ENGR 065 Electric Circuits

Lecture 10b: Maximum Power Transfer and Superposition

Today's Topics

- Maximum power transfer

  – under what condition that loads can obtain maximum power transferred from sources.

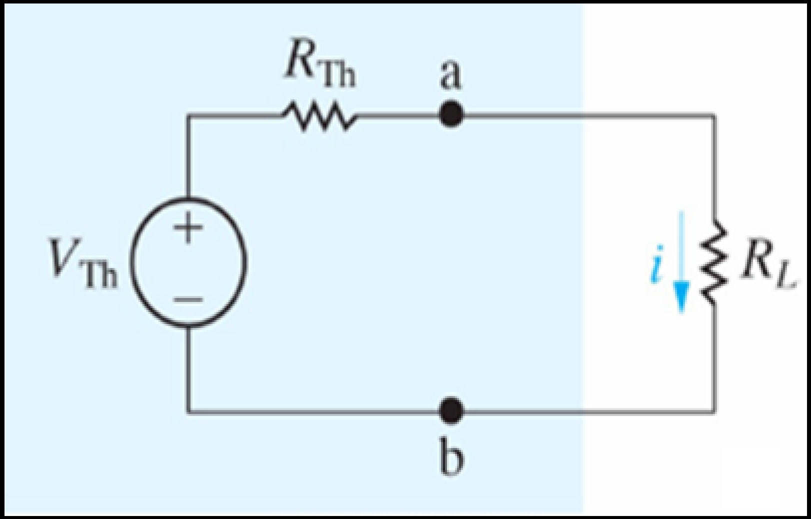
- The superposition principle

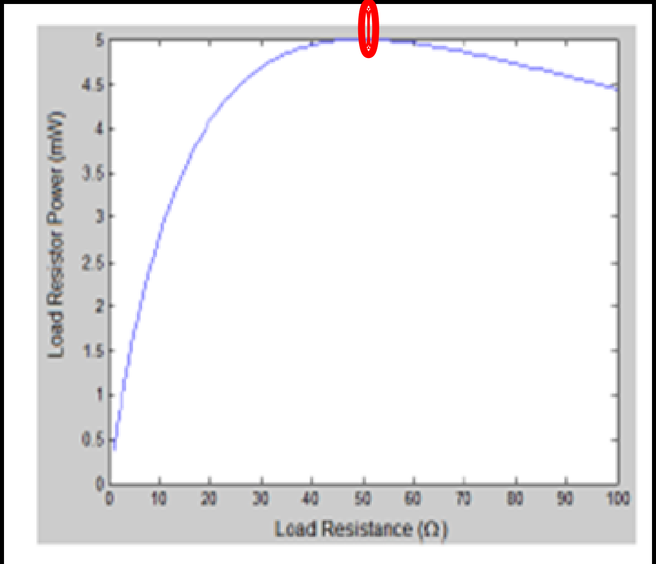
  – can only be applied to linear circuits

Covered in Sections 4.12 and 4.13

Power Transfer Calculation

The problem is to determine the value of R_L that permits maximum power delivery to R_L.





The power dissipated by R_L is

$$p = i^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 R_L$$

Maximum Power Transfer Calculation

Let $\frac{dp}{dR_L} = 0$, we have

$$\frac{dp}{dR_L} = \frac{d[(\frac{V_{Th}}{R_{Th}+R_L})^2 R_L]}{dR_L} = V_{Th}{}^2 \frac{(R_{Th}+R_L)^2 - 2(R_{Th}+R_L)R_L}{(R_{Th}+R_L)^4} = 0$$

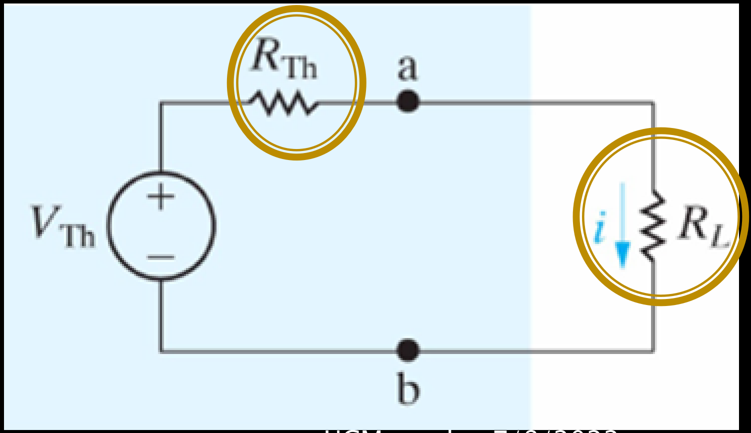
$$(R_{Th}+R_L)^2 - 2(R_{Th}+R_L)R_L = 0$$

$$\boxed{R_L = R_{Th}}$$

Which means when the load resistance is equal to the Thévenin equivalent resistance, the maximum power is transferred to the load.

