ENGR 65 Electric Circuits

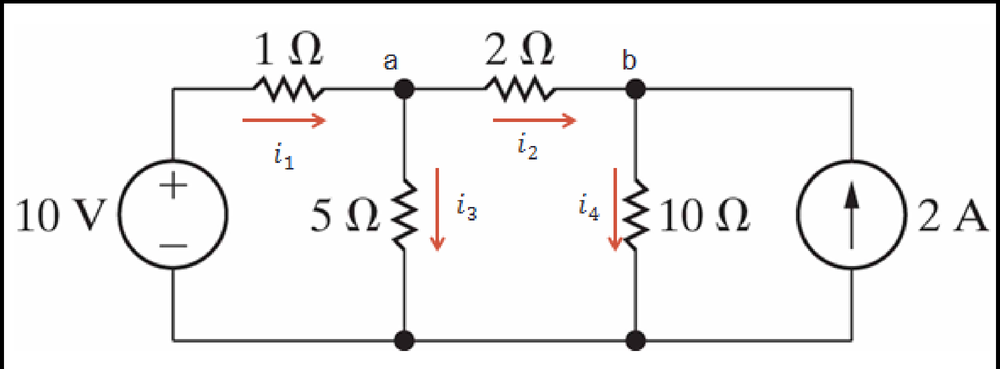
Lecture 6: The Node-Voltage Method (NVM)

Today's Topics

- Why node-voltage method and what is it?
- The steps for writing the node-voltage equations.
- The tips on selecting the reference node.
- How to deal with dependent sources?

- Covered in Sections 4.1, 4.2, 4.3 and 4.4

Why Node-Voltage Method?



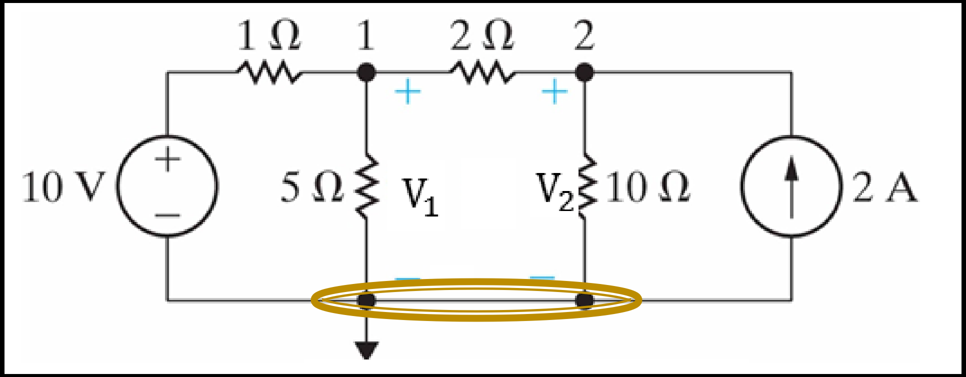
For the above circuit, if using the KCL, KVL, and Ohm's law, we need to write four equations for solving i_1, i_2, i_3, and i_4.

$$i_2 + i_3 - i_1 = 0 \text{ (KCL)}$$
$$i_4 - i_2 - 2 = 0 \text{ (KCL)}$$
$$1i_1 + 5i_3 - 10 = 0 \text{ (KVL+Ohm's Law)}$$
$$2i_2 + 10i_4 - 5i_3 = 0 \text{ (KVL+Ohm's Law)}$$

Are there any other methods that can reduce the number of equations for solving this circuit? One of these methods is called the node-voltage method.

The Node-Voltage Method

- Node-voltage method (also called nodal analysis) is a systematic way of determining the node voltages between essential nodes, based on the essential branch currents of an electric circuit.



Node-voltage equations:

$$\frac{V_1 - 10}{1} + \frac{V_1}{5} + \frac{V_1 - V_2}{2} = 0$$

$$\frac{V_2 - V_1}{2} + \frac{V_2}{10} - 2 = 0$$

The Node-Voltage Method

The steps used in the node-voltage method:

Step 1: Mark the essential nodes on the circuit.

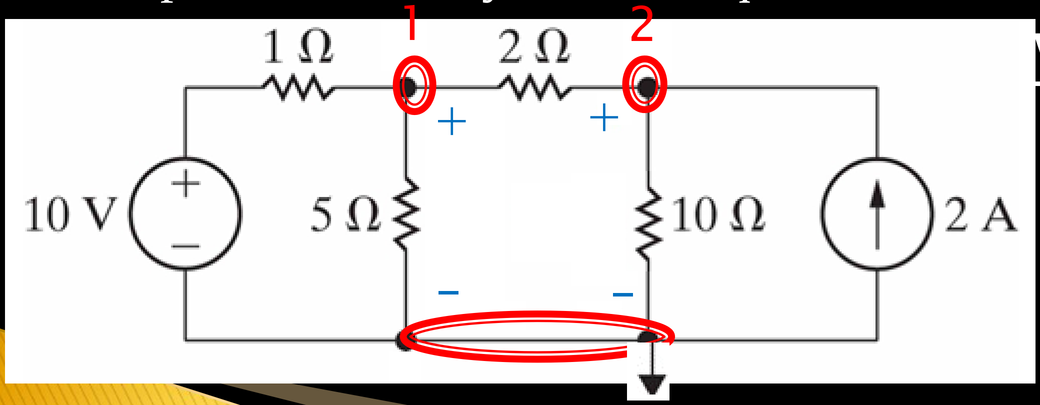
Step 2: Select one of the essential nodes as a reference node.

Step 3: Define node voltages as voltage rise from the reference node to the non-reference nodes.

Step 4: Apply KCL (assuming all currents leaving nodes) to each non-reference node. (Note: these equations are independent! Why?)

Step 5: Write an equation for each current or voltage on which dependent sources depend.

Step6: Solve the systems of equations.

