Distributed loads

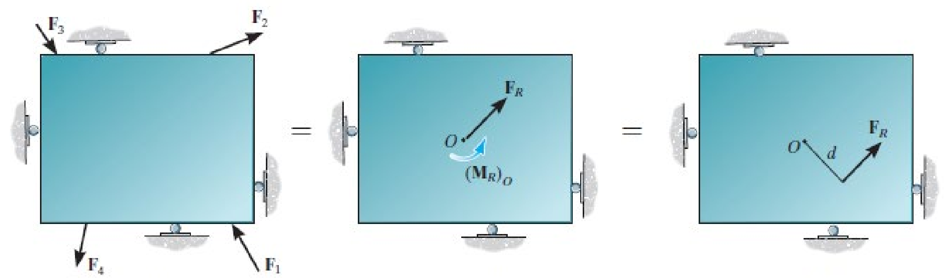
Instructor

Ingrid M. Padilla Espinosa, PhD

UNIVERSITY OF CALIFORNIA MERCED

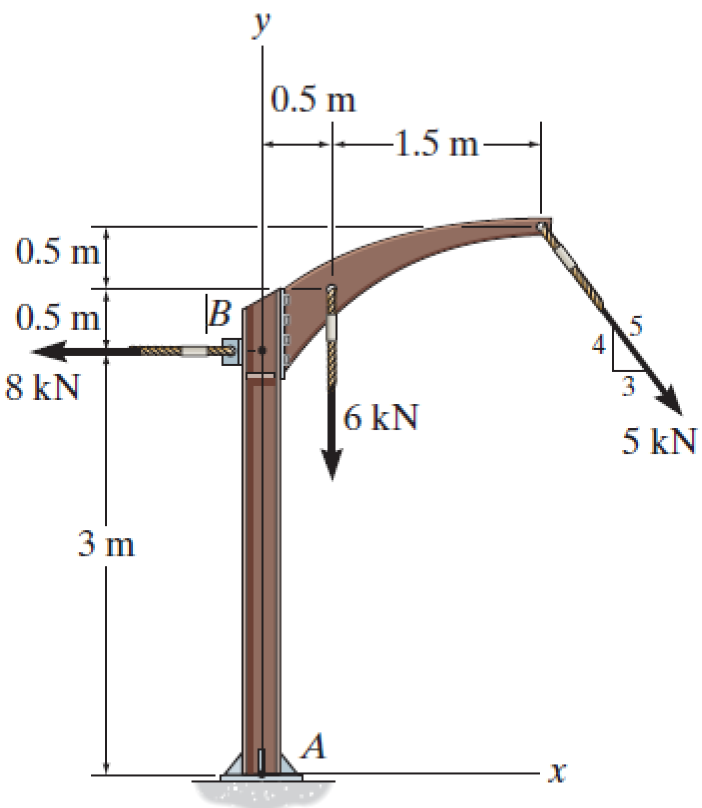
Simplification of a force and a couple system into an equivalent resultant force acting at a point and a resultant couple moment

1. Establish the coordinate axes with the origin located at point O and the axes having a selected orientation.
2. Force Summation. The resultant force is equal to the sum of all the forces in the system.
   - Coplanar or 2D: resolve each force into its *x* and *y* components.
   - 3D: represent each force as a Cartesian vector.
3. Moment Summation. The moment of the resultant force about point O is equal to the sum of all the couple moments in the system plus the moments of all the forces in the system about O.
   - 2D: use the principle of moments, i.e., determine the moments of the components of each force, rather than the moment of the force itself.
   - 3D: use the vector cross product to determine the moment of each force about point O. The position vectors extend from O to any point on the line of action of each force.



