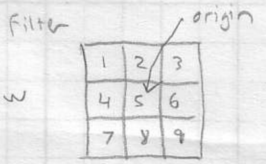
Spatial filtering example:

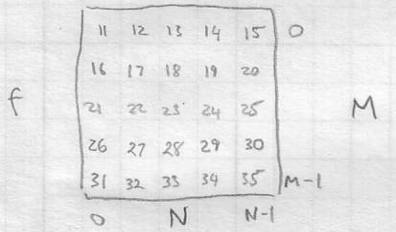
Filter



1	2	3
4	5	6
7	8	9

$w(-1,-1) = 1$
$w(-1,0) = 2$
$w(-1,1) = 3$
$w(0,-1) = 4$
$w(0,0) = 5$
$w(0,1) = 6$
$w(1,-1) = 7$
$w(1,0) = 8$
$w(1,1) = 9$

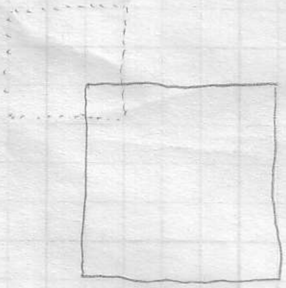
Image



11	12	13	14	15
16	17	18	19	20
21	22	23	24	25
26	27	28	29	30
31	32	33	34	35

Apply spatial filter w to image f to get filtered image g.

$$g(x,y) = \sum_{s=-1}^{1}\sum_{t=-1}^{1} w(s,t)\, f(x+s, y+t)$$



Need to extend image by 2 pixels along each edge.
→ Zero-pad

②

0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	11	12	13	14	15	6	0
0	0	16					0	0
0	0	⋮					0	0
0	0						0	0
0	0	31				35	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

$f(-2, *) = 0$
$f(-1, *) = 0$
$f((M-1)+1, *) = 0$ or $f(5, *) = 0$
$f((M-1)+2, *) = 0$ or $f(6, *) = 0$

$f(*, -2) = 0$
$f(*, -1) = 0$
$f(*, (N-1)+1) = 0$ or $f(*, 5) = 0$
$f(*, (N-1)+2) = 0$ or $f(*, 6) = 0$

First overlap when $x = -1$, $y = -1$

0	0	0	0	0
0	0	0	0	0
0	0	11	12	13
0	0	16	17	
0	0			

$$g(-1,-1) = \sum_{s=-1}^{1} \sum_{t=1}^{1} w(s,t)\, f(-1+s, -1+t)$$

$$= w(-1,-1) f(-2,-2) + w(-1,0) f(-2,-1) + w(-1,1) f(-2,0) + w(0,-1) f(-1,-2) + w(0,0) f(-1,-1) + w(0,1) f(-1,0) + w(1,-1) f(0,-2) + w(1,0) f(0,-1) + \underbrace{w(1,1) f(0,0)}_{(9)(11)}$$

$$= 99$$

all f's are zero here except f(0,0)

"shift w over by one pixel and repeat"

88
108
196

0	0	0	0	0
0	0	0	0	0
0	0	11	12	13
0	0	16	17	
0	0			

$$g(-1,0) = w(1,0) f(0,0) + w(1,1) f(0,1)$$
$$(8)(11) + (9)(12)$$
$$= 196$$

continue shifting to right by one pixel

0	0	0	0	0
0	0	0	0	0
	14	15	0	0
	19	20	0	0

$$g(-1,5) = w(1,-1) f(0,4)$$
$$(7)(15)$$
$$= 105$$

Now shift w down one pixel and start from left

$$\begin{array}{|cccccc} \hline 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 11 & 12 & 13. & \\ 0 & 0 & 16 & 17 & & \\ 0 & 0 & & & & \\ 0 & 0 & & & & \end{array}$$

$$g(0,-1) = w(0,1)\,f(0,0) + w(1,1)\,f(1,0)$$
$$(6)(11) + (9)(16)$$
$$= 210$$

Continue until get to $g(5,-1)$ through $g(5,5)$

$$g(x,y) = \begin{array}{|ccccccc|c} -1 & & & & & & M & \\ \hline 99 & 196 & 290 & 314 & 338 & 218 & 105 & -1 \\ 218 & 408 & 592 & 631 & 670 & 424 & 105 & \\ 318 & 606 & 861 & 906 & 951 & 588 & 270 & \\ & & & & & & & \\ & & & & & & & \\ 93 & & & & & & 35 & M \\ \hline \end{array}$$

Crop to same size as original

$$g(x,y) = \begin{array}{|ccccc|} \hline 408 & 592 & 631 & 670 & 424 \\ 606 & & & & \\ & & & & \\ & & & & \\ & & & & \\ \hline \end{array}$$