ENGR 65 Electric Circuits

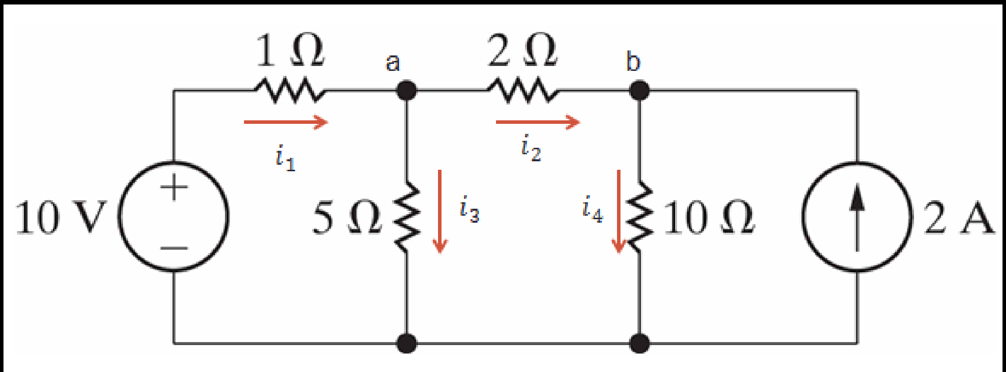
Lecture :7 The Mesh-Current Method (MCM)

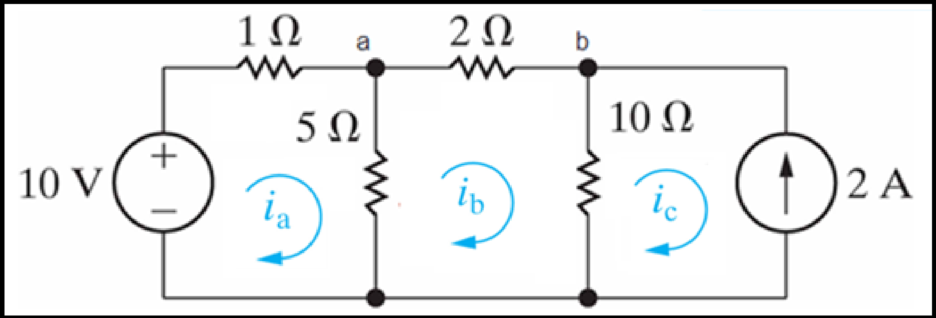
Today's Topics

- Why mesh-current method?
- The steps for writing mesh-current equations.
- What is the supermesh?
- How to deal with current sources in mesh-current method?

- Topics are covered in Sections 4.5, 4.6, 4.7, and 4.8

Why Mesh-Current Method?





Instead of finding $i_1, i_2, i_3,$ and i_4, we define the mesh currents $i_a, i_b,$ and i_c.

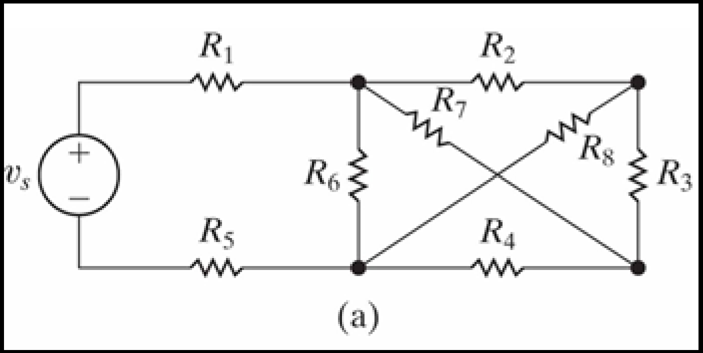
These currents are made up flowing through each mesh. Thus, instead of finding four currents, we need to solve for three currents. The number of simultaneous equations is reduced from 4 to 3.

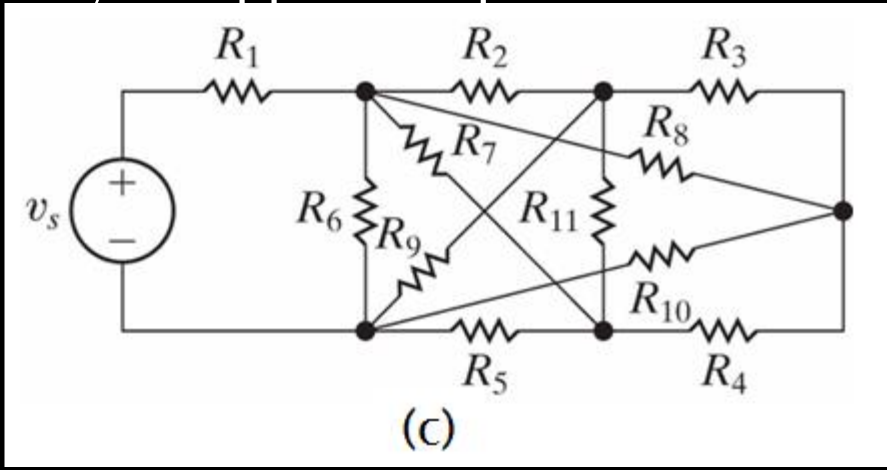
After solving the circuit for $i_a, i_b,$ and i_c, $i_1, i_2, i_3,$ and i_4 can be found by computing