(a) (b) (c)

Individual work (10 min)

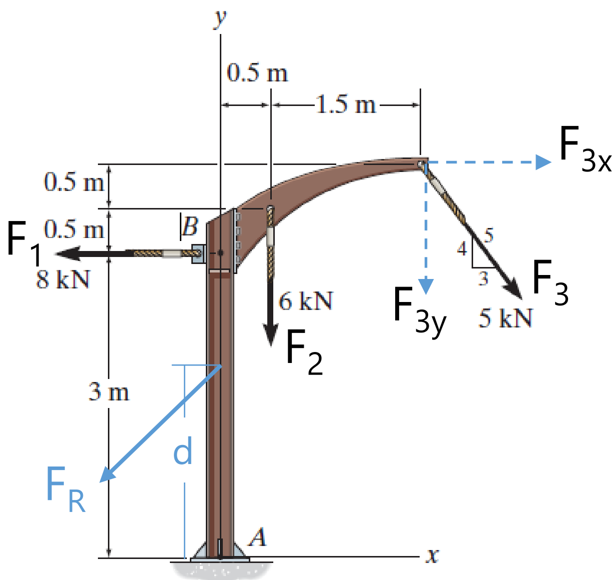


Replace the loading system by an equivalent resultant force and specify where the resultant's line of action intersects the member *AB* measured from *A*.

Solution:

Given: Force (5 kN, 6 kN, 8 kN) coordinates, angle

Asked: $\mathbf{F_R}$, d



Solution:

$\mathbf{F_1}$ = {-8 **i**} kN
$\mathbf{F_2}$ = {-6 **j**} kN
$\mathbf{F_3}$ = {5(3/5) **i** – 5(4/5) **j**} kN = {3 **i** – 4 **j**} kN

$\mathbf{F_r}$ = {(-8+3) **i** + (-6-4) **j**} kN = {-5 **i** – 10 **j**} kN

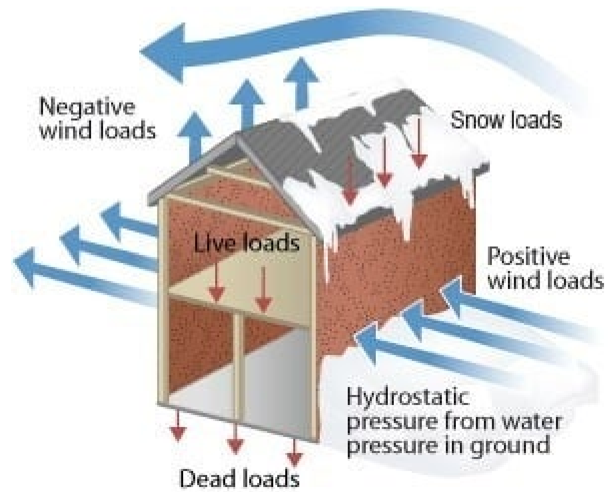
+↶ M_A : [8 kN (3 m) – 6 kN (0.5 m) – 3 kN (4 m) – 4 kN (2 m)] =5 kN (d)

d = 0.2 m

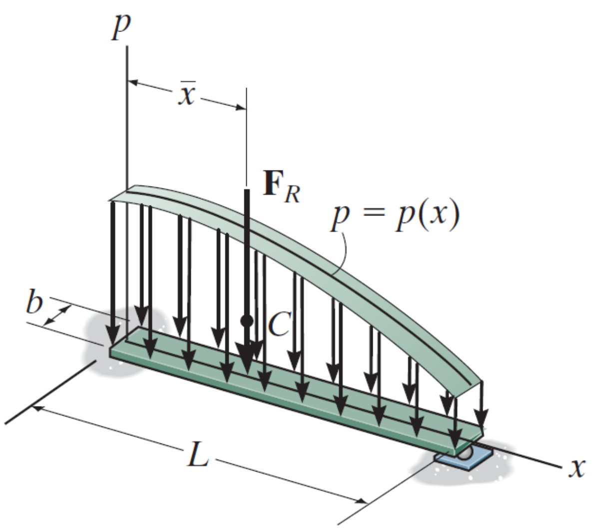
Distributed load

- Sometimes, forces are not applied at a single point. Instead, they are distributed over its surface.
- We can use statics methods to find an equivalent mechanical system in which the force is localized.
- A simplified mechanical system is much easier to handle and facilitates engineering design.

