CSE 185 | Chap. 2

Linear vs. non-linear operators

1) Is image average linear? $H[f(x,y)] = \text{average}(f(x,y)) = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)$

Two images $f1(x,y)$ $f2(x,y)$ $x = 0, \dots, M-1$, $y = 0, \dots, N-1$

$H[f1(x,y)] = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f1(x,y)$ $\quad H[f2(x,y)] = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f2(x,y)$

Consider $a1f1(x,y) + a2f2(x,y)$ → need to show $H[a1f1(x,y)+a2f2(x,y)] = a1H[f1(x,y)]+a2H[f2(x,y)]$

$$H[a1f1(x,y) + a2f2(x,y)] = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} a1f1(x,y) + a2f2(x,y)$$

$$= \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} a1f1(x,y) + \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} a2f2(x,y)$$

$$= a1\left(\frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f1(x,y)\right) + a2\left(\frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f2(x,y)\right)$$

$$= a1H[f1(x,y)] + a2H[f2(x,y)]$$

Yes, it is linear.

2) Is image max linear? $H[f(x,y)] = \max(f(x,y))$

To disprove need only show ∃ $a1, f1, a2, f2$ such that $H[a1f1(x,y)+a2f2(x,y)] \neq a1H[f1(x,y)]+a2H[f2(x,y)]$

pick $a1 = 1$, $a2 = -1$ $\quad f1 = \begin{bmatrix} 0 & 2 \\ 2 & 3 \end{bmatrix}$ and $f2 = \begin{bmatrix} 6 & 5 \\ 4 & 7 \end{bmatrix}$

$H[f1] = 3$ $\quad H[f2] = 7$

Consider $a1f1 + a2f2$ where $a1 = 1$, $a2 = -1$

$$H[a1f1 + a2f2] = H\begin{bmatrix} -6 & -3 \\ -2 & -4 \end{bmatrix} = -2 \neq a1H[f1] + a2H[f2] = -4$$

It is not linear.

Only need to show one counter example to prove not linear.