ENGR 065 Electric Circuits

Lecture 15: Poles and Zeros, and
Initial- and Final- Value Theorems

Today's Topics

- Poles and Zeros of F(s)
- Initial - and final - value theorems

- Covered in Sections 12.8 and 12.9

Poles and Zeros of F(s)

- The rational function also can be expressed as the ratio of two factored polynomials:

$$F(s) = \frac{K(s+z_1)(s+z_2)\ldots(s+z_n)}{(s+p_1)(s+p_2)\ldots(s+p_m)}$$

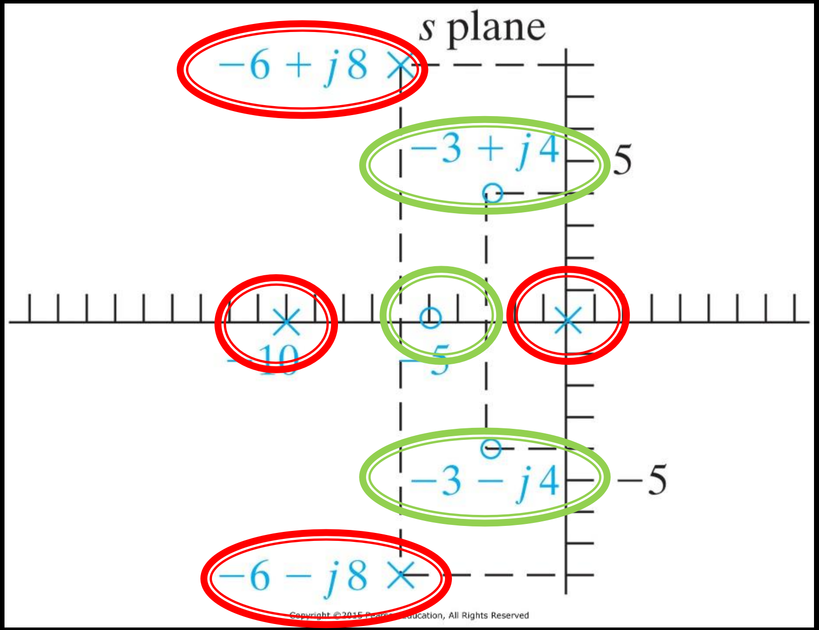
The roots of the denominator polynomial, $-p_1, -p_2, -p_3, \ldots\ -p_m$ are called the poles of *F(s)*.

The roots of the numerator polynomial, that is, $-z_1, -z_2, -z_3, \ldots\ -z_n$ are called the zeros of *F(s)*.

Poles and Zeros of F(s)

For example:

$$F(s) = \frac{10(s+5)(s+3-j4)(s+3+j4)}{s(s+10)(s+6-j8)(s+6+j8)}$$



The poles are 0, -10 rads/s, -6+j8 rads/s, -6-j8 rad/s.

The zeros are -5 rads/s, -3+j4 rads/s, -3-j4 rads/s.

In this course, we focus on the poles and zeros located in the finite s plane. $s = \infty$, for example, is not a zero.

Initial- and Final-Value Theorems

- They are used to determine the behavior of *f(t)* from *F(s)* at $t = 0^+$ and as $t \to \infty$.
- Initial-value theorem:

$$\lim_{t \to 0^+} f(t) = f(0^+) = \lim_{s \to \infty} sF(s)$$

- Final-value theorem:

$$\lim_{t \to \infty} f(t) = f(\infty) = \lim_{s \to 0^+} sF(s)$$

Example

$F(s) = \mathcal{L}\{f(t)\} = \frac{7s^2+63s+134}{(s+3)(s+4)(s+5)}$, find $f(0^+)$ and $f(\infty)$.

If using $f(t)$ to find $f(0^+)$ and $f(\infty)$, we have to do:

$$F(s) = \frac{7s^2+63s+134}{(s+3)(s+4)(s+5)} = \frac{k_1}{s+3} + \frac{k_2}{s+4} + \frac{k_3}{s+5}$$

$$k_1 = \mathrm{F(s)(s+3)}|_{s=-3} = 4$$

$$k_2 = \mathrm{F(s)(s+4)}|_{s=-4} = 6$$

$$k_3 = \mathrm{F(s)(s+5)}|_{s=-5} = -3$$

$$F(s) = \frac{k_1}{s+3} + \frac{k_2}{s+4} + \frac{k_3}{s+5} = \frac{4}{s+3} + \frac{6}{s+4} + \frac{-3}{s+5}$$

So $$f(t) = \left(4e^{-3t} + 6e^{-4t} - 3e^{-5t}\right)u(t)$$

$$f(0^+) = \lim_{t\to 0^+} f(t) = \lim_{t\to 0^+}\left(4e^{-3t} + 6e^{-4t} - 3e^{-5t}\right)u(t) = 4+6-3 = 7$$

$$f(\infty) = \lim_{t\to\infty} f(t) = \lim_{t\to\infty}\left(4e^{-3t} + 6e^{-4t} - 3e^{-5t}\right)u(t) = 0$$

Example –cont'd

$$F(s) = \frac{7s^2 + 63s + 134}{(s+3)(s+4)(s+5)}$$

If we use initial- and final-theorems, we have:

$$f(0^+) = \lim_{t\to 0^+} f(t) = \lim_{s\to\infty} sF(s) = \lim_{s\to\infty} s\frac{7s^2 + 63s + 134}{(s+3)(s+4)(s+5)} = 7$$

$$f(\infty) = \lim_{t\to\infty} f(t) = \lim_{s\to 0^+} sF(s) = \lim_{s\to 0^+} s\frac{7s^2 + 63s + 134}{(s+3)(s+4)(s+5)} = 0$$

Summary

- Two important concepts: the poles and zeros of F(s).
- Two important theorems:
  - The initial-value theorem: $\lim_{t \to 0^+} f(t) = \lim_{s \to \infty} sF(s)$
  - The final-value theorem: $\lim_{t \to \infty} f(t) = \lim_{s \to 0^+} sF(s)$

In next class, we will discuss

- Circuit elements in the s domain
- Circuit Analysis in the s domain