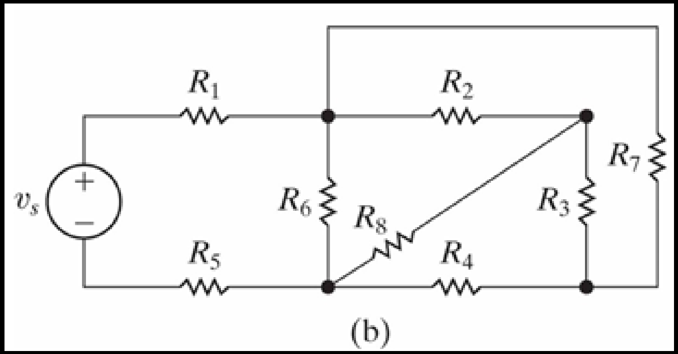
$$i_1 = i_a$$
$$i_2 = i_b$$
$$i_3 = i_a - i_b$$
$$i_4 = i_b - i_c$$

Terms for the Mesh-current Method

- Planar circuit: a circuit that can be drawn on a plane without crossing branches.
- Mesh-current method can only be applied to planar circuits.







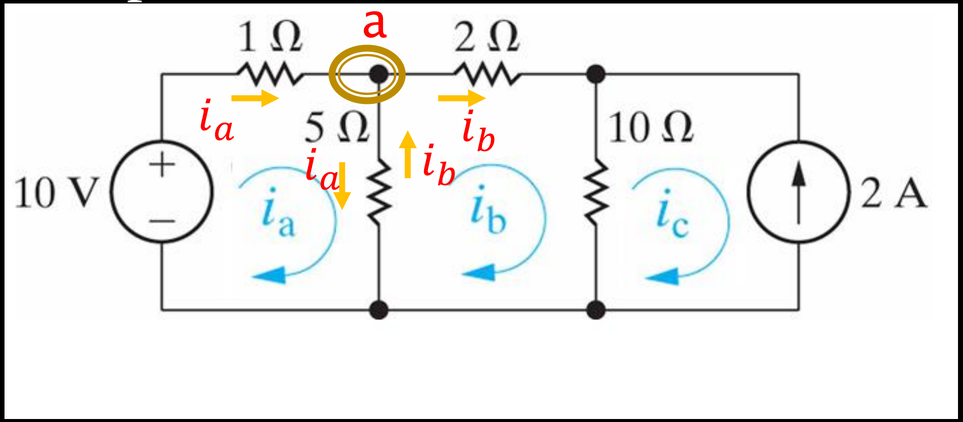
(a) is a planar circuit because the branch R7 can be redrawn as shown in (b). (c) is not a planar circuit because no matter how you redraw the circuit, R7 or R11 will cross R10 or R8

The Mesh-Current Method

- The Mesh-current method is a systematic way used to solve for the mesh currents in planar circuits.
- Mesh currents are fictitious quantities because they cannot be measured, but they automatically obey KCL at each node. For example, at the node a,

$$i_a - i_a + i_b - i_b = 0.$$

- Places where the mesh currents come together, both currents flow simultaneously. For example, in 5 Ω resistor, both i_a and i_b flow through it.

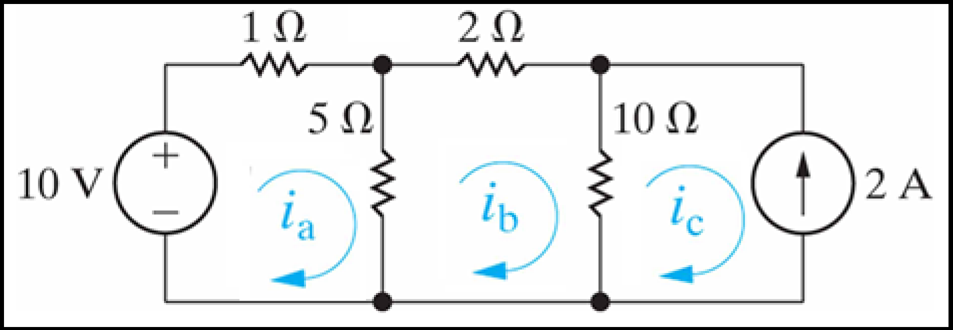


The Mesh-Current Method

The steps in the mesh-current method:

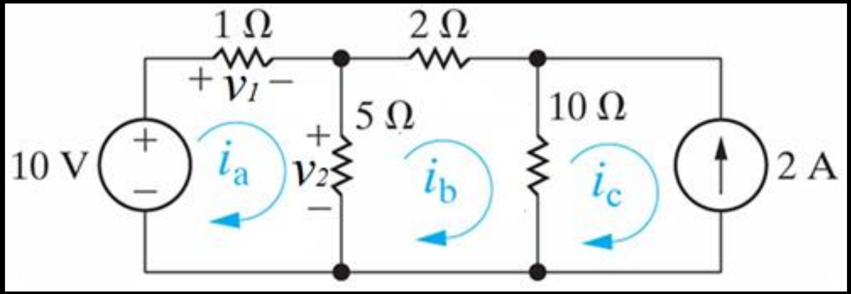
Step 1: Redraw the circuit in planar form if necessary.

