
# ENGR 057 Statics and Dynamics

*Moments*

**Instructor**
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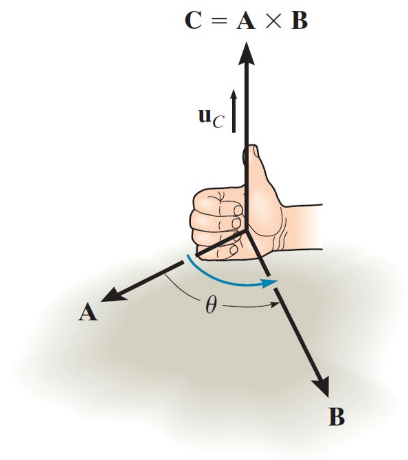
MERCED

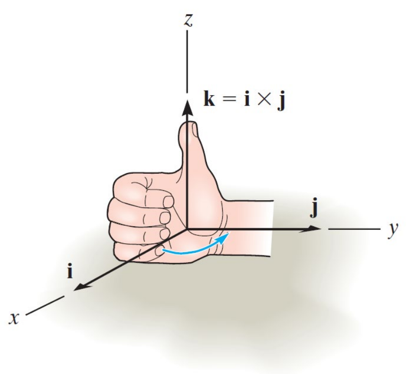


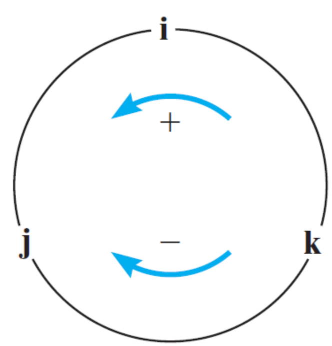

$$\boldsymbol{C} = \boldsymbol{A} \times \boldsymbol{B} = AB \sin\theta$$

# Vector (cross) product

- The vector (cross) product of vectors ***A***, ***B*** is another vector ***C*** perpendicular to both $\boldsymbol{C} = \boldsymbol{A} \times \boldsymbol{B}$
- In Cartesian space, the vector product represents the normal to the plane created by ***A***, ***B***.
- The vector product follows the so-called right-hand rule.
- the vector product of two parallel vectors is zero.

The right-hand rule is useful to determine the direction resulting from a vector product.




$$\hat{\boldsymbol{\imath}} \times \hat{\boldsymbol{\imath}} = \hat{\boldsymbol{\jmath}} \times \hat{\boldsymbol{\jmath}} = \widehat{\boldsymbol{k}} \times \widehat{\boldsymbol{k}} = \boldsymbol{0}$$

$$\hat{\boldsymbol{\imath}} \times \hat{\boldsymbol{\jmath}} = \widehat{\boldsymbol{k}} \qquad \hat{\boldsymbol{\jmath}} \times \hat{\boldsymbol{\imath}} = -\widehat{\boldsymbol{k}}$$

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# Vector (cross) product

The vector product can also be written as a determinant

$$\boldsymbol{A} \times \boldsymbol{B} = \begin{vmatrix} \hat{\boldsymbol{i}} & \hat{\boldsymbol{j}} & \hat{\boldsymbol{k}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Each component can be calculated in the following way

For element $\hat{\boldsymbol{i}}$

$$\begin{vmatrix} \hat{\boldsymbol{i}} & \hat{\boldsymbol{j}} & \hat{\boldsymbol{k}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \hat{\boldsymbol{i}}(A_y B_z - A_z B_y)$$

For element $\hat{\boldsymbol{j}}$

$$\begin{vmatrix} \hat{\boldsymbol{i}} & \hat{\boldsymbol{j}} & \hat{\boldsymbol{k}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = -\hat{\boldsymbol{j}}(A_x B_z - A_z B_x)$$

