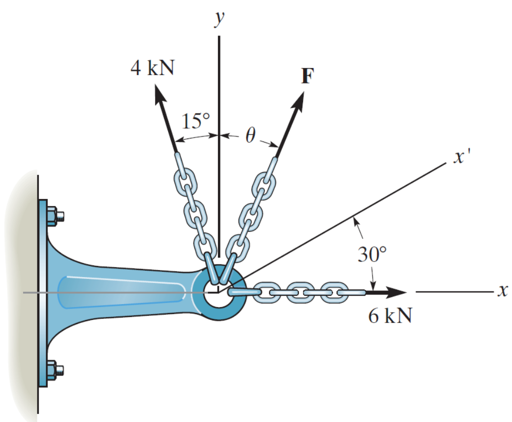
University of California, Merced

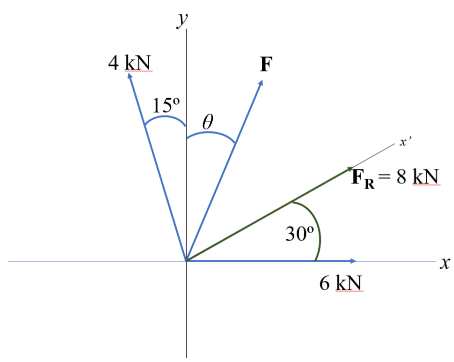
ENGR 057 Statics and Dynamics: Assignment #1

Summer - 2022

Due: June 28, 2022

Problem 1 (20 pts). Three forces act on the bracket. Determine the magnitude and direction θ of **F** so that the resultant force is directed along the positive *x*’ axis and has a magnitude of 8 kN.





$(F_R)_x = \sum F_x;$ $\quad$ $8\cos 30° = F\sin\theta + 6 - 4\sin 15°$

$F\sin\theta = 1.9635$ $\quad$ (1)

$(F_R)_x = \sum F_y;$ $\quad$ $8\sin 30° = F\cos\theta + 4\cos 15°$

$F\cos\theta = 0.1363$ $\quad$ (2)

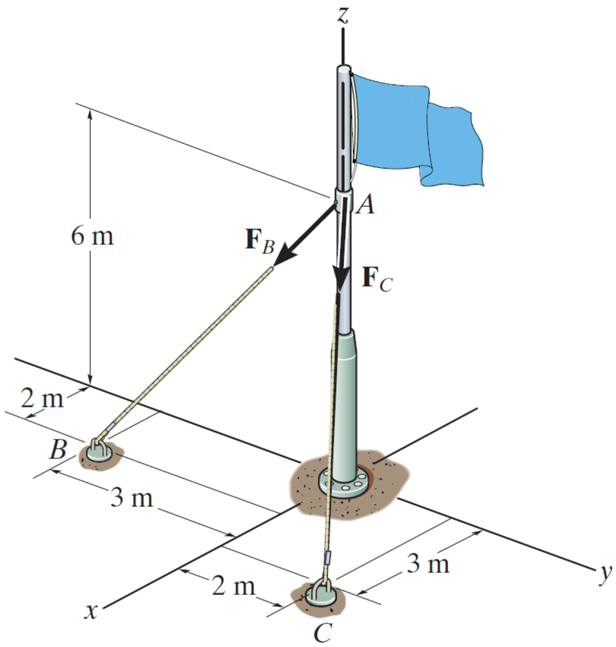
Dividing Eq (1) by (2)

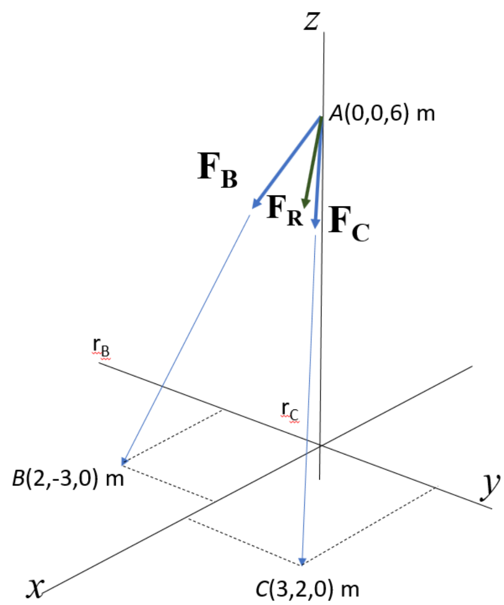
$\tan\theta = 14.406$ $\quad$ $\boxed{\theta = 86.03 = 86.0°}$

$F\sin 86.03° = 1.9635$

$\boxed{F = 1.968\text{ kN} = 1.97\text{ kN}}$

Problem 2 (20 pts). If $F_B = 700$ N, and $F_C = 560$ N, determine the magnitude and coordinate direction angles of the resultant force acting on the flagpole.