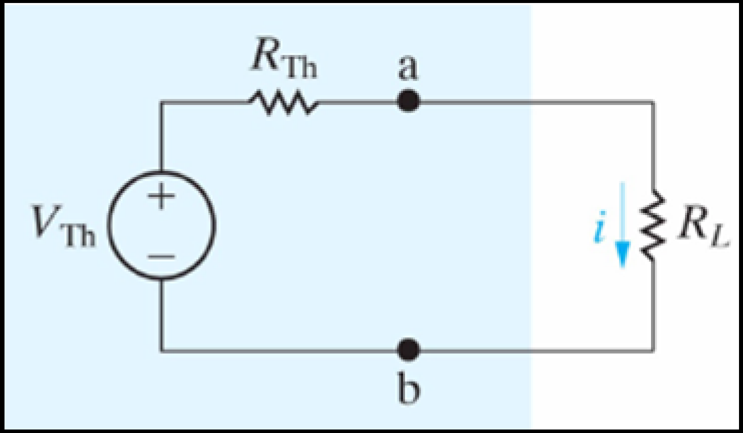
Maximum Power Transfer

- In an electrical system, maximum power transferred from a power source to a load is when the resistance of the load R_L is equal to the equivalent or internal resistance of the source, which is $R_L = R_{Th}$ or $R_L = R_{IN}$.
- The process to match $R_L = R_{Th}$ or $R_L = R_{IN}$ is called resistance or impedance matching.



The Maximum Power Delivered to Load

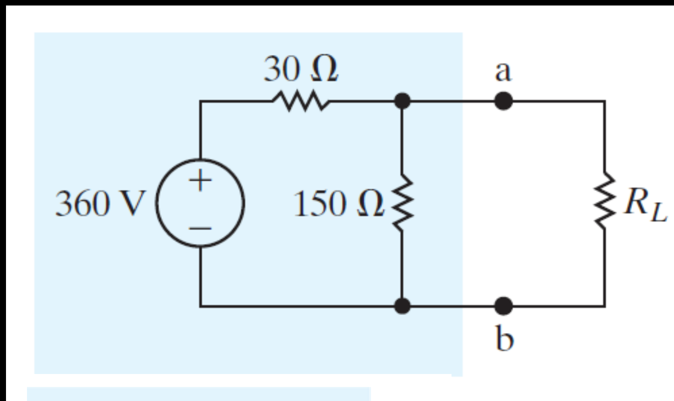
$$p_{max}|_{R_L=R_{Th}} = \left(\frac{V_{Th}}{R_{Th}+R_L}\right)^2 R_L = \left(\frac{V_{Th}}{R_L+R_L}\right)^2 R_L = \frac{V_{Th}^{\;2}}{4R_L^{\;2}} R_L = \frac{V_{Th}^{\;2}}{4R_L}$$

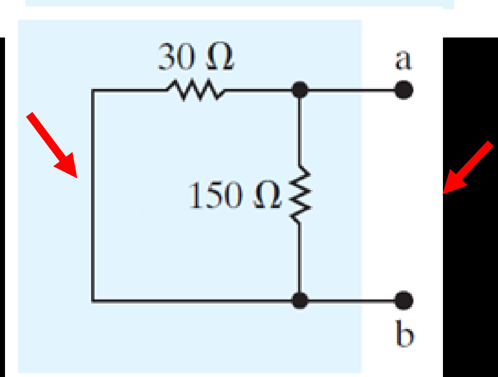


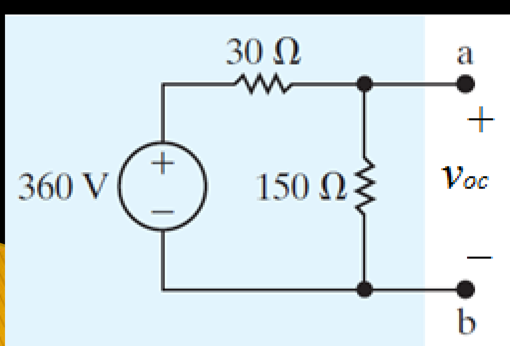
In addition, the power dissipated by the equivalent resistor ($R_{Th}, R_N or\ \ R_{IN}$) is same as the power absorbed by the load.

$$p_{R_{Th}} = p_{R_L} = \frac{V_{Th}^{\;2}}{4R_L}$$

Example #1







For the circuit shown on the left, find the value of R_L that results in the maximum power being transferred to R_L and this maximum power.

