ENGR 065 Electric Circuits

Lecture 16: Circuit Elements and Analysis in the s Domain

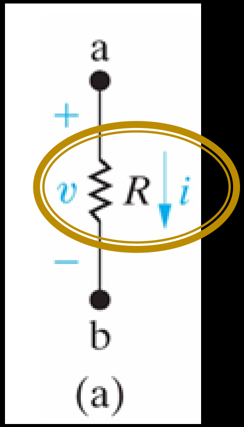
Today's Topics

- Circuit elements in the s domain
- Circuit analysis in the s domain
- Transfer functions
- Examples of circuit analysis in the s domain

Covered in Sections 13.1, 13.2, 13.3, 13.4, and 13.5

Circuit Elements in the s Domain–Resistors

- Resistors in the time domain

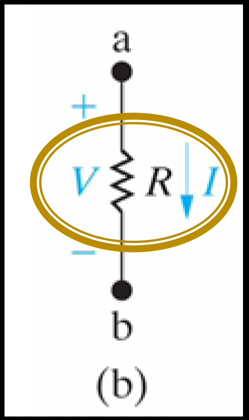


$v(t) = Ri(t)$ - Ohm's law

Applying Laplace transform to the above law, we have:

$$V(s) = RI(s)$$

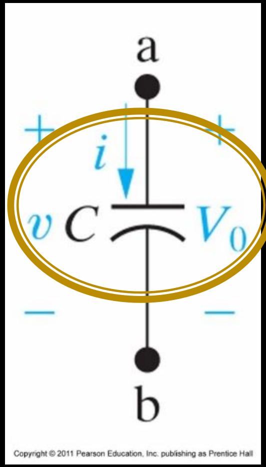
Which is Ohm's law in the s domain



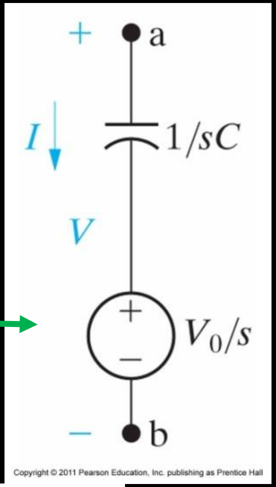
Circuit Elements in the s Domain–Capacitors

Capacitors in the time domain

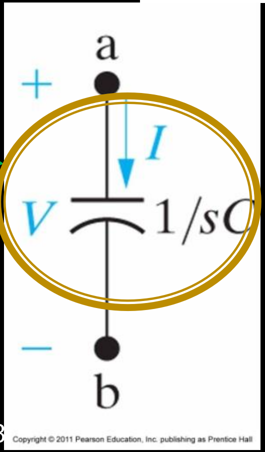
$$i(t) = C\frac{dv(t)}{dt}$$



Applying the Laplace transform to the above equation and rearranging, we have: $V(s) = \left(\frac{1}{sC}\right)I(s) + \frac{V_0}{s}$



If the initial voltage on the capacitor is zero, which is $V_0 = 0$, we have $\boxed{V(s) = \left(\frac{1}{sC}\right)I(s) = Z(s)I(s)}$

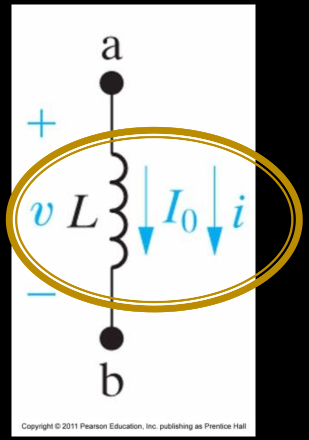


where $Z(s) = \frac{1}{sC}$ is called impedance, measured in ohms

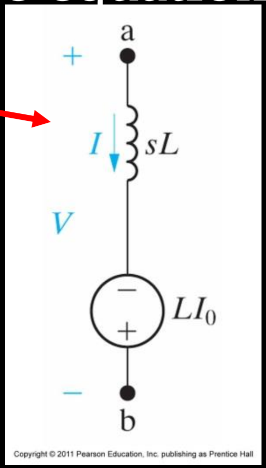
Circuit Elements in the s Domain-Inductors