The unit vectors $\mathbf{u}_B$ and $\mathbf{u}_C$ of $\mathbf{F}_B$ and $\mathbf{F}_B$:

$$\mathbf{r_B} = (2-0)\mathbf{i} + (-3-0)\mathbf{j} + (0-6)\mathbf{k}$$

$$r_B = \sqrt{2^2 + (-3)^2 + (-6)^2} = 7$$

$$\mathbf{u}_B = \frac{2}{7}\mathbf{i} - \frac{3}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

$$\mathbf{F}_B = 700\left(\frac{2}{7}\mathbf{i} - \frac{3}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right) = \{200\mathbf{i} - 300\mathbf{j} - 600\mathbf{k}\}\text{ N}$$

$$\mathbf{r_C} = (3-0)\mathbf{i} + (2-0)\mathbf{j} + (0-6)\mathbf{k}$$

$$r_C = \sqrt{3^2 + (2)^2 + (-6)^2} = 7$$

$$\mathbf{u}_C = \frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

$$\mathbf{F}_C = 560\left(\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right) = \{240\mathbf{i} - 160\mathbf{j} - 480\mathbf{k}\}\text{ N}$$

Resultant Force:

$$\mathbf{F}_R = \mathbf{F}_B + \mathbf{F}_C = (200\mathbf{i} - 300\mathbf{j} - 600\mathbf{k}) + (240\mathbf{i} - 160\mathbf{j} - 480\mathbf{k})$$
$$= \{440\mathbf{i} - 140\mathbf{j} - 1080\mathbf{k}\}\text{ N}$$

The magnitude of $\mathbf{F}_R$ is

$$\boxed{F_R = \sqrt{(440)^2 + (-140)^2 + (-1080)^2} = 1174.56\text{ N} = 1.17\text{ kN}}$$

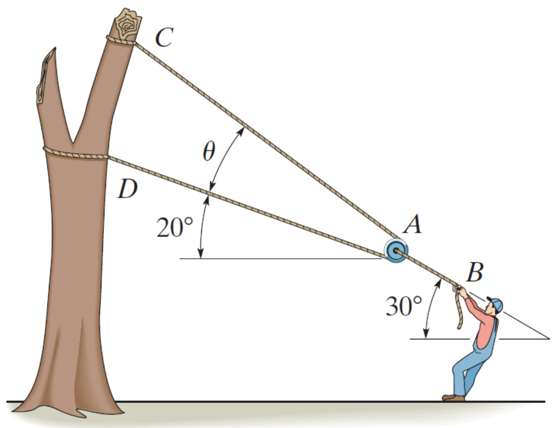
The coordinate direction angles of $\mathbf{F}_\mathrm{R}$ are

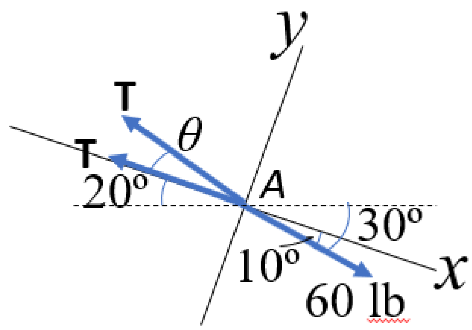
$$\alpha = \cos^{-1}\left(\frac{440}{1174.56}\right) = 68.0°$$

$$\beta = \cos^{-1}\left(\frac{-140}{1174.56}\right) = 96.8°$$

$$\gamma = \cos^{-1}\left(\frac{-1080}{1174.56}\right) = 157°$$

Problem 3 (20 pts). The man attempts to pull down the tree using the cable and *small* pulley arrangement shown. If the tension in *AB* is 60 lb, determine the tension in cable *CAD* and the angle θ which the cable makes at the pulley.





