ENGR 065 Electric Circuits

Lecture 11: The Operational Amplifier

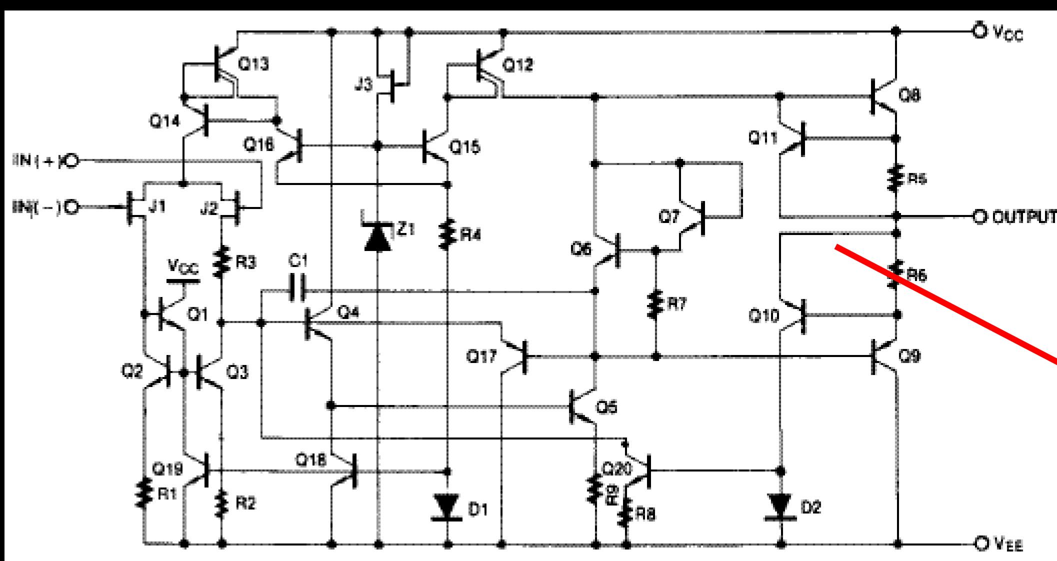
Today’s Topics

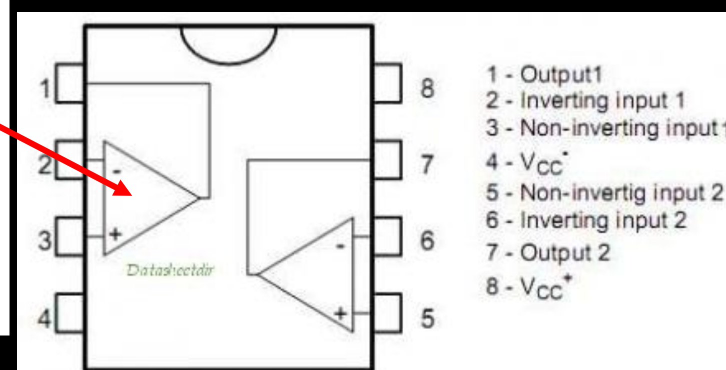
- Operational amplifier terminals
- The ideal operational amplifier
- The inverting-amplifier circuit
- The summing-amplifier circuit
- The noninverting-amplifier circuit
- The difference-amplifier circuit
    - The common mode rejection ratio (CMRR)

Covered in Sections 5.1, 5.2, 5.3, 5.4, 5.5, and 5.6

Introductions

Operational amplifiers (Op-amps) are made of many transistors, diodes, resistors and capacitors with integrated circuit technology.



