



ENGR 057 Statics and Dynamics

Free body diagram

Instructor

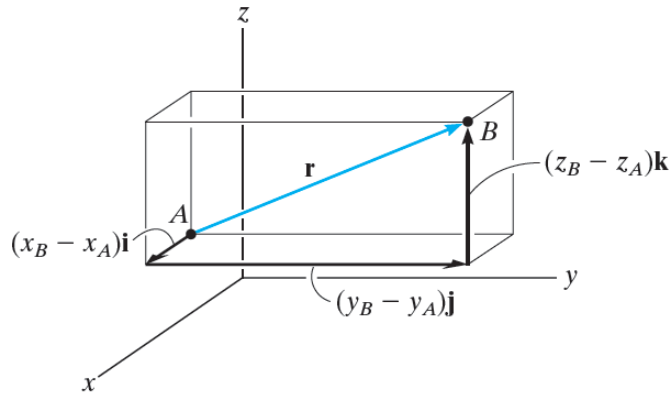
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MERCED

Applications



Position vector



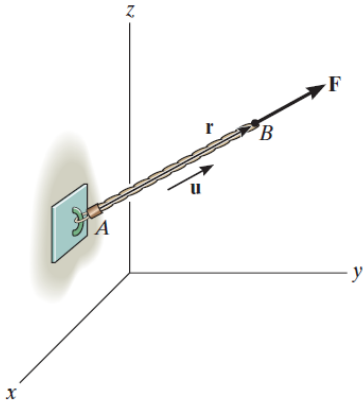
- A position vector gives the location of a point in cartesian space with respect to a certain reference point
- There is always at least one reference point: the origin
- Position vectors are essential to describe the mechanics of a problem

$$\mathbf{r}_{AB} = (x_B - x_A)\hat{\mathbf{i}} + (y_B - y_A)\hat{\mathbf{j}} + (z_B - z_A)\hat{\mathbf{k}}$$

ALWAYS subtract the “tail” coordinates from the “tip” coordinates!

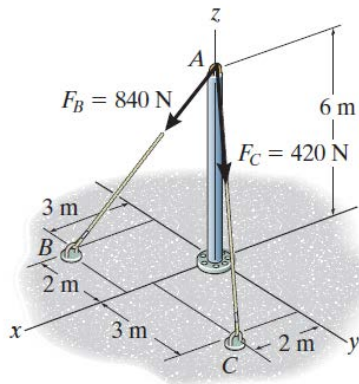
Note that \mathbf{r}_{AB} has the same magnitude of \mathbf{r}_{BA} but opposite direction

Force vector along a line



- Find the position vector, \mathbf{r}_{AB} , along two points on that line.
- Find the unit vector describing the line's direction, $\mathbf{u}_{AB} = (\mathbf{r}_{AB}/r_{AB})$.
- Multiply the unit vector by the magnitude of the force, $\mathbf{F} = F \mathbf{u}_{AB}$.

Example



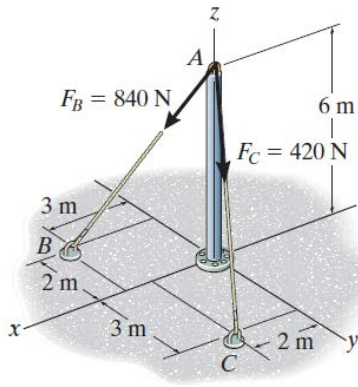
Determine the magnitude of the resultant force at A.