$\sum F_x = 0;$ $\qquad 60\cos 10^\circ - T - T\cos\theta = 0$

$\sum F_y = 0;$ $\qquad -60\sin 10^\circ - T\sin\theta = 0$

Then $\quad 60\cos 10^\circ = T(1+\cos\theta)$

$$60\cos 10^\circ = T(2\cos^2\tfrac{\theta}{2}) \qquad (1)$$

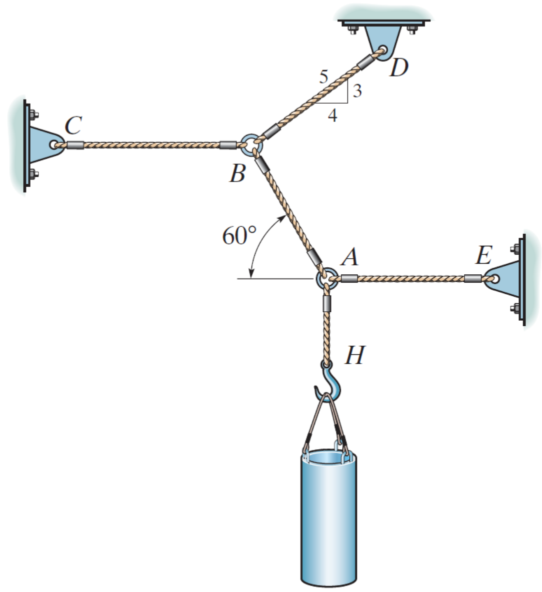
$$60\sin 10^\circ = 2T(\sin\tfrac{\theta}{2}\cos\tfrac{\theta}{2}) \qquad (2)$$

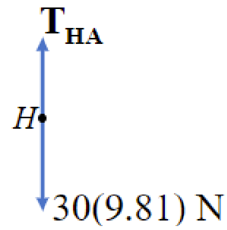
Dividing eq(2) by (1)

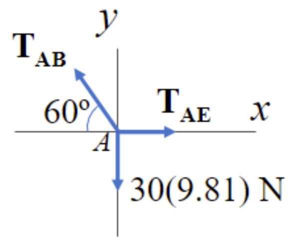
$$\tan\frac{\theta}{2} = \tan 10^\circ$$

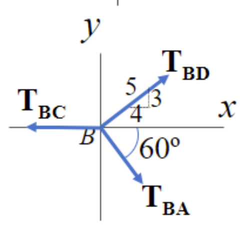
$$\boxed{\theta = 20^\circ \quad T = 30.5\text{ lb}}$$

Problem 4 (20 pts). The 30-kg pipe is supported at *A* by a system of five cords. De4termine the force in each cord for equilibrium.









At *H*

$\sum F_y = 0; \quad T_{HA} - 30(9.81) = 0$

$\boxed{T_{HA} = 294\ \text{N}}$

At *A*

$\sum F_y = 0; \quad T_{AB}\sin 60° - 30(9.81) = 0$

$\boxed{T_{AB} = 339.84 = 340\ \text{N}}$

$\sum F_x = 0; \quad T_{AE} - 339.83\cos 60° = 0$

$\boxed{T_{AE} = 170\ \text{N}}$

At *B*

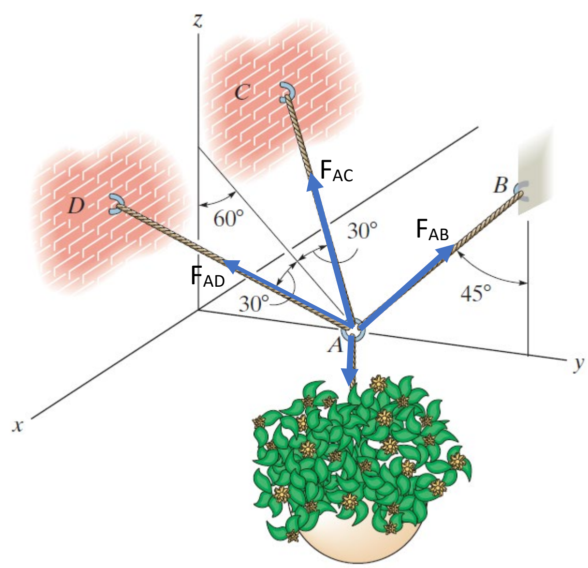
$\sum F_y = 0; \quad T_{BD}\left(\frac{3}{5}\right) - 339.83\sin 60° = 0$

$\boxed{T_{BD} = 490.50 = 490\ \text{N}}$

$\sum F_x = 0; \quad 490.50\left(\frac{4}{5}\right) + 339.83\cos 60° - T_{BC} = 0$

