Discussion Section: Week #11
Due: By 11:59pm the day of your Discussion Section

Instructions:

You are allowed to work alone or in small groups. But if you work in a group, the work you turn in should be your own. Each student will submit a physical write-up as a PDF document to CatCourses **by 11:59 pm of your discussion section day**.

Discussion assignments will be graded partially for completeness but a subset of the problems will be graded for correctness. Providing the correct answer, without justification, is not considered complete. For credit you **must** either show steps (if it's a calculation problem) or explain/justify your reasoning (if it's a short answer problem).

Problem Set:

1. Perform the Gran-Schmidt algorithm on the following set of vectors.

$$\begin{bmatrix}0\\0\\1\\1\end{bmatrix} \begin{bmatrix}0\\1\\1\\0\end{bmatrix} \begin{bmatrix}1\\1\\0\\0\end{bmatrix}$$

Solution: To begin the Gran-Schmidt algorithm, we note that all the given vectors are linearly independent, hence we can expect the algorithm to work.
First we normalize the first vector:
$\|v_1\| = \sqrt{0^2+0^2+1^2+1^2} = \sqrt{2}$. This then results in our first vector as $u_1 = \begin{bmatrix}0\\0\\1/\sqrt{2}\\1/\sqrt{2}\end{bmatrix}$.
Next we find u_2 by the formula: $u_2 = v_2 - (v_2 \cdot u_1)u_1$. This results in:

$$\begin{bmatrix}0\\1\\1\\0\end{bmatrix} - \left(\begin{bmatrix}0\\0\\1/\sqrt{2}\\1/\sqrt{2}\end{bmatrix} \cdot \begin{bmatrix}0\\1\\1\\0\end{bmatrix}\right)\begin{bmatrix}0\\0\\1/\sqrt{2}\\1/\sqrt{2}\end{bmatrix}$$

This results in $u_2 = \begin{bmatrix}0\\1\\1/2\\-1/2\end{bmatrix}$. Now, we must normalize this as a apart of the algorithm, hence we get: $u_2 = \sqrt{\frac{2}{3}}\begin{bmatrix}0\\1\\1/2\\-1/2\end{bmatrix}$. Finally, to get u_3 we perform a similar

operation of, $u_3 = v_3 - (v_3 \cdot u_1)u_1 - (v_3 \cdot u_2)u_2$ followed by normalization, which yields $u_3 = \frac{1}{2\sqrt{3}} \begin{bmatrix} 3 \\ 1 \\ -1 \\ 1 \end{bmatrix}$.
One can check that all u_i are orthogonal and have magnitude 1.

2. Find the QR decomposition of

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

Solution: We will perform the same operations as before, as well as keeping track of what dot products we perform in order to formulate the R matrix. To begin the Gran-Schmidt, we normalize v_1 to get $q_1 = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{bmatrix}$. Next we find q_2 as before, $q_2 = v_2 - (v_2 \cdot q_1)q_1$, which yields after normalization, $q_2 = \begin{bmatrix} 1/\sqrt{6} \\ -1/\sqrt{6} \\ 2/\sqrt{6} \end{bmatrix}$.
Finally, we find $q_3 = v_3 - (v_3 \cdot q_1)q_1 - (v_3 \cdot q_2)q_2$. After normalization, this yields $q_3 = \begin{bmatrix} -1/\sqrt{3} \\ 1/\sqrt{3} \\ 1\sqrt{3} \end{bmatrix}$.
As we can tell by the use of notation, the Q matrix can be formulated by the result of our Gran-Schmidt procedure, i.e., $Q = [q_1|q_2|q_3]$. To formulate the R matrix, we can use our previous work to form

$$R = \begin{bmatrix} (v_1 \cdot q_1) & (v_2 \cdot q_1) & (v_3 \cdot q_1) \\ 0 & (v_2 \cdot q_2) & (v_3 \cdot q_2) \\ 0 & 0 & (v_3 \cdot q_3) \end{bmatrix}$$

, which yields

$$R = \begin{bmatrix} 2/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 3/\sqrt{6} & 1/\sqrt{6} \\ 0 & 0 & 2/\sqrt{3} \end{bmatrix}.$$