Given: $F_B = 840\text{ N}$, $F_C = 420\text{ N}$, coordinates

Asked: F_A

Example

Determine the magnitude of the resultant force at A .



$$\mathbf{r}_{AC} = \{2\hat{i} + 3\hat{j} - 6\hat{k}\} \text{ m}$$

$$r_{AC} = \sqrt{2^2 + 3^2 + (-6)^2} = 7 \text{ m}$$

$$\mathbf{u}_{AC} = \frac{2}{7}\hat{i} + \frac{3}{7}\hat{j} - \frac{6}{7}\hat{k}$$

$$\mathbf{F}_C = F_C \mathbf{u}_{AC} = (420 \text{ N}) \left(\frac{2}{7}\hat{i} + \frac{3}{7}\hat{j} - \frac{6}{7}\hat{k} \right)$$

$$\mathbf{F}_C = \{120\hat{i} + 180\hat{j} - 360\hat{k}\} \text{ N}$$

$$\mathbf{r}_{AB} = \{3\hat{i} - 2\hat{j} - 6\hat{k}\} \text{ m}$$

$$r_{AB} = \sqrt{3^2 + (-2)^2 + (-6)^2} = 7 \text{ m}$$

$$\mathbf{u}_{AB} = \frac{3}{7}\hat{i} - \frac{2}{7}\hat{j} - \frac{6}{7}\hat{k}$$

$$\mathbf{F}_B = F_B \mathbf{u}_{AB} = (840 \text{ N}) \left(\frac{3}{7}\hat{i} - \frac{2}{7}\hat{j} - \frac{6}{7}\hat{k} \right)$$

$$\mathbf{F}_B = \{360\hat{i} - 240\hat{j} - 720\hat{k}\} \text{ N}$$

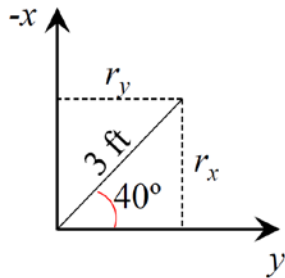
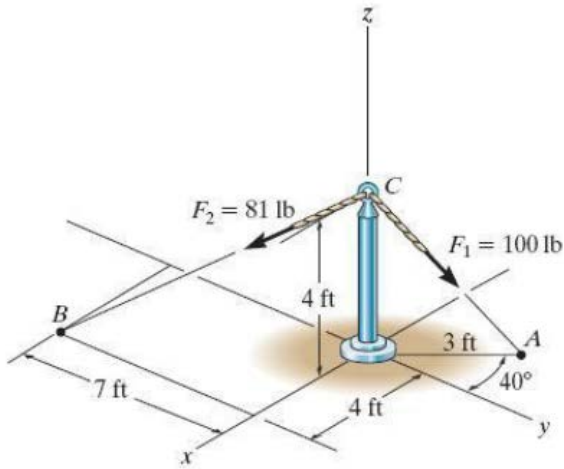
$$F_R = \sqrt{(120 + 360)^2 + (180 - 240)^2 + (-360 - 720)^2} \text{ N} = 1.18 \text{ kN}$$

Individual work (15 min)

Determine the resultant force acting on the pipe in Cartesian vector form

Given: $F_1 = 81 \text{ lb}$, $F_2 = 100 \text{ lb}$, coordinates, angle

Asked: \mathbf{F}_C



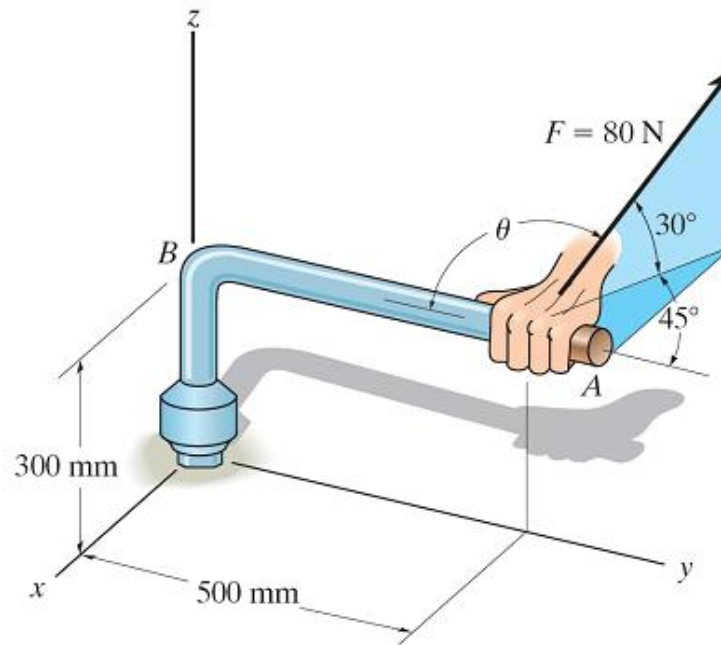
First we need to determine \mathbf{F}_{CA} and \mathbf{F}_{CB} as cartesian vectors

