Structural analysis - trusses

Instructor

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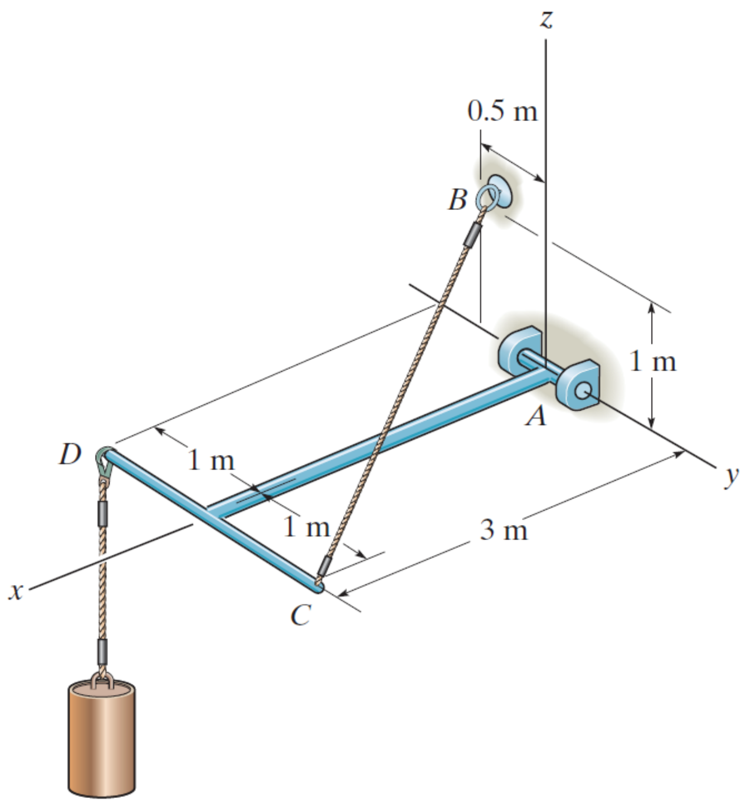
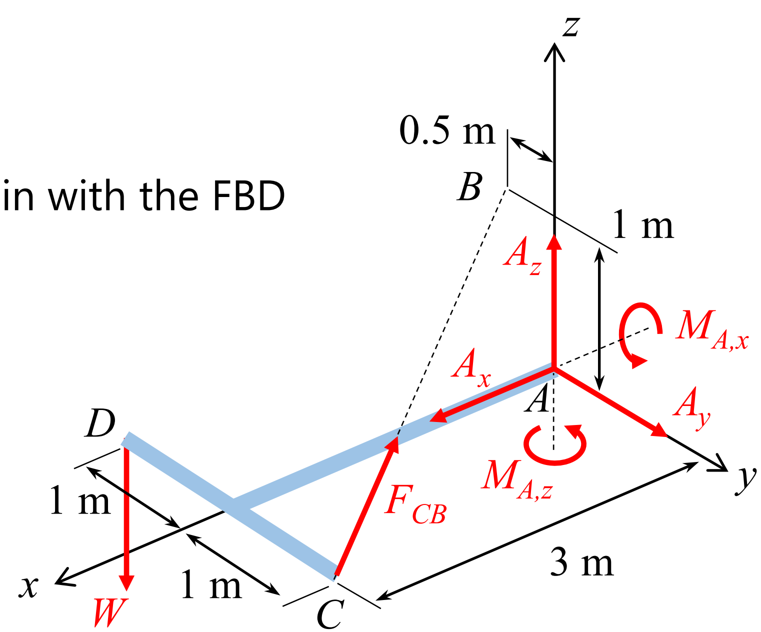
star-forming region called NGC 3324 in the Carina Nebula

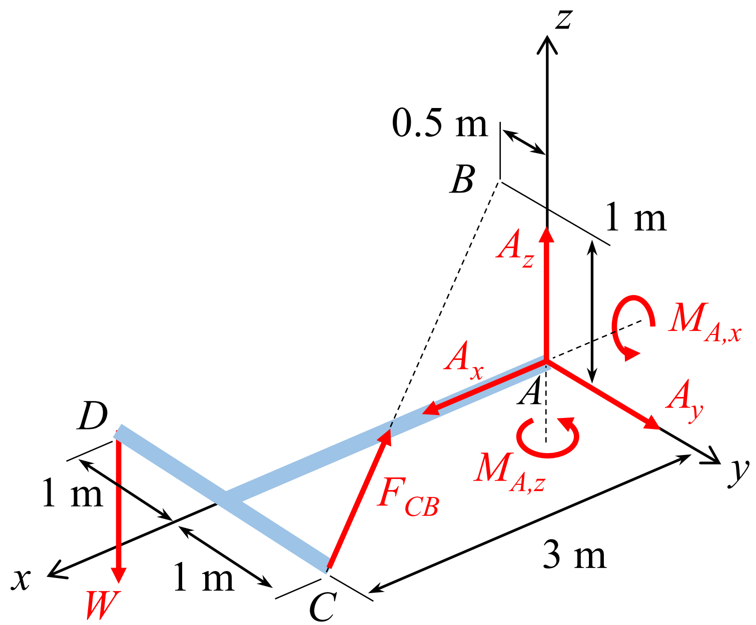
Individual problem - Equilibrium of a rigid body (15 minutes)



The member is supported by a pin at *A* and cable *BC*. Determine the reactions at these supports if the cylinder has a mass of 40 kg

Let's begin with the FBD





In order to find the components of F_{CB} we can use the position vector r_{CB} and divide by its magnitude to get a unit vector

$$\mathbf{r}_{CB} = (0-3)\hat{\mathbf{i}} + (-0.5-1)\hat{\mathbf{j}} + (1-0)\hat{\mathbf{k}}$$

$$\mathbf{r}_{\mathrm{CB}} = -3\hat{\mathbf{i}} - 1.5\hat{\mathbf{j}} + \hat{\mathbf{k}}$$

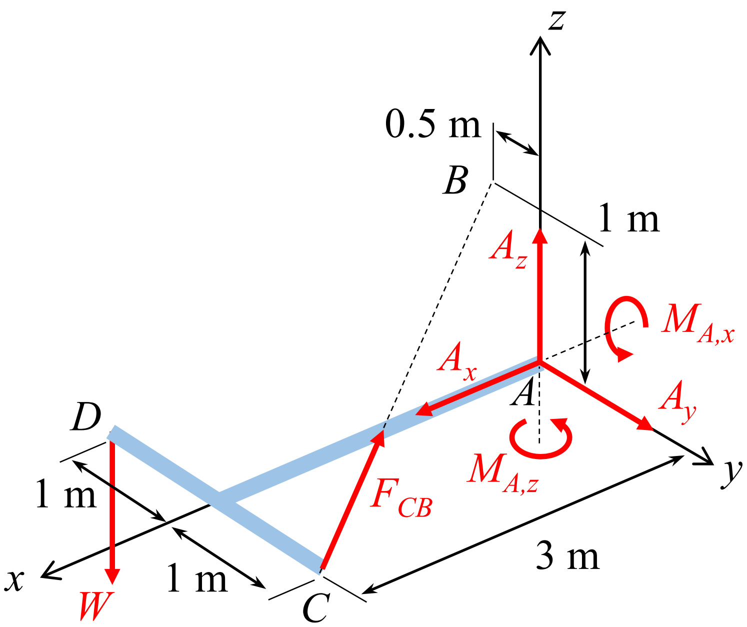
$$r_{CB} = \sqrt{(-3)^2 + (-1.5)^2 + (1)^2} = 3.5$$

$$\hat{\mathbf{u}}_{CB} = -0.8571\hat{\mathbf{i}} - 0.4286\hat{\mathbf{j}} + 0.2857\hat{\mathbf{k}}$$

