Particle kinematics

Instructor
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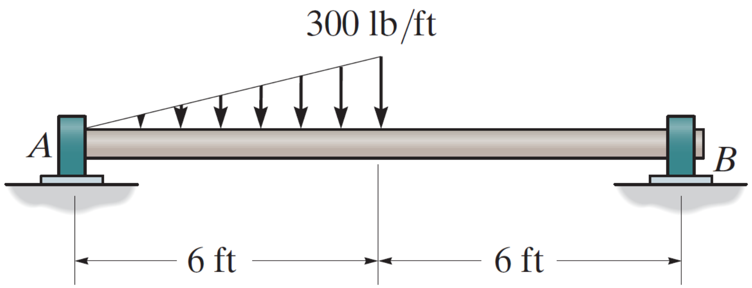
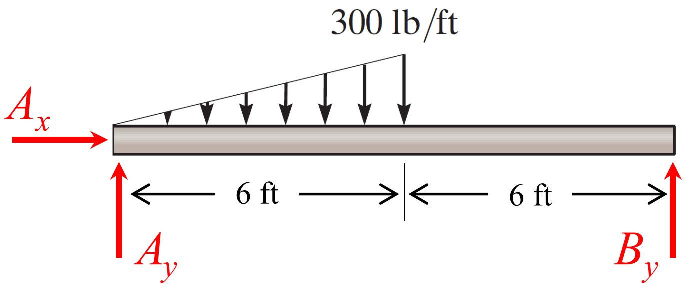
MERCED

Review



The shaft is supported by a thrust bearing at *A* and a journal bearing at *B*. Draw the shear and moment diagrams

The FBD is



The reactions are given by

$$\sum M_A = -4(6)(300)(0.5) + 12(B_y) = 0$$

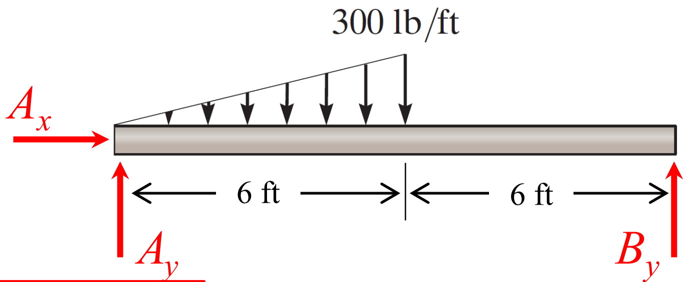
$$\boxed{B_y = 300 \text{ lb}}$$

$$\sum F_y = A_y - (6)(300)(0.5) + B_y = 0$$

$$\boxed{A_y = 600 \text{ lb}}$$

$$\sum F_x = A_x = 0$$

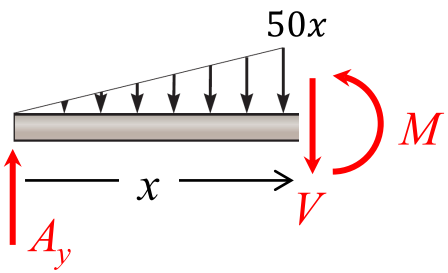
$$\boxed{A_x = 0 \text{ lb}}$$



$\boxed{A_y = 600 \text{ lb}}$ $\boxed{B_y = 300 \text{ lb}}$

With the reactions known, we need to divide the beam in sections. In this case $x = 6$ ft is the only "cut" point

$0 \leq x < 6$



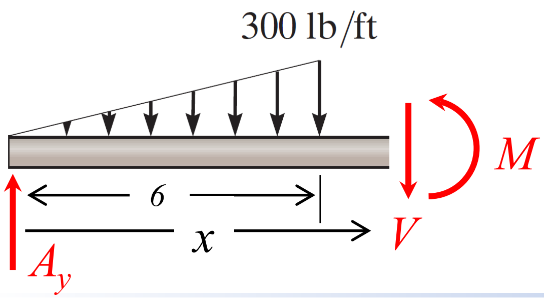
$$\sum F_y = A_y - (x)(50x)(0.5) - V = 0$$

$$\sum M_x = M + (x/3)(x)(50x)(0.5) - (x)(A_y)$$

$$\boxed{V = 600 - 25x^2}$$

$$\boxed{M = 600x - \frac{50}{6}x^3}$$

$6 < x \leq 12$



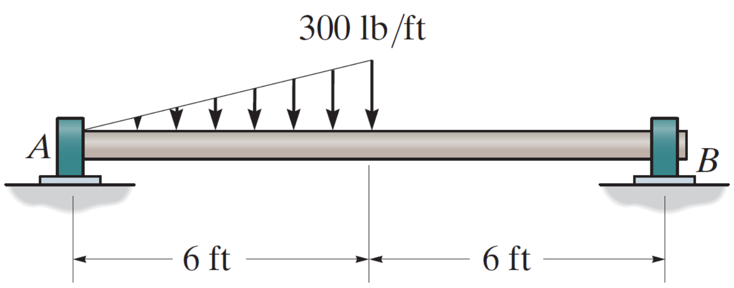
$$\sum F_y = A_y - (6)(300)(0.5) - V = 0$$

$$\sum M_x = M + (x - 4)(6)(300)(0.5) - (x)(A_y)$$

$$\boxed{V = -300}$$

$$\boxed{M = -300x + 3600}$$

Example



$0 \le x < 6$

$$\boxed{V = 600 - 25x^2}$$

$$\boxed{M = 600x - \frac{50}{6}x^3}$$

$6 < x \le 12$

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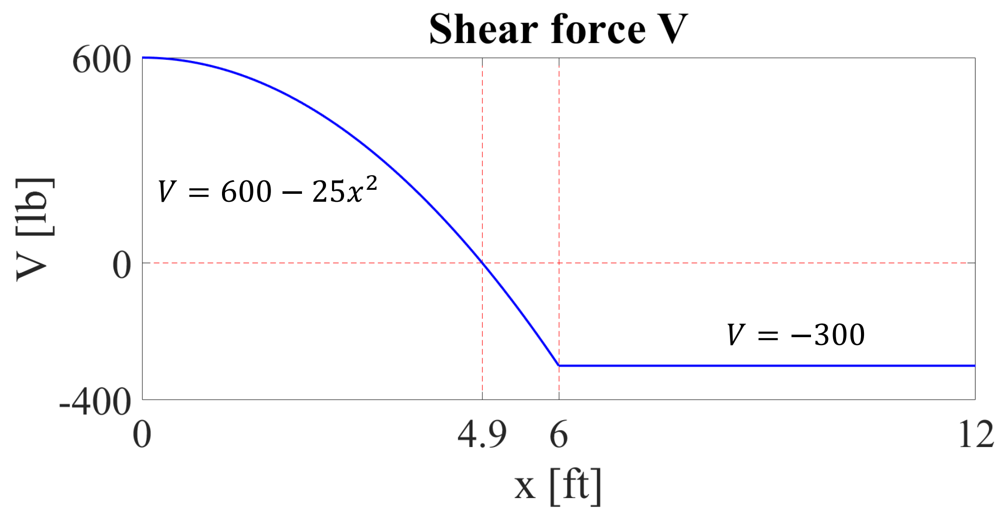
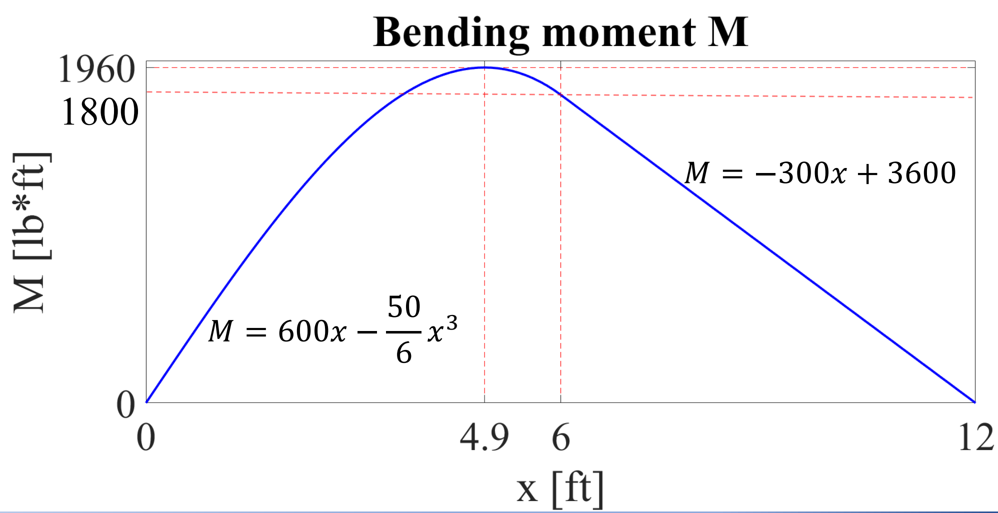
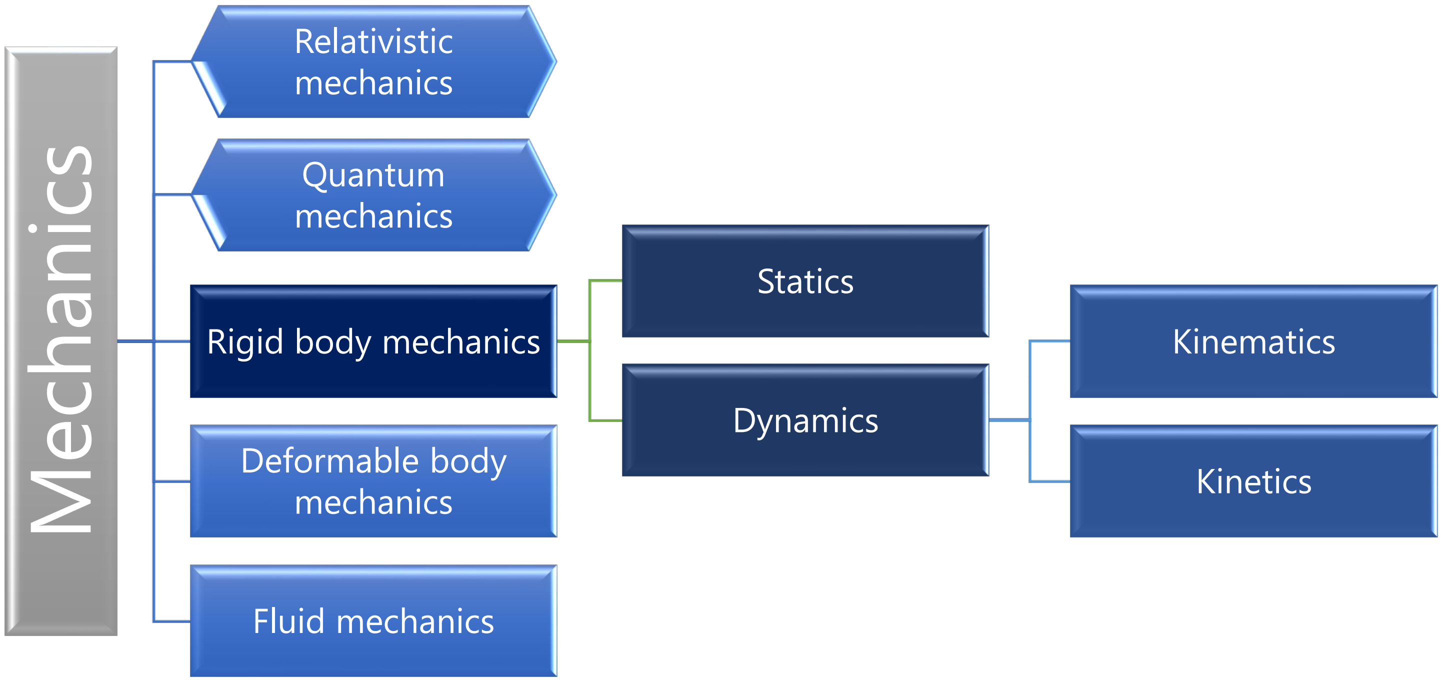




