ENGR 057 Statics and Dynamics

Free body diagram

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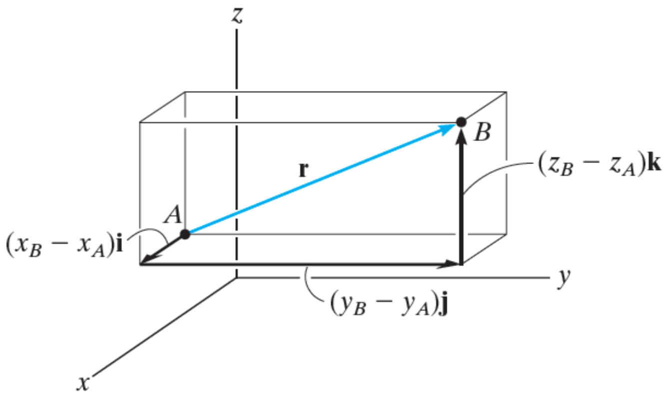
MERCED

Applications

Position vector

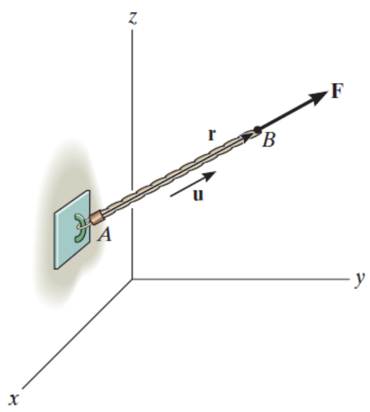


- A position vector gives the location of a point in cartesian space with respect to a certain <u>reference point</u>
- There is <u>always</u> at least one reference point: the origin
- Position vectors are essential to describe the mechanics of a problem

$$r_{AB} = (x_B - x_A)\hat{\mathbf{i}} + (y_B - y_A)\hat{\mathbf{j}} + (z_B - z_A)\hat{\mathbf{k}}$$

ALWAYS subtract the "tail" coordinates from the "tip" coordinates!

Note that $\mathbf{r}_{AB}$ has the same magnitude of $\mathbf{r}_{BA}$ but <u>opposite</u> direction

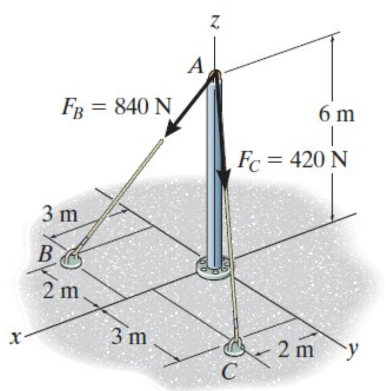


Force vector along a line

a) Find the position vector, $\mathbf{r}_{\mathbf{AB}}$, along two points on that line.

b) Find the unit vector describing the line's direction, $\mathbf{u}_{\mathbf{AB}}$ = ($\mathbf{r}_{AB}$/r_{AB}).

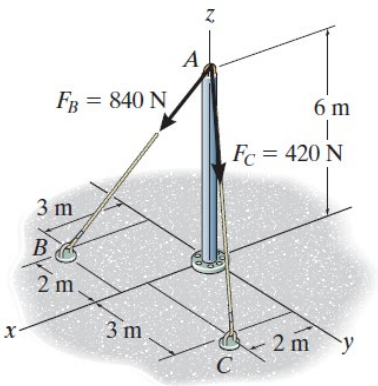
c) Multiply the unit vector by the magnitude of the force, $\mathbf{F}$ = F $\mathbf{u}_{\mathbf{AB}}$.



Example

Determine the magnitude of the resultant force at *A*.

