ENGR 065 Circuit Theory

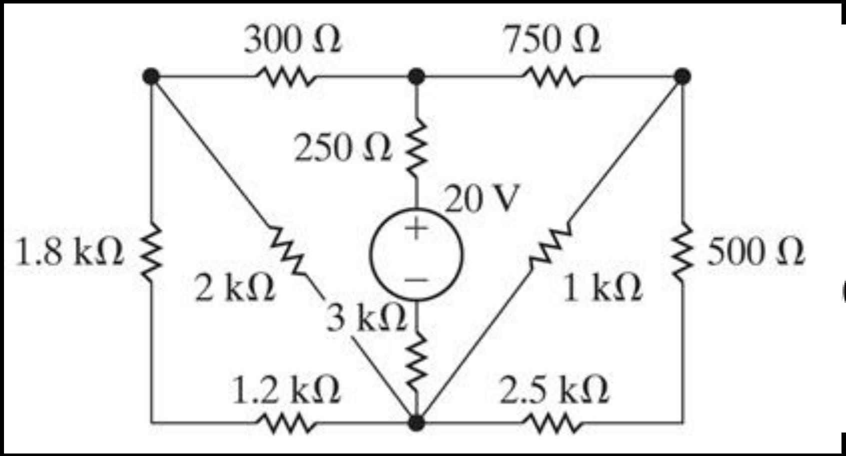
Lecture 5: Simple Resistive Circuits

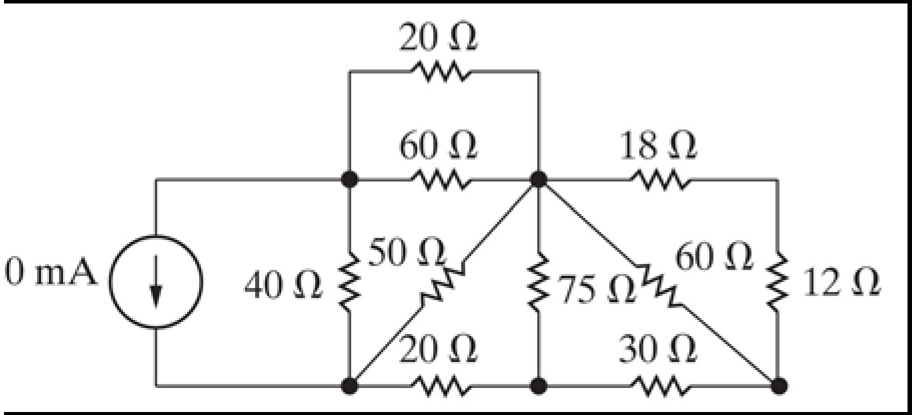
Today's Topics

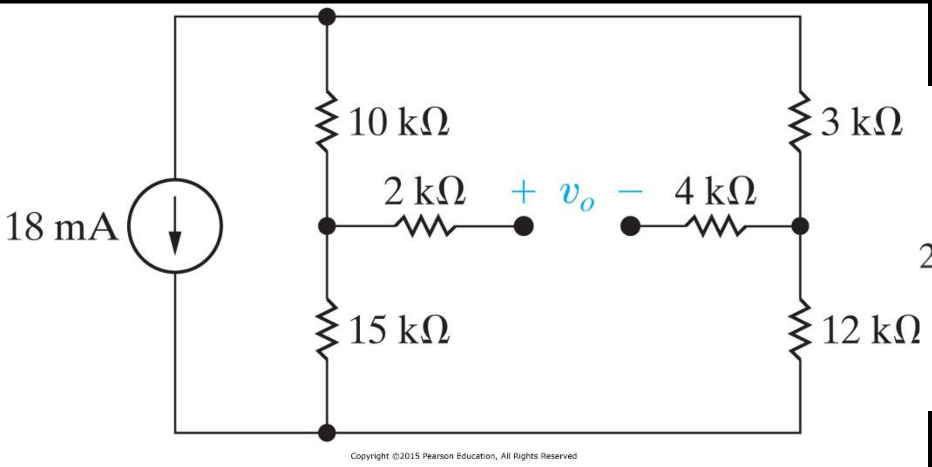
- Series- and parallel-connected resistive circuits
- How to simplify circuits using resistance combinations
- Voltage and current divider circuits

