ENGR 057 Statics and Dynamics

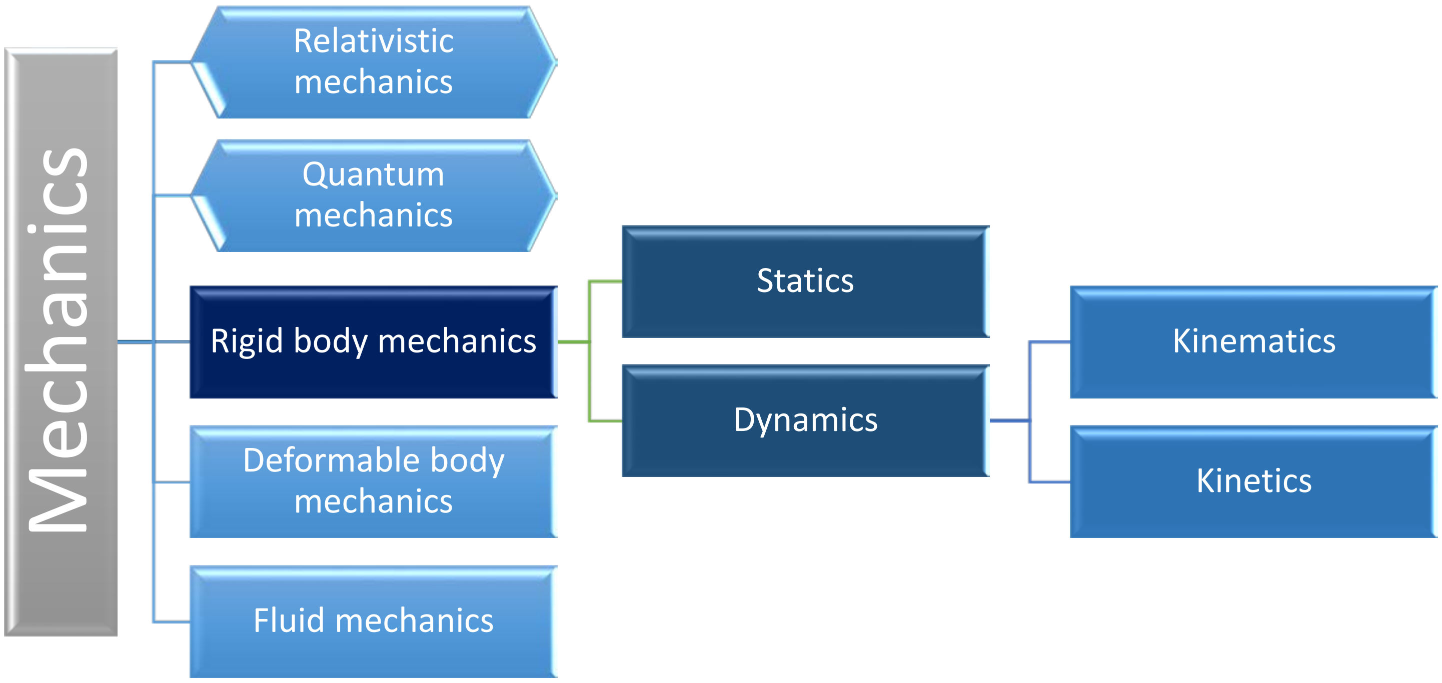
Introduction to the course

Instructor
Ingrid M. Padilla Espinosa, PhD

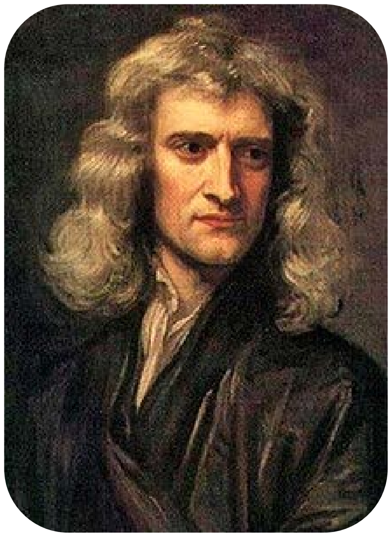
UNIVERSITY OF CALIFORNIA
MERCED

What is mechanics?

- From Greek mēchanikós, meaning “of machines”
- Machines are made up of physical objects (bodies) on which <u>forces</u> are applied
- Bodies respond to forces by either deforming or moving
- Mechanics is the analysis of the action of forces on bodies



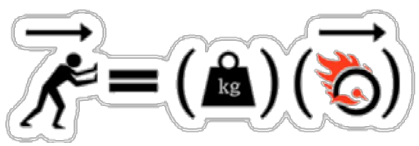
Fundamentals: Newton’s laws of motions



Portrait of Isaac Newton

1. In an inertial frame of reference, an object either remains at rest or continues to move at a constant velocity, unless acted upon by a force

2. In an inertial reference frame, the vector sum of the forces F on an object is equal to the mass m of that object multiplied by the acceleration a of the object: $F = ma$



3. When one body exerts a force on a second body, the second body simultaneously exerts a force equal in magnitude and opposite in direction on the first body

Fundamentals: Units of measurement

- Fundamental physical quantities:
  1. Length
  2. Mass
  3. Time
  4. Force (push or a pull)

- Special names are given to an amounts of these quantities, called units.

- We will mostly use the International System (SI) of units

- You must always express your physical quantities in consistent units

Systems of units

Name	Length	Time	Mass	Force
International System of Units SI	meter m	second s	kilogram kg	$\boxed{\text{newton*}}$ N $\left(\frac{\text{kg} \cdot \text{m}}{\text{s}^2}\right)$
U.S. Customary FPS	foot ft	second s	$\boxed{\text{slug*}}$ $\left(\frac{\text{lb} \cdot \text{s}^2}{\text{ft}}\right)$	pound lb

*Derived unit.

