Bryant Chon

Math 32 HW#1

1(a)

$$\int e^{-x/32}dx$$
$$u = -x/32$$
$$du = -dx/32$$
$$-32du = dx$$
$$-32\int e^u du$$
$$-32e^{-x/32}$$ +C

1(b)

$$\int xe^{-x/32}$$
$$u = -x/32$$
$$-32u = x$$
$$du = -dx/32$$
$$dx = -32du$$
$$1024\int ue^u du$$
$$integration\ by\ parts \int fg' = fg - \int f'g$$
$$f = u, g' = e^u$$
$$f' = 1, g = e^u$$
$$ue^u - \int e^u du$$
$$solve: \int e^u du$$
$$exponential rule \int a^u = a^u/ln(a), a = e$$
$$\int e^u du = e^u$$
$$ue^u - e^u$$
$$1024ue^u - 1024e^u$$
$$-32xe^{-x/32} - 1024e^{-x/32}$$
$$-32e^{-x/32}(x+32) + C$$

1(c)

$$\int x^2 e^{-x/32}$$

$$u = -x/32$$
$$-32u = x$$
$$du = -dx/32$$
$$dx = -32du$$
$$-32768 \int u^2 e^u du$$
$$integration\ by\ parts \int fg' = fg - \int f'g$$
$$f = u, g' = e^u$$
$$f' = 2u, g = e^u$$
$$u^2 e^u - \int 2ue^u du$$
$$solve : 2 \int ue^u du$$
$$integration\ by\ parts \int fg' = fg - \int f'g$$
$$f = u, g' = e^u$$
$$f' = 1, g = e^u$$
$$ue^u - \int e^u$$
$$exponentialrule \int a^u = a^u/ln(a), a = e$$
$$\int e^u du = e^u$$
$$ue^u - e^u$$
$$2ue^u - 2e^u$$
$$u^2 e^u - \int 2ue^u du$$
$$u^2 e^u - 2ue^u + 2e^u$$
$$-32768u^2 e^u + 65536ue^u - 65536e^u$$
$$-32x^2 e^{-x/32} - 2048xe^{-x/32} - 65536e^{-x/32}$$
$$-32e^{-x/32}(x^2 + 64x + 2048) + C$$

2(a)

$$-32e^{-x/32} + C|_0^{\infty}$$

$0 - -32$

32

2(b)

$$-32e^{-x/32}(x+32) + C|_0^{\infty}$$

$0 - -1024$

1024

2(c)

$$-32e^{-x/32}(x^2 + 64x + 2048) + C|_0^{\infty}$$

$0 - -65536$

65536

3)

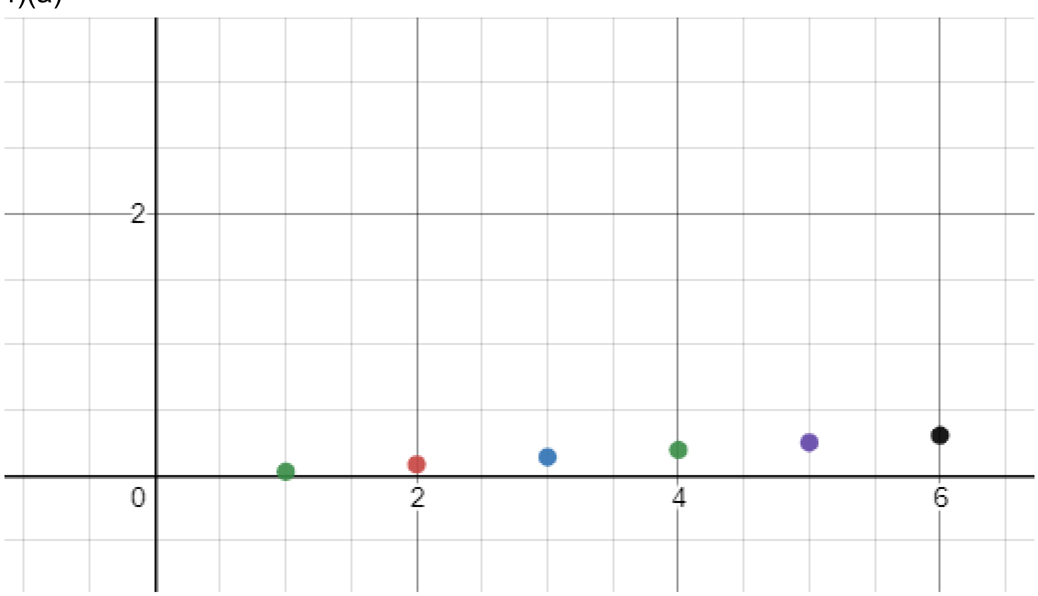
$-32e^{-x/32}(x^3 + 96x^2 + 6144x + 196608) + C$