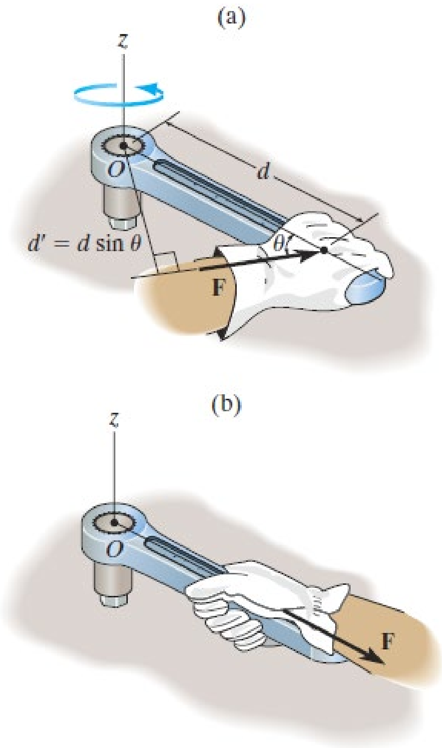
For element $\hat{\boldsymbol{k}}$

$$\begin{vmatrix} \hat{\boldsymbol{i}} & \hat{\boldsymbol{j}} & \hat{\boldsymbol{k}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \hat{\boldsymbol{k}}(A_x B_y - A_y B_x)$$

Remember the scalar and vector products are entirely different operations

The resulting units will be the product of the original two units

The resultant equation is also called the triple scalar product.

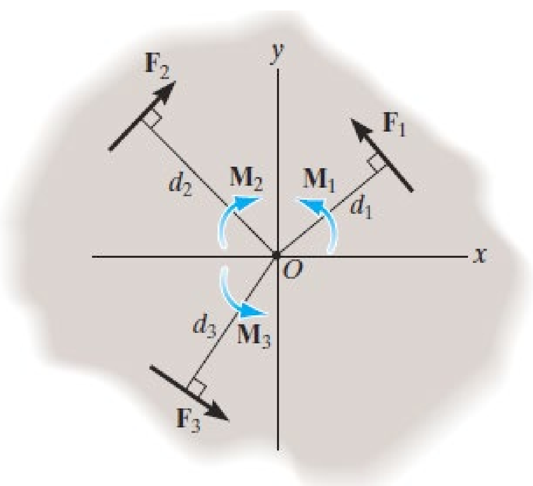



# Moment

If we apply a force at a certain distance from a fixed point, we will generate a moment, which creates rotation around that point

The moment of a force can be determined using the vector product

$$\mathbf{M_0} = \mathbf{r} \times \mathbf{F} = (rF\sin\theta)\mathbf{u_o} = (Fd)\mathbf{u_o}$$



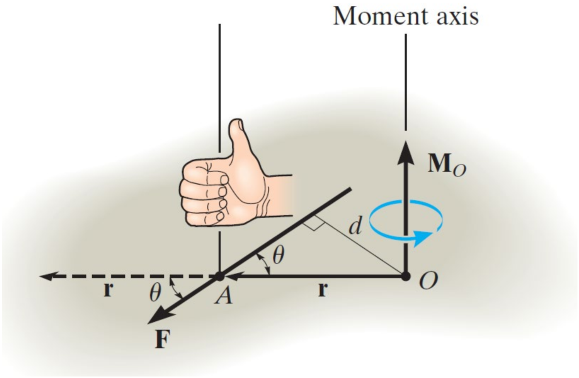

# Resultant Moment

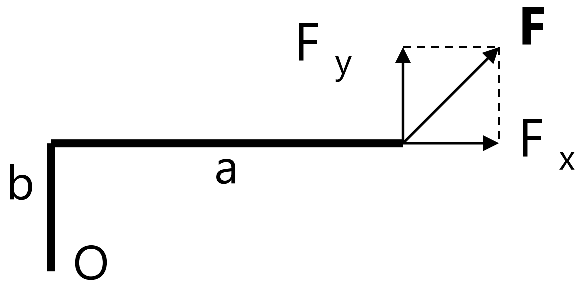
$\circlearrowleft + (M_R)_O = Fd;$ $\qquad (M_R)_O = F_1d_1 - F_2d_2 + F_3d_3$

If the numerical result is a <u>positive</u> scalar, the moment is <u>counterclockwise</u> (out of the page). If it is <u>negative</u>, then the moment is <u>clockwise</u>.

