CSE 015: Discrete Mathematics
Homework 3

Fall 2021
Provided Solution

1 Rules of Inference

Let be p is the proposition “Jane does not fly” and q be the proposition “Jane (she) is not a bird”. Then this is an argument of the type

$$\begin{array}{l} p \to q \\ \neg q \\ \hline \therefore \neg p \end{array}$$

Indeed, with this choice of propositions the first premise is “If Jane does not fly, then she (Jane) is not a bird“ ($p \to q$), and the second premise $\neg q$ is “It is not the case that Jane is not a bird” (i.e., “Jane is a bird”). The conclusion $\neg p$ reads “It is not the case that Jane does not fly” (i.e., “Jane flies”). The argument form is therefore *modus tollens*, and assuming the premises are true the conclusion is true.

2 More Rules of Inference

a) Bats can fly and are mammals. Therefore bats are mammals.
Simplification $(p \wedge q) \to q$

b) Pigs are mammals or birds. Pigs are not birds. Therefore pigs are mammals.
Disjunctive syllogism $((p \vee q) \wedge \neg p) \to q$

c) Jack is a CSE major. Jack is a freshmen. Therefore Jack is a CSE major and a freshmen.
Conjunction $((p) \wedge (q)) \to (p \wedge q)$

d) Mary is a CSE major. Therefore Mary is a CSE major or Mary is a History major.
Addition $p \to (p \vee q)$

e) If I go hiking, I will sweat a lot. If I sweat a lot, I will lose weight. Therefore, if I go hiking, I will lose weight.
Hypothetical syllogism $((p \to q) \wedge (q \to r)) \to (p \to r)$

3 Checking arguments

a) If it is sunny, then I will go swimming. It is not sunny. Therefore I will not go swimming.
The argument is not valid. The premises are $p \to q$ and $\neg p$. There is no argument form that allows to infer $\neg q$ from these premises. You can also think about the truth table for $p \to q$. When p is false, $p \to q$ is true irrespective of the value of q, i.e. when p is false, then q can be either true or false.

b) If it is Sunday, then I will go to the park. I will not go to the park. Therefore it is not Sunday. The premises are $p \to q$ and $\neg q$. The conclusion is $\neg p$. The argument is correct because it applies the *modus tollens* argument form.

c) I will pass the class if and only if I score at least 60% on the final exam. I scored 55% on the final exam. Therefore I will not pass the class. The premise includes $p \leftrightarrow q$ and $\neg q$ where p is "I will pass the class" and q is "I score at least 60% on the final." Recall that $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$. Then, by *simplification* we can derive $(p \to q)$. This implication, combined with $\neg q$ imply $\neg p$ ("I will not pass the class"), by using *modus tollens*. Therefore the argument is correct.

4 Proofs by Contraposition

The theorem is

if n is an integer and n^2 is odd, then n is odd.

Using quantifiers and the predicate $O(x)$ for "x is odd," the theorem could be stated as:

$$\forall n (O(n^2) \to O(n))$$

where the set of integers is the domain for the quantifiers. The part within parenthesis has the form $p \to q$ where p is the proposition "n^2 is odd" and q is the proposition "n is odd". The contrapositive is $\neg q \to \neg p$. It therefore reads "If n is an integer and n is not odd, then n^2 is not odd". Equally valid answers are "If n is not odd, then n^2 is not odd" or "If n is even, then n^2 is even."

Alternatively, one could look at the theorem statement as $p \to q$ where p is "n is an integer and n^2 is odd" and q is the proposition "n is odd'. In this case p is the conjunction (logical and) between the propositions "n is an integer" and "n^2 is odd." The contrapositive would still be $\neg q \to \neg p$, and in this case if would read "if n is not odd, then it is not the case that n is an integer and n^2 is odd". Furthermore, $\neg p$ could be rewritten using De Mogan's law $\neg(a \wedge b) \equiv \neg a \vee \neg b$. This would lead to the statement "if n is not odd, then n is not an integer or n^2 is not odd." All of these are equally valid answers.

5 Proof by Cases

For $n = 3$, the tautology to show is the following:

$$[(p_1 \vee p_2 \vee p_3) \to q] \leftrightarrow [(p_1 \to q) \wedge (p_2 \to q) \wedge (p_3 \to q)]$$

Recalling the definition of tautology, we need to write a truth table with four atomic propositions p_1, p_2, p_3, q. For typsetting reasons it may be more practical to write two truth tables rather than one. The conclusion follows observing that the last column in the first table and the last column in the second table display the same value for each combination of values for p_1, p_2, p_3, q.

p_1	p_2	p_3	q	$(p_1 \vee p_2 \vee p_3)$	$(p_1 \vee p_2 \vee p_3) \to q$
F	F	F	F	F	T
F	F	F	T	F	T
F	F	T	F	T	F
F	F	T	T	T	T
F	T	F	F	T	F
F	T	F	T	T	T
F	T	T	F	T	F
F	T	T	T	T	T
T	F	F	F	T	F
T	F	F	T	T	T
T	F	T	F	T	F
T	F	T	T	T	T
T	T	F	F	T	F
T	T	F	T	T	T
T	T	T	F	T	F
T	T	T	T	T	T

Table 1: Truth table for $(p_1 \vee p_2 \vee p_3) \to q$.

p_1	p_2	p_3	q	$p_1 \to q$	$p_2 \to q$	$p_3 \to q$	$(p_1 \to q) \wedge (p_2 \to q) \wedge (p_3 \to q)$
F	F	F	F	T	T	T	T
F	F	F	T	T	T	T	T
F	F	T	F	T	T	F	F
F	F	T	T	T	T	T	T
F	T	F	F	T	F	T	F
F	T	F	T	T	T	T	T
F	T	T	F	T	F	F	F
F	T	T	T	T	T	T	T
T	F	F	F	F	T	T	F
T	F	F	T	T	T	T	T
T	F	T	F	F	T	F	F
T	F	T	T	T	T	T	T
T	T	F	F	F	F	T	F
T	T	F	T	T	T	T	T
T	T	T	F	F	F	F	F
T	T	T	T	T	T	T	T

Table 2: Truth table for $(p_1 \to q) \wedge (p_2 \to q) \wedge (p_3 \to q)$.