Step 2: Define the mesh currents and label them. This includes assigning the reference direction of each mesh current.



The Mesh-Current Method

Step 3: Apply KVL to each mesh.



Assuming along the direction of i_a, both v_1 *and* v_2 *drop*. Applying KVL to mesh **a**:

$$v_1 + v_2 - 10 = 0 \quad (1)$$

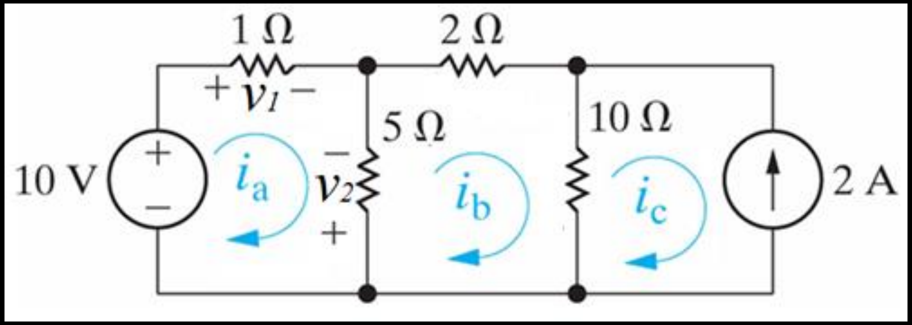
Applying Ohm's law to 1 Ω and 5 Ω resistors:

$$v_1 = 1 \times i_a \qquad v_2 = 5(i_a - i_b)$$

Plugging v_1 *and* v_2 to (1):

$$1 \times i_a + 5(i_a - i_b) - 10 = 0 \quad (2)$$

After Rearranging (2): $6i_a - 5i_b - 10 = 0$



Assuming along the direction *of* i_a, v_1 drops, *but* v_2 rises. Applying KVL to mesh **a**:

$$v_1 - v_2 - 10 = 0$$

$$v_1 = 1 \times i_a \qquad v_2 = -5(i_a - i_b)$$

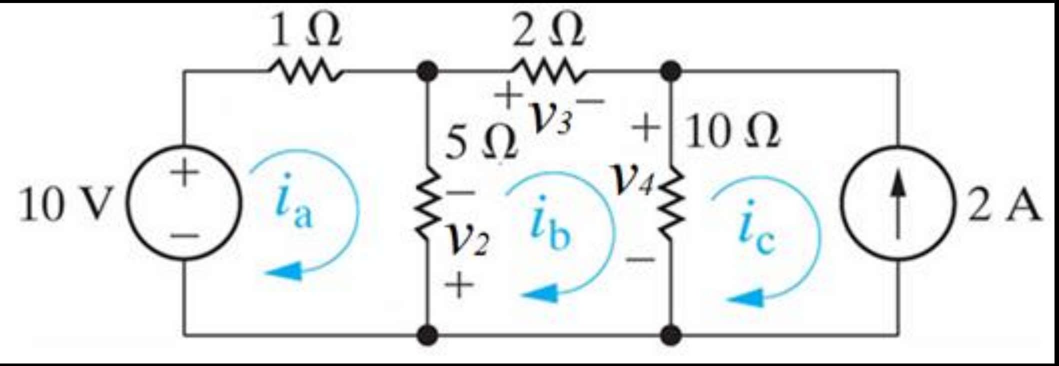
$$1 \times i_a - [-5(i_a - i_b)] - 10 = 0$$

Which reference assignment about v_1 and v_2 is better?

Can you directly write this equation for mesh a?

The Mesh-Current Method

Step 3: Apply KVL to each mesh (cont.)



Assuming along the direction $i_b, v_2, v_3\ and v_4\ all\ drop$. Applying KVL to mesh **b**:

$$v_3 + v_4 + v_2 = 0 \quad (1)$$

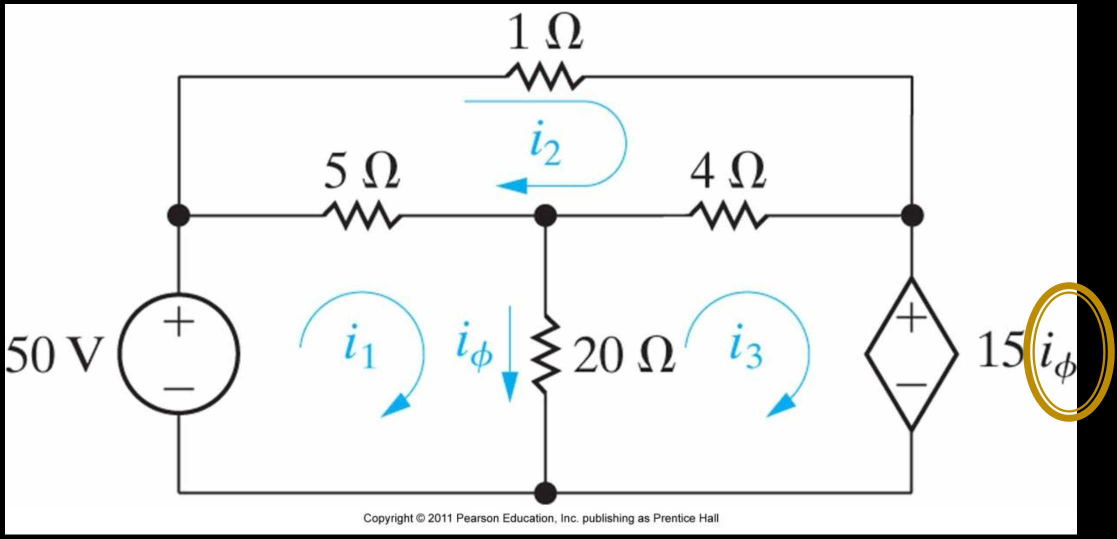
$$2i_b + 10(i_b - i_c) + 5(i_b - i_a) = 0 \quad (2)$$

After Rearranging (2): $-5\, i_a + 17\, i_b - 10\, i_c = 0$ ← $Can\ you\ directly\ write\ this\ equation?$

$$i_c = -2 \quad (3)$$

The Mesh-Current Method

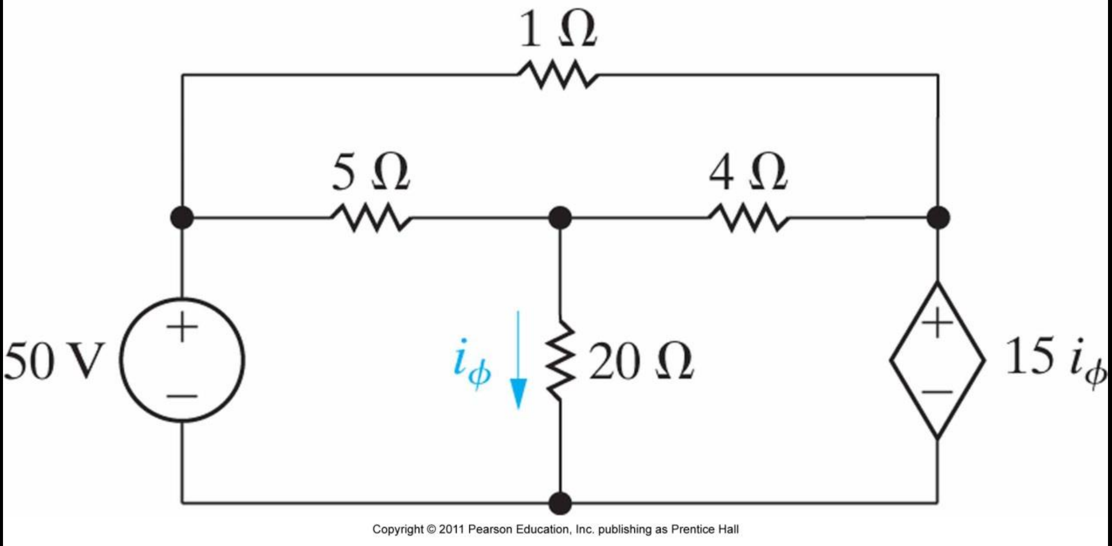
Step 4: Write an equation for each current or voltage upon which dependent sources depend.
Step 5: Solve the simultaneous equations.



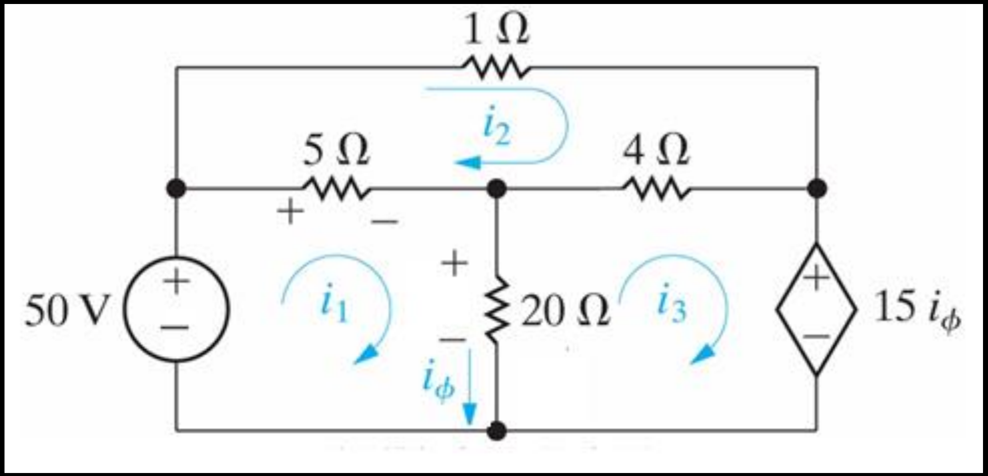
$$i_\phi = i_1 - i_3$$

The number of meshes in the circuit is

A. 1
B. 2
C. 3
D. 4



The mesh-current equation for mesh 1 is



The net current in 5 Ω resistor is $i_1 - i_2$. The net current in 20 Ω resistor is $i_1 - i_3$. So