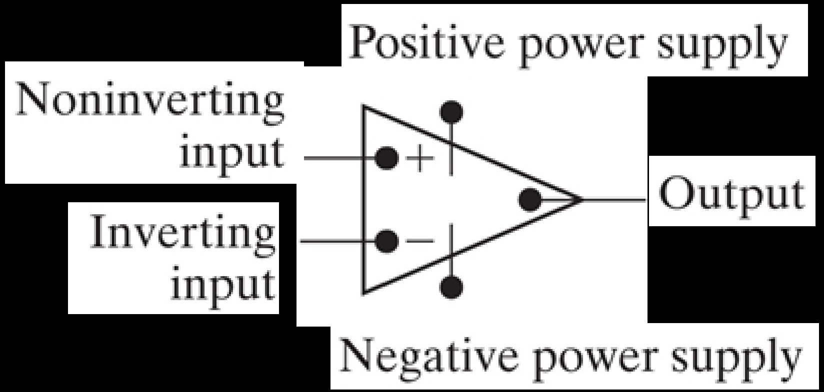


Introduction

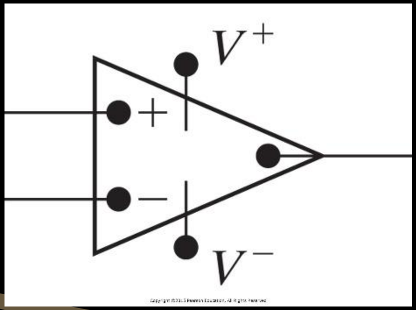
- Op-amps are the basic components used to build analog circuits.
- Most op-amps behave like voltage amplifiers. They take an input voltage and output a scaled version.
- The name "operational amplifier" comes from the fact that they were originally used to perform mathematical operations such as summing, subtracting, integration, and differentiation.
- The fabrication techniques of Integrated circuits make high-performance op-amps very cheap in comparison to older discrete devices.

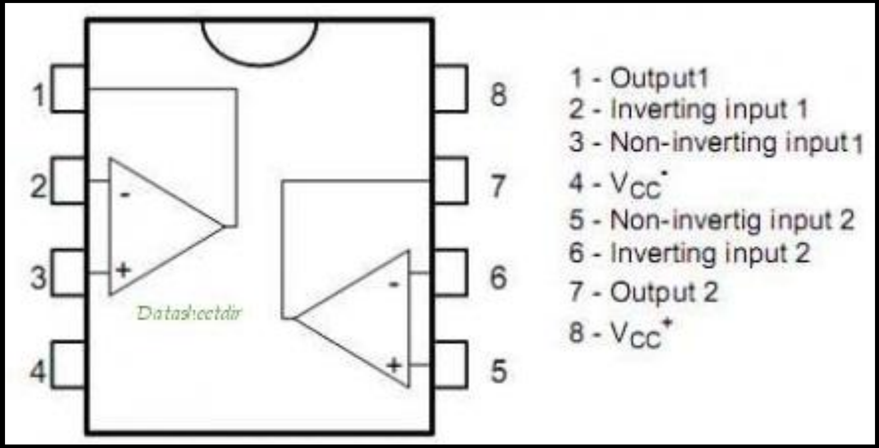
Terminal Voltages and Currents

2. There are five primary terminals:
- ➢ Inverting input
- ➢ Noninverting input
- ➢ Output
- ➢ Positive power supply
- ➢ Negative power supply



