ENGR 065 Electric Circuits

Lecture 12: Inductors and Capacitors

Today's Topics

- Inductors and inductance
- Capacitors and Capacitance
- The $v - i$ and $i - v$ equations for inductors and capacitors
- Power and energy in inductors and capacitors
- Series-parallel combinations of inductors and capacitors

These topics are covered in Sections 6.1, 6.2, and 6.3.

https://www.youtube.com/watch?v=ukBFPrXiKWA
https://www.youtube.com/watch?v=f_MZNsEqyQw

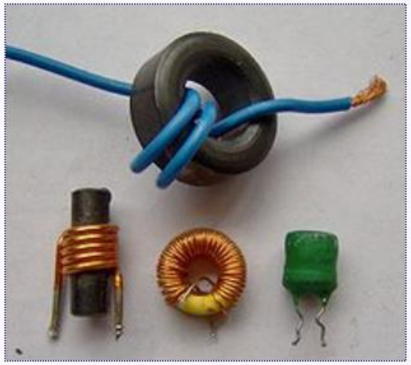
Introductions – Inductors, Capacitors, and Resistors

- Energy can be stored in both electric and magnetic fields.
- Inductors store energy in its magnetic field.
- Capacitors store energy in its electric filed.
- Resistors do not store energy, but dissipate energy as heat.

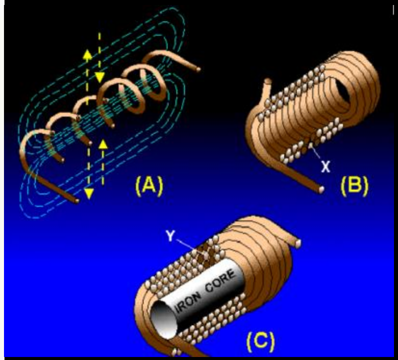
Introductions – Inductor and Inductance

- An inductor is a passive two-terminal electrical component used to store energy in magnetic field.
- An inductor's ability to store magnetic energy is measured by its inductance, measured in units of henries.
- Electric current flowing through conductors generate magnetic flux proportional to the current. A change in this current creates a corresponding change in magnetic flux. Faraday's law indicates that the change in magnetic flux generates electromotive force (voltage) that opposes this change in current.

Inductor



A selection of low-value inductors



Inductors

- The voltage across the terminals of an inductor is proportional to the time rate of change of the current in the inductor.
- Inductor $v - i$ equation:

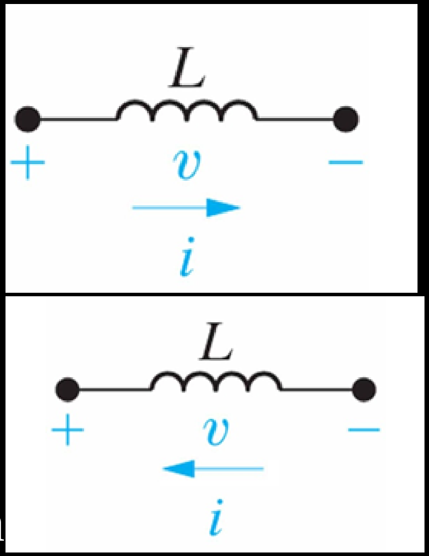
$$\boxed{v(t) = L\frac{d}{dt}i(t)}$$

where v is the voltage measured in volts (V),

i is the current measured in amperes (A),

L is the inductance measured in henries (H)

t is the time measured in seconds (s)



If the current reference direction is in the direction of th
equation is written with a negative sign:

$$\boxed{v(t) = -L\frac{d}{dt}i(t)}$$

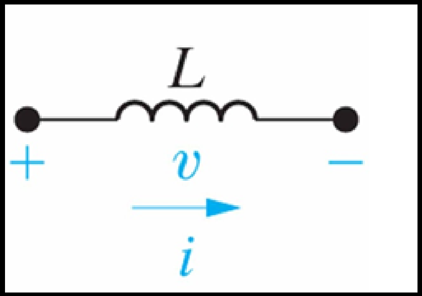
Inductors

From $v(t) = L\frac{d}{dt}i(t)$, we know:

- When the current in a circuit is constant, the voltage across an ideal inductor is zero. The inductor behaves as a short circuit.
- The current in an inductor cannot change instantaneously.

