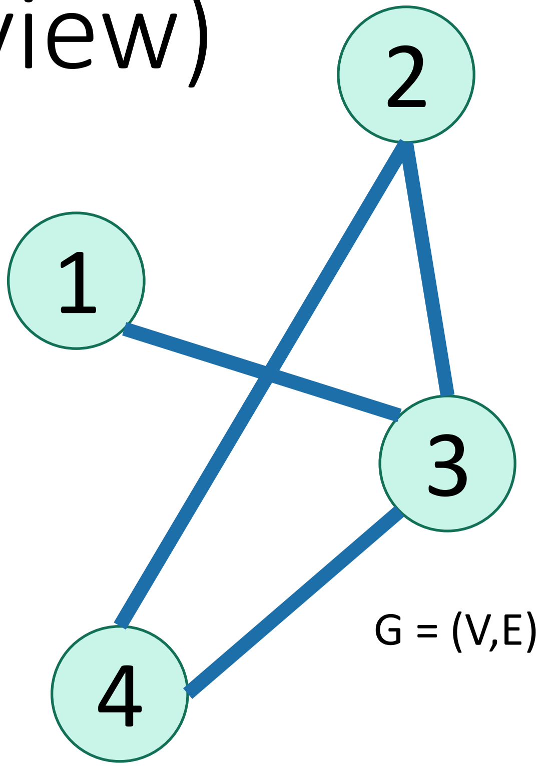
CSE100: Design and Analysis of Algorithms
Lecture 17 – Graphs, DFS and BFS (wrap up) and Strongly Connected Components

Mar 17th 2022

Graphs, Depth First Search, Breadth First Search and Finding strongly connected components

Undirected Graphs (review)

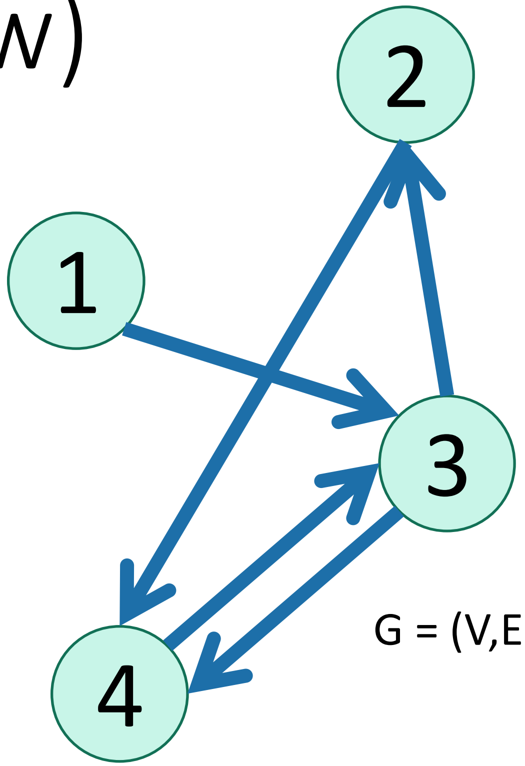
- Has vertices and edges
  - V is the set of vertices
  - E is the set of edges
  - Formally, a graph is G = (V,E)
- Example
  - V = {1,2,3,4}
  - E = { {1,3}, {2,4}, {3,4}, {2,3} }



- The **degree** of vertex 4 is 2.
  - There are 2 edges coming out.
- Vertex 4's **neighbors** are 2 and 3

Directed Graphs (review)

- Has vertices and edges
  - V is the set of vertices
  - E is the set of **DIRECTED** edges
  - Formally, a graph is G = (V,E)
- Example
  - V = {1,2,3,4}
  - E = { (1,3), (2,4), (3,4), (4,3), (3,2) }

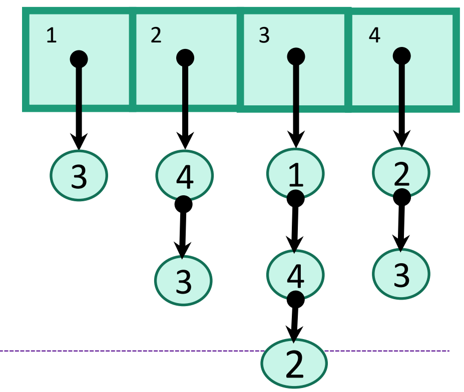


G = (V,E)

- The **in-degree** of vertex 4 is 2.
- The **out-degree** of vertex 4 is 1.
- Vertex 4's **incoming neighbors** are 2,3
- Vertex 4's **outgoing neighbor** is 3.

Adjacency Matrix vs Lists

Generally better for **sparse** graphs

Say there are n vertices and m edges.	$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$	
Edge membership Is e = {v,w} in E?	O(1)	O(deg(v)) or O(deg(w))
Neighbor query Give me v's neighbors.	O(n)	O(deg(v))
Space requirements	$O(n^2)$	O(n + m) **We'll assume this representation for the rest of the class**

Today

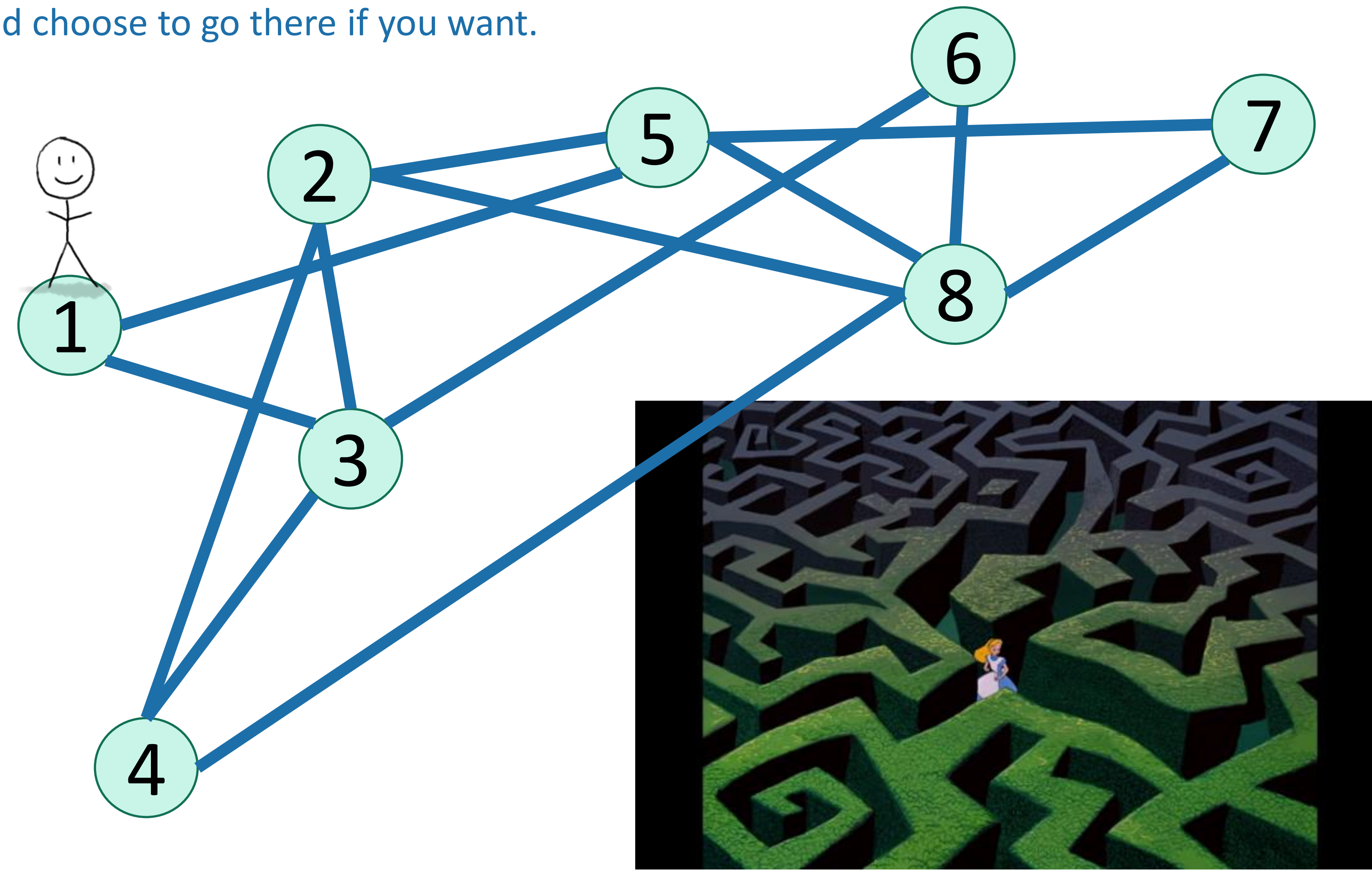
- Part A: Graphs and terminology

- Part B: Depth-first search
  - Application: topological sorting
  - Application: in-order traversal of BSTs
- Part C: Breadth-first search
  - Application: shortest paths
  - Application (if time): is a graph bipartite?

Part B: Depth-first search

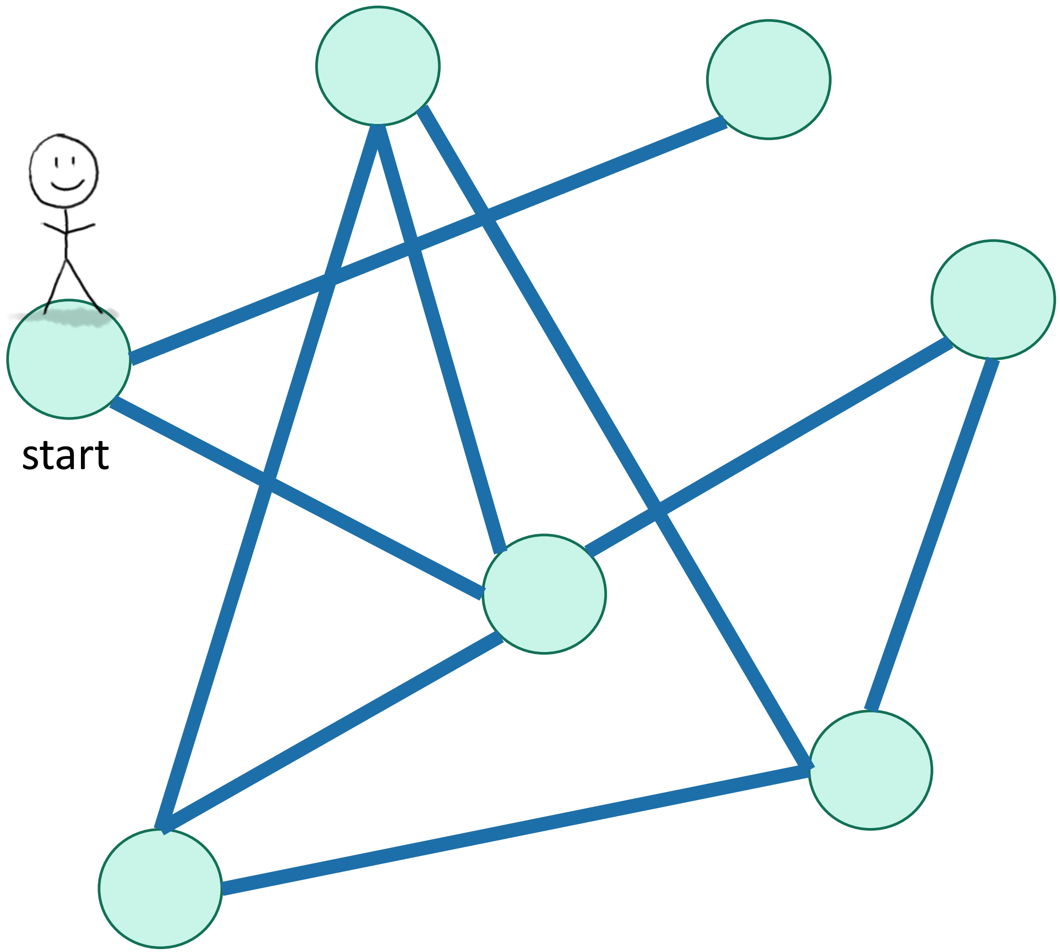
How do we explore a graph?

At each node, you can get a list of neighbors, and choose to go there if you want.



Depth First Search

Exploring a labyrinth with chalk and a piece of string



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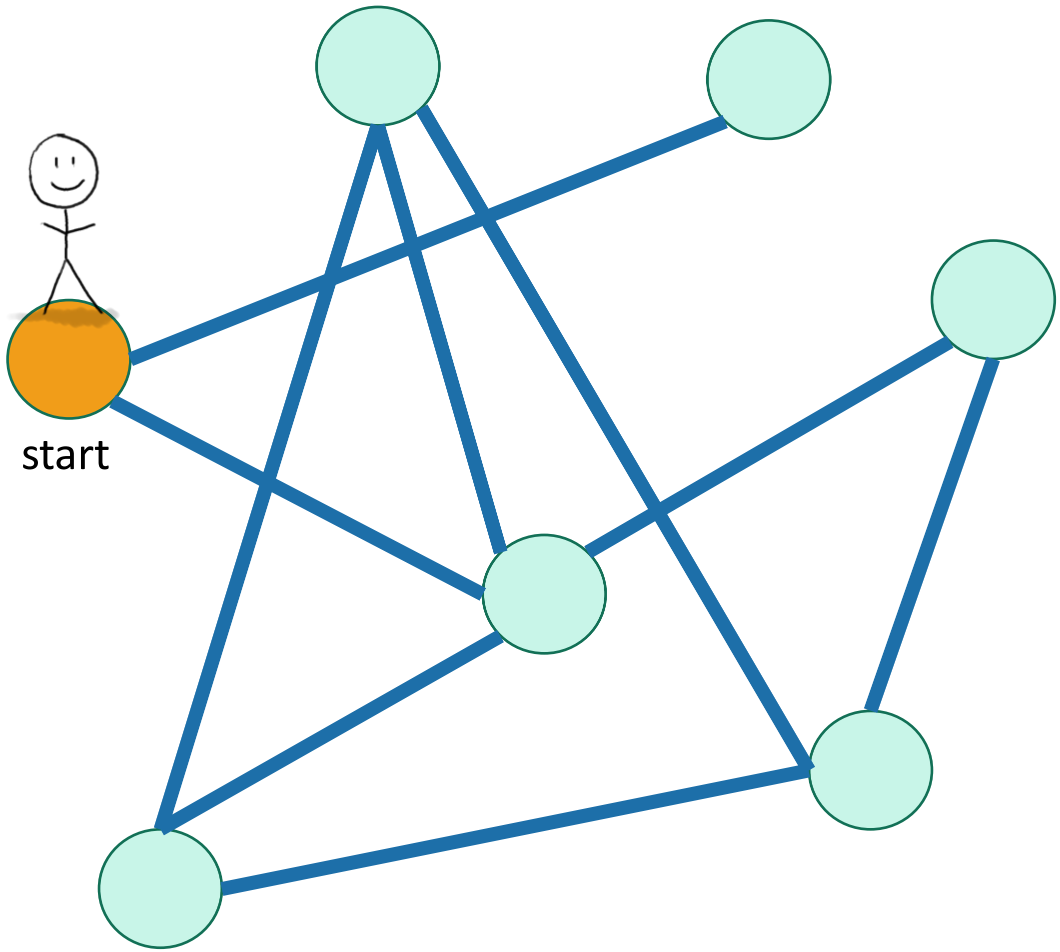

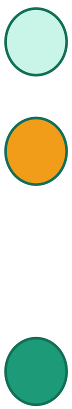

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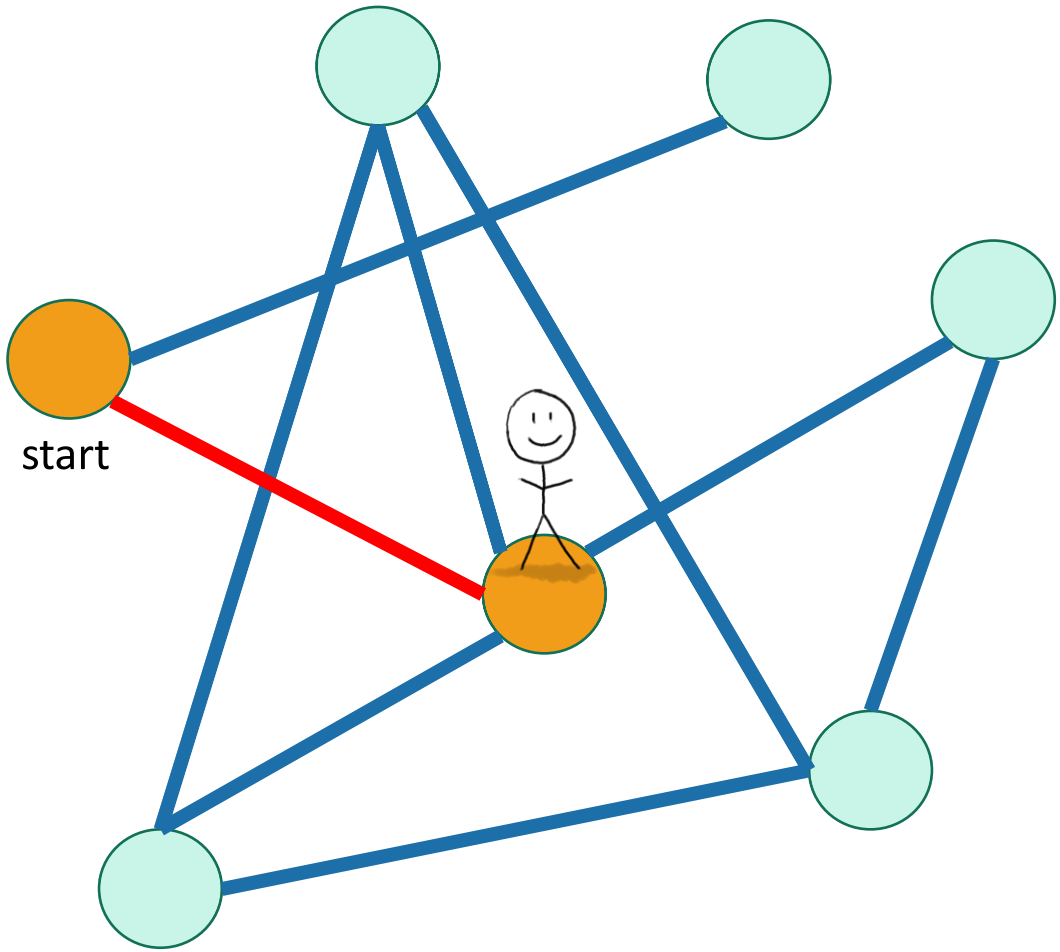

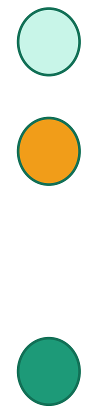

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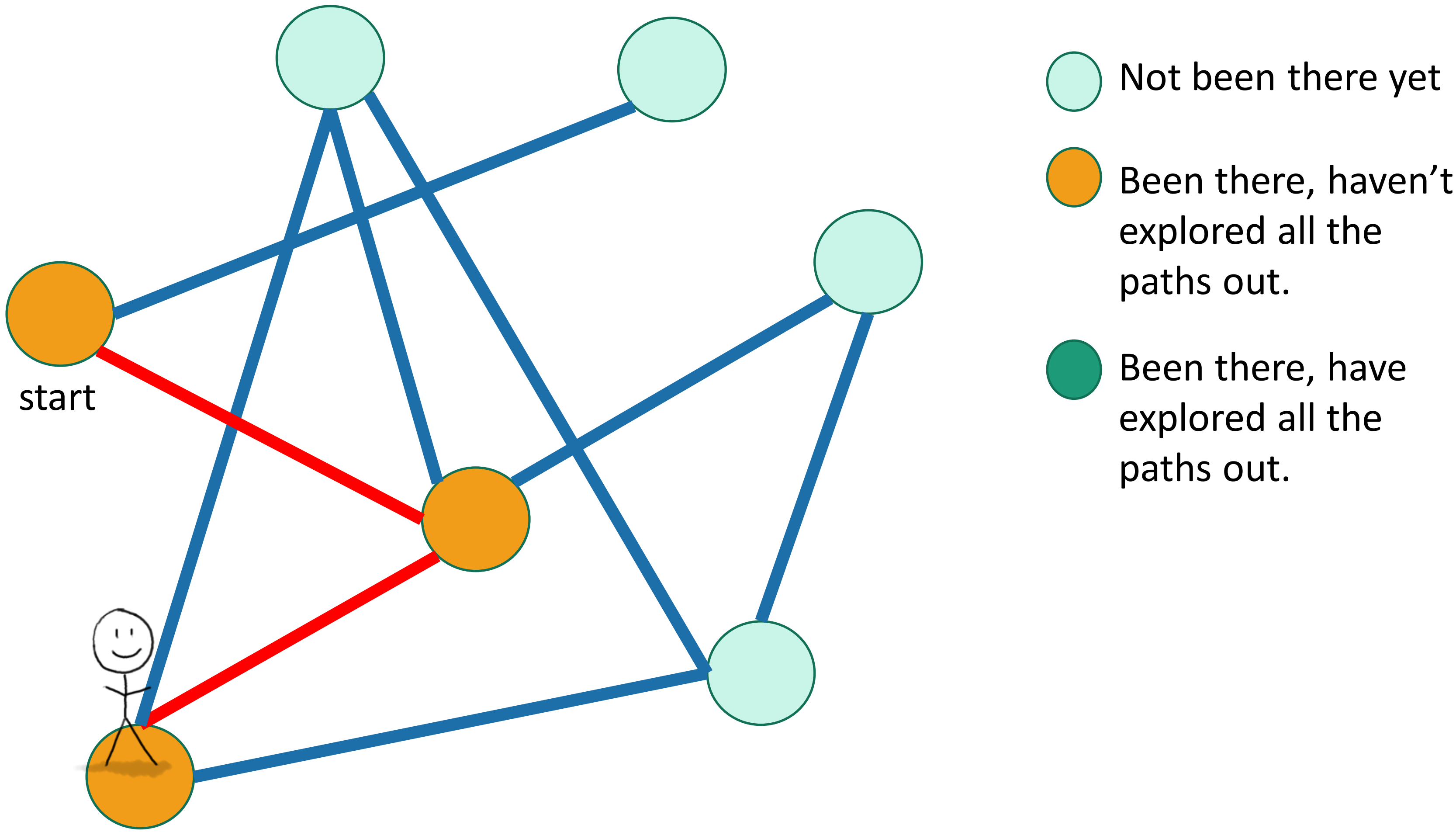

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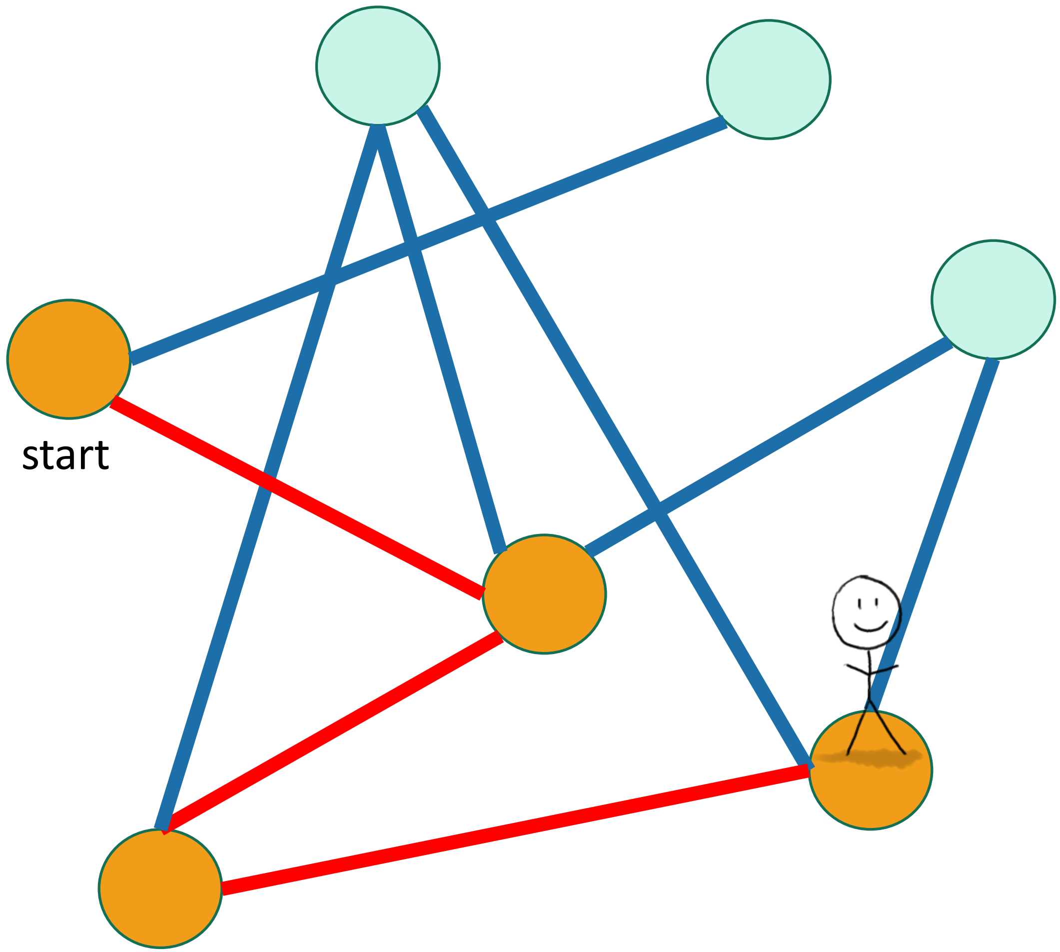

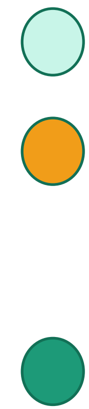

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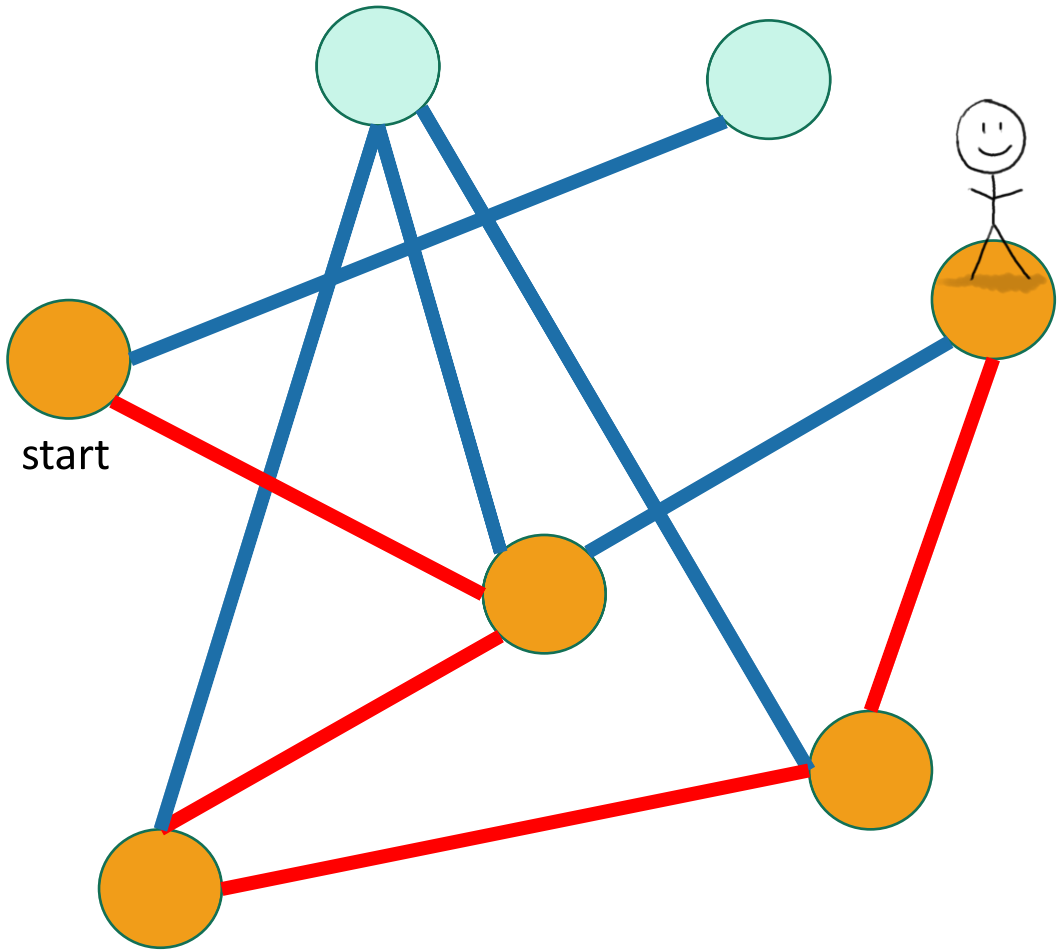

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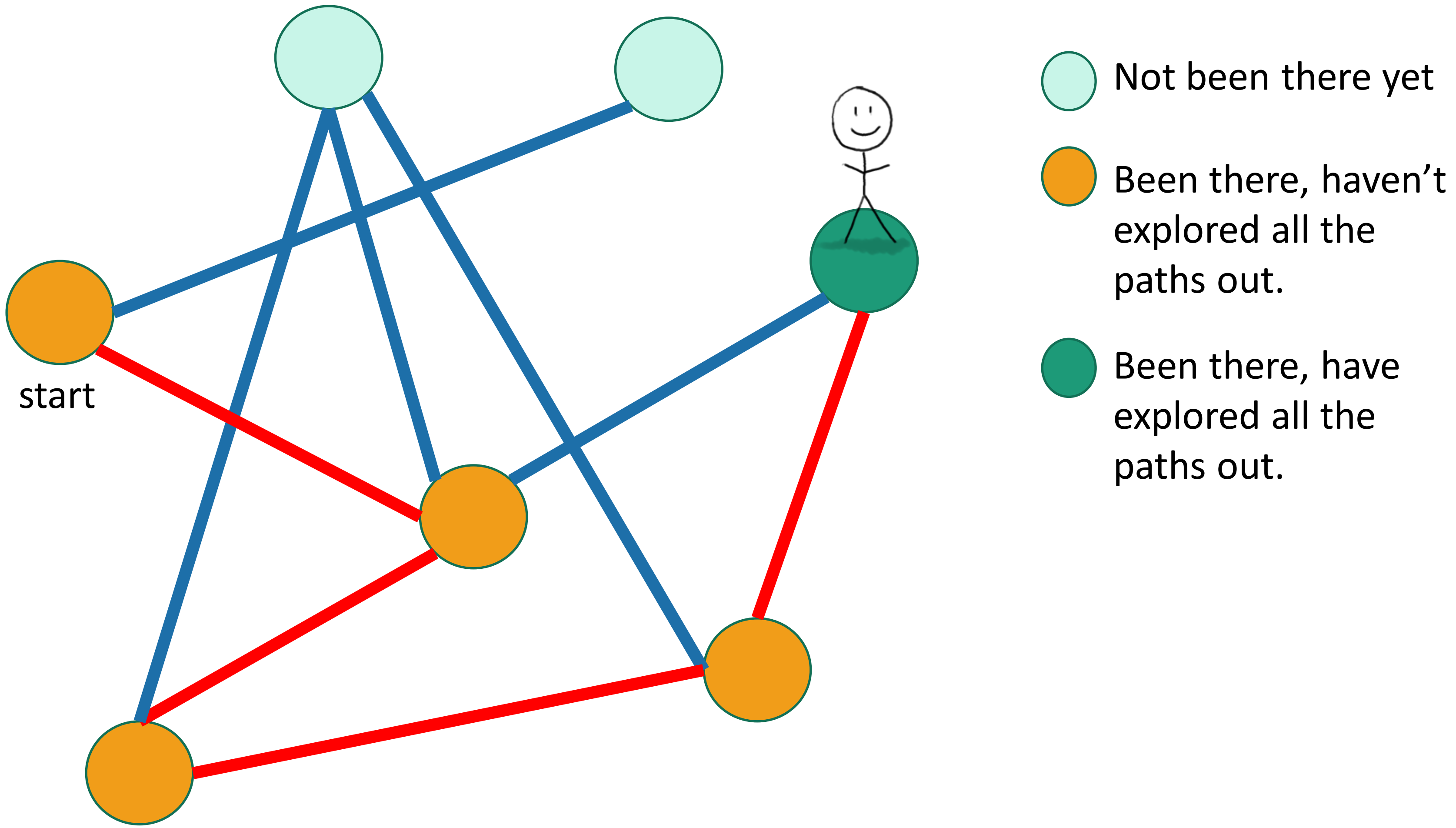

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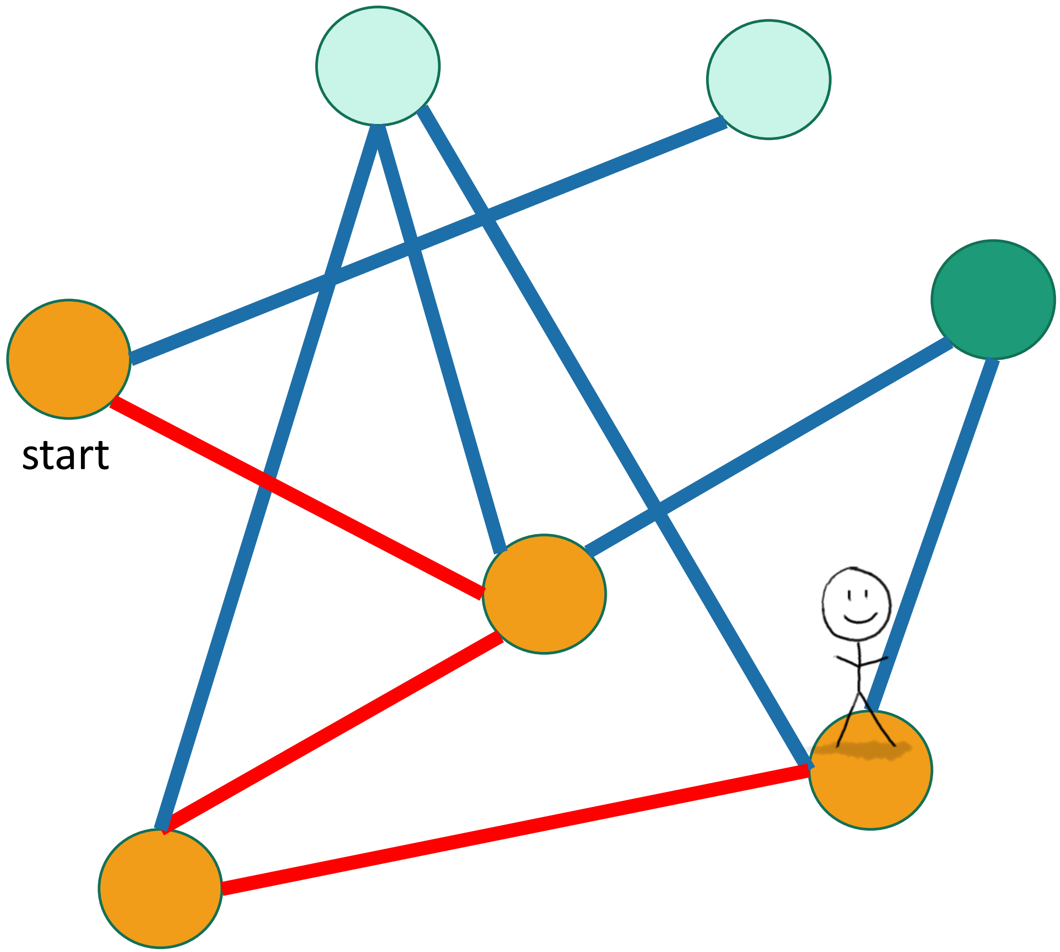

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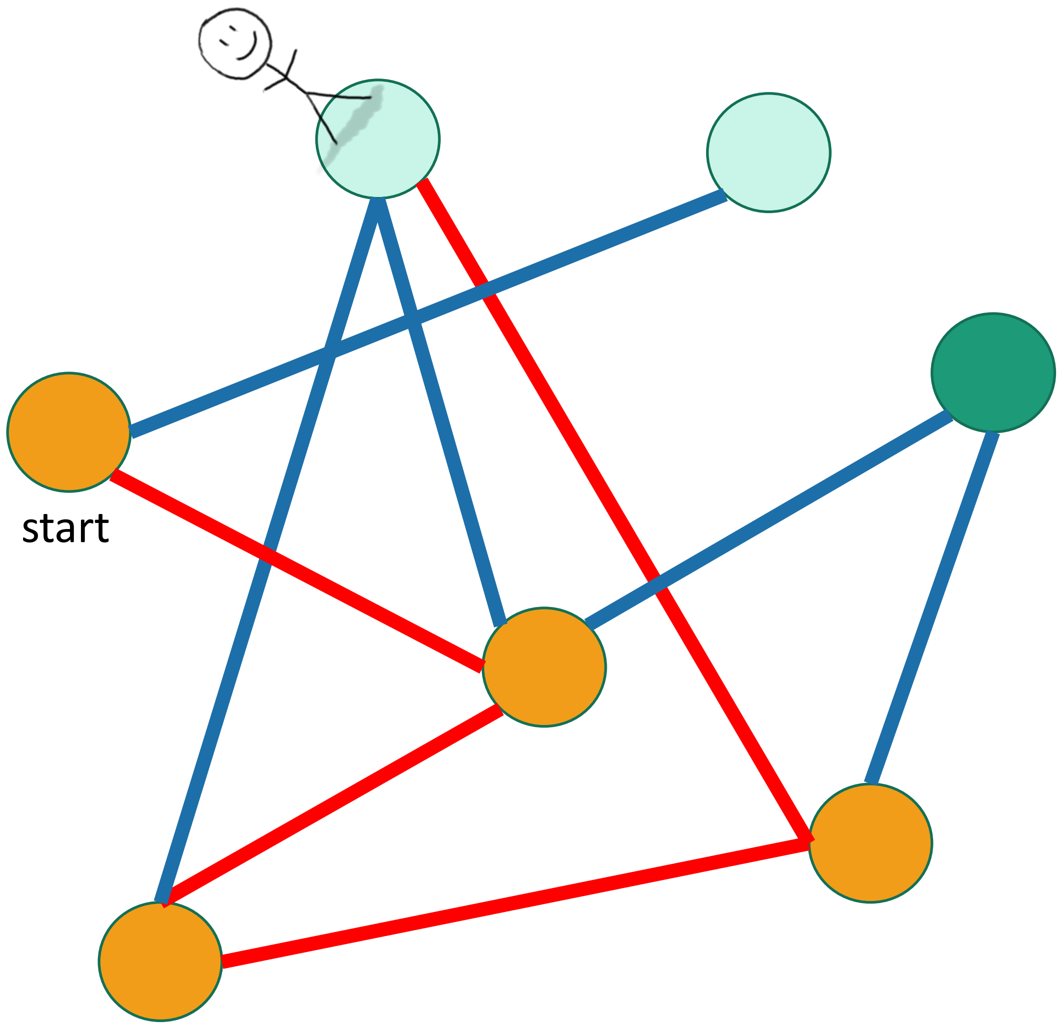