Using this unit vector, we can define the force components

$$\mathbf{F}_{\mathrm{CB}} = -0.86F_{CB}\,\hat{\mathbf{i}} - 0.43F_{CB}\,\hat{\mathbf{j}} + 0.29F_{CB}\,\hat{\mathbf{k}}$$

$$\mathbf{W} = -40\ \mathrm{kg}\left(9.81\,\frac{\mathrm{m}}{\mathrm{s}^2}\right)\hat{\mathbf{k}} = \{-392.4\,\hat{\mathbf{k}}\}\ \mathrm{N}$$



We can now use the equations of equilibrium of forces in *x, y, z*

$$A_x - 0.86F_{CB} = 0$$

$$A_y - 0.43F_{CB} = 0$$

$$A_z - 392.4 + 0.29F_{CB} = 0$$

Distance from A to C and D:

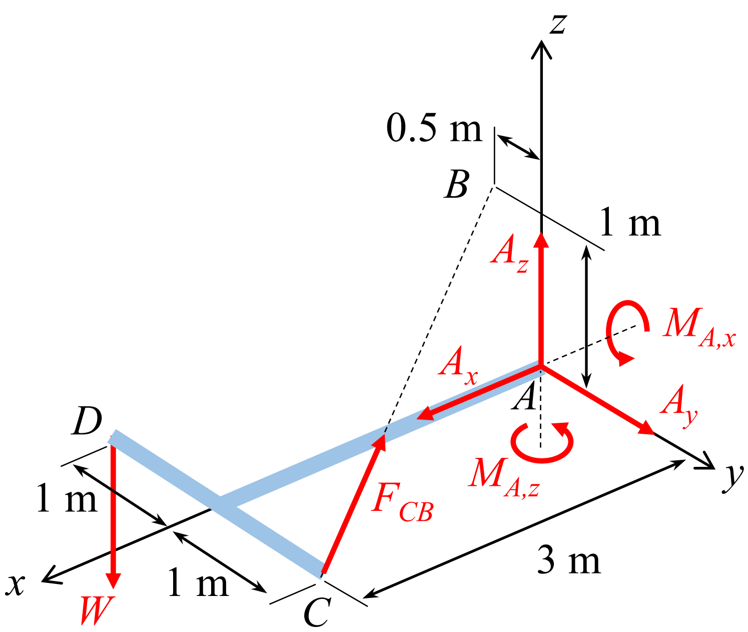
$$\mathbf{r}_{\mathrm{AC}} = \{3\hat{\mathbf{i}} + \hat{\mathbf{j}}\}\ \text{m}$$

$$\mathbf{r}_{\mathrm{AD}} = \{3\hat{\mathbf{i}} - \hat{\mathbf{j}}\}\ \text{m}$$

The equilibrium of moments equation is

$$\sum \boldsymbol{M}_A = \begin{vmatrix} \hat{\boldsymbol{\imath}} & \hat{\boldsymbol{\jmath}} & \widehat{\boldsymbol{k}} \\ 3 & 1 & 0 \\ -0.86F_{CB} & -0.43F_{CB} & 0.29F_{CB} \end{vmatrix} + \begin{vmatrix} \hat{\boldsymbol{\imath}} & \hat{\boldsymbol{\jmath}} & \widehat{\boldsymbol{k}} \\ 3 & -1 & 0 \\ 0 & 0 & -392.4 \end{vmatrix} + M_{A,x}\hat{\boldsymbol{\imath}} + M_{A,z}\widehat{\boldsymbol{k}} = \mathbf{0}$$

$$\sum \boldsymbol{M}_A = \left(0.29F_{CB} + 392.4 + M_{A,x}\right)\hat{\boldsymbol{\imath}} - \left(0.86F_{CB} - 1177\right)\hat{\boldsymbol{\jmath}} + \left(M_{A,z} - 0.43F_{CB}\right)\widehat{\boldsymbol{k}} = \mathbf{0}$$



Each component of $\Sigma \mathbf{M_A}$ is zero, then

$$0.29F_{CB} + 392.4 + M_{A,x} = 0$$

$$1177 - 0.86F_{CB} = 0$$

$$M_{A,z} - 0.43F_{CB} = 0$$

Immediately, we can solve for F_{CB}

$$\boxed{F_{CB} = 1374 \text{ N}}$$

Which in turn gives us $\boxed{M_{A,z} = 588.9 \text{ N} \cdot \text{m}}$ and $\boxed{M_{A,x} = -784.8 \text{ N} \cdot \text{m}}$

And, using the force equilibrium equations:

$$A_x - 0.86F_{CB} = 0 \qquad \boxed{A_x = 1177.3 \text{ N}}$$

$$A_y - 0.43F_{CB} = 0 \qquad \boxed{A_y = 588.9 \text{ N}}$$

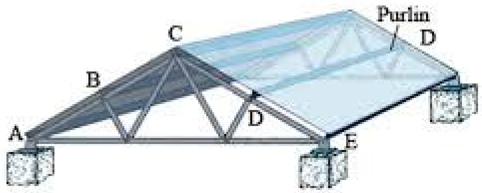
$$A_z - 392.4 + 0.29F_{CB} = 0 \qquad \boxed{A_z = 0}$$

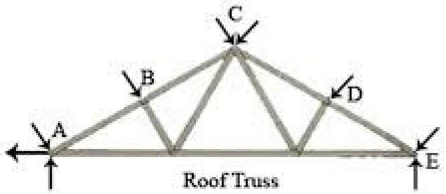
https://www.youtube.com/watch?v=4uuCdTK7CjI&t=22s

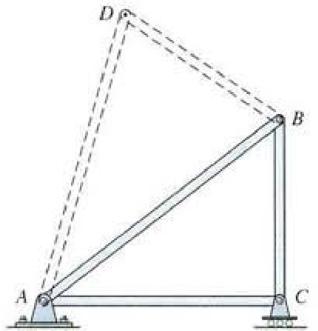
Truss structures

- A truss is a structure.
- Truss is composed are slender members joined together at their end points.
- Trusses are used to support roofs, bridges, cranes, frames, aircrafts, and other lightweight rigid structures.
- When trusses extend over large distances, a rocker or roller is commonly used for supporting one end, allowing freedom for expansion or contraction due to a change in temperature or application of loads.