$-32e^{-x/32}(x^3 + 96x^2 + 6144x + 196608) + C|_0^{\infty}$

0--6291456

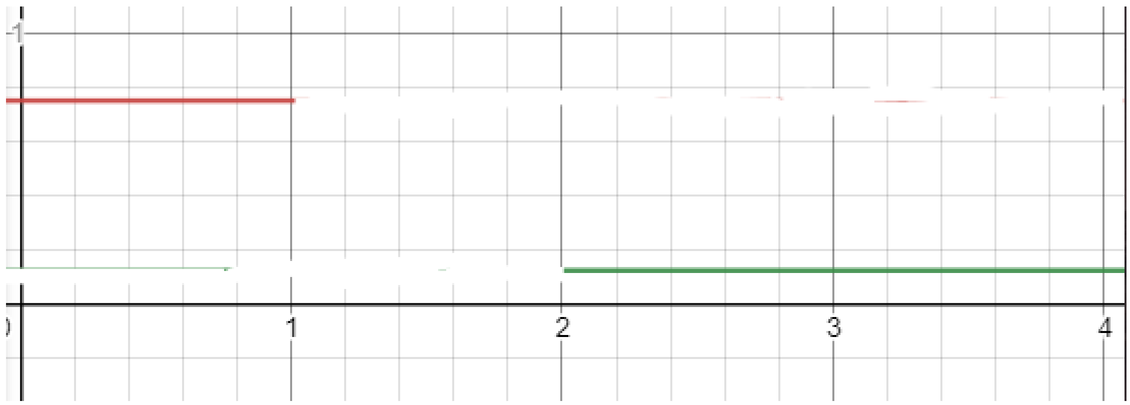
6291456

4)(a)

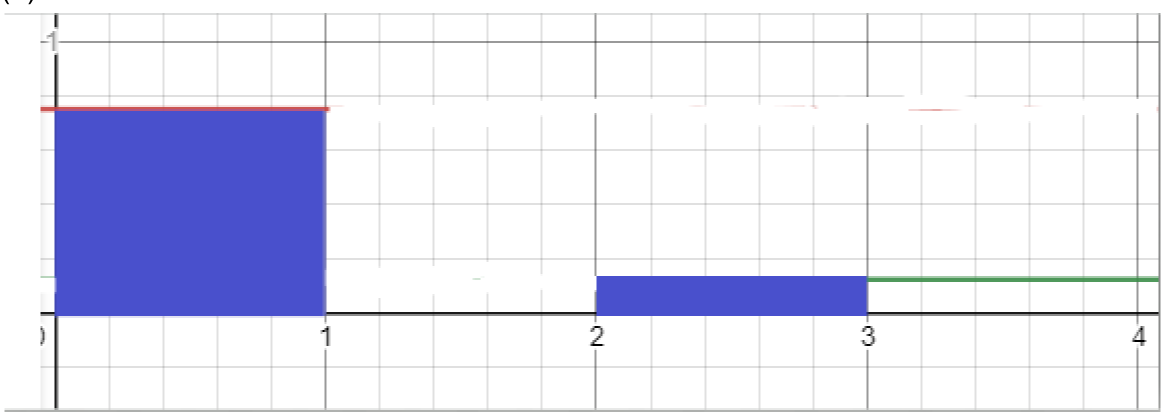


b) p(-.5) = 0, p(0) = 0, p(2) = 3/36, p(3.5) = 0, p(7) =0

5) (a)