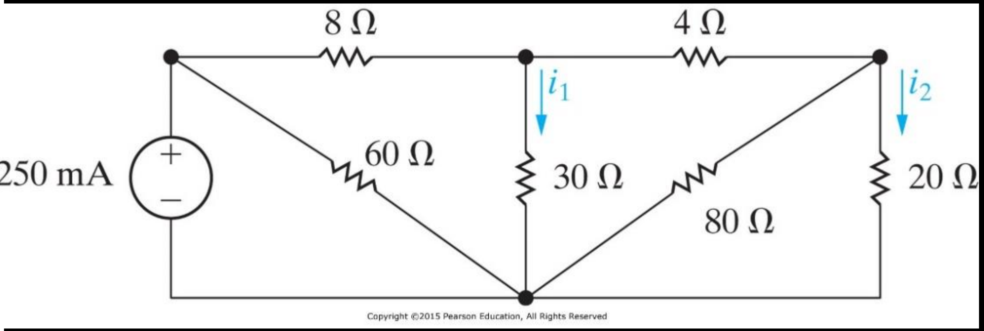
- Covered in Section 3.1, 3.2, 3.3, and 3.4

Some Examples of Resistive Circuits









Resistors in Series Connection

Resistors in series: two resistors connected at a single node and carry the same current.

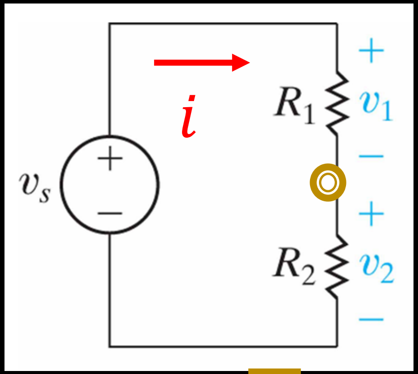
Applying KVL to the circuit *on the right*:

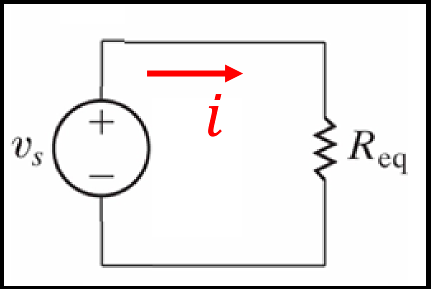
$$\boldsymbol{v_s} = v_1 + v_2 = iR_1 + iR_2 = i(R_1 + R_2)$$

So, the current in the circuit is: $i = \dfrac{v_s}{R_1 + R_2}$

Assuming: $R_{eq} = R_1 + R_2$, then: $i = \dfrac{v_s}{R_{eq}}$

R_{eq} is called equivalent resistance.





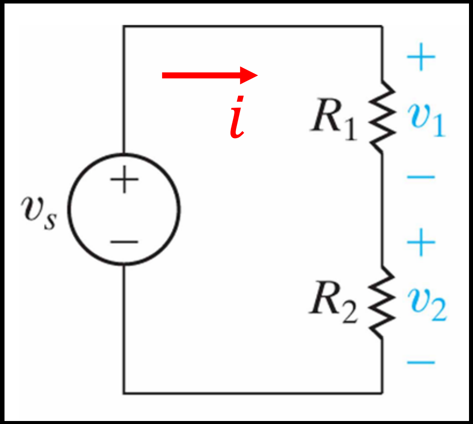
The solution can be extended to n series connected resistors:

$$R_{eq} = R_1 + R_2 + \cdots + R_{n-1} + R_n \quad \text{so} \quad i = \frac{v_s}{R_{eq}}$$

Note that the equivalent resistance is always greater than the largest resistance of the resistor in the series connection.

