Segmentation By Thresholding

Advantages:
- + intuitive
- + simple implementation
- + computationally efficient

Disadvantages
- − not effective for complex images

Intensity Thresholding Basis

Suppose image has intensity histogram of 10.35(a)
→ light objects on dark background
→ intensity values grouped into two dominant modes

Obvious way to "extract" objects from background is to select a threshold T that separates these two modes.

Any pt. (x,y) at which $f(x,y) > T$ is called an object pt.
Otherwise, a background pt.

Segmented image $g(x,y)$ given by

$$g(x,y) = \begin{cases} 1 & \text{if } f(x,y) > T \\ 0 & \text{if } f(x,y) \le T \end{cases}$$

If constant T is used for whole image then have <u>global thresholding</u>

When T changes have <u>variable thresholding</u>.
<u>Local</u> or <u>regional</u> thresholding refers to case where T at any pt. (x,y) depends on properties of neighborhood of (x,y). (ex: average of pixel values in neighborhood)

Consider 10.35(b). Now three dominant modes.
Two types of objects?

Now, can use multiple thresholding:

$$g(x,y) = \begin{cases} a & \text{if } f(x,y) > T_2 \\ b & \text{if } T_1 < f(x,y) \le T_2 \\ c & \text{if } f(x,y) \le T_1 \end{cases}$$

where a, b, c are distinct intensity values.

Picking more than two global thresholds is difficult and usually better using adaptive thresholding.

Success of intensity thresholding depends on width and depth of valleys separating the histogram modes.

Key factors affecting the properties of the valleys are

1) Separation between peaks

2) Noise content of images (modes broaden as noise increases)

3) Relative sizes of objects and background

4) Uniformity of illumination source

5) Uniformity of reflectance properties of image

Noise and Thresholding

Consider 10.36

Illumination / Reflectance and Thresholding

Consider 10.37

Basic Global Thresholding

Even if have an image whose intensity histogram has distinct modes, still need to algorithmically decide threshold value T.

Iterative algorithm for picking global threshold T:

1. Select an estimate for T
2. Segment image using T. This will result in two groups of pixels:

$$G_1 : \text{all pixels with intensity} > T$$
$$G_2 : \text{all pixels with intensity} \leq T$$

3. Compute mean pixel values m_1 and m_2 of G_1 and G_2
4. Compute new threshold

$$T = \frac{1}{2}(m_1 + m_2)$$

5. Repeat steps 2-4 until difference between values of T in successive iterations is smaller than predefined parameter ΔT.

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(3) Thresholding

Algorithm works well when clear valley between modes of histogram related to objects and background.

Example in 10.38

Final value for $T = 125.4$
3 iterations
T=mean initially

Optimum Global Thresholding Using Otsu's Method

Maximizes the between-class variance

class ~ group of pixels assigned to object

Idea: well-thresholded classes should be distinct with respect to the intensity values of their pixels

Uses histogram of image to compute T that results in maximum between-class variance.

Image with L intensity values $\{0, \dots, L-1\}$
$M \times N$ pixels
n_i = # of pixels with intensity i

So $MN = \sum_{i=0}^{L-1} n_i$

Normalized histogram components $p_i = \dfrac{n_i}{MN}$

Have $\sum_{i=0}^{L-1} p_i = 1 \qquad p_i \geq 0.$

Suppose pick threshold $T(k) = k$, $0 < k < L-1$ and threshold image into two classes C_1, C_2

C_1 = all pixels with value in range $[0, k]$
C_2 = " " " " " " $[k+1, L-1]$

Probability $P_1(k)$ that a pixel is assigned to class C_1 is given by cumulative sum

$$P_1(k) = \sum_{i=0}^{k} p_i$$

$P_1(k)$ = probability of class C_1 occurring.

Similarly

$$P_2(k) = \sum_{i=k+1}^{L-1} p_i = 1 - P_1(k) = \text{probability of class } C_2 \text{ occurring.}$$

Mean intensity of pixels assigned to class C_1 is

$$m_1(k) = \sum_{i=0}^{k} i\, P(i \mid C_1)$$

$P(i \mid C_1)$ = probability of pixel having intensity i given that it is in C_1.

Using Bayes' formula

$$P(i \mid C_1) = \frac{P(C_1 \mid i)\, P(i)}{P(C_1)}$$

so $$m_1(k) = \sum_{i=0}^{k} i\, \frac{P(C_1 \mid i)\, P(i)}{P(C_1)}$$

But $P(C_1 \mid i) = \underline{1}$ because only dealing with values i from class C_i.

$P(C_1) = P_1(k)$

so

$$m_1(k) = \frac{1}{P_1(k)} \sum_{i=0}^{k} i\, p_i$$

Similarly $$m_2(k) = \sum_{i=k+1}^{L-1} i\, P(i \mid C_2) = \frac{1}{P_2(k)} \sum_{i=k+1}^{L-1} i\, p_i$$

Cumulative mean up to level k is given by

$$m(k) = \sum_{i=0}^{k} i\, p_i \quad = m$$

Global mean

$$m_G = \sum_{i=0}^{L-1} i\, p_i$$

Can show

$$P_1(k)\,m_1(k) + P_2(k)\,m_2(k) = m_G \quad \text{or} \quad P_1 m_1 + P_2 m_2 = m_G$$

and

$$P_1(k) + P_2(k) = 1 \quad \text{or} \quad P_1 + P_2 = 1$$

Evaluate the "goodness" of threshold at level k through

$$\eta = \frac{\sigma_B^2}{\sigma_G^2} = \frac{\text{between class variance}}{\text{global variance}}$$

$$\sigma_G^2 = \sum_{i=0}^{L-1} (i - m_G)^2 p_i$$

$$\sigma_B^2 = P_1(m_1 - m_G)^2 + P_2(m_2 - m_G)^2$$

Can rewrite

$$\sigma_B^2 = P_1 P_2 (m_1 - m_2)^2 = \frac{(m_G P_1 - m)^2}{P_1(1-P_1)}$$

more efficient to evaluate for different threshold values k
→ only m, P_1 change

The further m_1 and m_2 are apart,

the larger σ_B^2 and larger η (σ_G^2 is constant)

→ Maximize σ_B^2 wrt k

To summarize

$$\eta(k) = \frac{\sigma_B^2(k)}{\sigma_G^2}$$

$$\sigma_B^2(k) = \frac{\left[m_G P_1(k) - m(k)\right]^2}{P_1(k)\left[1 - P_1(k)\right]}$$

Optimum threshold is value k^* that maximizes $\sigma_B^2(k)$

$$k^* = \arg\max_k \sigma_B^2(k).$$

To find k^*, simply evaluate $\sigma_B^2(k)$ for all integer values of k.

Once k^* is chosen, use to threshold image.

$\eta(k) = \dfrac{\sigma_B^2(k^*)}{\sigma_G^2}$ indicates how good segmentation is.

$0 \leq \eta(k^*) \leq 1$ larger = better

Ex. 10.39

Threshold by standard algorithm = 169
" " Otsu's method = 181

Summary of Otsu's algorithm:

1. Compute normalized histogram of input image : p_i, $i = 0, \dots, L-1$
2. Compute cumulative sums $P_1(k)$ for $k = 0, \dots, L-1$
3. Compute cumulative means $m(k)$ for $k = 0, \dots, L-1$
4. Compute global mean m_G
5. Compute between class variance $\sigma_B^2(k)$ for $k = 0, \dots, L-1$
6. Obtain k^* for which $\sigma_B^2(k)$ is max.
7. Obtain separability measure η^*.