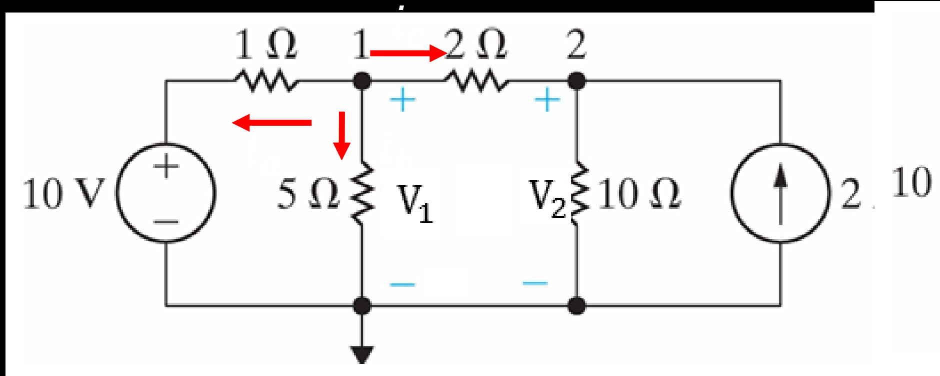


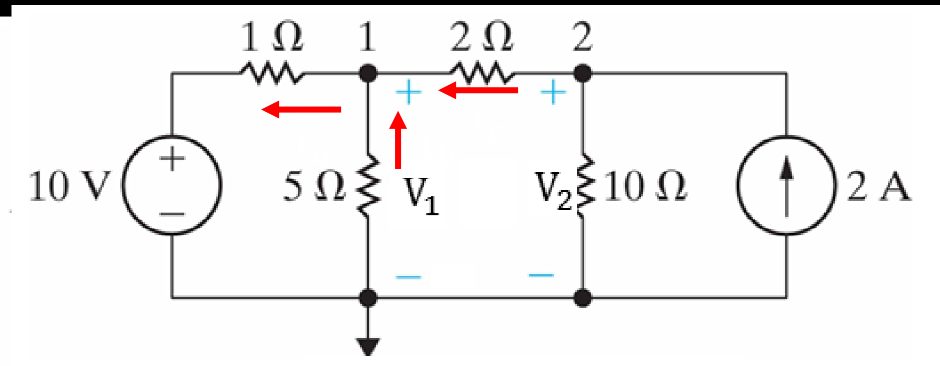
$$\frac{V_1-10}{1}+\frac{V_1}{5}+\frac{V_1-V_2}{2}=0 \quad (1)$$

$$\frac{V_2-V_1}{2}+\frac{V_2}{10}-2=0 \quad (2)$$

Reference node $V=0$

Step 4: Apply KCL (assuming all currents leaving nodes) to each non-reference node.





Applying KCL at node 1: $i_a + i_b + i_c = 0$

$$i_a = \frac{V_1 - 10}{1} ,$$

$$i_b = \frac{V_1}{5} ,$$

$$i_c = \frac{V_1 - V_2}{2}.$$

Plugging $i_a, i_b, i_c = 0 \; into \; the \; KCL \; equation$:

$$\frac{V_1 - 10}{1} + \frac{V_1}{5} + \frac{V_1 - V_2}{2} = 0$$

Applying KCL at node 1: $i_a - i_b - i_c = 0$

$$i_a = \frac{V_1 - 10}{1}$$

$$i_b = -\frac{V_1}{5}$$

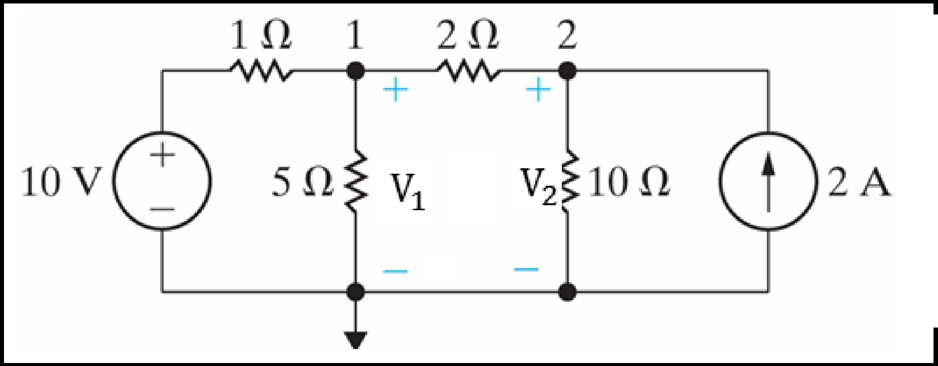
$$i_c = -\frac{V_1 - V_2}{2}.$$

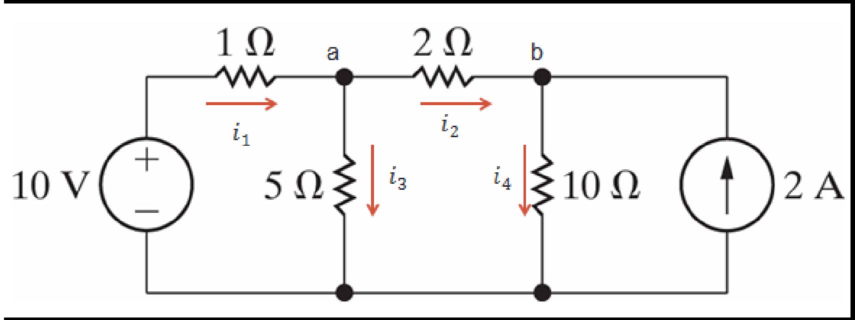
Plugging i_a, i_b, and i_c, we have the same equation

$$\frac{V_1 - 10}{1} + \frac{V_1}{5} + \frac{V_1 - V_2}{2} = 0$$

Which reference assignments are better?