Dynamics



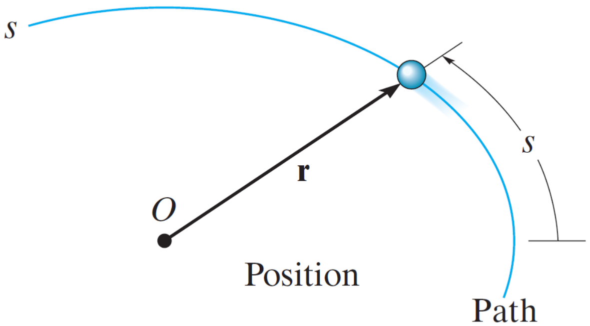
Statics study bodies in equilibrium.

Dynamics:

Kinematics studies geometric aspects of motion.

Kinetics concerned with the forces causing the motion.

Some definitions



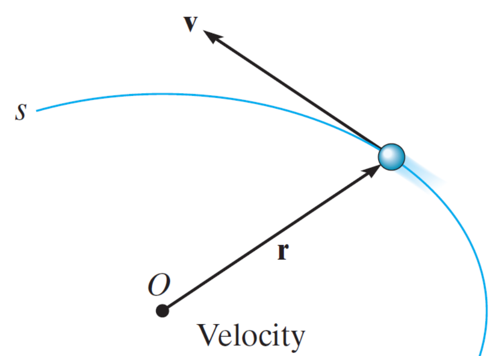
We can describe the motion of a particle using the concept of the position vector introduced in statics

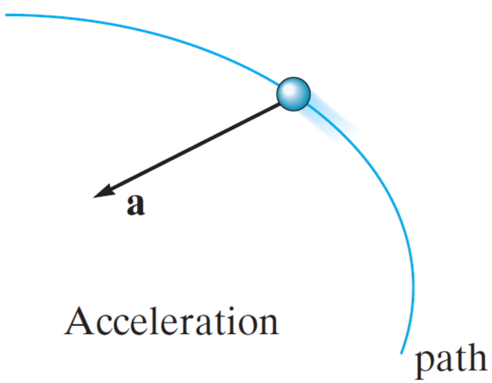
The transition from statics to dynamics can be summarized as: the position vector changes over time

The path described by a particle over time may be rectilinear, curvilinear, circular, etc.

Given $\boldsymbol{r} = \boldsymbol{r}(t)$ we can define the velocity of a particle as the change in position over time

$$\boldsymbol{v} = \frac{d\boldsymbol{r}}{dt}$$





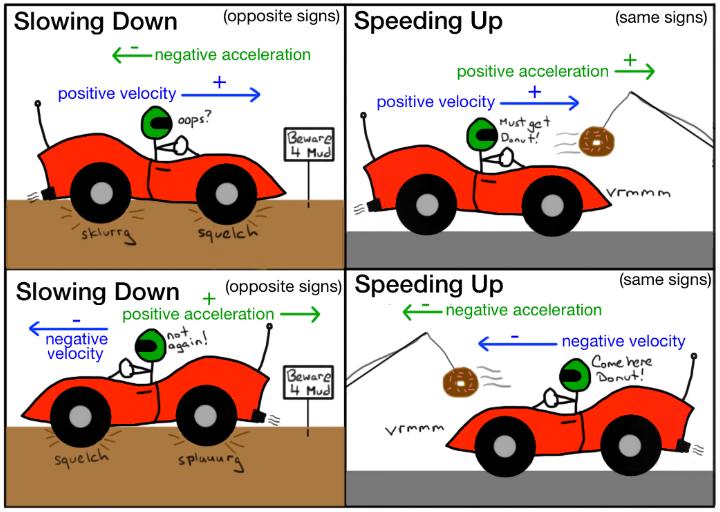
And given $\boldsymbol{v}(t)$ we can define the acceleration of a particle in a similar way

$$\boldsymbol{a} = \frac{d\boldsymbol{v}}{dt}$$

Applications for components design

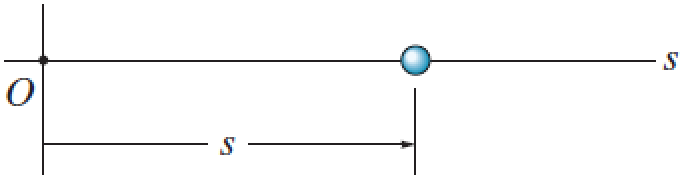
When objects move in such a way that their rotation can be dismissed, we can treat them as particles

If we measure the altitude of this rocket as a function of time, how can we determine its velocity and acceleration?



If you were designing any of these machines, why would it be important to know their velocity or acceleration? What about their position? How would you approach such problems?

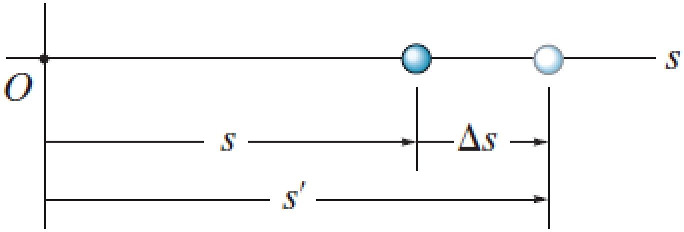
Rectilinear motion



Position

A particle travels along a <u>straight-line path</u> defined by the coordinate axis s.

The position of the particle at any instant, relative to the origin, O, is defined by the position vector ***r***, or the scalar s. Scalar *s* can be positive or negative. Typical units for ***r*** and *s* are meters (m) or feet (ft).



Displacement

The displacement of the particle is defined as its change in position.

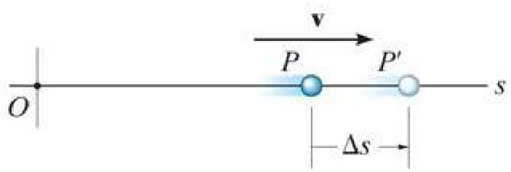
Vector form: Δ **r = r' - r**

Scalar form: $\Delta s = s' - s$

The total distance traveled by the particle, s_T, is a positive scalar that represents the total length of the path over which the particle travels.

