1. This data came from a pre-employment drug screening.

| | Positive Test Result (drug use is indicated) | Negative Test Result (drug use is not indicated) |
|---|---|---|
| **Subject Uses Drugs** | 44 (true positive) | 6 (false negative) |
| **Subject is Not a Drug User** | 90 (false positive) | 860 (true negative) |

(a) **False positive** Find the probability of selecting a subject with a positive test result given that the subject does not use drugs.

(b) **False negative** Find the probability of selecting a subject with a negative test result given that the subject uses drugs.

a) $P(T|D^c) = \frac{90}{950}$

b) $P(T^c | D) = \frac{6}{50}$

$$P(B|A) = \frac{P(B)\cdot P(A|B)}{P(B)P(A|B)+P(B^c)P(A|B^c)}$$

F = female resident

$P(F) = 0.503$

$P(B|F) = 0.0833$

$P(B|F^c) = .001$

$P(F^c) = 1 - .503 = .497$

$P(F|B) =$

$$\frac{(.503)(0.0833)}{(.503)(0.0833) + .497(.001)}$$

$\approx 0.988277$

2)

$P(V|E) = .7 = P(A|B)$ $E = B$, $V = A$

$P(V|E^c) = .02 = P(A|B^c)$

$P(E|V) = ? = P(B|A)$

$P(E) = .111 = P(B)$

$P(E^c) = .889 = P(B^c)$

$P(E|V)$

$$= \frac{.111 \cdot .7}{.111 \cdot .7 + .889 \cdot .02} = \frac{.0777}{.0777 + .1778}$$

$= 0.8138$

3)

a)

$P(A|B) = P(A^c|B^c) = x$

$P(A)$ → exceeded limit by breathalyzer

$P(B)$ → exceeded limit by blood

$P(B)$ → exceed limit = 5% = 0.05

$P(A|B)$ → exceeded breathalyzer given exceeded blood limit

a.) $P(B^c|A)$

↳ probability of being within legal limit in blood given exceeding legal limit in breathalyzer

b.) determine $P(B^c|A)$ if $x = 0.95$

$$P(B^c|A) = \frac{P(B^c) \cdot P(A|B^c)}{P(B) \cdot P(A|B^c) + P(B)P(A|B)}$$ Formula

Finding

$P(B) = 0.05$ $\quad$ $P(B^c) = 0.95$ $\quad$ $P(A) = 0.0875$

$P(A|B) = 0.95$ $\quad$ $P(A|B^c) = 0.05$ $\quad$ $P(A^c) = 0.9025$

$P(A^c|B^c) = 0.95$

$P(A|B^c) = (1 - P(A^c|B^c)) = 0.05$

$$= \frac{0.95 \times 0.05}{0.95 \times 0.05 + 0.05 \times 0.95}$$

$\boxed{= 0.5}$

**4)**

c) $P(B|A) = 0.9$ $\quad$ $x = ?$ $\quad$ $P(A|B) = P(A^c|B^c) = x$

$P(A^c|B^c) = 1 - x$

$$P(B|A) = \frac{P(B) \cdot P(A|B)}{P(B)P(A|B) + P(B^c)P(A|B^c)}$$

$$0.9 = \frac{0.05 \times x}{0.05 \times x + 0.95 \times (1-x)}$$

$\boxed{= 0.9942}$

## 5)

5. A tattoo enthusiast website[3] claims that

- 47% of Millennials have tattoos
- 36% of Generation X have tattoos
- 13% of Boomers have tattoos

whereas the population proportions are 22%, 20%, and 22% for those generations respectively.[4] Compute the probability that a person is a Millennial given that they have tattoos. (For homework brevity, let us assume that no one in other age groups have tattoos.)

$P(M) = 0.22$ $\quad P(T|M) = 0.47$

$P(G) = 0.2$ $\quad P(T|G) = 0.36$

$P(B) = 0.22$ $\quad P(T|B) = 0.13$

$$P(M|T) = \frac{P(M)P(T|M)}{P(M)P(T|M) + P(G)P(T|G) + P(B)P(T|B)} = \frac{(0.22)(0.47)}{(0.22)(0.47) + (0.2)(0.36) + (0.22)(0.13)}$$

$$= 0.50686$$

## 6)

6. A popular video game company sent their futuristic product to a couple of quality control operations in Arstotzka ($Q_A$) and Stardew ($Q_S$). The proportion of biomes sent to Artotzka is 0.32, and the rest went to Stardew. Given that the biome was sent to Arstotzka, the probability that the quality control operation finds a bug is 13 percent. Given that the biome was sent to Stardew, the probability that the quality control operation finds a bug is 55 percent. Compute the Bayesian odds of $Q_A$ to $Q_B$ given that a bug was found.[5]

$P(Q_A) = 0.32$ $\quad P(B|Q_A) = 0.13$

$P(Q_S) = 0.68$ $\quad P(B|Q_S) = 0.55$

Bayesian odds: $\frac{(0.32)(0.13)}{(0.68)(0.55)} = 0.1112$