If a truss lies on a single plane, it's called a planar truss





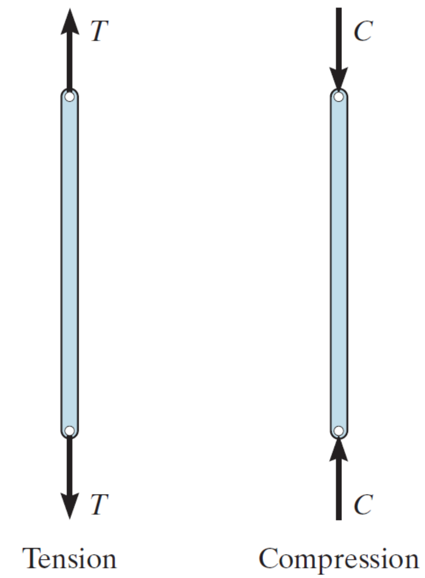


- A simple truss is a planar truss which begins with a triangular element and can be expanded by adding two members and a joint
- For these trusses the number of members (M) and the number of joints (J) are related by the equation $M = 2J - 3$

Force analysis

When designing both the members and the joints of a truss, it is necessary to determine the forces in each truss member. This is called the force analysis of a truss. Two assumptions are made:

1. All loads are applied at the joints. The weight of truss members is often neglected since it is small when compared to the forces supported by the members
2. The members are joined together by smooth pins. This assumption corresponds to most practical cases when joints are formed by bolting the members together

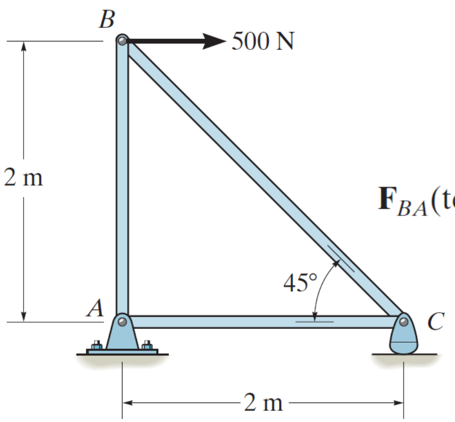


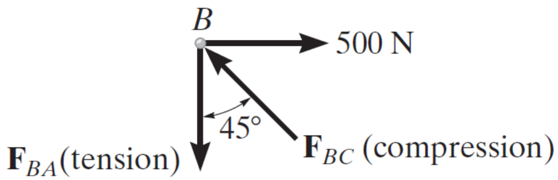
Under these assumptions, the members act as two-force members, which means they are loaded in either tension or compression, but no other loading modes are present

To perform the force analysis of a truss, we will apply the same method we have used so far: FBD, equations of equilibrium, and simultaneous solution of equations

As we have seen before, the choices made when posing a problem can greatly affect how easy (or difficult) it is to solve it. There are two general approaches in trusses: the method of joints and the method of sections

Method of Joints

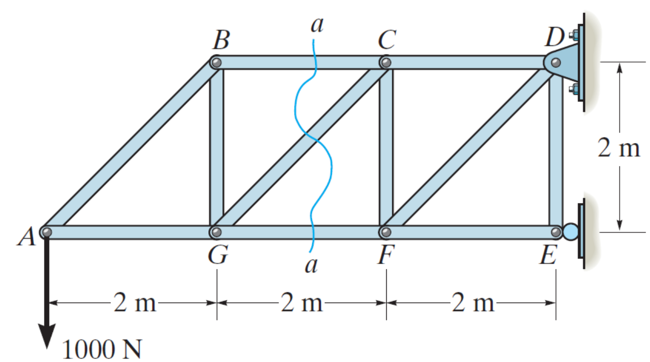




Each joint is analyzed separately, drawing a new FBD and applying a new set of equilibrium equations for each joint

This method is longer, but quite simple

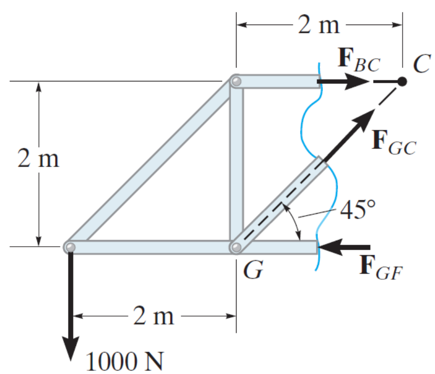
Method of Sections

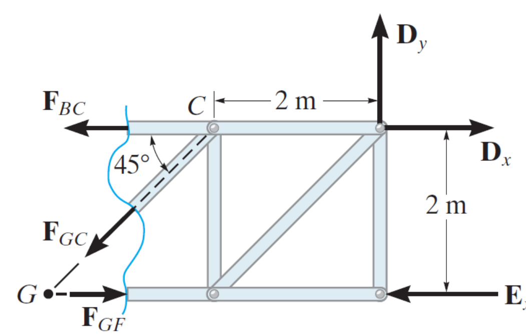


The truss is ”cut” into two parts and considering each part separately.

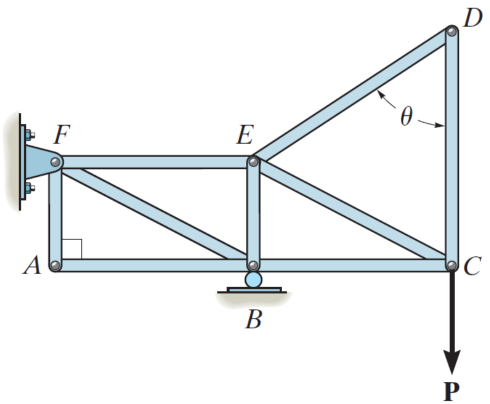
When “cutting” the truss members, <u>internal forces</u> will appear. These forces have equal and opposite counterparts in the other half

The choice of where to “cut” the truss is key to this method. We must select the cut to minimize the number of unknowns

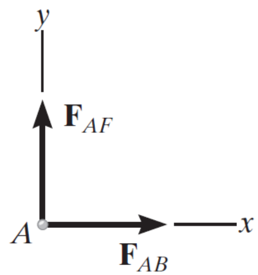




Zero-force members. Case 1



Consider this truss. We can use the joints method to determine the forces at each joint focusing on *A:*

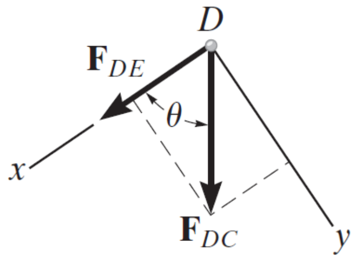


This gives

$$\sum F_x = 0 \quad \Rightarrow \quad F_{AB} = 0$$

$$\sum F_y = 0 \quad \Rightarrow \quad F_{AF} = 0$$

In a similar way, we can consider *D*

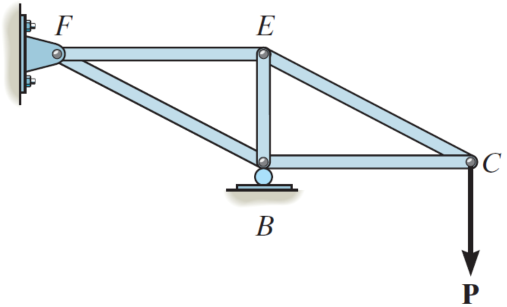


Which gives

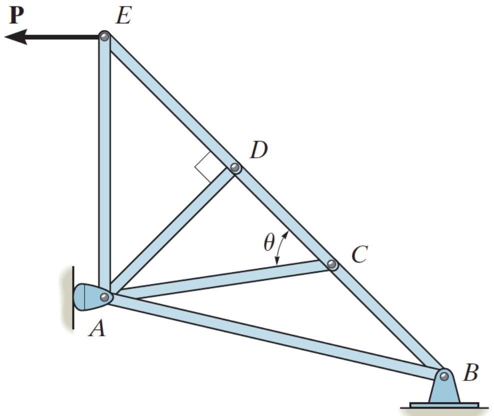
$$\sum F_y = F_{DC} \sin\theta = 0 \quad \Rightarrow \quad F_{DC} = 0$$

$$\sum F_x = 0 \quad \Rightarrow \quad F_{DE} = 0$$

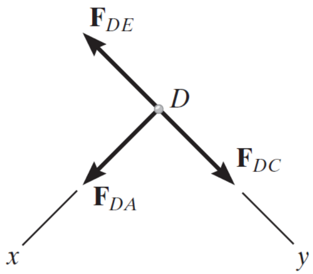
It is easy to see the same is true for any other such joint. We can conclude that if only <u>two non-collinear </u>members form a truss joint and <u>no external load </u>or support reaction is applied to the joint, <u>the two members must be zero-force members</u>. The equivalent frame is shown