TABLE 1–2 Conversion Factors

Quantity	Unit of Measurement (FPS)	Equals	Unit of Measurement (SI)
Force	lb		4.448 N
Mass	slug		14.59 kg
Length	ft		0.3048 m

TABLE 1–3 Prefixes

	Exponential Form	Prefix	SI Symbol
Multiple			
1 000 000 000	10^9	giga	G
1 000 000	10^6	mega	M
1 000	10^3	kilo	k
Submultiple			
0.001	10^{-3}	milli	m
0.000 001	10^{-6}	micro	μ
0.000 000 001	10^{-9}	nano	n

R. C. Hibbeler, *Statics and dynamics*, 2016

PROBLEM SOLVING STRATEGY:
A 3 Step Approach

1. **Interpret:** Read carefully and determine what is given and what is to be found/ delivered. Ask, if not clear. If necessary, make assumptions and indicate them.

2. **Plan:** Think about major steps (or a road map) that you will take to solve a given problem. Think of alternative/creative solutions and choose the best one.

3. **Execute:** Carry out your steps. Use appropriate diagrams and equations. Estimate your answers. Avoid simple calculation mistakes. Reflect on / revise your work.

Numerical calculations must have dimensional homogeneity. Dimensions must be the same on both sides of the equal sign

Use 3 significant figures to give your answer, but do not round off for intermediate calculations

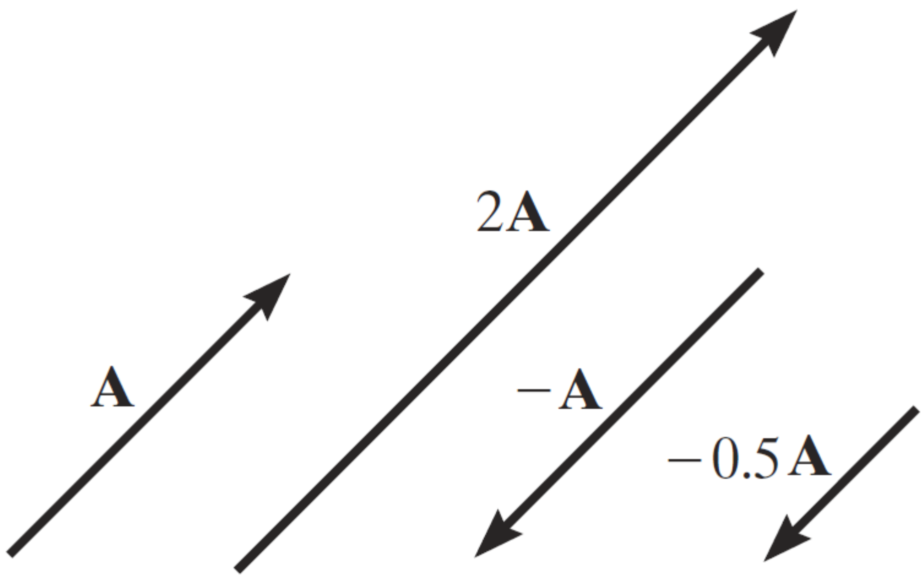
Scalars

- Magnitude only
- Arithmetic addition
- Mass, volume, time, charge, etc.
- Denoted with plain text

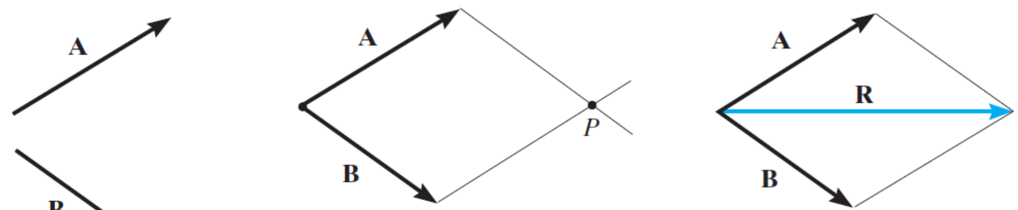
Vectors

- Magnitude and direction
- Vectorial addition
- Velocity, force, field, heat flux, etc.
- Denoted with **bold** or arrow $\vec{A}$

Multiplication and division by a scalar



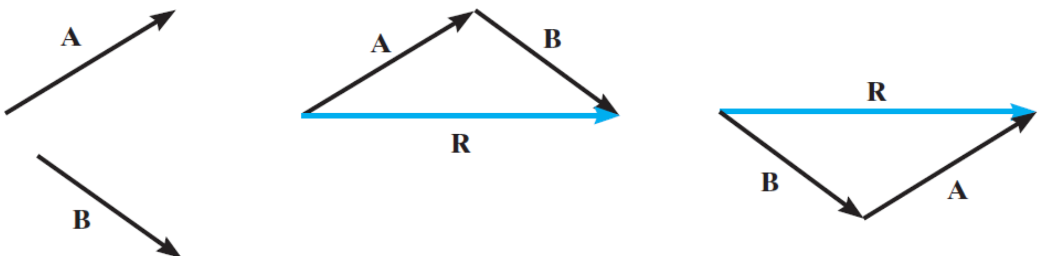
Vector addition – parallelogram law - triangle rule



$\mathbf{R} = \mathbf{A} + \mathbf{B}$

Parallelogram law

(a) (b) (c)



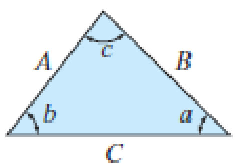
$\mathbf{R} = \mathbf{A} + \mathbf{B}$

Triangle rule

$\mathbf{R} = \mathbf{B} + \mathbf{A}$

Triangle rule

(a) (b) (c)



Cosine law:

$C = \sqrt{A^2 + B^2 - 2AB \cos c}$

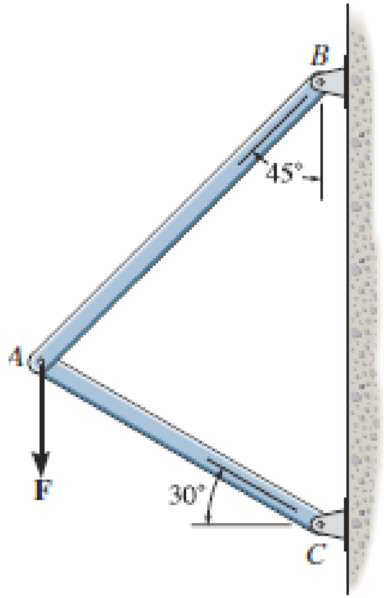
Sine law:

$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$

Subtraction can be performed by simply changing the <u>direction</u> of a vector and then performing addition

Proper graphical representation is essential to understand vector operations.

