



ENGR 057 Statics and Dynamics

Kinetics of a particle: Work, energy, impulse, impact

Instructor

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Work and energy

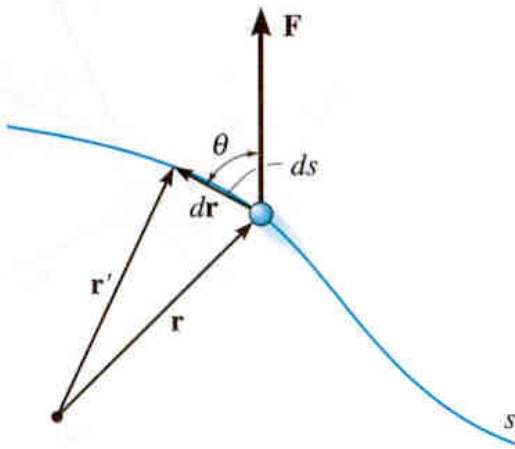


Another equation for working kinetics problems involving particles can be derived by integrating the equation of motion ($F = ma$) with respect to displacement.

By substituting $a_t = v (dv/ds)$ into $F_t = ma_t$, the result is integrated to yield an equation known as the principle of work and energy.

This principle is useful for solving problems that involve force, velocity, and displacement. It can also be used to explore the concept of power.

Work of a force



A force does work on a particle when the particle undergoes a displacement along the line of action of the force.

Work is defined as the product of force and displacement components acting in the same direction. So, if the angle between the force and displacement vector is θ , the increment of work dU done by the force is

$$dU = F ds \cos \theta$$

By using the definition of the dot product and integrating, the total work can be written as

$$U_{1-2} = \int_{r_1}^{r_2} \mathbf{F} \cdot d\mathbf{r}$$

If F is a function of position, this becomes

$$U_{1-2} = \int_{s_1}^{s_2} F \cos \theta ds$$

If both F and θ are constant ($F = F_c$), this equation simplifies to $U_{1-2} = F_c \cos \theta (s_2 - s_1)$

Work is positive if the force and the movement are in the same direction. If they are opposing, then the work is negative. If the force and the displacement directions are perpendicular, the work is zero.

Special cases

The work done by the gravitational force acting on a particle (or weight of an object) can be calculated by using

$$U_{1-2} = \int_{y_1}^{y_2} -W \, dy = -W (y_2 - y_1)$$

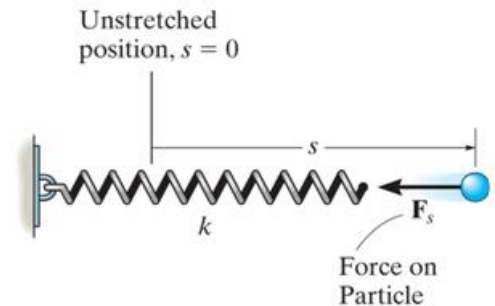
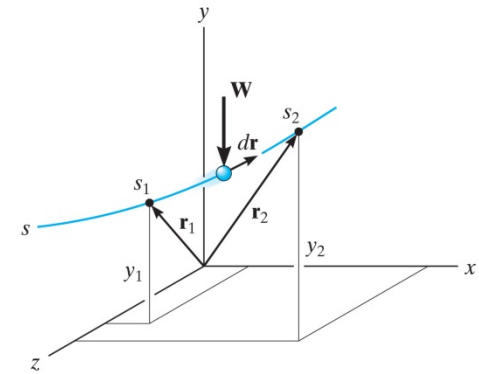
If the displacement is upward, the work is negative since the weight force always acts downward.

When stretched, a linear elastic spring develops a force of magnitude $F_s = ks$, where k is the spring stiffness and s is the displacement from the unstretched position.

The work of the spring force moving from position s_1 to position s_2 is

$$U_{1-2} = \int_{s_1}^{s_2} F_s \, ds = \int_{s_1}^{s_2} k s \, ds = 0.5 k (s_2)^2 - 0.5 k (s_1)^2$$

If a particle is attached to the spring, the force F_s exerted on the particle is opposite to that exerted on the spring. Thus, the work done on the particle by the spring force will be negative



Principle of work and energy

By integrating the equation of motion, $\sum F_t = ma_t = mv(dv/ds)$, the principle of work and energy can be written as

$$\sum U_{1-2} = 0.5 m (v_2)^2 - 0.5 m (v_1)^2 \quad \text{or} \quad T_1 + \sum U_{1-2} = T_2$$

$\sum U_{1-2}$ is the work done by all the forces acting on the particle as it moves from point 1 to point 2. Work can be either a positive or negative scalar.

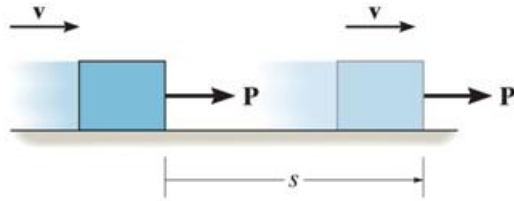
T_1 and T_2 are the kinetic energies of the particle at the initial and final position, respectively. Thus, $T_1 = 0.5 m (v_1)^2$ and $T_2 = 0.5 m (v_2)^2$. The kinetic energy is always a positive scalar.

The particle's initial kinetic energy plus the work done by all the forces acting on the particle as it moves from its initial to final position is equal to the particle's final kinetic energy.

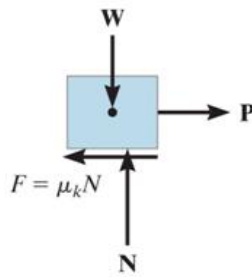
Both kinetic energy and work have the same units, that of energy! In the SI system, the unit for energy is called a joule (J), where $1 \text{ J} = 1 \text{ N}\cdot\text{m}$. In the FPS system, units are ft·lb.

The principle of work and energy can also be applied to a system of particles by summing the kinetic energies of all particles in the system and the work due to all forces acting on the system.