Given: F_B = 840 N, F_C = 420 N, coordinates
Asked: F_A



Example

Determine the magnitude of the resultant force at *A*.

$$\mathbf{r_{AC}} = \{2\hat{\imath} + 3\hat{\jmath} - 6\hat{k}\}\ \mathrm{m}$$

$$r_{AC} = \sqrt{2^2 + 3^2 + (-6)^2} = 7\ \mathrm{m}$$

$$\mathbf{u_{AC}} = \frac{2}{7}\hat{\imath} + \frac{3}{7}\hat{\jmath} - \frac{6}{7}\hat{k}$$

$$\mathbf{F_C} = F_C\mathbf{u_{AC}} = (420\ \mathrm{N})\left(\frac{2}{7}\hat{\imath} + \frac{3}{7}\hat{\jmath} - \frac{6}{7}\hat{k}\right)$$

$$\mathbf{F_C} = \{120\hat{\imath} + 180\hat{\jmath} - 360\hat{k}\}\ \mathrm{N}$$

$$\mathbf{r_{AB}} = \{3\hat{\imath} - 2\hat{\jmath} - 6\hat{k}\}\ \mathrm{m}$$

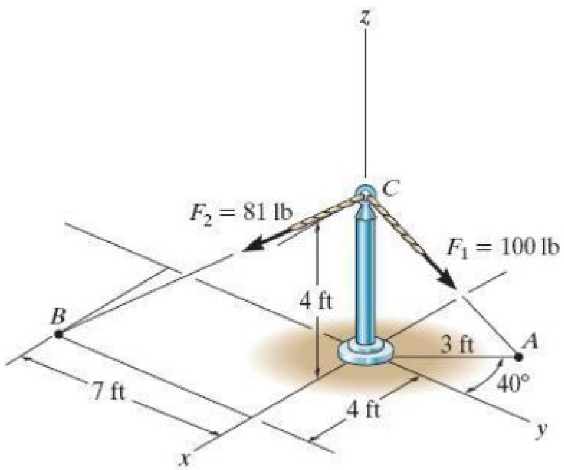
$$r_{AB} = \sqrt{3^2 + (-2)^2 + (-6)^2} = 7\ \mathrm{m}$$

$$\mathbf{u_{AB}} = \frac{3}{7}\hat{\imath} - \frac{2}{7}\hat{\jmath} - \frac{6}{7}\hat{k}$$

$$\mathbf{F_B} = F_B\mathbf{u_{AB}} = (840\ \mathrm{N})\left(\frac{3}{7}\hat{\imath} - \frac{2}{7}\hat{\jmath} - \frac{6}{7}\hat{k}\right)$$

$$\mathbf{F_B} = \{360\hat{\imath} - 240\hat{\jmath} - 720\hat{k}\}\ \mathrm{N}$$

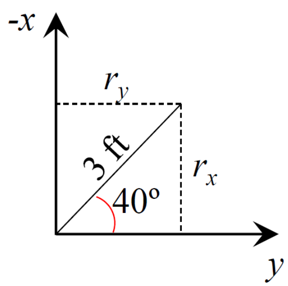
$$F_R = \sqrt{(120 + 360)^2 + (180 - 240)^2 + (-360 - 720)^2}\ \mathrm{N} = 1.18\ \mathrm{kN}$$



Individual work (15 min)

Determine the resultant force acting on the pipe in Cartesian vector form

Given: F_1 = 81 lb, F_2 = 100 lb, coordinates, angle
Asked: $\mathbf{F_C}$



First we need to determine $\mathbf{F_{CA}}$ and $\mathbf{F_{CB}}$ as cartesian vectors

$$\boldsymbol{F}_{CA} = F_1\left(\frac{\boldsymbol{r}_{CA}}{r_{CA}}\right)$$

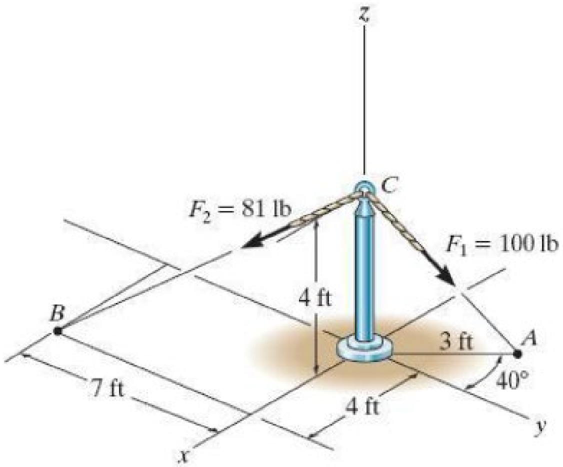
$$\mathbf{r_{AC}} = \{-3\sin 40\,\hat{\mathrm{i}} + 3\cos 40\,\hat{\mathrm{j}} - 4\hat{\mathrm{k}}\}\ \mathrm{m}$$

