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CSE 185

Hough Transform

A computationally efficient way to determine whether edge points lie on boundary of objects with particular shapes

- lines
- circles
- polygons

} some knowledge of what looking for (priors)

We will focus on line detection.

Given n points in an image, suppose we wanted to find subsets of points that lie on straight lines.

One approach
- Find all lines determined by every pair of points
- Find all subsets of points that are close to particular lines

Step one requires finding $n(n-1)/2 \sim n^2$ lines
Step two requires performing $n(n(n-1))/2 \sim n^3$ comparisons

computationally prohibitive!

Hough transform (1962) is alternate approach.

Consider pt. (x_i, y_i) and general equation of line in slope-intercept form

$$y_i = ax_i + b.$$

Infinitely many lines pass through (x_i, y_i) but all satisfy equation $y_i = ax_i + b$ for varying values of a, b.

If we write $b = -x_i a + y_i$ and considering the ab-plane (called parameter space) yields the equation for a single line for a fixed pair (x_i, y_i)

A second point (x_j, y_j) will also yield a line in parameter space.

These lines in parameter space will intersect at a point (a', b') which defines the line passing through (x_i, y_i) and (x_j, y_j):

$$y = a'x + b'$$

Fig. 10.31 All points on this line yield lines in parameter space that intersect at (a', b')

A strategy:

1) For each edge point (x_i, y_i), plot line in parameter space

$$b = -x_i a + y_i$$

2) Determine where large numbers of parameter space lines interesect. This will give the (a, b) values of the lines in image space.

Problem: as lines approach vertical (in image space), slope of line approaches infinity. a-axis in parameter space would have to go to infinity.

Solution: instead, use normal representation of a line:

$$x\cos\theta + y\sin\theta = \rho$$

Horizontal line has $\theta = 0°$ with ρ the positive x-intercept
Vertical line has $\theta = 90°$ with ρ the positive y-intercept

Fig. 10.32

Each sinusoidal curve in 10.32(b) represents the family of curves that pass through a particular point (x_k, y_k) in the xy-plane (image plane)

The intersection pt. (ρ', θ') in 10.32(b) corresponds to the line that passes through both (x_i, y_i) and (x_j, y_k) in 10.32(a)

Strategy:

1) Partition $\rho\theta$ parameter space into accumulator cells

where $-90° \leq \theta \leq 90°$
and
$-D \leq \rho \leq D$ where D is maximum distance between opposite corners of image.

Call this partitioning $A(i,j)$
in which cell at coordinates (i,j) corresponds to the square associated with parameter-space coordinates (ρ_i, θ_j)

Initially, set these accumulator cells to zero.

2) For every edge point in xy-plane, let θ equal each of the allowed subdivision values on θ-axis and solve for the corresponding ~~$\not{z}$~~ ρ using the equation

$$\rho = x_k \cos\theta + y_k \sin\theta$$

Resulting ρ values are rounded off ~~the~~ to the nearest allowed cell on the ρ axis.

If a choice of θ_p results in solution ρ, we let

$$A(p,q) = A(p,q) + 1$$

3) At the end of this procedure, a value of P in $A(i,j)$ means that P points in the xy-plane lie on the line

$$x\cos\theta_j + y\sin\theta_j = \rho_i$$

The number of subdivisions in the $\rho\theta$-plane determines the colinearity of these points.

This method is linear in the number of edge points.

Fig 10.33

Summary
1) Obtain a binary edge image (using Canny ED for example)
2) Specify subdivisions in $\rho\theta$-plane
3) For each edge point, accumulate values in $\rho\theta$-plane
4) Examine the counts of the accumulator cells for high pixel concentrations.
5) Examine the relationship (principally for continuity) between pixels in a chosen cell.

Fig. 10.34

Hough transform can be generalized to other shapes.

Example: circle

$$(x - c_1)^2 + (y - c_2)^2 = c_3^2$$