$$5(i_1 - i_2) + 20(i_1 - i_3) - 50 = 0$$

by combining like terms, we have

$$25i_1 - 5i_2 - 20i_3 = 50$$

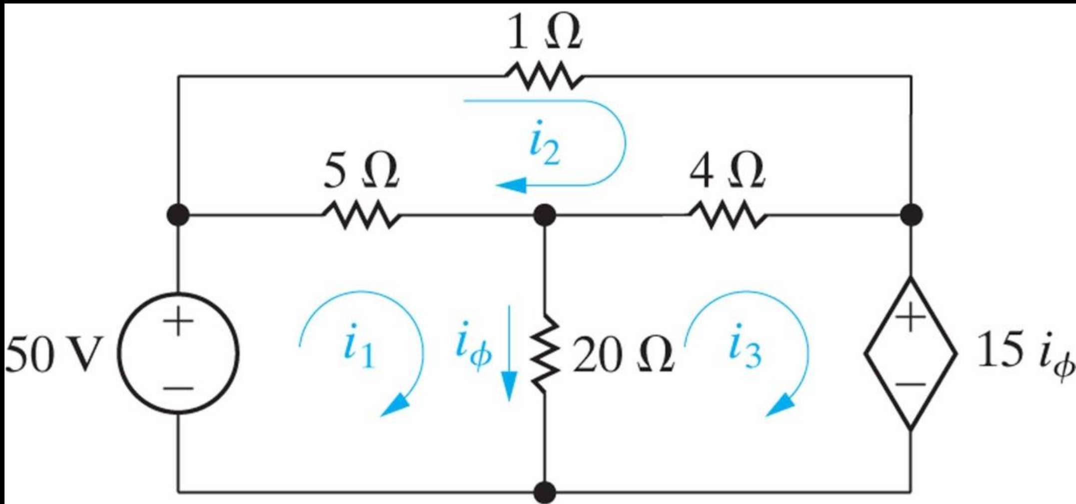
The mesh-current equation for mesh 2 is

A. $-5i_1 + 10i_2 - 4i_3 = 50$

B. $10i_2 - 4i_3 = 50$

C. $-5i_1 + 10i_2 - 4i_3 = 0$

D. $-5i_1 + 10i_2 = 0$



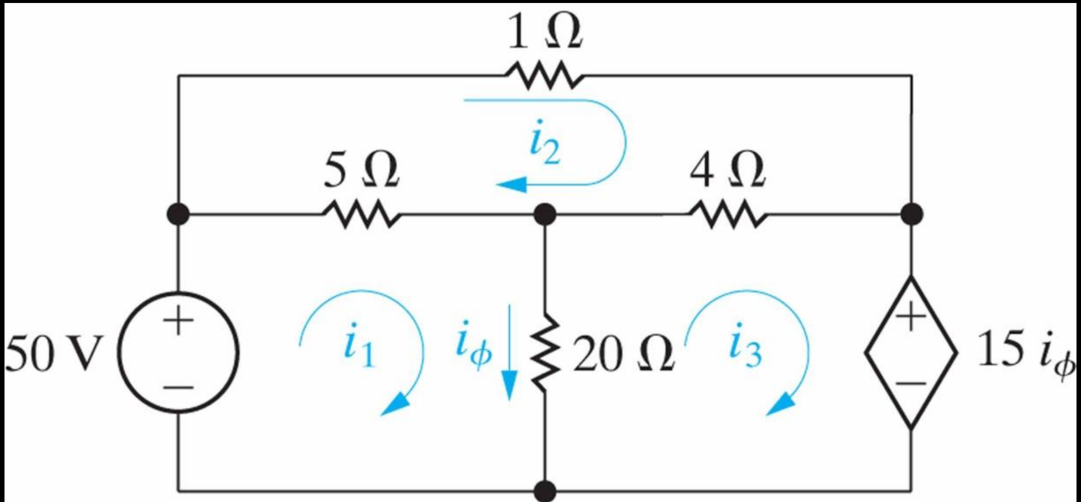
The mesh-current equation for mesh 3 is

A. $-20i_1 - 4i_2 + 24i_3 - 15i_\phi = 0$

B. $-20i_1 - 4i_2 + 24i_3 + 15i_\phi = 0$

C. $-20i_1 + 24i_3 + 15i_\phi = 0$

D. $-4i_2 + 24i_3 + 15i_\phi = 0$



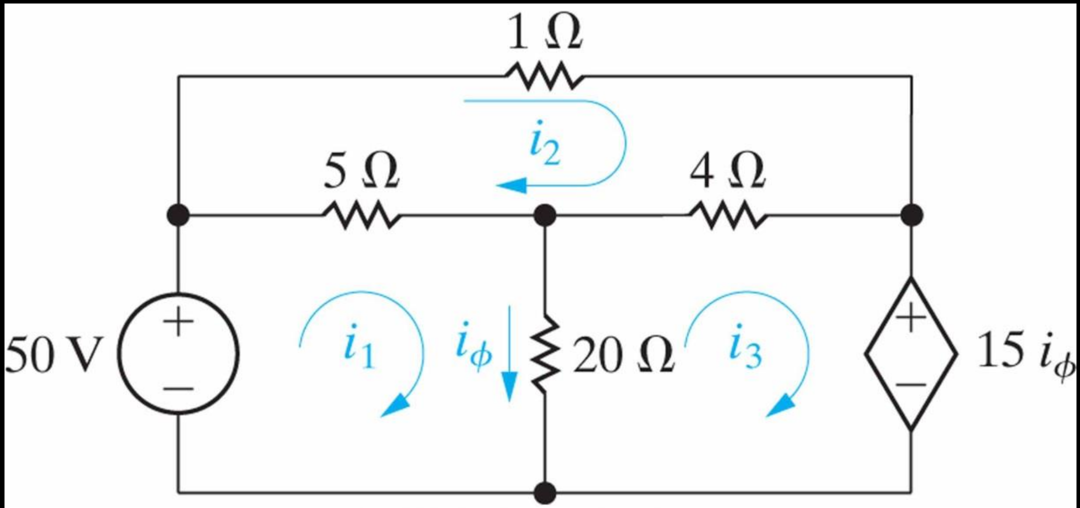
How many unknowns are in the circuits?