$$r_{AC} = \sqrt{(-3\sin 40)^2 + (3\cos 40)^2 + (-4)^2} = 5\ \mathrm{m}$$

This allows us to write

$$\boldsymbol{F}_{CA} = 100\ \mathrm{lb}\left(-3\sin 40\ \hat{\boldsymbol{\imath}} + 3\cos 40\ \hat{\boldsymbol{\jmath}} - 4\ \hat{\boldsymbol{k}}\right)/5$$

$$\boldsymbol{F}_{CB} = 81\ \mathrm{lb}\left(4\hat{\boldsymbol{\imath}} - 7\hat{\boldsymbol{\jmath}} - 4\hat{\boldsymbol{k}}\right)/(4^2 + 7^2 + 4^2)^{1/2}$$



We can now determine the resultant force vector

$$\mathbf{F}_\mathrm{R} = \mathbf{F}_\mathrm{CA} + \mathbf{F}_\mathrm{CB} = \left[-2.57\hat{\boldsymbol{\imath}} - 17.04\hat{\boldsymbol{\jmath}} - 116\hat{\boldsymbol{k}}\right] \mathrm{lb}$$

The magnitude of this vector is

$$F_R = (2.57^2 + 17.04^2 + 116^2)^{1/2} = 117.3 \text{ lb}$$

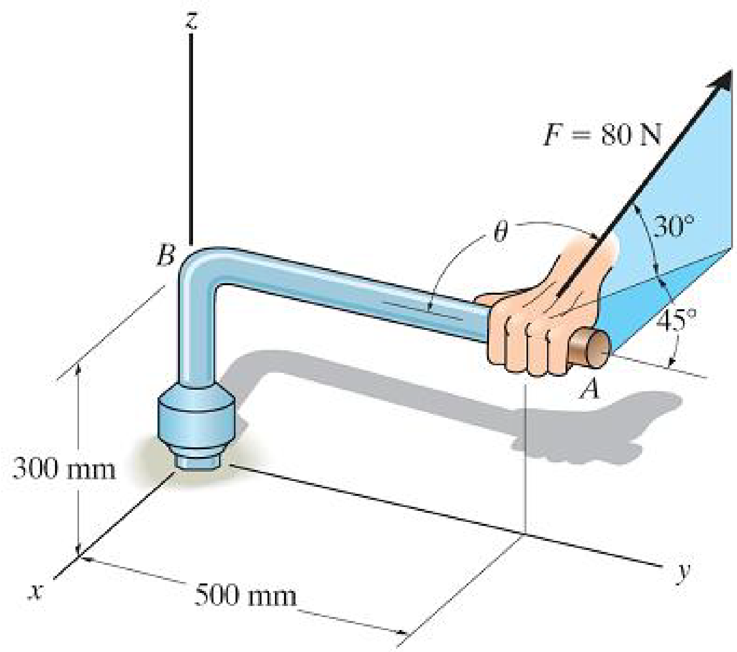
We can also calculate the direction angles

$$\alpha = \cos^{-1}\left({}^{-2.57}/_{117.3}\right) = 91.3$$

$$\beta = \cos^{-1}\left({}^{-17.04}/_{117.3}\right) = 98.4$$

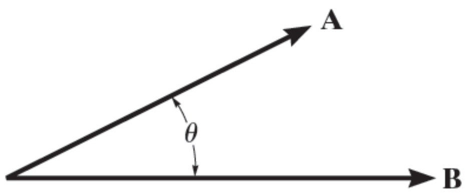
$$\gamma = \cos^{-1}\left({}^{-116}/_{117.3}\right) = 172$$

Scalar product - Applications



For the force **F** being applied to the wrench at Point *A*, what component of it actually helps turn the bolt (i.e., the force component acting perpendicular to the pipe)?