Zero-force members. Case 2



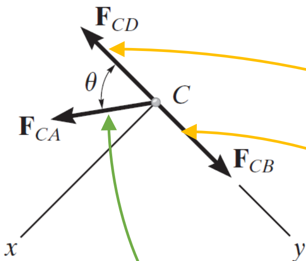
Lets consider this simple truss. We will focus our attention on joints D and C to prove they have zero-force members



This gives

$$\sum F_x = 0 \quad \Rightarrow \quad F_{DA} = 0$$

$$\sum F_y = 0 \quad \Rightarrow \quad F_{DE} = F_{DC}$$

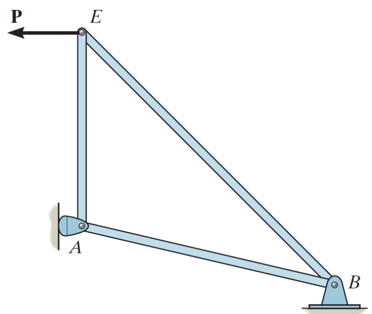


This gives

$$\sum F_x = F_{CA} \sin\theta = 0 \quad \Rightarrow \quad F_{CA} = 0$$

$$\sum F_y = F_{CD} + F_{CA}\cos\theta - F_{CB} = 0 \quad \Rightarrow \quad F_{CB} = F_{CD}$$

As before, this can be generalized: if three members form a truss joint for which two of the members are collinear, the third member is a zero-force member provided no external force or support reaction has a component that acts along this member. As such, the equivalent truss is:



Steps for analysis: Method of joints

1. If the truss's support reactions are not given, draw a FBD of the entire truss and determine the support reactions (typically using scalar equations of equilibrium).

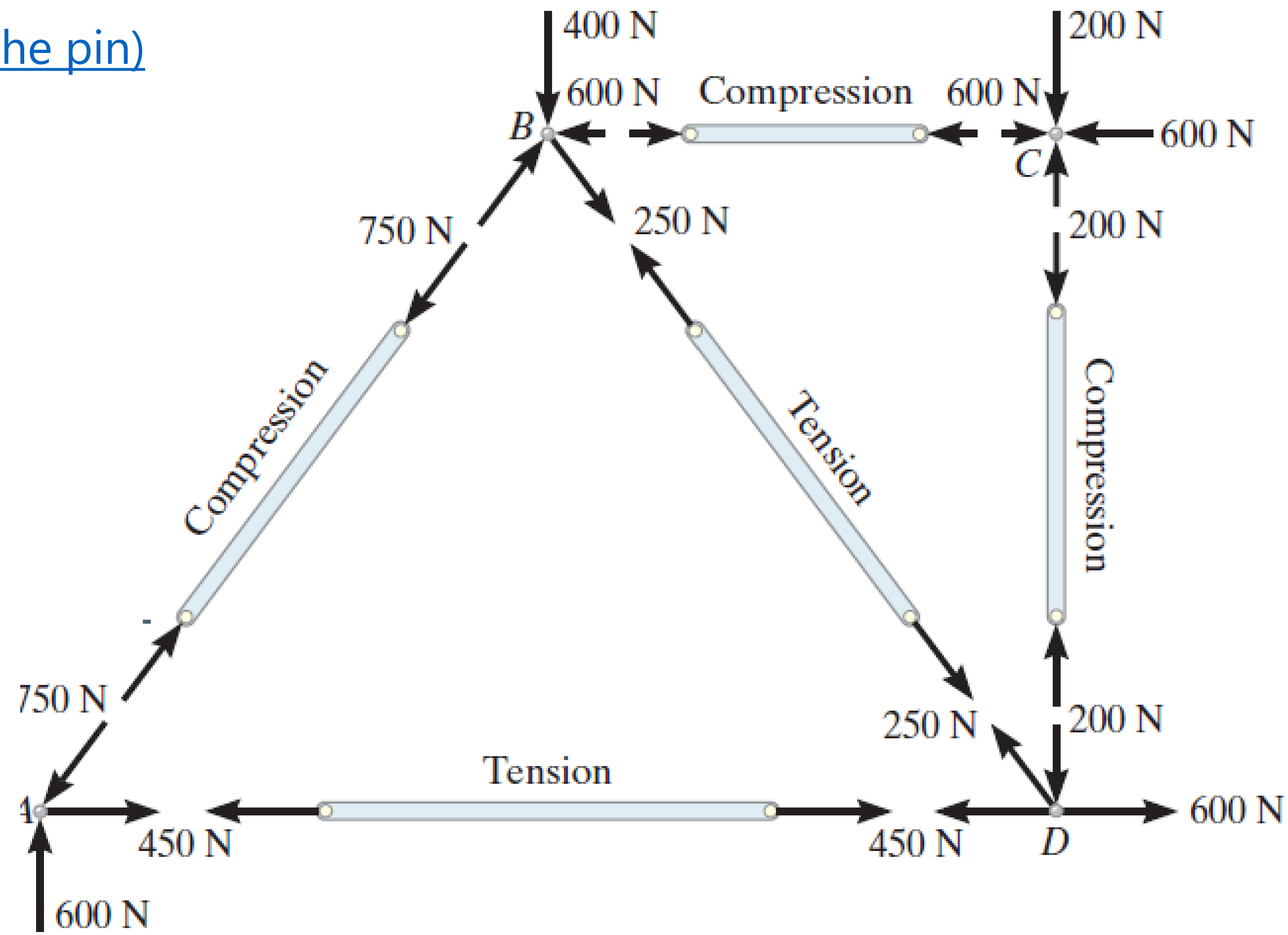
2. Draw the free-body diagram of a joint with one or two unknowns (at the least start at a joint with a least 1 known force). Assume that all unknown member forces act in tension (pulling the pin) unless you can determine by inspection that the forces are compression loads.

3. Apply the scalar equations of equilibrium, $\sum F_X = 0$ and $\sum F_Y = 0$, to determine the unknown(s). If the answer is positive, then the assumed direction (tension) is correct, otherwise it is in the opposite direction (compression).

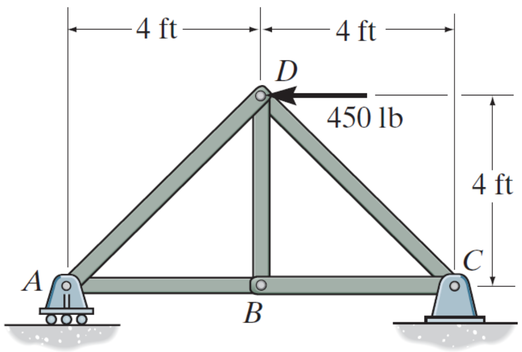
4. Repeat steps 2 and 3 at each joint in succession until all the required forces are determined.

Example: Tension - Compression

tension (pulling the pin)



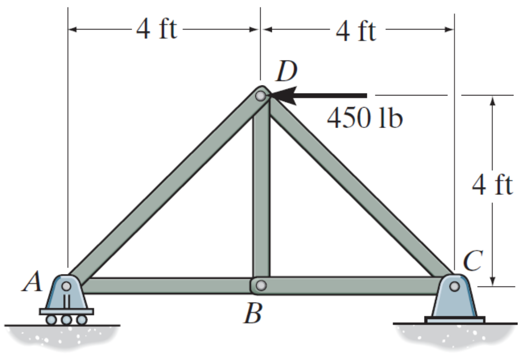
Example



Determine the force in each member of the truss. State if each member is in tension or compression.

