ENGR 65 Circuit Theory

Lecture 4: Kirchhoff's Current Law, Kirchhoff's Voltage Law, and Their Applications in Circuit Analysis

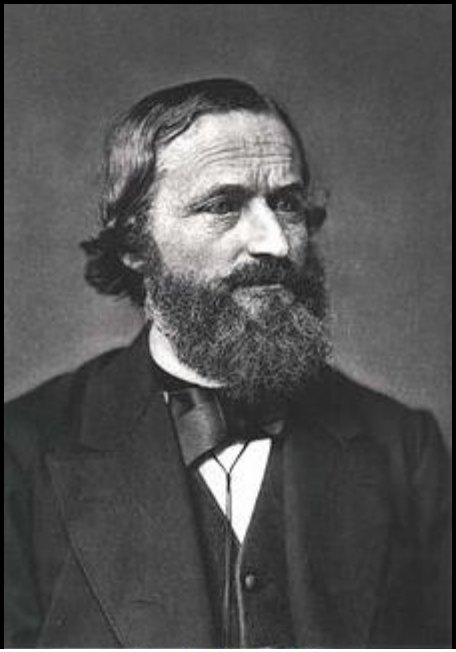
Today’s Topics

- What are nodes and closed loops?
- What is Kirchhoff’s Voltage Law (KVL)?
- What is Kirchhoff’s Current Law (KCL)?
- Applications of Ohm’s law, KVL, and KCL.

Covered in Sections 2.4 and 2.5.

https://www.youtube.com/watch?v=m4jzgqZu-4s

Kirchhoff's Circuit Laws



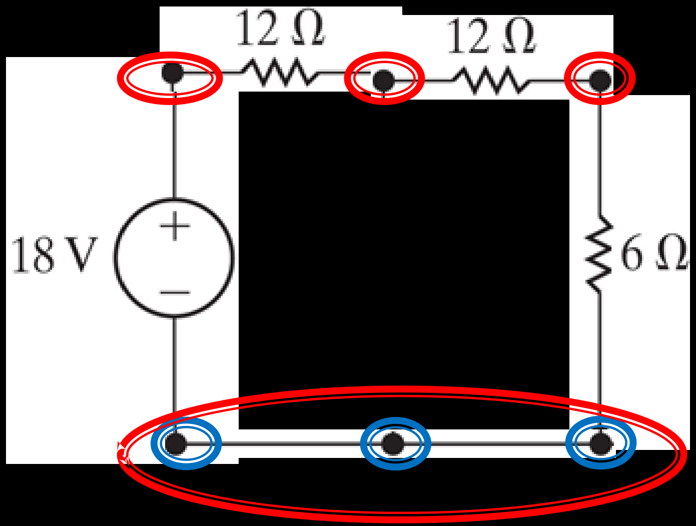
1824-1887

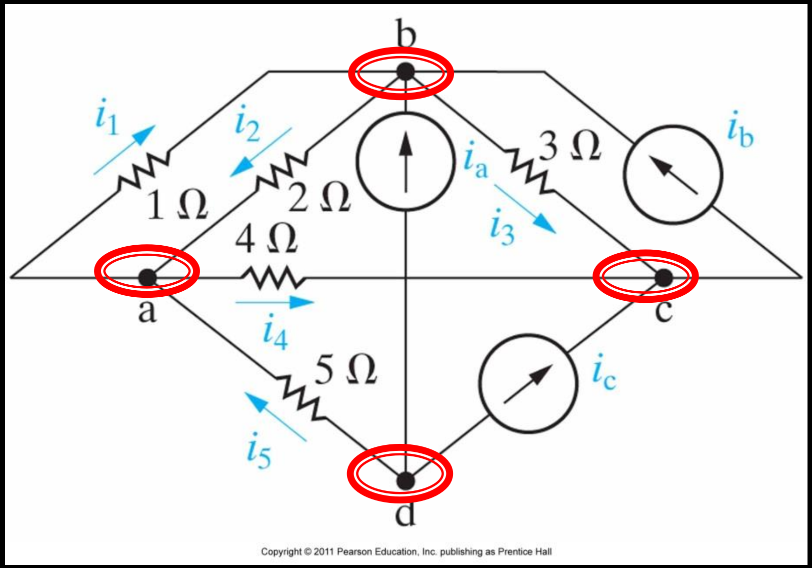
Gustav Robert Kirchhoff was a German physicist who made contribution to the fundamental understanding of electrical circuits, spectroscopy, and the emission of black body radiation by heated objects.

