ENGR 065 Electric Circuits

Lecture 9: The Thévenin and Norton Equivalents

Today's Topics

- Thévenin and Norton theorems
- Thévenin and Norton equivalents
- The applications of Thévenin and Norton theorems

The above topics are covered in Sections 4.10 and 4.11.

Léon Charles Thévenin and Edward Lawry Norton

Meaux (Paris) 30.March. 1857-Paris 21.September.1926

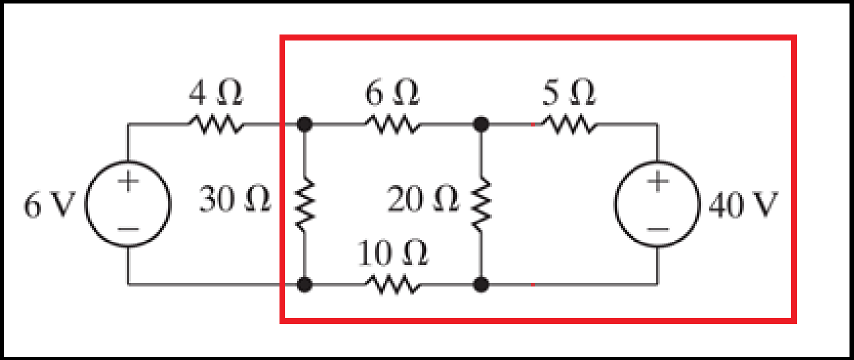
Léon Charles Thévenin (1857 – 1926) was a French telegraph engineer. He extended Ohm's law to the analysis of complex electrical circuits.

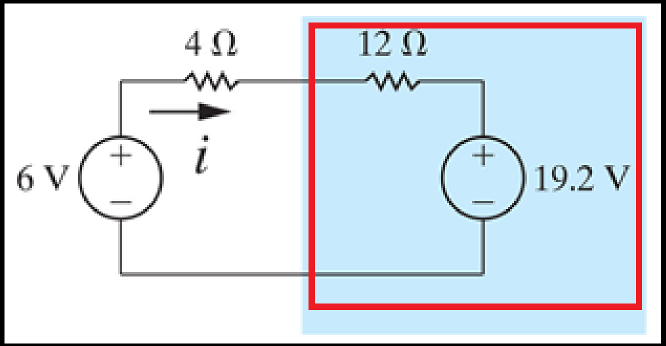
Edward Lawry Norton

(28 July 1898, Rockland, Maine – 28 January 1983, Chatham, New Jersey)

Edward Lawry Norton (1898 – 1983) was an engineer and scientist in Bell Labs. He is famous for developing the concept of Norton equivalent circuit.

Introduction



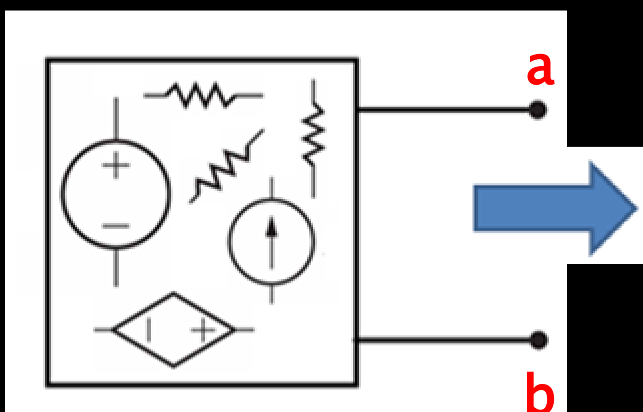


In the last lecture, we used the source transformation method to simplify the circuit shown above. The solution tells us that the circuit in the red box on the left circuit can be simplified to the circuit in the red box on the right.

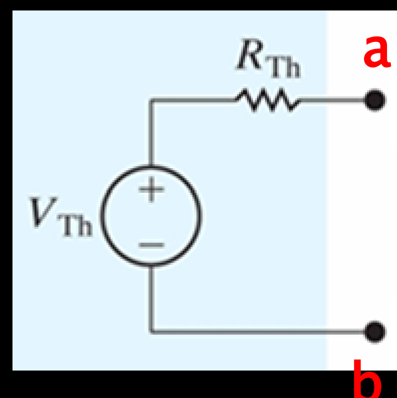
In other words, one of the circuits in the red box can be replaced by the other. They are equivalent! The right is called Thévenin equivalent of the left.

Thévenin and Norton Theorems

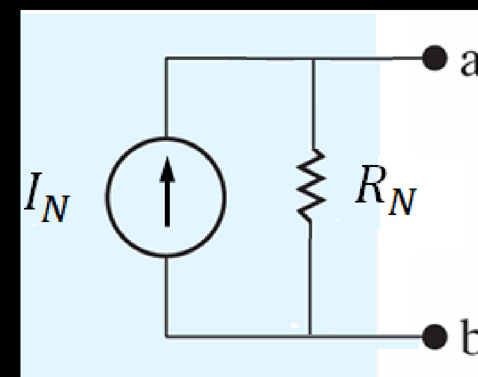
Theorems: Any linear circuit made up of resistors and sources, viewed from two terminals of that circuit, is equivalent to an ideal independent voltage source in series with a resistor (Thévenin) or an ideal independent current source in parallel with a resistor (Norton).