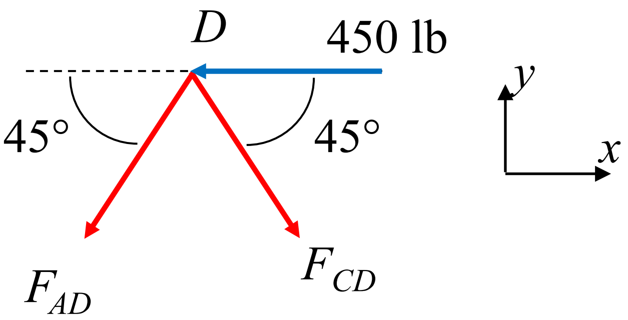
Solution steps

1. Check if there are any zero-force members.
2. Analyze pin *D* and then pin *A*, and then *C* if needed
3. Calculate the forces and reactions



1. The first thing we can do is notice that *BD* is a zero-force member

2. FBD at D:



Which gives the equilibrium equations:

$$\sum F_x = -450 \text{ lb} + F_{CD} \cos 45° - F_{AD} \cos 45° = 0$$

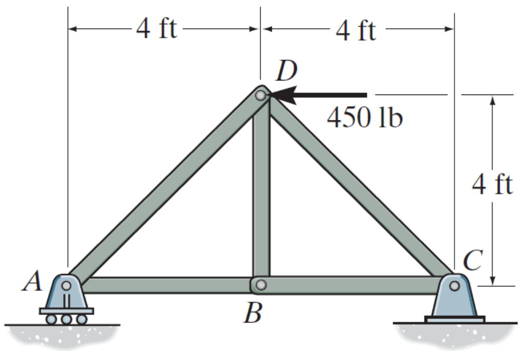
$$\sum F_y = -\, F_{CD} \sin 45° - F_{AD} \sin 45° = 0$$

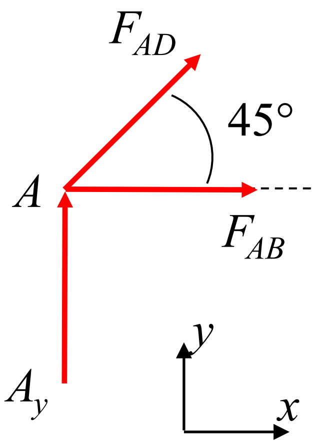
Solving these two equations we get

$F_{CD} = 318 \text{ lb}$ Tension

$F_{AD} = -318 \text{ lb}$ Compression

Similarly, we can focus on joint *A*. The FBD is





$$\sum F_x = F_{AD}\cos 45° + F_{AB} = 0$$

$$\sum F_y = F_{AD}\sin 45° + A_y = 0$$

$F_{AB} = 225 \text{ lb}$ Tension

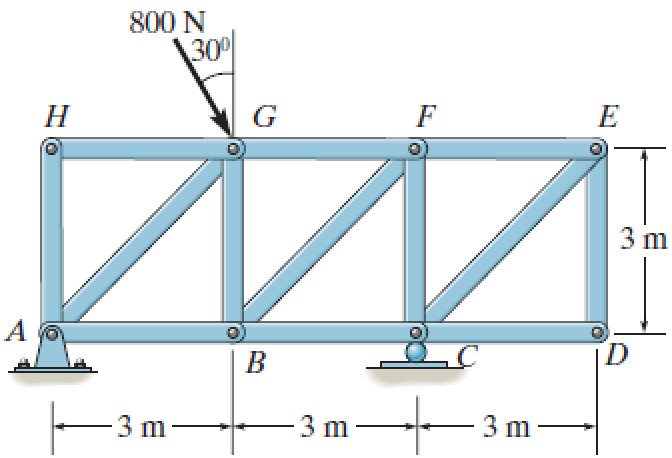
$A_y = 225 \text{ lb}$

Notice there is no need to solve for the joint *C*. The force F_{CD} is already known and, since *BD* is a zero-force member, $F_{CB} = F_{AB}$

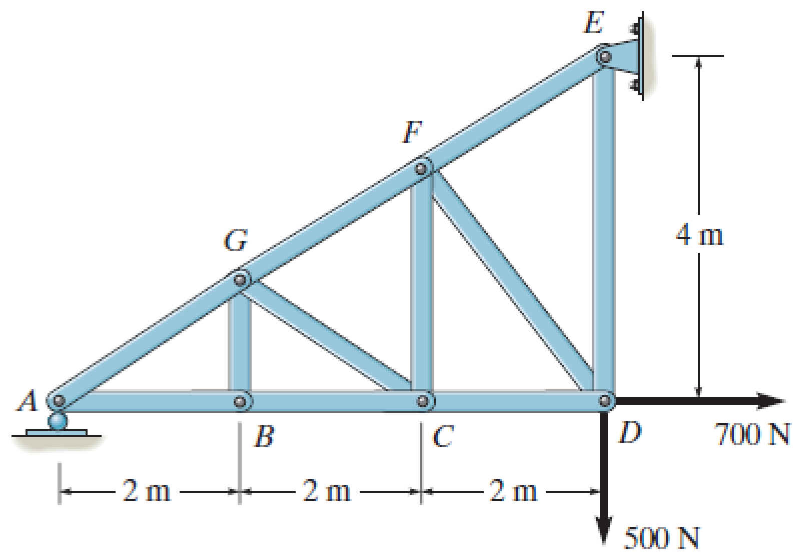
zero-force members are not useless. They give stability and robustness to a truss even if they have no effect on the joints. They should, however, be kept to a minimum when designing

Individual work (10 min)

Identify the zero-force members in each truss

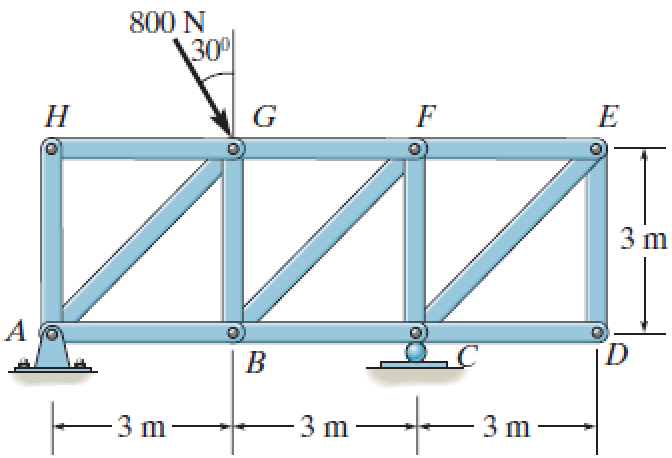


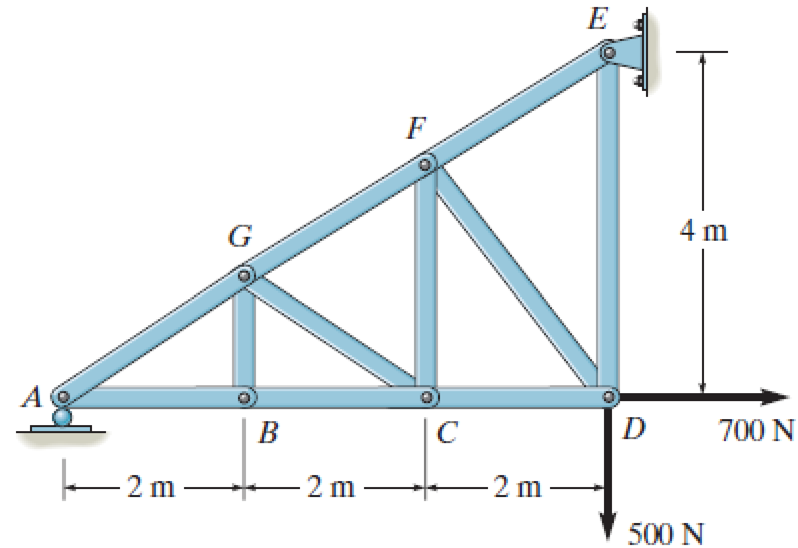
(a)