Velocity

Velocity is a measure of the rate of change in the position of a particle. It is a vector quantity. The magnitude of the velocity is called speed, with units of m/s or ft/s.



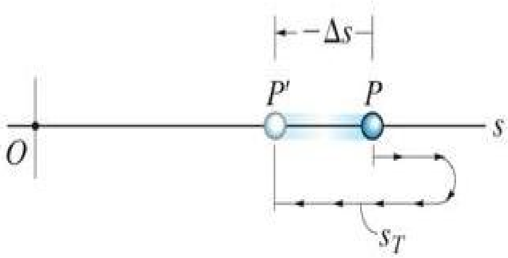
Velocity

The average velocity of a particle during a time interval Δt is

$$\boldsymbol{v}_{avg} = \boldsymbol{\Delta r} / \Delta t$$

The instantaneous velocity is the time-derivative of position.

$$\boldsymbol{v} = d\boldsymbol{r} / dt$$



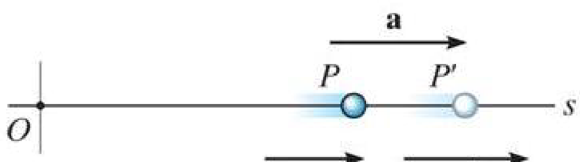
Average velocity and Average speed

Speed is the magnitude of velocity: $v = ds / dt$
(always positive)

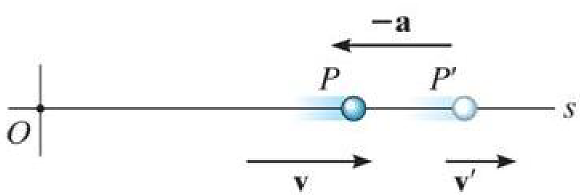
Average speed is the total distance traveled divided by elapsed time: $(v_{sp})_{avg} = s_T / \Delta t$

Acceleration

Acceleration is the rate of change in the velocity of a particle. It is a vector quantity. Typical units are m/s^2 or ft/s^2.



Acceleration



Deceleration

The instantaneous acceleration is the time derivative of velocity.

Vector form: $\mathbf{a} = d\boldsymbol{v} / dt$

Scalar form: $a = dv / dt = d^2s / dt^2$

Acceleration can be positive (speed increasing) or negative (speed decreasing).

Prove that: $a\,ds = v\,dv$

Summary of kinematic relations: Rectilinear motion

Differentiate position to get velocity and acceleration.

$$v = ds/dt \; ; \qquad a = dv/dt \quad \text{or} \quad a = v\, dv/ds$$

Integrate acceleration for velocity and position (constant acceleration).

Velocity:

$$\int_{v_o}^{v} dv = \int_{o}^{t} a\,dt \quad \text{or} \quad \int_{v_o}^{v} v\,dv = \int_{s_o}^{s} a\,ds$$

Position:

$$\int_{s_o}^{s} ds = \int_{o}^{t} v\,dt$$

Note that s_o and v_o represent the initial position and velocity of the particle at $t = 0$.

Constant acceleration

The three kinematic equations can be integrated for the special case when acceleration is constant ($a = a_c$) to obtain very useful equations. A common example of constant acceleration is gravity; i.e., a body freely falling toward earth. In this case, $a_c = g = 9.81\ m/s^2 = 32.2\ ft/s^2$ downward. These equations are:

$$\int_{v_0}^{v} \mathrm{d}v = \int_0^t a_c\, \mathrm{d}t \quad \Rightarrow \quad v = v_0 + a_c t$$

$$\int_{s_0}^{s} \mathrm{d}s = \int_0^t v\, \mathrm{d}t \quad \Rightarrow \quad s = s_0 + v_0 t + \frac{a_c t^2}{2}$$

$$\int_{v_0}^{v} v\, \mathrm{d}v = \int_{s_0}^{s} a_c\, \mathrm{d}s \quad \Rightarrow \quad v^2 = (v_0)^2 + 2a_c(s - s_0)$$

Example

A particle travels along a straight line to the right with a velocity of $v = (4t - 3t^2)$ m/s where t is in seconds. Also, $s = 0$ when $t = 0$.

Find: The position and acceleration of the particle when $t = 4$ s.

1) Take a derivative of the velocity to determine the acceleration.

$$a = dv / dt = d(4t - 3t^2) / dt = 4 - 6t$$

$\Rightarrow$ $a = -20$ m/s^2 (or in the $\leftarrow$ direction) when $t = 4$ s

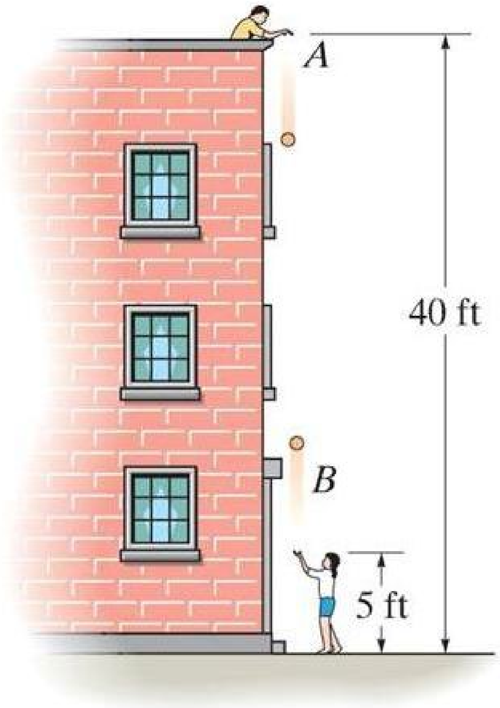
2) Calculate the distance traveled in 4s by integrating the velocity using $s_o = 0$:

$v = ds / dt \Rightarrow ds = v\,dt \Rightarrow \int_{s_o}^{s} ds = \int_{o}^{t} (4t - 3t^2)\,dt$

$\Rightarrow$ $s - s_o = 2t^2 - t^3$

$\Rightarrow$ $s - 0 = 2(4)^2 - (4)^3 \Rightarrow s = -32$ m (or $\leftarrow$)

Breakout room (15 min)



The ball A is released from rest at a height of 40 ft at the same time that ball B is thrown upward, 5 ft from the ground. The balls pass one another at a height of 20 ft.

Find the speed at which ball B was thrown upward. Gravity is equal to 32.2 ft/s^2

Both balls experience a constant downward acceleration of 32.2 ft/s^2 due to gravity. Apply the formulas for constant acceleration, with a_c = -32.2 ft/s^2.

Solution:

1) First consider ball A. With the origin defined at the ground, ball A is released from rest ($(v_A)_o = 0$) at a height of 40 ft : ($(s_A)_o = 40$ ft). Calculate the time required for ball A to drop to 20 ft ($s_A = 20$ ft) using a position equation.

$$s_A = (s_A)_o + (v_A)_o t + (1/2)\, a_c t^2$$

So,

$$20 \text{ ft} = 40 \text{ ft} + (0)(t) + (1/2)(-32.2)(t^2) \quad => \quad t = 1.115 \text{ s}$$

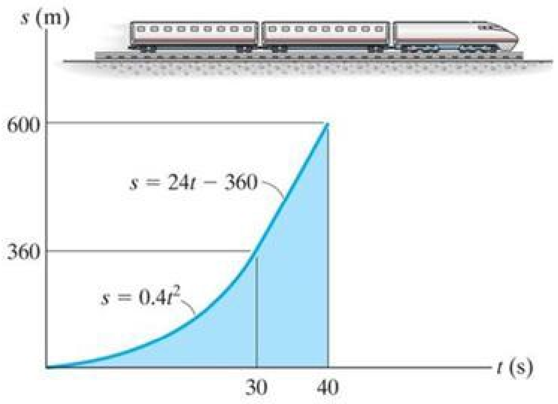
2) Now consider ball B. It is throw upward from a height of 5 ft ($(s_B)_o = 5$ ft). It must reach a height of 20 ft ($s_B = 20$ ft) at the same time ball A reaches this height (t = 1.115 s). Apply the position equation again to ball B using t = 1.115s.

$$s_B = (s_B)_o + (v_B)_o t + (1/2)\, a_c t^2$$

So,

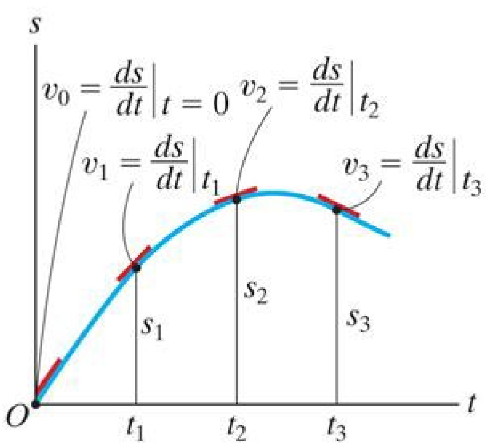
$$20 \text{ ft} = 5 + (v_B)_o(1.115) + (1/2)(-32.2)(1.115)^2$$