https://en.wikipedia.org/wiki/Gustav_Kirchhoff

Nodes

- A node is a place where two or more elements are connected.





- The places with the red circles are nodes.
- An essential node is a node where three or more elements are **connected.**

Kirchhoff's Current Law

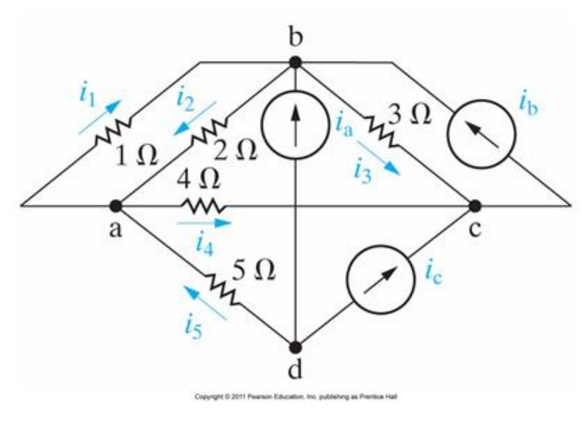
Kirchhoff's Current Law (KCL)

The algebraic (signed) sum of all the currents at any node in a circuit equals to zero. Or the sum of the currents that enter a node is equal to the sum of the currents that leave the node.

The law simply means that the charge does not build up at a node.

Mathematically, the law is expressed as, for any node,

$$\sum_{at\ any\ nodes} i = 0$$



$$or \sum i_{leaving} - \sum i_{entering} = 0\ \boldsymbol{at\ any\ nodes}$$

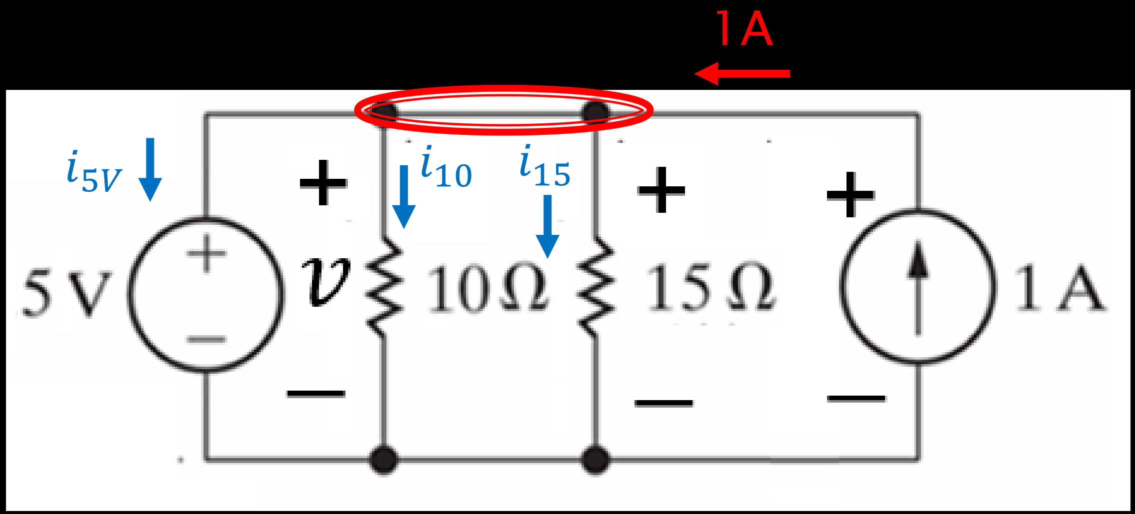
$or \sum i_{leaving} = \sum i_{entering}$ $\boldsymbol{at\ any\ nodes}$ For example: at $node\ a$: $i_1 + i_4 - i_2 - i_5 = 0$

How to apply KCL into a circuit analysis?