Scalar product



- The scalar product of vectors ***A***, ***B*** multiplies the two vectors to obtain a scalar result. It is also known as dot vector
- It is helpful to calculate the angle between two vectors
- In Cartesian space, the scalar product represents the projection of ***A*** over ***B***

We write the scalar product as $\boldsymbol{A} \cdot \boldsymbol{B} = AB\cos\theta$ where $0 \le \theta \le 180$

Note that the scalar product of two vectors perpendicular to each other is zero, since $\cos 90 = 0$

In particular the unit vectors $\hat{\boldsymbol{\imath}}$, $\hat{\boldsymbol{\jmath}}$, and $\widehat{\boldsymbol{k}}$ are perpendicular to each other so $\hat{\boldsymbol{\imath}} \cdot \hat{\boldsymbol{\jmath}} = \hat{\boldsymbol{\imath}} \cdot \widehat{\boldsymbol{k}} = \hat{\boldsymbol{\jmath}} \cdot \widehat{\boldsymbol{k}} = 0$

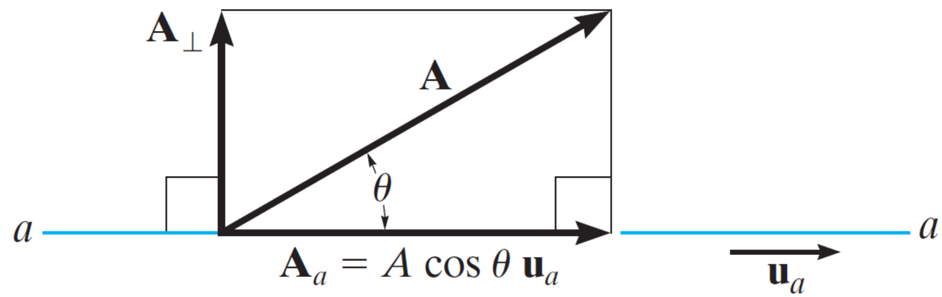
Similarly, it is easy to prove that $\hat{\boldsymbol{\imath}} \cdot \hat{\boldsymbol{\imath}} = \hat{\boldsymbol{\jmath}} \cdot \hat{\boldsymbol{\jmath}} = \widehat{\boldsymbol{k}} \cdot \widehat{\boldsymbol{k}} = 1$

Which gives us the following rule

$$\boldsymbol{A} \cdot \boldsymbol{B} = \left(A_x\hat{\boldsymbol{\imath}} + A_y\hat{\boldsymbol{\jmath}} + A_z\widehat{\boldsymbol{k}}\right) \cdot \left(B_x\hat{\boldsymbol{\imath}} + B_y\hat{\boldsymbol{\jmath}} + B_z\widehat{\boldsymbol{k}}\right)$$

$$\boldsymbol{A} \cdot \boldsymbol{B} = A_xB_x + A_yB_y + A_zB_z$$

Determining the projection of a vector



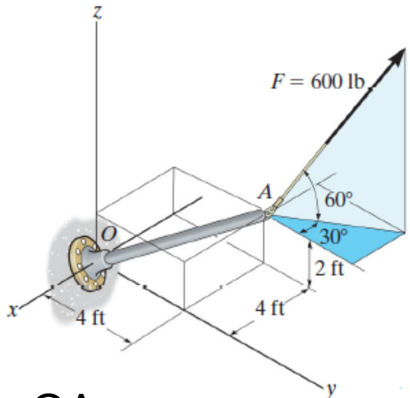
You can determine the components of a vector parallel and perpendicular to a line using the dot product.

Steps:

1. Find the unit vector, $\boldsymbol{u}_{aa}$ along line *aa*
2. Find the scalar projection of $\boldsymbol{A}$ along line aa by $A_{||} = \boldsymbol{A} \bullet \boldsymbol{u}_{aa} = A_x U_x + A_y U_y + A_z U_z$
3. The scalar and vector forms of the perpendicular component can easily be obtained by

$$A_\perp = (A^2 - A_{||}^2)^{½} \text{ and}$$
$$\boldsymbol{A}_\perp = \boldsymbol{A} - \boldsymbol{A}_{||}$$

(rearranging the vector sum of $\boldsymbol{A} = \boldsymbol{A}_\perp + \boldsymbol{A}_{||}$)



Example

Determine the components of the force acting parallel and perpendicular to the axis of the pole.