Thévenin equivalent



Norton equivalent

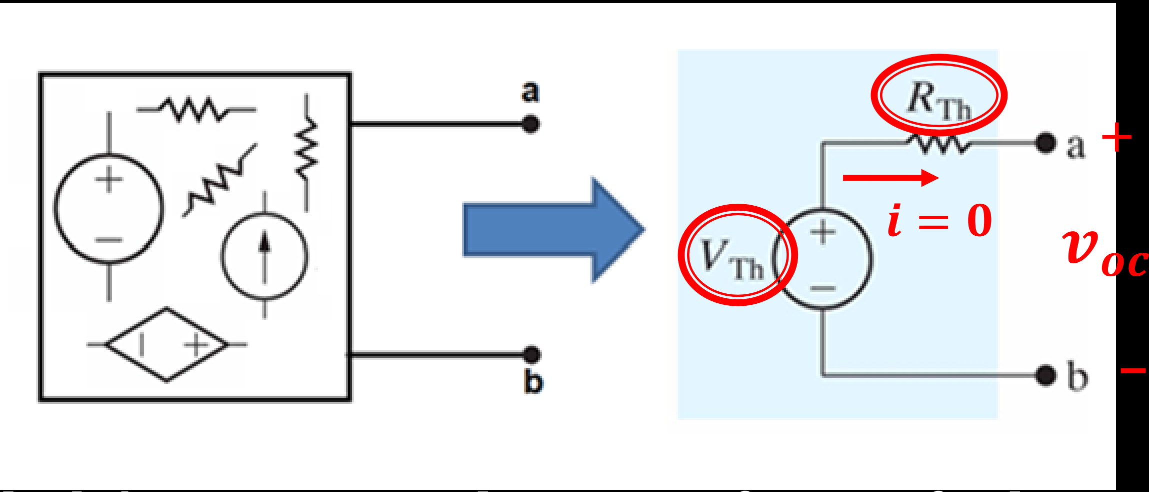


R_{Th} *and* R_N are called the Thévenin and Norton resistance of the circuit.

V_{Th} is called the Thévenin voltage and I_N is called the Norton current.

The Thévenin Equivalent

Original circuit Thévenin equivalent circuit

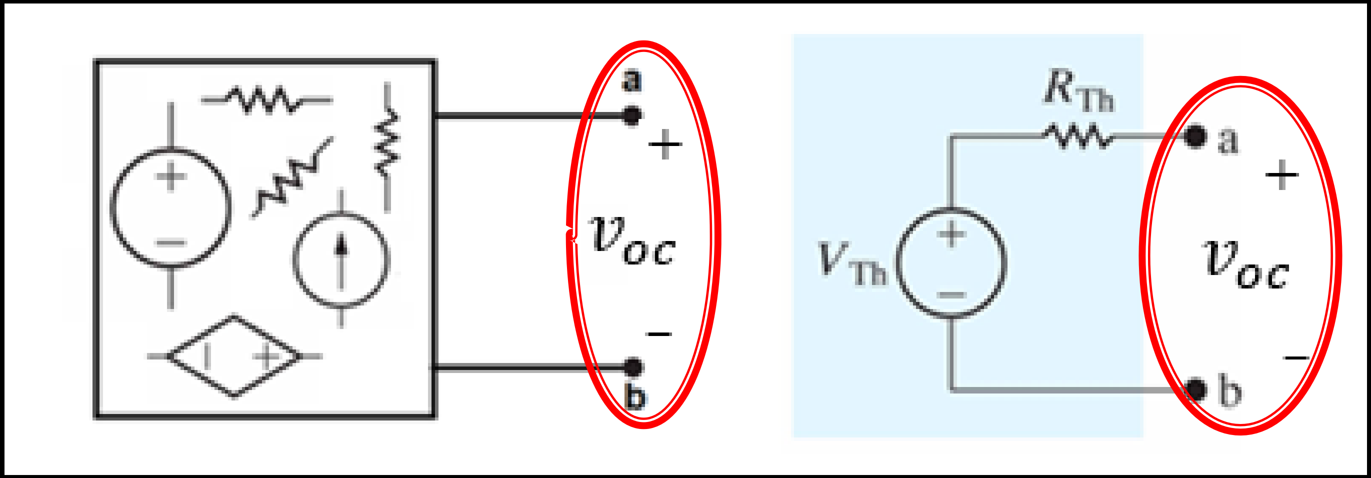


To find Thévenin equivalent is, in fact, to find V_{Th}, R_{Th}.

From the equivalent circuit on the right above, we know $i = 0$, so $V_{Th} = v_{oc}$. v_{oc} is called the open circuit voltage across the terminals a and b.

How to Find V_{Th}?

