Internal Loading in Structural Members

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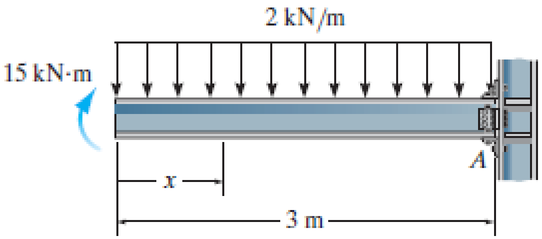
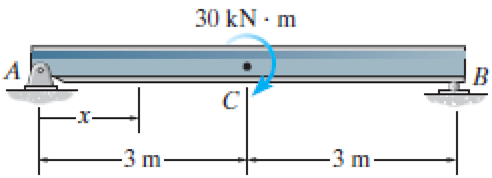
Beams:

- Slender structural element that supports transversal loads
- More common than trusses are more versatile, allowing for flexible design
- They are commonly subject to complex load systems. Beams are not two-force members.
- They usually have a constant cross-sectional area.
- They are often classified by the supports, e.g. simply supported beam or cantilever beam.



cantilever beam



simply supported beam

Beams:

Their performance depends on its cross section and its material. Some common cross sections are I, H, L, C, and Z

Their primary job is to support bending. Depending on the loading and the support provided, beams will bend in different ways.

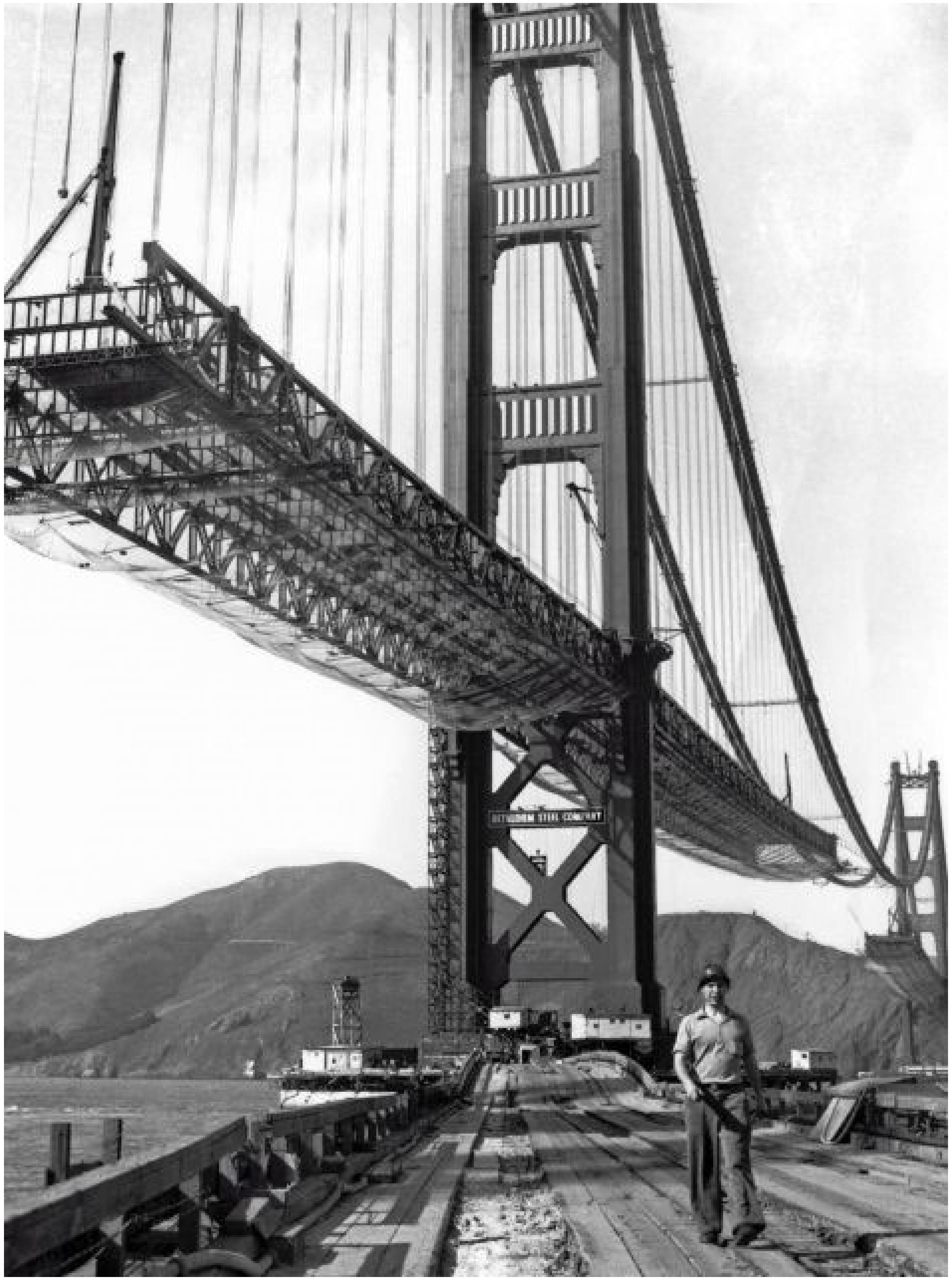
As an engineer, it will be your responsibility to choose not only the geometry (cross section) of the beam and its material, but also its supports. The decision is not trivial and is related to the other two choices

Example: Golden Gate

Structural components of the deck:
The deck of the Golden Gate Bridge is composed of steel beams supported by the trusses and hanged through suspender ropes.

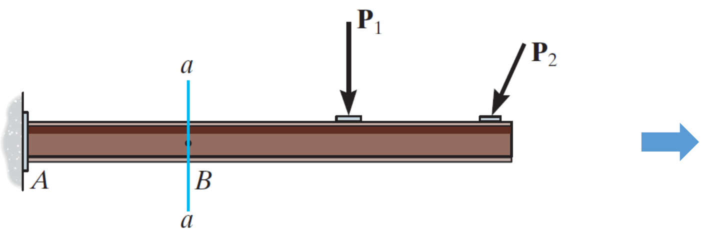
Loads Considered for Design of Golden Gate Bridge:
The bridge was designed for dead load, live load, and wind load.

https://theconstructor.org/case-study/golden-gate-bridge-construction/80548/ accessed July 2022

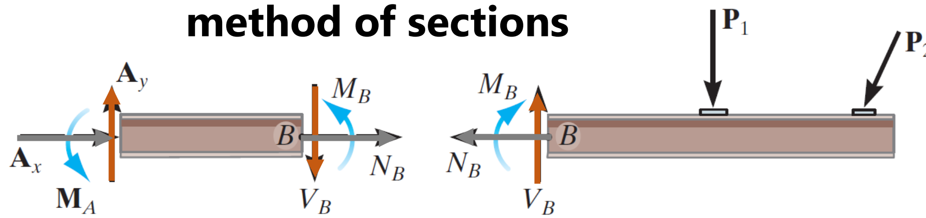


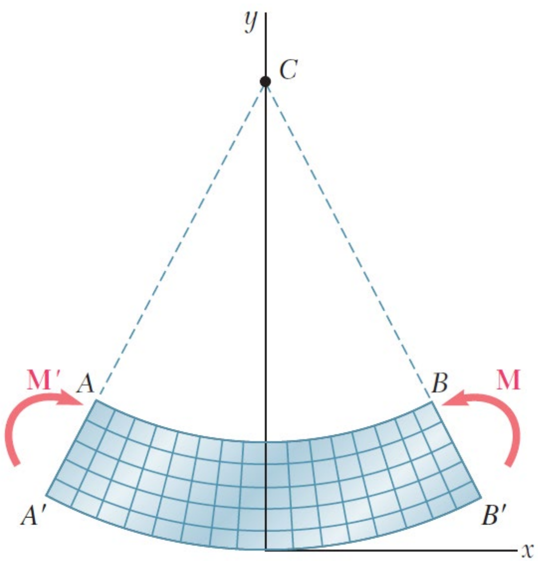
Beam loading

Generally speaking, beams are loaded transversally. However, each loading condition will give rise to internal forces as a mechanical reaction. These are **shear force**, **axial force**, and **bending moment**.