Special case: work of friction caused by sliding



Consider a block which is moving over a rough surface. If the applied force P just balances the resultant frictional force $\mu_k N$, a constant velocity v would be maintained.

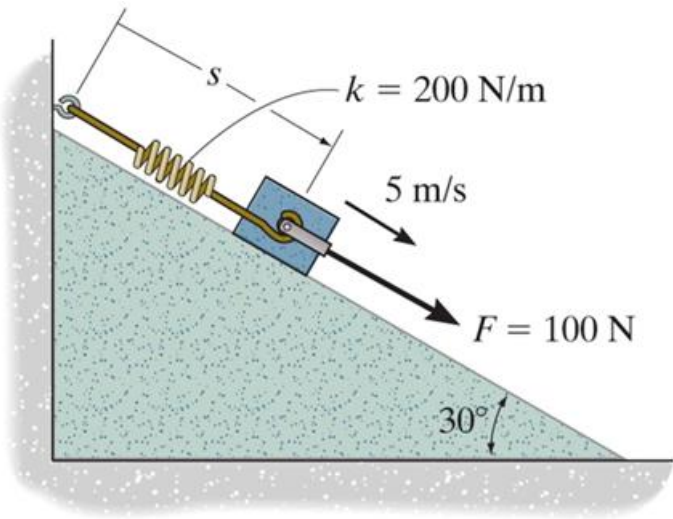


The principle of work and energy would be applied as

$$0.5m (v)^2 + P s - (\mu_k N) s = 0.5m (v)^2$$

This equation is satisfied if $P = \mu_k N$. However, we know from experience that friction generates heat, a form of energy that does not seem to be accounted for in this equation. It can be shown that the work term $(\mu_k N)s$ represents both the external work of the friction force and the internal work that is converted into heat.

Example



When $s = 0.6 \text{ m}$, the spring is not stretched or compressed, and the 10 kg block, which is subjected to a force of $F = 100 \text{ N}$, has a speed of 5 m/s down the smooth plane.

Find the distance s when the block stops.

Solution

Since this problem involves forces, velocity and displacement, apply the principle of work and energy to determine s .

Apply the principle of work and energy between position 1 ($s_1 = 0.6$ m) and position 2 (s_2). Note that the normal force (N) does no work since it is always perpendicular to the displacement.

$$T_1 + \sum U_{1-2} = T_2$$

There is work done by three different forces;

1) work of the force $F = 100$ N;

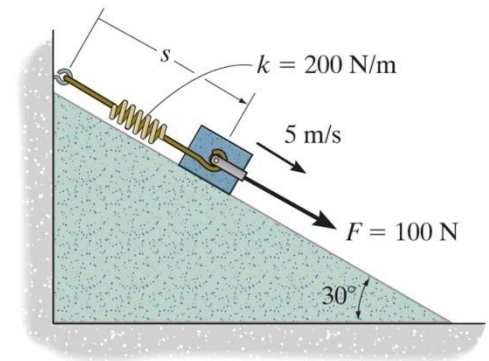
$$U_F = 100 (s_2 - s_1) = 100 (s_2 - 0)$$

2) work of the block weight;

$$U_W = 10 (9.81) (s_2 - s_1) \sin 30^\circ = 49.05 (s_2 - 0)$$

3) and work of the spring force.

$$U_S = -0.5 (200) (s_2^2 - 0^2) = -100 (s_2^2 - 0^2)$$



The work and energy equation will be

$$T_1 + \sum U_{1-2} = T_2$$

$$0.5 (10) 5^2 + 100(s_2) + 49.05(s_2) - 100(s_2^2) = 0$$

$$\Rightarrow 125 + 149.05s_2 - 100s_2^2 = 0$$

Solving for (s_2) ,

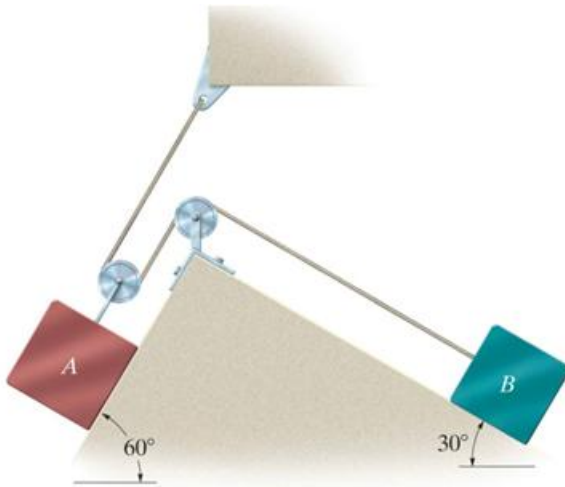
$$s_2 = \{-149.05 \pm (149.05^2 - 4 \times (-100) \times 125)^{0.5}\} / 2(-100)$$

Selecting the positive root, indicating a positive spring deflection,

$$s_2 = 2.08$$

$$S = 2.08 + 0.6 = 2.68 \text{ m}$$

Individual work (15 min)



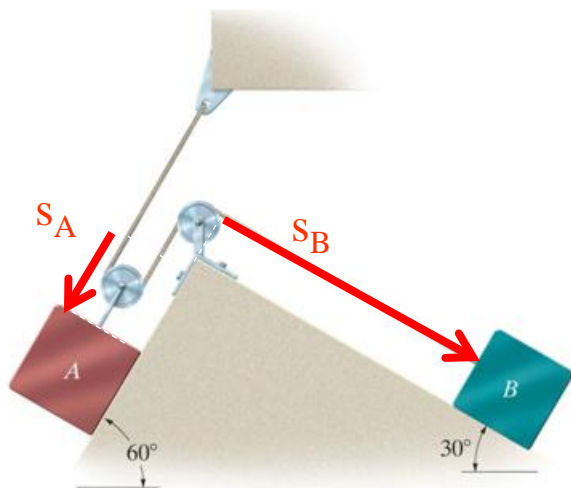
Block A has a weight of 60 lb and block B has a weight of 40 lb. The coefficient of kinetic friction between the blocks and the incline is $\mu_k = 0.1$. Neglect the mass of the cord and pulleys.