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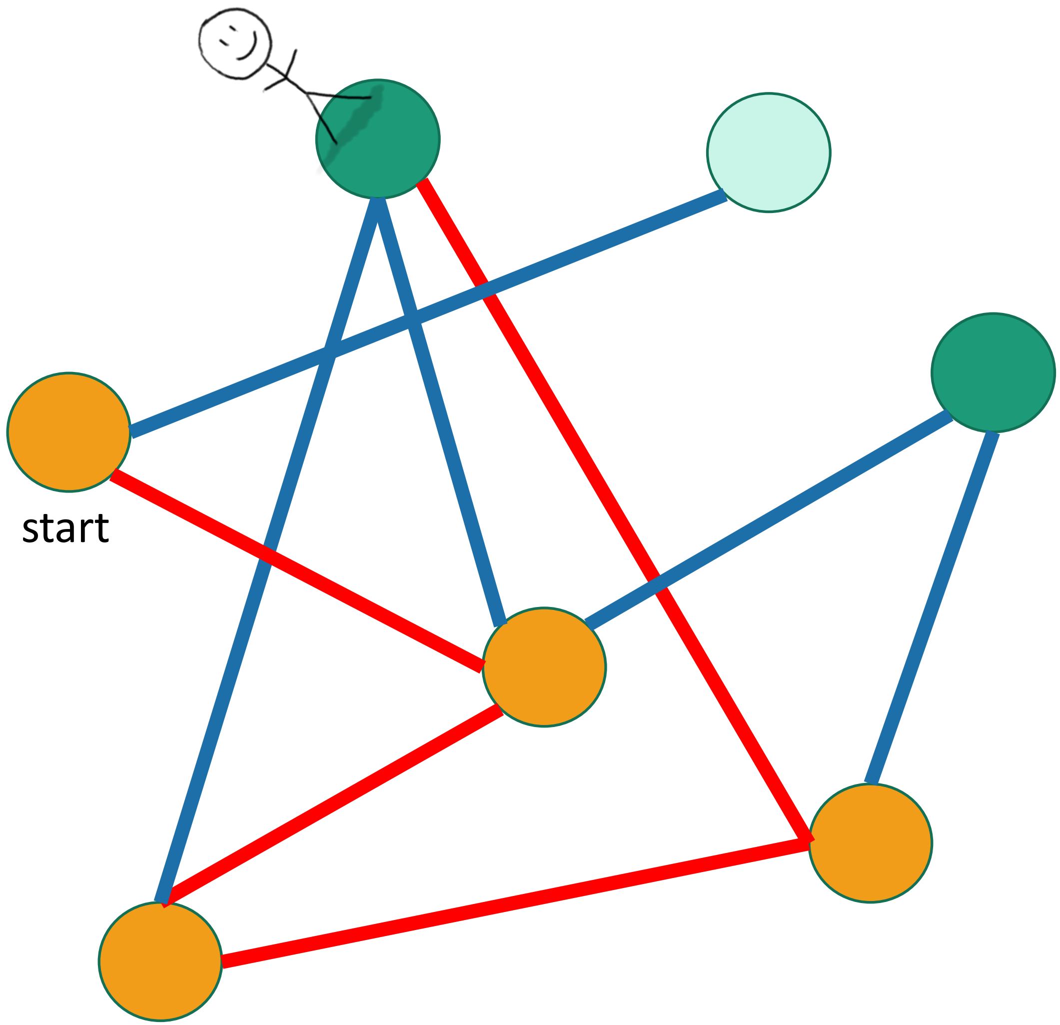

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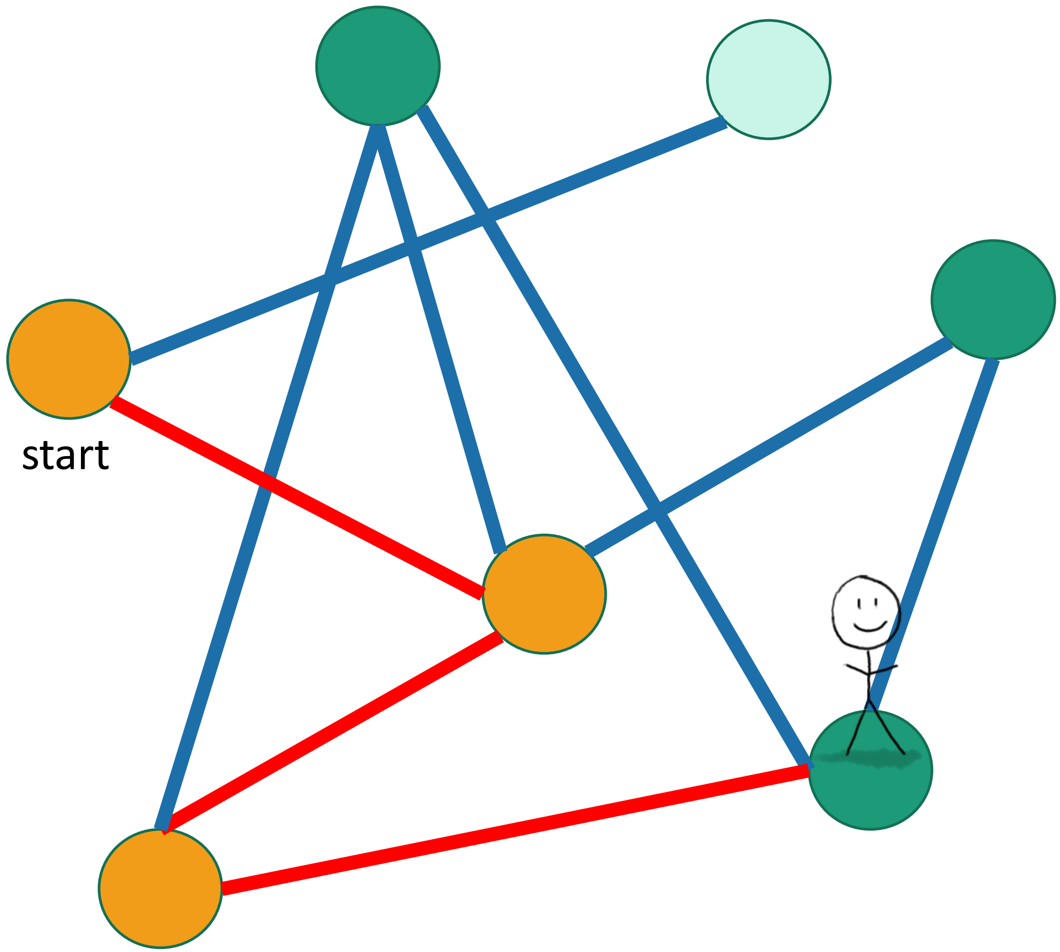

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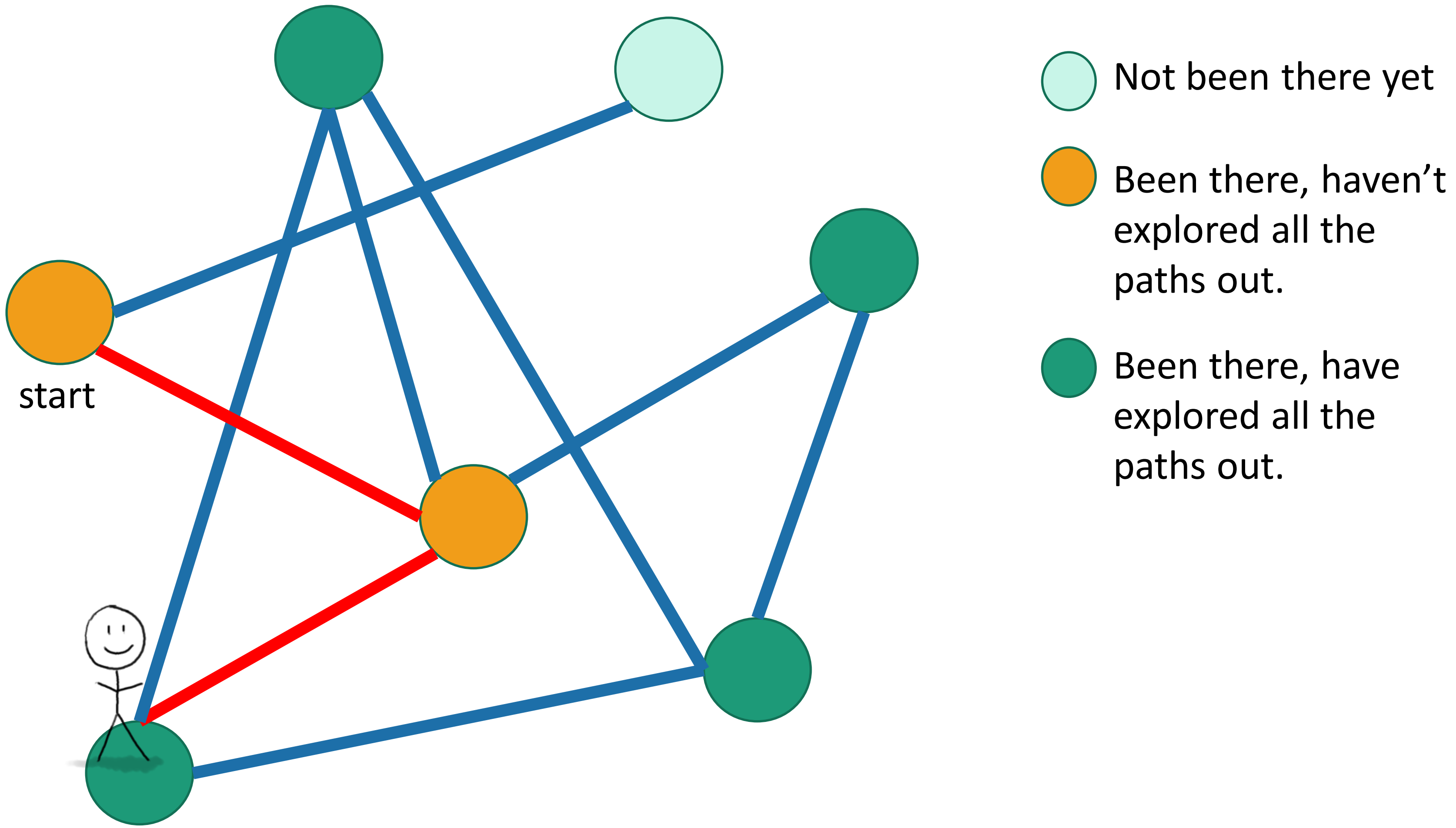

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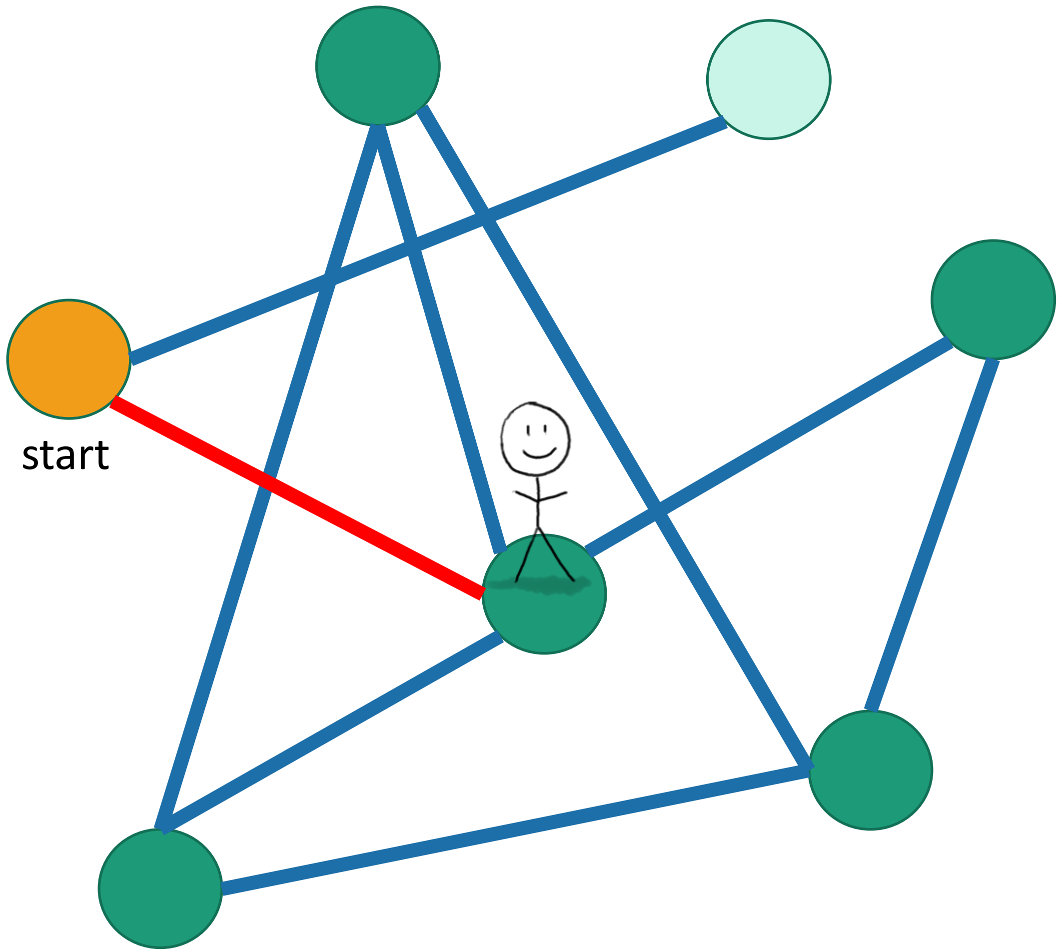

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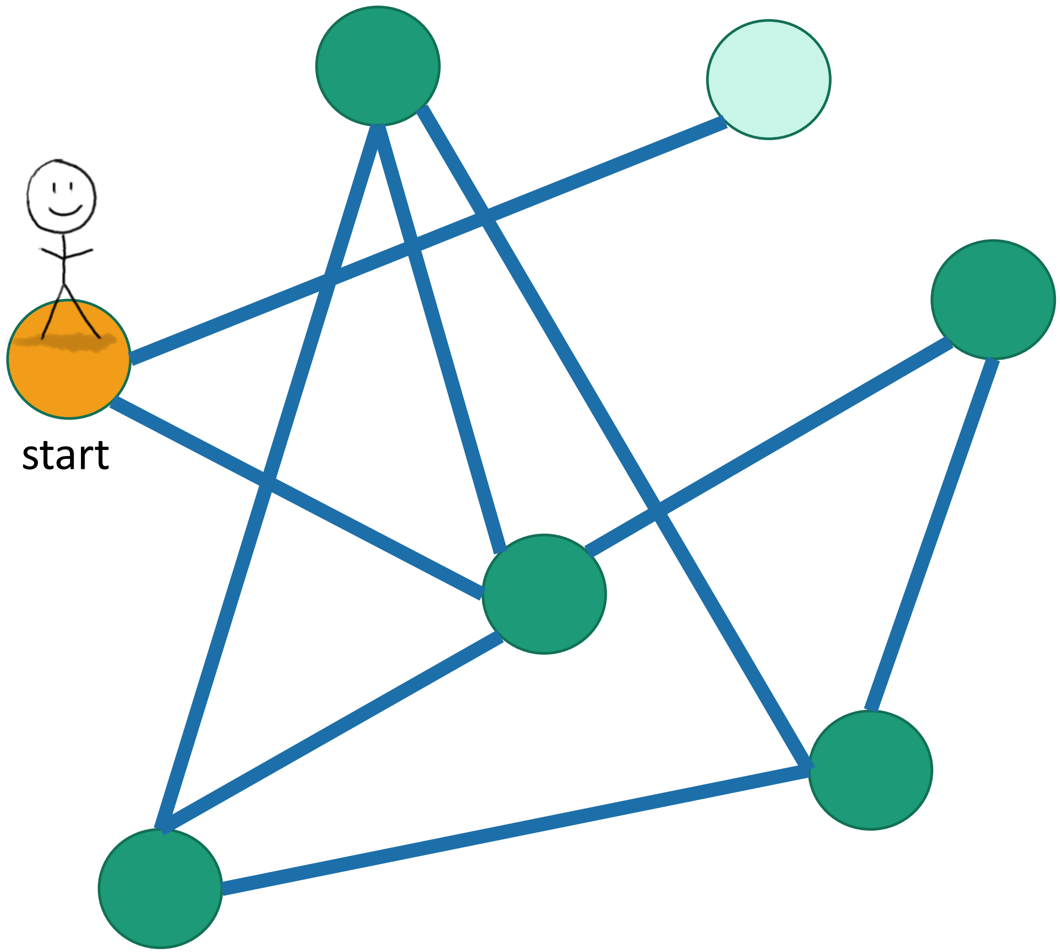

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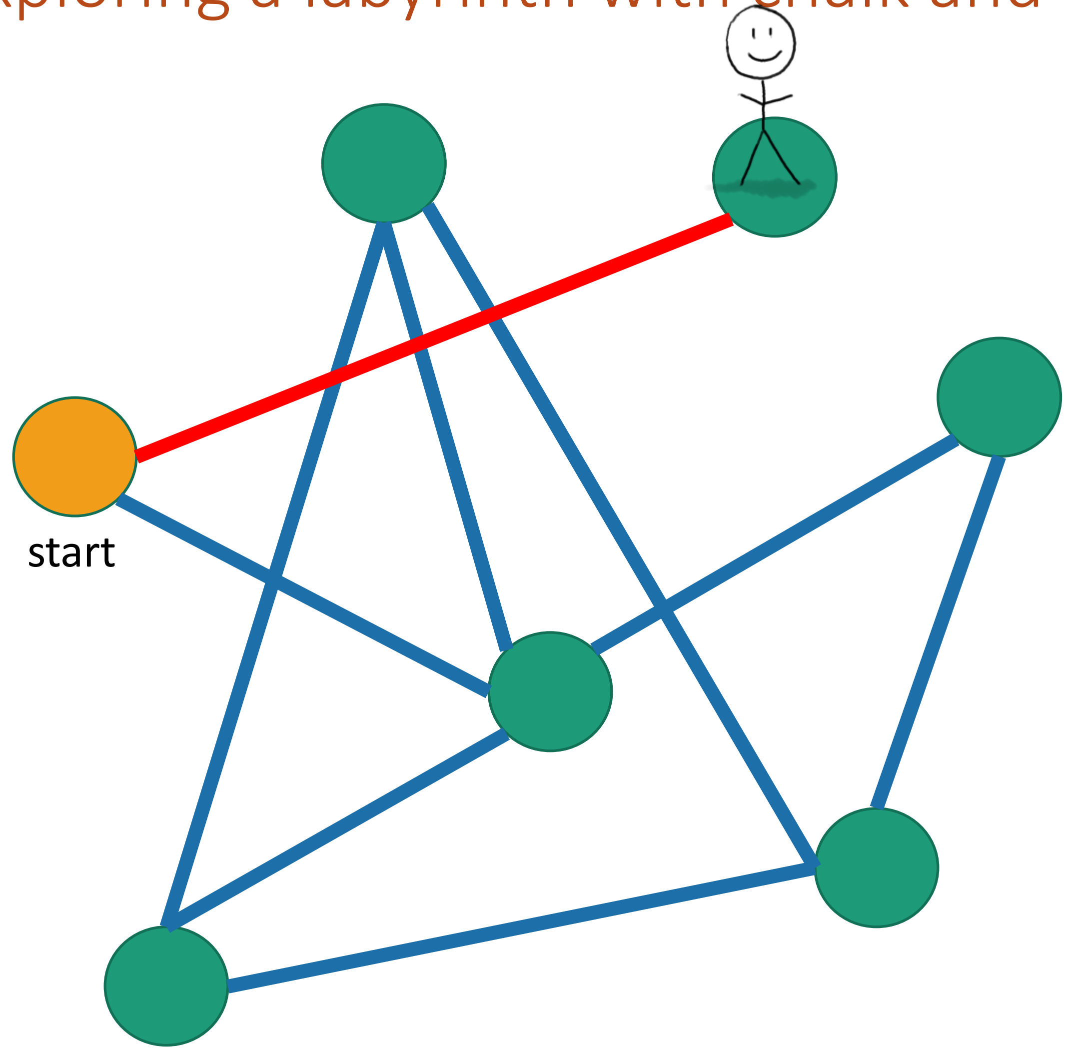

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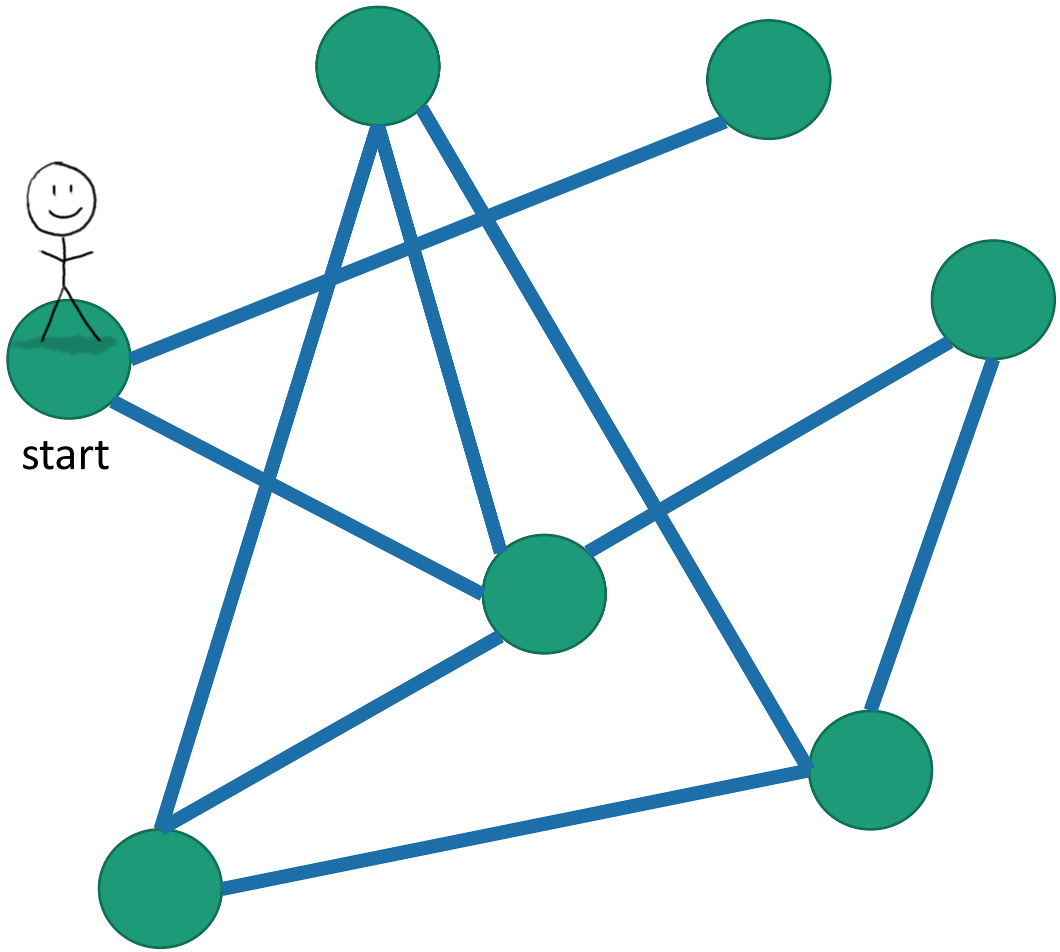

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Labyrinth:
explored!

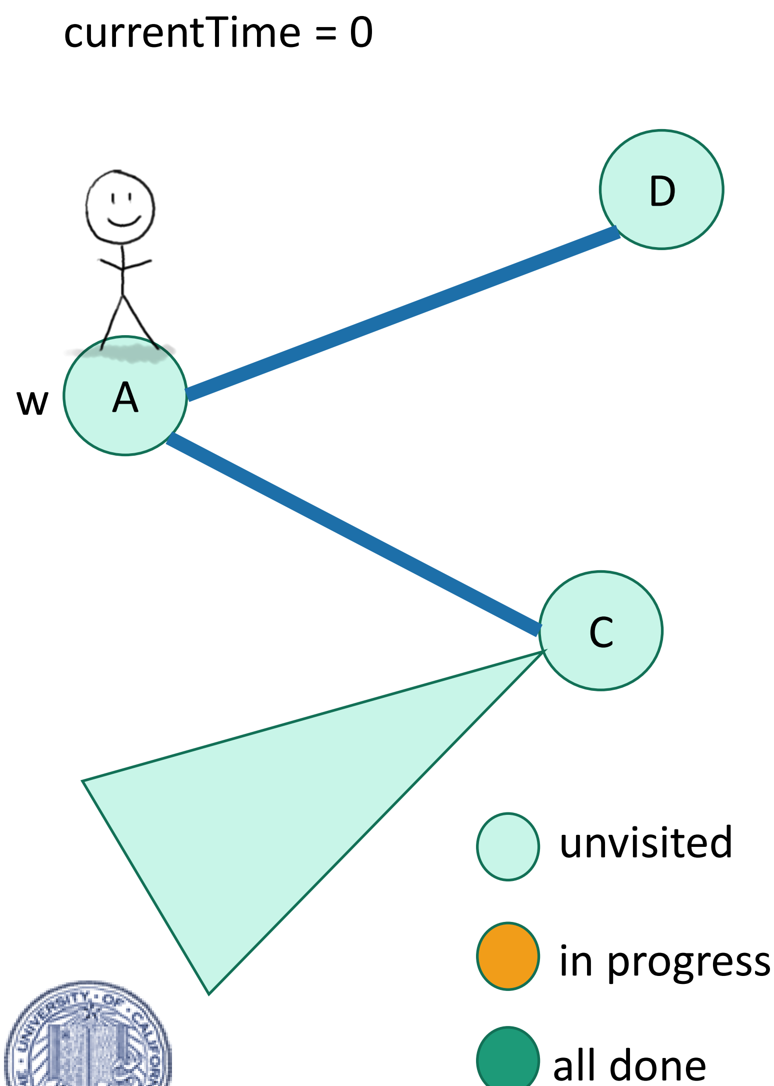
Depth First Search

Exploring a labyrinth with pseudocode

- Each vertex keeps track of whether it is:
  - Unvisited
  - In progress
  - All done
- Each vertex will also keep track of:
  - The time we **first enter it**.
  - The time we finish with it and mark it **all don**

You might have seen other ways to implement DFS than what we are about to go through. This way has more bookkeeping – the bookkeeping will be useful later!

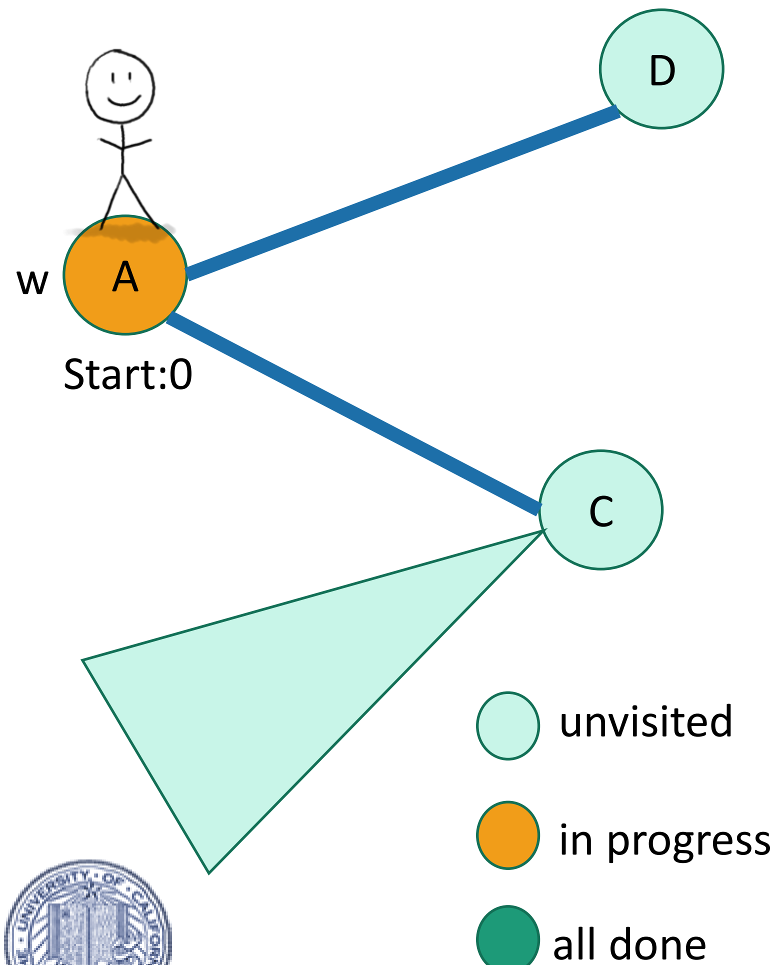
Depth First Search



- **DFS**(w, currentTime):
  - w.startTime = currentTime
  - currentTime ++
  - Mark w as **in progress**.
  - **for** v in w.neighbors:
    - **if** v is **unvisited**:
      - currentTime = **DFS**(v, currentTime)
      - currentTime ++
  - w.finishTime = currentTime
  - Mark w as **all done**
  - **return** currentTime

Depth First Search

currentTime = 1



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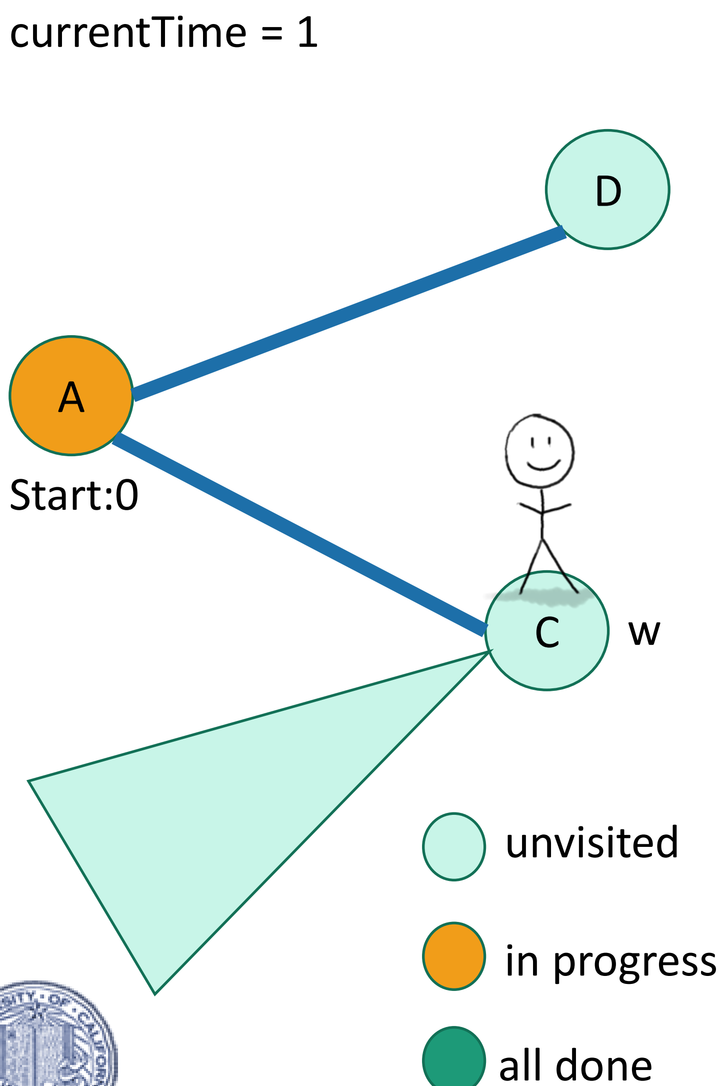
Depth First Search



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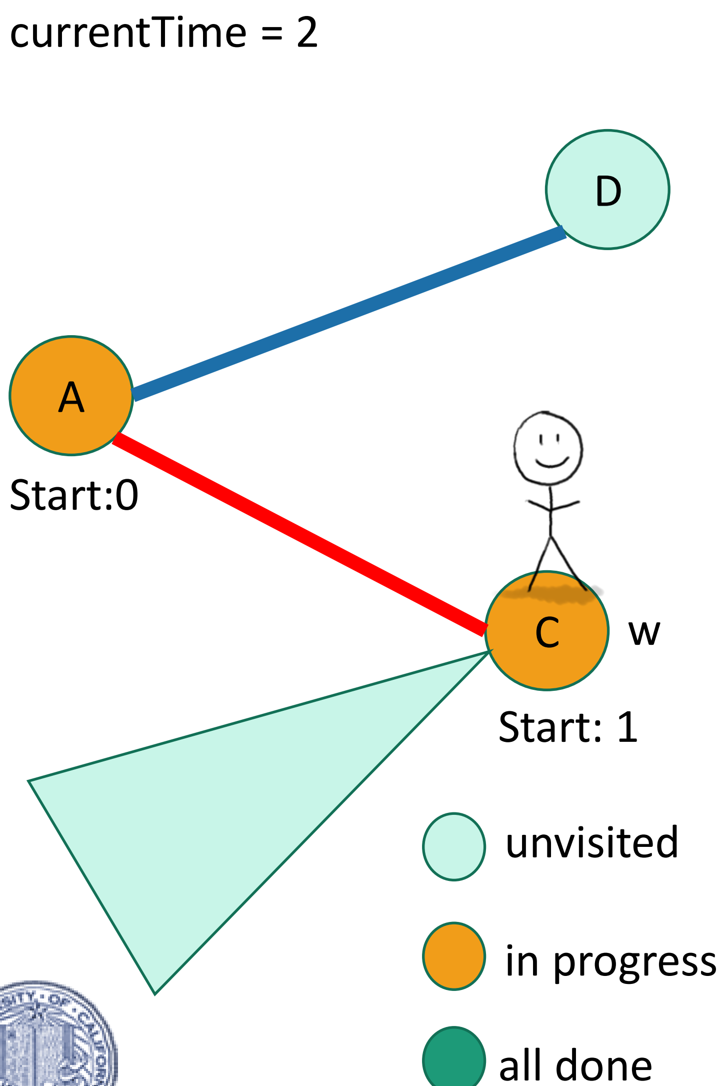
Depth First Search



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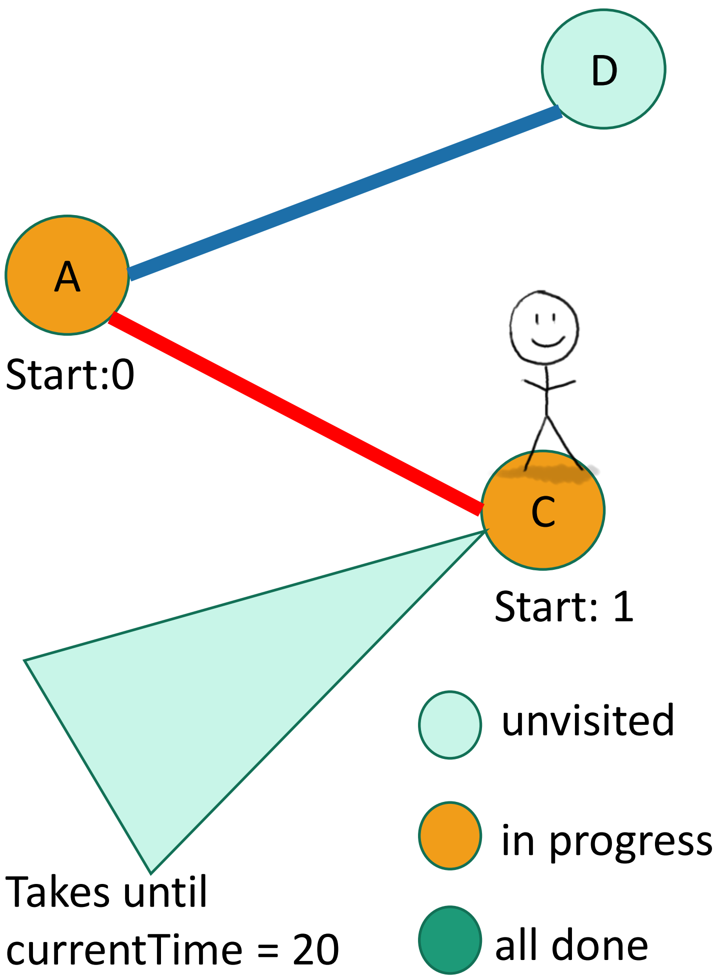
Depth First Search

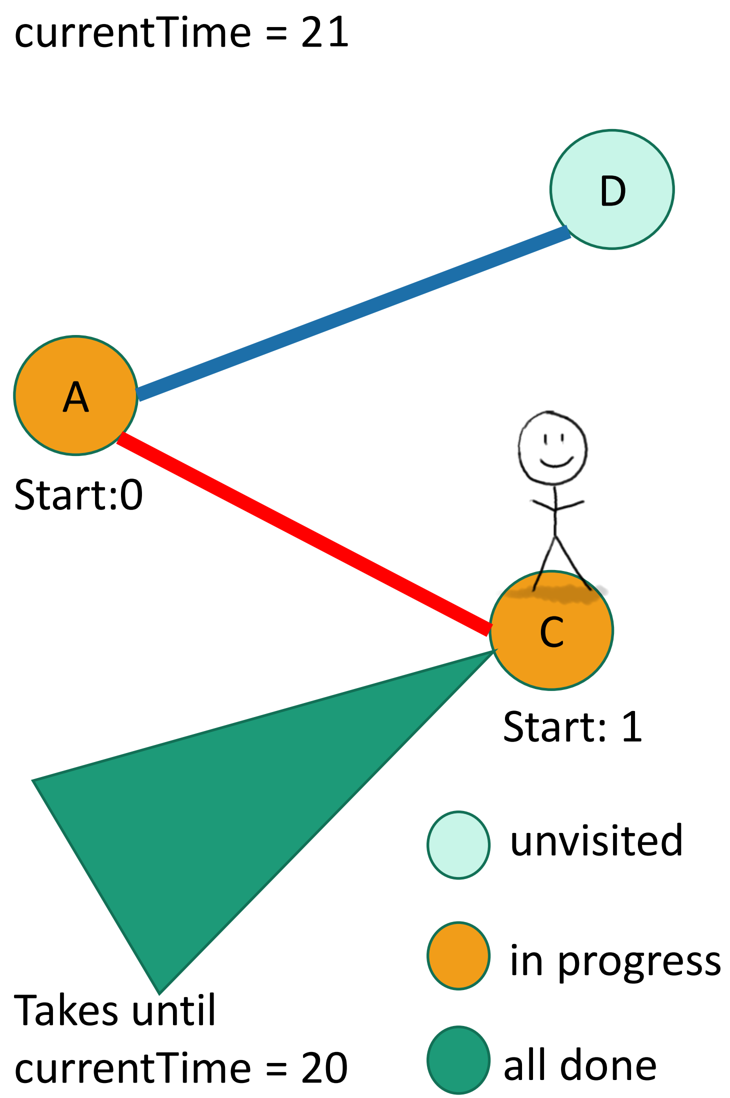
currentTime = 20



Takes until
currentTime = 20

- **DFS**(w, currentTime):
  - w.startTime = currentTime
  - currentTime ++
  - Mark w as **in progress**.
  - **for** v in w.neighbors:
    - **if** v is **unvisited**:
      - currentTime
        = **DFS**(v, currentTime)
      - currentTime ++
  - w.finishTime = currentTime
  - Mark w as **all done**
  - **return** currentTime

Depth First Search



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      - currentTime
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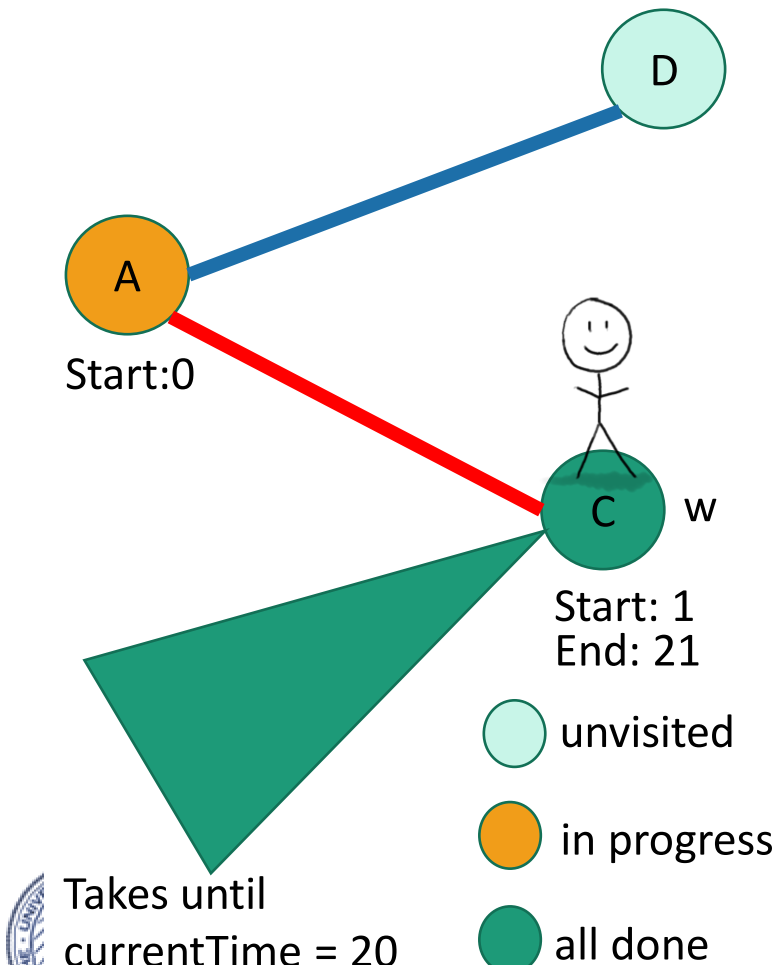
  - w.finishTime = currentTime
  - Mark w as **all done**
  - **return** currentTime

Depth First Search

currentTime = 21



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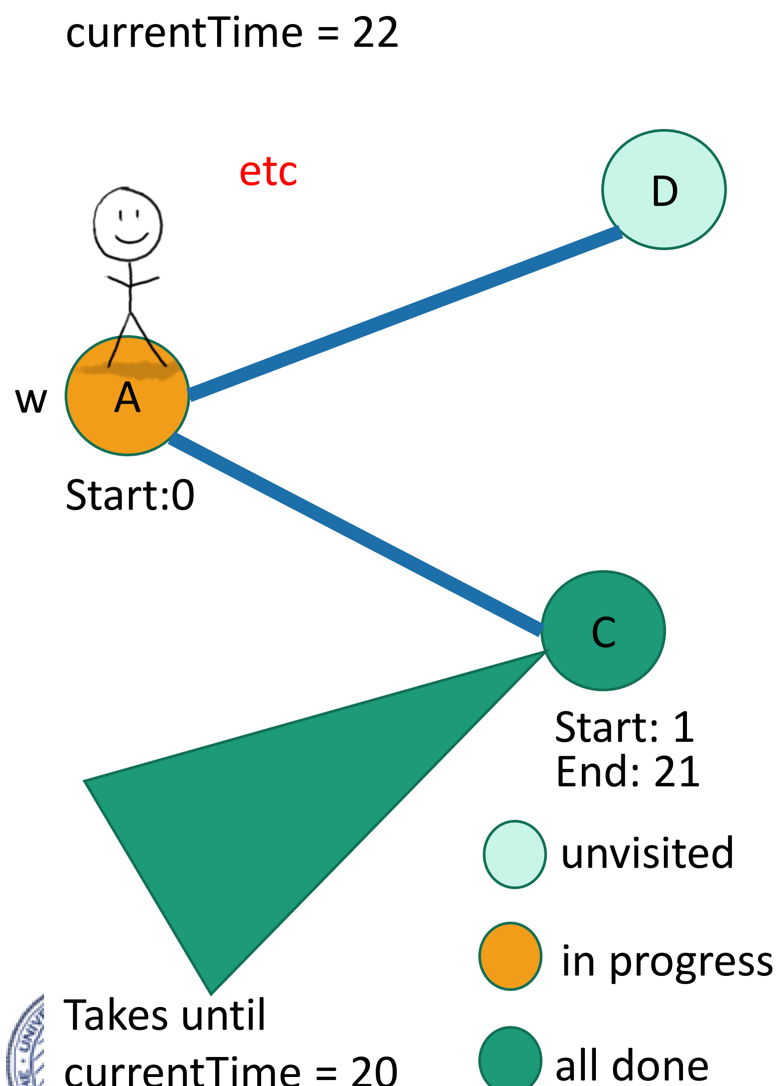
Depth First Search



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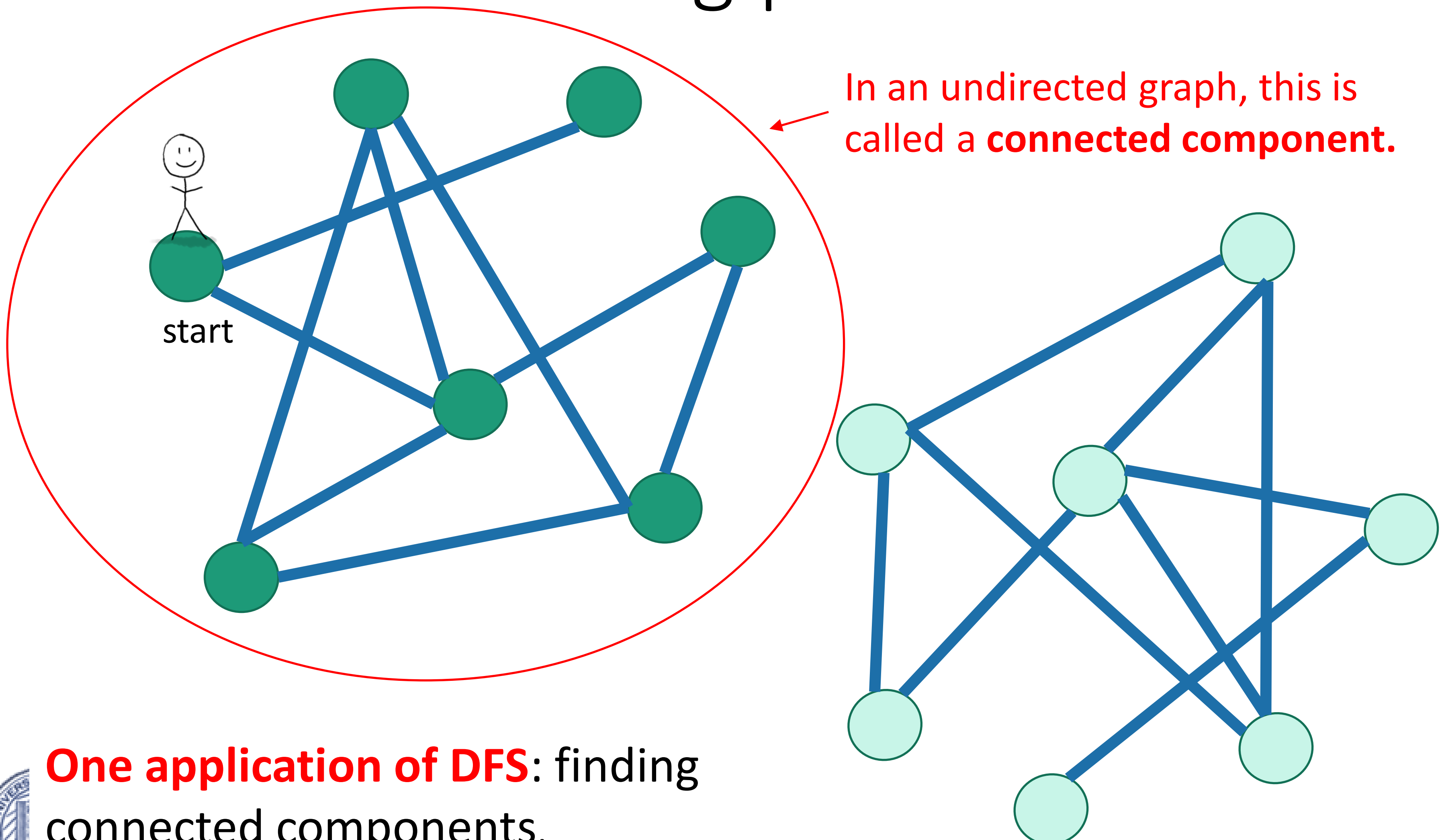
  - **return** currentTime

Fun exercise

- Write pseudocode for an iterative version of DFS.

