1)

a) $F(k) = P(X \le k) =$

$P(X=0) + P(X=1) + \dots + P(X=k)$

$= (1-p)^0 + (1-p)^1 p + \dots + (1-p)^k p$

$= p((1-p)^0 + (1-p)^1 + \dots + (1-p)^k)$

$= p\left(\sum_{i=0}^{k} (1-p)^i\right)$

$= p\left(\dfrac{1-(1-p)^{k+1}}{1-(1-p)}\right)$

$a + ar + ar^2 + \dots + ar^{n-1}$

$\sum_{k=0}^{n-1} (ar^k) = a\left(\dfrac{1-r^n}{1-r}\right)$

$F(k) = P(X \le k) = 1-(1-p)^{k+1}$

b) i)

$\sum_{k=0}^{\infty} f(k)$

$= p[1 + (1-p) + (1-p)^2 + \dots \infty]$

$= p\left[\dfrac{1}{1-(1-p)}\right]$

$= 1$

sum of infinite terms

$a + ar^1 + ar^2 + \dots \infty$

$= \dfrac{a}{1-r}, r < 1$

ii) $\lim_{k \to \infty} F(k) = \lim_{k \to \infty} [1-(1-p)^{k+1}]$

$\boxed{0 < p < 1, (1-p) < 0}$

$= 1 - 0$

$= 1$

1) c) $\frac{d}{dx}\left[\sum_{k=0}^{\infty} x^k\right] = \frac{d}{dx}\left[x^0+x^1+x^2+x^3+\dots\right] = \frac{d}{dx}\left[\frac{1}{1-x}\right] = \frac{d}{dx}(1-x)^{-1}$

$\sum_{k=1}^{\infty} kx^{x-1} = 0+1x^1+2x^1+3x^2+\dots = -1(1-x)^{-2}(-1) = \frac{1}{(1-x)^2}$

$$\boxed{\sum_{k=1}^{\infty} kx^{k-1} = \frac{1}{(1-x)^2}}$$

d) $E[X] = \sum_{x=0}^{\infty} x f(x) = \sum_{x=0}^{\infty} x(1-p)^x p = p\sum_{x=0}^{\infty} x(1-p)^x$

$= p\sum_{x=0}^{\infty} x(1-p)^{k-1}(1-p) = p(1-p)\left(\frac{1}{(1-(1-p))^2}\right) = p(1-p)\frac{1}{p^2}$

$$= \boxed{E(x) = \frac{1-p}{p}}$$

e) $1-F(k) = (1-p)^{k+1}$

$P(x>a+b \mid x>b) = P\left[\frac{(x>a+b)\cap(x>b)}{P(x>b)}\right]$

$= \frac{P(x>a+b)}{P(x>b)} = \frac{1-P(x\le a+b)}{1-P(x\le b)}$

$= \frac{1-F(a+b)}{1-F(b)} = \frac{(1-p)^{a+b+1}}{(1-p)^{b+1}} = (1-p)^a$

$$\boxed{P(x>a+b \mid x>b) = P(X>a)}$$

2) a) $P(D_r)P(D_r)P(D_r)P(\bar{D}_r) = .49 \times .49 \times .49 \times .51$
$= .06$

b) at most 3 $\quad P(k\le 4) = P(k=0)+P(k=1)+P(k=2)+P(k=3)+P(k=4)$

$k-1\le 3 \quad = F(4)$ not addressed by doctor

$k\le 4 \quad = 1-(1-.51)^4 = .9424$

3) a) $\binom{25}{23}(.421)^{23}(.579)^2 = 2.2960\times 10^{-7}$

b) $(.421)^4 = .0314$

c) $\frac{P(x\ge 9 \cap x\ge 5)}{P(x\ge 5)} = \frac{P(x\ge 9)}{P(x\ge 5)} = \frac{(.421)^9}{(.421)^5} = (.421)^4$

$= .0314$