Original circuit	Thévenin equivalent circuit

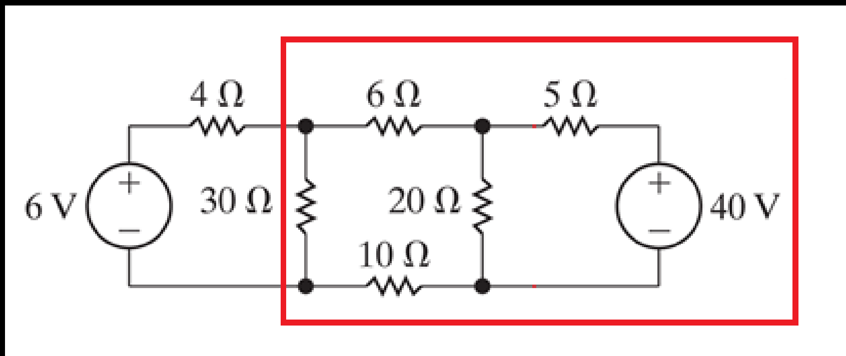


From the above Thévenin equivalent, we know: $V_{Th} = v_{oc}$

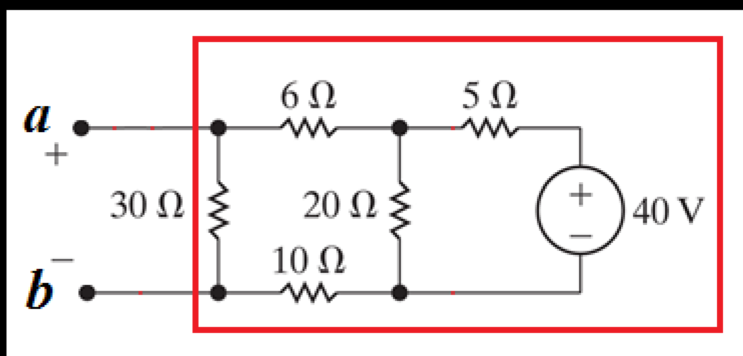
Then, how to find v_{oc}?

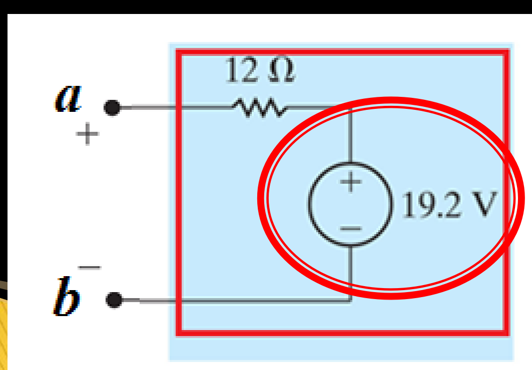
1. Go to the original circuit,
2. Open the circuit at the terminals *a* and *b* if it is not opened,
3. Find v_{oc} by using the methods discussed before.

Example #1



v_{oc}





Find V_{Th} of the circuit in the red box.

1. Remove the 6 V voltage source and 4 Ω resistor. Open the circuit at the terminals **a** and **b**.

Since 6 Ω, 30 Ω, 10 Ω are in series, we have

$$6 + 30 + 10 = 46\ \Omega$$

Then, 46 Ω resistor is in parallel with 20 Ω. we combine 46 Ω and 20 Ω.

$$\frac{46 \times 20}{46 + 20} = 13.94\ \Omega$$

Applying voltage division, we find the voltage across 13.94 Ω is

$$\frac{40 \times 13.94}{13.94 + 5} = 29.44\ V$$

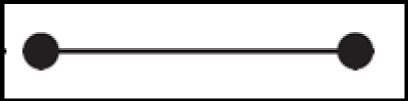
Applying voltage division again, we find the voltage across 30 Ω is v_{oc}

$$v_{oc} = \frac{29.44 \times 30}{6 + 30 + 10} = 19.2\ V$$

How to Find R_{Th}?

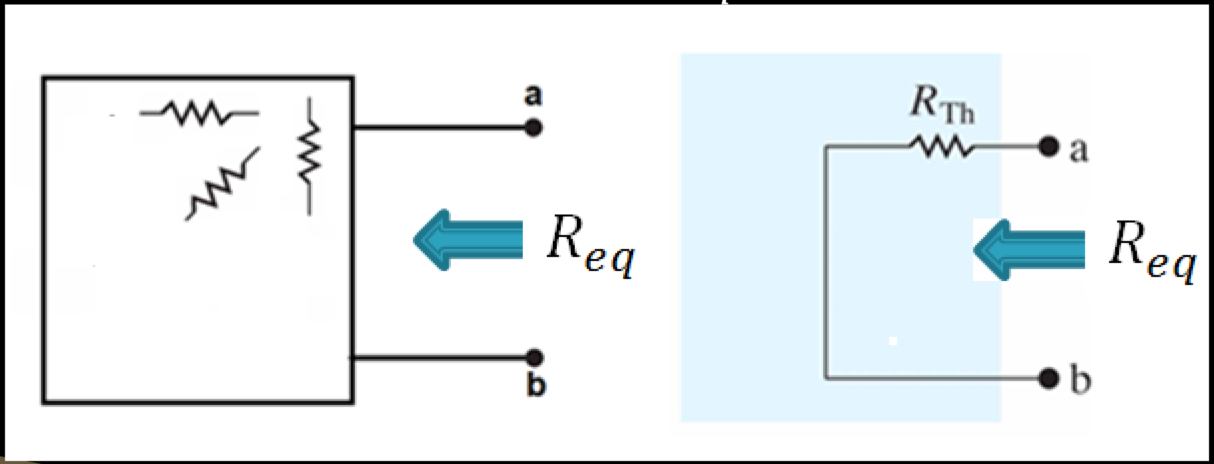
1. If circuits only contain independent sources:

a. Set all independent sources to zero by replacing

- voltage sources with short circuits,  $v = 0$
- current sources with open circuits  $i = 0$