1. Assign reference direction for each current at a node.
2. Give positive sign to a current leaving the node and negative sign to a current entering the node.

Example #1

Find p_{5V}



$$From\ KCL\!:\ \ i_{5V} + i_{10} + i_{15} - 1 = 0$$

$$From\ Ohm's\ law\!:\ \ i_{10} = \frac{5}{10} = 0.5\ A, \qquad i_{15} = \frac{5}{15} = 0.333\ A$$

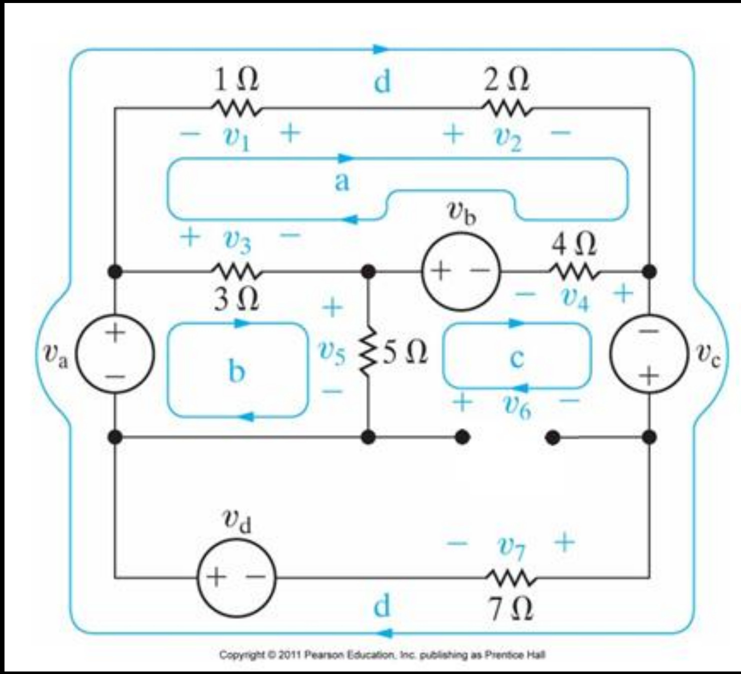
Therefore: $i_{5V} = 1 - 0.5 - 0.333 = 0.167\ A$

$$p_{5V} = 0.167 \times 5 = 0.835\ W$$

Closed Loops

A closed loop is defined in this way: Starting at an arbitrarily selected node, trace a closed path in a circuit and return to the original node without passing through any intermediate node more than once. The loops a, b, c, and d are partial of the closed loops in this circuit.

A mesh is a closed loop that does not include any other closed loops. The closed loops a, b, and c are meshes.



Kirchhoff's Voltage Law

Kirchhoff's Voltage Law (KVL)

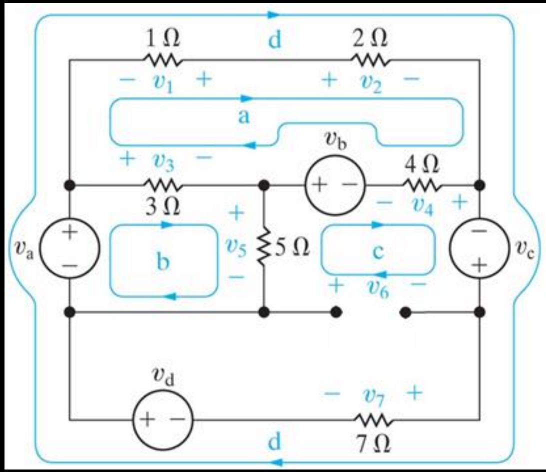
The algebraic (signed) sum of all the voltages around any closed loop in a circuit equals zero. Or the sum of the voltages drop along any closed loop is equal to the sum of the voltages rise in the circuit.

