $\sigma = 3.5$ area $= .99$



using calculator: inverse normal $= \boxed{20.1422}$

from table of SND of 99th percentile $\Rightarrow$ $Z = \frac{x-\mu}{\sigma}$

$2.33 = \frac{x-12}{3.5} \Rightarrow x = 20.155$ ⇐ another way of getting answer

3) $\mu = 1.184$, $\sigma = .587$

a) $P(x < 2)$ $Z = \frac{2-1.184}{.587} = 1.3901$

from Z-score table $P(Z < 1.3901) = .9177 = 91.77\%$

b) $P(x > 4)$

$Z = \frac{4-1.184}{.587} = 4.7973$

$P(Z > 4.7973) = 8.0420 \times 10^{-5}\ \%$

c) $P\left(\frac{x-1.184}{.587}\right) = 1.64$ Z-score of 95% = 1.64

$x = 2.146$ using calc inverse normal: 2.1495

$\boxed{\text{2.1495 Will not cause items to shake}}$

4) a) $P(X < 105) = P\left(Z < \frac{x-\mu}{\sigma}\right) = P\left(Z < \frac{105-104}{5}\right)$

$= P(Z < .2) = \boxed{.5793}$

b) $P(Z > 1) + P(Z < -1) = (1 - P(Z < 1)) + P(Z < -1)$

$= (1 - .8413) + .1587 = \boxed{.3173}$

using Z-score table

c) $.1\% = .001 \Rightarrow P(X < C_1) = .0005$ $P(X > C_2) = .0005$

$C_1 = \mu + \sigma(.05^{th}\ \text{percentile})$ $\hookrightarrow P(X \le C_2) = .9995$

$C_1 = 104 + 5(-3.29)$ $C_2 = \mu + \sigma(.9995)$

$C_1 = 87.55$ $C_2 = 104 + 5(3.29)$

$\boxed{\text{below 87.55 or above 120.45}}$ $C_2 = 120.45$

5) $\mu = .499$ $\sigma = .002$ $P(\text{not acceptable}) = 1 - P(.496 < X < .504)$

$= 1 - P\left(\frac{.496-.499}{.002} < Z < \frac{.504-.499}{.002}\right)$

$= 1 - P(-1.5 < Z < 2.5)$

$= 1 - (P(Z < 2.5) - P(Z < -1.5))$

$= 1 - (.9938 - .0668)$

$= .073$

$= 7.3\%$

W \ T	0	1	2	3	Marginal
0	.31	.22	.09	.04	.66
1	.15	.05	.04	.01	.25
2	.04	.03	.01	0	.08
Marginal	.5	.3	.14	.05	

6) a) $P(W=0, T=0) = .31$, $P[T=0] = .5$, $P[W=0] = .66$

$P[T=0] + P[V=.66] - P[W=0, T=0]$

$.5 + .66 - .31 = \boxed{.85}$

b) $E[T] \sum_{i=0}^{3} P_i T_i = .5 \cdot 0 + .3 \cdot 1 + .14 \cdot 2 + .05 \cdot 3$

$E[T] = 0 + .3 + .28 + .15 = \boxed{.73 \text{ packages of toilet paper}}$

c) $P[W=w/T=1] = P\left[\frac{T=1, W=w}{P[T=1]}\right]$

W	$P[W=w/T=1]$
0	$.22/.3$
1	$.05/.3$
2	$.03/.3$
Total	$= 1$

$\Rightarrow E[W/T=1] = \sum_{w=0}^{2} w \cdot P[W=w/T=1]$

$= 0 + (1 \cdot .05/.3) + (2 \cdot \frac{.03}{.3})$

$= \frac{.11}{.3} = \frac{11}{30}$

$\Downarrow$

$E[W^2/T=1] = \sum_{w=0}^{2} w^2 P[W=w/T=1]$

$= 0 + (1 \cdot \frac{.05}{.3}) + (4 \cdot \frac{.03}{.3})$

$= \frac{.17}{.3} = \frac{17}{30}$

$V(x) = E(x^2) - (E(x))^2 = E(W^2/T=1) - E^2(W/T=1)$

$= \frac{17}{30} - \left(\frac{11}{30}\right)^2 = \frac{510}{900} - \frac{121}{900} = \frac{389}{900}$

$\sigma = \sqrt{V(x)}$

$\sigma = \sqrt{389/900} = \boxed{.6574 \text{ packages of disinfectant wipes}}$