A. 1

B. 2

C. 3

D. 4



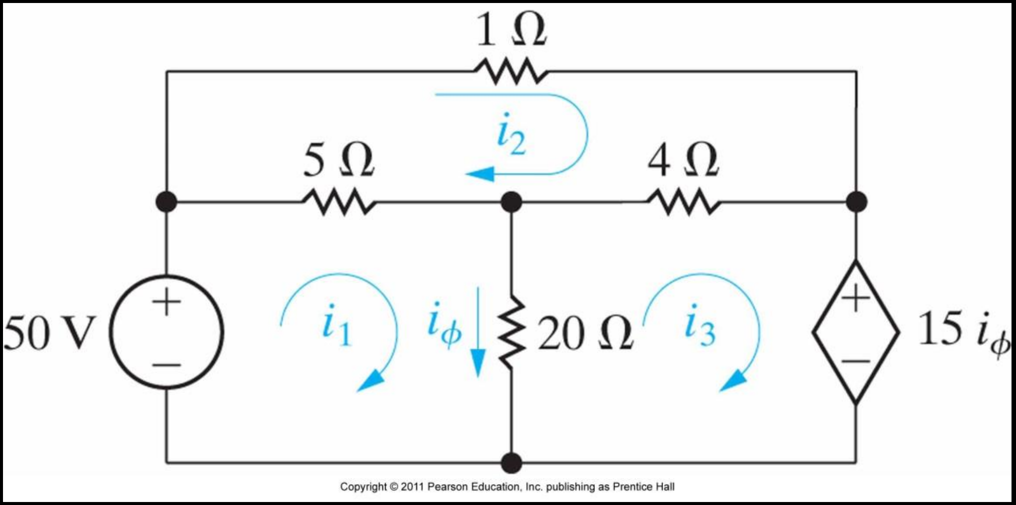
The i_ϕ in the circuit is equal to

A. i_1

B. i_2

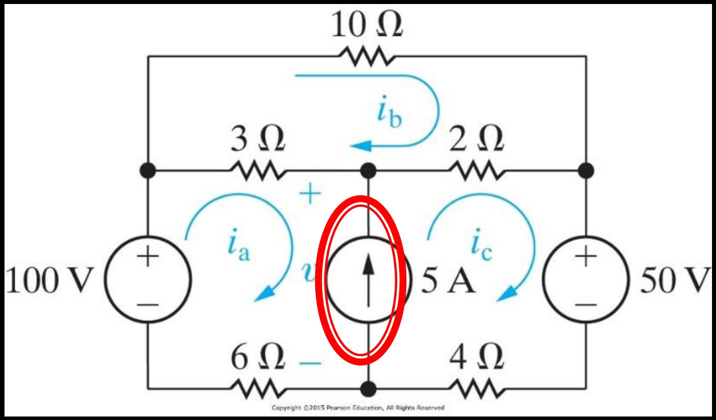
C. $-i_3$

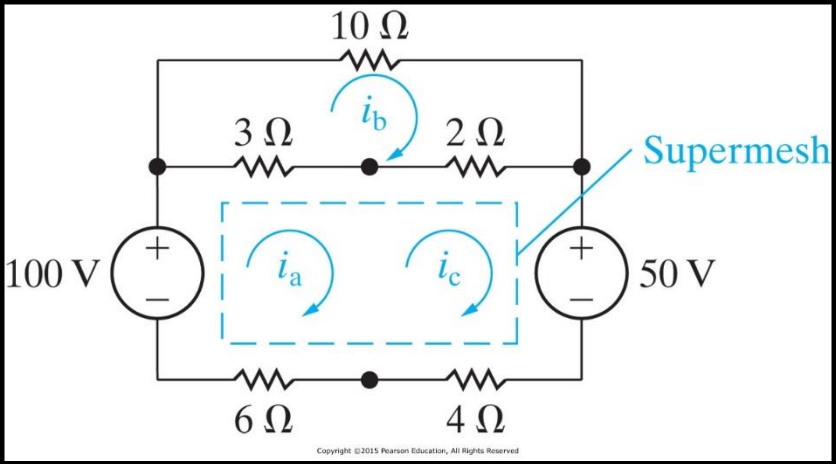
D. $i_1 - i_3$



If a circuit has dependent sources, write an equation for each current or voltage upon which the dependent sources depend. Write the equations in terms of mesh currents already defined in the circuit.

The Supermesh





A current (dependent/independent) source between two meshes forms a supermesh. Assume the voltage across the current source is v. The mesh current equations for mesh a and c are:

$$3(i_a - i_b) + v + 6i_a - 100 = 0 \quad (1)$$

$$2(i_c - i_b) + 50 + 4i_c - v = 0 \quad (2)$$

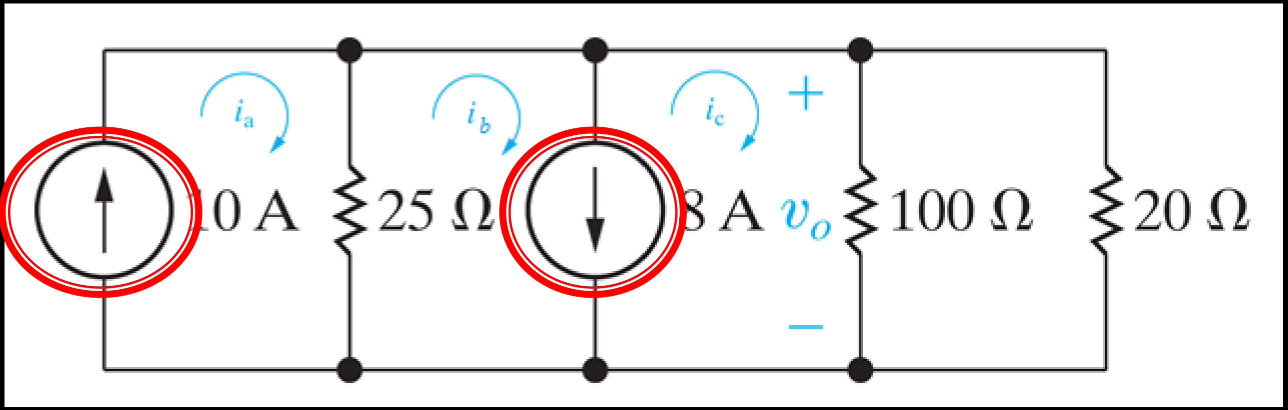
(1)+(2), v is cancelled out:

$$3(i_a - i_b) + 6i_a - 100 + 2(i_c - i_b) + 50 + 4i_c = 0$$

$$\text{or} \quad 9i_a - 5i_b + 6i_c = 50 \quad (3)$$

We can obtain the above equation (3) by using the mesh shown on the top right circuit. We call the mesh a **supermesh**.

How to Deal with Current Sources in Mesh-Current Method



There are two possible connections:

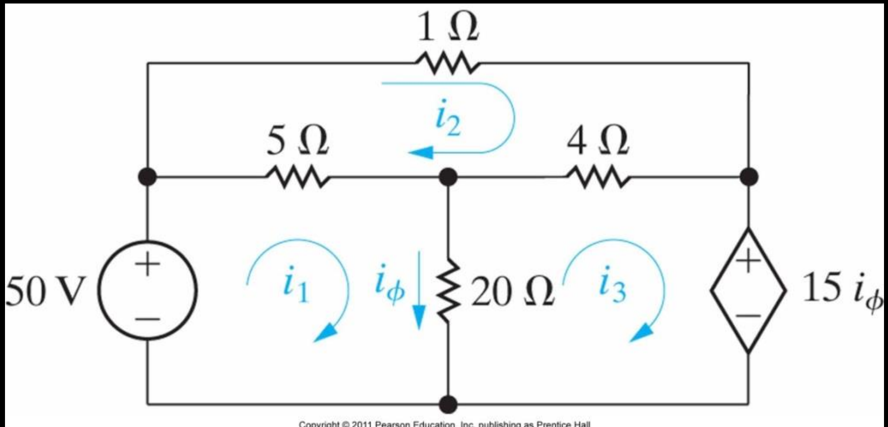
1. The current source appears as a part of only one mesh, such as 10 A current source. The mesh-current equation is:
$$i_a = 10$$
2. The current source is connected between two meshes, such as 8 A current source (supermesh). The mesh-current equation is:
$$25(i_b - i_a) + (100//20)i_c = 0$$

Note: 100 Ω and 20 Ω resistors are parallel-connected

How Many Mesh-Current Equations Do We Need to Write?

