Discussion Section: Week #4
Due: By 11:59pm the day of your Discussion Section

Instructions:

You are allowed to work alone or in small groups. But if you work in a group, the work you turn in should be your own. Each student will submit a physical write-up as a PDF document to CatCourses **by 11:59 pm of your discussion section day**.

Discussion assignments will be graded partially for completeness but a subset of the problems will be graded for correctness. Providing the correct answer, without justification, is not considered complete. For credit you **must** either show steps (if it's a calculation problem) or explain/justify your reasoning (if it's a short answer problem).

Problem Set:

1. Given the matrix

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 1 \\ 0 & 1 & 0 \end{bmatrix},$$

compute A^{-1}.

Solution:

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 2 & -2 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_2^*=-2R_1+R_2} \left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & -2 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow[R_2^*=R_3]{R_3^*=-R_2} \left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 2 & -1 & 0 \end{array}\right]$$

$$\xrightarrow[R_1^{**}=-R_3+R_1^*]{R_1^*=R_1+R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 2 & -1 & 0 \end{array}\right].$$

Hence the solution is A^{-1} =

$$\begin{bmatrix} -1 & 1 & 1 \\ 0 & 0 & 1 \\ 2 & -1 & 0 \end{bmatrix}.$$

One can verify that $AA^{-1} = A^{-1}A$ = I

2. Given the matrix

$$A = \begin{bmatrix} e^\pi & e^\pi & 10^i \\ e^\pi & 0 & 10^i \\ 0 & e^\pi & 0 \end{bmatrix},$$

compute A^{-1}.

Solution:

$$\left[\begin{array}{ccc|ccc} e^\pi & e^\pi & 10^i & 1 & 0 & 0 \\ e^\pi & 0 & 10^i & 0 & 1 & 0 \\ 0 & e^\pi & 0 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_2^*=-R_1+R_2} \left[\begin{array}{ccc|ccc} e^\pi & e^\pi & 10^i & 1 & 0 & 0 \\ 0 & -e^\pi & 0 & -1 & 1 & 0 \\ 0 & e^\pi & 0 & 0 & 0 & 1 \end{array}\right]$$
$$\xrightarrow{R_3^*=R_2+R_3} \left[\begin{array}{ccc|ccc} e^\pi & e^\pi & 10^i & 1 & 0 & 0 \\ 0 & -e^\pi & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 & 1 \end{array}\right]$$

Hence, we see that there is no inverse to the system, since we have a row of 0's on the LHS, meaning it is impossible to make this the identity. We also could have seen this by seeing that $R_1$is a linear combination of $R_2 + R_3$, meaning that the system is linearly dependent, therefore not invertible.

3. Given both systems from questions 1 and 2, solve the system $Ax = b$ for b = $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$.

Solution: Since the system in question 1 is invertible, we can merely compute that the solution is given by $x = A^{-1}b$, giving us x = $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$. Since the system in 2 is not invertible, we can compute the solution through an augmented matrix. Luckily, $b \in R(A)$, so we know a solution will exist.

$$\left[\begin{array}{ccc|c} e^\pi & e^\pi & 10^i & 1 \\ e^\pi & 0 & 10^i & 1 \\ 0 & e^\pi & 0 & 0 \end{array}\right] \xrightarrow{R_2^*=-R_1+R_2} \left[\begin{array}{ccc|c} e^\pi & e^\pi & 10^i & 1 \\ 0 & -e^\pi & 0 & 0 \\ 0 & e^\pi & 0 & 0 \end{array}\right]$$
$$\xrightarrow{R_3^*=R_2+R_3} \left[\begin{array}{ccc|c} e^\pi & e^\pi & 10^i & 1 \\ 0 & -e^\pi & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Hence, we see that x_3 is free, $x_2 = 0$, and as a result of this, taking $x_3 = 1$, we get $x_1 = \frac{1-10^i}{e^\pi}$.