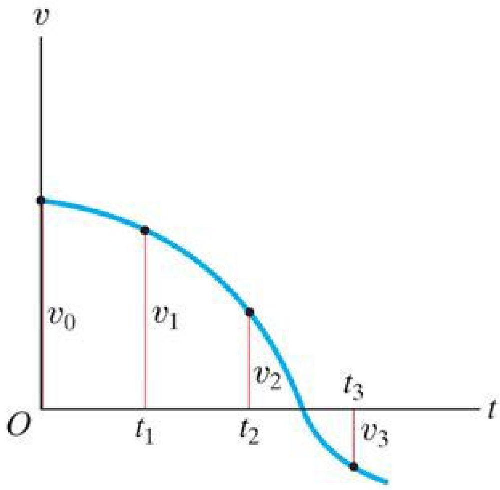
$$=> (v_B)_o = 31.4 \text{ ft/s}$$



Motion graphs: S-t graph

Graphs provides a good way to handle complex motions that would be difficult to describe with formulas
Graphs also provide a visual description of motion.
Slope and differentiation are linked and that integration can be thought of as finding the area under a curve

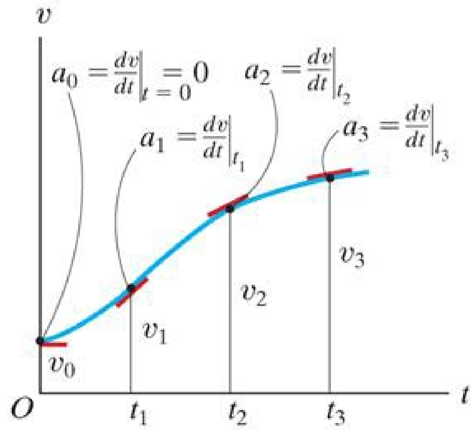




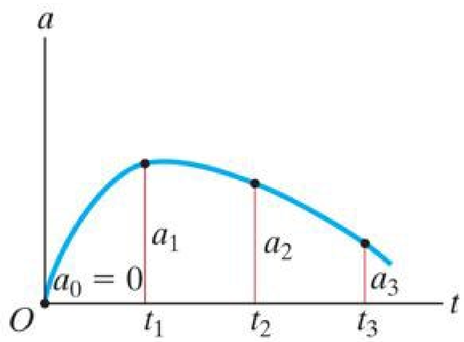
Plots of position vs. time can be used to find velocity vs. time curves. Finding the slope of the line tangent to the motion curve at any point is the velocity at that point (or v = ds/dt).

Therefore, the v-t graph can be constructed by finding the slope at various points along the s-t graph.

Motion graphs: **V-t graph**



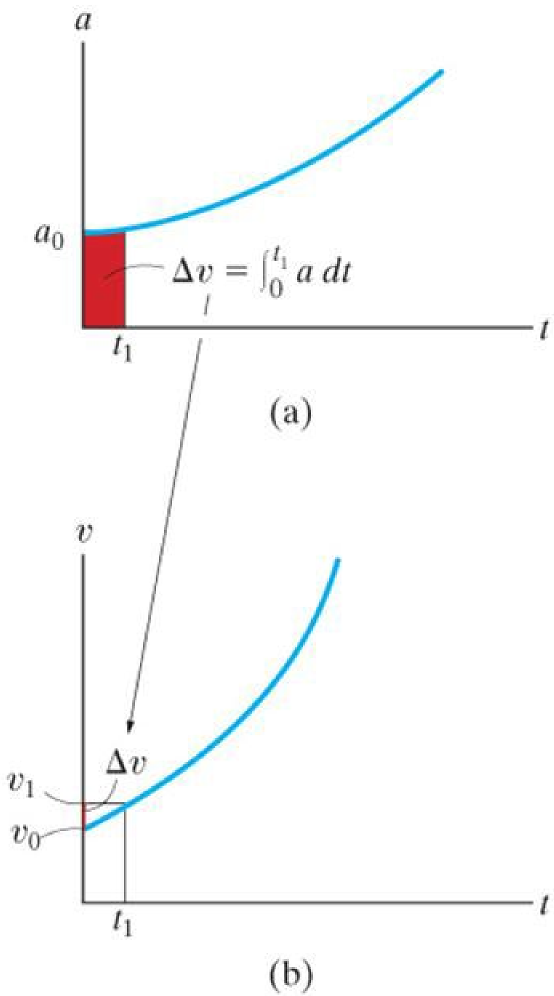
Plots of velocity vs. time can be used to find acceleration vs. time curves. Finding the slope of the line tangent to the velocity curve at any point is the acceleration at that point (or a = dv/dt).



Therefore, the acceleration vs. time (or a-t) graph can be constructed by finding the slope at various points along the v-t graph.

Also, the distance moved (displacement) of the particle is the area under the v-t graph during time Δt.

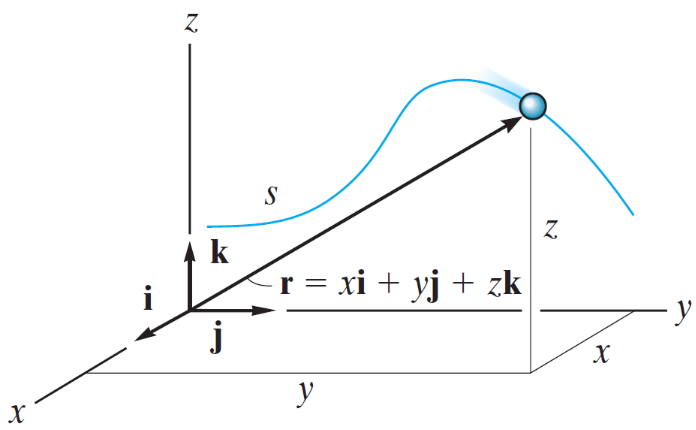
Motion graphs: **A-t graph**



Given the acceleration vs. time or a-t curve, the change in velocity (Δv) during a time period is the area under the a-t curve.

So we can construct a v-t graph from an a-t graph if we know the initial velocity of the particle.

Problems to be discussed on Friday



Rectangular components

Sometimes, it is useful to describe the motion of a particle in terms of the cartesian *x,y,z* components of ***r***

In this case, the *x,y,z* components are all functions of time. We can obtain ***v*** and ***a*** from those functions

We know that $\boldsymbol{r} = x\hat{\boldsymbol{\imath}} + y\hat{\boldsymbol{\jmath}} + z\widehat{\boldsymbol{k}}$ and $r = \sqrt{x^2 + y^2 + z^2}$

So, the first derivative of ***r*** gives

$$\boldsymbol{v} = \frac{d\boldsymbol{r}}{dt} = \frac{d}{dt}(x\hat{\boldsymbol{\imath}}) + \frac{d}{dt}(y\hat{\boldsymbol{\jmath}}) + \frac{d}{dt}(z\widehat{\boldsymbol{k}})$$

If we assume the vectors $\hat{\boldsymbol{\imath}}, \hat{\boldsymbol{\jmath}}, \widehat{\boldsymbol{k}}$ do not change over time, we get

$$\boldsymbol{v} = v_x\hat{\boldsymbol{\imath}} + v_y\hat{\boldsymbol{\jmath}} + v_z\widehat{\boldsymbol{k}}$$

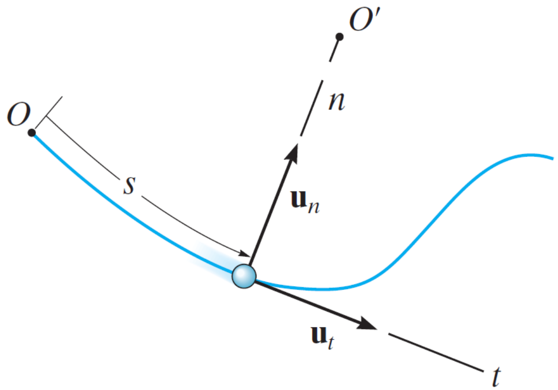
where $v_x = \dot{x}, v_y = \dot{y}, v_z = \dot{z}$

the dot notation represents the first time-derivative

Similarly, for the acceleration

$$\boldsymbol{a} = a_x\hat{\boldsymbol{\imath}} + a_y\hat{\boldsymbol{\jmath}} + a_z\widehat{\boldsymbol{k}}$$

where $a_x = \ddot{x}, a_y = \ddot{y}, a_z = \ddot{z}$



Normal and tangential components

If we know the path a particle will follow, it can be convenient to describe the motion with respect to that path using the *n* (normal) and *t* (tangential) coordinates

