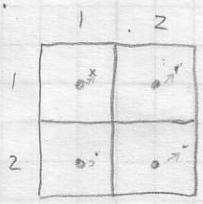
CSE 185

Affine Transforms



pixels at (1,1)
(1,2)
(2,1)
(2,2)

Want to rotate counter-clockwise by 10°.

Rotation transformation matrix:

$$T = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos 10 & \sin 10 & 0 \\ -\sin 10 & \cos 10 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.985 & 0.174 & 0 \\ -0.174 & 0.985 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Map location of each pixel:

$$[x \; y \; 1] = [v \; w \; 1]\, T$$

4 pixels

$$\begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \\ x_4 & y_4 & 1 \end{bmatrix} = \begin{bmatrix} v_1 & w_1 & 1 \\ v_2 & w_2 & 1 \\ v_3 & w_3 & 1 \\ v_4 & w_4 & 1 \end{bmatrix} T$$

In our case

$$\begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \\ x_4 & y_4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 2 & 1 & 1 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} 0.985 & -0.174 & 0 \\ -0.174 & 0.985 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$(x_1, y_1) = (0.811, 1.159)$
$(x_2, y_2) = (0.637, 2.144)$
$(x_3, y_3) = (1.796, 1.333)$
$(x_4, y_4) = (1.622, 2.318)$

Benefits of affine transforms (and their matrix representations) is that can combine multiple transformations into a single operation.

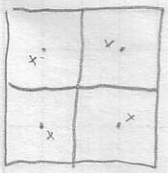
[translate center to (0,0)] [rotate by 10° counterclockwise] [translate center to (1.5, 1.5)]

$$T_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1.5 & -1.5 & 1 \end{bmatrix} \qquad T_2 = \begin{bmatrix} \cos 10 & \sin\theta & 0 \\ -\sin 10 & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad T_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1.5 & 1.5 & 1 \end{bmatrix}$$

$$T = T_1 T_2 T_3 = \begin{bmatrix} 0.985 & 0.174 & 0 \\ -0.174 & 0.985 & 0 \\ 0.2835 & 0.2385 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \\ x_4 & y_4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 2 & 1 & 1 \\ 2 & 2 & 1 \end{bmatrix} T$$

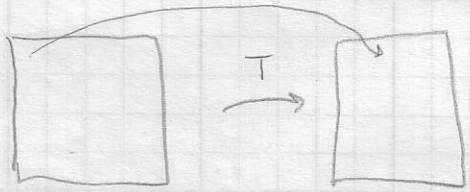
$$(x_1, y_1) = (1.0945,\ 0.925)$$
$$(x_2, y_2) = (0.9205,\ 1.9055)$$
$$(x_3, y_3) = (2.0795,\ 1.0945)$$
$$(x_4, y_4) = (1.9055,\ 2.0795)$$



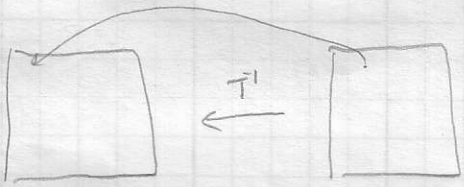
Still need to determine values at pixel locations

→ interpolation

Better to use backward mapping for interpolation



instead



$$(x, y) = (v, w)\, T$$

$$\Downarrow$$

$$(v, w) = (x, y)\, T^{-1}$$

- Affine transforms usually introduce distortions.

  See figure from text.