Homework 6

Answer all the required problems for each assignment and round your final solutions to two decimal places, if needed.

Assignment 1

(Around your solutions to two decimal places if necessary)

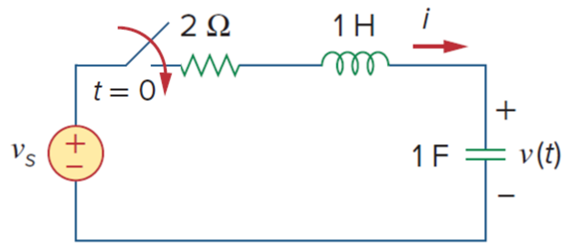
1. Determine the inverse Laplace transform of each of the following functions. **(9 pts/each = 81 pts)**
   a. $F(s) = \frac{1}{s} + \frac{2}{s+1}$
   b. $F(s) = \frac{e^{-4s}}{s+2}$
   c. $F(s) = \frac{3s+1}{s+4}$
   d. $F(s) = \frac{4}{(s+1)(s+3)}$
   e. $F(s) = \frac{6s}{(s+1)(s+2)}$
   f. $F(s) = \frac{s^2+2}{s^3+2s^2+2s}$
   g. $F(s) = \frac{10}{(s+1)(s^2+4s+8)}$
   h. $F(s) = \frac{2}{s(s+1)^2}$
   i. $F(s) = \frac{8}{s(s+1)^3}$
2. Given that $v(0^-) = 5,\ v'(0^-) = 10$, solve the following equation for the $v(t)$. **(19 pts)**

$$\frac{d^2v(t)}{dt^2} + 5\frac{dv(t)}{dt} + 6v(t) = 25e^{-t}u(t)$$

Assignment 2

(Around your solutions to two decimal places if necessary)

1. The initial energy stored in the following circuit is zero. The switch is closed at $t = 0$. Assume $V(s) = \mathcal{L}\{v(t)\}$ and $I(s) = \mathcal{L}\{i(t)\}$. **(50 pts)**



If $v_s(t) = 10\ V$,

a. Write the differential equations in terms of $v(t)$ and $i(t)$.
b. Find $V(s)$ and $I(s)$.
c. Find $v(t)$ and $i(t)$.
d. Find zeros and poles of $V(s)$ and $I(s)$.
e. Use the initial value theorem to find $v(0^+)$ and $i(0^+)$ from $V(s)$ and $I(s)$.
f. Use the final value theorem to find $v(\infty)$ and $i(\infty)$ from $V(s)$ and $I(s)$.
g. Do your answers in (e) and (f) make sense in the terms of the above circuit behavior? Please explain.

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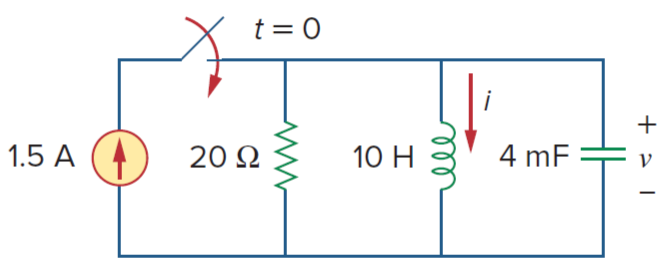

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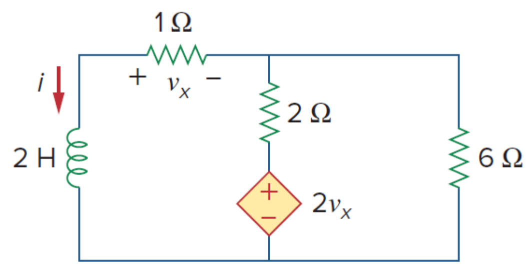
f. Use the final value theorem to find $v(\infty)$ and $i(\infty)$ from $V(s)$ and $I(s)$.

g. Do your answers in (e) and (f) make sense in the terms of the above circuit behavior? Please explain.

