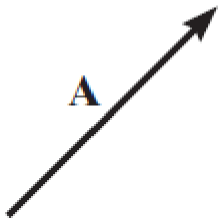
ENGR 057 Statics and Dynamics

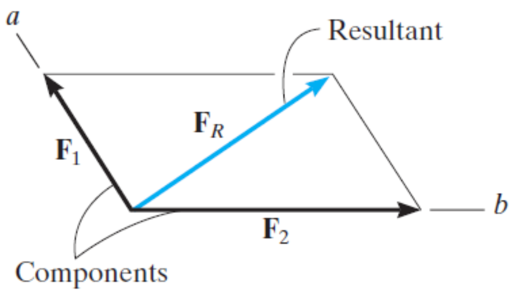
Review pre-exam 1

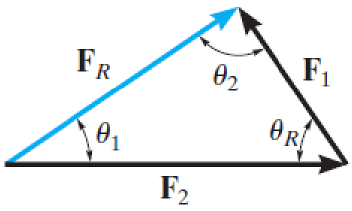
Summer 2022

A scalar is a positive or negative number;
A vector has a magnitude and direction, where the arrowhead represents the sense of the vector.



Parallelogram Law
Two forces add according to the parallelogram law. The *components* form the sides of the parallelogram and the *resultant* is the diagonal.





$$F_R = \sqrt{F_1^2 + F_2^2 - 2\,F_1F_2\cos\theta_R}$$

$$\frac{F_1}{\sin\theta_1} = \frac{F_2}{\sin\theta_2} = \frac{F_R}{\sin\theta_R}$$

Cartesian Vectors

The unit vector **u** has a length of 1, no units, and it points in the direction of the vector **F**.

$$\mathbf{u} = \frac{\mathbf{F}}{F}$$

A force can be resolved into its Cartesian components along the *x*, *y*, *z* axes so that $\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j} + F_z\mathbf{k}$.

The magnitude of **F** is determined from the positive square root of the sum of the squares of its components.

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

The coordinate direction angles α, β, γ are determined by formulating a unit vector in the direction of **F**. The *x*, *y*, *z* components of **u** represent $\cos\alpha$, $\cos\beta$, $\cos\gamma$.

$$\mathbf{u} = \frac{\mathbf{F}}{F} = \frac{F_x}{F}\mathbf{i} + \frac{F_y}{F}\mathbf{j} + \frac{F_z}{F}\mathbf{k}$$

$$\mathbf{u} = \cos\alpha\,\mathbf{i} + \cos\beta\,\mathbf{j} + \cos\gamma\,\mathbf{k}$$

To find the resultant of a concurrent force system, express each force as a Cartesian vector and add the **i**, **j**, **k** components of all the forces in the system.

$$\mathbf{F}_R = \Sigma\mathbf{F} = \Sigma F_x\mathbf{i} + \Sigma F_y\mathbf{j} + \Sigma F_z\mathbf{k}$$

Position and Force Vectors

A position vector locates one point in space
relative to another. The easiest way to formulate the components of a position vector is to determine the distance and direction that one must travel along the *x*, *y*, and *z* directions—going from the tail to the head of the vector.

$$\mathbf{r} = (x_B - x_A)\mathbf{i} + (y_B - y_A)\mathbf{j} + (z_B - z_A)\mathbf{k}$$

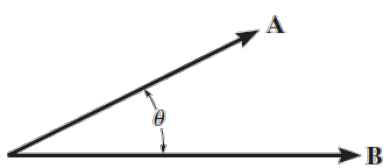
$$\mathbf{F} = F\mathbf{u} = F\left(\frac{\mathbf{r}}{r}\right)$$

Dot Product

The dot product between two vectors **A** and **B** yields a scalar. If **A** and **B** are expressed in Cartesian vector form, then the dot product is the sum of the products of their *x*, *y*, and *z* components.

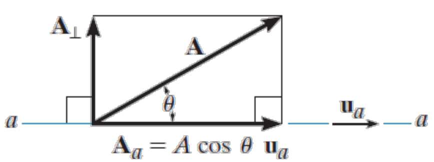
$$\mathbf{A} \cdot \mathbf{B} = AB\cos\theta = A_xB_x + A_yB_y + A_zB_z$$

The dot product can be used to determine the angle between **A** and **B**.



$$\theta = \cos^{-1}\left(\frac{\mathbf{A} \cdot \mathbf{B}}{AB}\right)$$

The dot product is also used to determine the projected component of a vector **A** onto an axis *aa* defined by its unit vector $\mathbf{u}_a$.