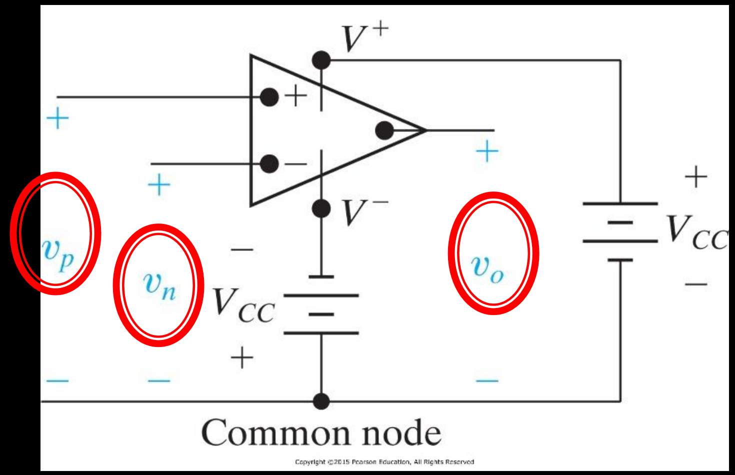
1. The circuit symbol for an op-amp



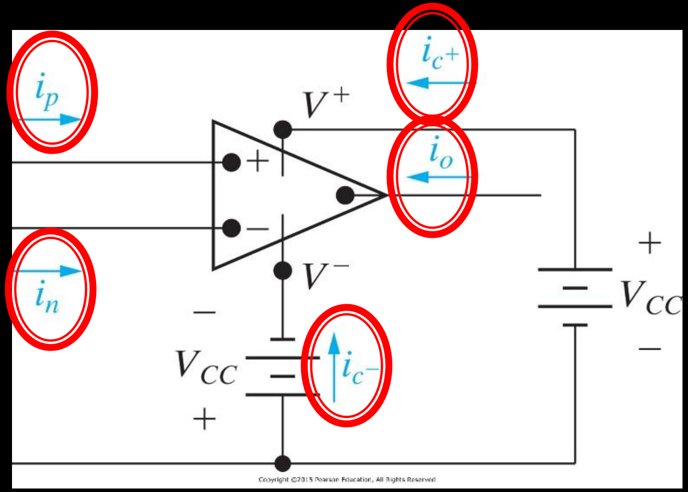


Terminal Voltages and Currents

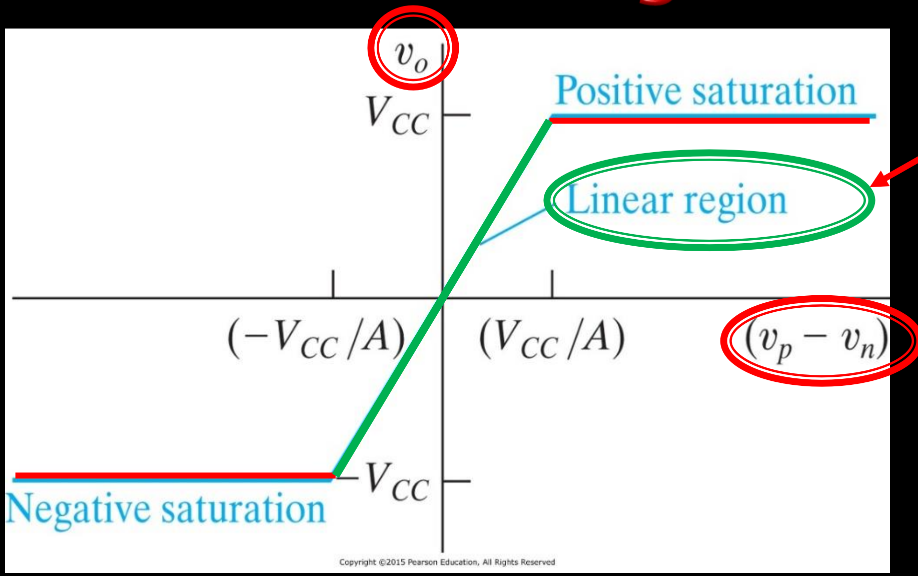
3. Terminal voltage variables
v_n: the voltage between the inverting input terminal and the common(reference) node.
v_p: the voltage between the noninverting input terminal and the common node.
v_o: the voltage between the output terminal and the common node.



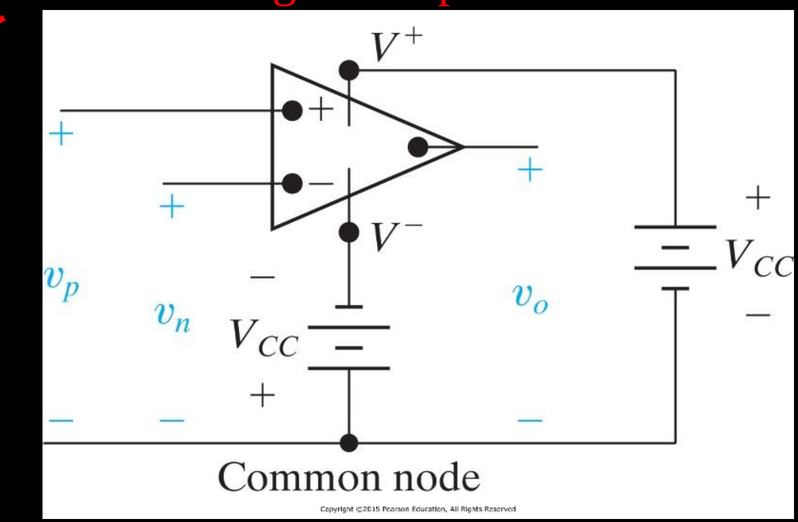
4. Terminal current variables
i_n: the current into the inverting input terminal.
i_p: the current into the noninverting input terminal.
i_o: the current into the output terminal.
i_{c^+}: the current into the positive power supply terminal.
i_{c^-}: the current into the negative power supply terminal.



Terminal Voltages and Currents(A≠ ∞)



This is the region we prefer

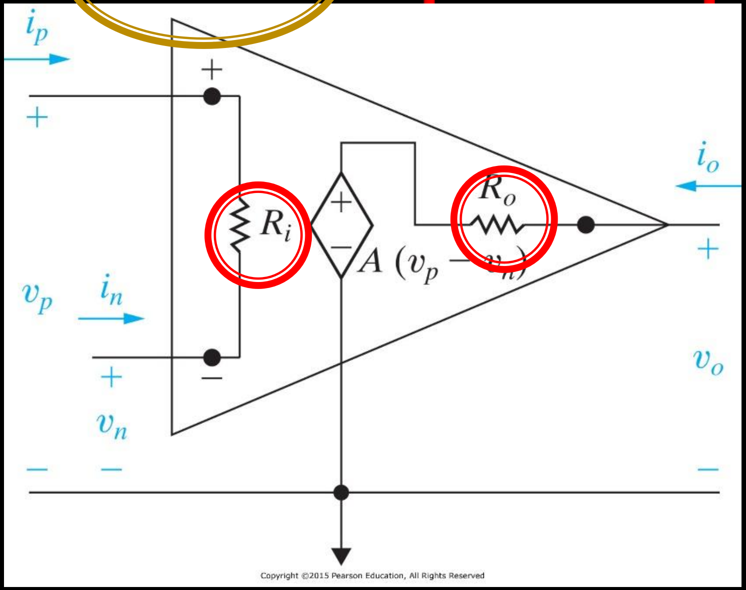


The output voltage of the op- amp is:

