Homework Assignment #5

Remember, this Homework Assignment is **not collected or graded**! But you are advised to do it anyway because this is a Review for Exam #1. In addition, Homework Quiz #5 will be heavily based on these problems!

1. For which values of a will the following vector matrix system fail to have 3 pivots?

$$\begin{aligned} ax + 2y + 3z &= b_1 \\ ax + ay + 4z &= b_2 \\ ax + ay + az &= b_3. \end{aligned}$$

2. Write down the 3 by 3 elementary row matrices that perform the following elementary row operations:
   (a) Subtracts 5 times row 1 from row 2.
   (b) Subtracts -7 times row 2 from row 3.
   (c) P exchanges rows 1 and 2.

3. Consider the following:
   (a) If A is invertible, what is the inverse of A^T
   (b) If A is invertible and symmetric, what is the transpose of A^{-1}?
   (c) Illustrate both formulas for the matrix:

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}.$$

4. Suppose A is the following matrix:

$$A = \begin{bmatrix} 1 & v_1 & 0 & 0 \\ 0 & v_2 & 0 & 0 \\ 0 & v_3 & 1 & 0 \\ 0 & v_4 & 0 & 1 \end{bmatrix}$$

   (a) Factor $A = LU$ assuming $v_2 \neq 0$.
   (b) Find A^{-1}. You'll see is has the same form as A.

5. Use Gauss Jordan Elimination to calculate the inverse of:

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}.$$

6. The trace of a matrix tr(A) is defined as the sum of entries along the diagonal. So, for a 4×4 matrix A we have:

$$\text{tr}(A) = a_{11} + a_{22} + a_{33} + a_{44}.$$

   In the following, you will consider S_4, the set of all 4×4 matrices with tr$(A) = 0$.
   (a) Show that S_4 is a subspace.
   (b) Find a basis for S_4 and determine its dimension.

7. Find a basis for the following subspaces of $\mathbb{R}^4$:
   (a) The vectors satisfying $x_1 = 2x_4$.

(b) The vectors for which:

$$\begin{aligned} x_1 + x_2 + x_3 &= 0 \\ x_3 + x_4 &= 0. \end{aligned}$$

(c) The subspace spanned by the vectors:

$$\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 4 \\ 5 \end{bmatrix} \right\}.$$

8. Suppose A is an $m \times n$ matrix with rank r. Suppose that there **some** choices for $\vec{b}$ where:

$$A\vec{x} = \vec{b}$$

has **no solution**.

(a) What inequalities ($<$ or $\leq$) must be true between m, n and r?

(b) How do we know that $A^T\vec{y} = 0$ has a non-zero solution?

9. Find dimensions and bases for the four fundamental subspaces for:

(a) $A = \begin{bmatrix} 0 & 3 & 3 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$.

(b) $B = \begin{bmatrix} 1 & 1 \\ 4 & 4 \\ 5 & 5 \end{bmatrix}$.

Hint: You should be able to figure these out without extensive calculations.

10. Consider the following matrix A and vector $\vec{b}$:

$$A = \begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 0 & 0 & 0 \\ 2 & 4 & 0 & 1 \end{bmatrix} \text{ and } \vec{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}.$$

(a) Find a basis for the nullspace of A.

(b) Find the general solution to $A\vec{x} = \vec{b}$ when a solution exists.

(c) Find a basis for the column space of A.

(d) Find the rank of A and A^T.