# Moments in 2D

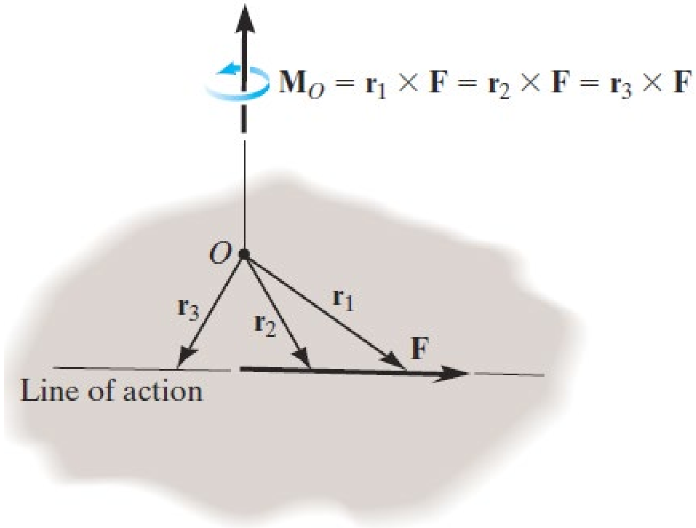
In planar mechanics, the calculation of the moment is trivial!




Often it is easier to determine $\mathbf{M_o}$ by using the components of **F** as shown.

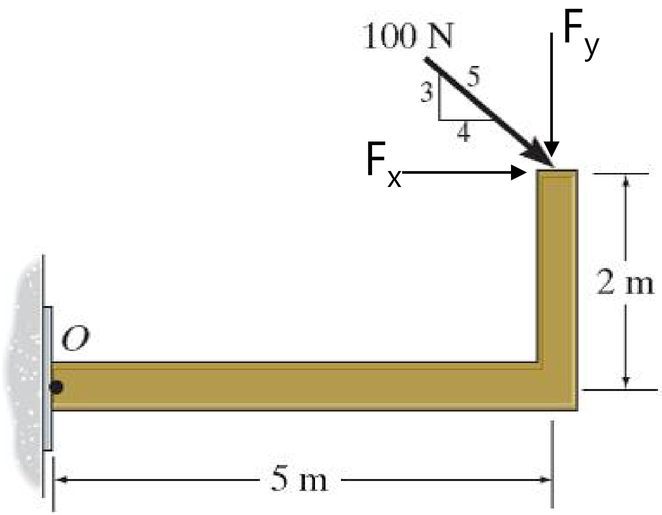
Magnitude $\mathrm{M}_0 = (rF \sin\theta)$




**Principle of Transmissibility.** we can use any position vector **r** measured from point *O* to any point on the line of action of the force **F**.

**F** can be considered a sliding vector.

# Example




**Given:** A 100 N force is applied to the frame.

**Find:** The moment of the force at point *O*

**Solution**

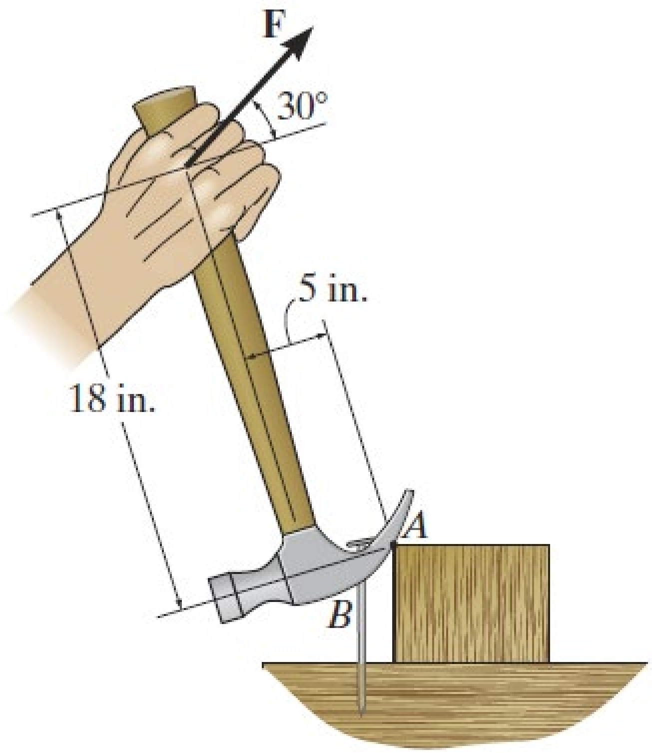
- Resolve the 100 N force along the x and y axes.
- Determine $M_O$ using a scalar analysis for the two force components and add those two moments together.