Graphical operations can be cumbersome in three-dimensional space or if there are many vectors. An <u>algebraic</u> alternative would be useful



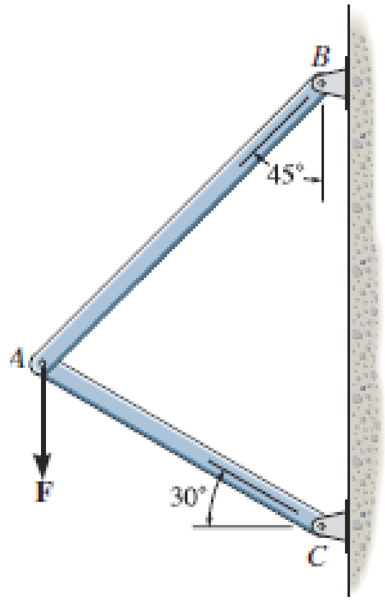
Example

The vertical force acts downward at *A* on the two-membered frame. Determine the magnitudes of the two components of **F** directed along the axes of *AB* and *AC*. Set F= 500 N.

1. Interpret: what is given and what is to be found?

2. Plan: Think about major steps.

3. Execute: Carry out your steps.



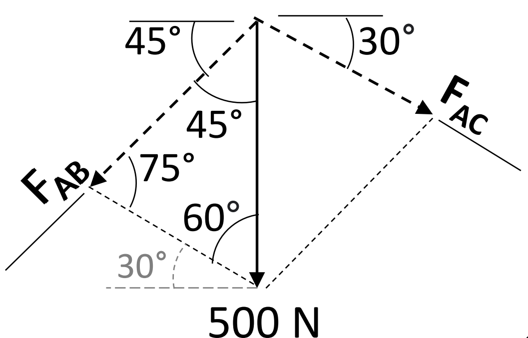
Example

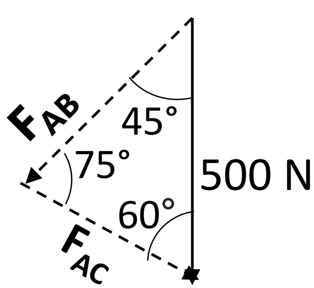
The vertical force acts downward at *A* on the two-membered frame. Determine the magnitudes of the two components of **F** directed along the axes of *AB* and *AC*. Set F= 500 N.

Solution

Given F = 500 N, 2 angles

Asked: F_{AC} and F_{AB}





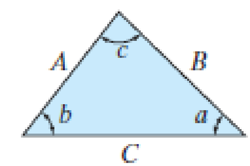
$$\frac{F_{AB}}{\sin 60°} = \frac{500}{\sin 75°}$$

$$F_{AB} = 448\text{ N}$$

$$\frac{F_{AC}}{\sin 45°} = \frac{500}{\sin 75°}$$

$$F_{AC} = 366\text{ N}$$

Remember:



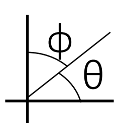
Cosine law:

$$C = \sqrt{A^2 + B^2 - 2AB\cos c}$$

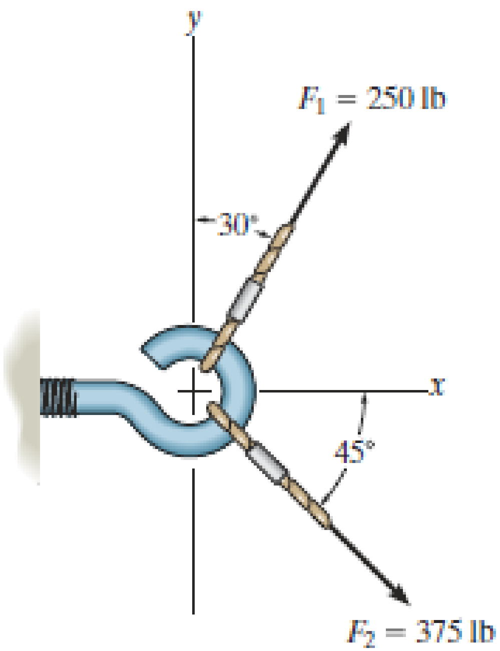
Sine law:

$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$

a+b+c = 180 °



Θ+ϕ = 90°



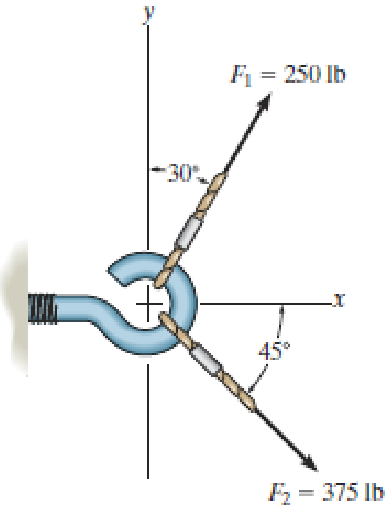
Individual work (15 min)

Determine the magnitude of the resultant force $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$ and its direction, measured counterclockwise from the positive x axis.