Find the speed of block A after block B moves 2 ft up the plane, starting from rest.

Solution

- 1) Define the kinematic relationships between the blocks.
- 2) Draw the FBD of each block.
- 3) Apply the principle of work and energy to the system of blocks. Why choose this method?

- 1) The kinematic relationships can be determined by defining position coordinates s_A and s_B , and then differentiating.



Since the cable length is constant:

$$2s_A + s_B = l$$

$$2\Delta s_A + \Delta s_B = 0$$

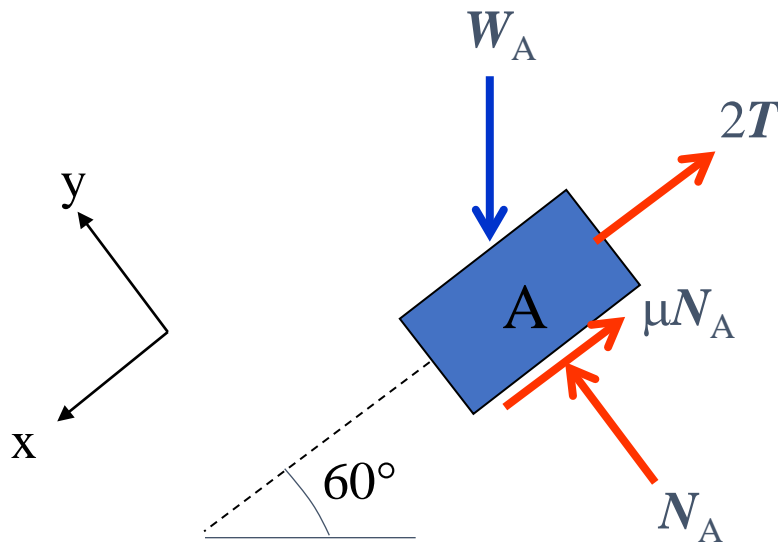
$$\text{When } \Delta s_B = -2 \text{ ft} \Rightarrow \Delta s_A = 1 \text{ ft}$$

$$\text{and } 2v_A + v_B = 0$$

$$\Rightarrow v_B = -2v_A$$

Note that, by this definition of s_A and s_B , positive motion for each block is defined as downwards.

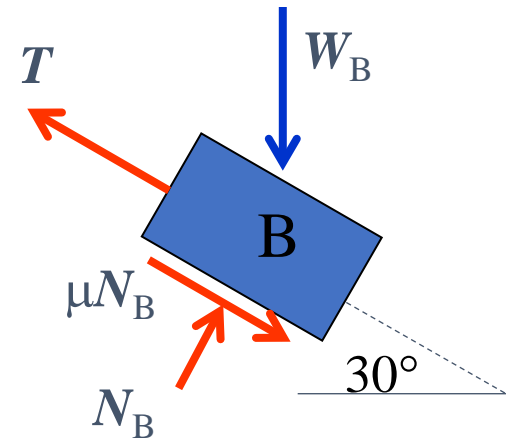
2) Draw the FBD of each block.



Sum forces in the y-direction for block A (note that there is no motion in y-direction):

$$\sum F_y = 0: N_A - W_A \cos 60^\circ = 0$$

$$N_A = W_A \cos 60^\circ$$



Similarly, for block B:

$$N_B = W_B \cos 30^\circ$$

- 3) Apply the principle of work and energy to the system (the blocks start from rest).

$$\Sigma T_1 + \Sigma U_{1-2} = \Sigma T_2$$

$$[0.5m_A(v_{A1})^2 + 0.5m_B(v_{B1})^2] + [W_A \sin 60^\circ - 2T - \mu N_A]\Delta s_A \\ + [W_B \sin 30^\circ - T + \mu N_B]\Delta s_B = [0.5m_A(v_{A2})^2 + 0.5m_B(v_{B2})^2]$$

where $v_{A1} = v_{B1} = 0$, $\Delta s_A = 1 \text{ ft}$, $\Delta s_B = -2 \text{ ft}$, $v_B = -2v_A$,
 $N_A = W_A \cos 60^\circ$, $N_B = W_B \cos 30^\circ$

$$\Rightarrow [0 + 0] + [60 \sin 60^\circ - 2T - 0.1(60 \cos 60^\circ)] (1) \\ + [40 \sin 30^\circ - T + 0.1(40 \cos 30^\circ)] (-2) \\ = [0.5(60/32.2)(v_{A2})^2 + 0.5(40/32.2)(-2v_{A2})^2]$$

Again, the Work and Energy equation is:

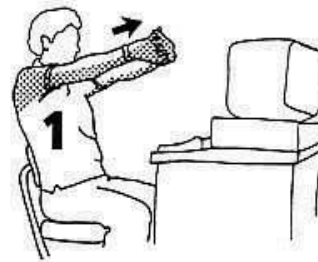
$$\begin{aligned} \Rightarrow & [0 + 0] + [60 \sin 60^\circ - 2T - 0.1(60 \cos 60^\circ)] (1) \\ & + [40 \sin 30^\circ - T + 0.1(40 \cos 30^\circ)] (-2) \\ & = [0.5(60/32.2)(v_{A2})^2 + 0.5(40/32.2)(-2v_{A2})^2] \end{aligned}$$

Solving for the unknown velocity yields

$$\Rightarrow v_{A2} = 0.771 \text{ ft/s}$$

Note that the work due to the cable tension force on each block cancels out.

Stretch break!



