

**Probability and Statistics—Exam 1**  
**Thursday, September 30, 2021**

Full Name:	Section 02D Mon., 1130 AM - 120 PM TA: Julio	Section 03D Mon., 130 PM - 320 PM TA: Li
Student ID Number:	Section 04D Mon., 330 PM - 520 PM TA: Li	Section 05D Wed., 1130 AM - 120 PM TA: Julio

- Write your full name and discussion section number on every page of this packet.
- **Show all work!** ... unless otherwise instructed. Partial credit can only be awarded for presented work. Full credit can only be awarded with presented work.
- You may use any calculator that does not have internet access (i.e. no smart phones, laptops, or tablets). Round approximate results to 4 decimal places.
- Box your final answers.
- Uniformly distributed, each question is worth 10 points.
- You may use the back of this exam as scratch paper/additional space.
- Pages of formulas have been provided.

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1. Curators at Katmai National Park are currently running a tournament<sup>1</sup> asking website visitors to vote on their favorite bear. For the sample of tag numbers below, compute the median and the coefficient of variation  $\frac{s}{\bar{x}}$

435, 128, 634, 151, 812, 131, 402, 507

**Solution:** (of course, the student should still show work)

$$\text{sample mean: } \bar{x} = 400$$

$$\text{sample median} = 418.5$$

$$\text{sample standard deviation: } s \approx 252.2822$$

$$\text{and the coefficient of variation is } \frac{s}{\bar{x}} \approx 0.6307$$

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<sup>1</sup>Source: <https://explore.org/meet-the-bears>

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2. The table below shows the counts for a sample of Pokemon types from generations 5 and 6. From this data, do the requested computations below.

Generation	Type					
		Bug	Fire	Grass	Psychic	Water
	Five	18	9	15	14	18
	Six	3	8	5	5	5

- (a) What is the probability of randomly selecting a fire-type Pokemon or a generation-five Pokemon?

**Solution:** There are 100 Pokemon in this sample. Let  $F$  be the event of selecting a fire-type Pokemon. Let  $G$  be the event of selecting a generation-five Pokemon. Then, by the inclusion-exclusion principle:

$$\begin{aligned}
 P(F \text{ or } G) &= P(F) + P(G) - P(F \text{ and } G) \\
 &= \frac{17}{100} + \frac{74}{100} - \frac{9}{100} \\
 &= \frac{82}{100}
 \end{aligned}$$

- (b) What is the probability of randomly selecting a grass-type Pokemon given that the Pokemon was introduced in generation six?

**Solution:** There are 100 Pokemon in this sample. Let  $R$  be the event of selecting a grass-type Pokemon. Let  $S$  be the event of selecting a generation-five Pokemon. Then, by conditional probability:

$$\begin{aligned}
 P(R|S) &= \frac{P(S \text{ and } R)}{P(S)} \\
 &= \frac{\frac{5}{100}}{\frac{18}{100}} \\
 &= \frac{5}{18}
 \end{aligned}$$

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3. Suppose that in the AOA building, we observe the presence of nine people at work. Overall on campus, 85 percent of people wearing masks while working.

(a) What is the probability that exactly 3 people in AOA are wearing a mask.

**Solution:** With a sample size of  $n = 9$  and proportion 0.85, the probability of exactly 3 mask wearers is

$$P(k = 3) = \binom{9}{3}(0.85)^3(0.15)^6 \approx 0.0006$$

(b) What is the probability that at least two people in AOA are not currently wearing a mask?

**Solution:** With a sample size of  $n = 9$  and proportion 0.15, the probability of at least 2 maskless persons is

$$\begin{aligned} P(k \geq 2) &= 1 - [P(k = 0) + P(k = 1)] \\ &= 1 - \left[ \binom{9}{0}(0.15)^0(0.85)^9 + \binom{9}{1}(0.15)^1(0.85)^8 \right] \\ &\approx 0.4005 \end{aligned}$$

(c) Compute the range-rule-of-thumb interval  $(\mu - 2\sigma, \mu + 2\sigma)$  for the number of people that are wearing a mask.

**Solution:** With a sample size of  $n = 9$  and proportion 0.85, the sample statistics include

$$\mu = np = (9)(0.85) = 7.65, \quad \sigma = \sqrt{np(1-p)} \approx 1.0712$$

and the range-rule-of-thumb interval is

$$(5.5076, 9.7924)$$

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4. A glitch in the Microsoft Outlook e-mail client affected some of the faculty, and I will refer to one of my colleagues in particular. Let  $A$  be the event of an archived e-mail message. Let  $U$  be the event of an unread e-mail message. The professor has read 77 percent of their e-mails. The probability that a message was archived given that the message was read is 84 percent.

- (a) Write a complete sentence to describe the meaning of the math expression:  $P(U^c|A^c)$

**Solution:**  $P(U^c|A^c)$  is the probability of selecting an already ready e-mail message given that the e-mail was not archived.

- (b) What should the sensitivity  $P(A|U)$  and the specificity  $P(A^c|U^c)$  be so that the probability of randomly selecting an unread message among the archived messages is over 95 percent? Assume that the sensitivity and specificity have the same value.

**Solution:** Let  $x = P(A|U)$  and  $x = P(A^c|U^c) = 1 - P(A|U^c)$

$$0.95 \leq P(U|A)$$

$$0.95 \leq \frac{P(A|U) \cdot P(U)}{P(A|U) \cdot P(U) + P(A|U^c) \cdot P(U^c)}$$

$$0.95 \leq \frac{0.23x}{0.23x + (1-x)(0.77)}$$

$$0.2185x + 0.7315(1-x) \leq 0.23x$$

$$0.7315 \leq 0.743x$$

$$0.9845 \leq x$$