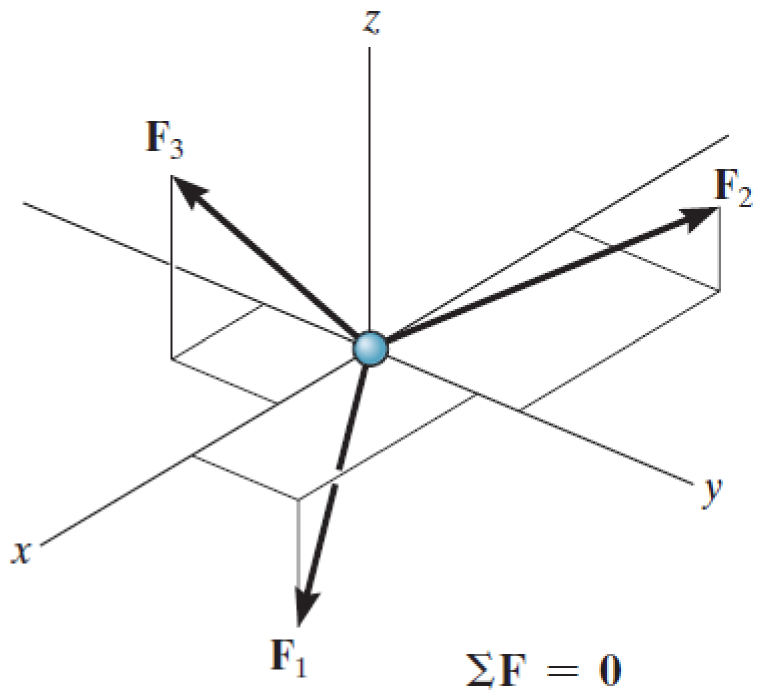


$$\mathbf{A}_a = A\cos\theta\,\mathbf{u}_a = (\mathbf{A} \cdot \mathbf{u}_a)\mathbf{u}_a$$

Particle Equilibrium

When a particle is at rest or moves with constant velocity, it is in equilibrium. This requires that all the forces acting on the particle form a zero resultant force.

If the three-dimensional geometry is difficult to visualize, then the equilibrium equation should be applied using a Cartesian vector analysis. This requires first expressing each force on the free-body diagram as a Cartesian vector. When the forces are summed and set equal to zero, then the **i**, **j**, and **k** components are also zero.



$$\Sigma \mathbf{F} = \mathbf{0}$$

$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$
$$\Sigma F_z = 0$$

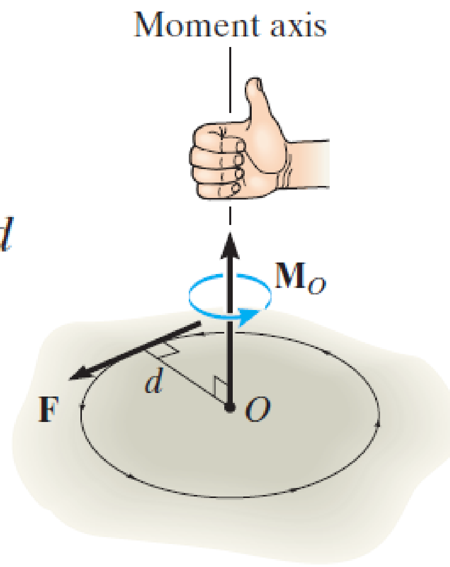
Moment of Force—Scalar Definition

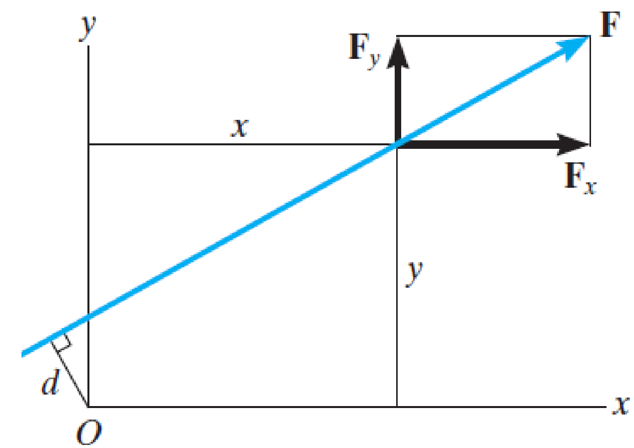
A force produces a turning effect or moment about a point *O* that does not lie on its line of action. In scalar form, the moment *magnitude* is the product of the force and the moment arm or perpendicular distance from point *O* to the line of action of the force.