1. Find R_{Th}

$$R_{Th} = \frac{30 \times 150}{30 + 150} = 25\ \Omega$$

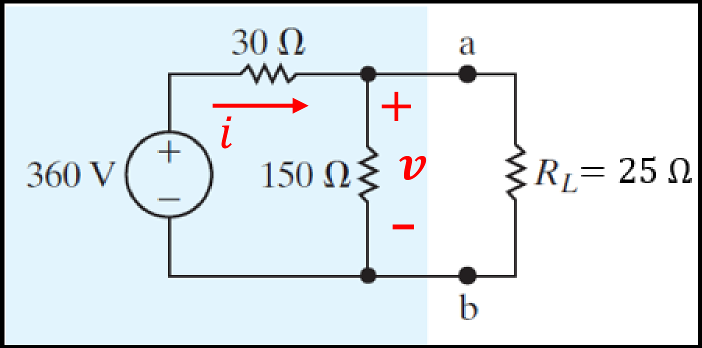
$$R_L = R_{Th} = 25\ \Omega$$

2. Find V_{Th}

$$V_{Th} = v_{oc} = \frac{360 \times 150}{30 + 150} = 300\ V$$

$$p_{max} = \frac{{V_{Th}}^2}{4R_L} = \frac{300^2}{4 \times 25} = 900\ W$$

Example #1 – cont'



Let's find the power associated with all elements in the circuit.

$$i = \frac{360}{30 + \frac{150 \times 25}{150 + 25}} = 7\ A$$

$$p_{360\ V} = -7 \times 360 = -2520\ W$$

$$p_{30} = i^2 \times R = 7^2 \times 30 = 1470\ W$$

$$v = 360 - 30 \times 7 = 150\ V$$

$$p_{150} = \frac{150^2}{150} = 150\ W$$

$$p_{25} = \frac{150^2}{25} = 900\ W$$

What do the results tell you?

The Superposition Principle

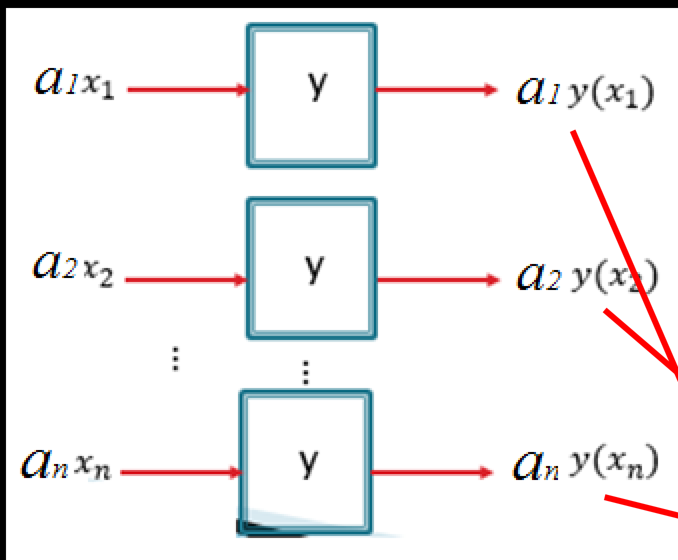
- The superposition principle can only be applied to linear circuits (systems).
- What is a linear circuit or system?

- If a circuit or system is linear, it must hold the following two properties:
  - Homogeneity: $y(ax) = ay(x)$
  - Additivity: $y(x_1 + x_2 + \cdots + x_n) = y(x_1) + y(x_2) + \cdots + y(x_n)$