$$\mathbf{F}_{CA} = F_1 \left(\frac{\mathbf{r}_{CA}}{r_{CA}} \right)$$

$$\mathbf{r}_{AC} = \{-3 \sin 40^\circ \hat{i} + 3 \cos 40^\circ \hat{j} - 4 \hat{k}\} \text{ m}$$

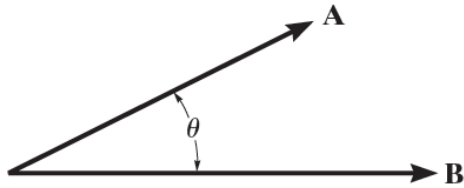
$$r_{AC} = \sqrt{(-3 \sin 40^\circ)^2 + (3 \cos 40^\circ)^2 + (-4)^2} = 5 \text{ m}$$

APPLICATIONS



For the force F being applied to the wrench at Point A , what component of it actually helps turn the bolt (i.e., the force component acting perpendicular to the pipe)?

Scalar product



- The scalar product of vectors \mathbf{A} , \mathbf{B} multiplies the two vectors to obtain a scalar result. It is also known as dot vector
- It is helpful to calculate the angle between two vectors
- In Cartesian space, the scalar product represents the projection of \mathbf{A} over \mathbf{B}

We write the scalar product as

$$\mathbf{A} \cdot \mathbf{B} = AB \cos \theta \quad \text{where } 0 \leq \theta \leq 180$$

Note that the scalar product of two vectors perpendicular to each other is zero, since $\cos 90 = 0$

In particular the unit vectors \hat{i} , \hat{j} , and \hat{k} are perpendicular to each other so $\hat{i} \cdot \hat{j} = \hat{i} \cdot \hat{k} = \hat{j} \cdot \hat{k} = 0$

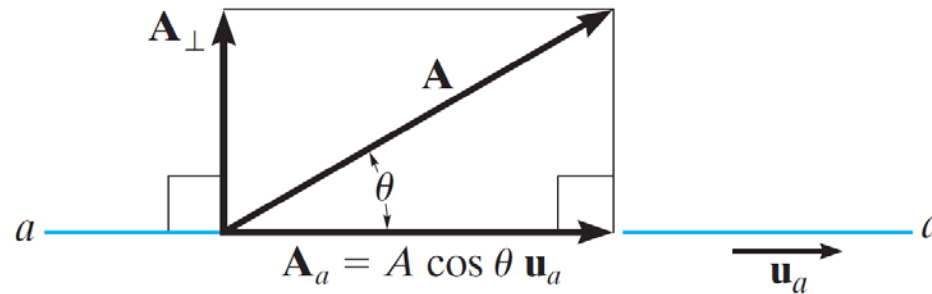
Similarly, it is easy to prove that $\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$

Which gives us the following rule

$$\mathbf{A} \cdot \mathbf{B} = (A_x \hat{i} + A_y \hat{j} + A_z \hat{k}) \cdot (B_x \hat{i} + B_y \hat{j} + B_z \hat{k})$$

$$\mathbf{A} \cdot \mathbf{B} = A_x B_x + A_y B_y + A_z B_z$$

Determining the projection of a vector



You can determine the components of a vector parallel and perpendicular to a line using the dot product.

