Complete the following tasks. You need to show work for full credit, and you may use a calculator such as RStudio to finish the calculations. In particular, for integrals, you may use resources like *Wolfram Alpha* to check your answers, but you need to show your work during Math 32 homework and exams. Some answers have been provided. Assemble your work into one PDF document and upload the PDF back into our CatCourses page.

1. The FBI was investigating the college admissions scandal of 2019 in "Operation Varsity Blues". Suppose that investigations of this magnitude are exponentially distributed with a mean length of 4.5 months. Compute the probability that this investigation will take over 6.75 months to complete.[1]

2. At the grocery store, there was a student who it seems takes a very long time to select a bottle of shampoo. Her coworker claims that she takes an average of 45 minutes to make a selection. Use an exponential distribution to predict the chance that we waited for over an hour to watch the student pick a shampoo.[2]

3. While the Pareto distributions are continuous, they tend to be used to model discrete data in humanities and actuarial sciences. Moreover, with its roots in power functions, Pareto distributions may be used in the growing popularity of the studies of networks. The probability density function (PDF) for a Pareto distribution is

$$f(x) = \frac{\alpha}{x^{\alpha+1}}, \quad 1 \leq x$$

where $\alpha > 0$ is the *shape parameter*.

   (a) Derive the cumulative distribution function (CDF) $F(x) = 1 - \dfrac{1}{x^{\alpha}}, \quad 1 \leq x$

   (b) Derive the expected value $\mu = \dfrac{\alpha}{\alpha - 1}$ for $\alpha > 1$

   (c) Derive the standard deviation $\sigma = \sqrt{\dfrac{\alpha}{(\alpha-1)^2(\alpha-2)}}$ for $\alpha > 2$

[1] This was an exam question during the Spring 2019 semester. Data source: `https://en.wikipedia.org/wiki/2019_college_admissions_bribery_scandal`

[2] This was an exam question during the Fall 2019 semester, and yes, I was picking on a student.

If the Pareto distribution is shifted so that its support starts at *scale parameter* x_m, then the support is $x_m \le x$, and the formulas become

$$f(x) = \begin{cases} \dfrac{\alpha x_m^\alpha}{x^{\alpha+1}}, & x \ge x_m \\ 0, & x < x_m \end{cases} \qquad F(x) = \begin{cases} 1 - \left(\dfrac{x_m}{x}\right)^\alpha, & x \ge x_m \\ 0, & x < x_m \end{cases}$$

$$\mu = \begin{cases} \infty & \alpha \le 1 \\ \dfrac{\alpha x_m}{\alpha - 1} & \alpha > 1 \end{cases}, \quad \sigma^2 = \begin{cases} \infty & \alpha \le 2 \\ \dfrac{\alpha x_m^2}{(\alpha-1)^2(\alpha-2)} & \alpha > 2 \end{cases}$$

4. Derive the median of a Pareto distribution with shape parameter α and scale parameter x_m.

5. **Cell Phone Consumption** The number of UC Merced students (Y, in thousands) who need X smart phones during their time at UC Merced can be loosely modeled by the Pareto distribution with PDF

$$f(x) = \frac{4}{x^5}$$

   (a) What is the shape parameter α?

   (b) Compute the expected number of cell phones and the standard deviation.

   (c) Build the "range rule of thumb" interval $(\mu - 2\sigma, \mu + 2\sigma)$

   (d) If a student goes through 3 cell phones during their time at UC Merced, is that amount of cell phones unusual?

Here are some of answers. Note that numbers may slightly vary depending on when and where the rounding took place.

1. 0.2231
2. 0.2636
3. (a) Recall $F(x) = \int_{-\infty}^{x} f(t)\,dt$

   (b)

   (c)
4. median $= x_m \left(2^{1/\alpha}\right)$
5. (a)

   (b) $\mu = \frac{4}{3}$, $\sigma = \frac{2}{3}\sqrt{\frac{1}{2}}$

   (c)

   (d) Yes, 3 cell phones is unusually high compared to the range rule of thumb