ENGR 057 Statics and Dynamics

Kinetics of a particle: Force and acceleration

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Applications

The motion of an object depends on the forces acting on it.

A parachutist relies on the atmospheric drag resistance force of the parachute to limit the velocity. How can we determine the velocity of the parachutist at any point in time? This has some importance when landing!

Can we determine the horizontal force acting on the coupling between the truck and the carts? This is needed when designing the coupling (or understanding why it failed).

Newton's laws of motion

The motion of a particle is governed by **Newton's three laws of motion**.

First Law: A particle originally at rest, or moving in a straight line at constant velocity, will remain in this state if the resultant force acting on the particle is zero.

statics

Second Law: If the resultant force on the particle is not zero, the particle experiences an acceleration in the same direction as the resultant force. This acceleration has a magnitude proportional to the resultant force.

dynamics

Third Law: Mutual forces of action and reaction between two particles are equal, opposite, and collinear.

statics

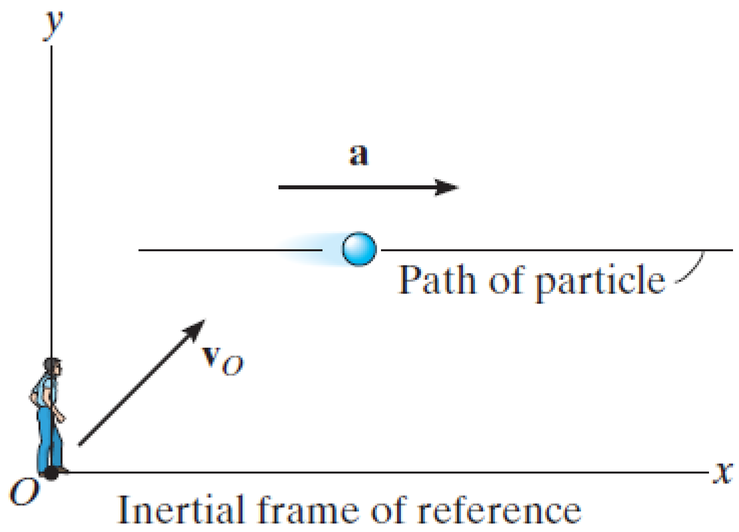
Newton's second law of motion

Mathematically, Newton's second law of motion can be written

$$\mathbf{F} = m\mathbf{a}$$

where **F** is the resultant unbalanced force acting on the particle, and ***a*** is the acceleration of the particle. The positive scalar m is the mass of the particle.

Newton's second law cannot be used when the particle's speed approaches the speed of light, or if the size of the particle is extremely small (~ size of an atom).



Newtonian or *inertial reference frame*

Newton's law of gravitational attraction

Any two particles or bodies have a mutually attractive gravitational force acting between them. Newton postulated the law governing this gravitational force as

$$\mathbf{F} = G\,\frac{m_1\,m_2}{\mathbf{r}^2}$$

where $\mathbf{F}$ = force of attraction between the two bodies,
G = universal constant of gravitation ,
m_1, m_2 = mass of each body, and
$\mathbf{r}$ = distance between centers of the two bodies.

When near the surface of the earth, the only gravitational force having any sizable magnitude is that between the earth and the body. This force is called the weight of the body defined as $W = mg$.

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Units: SI system vs. FPS system

SI system: In the SI system of units, mass is a base unit and weight is a derived unit. Typically, mass is specified in kilograms (kg), and weight is calculated from W = mg. If the gravitational acceleration (g) is specified in units of m/s2, then the weight is expressed in newtons (N). On the earth's surface, g can be taken as $g = 9.81\ m/s^2$.

$$W\ (N) = m\ (kg)\ g\ (m/s^2) \;\; => \;\; N = kg \cdot m/s^2$$

FPS System: In the FPS system of units, weight is a base unit and mass is a derived unit. Weight is typically specified in pounds (lb), and mass is calculated from $m = W/g$. If g is specified in units of ft/s2, then the mass is expressed in slugs. On the earth's surface, g is approximately $32.2\ ft/s^2$.

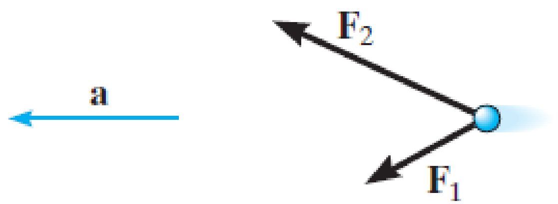
$$m\ (slugs) = W\ (lb)/g\ (ft/s^2) \;\; => \;\; slug = lb \cdot s^2/ft$$

Equation of motion

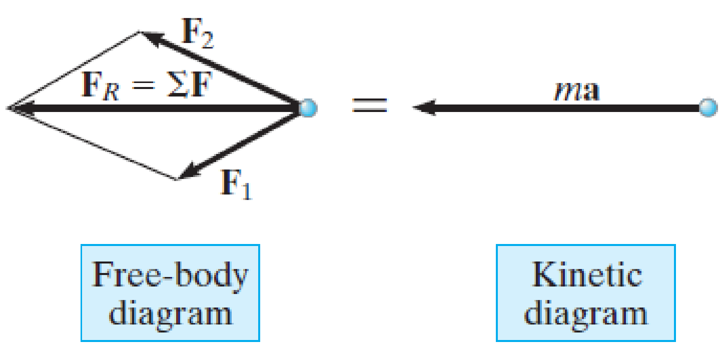
If more than one force acts on the particle, the equation of motion can be written

$$\sum \mathbf{F} = \mathbf{F}_R = m\boldsymbol{a}$$

where $\mathbf{F}_R$ is the resultant force, which is a vector summation of all the forces.



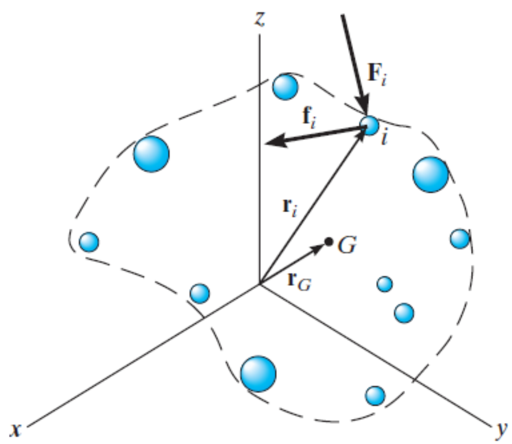
To illustrate the equation, consider a particle acted on by two forces.



1. Draw the particle's free-body diagram, showing all forces acting on the particle.
2. Draw the kinetic diagram, showing the inertial force $m\boldsymbol{a}$ acting in the same direction as the resultant force $\mathbf{F}_R$.

Equation of motion for a system of particles

The equation of motion can be extended to include systems of particles. This includes the motion of solids, liquids, or gas systems.



Inertial coordinate system

As in statics, there are internal forces and external forces acting on the system. What is the difference between them?

Using the definitions of $\mathrm{m} = \sum \mathrm{m_i}$ as the total mass of all particles and $\boldsymbol{a}_\mathrm{G}$ as the acceleration of the center of mass G of the particles, then $\mathrm{m}\,\boldsymbol{a}_\mathrm{G} = \sum \mathrm{m_i}\,\boldsymbol{a}_\mathrm{i}$.

$\sum \mathbf{F} = \mathrm{m}\,\boldsymbol{a}_\mathrm{G}$ where $\sum \mathbf{F}$ is the sum of the external forces acting on the entire system.

Summary of key points

1) Newton's second law is a "law of nature"-- experimentally proven, not the result of an analytical proof.

2) Mass (a property of an object) is a measure of the resistance to a change in velocity (*inertia)* of the object.

3) Weight (a force) depends on the local gravitational field. Calculating the weight of an object is an application of $F = ma$, i.e., $W = mg$.

4) Unbalanced forces cause the acceleration of objects. This condition is fundamental to all dynamics problems!

Procedure to solve problems

1) Select a convenient inertial coordinate system. Rectangular, normal/tangential, or cylindrical coordinates may be used.