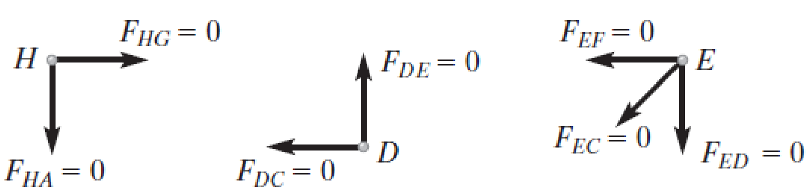


Individual work (10 min)

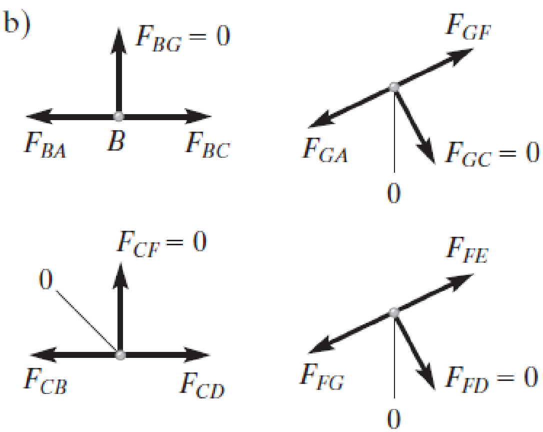
Identify the zero-force members in each truss





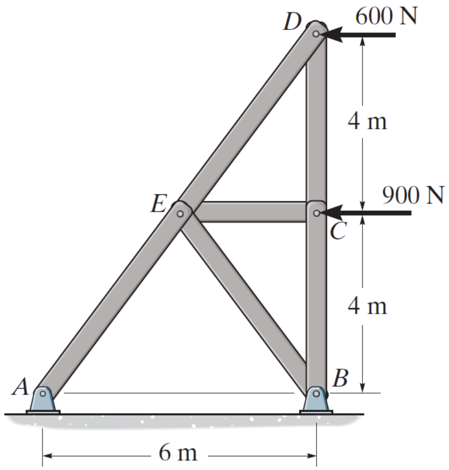


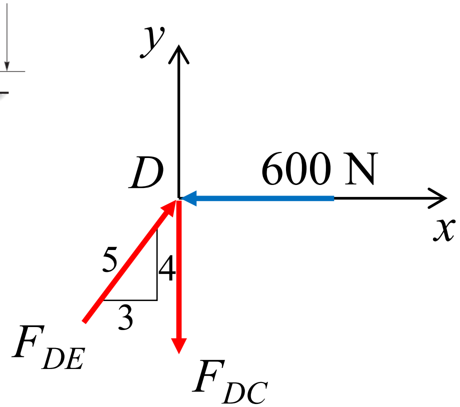
b)



Breakout room (15 min)

Determine the force in each member of the truss. State if each member is in tension or compression.



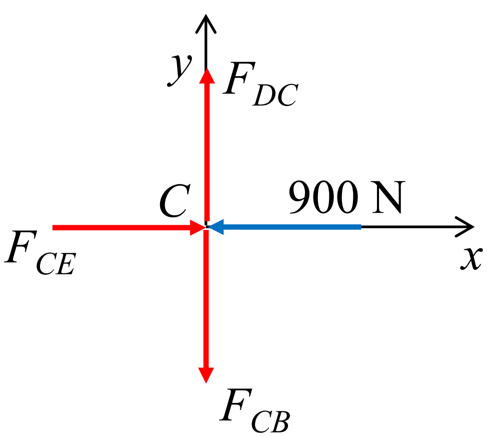


$$\sum F_x = F_{DE}\left(\frac{3}{5}\right) - 600 = 0 \quad \Rightarrow \quad F_{DE} = 1000\ N$$

Compression

$$\sum F_y = F_{DE}\left(\frac{4}{5}\right) - F_{DC} = 0 \quad \Rightarrow \quad F_{DC} = 800\ N$$

Tension

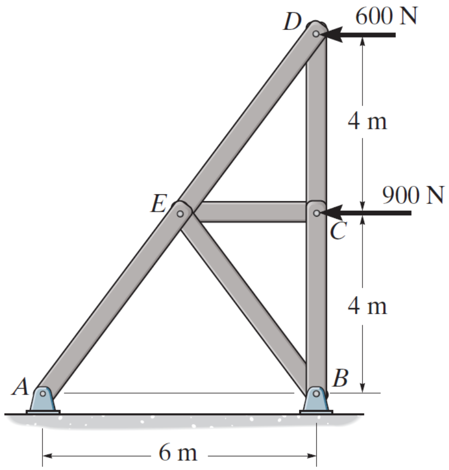


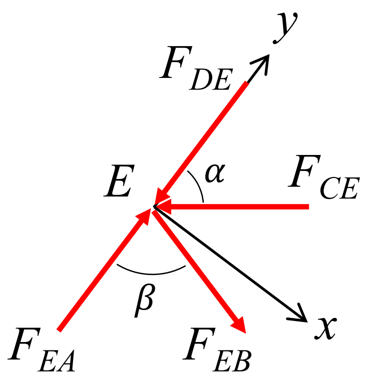
$$\sum F_x = F_{CE} - 900 = 0 \quad \Rightarrow \quad F_{CE} = 900\ N$$

Compression

$$\sum F_y = F_{DC} - F_{CB} = 0 \quad \Rightarrow \quad F_{CB} = 800\ N$$

Tension





The final joint will require a bit of trigonometry. We can take advantage of the similar triangles present

$$\alpha = \tan^{-1}\left(\frac{8}{6}\right) \quad \Rightarrow \quad \alpha = 53.13°$$

