Probability and Statistics—Exam 1
Thursday, September 30, 2021

Full Name:	Section 11D Mon., 530 PM - 720 PM TA: Li	Section 12D Mon., 130 PM - 320 PM TA: Natalie
Student ID Number:	Section 13D Mon., 1130 AM - 120 PM TA: Li	Section 14D Fri., 730 AM - 920 AM TA: Natalie

- Write your full name and discussion section number on every page of this packet.
- **Show all work!** ... unless otherwise instructed. Partial credit can only be awarded for presented work. Full credit can only be awarded with presented work.
- You may use any calculator that does not have internet access (i.e. no smart phones, laptops, or tablets). Round approximate results to 4 decimal places.
- $\boxed{\text{Box}}$ your final answers.
- Uniformly distributed, each question is worth 10 points.
- You may use the back of this exam as scratch paper/additional space.
- Pages of formulas have been provided.

Full name: _____ Section: _____

1. Curators at Katmai National Park are currently running a tournament[1] asking website visitors to vote on their favorite bear. For the sample of tag numbers below, compute the median and the range-rule-of-thumb interval $(\bar{x} - 2s, \bar{x} + 2s)$

$$435, 128, 634, 151, 812, 131, 402, 507$$

Solution: (of course, the student should still show work)

$$\begin{aligned} \text{sample mean: } \bar{x} &= 400 \\ \text{sample median} &= 418.5 \\ \text{sample standard deviation: } s &\approx 252.2822 \end{aligned}$$

and the range-rule-of-thumb interval is $(-104.5644, 904.5644)$

[1] Source: `https://explore.org/meet-the-bears`

Full name: _____ Section: _____

2. Suppose that Derek wants to make "old campus" (referring to the KL, COB1, COB2, S&E1, S&E2, SSM, and SSB buildings) a thing, and that the table below shows how many classes students have in "old campus".

classes	0	1	2	3	4
proportion	0.19	0.24	0.16	0.09	α

(a) Compute the value of α so that the information is a discrete probability distribution.

Solution:

$$0.19 + 0.24 + 0.16 + 0.09 + \alpha = 1.00 \quad \Rightarrow \quad \alpha = 0.32$$

(b) Compute the standard deviation for the number of classes in "old campus".

Solution:

$$\begin{aligned}
\mathrm{E}[X] &= (0)(0.19) + (1)(0.24) + (2)(0.16) + (3)(0.09) + (4)(0.32) &&= 2.11 \\
\mathrm{E}[X^2] &= (0)^2(0.19) + (1)^2(0.24) + (2)^2(0.16) + (3)^2(0.09) + (4)^2(0.32) &&= 6.81 \\
\mathrm{Var}(X) &= \mathrm{E}[X^2] - (\mathrm{E}[X])^2 &&= 2.3579 \\
\mathrm{SD}(X) &= \sqrt{\mathrm{Var}(X)} &&\approx 1.5355
\end{aligned}$$

(c) Compute the weighted mean of classes in "old campus" for the students who have at most 3 classes in "old campus".

Solution:

$$\bar{x} = \frac{(0)(0.19) + (1)(0.24) + (2)(0.16) + (3)(0.09)}{0.19 + 0.24 + 0.16 + 0.09} \approx 1.2206$$

Full name: _____ Section: _____

3. Suppose that in the AOA building, we observe the presence of nine people at work. Overall on campus, 85 percent of people wearing masks while working.

   (a) What is the probability that exactly 3 people in AOA are wearing a mask.

   Solution: With a sample size of $n = 9$ and proportion 0.85, the probability of exactly 3 mask wearers is

   $$P(k = 3) = \binom{9}{3}(0.85)^3(0.15)^6 \approx 0.0006$$

   (b) What is the probability that at least two people in AOA are not currently wearing a mask?

   Solution: With a sample size of $n = 9$ and proportion 0.15, the probability of at least 2 maskless persons is

   $$\begin{aligned} P(k \geq 2) &= 1 - [P(k = 0) + P(k = 1)] \\ &= 1 - \left[\tbinom{9}{0}(0.15)^0(0.85)^9 + \tbinom{9}{1}(0.15)^1(0.85)^8\right] \\ &\approx 0.4005 \end{aligned}$$

   (c) Compute the coefficient of variation $\dfrac{\sigma}{\mu}$ for the number of people that are wearing a mask.

   Solution: With a sample size of $n = 9$ and proportion 0.88, the sample statistics include

   $$\mu = np = (9)(0.85) = 7.65, \quad \sigma = \sqrt{np(1-p)} \approx 1.0712$$

   and the coefficient of variation is

   $$\frac{\sigma}{\mu} \approx 0.1400$$

Full name: _____ Section: _____

4. A glitch in the Microsoft Outlook e-mail client affected some of the faculty, and I will refer to one of my colleagues in particular. Let A be the event of an archived e-mail message. Let U be the event of an unread e-mail message. The professor has read 77 percent of their e-mails. The probability that a message was archived given that the message was read is 84 percent.

   (a) Write a complete sentence to describe the meaning of the math expression: $P(U^c|A^c)$

   Solution: $P(U^c|A^c)$ is the probability of selecting an already ready e-mail message given that the e-mail was not archived.

   (b) What should the sensitivity $P(A|U)$ and the specificity $P(A^c|U^c)$ be so that the probability of randomly selecting an unread message among the archived messages is over 95 percent? Assume that the sensitivity and specificity have the same value.

   Solution: Let $x = P(A|U)$ and $x = P(A^c|U^c) = 1 - P(A|U^c)$

$$\begin{aligned}
0.95 &\leq P(U|A) \\
0.95 &\leq \frac{P(A|U) \cdot P(U)}{P(A|U) \cdot P(U) + P(A|U^c) \cdot P(U^c)} \\
0.95 &\leq \frac{0.23x}{0.23x + (1-x)(0.77)} \\
0.2185x + 0.7315(1-x) &\leq 0.23x \\
0.7315 &\leq 0.743x \\
0.9845 &\leq x
\end{aligned}$$