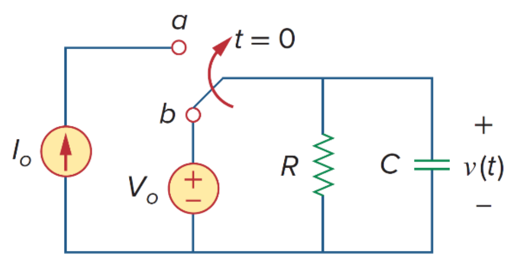
Assignment 3

1. Find $i(t)$ and $v_x(t)$ in the circuit below. Assume $I_0 = i(0^-) = 12\ A$. **(20 pts)**



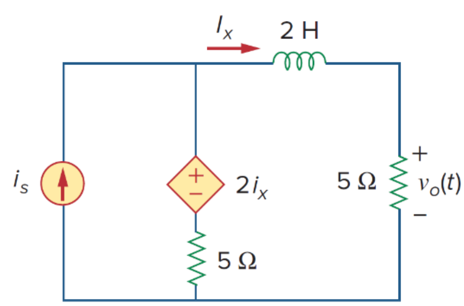
Answer: $\boldsymbol{i(t) = 12e^{-2t}\ A, v_x(t) = -12e^{-2t}\ V\ for\ t > 0.}$

2. The switch in the following circuit has been in position **b** for a long time. It is moved to position **a** at $t = 0$. Determine $v(t) for\ t > 0$. **(25 pts)**



Answer: $\boldsymbol{v(t) = (V_0 - I_0R)e^{-\frac{t}{\tau}} + I_0R, t > 0, where\ \tau = RC.}$

3. Assume there is no initial energy stored in the circuit below at $t = 0$ and that $i_s = 10\ u(t)\ A$. **(30 pts)**



   a. Use Thevenin's theorem to find $V_o(s)$.

   (Hints: Remove 5 Ω resistor and find $V_{Th} = V_{oc}$. Short 5 Ω resistor to find I_{sc} by using the node-voltage method, then $Z_{Th} = \frac{V_{Th}}{I_{sc}}$.

   b. Find the transfer function of $H(s) = \frac{V_o(s)}{I_s(s)}$

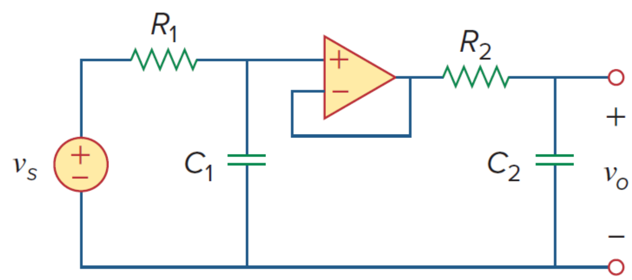
   c. Applying the initial- and final- value theorems to find $v_0(0^+)\ and\ v_o(\infty)$.

   d. Determine $v_o(t)$.

e. If $i_s = 20\cos(4t + 30^0)\,u(t)$ A, determine the steady-state response $v_{oss}(t)$.

Answer: $\boldsymbol{v_o(t) = 31.25(1 - e^{-4t})u(t)}$ **V;** $\boldsymbol{v_{oss}(t) = 31.25\sqrt{2}\cos(4t - 15^0)\,V}$

4. In the op-amp circuit below, $v_s(t) = 10u(t)$. Assume that $R_1 = R_2 = 10\ k\Omega$, $C_1 = 20\ \mu F, and\ C_2 = 100\ \mu F$. The op-amp in the circuit is ideal. The initial anergy stored in the circuit is zero. **(25 pts)**
   a. Find the transfer function $H(s) = \frac{V_o(s)}{V_s(s)}$.
   b. Determine the type of the circuit response based on the transfer function.
   c. Determine $v_o(t)\ for\ t > 0$.



Answer: $\boldsymbol{v_o(t) = (10 - 12.5e^{-t} + 2.5e^{-5t})u(t)\,V}$