

You're expected to work on the problems before coming to the lab. Discussion session is not meant to be a one-way lecture. The TA will lead the discussion and correct your solutions if needed. For many problems, we will not release 'official' solutions. If you're better prepared for discussion, you will learn more. The TA is allowed to give some bonus points to students who actively engage in discussion and report them to the instructor. The bonus points earned will be factored in the final grade.

In the following, you can assume that all elements in the input are distinct.

1. (Basic) Using Figure 7.1 as a model, illustrate the operation of Partition on the array $A = \langle 13, 19, 9, 5, 12, 8, 7, 4, 21, 2 \rangle$. Do the same thing when we have $A = \langle 13, 19, 9, 5, 12, 8, 7, 4, 21, 11 \rangle$
Sol. We only show the array at the end. For the first example, we have $\langle 2, 19, 9, 5, 12, 8, 7, 4, 21, 13 \rangle$; and the function returns 1 as the pivot location index. For the second example, we have $\langle 9, 5, 8, 7, 4, 11, 19, 12, 21, 13 \rangle$; and the function returns 6 as the pivot location index.
2. (Basic) Is Quick-Sort an in-place sorting algorithm? How about Merge-sort, Heap-sort, Insertion-sort?
Sol. All are in-place sorting algorithms except Merge sort.
3. (Basic) The running time of Randomized-Quicksort that picks the pivot uniformly at random is always $O(n \log n)$ for any input of size n . True or False?
Sol. False. For some badly chosen pivots, the running time could be as high as $\Omega(n^2)$.
4. (Basic) The expected running time of Randomized-Quicksort that picks the pivot uniformly at random is $O(n \log n)$ for any input of size n . True or False?
Sol. True.
5. (Basic) The running time of (Deterministic) Quicksort is $\Theta(n^2)$. True or False?
Sol. True. For sure, it makes at most $O(n^2)$ comparisons for any input. Consider an already sorted array. The algorithm picks $n, n-1, n-2, \dots, 2$ as pivot in this order, where $n-1, n-2, \dots, 1$ comparisons are made, respectively. So, it still makes $(n-1) + (n-2) + \dots + 1 = \Omega(n^2)$ comparisons. So, the running time is $\Omega(n^2)$. Thus, we can conclude that the RT is $\Theta(n^2)$.
6. (Basic) Suppose the input is a random permutation of integers 1 through n . The average-case running time of (Deterministic) Quicksort is $\Theta(n^2)$. True or False?
Sol. False. It is $\Theta(n \log n)$.
7. (Basic) Solve $T(n) = T(0.9n) + T(0.1n) + \Theta(n)$. What is your answer? No need to give any explanations. Solve $T(n) = T((2/3)n) + T((1/3)n) + \Theta(n)$. What is your answer?
Sol. $\Theta(n)$ in both cases.