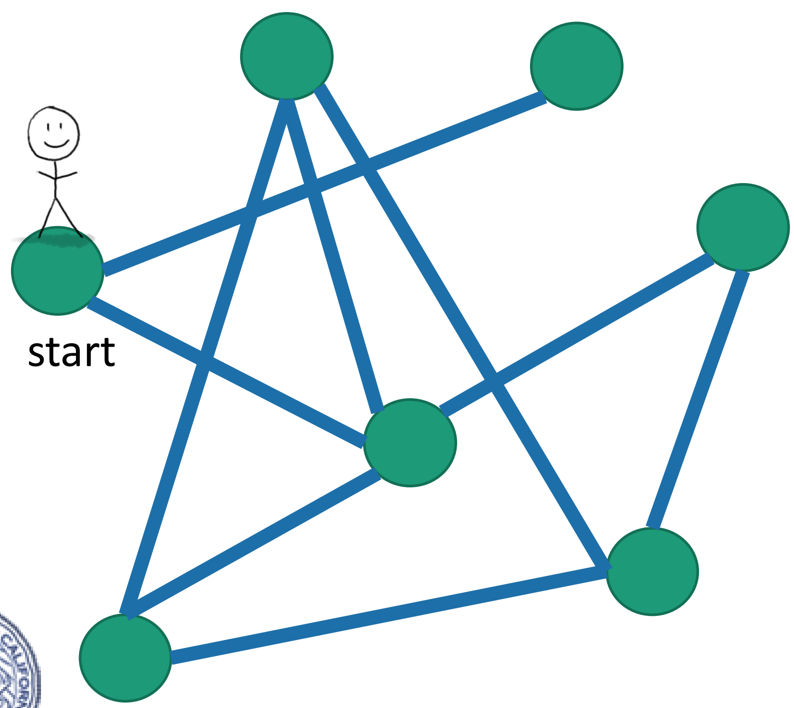
DFS finds all the nodes reachable from the starting point

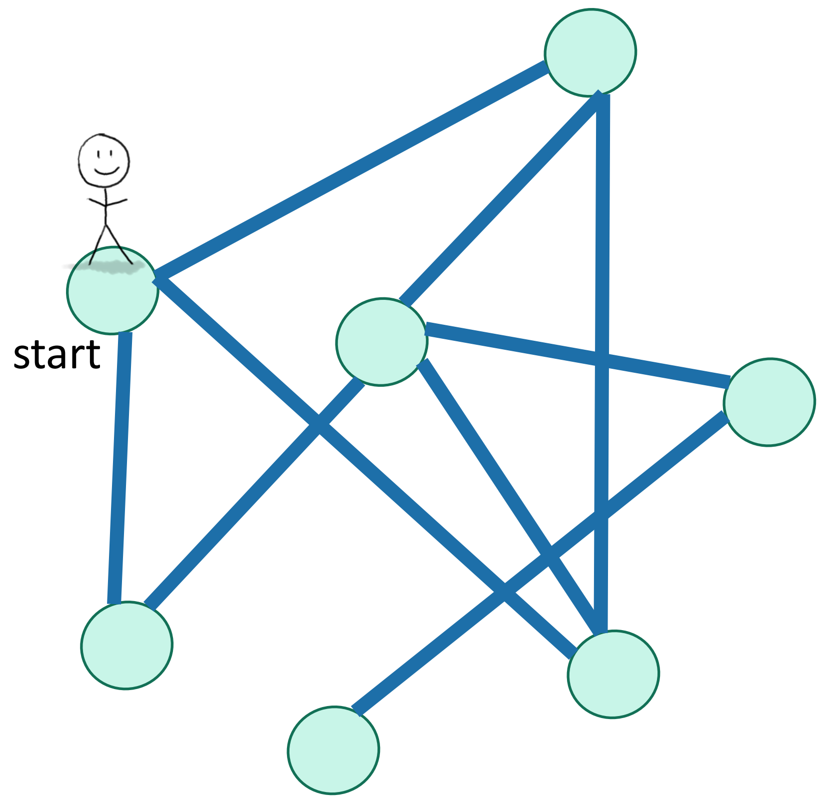


One application of DFS: finding connected components.

To explore the whole graph

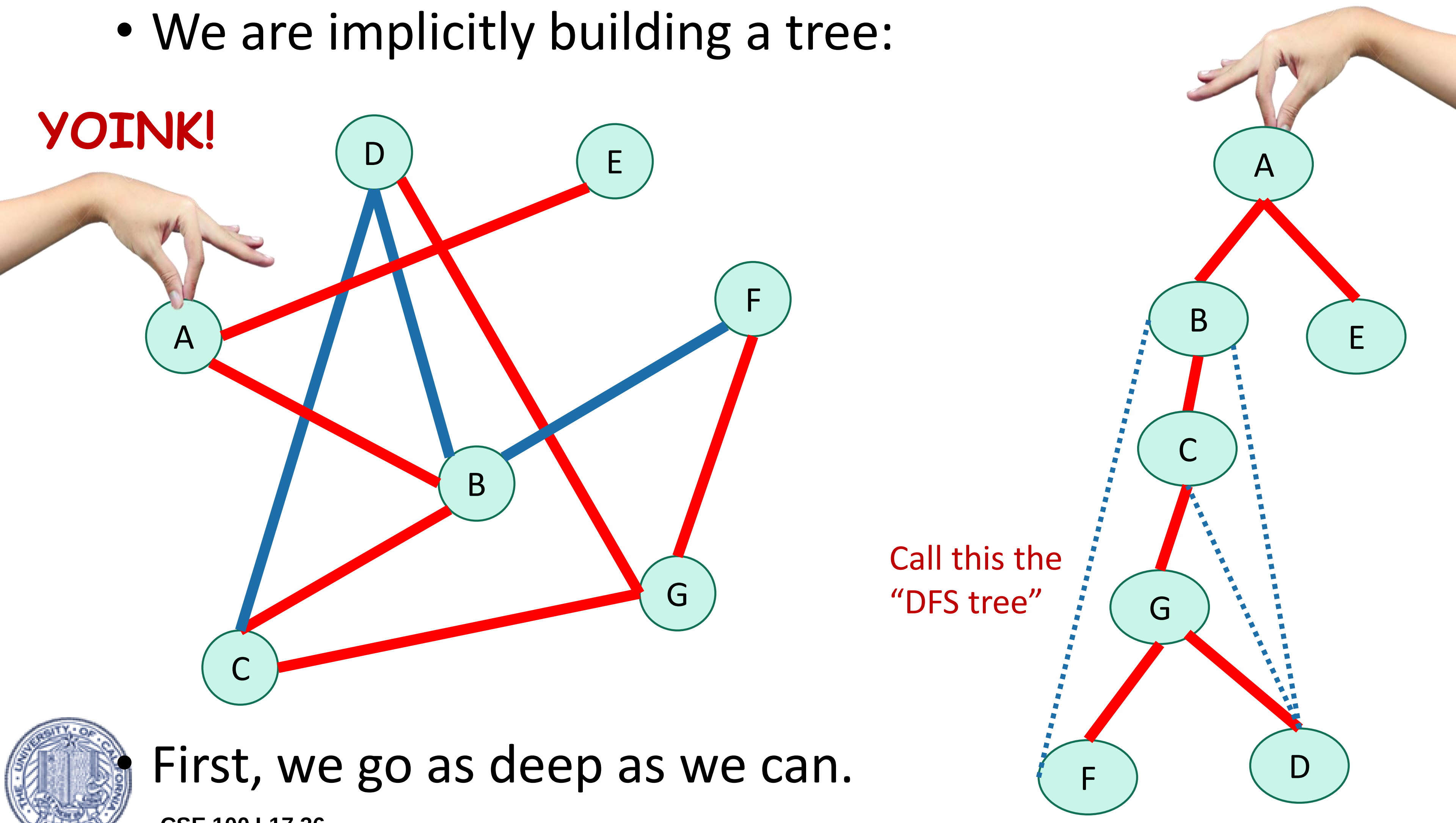
- Do it repeatedly!





Why is it called depth-first?

- We are implicitly building a tree:



- First, we go as deep as we can.

Running time

To explore **just the connected component** we started in

- We look at each edge at most twice.
  - Once from each of its endpoints
- And basically we don't do anything else.
- So…

$$O(m)$$

Running time

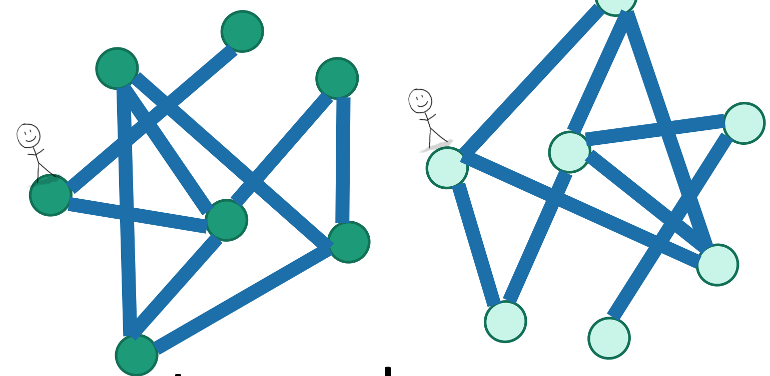
To explore just the connected component we started in

- Assume we are using the linked-list format for G.
- Say C = (V', E') is a connected component.
- We visit each vertex in C exactly once.
  - Here, "visit" means "call DFS on"
- At each vertex w, we:
  - Do some book-keeping: O(1)
  - Loop over w's neighbors and check if they are visited (and then potentially make a recursive call): O(1) per neighbor or O(deg(w)) total.
- Total time:
  - $\sum_{w \in V'} (O(\deg(w)) + O(1))$
  - $= O(|E'| + |V'|)$
  - $= O(|E'|)$

In a connected graph, $|V'| \leq |E'| + 1$.

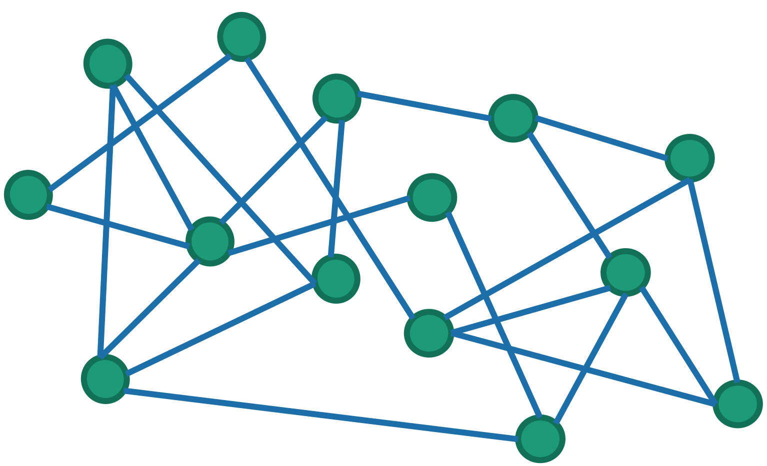
Running time

To explore the whole graph



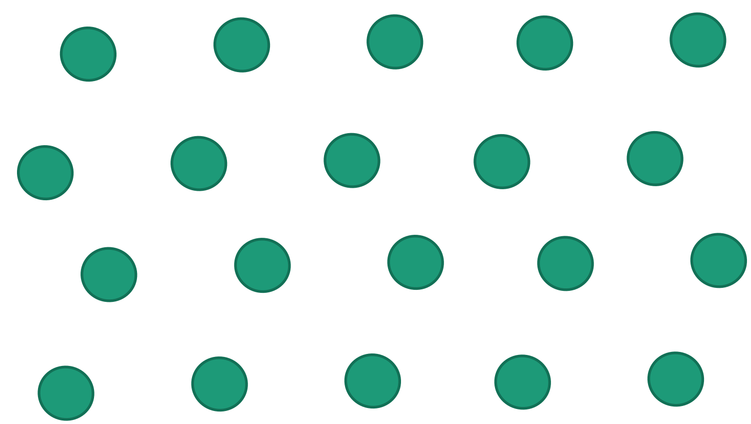
- Explore the connected components one-by-one.
- This takes time O(n + m)
  - Same computation as before:

$$\sum_{w \in V} (O(\deg(w)) + O(1)) = O(|E| + |V|) = O(n + m)$$



Here the running time is O(m) like before

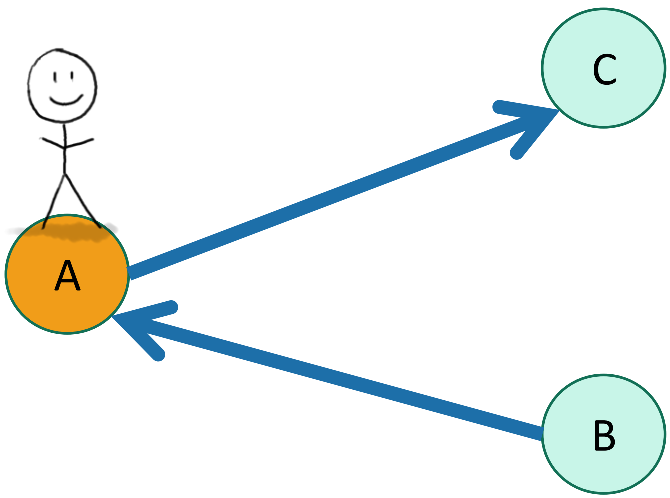
or



Here m=0 but it still takes time O(n) to explore the graph.

You check:

DFS works fine on directed graphs too!



Only walk to C, not to B.

Siggi the studious stork

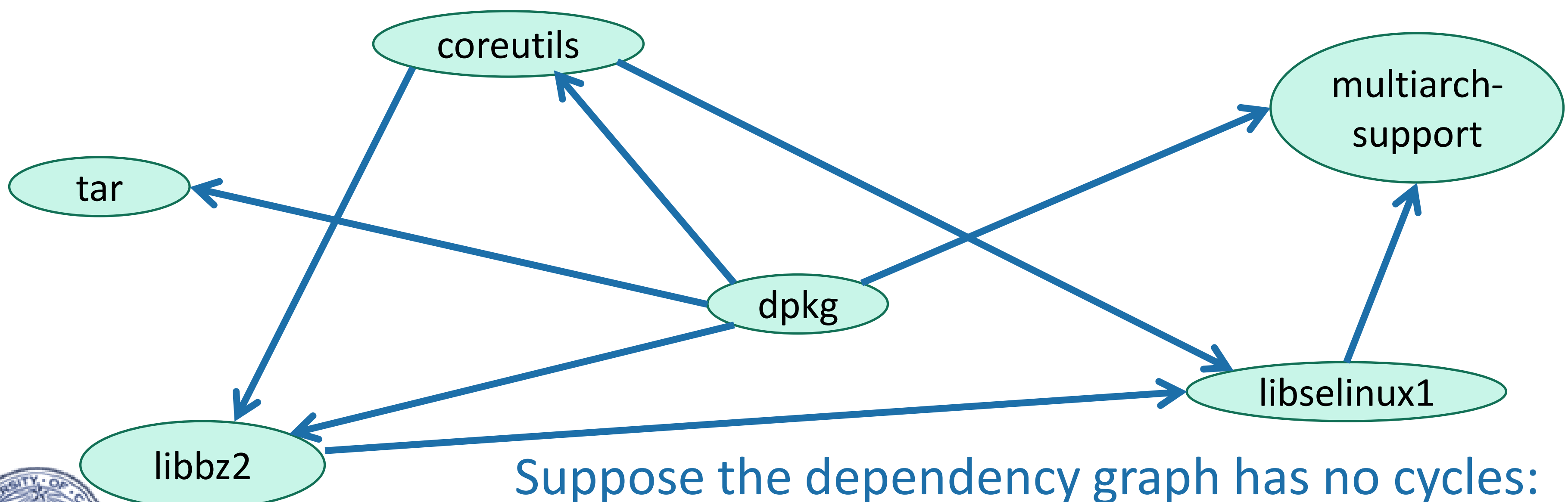
Some exercises

- How can you sign up for classes so that you never violate the pre-req requirements?

- More practically, how can you install packages without violating dependency requirements?

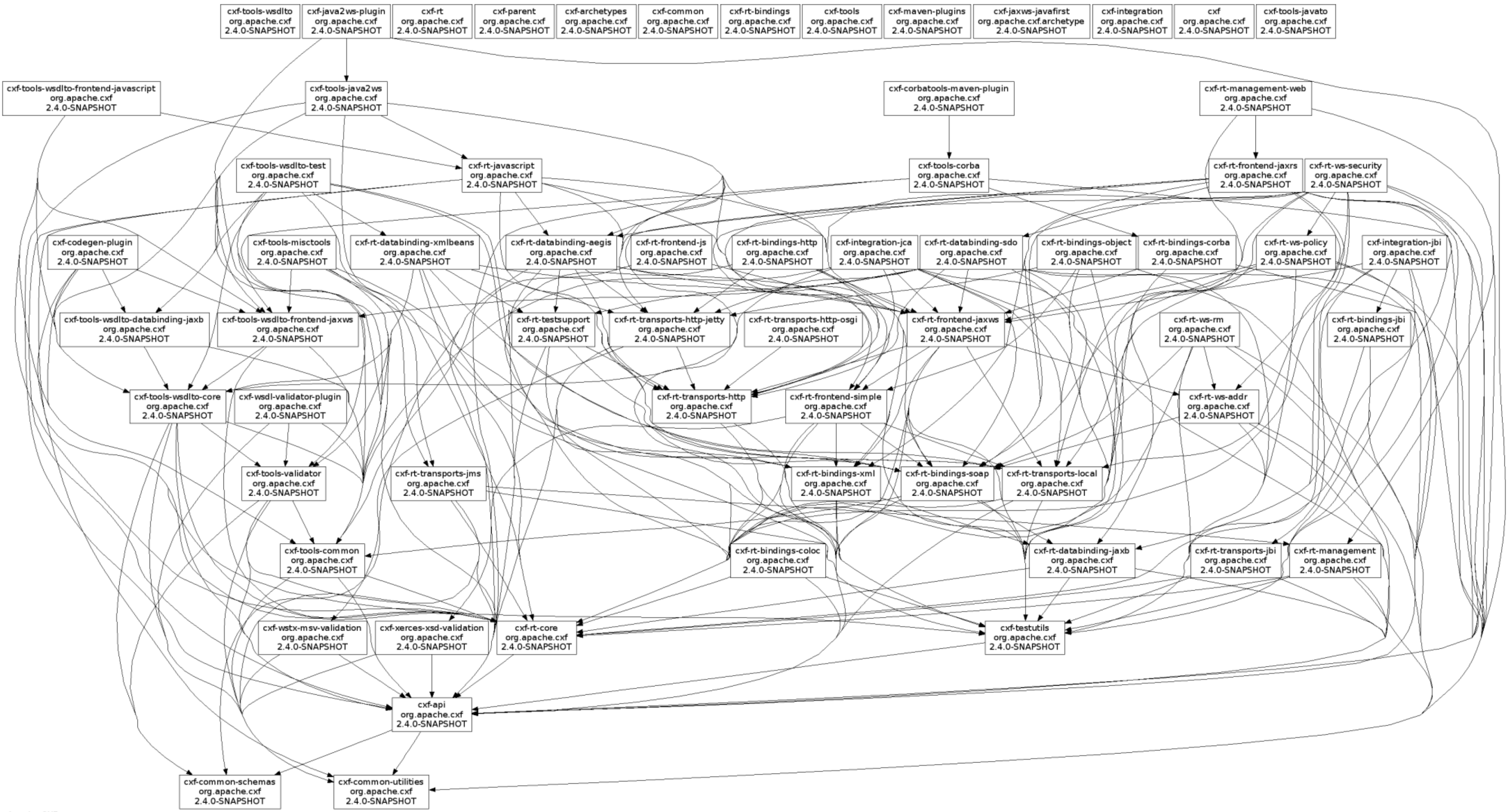
Application of DFS: topological sorting

- Find an ordering of vertices so that all of the dependency requirements are met.
  - Aka, if v comes before w in the ordering, there is not an edge from w to v.



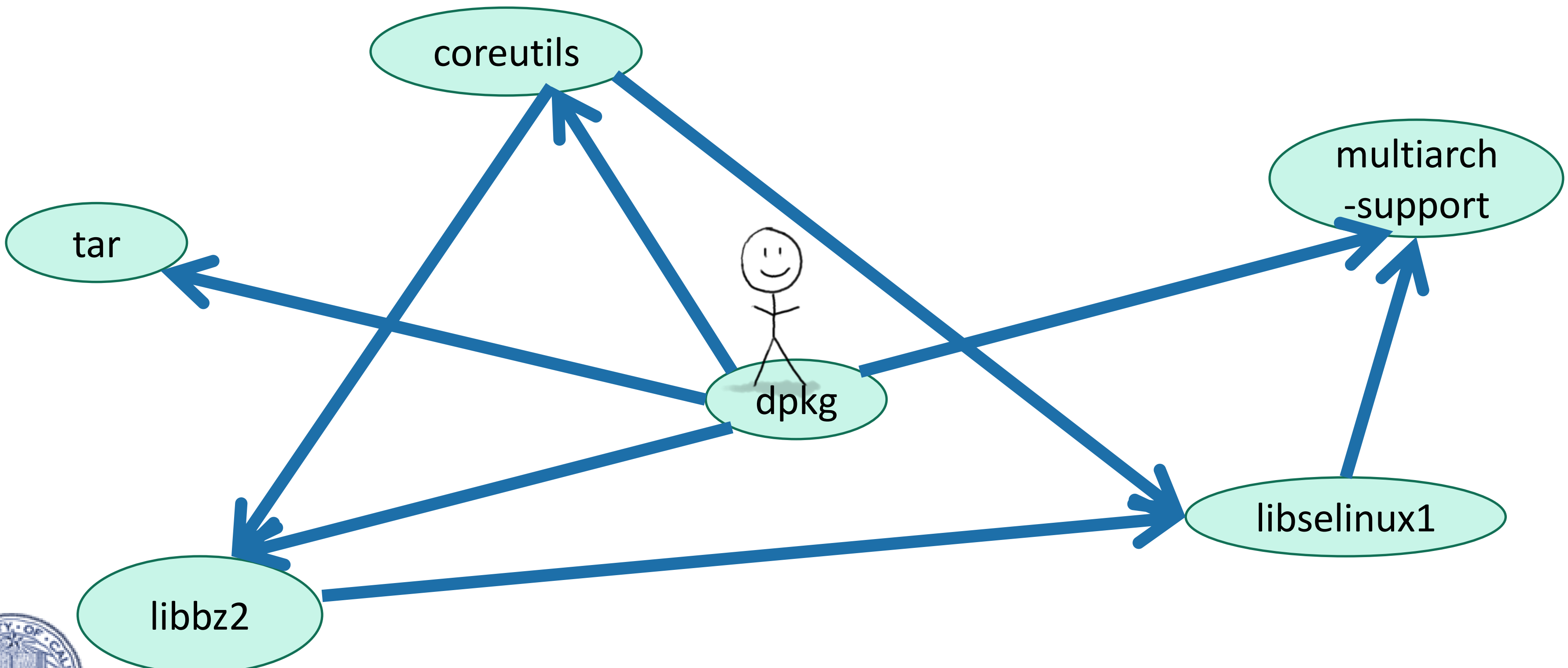
Suppose the dependency graph has no cycles: it is a **Directed Acyclic Graph (DAG)**

Can’t always eyeball it.



Apache CXF

Let's do DFS



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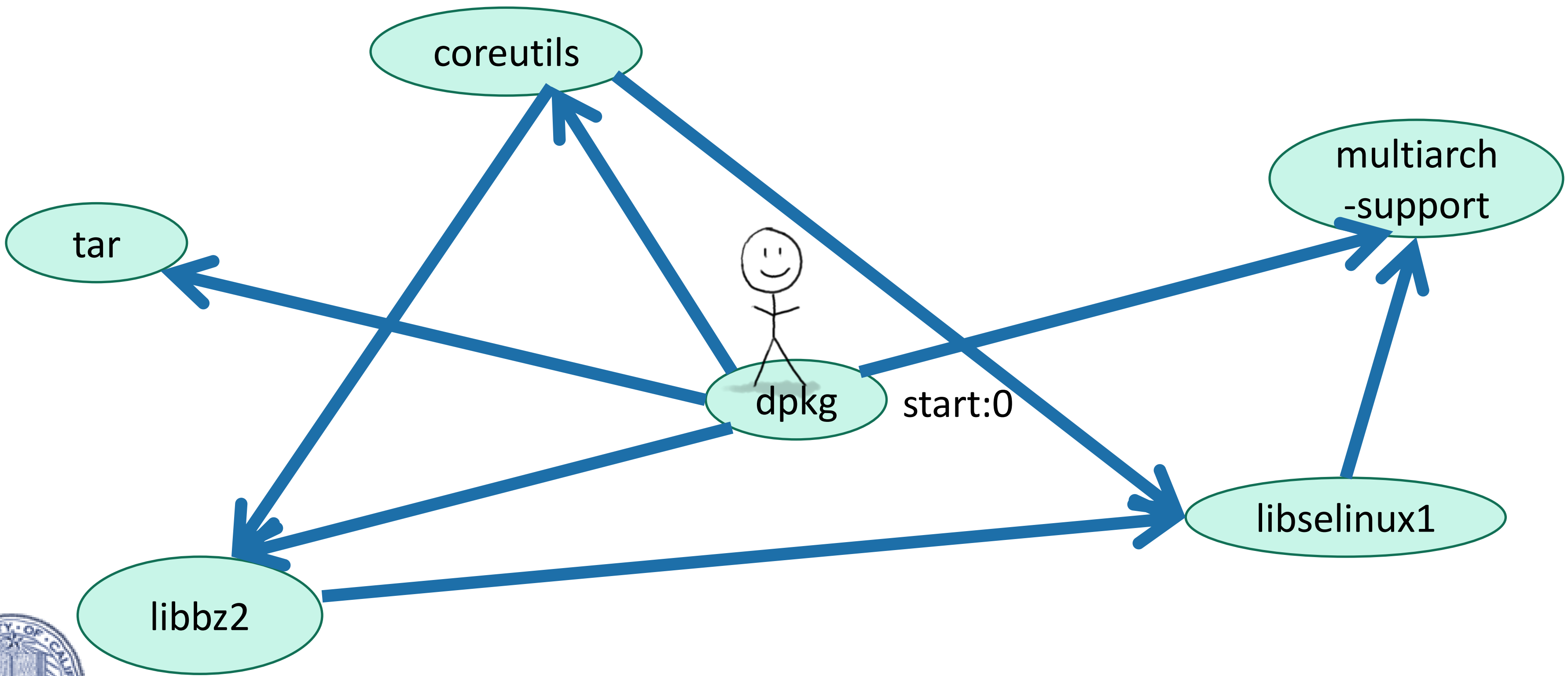
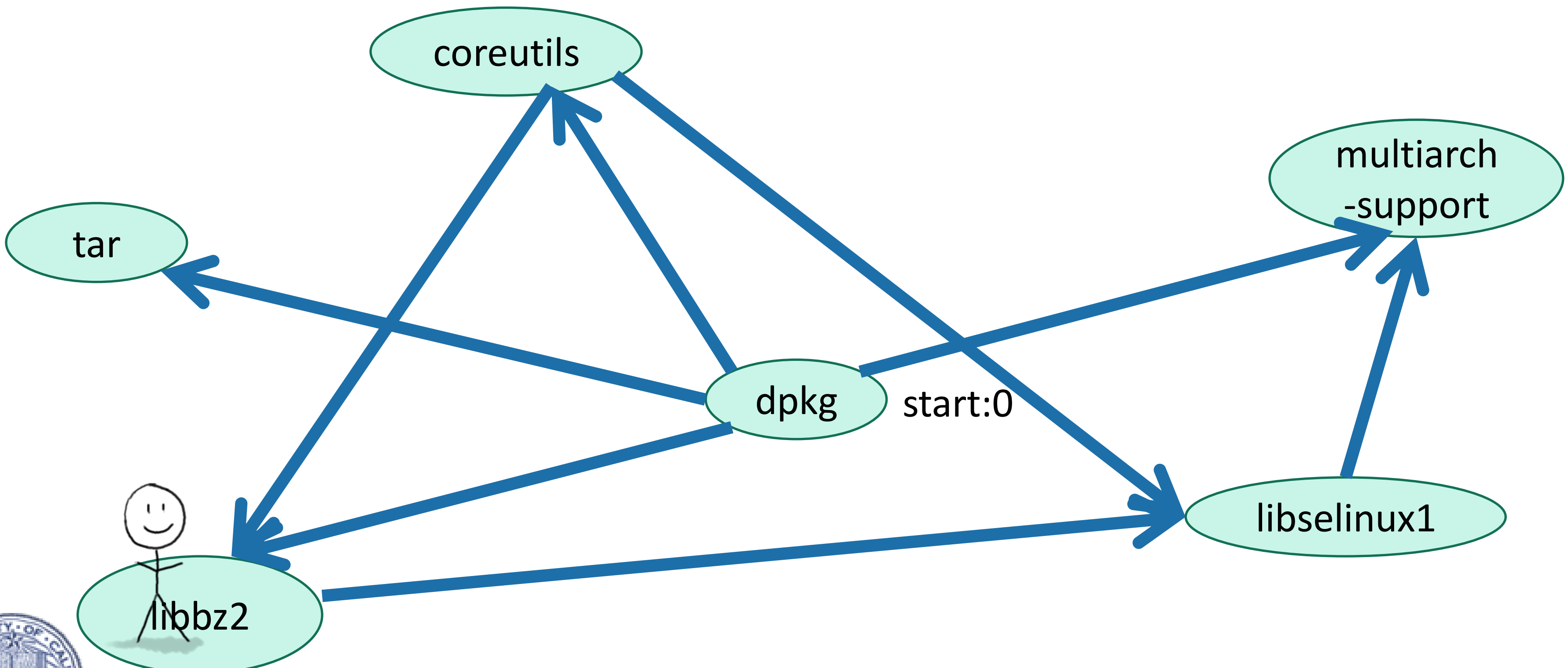