- *If there are n_m meshes*, we need to write n_m independent equations.
- *If there are dependent sources* presented in the circuit, the number of independent equations increases. If the number of variables that dependent sources depend on is *v*, we need to write n_m+v equations.

$n_m = 3 \quad v = 1$



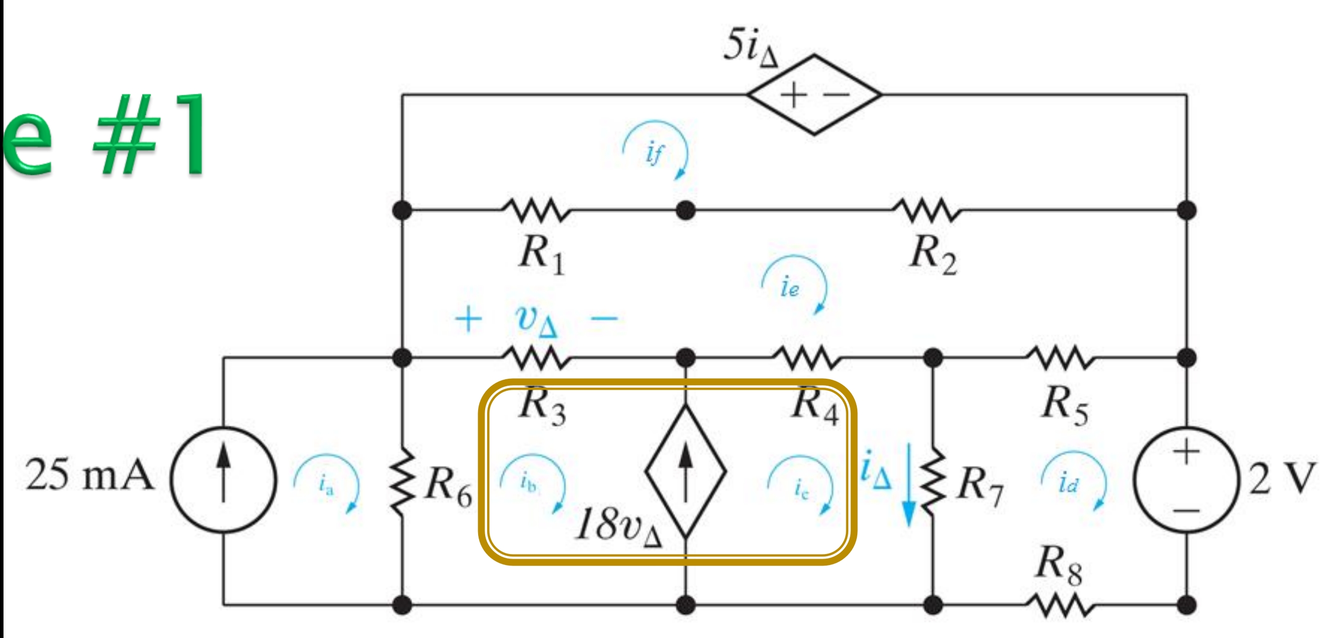
$$25i_1 - 5i_2 - 20i_3 = 50$$

$$-5i_1 + 10i_2 - 4i_3 = 0$$

$$-20i_1 - 4i_2 + 24i_3 + 15i_\phi = 0$$

$$i_\emptyset = i_1 - i_3$$

Example #1



(1). $i_a = 0.025$; The mesh current is the same as the current source

(2). $R_6(i_b - i_a) + R_3(i_b - i_e) + R_4(i_c - i_e) + R_7(i_c - i_d)=0$; (A supermesh)

(3). $R_7(i_d - i_c) + R_5(i_d - i_e) + R_8 i_d + 2 = 0$;

(4). $(R_1 + R_2)\left(i_e - i_f\right) + R_5(i_e - i_d) + R_4(i_e - i_c) + R_3(i_e - i_b) = 0$; R1 and R2 are in series. Combine them before writing the equation.

(5). $5i_\Delta + (R_1+R_2)\left(i_f - i_e\right) = 0$; A current controlled voltage source

(6). $18v_\Delta = i_c - i_b$; The current source is between two meshes

(7). $v_\Delta = R_3(i_b - i_e)$; A dependent variable

(8). $i_\Delta = i_c - i_d$; Another dependent variable

Summary

- The mesh-current method can also reduce the number of simultaneous equations of a circuit. It can only be applied to planar circuits.
- Once the mesh currents are found in the circuit, everything else in the circuit such as the currents in every branch can be found too.

In next class, I will be talking about:

- The source transformation
- Thévenin equivalents