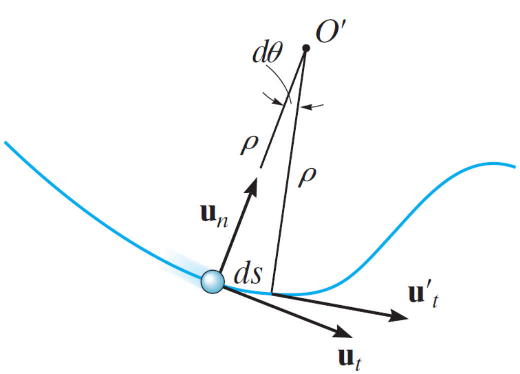
The *n* and *t* coordinates are always defined with respect to the particle instead of a fixed point. They <u>change over time</u>

According to our definitions, the position *s* is a function of time, its derivative gives the velocity:

$$\boldsymbol{v} = v\boldsymbol{u}_t$$

Where $\boldsymbol{u}_t$ is the unit vector in the *t* (tangential) direction and $v = \dot{s}$

Differentiating the velocity will get the acceleration $\boldsymbol{a} = \dot{\boldsymbol{v}} = \dot{v}\boldsymbol{u}_t + v\dot{\boldsymbol{u}}_t$

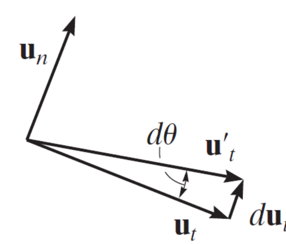


Considering an infinitesimal displacement *ds*

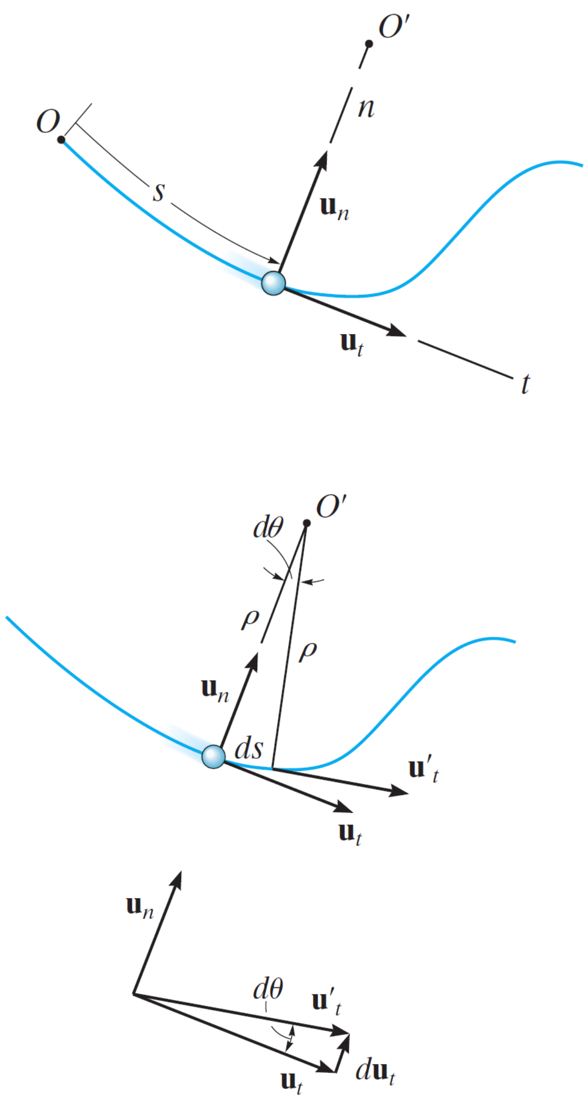
This gives the magnitude of $d\boldsymbol{u}_t$ as $du_t = (1)d\theta$

And the direction of $d\boldsymbol{u}_t$ is given by $\boldsymbol{u}_n$, thus

$$\dot{\boldsymbol{u}}_t = \dot{\theta}\boldsymbol{u}_n$$



Normal and tangential components



We also know that $ds = \rho\, d\theta$ which gives $\dot{\theta} = \dot{s}/\rho$ which means

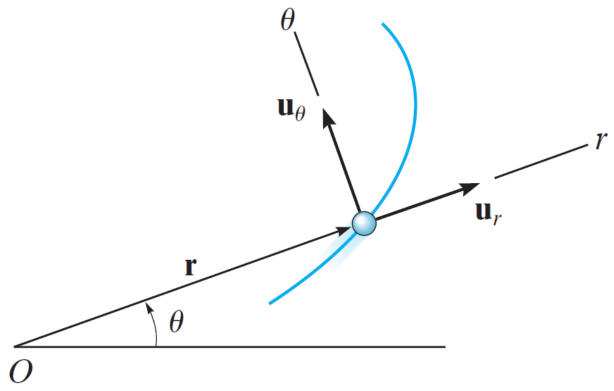
$$\dot{\boldsymbol{u}}_t = \dot{\theta}\boldsymbol{u}_n = \frac{\dot{s}}{\rho}\boldsymbol{u}_n = \frac{v}{\rho}\boldsymbol{u}_n$$

In summary, the acceleration is given by

$$a = a_t\boldsymbol{u}_t + a_n\boldsymbol{u}_n$$

With the tangential and normal components of the acceleration:

$$a_t = \dot{v} \qquad\qquad a_n = \frac{v^2}{\rho}$$



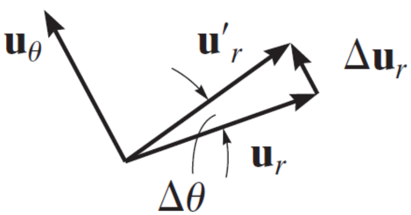
Cylindrical components

Sometimes, motion is most conveniently described using cylindrical coordinates or (in the plane) polar coordinates

In this case, the radial coordinate *r* extends outward from the origin and the transversal coordinate *q* is the counterclockwise angle between a fixed reference line and the *r* axis

At any instant the position is given by $$\boldsymbol{r} = r\boldsymbol{u}_r$$

The velocity is obtained by differentiation of $\boldsymbol{r}$ thus $$\boldsymbol{v} = \dot{\boldsymbol{r}} = \dot{r}\boldsymbol{u}_r + r\dot{\boldsymbol{u}}_r$$

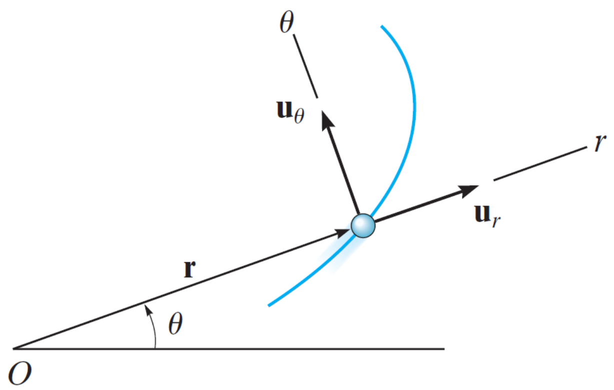


Following a similar rationale as before, the change $\dot{\boldsymbol{u}}_r$ is a change in direction given by the angular velocity $\dot{\theta}$ $$\dot{\boldsymbol{u}}_r = \dot{\theta}\boldsymbol{u}_\theta$$

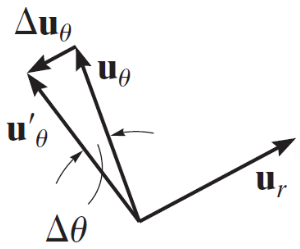
The velocity is then $\boldsymbol{v} = v_r\boldsymbol{u}_r + v_\theta\boldsymbol{u}_\theta$ with $v_r = \dot{r}$ and $v_\theta = r\dot{\theta}$

Finally, the acceleration is given by $$\boldsymbol{a} = \dot{\boldsymbol{v}} = \ddot{r}\boldsymbol{u}_r + \dot{r}\dot{\boldsymbol{u}}_r + \dot{r}\dot{\theta}\boldsymbol{u}_\theta + r\ddot{\theta}\boldsymbol{u}_\theta + r\dot{\theta}\dot{\boldsymbol{u}}_\theta$$

Cylindrical components



To determine $\dot{\boldsymbol{u}}_\theta$ we can follow the same procedure as before. In this case, the infinitesimal change in direction has a magnitude $d\theta$ and is in the direction $-\boldsymbol{u}_r$ thus

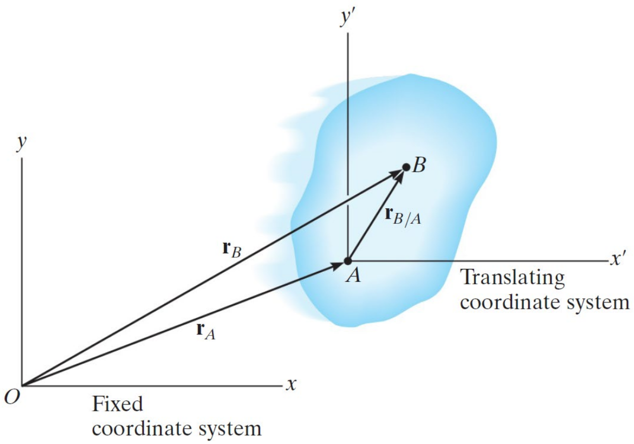


$$\dot{\boldsymbol{u}}_\theta = -\dot{\theta}\boldsymbol{u}_r$$

This gives $\boldsymbol{a} = a_r\boldsymbol{u}_r + a_\theta\boldsymbol{u}_\theta$ where

$$a_r = \ddot{r} - r\dot{\theta}^2$$
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$