Inductors in the time domain

$$v(t) = L\frac{di(t)}{dt}$$



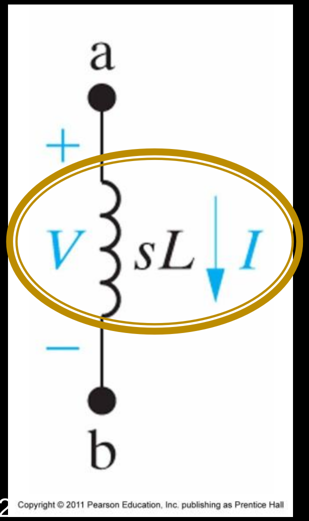
Applying the Laplace transform to the above equation and rearranging, we have: $V(s) = sL\mathrm{I}(s) - \mathrm{L}I_0$



If the initial current in the inductor is zero, which is $I_0 = 0$, we have:

$$V(s) = sLI(s) = Z(s)I(s)$$

where $Z(s) = \mathrm{sL}$ is called impedance, measured in ohms



Circuit Analysis in the s Domain

- If the initial voltage on the capacitors and the initial current in the inductors are zero, the $v - i$ relationship for all passive elements has the form:

$V(s) = ZI(s)$ - Ohm's law in the s domain

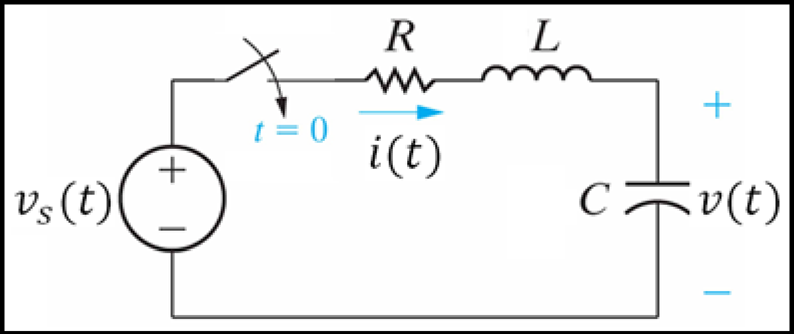
where Z refers to the s-domain impedance of elements. It has the unit of ohms (Ω)

- All the techniques of circuit analysis developed for pure resistive circuits can be used in s-domain circuit analysis.

1. The series-parallel simplifications
2. KCL and KVL
3. Source transformations
4. Thévenin and Norton equivalents
5. Node-voltage and mesh-current methods
6. Superpositions (if circuits are linear)

What you need to do is to turn a circuit model in the time domain to the one in the s domain by changing $v(t) \rightarrow V(s), i(t) \rightarrow I(s), L \rightarrow sL, C \rightarrow \frac{1}{sC}$ and remaining R the same.