$\boxed{T_{BC} = 562\ \text{N}}$

Problem 5 (20 pts). If each cord can sustain a maximum tension of 50 N before it fails, determine the greatest weight of the flowerpot the cords can support.



$$\mathbf{F}_{AD} = F_{AD}(\sin 30°\,\mathbf{i} - \cos 30° \sin 60°\,\mathbf{j} + \cos 30° \cos 60°\,\mathbf{k}) = F_{AD}(0.5\mathbf{i} - 0.75\mathbf{j} + 0.4330\mathbf{k})$$

$$\mathbf{F}_{AC} = F_{AC}(-\sin 30°\,\mathbf{i} - \cos 30° \sin 60°\,\mathbf{j} + \cos 30° \cos 60°\,\mathbf{k})$$
$$= F_{AC}(-0.5\mathbf{i} - 0.75\mathbf{j} + 0.4330\mathbf{k})$$

$$\mathbf{F}_{AB} = F_{AB}(\sin 45°\,\mathbf{j} + \cos 45°\,\mathbf{k}) = F_{AB}(0.7071\mathbf{j} + 0.7071\mathbf{k})\ \mathrm{N}$$

$\mathbf{W} = -W\mathbf{k}$

$\sum F_x = 0;\qquad 0.5F_{AD} - 0.5F_{AC} = 0$

$F_{AD} = F_{AC}$

$\sum F_y = 0;\qquad -0.75F_{AD} + 0.7071F_{AB} - 0.75F_{AC} = 0$

$0.7071F_{AB} = 1.5F_{AC}$

$\sum F_z = 0;\qquad 0.4330F_{AD} + 0.7071F_{AB} + 0.4330F_{AC} - W = 0$

$0.8660F_{AC} + 1.5F_{AC} - W = 0$

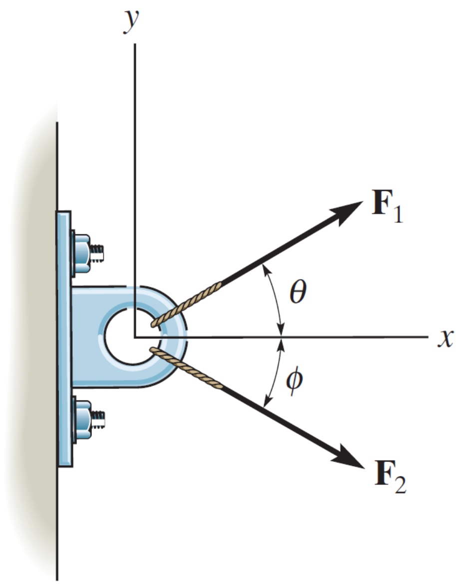
$2.366F_{AC} = W$

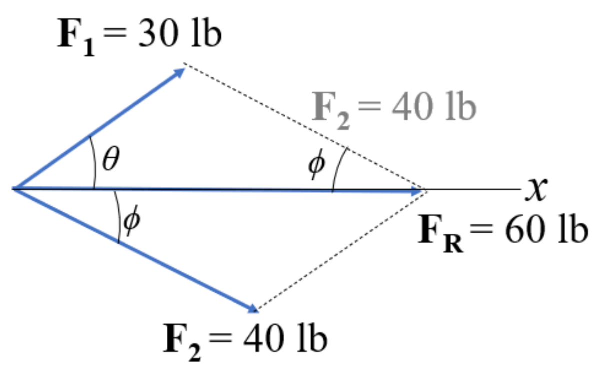
If F_{AC} = 50 N then F_{AB} = 106.07 N which exceeds the maximum tension of 50 N.

If F_{AB} = 50 N then F_{AC} = 23.57 N. And then $\boxed{W = 2.366(23.57) = 55.767 = 55.8\ \mathrm{N}}$

Bonus Problem (10 points). Use sine law or cosine law to solve the bonus problem:

If F_1 = 30 lb and F_2 = 40 lb, determine the angles θ and ϕ so that the resultant force is directed along the positive x-axis and has a magnitude of F_R = 60 lb.





Using the law of cosines

$40^2 = 30^2 + 60^2 - 2(30)(60)\cos\theta$

$\boxed{\theta = 36.3°}$

And

$30^2 = 40^2 + 60^2 - 2(40)(60)\cos\phi$

$\boxed{\phi = 26.4°}$