The Node-Voltage Method





After solving the circuit for V_1 and V_2, we can basically find everything else in the circuit, such as $i_1, i_2, i_3,$ and i_4.

$$i_1 = \frac{10-V_1}{1}, \quad i_2 = \frac{V_1-V_2}{2}, \quad i_3 = \frac{V_1}{5}, \quad i_4 = \frac{V_2}{10}.$$

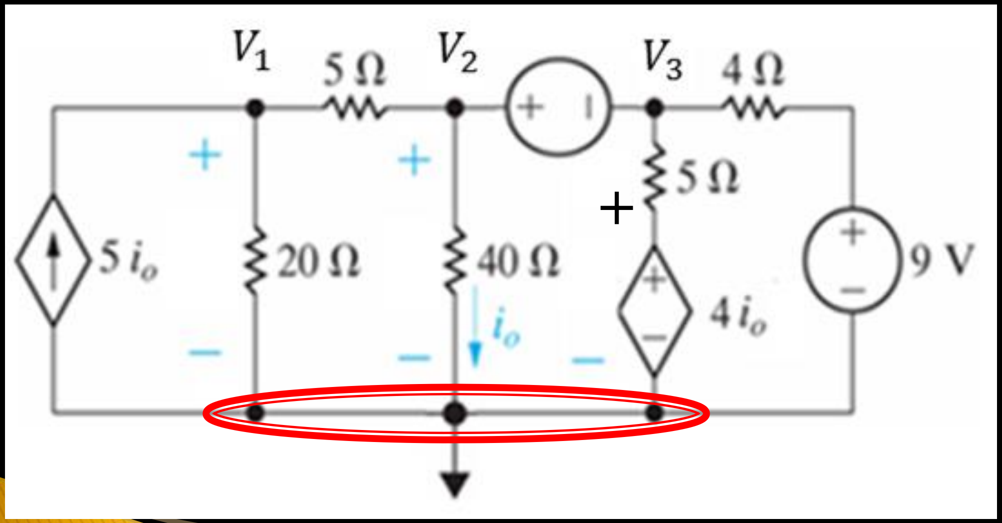
$$p_{1\Omega} = 1 \times i_1^2,\ p_{2\Omega} = 2 \times i_2^2,\ p_{5\Omega} = 5 \times i_3^2,\ p_{10\Omega} = 10 \times i_4^2$$

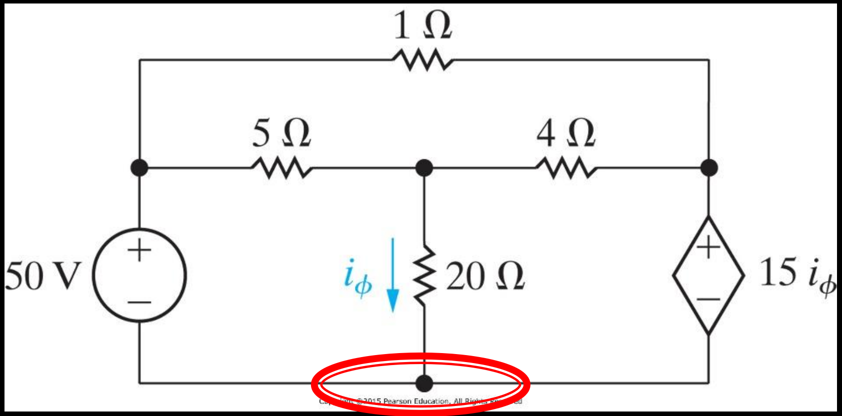
$$p_{10\,V} = -10 \times i_1, \qquad p_{2A} = -2 \times V_2$$

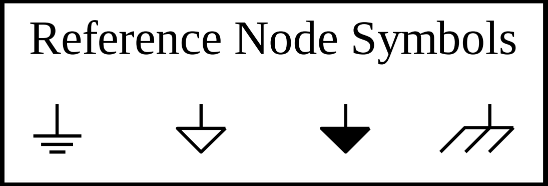
How to Choose a Reference Node

You can choose any essential node as a reference node. However, carefully choosing the reference node can greatly simplify writing the node-voltage equations.

The best way to choose a reference node is to pick the node with the most connections in order to eliminate the most difficult equation or pick the node that connects the most sources.







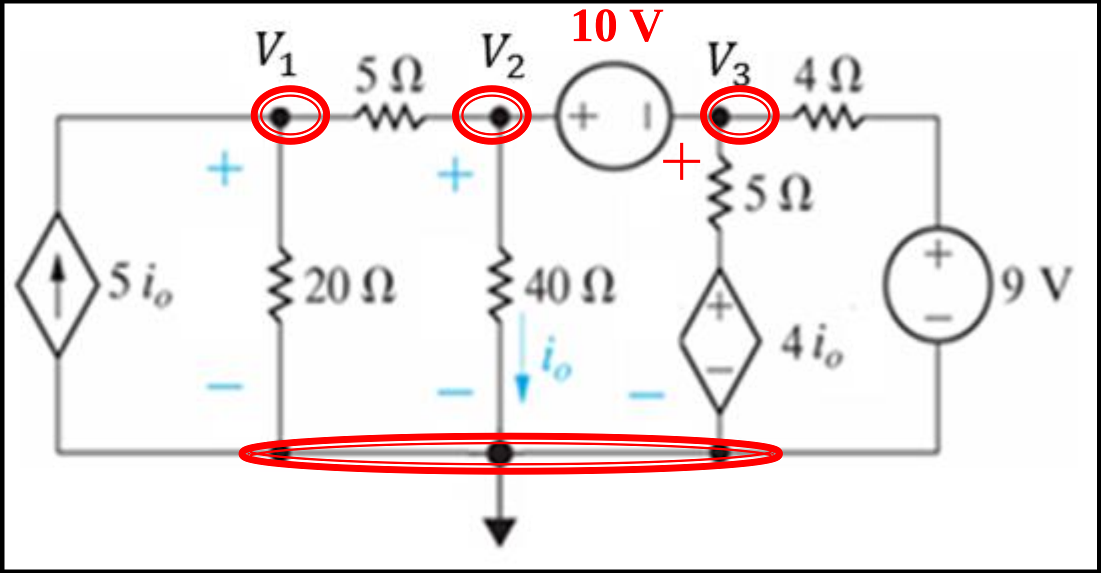
The number of essential nodes in this circuit is