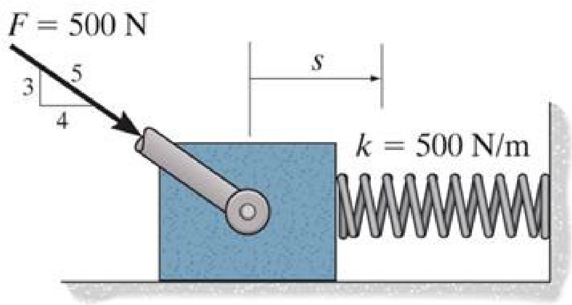
2) Draw a free-body diagram showing all external forces applied to the particle. Resolve forces into their appropriate components.

3) Draw the kinetic diagram, showing the particle's inertial force, $m\boldsymbol{a}$. Resolve this vector into its appropriate components.

4) Apply the equations of motion in their scalar component form and solve these equations for the unknowns.

5) It may be necessary to apply the proper kinematic relations to generate additional equations.

Example



A 10-kg block is subjected to the force F=500 N. A spring of stiffness k=500 N/m is mounted against the block. When s=0, the block is at rest and the spring is uncompressed. The contact surface is smooth.

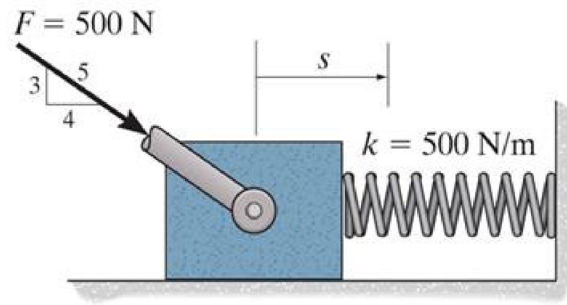
Draw the free-body and kinetic diagrams of the block.

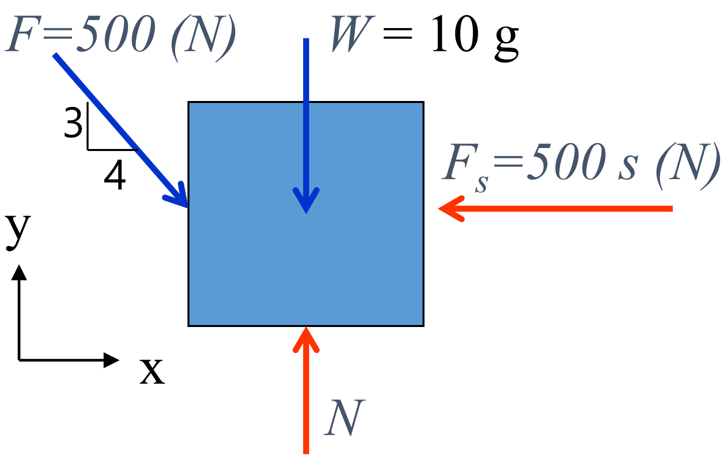
Solution

1) Define an inertial coordinate system.
2) Draw the block's free-body diagram, showing all external forces applied to the block in the proper directions.
3) Draw the block's kinetic diagram, showing the inertial force vector m**a** in the proper direction.

1) An inertial x-y frame can be defined as fixed to the ground.

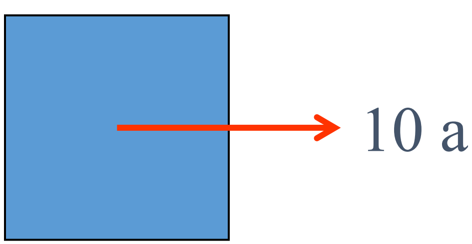
2) Draw the free-body diagram of the block:





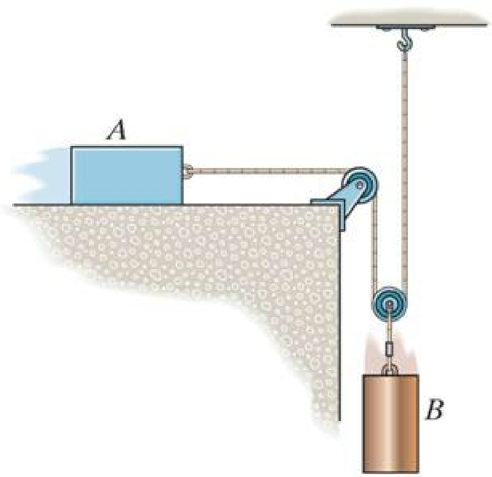
The weight force (W) acts through the block's center of mass. F is the applied load and F_s =500s (N) is the spring force, where s is the spring deformation. The normal force (N) is perpendicular to the surface. There is no friction force since the contact surface is smooth.

3) Draw the kinetic diagram of the block:



The block will be moved to the right. The acceleration can be directed to the right if the block is speeding up or to the left if it is slowing down.

Individual work (10 min)



The block and cylinder have a mass of m. The coefficient of kinetic friction at all surfaces of contact is μ. Block A is moving to the right.

Draw the free-body and kinetic diagrams of each block.

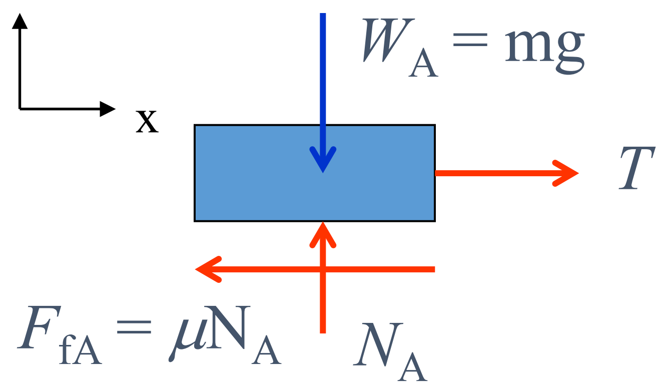
Solution

1) Define an inertial coordinate system.
2) Draw the free-body diagrams for each block, showing all external forces.
3) Draw the kinetic diagrams for each block, showing the inertial forces.

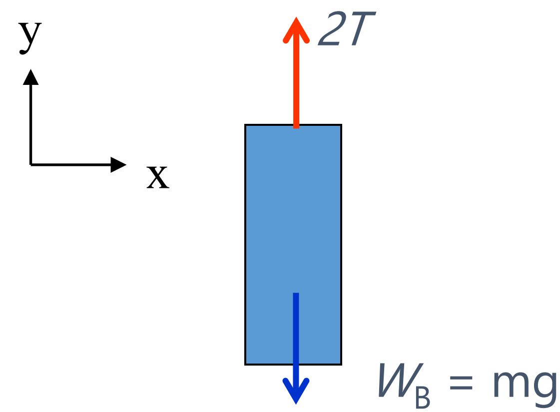
1) An inertial x-y frame can be defined as fixed to the ground.

2) Draw the free-body diagram of each block:

Block A:



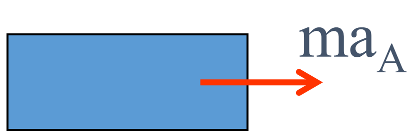
Block B:



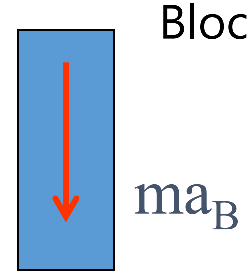
The friction force opposes the motion of block A relative to the surfaces on which it slides.

3) Draw the kinetic diagram of each block:

Block A:



Block B:



- Time for a 5 minutes stretching break!