Applications in Series- Connected RLC Circuits



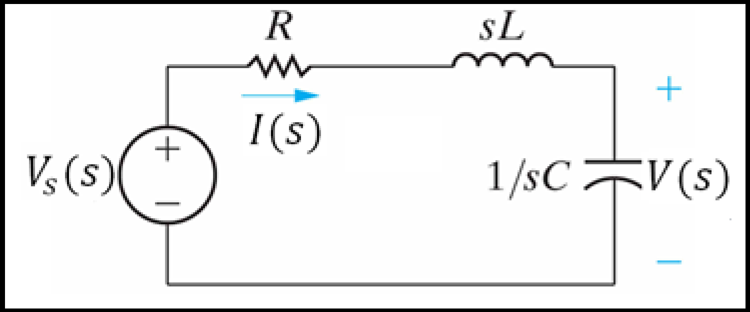
The time domain

By applying KVL to the circuit in the s domain, we have

$$RI(s) + sLI(s) + V(s) = V_s(s)$$

$$V(s) = \frac{1}{sC} I(s)$$

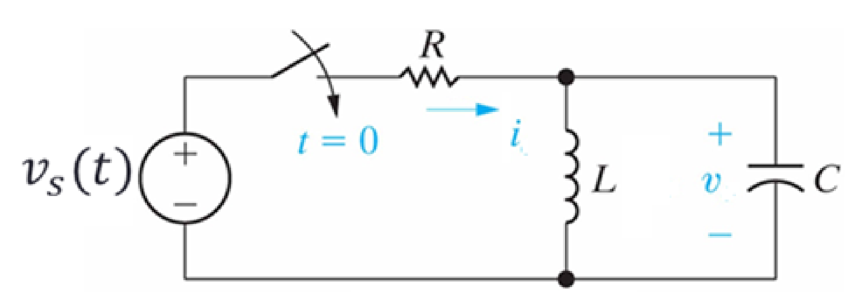
If the initial energy stored in the circuit is zero. The above circuit in the s domain is



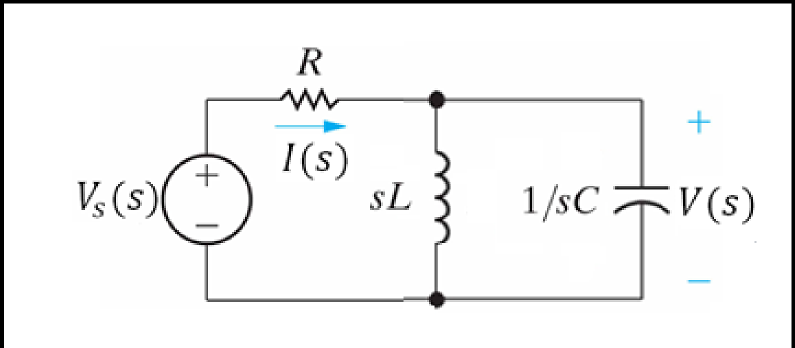
The s domain

$$V(s) = \frac{V_s(s)/LC}{s^2 + \left(\frac{R}{L}\right)s + (1/LC)}$$

Applications in Parallel-Connected RLC Circuits



The time domain



The s domain

Node-Voltage method:

$$\frac{V(s)-V_s(s)}{R}+\frac{V(s)}{sL}+\frac{V(s)}{\frac{1}{sC}}=0$$

$$V(s)=\frac{sV_s(s)/RC}{s^2+\left(\frac{1}{RC}\right)s+\left(\frac{1}{LC}\right)}$$

$$I(s)=\frac{V_s(s)-V(s)}{R}$$

or $I(s)=\frac{V(s)}{sL}+\frac{V(s)}{\frac{1}{sC}}$

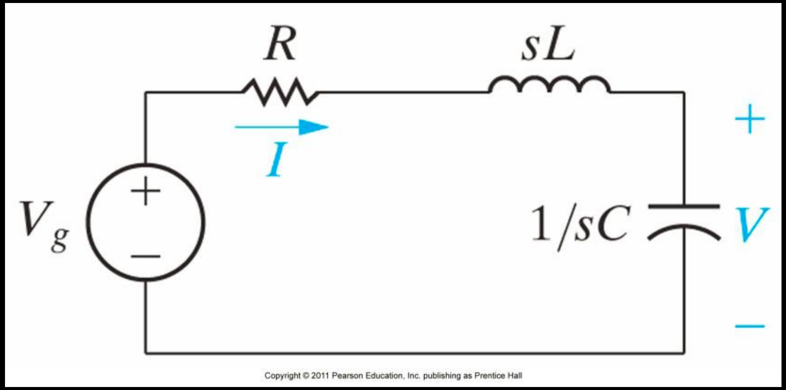
$$I(s)=\frac{V_s(s)\left(\frac{1}{R}s^2+\frac{1}{RLC}\right)}{s^2+\left(\frac{1}{RC}\right)s+\left(\frac{1}{LC}\right)}$$