method of sections





When a beam bends, one side is stretched and the other compressed. In between, there is a layer that remains undeformed. This is the <u>neutral axis</u>

We can follow a simple procedure to study beams

1. Draw a FBD
2. Determine the support reactions on the beam
3. Use equilibrium equations to calculate unknowns

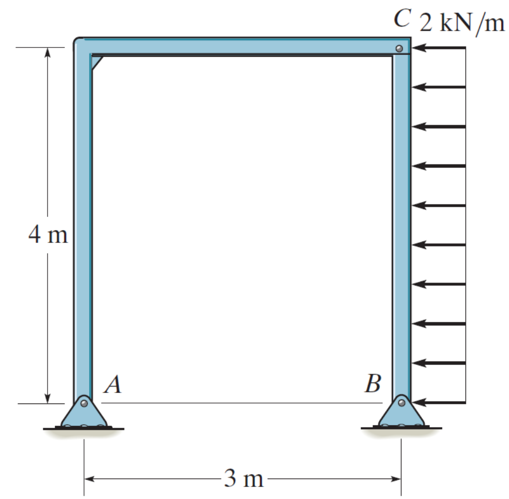
In a structure with more than one beam (frame), we can use these steps for each beam separately while respecting internal reactions

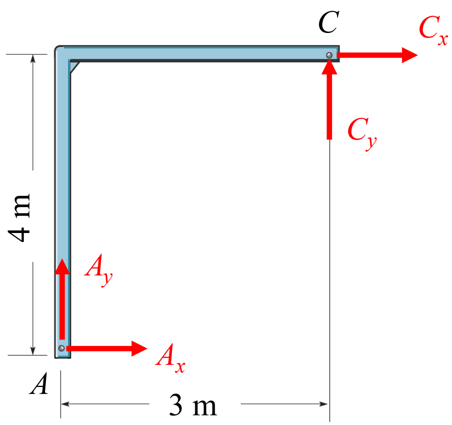
Example – External loads

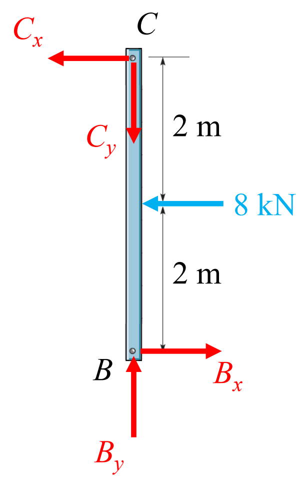
Determine the reactions at pins A and B in the frame shown

Solution

1. Draw the FBD of each member separately
2. Write equilibrium equations in each case







$$\sum F_x = A_x + C_x = 0$$

$$\sum F_y = A_y + C_y = 0$$

$$\sum M_C = 4A_x - 3A_y = 0$$

$$\sum M_C = -(2)(8) + 4B_x = 0$$

$$\sum F_x = B_x - C_x - 8 = 0$$

$$\sum F_y = B_y - C_y = 0$$

$$\boxed{B_x = 4 \text{ kN}} \qquad C_x = -4 \text{ kN}$$

$$\boxed{A_x = 4 \text{ kN}} \qquad C_y = -5.33 \text{ kN}$$

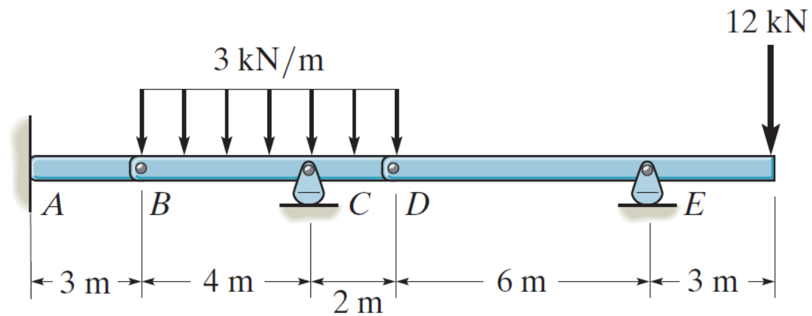
$$\boxed{A_y = 5.33 \text{ kN}}$$

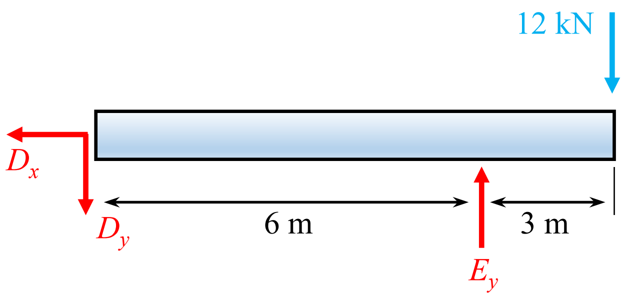
$$\boxed{B_y = -5.33 \text{ kN}}$$

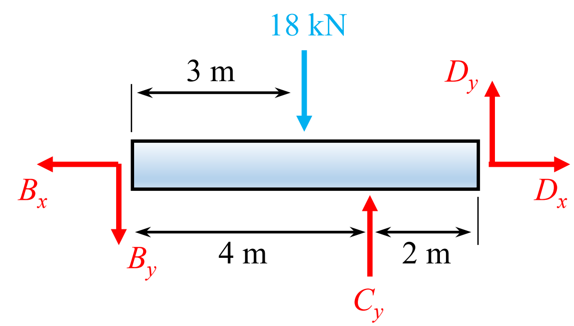
Group work (15 min)

Determine the reactions at the supports *A*, *C*, and *E* of the compound beam shown

1. Draw the FBD of each member separately.
2. Use equilibrium equations







$$\sum M_D = 6E_y - (9)(12) = 0$$

$$\sum F_y = -D_y + E_y - 12 = 0$$

$$\sum F_x = -D_x = 0$$

$$\sum M_B = -(3)(18) + 4C_y + 6D_y = 0$$

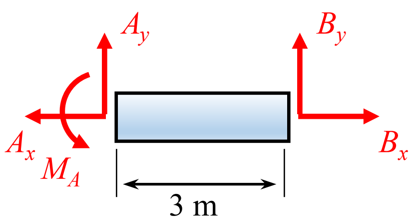
$$\sum F_y = -B_y - 18 + C_y + D_y = 0$$