Absolute motion



Generally we are not concerned with the motion of particles but of entire bodies, which translate and rotate

Translation can be understood using two coordinate systems: one fixed at the origin and one fixed in the body

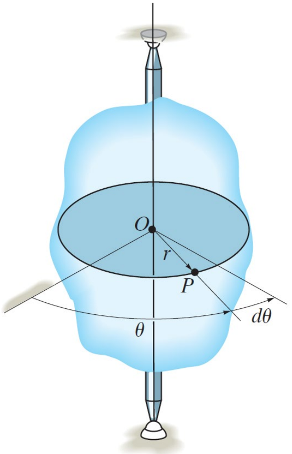
In this case, the position of any point in the body is given by the sum of the absolute plus relative positions:

$$\boldsymbol{r}_B = \boldsymbol{r}_A + \boldsymbol{r}_{B/A}$$

Taking the time derivatives of this equation $\quad \boldsymbol{v}_B = \boldsymbol{v}_A \quad \boldsymbol{a}_B = \boldsymbol{a}_A$

Since $\boldsymbol{r}_{B/A}$ is constant (no deformation)

To consider rotation, we can assume an axis fixed in the body



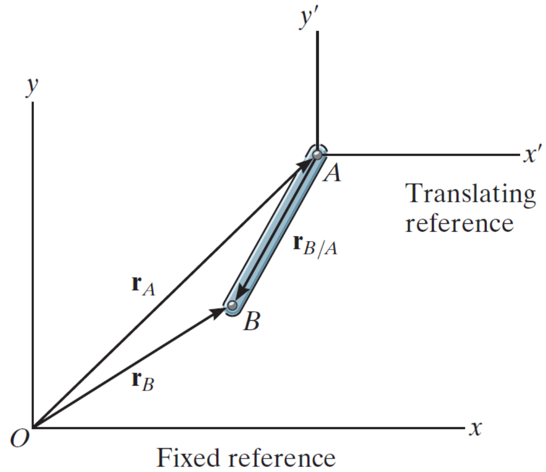
In this case, we can define the angular velocity and angular acceleration

$$\boldsymbol{\omega} = \frac{d\boldsymbol{\theta}}{dt} \qquad \boldsymbol{\alpha} = \frac{d\boldsymbol{\omega}}{dt}$$

As with particles, the challenge is in the definition of $\boldsymbol{r}$ and $\boldsymbol{q}$

Once the geometry is defined, everything follows naturally

Relative motion



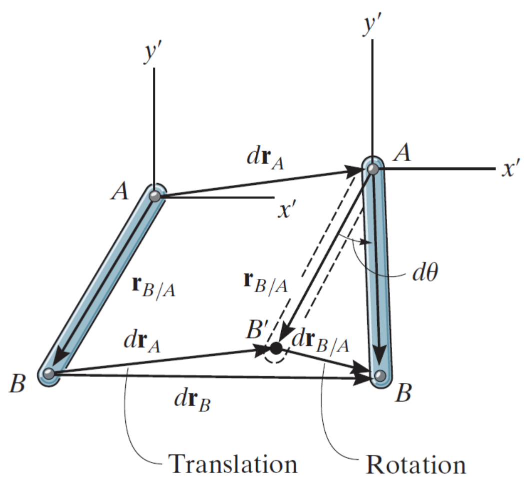
Normally, we cannot consider translation and rotation separately. Rigid body motion is a combination of the two
To consider them together, we must understand the motion of point *B* relative to point *A*

Under this analysis, we begin with the same equation

$$\boldsymbol{r}_B = \boldsymbol{r}_A + \boldsymbol{r}_{B/A}$$

But in this case, $\boldsymbol{r}_{B/A}$ is not constant. Point *B* has moved relative to *A* during the

We instead have $$\frac{d\boldsymbol{r}_B}{dt} = \frac{d\boldsymbol{r}_A}{dt} + \frac{d\boldsymbol{r}_{B/A}}{dt}$$

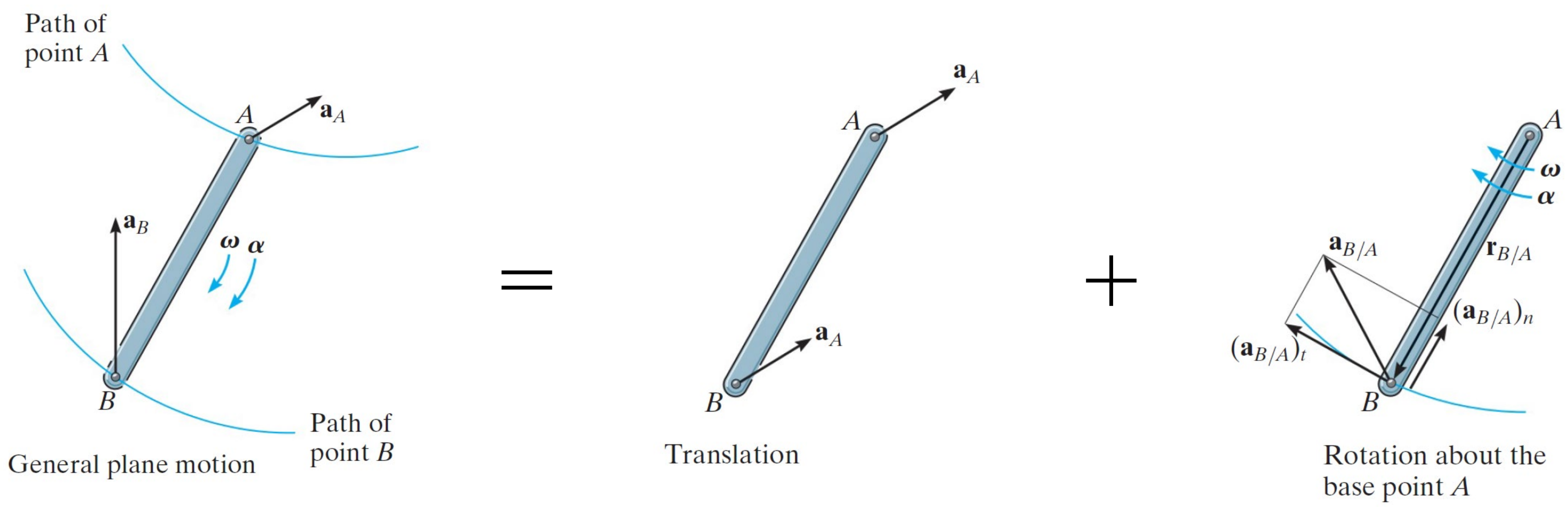


For a rotation *dq* we have $$\frac{dr_{B/A}}{dt} = r_{B/A}\frac{d\theta}{dt} = r_{B/A}\,\omega$$

In other words, the body appears to rotate with an angular velocity *w* about the axis passing through *A*

In vector terms we have $$\boldsymbol{v}_B = \boldsymbol{v}_A + \boldsymbol{\omega} \times \boldsymbol{r}_{B/A}$$

Relative motion



In the case of acceleration, we again consider motion as a combination of translation and rotation

$$\frac{d\boldsymbol{v}_B}{dt} = \frac{d\boldsymbol{v}_A}{dt} + \frac{d\boldsymbol{v}_{B/A}}{dt}$$

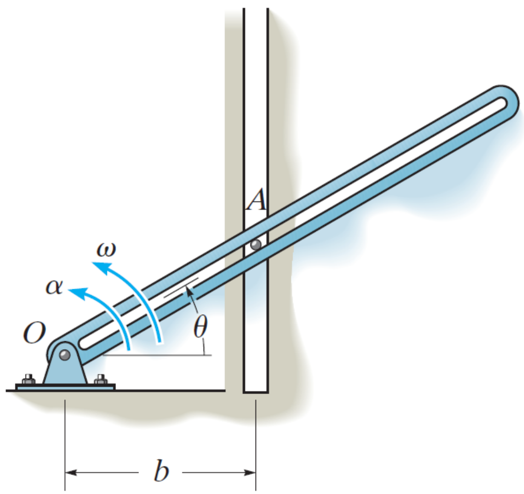
$\boldsymbol{a}_{B/A}$ will have <u>tangential</u> and <u>normal</u> components, since point B is rotating in a circular arc about A

We already know the magnitude of these components from our circular motion kinematics

$$a_{B/A} = \left(a_{B/A}\right)_t + \left(a_{B/A}\right)_n = \alpha r_{B/A} + \omega^2 r_{B/A}$$