A. 3

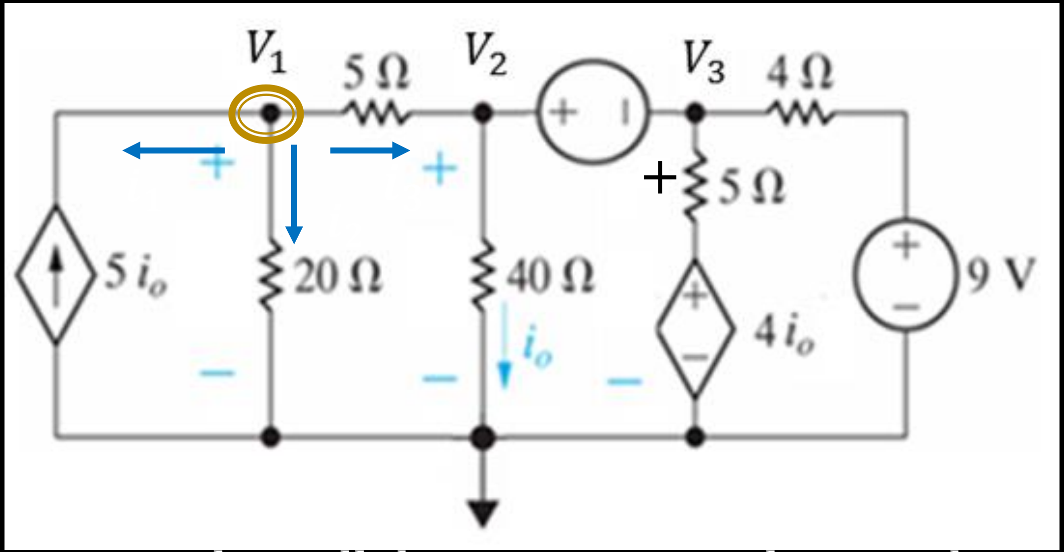
B. 4

C. 5

D. 6



The node-voltage equation at node 1 is



Assuming at node 1, all three currents are leaving the node, so

$$\boldsymbol{i_1} = -5i_0, \qquad \boldsymbol{i_2} = \frac{V_1}{20}, \quad \boldsymbol{i_3} = \frac{V_1 - V_2}{5}$$