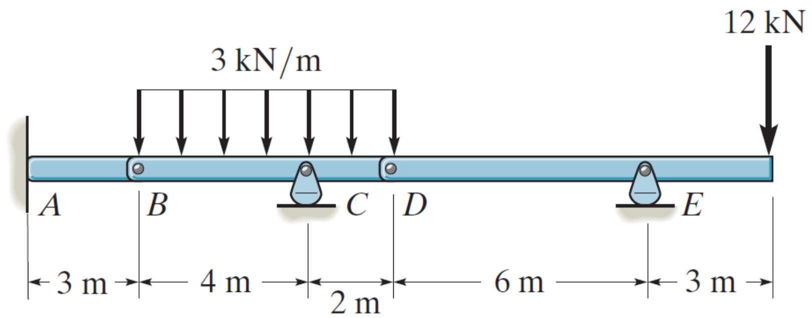
$$\sum F_x = -B_x + D_x = 0$$

$\boxed{E_y = 18 \text{ kN}}$ $\boxed{D_y = 6 \text{ kN}}$ $\boxed{D_x = 0 \text{ kN}}$ $\boxed{C_y = 4.5 \text{ kN}}$ $\boxed{B_y = -7.5 \text{ kN}}$ $\boxed{B_x = 0 \text{ kN}}$

$\boxed{B_y = -7.5 \text{ kN}}$ $\boxed{B_x = 0 \text{ kN}}$

Finally, we have member *AB*





$$\sum F_y = A_y + B_y = 0$$

$$\sum F_x = -A_x + B_x = 0$$

$$\sum M_B = -3A_y + M_A = 0$$

$\boxed{A_y = 7.5 \text{ kN}}$ $\boxed{A_x = 0 \text{ kN}}$ $\boxed{M_A = 22.5 \text{ kN} \cdot \text{m}}$

Beams – Internal Forces

Beams are often used to support the span of bridges. They can be thicker at the supports than at the center of the span

In a similar way, columns are usually wider/thicker at the bottom than at the top

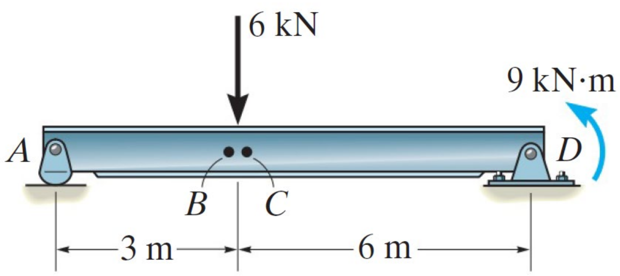
If the forces applied to the structure are, in principle, constant, why is the geometry variable?

In this crane, we not only see a tapered profile, but a significant amount of extra material at the base

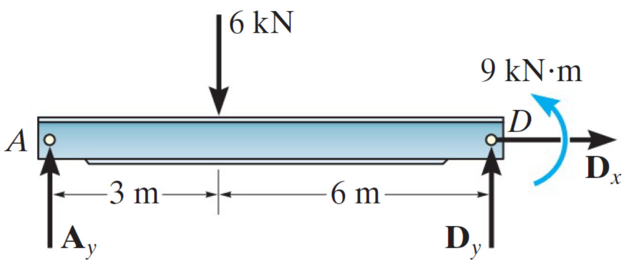
Any set of external loads will produce internal forces according to the laws of equilibrium

In general, the internal loads are not uniform, so structures are design to efficiently resist those internal loads

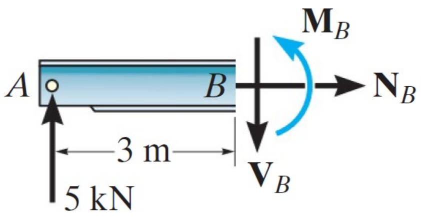
Beam loading



The design of any structural element requires finding the internal forces to make sure the material can resist

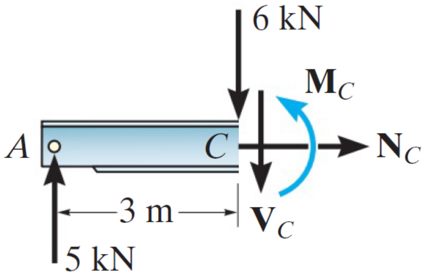


For example, if we wanted to find the internal forces at *B* the first step would be finding the support reactions

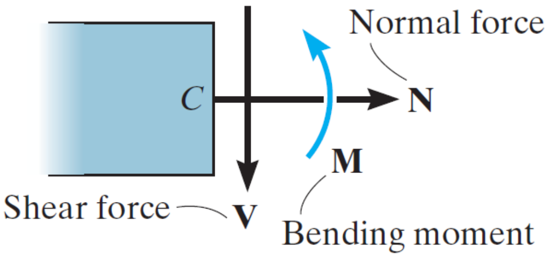


Now we can "cut" the beam at *B* and draw a FBD of the piece on the left. This includes internal reactions

Once we know the internal forces at *B*, we can repeat the procedure at another point such as *C*

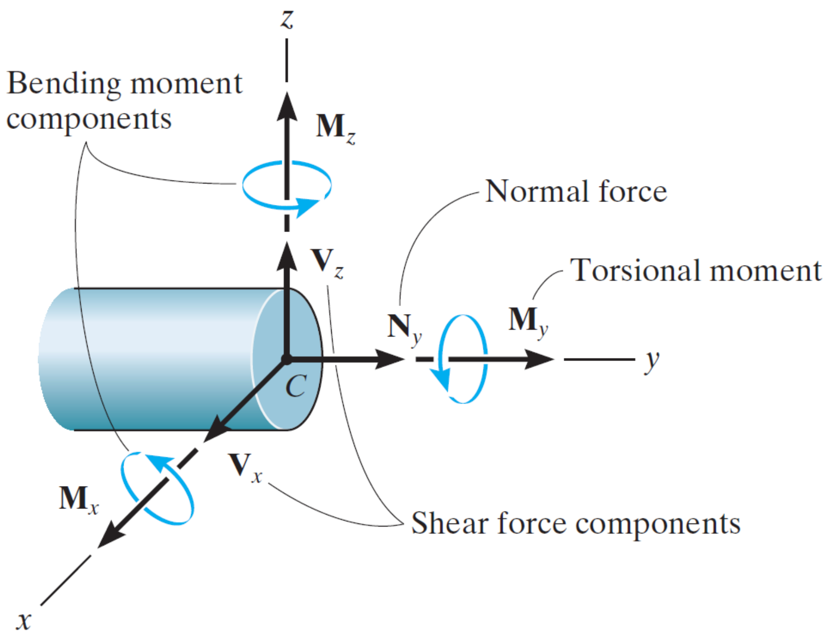


Each time we will get different values for the internal forces. We can then find critical points for design



Beam loading

In the context of beams, internal loads are axial forces, (*N*, acting along the neutral axis), shear forces (*V*, acting perpendicular to the neutral axis), and bending moment (*M*, normal to the previous two)



The orientations shown constitute the convention for positive axial, shear, and bending loads

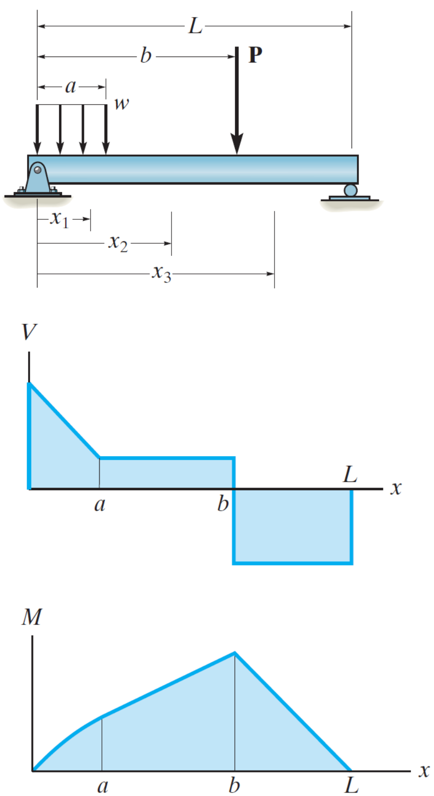
We should remember the internal loads on both sides of a "cut" section are equal and opposite

Since the beam is in equilibrium, internal forces must converge to external forces at the supports

Under this system, the design of beams is reduced to finding minima or maxima for internal forces

Internal forces diagram

Internal forces are not constant. They will change depending on where we "cut". In order to get detailed knowledge of their variation along the axis of the beam, we need to plot them.