10–20 seconds
2 times



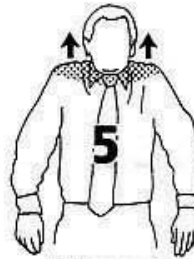
10–15 seconds



8–10 seconds
each side



15–20 seconds



3–5 seconds
3 times



10–12 seconds
each arm



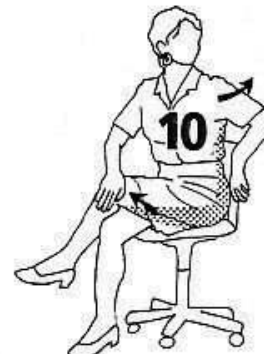
10 seconds



10 seconds



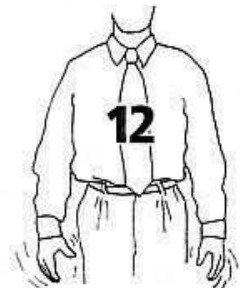
8–10 seconds
each side



8–10 seconds
each side



10–15 seconds
2 times



Shake out hands
8–10 seconds

Power and efficiency

Power is defined as the amount of work performed per unit of time.

If a machine or engine performs a certain amount of work, dU , within a given time interval, dt , the power generated can be calculated as

$$P = dU/dt$$

Since the work can be expressed as $dU = \mathbf{F} \cdot d\mathbf{r}$, the power can be written

$$P = dU/dt = (\mathbf{F} \cdot d\mathbf{r})/dt = \mathbf{F} \cdot (d\mathbf{r}/dt) = \mathbf{F} \cdot \mathbf{v}$$

Thus, power is a scalar defined as the product of the force and velocity components acting in the same direction.

Using scalar notation, power can be written

$$P = \mathbf{F} \cdot \mathbf{v} = F v \cos \theta$$

where θ is the angle between the force and velocity vectors.

The unit of power in the SI system is the Watt (W) where

$$1 \text{ W} = 1 \text{ J/s} = 1 (\text{N} \cdot \text{m})/\text{s} .$$

In the FPS system, power is usually expressed in units of horsepower (hp) where

$$1 \text{ hp} = 550 (\text{ft} \cdot \text{lb})/\text{s} = 746 \text{ W} .$$

The mechanical efficiency of a machine is the ratio of the useful power produced (output power) to the power supplied to the machine (input power) or

$$\varepsilon = (\text{power output}) / (\text{power input})$$

If energy input and removal occur at the same time, efficiency may also be expressed in terms of the ratio of output energy to input energy or

$$\varepsilon = (\text{energy output}) / (\text{energy input})$$

Procedure for analysis

- Find the **resultant external force** acting on the body causing its motion. It may be necessary to **draw a free-body diagram**.
 - Determine the **velocity** of the point on the body at which the force is applied. **Energy methods or the equation of motion** and appropriate **kinematic relations** may be necessary.
 - Multiply the **force magnitude** by the **component of velocity** acting in the **direction of F** to determine the power supplied to the body ($P = F v \cos \theta$).
 - **Power** may be found by calculating the **work done per unit of time** ($P = dU/dt$).
 - If the **mechanical efficiency** of a machine is known, either the **power input or output** can be determined.
-

Breakout room (15 min)

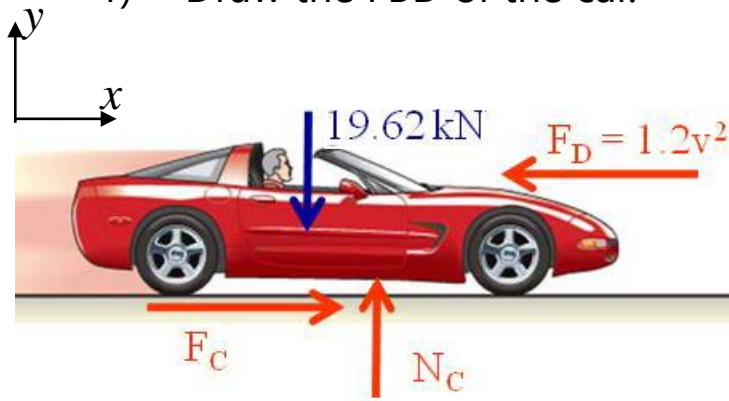


A sports car has a mass of 2000 kg and an engine efficiency of $\varepsilon = 0.65$. Moving forward, the wind creates a drag resistance on the car of $F_D = 1.2v^2$ N, where v is the velocity in m/s. The car accelerates at 5 m/s^2 , starting from rest.

Find the engine's input power when $t = 4 \text{ s}$.

- 1) Draw a free-body diagram of the car.
- 2) Apply the equation of motion and kinematic equations to find the car's velocity at $t = 4 \text{ s}$.
- 3) Determine the output power required for this motion.
- 4) Use the engine's efficiency to determine input power.

- 1) Draw the FBD of the car.



The drag force and weight are known forces. The normal force N_c and frictional force F_c represent the resultant forces of all four wheels. The frictional force between the wheels and road pushes the car forward.

- 2) The equation of motion can be applied in the x-direction, with $a_x = 5 \text{ m/s}^2$:

$$\rightarrow \sum F_x = ma_x \Rightarrow F_c - 1.2v^2 = (2000)(5) \Rightarrow F_c = (10,000 + 1.2v^2) \text{ N}$$

- 3) The constant acceleration equations can be used to determine the car's velocity.

$$v_x = v_{x0} + a_x t = 0 + (5)(4) = 20 \text{ m/s}$$

- 4) The power output of the car is calculated by multiplying the driving (frictional) force and the car's velocity:

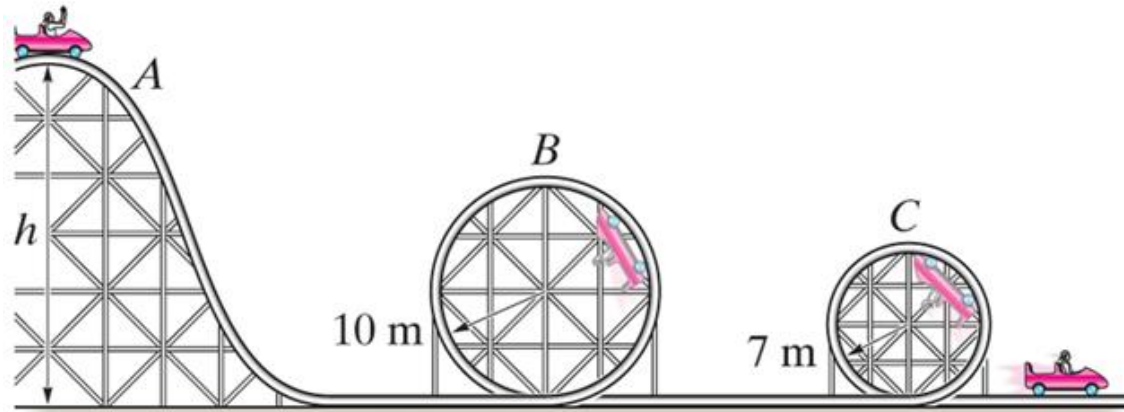
$$P_o = (F_c)(v_x) = [10,000 + (1.2)(20)^2](20) = 209.6 \text{ kW}$$

- 5) The power developed by the engine (prior to its frictional losses) is obtained using the efficiency equation.

$$P_i = P_o / e = 209.6 / 0.65 = 322 \text{ kW}$$



Conservation of forces



The roller coaster is released from rest at the top of the hill. As the coaster moves down the hill, potential energy is transformed into kinetic energy.

What is the velocity of the coaster when it is at B and C?

Also, how can we determine the minimum height of the hill so that the car travels around both inside loops without leaving the track?

Conservative force

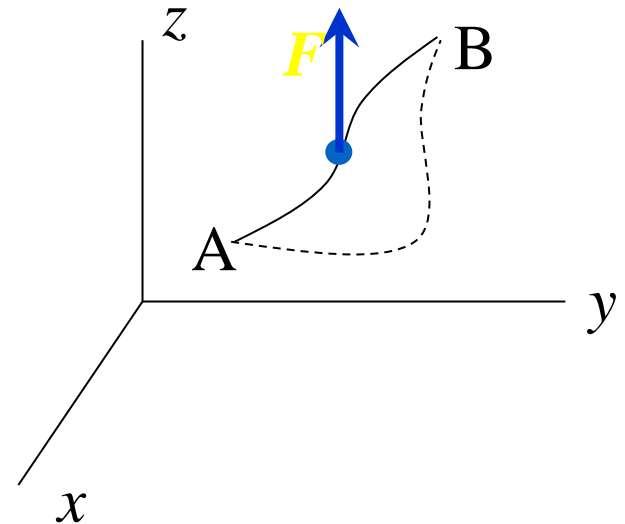
A force \mathbf{F} is conservative if the work done is independent of the path followed by the force acting on a particle as it moves from A to B . This also means that the work done by the force \mathbf{F} in a closed path (i.e., from A to B and then back to A) is zero.

The work done by a conservative force depends only on the positions of the particle, and is independent of its velocity or acceleration.

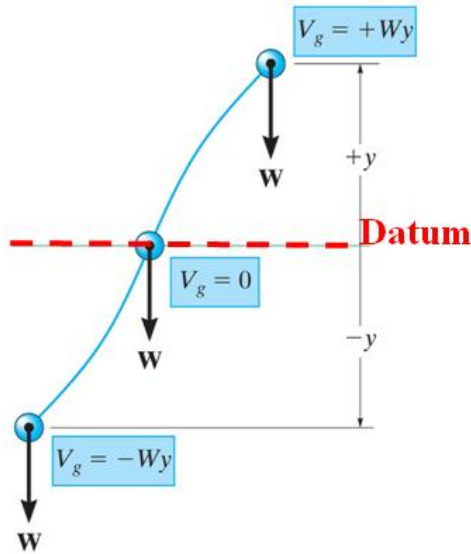
$$\oint \mathbf{F} \cdot d\mathbf{r} = 0$$

Potential energy is a measure of the amount of work a conservative force will do when a body changes position.

In general, for any conservative force system, we can define the potential function (V) as a function of position. The work done by conservative forces as the particle moves equals the change in the value of the potential function (e.g., the sum of V_{gravity} and V_{springs}).



Potential energy - Special cases



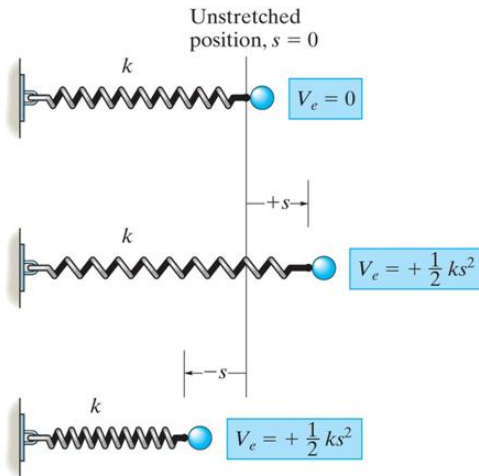
Gravitational potential energy

The potential function (formula) for a gravitational force, e.g., weight ($W = mg$), is the force multiplied by its elevation from a datum. The datum can be defined at any convenient location.

$$V_g = \pm W y$$

V_g is positive if y is above the datum and negative if y is below the datum. Remember, YOU get to set the datum.

The force of an elastic spring is $F = ks$.



Elastic potential energy

The potential energy of the spring is V_e (where 'e' denotes an elastic spring) has the distance "s" raised to a power (the result of an integration) or

$$V_e = \frac{1}{2} k s^2$$

Notice that the potential function V_e always yields positive energy.