Applying KCL to node 1, we have $i_1 + i_2 + i_3 = 0$

Plugging i_1, i_2, i_3 into above KCL equation, we have

$$-5i_0 + \frac{V_1}{20} + \frac{V_1 - V_2}{5} = 0$$

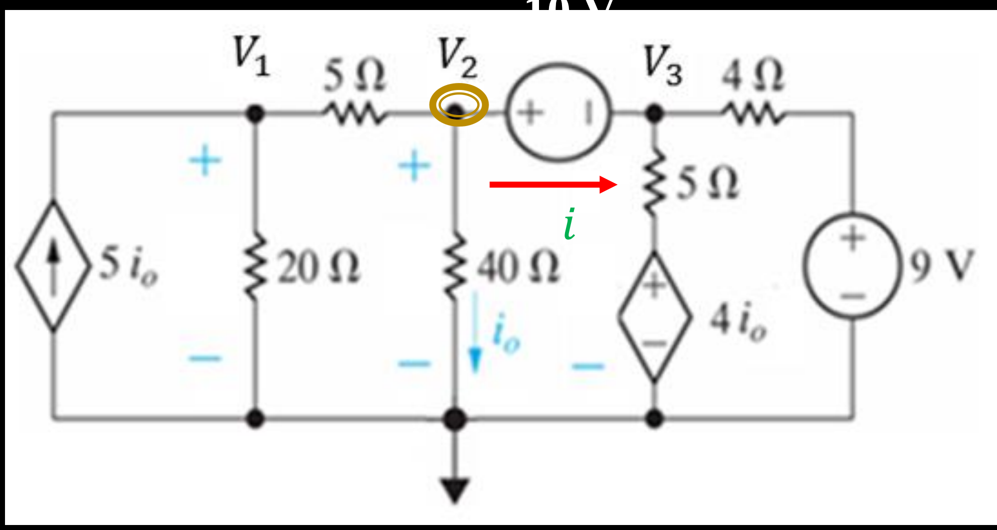
The node-voltage equation at node 2 is

A. $\frac{V_2+V_1}{5} - \frac{V_2}{40} - i = 0$

B. $\frac{V_2-V_1}{5} + \frac{V_2}{40} + i = 0$

C. $\frac{V_2-V_1}{5} + V_2 + i = 0$

D. $\frac{V_2-V_1}{5} - V_2 - i = 0$



The node-voltage equation at node 3 is

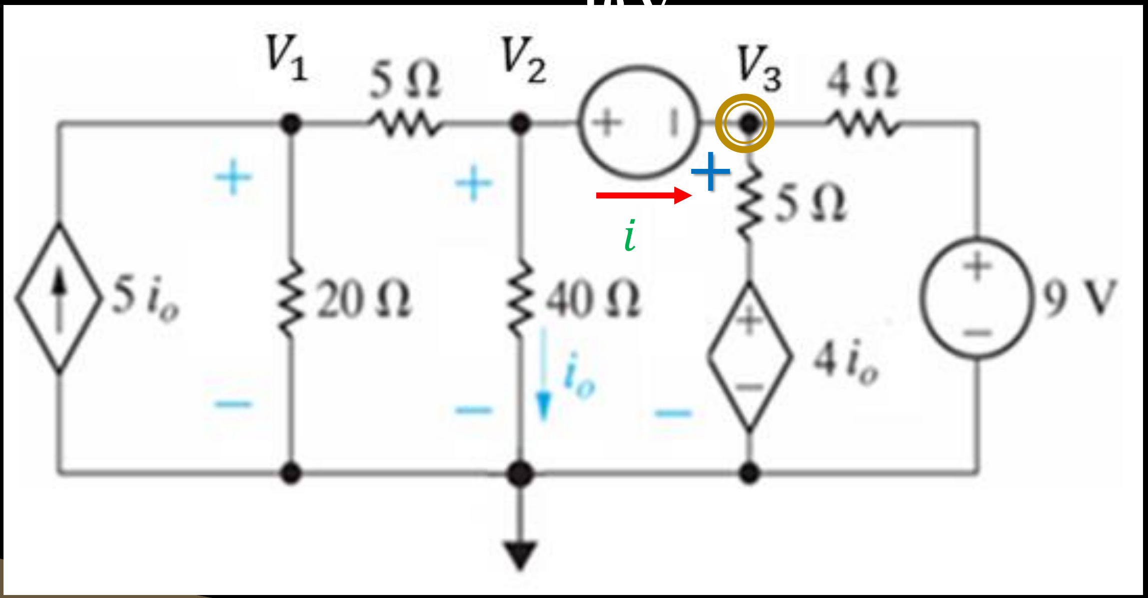
A. $i + 4i_0 + \frac{V_3 - 9}{4} = 0$

B. $i - 4i_0 + \frac{V_3 - 9}{4} = 0$

C. $-i + \frac{V_3 - 4i_0}{5} + \frac{V_3 - 9}{4} = 0$

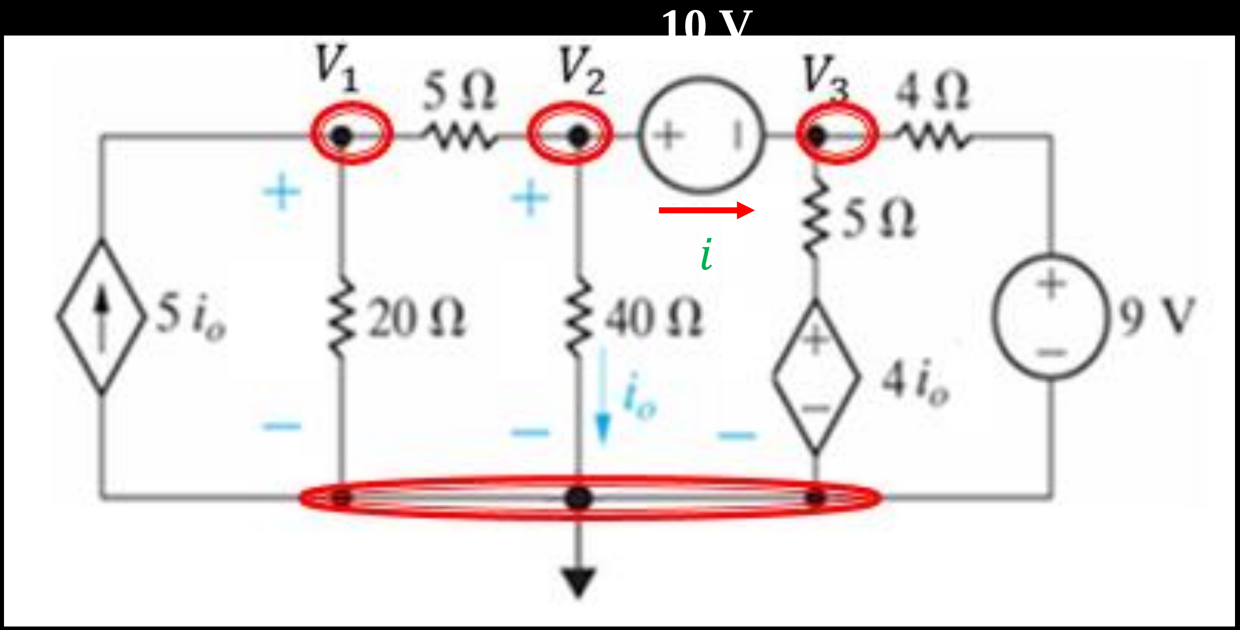
D. $-i + \frac{V_3 + 4i_0}{5} + \frac{V_3 - 9}{4} = 0$

Question: Here, why is the reference direction of the current *i* in the 10 V voltage source assigned going to the node rather than leaving the node?