Voltage-Divider Circuits

A series connected resistive circuit can be used as a voltage divider. A voltage-divider circuit is used to develop more than one voltage level from a single voltage source.



$$i = \frac{v_s}{R_1 + R_2}$$ ***Memorize me!***

$$v_1 = R_1 i = \frac{R_1 v_s}{R_1 + R_2} = \frac{R_1}{R_{eq}} v_s$$ *Memorize this!*

$$v_2 = R_2 i = \frac{R_2 v_s}{R_1 + R_2} = \frac{R_2}{R_{eq}} v_s$$ *Memorize this!*

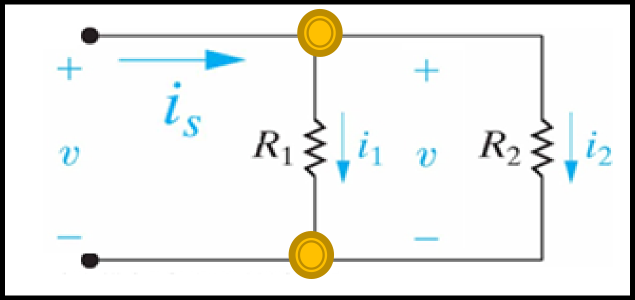
If n resistors are series-connected, the voltage across the jth resistor is obtained by:

$$v_j = \frac{R_j}{R_{eq}} v_s$$

Resistors in Parallel Connection

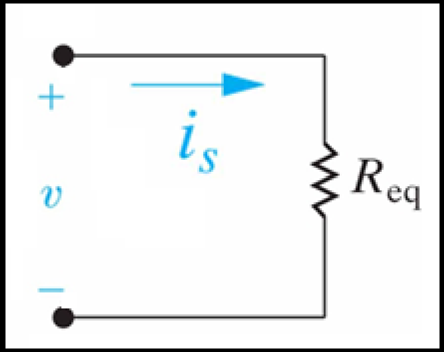
When two resistors are connected at a node pair, they are in parallel. Parallel-connected resistors have the same voltage across their terminals.

a



Applying KCL at node a:

$$i_s = i_1 + i_2 = \frac{v}{R_1} + \frac{v}{R_2} = v\,(\frac{1}{R_1} + \frac{1}{R_2})$$



Assuming $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$, we have:

$$i_s = v\frac{1}{R_{eq}} \qquad v = R_{eq} i_s$$

R_{eq} is called equivalent resistance

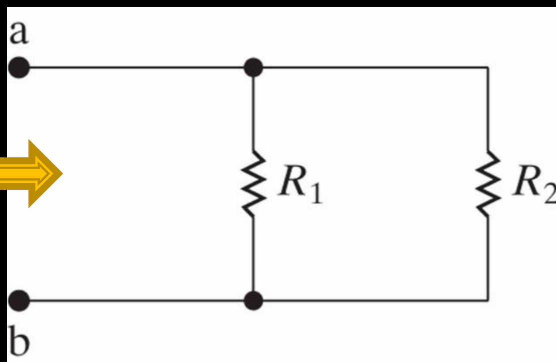
If n resistors are parallel-connected, then $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_{n-1}} + \frac{1}{R_n}$

$$v = R_{eq} i_s$$

Note that the equivalent resistance is always smaller than the smallest resistance (not zero) of the resistor in the parallel connection.