1. Interpret: what is given and what is to be found?

2. Plan: Think about major steps.

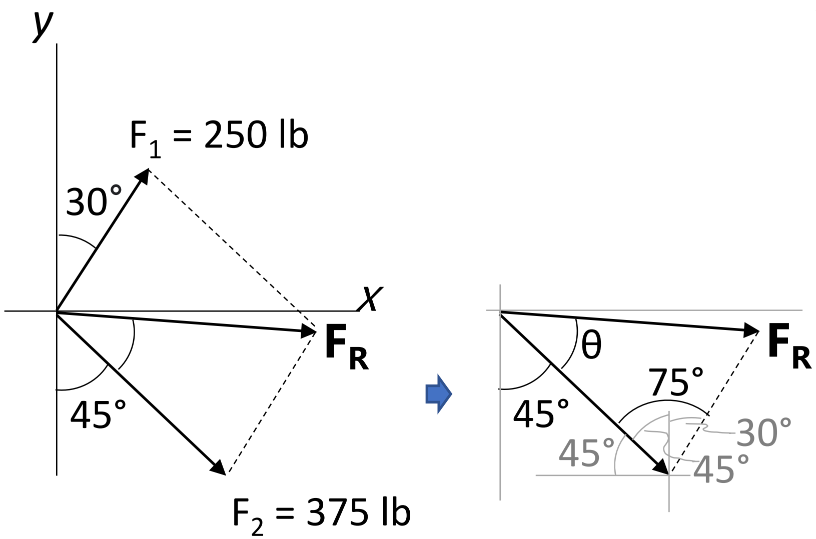
3. Execute: Carry out your steps.



Determine the magnitude of the resultant force $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$ and its direction, measured counterclockwise from the positive x-axis.

Solution

Given F_1 = 250 lb, angle of F_1 with y-axis, F_2 = 375 lb, angle of F_2 with x-axis
Asked: $\mathbf{F}_R$ (magnitude and angle ϕ with x-axis)



$$F_R = \sqrt{(250)^2 + (375)^2 - 2(250)(375)\cos 75°}$$

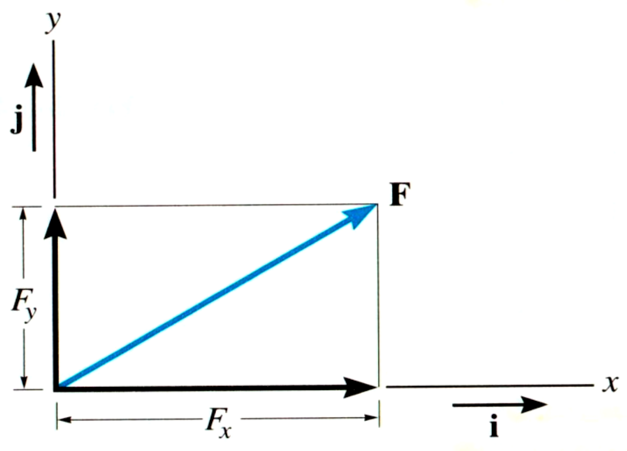
$$F_R = 393.2 = 393 \text{ lb}$$

$$\frac{393.2}{\sin 75°} = \frac{250}{\sin \theta}$$

$$\theta = 37.89°$$

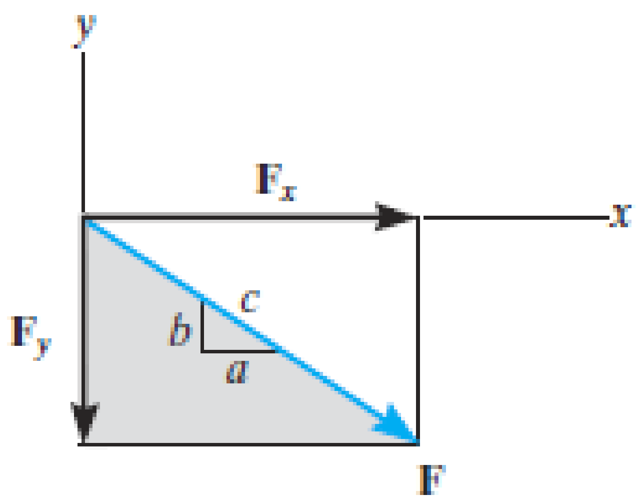
$$\emptyset = 360° - 45° + 37.89° = 353°$$

ADDITION OF A SYSTEM OF COPLANAR FORCES



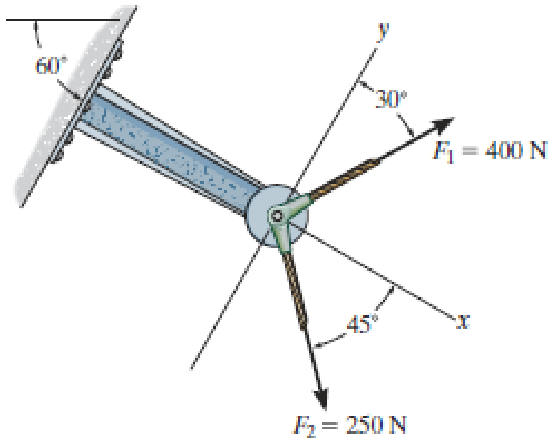
(a)

- We ‘resolve’ vectors into components using the x and y axis system.
- Each component of the vector is shown as a magnitude and a direction.
- The directions are based on the x and y axes. We use the “unit vectors” ***i*** and ***j*** to designate the x and y axes.