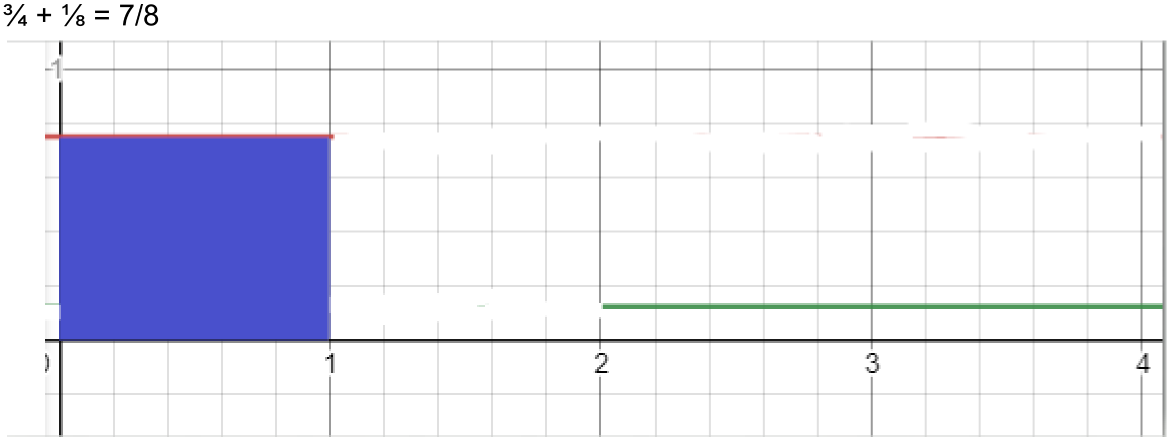


(b)