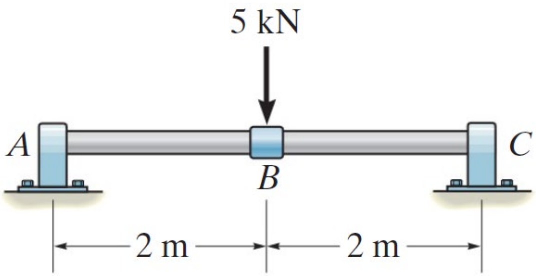
Following the procedure outlined before, we can always obtain a plot of either *N*, *V*, or *M*. This diagram is the basis for beam design

Generally speaking, *V* and *M* will be discontinuous at points where a force or moment is applied or where a distributed load changes

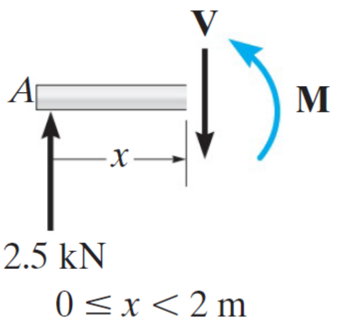
Since the shear force and bending moment diagrams originate from the FBD, they must be consistent with the equilibrium equations

A sure way to check is we have the correct shear force or bending moment diagram is to see if it satisfies the conditions of the FBD

Steps

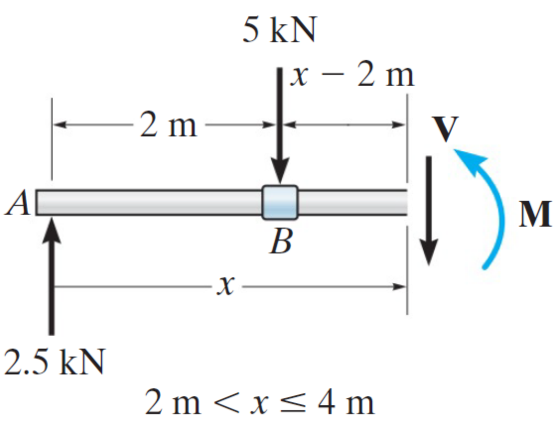


1. Determine the support reactions



2. Specify separate coordinates of x having an origin at the beam's left end and extending to regions of the beam between concentrated forces or moments, or where a distributed load is discontinuous

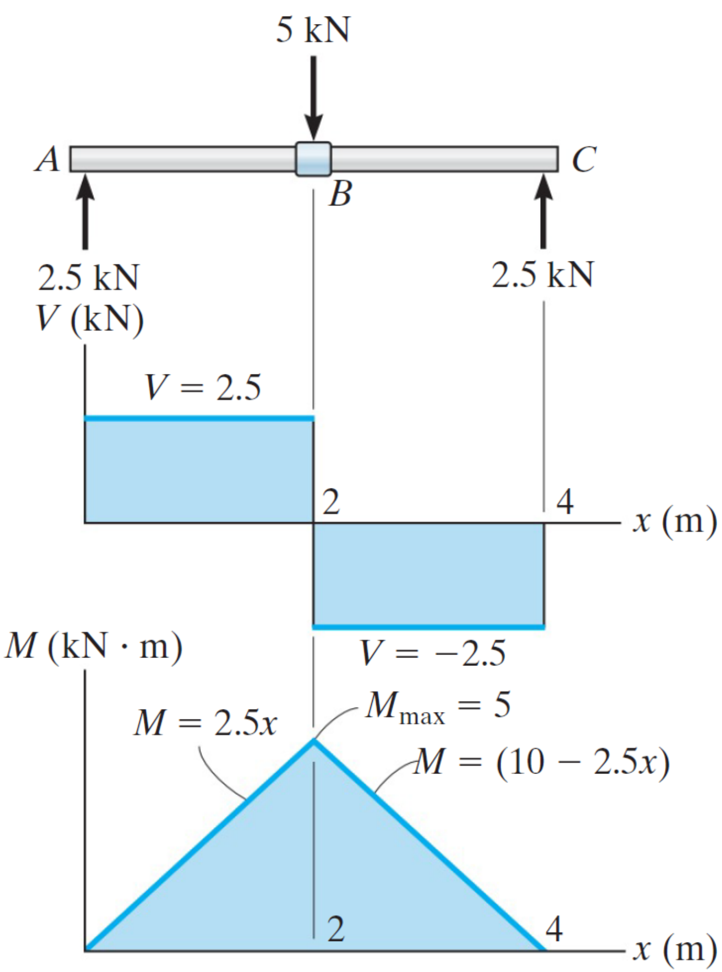
3. "cut" the beam at the first specific x coordinate and draw the FBD of the segment. In each case, V and M will show in the positive sense



4. Obtain V and M at the first specific x coordinate using the equilibrium equations. Either V or M may be functions of x.

5. Continue to the next specific x coordinate and repeat the procedure. We will do this until we reach the end of the beam

Procedure

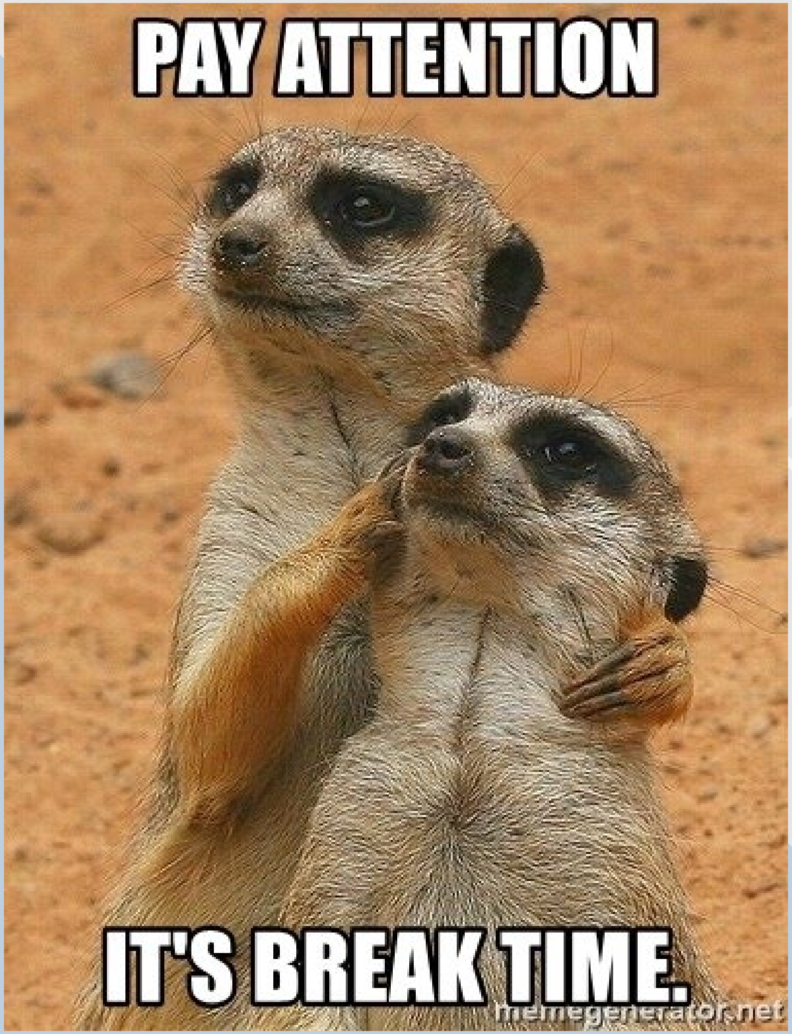


Drawing the shear force and bending moment diagrams is a dynamic process. You must make sure equilibrium conditions are satisfied everywhere along the beam