The inductor $i - v$ equation:

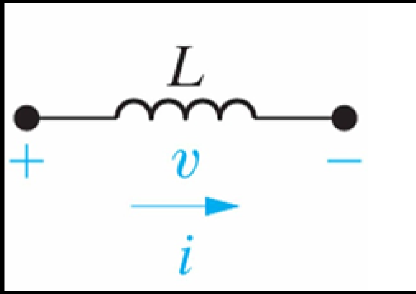
$$i(t) = \frac{1}{L}\int_{t_0}^{t} v(\tau)d\tau + i(t_0)$$



In many applications, $t_0 = 0$, we have:

$$i(t) = \frac{1}{L}\int_{0}^{t} v(\tau)d\tau + i(0)$$

Power and Energy in Inductors



- Power in an inductor:

$$p = vi, \qquad p = Li\frac{di}{dt}, \qquad p = v[\frac{1}{L}\int_{t_0}^{t} v d\tau + i(t_0)]$$

- Energy in an inductor:

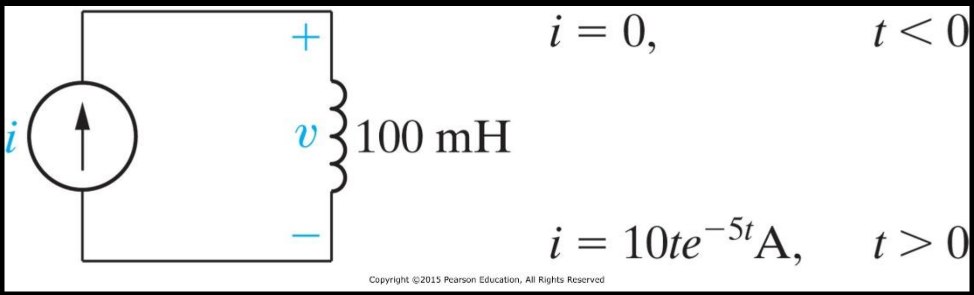
$$p = \frac{dw}{dt} = Li\frac{di}{dt} \text{ and } dw = Lidi$$

- If assuming zero energy corresponds to zero current in the inductor, we have, at $t_0 = 0$:

$$\int_0^w dx = L\int_0^i y dy$$

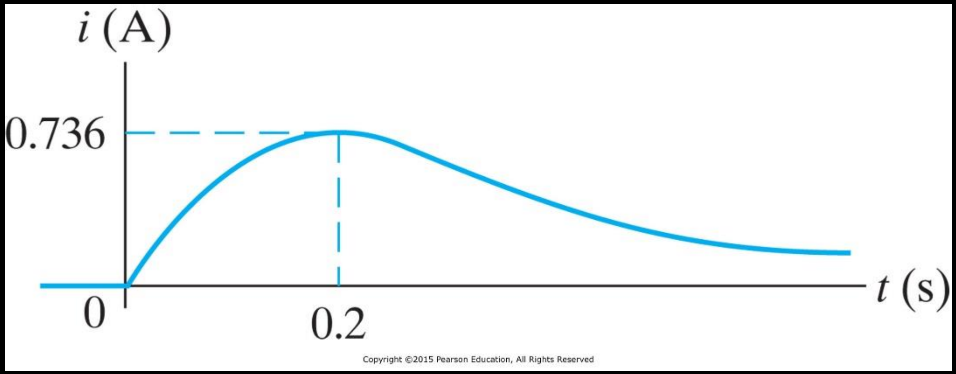
$$\boxed{w = \frac{1}{2}Li^2}$$

Example #1



The independent current source in the circuit shown on the left generates zero current for $t < 0$ and a pulse $10te^{-5t}$ A, for $t > 0$.

a) Sketch the current waveform.