$$F_x = F\cos\theta \qquad F_x = F\left(\frac{a}{c}\right)$$

$$F_y = F\sin\theta \qquad F_y = -F\left(\frac{b}{c}\right)$$



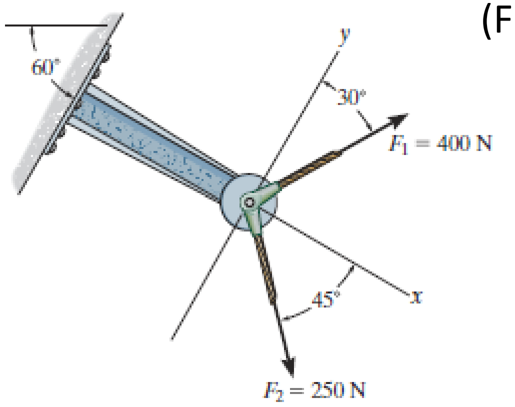
Example

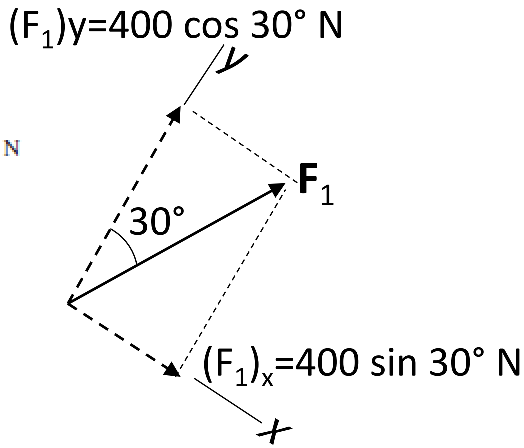
Determine the magnitude of the resultant force and its direction measured counterclockwise from the positive x axis.

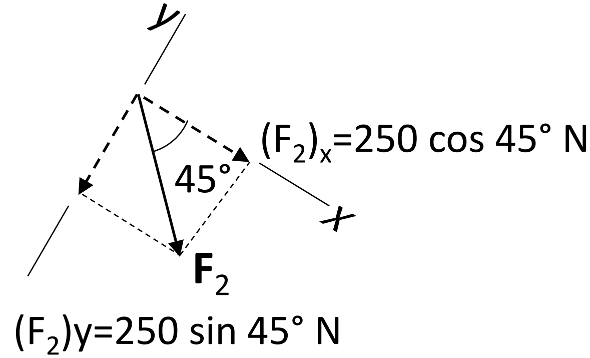
Solution

Given F_1 = 400 N, angle of F_1 with y-axis, F_2 = 250 N, angle of F_2 with x-axis
Asked: $\mathbf{F}_R$ (magnitude and angle ϕ with x-axis)

Example







$(F_1)_x$ = 400 sin 30° N = 200 N

$(F_1)_y$ = 400 cos 30° N = 346.4 N

$(F_2)_x$ = 250 cos 45° N = 176.8 N

$(F_2)_y$ = 250 sin 45° N = 176.8 N

Resultant force $\mathbf{F}_R$

$$(F_R)_x = \sum F_x = 200 + 176.8 = 376.8 \text{ N} \rightarrow$$

$$(F_R)_y = \sum F_y = 346.4 - 176.8 = 169.6 \text{ N} \uparrow$$

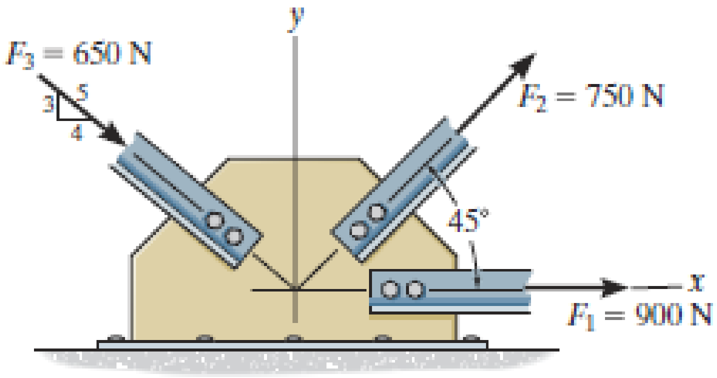
$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{376.8^2 + 169.6^2}$$

$$F_R = 413\ N$$

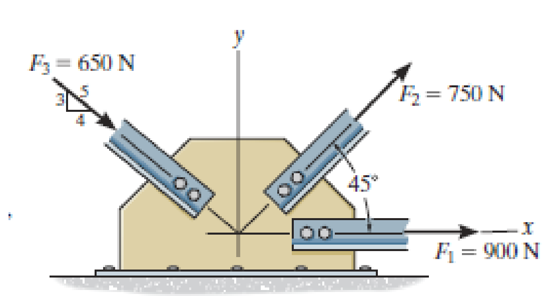
$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left[\frac{169.6}{376.8}\right] = 24.2°$$

Individual work (15 min)

Determine the magnitude of the resultant force acting on the plate and its direction, measured counterclockwise from the positive x axis.



Individual work (15 min)

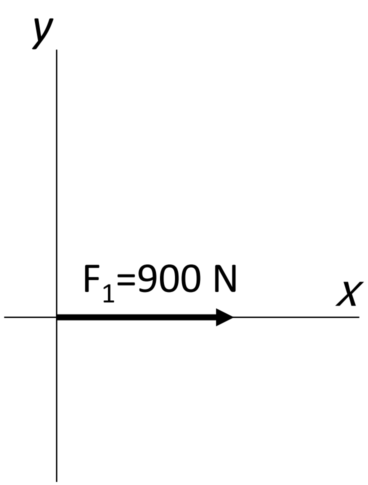


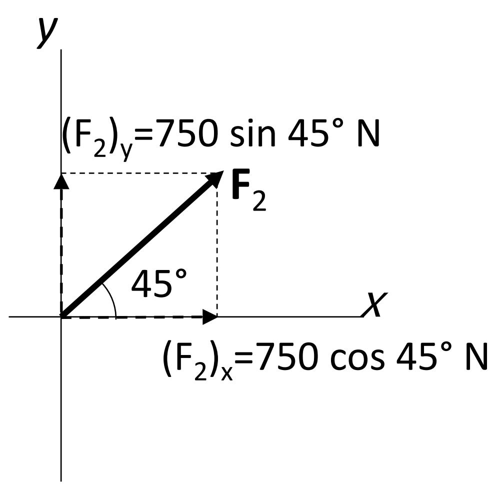
Determine the magnitude of the resultant force acting on the plate and its direction, measured counterclockwise from the positive x axis.