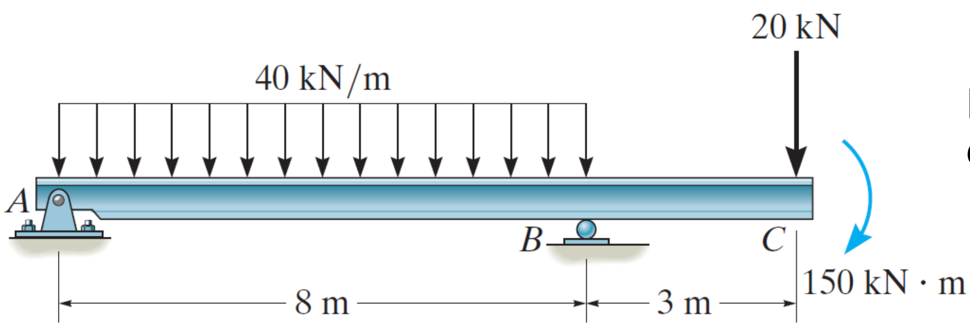
Remember the values of V and M match the external loads wherever they are applied. This is a sign your diagrams are correct. This includes the end points

Once the diagrams are finished, it is easy to see where the critical points are. Those locations with maximum shear force or bending moment require a stronger beam

It is easy now to see why some beams are tapered or why columns are thicker at the base. More material is used where the force and moment diagrams show peak values



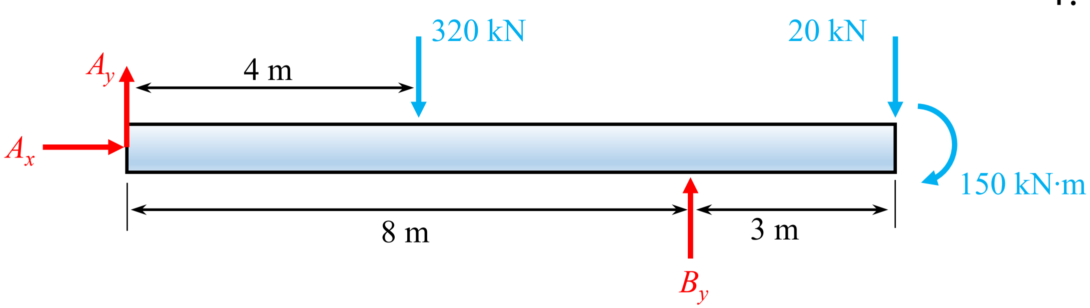
Time for a 5 minutes stretching break!



Example

Draw the shear force and bending moment diagrams

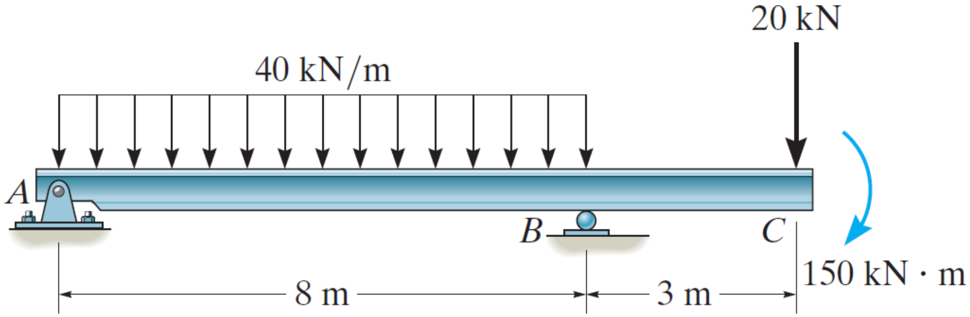
1. calculate the reactions



$$\sum F_x = A_x = 0$$

$$\sum M_A = -(4)(320) + (8)(B_y) - (11)(20) - 150 = 0 \quad \Rightarrow \quad B_y = 206.25\ \mathrm{kN}$$

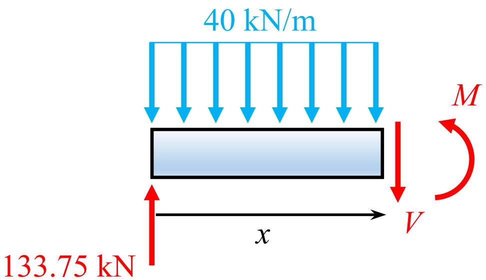
$$\sum F_y = A_y - 320 + B_y - 20 = 0 \quad \Rightarrow \quad A_y = 133.75\ \mathrm{kN}$$



2. Find coordinates where there are forces or moments applied or where a distributed load is discontinuous

In this case, x = 8 m is the only one

To the left of x = 8 m we have



The sum of forces in y gives

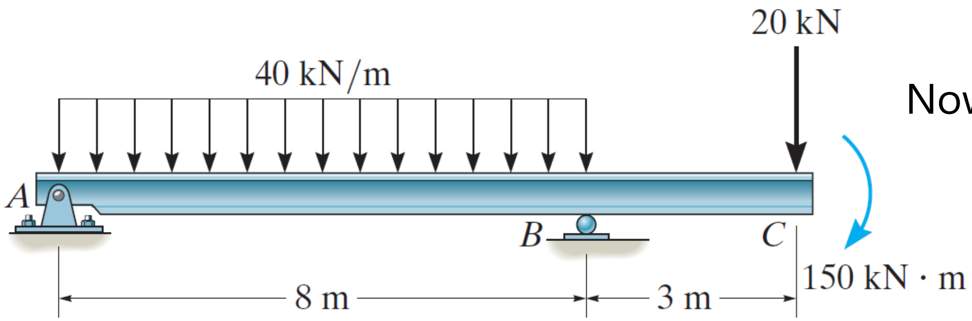
$$\sum F_y = 133.75 - 40x - V = 0 \quad \Rightarrow \quad \boxed{V = [133.75 - 40x]\ \mathrm{kN}}$$

This is the shear force function!