The Transfer Function

The transfer (system) function is the ratio of the output(response) to the input(source) in the frequency (s) domain. There are two premises: the circuit

1. is a linear time-invariant circuit
2. has zero initial conditions

Examples:

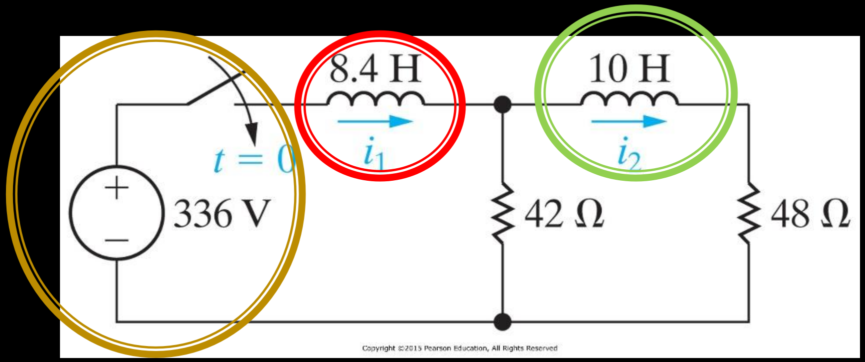


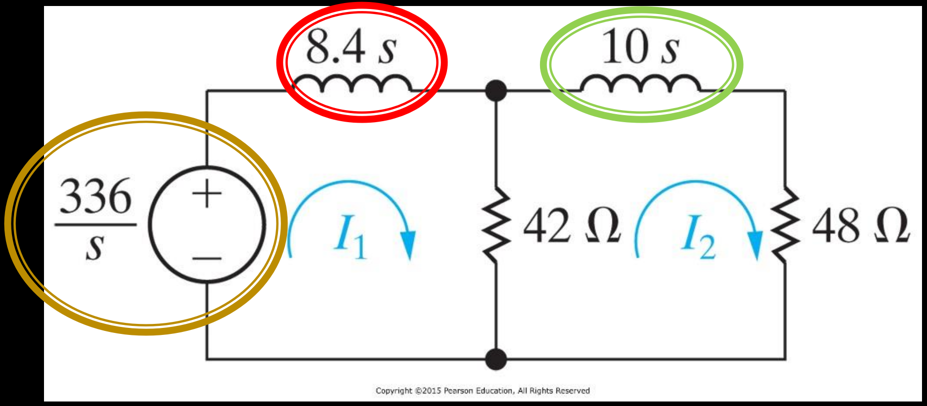
$$H(s) = \frac{V(s)}{V_g(s)} = \frac{1/LC}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

$$H(s) = \frac{I(s)}{V_g(s)} = \frac{\frac{1}{L}s}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

Example #1 (P.493)

Example 1: (P. 493)





No initial energy. Find the step responses of $i_1(t) and\ i_2(t)$.

Mesh-current method:

$$(42 + 8.4s)I_1 - 42I_2 = \frac{336}{s}$$

$$-42I_1 + (90 + 10s)I_2 = 0$$

$$\Delta = \begin{vmatrix} 42 + 8.4s & -42 \\ -42 & 90 + 10s \end{vmatrix}$$

$$N_1 = \begin{vmatrix} \frac{336}{s} & -42 \\ 0 & 90 + 10s \end{vmatrix}$$

$$N_2 = \begin{vmatrix} 42 + 8.4s & \frac{336}{s} \\ -42 & 0 \end{vmatrix}$$

Example #1 (P.493) –cont'd

$$I_1 = \frac{N_1}{\Delta} = \frac{40(s+9)}{s(s+2)(s+12)} \qquad I_2 = \frac{N_2}{\Delta} = \frac{168}{s(s+2)(s+12)}$$