$+\uparrow F_y = -100\ (3/5)\ \text{N}$

$+\rightarrow F_x = 100\ (4/5)\ \text{N}$

$+\circlearrowleft M_O = \{-100\ (3/5)\text{N}\ (5\ \text{m}) - (100)(4/5)\text{N}\ (2\ \text{m})\}\ \text{N·m} = -460\ \text{N·m}$

# Individual work (10 min)

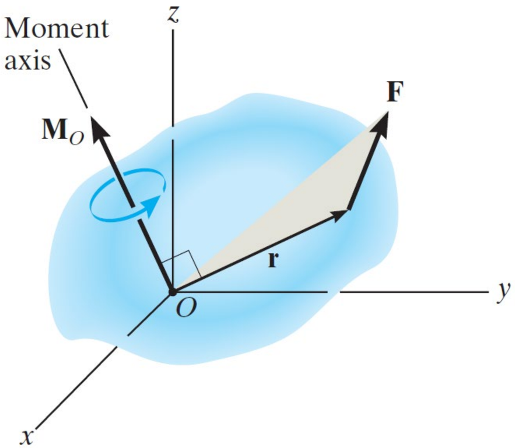



In order to pull out the nail at *B*, the force **F** exerted on the handle of the hammer must produce a clockwise moment of 500 lb.in about point *A*. Determine the required magnitude of force **F**.

**Solution**:
Given: $M_A$ = 500 lb.in, coordinates, angle
Asked: **F**

# Moments in 3D

In three dimensions:

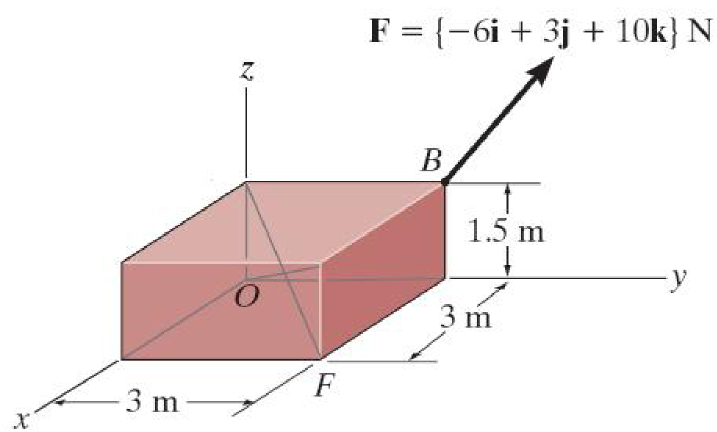



$$\boldsymbol{M_0} = \boldsymbol{r} \times \boldsymbol{F} = \begin{vmatrix} \hat{\boldsymbol{\imath}} & \hat{\boldsymbol{\jmath}} & \hat{\boldsymbol{k}} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

$$\boldsymbol{M_0} = \hat{\boldsymbol{\imath}}\left(r_y F_z - r_z F_y\right) - \hat{\boldsymbol{\jmath}}(r_x F_z - r_z F_x) + \hat{\boldsymbol{k}}\left(r_x F_y - r_y F_x\right)$$

Note this is **not** commutative!

# Example




A force **F** acts along the diagonal of the parallelepiped. Determine the moment of **F** about the point *O*

**Solution**:
Given: **F,** coordinates
Asked: $\mathbf{M}_O$

First, find the position vector $\mathbf{r}_{OB}$

$$\mathbf{r}_{OB} = \{0\,\mathbf{i} + 3\,\mathbf{j} + 1.5\,\mathbf{k}\}\text{ m}$$

Then find the moment by using the cross product.