coreutils
multiarch
-support
tar
dpkg
start:0
libselinux1
libbz2

Let's do DFS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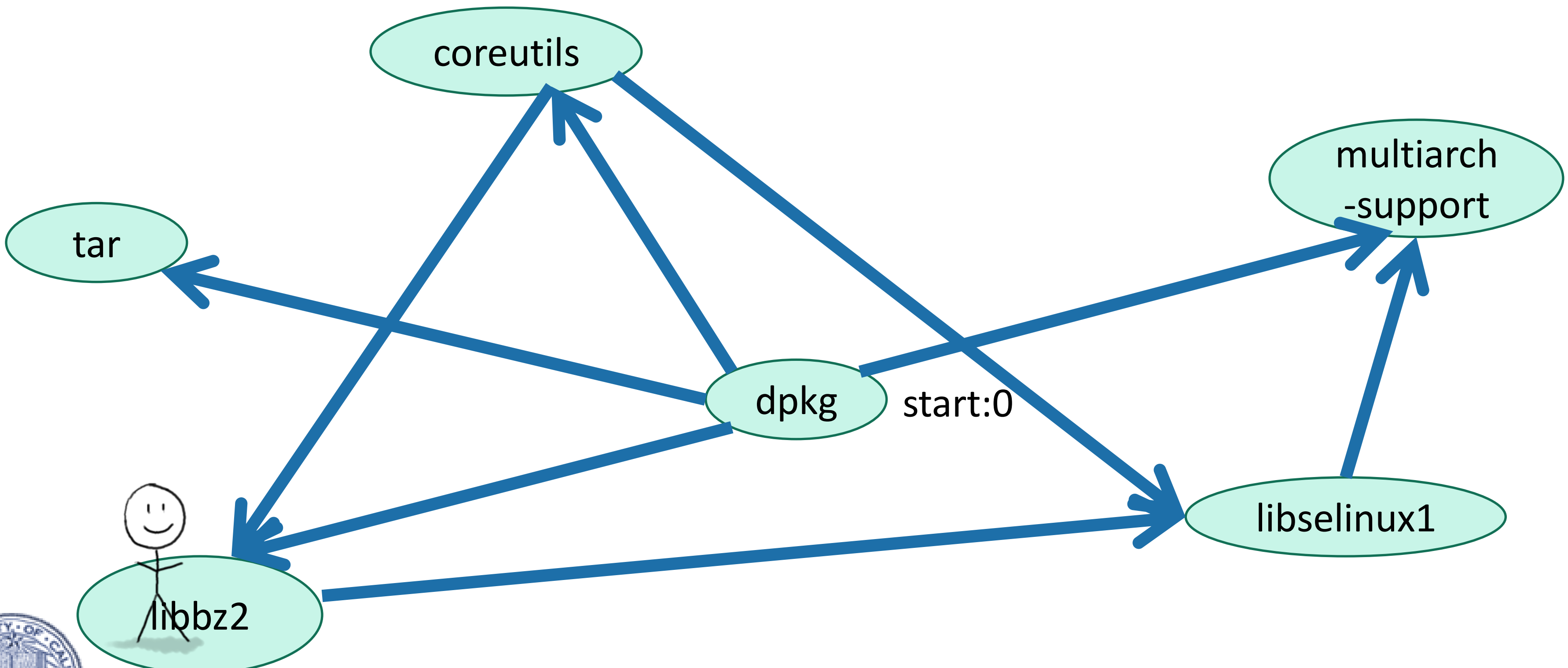
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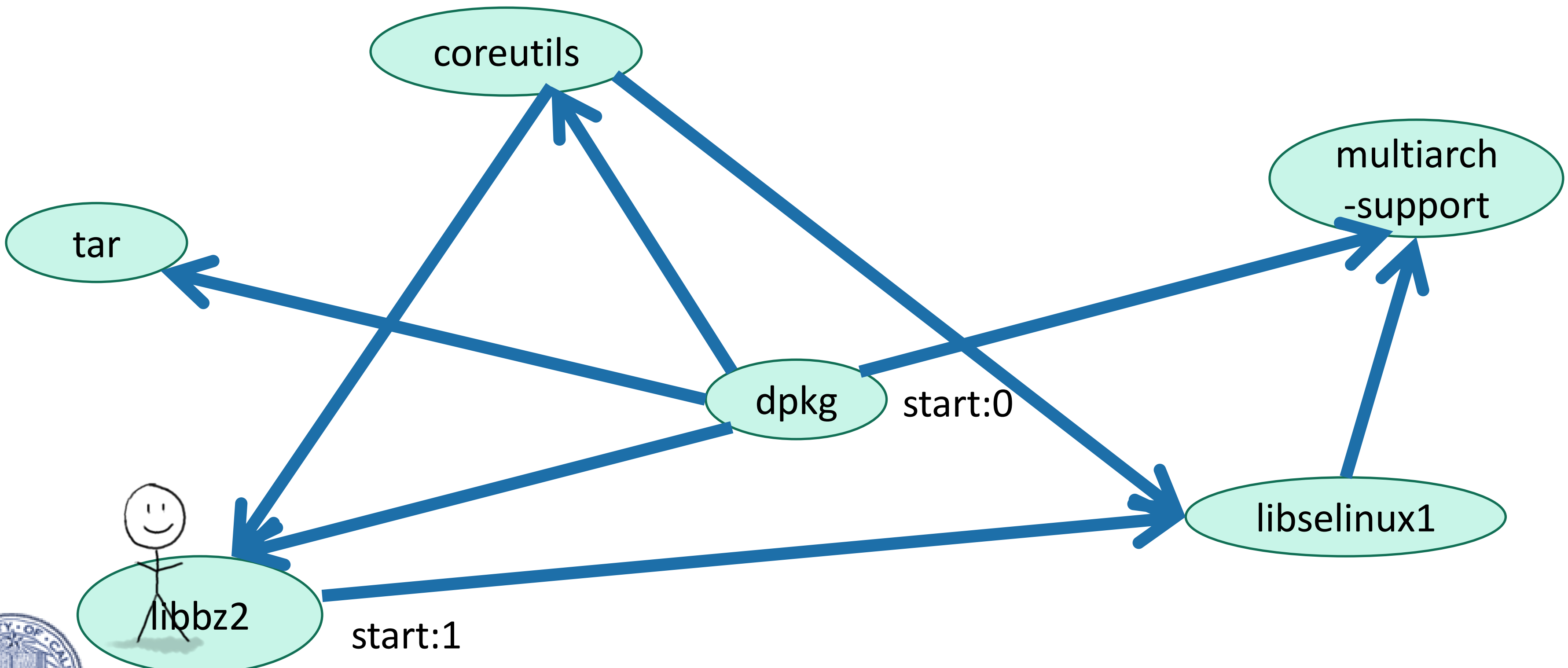
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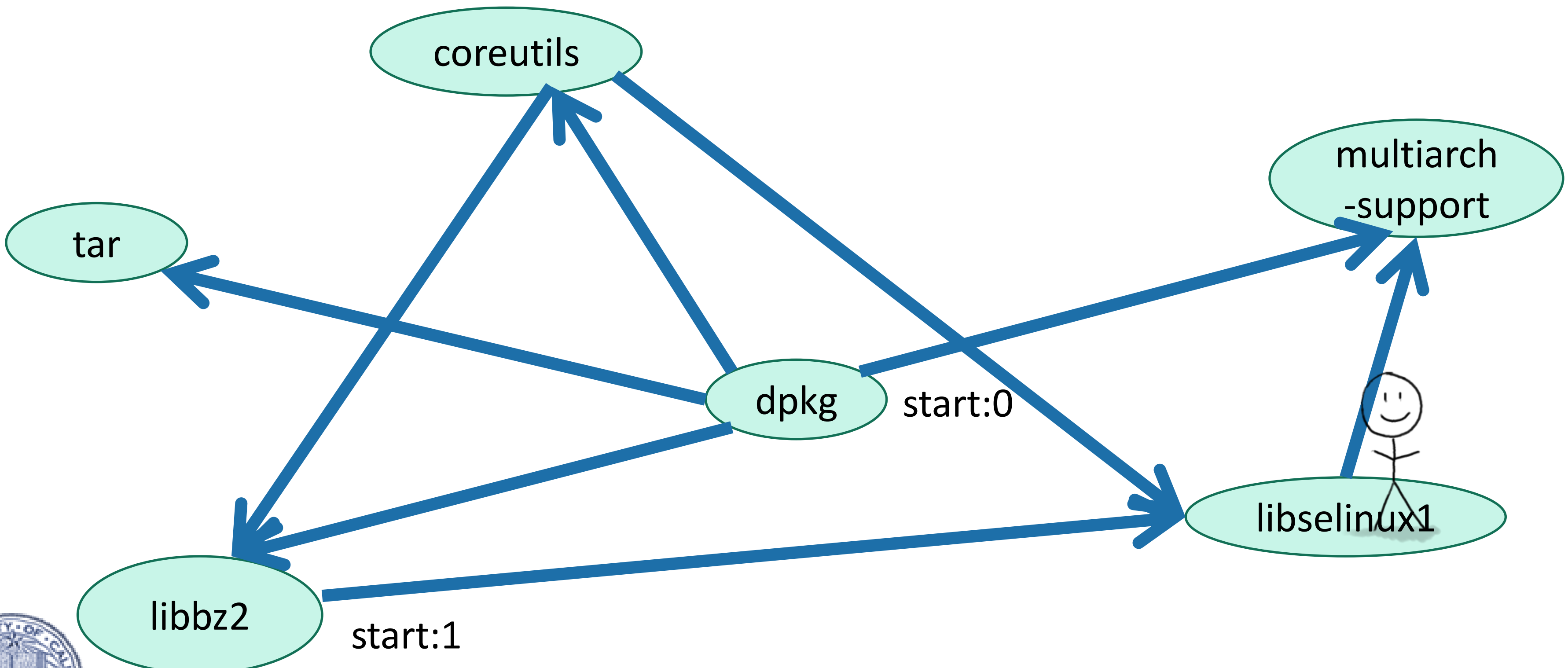
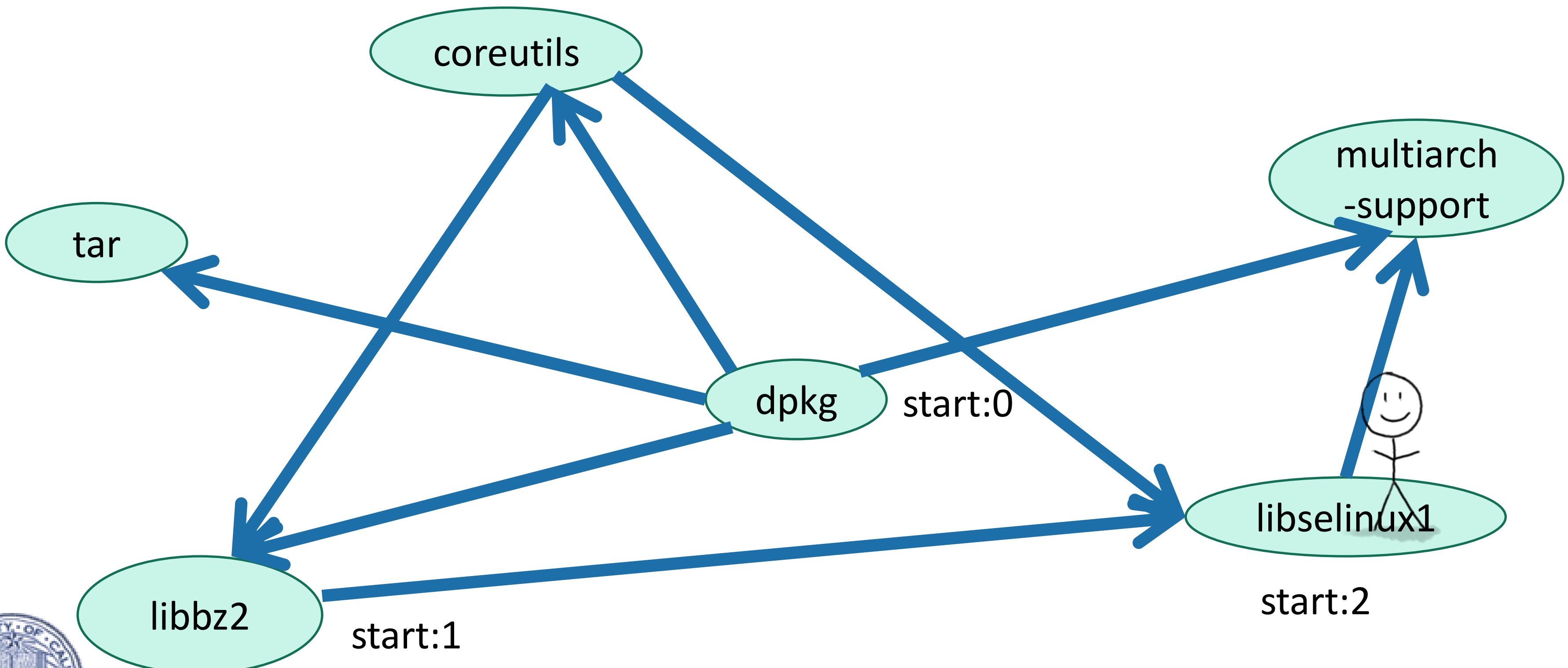
start:1

Let's do DFS



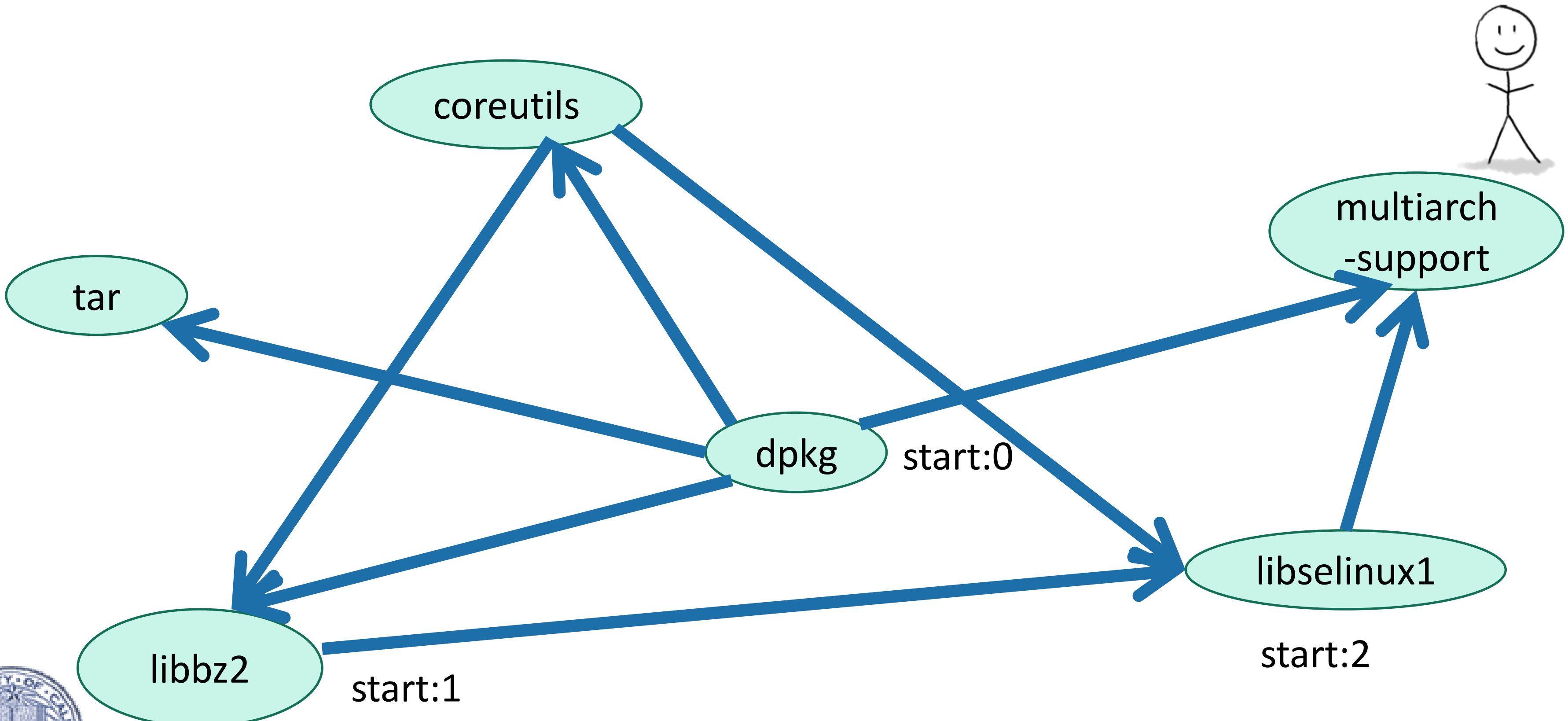
coreutils
multiarch
-support
tar
dpkg
start:0
libselinux1
libbz2
start:1

Let's do DFS

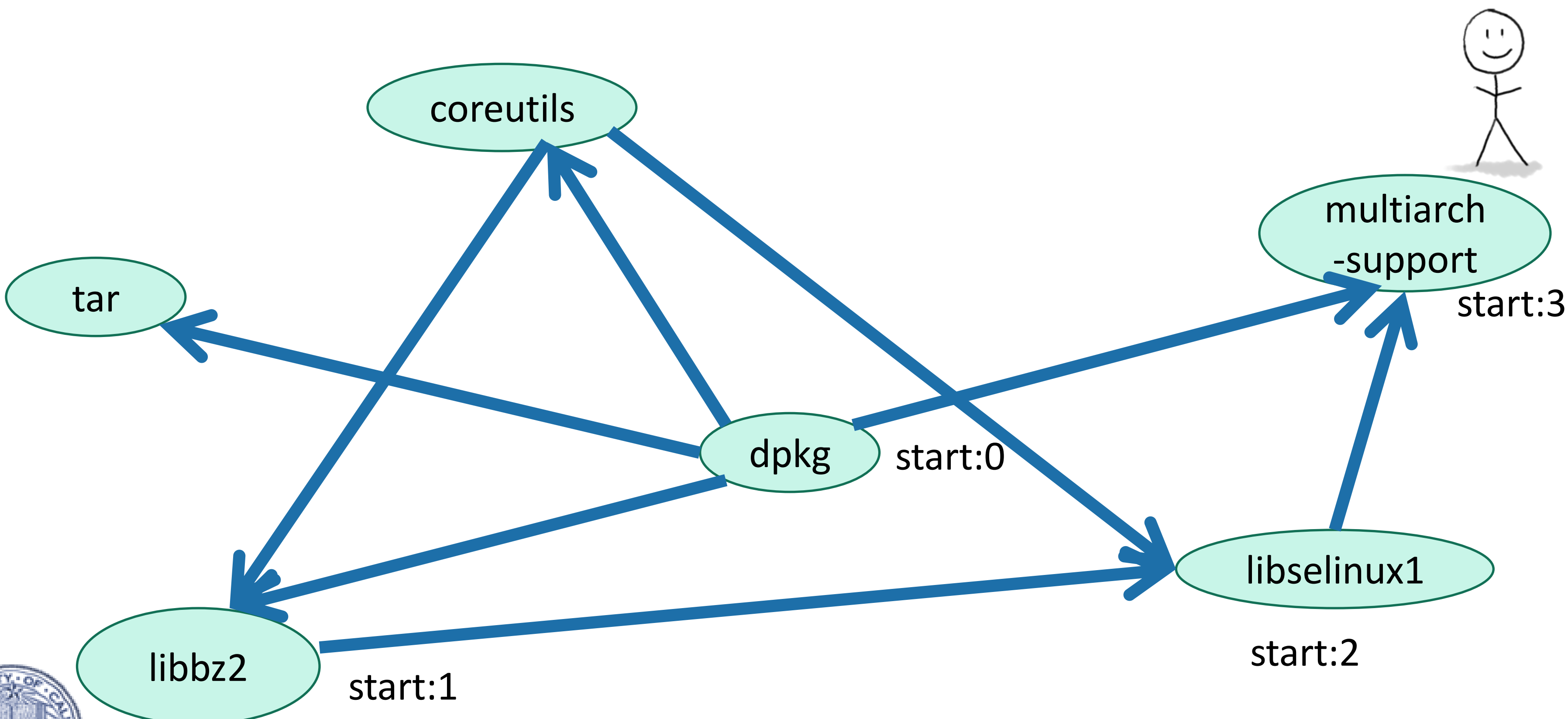


coreutils
multiarch
-support
tar
dpkg
start:0
libselinux1
start:2
libbz2
start:1

Let’s do DFS

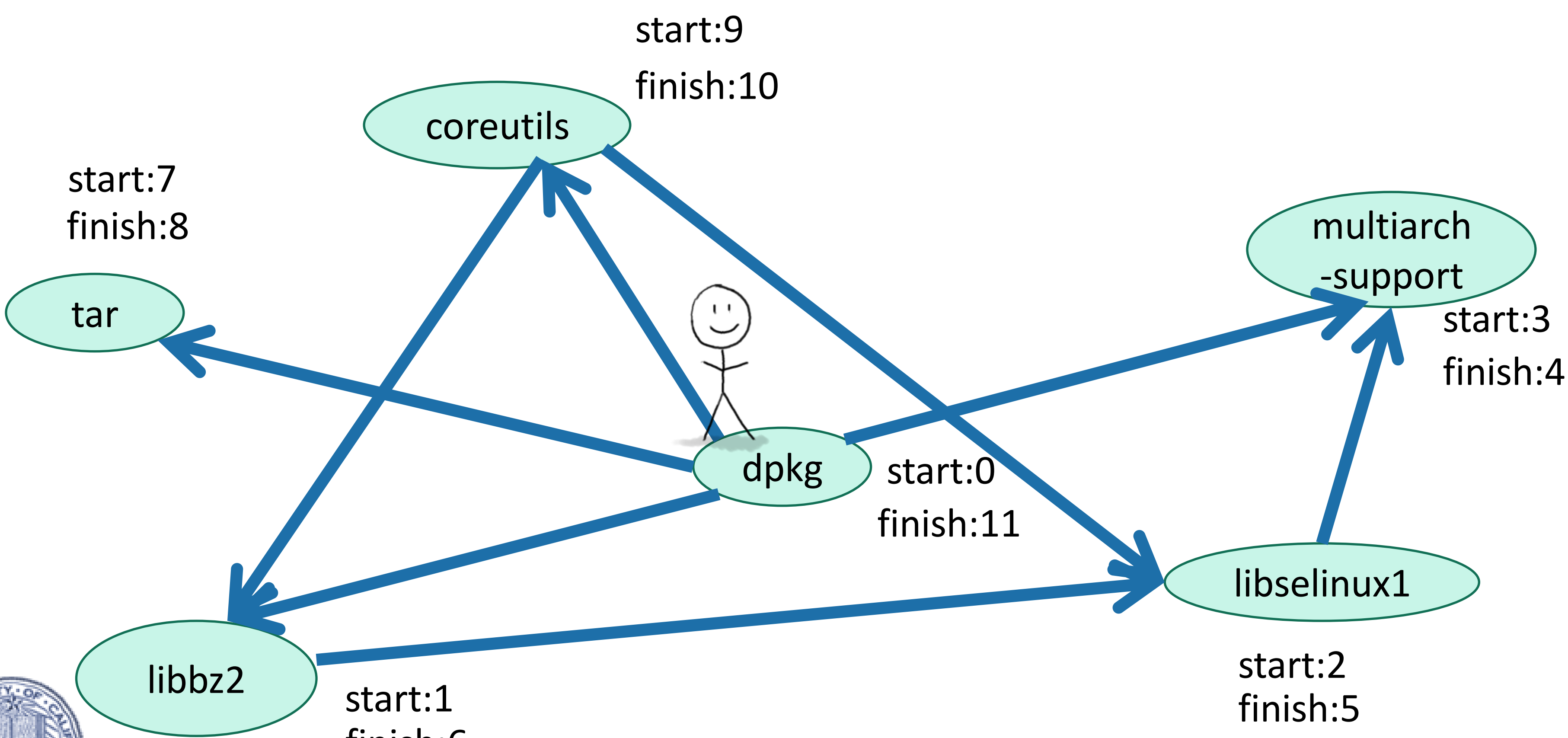

coreutils
multiarch
-support
tar
dpkg
start:0
libselinux1
libbz2
start:1
start:2

Let’s do DFS



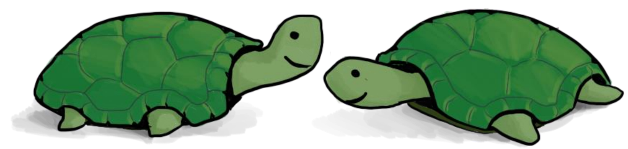
coreutils
tar
multiarch
-support
start:3
dpkg
start:0
libselinux1
libbz2
start:1
start:2

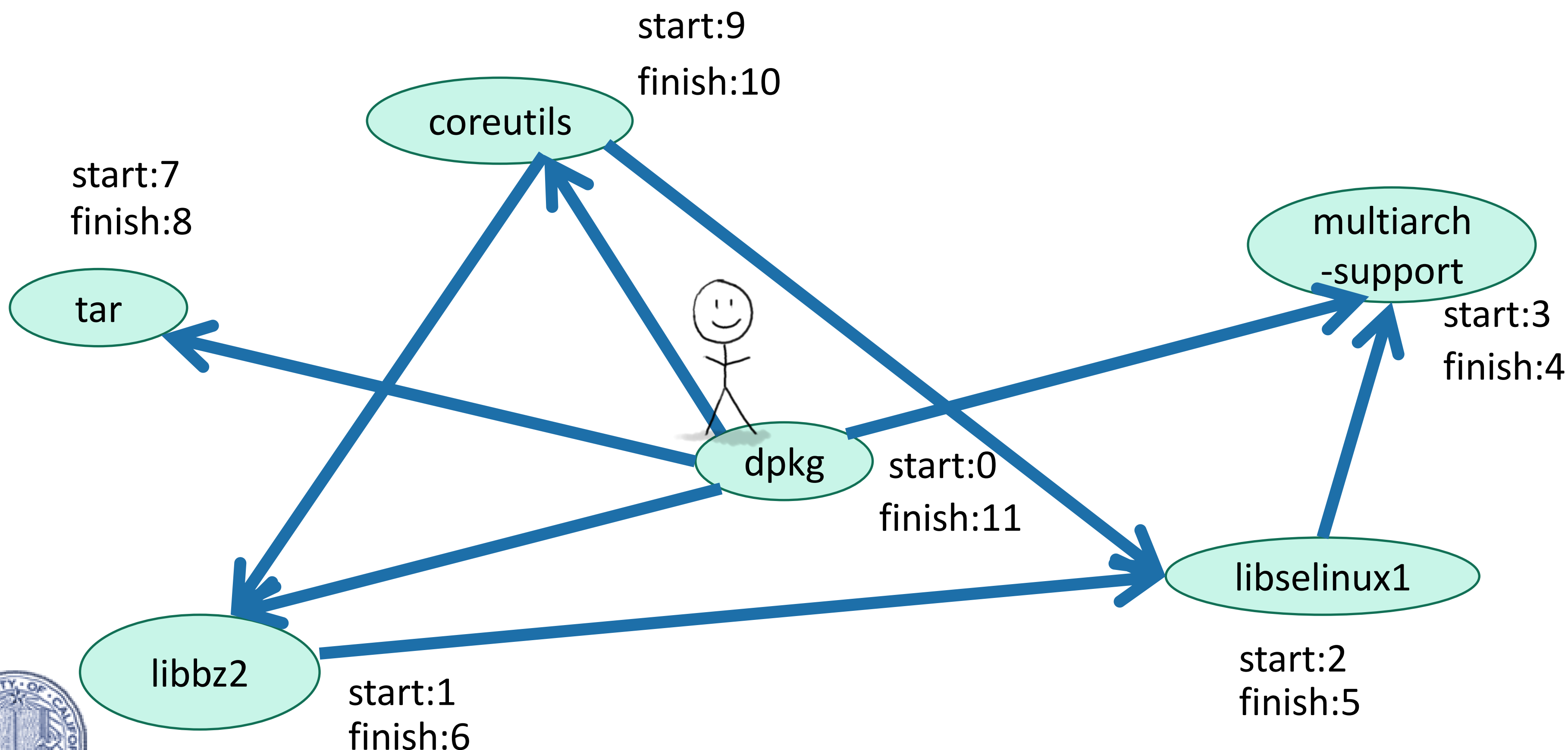
Let’s do DFS


start:9
finish:10
coreutils
start:7
finish:8
tar
multiarch
-support
start:3
finish:4
dpkg
start:0
finish:11
libselinux1
libbz2
start:1
finish:6
start:2
finish:5

Let’s do DFS

What do you notice about the finish times? Any ideas for how we should do topological sort?





Suppose the underlying graph has no cycles

Finish times seem useful

Claim: In general, we'll always have:

finish: [larger]   finish: [smaller]

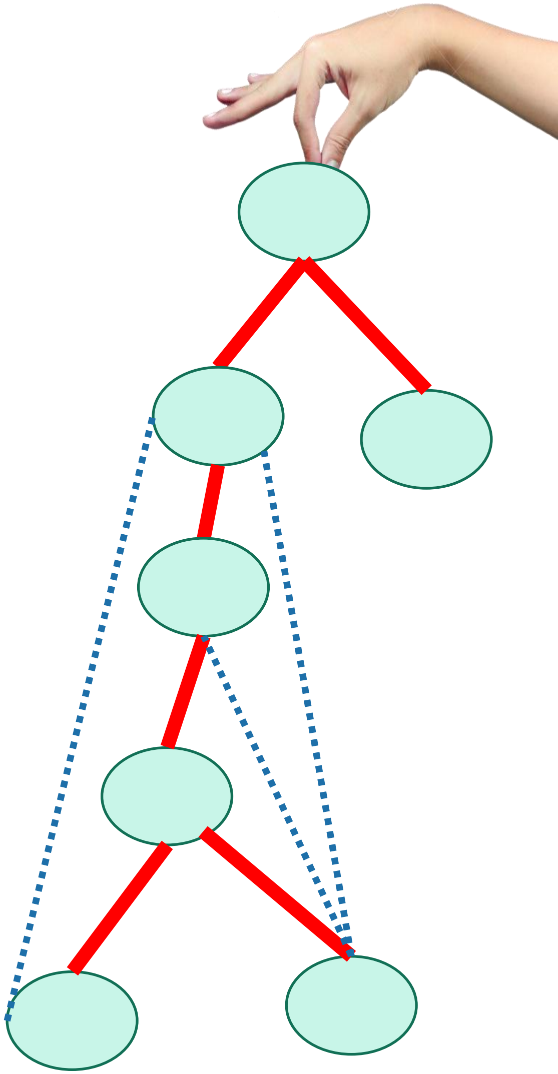
To understand why, let's go back to that DFS tree.

A more general statement

(this holds even if there are cycles)

This is called the “parentheses theorem” in CLRS

(check this statement carefully!)



A more general statement

(check this statement carefully!)

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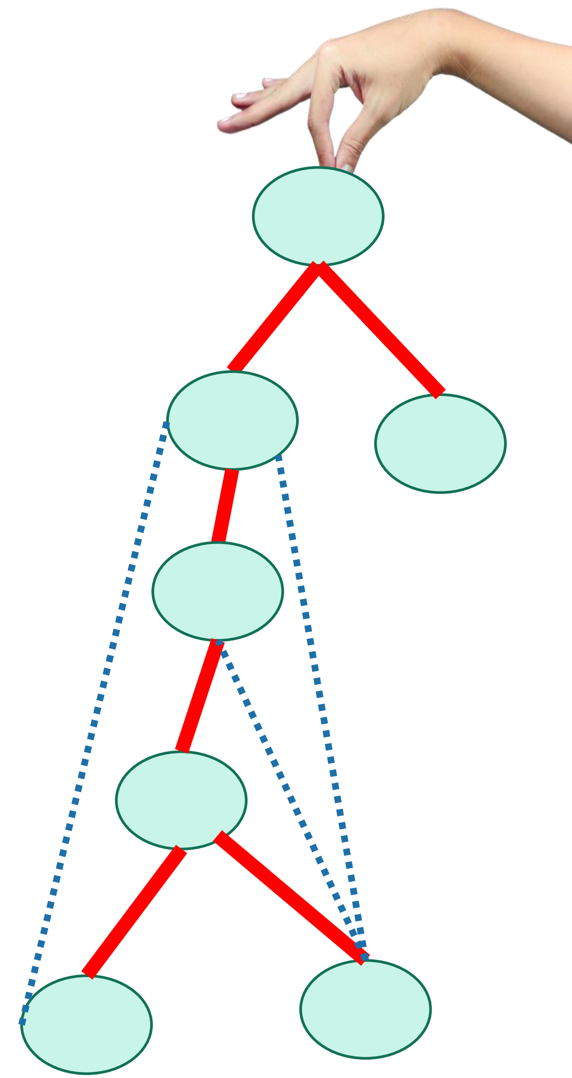
- If v is a descendant of w in this tree:



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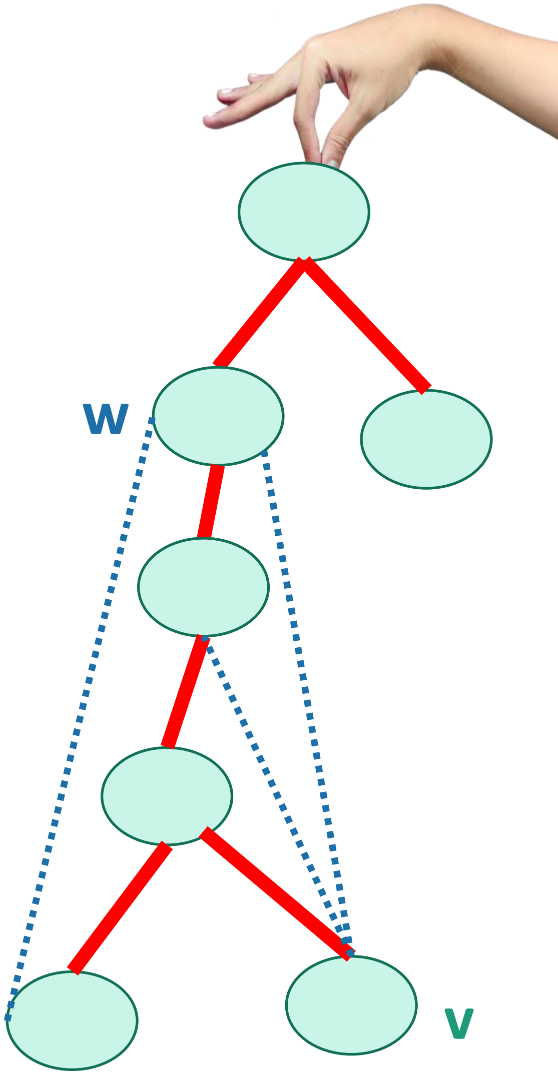

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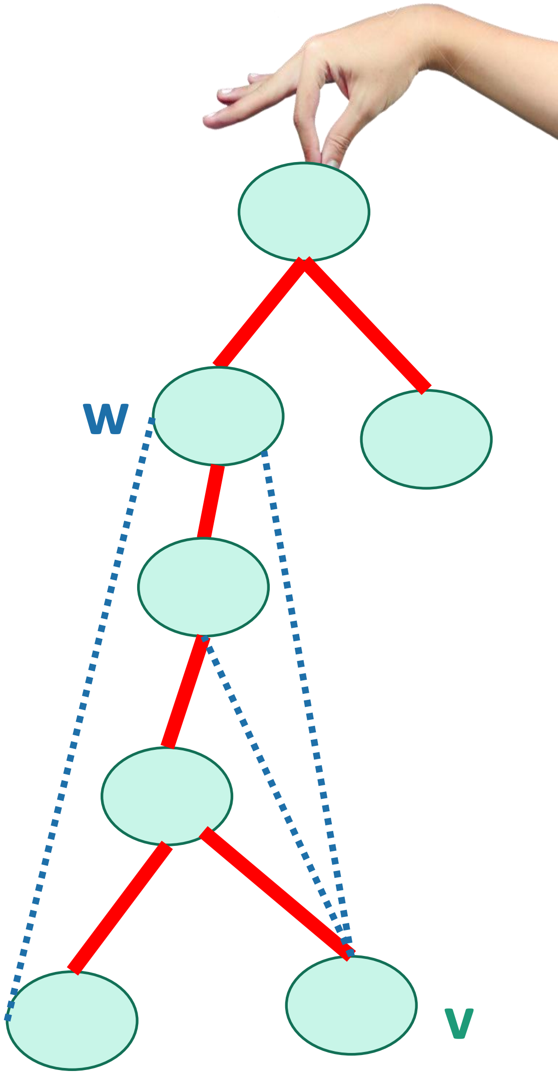

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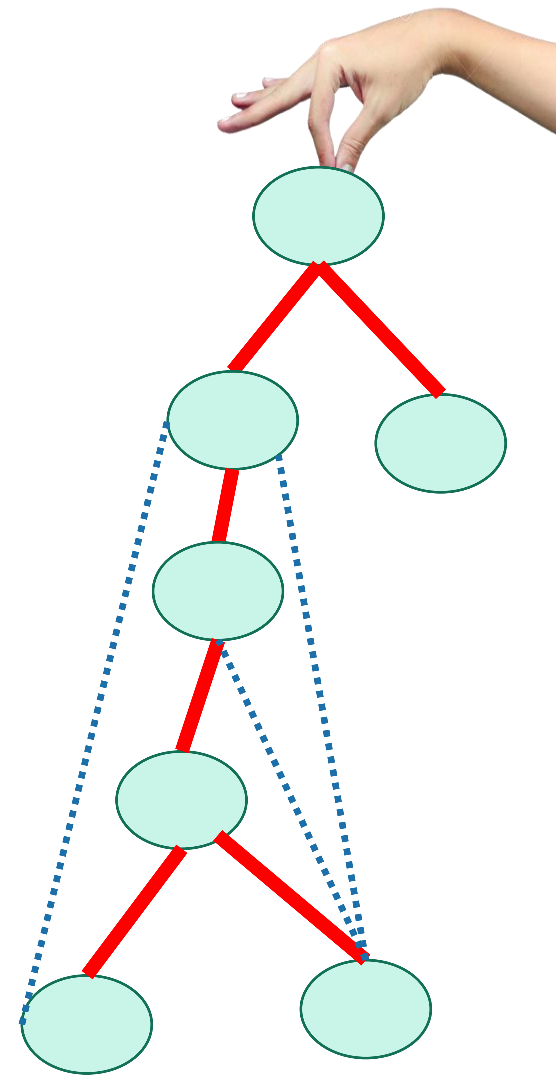


A more general statement

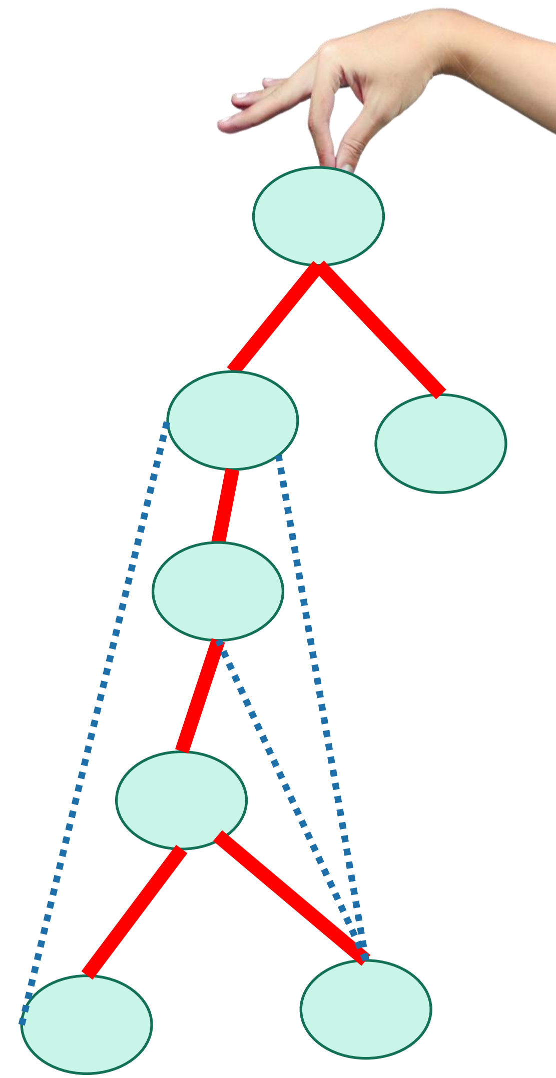
(this holds even if there are cycles)

This is called the “parentheses theorem” in CLRS

(check this statement carefully!)

- If v is a descendant of w in this tree:

- If w is a descendant of v in this tree:



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(check this statement carefully!)

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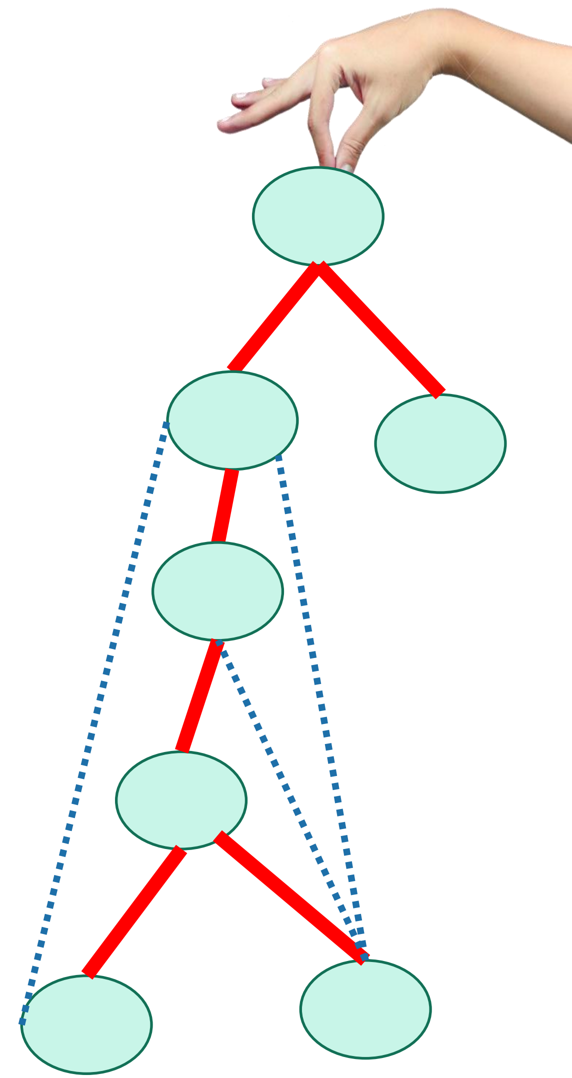
- If w is a descendant of v in this tree:



A more general statement

(this holds even if there are cycles)

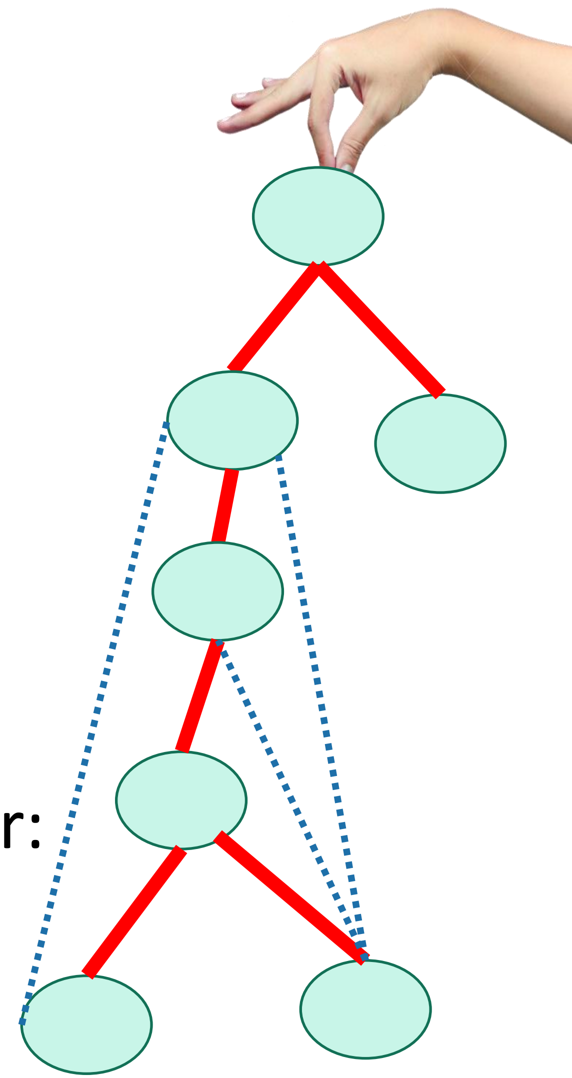
This is called the “parentheses theorem” in CLRS

(check this statement carefully!)

- If v is a descendant of w in this tree:

- If w is a descendant of v in this tree:

- If neither are descendants of each other:



A more general statement

(this holds even if there are cycles)

This is called the “parentheses theorem” in CLRS

(check this statement carefully!)