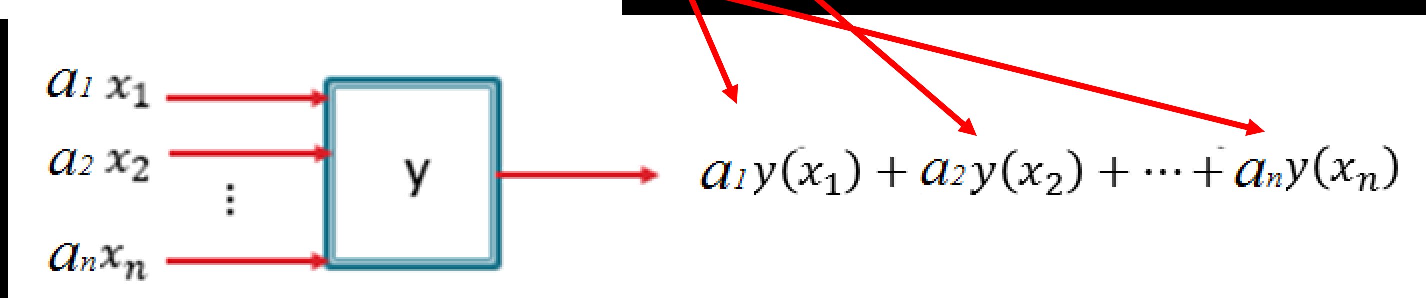
The Superposition Principle

If a circuit (system) is linear (homogeneity + additivity), we have

$$y(ax) = ay(x) \text{ and } y(x_1 + x_2 + \cdots + x_n) = y(x_1) + y(x_2) + \cdots + y(x_n)$$



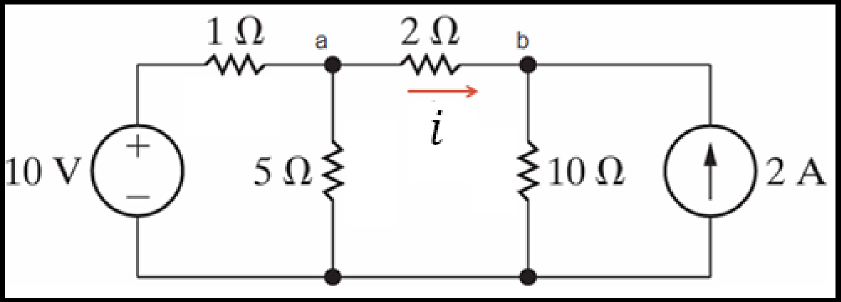
Whenever a linear circuit is driven by more than one **independent** sources, the response caused by these multiple sources is the sum of the individual responses.

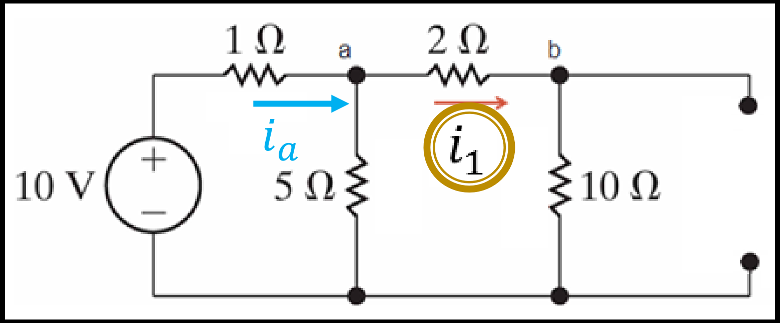


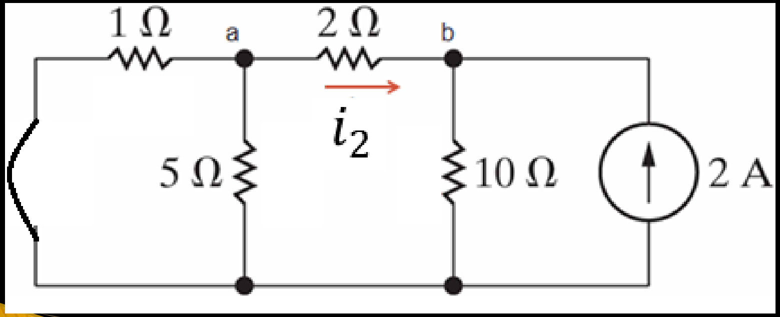
Steps in Applying the Superposition Principle

1. If there are multiple independent sources in a circuit, keep ONE independent source in the circuit and remove all other independent sources. The removed voltage sources are replaced with short circuits and current sources replaced with open circuits.
2. Find the response driven by the kept independent source.
3. Repeat Step 1 and 2 for each independent source.
4. Algebraically add the responses driven by all the individual independent sources found in Steps 1 to 3 above.