Conservation of energy

When a particle is acted upon by a system of conservative forces, the work done by these forces is conserved and the sum of kinetic energy and potential energy remains constant. In other words, as the particle moves, kinetic energy is converted to potential energy and vice versa. This principle is called the principle of conservation of energy and is expressed as

$$T_1 + V_1 = T_2 + V_2 = \text{Constant}$$

T_1 stands for the kinetic energy at state 1 and V_1 is the potential energy function for state 1. T_2 and V_2 represent these energy states at state 2. Recall, the kinetic energy is defined as $T = \frac{1}{2} mv^2$.

Principle of linear impulse and momentum

This principle is useful for solving problems that involve force, velocity, and time. It can also be used to analyze the mechanics of impact.

The principle of linear impulse and momentum is obtained by integrating the equation of motion with respect to time. The equation of motion can be written

$$\sum \mathbf{F} = m \mathbf{a} = m (d\mathbf{v}/dt)$$

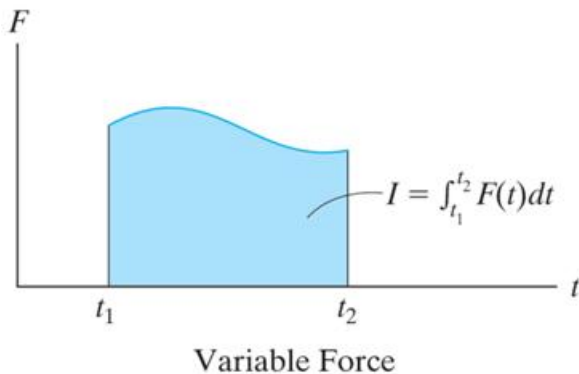
Separating variables and integrating between the limits $\mathbf{v} = \mathbf{v}_1$ at $t = t_1$ and $\mathbf{v} = \mathbf{v}_2$ at $t = t_2$ results in

$$\sum \int_{t_1}^{t_2} F dt = m \int_{v_1}^{v_2} dv = mv_2 - mv_1$$

This equation represents the principle of linear impulse and momentum. It relates the particle's final velocity (v_2) and initial velocity (v_1) and the forces acting on the particle as a function of time.

The vector $m\mathbf{v}$ is called the linear momentum, denoted as \mathbf{L} . This vector has the same direction as \mathbf{v} . The linear momentum vector has units of $(\text{kg}\cdot\text{m})/\text{s}$ or $(\text{slug}\cdot\text{ft})/\text{s}$.

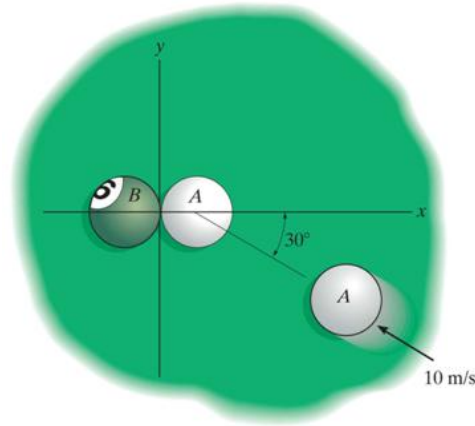
The integral $\int \mathbf{F} dt$ is the linear impulse, denoted \mathbf{I} . It is a vector quantity measuring the effect of a force during its time interval of action. \mathbf{I} acts in the same direction as \mathbf{F} and has units of $\text{N}\cdot\text{s}$ or $\text{lb}\cdot\text{s}$.



The impulse may be determined by direct integration. Graphically, it can be represented by the area under the force versus time curve. If F_c is constant, then

$$I = F_c (t_2 - t_1) .$$

Impact - application



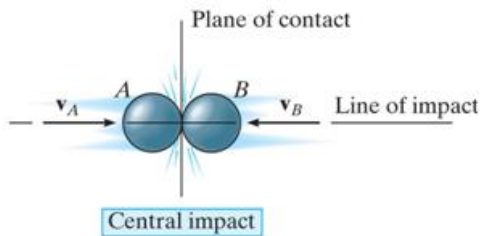
In the game of billiards, it is important to be able to predict the trajectory and speed of a ball after it is struck by another ball.

If we know the velocity of ball A before the impact, how can we determine the magnitude and direction of the velocity of ball B after the impact?

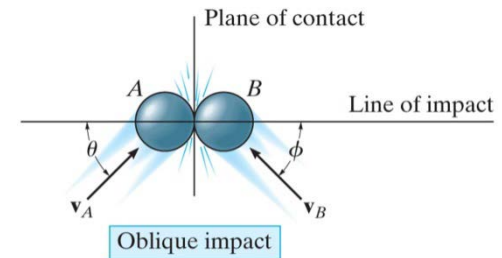
What parameters do we need to know for this?

Impact - application

Impact occurs when two bodies collide during a very short time period, causing large impulsive forces to be exerted between the bodies. Common examples of impact are a hammer striking a nail or a bat striking a ball. The line of impact is a line through the mass centers of the colliding particles. In general, there are two types of impact:



Central impact occurs when the directions of motion of the two colliding particles are along the line of impact.



Oblique impact occurs when the direction of motion of one or both of the particles is at an angle to the line of impact.

In most problems, the initial velocities of the particles, $(\mathbf{v}_A)_1$ and $(\mathbf{v}_B)_1$, are known, and it is necessary to determine the final velocities, $(\mathbf{v}_A)_2$ and $(\mathbf{v}_B)_2$. So the first equation used is the conservation of linear momentum, applied along the line of impact.

$$(m_A \mathbf{v}_A)_1 + (m_B \mathbf{v}_B)_1 = (m_A \mathbf{v}_A)_2 + (m_B \mathbf{v}_B)_2$$

This provides one equation, but we have two unknowns, $(\mathbf{v}_A)_2$ and $(\mathbf{v}_B)_2$. So another equation is needed. The principle of impulse and momentum is used to develop this equation, which involves the coefficient of restitution, or e

The coefficient of restitution is an indicator of the energy lost during the impact.

$$e = \frac{(\mathbf{v}_B)_2 - (\mathbf{v}_A)_2}{(\mathbf{v}_A)_1 - (\mathbf{v}_B)_1}$$