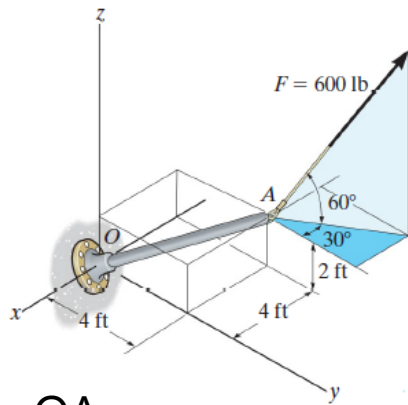
Steps:

1. Find the unit vector, \mathbf{u}_{aa} along line aa
2. Find the scalar projection of \mathbf{A} along line aa by $A_{||} = \mathbf{A} \cdot \mathbf{u}_{aa} = A_x U_x + A_y U_y + A_z U_z$
3. The scalar and vector forms of the perpendicular component can easily be obtained by

$$A_\perp = (A^2 - A_{||}^2)^{1/2} \text{ and}$$

$$\mathbf{A}_\perp = \mathbf{A} - \mathbf{A}_{||}$$

(rearranging the vector sum of $\mathbf{A} = \mathbf{A}_\perp + \mathbf{A}_{||}$)



Example

Determine the components of the force acting parallel and perpendicular to the axis of the pole.

Given: $F = 600 \text{ N}$, coordinates

Asked: $F_{||}$ and F_{\perp}

Bar OA:

$$\mathbf{r}_A = \{-4\hat{i} + 4\hat{j} + 2\hat{k}\} \text{ ft}$$

$$r_A = \sqrt{(-4)^2 + 4^2 + 2^2} = 6 \text{ ft}$$

$$\mathbf{u}_A = \frac{-2}{3}\hat{i} + \frac{2}{3}\hat{j} + \frac{1}{3}\hat{k}$$

$$\mathbf{F} = \{[-600 \cos 60^\circ \sin 30^\circ]\hat{i} + [600 \cos 60^\circ \cos 30^\circ]\hat{j} + [600 \sin 60^\circ]\hat{k}\} \text{ lb}$$

$$\mathbf{F} = \{-150\hat{i} + 259.81\hat{j} - 519.62\hat{k}\} \text{ lb}$$

$$\mathbf{F}_{||} = F \cdot \mathbf{u}_A = \left(-\frac{2}{3}(-150) + \frac{2}{3}(259.81) + \frac{1}{3}(519.62)\right) \text{ lb} = 446.41 \text{ lb} = 446 \text{ lb}$$

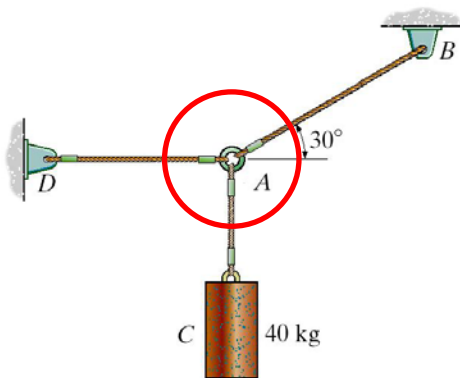
$$\mathbf{F}_{\perp} = \sqrt{(600)^2 - (446.41)^2} = 401 \text{ lb}$$

Free body diagram

While mechanics analysis is generally algebraic, it has a geometric basis. For this reason a graphical representation of the problem at hand would be extremely useful. This representation is the Free Body Diagram (FBD)

The FBD is the most useful tool in mechanics analysis. A well-done and complete FBD does half the work for us, showing exactly the steps that must be taken and the equations that must be written