$$I_1 = \frac{k_1}{s} + \frac{k_2}{s+2} + \frac{k_3}{s+12} = \frac{15}{s} - \frac{14}{s+2} - \frac{1}{s+12}$$

$$\Rightarrow \quad i_1(t) = \left(15 - 14e^{-2t} - e^{-12t}\right)u(t)$$

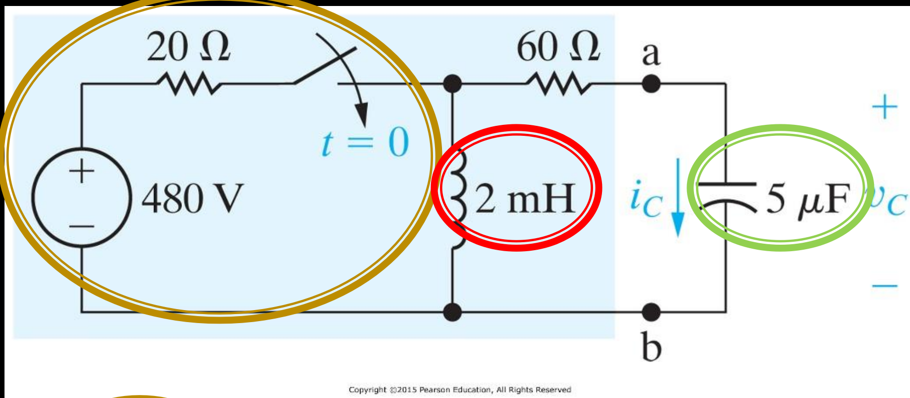
$$I_2 = \frac{k_1}{s} + \frac{k_2}{s+2} + \frac{k_3}{s+12} = \frac{7}{s} - \frac{8.4}{s+2} + \frac{1.4}{s+12}$$

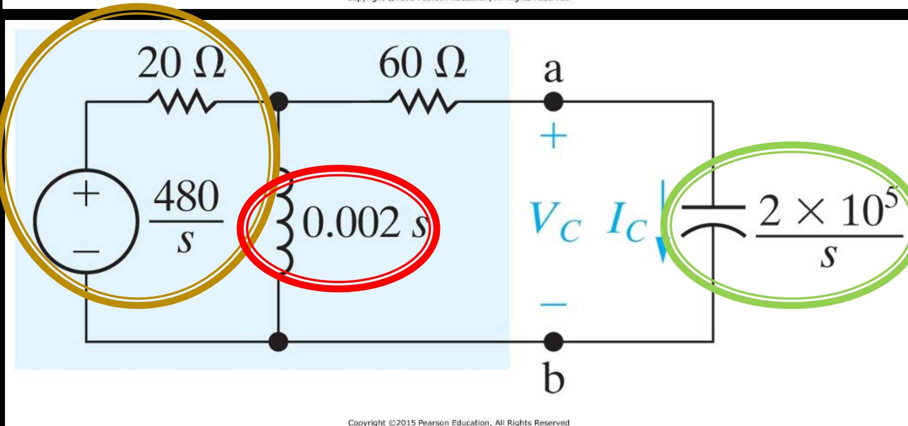
$$\Rightarrow \quad i_2(t) = \left(7 - 8.4e^{-2t} + 1.4e^{-12t}\right)u(t)$$

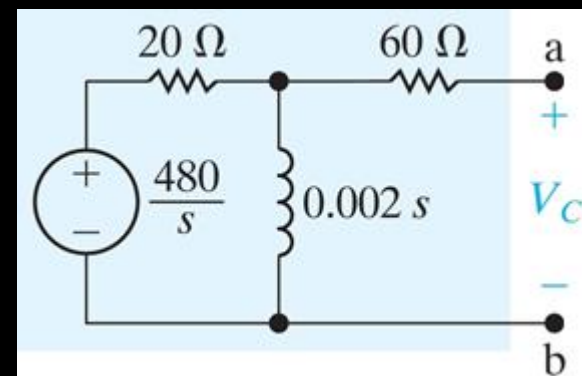
Steps of Circuit Analysis in the s Domain