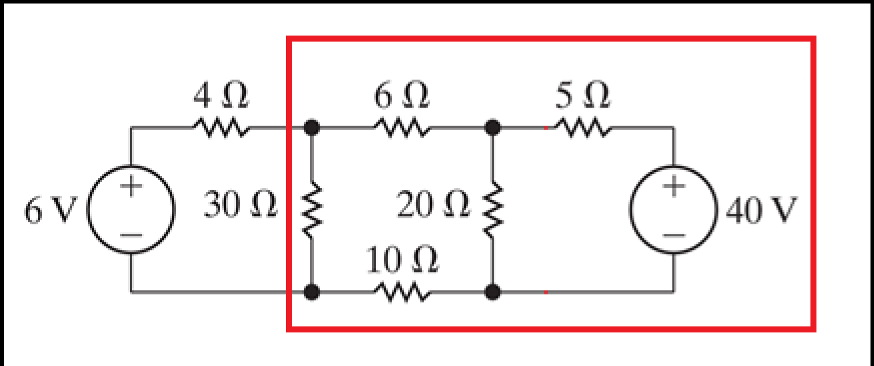
b. Find equivalent resistance seen by the terminals **a** and **b**

$$R_{Th} = R_{eq}$$

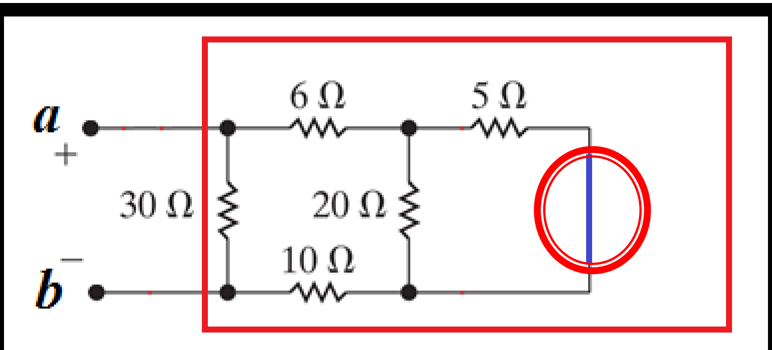


Example #2

Find R_{Th} of the circuit in the top red box.

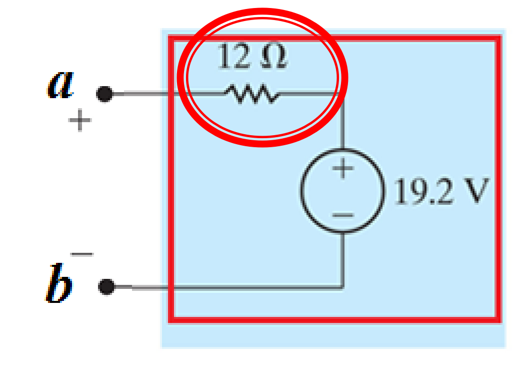


Because the circuit in the red box only has an independent voltage source, we replace the voltage source with a short circuit, as shown in the middle circuit.



The equivalent resistance seen by the terminals ***a*** and ***b*** is

$$R_{ab} = (20//5+6+10)//30=12\ \Omega$$



It is the same as the resistance in the bottom red box.

How to Find R_{Th}?

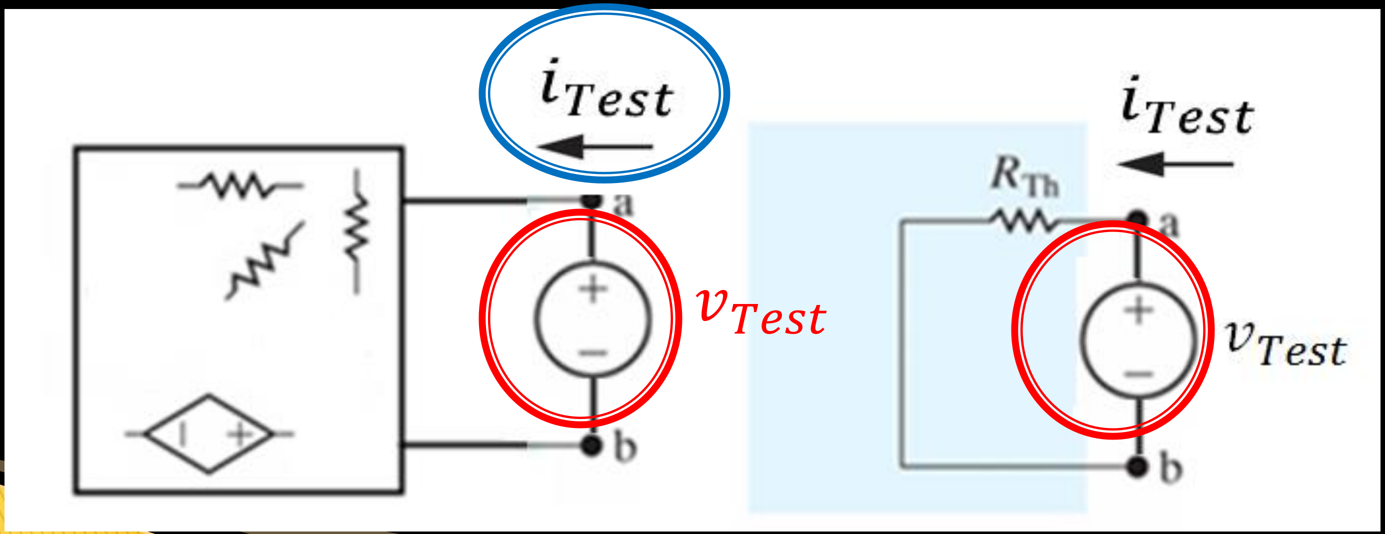
2. If circuits contain dependent sources:

   a. Remove all independent sources

   b. Apply a test voltage source v_{Test}

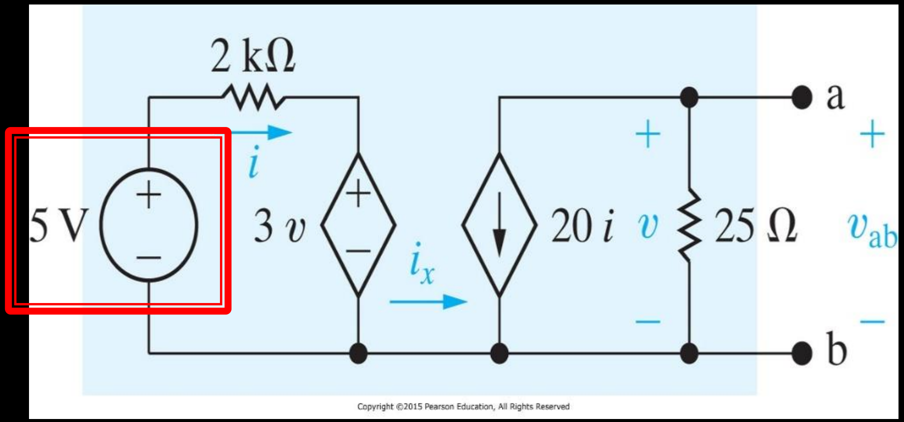
   c. Calculate i_{Test}

   d. Calculate $R_{Th} = \frac{v_{Test}}{i_{Test}}$



Example #3

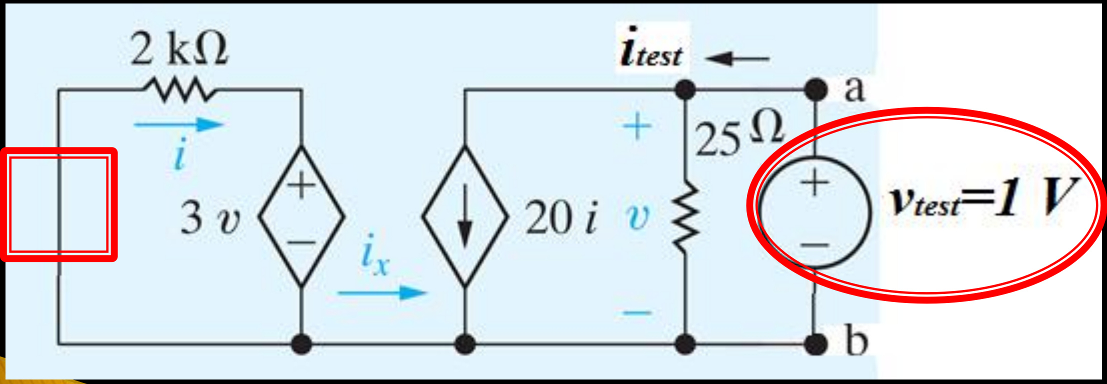
Find the Thévenin equivalent resistance of the circuit seen by the terminals **a** and **b**.



1. short the 5 V voltage source.
2. add a test voltage source across the terminals **a** and **b**.

so the current in 2 kΩ is

$$i = -\frac{3v}{2\times10^3} = -\frac{3v_{test}}{2\times10^3} = -\frac{3\times1}{2\times10^3} = -1.5\ mA$$



$$i_{test} = 20i + \frac{v_{test}}{25}$$

$$= 20(-1.5\times10^{-3}) + \frac{1}{25} = 10\ mA$$

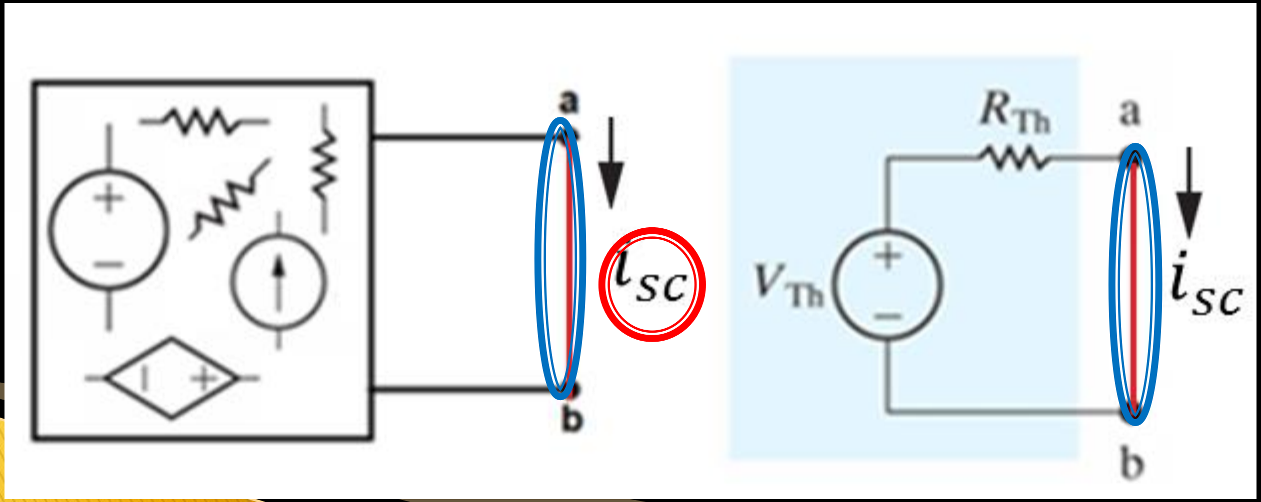
$$R_{Th} = \frac{v_{test}}{i_{test}} = \frac{1}{10\times10^{-3}} = 100\ \Omega$$

How to Find R_{Th}?