- This approach can be used in 2D and 3D. Forces can be distributed over lines, surfaces, or volumes.
- Most real-life loads considered in engineering are distributed loads, but they all need to be simplified.
- Statically equivalent systems of loads are central to computer-aided engineering methods such as FEM

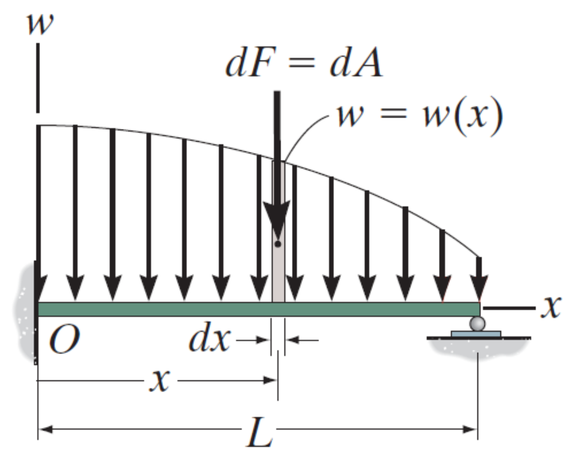


Distributed load

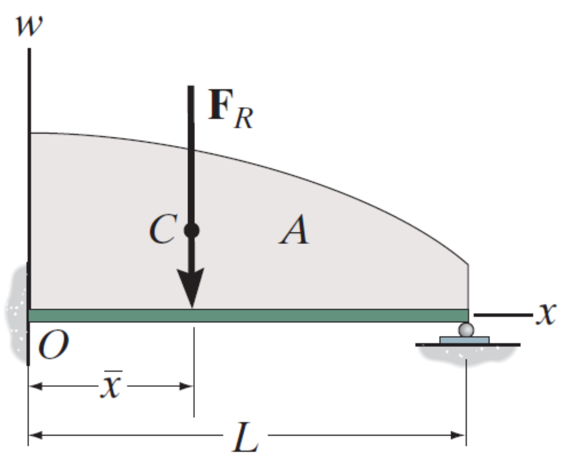


- Usually, distributed loads can be represented along a single axis.
- In a beam, transversal loads can vary along the length
- We can simplify this problem to a 2D case:

$$w(x) = p(x)b \text{ N/m}$$



Our objective is to find a single load $\mathbf{F}_R$ to represent the distributed load:

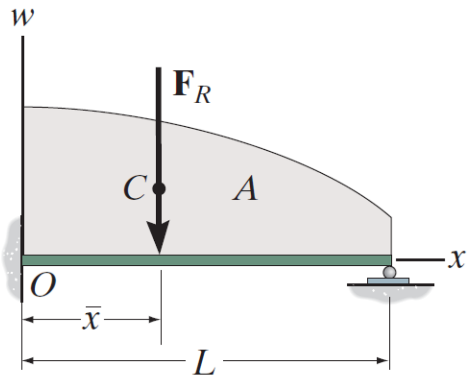


$\mathbf{F}_R$ is the <u>sum</u> of all differential forces *dF*

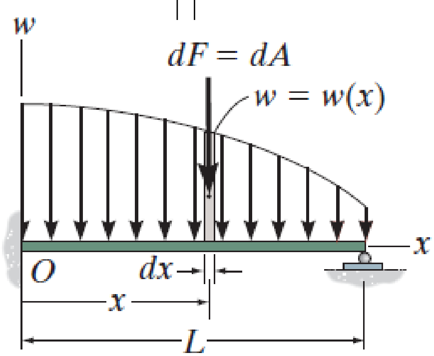
$$dF = w(x)dx \qquad F_R = \int_0^L w(x)dx$$

We can also find the moment created by this force!

$$M_{R,O} = \int_0^L xw(x)dx$$



||



Distributed load

Since the two systems (distributed vs local) are mechanically equivalent, they must have the same total moment around O

In the localized case, we have $M_{R,O} = \bar{x}F_R$ where $\bar{x}$ is unknown

In the distributed case, $M_{R,O} = \int_0^L xw(x)dx$

Since these two are equal, we have $\bar{x}F_R = \int_0^L xw(x)dx$ And, using $F_R = \int_0^L w(x)dx$