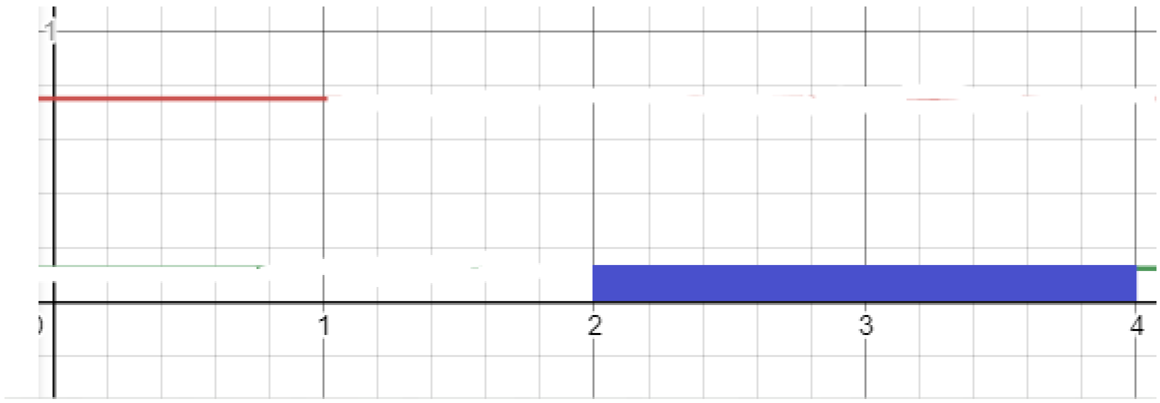


$$\int_0^3 f(x)\,dx = \frac{7}{8}$$

¾ + ⅛ = 7/8



$$\int_{-\infty}^{1} f(x)\, dx = \frac{3}{4};$$



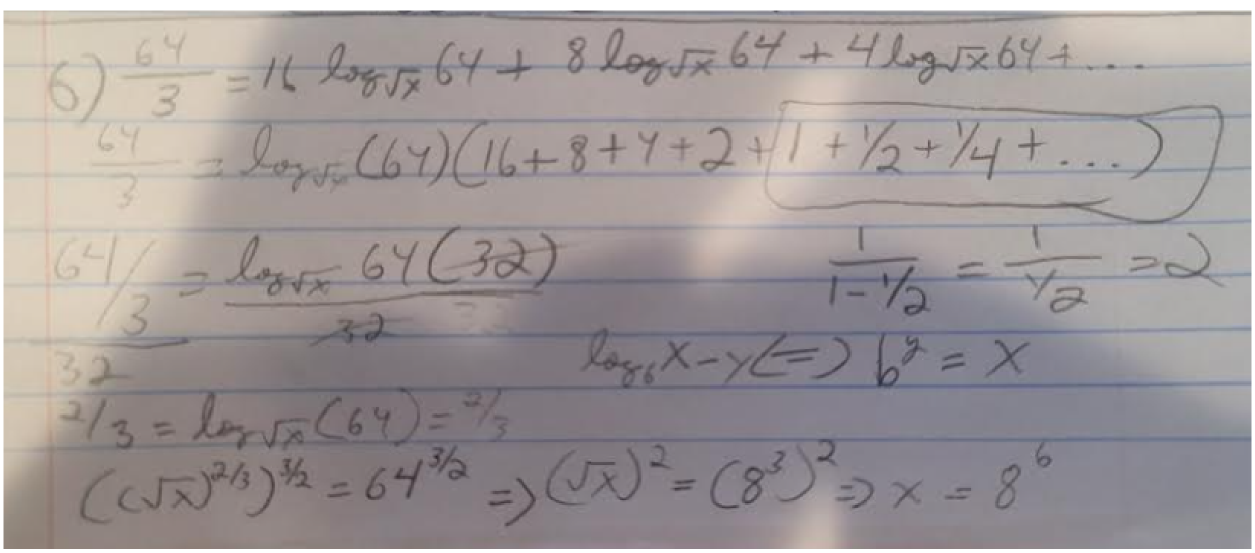
$$\int_{1.5}^{\infty} f(x)\, dx = \frac{1}{4}$$

2 * ⅛ = ¼

6)

6. Solve for x

$$\frac{64}{3} = 16\log_{\sqrt{x}} 64 + 8\log_{\sqrt{x}} 64 + 4\log_{\sqrt{x}} 64 + \cdots$$



$6)\ \frac{64}{3} = 16\log_{\sqrt{x}} 64 + 8\log_{\sqrt{x}} 64 + 4\log_{\sqrt{x}} 64 + \ldots$

$\frac{64}{3} = \log_{\sqrt{x}}(64)\left(16+8+4+2+\boxed{1+\tfrac{1}{2}+\tfrac{1}{4}+\ldots}\right)$

$\frac{64/3}{32} = \frac{\log_{\sqrt{x}} 64\,(32)}{32}$ $\qquad \frac{1}{1-\frac{1}{2}} = \frac{1}{\frac{1}{2}} = 2$

$\log_b x = y \iff b^y = x$

$2/3 = \log_{\sqrt{x}}(64) = 2/3$

$\left((\sqrt{x})^{2/3}\right)^{3/2} = 64^{3/2} \Rightarrow (\sqrt{x})^2 = (8^3)^2 \Rightarrow x = 8^6$

x=262144

7. Compute the derivative of

$$y = \sqrt{\frac{x(x+2)}{(2x+1)(3x+2)}}$$

(a) directly using the Chain and Product Rules

(b) via logarithmic differentiation

a)