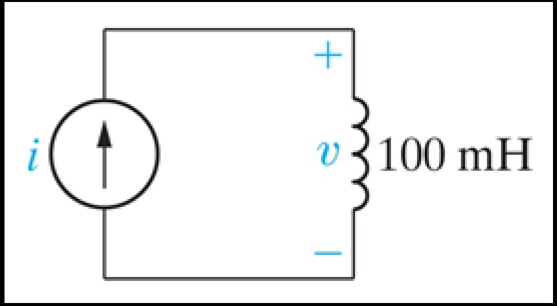


b) At which instant of time is the current maximum?

$$\frac{di}{dt} = 10\left(-5te^{-5t} + e^{-5t}\right) = 10e^{-5t}(1 - 5t)\ A/s$$

Let $\frac{di}{dt} = 0$, we have $t = 0.2s\ and\ i_{max} = 10te^{-5t} = 0.736\ A$

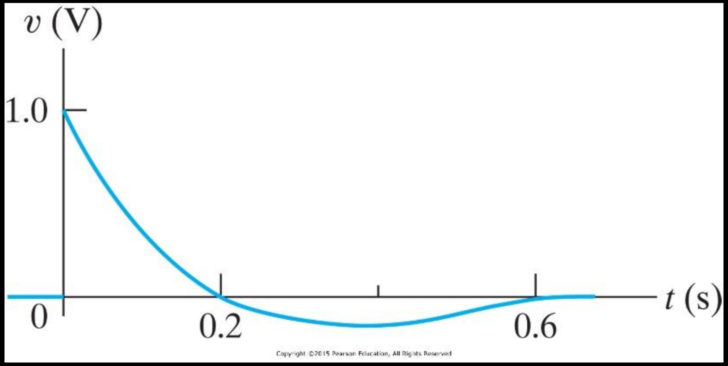
Example #1 (ctn'd)



c) Express the voltage across the terminals of the 100 mH inductors as a function of time. $v = 0, t < 0$

$$v = L\frac{di}{dt} = 0.1 \times 10 \times e^{-5t}(1 - 5\mathrm{t}) = e^{-5t}(1 - 5\mathrm{t})\ \mathrm{V}, t \geq 0;$$

d) Sketch the voltage waveform.



e) Are the voltage and current at a maximum at the same time? No.

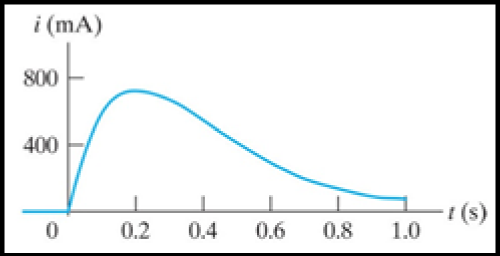
f) At what instant time does the voltage change polarity?

At 0.2 s. It is the time when $\frac{di}{dt}$ changes the sign.

g) Is there ever an instantaneous change in voltage across the inductor?

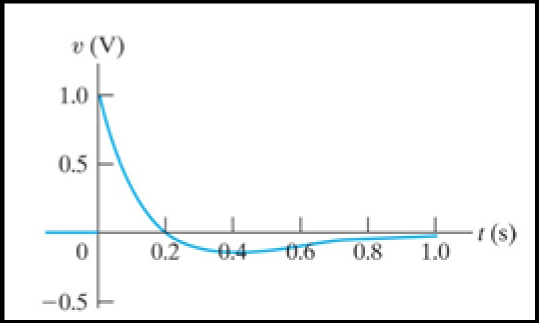
Yes. At $t = 0$. So not like the current, the voltage can change instantaneously across the terminals of an inductor.