In vector terms, we can write $\boldsymbol{a}_B = \boldsymbol{a}_A + \boldsymbol{\alpha} \times \boldsymbol{r}_{B/A} + \omega^2 \boldsymbol{r}_{B/A}$

Example



Determine the velocity and acceleration of the peg *A* which is confined between the vertical guide and the rotating slotted rod

All kinematic problems follow the same general procedure:

1. Use the geometry of the problem to establish an equation for your variable of interest
2. Apply the kinematic relations (time derivatives) to find velocities and accelerations
3. Solve any system of equations to find your unknown

In this case, we can write the vertical displacement of *A* as

$$y = b\tan\theta$$

Its velocity is given by

$$\dot{y} = b(\sec^2\theta)\dot{\theta}$$

And its acceleration is

$$\ddot{y} = b\left[2\sec\theta\left(\sec\theta\tan\theta\,\dot{\theta}\right)\dot{\theta} + \sec^2\theta\,\ddot{\theta}\right]$$

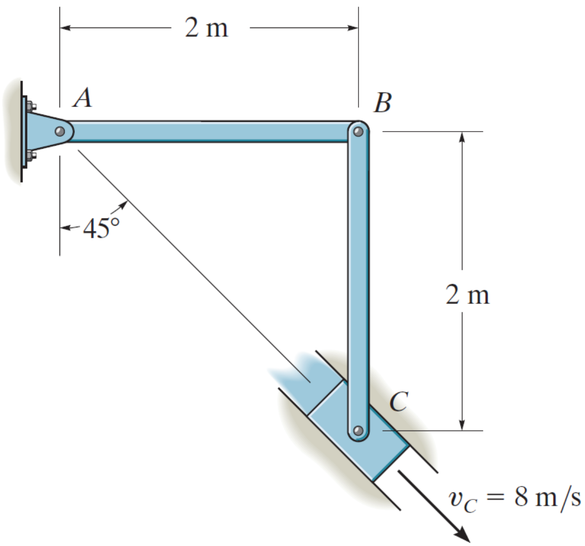
Or, using the appropriate substitutions

$$\dot{y} = v, \ddot{y} = a, \dot{\theta} = \omega, \ddot{\theta} = \alpha$$

$$\boxed{v = b\omega\sec^2\theta}$$

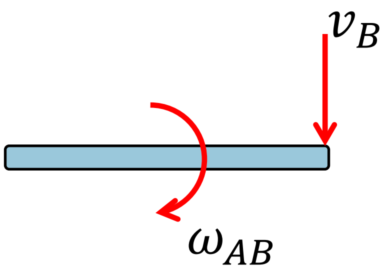
$$\boxed{a = b\sec^2\theta(2\omega^2\tan\theta + \alpha)}$$

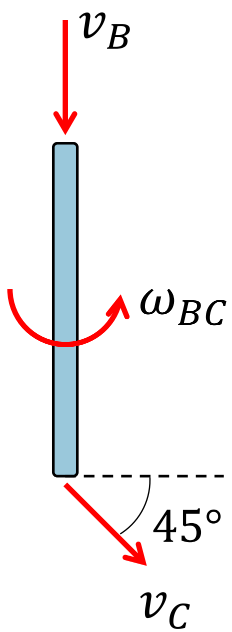
Example

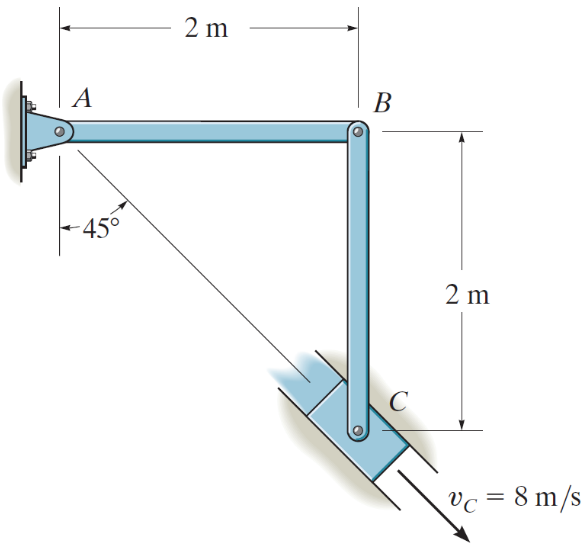


The slider block *C* moves at 8 m/s down the inclined groove. Determine the angular velocities of links *AB* and *BC*

It is useful to draw a kinetic diagram showing the velocities of all the bodies involved in the problem:





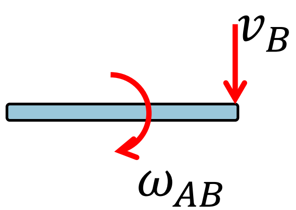


Solution

For link AB we have

$$\boldsymbol{v}_B = \boldsymbol{\omega}_{AB} \times \boldsymbol{r}_{AB}$$

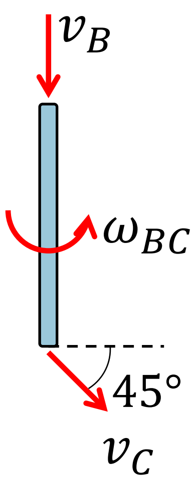
$$\boldsymbol{v}_B = \left(-\omega_{AB}\hat{\boldsymbol{k}}\right) \times (2\hat{\boldsymbol{\imath}}) \quad \Rightarrow \quad \boldsymbol{v}_B = -2\omega_{AB}\hat{\boldsymbol{\jmath}}$$



In the case of link BC we have $\boldsymbol{v}_B = \boldsymbol{v}_C + \boldsymbol{\omega}_{BC} \times \boldsymbol{r}_{B/C}$

$$-2\omega_{AB}\hat{\boldsymbol{\jmath}} = (8\cos 45^\circ\,\hat{\boldsymbol{\imath}} - 8\sin 45^\circ\,\hat{\boldsymbol{\jmath}}) + \left(\omega_{BC}\hat{\boldsymbol{k}}\right) \times (2\hat{\boldsymbol{\jmath}})$$

$$-2\omega_{AB}\hat{\boldsymbol{\jmath}} = (8\cos 45^\circ\,\hat{\boldsymbol{\imath}} - 8\sin 45^\circ\,\hat{\boldsymbol{\jmath}}) - 2\omega_{BC}\hat{\boldsymbol{\imath}}$$

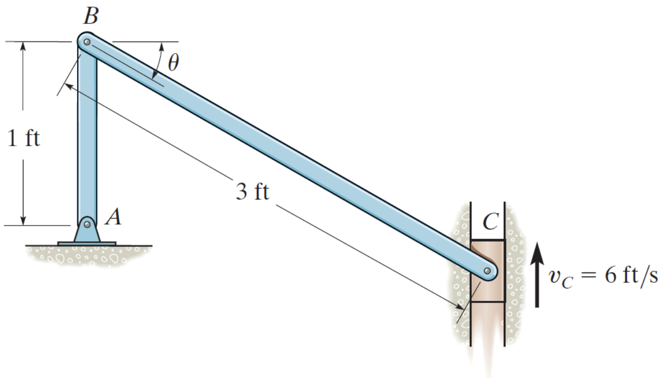


This gives two equations ($\hat{\boldsymbol{\imath}}$ and $\hat{\boldsymbol{\jmath}}$)

$$8\cos 45^\circ - 2\omega_{BC} = 0 \quad \Rightarrow \quad \boxed{\omega_{BC} = 2.83 s^{-1}}$$

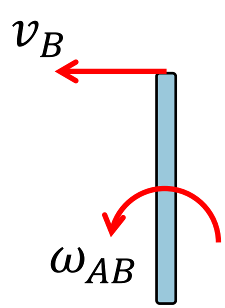
$$-8\sin 45^\circ = -2\omega_{AB} \quad \Rightarrow \quad \boxed{\omega_{AB} = 2.83 s^{-1}}$$

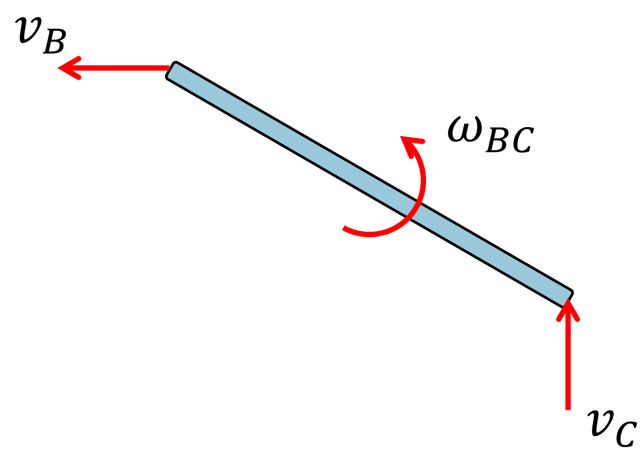
Breakout room (10 min)