Given: F = 600 lb, coordinates
Asked: $F_{//}$ and $F_{\perp}$

Bar OA:

$$\mathbf{r_A} = \{-4\hat{\imath} + 4\hat{\jmath} + 2\hat{k}\}\ \text{ft}$$

$$r_A = \sqrt{(-4)^2 + 4^2 + 2^2} = 6\ \text{ft}$$

$$\mathbf{u_A} = \frac{-2}{3}\hat{\imath} + \frac{2}{3}\hat{\jmath} + \frac{1}{3}\hat{k}$$

$$\mathbf{F} = \{[-600\cos 60°\sin 30°]\hat{\imath} + [600\cos 60°\cos 30°]\hat{\jmath} + [600\sin 60°]\,\hat{k}\}\text{lb}$$

$$\mathbf{F} = \{-150\hat{\imath} + 259.81\hat{\jmath} + 519.62\hat{k}\}\ \text{lb}$$

$$\mathbf{F}_{//} = \mathbf{F}.\mathbf{u_A} = \left(-\tfrac{2}{3}(-150) + \tfrac{2}{3}(259.81) + \tfrac{1}{3}(519.62)\right)\ \text{lb} = 446.41\ \text{lb} = 446\ \text{lb}$$

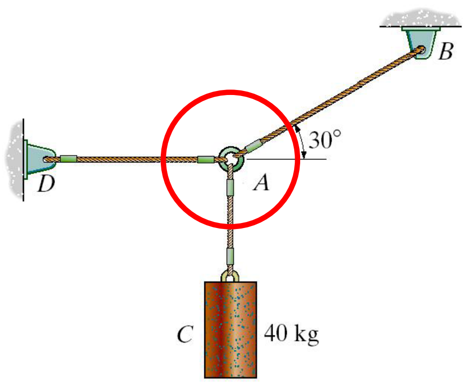
$$\mathbf{F}_{\perp} = \sqrt{(600)^2 - (446.41)^2} = 401\ \text{lb}$$

Free body diagram

While mechanics analysis is generally algebraic, it has a geometric basis. For this reason a graphical representation of the problem at hand would be extremely useful. This representation is the Free Body Diagram (FBD)

The FBD is the most useful tool in mechanics analysis. A well-done and complete FBD does half the work for us, showing exactly the steps that must be taken and the equations that must be written

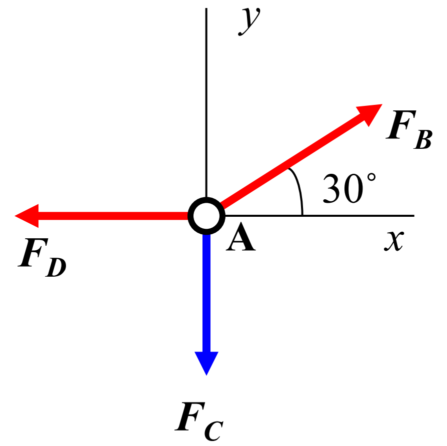
Consider the following problem



We are interested in the forces on A

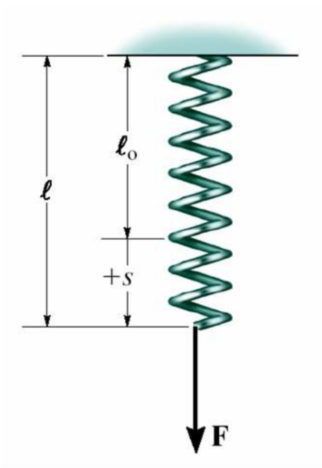
Imagine A as an isolated particle

Draw all forces acting on A as vectors



Once we have completed the FBD, it is clear which forces are known and which forces need to be determined. We also have and idea of the procedure to determine such forces