Example #1 (ctn'd)



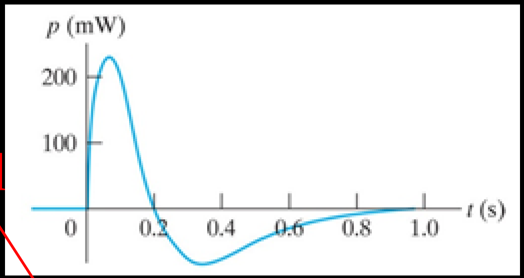
h) Sketch the power and energy versus time

$$p = vi \text{ and } w = \frac{1}{2}Li^2$$



i) In what time interval is energy being stored in the inductor?

When the energy curve is increasing, the energy is stored.

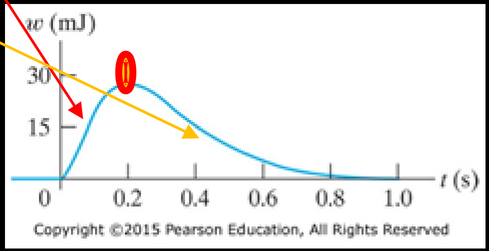


j) In what time interval is energy being extracted from the inductor?

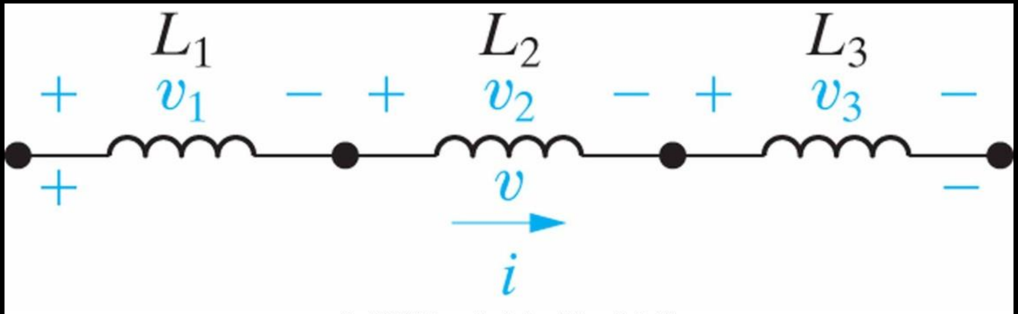
When the energy curve is decreasing, the energy is extracted

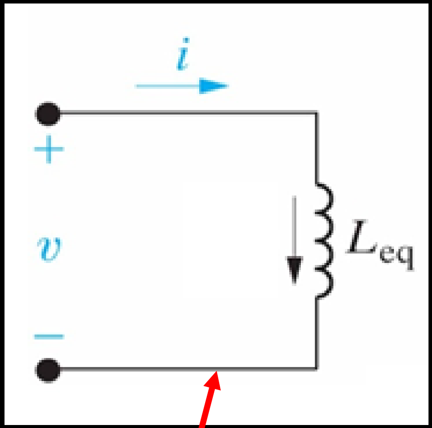
k) What is the maximum energy stored in the inductor?

From $w = \frac{1}{2}Li^2$, the energy reaches its maximum when the current reaches its maximum value, $i = 0.736\ A$, which is 27.07 mJ



Series Connections of Inductors





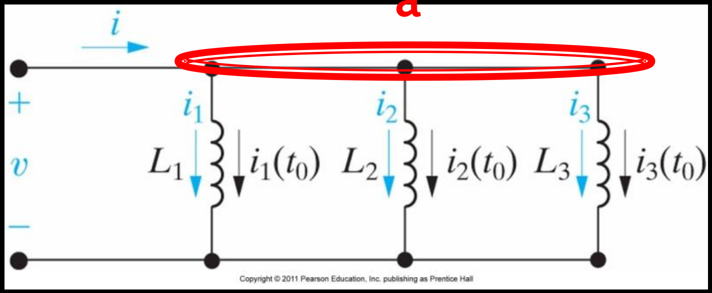
The inductors connected in series have the same current flowing through the inductors. The voltage drop across each inductor is:

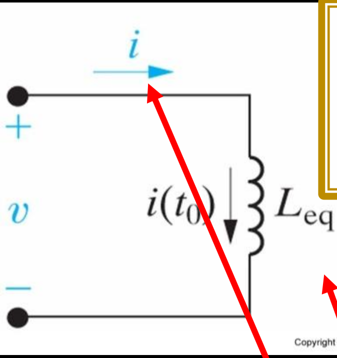
$$v_1 = L_1 \frac{di}{dt}, \qquad v_2 = L_2 \frac{di}{dt}, \qquad \text{and} \qquad v_3 = L_3 \frac{di}{dt}$$

$$v = v_1 + v_2 + v_3 = (L_1 + L_2 + L_3)\frac{di}{dt} = L_{eq}\frac{di}{dt}$$

Equivalent inductance is $\boxed{L_{eq} = L_1 + L_2 + L_3}$

Parallel Connections of Inductors





$$\boxed{\begin{aligned}\frac{1}{L_{eq}} &= \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} \\ i(t_0) &= i_1(t_0) + i_2(t_0) + i_3(t_0)\end{aligned}}$$

The current for each inductor is:

$$i_1 = \frac{1}{L_1}\int_{t_0}^{t} v d\tau + i_1(t_0)$$

$$i_2 = \frac{1}{L_2}\int_{t_0}^{t} v d\tau + i_2(t_0)$$

$$i_3 = \frac{1}{L_3}\int_{t_0}^{t} v d\tau + i_3(t_0)$$

Applying KCL at node a: $\quad i = i_1 + i_2 + i_3$

$$= \left(\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}\right)\int_{t_0}^{t} v d\tau + i_1(t_0) + i_2(t_0) + i_3(t_0)$$

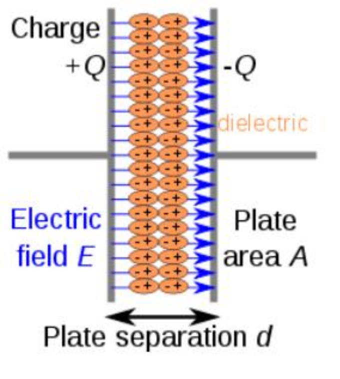
$$= \frac{1}{L_{eq}}\int_{t_0}^{t} v d\tau + i(t_0)$$

$$i = \frac{1}{L_{eq}}\int_{t_0}^{t} v d\tau + i(t_0)$$

Capacitors

- A capacitor is also a passive two-terminal electrical element that consists of two conductors separated by an insulator or dielectric material.
- Capacitance is the ability of a body to store electrical charge.
- Any body or structure that is capable of being charged, either with static electricity or by an electric current exhibits capacitance.

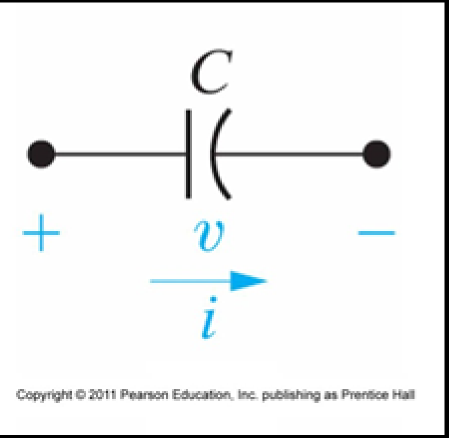
Miniature low-voltage capacitors, by a cm ruler



Capacitors

- The current is proportional to the rate at which the voltage across the capacitor varies with time:
- Capacitor $i - v$ *equation*:

$$\boxed{i(t) = C\frac{d}{dt}v(t)}$$



where i is current measured in amperes,

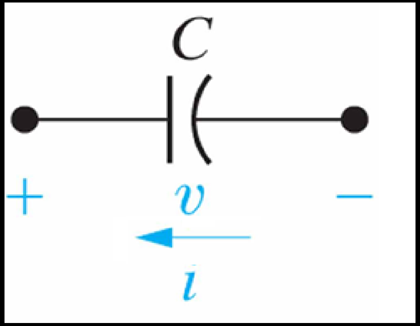
C is capacitance in farads,

v is voltage in volts, and

t is time in seconds.