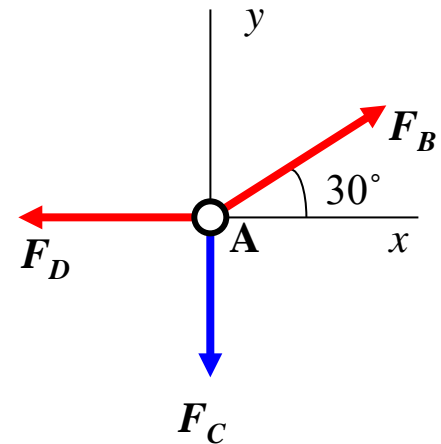
Consider the following problem



We are interested in the forces on A

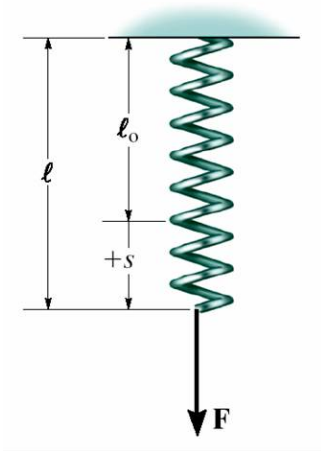
Imagine A as an isolated particle

Draw all forces acting on A as vectors



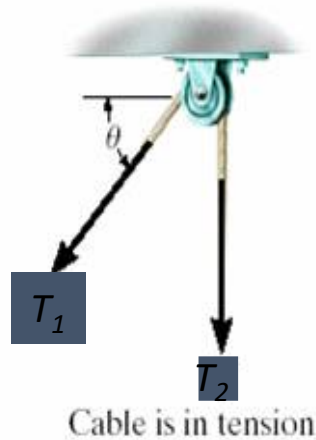
Once we have completed the FBD, it is clear which forces are known and which forces need to be determined. We also have an idea of the procedure to determine such forces

Springs, cables, and pulleys

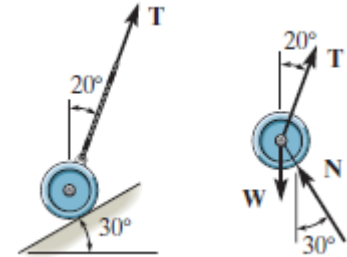


Spring Force =
spring constant * deformation
or

$$F = k * s$$



With a frictionless pulley,
 $T_1 = T_2$.



In a smooth surface, the
surface exerts a force on
the object that is normal
to the surface at
the point of contact.

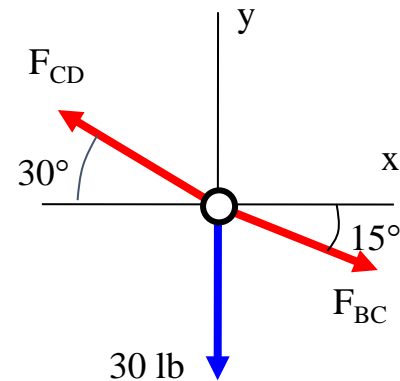
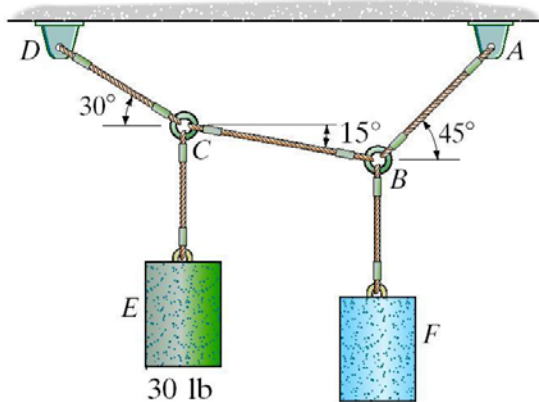
Example

Determine the forces on the cables and the weight of cylinder F

Solution

The FBD at C is

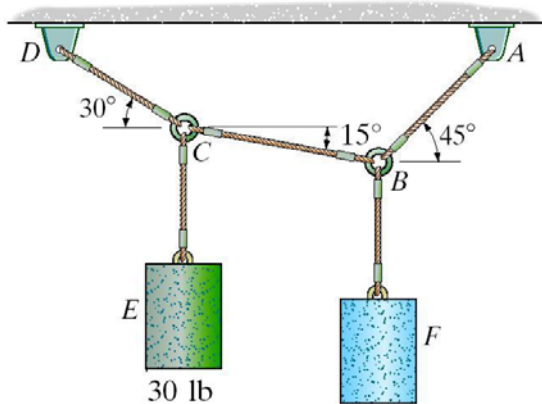
Notice we are assuming directions for F_{CD} and F_{BC}