Springs, cables, and pulleys

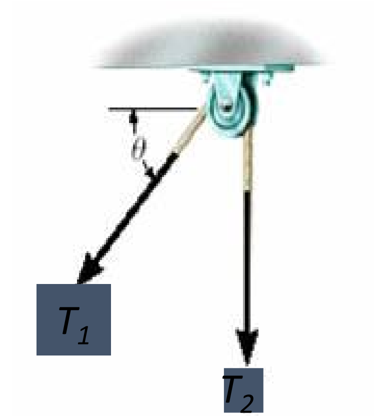


Spring Force =

spring constant * deformation

or

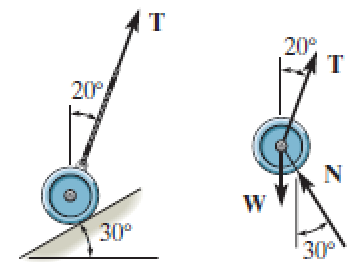
$$F = k * s$$



Cable is in tension

With a frictionless pulley,

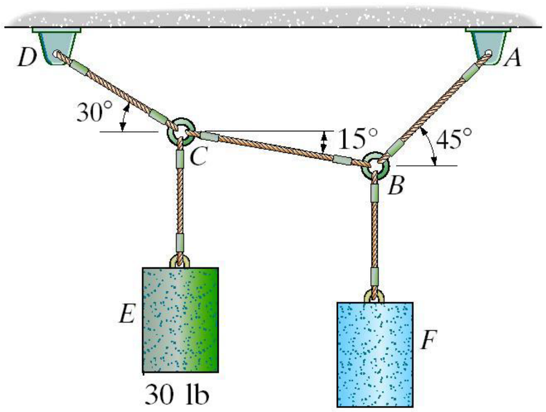
$T_1 = T_2$.



In a smooth surface, the surface exerts a force on the object that is normal to the surface at the point of contact.

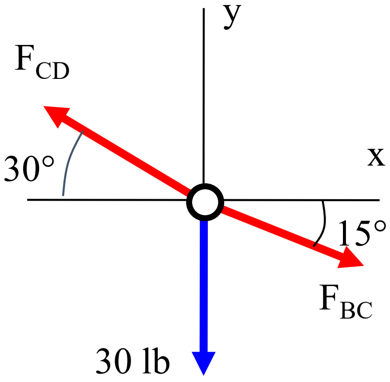
Example

Determine the forces on the cables and the weight of cylinder F



Solution

The FBD at C is

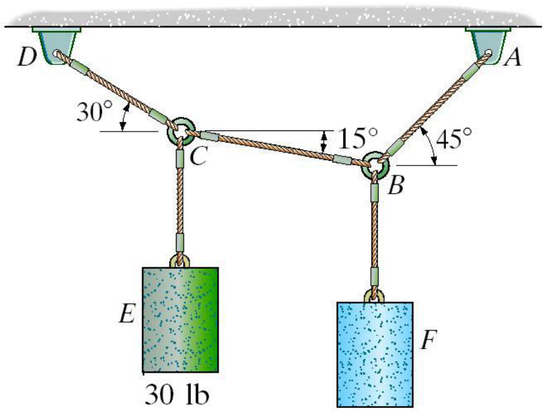


Notice we are <u>assuming</u> directions for $\boldsymbol{F}_{CD}$ and $\boldsymbol{F}_{BC}$

Since our system is in equilibrium, any part of it must also be in equilibrium

In particular, point C is in equilibrium, hence (see Newton's 2nd law) the resultant force $\boldsymbol{F} = 0$

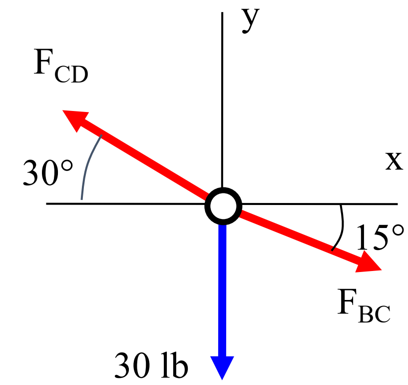
This equation holds true in all directions. This means we have three (x, y, z) independent equilibrium equations