We can solve for $\bar{x}$

$$\bar{x} = \frac{\int_0^L xw(x)dx}{\int_0^L w(x)dx}$$

Or, in terms of area

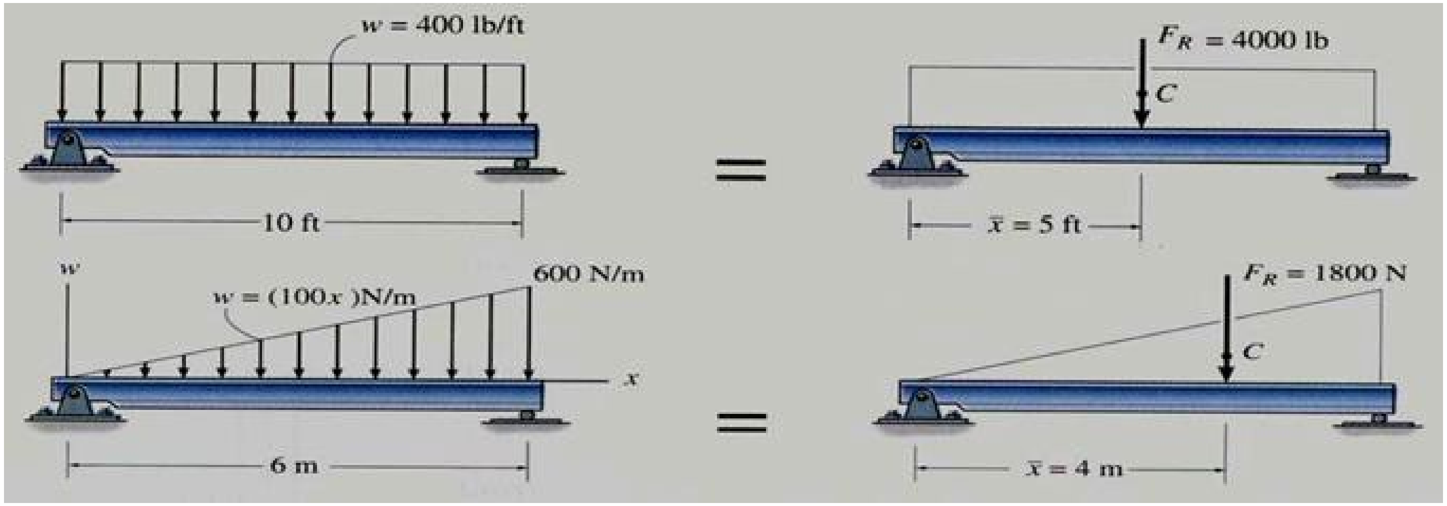
$$\bar{x} = \frac{\int_A xdA}{\int_A dA}$$

The total resultant force F_R is the <u>area under the curve</u> $W(X)$

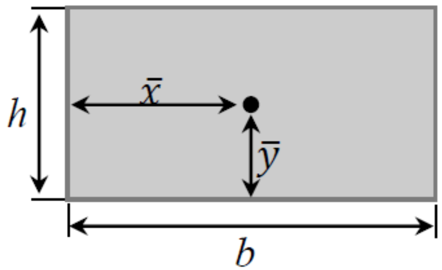
The coordinate $\bar{x}$ is known as the <u>centroid</u> of the area under the curve $W(X)$

Distributed loads

A few notable cases such as those shown below

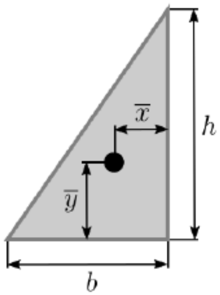


Generally speaking, the resultant force will be equal to the area under the load curve. This force will be applied at the centroid. We have not defined the centroid for the general case, but you can find it tabulated for common shapes