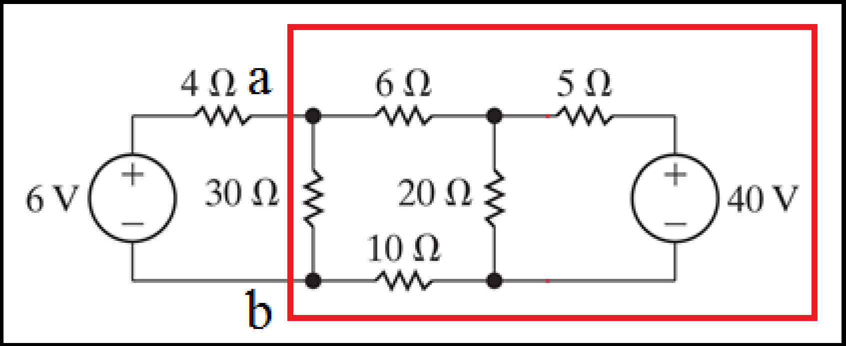
3. If a circuit contains dependent sources and $\boldsymbol{V_{Th}}$ **has been found or given:**

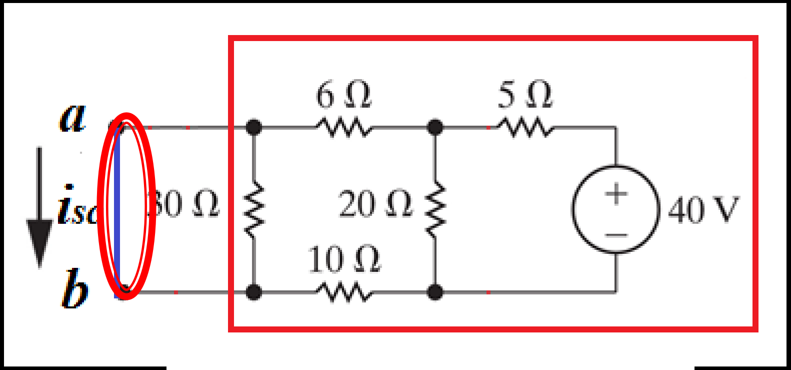
- Short the terminals a and b,
- Calculate the short-circuit current, i_{sc},
- Find R_{Th} by using:

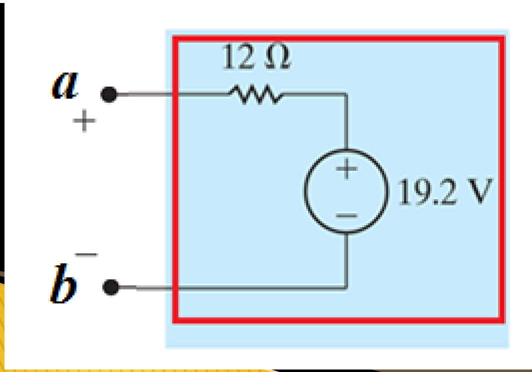
$$R_{Th} = \frac{V_{Th}}{i_{sc}}$$



Example #4







Find R_{Th} seen by the terminals **a** and **b.**
We know $V_{Th} = 19.2$ V.

1. Short the terminals a and b.
Because the 30 Ω is shorted, the equivalent resistance seen by the 40 V voltage source is: (10+6) //20+5 =13.89 Ω
The current in the voltage source is