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\sqrt{\frac{x\,(x+2)}{(2x+1)\,(3x+2)}}\right]$$

$$= \frac{1}{2}\left(\frac{x\,(x+2)}{(2x+1)\,(3x+2)}\right)^{\frac{1}{2}-1} \cdot \frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{x\,(x+2)}{(2x+1)\,(3x+2)}\right]$$

$$= \frac{\frac{\frac{\mathrm{d}}{\mathrm{d}x}[x(x+2)]\cdot(2x+1)(3x+2) - x(x+2)\cdot\frac{\mathrm{d}}{\mathrm{d}x}[(2x+1)(3x+2)]}{((2x+1)(3x+2))^2}}{2\sqrt{\frac{x(x+2)}{(2x+1)(3x+2)}}}$$

$$= \frac{\left(\frac{\mathrm{d}}{\mathrm{d}x}[x]\cdot(x+2) + x\cdot\frac{\mathrm{d}}{\mathrm{d}x}[x+2]\right)(2x+1)\,(3x+2) - \left(\frac{\mathrm{d}}{\mathrm{d}x}[2x+1]\cdot(3x+2) + (2x+1)\cdot\frac{\mathrm{d}}{\mathrm{d}x}[3x+2]\right)x\,(x+2)}{2(2x+1)^2(3x+2)^2\sqrt{\frac{x(x+2)}{(2x+1)(3x+2)}}}$$

$$= \frac{\left(1\,(x+2) + x\left(\frac{\mathrm{d}}{\mathrm{d}x}[x] + \frac{\mathrm{d}}{\mathrm{d}x}[2]\right)\right)(2x+1)\,(3x+2) - \left(\left(2\cdot\frac{\mathrm{d}}{\mathrm{d}x}[x] + \frac{\mathrm{d}}{\mathrm{d}x}[1]\right)(3x+2) + (2x+1)\left(3\cdot\frac{\mathrm{d}}{\mathrm{d}x}[x] + \frac{\mathrm{d}}{\mathrm{d}x}[2]\right)\right)x\,(x+2)}{2(2x+1)^2(3x+2)^2\sqrt{\frac{x(x+2)}{(2x+1)(3x+2)}}}$$

$$= \frac{(x\,(1+0) + x + 2)\,(2x+1)\,(3x+2) - ((2\cdot 1+0)\,(3x+2) + (2x+1)\,(3\cdot 1+0))\,x\,(x+2)}{2(2x+1)^2(3x+2)^2\sqrt{\frac{x(x+2)}{(2x+1)(3x+2)}}}$$

$$= \frac{(2x+1)\,(2x+2)\,(3x+2) - x\,(x+2)\,(2\,(3x+2) + 3\,(2x+1))}{2(2x+1)^2(3x+2)^2\sqrt{\frac{x(x+2)}{(2x+1)(3x+2)}}}$$

$$-\frac{5x^2 - 4x - 4}{2(2x+1)^2(3x+2)^2\sqrt{\frac{x(x+2)}{(2x+1)(3x+2)}}}$$

b)

b) $y = \left(\frac{x(x+2)}{(2x+1)(3x+2)}\right)^{\frac{1}{2}}$

$\ln y = \frac{1}{2}\ln\left(\frac{x(x+2)}{(2x+1)(3x+2)}\right)$

$= \frac{1}{2}\ln(x) + \frac{1}{2}\ln(x+2) - \frac{1}{2}\ln(2x+1) - \frac{1}{2}\ln(3x+2)$

$\left(\frac{x(x+2)}{(2x+1)(3x+2)}\right)^{\frac{1}{2}} \cdot \frac{dy}{y} = \frac{1}{2x} + \frac{1}{2(x+2)} - \frac{2}{2(2x+1)} - \frac{3}{2(3x+2)} \cdot \left(\frac{x(x+2)}{(2x+1)(3x+2)}\right)^{\frac{1}{2}}$