ENGR 65 Circuit Theory

Lecture 2: Voltage, Current, Power, Energy, and Passive Sign Convention

Topics

- Definitions of current, voltage, power, and energy
- The reference polarities and directions
- The passive sign convention

Covered in Sections 1.4, 1.5, and 1.6

- http://www.youtube.com/watch?v=D2monVkCkX4
- https://www.youtube.com/watch?v=1xPjES-sHwg
- https://www.youtube.com/watch?v=8gvJzrjwjds
- https://www.youtube.com/watch?v=J4Vq-xHqUo8

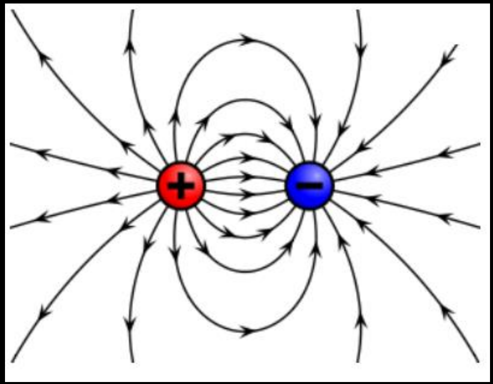
Electric Charge

Some properties of electric charge:

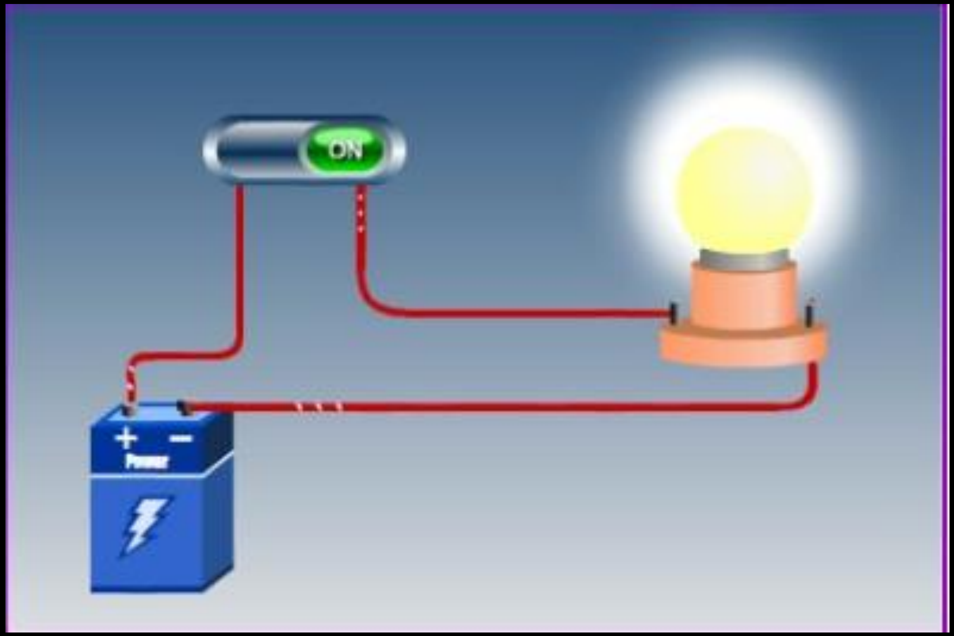
1. The electric charge is bipolar: positive and negative.
2. Like charges repel each other while unlike charges attract each other
3. The electric charge is quantized. Its SI unit is coulomb (C). It is integral multiples of the elementary charge (an electron or a proton, for example: p $= 1.6022 \times 10^{-19} C \; or$ e $= -1.6022 \times 10^{-19}\, C$)

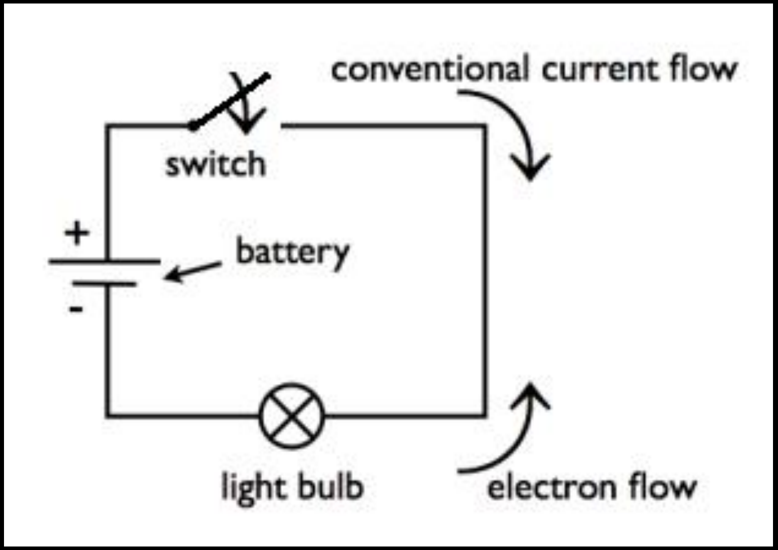
$$1\, coulomb = \frac{1}{1.6022\times10^{-19}} = 6.2414 \times 10^{18}\;\; electron\ (proton).$$

