CSE 015: Discrete Mathematics
Fall 2021
Homework #3
Solution

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1. **Question 1:**
   The conclusion is true
   Jane Flies = p
   Jane is a bird = q
   Modus Ponens
   $\neg p \to \neg q$
   q
   ____
   $\therefore p$

2. **Question 2:**

   (a) p: Bats can fly
   q: Bats are mammals
   $p \wedge q$
   ___________ Conjunction Elimination
   $\therefore q$

   (b) p: Pigs are mammals
   q: Pigs are birds
   $\neg q$
   ___________ Disjunctive Syllogism
   $\therefore p$

   (c) p: Jack is a CSE major
   q: Jack is a freshman
   p
   q
   ___________ Conjunction
   $\therefore p \wedge q$

   (d) p: Mary is a CSE major
   q: Mary is a History major

p

——————— Addition

$\therefore p \vee q$

(e) p: I go hiking
q: I will sweat a lot
r: I will lose weight
$p \to q$
$q \to r$

——————— Hypothetical Syllogism

$\therefore p \to r$

3. **Question 3:**

(a) p: It is sunny
q: I will go swimming
$p \to q$
$\neg p$

———————

$\therefore \neg q$

		premise	premise	conclusion
p	q	$p \to q$	$\neg p$	$\neg q$
T	T	T	F	F
T	F	F	F	T
F	T	T	T	F
F	F	T	T	T

Invalid
Third row, both premises are True but the conclusion is False

(b) p: It is Sunday
q: I will go to the park
$p \to q$
$\neg q$

———————

$\therefore \neg p$

		premise	premise	conclusion
p	q	$p \to q$	$\neg q$	$\neg p$
T	T	T	F	F
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

Valid
In the last row when both premises are true, the conclusion is true.

(c) p: I will pass the class
q: I score at least 60 on the final exam

$p \iff q$
$\neg q$

$\therefore \neg p$

		premise	premise	conclusion
p	q	$p \iff q$	$\neg q$	$\neg p$
T	T	T	F	F
T	F	F	T	F
F	T	F	F	T
F	F	T	T	T

Valid
In the last row when both premises are true, the conclusion is true.

4. **Question 4:**
if n is an integer and n^2 is odd, then n is odd
p: n^2 is odd
q: n is odd
$p \to q$

Contrapositive:
$\neg p$: n^2 is even
$\neg q$: n is even
$\neg p \to \neg q$

Proof:
Let n be any even integer and k be any integer.
n = 2k
$n^2 = (2k)^2$
$n^2 = 4k^2$
$n^2 = 2(2k^2)$
The proof shows that if n is even then n^2 is even.

5. **Question 5:**
For n = 3
$[(p_1 \vee p_2 \vee p_3) \to q] \iff [(p_1 \to q) \wedge (p_2 \to q) \wedge (p_3 \to q)]$

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

p_1	p_2	p_3	q	$p_1 \to q$	$p_2 \to q$	$p_3 \to q$	$p_1 \vee p_2 \vee p_3$	$(p_1 \vee p_2 \vee p_3) \to q$	$(p_1 \to q) \wedge (p_2 \to q) \wedge p_3 \to q$
T	T	T	T	T	T	T	T	T	T
T	T	T	F	F	F	F	T	F	F
T	T	F	T	T	T	T	T	T	T
T	T	F	F	F	F	T	T	F	F
T	F	T	T	T	T	T	T	T	T
T	F	T	F	F	T	F	T	F	F
T	F	F	T	T	T	T	T	T	T
T	F	F	F	F	T	T	T	F	F
F	T	T	T	T	T	T	T	T	T
F	T	T	F	T	F	F	T	F	F
F	F	T	T	T	T	T	T	T	T
F	F	T	F	T	T	F	T	F	F
F	F	F	T	T	T	T	T	T	T
F	F	F	F	T	T	T	F	T	T

$[p_1 \vee p_2 \vee p_3 \to q] \iff (p_1 \to q) \wedge (p_2 \to q) \wedge (p_3 \to q)$	
T	
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T	Tautology proved by truth table
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