Rectangular coordinates

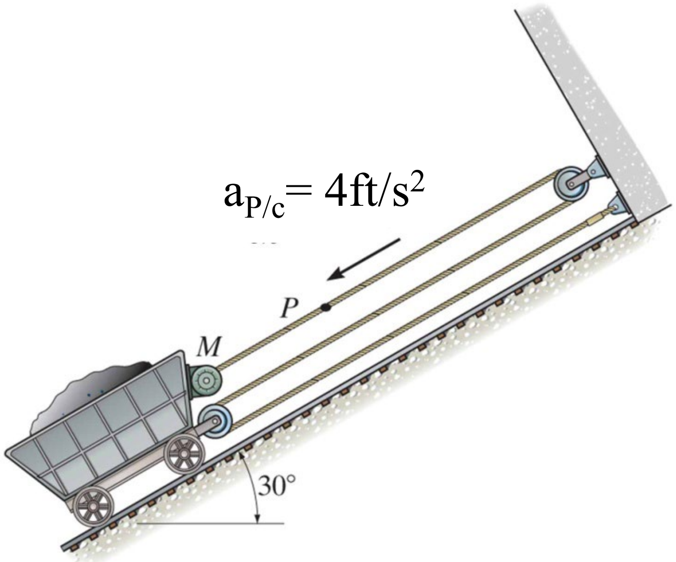
The equation of motion, $F = m\,a$, is best used when the problem requires finding forces (especially forces perpendicular to the path), accelerations, velocities, or mass. Remember, unbalanced forces cause acceleration!

Three scalar equations can be written from this vector equation. The equation of motion, being a vector equation, may be expressed in terms of its three components in the Cartesian (rectangular) coordinate system as

$$\sum \boldsymbol{F} = \mathrm{m}\boldsymbol{a} \quad \text{or} \quad \sum \mathrm{F_x}\,\boldsymbol{i} + \sum \mathrm{F_y}\,\boldsymbol{j} + \sum \mathrm{F_z}\,\boldsymbol{k} = \mathrm{m}(\mathrm{a_x}\,\boldsymbol{i} + \mathrm{a_y}\,\boldsymbol{j} + \mathrm{a_z}\,\boldsymbol{k})$$

or, as scalar equations, $\sum \mathrm{F_x} = \mathrm{ma_x}$, $\sum \mathrm{F_y} = \mathrm{ma_y}$, and $\sum \mathrm{F_z} = \mathrm{ma_z}$.

Example



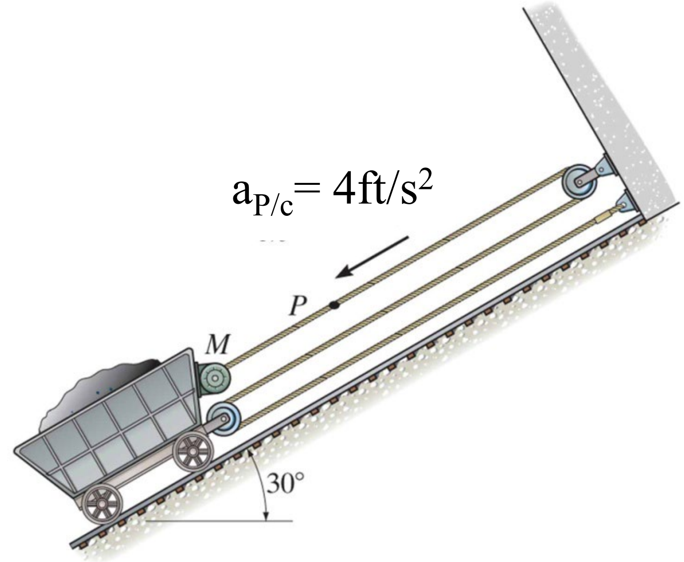
The 200 lb mine car is hoisted up the incline. The motor M pulls in the cable with an acceleration of 4 ft/s^2.
Find the acceleration of the mine car and the tension in the cable.

Solution

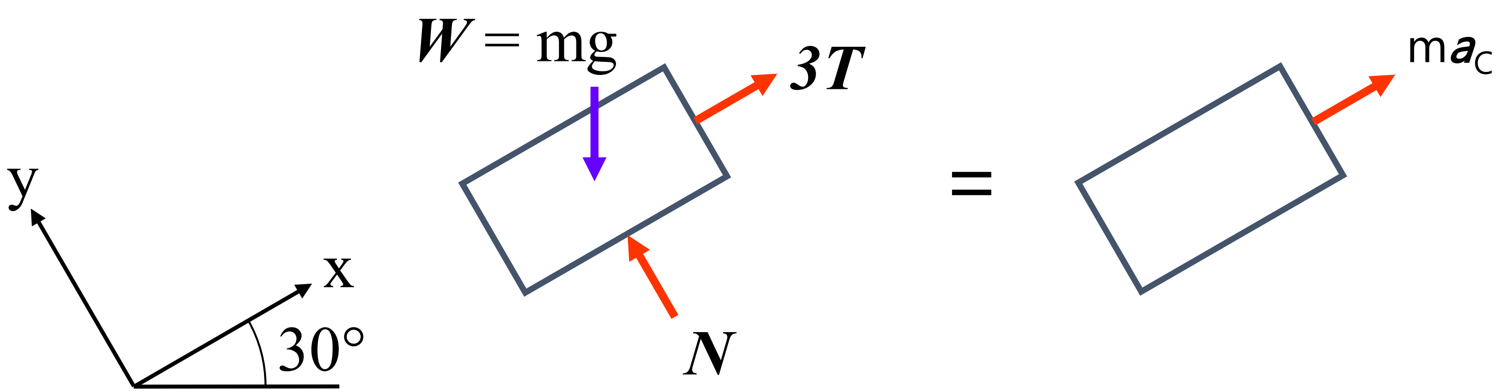
1. Draw the free-body and kinetic diagrams of the car.

2 Using a dependent motion equation, determine an acceleration relationship between cable and mine car.

3. Apply the equation of motion to determine the cable tension.



1) Draw the free-body and kinetic diagrams of the mine car:



Since the motion is up the incline, rotate the x-y axes.
Motion occurs only in the x-direction. We are also neglecting any friction in the wheel bearings, etc., on the cart.

2) The cable equation results in

$$s_p + 2\, s_c = l_t$$

Taking the derivative twice yields

$$a_p + 2\, a_c = 0 \quad \text{(eqn. 1)}$$

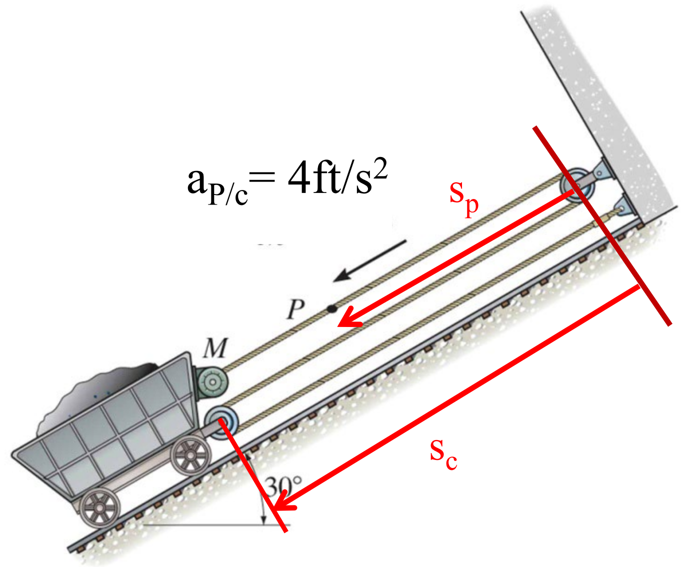
The relative acceleration equation is

$$a_p = a_c + a_{p/c}$$

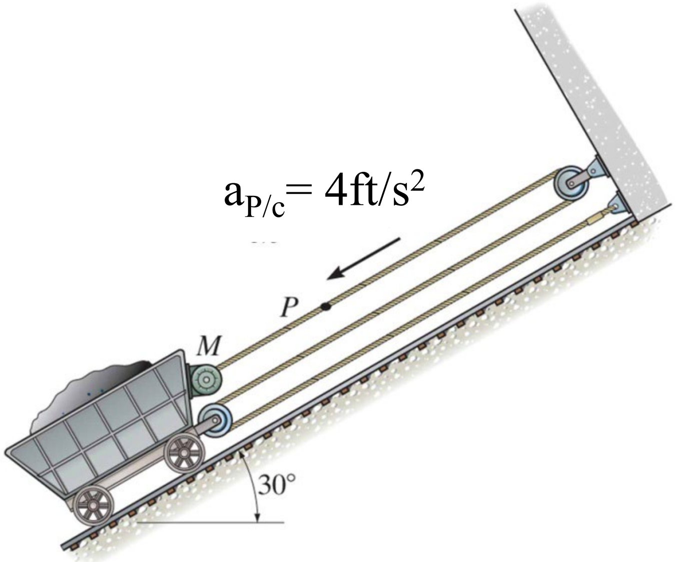
As the motor is mounted on the car,

$$a_{p/c} = -4\ \text{ft/s}^2$$

So, $a_p = a_c - 4\ \text{ft/s}^2$ (eqn. 2)