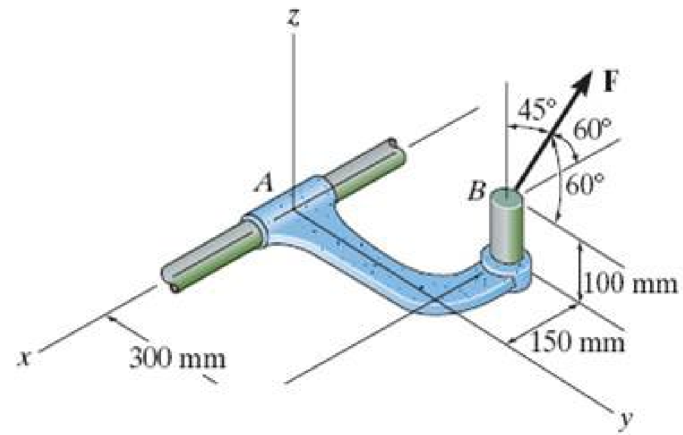
$$\mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 3 & 1.5 \\ -6 & 3 & 10 \end{vmatrix} = [\{3(10) - 1.5(3)\}\mathbf{i} - \{0(10) - 1.5(-6)\}\mathbf{j} + \{0(3) - 3(-6)\}\mathbf{k}]\text{ N}\cdot\text{m}$$

$$= \{25.5\,\mathbf{i} - 9\,\mathbf{j} + 18\,\mathbf{k}\}\text{ N}\cdot\text{m}$$

## Individual work (15 min)

Sleeve A can provide a maximum resisting moment of 125 N·m about the x-axis.

**Find**: The maximum magnitude of F before slipping occurs at A (the sleeve rotating around the x-axis).



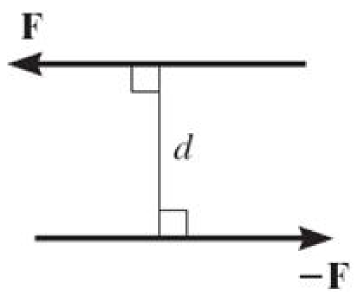

## Solution

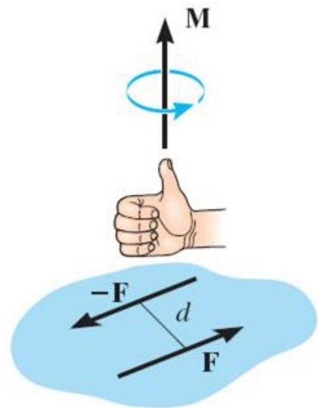
1) We need to use $\mathbf{M_X} = (\mathbf{r_{AB}} \times \mathbf{F})$
2) Find $\mathbf{r_{AB}}$

3) Find **F** in Cartesian vector form.

4) Complete the triple scalar product & solve for **F**

# Moment of a couple




A couple is defined as two parallel forces with the same magnitude but opposite in direction separated by a perpendicular distance d.

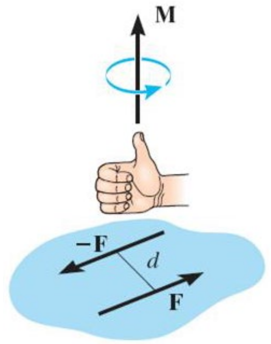
The moment of a couple is defined as

$M_O = F\ d$ (using a scalar analysis) or as

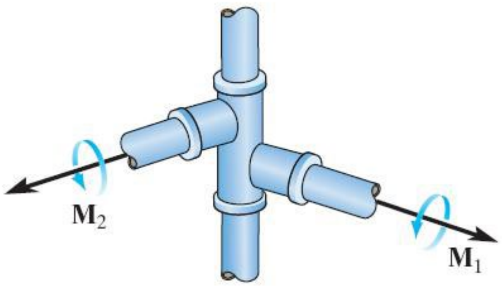
$\mathbf{M_O} = \mathbf{r} \times \mathbf{F}$ (using a vector analysis).

Here **r** is any position vector from the line of action of **–F** to the line of action of **F**.

