Homework Assignment #3

Remember, this Homework Assignment is **not collected or graded**! But you are advised to do it anyway because the problems for Homework Quiz #3 will be heavily based on these problems!

1. Recall that an $n \times n$ matrix A is **symmetric** if $A^T = A$.

   Let R be an $m \times n$ matrix, prove that RR^T and $R^T R$ are both symmetric matrices.

2. We learned that $n \times n$ matrices A can be factored into LDU where L is a lower triangular matrix, D is a diagonal matrix and U is an upper triangular matrix.

   But if A is symmetric it turns out that this the same LDU factorization process produces an upper triangular matrix U that is actually the transpose of the lower triangular matrix L. As such, we write the factorization as:
$$A = LDL^T.$$

   Using elementary row matrices, determine the symmetric LDL^T factorization of:
$$A = \begin{bmatrix} 1 & 3 & 5 \\ 3 & 12 & 18 \\ 5 & 18 & 30 \end{bmatrix}.$$

3. Invert these matrices A by the Gauss-Jordan method:

   (a) $A_1 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

   (b) $A_2 = \begin{bmatrix} 2 & -1 & 0 \\ -1 & -2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$

   (c) $A_3 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$.

4. The matrix:
$$A = \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$$
   has a special property. When you multiply A by itself you get the matrix of all 0's: $A^2 = 0$.

   Is it possible for a non-zero **symmetric** matrix B to have the second property $B^2 = 0$? Prove your answer. (Hint: You might need to remember what $i = \sqrt{-1}$ is.)

5. A matrix A is said to be **skew symmetric** if $A^T = -A$. Show that if a matrix A is skew-symmetric then all it's diagonal entries must be 0.