UNIVERSITY OF CALIFORNIA, MERCED

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CSE 015: Discrete Mathematics
Final Exam – Fall 2021

Preliminary Notes

- **This exam must be solved individually.** No exceptions. Be reminded that the CSE academic honesty policy discussed in class will be enforced.
- Your solution must be exclusively submitted via CatCourses. Pay attention to the posted deadline because **the system automatically stops accepting submissions when the deadline passes. Late submissions will receive a 0**. You only need to submit the PDF and you have to use the template file provided in CatCourses. Please note that the system does not allow to submit any other file format. Do not submit the LaTeXsource of your solution. **If in your LaTeXsubmission you embed screenshots or scans of your handwritten solution, those will not be graded.**
- Most questions require to provide a justification. If you do not provide a justification you will get no credit even if your answer is correct.

1 Logical Equivalences

Using a truth table, determine whether the expression $(p \to q) \wedge (\neg r \to \neg q)$ is equivalent to $\neg r \to \neg p$ or not (as usual, assume that the operator $\neg$ has the highest priority). Important: your truth table cannot just show the columns for $(p \to q) \wedge (\neg r \to \neg q)$ and $\neg r \to \neg p$. You also have to show the intermediate steps, i.e. some of the intermediate columns.

2 Quantifiers

Assuming the domain for each of the variables is the set of real numbers, determine the truth value of each of these statements. Justify each answer. If you just state True/False without explanation you will get no credit. Your explanation must be substantiative, and generic explanations will not give any credit.

a) $\forall x(x^2 - 1 > 0)$

b) $\exists x(x^3 - 1 = 0)$

c) $\forall x \exists y(x^2 + y^2 = 1)$

d) $\exists x \exists y(x^4 + y^4 = 0)$

3 Negation of Complex Sentences

Rewrite each of the following sentences so that the negation operator appears only within the predicates (i.e., no negation is outside a quantifier or an expression involving logical connectives). Explain how you obtain the results (generic explanations will give no credit).

a) $\neg\exists y \exists x P(x, y)$

b) $\neg\forall x \exists y (P(x, y) \wedge Q(x, y))$

c) $\neg\exists y (\exists x R(x, y) \vee \forall x S(x, y))$

4 Cartesian Products

Consider the sets $A = \{a, b, c\}$, $B = \{dog, cat, mouse\}$, $C = \{True, False\}$. Answer the following questions.

a) What is the set $A \times C$?

b) How many elements are in $A \times B \times C \times A$? Justify your answer.

c) Write one element of $A \times B \times C \times A$.

d) Is $A \times B = B \times A$? Justify your answer.

e) Is $(True, True, Dog)$ an element of $C \times B \times C$? Justify your answer.

5 Arguments

Determine if the following arguments are correct or not (you must justify your answer).

a) Every student enrolled in CSE015 has access to computer Lab #2. Jennifer does not have access to computer Lab #2. Therefore Jennifer is not enrolled in CSE015.

b) Every student majoring in computer science must take CSE120. Jake is taking CSE120. Therefore Jake is majoring in computer science.

6 Functions

Consider the following functions from $\mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$. For each function state if the function is injective, surjective, bijective, or none of the former. Justify each answer. Correct answers without justification will give no credit.

a) $f(x, y) = x + y$

b) $f(x, y) = |x + y|$ (here $|\cdot|$ is the absolute value function)

c) $f(x, y) = x^2 - y$

7 Order of Growth

For each of the following couple of functions, say if 1) the first function is big-O of the second function, but the second is not big-O of the first function; 2) the second function is big-O of the first function, but the first function is not big-O of the second function; 3) both functions big-O of each other; 4) none of the former statements is true. Motivate your choice. Answers without explanation will give no credit even if correct.

a) $f_1(n) = 4n^2 + \log n \qquad f_2(n) = 21n + \sqrt{n}$

b) $f_1(n) = 3n^2 + 2^n \qquad f_2(n) = 4n^2 + 45n$

c) $f_1(n) = 2n^3 + 4n \log n \qquad f_2(n) = n \log n + 3n^2 + 12n^3$

d) $f_1(n) = 4n^2 \log n^2 + 21n \qquad f_2(n) = 0.002n^3 + n$

8 Induction

Using the principle of induction, prove that the following formula is true for each $n \geq 1$:

$$\sum_{i=1}^{n} 2^{i-1} \cdot i = 2^n(n-1) + 1$$

9 Modular Arithmetic

Consider arithmetic modulo m with $m = 13$. Answer the following questions and explain how you obtain your results.

a) What is the result of $5 +_m 8$?

b) What is the additive inverse of 9 (modulo m)?

c) Is $4 \equiv 38 (\mathrm{mod} m)$?

d) Using the division algorithm, determine $-4 \bmod m$.

10 Cryptography

Consider the case where plaintext messages are only composed of upper case letters of the English alphabet and spaces. Therefore there are 27 different symbols to represent (26 letters and the space). Consider an encoding where the space is 0, A is 1, B is 2, ... , Z is 26. Using a shift cipher with key $k = 4$ a plain text message was encrypted, and the following ciphertext was obtained:

HMWGVIXIDQEXL

What was the original plaintext message? Explain how you obtain the result. Simply giving the correct answer will not give any credit.