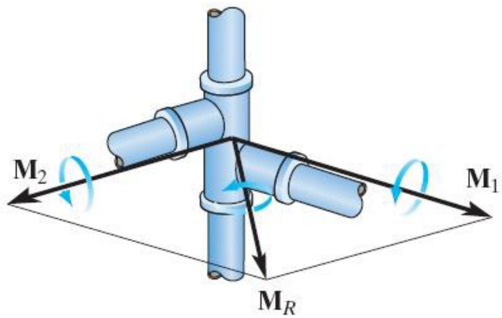
# Moment of a couple




The net external effect of a couple is that the net force equals zero and the magnitude of the net moment equals F *d.

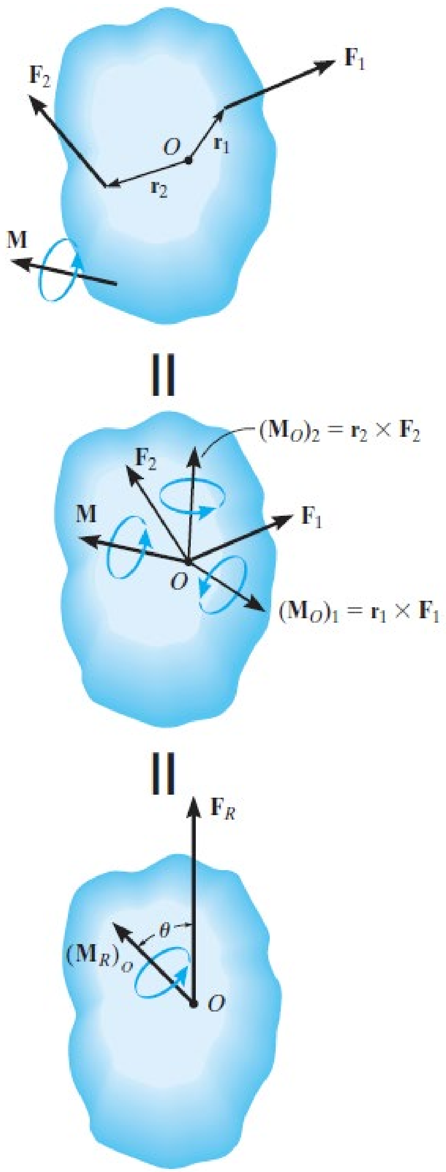



Since the moment of a couple depends only on the distance between the forces, the moment of a couple is a free vector. It can be moved anywhere on the body and have the same external effect on the body.




Moments due to couples can be added together using the same rules as adding any vectors.

# Force-moment resultant




When several forces and couple moments act on a body, you can move each force and its associated couple moment to a common point O.

Now you can add all the forces and couple moments together and find one resultant force-couple moment pair.

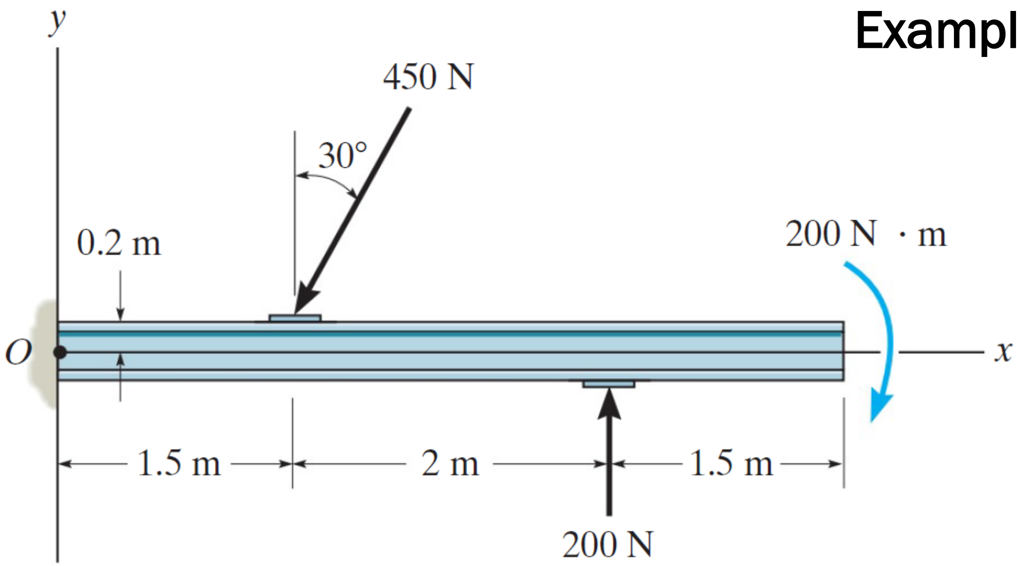
$$\boldsymbol{F}_R = \sum \boldsymbol{F} \qquad \boldsymbol{M}_{R,O} = \sum \boldsymbol{M}_C + \sum \boldsymbol{M}_0$$

