CSE 015: Discrete Mathematics
Homework 1

Fall 2021
Provided Solution

1. **Question 1:**

   (a) "It is not the case that XYZ is a CSE major". Alternative answer: "XYZ is not a CSE major."

   (b) "It is not the case that XYZ scored at least 90% in the labs." Alternative answer "XYZ scored less than 90% in the labs."

   (c) "If XYZ scored 100% in the CSE015 final or XYZ scored at least 90% in the labs, then XYZ receives an A+ in CSE015."

   (d) "If XYZ scored 100% in the CSE015 final and XYZ scored at least 90% in the labs, then XYZ receives an A+ in CSE015."

   (e) "It is not the case that if XYZ is a CSE major then XYZ receives an A+ in CSE015."

2. **Question 2:**

   Throughout this exercise and the following ones, we introduce intermediate columns in truth tables to aid with the computation of the final results. Intermediate columns are not strictly required, but they are helpful to avoid making mistakes when manually computing truth tables for complex compound propositions.

   (a) The truth table for $p \oplus (q \vee \neg r)$ is given in table 1. It is convenient to introduce an intermediate column to display the result of the subexpression $(q \vee \neg r)$. The final result is then obtained by recalling the truth table for the exclusive or operator ($\oplus$), i.e., the overall expression is false when p and $(q \vee \neg r)$ have the same truth value, and true otherwise.

p	q	r	$q \vee \neg r$	$p \oplus (q \vee \neg r)$
F	F	F	T	T
F	F	T	F	F
F	T	F	T	T
F	T	T	T	T
T	F	F	T	F
T	F	T	F	T
T	T	F	T	F
T	T	T	T	F

Table 1: Truth table for $p \oplus (q \vee \neg r)$.

   (b) The truth table for $(p \vee q) \rightarrow (\neg r \vee p)$ is given in table 2. In this case it is convenient to introduce two intermediate columns for $(p \vee q)$ and $(\neg r \vee p)$. The truth values for the compound proposition are then obtained by applying the truth table for the conditional statement (implication) operator. For the compound proposition, $(p \lor q)$ is the premise (or antecedent, or hypothesis), and $(\neg r \lor p)$ is the consequence (or conclusion).

p	q	r	$(p \lor q)$	$(\neg r \lor p)$	$(p \lor q) \to (\neg r \lor p)$
F	F	F	F	T	T
F	F	T	F	F	T
F	T	F	T	T	T
F	T	T	T	F	F
T	F	F	T	T	T
T	F	T	T	T	T
T	T	F	T	T	T
T	T	T	T	T	T

Table 2: Truth table for $(p \lor q) \to (\neg r \lor p)$.

(c) The truth table for $((p \to q) \land p) \to q$ is given in table 3. In this case it is convenient to introduce two intermediate columns, one for $(p \to q)$ and one for $((p \to q) \land p)$. The truth values for the compound proposition are then obtained by applying the truth table for the conditional statement (implication) operator. For the compound proposition, $((p \to q) \land p)$ is the premise (or antecedent, or hypothesis), and q is the consequence (or conclusion).

p	q	$p \to q$	$(p \to q) \land p$	$((p \to q) \land p) \to q$
F	F	T	F	T
F	T	T	F	T
T	F	F	F	T
T	T	T	T	T

Table 3: Truth table for $((p \to q) \land p) \to q$.

Note: this compound proposition is a *tautology*, i.e., it is always true for each possible truth assignment of the atomic propositions p and q.

3. **Question 3:**

To prove logical equivalences using truth tables, it is necessary to compute the truth tables for both compound propositions and show that they have the same value for all possible truth values assigned to the atomic propositions. As in the previous exercise, when computing truth tables it is convenient to introduce intermediate columns.

(a) The truth table to establish the logical equivalence $p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$ is given in table 4. The equivalence is established by observing that the fifth and eight columns display the same truth values.

(b) The truth table to establish the logical equivalence $(p \to q) \land (p \to r) \equiv p \to (q \land r)$ is given in table 5. The equivalence is established by observing that the sixth and eight columns display the same truth values.