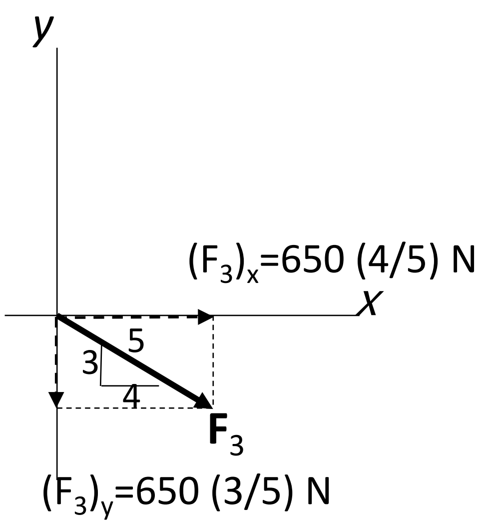
Solution

Given $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$

Asked: $\mathbf{F}_R$ (magnitude and angle ϕ with x-axis)

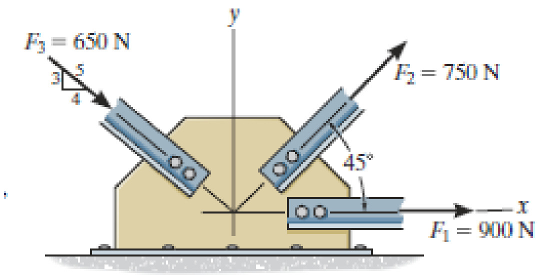






$$(F_R)_x = \sum F_x = 900 + 750 \cos 45° + 650\ (4/5) = 1950.33 \text{ N} \rightarrow$$

$$(F_R)_y = \sum F_y = 750 \sin 45° - 650\ (3/5) = 140.33 \text{ N} \uparrow$$



Individual work (15 min)

Determine the magnitude of the resultant force acting on the plate and its direction, measured counterclockwise from the positive x axis.

Solution

Given $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$
Asked: $\mathbf{F}_R$ (magnitude and angle ϕ with x-axis)

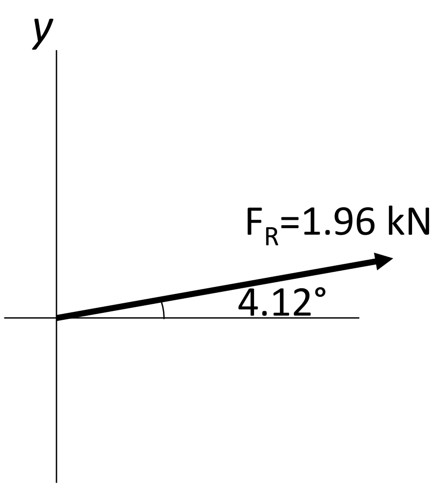
$$(F_R)_x = \sum F_x = 900 + 750 \cos 45° + 650\,(4/5) = 1950.33 \text{ N} \rightarrow$$

$$(F_R)_y = \sum F_y = 750 \sin 45° - 650\,(3/5) = 140.33 \text{ N} \uparrow$$

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{1950.33^2 + 140.33^2}$$

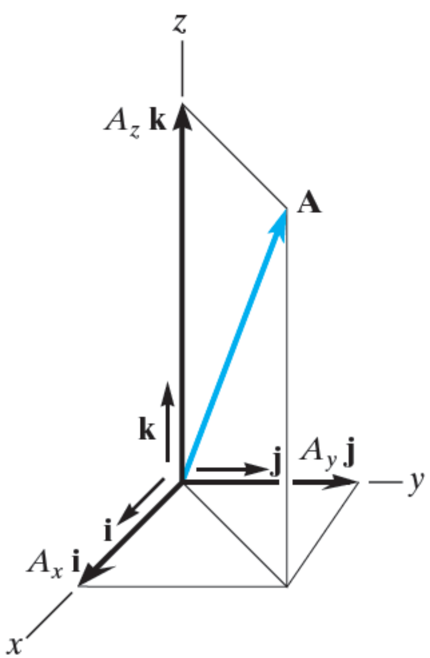
$$F_R = 1955\ N = 1.96\ kN$$

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left[\frac{140.33}{1950.33}\right] = 4.12°$$



Cartesian vector notation

- It is often convenient to divide a vector $\boldsymbol{v}$ by its own magnitude v. This operation is called normalization and its result $\widehat{\boldsymbol{v}}$ is called a unit vector
- The magnitude of this unit vector $\widehat{\boldsymbol{v}}$ is 1. It has no dimensions, and it has the same direction as the original vector $\boldsymbol{v}$ before normalization
- The most important unit vectors are $\hat{\boldsymbol{\imath}}, \hat{\boldsymbol{\jmath}}, \widehat{\boldsymbol{k}}$. These are direction vectors along the positive *x, y,* and *z* axes respectively
- Any vector can be expressed as a linear combination of $\hat{\boldsymbol{\imath}}, \hat{\boldsymbol{\jmath}}, \widehat{\boldsymbol{k}}$