1. Check the initial energy in a circuit.
2. Apply the Laplace transform to the elements and sources in the circuit.
3. Redraw the circuit in the s domain.
4. Use the circuit analysis techniques discussed in the first four chapters to find the voltages/currents in the s domain.
5. Expand the voltages/currents into a sum of partial fractions.
6. Apply the inverse Laplace transform and find the voltages/currents in the time domain.
7. Use the initial-value and final-value theorems to verify the solutions found in step 6.

Example #2 (P.495)



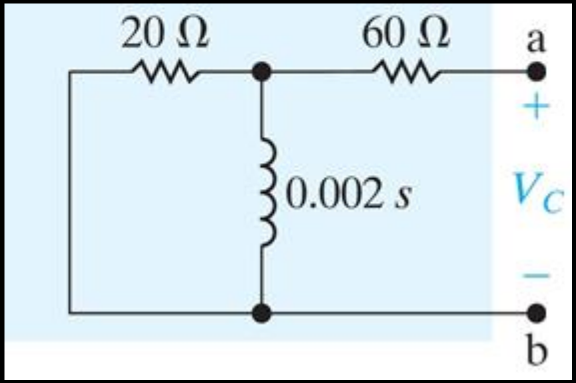




$$V_{Th} = \frac{(480/s)(0.002s)}{20 + 0.002s} = \frac{480}{s + 10^4}$$

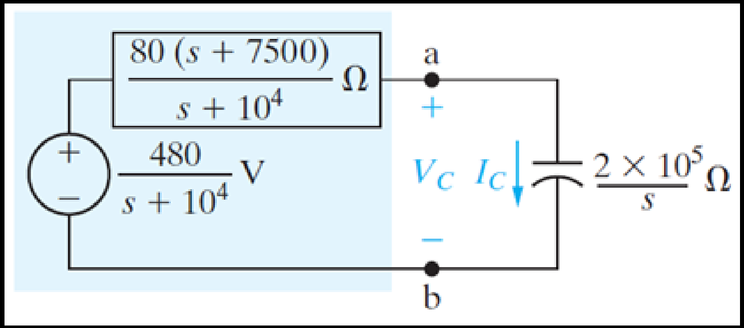
No initial energy. Use Thévenin equivalent to find $i_c(t)$.

Example #2 (P.495) – cont'd



$$Z_{Th} = 60 + \frac{0.002s(20)}{20 + 0.002s} = \frac{80(s + 7500)}{s + 10^4}$$

$$I_C(s) = \frac{\frac{480}{s + 10^4}}{\frac{80(s + 7500)}{s + 10^4} + \frac{2 \times 10^5}{s}} = \frac{6s}{(s + 5000)^2}$$



$$I_C(s) = \frac{k_1}{(s + 5000)^2} + \frac{k_2}{s + 5000} = \frac{-30{,}000}{(s + 5000)^2} + \frac{6}{s + 5000}$$

$$i_c(t) = \left(-30{,}000te^{-5000t} + 6e^{-5000t}\right)u(t)\ \text{A}$$

Summary

- The Laplace transform of the voltage - current equation for each element:
    a. Resistors: $V(s) = RI(s)$
    b. Inductors: $V(s) = sLI(s) - LI_0$
    c. Capacitors: $V(s) = \left(\frac{1}{sC}\right) I(s) - V_0/s$
- Several examples of circuits analysis in the s domain are given in this lecture.
    a. How to find the transfer function of a circuit.
    b. How to use circuit elements in the s domain to build a circuit model.
    c. How to apply circuit analysis techniques to solve the quantities for a circuit in which inductors and capacitors are included.

In next lecture, we will discuss the steady-state sinusoidal response