This can be very useful to evaluate the performance of a mechanical element


How many traffic signals can we hang from this pole?

Does it make a difference where we hang them?

If you had to reinforce a section of the pole, which would it be? Why?



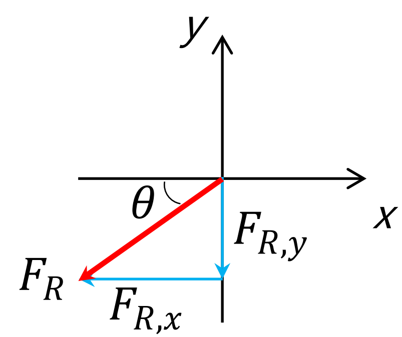

# Example

Determine the force-moment resultant at $O$ to verify this beam will function safely

We already have a FBD. We can proceed to calculate the resultants

$$F_{R,x} = \sum F_x = -450\sin(30°) = -225\text{ N}$$

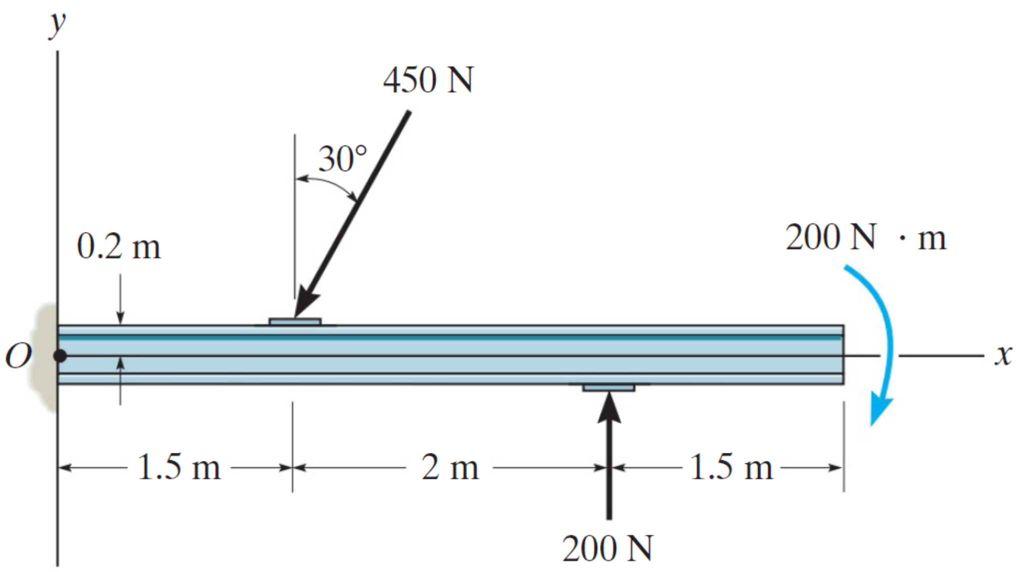
$$F_{R,y} = \sum F_y = -450\cos(30°) + 200 = -189.7\text{ N}$$




The magnitude of the resultant force is

$$F_R = \sqrt{(-225)^2 + (-189.7)^2} = 294\text{ N}$$

$$\theta = \tan^{-1}\left(\frac{189.7}{225}\right) = 40.1°$$




We can find the resultant moment $\boldsymbol{M}_{R,O}$ by adding all moments around $O$

$$M_{R,O} = \sum M_O = -450\cos(30°)\,(1.5) + 450\sin(30°)\,(0.2) + 200(3.5) - 200$$

$$M_{R,O} = 39.6\ \mathrm{N\cdot m}$$