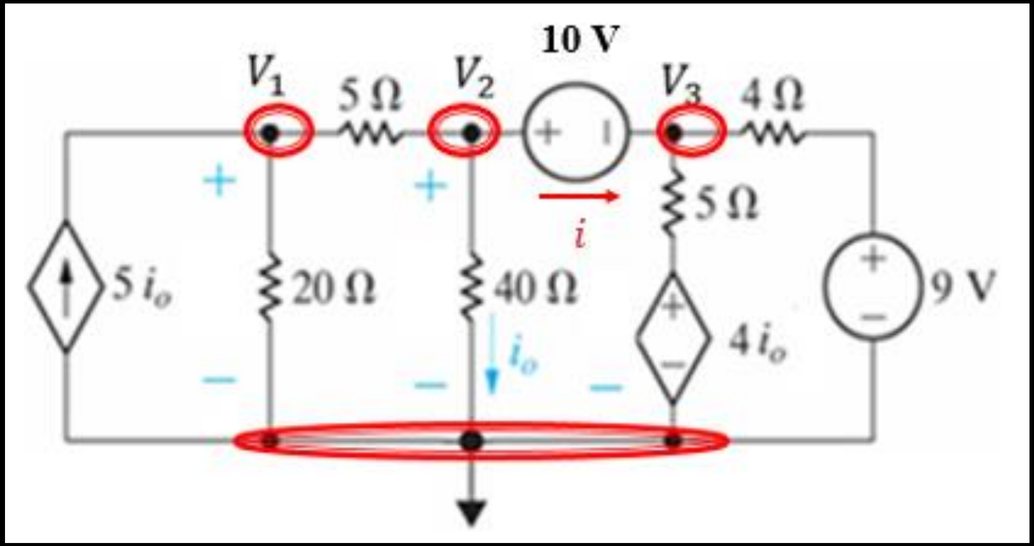


The number of unknown variables in the simultaneous equations is

A. 2
B. 3
C. 4
D. 5



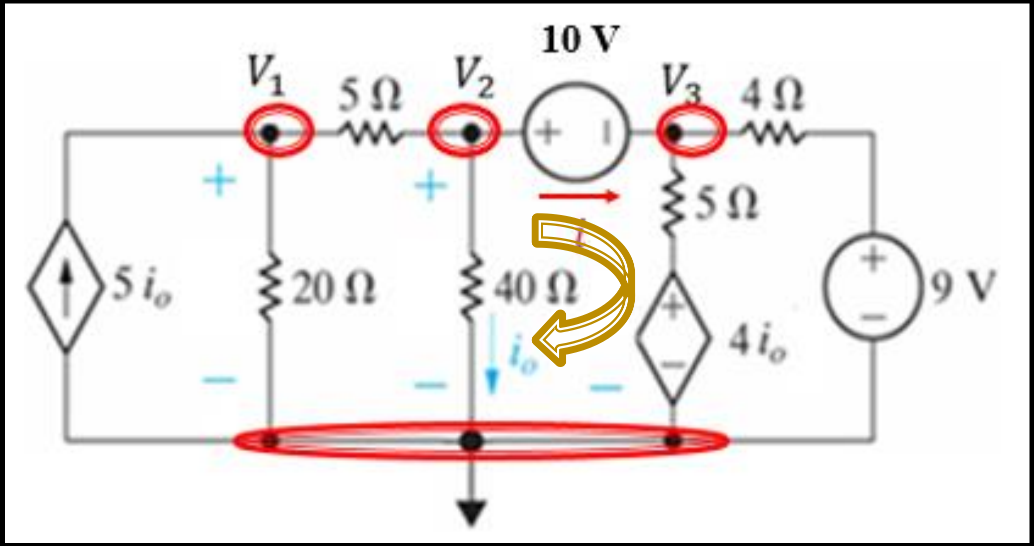
How to Deal with Dependent Sources



Write the equation for i_0 (dependent variable) in terms of node-voltages by using KCL, KVL, and Ohm’s law.

From Ohm′s law: $$i_0 = \frac{V_2}{40}$$

The Equation about Voltage Sources between Two Non-Reference Nodes

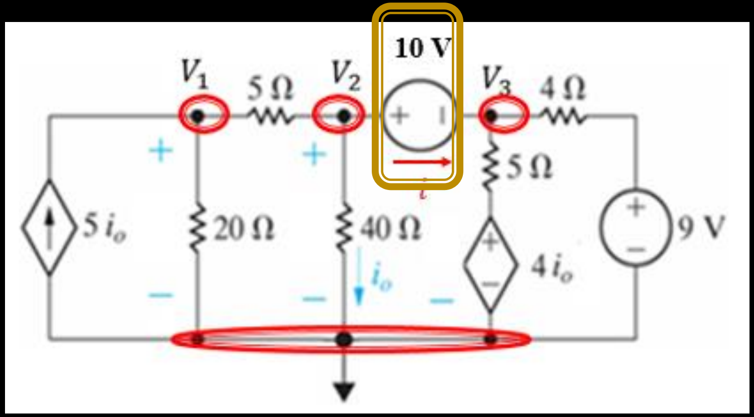


It is the difference between the two node-voltages

Applying KVL to the mesh: $V_2 - V_3 = 10$

Up to now, the total number of independent equations is 5, which is equal to the number of the unknown variables in the simultaneous equations.

The Supernode



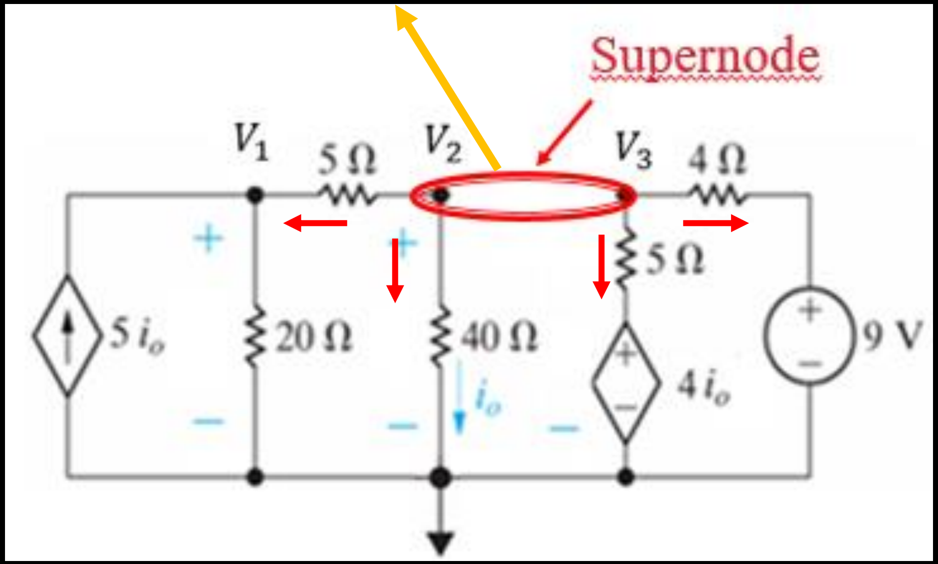
The supernode occurs when a voltage source is located between two non-reference nodes.