If the reference direction of the current is in the direction of the voltage rise, the equation is written with a negative sign.

$$\boxed{i(t) = -C\frac{d}{dt}v(t)}$$



Capacitors

From $i(t) = C\frac{d}{dt}v(t)$

- If the voltage across the terminals of a capacitor is constant, the capacitor current is zero. The capacitor behaves as an open circuit.
- The voltage across the terminals of a capacitor cannot change instantaneously.

  - The capacitor $v{-}i$ equation:

$$v(t) = \frac{1}{C}\int_{t_0}^{t} i(\tau)d\tau + v(t_0)$$

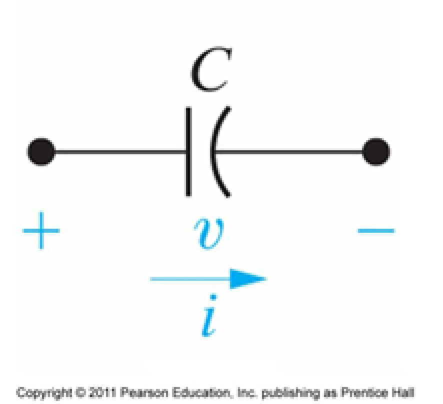
In many applications, $t_0 = 0$, we have:

$$v(t) = \frac{1}{C}\int_{0}^{t} i(\tau)d\tau + v(0)$$

Power and Energy in Capacitors

The power in a capacitor:

$$p = vi,\; p = Cv\frac{dv}{dt},\;\; p = i[\tfrac{1}{C}\int_{t_0}^{t} i d\tau + v(t_0)]$$



The energy stored in a capacitor:

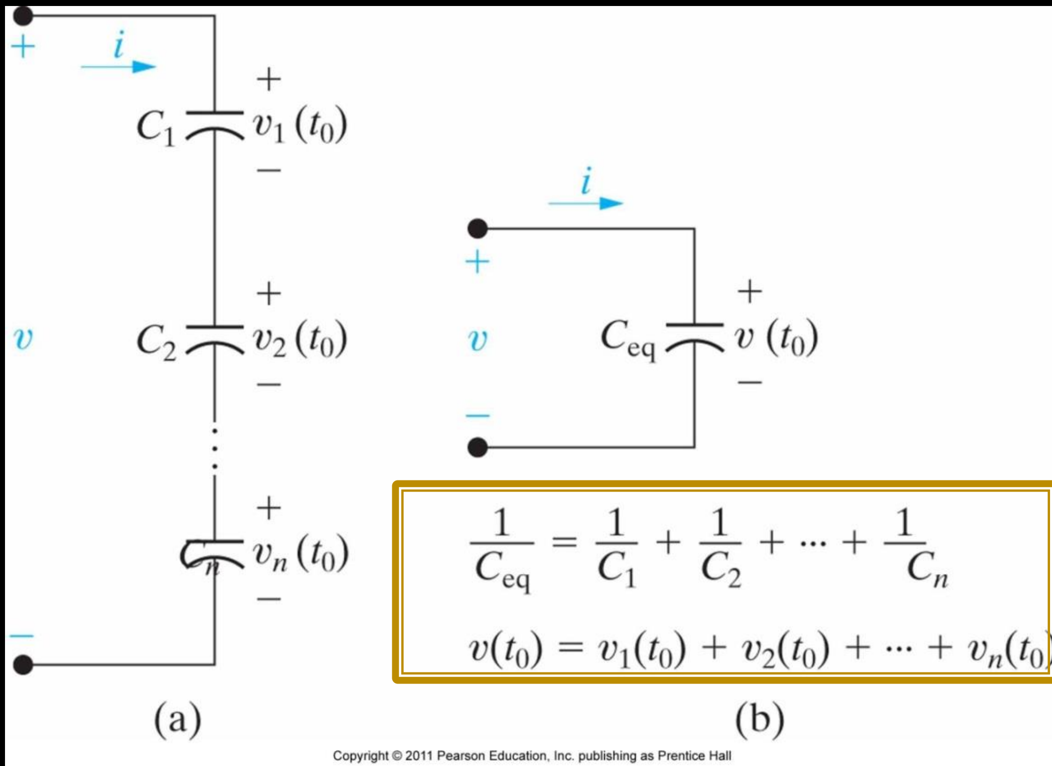
$$p = \frac{dw}{dt} = Cv\frac{dv}{dt},\;\; dw = Cvdv$$