The *direction* of the moment is defined using the right-hand rule. **M***O* always acts along an axis perpendicular to the plane containing **F** and *d*, and passes through the point *O*.

Rather than finding *d*, it is normally easier to resolve the force into its *x* and *y* components, determine the moment of each component about the point, and then sum the results. This is called the principle of moments.

$$M_O = Fd$$





$$M_O = Fd = F_x y - F_y x$$

Moment of a Force—Vector Definition

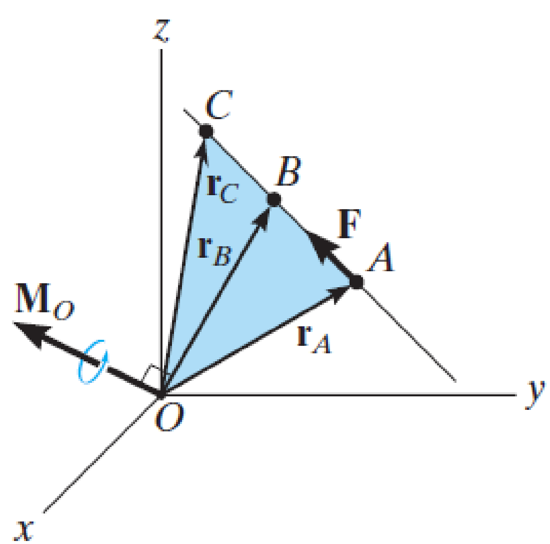
Since three-dimensional geometry is generally more difficult to visualize, the vector cross product should be used to determine the moment.

Here $\mathbf{M}_O$ = **r** x **F**, where **r** is a position vector that extends from point *O* to any point *A, B, or C* on the line of action of **F**.

$$\mathbf{M}_O = \mathbf{r}_A \times \mathbf{F} = \mathbf{r}_B \times \mathbf{F} = \mathbf{r}_C \times \mathbf{F}$$

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

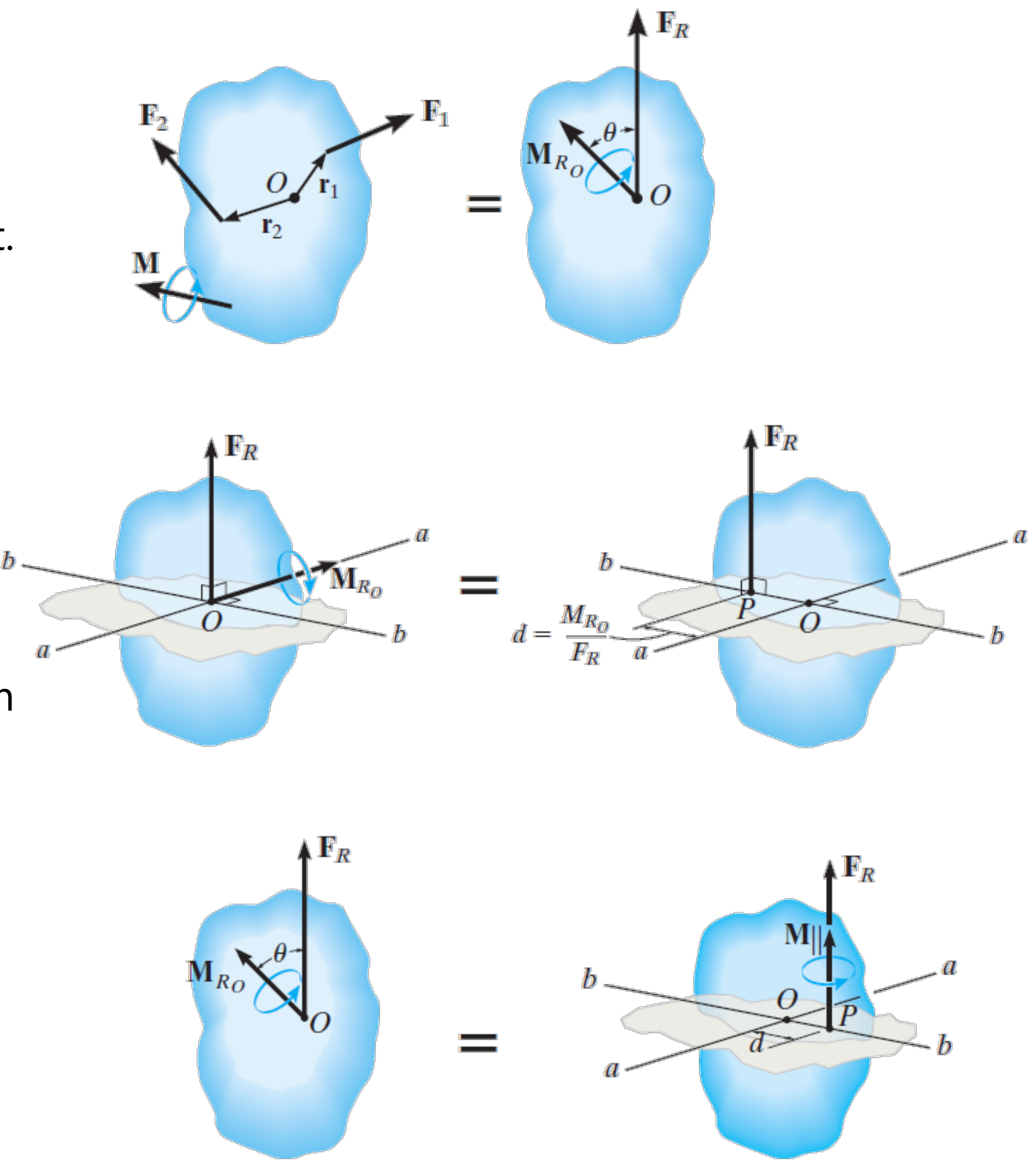
If the position vector **r** and force **F** are expressed as Cartesian vectors, then the cross product results from the expansion of a determinant.