Solving equations 1 and 2, yields a_C=1.333 ft/s²



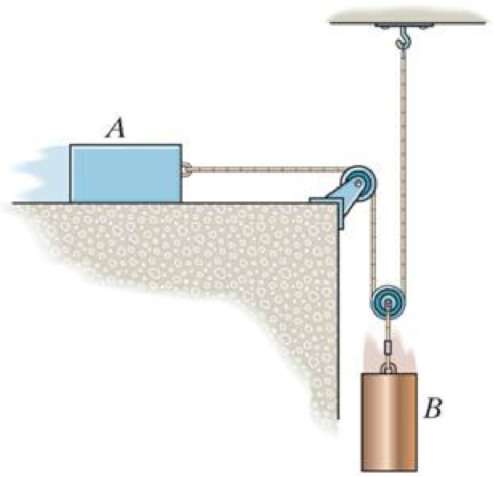
3) Apply the equation of motion in the x-direction:

$$\nearrow + \quad \sum F_x = ma_x \quad \Rightarrow \quad 3T - mg(\sin 30^\circ) = ma_x$$

$$\Rightarrow \quad 3T - (200)(\sin 30^\circ) = (200/32.2)(1.333)$$

$$\Rightarrow \quad T = 36.1 \ lb$$

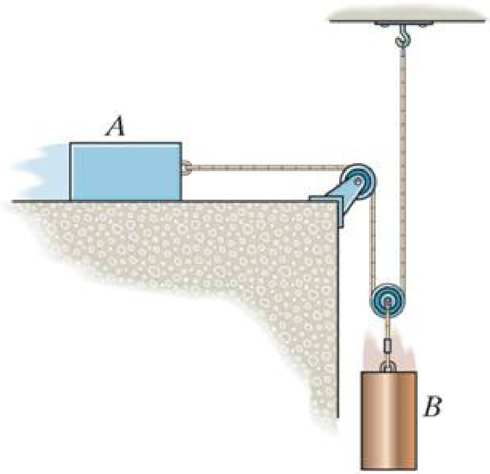
Group work breakout rooms (5 min discussion)



$$W_A = 10 \ \text{lb}$$
$$W_B = 20 \ \text{lb}$$
$$v_{oA} = 2 \ \text{ft/s}$$
$$\mu_k = 0.2$$

Find: v_A when A has moved 4 feet to the right.

Discussion: This is not an easy problem, so think carefully about how to approach it!



$$W_A = 10 \text{ lb}$$
$$W_B = 20 \text{ lb}$$
$$v_{oA} = 2 \text{ ft/s} \longrightarrow$$
$$\mu_k = 0.2$$

Find: v_A when A has moved 4 feet to the right.

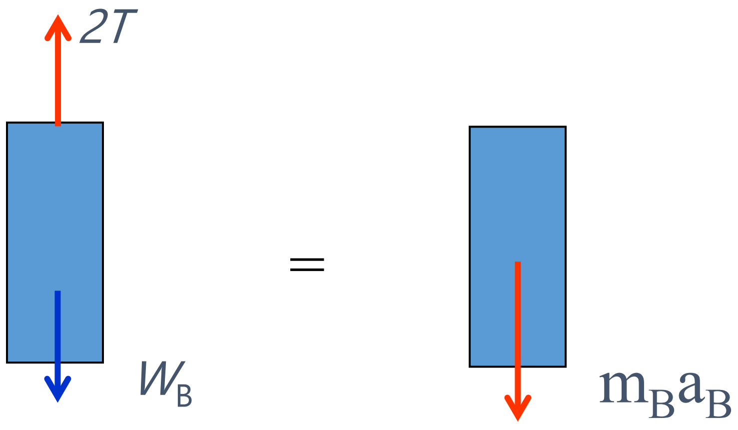
Solution

Since both forces and velocity are involved, this problem requires both the equation of motion and kinematics.

First, draw free body diagrams of A and B. Apply the equation of motion to each.

Using dependent motion equations, derive a relationship between $\mathbf{a}_A$ and $\mathbf{a}_B$ and use with the equation of motion formulas.

1-3a. Free-body and kinetic diagrams of B:



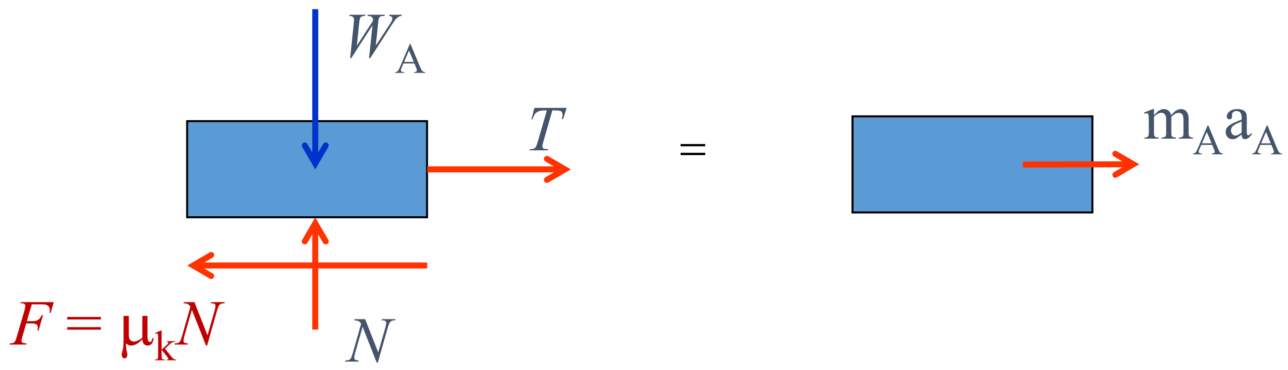
4a. Apply the equation of motion to B:

$$+\downarrow \sum F_y = m\, a_y$$

$$W_B - 2T = m_B\, a_B$$

$$20 - 2T = \frac{20}{32.2}\, a_B \qquad (1)$$

1-3b. Free-body and kinetic diagrams of A:



Friction is the force resisting the relative motion of solid surfaces, fluid layers, and material elements sliding against each other.

4b. Apply the equations of motion to A:

$$+\uparrow \sum F_y = m\, a_y = 0$$

$$N = W_A = 10 \text{ lb}$$

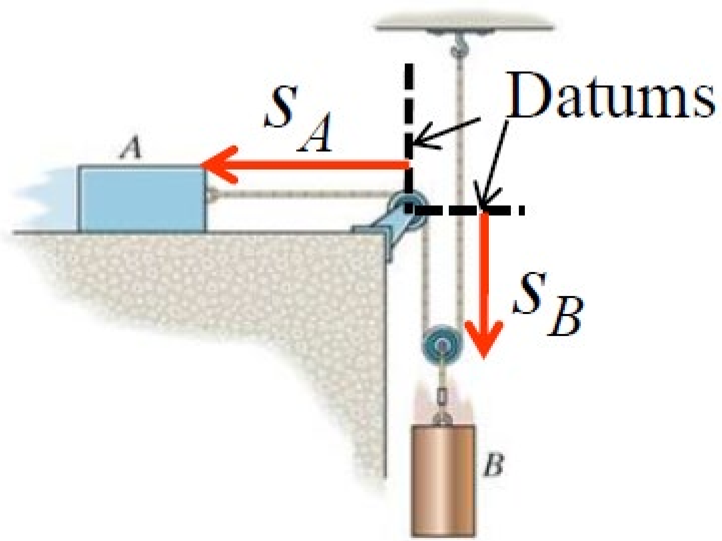
$$F = \mu_k N = 2 \text{ lb}$$

$$\overset{+}{\leftarrow} \sum F_x = m\, a_x$$

$$F - T = m_A\, a_A$$

$$2 - T = \frac{10}{32.2}\, a_A \qquad (2)$$

Now consider the kinematics.