$$v_o = \begin{cases} -V_{CC} & A(v_p - v_n) < -V_{CC} \\ A(v_p - v_n) & -V_{CC} \le A(v_p - v_n) \le +V_{CC} \\ +V_{CC} & A(v_p - v_n) > +V_{CC} \end{cases}$$

where A is called the open-loop gain of the op amp.

The Constraints of Ideal Op-Amps



1. The input resistance is infinite

$$R_i = \infty$$

2. The output resistance is zero

$$R_o = 0$$

3. The open-loop gain is infinite

$$A = \infty$$

4. The frequency bandwidth is infinite

5. Input voltage constraint for ideal op-amps (virtual short）

$$\boxed{v_p = v_n,}\ v_p - v_n = 0 \quad (\text{because } A = \infty)$$

6. Input current constraint for idea op amps

$$\boxed{i_p = i_n = 0}\ (\text{because } R_i = \infty)$$

7. From KCL, we know:

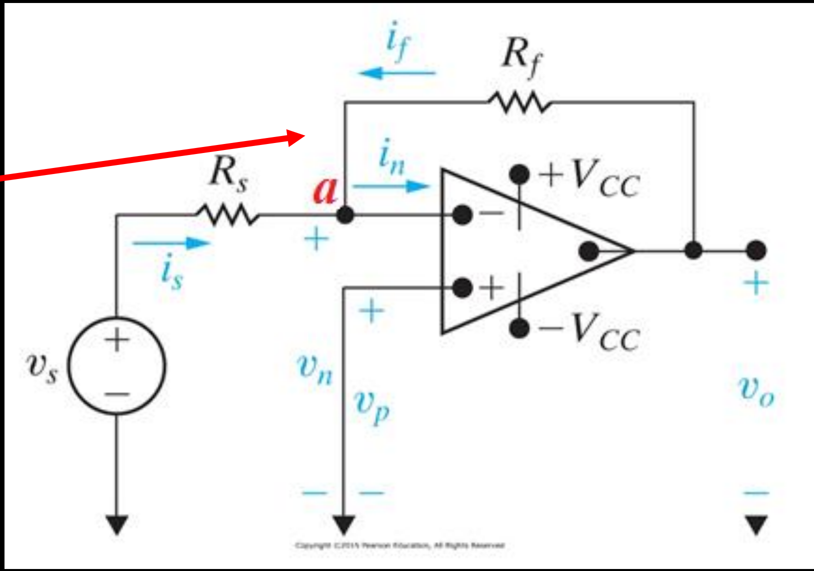
$$i_p + i_n + i_o + i_{c^+} + i_{c^-} = 0$$

Substituting the current constraint, we have:

$$i_o = -(i_{c^+} + i_{c^-})$$

The Inverting-Amplifier Circuit

The output signal is fed back from the output terminal to the inverting input terminal. This configuration is called negative feedback connection.



Because the noninverting terminal is connected to ground, $v_p = 0$, so $v_n = 0$

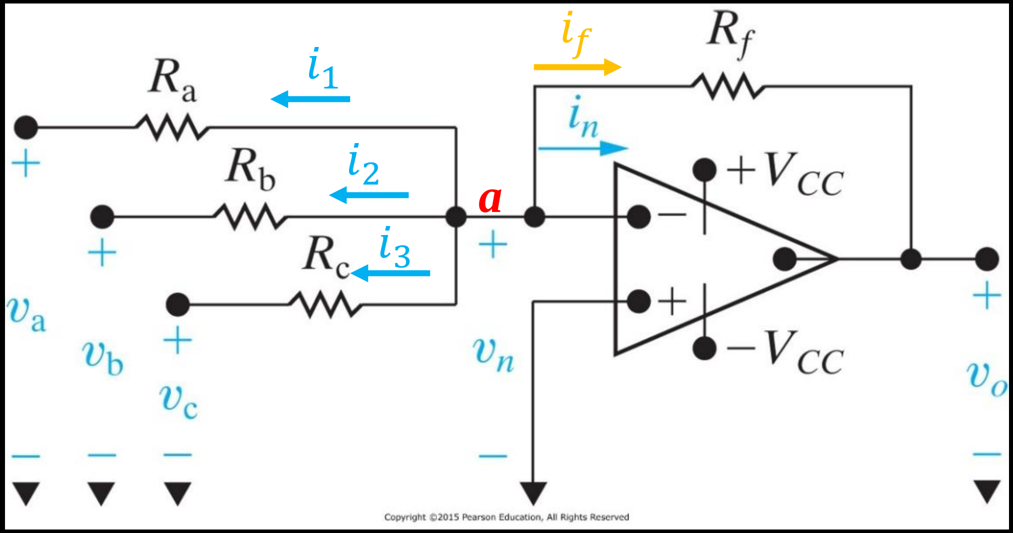
$$i_s = \frac{v_s - v_n}{R_s} = \frac{v_s}{R_s}, \qquad i_f = \frac{v_o - v_n}{R_f} = \frac{v_o}{R_f}$$