Mathematically, the law is expressed as

$$\sum_{along\ any\ closed\ loops} v = 0$$

$$or \sum v_{drop} - \sum v_{rise} = 0 \quad along\ any\ closed\ loops$$

$$or \sum v_{drop} = \sum v_{rise} \quad along\ any\ closed\ loops$$

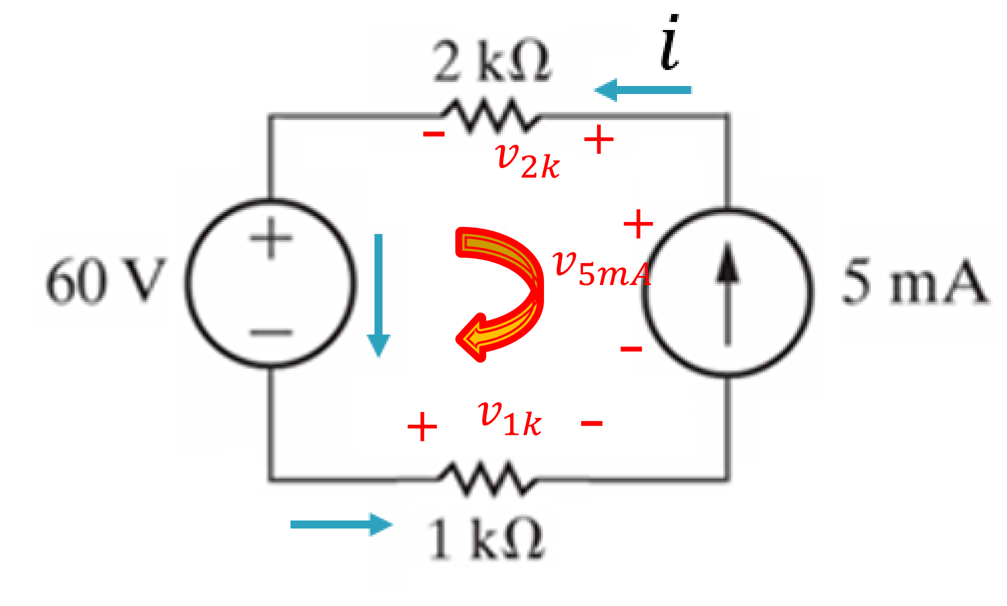


Fox example: loop c: $v_b - v_4 - v_c - v_6 - v_5 = 0$

How to apply KVL into a circuit analysis:

- Assign reference polarity to each voltage in a closed loop.
- Give positive sign to a voltage drop, a negative sign to a voltage rise.

Example #2



1. Assign reference polarity to each voltage in a closed loop.

2. Place positive sign to a voltage drop, negative sign to voltage rise and write KVL equations.

KVL equation:

$$-v_{2k} + v_{5mA} - v_{1k} - 60 = 0 \quad (1)$$

Ohm's law: $v_{2k} = 0.005 \times 2000 = 10\ V$

Ohm′s law: $v_{1k} = 0.005 \times 1000 = 5\ V$

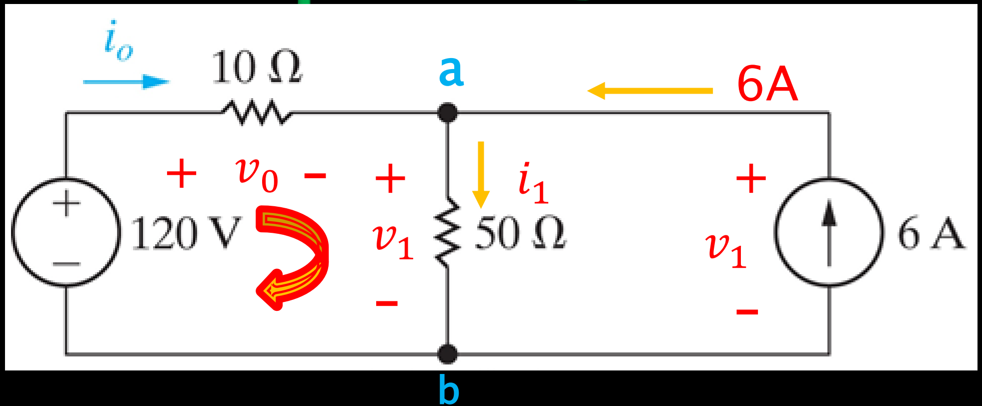
From (1): $v_{5mA} = 60 + v_{2k} + v_{1k} = 60 + 10 + 5 = 75\ V$

$$p_{5mA} = -0.005 \times v_{5mA} = -0.005 \times 75 = -375\ mW < 0 \ \ delivering\ power$$

Steps in Solving a Circuit Using Ohm's Law, KCL, and KVL

- Step 1: Define the currents and voltages in a circuit
- Step 2: Assign the references for currents and voltages.
- Step 3: Apply KCL to essential nodes in the circuit.
- Step 4: Apply KVL to meshes in the circuit.
- Step 5: Apply Ohm's Law to each resistor.
- Step 6: Check if the number of Independent equations equals the number of unknown variables.
- Step 7: Solve the simultaneous equations obtained above.

Example #3



Using KCL, KVL, and Ohm's law to find i_0 in the circuit.

1. Define currents and voltages in a circuit
2. Assign the references for the currents and voltages

3: Apply KCL to the essential nodes in the circuit.

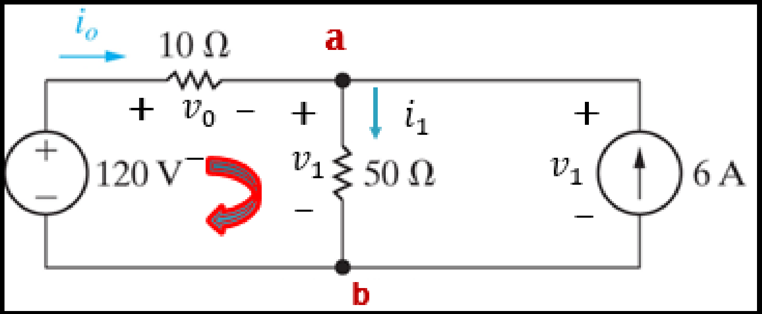
at node a: $i_1 - i_0 - 6 = 0 \quad (1)$

at node b: $-i_1 + i_0 + 6 = 0$ **Redundant. Get rid of it**

4. Apply KVL to the meshes in the circuit.

$$v_0 + v_1 - 120 = 0 \quad (2)$$

Example #3 (cont.)