4. **Question 4:**

To answer this question it is necessary to write the truth tables for the compound propositions and recall the definitions of tautology, contradiction, and contingency.

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$	$p \vee q$	$p \vee r$	$(p \vee q) \wedge (p \vee r)$
F	F	F	F	F	F	F	F
F	F	T	F	F	F	T	F
F	T	F	F	F	T	F	F
F	T	T	T	T	T	T	T
T	F	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	T	F	F	T	T	T	T
T	T	T	T	T	T	T	T

Table 4: Truth table to establish $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$.

p	q	r	$p \rightarrow q$	$p \rightarrow r$	$(p \rightarrow q) \wedge (p \rightarrow r)$	$q \wedge r$	$p \rightarrow (q \wedge r)$
F	F	F	T	T	T	F	T
F	F	T	T	T	T	F	T
F	T	F	T	T	T	F	T
F	T	T	T	T	T	T	T
T	F	F	F	F	F	F	F
T	F	T	F	T	F	F	F
T	T	F	T	F	F	F	F
T	T	T	T	T	T	T	T

Table 5: Truth table to establish $(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r)$.

(a) The truth table for $p \rightarrow (p \vee q)$ is shown in table 6. The compound proposition is therefore a <u>tautology</u> because it is always true.

p	q	$p \vee q$	$p \rightarrow (p \vee q)$
F	F	F	T
F	T	T	T
T	F	T	T
T	T	T	T

Table 6: Truth table for $p \rightarrow (p \vee q)$.

(b) The truth table for $(p \wedge q) \rightarrow \neg p$ is shown in table 7. The compound proposition is therefore a <u>contingency</u> because it is true for some values of p and q and false for others. Stated differently, it is neither a tautology nor a contradiction and therefore it is a contingency.

p	q	$p \wedge q$	$\neg p$	$(p \wedge q) \rightarrow \neg p$
F	F	F	T	T
F	T	F	T	T
T	F	F	F	T
T	T	T	F	F

Table 7: Truth table for $(p \wedge q) \rightarrow \neg p$.

(c) The truth table for $(p \rightarrow (q \vee r)) \rightarrow (\neg q \vee p)$ is shown in table 8. The compound proposition is a <u>contingency</u>. Note that to establish this fact you do not even need to complete the truth table, because after filling the third row in the last column you have established that the proposition is true for some values of p, q, r, and false for others. Therefore it can neither be a tautology nor a contradiction.

p	q	r	$q \vee r$	$p \to (q \vee r)$	$\neg q \vee p$	$(p \to (q \vee r)) \to (\neg q \vee p)$
F	F	F	F	T	T	T
F	F	T	T	T	T	T
F	T	F	T	T	F	F
F	T	T	T	T	F	
T	F	F	F	F	T	
T	F	T	T	T	T	
T	T	F	T	T	T	
T	T	T	T	T	T	

Table 8: Truth table for $(p \to (q \vee r)) \to (\neg q \vee p)$.

5. **Question 5:**

   (a) "You cannot be late and you cannot smoke." Let p be the proposition "You can be late" and q be the proposition "You can smoke". Therefore the given compound proposition has the form $\neg p \wedge \neg q$. One of De Morgan's laws states the equivalence $\neg p \wedge \neg q \equiv \neg(p \vee q)$. The proposition $p \vee q$ is "You can be late or you can smoke" and therefore the original proposition is logically equivalent to "It is not the case that you can be late or you can smoke".

   (b) "It is not the case that you can take an annuity and you can take a lump sum." Let p be the proposition "You can take an annuity" and q be the proposition "You can take a lump sum." Therefore the original proposition has the form $\neg(p \wedge q)$ were $p \wedge q$ is "You can take an annuity and you can take a lump sum." One of De Morgan's laws states the equivalence $\neg(p \wedge q) \equiv \neg p \vee \neg q$. Therefore the original proposition is logically equivalent to "It is not the case you can take an annuity or it is not the case you can take a lump sum."