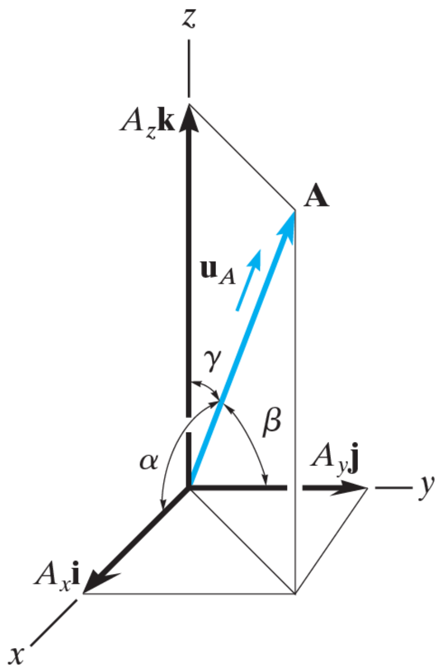
$$\boldsymbol{A} = A_x\hat{\boldsymbol{\imath}} + A_y\hat{\boldsymbol{\jmath}} + A_z\widehat{\boldsymbol{k}}$$

A_x, A_y, A_z are projections of $\boldsymbol{A}$

Direction vectors help us easily perform vector addition

$$\boldsymbol{A} + \boldsymbol{B} = \left(A_x\hat{\boldsymbol{\imath}} + A_y\hat{\boldsymbol{\jmath}} + A_z\widehat{\boldsymbol{k}}\right) + \left(B_x\hat{\boldsymbol{\imath}} + B_y\hat{\boldsymbol{\jmath}} + B_z\widehat{\boldsymbol{k}}\right)$$

$$\boldsymbol{A} + \boldsymbol{B} = (A_x + B_x)\hat{\boldsymbol{\imath}} + \left(A_y + B_y\right)\hat{\boldsymbol{\jmath}} + (A_z + B_z)\widehat{\boldsymbol{k}}$$



Cartesian vector notation

- Vectors can be expressed in trigonometrical terms
- The direction of a vector is defined by angles α, β, γ
- These angles are measured from the <u>positive</u> *x, y,* and *z* axes
- These angles are <u>not independent</u>

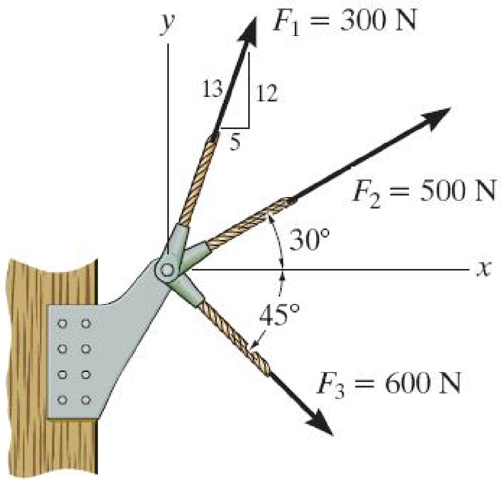
$$\cos\alpha = \frac{A_x}{A} \qquad \cos\beta = \frac{A_y}{A} \qquad \cos\gamma = \frac{A_z}{A}$$

From these definitions it follows that

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

For every vector $\boldsymbol{A}$ there is a corresponding unit vector $\hat{\boldsymbol{u}}_A$

$$\hat{\boldsymbol{u}}_A = \cos\alpha\,\hat{\boldsymbol{\imath}} + \cos\beta\,\hat{\jmath} + \cos\gamma\,\widehat{\boldsymbol{k}}$$



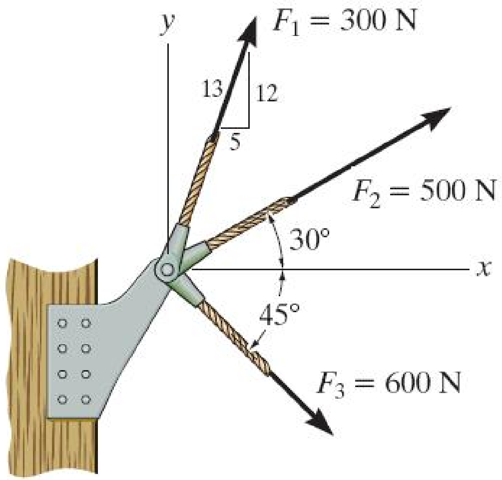
Example

A bracket is subject to multiple forces, all in the same plane

If the force on the bracket exceeds a limit, it will break

Calculate the resultant force

Example

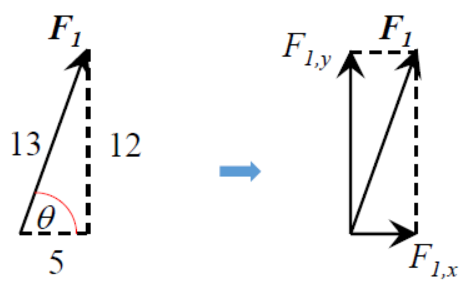


A bracket is subject to multiple forces, all in the same plane

If the force on the bracket exceeds a limit, it will break

Calculate the resultant force

First we must write ***F₁***, ***F₂***, and ***F₃*** in cartesian form



$$\cos\theta = \frac{F_{1,x}}{F1} = \frac{5}{13} \qquad F_{1,x} = \frac{5}{13} F_1$$

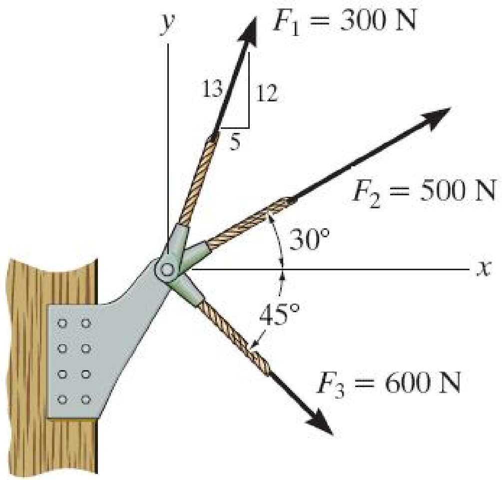
Similarly, $$F_{1,y} = \frac{12}{13} F_1$$