Constraint equation:

$$s_A + 2\,s_B = \text{constant}$$

or

$$v_A + 2\,v_B = 0$$

Therefore

$$a_A + 2\,a_B = 0$$

$$a_A = -\,2\,a_B \qquad (3)$$

(Notice a_A is considered positive to the left and a_B is positive downward.)

Now combine equations (1), (2), and (3).

$$T = \frac{22}{3} = 7.33 \text{ lb}$$

$$a_A = -17.16 \text{ ft/s}^2 = 17.16 \text{ ft/s}^2 \rightarrow$$

Now use the kinematic equation:

$$(v_A)^2 = (v_{0A})^2 + 2\,a_A(s_A - s_{0A})$$

$$(v_A)^2 = (2)^2 + 2\,(17.16)(4)$$

$$v_A = 11.9 \text{ ft/s} \quad \rightarrow$$


Have
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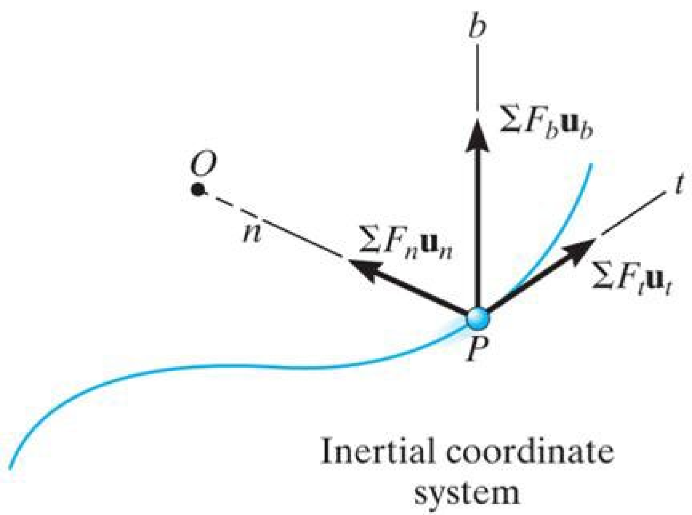
Normal and tangential coordinates applications

Race tracks are banked in the turns to reduce the frictional forces required to keep the cars from sliding up to the outer rail at high speeds.
If the car's maximum velocity and a minimum coefficient of friction between the tires and track are specified, how can we determine the minimum banking angle required to prevent the car from sliding up the track?

The hydraulically-powered arms turn at a constant rate, which creates a centrifugal force on the riders.
We need to determine the smallest angular velocity of the cars A and B so that the passengers do not loose contact with the seat. What parameters do we need for this calculation?

Normal & tangential coordinates

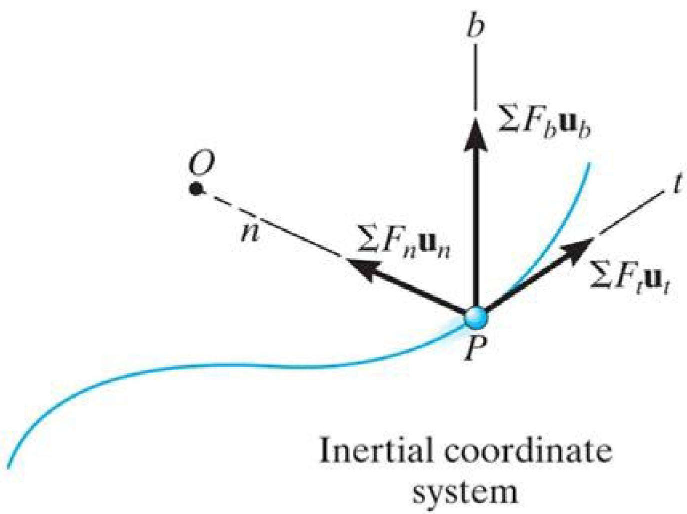


When a particle moves along a curved path, it may be more convenient to write the equation of motion in terms of normal and tangential coordinates.

The normal direction (n) *always* points toward the path's center of curvature. In a circle, the center of curvature is the center of the circle.

The tangential direction (t) is tangent to the path, usually set as positive in the direction of motion of the particle.

Equations of motion



Since the equation of motion is a vector equation , $\sum \mathbf{F} = \text{ma},$
it may be written in terms of the n and t coordinates as

$$\sum F_t u_t + \sum F_n u_n + \sum F_b u_b = ma_t + ma_n$$

Here $\sum F_t$ and $\sum F_n$ are the sums of the force components acting in the t and n directions, respectively.

This vector equation will be satisfied provided the individual components on each side of the equation are equal, resulting in the two scalar equations: $\sum F_t = ma_t$ and $\sum F_n = ma_n$.

Since there is no motion in the binormal (b) direction, we can also write $\sum F_b = 0$.

Normal and tangential accelerations

The tangential acceleration, $a_t = dv/dt$, represents the time rate of change in the magnitude of the velocity. Depending on the direction of $\sum F_t$, the particle's speed will either be increasing or decreasing.

The normal acceleration, $a_n = v^2/r$, represents the time rate of change in the direction of the velocity vector. Remember, a_n always acts toward the path's center of curvature. Thus, $\sum F_n$ will always be directed toward the center of the path.

If the path of motion is defined as $y = f(x)$, the radius of curvature at any point can be obtained from

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}$$

Solving problems with n-t coordinates

- Use n-t coordinates when a particle is moving along a known, curved path.

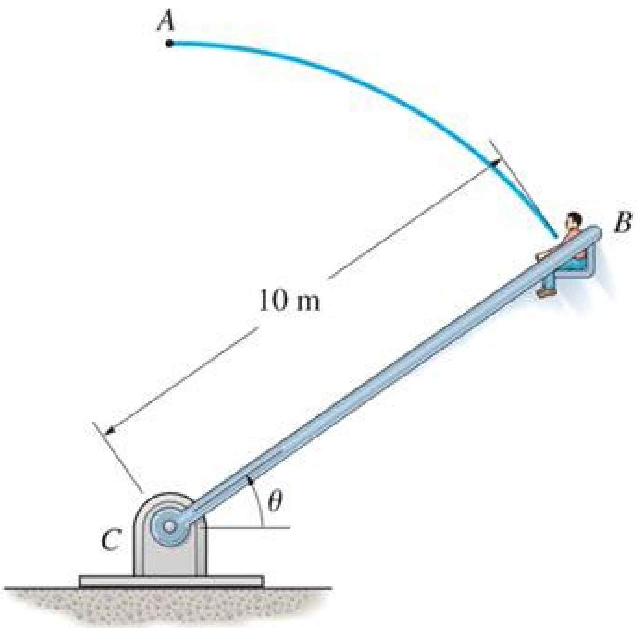
- Establish the n-t coordinate system on the particle.

- Draw free-body and kinetic diagrams of the particle. The normal acceleration (a_n) always acts "inward" (the positive n-direction). The tangential acceleration (a_t) may act in either the positive or negative t direction.

- Apply the equations of motion in scalar form and solve.

- It may be necessary to employ the kinematic relations:

$$a_t = dv/dt = v\, dv/ds \qquad a_n = v^2/\rho$$



Example

At the instant $\theta = 45°$, the boy with a mass of $75\ \mathrm{kg}$, moves a speed of $6\ \mathrm{m/s}$, which is increasing at $0.5\ \mathrm{m/s^2}$.

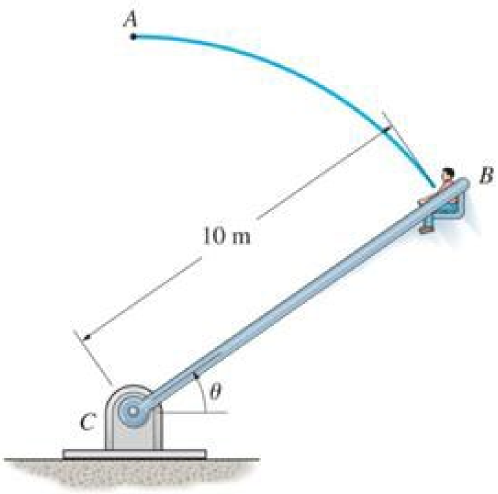
Neglect his size and the mass of the seat and cords. The seat is pin connected to the frame BC.

