Complete the following tasks. You need to show work for full credit, and you may use a calculator such as RStudio to finish the calculations. Some answers have been provided. Assemble your work into one PDF document and upload the PDF back into our CatCourses page.

1. **Geometric Thoughts** Starting with the probability mass function for the geometric distribution ("success" on the $k + 1$ trial)

$$f(k) = (1 - p)^k p, \quad k = 0, 1, 2, 3, ..., \quad 0 < p \leq 1$$

   (a) Derive the cumulative mass function $F(k) = 1 - (1 - p)^{k+1}$

   (b) Show that all of the probabilities do indeed add up to 100% in the following ways:

   i. $\displaystyle\sum_{k=0}^{\infty} f(k)$

   ii. $\displaystyle\lim_{k \to \infty} F(k)$

   (c) Given $|x| < 1$, take the derivative of both sides of $\displaystyle\sum_{k=0}^{\infty} x^k = \frac{1}{1 - x}$ with respect to x.

   (d) Use the above result to derive the expected value for the geometric distribution:

   (e) Using the complementary CMF $P(X > k) = 1 - F(k) = (1 - p)^{k+1}$, prove the *memoryless property*

2. Even at conferences full of physicians, female doctors noticed that they were introduced as "Doctor" about 49% of the time.[1] Visualize a female physician networking at such an event.

   (a) What is the probability that she will be introduced as "Doctor" 3 times before not being addressed by the title on the fourth introduction?

   (b) What is the probability that she will be introduced as "Doctor" at most 3 times before not being addressed by the title?

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[1]Source: https://www.washingtonpost.com/national/health-science/
why-are-female-doctors-introduced-by-first-name   while-men-are-called-doctor/2017/06/
23/b790ddf2-4572-11e7-a196-a1bb629f64cb_story.html

3. During the 2020-2021 NBA season, Stephen Curry had a 3-point shooting success rate of 0.421.

   (a) During the 3-point shootout[2], each contestant has 25 shots. Using the rate above, what was the probability of Curry's 23-basket round?

   (b) During the season, what is the probability that Stephen Curry will shoot at least 4 consecutive 3-point shots before he misses?

   (c) What is the probability that Curry will shoot at least 9 consecutive 3-point shots given that he has already made 5?

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[2]Source: `https://www.basketball-reference.com/players/c/curryst01.html`

Here are some of answers. Note that numbers may slightly vary depending on when and where the rounding took place.

1.

2. (a) 0.0600
   (b) 0.9424

3. (a) 2.2960×10^{-7}
   (b) 0.0314
   (c)