

You're expected to work on the discussion problems before coming to the lab. Discussion session is not meant to be a lecture. TA will guide the discussion and correct your solutions if needed. We will not release 'official' solutions. If you're better prepared for discussion, you will learn more. TAs will record names of the students who actively engage in discussion and report them to the instructor; they are also allowed to give some extra points to those students at their discretion. The instructor will factor in participation in final grade.

1. (Advanced) Let f be a flow in a network, and let α be a real number. The scalar flow product, denoted αf , is a function from $V \times V$ to \mathbb{R} defined by

$$(\alpha f)(u, v) = \alpha \cdot f(u, v).$$

Prove that the flows in a network form a *convex set*. That is, show that if f_1 and f_2 are flows, then so is $\alpha f_1 + (1 - \alpha)f_2$ for all α in the range $0 \leq \alpha \leq 1$.

2. (Basic) Professor Adam has two children who, unfortunately, dislike each other. The problem is so severe that not only do they refuse to walk to school together, but in fact each one refuses to walk on any block that the other child has stepped on that day. The children have no problem with their paths crossing at a corner. Fortunately both the professors house and the school are on corners, but beyond that he is not sure if it is going to be possible to send both of his children to the same school. The professor has a map of his town. Show how to formulate the problem of determining whether both his children can go to the same school as a maximum-flow problem.
3. (Basic) In Figure 26.1(b) (see CLRS page 710), what is the flow across the cut $(\{s, v_2, v_4\}, \{v_1, v_3, t\})$? What is the capacity of this cut?
4. (Intermediate) Suppose that we redefine the residual network to disallow edges into s . Argue that the procedure FORD-FULKERSON still correctly computes a maximum flow.
5. (Advanced) Suppose that you wish to find, among all minimum cuts in a flow network G with integral capacities, one that contains the smallest number of edges. Show how to modify the capacities of G to create a new flow network G' in which any minimum cut in G' is a minimum cut with the smallest number of edges in G .
6. (Basic) Let $G = (V, E)$ be a bipartite graph with vertex partition $V = L \cup R$, and let G' be its corresponding flow network. Give a good upper bound on the length of any augmenting path found in G' during the execution of FORD-FULKERSON.