- If v is a descendant of w in this tree:

- If w is a descendant of v in this tree:

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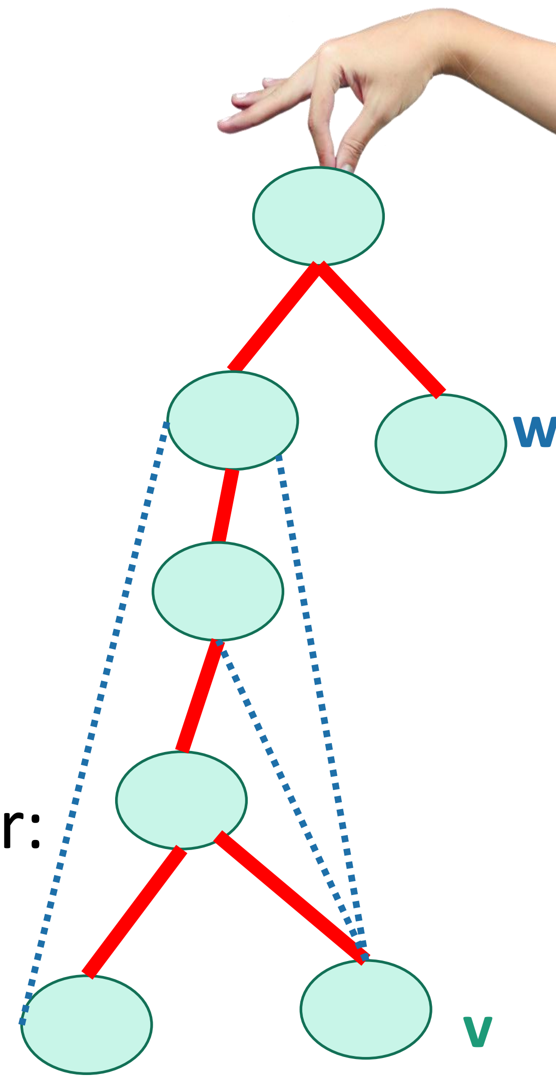

A more general statement

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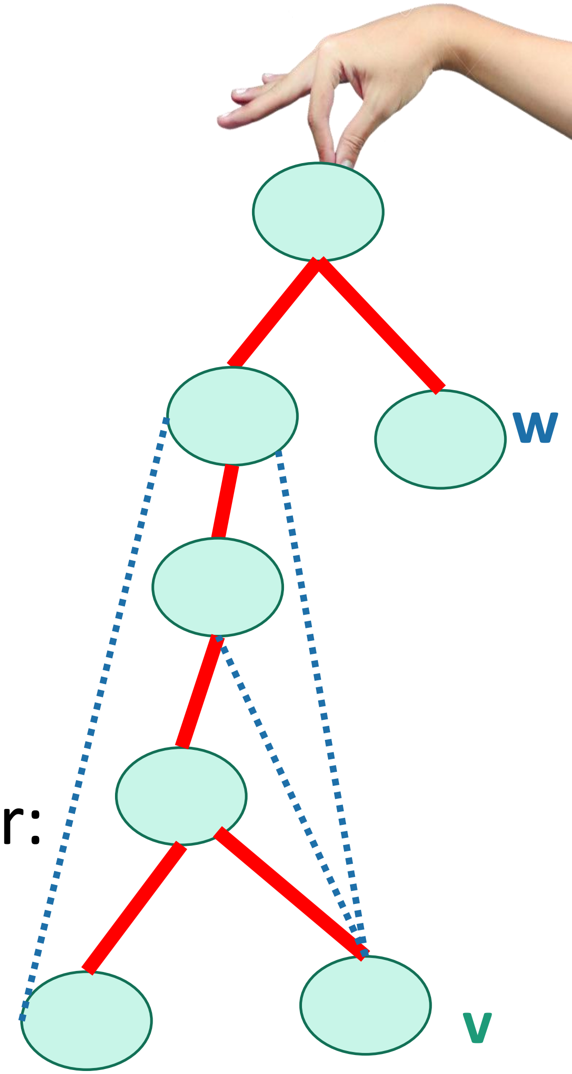
This is called the “parentheses theorem” in CLRS

(check this statement carefully!)



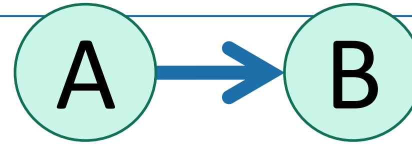
- If v is a descendant of w in this tree:

- If w is a descendant of v in this tree:

- If neither are descendants of each other:

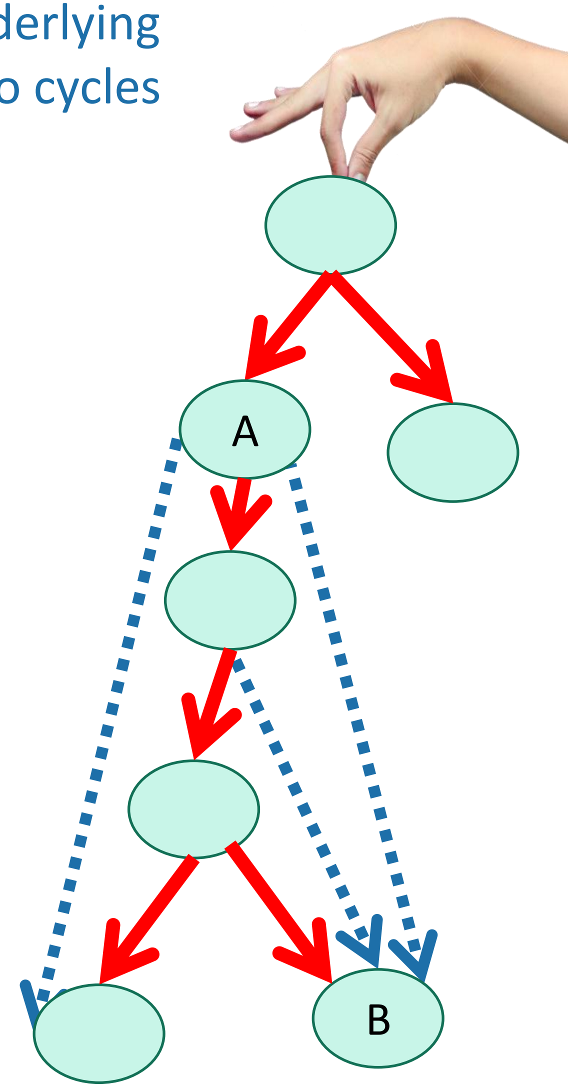
(or the other way around)

So to prove this →

If


Then B.finishTime < A.finishTime

Suppose the underlying graph has no cycles

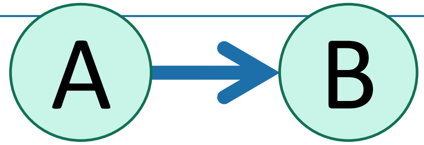


- **Case 1**: B is a descendant of A in the DFS tree.

- Then

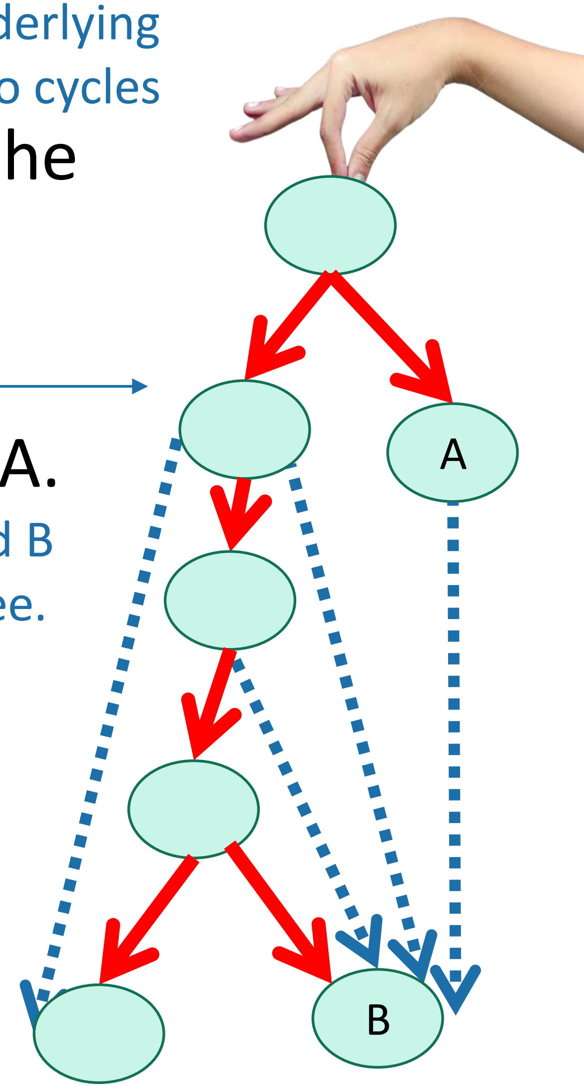
- aka, B.finishTime < A.finishTime.

So to prove this →

If 

Then B.finishTime < A.finishTime

Suppose the underlying graph has no cycles

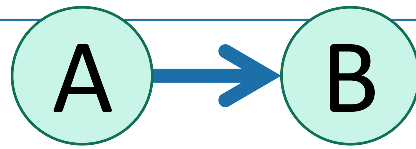


- **Case 2**: B is a NOT descendant of A in the DFS tree.
  - Notice that A can't be a descendant of B or else there'd be a cycle; so it looks like this
- Then we must have explored B before A.
  - Otherwise we would have gotten to B from A, and B would have been a descendant of A in the DFS tree.
- Then

- aka, B.finishTime < A.finishTime.

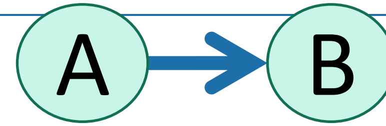
Theorem

- If we run DFS on a directed acyclic graph,

If 

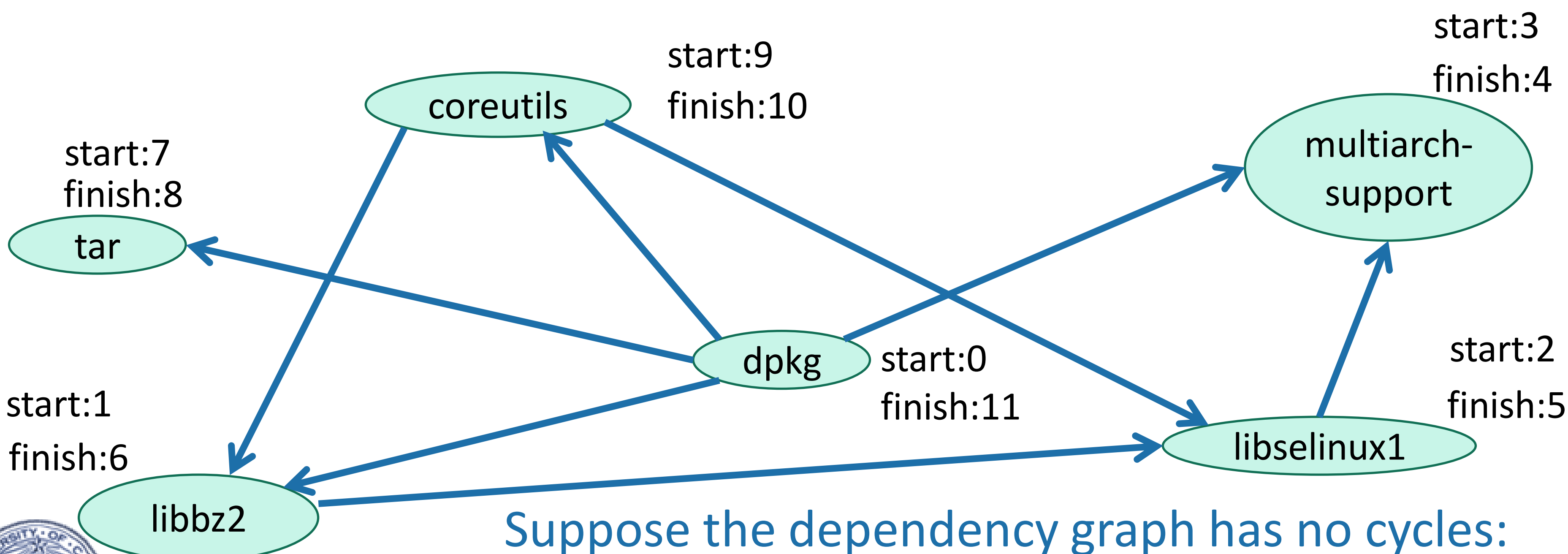
Then B.finishTime < A.finishTime

Back to topological sorting

If 

Then B.finishTime < A.finishTime

- In what order should I install packages?
- In reverse order of finishing time in DFS!

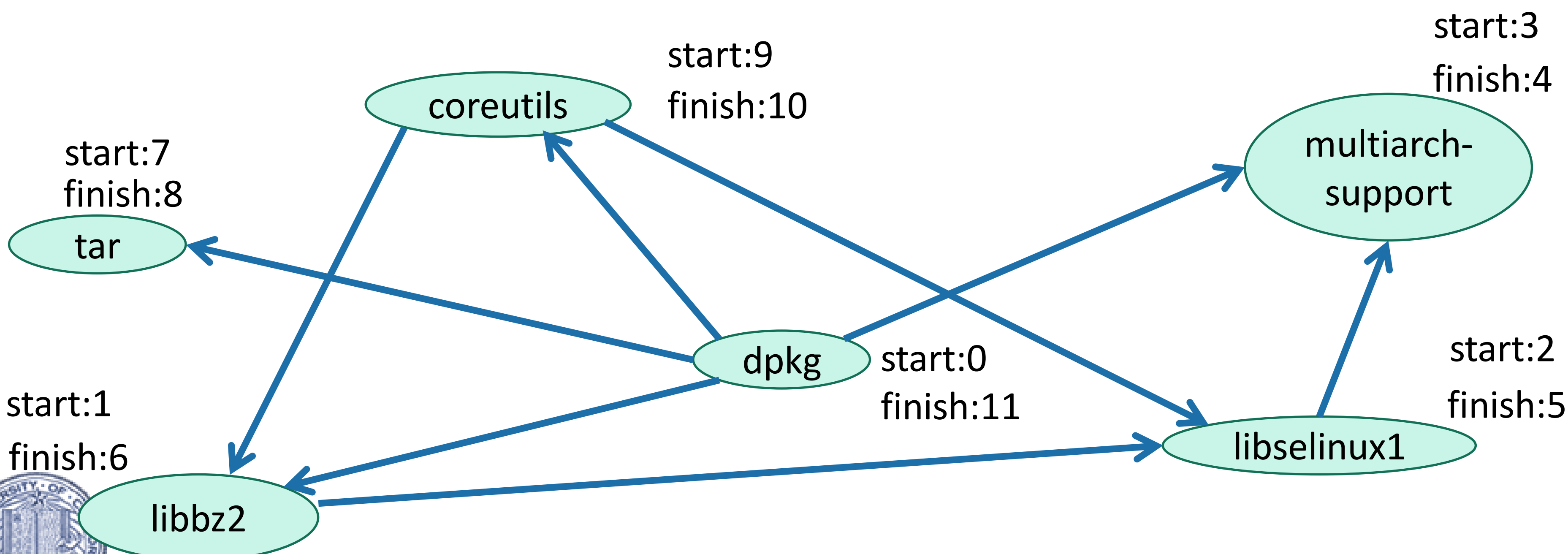


Suppose the dependency graph has no cycles: it is a **Directed Acyclic Graph (DAG)**

Topological Sorting (on a DAG)

- Do DFS
- When you mark a vertex as **all done,** put it at the **beginning** of the list.

- `dpkg`
- `coreutils`
- `tar`
- `libbz2`
- `libselinux1`
- `multiarch_support`

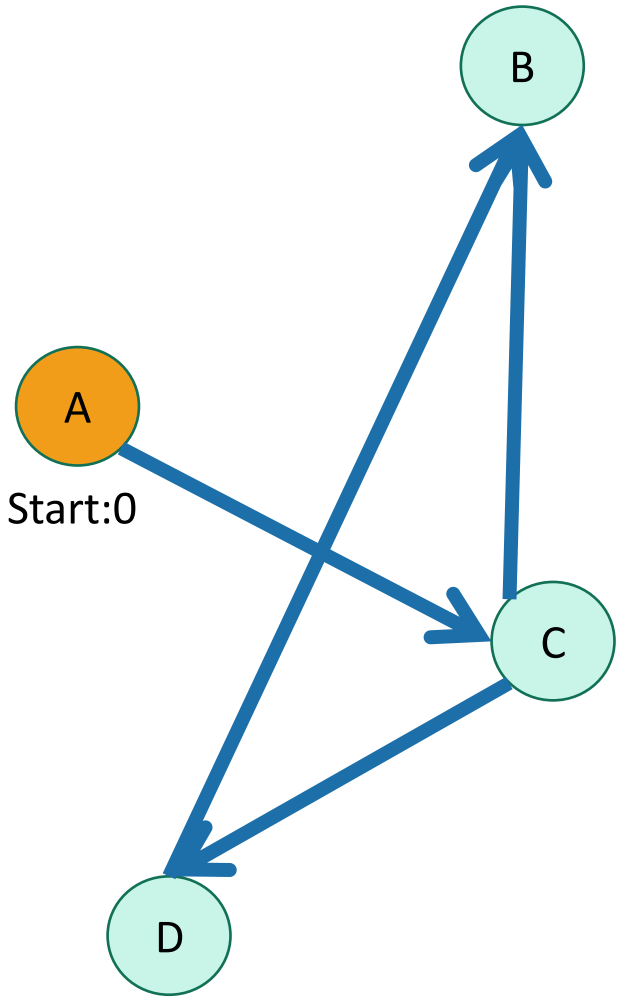


What did we just learn?

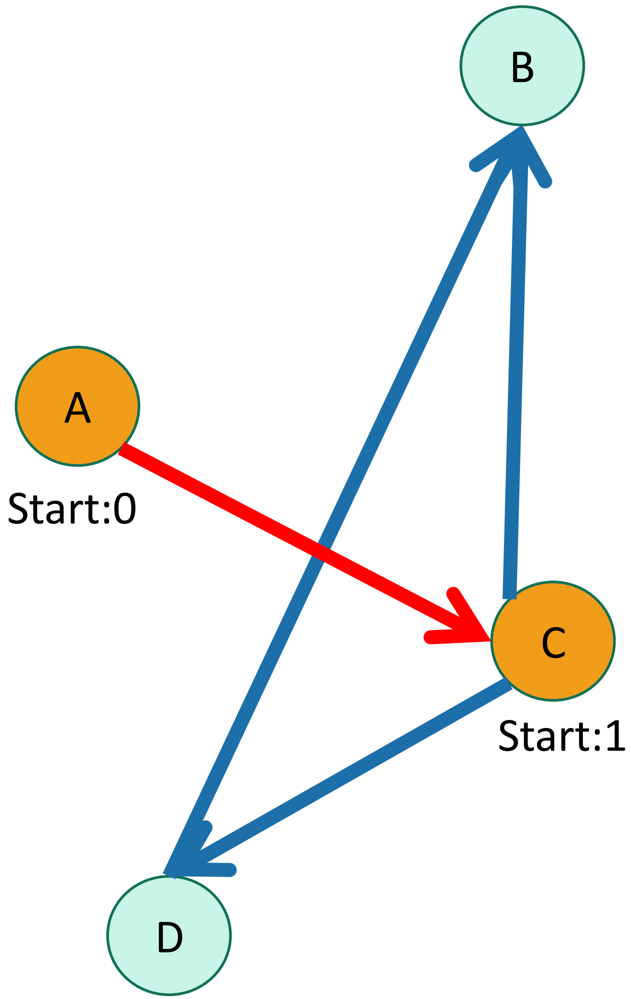
- DFS can help you solve the **topological sorting problem**
  - That's the fancy name for the problem of finding an ordering that respects all the dependencies

- Thinking about the DFS tree is helpful.

Example:



Example

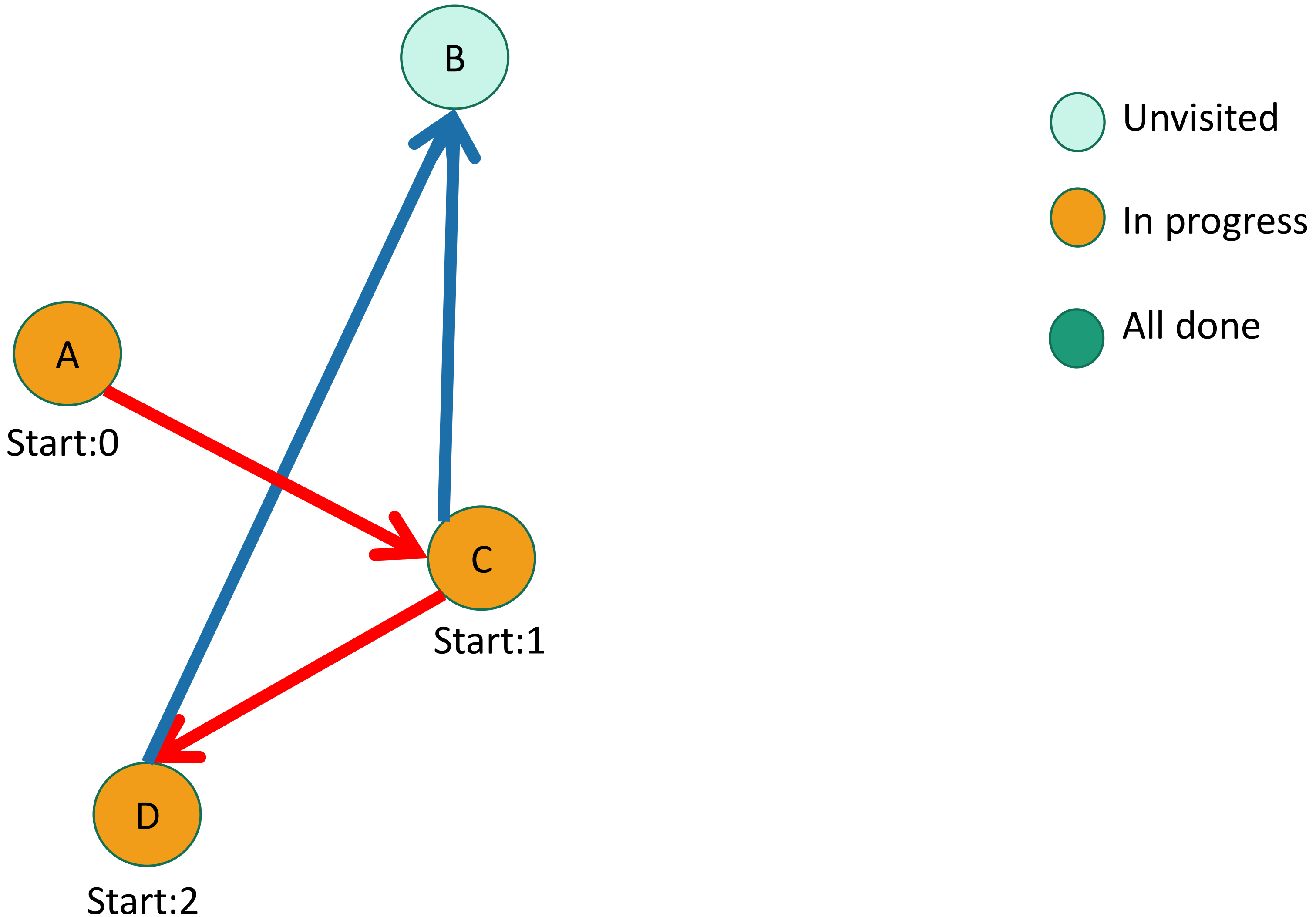


Unvisited

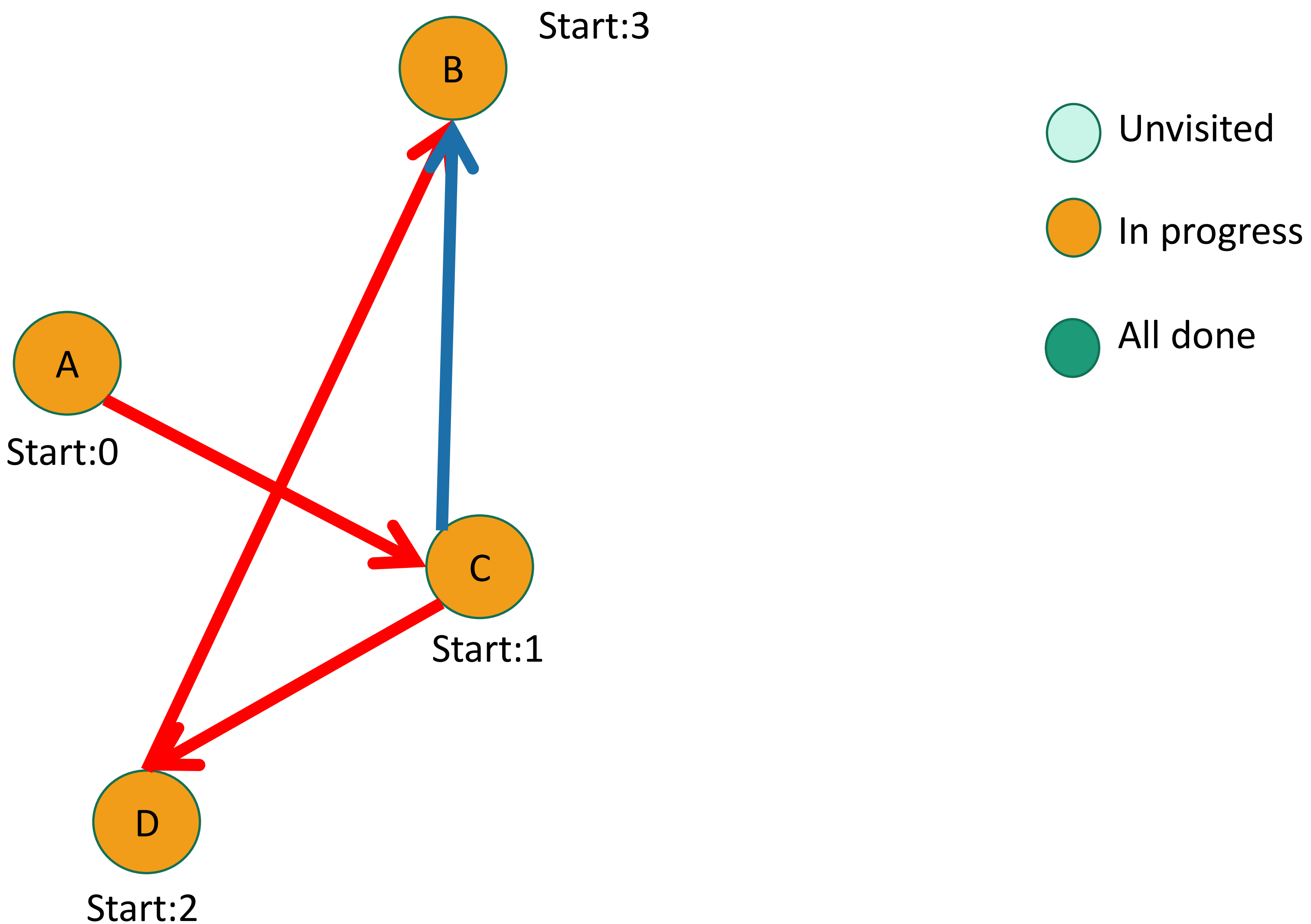
In progress

All done

Example

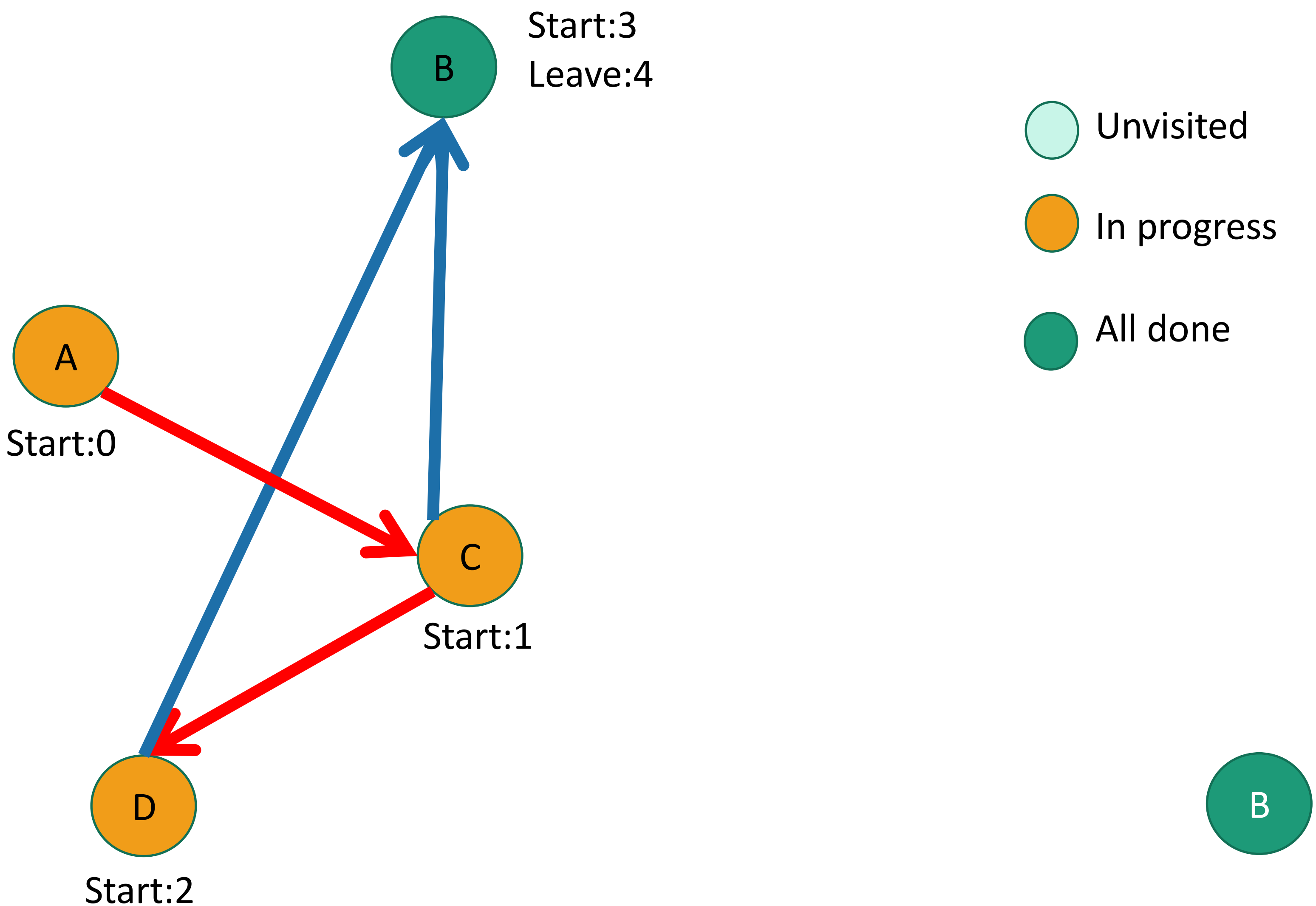


Example



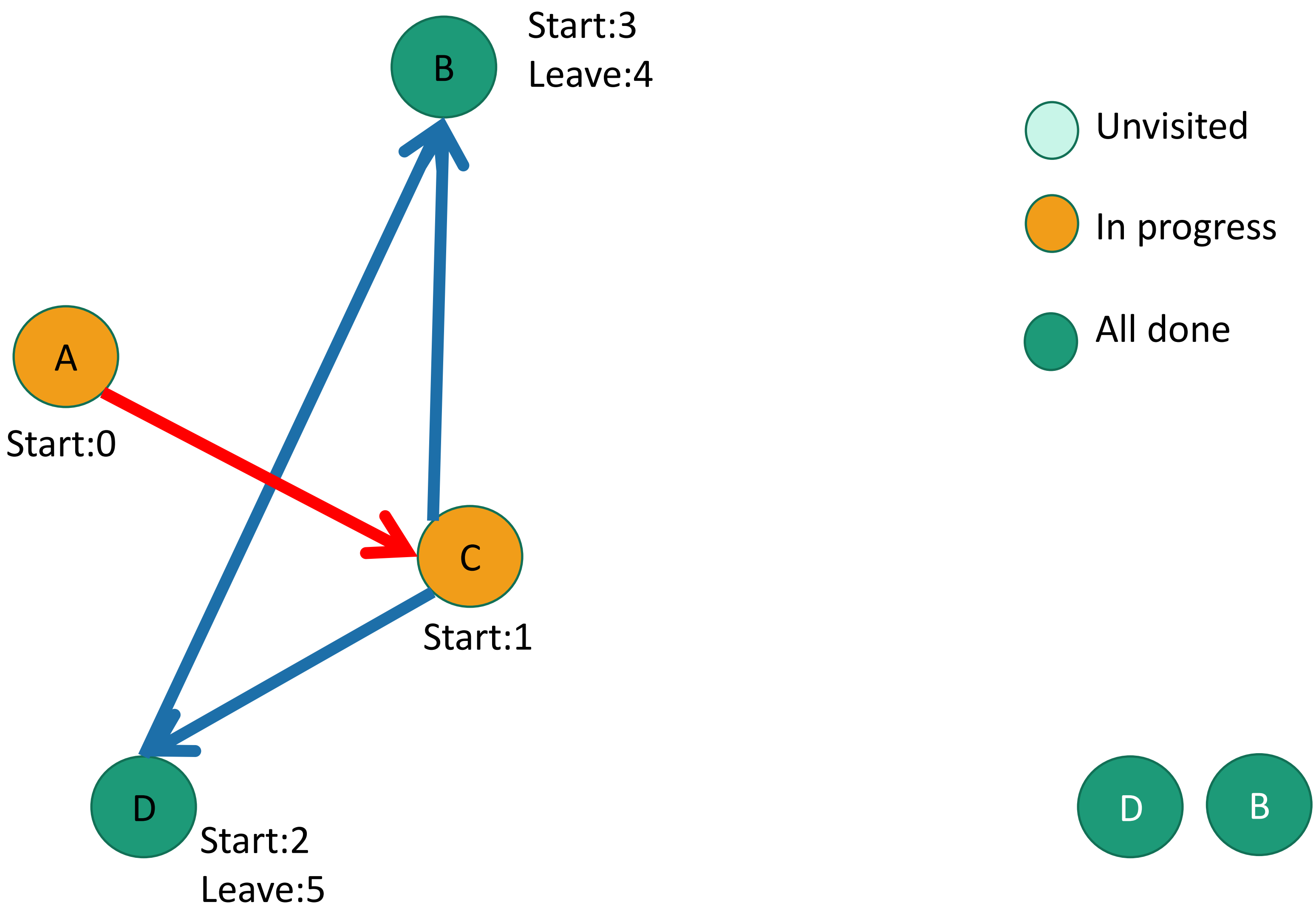
Start:3
B
A
Start:0
C
Start:1
D
Start:2
Unvisited
In progress
All done

Example

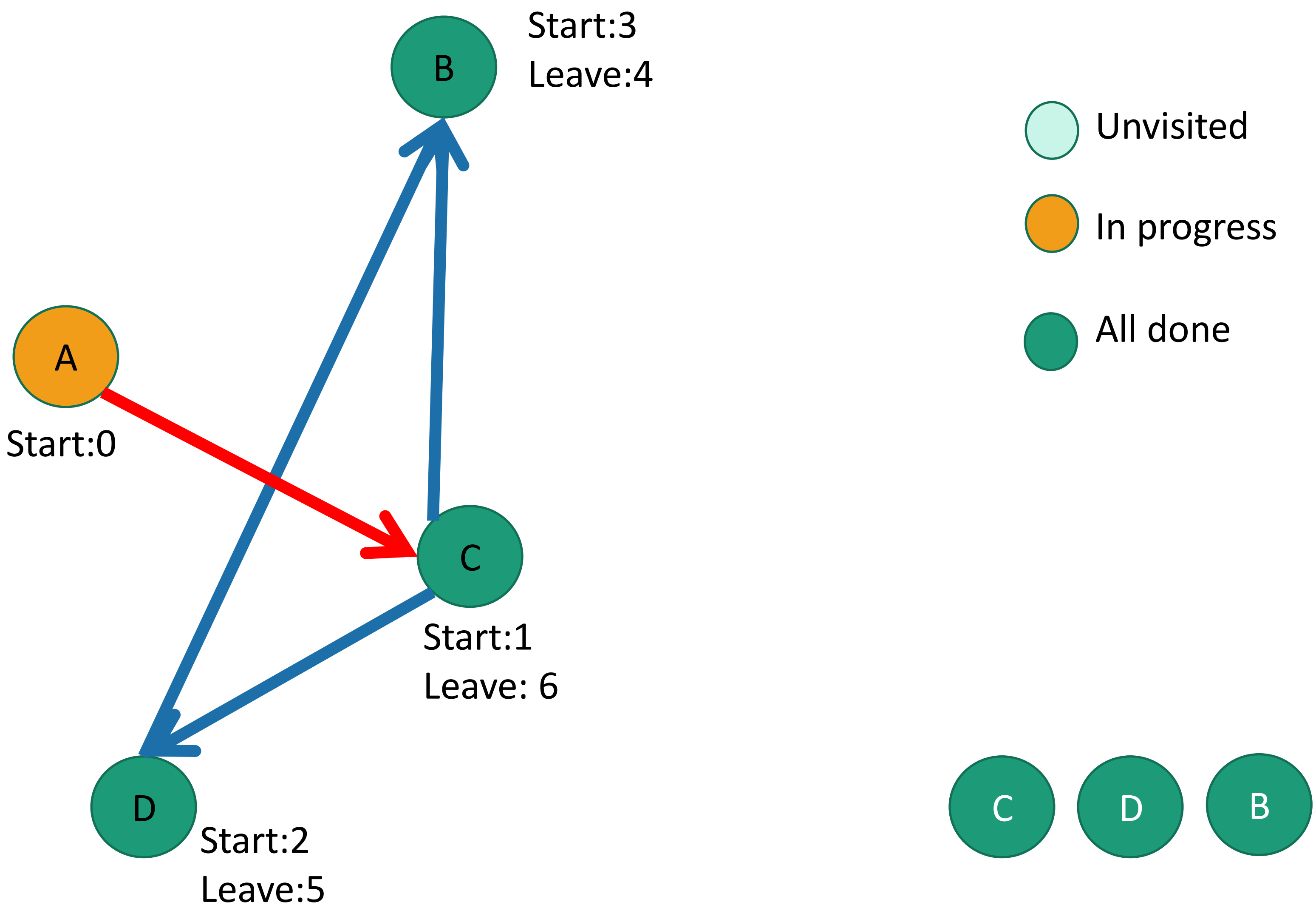


Start:3
Leave:4
B
A
Start:0
C
Start:1
D
Start:2
Unvisited
In progress
All done
B

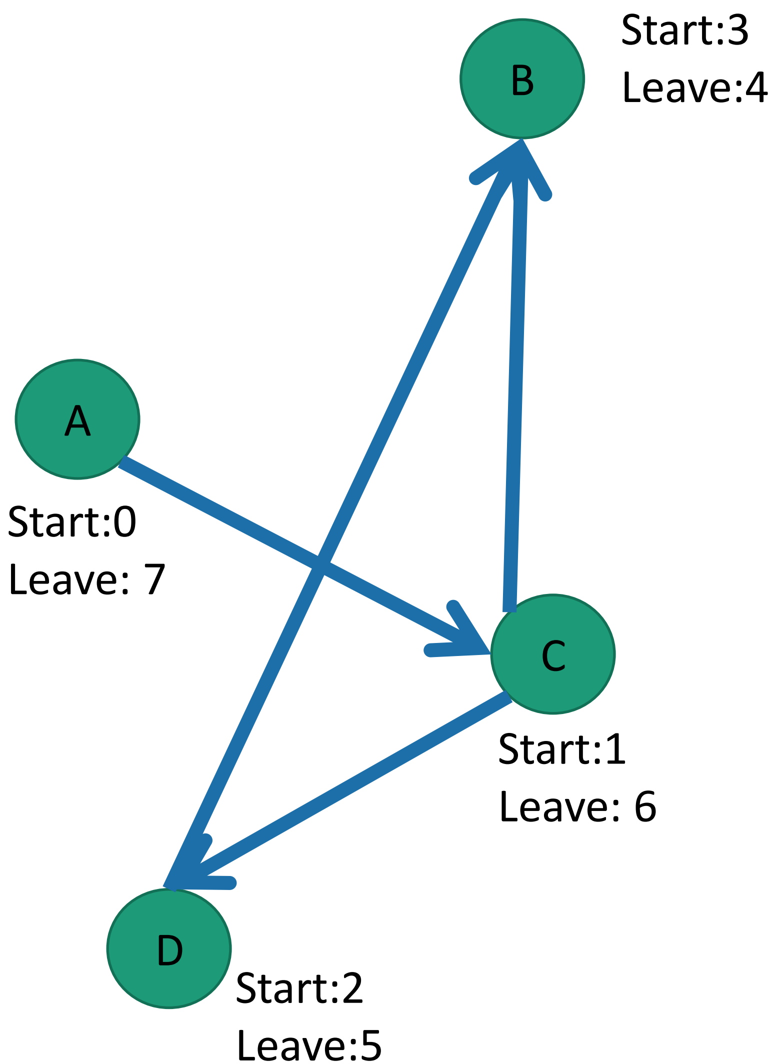
Example


B
Start:3
Leave:4
A
Start:0
C
Start:1
D
Start:2
Leave:5
Unvisited
In progress
All done
D
B