Two Resistors in Parallel

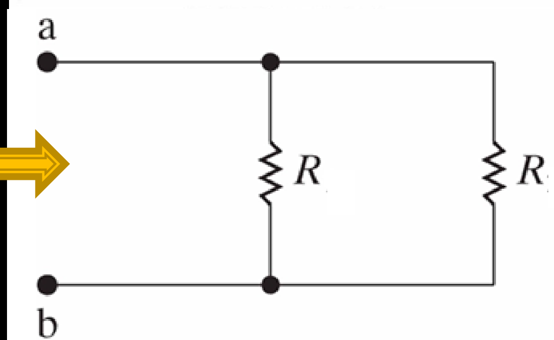
R_{eq}



From : $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$

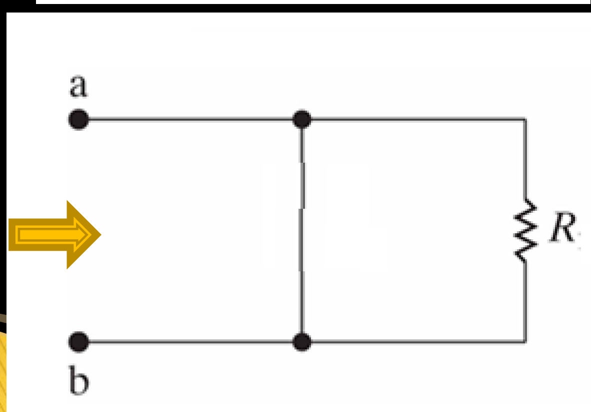
$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$ Memorize this!

R_{eq}



If $R_1 = R_2 = R, R_{eq} = \frac{R}{2}$

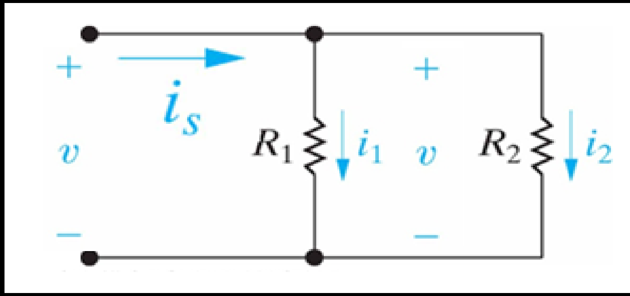
R_{eq}



If $R_1 = 0$ *or* $R_2 = 0, R_{eq} = 0$

Current-Divider Circuits

A parallel connected resistive circuit can be used as a current-divider. The circuit is designed to divide a current level into more than one current levels.



$$v = R_{eq} i_s = \frac{R_1 R_2}{R_1 + R_2} i_s$$

Memorize these!

$$i_1 = \frac{v}{R_1} = \frac{R_{eq}}{R_1} i_s = \frac{R_2}{R_1 + R_2} i_s$$

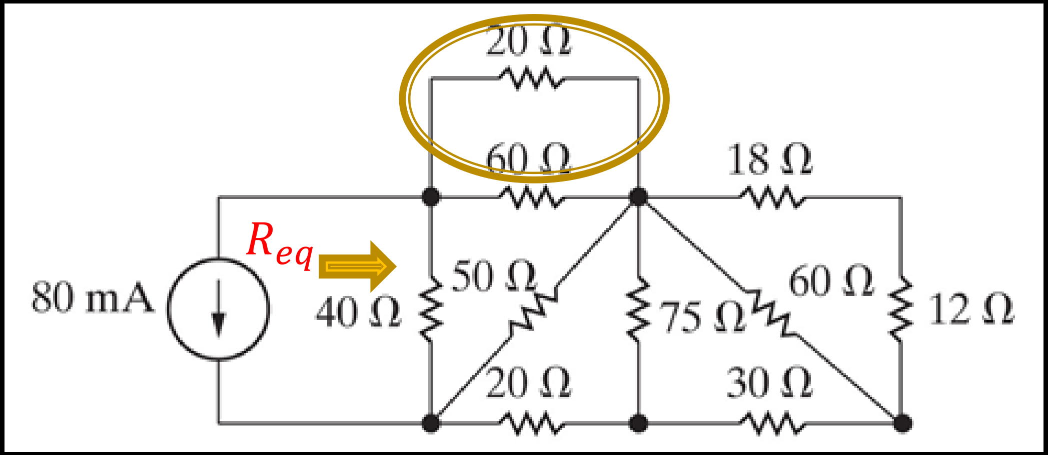
$$Where\ R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

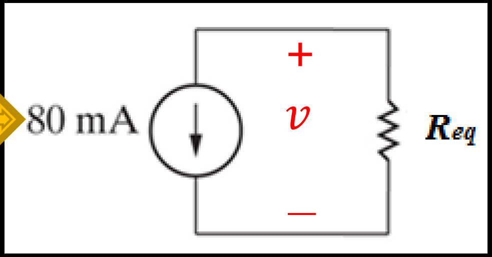
$$i_2 = \frac{v}{R_2} = \frac{R_{eq}}{R_2} i_s = \frac{R_1}{R_1 + R_2} i_s$$