Because $i_n = 0$, Applying KCL at node ***a***, we know: $i_s + i_f = 0$, which means

$$\frac{v_s}{R_s} + \frac{v_o}{R_f} = 0, \qquad so \quad \boxed{v_o = -\frac{R_f}{R_s} v_s} \qquad \boxed{G = \frac{v_0}{v_s} = -\frac{R_f}{R_s}}$$

G is called the closed-loop gain of the op-amp.

The Summing-Amplifier Circuit

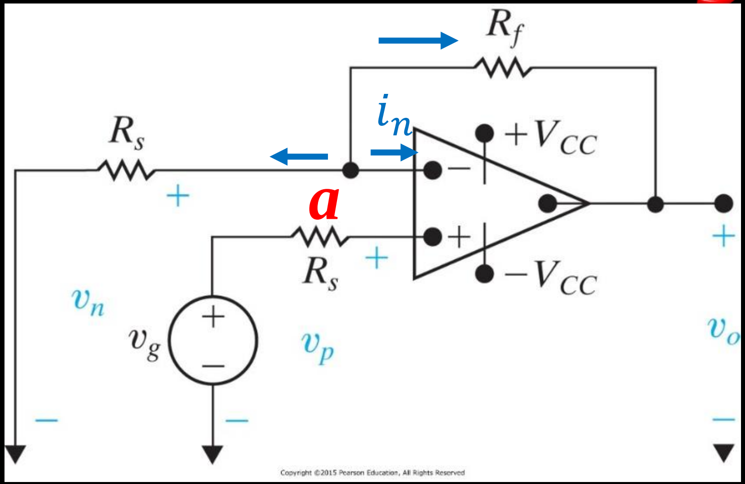


$$i_1 = \frac{v_n - v_a}{R_a}, \quad i_2 = \frac{v_n - v_b}{R_b}, \quad i_3 = \frac{v_n - v_c}{R_c}, \quad i_f = \frac{v_n - v_o}{R_f}, \quad i_n = 0, \quad v_n = v_p = 0$$

Applying KCL at node ***a***, $\frac{v_n - v_a}{R_a} + \frac{v_n - v_b}{R_b} + \frac{v_n - v_c}{R_c} + \frac{v_n - v_o}{R_f} + i_n = 0$

$$\boxed{v_o = -\left(\frac{R_f}{R_a} v_a + \frac{R_f}{R_b} v_b + \frac{R_f}{R_c} v_c\right)}$$

The Noninverting-Amplifier Circuit



It is still a negative feedback connection. However, the source is connected to the non-inverting input terminal.

Because $i_p = 0\ (ideal\ op - amp)$, the voltage on R_s is zero and $v_p = v_g$. In addition, because $v_p = v_n\ (ideal\ op - amp), v_n = v_g$.

As we know, at node ***a***, $\dfrac{v_n - 0}{R_s} + \dfrac{v_n - v_0}{R_f} + i_n = 0$ and $i_n = 0$ (1)