The equilibrium equation on x

$$\sum \boldsymbol{F}_x = 0$$

$$F_{BC}\cos 15 - F_{CD}\cos 30 = 0$$

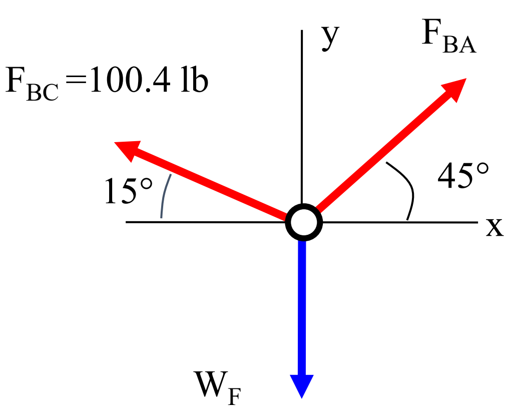


Similarly, for y $F_{CD}\sin 30 - F_{BC}\sin 15 - 30 = 0$

$$F_{BC} = 100.38 \text{ lb}$$

$$F_{CD} = 112.2 \text{ lb}$$

The FBD at B instead is



This allows us to write the equilibrium equations

$$F_{BA}\cos 45 - F_{BC}\cos 15 = 0$$

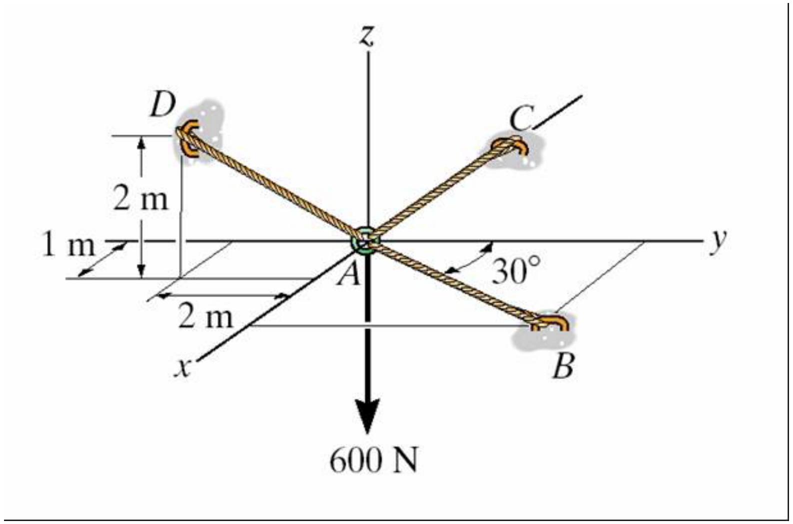
$$F_{BA}\sin 45 + F_{BC}\sin 15 - W_F = 0$$

$$F_{BA} = 137 \text{ lb}$$

$$W_F = 123 \text{ lb}$$

Individual work (15 min)

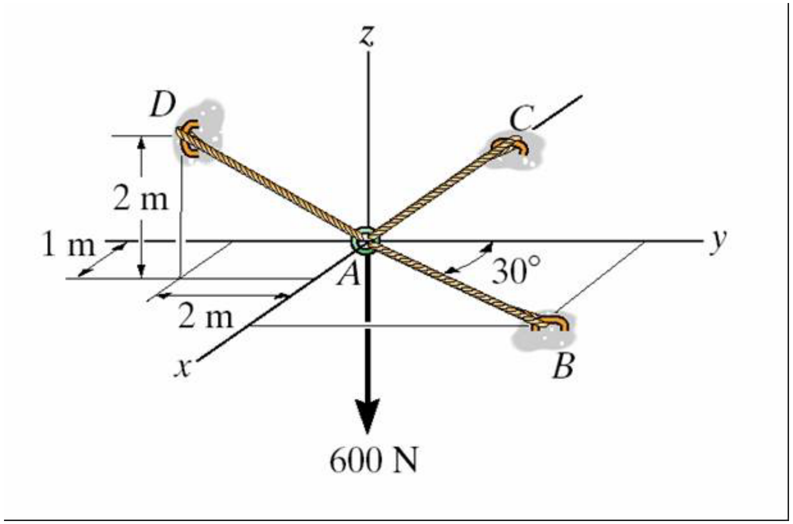
Determine the tension developed in cables *AB*, *AC*, and *AD*.