If n resistors are parallel-connected, the current in the jth resistor is obtained by:

$$i_j = \frac{R_{eq}}{R_j} i_s$$

Example #1





Find the power delivered by the source

$$18 + 12 = 30\ \Omega$$

$$60||30 = \frac{60\times30}{60+30} = 20\ \Omega$$

$$20 + 30 = 50\ \Omega$$

$$50||75 = \frac{50\times75}{50+75} = 30\ \Omega$$

$$30 + 20 = 50\ \Omega$$

$$50||50 = 25\ \Omega$$

$$20||60 = \frac{20\times60}{20+60} = 15\ \Omega$$

$$25 + 15 = 40\ \Omega$$

$$R_{eq} = 40||40 = 20\ \Omega$$

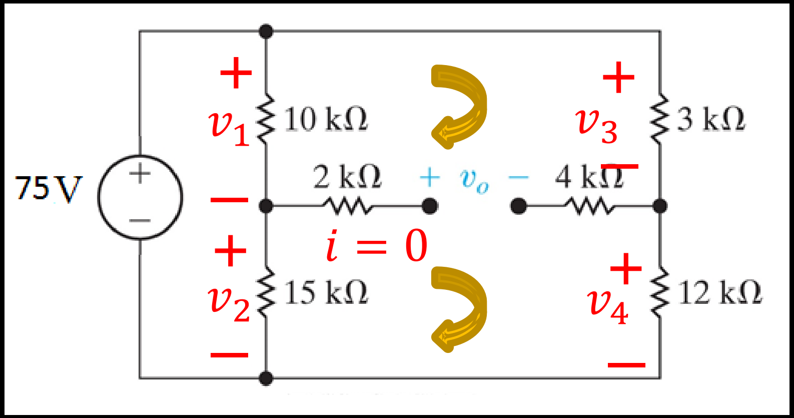
From Ohm's law: $v = -0.08 \times 20 = -1.6\ V$

$$p_{80mA} = 0.08v = (0.08)(-1.6) = -128\ mW$$

Or $\quad p_{80mA} = -0.08^2 \times 20 = -128\ mW < 0$

$because\ the\ current\ source\ delieves\ power$

Example #2



Find the voltage v_0 by using the voltage division or current division.

Applying KVL to the top loop: $v_3 - v_0 - v_1 = 0$ So $v_0 = v_3 - v_1$ (1)

Or applying KVL to the bottom loop: $v_4 - v_2 + v_0 = 0$ So $v_0 = v_2 - v_4$ (2)

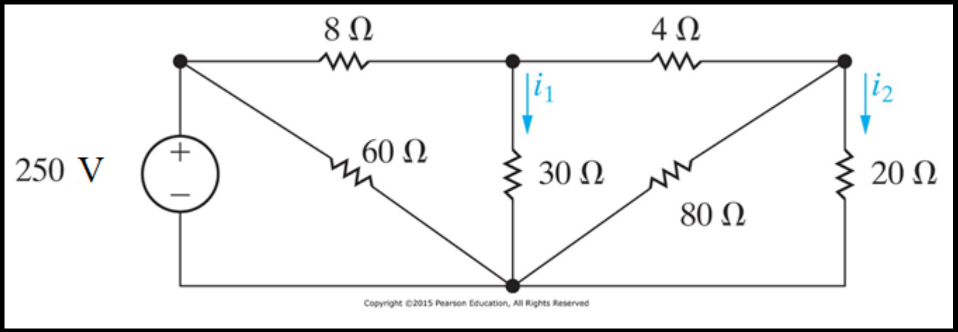
Voltage division: $v_1 = \dfrac{10000 \times 75}{10000 + 15000} = 30\ V$ $From\ KVL\colon\ v_2 = 75 - 30 = 45\ V$

$v_3 = \dfrac{3000 \times 75}{3000 + 12000} = 15\ V$ $From\ KVL\colon\ v_4 = 75 - 15 = 60\ V$