4. The separation of charge (voltage) and charge in motion (current) cause the electrical effects.



A Simple Electric Circuit





Current

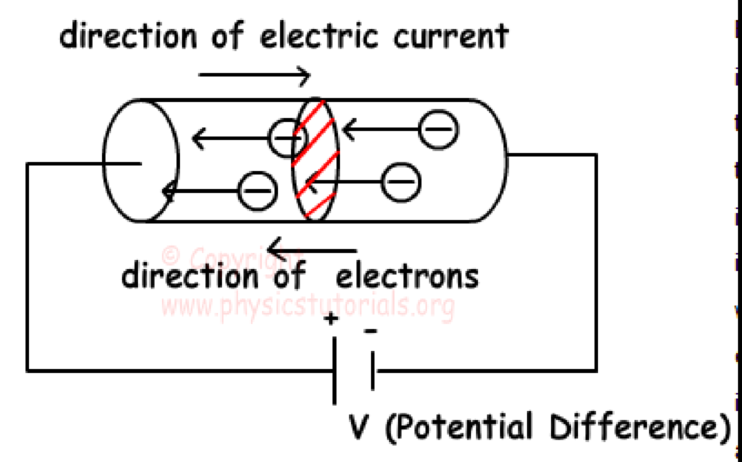
- Current is the net flow of **positive charge** per unit time passing through a cross section, perpendicular to the flow direction, of conductive materials.

$$i(t) = \frac{dq(t)}{dt}, \qquad (\Delta i(t) = \frac{\Delta q(t)}{\Delta t})$$

i: the current in amperes (A),

q: the charges in coulombs (C),

t: the time in seconds **(s).**

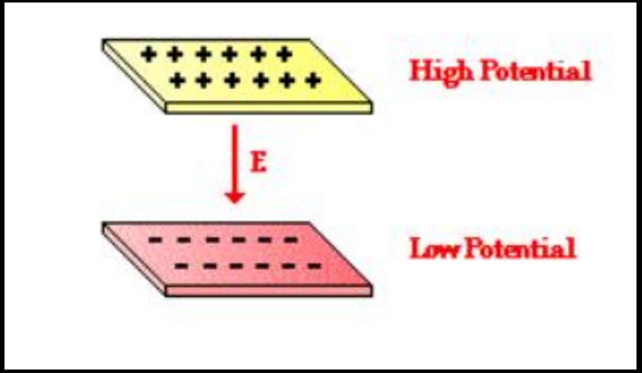


http://www.physicstutorials.org/home/electric-current/electric-current-and-flow-of-charge

Voltage

Voltage is the electric potential difference. It is the energy expended per unit of charge against a static electric field to move the charge between two points. It is defined as the ratio of the energy per unit charge.

$$v(t) = \frac{dw(t)}{dq}$$



v: the voltage in volts (V),
w: the energy in joules (J),
q: the charge in coulombs(C).

http://www.physicsclassroom.com/class/circuits/u9l2a.cfm

.

Power

Power is the time rate of delivering or absorbing energy in an object.

$$p(t) = \frac{dw(t)}{dt}$$

p: the power in watts (W),
w: the energy in joules (J),
t: the time in seconds **(s).**

Let's see how the electric power is related to current and voltage,

$$p(t) = \frac{dw}{dt} = \left(\frac{dw}{dq}\right)\left(\frac{dq}{dt}\right) = v(t)i(t),$$

p: the power in watts (W),
v: the voltage in volts (V),
i: the current in amperes(A).