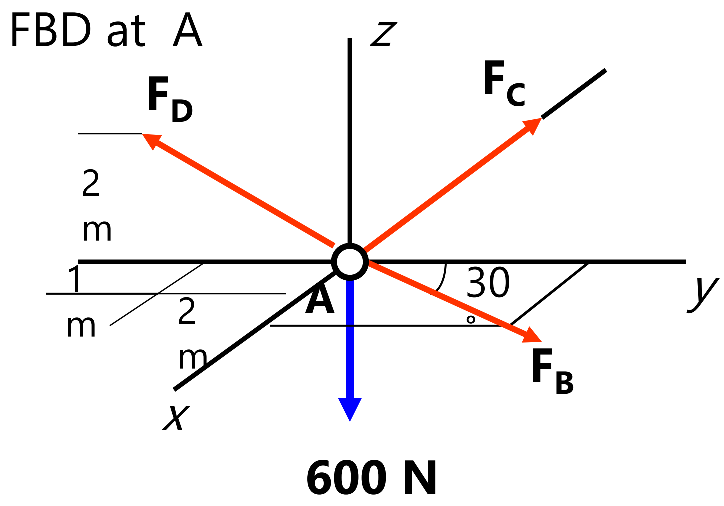


1) Draw a free body diagram of Point A. Let the unknown force magnitudes be F_B, F_C, F_D.

2) Represent each force in the Cartesian vector form.

3) Apply equilibrium equations to solve for the three unknowns.





$$\mathbf{F_B} = F_B(\sin 30^\circ\, \mathbf{i} + \cos 30^\circ\, \mathbf{j})\ \text{N}$$

$$= \{0.5\, F_B\, \mathbf{i} + 0.866\, F_B\, \mathbf{j}\}\ \text{N}$$

$$\mathbf{F_C} = -F_C\, \mathbf{i}\ \ \text{N}$$

$$\mathbf{F_D} = F_D(\mathbf{r_{AD}}/r_{AD})$$

$$= F_D\{(1\,\mathbf{i} - 2\,\mathbf{j} + 2\,\mathbf{k}) / (1^2 + 2^2 + 2^2)^{½}\}\ \text{N}$$

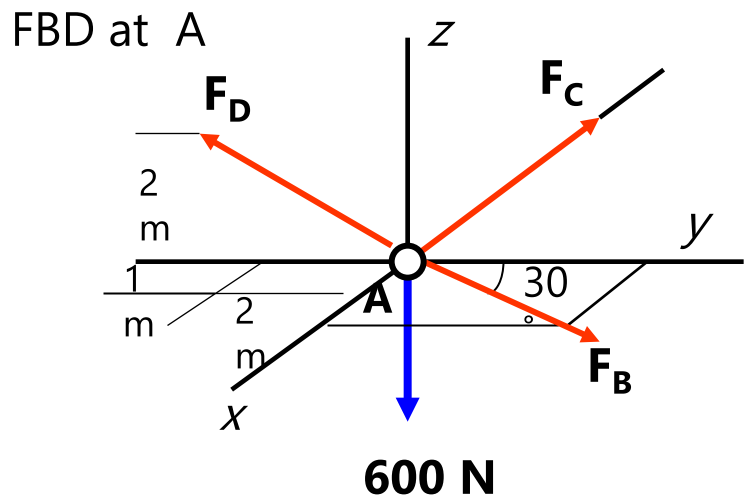
$$= \{0.333\, F_D\, \mathbf{i} - 0.667\, F_D\, \mathbf{j} + 0.667\, F_D\, \mathbf{k}\}\ \text{N}$$

Now equate the respective **i** , **j** , **k** components to zero.

$$\sum F_x = 0.5\ F_B - F_C + 0.333\ F_D = 0$$

$$\sum F_y = 0.866\ F_B - 0.667\ F_D = 0$$

$$\sum F_z = 0.667\ F_D - 600 = 0$$



Solving the three simultaneous equations yields

F_C = 646 N

F_D = 900 N

F_B = 693 N