Example #2







Find the current i

1. There are two independent sources in the circuit.
2. Keep the 10V voltage source in the circuit and remove the 2A current source.
3. Find i_1. (2+10)//5+1= 4.53 Ω

$$i_a = \frac{10}{4.53} = 2.21\ A$$

$$i_1 = \frac{i_a \times 5}{5+12} = 0.65\ A\ (cuurent\ division)$$

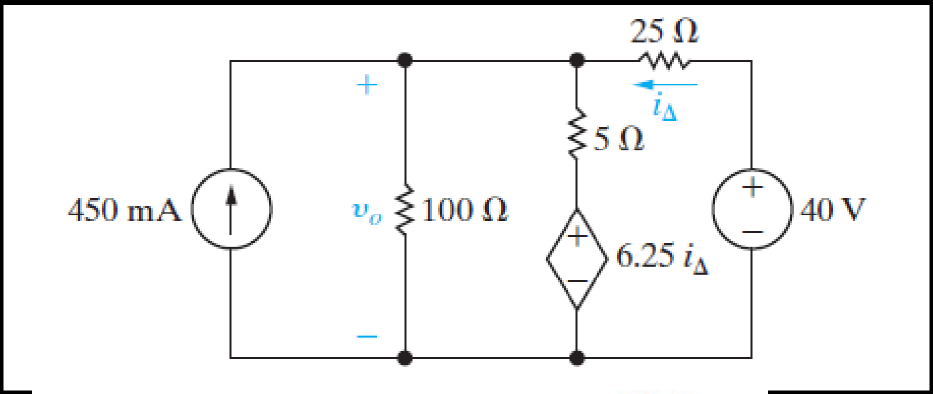
4. Keep the 2A current source in the circuit and remove the 10V voltage source.

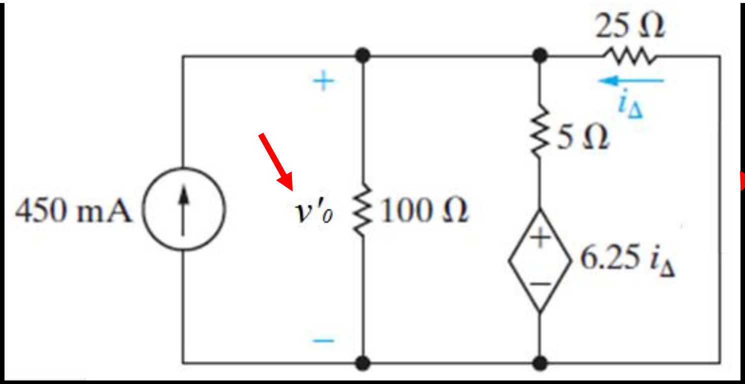
1//5+2 = 2.83 Ω

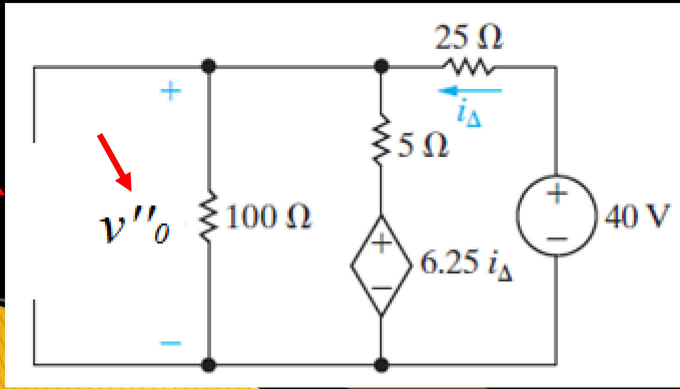
$$i_2 = -\frac{2 \times 10}{10 + 2.83} = -1.56\ A$$
$$(cuurent\ division)$$

$$i = i_1 + i_2 = 0.65 - 1.56 = -\ 0.91\ A$$

Example #3 (P.4.20)







Use the superposition to find v_0.

1. Remove the 40 V voltage source
2. Use node-voltage method to find v_0'.

$$-0.45 + \frac{v_0'}{100} + \frac{v_0' - 6.25 i_\Delta}{5} + \frac{v_0'}{25} = 0$$

$$i_\Delta = -\frac{v_0'}{25} \quad (Ohlm's\ law)$$

Solve above equations for $v_0', we\ have$:

$$v_0' = 1.5\ V$$

3. Remove the 450 mA current source
4. Use node-voltage method to find v_0''.

$$\frac{v_0''}{100} + \frac{v_0'' - 6.25 i_\Delta}{5} + \frac{v_0'' - 40}{25} = 0$$

$$i_\Delta = -\frac{v_0'' - 40}{25}$$

Solve above equations for $v_0'', we\ have$:

$$v_0'' = 12\ V$$

5. $v_0 = v_0' + v_0'' = 1.5 + 12 = 13.5\ V$

Summary

- In this chapter, we learned four important techniques that can be used in circuit analysis:
  1. Node-voltage and mesh-current methods
  2. Source transformations
  3. Thévenin and Norton equivalents
  4. Superposition

In the next lecture, we will discuss the applications of these techniques:

The operational amplifier