Determine the angular velocities of links *AB* and *BC* at the instant $\theta = 30°$

The first thing we can do is draw our kinematic diagrams





We can begin from *A* since it is a fixed point and study the motion of *B*

$$\boldsymbol{v}_B = \boldsymbol{\omega}_{AB} \times \boldsymbol{r}_{AB}$$

$$\boldsymbol{v}_B = \left(\omega_{AB}\hat{\boldsymbol{k}}\right) \times (\hat{\boldsymbol{j}}) = -\omega_{AB}\hat{\boldsymbol{\imath}}$$

We can now study the motion of *B* from point *C* since it has a known velocity

$$\boldsymbol{v}_B = \boldsymbol{v}_C + \boldsymbol{\omega}_{BC} \times \boldsymbol{r}_{CB}$$

$$\boldsymbol{v}_B = 6\hat{\boldsymbol{j}} + \left(\omega_{BC}\hat{\boldsymbol{k}}\right) \times (-3\cos\theta\,\hat{\boldsymbol{\imath}} + 3\sin\theta\,\hat{\boldsymbol{j}})$$

We can now relate the two equations for $\boldsymbol{v}_B$

$$-\omega_{AB}\hat{\boldsymbol{\imath}} = 6\hat{\boldsymbol{j}} - 3\omega_{BC}\cos\theta\,\hat{\boldsymbol{j}} - 3\omega_{BC}\sin\theta\,\hat{\boldsymbol{\imath}}$$

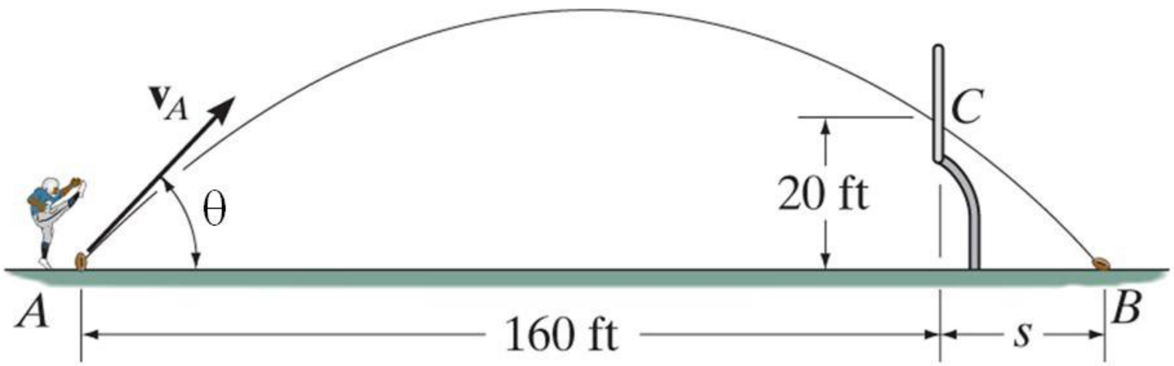
This gives

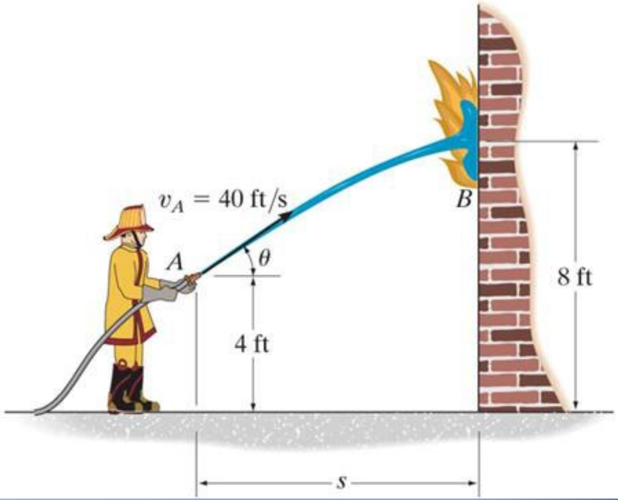
$$-\omega_{AB} = -3\omega_{BC}\sin\theta \quad \Rightarrow \quad \boxed{\omega_{AB} = 3.46\ s^{-1}}$$

$$0 = 6 - 3\omega_{BC}\cos\theta \quad \Rightarrow \quad \boxed{\omega_{BC} = 2.31\ s^{-1}}$$

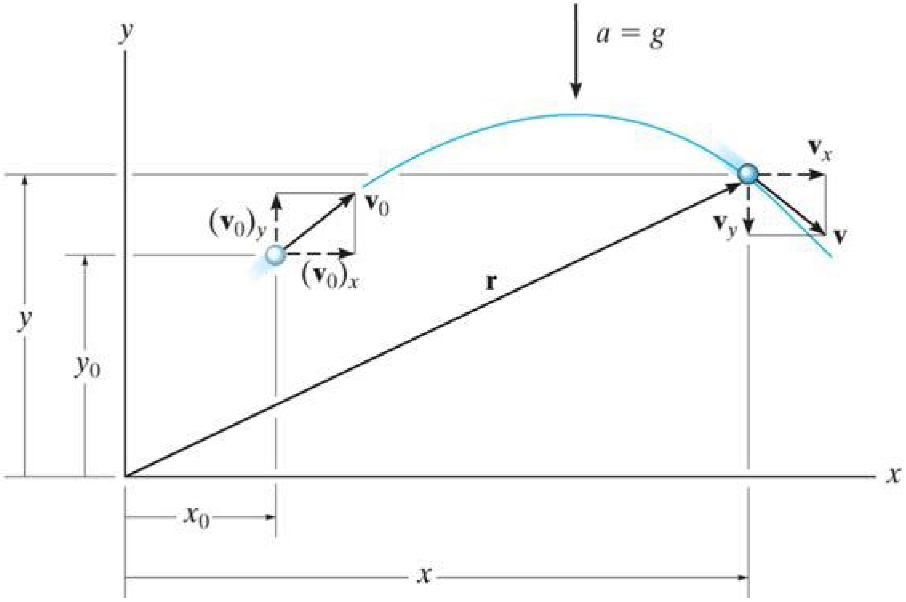
Motion of a projectile

Projectile motion can be treated as two rectilinear motions, one in the horizontal direction experiencing zero acceleration and the other in the vertical direction experiencing constant acceleration (i.e., from gravity).





Kinematic equations: horizontal motion



Since $a_x = 0$, the velocity in the horizontal direction remains constant ($v_x = v_{ox}$) and the position in the x direction can be determined by:

$$x = x_o + (v_{ox})\ t$$

Why is a_x equal to zero (assuming movement through the air)?

Kinematic equations: vertical motion

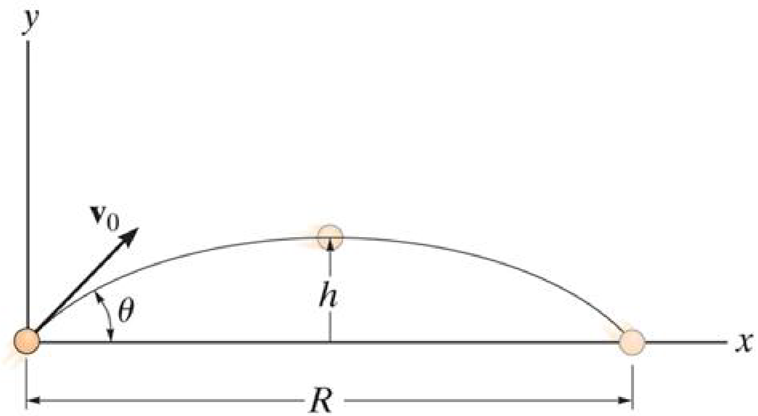
Since the positive y-axis is directed upward, $a_y = -g$. Application of the constant acceleration equations yields:

$$v_y = v_{oy} - g\,t$$

$$y = y_o + (v_{oy})\,t - \tfrac{1}{2}\,g\,t^2$$

$$v_y^2 = v_{oy}^2 - 2\,g\,(y - y_o)$$

Example



Given: v_o and θ

Find: The equation that defines y as a function of x.

Plan: "Eliminate time from the kinematic equations."

Solution: Using $v_x = v_o \cos\theta$ and $v_y = v_o \sin\theta$

We can write: $x = (v_o \cos\theta)t$ or $t = \dfrac{x}{v_o \cos\theta}$

$$y = (v_o \sin\theta)\, t - ½\, g\, (t)^2$$

By substituting for t:

$$y = (v_o \sin\theta) \left\{\frac{x}{v_o \cos\theta}\right\} - \frac{g}{2} \left\{\frac{x}{v_o \cos\theta}\right\}^2$$

Simplifying the last equation, we get:

$$y = (x \tan\theta) - \frac{g\,x^2}{2{v_o}^2}(1 + \tan^2\theta)$$

The above equation is called the “path equation” which describes the path of a particle in projectile motion.

The equation shows that the path is parabolic.