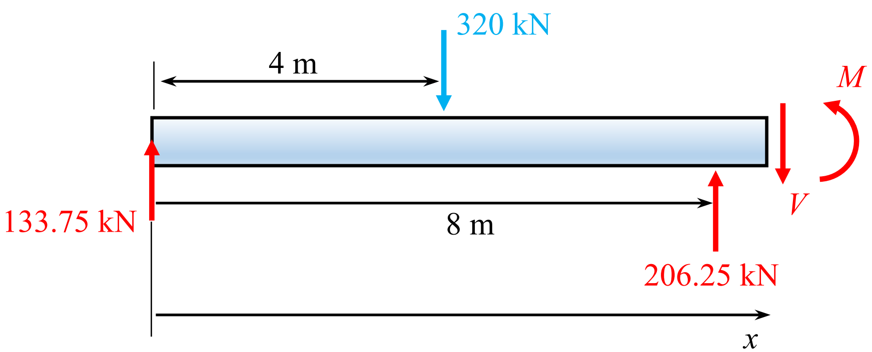
The sum of moments at the cut gives

$$\sum M_x = M + 40x\left(\frac{x}{2}\right) - 133.75x = 0 \quad \Rightarrow \quad \boxed{M = [-20x^2 + 133.75x]\ \mathrm{kN \cdot m}}$$

This is the bending moment function!



Now we repeat the process to the right of x = 8 m

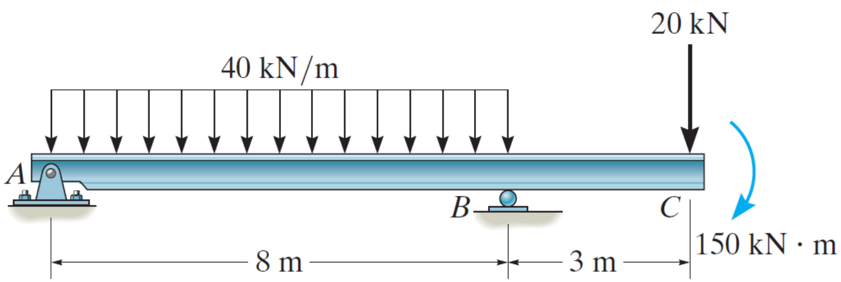


$$\sum F_y = 133.75 - 320 + 206.25 - V$$

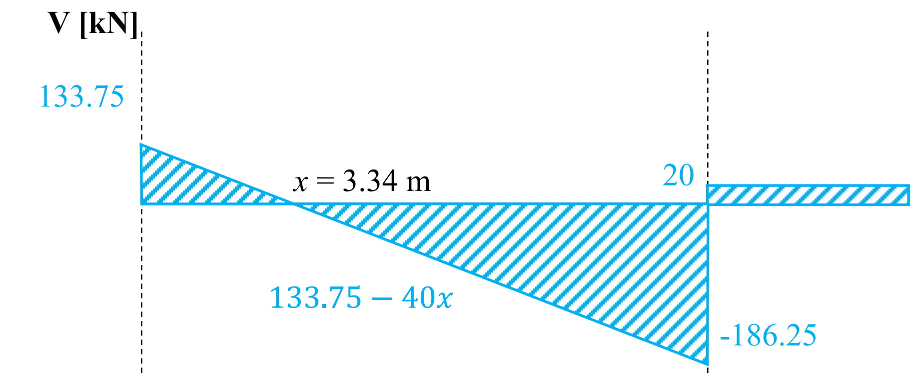
$$\boxed{V = 20 \text{ kN}}$$

$$\sum M_x = M - 133.75x + 320(x-4) - 206.25(x-8) = 0$$

$$\boxed{M = [20x - 370] \text{ kN} \cdot \text{m}}$$

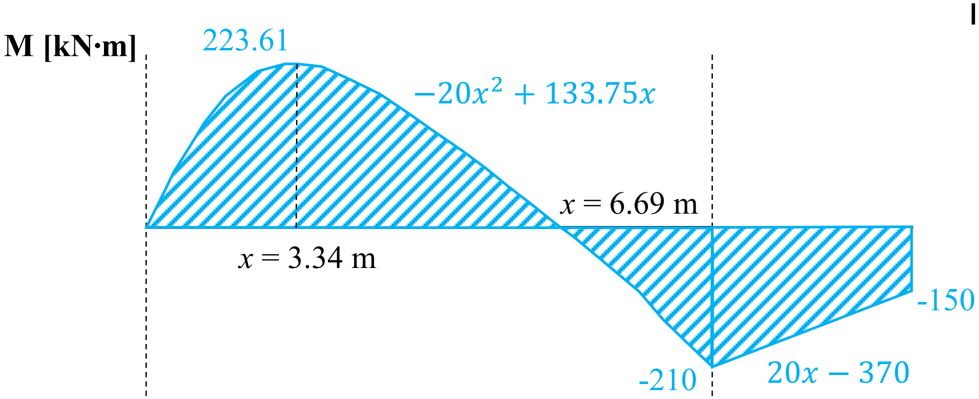


Our final step is to plot these functions



Notice that the shear force and bending moment diagrams give the exact values for the reactions

Do you notice a mathematical relation between the functions of *V* and *M*? What is it? Why?

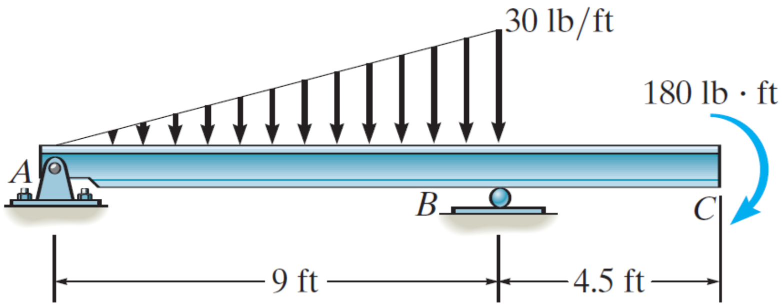


$$\Delta M = \int V\,dx$$

Change in moment = Area under shear diagram

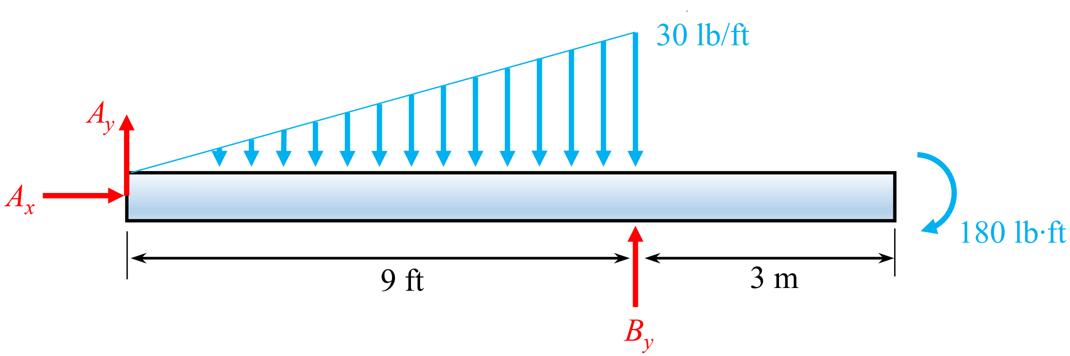
A maximum or minimum bending moment occurs at the point where the shear is equal to zero.