$$From\ (1)\ v_n = v_g = \frac{v_o R_s}{R_s + R_f}$$

$$\boxed{v_o = \frac{R_s + R_f}{R_s} v_g}$$

$$\boxed{G = \frac{R_s + R_f}{R_s}}$$

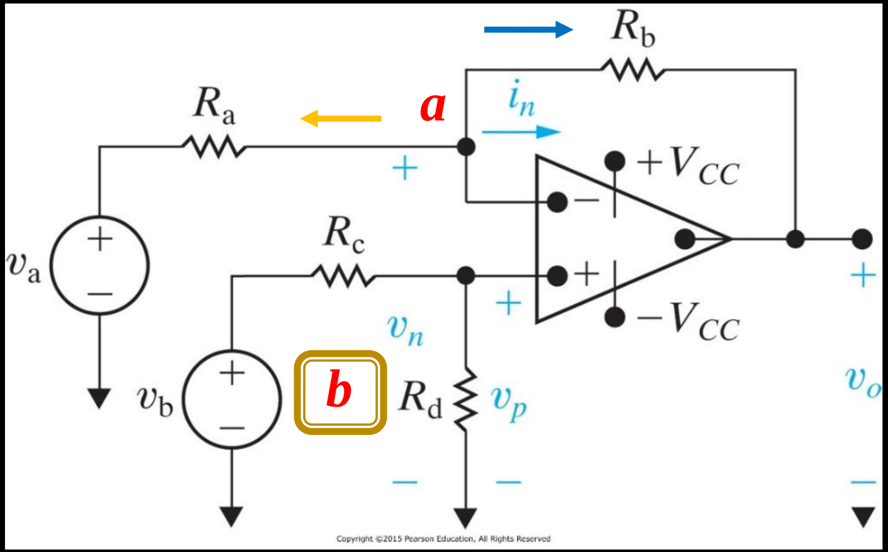
The Difference-Amplifier Circuit

Applying KCL at node ***a***

$$\frac{v_n - v_a}{R_a} + \frac{v_n - v_o}{R_b} + i_n = 0 \quad (1)$$

Because $i_p = i_n = 0$, applying the voltage division to the loop ***b***

$$v_n = v_p = \frac{R_d}{R_c + R_d} v_b \quad (2)$$



It is still a negative feedback connection. However, it has two sources, v_a *and* v_b.

Solving (1) and (2) for v_o ,

$$v_o = \frac{R_d(R_a + R_b)}{R_a(R_c + R_d)} v_b - \frac{R_b}{R_a} v_a \quad (3)$$

If $\frac{R_a}{R_b} = \frac{R_c}{R_d}$,

$$v_o = \frac{R_b}{R_a}(v_b - v_a) \quad (4)$$

$$G = \frac{R_b}{R_a}$$

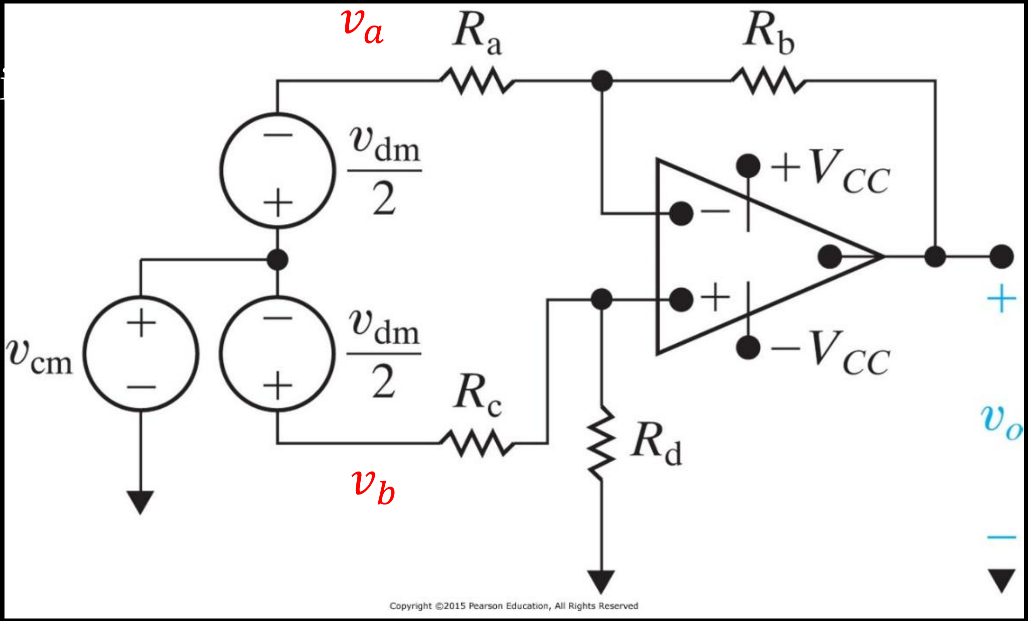
The Difference-Amplifier Circuit

If we define the differential mode input i

$$v_{dm} = v_b - v_a \quad (5)$$

Also , the common mode input is:

$$v_{cm} = (v_a+v_b)/2 \quad (6)$$



From (5) and (6), we have：

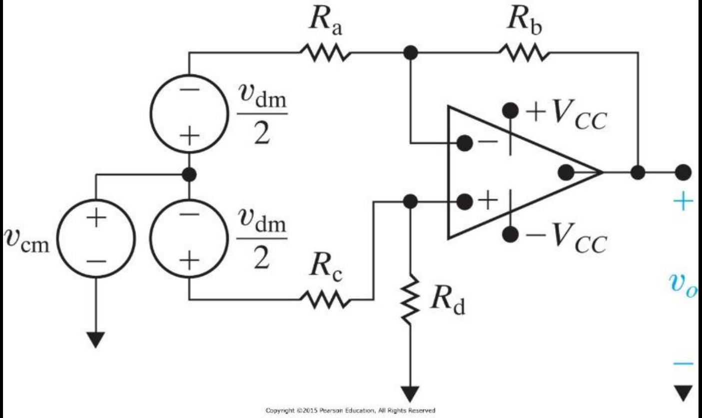
$$v_a = v_{cm} - \frac{1}{2}v_{dm}, \ (7) \quad v_b = v_{cm} + \frac{1}{2}v_{dm} \quad (8)$$