Since our system is in equilibrium, any part of it must also be in equilibrium

In particular, point C is in equilibrium, hence (see Newton's 2nd law) the resultant force $\mathbf{F} = 0$

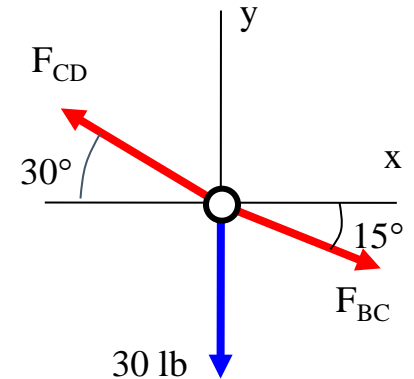
This equation holds true in all directions. This means we have three (x, y, z) independent equilibrium equations



The equilibrium equation on x

$$\sum F_x = 0$$

$$F_{BC} \cos 15 - F_{CD} \cos 30 = 0$$

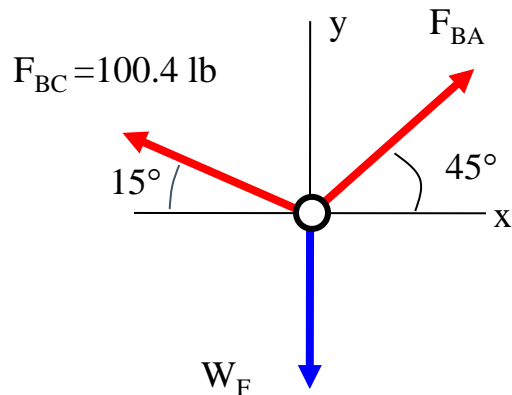


Similarly, for y $F_{CD} \sin 30 - F_{BC} \sin 15 - 30 = 0 \Rightarrow$

$$F_{BC} = 100.38 \text{ lb}$$

$$F_{CD} = 112.2 \text{ lb}$$

The FBD at B instead is



This allows us to write the equilibrium equations

$$F_{BA} \cos 45 - F_{BC} \cos 15 = 0$$

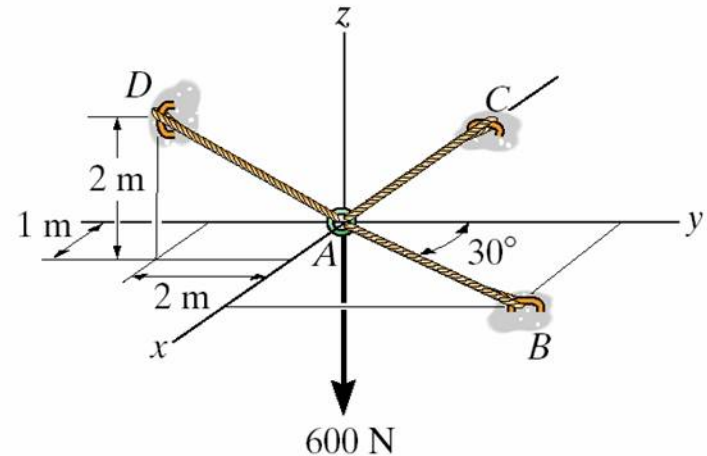
$$F_{BA} = 137 \text{ lb}$$

$$F_{BA} \sin 45 + F_{BC} \sin 15 - W_F = 0$$

$$W_F = 123 \text{ lb}$$

Individual work (15 min)

Determine the tension developed in cables AB , AC , and AD .



- 1) Draw a free body diagram of Point A. Let the unknown force magnitudes be F_B , F_C , F_D .
- 2) Represent each force in the Cartesian vector form.
- 3) Apply equilibrium equations to solve for the three unknowns.