If $p = \frac{dw}{dt}$, which of statement below is correct:

A. $dw = pdt$

B. $\int_{w(t_0)}^{w(t)} dy = \int_{t_0}^{t} pdx$

C. $w(t) = \int_{t_0}^{t} pdx + w(t_0)$

D. All of the above

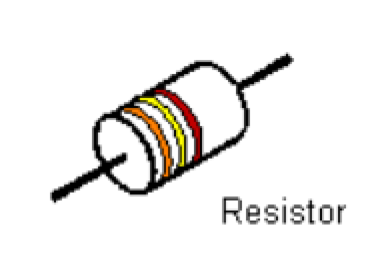
Energy

- **Electrical energy** is energy derived from electric potential energy before it is delivered to the end-use.

$$w(t) = \int_{t_0}^{t} p dx + w(t_0)$$

p: the power in watts (W),
w: the energy in joules (J),
t: the time in seconds **(s).**

Circuit Elements



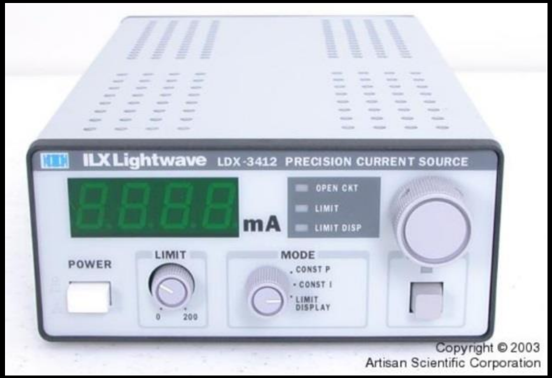
5 basic circuit elements

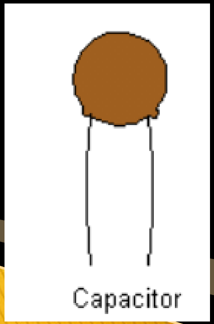
- Passive elements

(do not generate power) { Resistors, Inductors, Capacitors }

- Active elements

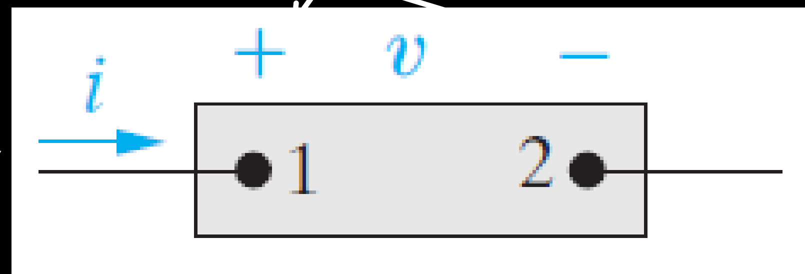
(If generating power) { Voltage sources, Current sources }





Ideal Basic Circuit Elements

The *reference* polarity of the voltage



The *reference* direction of the current

Three attributes:

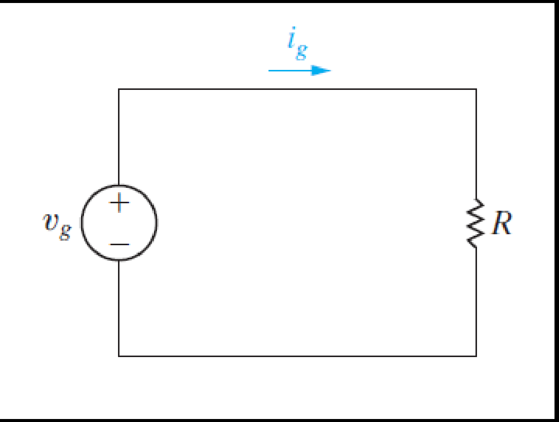
1. Has two terminals
2. Be described in terms of current and/or voltage
3. Cannot be divided into other elements

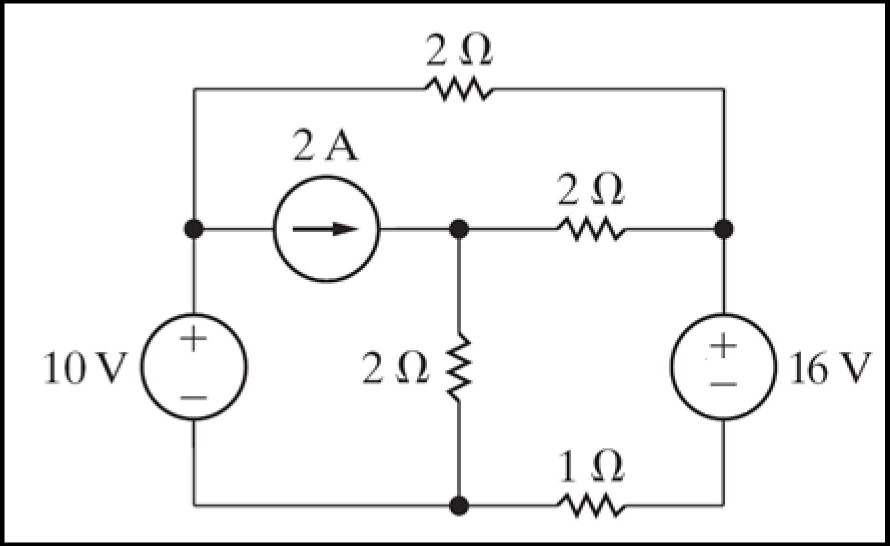
Why Do We Need a Reference Polarity or Direction?