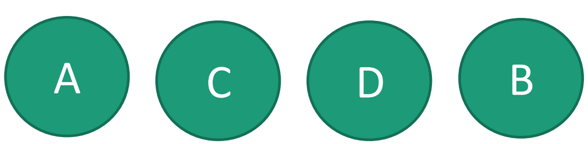
Example


B
Start:3
Leave:4
A
Start:0
C
Start:1
Leave: 6
D
Start:2
Leave:5
Unvisited
In progress
All done
C
D
B

Example

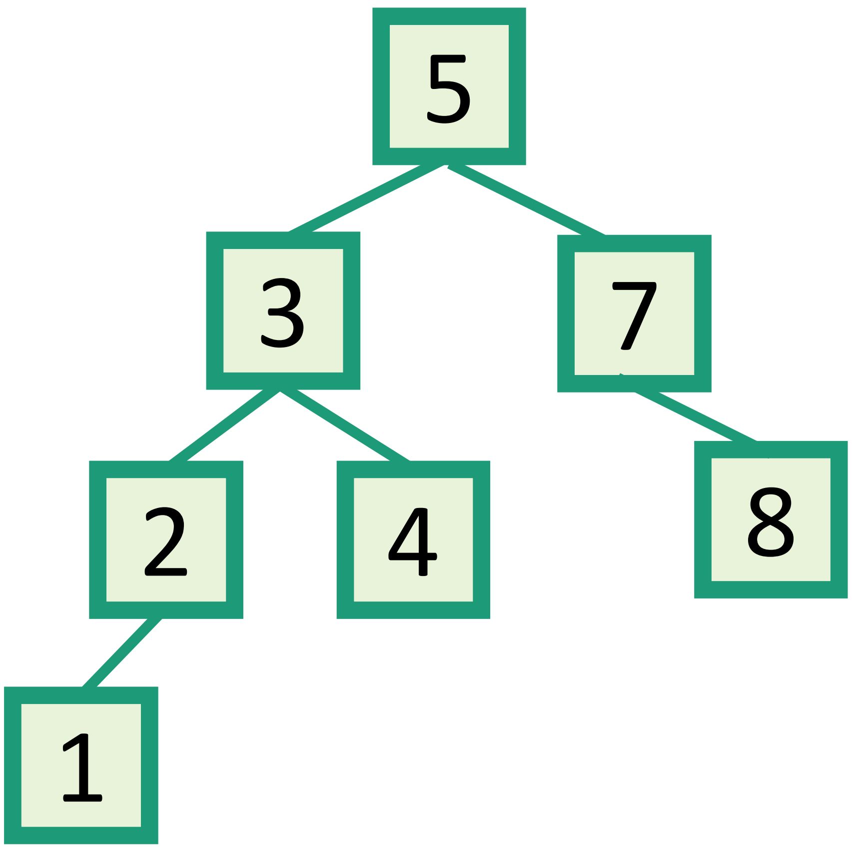


Do them in this order:



Another use of DFS that we've already seen

- In-order enumeration of binary search trees

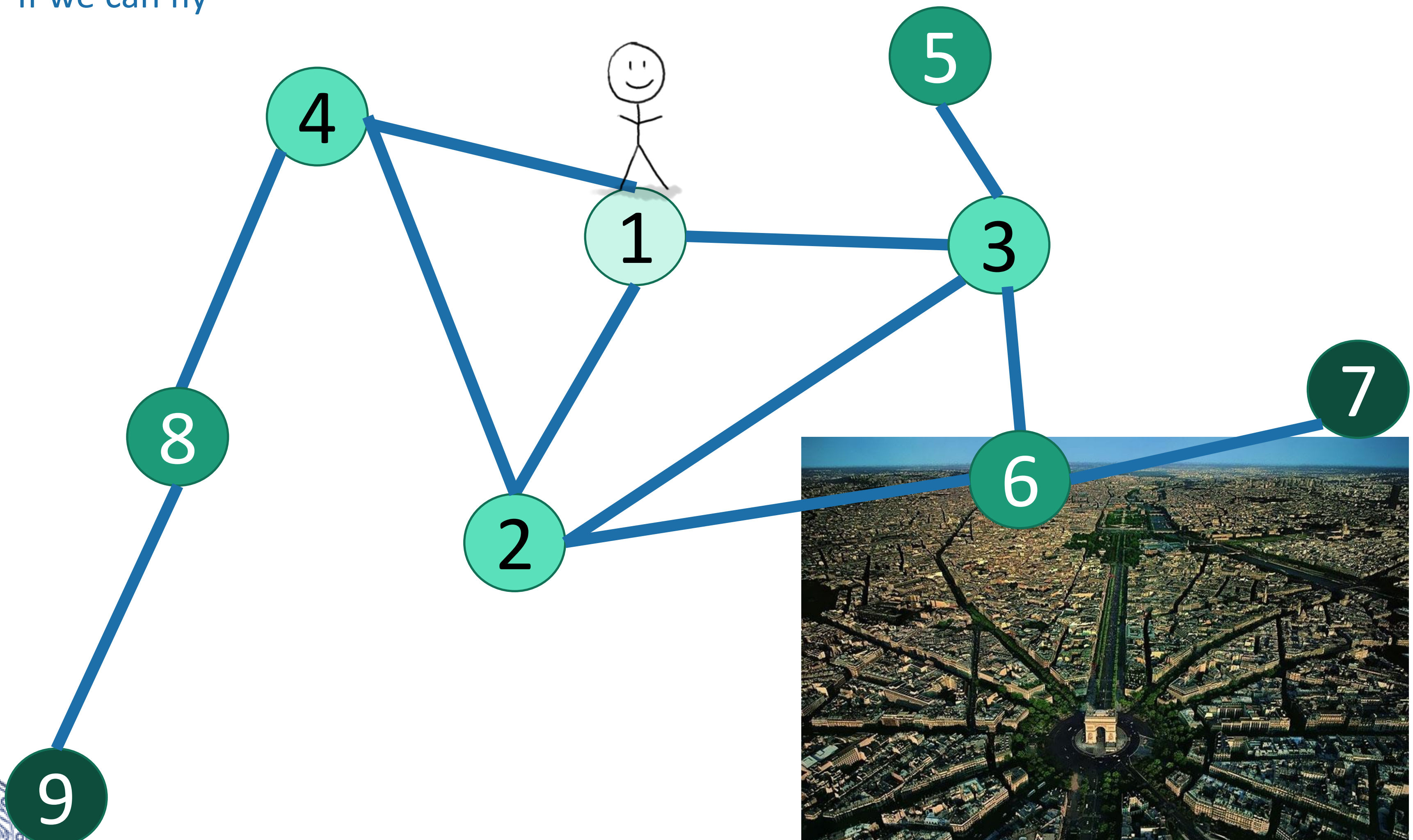


Do DFS and print a node's label when you are done with the left child and before you begin the right child.

Part C: breadth-first search

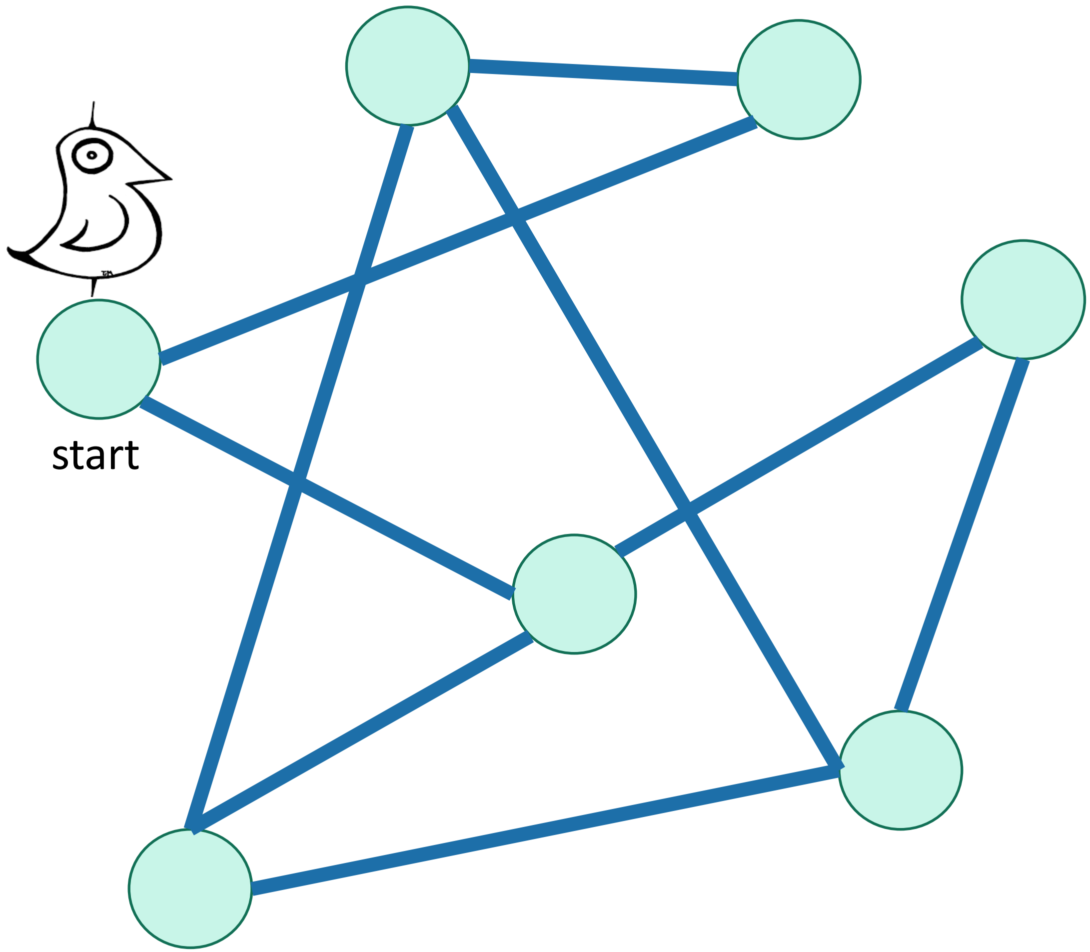
How do we explore a graph?

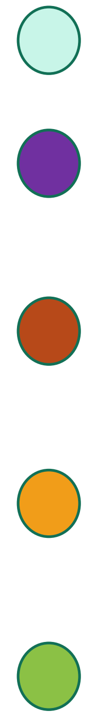
If we can fly



Breadth-First Search

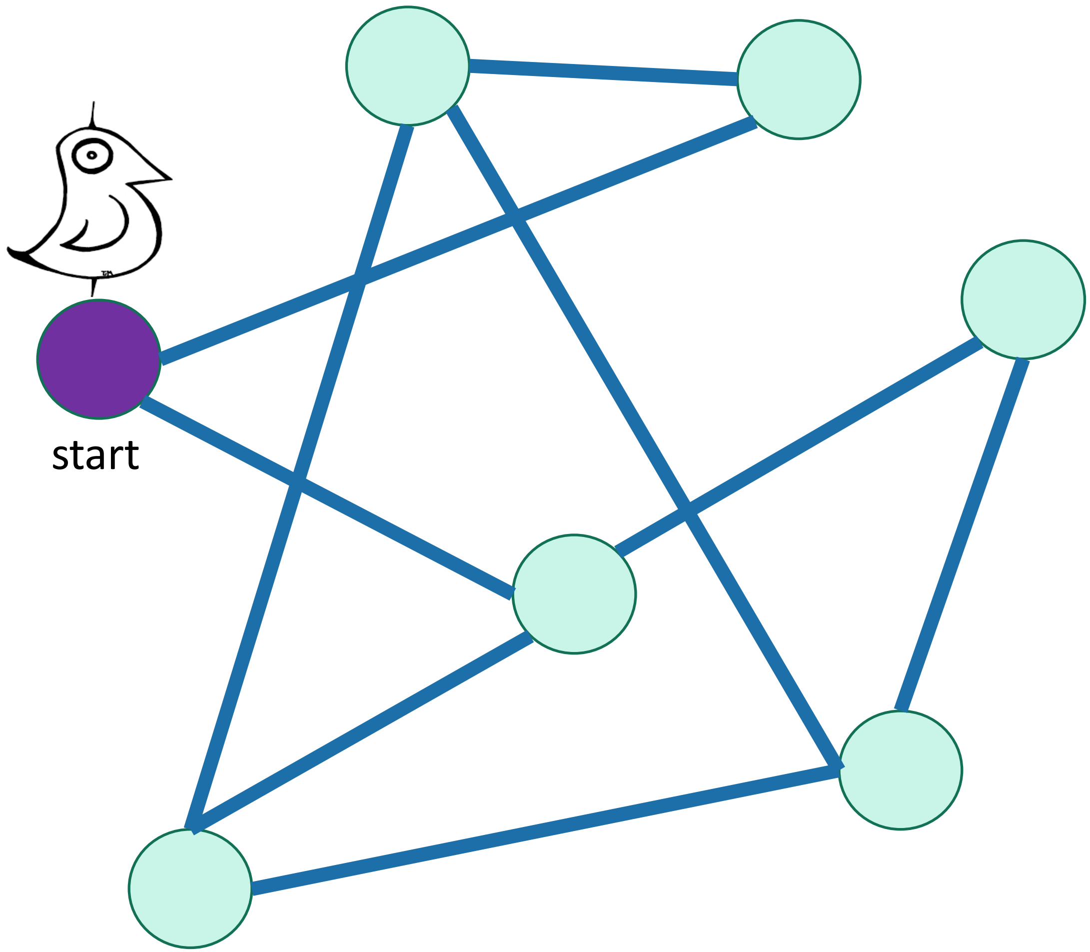
Exploring the world with a bird's-eye view





Breadth-First Search

Exploring the world with a bird's-eye view



Not been there yet

Can reach there in zero steps

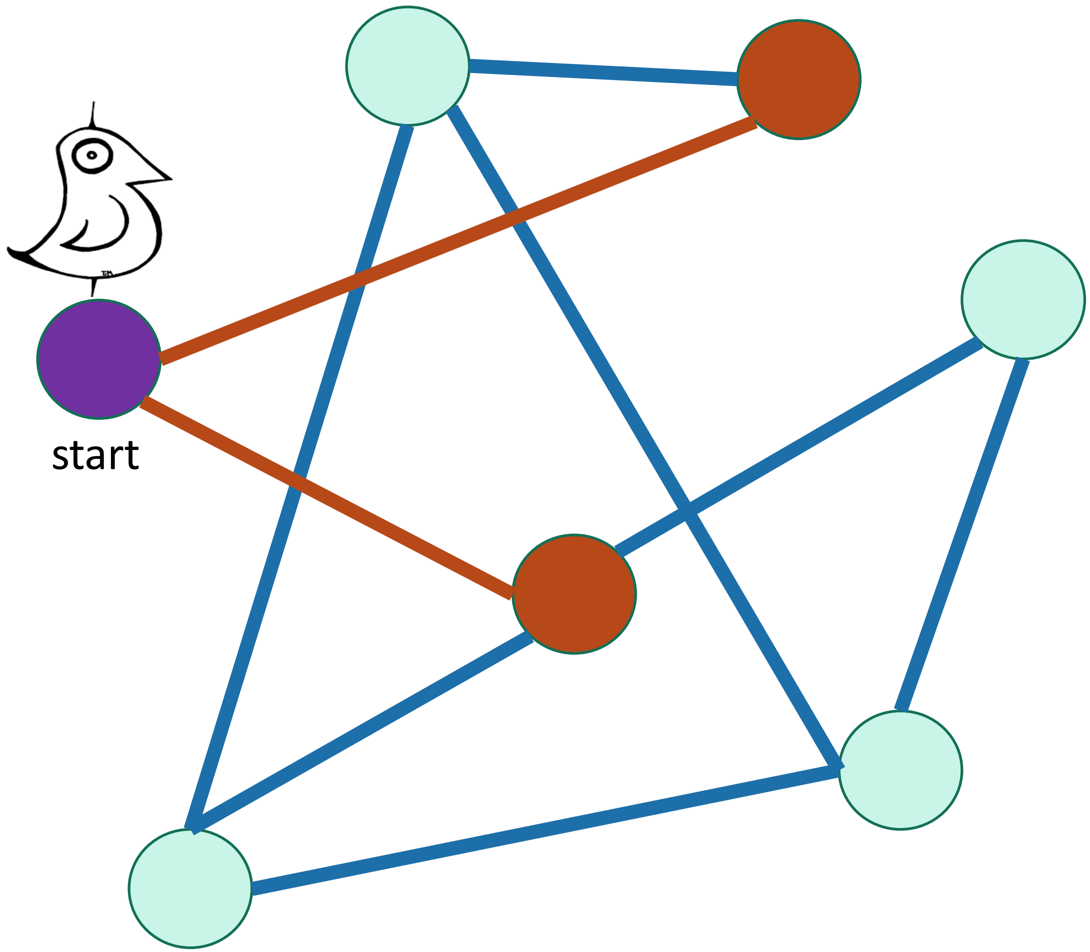
Can reach there in one step

Can reach there in two steps

Can reach there in three steps

Breadth-First Search

Exploring the world with a bird's-eye view



Not been there yet

Can reach there in zero steps

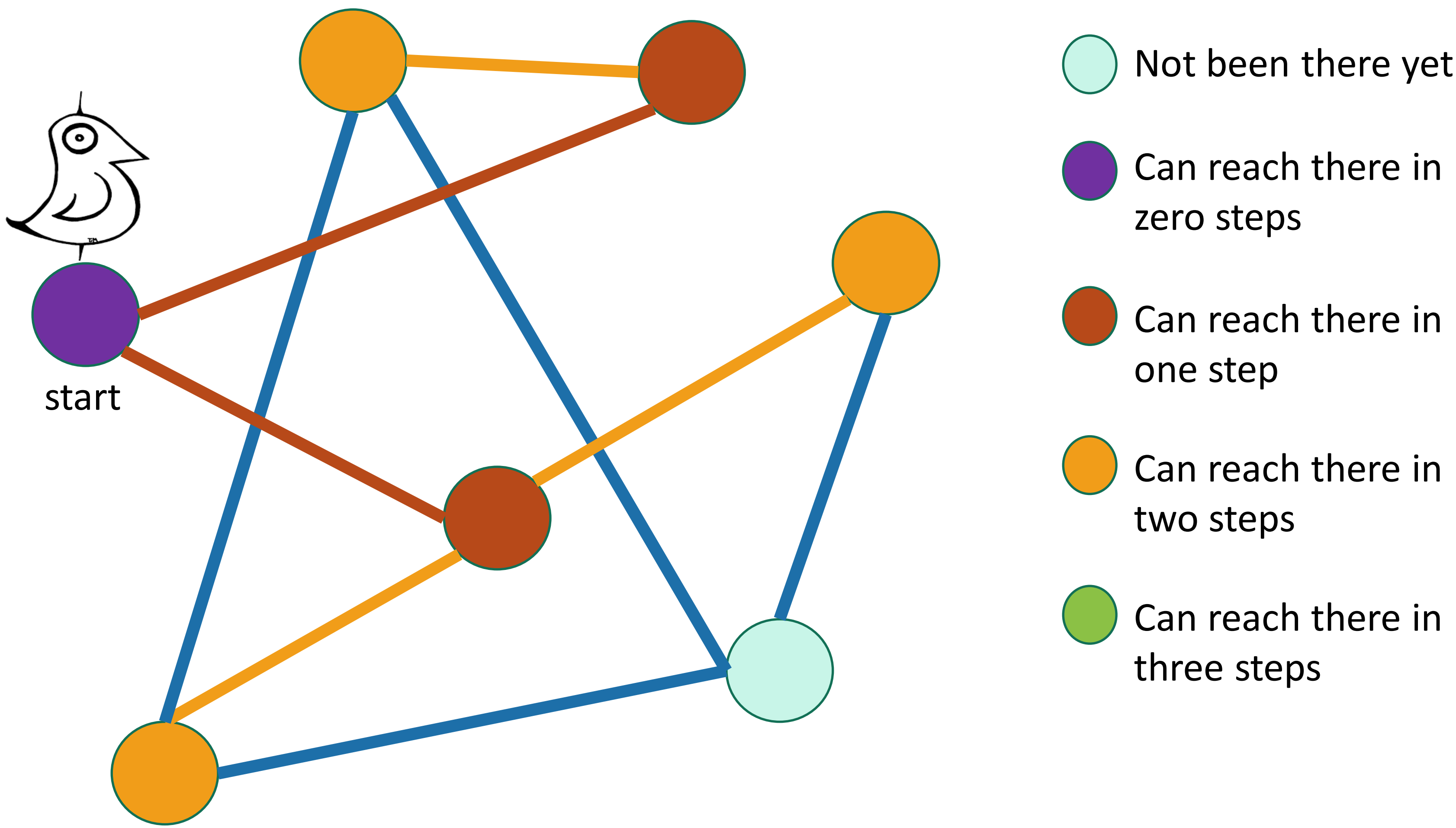
Can reach there in one step

Can reach there in two steps

Can reach there in three steps

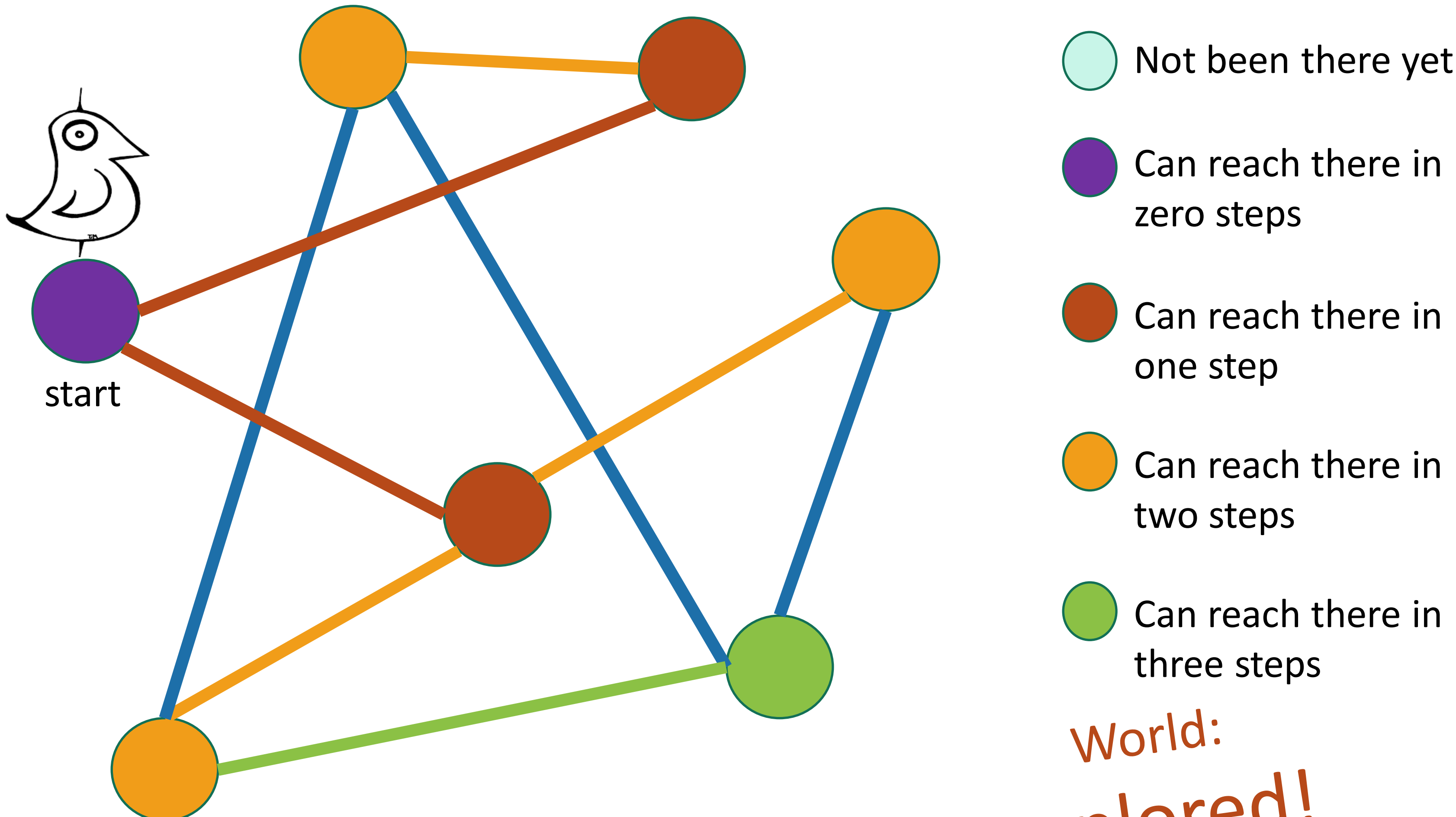
Breadth-First Search

Exploring the world with a bird's-eye view



Breadth-First Search

Exploring the world with a bird's-eye view



World: explored!

Same disclaimer as for DFS: you may have seen other ways to implement this, this will be convenient for us.

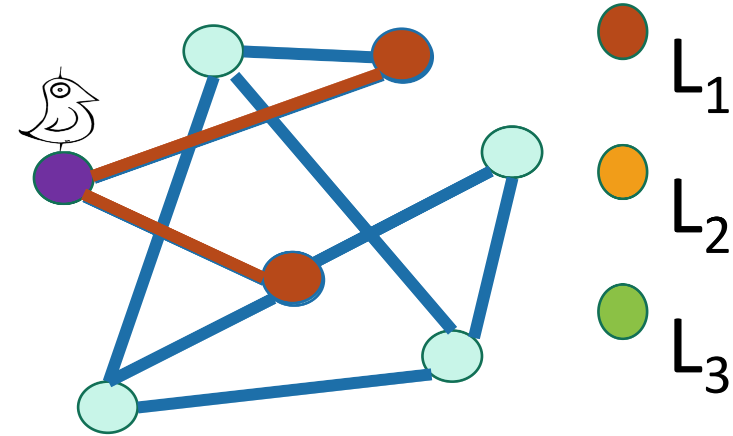
Breadth-First Search

Exploring the world with pseudocode

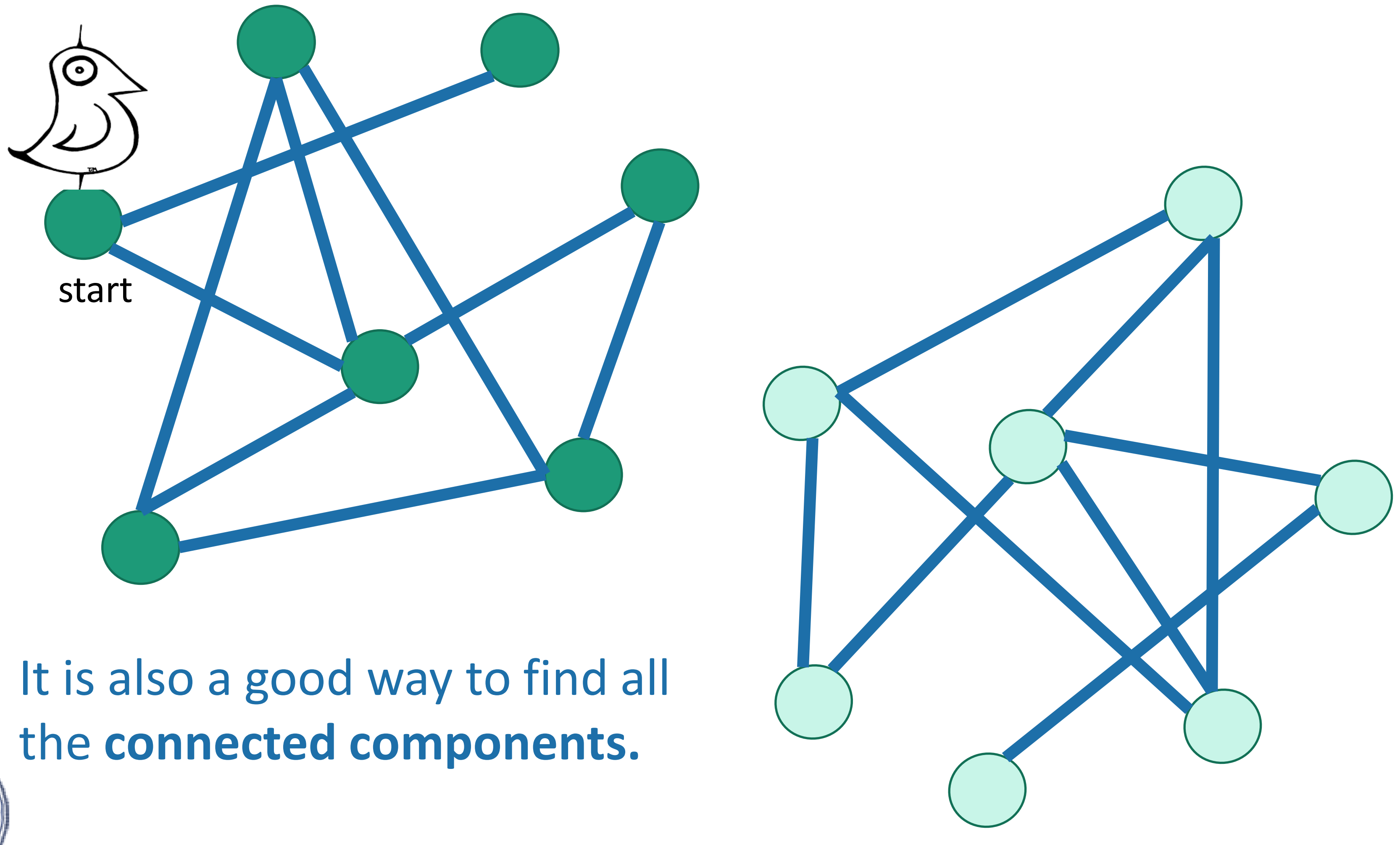
- Set L_i = [] for i=1,…,n
- L_0 = [w], where w is the start node
- Mark w as visited
- **For** i = 0, …, n-1:
    - **For** u in L_i:
        - **For** each v which is a neighbor of u:
            - **If** v isn't yet visited:
                - mark v as visited, and put it in L_{i+1}

L_i is the set of nodes we can reach in i steps from w

Go through all the nodes in L_i and add their unvisited neighbors to L_{i+1}



BFS also finds all the nodes reachable from the starting point



It is also a good way to find all the **connected components.**

Running time and extension to directed graphs

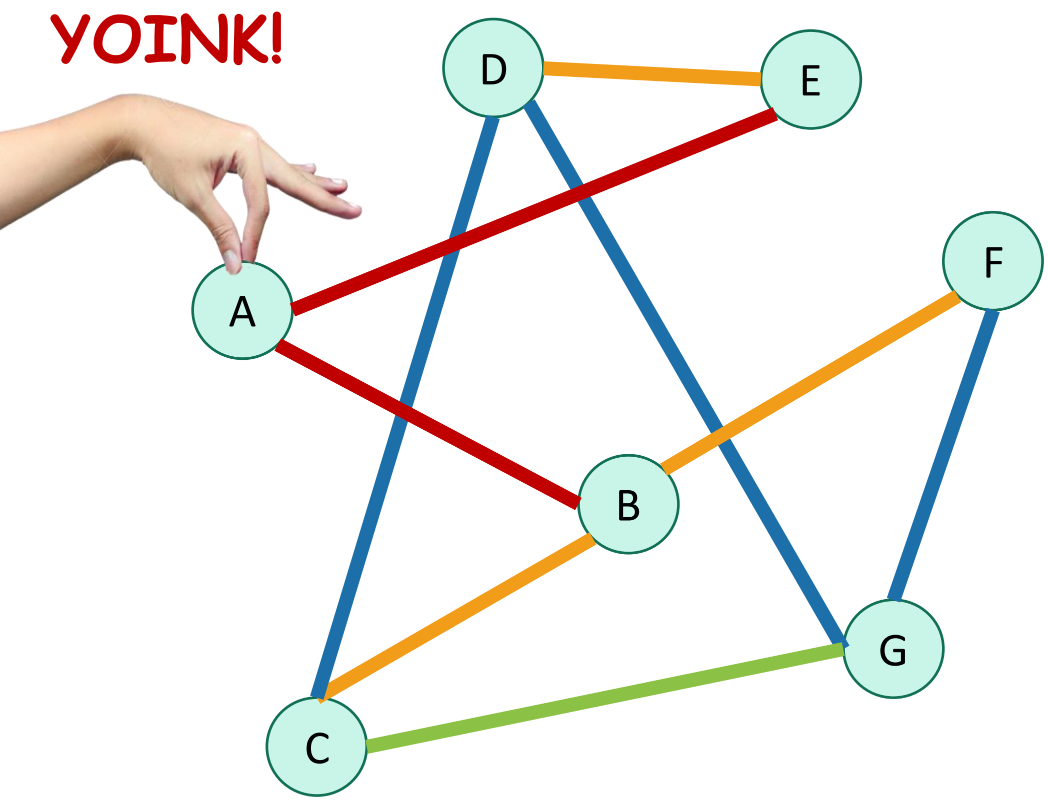
- To explore the whole graph, explore the connected components one-by-one.
  - Same argument as DFS: BFS running time is O(n + m)
- Like DFS, BFS also works fine on directed graphs.

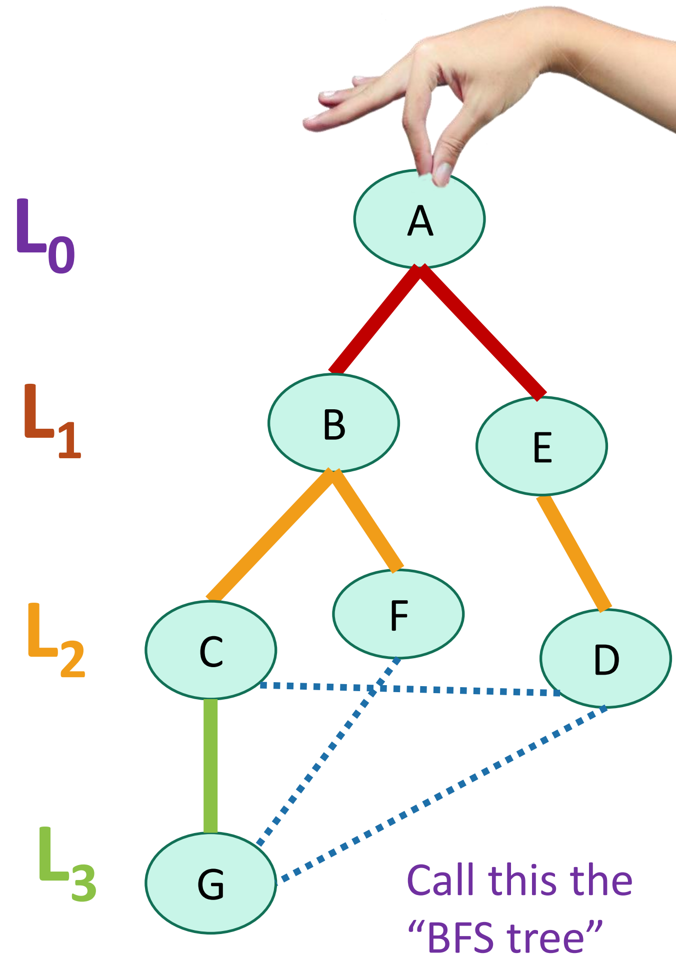
Verify these!

Siggi the Studious Stork

Why is it called breadth-first?

- We are implicitly building a tree:

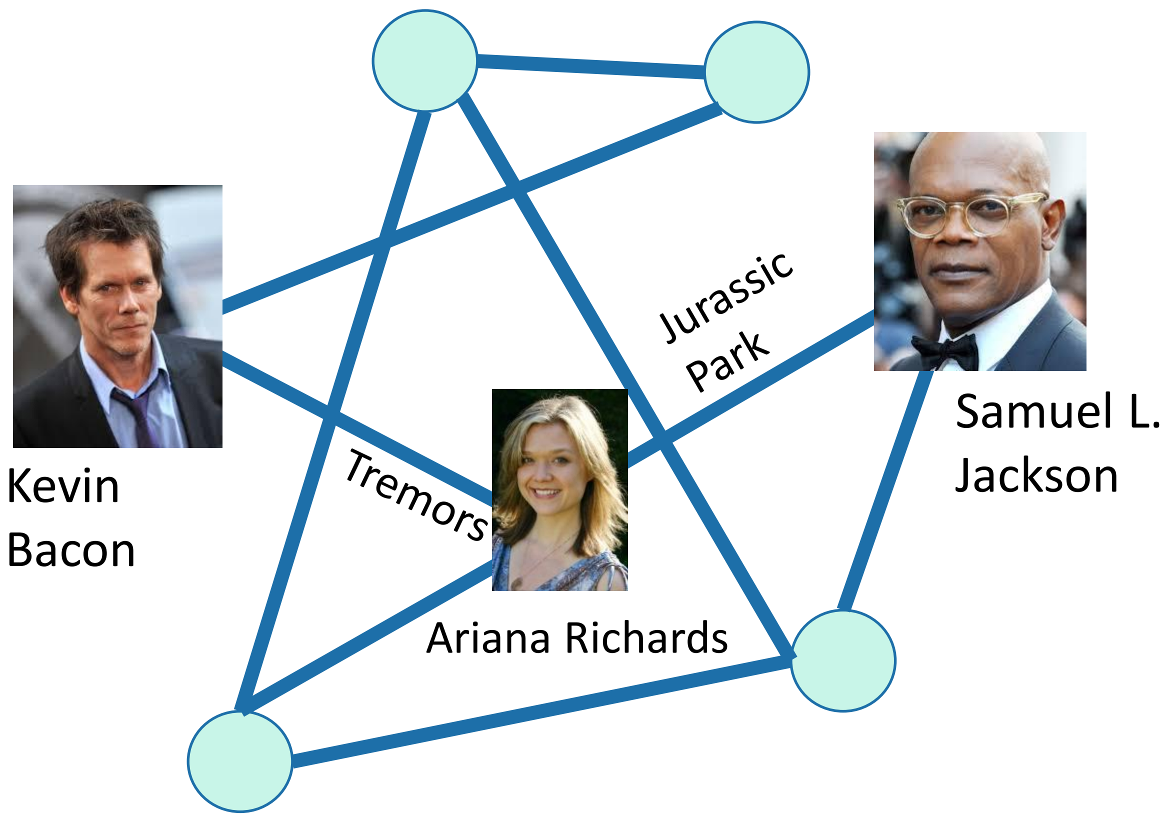




- First we go as broadly as we can.

Fun exercises

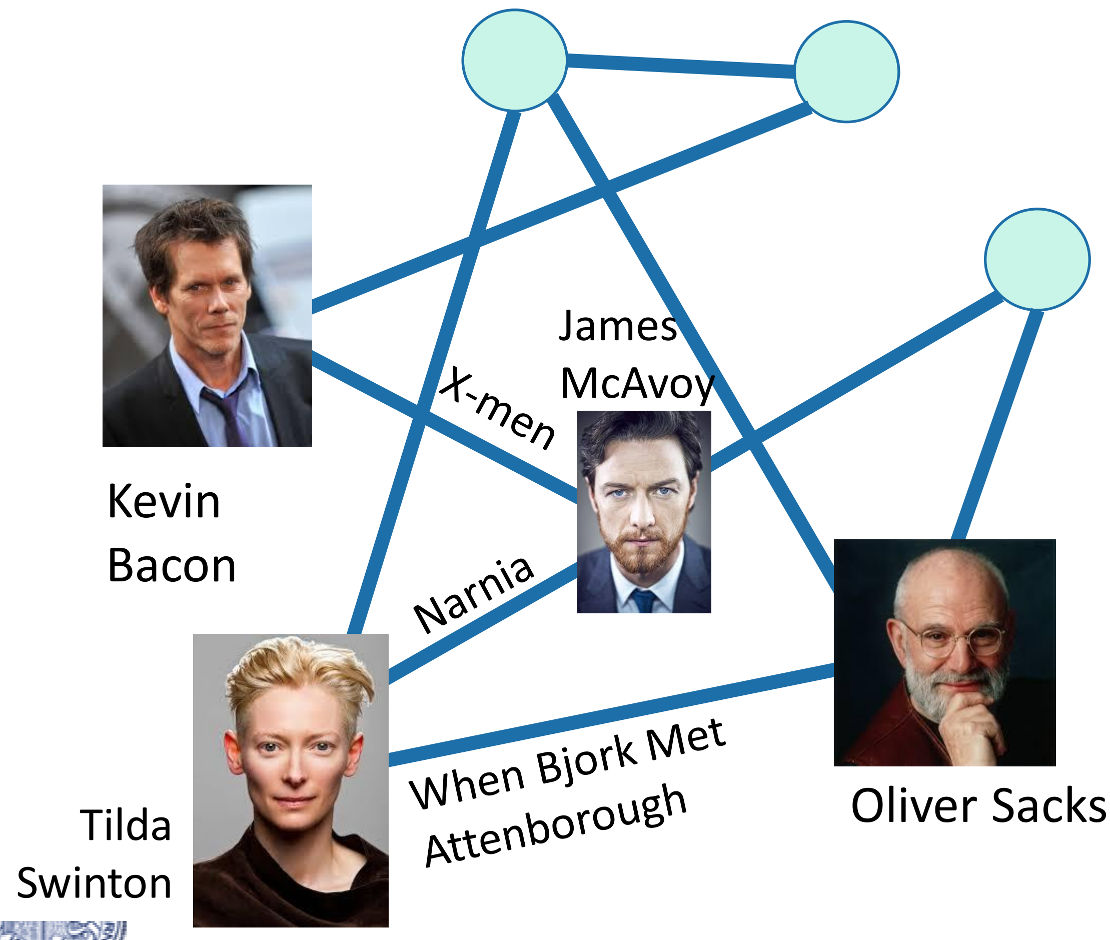
- What is Samuel L. Jackson's (Kevin) Bacon number?