Find horizontal and vertical reactions of the seat on the boy.

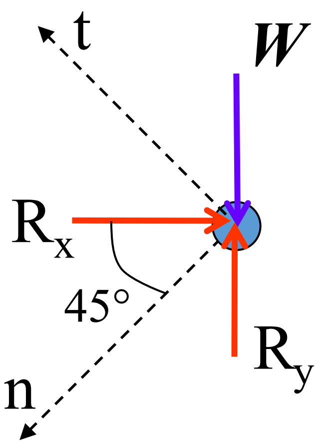
Solution:

1) Since the problem involves a curved path and requires finding the force perpendicular to the path, use n-t coordinates. Draw the boy's free-body and kinetic diagrams.
2) Apply the equation of motion in the n-t directions.

1) The n-t coordinate system can be established on the boy at angle 45°. Approximating the boy and seat together as a particle, the free-body and kinetic diagrams can be drawn.



Free-body diagram



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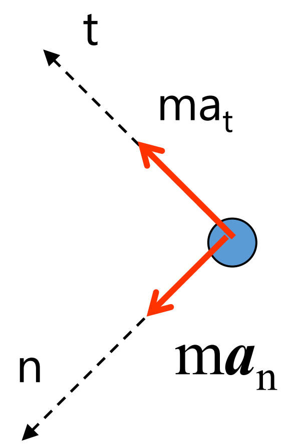
Kinetic diagram



2) Apply the equations of motion in the n-t directions.

(a) $\sum F_n = ma_n \Rightarrow -R_x \cos 45° - R_y \sin 45° + W \sin 45° = ma_n$

Using $a_n = v^2/\rho = 6^2/10$, $W = 75(9.81)$ N, and $m = 75$ kg,

we get: $-R_x \cos 45° - R_y \sin 45° + 520.3 = (75)(6^2/10)$ (1)

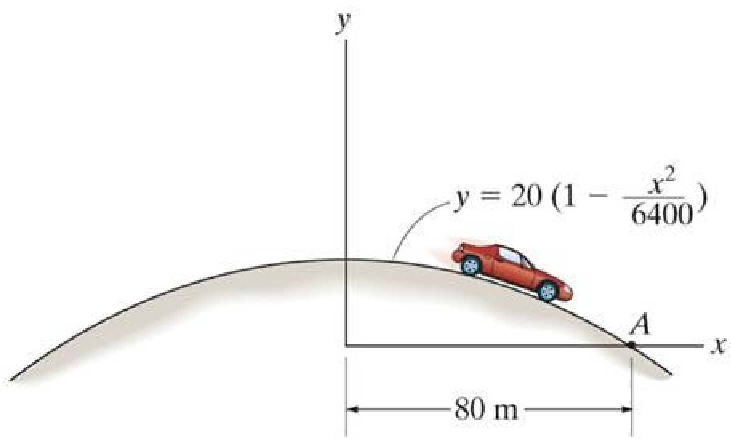
(b) $\sum F_t = ma_t \Rightarrow -R_x \sin 45° + R_y \cos 45° - W \cos 45° = ma_t$

we get: $-R_x \sin 45° + R_y \cos 45° - 520.3 = 75\ (0.5)$ (2)

Using equations (1) and (2), solve for R_x, R_y.

$R_x = -217$ N, $R_y = 572$ N

Group problem (15 min)



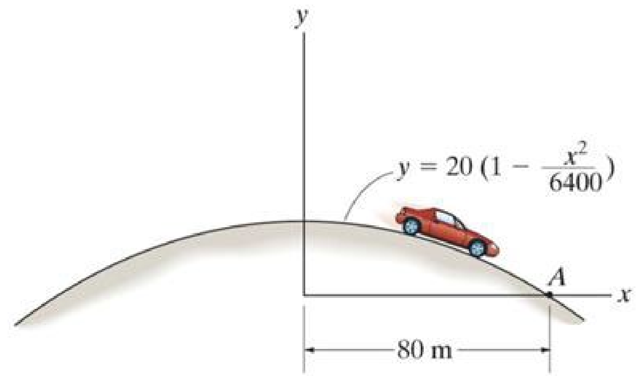
A 800 kg car is traveling over the hill having the shape of a parabola. When it is at point A, it is traveling at 9 m/s and increasing its speed at 3 m/s^2.

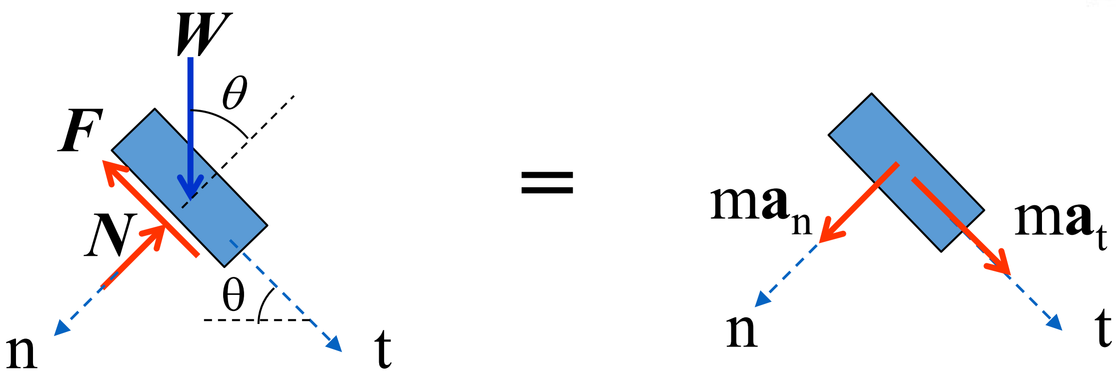
Find the resultant normal force and resultant frictional force exerted on the road at point A.

Solution

1) Treat the car as a particle. Draw the free-body and kinetic diagrams.
2) Apply the equations of motion in the n-t directions.
3) Use calculus to determine the slope and radius of curvature of the path at point A.

1) The n-t coordinate system can be established on the car at point A. Treat the car as a particle and draw the free-body and kinetic diagrams:





W = mg = weight of car

N = resultant normal force on road

F = resultant friction force on road

2) Apply the equations of motion in the n-t directions:

$\sum F_n = ma_n \;\Rightarrow\; W \cos\theta - N = ma_n$

Using $W = mg$ and $a_n = v^2/\rho = (9)^2/\rho$

$\Rightarrow\; (800)(9.81)\cos\theta - N = (800)(81/\rho)$

$\Rightarrow\; N = 7848\cos\theta - 64800/\rho \qquad (1)$

$\sum F_t = ma_t \;\Rightarrow\; W \sin\theta - F = ma_t$

Using $W = mg$ and $a_t = 3\ \text{m/s}^2$ (given)

$\Rightarrow\; (800)(9.81)\sin\theta - F = (800)(3)$

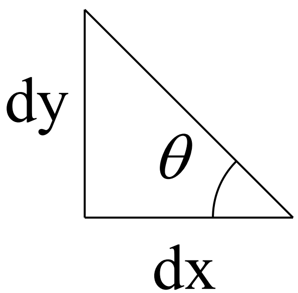
$\Rightarrow\; F = 7848\sin\theta - 2400 \qquad (2)$

3) Determine ρ by differentiating $y = f(x)$ at $x = 80$ m:

$$y = 20(1 - x^2/6400) \;\; \Rightarrow \;\; dy/dx = (-40)\, x \,/\, 6400$$
$$\Rightarrow \;\; d^2y/dx^2 = (-40) \,/\, 6400$$

$$\rho\Big|_{x = 80\text{ m}} = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{[1 + (-0.5)^2]^{3/2}}{|0.00625|} = 223.6 \text{ m}$$

Determine θ from the slope of the curve at A:



$$\tan\,\theta = dy/dx\Big|_{x = 80\text{ m}}$$

$$\theta = \left|\tan^{-1}(dy/dx)\right| = \left|\tan^{-1}(-0.5)\right| = 26.6°$$

