Homework #4
PDF version due Mon. December 12 11:59pm through CatCourses
NO LATE SUBMISSIONS ACCEPTED
SOLUTION

1) This problem is about the Hough Transform.

a) Given an edge point at pixel location (0,0) in image space, provide the equation for the corresponding sinusoid in parameter space. (This is an equation involving θ and ρ.)

SOLUTION:

Plugging in x=0, y=0 into the equation

$$x\cos\theta + y\sin\theta = \rho$$

yields the equation

$$0 = \rho$$

or

$$\rho = 0.$$

(This is a "degenerate" sinusoid since it is just a horizontal line.)

b) Are there any other points (at locations other than (0,0)) that correspond to this particular sinusoid?

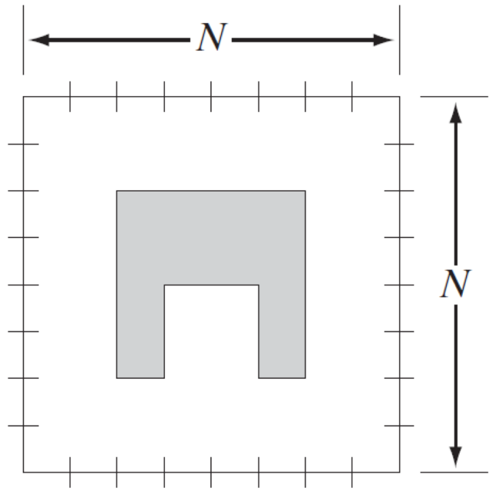
SOLUTION:

No, no other points correspond to this sinusoid. (There are no other pairs (x,y) that result in the sinusoid $\rho = 0$.)

2) Segment the image below using the split and merge procedure described in lecture and in the text (Section 10.4.2 in the 3rd edition and the section titled "REGION SPLITTING AND MERGING" in Chapter 10 of the 4th edition). For simplicity, you can assume N=8. Let $Q(R_i) = TRUE$ if all the pixels in R_i have the same intensity. The image has pixels with two different intensities.

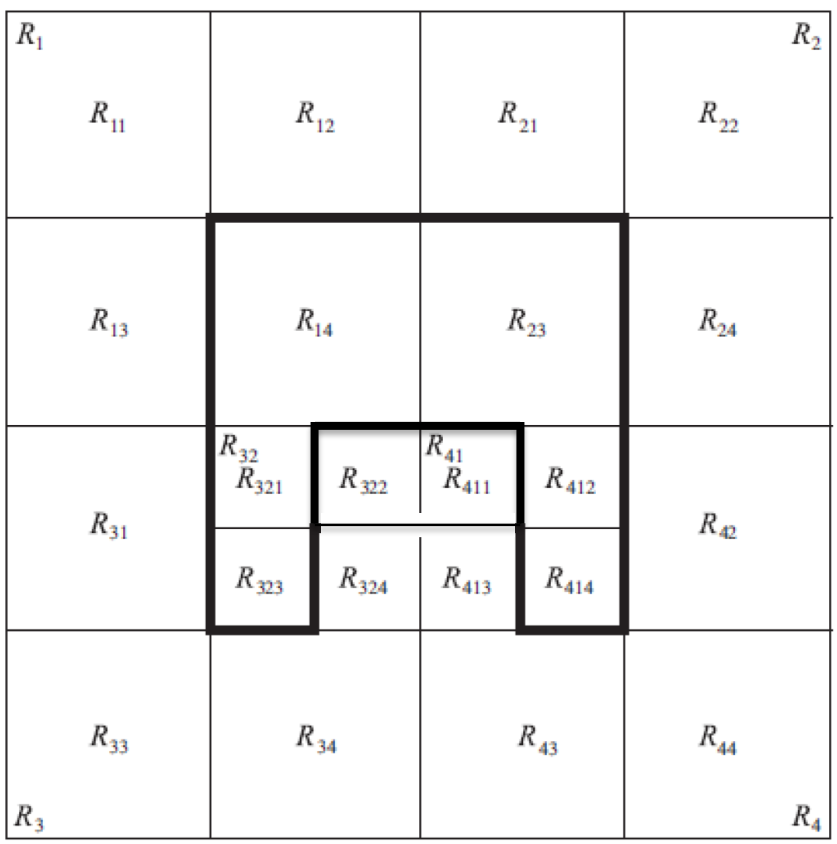
Show your results using two figures:

a) The segmented image with the various quandrants labeled (e.g., R_1, R_{23}, R_{414}, etc.).
b) The quadtree corresponding to the segmentation.

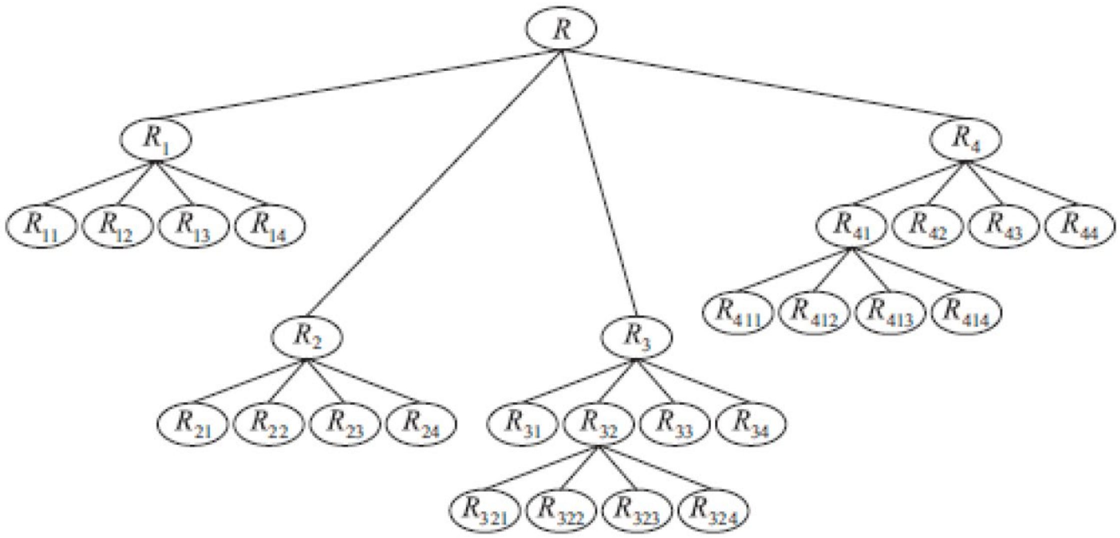


SOLUTION:

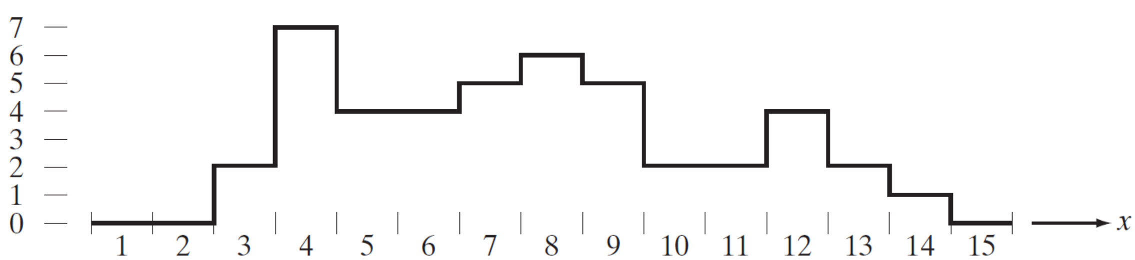
a)



b)



3) This problem is about watershed segmentation. Consider the one-dimensional intensity cross section of an image below. Give a step-by-step implementation of watershed segmentation for this cross section. Show a drawing of the cross section at each step, showing the "water" levels and the dams constructed. You can assume there are "dams" of sufficient height at the edges of the cross section to prevent the water from "flowing" off the edges..



