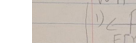
WH 11 Y

1) 

$P(|Y-E[Y]| \geq a) \leq \frac{1}{a^2} Var(Y)$

$1-P(|Y-E[Y]| \geq a) \geq 1 - \frac{1}{a^2} Var(Y)$

$P(|Y-E[Y]| \leq a) \geq 1 - \frac{1}{a^2} Var(Y)$

$X \sim U(a,b)$

$E[X] = \frac{a+b}{2}$ $\quad Var(X) = \frac{(b-a)^2}{12}$

$E[D] = \frac{5+21}{2} = 13$ $\quad Var[D] = \frac{(21-5)^2}{12}$

$= \frac{16^2}{12} = \frac{64}{3}$

2) $\mu = \frac{5+5}{2} = 4$

$\sigma = \sqrt{\frac{(5-3)^2}{12}} = .5774$

D - diameter of saliva particles

$D \sim U(5,21)$

$P[|\bar{x}-\mu| < \sigma \cdot a] \geq 1 - \frac{1}{a^2}$

$1 - \frac{1}{a^2} \geq .90$

$a = 3.1623$

$\sigma \cdot a = .02 \cdot \mu$

$\frac{\sigma}{\sqrt{n}} \cdot 3.1623 = .02 \cdot 4$

$\sqrt{n} = \frac{.5774 \cdot 3.1623}{4 \cdot .02}$

$n \geq 521$ doctors

$P(|\bar{D}_n - E[\bar{D}_n]| \leq 3.2) \geq 1 - \frac{1}{(3.2)^2} Var(\bar{D}_n) \geq .99$

$\geq 1 - \frac{1}{(3.2)^2} \frac{.0833}{n} \geq .99$

$1 - \frac{.0833}{(3.2)^2 n} \geq .99$

$-\frac{.0833}{(3.2)^2 n} \geq -.01$

$n \geq 209$

3) a) $X_i \sim Ber(p)$ $\quad E(X_i) = p$; $V(X_i) = p(1-p)$

$E(\bar{X}_n) = p$; $V(\bar{X}_n) = \frac{p(1-p)}{n}$

$P(|\bar{X}_n - p| \geq .2) \leq \frac{p(1-p)/n}{(.2)^2} = \frac{.25p(1-p)}{n}$

$p = 1/2 \Rightarrow \frac{6.25}{n} \geq .1$ $\quad n \geq 63$

$n = 62.5$

b) $P(|\bar{x}_n - p| \geq a) \leq \frac{p(1-p)/n}{a^2} = \frac{1/2(1-1/2)}{n(.1)^2} \leq \frac{1}{4na^2}$

$\frac{1}{4na^2} \geq .1 \Rightarrow \frac{1}{4 \cdot n \cdot .1^2} \geq .1 \Rightarrow n \geq 250$

c) $\frac{6.25}{n} \geq .05 \Rightarrow n \geq 125$

d) $\frac{.6(1-.6)}{n(.1)^2} \geq .1 \Rightarrow n \geq 240$

4) $E(x) = \int_0^1 x(3(1-x)^2)\,dx$

$= \int_0^1 x(3(1-x-x+x^2))\,dx$

$= \int_0^1 3x(1-2x+x^2))\,dx$

$= \int_0^1 3x - 6x^2 + 3x^3\,dx$

$= \frac{3x^2}{2} - \frac{6x^3}{3} + \frac{3x^4}{4}\Big]_0^1 = .25$

$E(x^2) = \int_0^1 x^2(3(1-x)^2)\,dx$

$= \int_0^1 3x^2(1-2x+x^2)\,dx$

$= \int_0^1 3x^2 - 6x^3 + 3x^4\,dx$

$= \frac{3x^3}{3} - \frac{6x^4}{4} + \frac{3x^5}{5}\Big]_0^1 = .1$

$\sigma = \sqrt{E(x^2) - E(x)^2} = \sqrt{.1 - (.25)^2} = .1936$

$\mu = 625 \cdot .25 = 156.25$

Standard error $= \frac{\sigma}{\sqrt{n}} = \frac{.1936}{\sqrt{625}} = 4.84$

$\therefore z = \frac{x-\mu}{\sigma} = \frac{170 - 156.25}{4.84} = 2.84$

$P(z < 2.84) = .9977$ from z table

$P(X < 170)$

5) $\mu = 1$ $\quad \sigma = 10$

a) $E(x) = 32$ $\quad \sigma = \sqrt{32} \times \frac{10}{60} = .9428$

$P(x > 34) = P\left(z > \frac{34-32}{.9428}\right) = P(z > 2.1213)$

$= .0169$

z-table

b) $\mu = 60$ min $\quad \sigma = 10$ $\quad n = 28$

$P(\bar{x} < 57) = P\left(z < \frac{57-60}{10/\sqrt{28}}\right) = P(z < -1.5875)$

$= .0562$