Complete the following tasks. To show work for the double-integrals, demonstrate that you can set up the integrals. From there, you may use software such as *Wolfram Alpha* to compute the values of the double integrals. Some answers have been provided. Assemble your work into one PDF document and upload the PDF back into our CatCourses page.

1. Your math teacher intentionally misinterprets the definition of N99 masks to bring you this challenge. If the diameters of saliva particles are uniformly distributed between 5 and 21 micrometers, use Chebyshev's Inequality to compute how many particles are needed so that the average diameter of the saliva particles is within 3.2 micrometers of the true population mean with at least 99 percent probability?[1]

2. Today, the student chefs are preparing the "Tortilla de Merced", a traditional Spanish egg dish with an infusion of spicy hot potato chips whose mascot is a cheetah. Let us model the radius (in inches) of the dish with $R \sim U(3, 5)$. Use Chebyshev's Inequality to compute how many dishes need to be measured so that their average radius is within 2 percent error with at least 90 percent probability.[2]

3. The voters in California are voting for a new governor. Of the voters in California, a proportion p will vote for Frank, and a proportion $1 - p$ will vote for Tony. In an election poll, a number of voters are asked for whom they will vote. Let X_i be the indicator random variable for the event "the ith person interviewed will vote for Frank."[3] A model for the election poll is that the people to be interviewed are selected in such a way that the indicator random variables $X_1, X_2, ...$ are independent and have a $Ber(p)$ distribution.

   (a) Suppose we use $\bar{X}_n$ to predict p. According to Chebyshev's inequality, how large should n be (how many people should be interviewed) such that the probability that $\bar{X}_n$ is within 0.2 of the "true" p is at least 0.9? (**Hint.** solve this first for $p = 1/2$, and use the fact that $p(1 - p) \leq 1/4$ for all $0 \leq p \leq 1$.)

   (b) Answer the same question, but now $\bar{X}_n$ should be within 0.1 of p.

   (c) Answer the question from part **(a)**, but now the probability should be at least 0.95.

   (d) If $p > 1/2$, then Frank wins; if $\bar{X}_n > 1/2$, you predict that Frank will win. Find an n (as small as you can) such that the probability that you predict correctly is at least 0.9, if in fact $p = 0.6$.

[1] Source: `https://www.envirosafetyproducts.com/resources/dust-masks-whats-the-difference.html` This was an example question during the Spring 2020 semester

[2] This was an exam question during the Summer 2020 session.

[3] That is, $X_i = 1$ if the ith person interviewed will vote for Frank, and $X_i = 0$ if the ith person interviewed will not vote for Frank.

4. Let $X_1, X_2, ..., X_{625}$ be independent and identically distributed random variables, with probability density function f given by

$$f(x) = \begin{cases} 3(1-x)^2 & \text{for } 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

   Use the central limit theorem to approximate $P(X_1 + X_2 + \cdots + X_{625} < 170)$.

5. For a request to use the campus computing cluster, and knowing that your jobs' durations are normally distributed with a mean of one hour and a standard deviation of 10 minutes, answer the following inquiries.[4]

   (a) What is the probability that the total duration of your 32 jobs is more than 34 hours?

   (b) What is the probability that the average duration of your 28 jobs is less than 57 minutes?

[4]This was an exam question during the Spring 2021 semester.

Here are some of answers. Note that numbers may slightly vary depending on when and where the rounding took place.

1. at least 209 particles[5]
2. at least 521 dishes
3. (a) 63
   (b) 250
   (c) 125
   (d) 240
4. 0.9977
5. (a) 0.0169
   (b) 0.0562

[5]Optional: "209" is Merced's area code!