$$\frac{V_2-V_1}{5}+\frac{V_2}{40}+i=0 \quad \text{(at node 2)}$$

$$-i+\frac{V_3-4i_0}{5}+\frac{V_3-9}{4}=0 \quad \text{(at node 3)}$$

Summing the two equations, we have

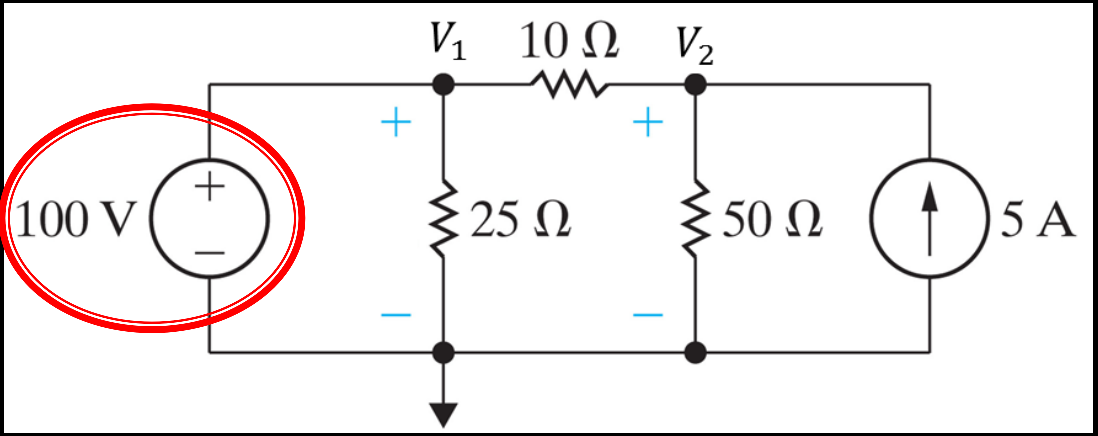
$$\frac{V_2-V_1}{5}+\frac{V_2}{40}+\frac{V_3-4i_0}{5}+\frac{V_3-9}{4}=0$$



How to Deal with Voltage Sources?

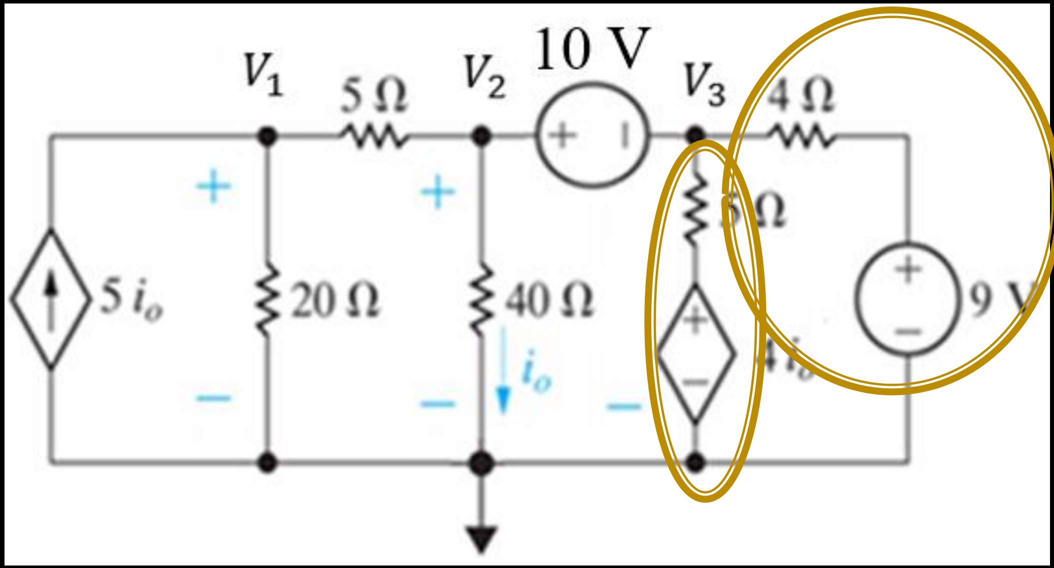
In general, a voltage source requires some special attentions since the current through it entirely depends on what it is connected to.

1. If the voltage source is between a non-reference node and a reference node;



$$V_1 = 100\ V$$

How to Deal with Voltage Sources?

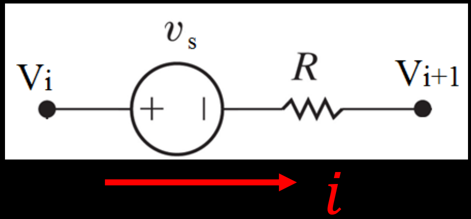


2. If a voltage source is in series with a resistor, the current in the corresponding branch, for example, is

$$\frac{V_3 - 4i_0 - V_f}{5} \ or \ \frac{V_3 - 9 - V_f}{4}.$$

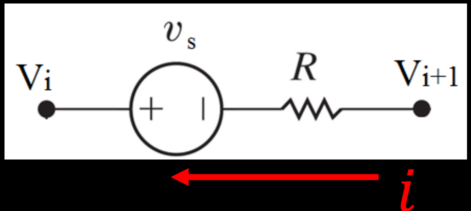
$$V_f = 0$$

In general,



The current in the above branch can be found as

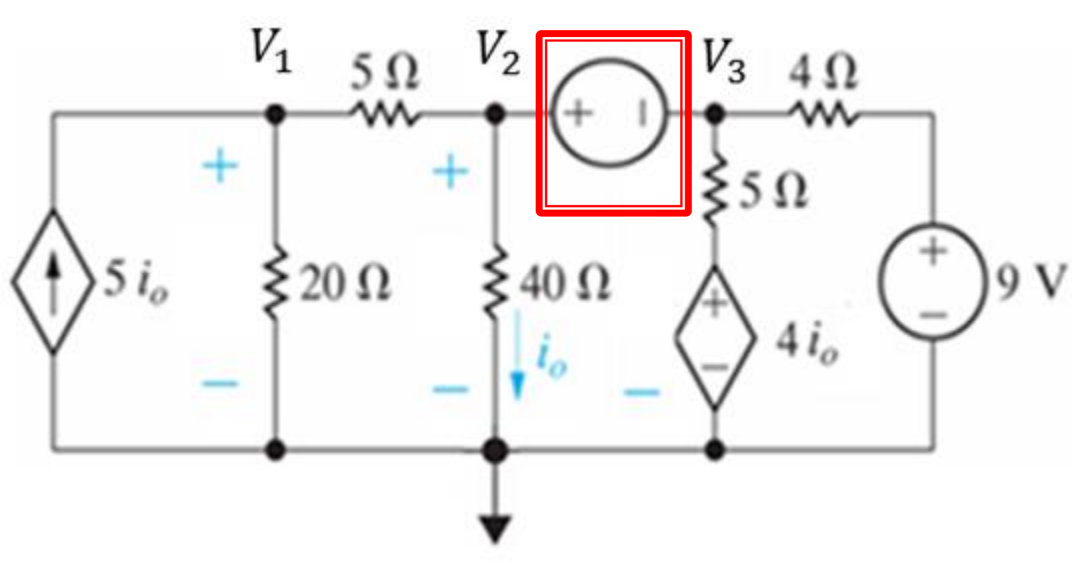
$$i = \frac{V_i - v_s - V_{i+1}}{R}$$



The current in the above branch can be found as

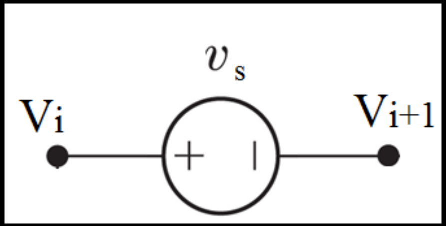
$$i = \frac{V_{i+1} + v_s - V_i}{R}$$

How to Deal with Voltage Sources?