From Eq.(1): $N = 7848 \cos \theta - 64800 / \rho$

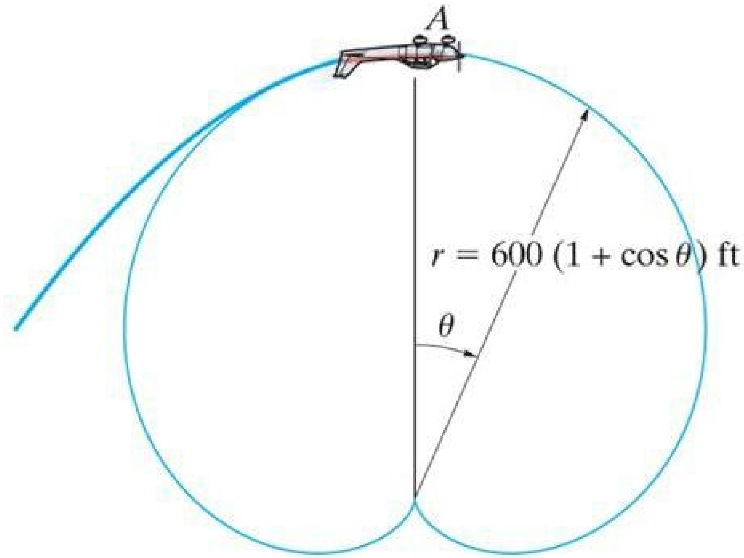
$$= 7848 \cos (26.6°) - 64800 / 223.6 = 6728 \text{ N}$$

From Eq.(2): $F = 7848 \sin \theta - 2400$

$$= 7848 \sin (26.6°) - 2400 = 1114 \text{ N}$$


Have
a
Break

Cylindrical Coordinates applications



The forces acting on the 100-lb boy can be analyzed using the cylindrical coordinate system.

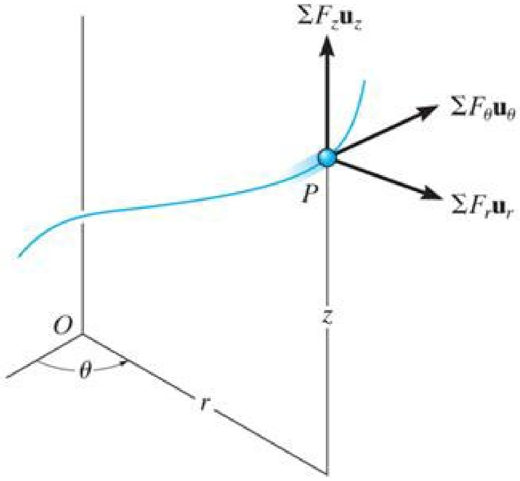
How would you write the equation describing the frictional force on the boy as he slides down this helical slide?

When an airplane executes the vertical loop shown above, the centrifugal force causes the normal force (apparent weight) on the pilot to be smaller than her actual weight.

How would you calculate the velocity necessary for the pilot to experience weightlessness at A?

Cylindrical coordinates

This approach to solving problems has some external similarity to the normal & tangential method just studied. However, the path may be more complex or the problem may have other attributes that make it desirable to use cylindrical coordinates.



Inertial coordinate system

Equilibrium equations or "Equations of Motion" in cylindrical coordinates (using r, θ, and z coordinates) may be expressed in scalar form as:

$$\sum F_r = ma_r = m(\ddot{r} - r\dot{\theta}^2)$$

$$\sum F_\theta = ma_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$$

Angular velocity

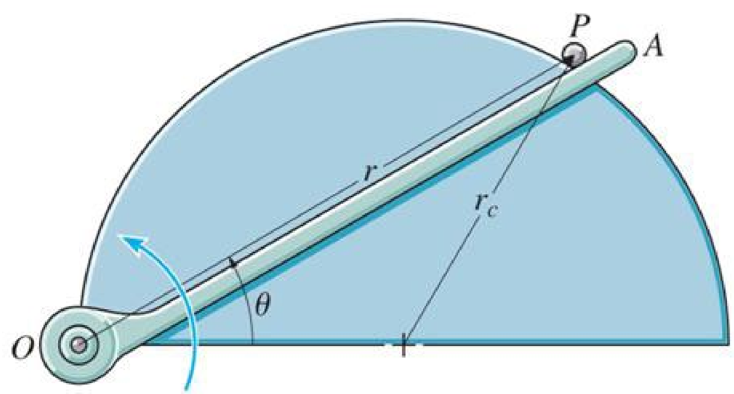
$$\sum F_z = ma_z = m\ddot{z}$$

If the particle is constrained to move only in the $r - \theta$ plane (i.e., the z coordinate is constant), then only the first two equations are used (as shown below). The coordinate system in such a case becomes a polar coordinate system. In this case, the path is only a function of θ.

$$\sum F_r = ma_r = m(\ddot{r} - r\dot{\theta}^2)$$
$$\sum F_\theta = ma_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$$

Note that a fixed coordinate system is used, not a “body-centered” system as used in the n – t approach.

Example



The ball (P) is guided along the vertical circular path.

$$W = 0.5 \text{ lb}, \ \dot{\theta} = 0.4 \text{ rad/s},$$
$$\ddot{\theta} = 0.8 \text{ rad/s}^2, \ r_c = 0.4 \text{ ft}$$

Find the force normal to the path and Force of the arm OA on the ball when $\theta = 30°$.

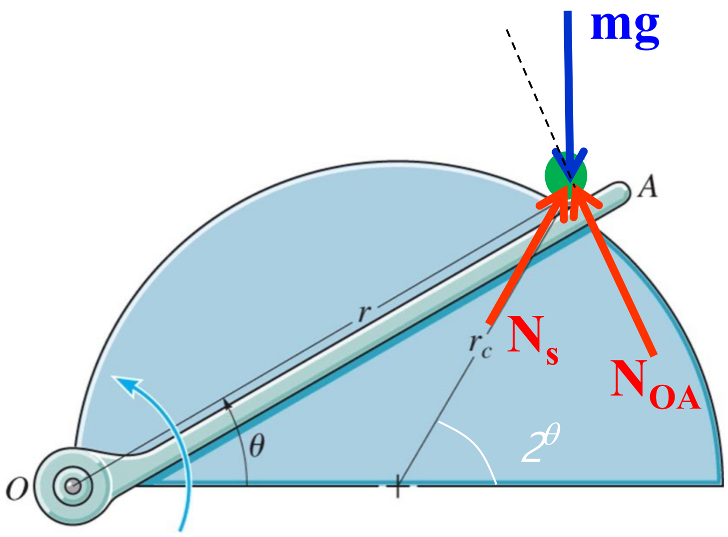
Solution

Draw a FBD. Then develop the kinematic equations and finally solve the kinetics problem using cylindrical coordinates.

Notice that $r = 2r_c \cos \theta$, therefore:

$$\dot{r} = -2r_c \sin \theta \, \dot{\theta}$$
$$\ddot{r} = -2r_c \cos \theta \, \dot{\theta}^2 - 2r_c \sin \theta \, \ddot{\theta}$$

Free Body Diagram and Kinetic Diagram : Establish the r, θ inertial coordinate system and draw the particle's free body diagram.