Simplification of a Force and Couple System

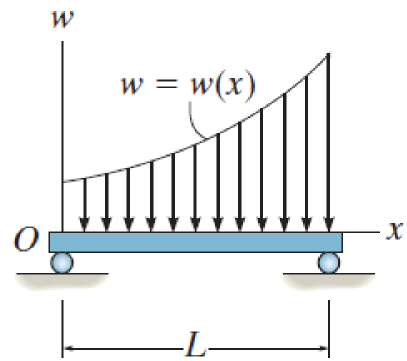
Any system of forces and couples can be reduced to a single resultant force and resultant couple moment acting at a point. The resultant force is the sum of all the forces in the system, $\mathbf{F}_R = \Sigma \mathbf{F}$, and the resultant couple moment is equal to the sum of all the moments of the forces about the point and couple moments. $\mathbf{M}_{RO} = \mathbf{M}_O + \Sigma \mathbf{M}$.

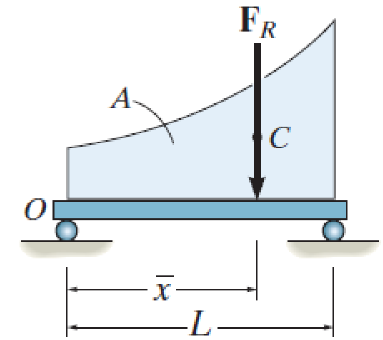
Further simplification to a single resultant force is possible provided the force system is concurrent, coplanar, or parallel. To find the location of the resultant force from a point, it is necessary to equate the moment of the resultant force about the point to the moment of the forces and couples in the system about the same point.



Coplanar Distributed Loading

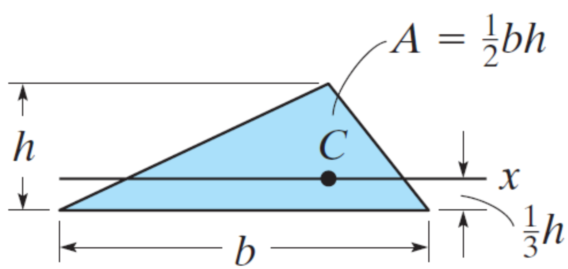
A simple distributed loading can be represented by its resultant force, which is equivalent to the *area* under the loading curve. This resultant has a line of action that passes through the *centroid* or geometric center of the area or volume under the loading diagram.



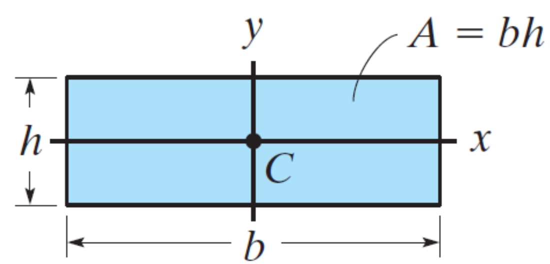


$$F_R = \int_L w(x)\,dx = \int_A dA = A$$

$$\bar{x} = \frac{\int_L x w(x)\,dx}{\int_L w(x)\,dx} = \frac{\int_A x\,dA}{\int_A dA}$$



Triangular area



Rectangular area