Homework Quiz #1

1. If A and B are both $n \times n$ matrices, then we know:

$$AB = BA.$$

Solution: FALSE. In general matrix multiplication does not commute.
Here is just one example:

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix}.$$

We have:

$$AB = \begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix} \text{ and } BA = \begin{bmatrix} 2 & 4 \\ 0 & 0 \end{bmatrix}.$$

2. Suppose A, B and C are $n \times n$ matrices. When does the following hold:

$$A(BC) = (AB)C.$$

Solution: For all choices of matrices A, B and C. Matrix multiplication is associative.

3. Choose a value for b that guarantees that the following system has infinitely many solutions:

$$\begin{aligned} 3x + 2y &= 10 \\ 6x + 4y &= b. \end{aligned}$$

Solution:

$$\left[\begin{array}{cc|c} 3 & 2 & 10 \\ 6 & 4 & b \end{array}\right] \xrightarrow{R_2 \to R_2 - 2R_1} . \left[\begin{array}{cc|c} 3 & 2 & 10 \\ 0 & 0 & b-20 \end{array}\right].$$

If $b = 20$ then we will have a free variable, and the system will have infinitely many solutions:

$$x_2 = t \text{ and } 3x_1 + 2x_2 = 10 \implies 3x_1 + 2t = 10 \implies x_1 = \frac{1}{3}(10 - 2t).$$

4. Choose a value for b that guarantees that the following system has no many solutions:

$$\begin{aligned} 3x + 2y &= 10 \\ 6x + 4y &= b. \end{aligned}$$

Solution:

$$\left[\begin{array}{cc|c} 3 & 2 & 10 \\ 6 & 4 & b \end{array}\right] \xrightarrow{R_2 \to R_2 - 2R_1} . \left[\begin{array}{cc|c} 3 & 2 & 10 \\ 0 & 0 & b-20 \end{array}\right].$$

If $b \neq 20$ then we have no solutions. So, for example $b = 4$.

5. **Prove** that the product of two 2×2 upper triangular matrices A and B is again a upper triangular matrix.

> **Solution:** We first define A and B to be 2×2 upper triangular matrices which means that, $a_{1,2} = b_{1,2} = 0$.
> We will prove the matrix $C = AB$ is also an upper triangular matrix.by showing $c_{1,2} = 0$.
> By the definition of matrix multiplication we know that:
>
> $$c_{1,2} = \sum_{k=1}^{2} a_{1,k} b_{k,2} = a_{1,1} b_{1,2} + a_{1,2} b_{2,2} = a_{1,1}(0) + (0) b_{2,2} = 0.$$
>
> Since all other terms for C can be non-zero, we have shown that C must be an upper triangular matrix.

6. Consider the two following systems:

$$\begin{array}{rcrcrcr} 3x_1 & + & 2x_2 & - & x_3 & = & -2 \\ & & x_2 & & & = & 3 \\ & & & & 2x_3 & = & 4. \end{array} \tag{1}$$

$$\begin{array}{rcrcrcr} 3x_1 & + & 2x_2 & - & x_3 & = & -2 \\ -3x_1 & - & x_2 & + & x_3 & = & 5 \\ 3x_1 & + & 2x_2 & + & x_3 & = & 2. \end{array} \tag{2}$$

Which of the following row operations translates the Hard system (2) to the Easy system (1).

> **Solution:** In order to do this, we're going to write (2) as an augmented matrix and carry out the row operations:
>
> $$\left[\begin{array}{ccc|c} 3 & 2 & -1 & -2 \\ -3 & -1 & 1 & 5 \\ 3 & 2 & 1 & 2 \end{array}\right] \xrightarrow{R_2 \to R_2 + R_1} \left[\begin{array}{ccc|c} 3 & 2 & -1 & -2 \\ 0 & 1 & 0 & 3 \\ 3 & 2 & 1 & 2 \end{array}\right] \xrightarrow{R_3 \to R_3 - R_1} \left[\begin{array}{ccc|c} 3 & 2 & -1 & -2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 2 & 4 \end{array}\right].$$
>
> Thus we arrive at the same system as (1).