Group work (20 min)



Draw the shear force and bending moment diagrams

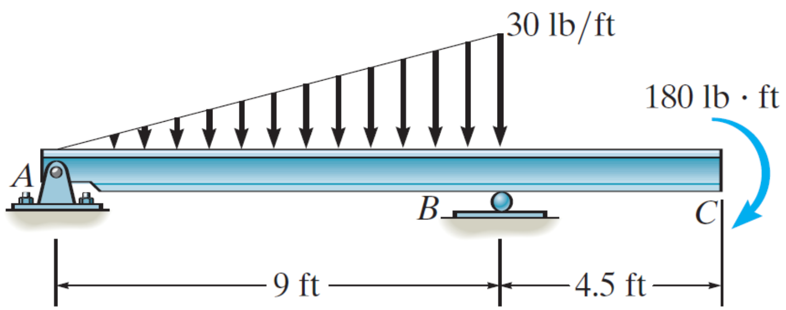
1. Calculate the reactions



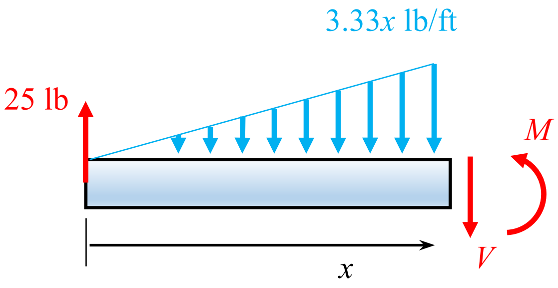
$$\sum F_x = A_x = 0$$

$$\sum M_A = -(6)\left[\frac{(9)(30)}{2}\right] + (9)(B_y) - 180 = 0 \quad \Rightarrow \quad B_y = 110 \text{ lb}$$

$$\sum F_y = A_y - \frac{(9)(30)}{2} + B_y = 0 \quad \Rightarrow \quad A_y = 25 \text{ lb}$$



To the left of x = 9 ft we have



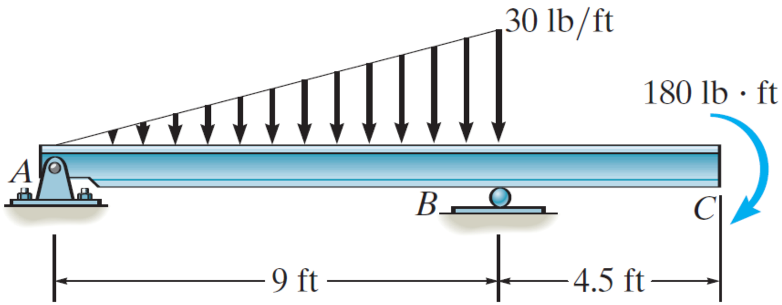
$$\sum F_y = 25 - \frac{x(3.33x)}{2} - V$$

$$\boxed{V = [25 - 1.67x^2]\ \text{lb}}$$

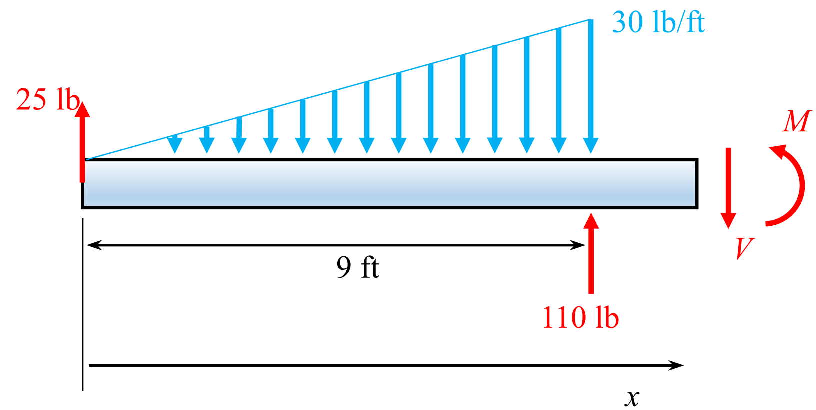
In this case, the maximum value of the distributed load depends on x

$$\sum M_x = M - 25x + \left(\frac{x(3.33x)}{2}\right)\left(\frac{x}{3}\right) = 0$$