SOLUTION:

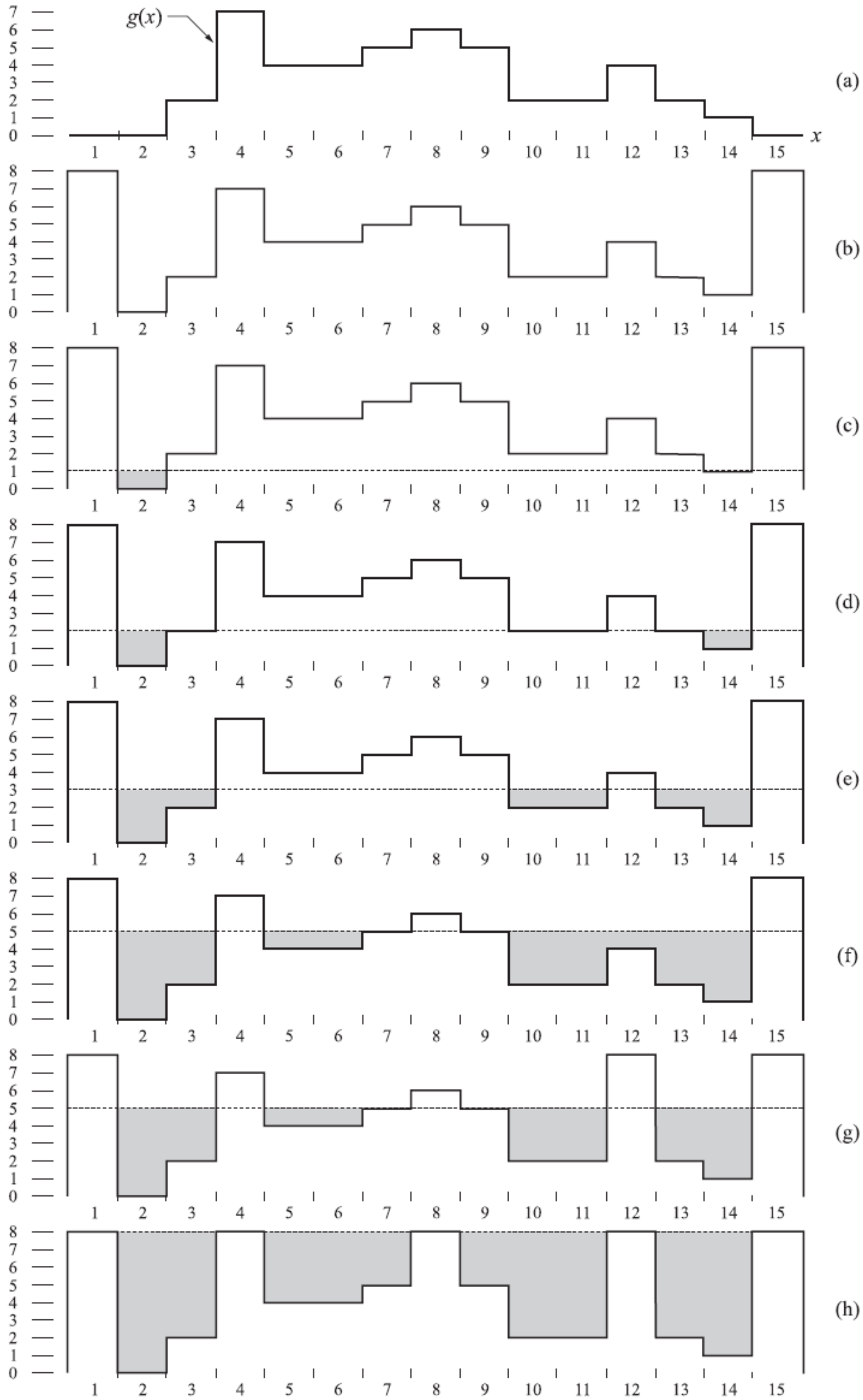


Figure P10.43

The first step in the application of the watershed segmentation algorithm is to build a dam of height $\max + 1$ to prevent the rising water from running off the ends of the function, as shown in Fig. P10.43(b). For an image function we would build a box of height $\max + 1$ around its border. The algorithm is initialized by setting $C[1] = T[1]$. In this case, $T[1] = \{g(2)\}$, as shown in Fig. P10.43(c) (note the water level). There is only one connected component in this case: $Q[1] = \{q_1\} = \{g(2)\}$.

Next, we let $n = 2$ and, as shown in Fig. P10.43(d), $T[2] = \{g(2), g(14)\}$ and $Q[2] = \{q_1; q_2\}$, where, for clarity, different connected components are separated by semicolons. We start construction of $C[2]$ by considering each connected component in $Q[2]$. When $q = q_1$, the term $q \cap C[1]$ is equal to $\{g(2)\}$, so condition 2 is satisfied and, therefore, $C[2] = \{g(2)\}$. When $q = q_2$, $q \cap C[1] = \emptyset$ (the empty set) so condition 1 is satisfied and we incorporate q in $C[2]$, which then becomes $C[2] = \{g(2); g(14)\}$ where, as above, different connected components are separated by semicolons.

When $n = 3$ [Fig. P10.43(e)], $T[3] = \{2, 3, 10, 11, 13, 14\}$ and $Q[3] = \{q_1; q_2; q_3\} = \{2, 3; 10, 11; 13, 14\}$ where, in order to simplify the notation we let k denote $g(k)$. Proceeding as above, $q_1 \cap C[2] = \{2\}$ satisfies condition 2, so q_1 is incorporated into the new set to yield $C[3] = \{2, 3; 14\}$. Similarly, $q_2 \cap C[2] = \emptyset$ satisfies condition 1 and $C[3] = \{2, 3; 10, 11; 14\}$. Finally, $q_3 \cap C[2] = \{14\}$ satisfies condition 2 and $C[3] = \{2, 3; 10, 11; 13, 14\}$. It is easily verified that $C[4] = C[3] = \{2, 3; 10, 11; 13, 14\}$.