The Difference-Amplifier Circuit

Substituting (7) and (8) into (3) , we have:

$$v_o = \frac{R_a R_d - R_b R_c}{R_\mathrm{a}(R_c + R_d)} v_{cm} + \frac{R_d(R_a + R_b) + R_b(R_c + R_d)}{2R_\mathrm{a}(R_c + R_d)} v_{dm}$$

$$= A_{cm} v_{cm} + A_{dm} v_{dm}$$



Where $A_{cm} = \frac{R_a R_d - R_b R_c}{R_\mathrm{a}(R_c + R_d)}$ (9) is called the common mode gain

$A_{dm} = \frac{R_d(R_a + R_b) + R_b(R_c + R_d)}{2R_\mathrm{a}(R_c + R_d)}$ (10) is called the differential mode gain

If $R_a = R_c$ and $R_b = R_d$, $\boxed{v_o = 0 v_{cm} + \frac{R_b}{R_a} v_{dm} = \frac{R_b}{R_a} v_{dm} \quad (11)}$

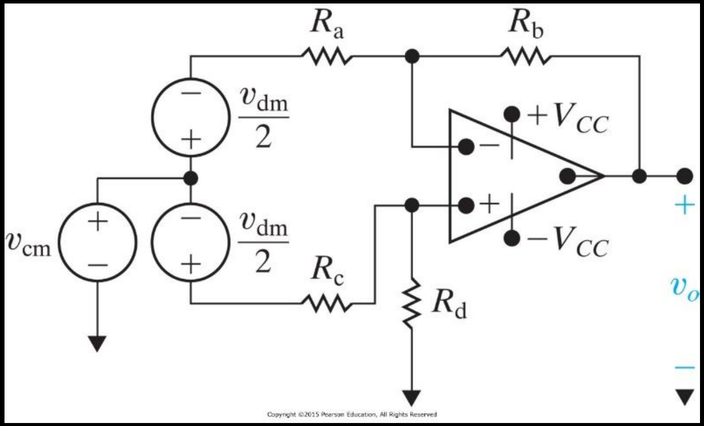
The Common Mode Rejection Ratio(CMRR)

Let's assume $\frac{R_a}{R_b} = (1-\epsilon)\frac{R_c}{R_d}$, which means the resistance mismatches. Let us choose

$$R_a = (1-\epsilon)\ R_c,\ R_b{=}R_d \qquad (12)$$

or $$R_d = (1-\epsilon)\ R_b,\ R_a{=}R_c \qquad (13)$$

Substituting (13) into (9)



$$A_{cm} = \frac{R_a(1-\epsilon)\ R_b - R_bR_a}{R_\mathrm{a}[R_a + (1-\epsilon)\ R_b]}$$

$$= \frac{-\epsilon R_b}{R_a + (1-\epsilon)\ R_b} \approx \frac{-\epsilon R_b}{R_a + R_b}$$

The Common Mode Rejection Ratio

Substituting (13) into (10), we have

$$A_{dm} = \frac{(1-\epsilon)\ R_b(R_a + R_b) + R_b(R_c + \ (1-\epsilon)\ R_b)}{2R_{\mathrm{a}}[R_c + (1-\epsilon)\ R_b]}$$

$$= \frac{R_b}{R_a}\left[1 - \frac{{}^{\epsilon}/_{2}\, R_a}{R_a + (1-\epsilon)R_b}\right] \approx \frac{R_b}{R_a}\left[1 - \frac{{}^{\epsilon}/_{2}\, R_a}{R_a + R_b}\right]$$

