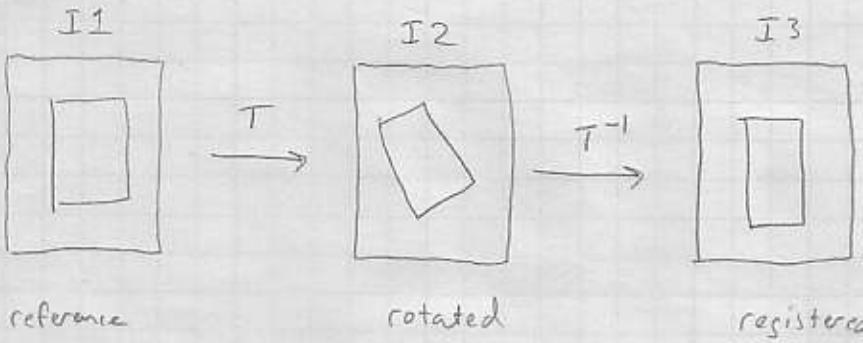
Chap. 2 Rectifying Affine Transformations



Suppose I2 is an affine transformed version of I1 and we would like to create I3 from I2 that "matches" I1
"is registered to"

Steps:

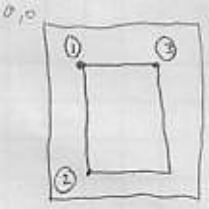
1) Estimate transformation from I1 to I2: T.

2) Apply T^{-1} to I2 to get I3.

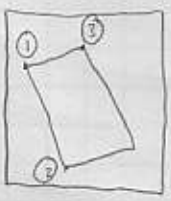
1) Estimate T

$$[x \;\; y \;\; 1] = [v \;\; w \;\; 1] \begin{bmatrix} t_{11} & t_{12} & 0 \\ t_{21} & t_{22} & 0 \\ t_{31} & t_{32} & 1 \end{bmatrix}$$

Pick 3 points in I1 and corresponding points in I2:



1: (136, 168)
2: (399, 162)
3: (137, 365)

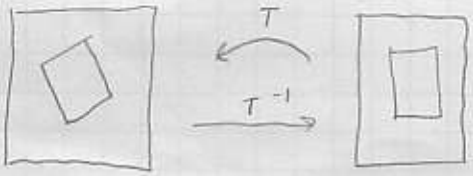


1: (170, 127)
2: (418, 212)
3: (101, 301)

$$\underset{B}{\begin{bmatrix} 170 & 127 & 1 \\ 418 & 212 & 1 \\ 101 & 301 & 1 \end{bmatrix}} = \underset{A}{\begin{bmatrix} 136 & 168 & 1 \\ 399 & 162 & 1 \\ 137 & 365 & 1 \end{bmatrix}} \underset{T}{\begin{bmatrix} t_{11} & t_{12} & 0 \\ t_{21} & t_{22} & 0 \\ t_{31} & t_{32} & 1 \end{bmatrix}}$$

$$T = A^{-1}B = \begin{bmatrix} 0.9349 & 0.3433 & 0 \\ -0.3550 & 0.8815 & 0 \\ 102.4980 & -67.7824 & 1 \end{bmatrix}$$

2) Now, create $I3$ which is a rectified version of $I2$



For every pixel in $I3$, use "inverse" mapping T to determine where pixel "came from" in $I2$. Then, use interpolation to determine its value.

Let's look closer at T:
Assume only translation & rotation (no scaling or shearing)

T = [translate] [rotate] [translate]

$$T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ c & d & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ a\cos\theta - b\sin\theta + c & a\sin\theta + b\cos\theta + d & 1 \end{bmatrix}$$

$$\cos\theta = \frac{0.9349 + 0.8815}{2} = 0.9082 \Rightarrow \theta = 24.7^\circ$$

$$\sin\theta = \frac{0.3433 + 0.3550}{2} = 0.3486 \Rightarrow \theta = 20.4^\circ$$

Assume $a = 275$, $b = 270$ $\Rightarrow$ $c = 258$, $d = 273$