The reason is that the actual polarity of a voltage or the actual direction of a current in a circuit may not be known before solving the circuit. We need to assign a reference polarity/direction for each variable to help us figure out what the actual polarity and direction would be. The reference polarity and direction can be chosen arbitrarily (your preference).

After the circuit is solved, for example, if a current has negative value, which means that the actual current direction through the circuit element is opposite the chosen reference direction. The positive value means that the current actual flow direction is same as the assigned reference direction. This fact can also be applied to voltage.

Why Do We Need a Reference Polarity or Direction?





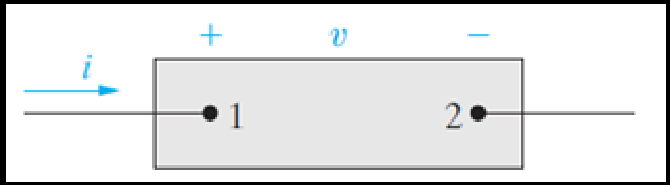
In above circuit, it is easy for us to identify the actual current direction.

However, in the circuit above, what are current actual directions in these 2Ω resistors?

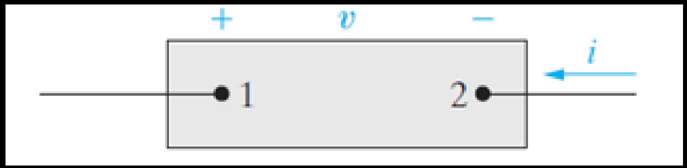
The Passive Sign Convention

Passive sign convention

Whenever the reference direction of the current in an element is in the direction of the reference of voltage drop across the element, use a positive sign in any expression that relates the voltage to the current. Otherwise, use a negative sign.