F₂, and ***F₃*** are a bit easier

$$F_{2,x} = F_2 \cos 30° \qquad F_{2,y} = F_2 \sin 30°$$

$$F_{3,x} = F_3 \cos 45° \qquad F_{3,y} = -F_3 \sin 45°$$

Example



We can now add all the cartesian components following the vector addition rule. This is

$$\boldsymbol{F}_R = \left(F_{1,x} + F_{2,x} + F_{3,x}\right)\hat{\boldsymbol{\imath}} + \left(F_{1,y} + F_{2,y} + F_{3,y}\right)\hat{\boldsymbol{\jmath}}$$

And substituting for the numerical values we get

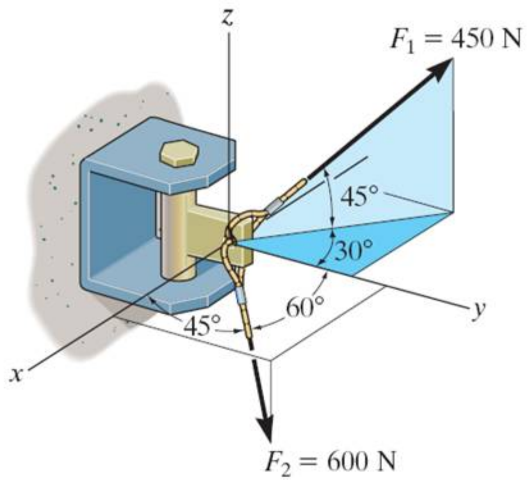
$$\boldsymbol{F}_R = \left(\frac{5}{13}F_1 + F_2\cos(30°) + F_3\cos(45°)\right)\hat{\boldsymbol{\imath}} + \left(\frac{12}{13}F_1 + F_2\sin(30°) - F_3\sin(45°)\right)\hat{\boldsymbol{\jmath}}$$

Or in resolved terms $\boldsymbol{F}_R = [972.7\ \hat{\boldsymbol{\imath}} + 102.7\ \hat{\boldsymbol{\jmath}}]N$

We can also calculate the <u>magnitude</u> of this vector $F_R = ((972.7)^2 + (102.7)^2)^{1/2} = 978.1\ N$

And finally we can determine the <u>direction</u> of the vector $\alpha = \cos^{-1}\left(\frac{972.7}{978.1}\right) = 6.03$

Example



In this case, we have two forces in different planes
As before, we must calculate the resultant

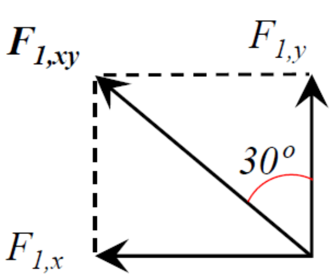
We can begin with $\boldsymbol{F_1}$ since it has an easy vertical component

$$F_{1,z} = F_1 \sin(45°)$$

The two horizontal components require a bit more work. Let us first find the projection of $\boldsymbol{F_1}$ in the *xy* plane

$$F_{1,xy} = F_1 \cos(45)$$

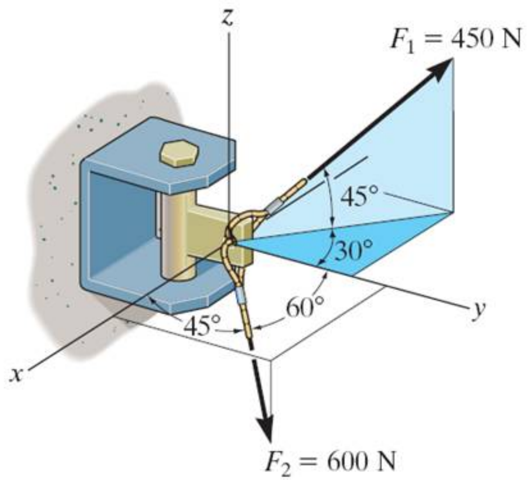
This in-plane projection can be now decomposed into *x* and *y*



$$F_{1,x} = -F_1 \cos(45°) \sin(30°)$$

$$F_{1,y} = F_1 \cos(45°) \cos(30°)$$

Example



To resolve $\boldsymbol{F_2}$ we can use the director cosines

$$\cos^2(45°) + \cos^2(60°) + \cos^2\gamma = 1 \qquad \gamma = 120\ °$$

With all the angles known, we can write the cartesian vector $\boldsymbol{F_2}$

$$\boldsymbol{F}_2 = F_2\big(\cos(45°)\ \hat{\boldsymbol{\imath}} + \cos(60°)\ \hat{\boldsymbol{\jmath}} + \cos(120°)\ \widehat{\boldsymbol{k}}\big)$$

The total force is $\boldsymbol{F_R} = \boldsymbol{F_1} + \boldsymbol{F_2}$

$$\boldsymbol{F}_R = [F_2\cos(45°) - F_1\cos(45)\sin(30°)]\hat{\boldsymbol{\imath}} + [F_2\cos(60°) + F_1\cos(45°)\cos(30°)]\hat{\boldsymbol{\jmath}} + [F_2\cos(120°) + F_1\sin(45°)\ ]\widehat{\boldsymbol{k}}$$

$$\boldsymbol{F}_R = \big[265.2\ \hat{\boldsymbol{\imath}} + 575.6\ \hat{\boldsymbol{\jmath}} + 18.2\ \widehat{\boldsymbol{k}}\big]N \qquad F_R = ((265.2)^2 + (575.6)^2 + (18.2)^2)^{1/2} = 634\ N$$

$$\alpha = \cos^{-1}\left(\frac{265.2}{634}\right) = 65.3° \qquad \beta = \cos^{-1}\left(\frac{575.6}{634}\right) = 24.8\ ° \qquad \gamma = \cos^{-1}\left(\frac{18.2}{634}\right) = 88.4°$$