$$\bar{x} = \frac{b}{2}$$

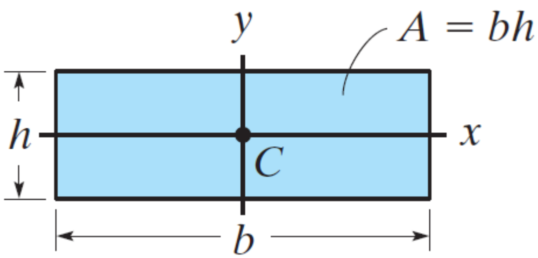
$$\bar{y} = \frac{h}{2}$$



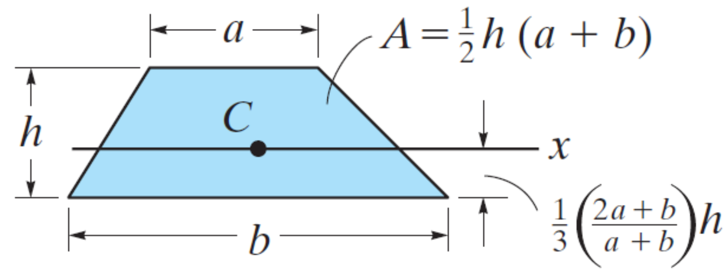
$$\bar{x} = \frac{b}{3}$$

$$\bar{y} = \frac{h}{3}$$

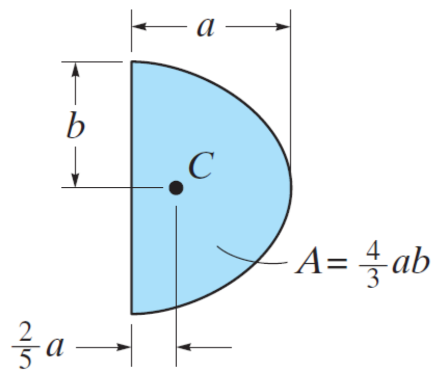
Some centroid locations



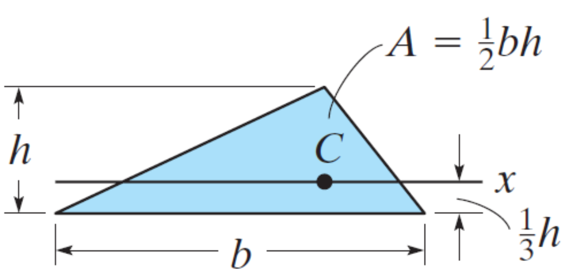
Rectangular area



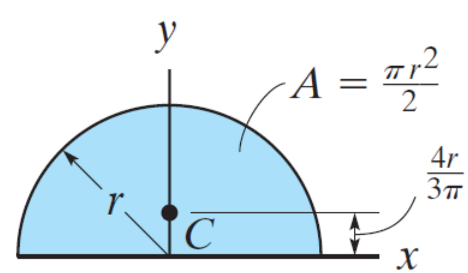
Trapezoidal area



Parabolic area

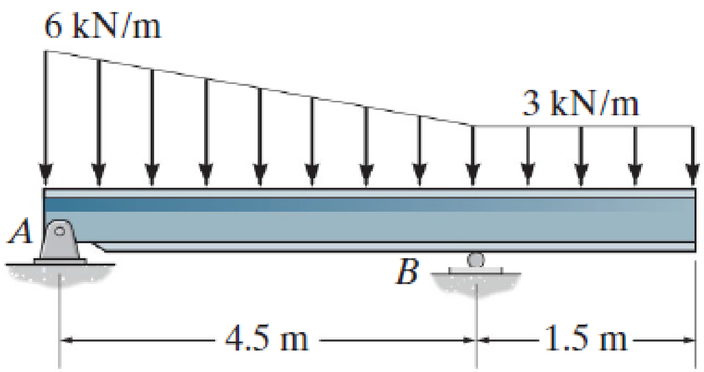


Triangular area



Semicircular area

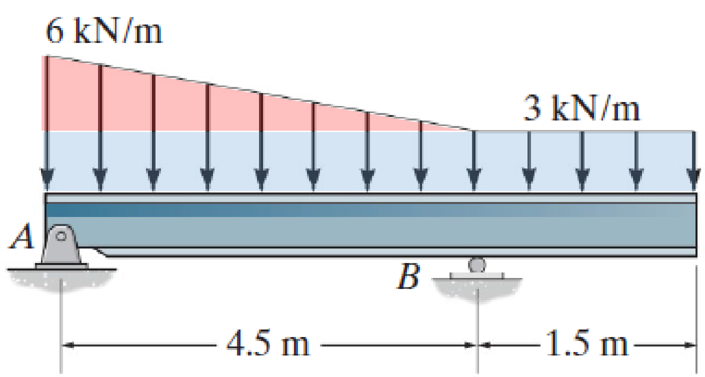
Example



Determine the resultant force and specify where it acts on the beam measured from *A*.