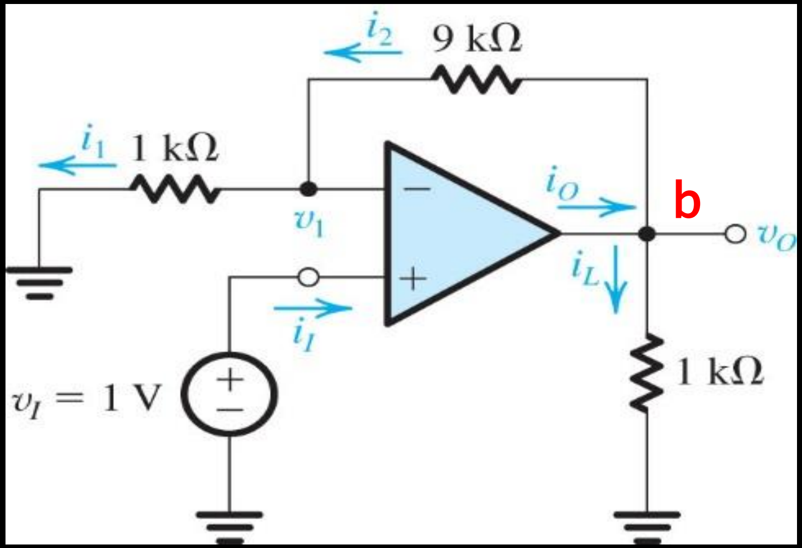
The common mode rejection ratio is defined as

$$\boxed{CMRR = \left|\frac{A_{dm}}{A_{cm}}\right|} = \left|\frac{\frac{R_b}{R_a}[1 - \frac{{}^{\epsilon}/_{2}\, R_a}{R_a + R_b}]}{\frac{-\epsilon R_b}{R_a + R_b}}\right| \approx \left|\frac{1 + R_b/R_a}{-\epsilon}\right|$$

CMRR can be used to measure how excellent a difference amplifier circuit is. The larger the CMRR, the better the difference op-amp performs.

Example #1

For the circuit below, find $i_I, v_1, i_1, i_2, v_0, i_L, and\ i_0$.



$$i_I = 0$$

$$v_1 = v_I = 1\,V$$

$$i_1 = \frac{v_1}{1000} = 1\,mA$$

$$i_2 = i_1 = 1\text{ mA}$$

$$from\ \frac{v_0 - v_1}{9000} = i_2 = 1\,mA$$

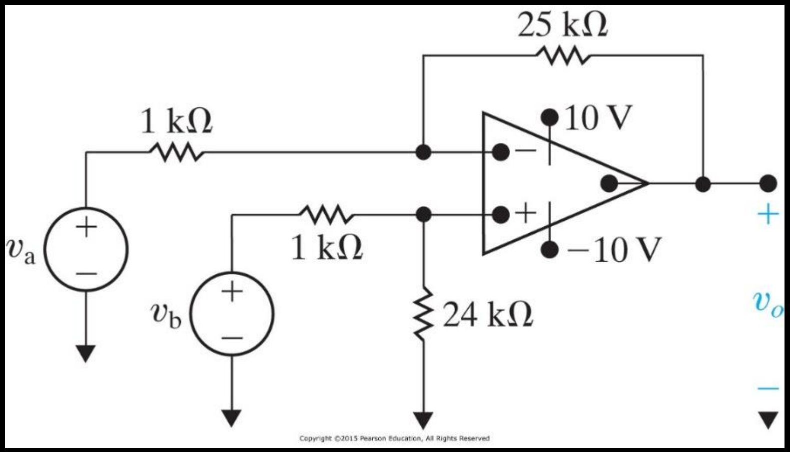
$$v_0 = 10\,V$$

$$i_L = \frac{v_0}{1000} = 10\,mA$$

At node b, applying KCL:

$i_L + i_2 = i_0,$ So, $i_0 = 10 + 1 = 11\,mA$

Example #2



In the difference amplifier on the left, find

(a) The differential mode gain

(b) The common mode gain

(c) The CMRR

$R_a = 1\ k\Omega,\quad R_b = 25\ k\Omega$

$R_c = 1\ k\Omega \quad R_d = 24\ k\Omega$

$$A_{dm} = \frac{R_d(R_a + R_b) + R_b(R_c + R_d)}{2R_a(R_c + R_d)} = \frac{24(1 + 25) + 25(1 + 24)}{2 \times 1 \times (1 + 24)} = 24.98$$

$$A_{cm} = \frac{R_a R_d - R_b R_c}{R_a(R_c + R_d)} = \frac{1 \times 24 - 25 \times 1}{1 \times (1 + 24)} = -0.04$$

$$CMRR = \left|\frac{A_{dm}}{A_{cm}}\right| = \left|\frac{24.98}{-0.04}\right| = 624.5$$

Summary

- In this lecture, a simplified model is used to analyze an op-amp. The op-amp is considered as an ideal op-amp, which means it has infinite input resistance, infinite open-loop gain, and zero output resistance.
- The inverting, summing, noninverting, and difference amplifier are discussed.
- The common mode rejection mode(CMRR) is introduced. An ideal difference amplifier has an infinite CMRR.

In the next lecture, we will discuss:

1. The inductor and inductance
2. The capacitor and capacitance