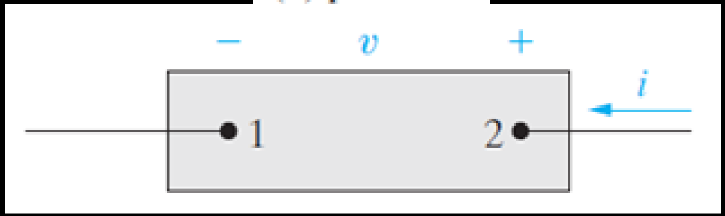


(a) $p = vi$



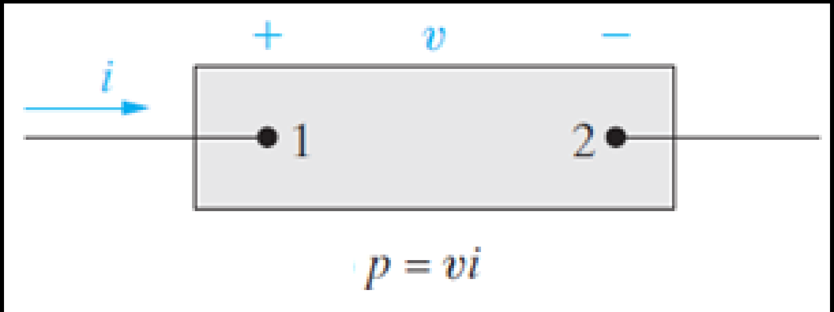
(b) $p = -vi$

(c) $p = -vi$



(d) $p = vi$

Example #1



Given $\begin{cases} v(t) = 12\ V \\ i(t) = 5e^{-t}\ A \end{cases}$

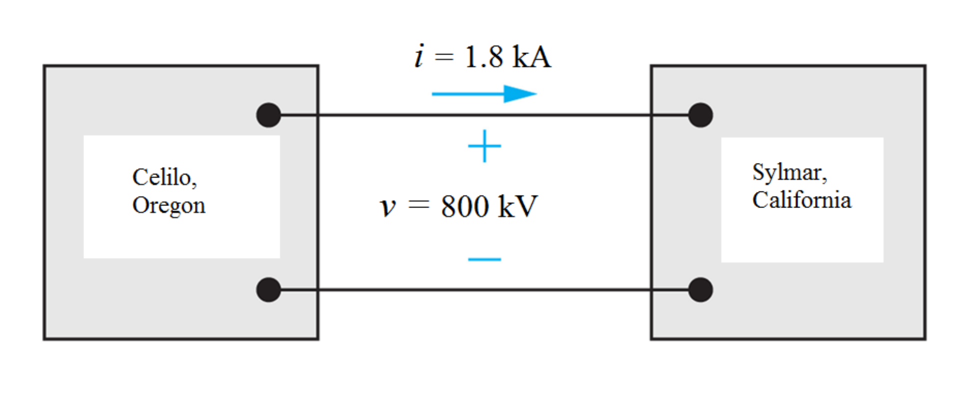
Find $p(t)$ and energy $\Delta w(t)$ that is absorbed by the element for $3s < t < 4\ s$.

$$p(t) = +v(t)i(t) = 12 \times 5e^{-t} = 60e^{-t}\ W$$

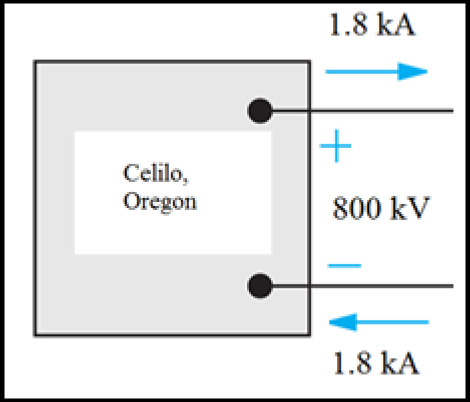
$$\Delta w(t) = w(4) - w(3) = \int_3^4 pdt = \int_3^4 60e^{-t}dt$$

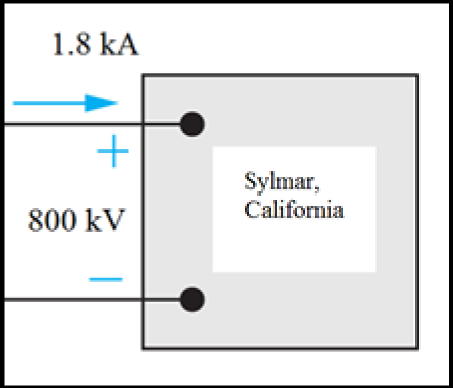
$$= -60e^{-t}\big|_3^4 = 1.89\ J$$

Example #2



A high-voltage direct-current (DC) transmission line between Celilo, Oregon and Sylmar, California is operating at 800 kV and carrying 1800 A. Find power in megawatts at the end of the lines of both Oregon and California.





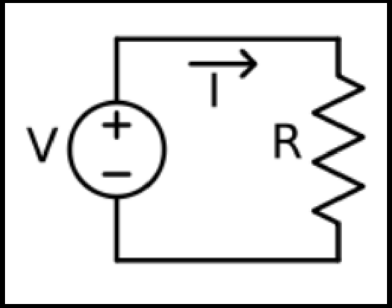
$$p_c = -vi = -800 \times 1800 = -1.44\ MW$$

$$p_c = vi = 800 \times 1800 = 1.44\ MW$$

Power (Energy) Absorbed and Delivered

- Power absorbed means the power is absorbed by an electrical system and converted to nonelectrical power. **The value of the power is greater than 0.**
- Power delivered means that power is taken from somewhere else, converted, and delivered to the electrical system. **The value of the power is less than 0.**

