You're expected to work on the problems before coming to the lab. Discussion session is not meant to be a one-way lecture. The TA will lead the discussion and correct your solutions if needed. For many problems, we will not release 'official' solutions. If you're better prepared for discussion, you will learn more. The TA is allowed to give some bonus points to students who actively engage in discussion and report them to the instructor. The bonus points earned will be factored in the final grade.

1. (Basic) Rank the following functions by order of growth; that is, find an ordering $g_1, g_2, \cdots, g_k$ (here k is the number of functions given) such that $g_1 = O(g_2)$, $g_2 = O(g_3), \cdots, g_{k-1} = O(g_k)$. (For example, if you are given functions, $n^2, n, 2n$, your solution should be either $n, 2n, n^2$ or $2n, n, n^2$.)

$n^2 + 2^n$	$n \log n$	$n^2 \log^2 n$	$n \log^2 n$	n^2	$\log^{100} n$
$\log \log n$	n^3	1	$\log n$	$\log n / \log \log n$	$n^2 / \log n$
$n^{10} \cdot 3^n$	4^n				

Sol.

1	$\log \log n$	$\log n / \log \log n$	$\log n$	$\log^{100} n$	$n \log n$
$n \log^2 n$	$n^2 / \log n$	n^2	$n^2 \log n$	$n^2 \log^2 n$	$n^2 + 2^n$
$n^{10} \cdot 3^n$	4^n				

2. (Basic) What is $\lim_{n\to\infty} \frac{n^{10}\cdot 3^n}{4^n}$? Which one is asymptotically no smaller between the two functions, $n^{10} \cdot 3^n$ and 4^n?

   Sol. 0, $n^{10 \cdot 3^n} = O(4^n)$.

3. (Basic) Formally prove that $n^2 + 100 = O(n^2)$ using the definition of $O(\cdot)$.

   Sol. Use $c = 101$ and $n_0 = 1$. Then for all $n \geq 1$, we have $n^2 + 100 \leq 101n^2$.

4. (Basic) Formally prove that $n^2 = \Omega(n^2 + 100)$ using the definition of $\Omega(\cdot)$.

   Sol. Use $c = 101$ and $n_0 = 1$. Then for all $n \geq 1$, we have $\frac{1}{101}(n^2 + 100) \leq n^2$.

5. (Basic) Formally prove that $n^2 = \Omega(n \log_2 n + 100)$ using the definition of $\Omega(\cdot)$.

   Sol. Use $c = 1$ and $n_0 = 200$. Then for all $n \geq 200$, we have $n^2 \geq n \log_2 n + 100$.

6. (Basic) Formally prove that $n + 10 = \Theta(50n + 1)$ using the definition of $\Theta(\cdot)$.

   Sol. Set $c_1 = 1/50$, $c_2 = 10$, and $n_0 = 1$. Then for all $n \geq 1$, we have $\frac{1}{50}(50n + 1) \leq n + 10 \leq 10(50n + 1)$

7. (Intermediate) Prove that $f = O(g)$ implies $g = \Omega(f)$.

   Sol. By def of O, there exists $c > 0$ and n_0 such that $f \leq cg$ for all $n \geq n_0$. In other words, $g \geq (1/c)f$ for all $n \geq n_0$, and $1/c > 0$. This implies $g = \Omega(f)$ as desired.

8. (Intermediate) Prove that $f = \Omega(g)$ and $g = \Omega(h)$ implies $f = \Omega(h)$.

9. (Basic) Order the following functions in asymptotically non-decreasing order using $\leq$ or $=$.[1].

   $n^n, 2^n - n^2, \lg n, \lg\lg n, 7n^3 + 10n^2, n^3 \lg n, 3^n, n \lg n, n^{\lg n}, 50000$.

   Sol. $50000 \leq \lg\lg n \leq \lg n \leq n \lg n \leq 7n^3 + 10n^2 \leq n^3 \lg n \leq n^{\lg n} \leq 2^n - n^2 \leq 3^n \leq n^n$

10. (Basic) Is the following correct? $O(n) \cdot O(n) = O(n^2)$. If it is correct, what is the meaning?

    Sol. Yes. For any $f \in O(n)$ and $g \in O(n)$, fg is in $O(n^2)$. More precisely: Look at the LHS. It's a product of two functions f and g which are both in $O(n)$. The product is always a function in $O(n^2)$.

[1] For example, $n \leq 3n^2 + 5 = n^2 - 3 \leq n^3$