$$\beta = 180 - 2\alpha \quad \Rightarrow \quad \beta = 73.74°$$

With this information, we can now write our equilibrium equations

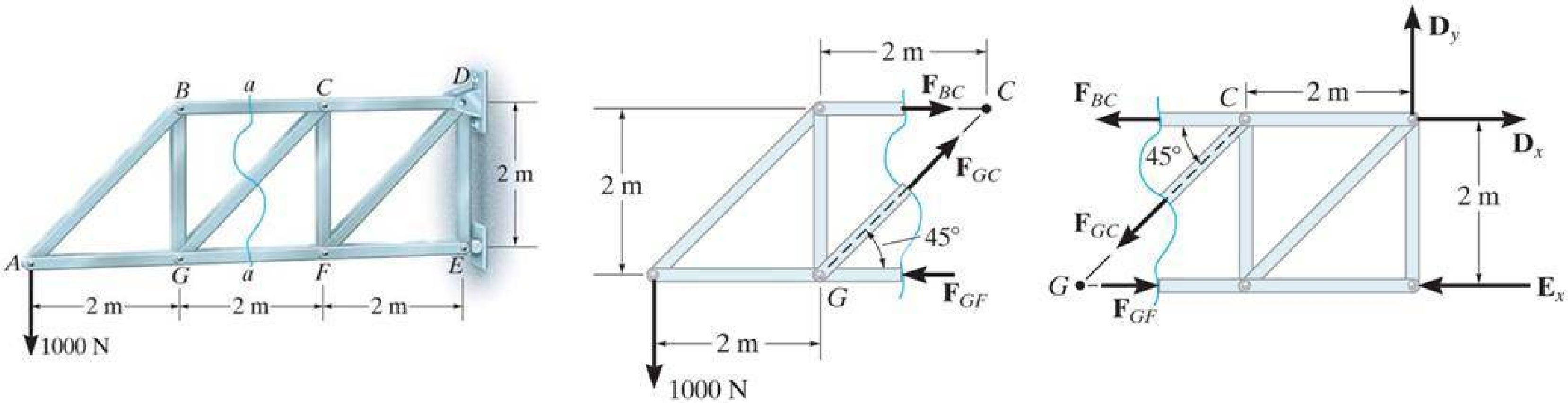
$$\sum F_x = F_{EB}\cos(90-\beta) - F_{CE}\cos(90-\alpha) = 0 \quad \Rightarrow \quad F_{EB} = 750\ N$$

Tension

$$\sum F_y = F_{EA} - F_{DE} - F_{CE}\sin(90-\alpha) - F_{EB}\sin(90-\beta) = 0 \quad \Rightarrow \quad F_{EA} = 1750\ N$$

Compression

Steps for analysis: Method of sections

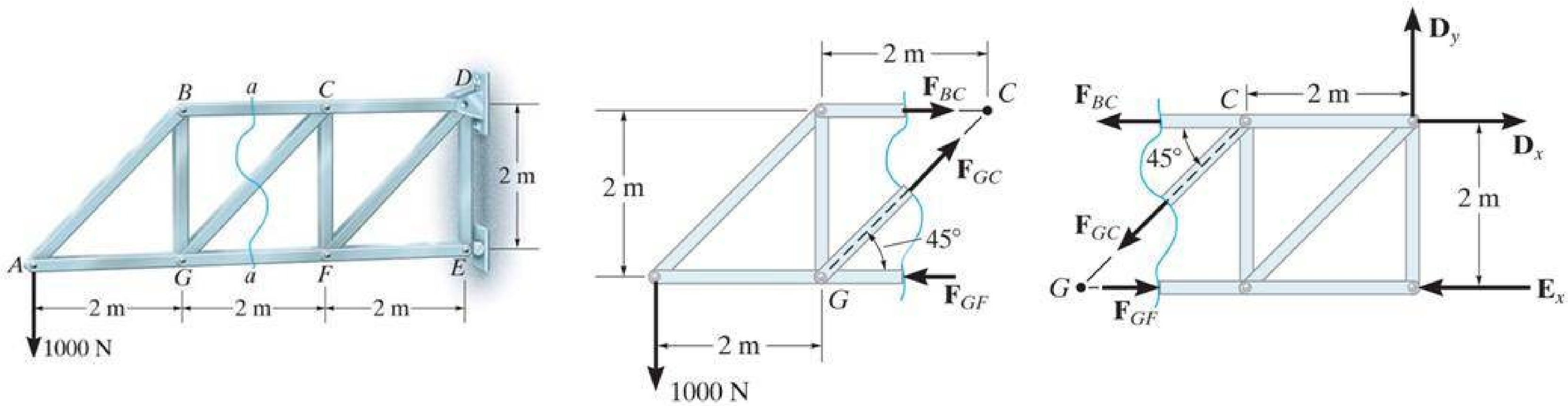


1. Decide how you need to “cut” the truss. This is based on:
   a) where you need to determine forces, and, b) where the total number of unknowns does not exceed three (in general).

2. Decide which side of the cut truss will be easier to work with (minimize the number of reactions you have to find).

3. If required, determine any necessary support reactions by drawing the FBD of the entire truss and applying the E-of-E.

Steps for analysis: Method of sections (Continuation)

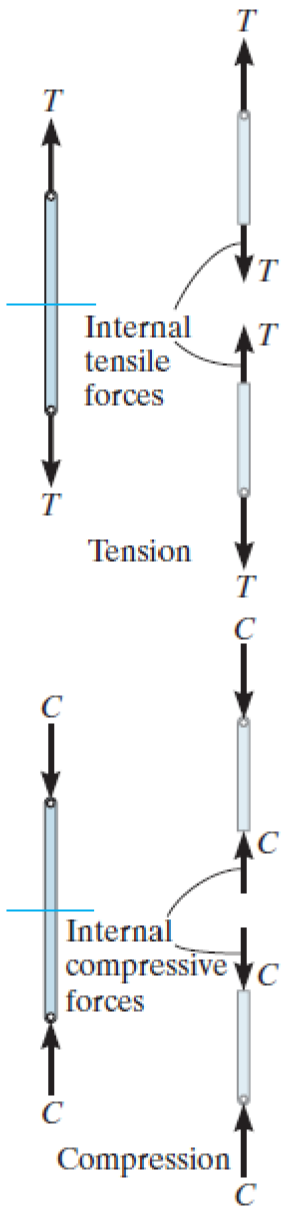


4. Draw the FBD of the selected part of the cut truss. Indicate the unknown forces at the cut members. Initially we may assume all the members are in tension, as we did when using the method of joints. Upon solving, if the answer is positive, the member is in tension as per our assumption. If the answer is negative, the member must be in compression. (Please note that you can also assume forces to be either tension or compression by inspection as was done in the figures above.)

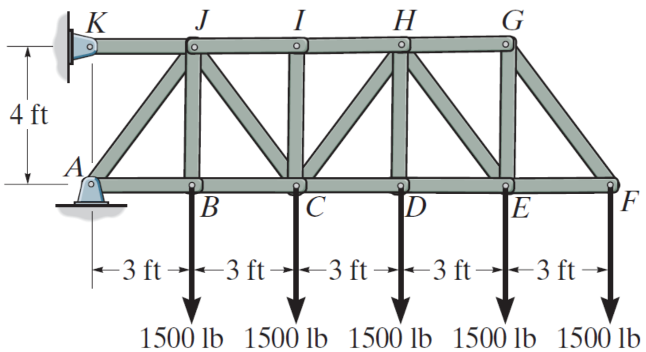
Internal Forces: Tension - Compression

Members are assumed to be in *tension when* they are subjected to a “pull”.

Members are in *compression* when they are subjected to a “push”.



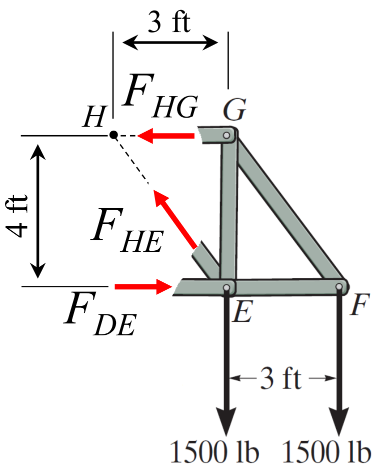
Example



Determine the forces in members *HG*, *HE*, and *DE* of the truss and state if the members are in tension or compression

In this case, the sections method is useful. The question is, where shall we "cut" the truss?