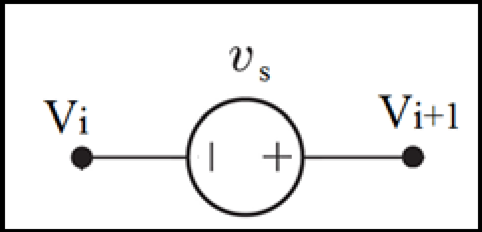


3. If the voltage source is between two non-reference nodes, $V_2 - V_3 = 10$ and a supernode is formed.

In General,



$V_i - V_{i+1} = v_s$



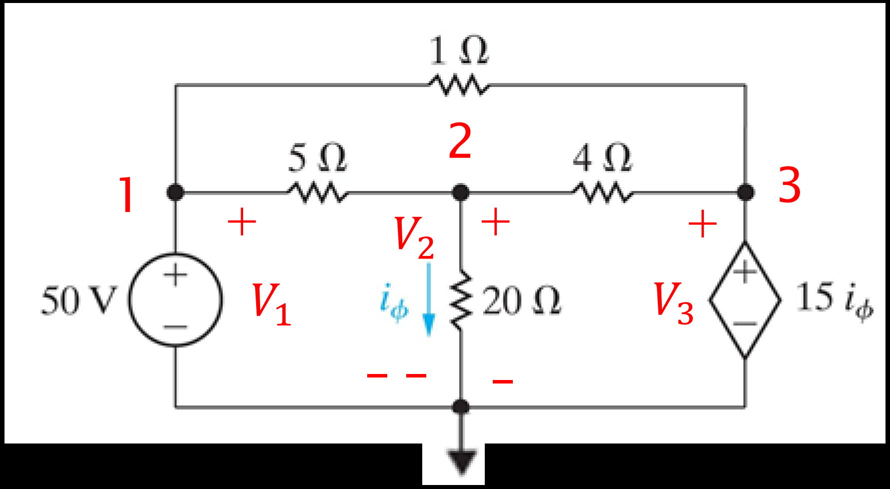
$V_i - V_{i+1} = -v_s$

How Many Node-Voltage Equations Do You Need to Write?

- If there are n_e essential nodes, you need to write n_e-1 equations because one essential node is used as a reference node, and we do not write a KCL equation for the reference node.
- If there are dependent sources in the circuit, the number of equations increases. In general, each dependent source introduces a variable which is unknown. If v is the number of variables that dependent sources depend on,the total number of equations you need to write is n_e -1+v.

Example #1

Write the node-voltage equations for the circuit shown below.



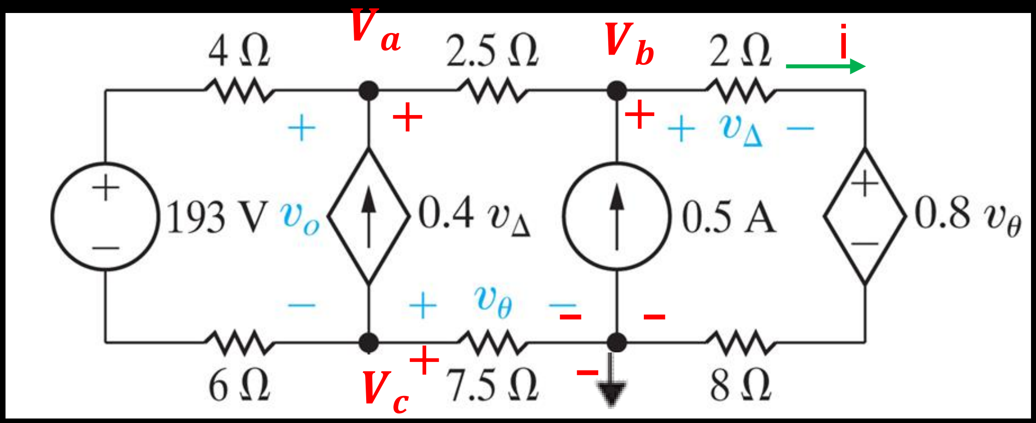
At node 1, $V_1 = 50$

At node 2, $\frac{V_2 - V_1}{5} + \frac{V_2}{20} + \frac{V_2 - V_3}{4} = 0$

$i_\phi = \frac{V_2}{20}$

At node 3, $V_3 = 15 i_\phi$

Example #2



Write the node-voltage equations for the circuit.

$$i = \frac{V_b - 0.8v_\theta}{2+8}$$

$$at\ V_a: \frac{V_a - 193 - V_c}{4+6} - 0.4v_\Delta + \frac{V_a - V_b}{2.5} = 0$$

$$v_\Delta = 2i = \frac{V_b - 0.8v_\theta}{10} \times 2 = \frac{V_b - 0.8v_\theta}{5}$$

$$at\ V_b: \frac{V_b - V_a}{2.5} - 0.5 + \frac{V_b - 0.8v_\theta}{2+8} = 0$$

$$v_\theta = V_c$$

$$at\ V_c: \frac{V_c + 193 - V_a}{4+6} + 0.4v_\Delta + \frac{V_c}{7.5} = 0$$

Summary

The node-voltage method can reduce the number of independent equations in most of the real applications. Based on these node voltages, you can find the solutions of all other variables in the circuits

- In next class, I will be talking about:
- The mesh-current method