Homework Assignment #8

Remember, this Homework Assignment is **not collected or graded**! But it is in your best interest to do it as the Homework Quiz will be based on it and it is the best way to ensure you know the material.

Section 4.2: Properties of the Determinant

1. If a 4×4 matrix A has $\det(A) = 1/2$. What is the value of:
   (a) $\det(2A)$
   (b) $\det(-A)$
   (c) $\det(A^2)$
   (d) $\det(A^{-1})$

   Hint: It might be helpful to remember the Properties of the Determinant we discussed in class (Week 10, Tuesday) and that are given in your Textbook in Section 4.1.

2. Recall that if Q is an orthogonal matrix, that is an $n \times n$ matrix with orthonormal columns, then Q is invertible and we have: $Q^{-1} = Q^T$. This means $Q^T Q = QQ^T = I$.

   Use properties of the Determinant to show that $\det(Q) = \pm 1$. (Hint: In particular you'll need Properties 9 and 10).

3. Use row operations to show that the 3×3 matrix given has the following determinant.

$$A = \begin{bmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{bmatrix}.$$

$$\det(A) = (b-a)(c-a)(c-b)$$

Section 5.1: Introduction to Eigenvalues and Eigenvectors

4. Find the eigenvalues and eigenvectors of A:

$$A = \begin{bmatrix} 0 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 0 & 0 \end{bmatrix}.$$

   Check that $\lambda_1 + \lambda_2 + \lambda_3$ equals the trace of A and $(\lambda_1 \lambda_2 \lambda_3)$ equals the determinant of A.

   Hint: To calculate the 3×3 determinant, you might find it helpful to remember your Properties of Determinants, the basket-weaving method which I've now learned should more likely be credited to French mathematician Pierre Sarrus.

The Method of Sarrus

Pierre Frédéric Sarrus (10 March 1798, Saint-Affrique - 20 November 1861) was a French mathematician. Sarrus was professor at the University of Strasbourg, France (1826-1856) and member of the Academy of Sciences in Paris (1842). He discovered a mnemonic rule for solving the determinant of a 3-by-3 matrix, named Sarrus' scheme, which provides an easy-to-remember method of working out the determinant of a 3-by-3 matrix (as illustrated below)

$$\det(M) = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \left(a_{11}a_{22}a_{33} + a_{21}a_{32}a_{13} + a_{31}a_{12}a_{33}\right) - \left(a_{13}a_{22}a_{31} + a_{23}a_{32}a_{11} + a_{33}a_{12}a_{31}\right)$$