When $n = 5$ [Fig. P10.43(f)], we have, $T[5] = \{2, 3, 5, 6, 10, 11, 12, 13, 14\}$ and $Q[5] = \{q_1; q_2; q_3\} = \{2, 3; 5, 6; 10, 11, 12, 13, 14\}$ (note the merging of two previously distinct connected components). Is is easily verified that $q_1 \cap C[4]$ satisfies condition 2 and that $q_2 \cap C[4]$ satisfied condition 1. Proceeding with these two connected components exactly as above yields $C[5] = \{2, 3; 5, 6; 10, 11; 13, 14\}$ up to this point. Things get more interesting when we consider q_3. Now, $q_3 \cap C[4] = \{10, 11; 13, 14\}$ which, becuase it contains two connected components of $C[4]$, satisfies condition 3. As mentioned previously, this is an indication that water from two different basins has merged and a dam must be built to prevent this condition. Dam building is nothing more than separating q_3 into the two original connected components. In this particular case, this is accomplished by the dam shown in Fig. P10.43(g), so that now $q_3 = \{q_{31}; q_{32}\} = \{10, 11; 13, 14\}$. Then, $q_{31} \cap C[4]$ and $q_{32} \cap C[4]$ each satisfy condition 2 and we have the final result for $n = 5$, $C[5] = \{2, 3; 5, 6; 10, 11; 13; 14\}$.

Continuing in the manner just explained yields the final segmentation result shown in Fig. P10.43(h), where the "edges" are visible (from the top) just above the water line. A final post-processing step would remove the outer dam walls to yield the inner edges of interest.