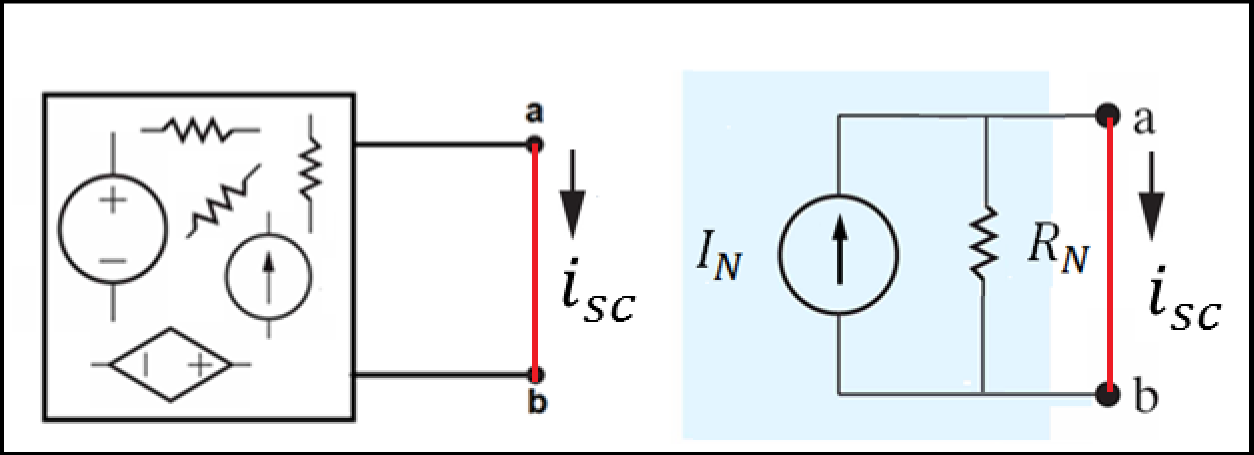
$$\frac{40}{13.89} = 2.88\ A$$

2. The short circuit current is:

$$i_{sc} = \frac{20 \times 2.88}{6 + 10 + 20} = 1.60\ A$$

3. $R_{Th} = \frac{V_{Th}}{i_{sc}} = \frac{19.2}{1.60} = 12\ \Omega$

The Norton Equivalent



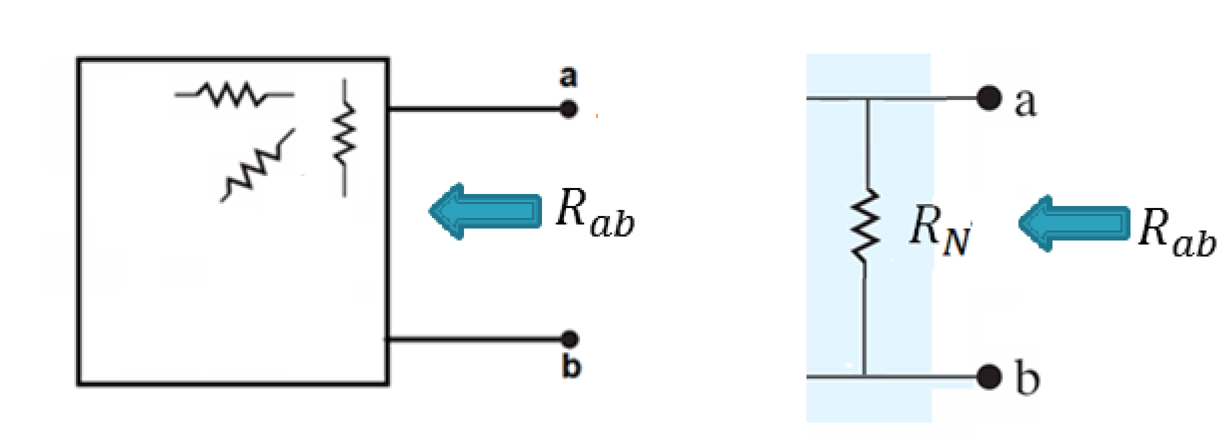
How to Find $\boldsymbol{I_N}$?

From the Norton equivalent circuit, we know, when the terminals a and b are shorted, $I_N = i_{sc}$. Then, go back to the original circuit, short the terminals a and b, find i_{sc}.

How to Find R_N?

➢ If circuits only contain independent sources:

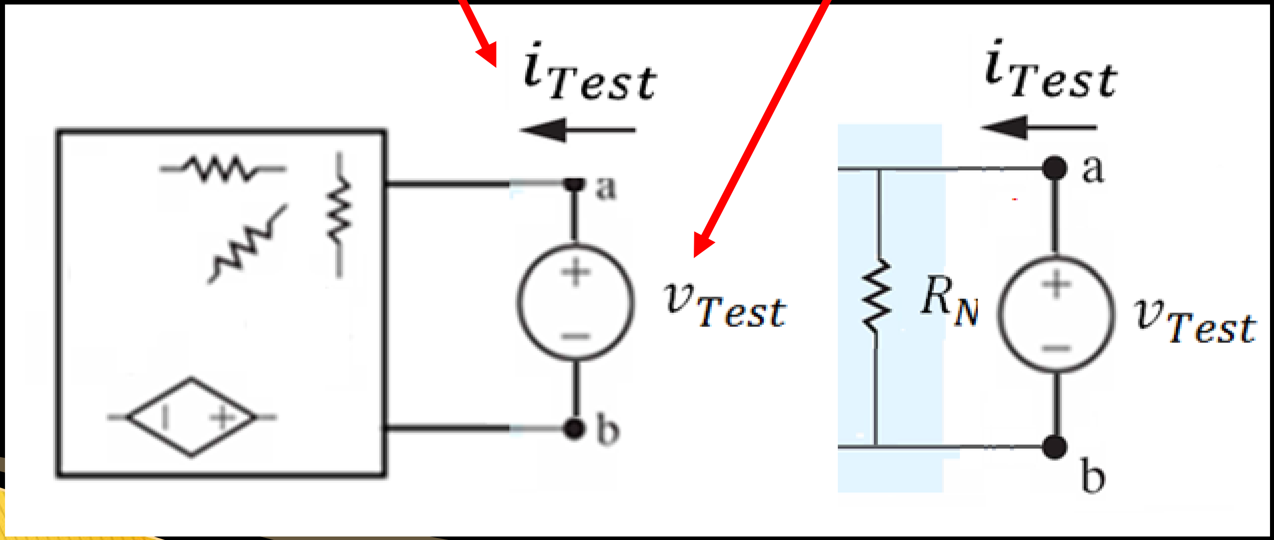
1. Remove all independent sources by replacing
   a. voltage sources ⇨ short circuits  $v = 0$
   b. current sources ⇨ open circuits  $i = 0$
2. Find equivalent resistance between the terminals $R_N = R_{ab}$



How to Find R_N?