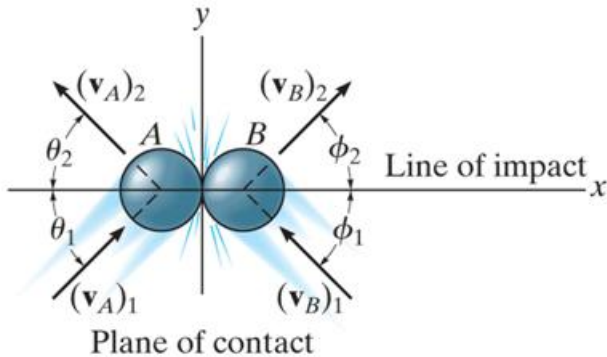
If a value for e is specified, this relation provides the second equation necessary to solve for $(\mathbf{v}_A)_2$ and $(\mathbf{v}_B)_2$

In general, e has a value between 0 and 1. The two limiting conditions can be considered:

Elastic impact ($e = 1$): In a perfectly elastic collision, no energy is lost and the relative separation velocity equals the relative approach velocity of the particles. In practical situations, this condition cannot be achieved.

Plastic impact ($e = 0$): In a plastic impact, the relative separation velocity is zero. The particles stick together and move with a common velocity after the impact.

During a collision, some of the particles' initial kinetic energy will be lost in the form of heat, sound, or due to localized deformation.



Oblique impact

In an oblique impact, one or both of the particles' motion is at an angle to the line of impact. Typically, there will be four unknowns: the magnitudes and directions of the final velocities.

The four equations required to solve for the unknowns are:

$$\begin{array}{c}
 \begin{array}{c} m_A(\mathbf{v}_{Ax})_1 \rightarrow \\ m_A(\mathbf{v}_{Ay})_1 \uparrow \end{array} \text{A} + \begin{array}{c} \leftarrow \\ \int \mathbf{F} dt \end{array} = \begin{array}{c} m_A(\mathbf{v}_{Ax})_2 \leftarrow \\ m_A(\mathbf{v}_{Ay})_2 \uparrow \end{array} \text{A} \\
 \\
 \begin{array}{c} m_B(\mathbf{v}_{Bx})_1 \leftarrow \\ m_B(\mathbf{v}_{By})_1 \uparrow \end{array} \text{B} + \begin{array}{c} \rightarrow \\ \int \mathbf{F} dt \end{array} = \begin{array}{c} m_B(\mathbf{v}_{Bx})_2 \rightarrow \\ m_B(\mathbf{v}_{By})_2 \uparrow \end{array} \text{B}
 \end{array}$$

Conservation of momentum and the coefficient of restitution equation are applied along the line of impact (x-axis):

$$m_A(\mathbf{v}_{Ax})_1 + m_B(\mathbf{v}_{Bx})_1 = m_A(\mathbf{v}_{Ax})_2 + m_B(\mathbf{v}_{Bx})_2$$

$$e = [(\mathbf{v}_{Bx})_2 - (\mathbf{v}_{Ax})_2] / [(\mathbf{v}_{Ax})_1 - (\mathbf{v}_{Bx})_1]$$

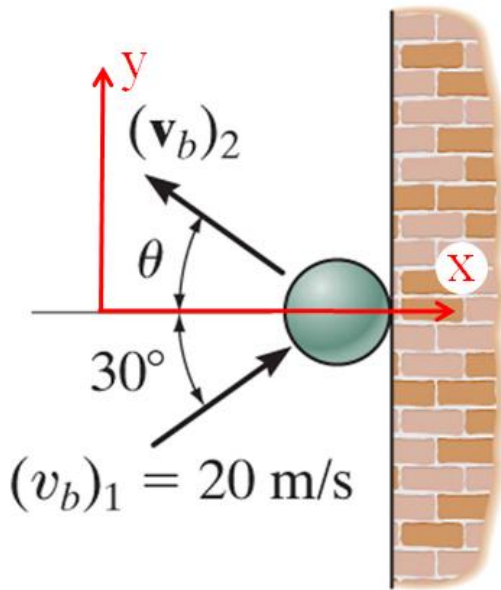
Momentum of each particle is conserved in the direction perpendicular to the line of impact (y-axis):

$$m_A(\mathbf{v}_{Ay})_1 = m_A(\mathbf{v}_{Ay})_2 \quad \text{and} \quad m_B(\mathbf{v}_{By})_1 = m_B(\mathbf{v}_{By})_2$$

Procedure for analysis

- In most impact problems, the initial velocities of the particles and the coefficient of restitution, e , are known, with the final velocities to be determined.
- Define the x-y axes. Typically, the **x-axis** is defined **along** the line of impact and the **y-axis** is in the plane of contact **perpendicular** to the x-axis.
- For both **central and oblique** impact problems, the following equations apply **along** the line of impact (x-dir.):
$$\sum m(v_x)_1 = \sum m(v_x)_2 \quad \text{and} \quad e = [(v_{Bx})_2 - (v_{Ax})_2] / [(v_{Ax})_1 - (v_{Bx})_1]$$
- For **oblique** impact problems, the following equations are also required, applied **perpendicular** to the line of impact (y-dir.):
$$m_A(v_{Ay})_1 = m_A(v_{Ay})_2 \quad \text{and} \quad m_B(v_{By})_1 = m_B(v_{By})_2$$

Example



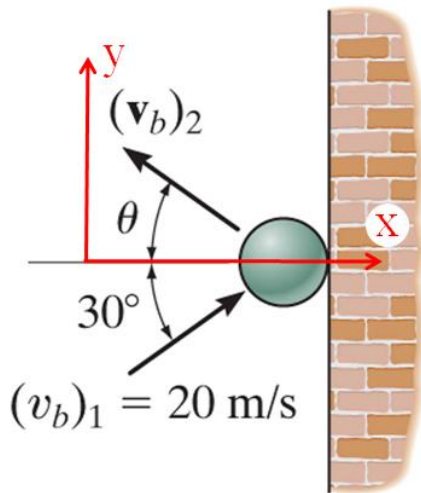
The ball strikes the smooth wall with a velocity $(\mathbf{v}_b)_1 = 20 \text{ m/s}$. The coefficient of restitution between the ball and the wall is $e = 0.75$.