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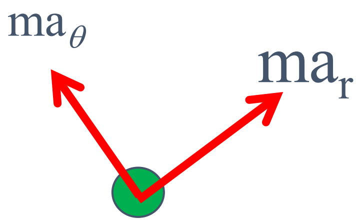


Kinematics: at $\theta = 30°$

$$r = 2(0.4)\cos(30°) = 0.693 \text{ ft}$$

$$\dot{r} = -\,2(0.4)\sin(30°)(0.4) = -\,0.16 \text{ ft/s}$$

$$\ddot{r} = -\,2(0.4)\cos(30°)(0.4)^2 - 2(0.4)\sin(30°)(0.8) = -\,0.431 \text{ ft/s}^2$$

Acceleration components are

$$a_r = \ddot{r} - r\dot{\theta}^2 = -\,0.431 - (0.693)(0.4)^2 = -\,0.542 \text{ ft/s}^2$$

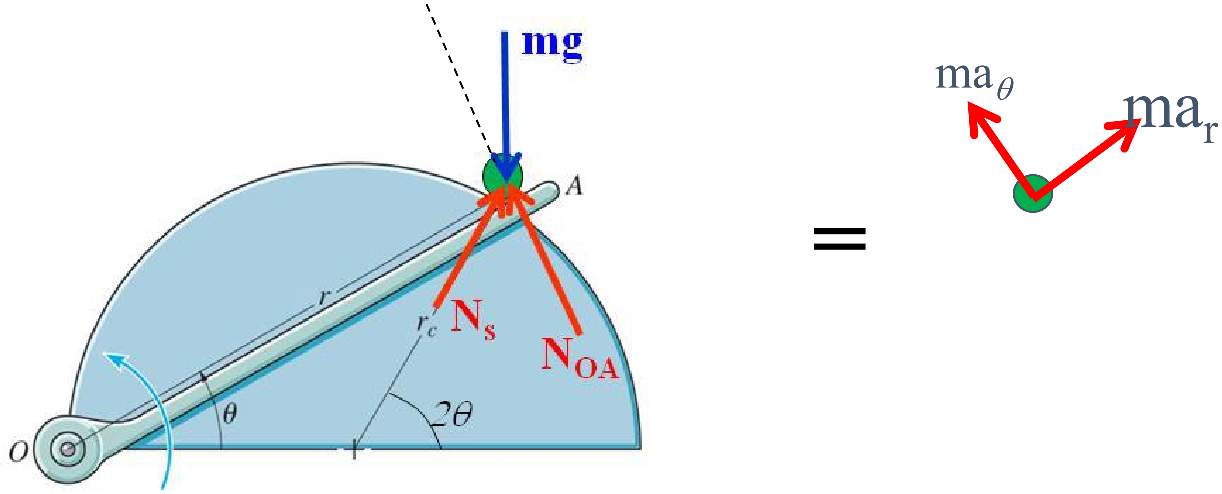
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = (0.693)(0.8) + 2(-0.16)(0.4) = 0.426 \text{ ft/s}^2$$

Equation of motion: r direction

$$\sum F_r = ma_r$$

$$N_s \cos(30°) - 0.5 \sin(30°) = \frac{0.5}{32.2}(-0.542)$$

$$N_s = 0.279 \text{ lb}$$

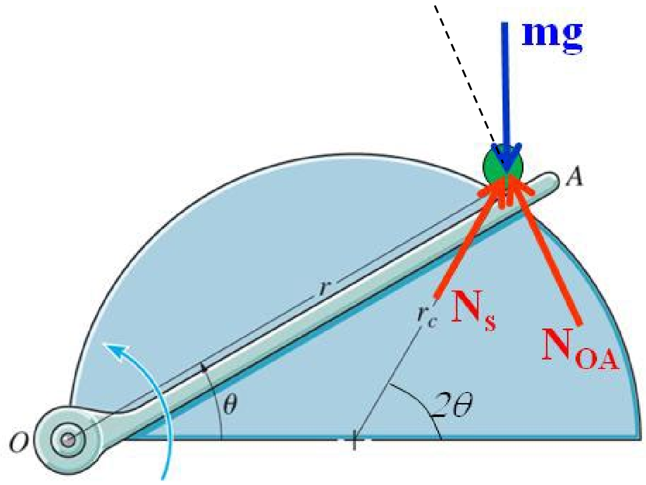


Equation of motion: θ direction

$$\sum F_\theta = ma_\theta$$

$$N_{OA} + 0.279\sin(30°) - 0.5\cos(30°) = \frac{0.5}{32.2}(0.426)$$

$$N_{OA} = 0.3 \text{ lb}$$



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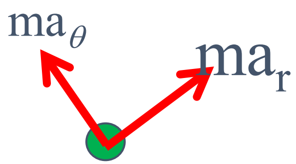
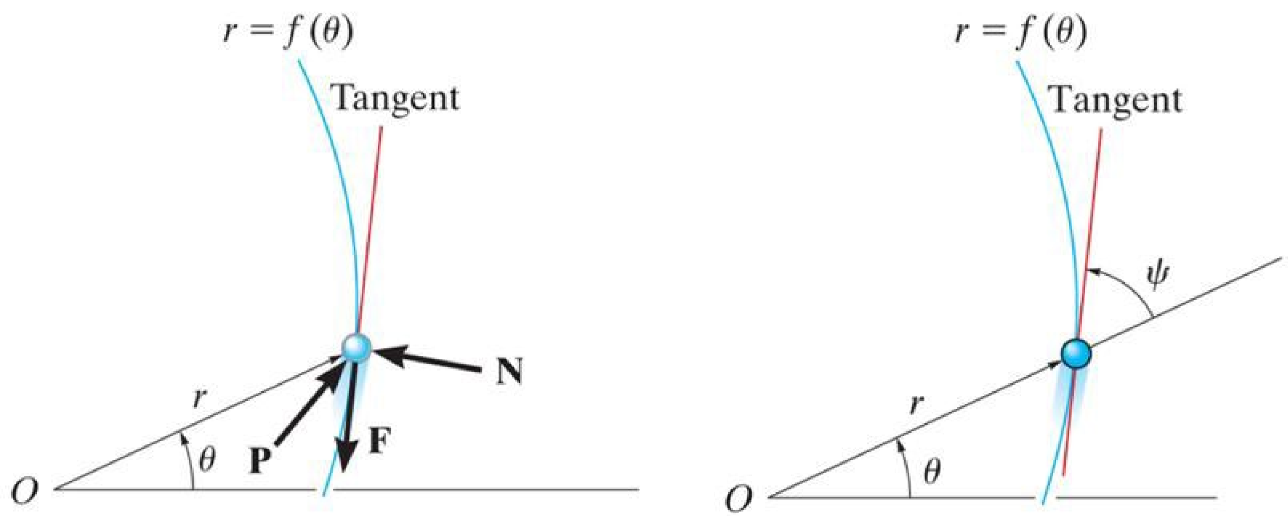


Tangential and normal forces

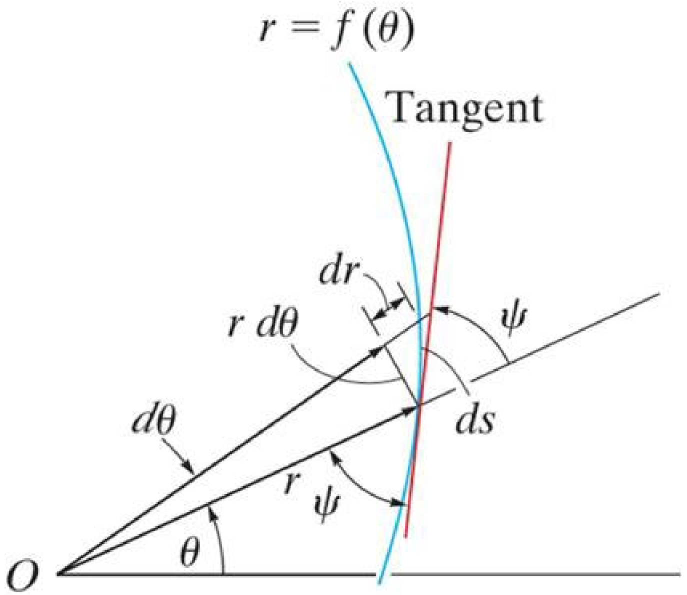
If a force $\boldsymbol{P}$ causes the particle to move along a path defined by $r = f(\theta)$, the normal force $\boldsymbol{N}$ exerted by the path on the particle is always perpendicular to the path's tangent. The frictional force $\boldsymbol{F}$ always acts along the tangent in the opposite direction of motion. The directions of $\boldsymbol{N}$ and $\boldsymbol{F}$ can be specified relative to the radial coordinate by using angle ψ.



Determination of angle ψ

The angle ψ, defined as the angle between the extended radial line and the tangent to the curve, can be required to solve some problems. It can be determined from the following relationship.

$$\tan \psi = \frac{r\,d\theta}{dr} = \frac{r}{dr/d\theta}$$



If ψ is positive, it is measured counterclockwise from the radial line to the tangent. If it is negative, it is measured clockwise.