We can always use integration, but it possible we can take advantage of already-known figures

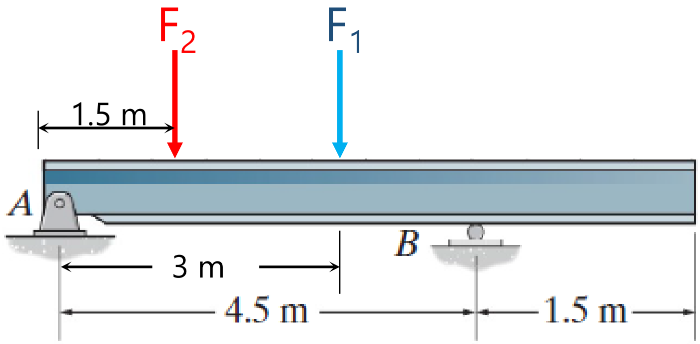
The load shown can be divided into a rectangle and a triangle. Their resultants then can be added



The rectangular blue area gives $F_1 = -(3\ \mathrm{kN/m})(6\ \mathrm{m}) = -18\ \mathrm{kN}$

The triangular pink area gives $F_2 = -\frac{1}{2}\left(3\,\frac{\mathrm{kN}}{\mathrm{m}}\right)(4.5\ \mathrm{m})$

$= -6.75\ \mathrm{kN}$

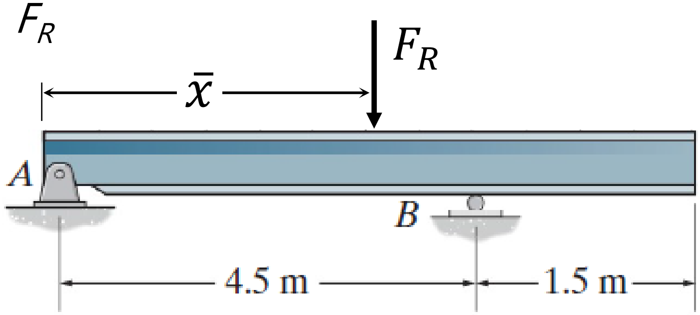


The lines of action of these parallel forces act through the respective *centroids*

The rectangular blue area centroid $\overline{x_1} = \frac{1}{2}(6\text{ m}) = 3\text{ m}$

The triangular pink area centroid $\overline{x_2} = \frac{1}{3}(4.5\text{ m}) = 1.5\text{ m}$

We want a single force resultant F_R



The two cases must be <u>mechanically equivalent</u>

$$F_R = F_1 + F_2$$

$$M_{R,A} = -\bar{x}F_R = -3F_1 - 1.5F_2$$

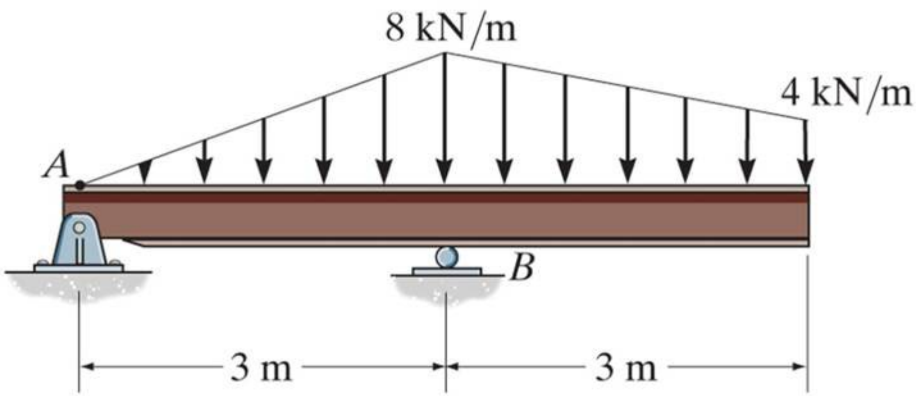
$$F_R = (-18 - 6.75)\text{ kN}$$

$$\bar{x}(-24.75) = -3(18) - 1.5(6.75)$$

$$F_R = -24.75\text{ kN}$$

$$\bar{x} = 2.59\text{ m}$$

Individual work (15 min)