Find the velocity and direction of the ball just after the impact.

Solution

The collision is an oblique impact, with the line of impact perpendicular to the plane (through the relative centers of mass).

Thus, the coefficient of restitution applies perpendicular to the wall and the momentum of the ball is conserved along the wall.



The momentum of the ball is conserved in the y -dir:

$$m(v_b)_1 \sin 30^\circ = m(v_b)_2 \sin \theta$$

$$(v_b)_2 \sin \theta = 10 \text{ m/s} \quad (1)$$

The coefficient of restitution applies in the x -dir:

$$e = [0 - (v_{bx})_2] / [(v_{bx})_1 - 0]$$

$$\Rightarrow 0.75 = [0 - (-v_b)_2 \cos \theta] / [20 \cos 30^\circ - 0]$$

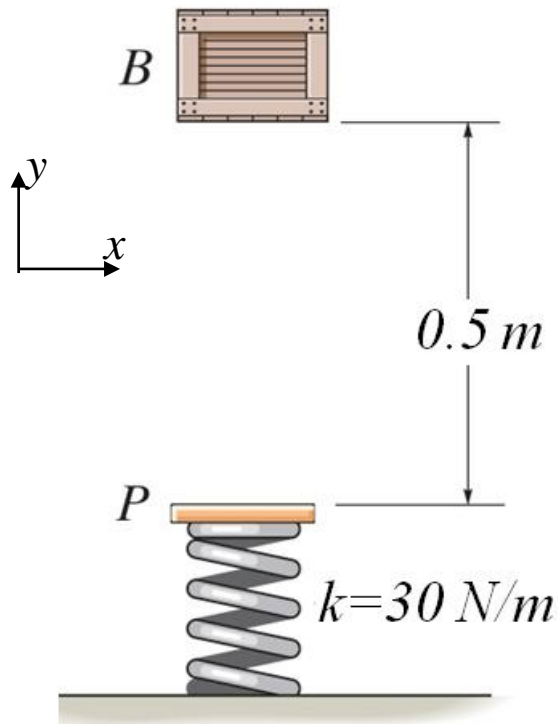
$$\Rightarrow (v_b)_2 \cos \theta = 12.99 \text{ m/s} \quad (2)$$

Using Eqs. (1) and (2) and solving for the velocity and θ yields:

$$(v_b)_2 = (12.99^2 + 10^2)^{0.5} = 16.4 \text{ m/s}$$

$$\theta = \tan^{-1}(10/12.99) = 37.6^\circ$$

Breakout rooms (15 min)



A 2 kg crate B is released from rest, falls a distance $h = 0.5\text{ m}$, and strikes plate P (3 kg mass). The coefficient of restitution between B and P is $e = 0.6$, and the spring stiffness is $k = 30\text{ N/m}$.

Find the velocity of crate B just after the collision.

Solution

- 1) Determine the speed of the crate just before the collision using projectile motion or an energy method.
- 2) Analyze the collision as a central impact problem.

- 1) Determine the speed of block B just before impact by using conservation of energy (why?). Define the gravitational datum at the initial position of the block ($h_1 = 0$) and note the block is released from rest ($v_1 = 0$):

$$T_1 + V_1 = T_2 + V_2$$

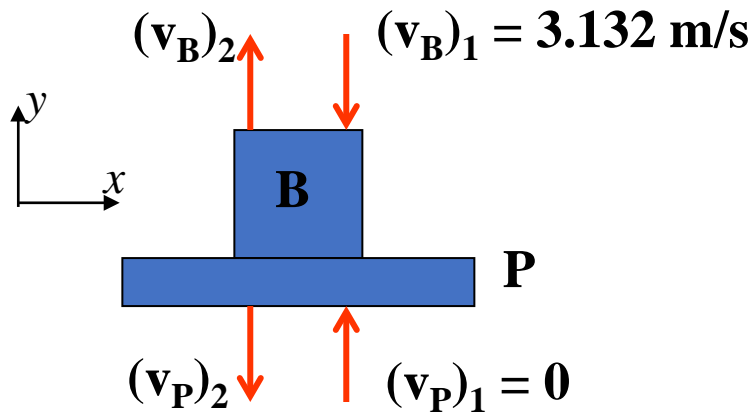
$$0.5m(v_1)^2 + mgh_1 = 0.5m(v_2)^2 + mgh_2$$

$$0 + 0 = 0.5(2)(v_2)^2 + (2)(9.81)(0.5)$$

$$v_2 = 3.132 \text{ m/s} \downarrow$$

This is the speed of the block just before the collision. Plate (P) is at rest, velocity of zero, before the collision.

2) Analyze the collision as a central impact problem.



Apply conservation of momentum to the system in the vertical direction:

$$+\uparrow m_B(v_B)_1 + m_P(v_P)_1 = m_B(v_B)_2 + m_P(v_P)_2$$

$$(2)(-3.132) + 0 = (2)(v_B)_2 + (3)(v_P)_2$$

Using the coefficient of restitution:

$$+\uparrow e = [(v_P)_2 - (v_B)_2] / [(v_B)_1 - (v_P)_1]$$

$$\Rightarrow 0.6 = [(v_P)_2 - (v_B)_2] / [-3.132 - 0] \Rightarrow -1.879 = (v_P)_2 - (v_B)_2$$

Solving the two equations simultaneously yields

$$(v_B)_2 = -0.125 \text{ m/s} \downarrow \text{ and } (v_P)_2 = -2.00 \text{ m/s} \downarrow$$

Both the block and plate will travel down after the collision.