CSE 015: Discrete Mathematics
Homework 4

Fall 2021
Provided Solution

1 Set Operations

a) $A \cup B$ is the set of UCM students that are registered in CSE015 or live in Merced county. Note that here the "or" is the inclusive or.

b) $A \cap C$ is the set of UCM students that are registered in CSE015 and who are freshmen.

c) $C \setminus B$ is the set of UCM students who are freshmen and do not live in Merced county.

d) $\overline{A}$ is the set of UCM students who are not registered for CSE015.

e) $A \cap B \cap C$ is the set of UCM students who are registered for CSE015, live in Merced county and are freshmen.

2 Cartesian Product

a) $C \times A = \{(True, 1), (True, 2), (True, 3), (True, 4), (False, 1), (False, 2), (False, 3), (False, 4)\}$

b) $B \times B = \{(a, a), (a, b), (a, c), (b, a), (b, b), (b, c), (c, a), (c, b), (c, c)\}$

c) $B \times A \times C = \{(a, 1, True), (a, 1, False), (a, 2, True), (a, 2, False), (a, 3, True), (a, 3, False), (a, 4, True), (a, 4, False), (b, 1, True), (b, 1, False), (b, 2, True), (b, 2, False), (b, 3, True), (b, 3, False), (b, 4, True), (b, 4, False), (c, 1, True), (c, 1, False), (c, 2, True), (c, 2, False), (c, 3, True), (c, 3, False), (c, 4, True), (c, 4, False), \}$

3 Composite Cartesian Products

The equality is true and this is shown by proving the two sets are equal. As we have shown in class, to show that two sets A and B are equal we need to show that $A \subseteq B$ and $B \subseteq A$.

We first show $A \times (B \cup C) \subseteq (A \times B) \cup (A \times C)$. This is shown as follows. A generic element of $A \times (B \cup C)$ is (x, y), where $x \in A$ and $y \in B \cup C$. If $y \in B$, then $(x, y) \in A \times B$ and therefore $(x, y) \in (A \times B) \cup (A \times C)$. Likewise, if $y \in C$, then $(x, y) \in A \times C$ and therefore $(x, y) \in (A \times B) \cup (A \times C)$. This concludes the first part of the proof.

Let us next show $(A \times B) \cup (A \times C) \subseteq A \times (B \cup C)$. Let $(x, y) \in (A \times B) \cup (A \times C)$. Then $(x, y) \in A \times B$ or $(x, y) \in (A \times C)$. If $(x, y) \in A \times B$, then $y \in B \cup C$, and therefore $(x, y) \in A \times (B \cup C)$. Likewise, if $(x, y) \in A \times C$, then $y \in B \cup C$, and therefore $(x, y) \in A \times (B \cup C)$. This concludes the second part of the proof and the theorem.

4 Relations

a) $R_1 = \{(a,b),(a,c),(a,a),(b,a),(c,a)\}$
Symmetric.

b) $R_2 = \{(a,b),(b,b),(b,c),(c,c),(a,c)\}$
Transitive, antisymmetric.

c) $R_3 = \{(a,b),(d,c),(c,a),(c,d),(a,b)\}$
In this case there was a typo because the couple (a,b) appears twice. Therefore one could say R_3 is not a set, and therefore it does not represent a relation and the question is ill posed. Alternatively, one could answer "None of the former".

d) $R_4 = \{(a,a),(b,b),(c,c)\}$
Symmetric, antysimmetric, transitive. Note that this is not reflexive because it is missing (d,d) (check the definition.)

5 Functions

Recall that a function is surjective if each element of the codomain is the image of at least one element of the domain.

a) $f(m,n) = 2m - n$
This function is **surjective** because for every possible $k \in \mathbb{Z}$ there exists at least one couple $(m,n) \in \mathbb{Z} \times \mathbb{Z}$ such that $k = f(m,n)$. In particular $k = f(0,-k)$.

b) $f(m,n) = m^2 - n^2$
This function is **not surjective** because 2 cannot be expressed as the difference between two squares. To see this, consider the following cases.

   (a) If $m = n$, then $f(m,n) = 0 \neq 2$
   (b) If $|n| > |m|$, then $f(m,n) < 0$ because $n^2 > m^2$ and so $f(m,n) \neq 2$.
   (c) If $|m| > |n|$ we distinguish various subcases. First assume $m > n \geq 0$. For $n = 0$, the function is $f(m,0) = m^2$ and we know that $m^2 \neq 2$ for every integer. For $m > n \geq 1$ we can then write $m = n + k$ with $k \geq 1$. We can then write $f(m,n) = m^2 - n^2 = (n+k)^2 - n^2 = 2k + k^2$ and this is larger than 2 (recall $k > 1$.) A symmetric reasoning can be used for the case where $m < n < 0$. This concludes all possible cases.

c) $f(m,n) = |m| - |n|$ (here $|x|$ is the absolute value of x)
This function is **surjective** because for every possible $k \in \mathbb{Z}$ there exists at least one couple $(m,n) \in \mathbb{Z} \times \mathbb{Z}$ such that $k = f(m,n)$. In particular, for $k \geq 0$ we have $k = f(k,0)$, while for $k < 0$ we have $k = f(0,k)$.

d) $f(m,n) = m^2 - 4$
Note that this function, even though it is defined of $\mathbb{Z} \times \mathbb{Z}$, does not depend on the second parameter n. This function is **not surjective** because the set of images does not contain any integer number smaller than -4. Another way to see is this is that $f(m,n) \geq -4$ for every possible couple (m,n).