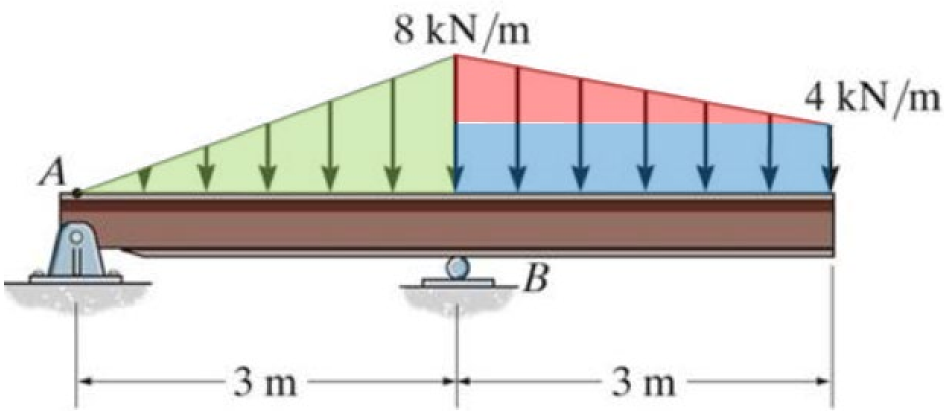
Replace the loading system by an equivalent resultant force and specify where the resultant's line of action intersects the member *AB* measured from *A*.

Solution:

1) The distributed loading can be divided into three parts. (one rectangular loading and two triangular loadings).

2) Find F_R and its location for each of these three distributed loads.

3) Determine the overall F_R of the three points loadings and its location.



For the left triangular loading, of height 8 kN/m and width 3 m.

F_{R1} = -(0.5) (8 kN/m) (3 m) = -12 kN

x_1 = (2/3)(3m) = 2 m from A

For the top right triangular loading of height 4 kN/m and width 3 m.

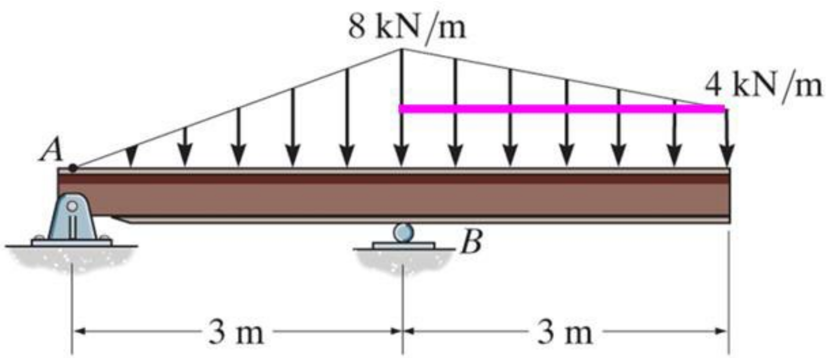
F_{R2} = -(0.5) (4 kN/m) (3 m) = -6 kN

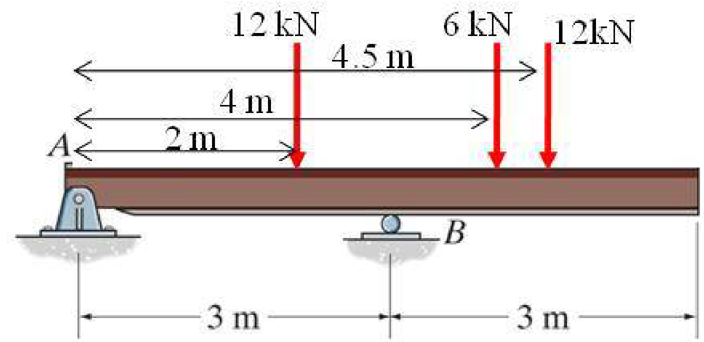
x_2= 3 + (1/3)(3m) = 4 m from A

For the rectangular loading of height 4 kN/m and width 3 m:

F_{R3} = -(4 kN/m) (3 m) = -12 kN

x_3 = 3 + (1/2)(3m) = 4.5 m from A





For the combined loading of the three forces:

F_R = -12 kN - 6 kN - 12 kN = -30 kN

$+\circlearrowleft M_{RA}$ = -(2)(12) - (4)(6) - (4.5)(12) = -102 kN • m

Now, $-\bar{x}F_R$ = -102 kN • m

Hence, $\bar{x}$ = (-102 kN • m) / (-30 kN) = 3.4 m from A.