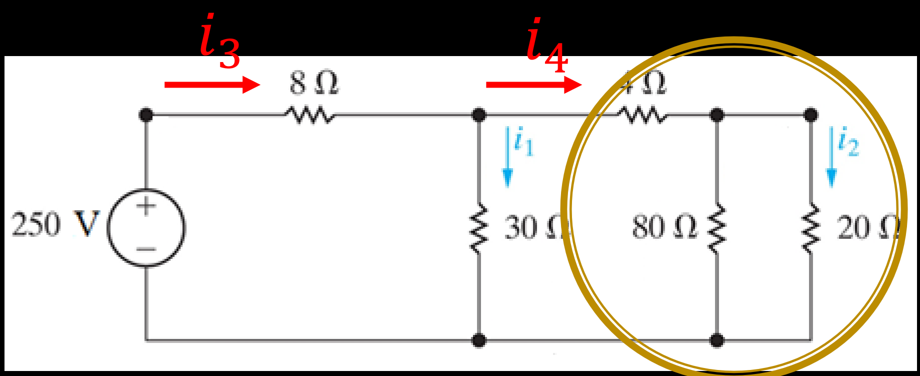
From (1): $v_0 = v_3 - v_1 = 15 - 30 = -15\ V$

From (2): $v_0 = v_2 - v_4 = 45 - 60 = -15\ \text{V}$

Example #3

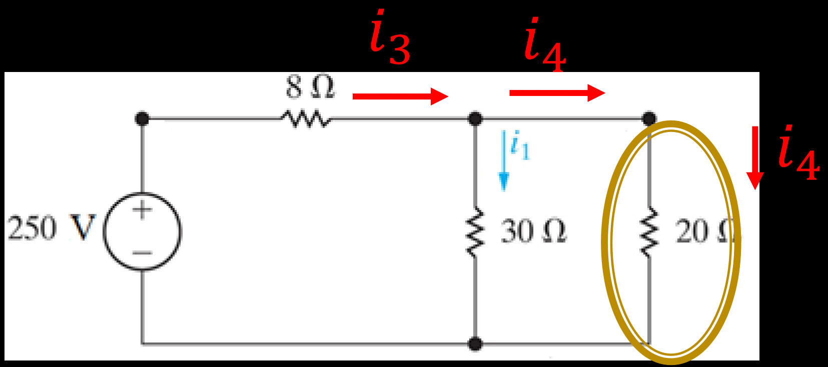


Find i_1 and i_2 shown in the circuit above

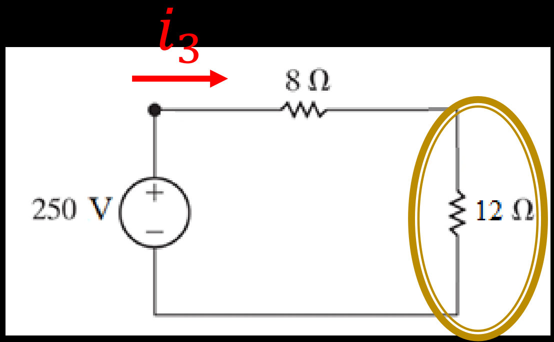


$$20||80 = \frac{20 \times 80}{20 + 80} = 16\ \Omega$$

$$16 + 4 = 20\ \Omega$$



$$20||30 = \frac{20 \times 30}{20 + 30} = 12\ \Omega$$



$$i_3 = \frac{250}{8 + 12} = 12.5\ A$$

$$i_1 = \frac{20 \times i_3}{30 + 20} = 5\ A$$

$$i_4 = \frac{30 \times i_3}{30 + 20} = 7.5\ A$$

$$i_2 = \frac{80 \times i_4}{80 + 20} = 6\ A$$

Summary

- Circuit simplification is one of the most important circuit analysis techniques.
- Two types of resistor connections: series and parallel connections.
- Series-connected resistive circuits can be used as the voltage dividers. Parallel-connected circuits can be used as the current dividers.

In next lecture, we will discuss:
The node-voltage method.