The most convenient "cut" is



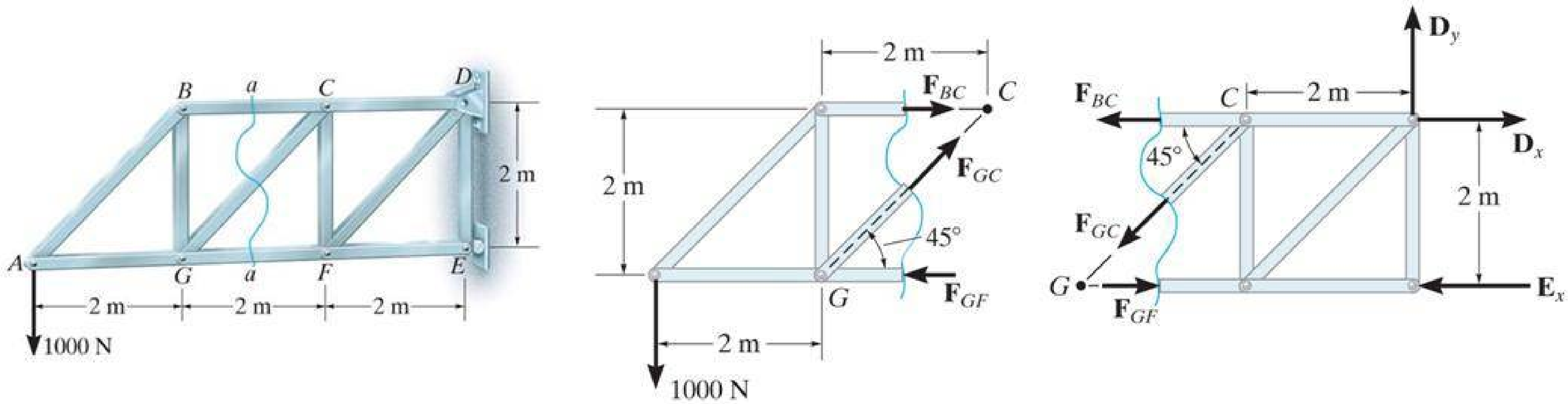
Using this FBD, we can easily calculate the unknown forces

$$\sum F_y = \frac{4}{5}F_{HE} - 1500 - 1500 = 0 \quad \Rightarrow \quad F_{HE} = 3750 \text{ lb} \quad \text{Tension}$$

$$\sum M_H = 4F_{DE} - (3)(1500) - (6)(1500) = 0 \quad \Rightarrow \quad F_{DE} = 3375 \text{ lb} \quad \text{Compression}$$

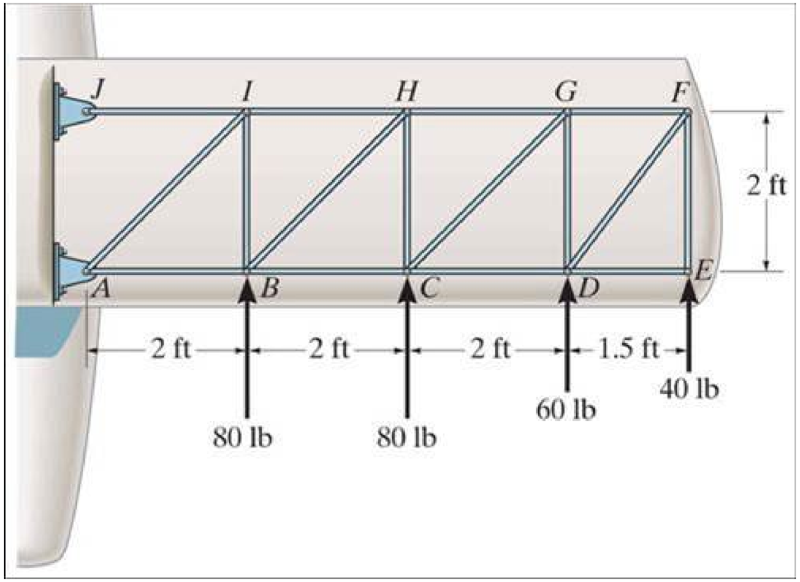
$$\sum F_x = F_{DE} - \frac{3}{5}F_{HE} - F_{HG} = 0 \quad \Rightarrow \quad F_{HG} = 1125 \text{ lb} \quad \text{Tension}$$

Steps for analysis: Method of sections (Continuation)



5. Apply the equations of equilibrium (E-of-E) to the selected cut section of the truss to solve for the unknown member forces. Please note, in most cases it is possible to write one equation to solve for one unknown directly. So look for it and take advantage of such a shortcut!

Breakout room (15 min)



Given:The internal drag truss for the wing of a airplane is subjected to the forces shown.

Find:The force in members IH,BH, and BC.

Plan:

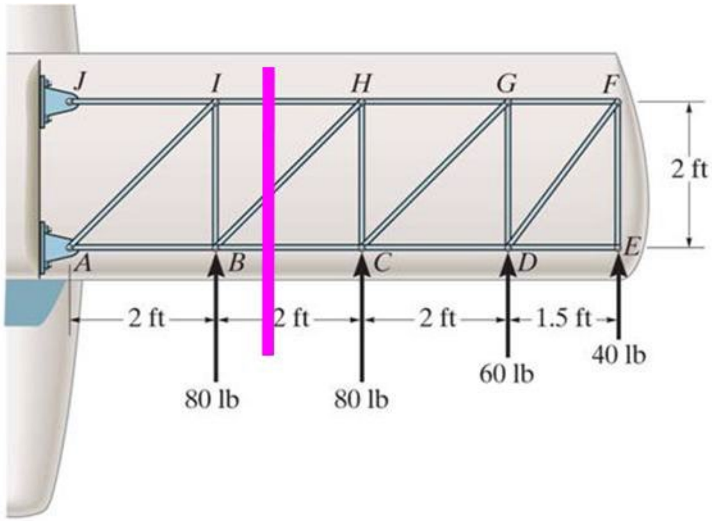
a) Take a cut through the members IH, BH, and BC.

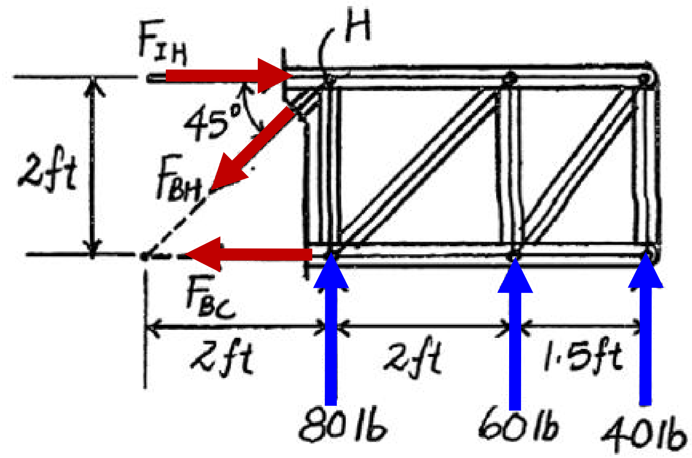
b) Analyze the right section (no support reactions!).

c) Draw the FBD of the right section.

d) Apply the equations of equilibrium (if possible, try to do it so that every equation yields an answer to one unknown.

Solution





$$+\uparrow \Sigma F_Y = 80 + 60 + 40 - F_{BH} \sin 45° = 0;$$

$\underline{F_{BH} = 255 \text{ lb}}$ Tension

$$\circlearrowright + \Sigma M_H = - F_{BC} (2) + 60 (2) + 40 (3.5) = 0;$$

$\underline{F_{BC} = 130 \text{ lb}}$ Tension

$$+ \rightarrow \Sigma F_X = F_{IH} - 130 - 255 \cos 45° = 0;$$ $\underline{F_{IH} = 310 \text{ lb}}$ Compression