(Answer: 2)

Fun exercises

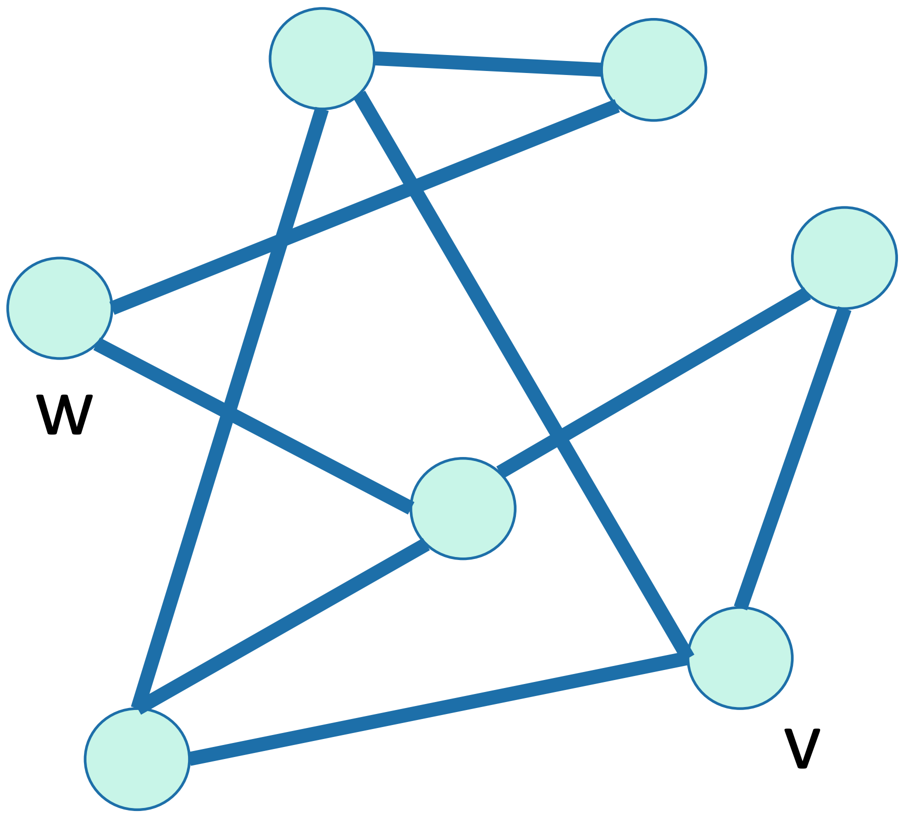
- What is Oliver Sacks' (Kevin) Bacon number?



It is really hard to find people with Bacon number 3!

Application of BFS: shortest path

- How long is the shortest path between w and v?



Application of BFS: shortest path

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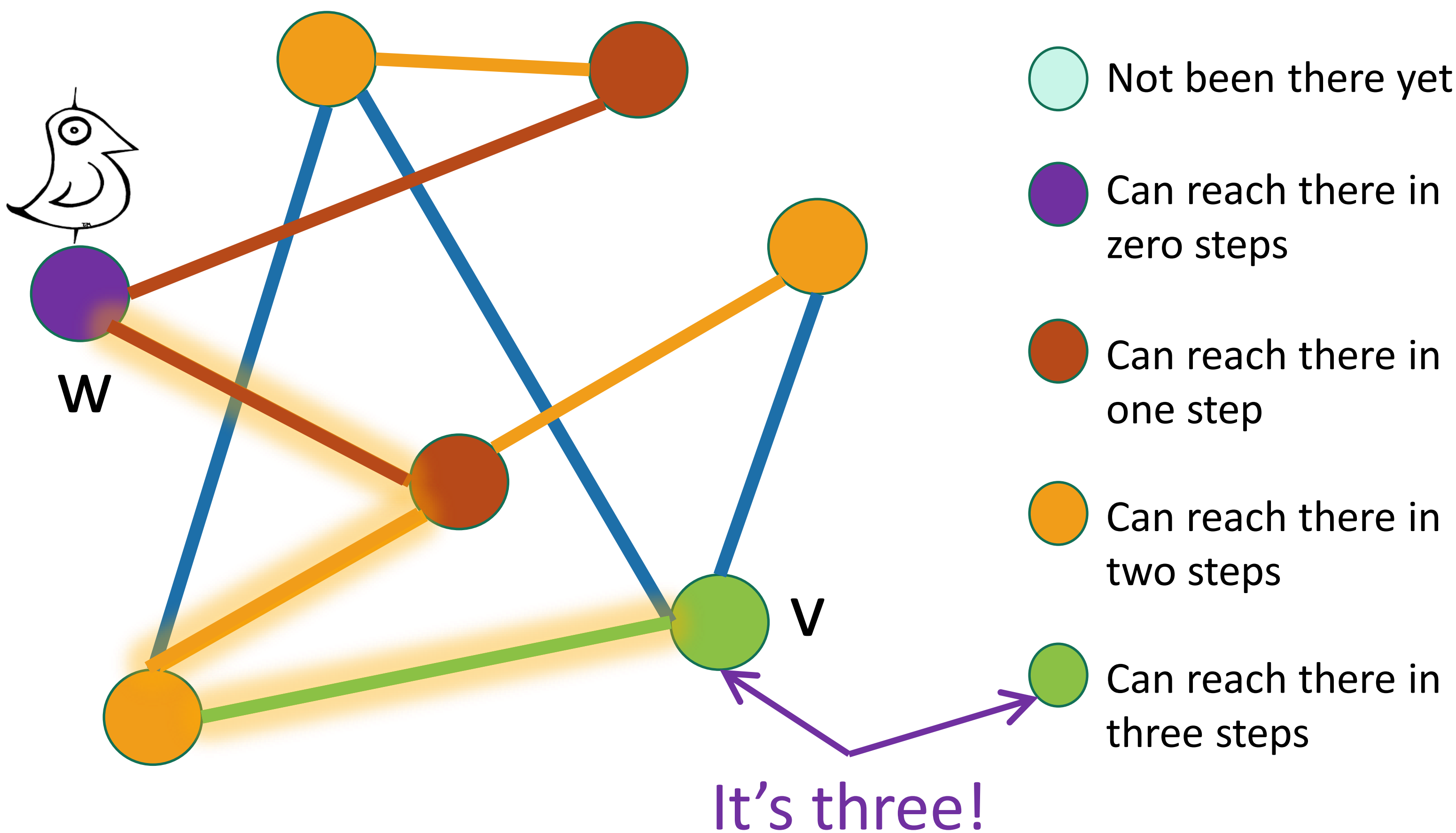
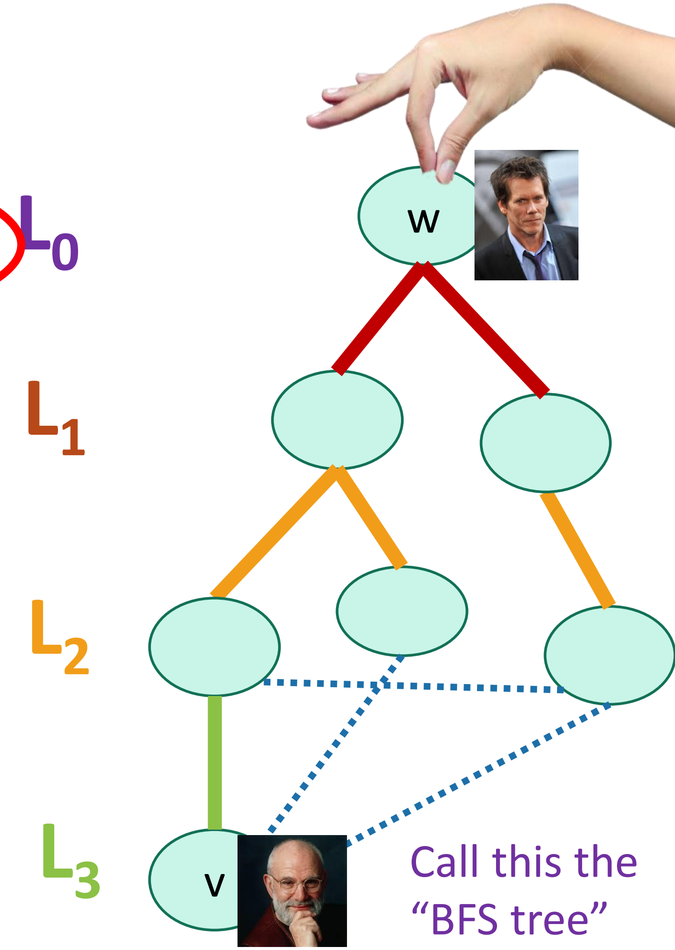


To find the distance between w and all other vertices v

The **distance** between two vertices is the number of edges in the shortest path between them.

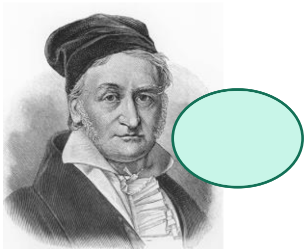
- Do a BFS starting at w
- For all v in L_i
  - The shortest path between w and v has length i
  - A shortest path between w and v is given by the path in the BFS tree.
- If we never found v, the distance is infinite.

This requires some proof! See next slide.



Modify the BFS pseudocode to return shortest paths!

Gauss has no Bacon number



Proof overview
that the BFS tree behaves like it should

See CLRS for details!

- Proof by induction.
- Inductive hypothesis for j:
  - For all i<j the vertices in L_i have distance i from v.
- Base case:
  - L_0 = {v}, so we're good.
- Inductive step:
  - Let w be in L_j. Want to show dist(v,w) = j.
  - We know dist(v,w) $\leq$ j, since dist(v, w's parent in L_{j-1}) = j-1 by induction, so that gives a path of length j from v to w.
  - On the other hand, dist(v,w) $\geq$ j, since if dist(v,w) < j, w would have shown up in an earlier layer.
  - Thus, dist(v,w) = j.
- Conclusion:
  - For each vertex w in V, if w is in L_j, then dist(v,w) = j.

What have we learned?

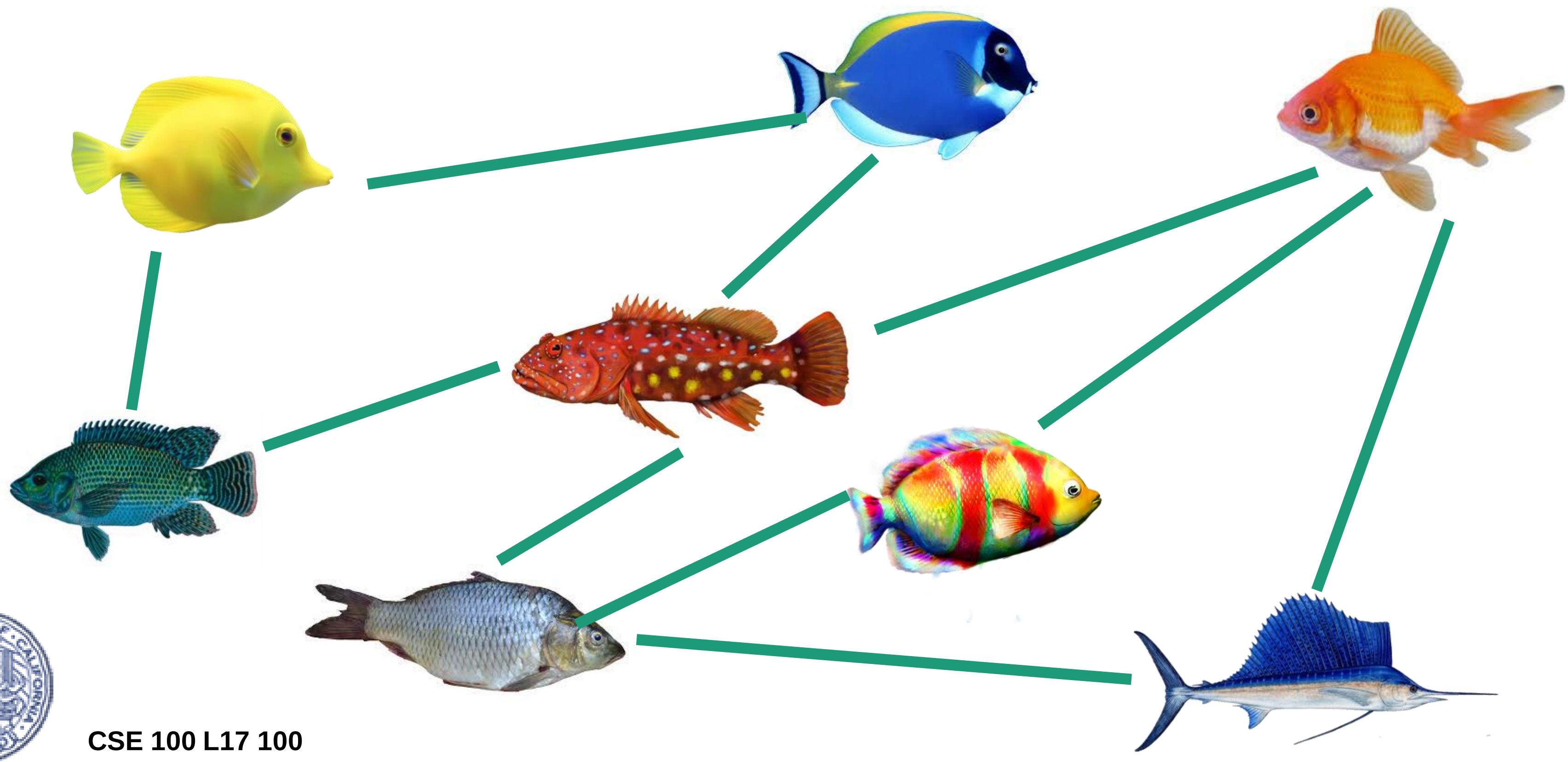
- The BFS tree is useful for computing distances between pairs of vertices.
- We can find the shortest path between u and v in time O(m).

Another application of BFS

- Testing bipartite-ness

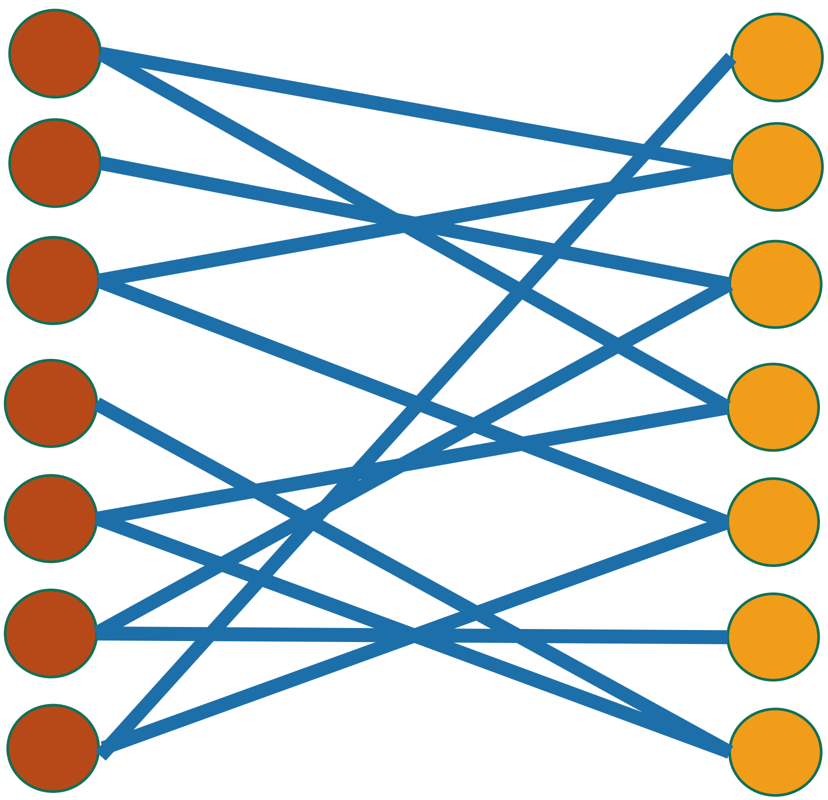
Another exercise: fish

- You have a bunch of fish and two fish tanks.
- Some pairs of fish will fight if put in the same tank.
    - Model this as a graph: connected fish will fight.
- Can you put the fish in the two tanks so that there is no fighting?



Bipartite graphs

- A bipartite graph looks like this:

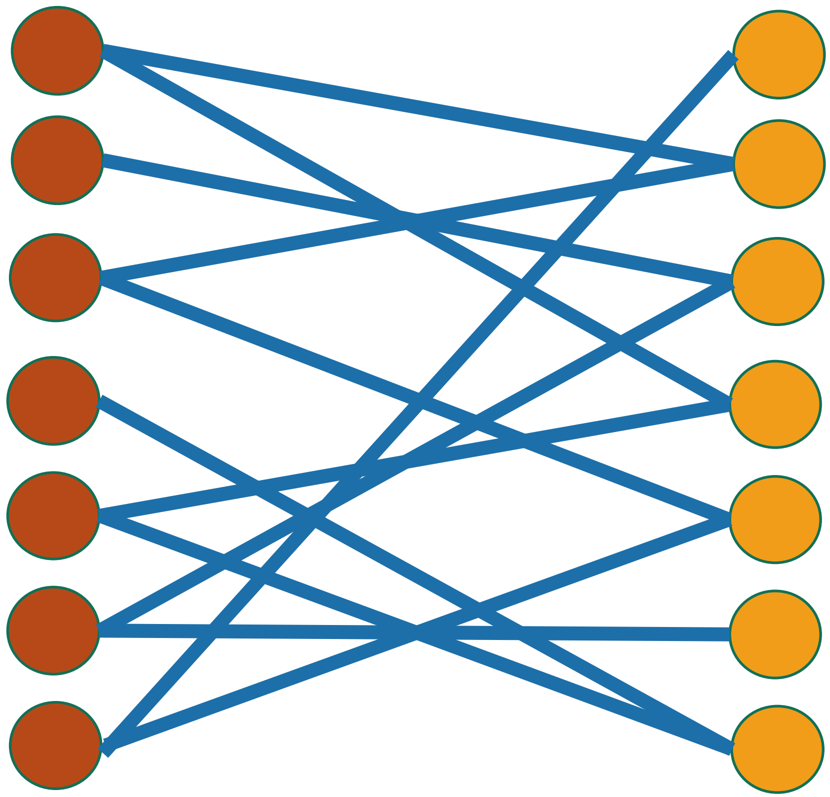


Can color the vertices red and orange so that there are no edges between any same-colored vertices

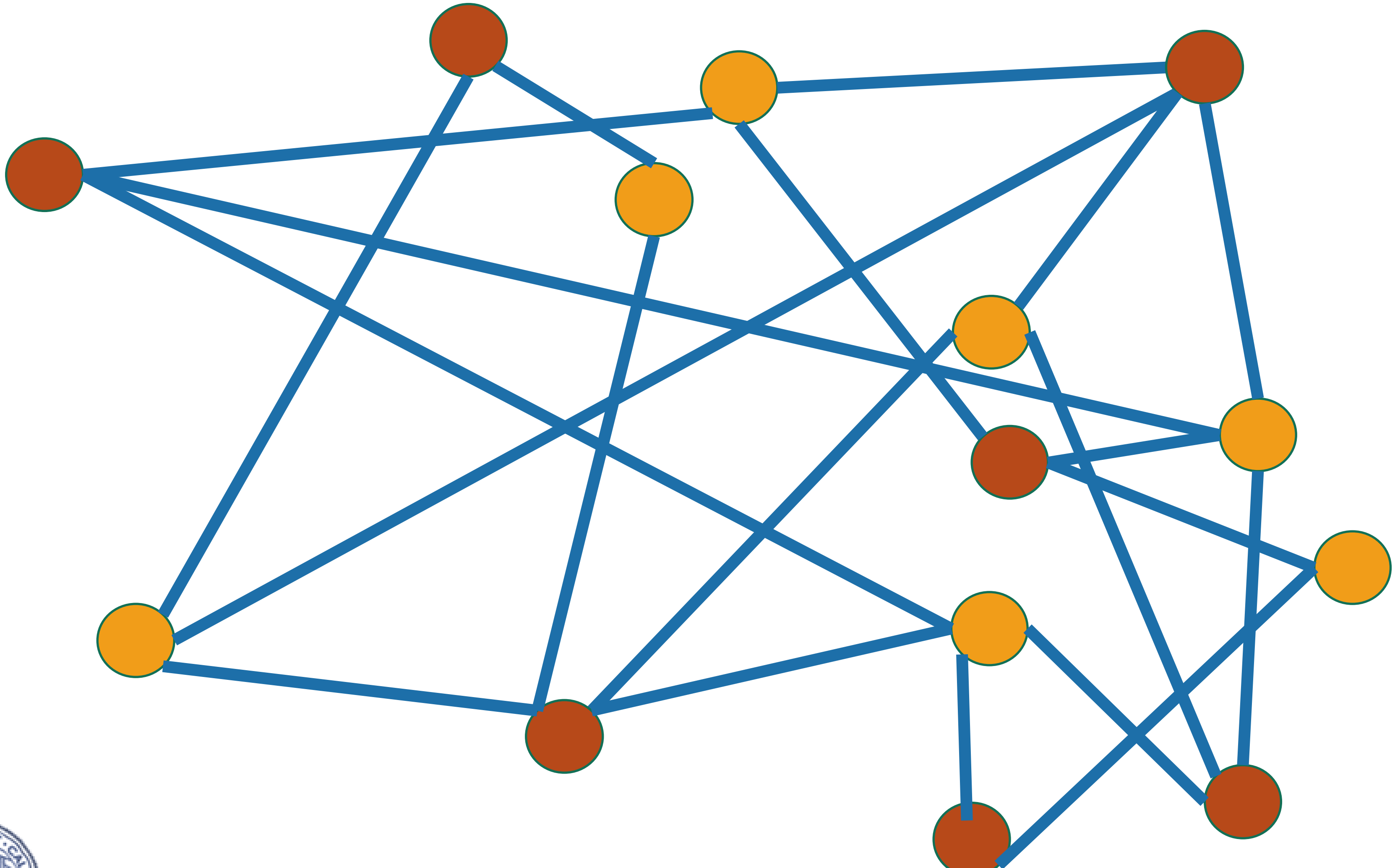
Example:
are in tank A
are in tank B
if the fish fight

Example:
are students
are classes
if the student is enrolled in the class

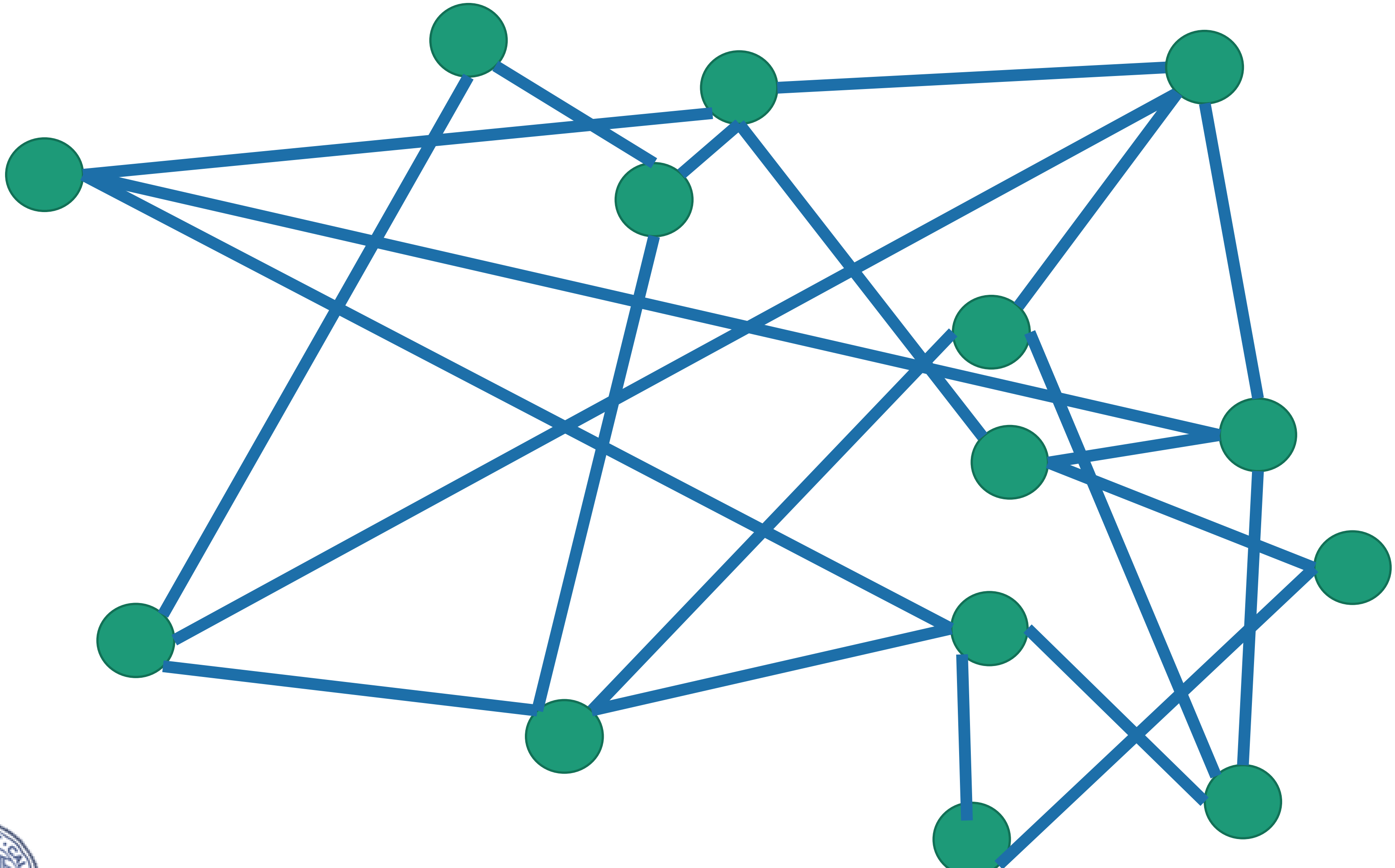
Is this graph bipartite?



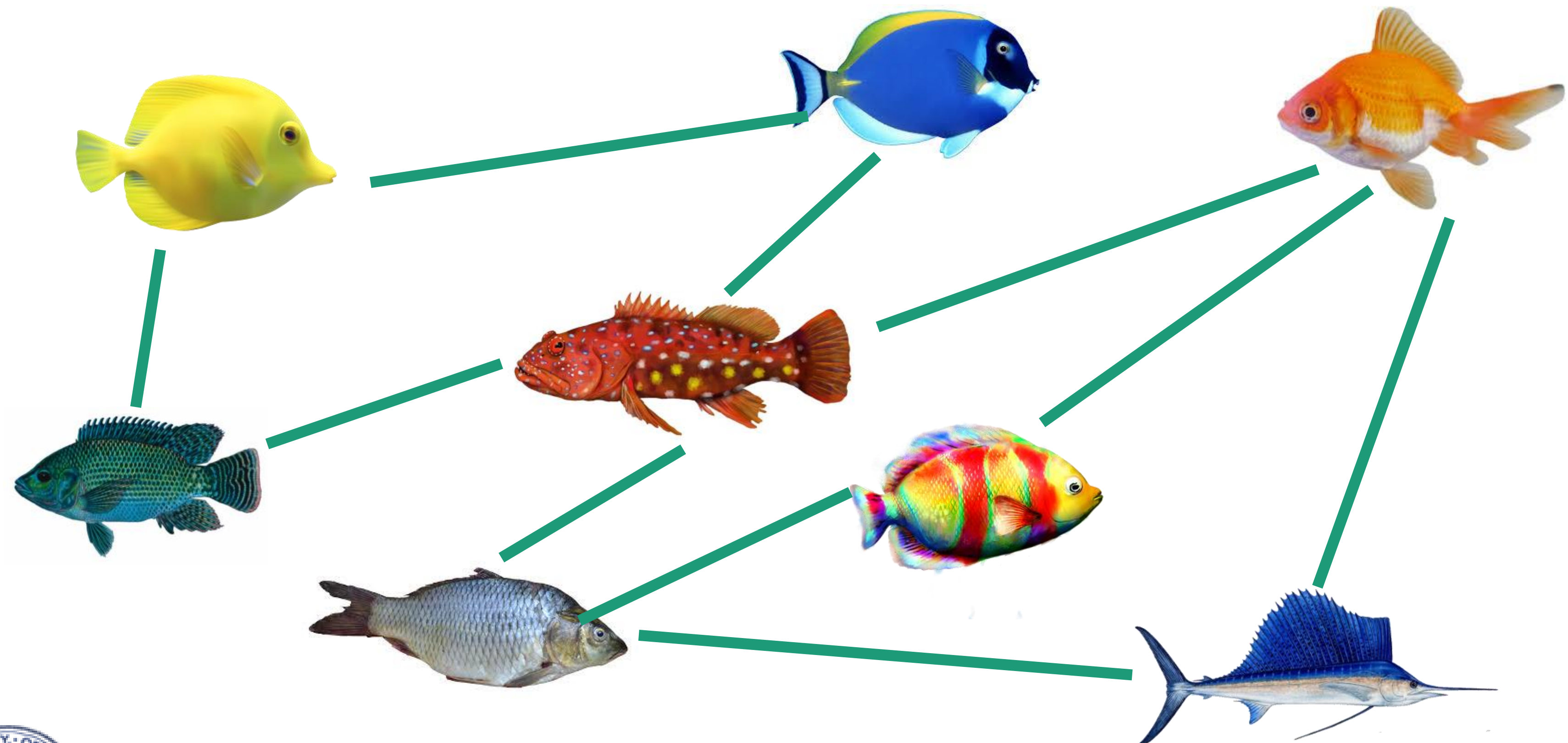
How about this one?



How about this one?

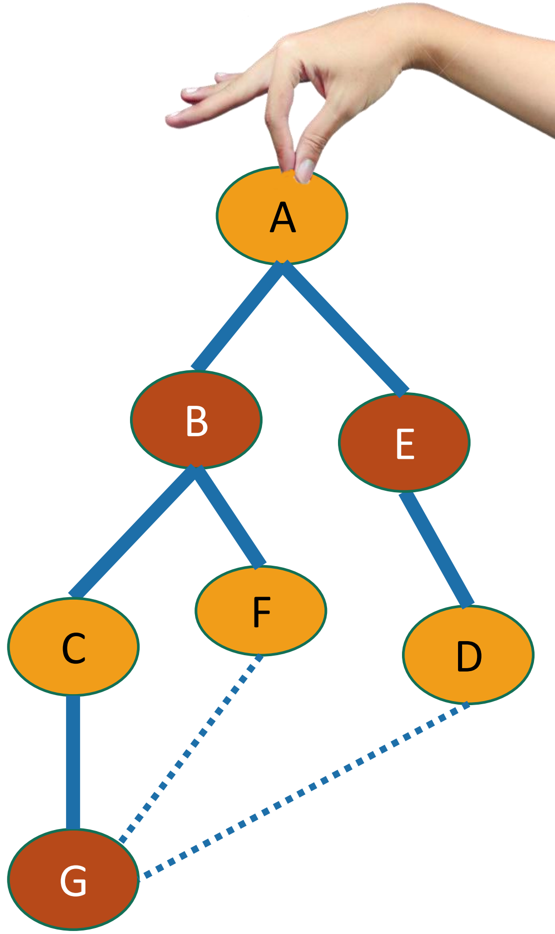


This one?



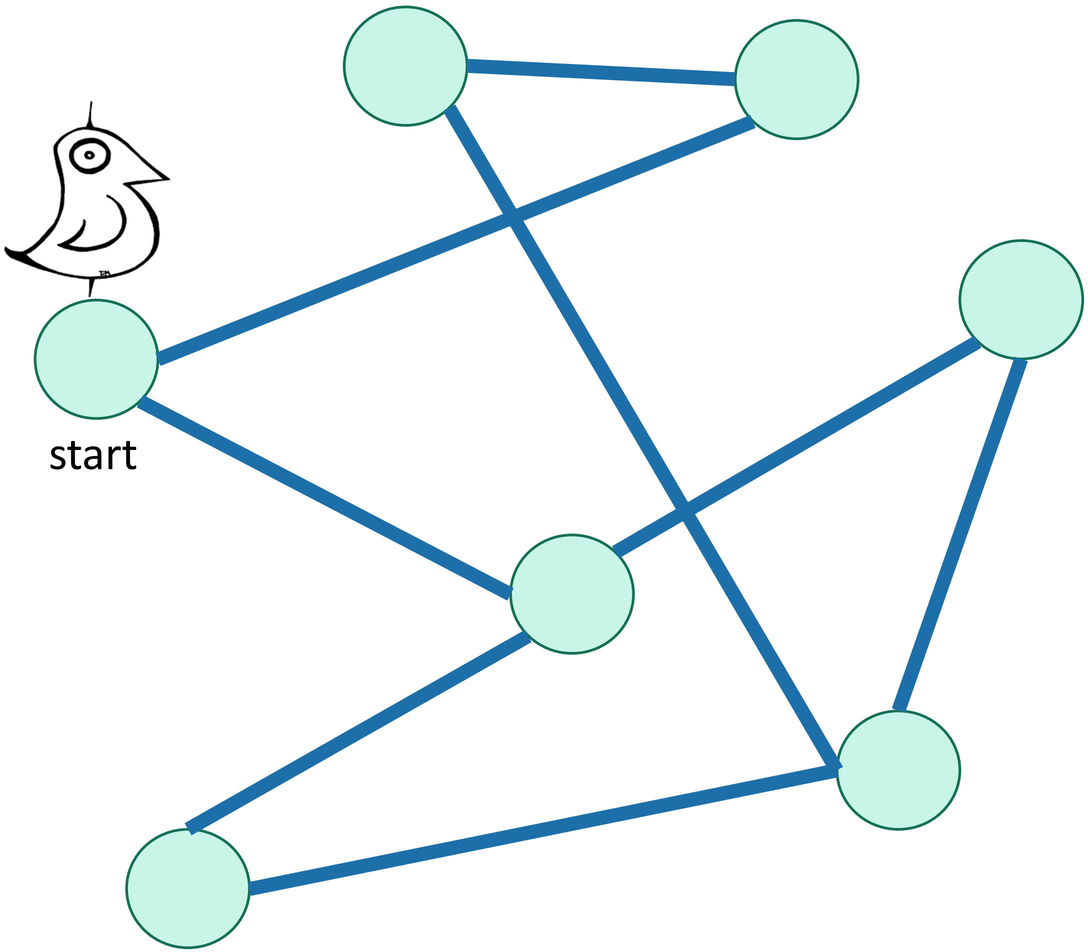
Application of BFS:
Testing Bipartiteness

- Color the levels of the BFS tree in alternating colors.
- If you never color two connected nodes the same color, then it is bipartite.
- Otherwise, it's not.