- If circuits contain dependent sources:

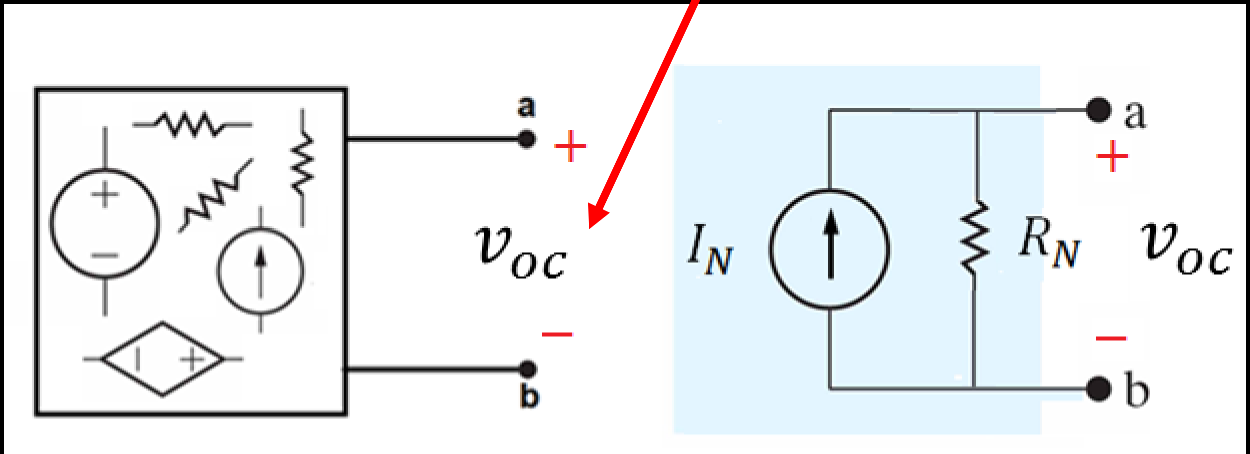
1. Remove all independent sources.
2. Apply a test voltage source v_{Test}
3. Calculate i_{Test}
4. Calculate $R_N = \frac{v_{Test}}{i_{Test}}$



How to Find R_N?

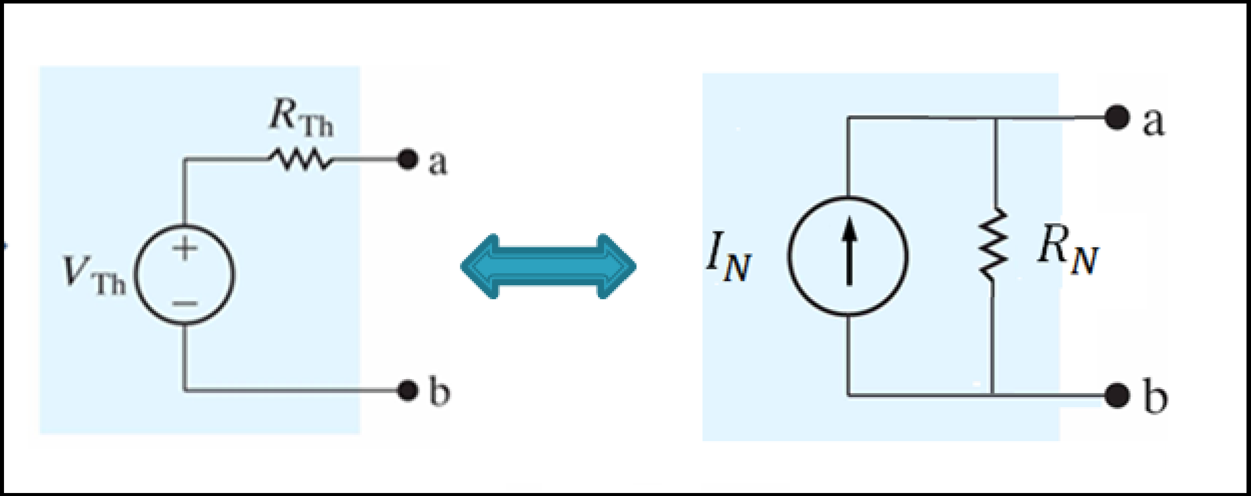
- If circuits contain dependent sources and I_N is known
  - Calculate the open-circuit voltage, v_{oc}, then

$$R_N = \frac{v_{oc}}{I_N}$$



The Thévenin and Norton Equivalent

By simply using the source transformation, we can find the Norton equivalent from its corresponding Thévenin equivalent.



$$I_N = \frac{V_{Th}}{R_{Th}}, \text{ or } V_{Th} = R_{Th} I_N$$

and $R_N = R_{Th}$

Summary

- Any linear circuits comprised of linear sources and linear elements can be simplified into their equivalent circuits consisting of an independent voltage source and a series resistor (Thévenin) or an independent current source and a parallel resistor (Norton).
- The Thévenin voltage $V_{Th} = v_{oc}$ which is the open-circuit voltage across the terminals of the original circuit. The Thévenin resistance can be found in three ways.
- The Norton equivalent of a circuit can also be found by transforming its corresponding Thévenin equivalent.

In the next lecture, we are going to discuss:

- The maximum power transfer and the superposition