5. Applying Ohm's Law to each resistor.

$$v_0 = 10i_0 \quad (3)$$

$$v_1 = 50i_1 \quad (4)$$

6. Check if the number of Independent equations equals the number of unknown variables.

7. Solving the simultaneous equations obtained above.

$$i_1 - i_0 - 6 = 0 \quad (1)$$

$$v_0 + v_1 - 120 = 10i_0 + 50i_1 - 120 = 0 \quad (2)$$

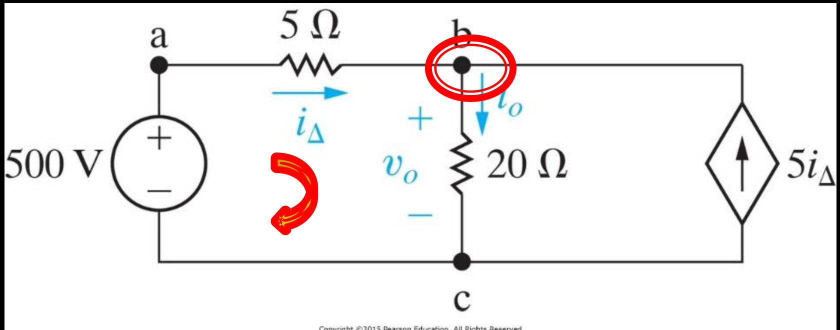
Solving the above equations for i_1 and i_0, *we have*

$i_0 = -3\ A$: The actual current direction is opposite to the reference

$i_1 = 3\ A$: The actual current direction is same as the reference

Note: Two equations are required if there are two resistors in the circuit

Example #4: Analysis of a Circuit Containing Dependent Sources



Using KCL, KVL, and Ohm's law to find

1. i_0.
2. The power associated with the dependent source in the circuit.

Using KCL: $i_0 - i_\Delta - 5i_\Delta = 0 \quad (1)$

Using KVL: $5i_\Delta + 20i_0 - 500 = 0 \quad (2)$

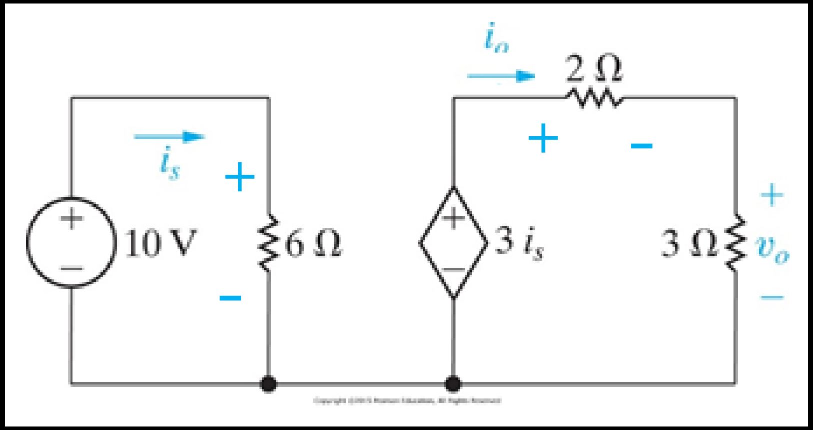
Solving (1) and (2) for i_Δ and i_0 : $i_\Delta = 4\ A$

$$i_0 = 24\ A$$

The power associated with the dependent source in the circuit.

$$p_{5i_\Delta} = -v_0(5i_\Delta) = -(20i_0)(5i_\Delta) = -9600\ W\ (Delivering)$$

Example #5: Analysis of a Circuit Containing Dependent Sources



Using KCL, KVL, and Ohm's law to find

1. i_0.
2. the power associated with the dependent source in the circuit.

1. Do not need to write KCL equations because there are no essential nodes in the circuit.

2. Applying KVL to the left side of the circuit, $6i_s = 10, so\ i_s = \frac{10}{6} = \frac{5}{3}\ A$

3. Applying KVL to the right side of the circuit, $2i_0 + 3i_0 = 3\ i_s, \quad i_0 \quad = \frac{3 \times \frac{5}{3}}{5} = 1\ A$

$$p_{3i_s} = \quad -3i_s(i_0) = -3 \times \frac{5}{3} \times 1 = -5\ W$$

Summary

- A node is a place where two or more elements are connected. A closed loop is a loop when tracing through connecting elements, starting and ending at the same node and passing through intermediate nodes only once.
- The currents at any node and voltages along any closed-loop in a circuit obey Kirchhoff's current law and Kirchhoff's voltage law. By applying Ohm's law, KCL, and KVL, simple circuits can be solved.

- In next class, we will discuss:
- Resistors in series and parallel connections
- The voltage-divider and current divider circuits