# Homework #2
## PDF version due Mon. Oct. 24 11:59pm through CatCourses
## (Upload a PDF of a legible scan of your written solution or a PDF of it typeset in Word/Latex/etc.)

**PROBLEM 1:**

Consider the two image subsets, $S_1$ and $S_2$, shown in the following figure. For $V = \{1\}$, determine whether these two subsets are (a) 4-adjacent, (b) 8-adjacent, or (c) $m$-adjacent.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | $S_1$ | | | | $S_2$ | | |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 |

**PROBLEM 2:**

Consider the image segment shown.

★**(a)** Let $V = \{0, 1\}$ and compute the lengths of the shortest 4-, 8-, and $m$-path between $p$ and $q$. If a particular path does not exist between these two points, explain why.

**(b)** Repeat for $V = \{1, 2\}$.

| | | | |
|---|---|---|---|
| 3 | 1 | 2 | 1 $(q)$ |
| 2 | 2 | 0 | 2 |
| 1 | 2 | 1 | 1 |
| $(p)$ 1 | 0 | 1 | 2 |

**PROBLEM 3:**

Let $H[\cdot]$ be the operator that determines the minimum pixel value in an image. That is, if the image $f(x,y)$ has the following pixel values

$f(x,y)=$

| | | |
|---|---|---|
| 30 | 23 | 6 |
| 110 | 128 | 234 |
| 12 | 4 | 175 |

then

$$H[f] = 4.$$

Prove that $H[\cdot]$ is non-linear.

(Remember that to prove non-linearity, you just need to come up with a counter-example; i.e., images $f1$ and $f2$ and constants $a$ and $b$ such that $H[a*f1+b*f2] \neq a*H[f1]+b*H[f2]$.)

**PROBLEM 4:**

In general, do affine transformations commute?

That is, given two affine transformations $T_1$ and $T_2$, does the transformation $T_1T_2$ give the same result as $T_2T_1$? ($T_1$ and $T_2$ are the matrix representations of the transformations. You can interpret $T_1T_2$ as first applying $T_1$ and then applying $T_2$ or you can interpret it as the matrix multiplication of $T_1$ and $T_2$.)

If not, provide a counter example. That is, provide affine transformation matrices $T_1$ and $T_2$ such that a point mapped by $T_1T_2$ is different from the same point mapped by $T_2T_1$.

**PROBLEM 5:**

Suppose that only pixels with values 5, 10, 30, and 150 occur in a grayscale image. And suppose that these pixels occur with the following probabilities in the image:

$$p(5) = 0.10$$
$$p(10) = 0.55$$
$$p(30) = 0.05$$
$$p(150) = 0.30$$

(You can assume the usual case where grayscale images have pixels with values 0 through 255.)

(a) Compute the mean of the pixel values in the image.

(b) Compute the variance ($\sigma^2$) of the pixel values in the image.