$$\boxed{M = [25x - 0.56x^3]\ \text{lb} \cdot \text{ft}}$$



To the right of $x = 9$ ft we have

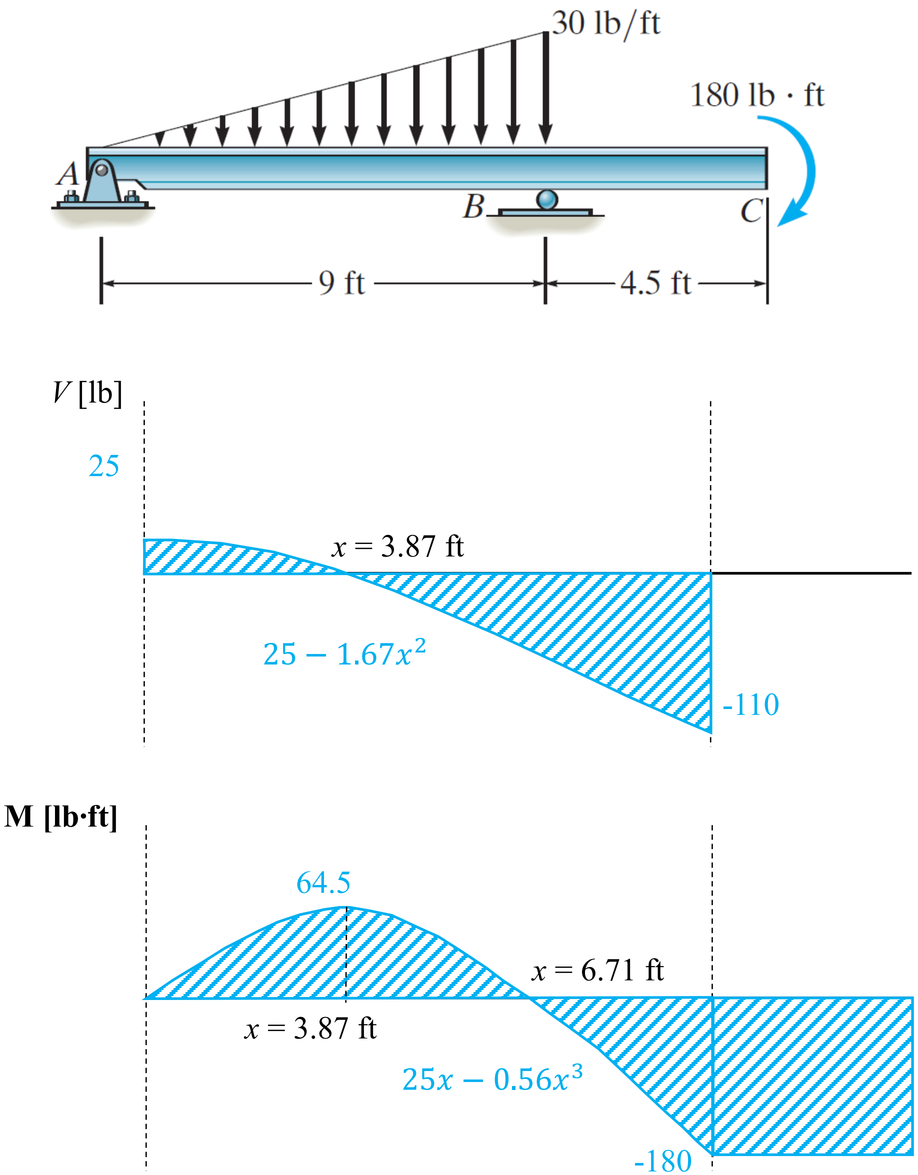


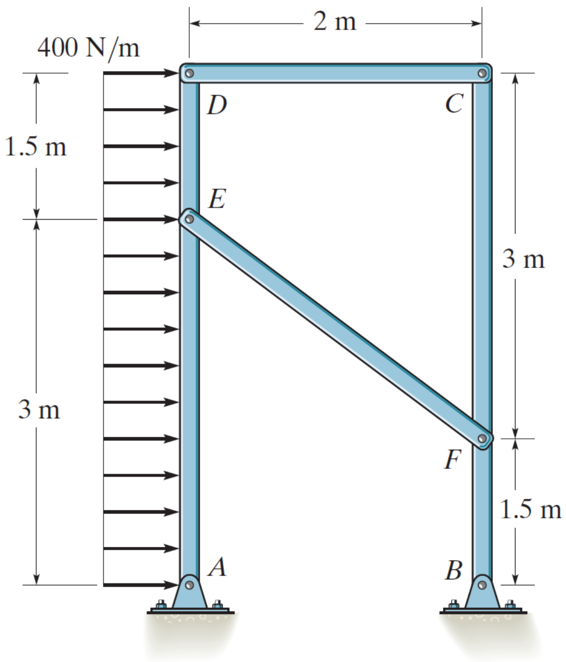
$$\sum F_y = 25 - (9)(30)(0.5) + 110 - V$$

$$\boxed{V = 0 \text{ lb}}$$

$$\sum M_x = M - 25x + \left(\frac{(9)(30)}{2}\right)(x-6) - (110)(x-9) = 0$$

$$\boxed{M = -180 \text{ lb} \cdot \text{ft}}$$

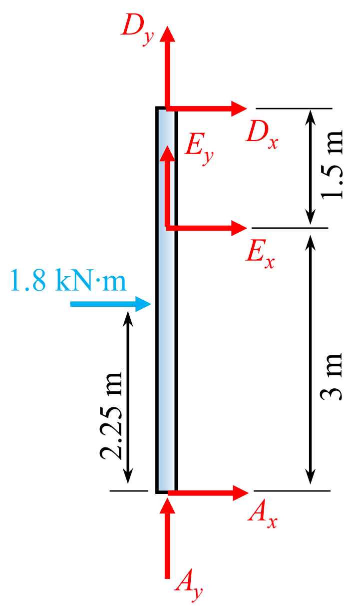

30 lb/ft
180 lb · ft
A
B
C
9 ft
4.5 ft
V [lb]
25
x = 3.87 ft
25 − 1.67x²
-110
M [lb·ft]
64.5
x = 6.71 ft
x = 3.87 ft
25x − 0.56x³
-180



Example

Determine the reactions at the supports A and B

Beginning with AED

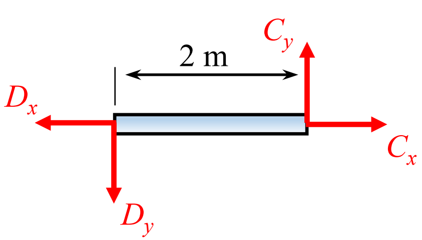


$$\sum F_x = A_x + E_x + D_x + 1.8 = 0$$

$$\sum F_y = A_y + E_y + D_y = 0$$

$$\sum M_E = (0.75)(1.8) + 3A_x - 1.5D_x = 0$$

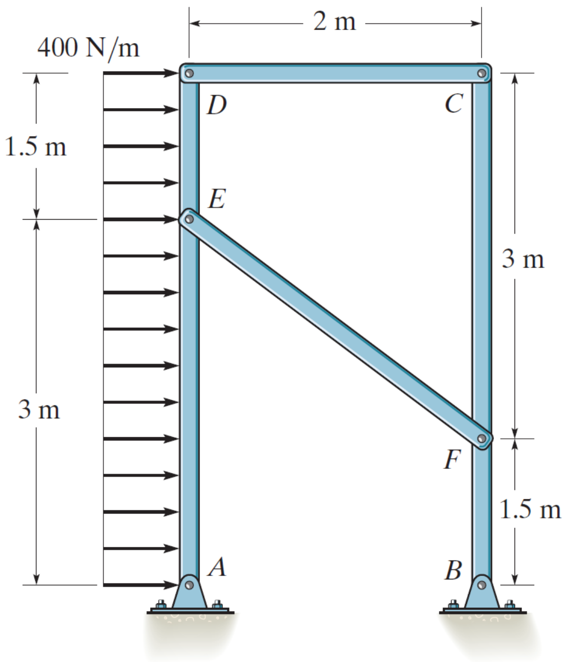
Since we can't solve for any unknown, let us move on to DC



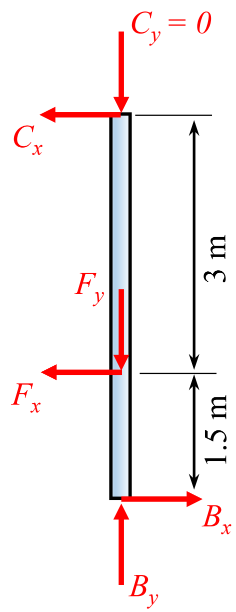
$$\sum F_x = -D_x + C_x = 0$$

$$\sum M_C = 2D_y = 0 \Rightarrow \boxed{D_y = 0}$$

$$\sum F_y = -D_y + C_y = 0 \Rightarrow \boxed{C_y = 0}$$



To complete the picture, we need the other two members. For *BFC* we have

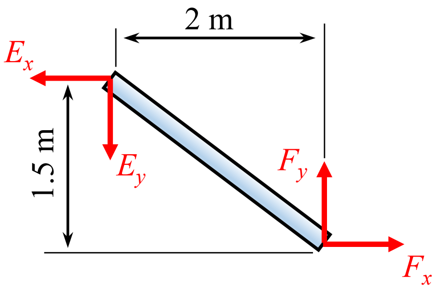


$$\sum F_x = B_x - F_x - C_x = 0$$

$$\sum F_y = B_y - F_y = 0$$

$$\sum M_F = 3C_x + 1.5B_x = 0$$

And in the case of *EF*



$$\sum F_x = -E_x + F_x = 0$$

$$\sum F_y = -E_y + F_y = 0$$

$$\sum M_F = 2E_y + 1.5E_x = 0$$

Then our equations are:

1 $A_x + E_x + D_x + 1.8 = 0$

2 $A_y + E_y = 0$

3 $(0.75)(1.8) + 3A_x - 1.5D_x = 0$

4 $C_x = D_x$

5 $B_x - F_x - C_x = 0$

6 $B_y = F_y$

7 $3C_x + 1.5B_x = 0$

8 $F_x = E_x$

9 $F_y = E_y$

10 $2E_y + 1.5E_x = 0$

We can use 7 and 8 in 5 to get $E_x = -3C_x$

And if we substitute that in 1 together with 4

$A_x - 2D_x + 1.8 = 0$

Which, combined with 3 gives

$\boxed{D_x = 0.9 \text{ kN}}$ $\boxed{A_x = 0 \text{ kN}}$

Which in turn gives $\boxed{C_x = 0.9 \text{ kN}}$ $\boxed{E_x = -2.7 \text{ kN}}$ $\boxed{F_x = -2.7 \text{ kN}}$

And, using 5 $\boxed{B_x = -1.8 \text{ kN}}$ Also, with 10 $\boxed{E_y = 2.025 \text{ kN}}$

Which gives $\boxed{F_y = 2.025 \text{ kN}}$ and $\boxed{B_y = 2.025 \text{ kN}}$

Finally, we can use 2 to get $\boxed{A_y = -2.025 \text{ kN}}$