Breadth-First Search

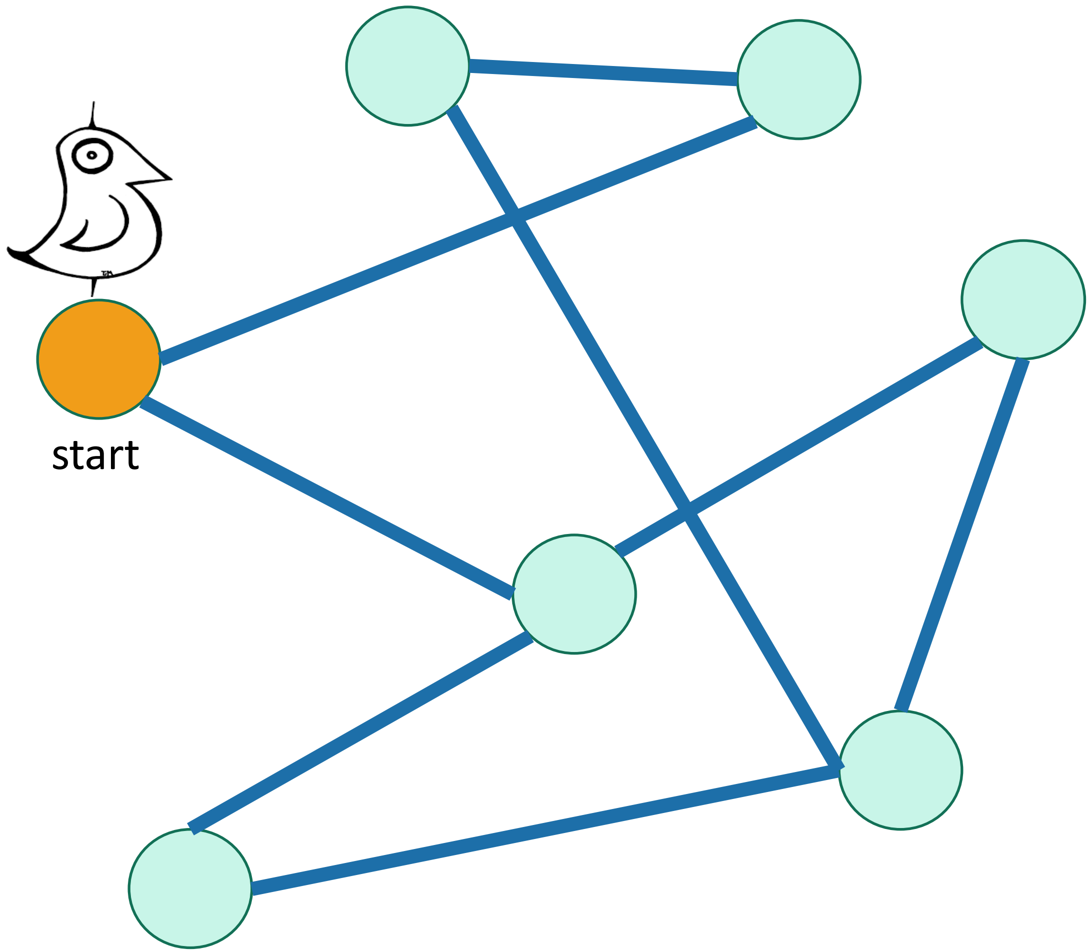
For testing bipartite-ness



- Not been there yet
- Can reach there in zero steps
- Can reach there in one step
- Can reach there in two steps
- Can reach there in three steps

Breadth-First Search

For testing bipartite-ness



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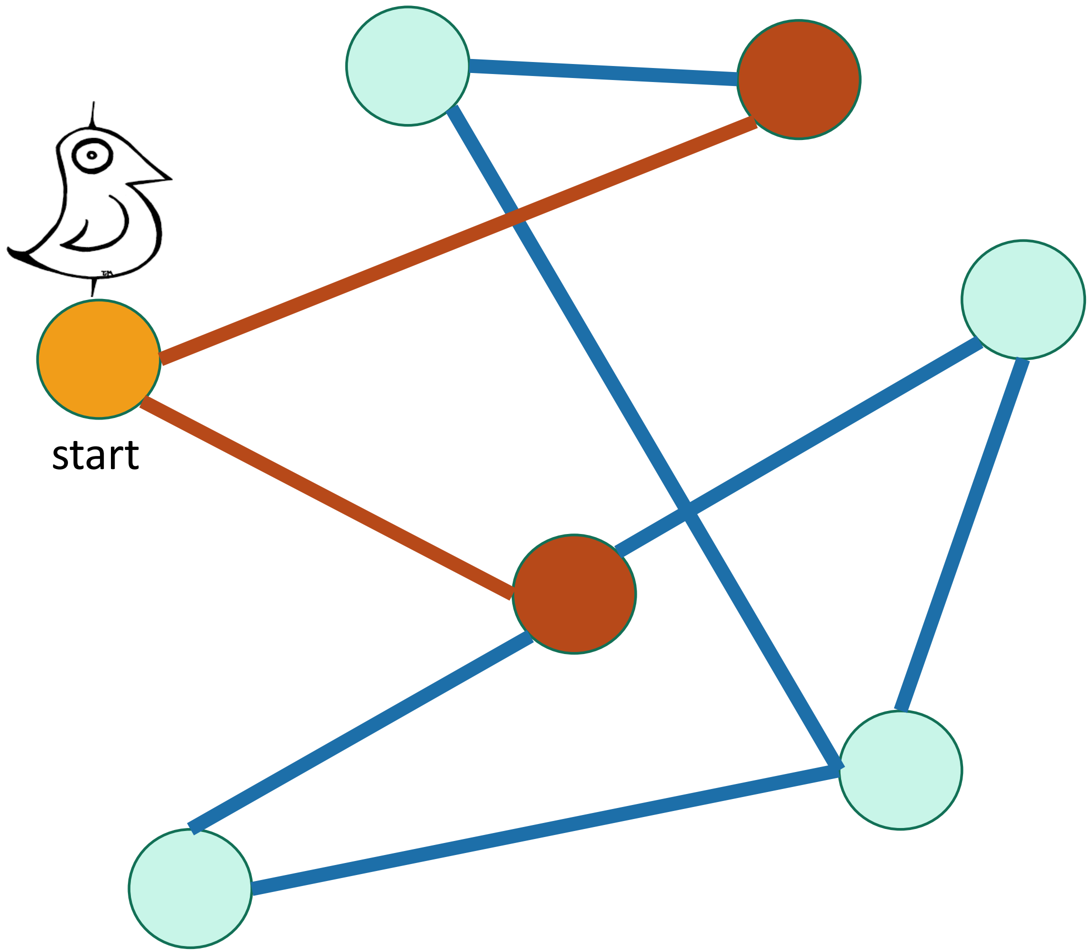
Breadth-First Search

For testing bipartite-ness



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Breadth-First Search

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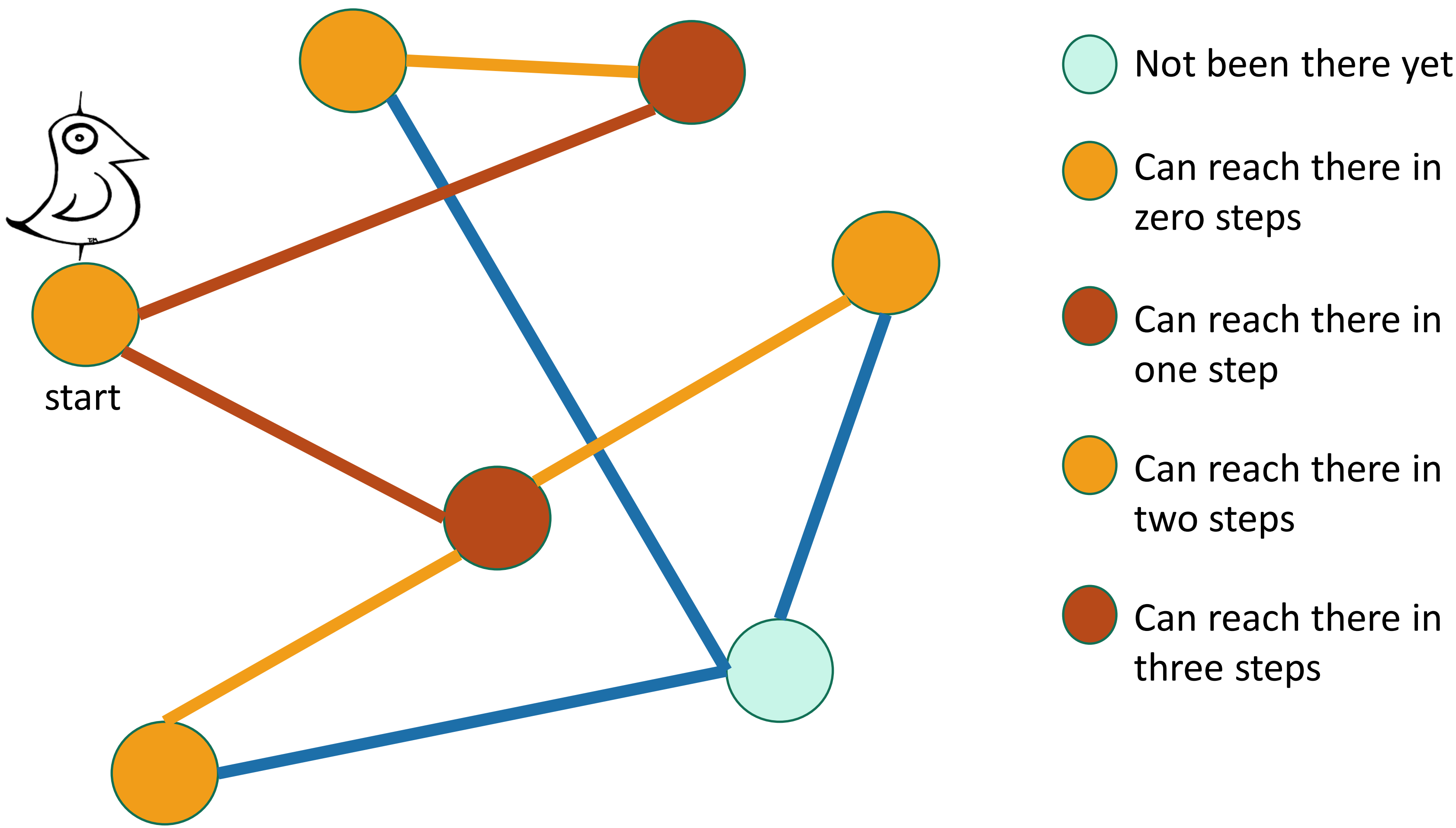
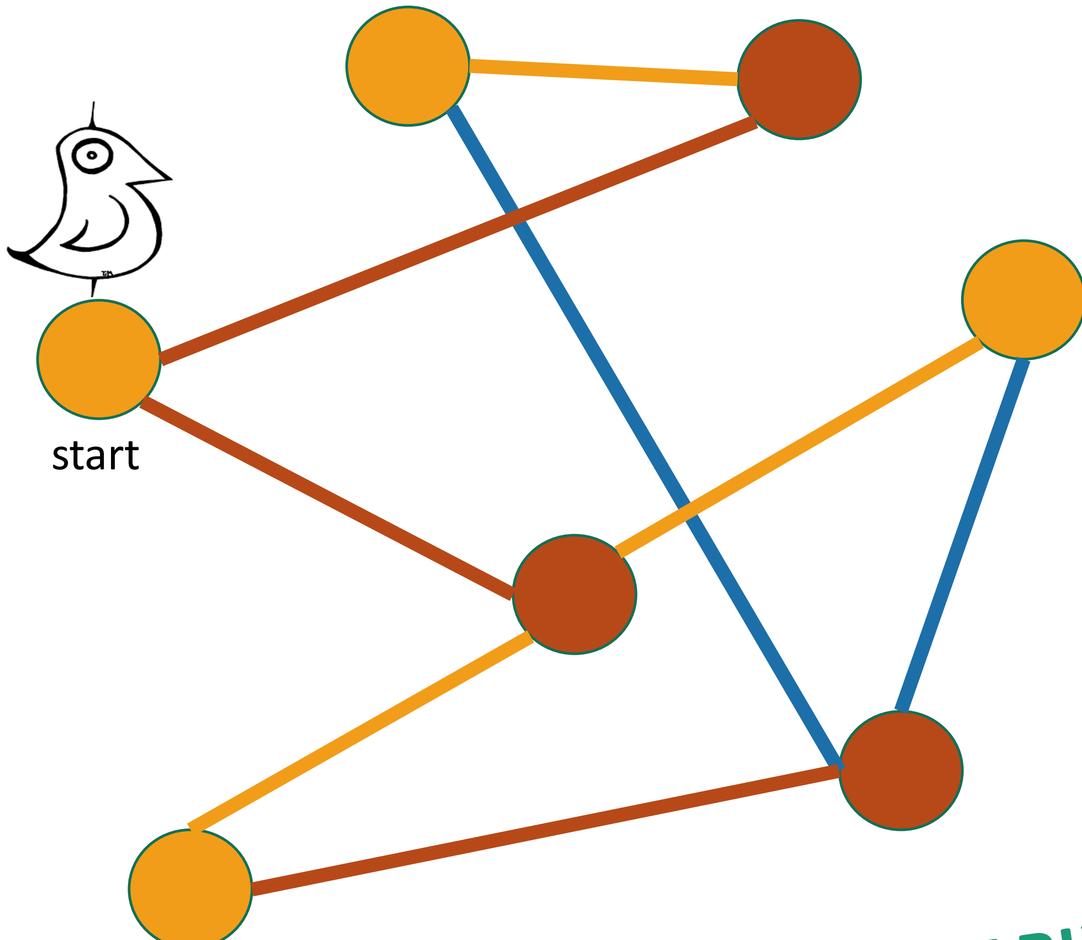


Breadth-First Search

For testing bipartite-ness



- Not been there yet
- Can reach there in zero steps
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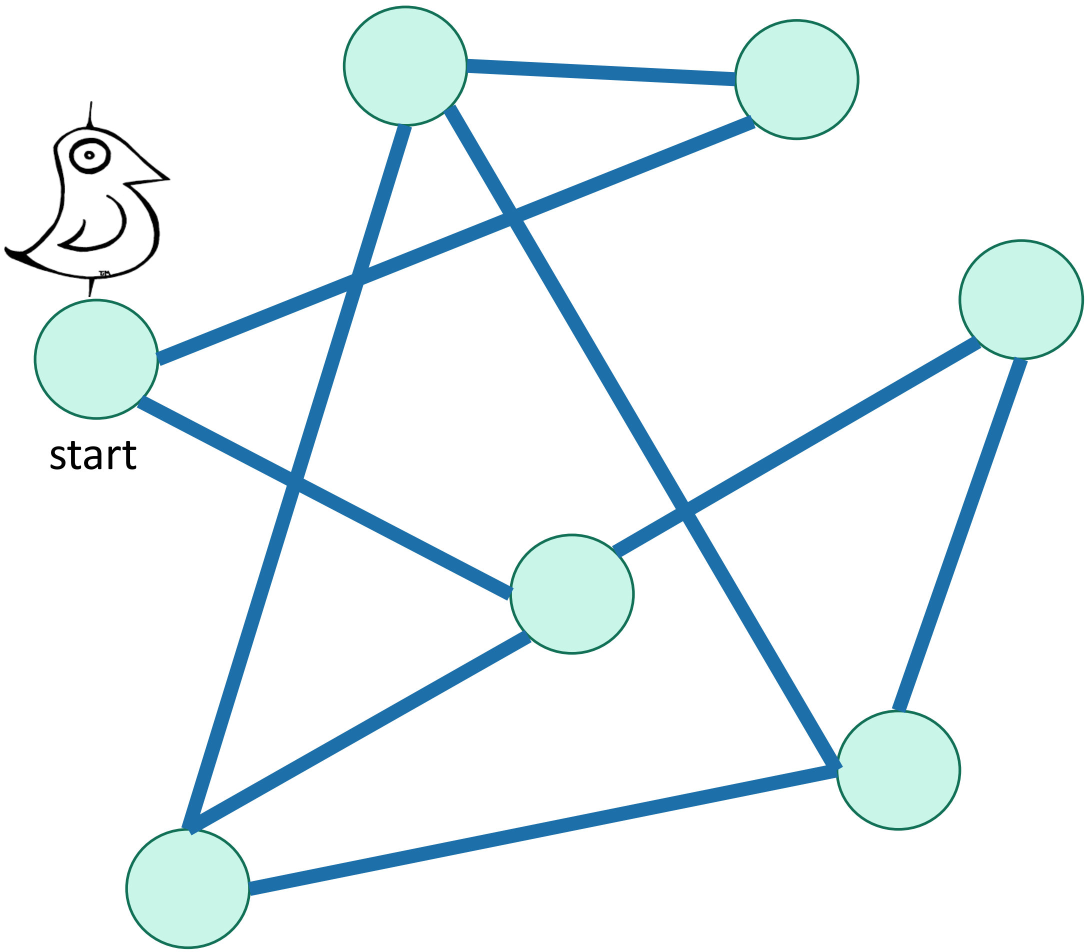
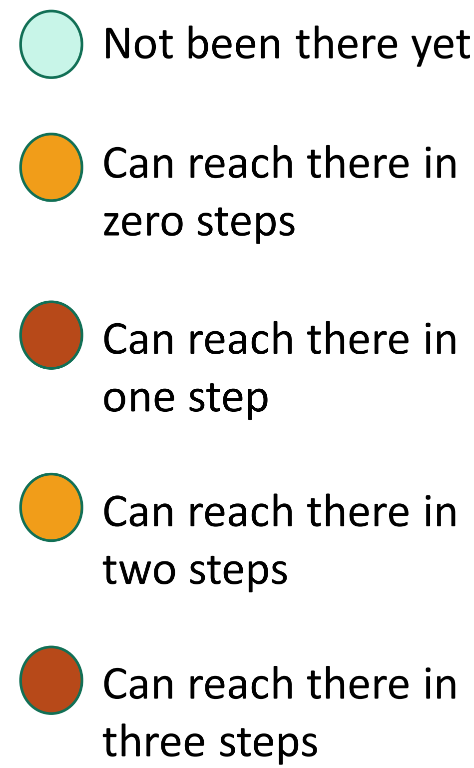
CLEARLY BIPARTITE!

Breadth-First Search

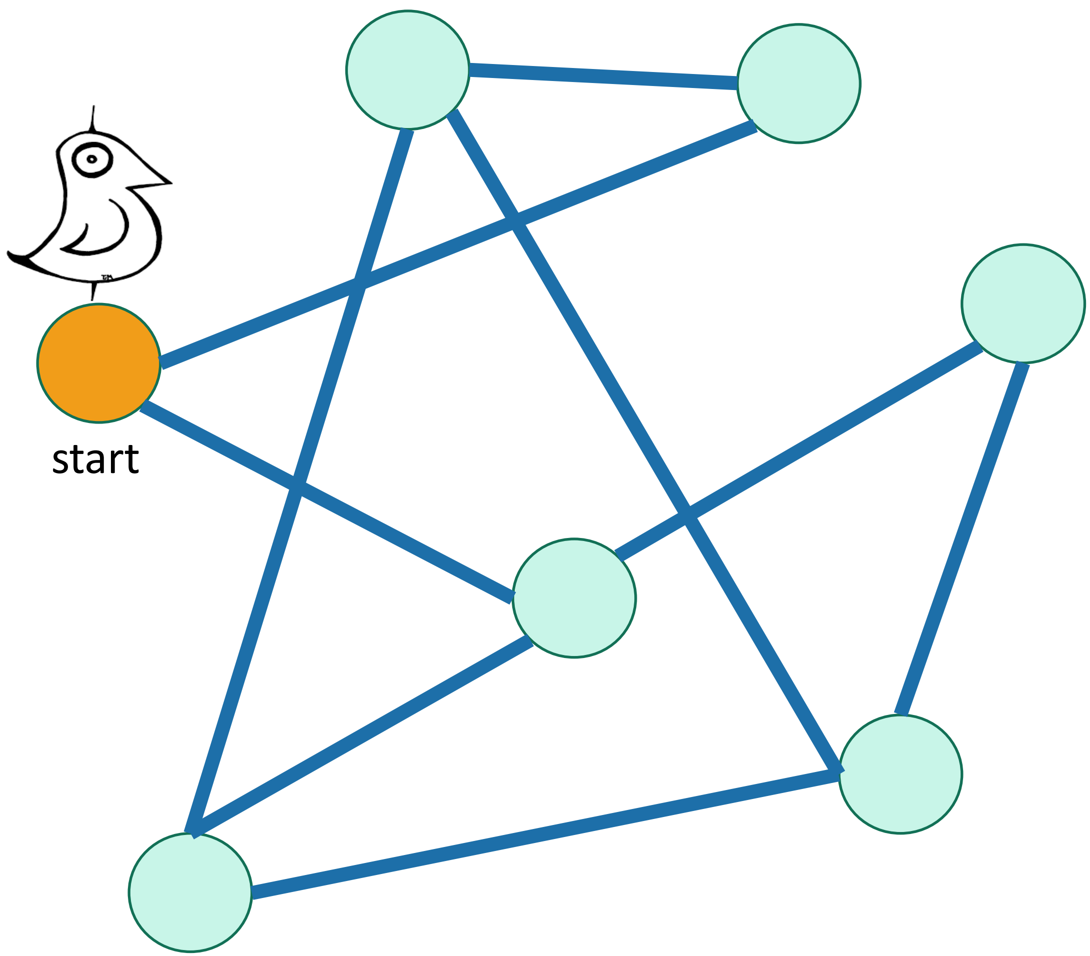
For testing bipartite-ness

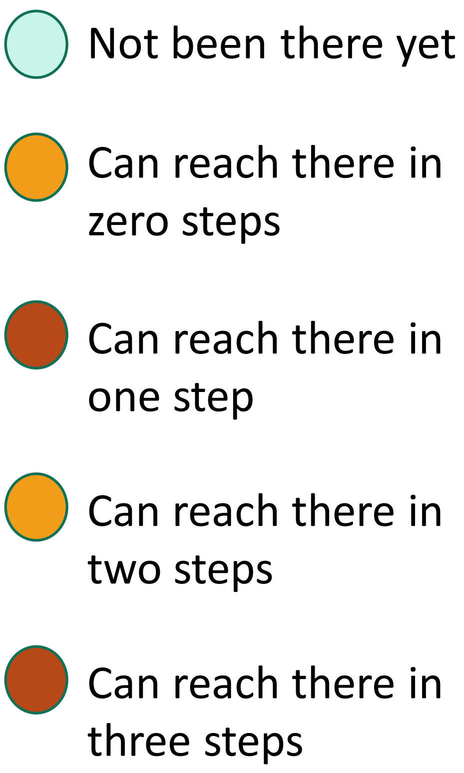




Breadth-First Search

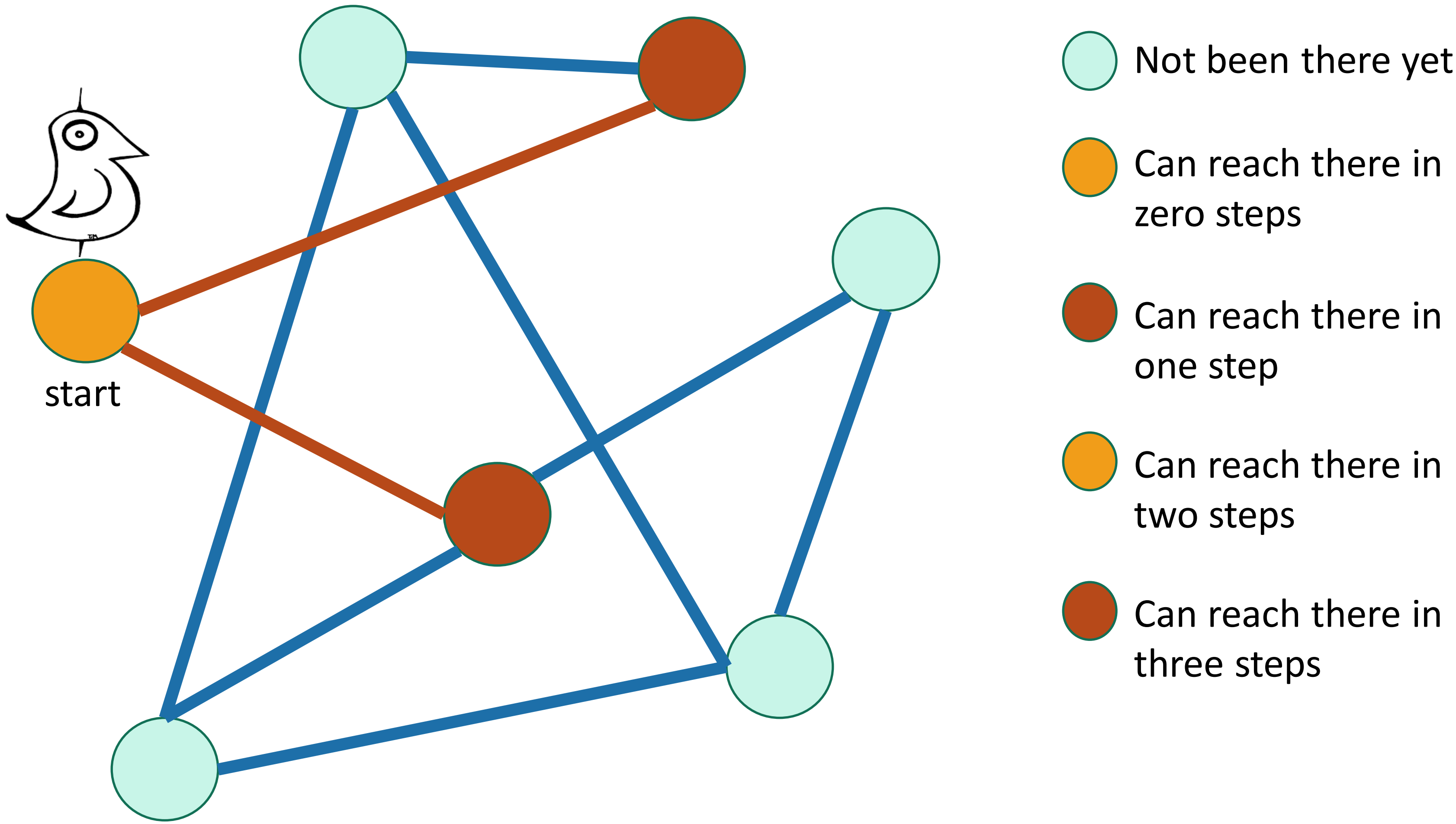
For testing bipartite-ness





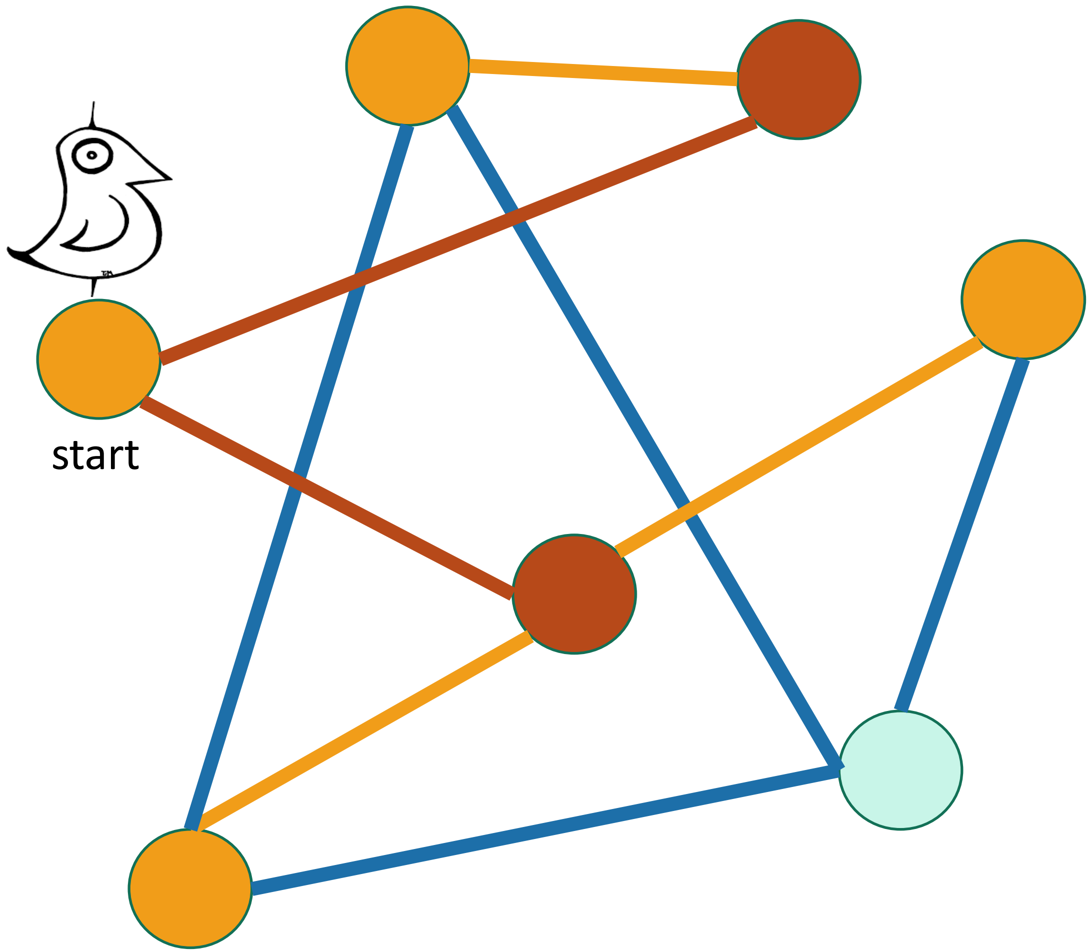
Breadth-First Search

For testing bipartite-ness



Breadth-First Search

For testing bipartite-ness



Not been there yet

Can reach there in zero steps

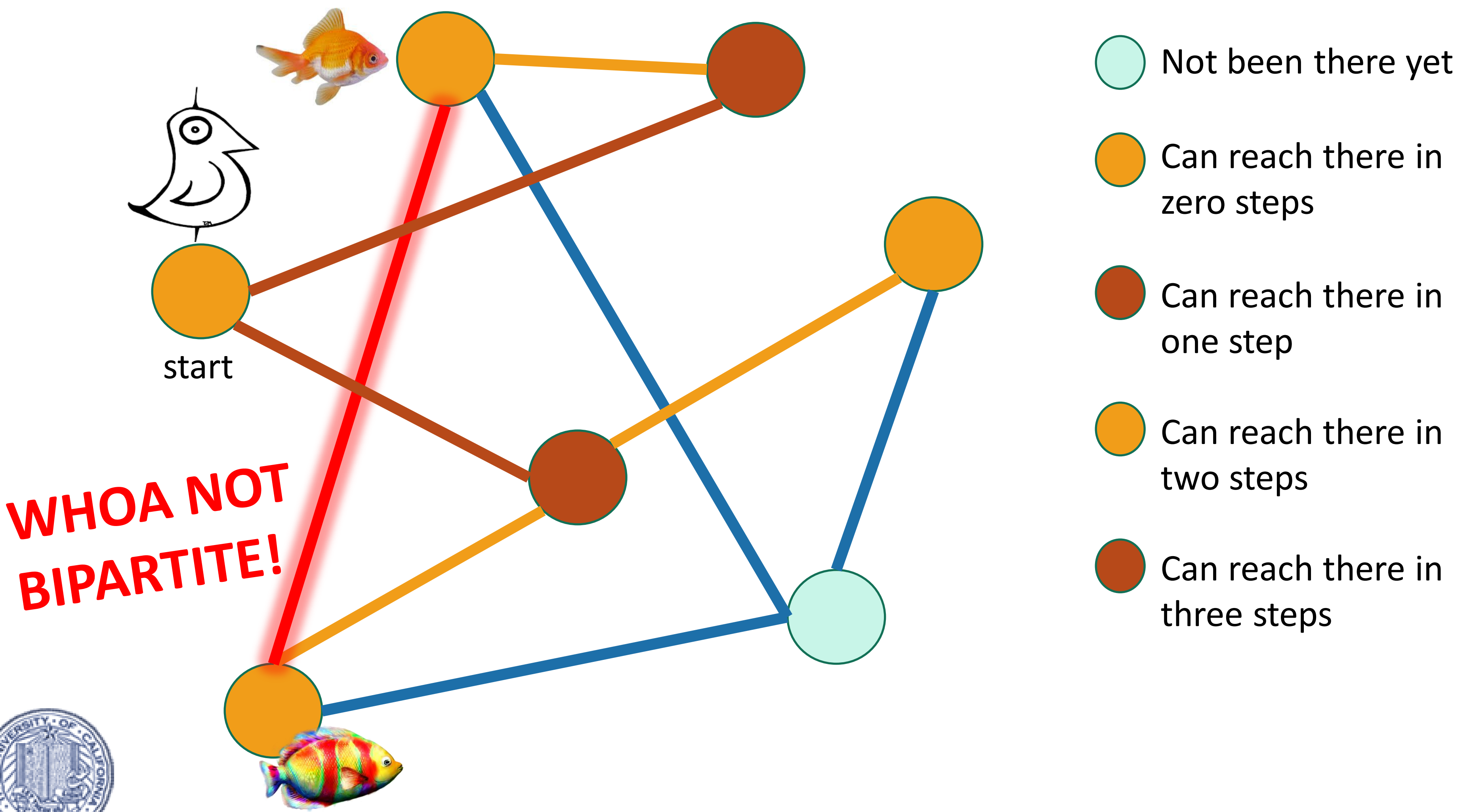
Can reach there in one step

Can reach there in two steps

Can reach there in three steps

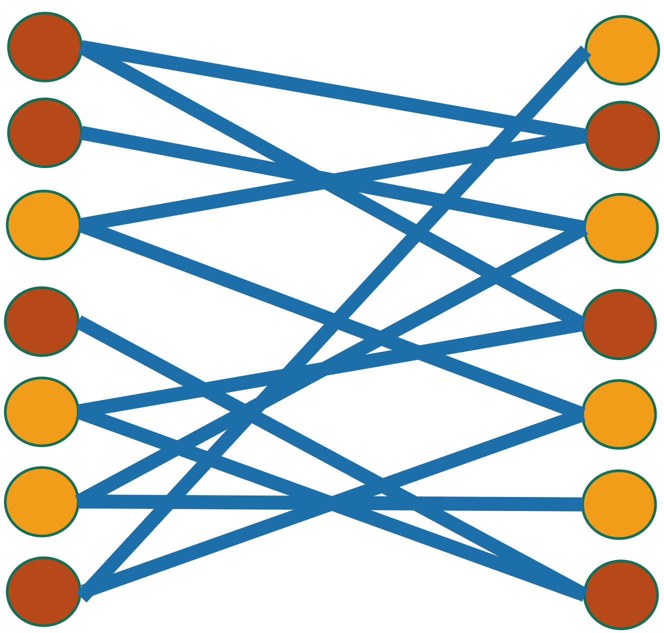
Breadth-First Search

For testing bipartite-ness



Hang on now.

- Just because **this** coloring doesn't work, why does that mean that there is **no** coloring that works?

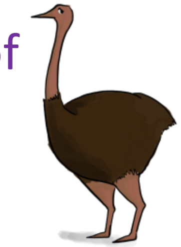


I can come up with plenty of bad colorings on this legitimately bipartite graph…

Plucky the pedantic penguin

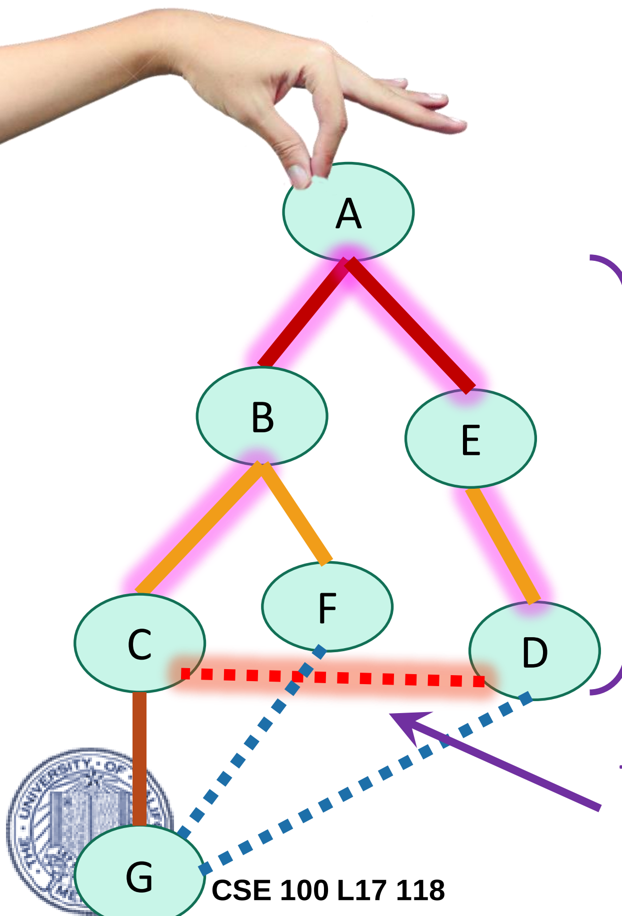
Some proof required

Make this proof sketch formal!



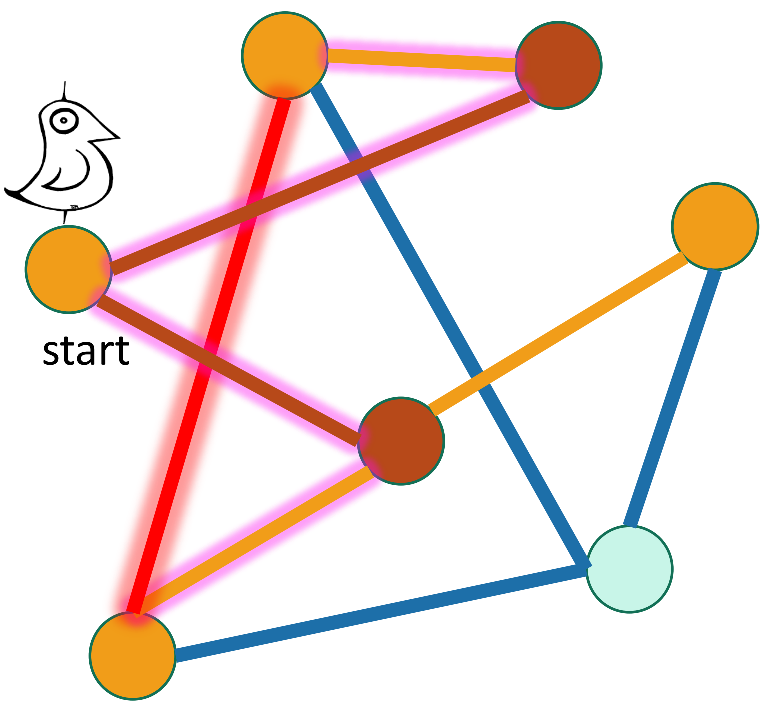
Ollie the over-achieving ostrich

- If BFS colors two neighbors the same color, then it's found a cycle of odd length in the graph.



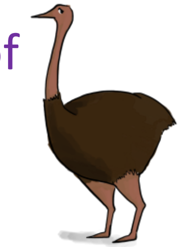
There must be an even number of these edges

This one extra makes it odd



Some proof required

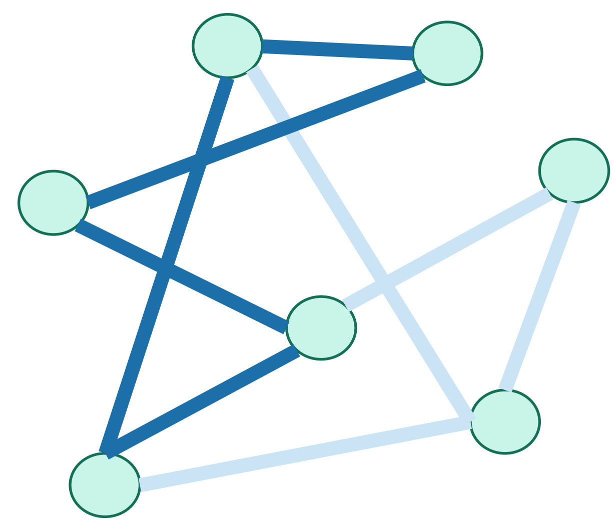
Make this proof sketch formal!



Ollie the over-achieving ostrich

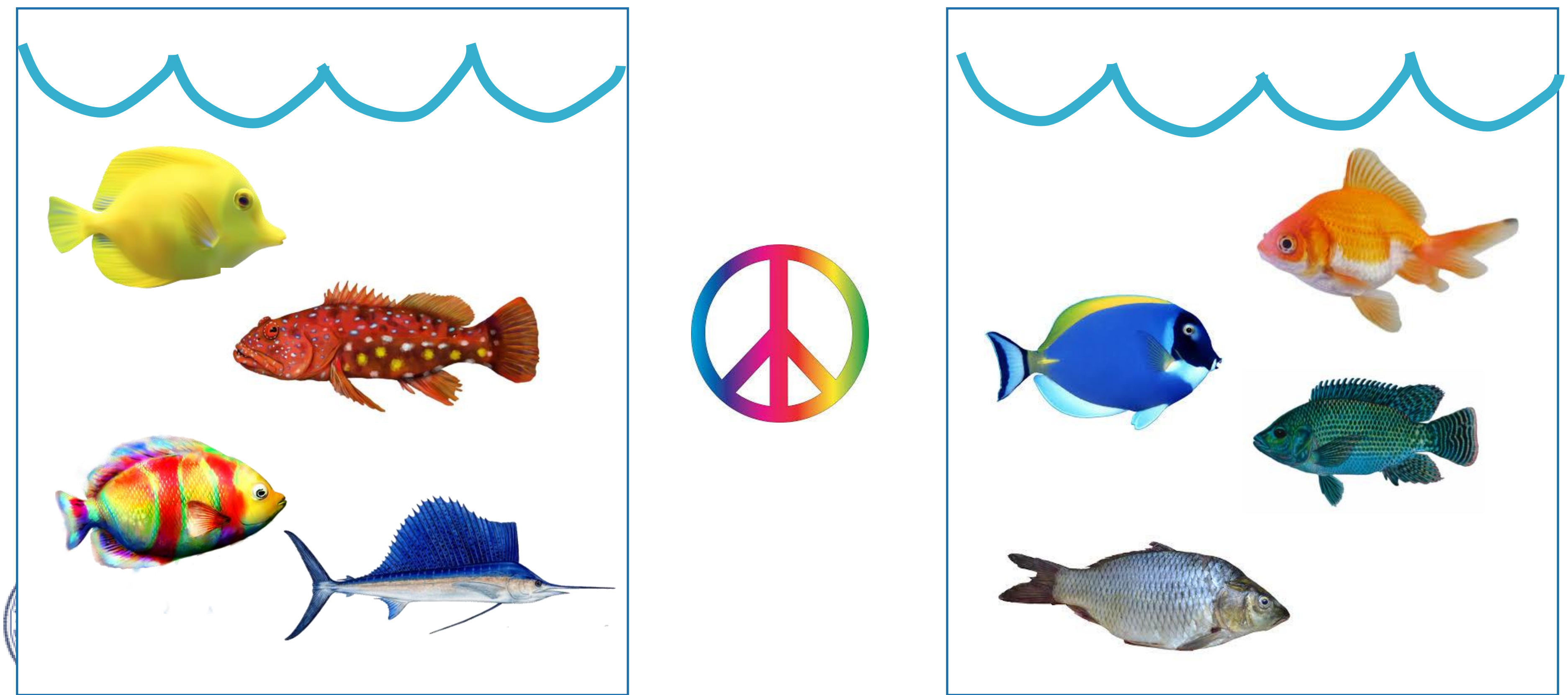
- If BFS colors two neighbors the same color, then it's found a cycle of odd length in the graph.
- But you can **never** color an odd cycle with two colors so that no two neighbors have the same color.
  - [Fun exercise!]

- So you can't legitimately color the whole graph either.
- **Thus it's not bipartite.**



What have we learned?

BFS can be used to detect bipartite-ness in time $O(n + m)$.



Outline

- Part 0: Graphs and terminology
- Part 1: Depth-first search
  - Application: topological sorting
  - Application: in-order traversal of BSTs
- Part 2: Breadth-first search
  - Application: shortest paths
  - Application (if time): is a graph bipartite?

Recap

Recap

- Depth-first search
  - Useful for topological sorting
  - Also in-order traversals of BSTs
- Breadth-first search
  - Useful for finding shortest paths
  - Also for testing bipartiteness
- Both DFS, BFS:
  - Useful for exploring graphs, finding connected components, etc

Still open (next few classes)

- We can now find components in undirected graphs…
  - What if we want to find strongly connected components in directed graphs?
- How can we find shortest paths in weighted graphs?
- What is Samuel L. Jackson’s Erdos number?
  - (Or, what if I want everyone’s everyone-else number?)

Next Lecture

- Strongly Connected Components