Complete the following tasks. To show work for the double-integrals, demonstrate that you can set up the integrals. From there, you may use software such as *Wolfram Alpha* to compute the values of the double integrals. Some answers have been provided. Assemble your work into one PDF document and upload the PDF back into our CatCourses page.

1. $X = \{0, 1, 2\}$ and $Y = \{-1, 1\}$ are discrete random variables with

   - $P(X = 0, Y = -1) = P(X = 1, Y = -1) = P(X = 2, Y = -1)$
   - $P(X = 0, Y = 1) = 0, P(X = 1, Y = 1) = 0.5, P(X = 2, Y = 1) = 0$

   (a) Write out the discrete probability distribution table

   (b) Are X and Y independent?

   (c) Compute the correlation of X and Y.

   (d) Does zero correlation guarantee independence?

2. Show that if X and Y are independent random variables, then

   (a) $\mathrm{E}[XY] = \mathrm{E}[X] \cdot \mathrm{E}[Y]$

   (b) $\mathrm{Cov}(X, Y) = 0$

   (c) Does independence imply zero correlation?

3. Our story begins at the Pearson Cooking Academy where many new students are tasked with cooking eggs on a rectangular pan. An analyst at Cal Kulas Consulting recommends the following function form to model the locations of the eggs

$$f(x) = k(4x^2 + y^2), \quad -2 \leq X \leq 2, \quad -1 \leq Y \leq 1$$

Compute $\mathrm{Var}(X)$, $\mathrm{Var}(Y)$, the correlation between X and Y, and describe the correlation with a complete sentence.[1]

[1] Hint: leave intermediate calculations in terms of k. This was an exam question during the Summer 2020 session.

4. Let X be a random variable that represents the number of processes currently running in your computer—between 50 and 200 processes. Let Y be a random variable that represents your computer's internal temperature—between 20 and 100 degrees Celsius. Compute the correlation between these two random variables if we model the situation with the joint density function

$$f(x, y) = \frac{k \ln x}{\sqrt{y}}$$

where k is a scaling constant. To show your work, set up all of the integrals correctly. You may then use software to perform the computations. [2]

[2] This was an exam question during the Spring 2021 semester.

5. Let X be a continuous random variable with probability density function:

$$f_X(x) = \begin{cases} \frac{3}{4}x\,(2-x) & 0 \leq x \leq 2 \\ \\ 0 & \text{otherwise.} \end{cases}$$

(a) Determine the cumulative distribution function F_X.

(b) Let $Y = \sqrt{X}$. Determine the cumulative distribution function F_Y.

(c) Determine the probability density of Y.

6. Let X have an exponential distribution with parameter $\lambda = 1/2$.
   (a) Determine the cumulative distribution function of $Y = \frac{1}{2}X$.
   (b) Determine the probability distribution function of Y.
   (c) What kind of distribution does Y have?

Here are some of answers. Note that numbers may slightly vary depending on when and where the rounding took place.

1. (a)
   (b) No (but how do you know?)
   (c) Zero
   (d) No
2. (a)
   (b)
   (c) Yes (but how do you know?)
3. (a) $\text{Var}(X) = \dfrac{4768k}{45} \approx 105.956k$ or $\dfrac{596}{255} \approx 2.3373$
   (b) $\text{Var}(Y) = \dfrac{712k}{45} \approx 15.8222k$ or $\dfrac{89}{255} \approx 0.3490$
   (c) The random variables X and Y are uncorrelated
4. $r \approx 2.3912 \times 10^{-5}$
5. (a)
   (b) $F_Y(y) = \begin{cases} 0 & y < 0 \\ \dfrac{3}{4}\left(y^4 - \dfrac{1}{3}y^6\right) & 0 \le y \le \sqrt{2} \\ 1 & y > \sqrt{2} \end{cases}$
   (c)
6. (a)
   (b)
   (c) $Y \sim Exp(1)$