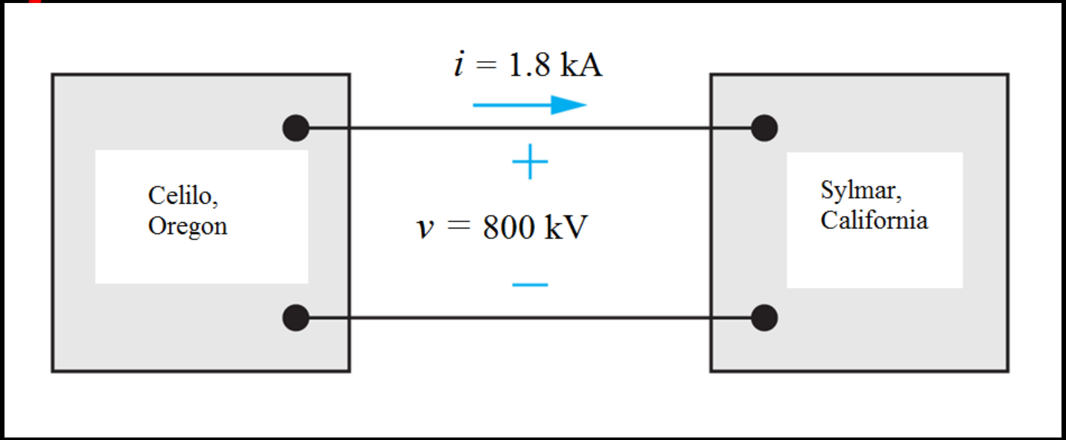
Deliver or supply power



Absorb or dissipate power

In a circuit, if the power delivered to the circuit is equal to the power absorbed by the circuit, the circuit is called **balanced.**

Example #3



State the direction of power flow.

$p_c = -1.44\ MW\ < 0.$ It delivers power to the system

$p_s = 1.44\ MW\ > 0.$ It absorbs power from the system.

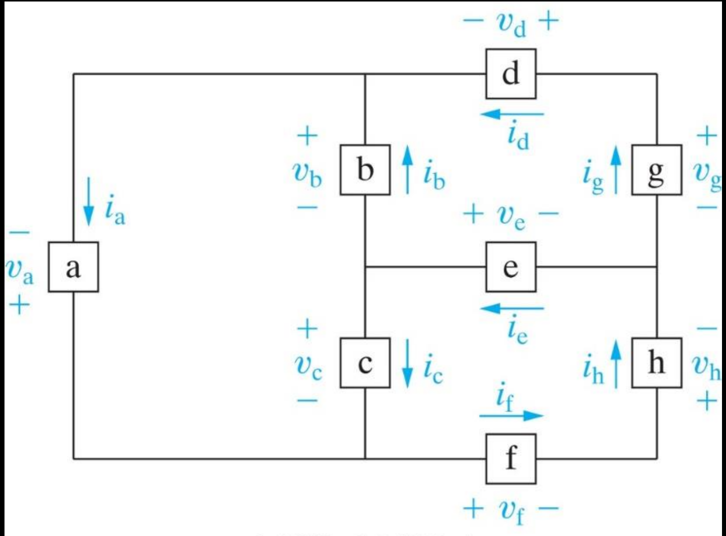
$p_c = p_s.$ Therefore, the system is balanced.

The power flows from Celilo to Sylmar

Example #4

TABLE 1.4 Voltage and current values for the circuit in Fig. 1.7.

Component	v(V)	i(A)
a	120	−10
b	120	9
c	10	10
d	10	1
e	−10	−9
f	−100	5
g	120	4
h	−220	−5



Use the passive sign convention to calculate the power associated with each component.

$$p_a = -v_a i_a = -(-10)(120) = 1200\ W$$

$$p_b = -v_b i_b = -(9)(120) = -1080\ W$$

$$p_c = +v_c i_c = (10)(10) = 100\ W$$

$$p_d = +v_d i_d = (10)(1) = 10\ W$$

$$p_e = -v_e i_e = -(-10)(-9) = -90\ W$$

$$p_f = +v_f i_f = (-100)(5) = -500\ W$$

$$p_g = -v_g i_g = -(120)(4) = -480\ W$$

$$p_h = +v_h i_h = (-220)(-5) = 1100\ W$$

$$p_{abs} = 1200 + 100 + 10 + 1100 = 2410\ W$$

$$p_{del} = -1080 - 90 - 500 - 480 = -2150\ W$$

UPPERCASE vs lowercase

- In this course, lowercase variables are used for quantities that vary with time. For example, voltage, current, energy, and power are assumed to be able to change with time, and so are represented as lowercase variables, for example, v$, i, p$.
- Uppercase variables are used for quantities that do not vary with time. Resistance, capacitance, and inductance are assumed to be constant in this course, and so are represented as uppercase, for example, $R, L,$ and C.

Summary

- How to express voltage, current, power, and energy quantitatively
- The reference and actual polarities and directions
- The passive sign convention

- In next class, we are going to discuss:
- Dependent and independent power sources
- Resistors and resistance
- Ohm’s law