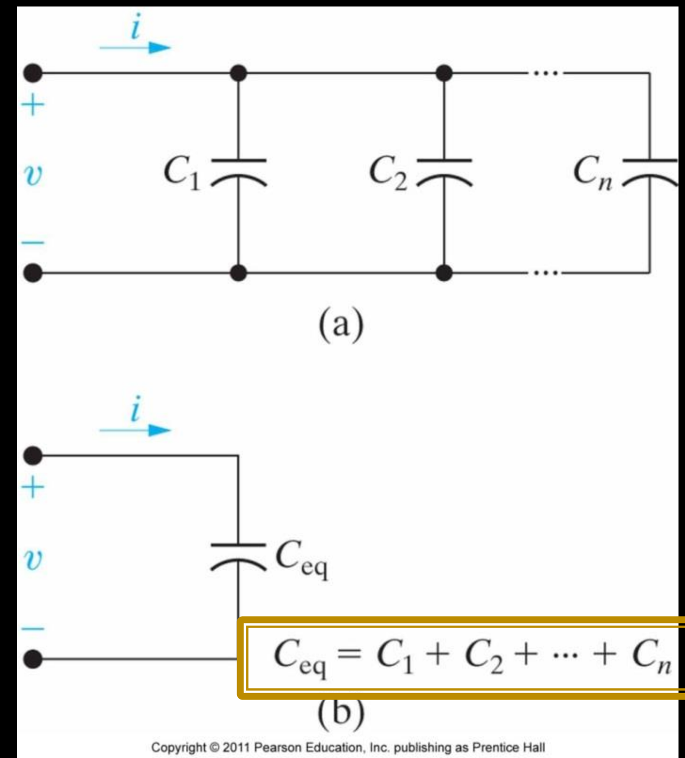
Taking integration on the both sides of $dw = Cvdv$, we have $\int_0^{w} dx = C\int_0^{v} ydy$

$$\boxed{w = \frac{1}{2}Cv^2}$$

Here we assume that the zero energy in the capacitor corresponds to zero voltage $at\ t_0 = 0$.

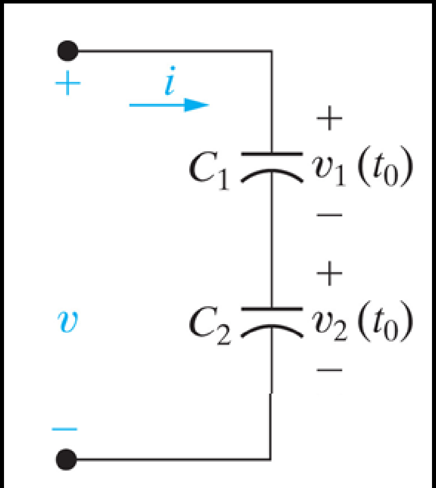
Series and Parallel Connections of Capacitors





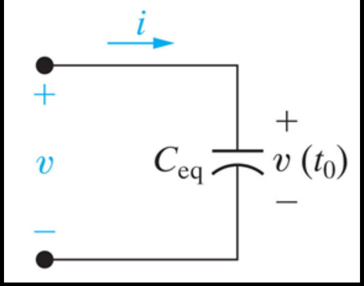
C_{eq} *is* called equivalent capacitance

Example #2



The current at the terminals of the two capacitors shown in the circuit is $240e^{-10t}\,\mu A$ for $t \geq 0$. $C_1 = 2\ \mu F, C_2 = 8\ \mu F$. The initial values of v_1 *and* v_2 are -10 V and -5 V, respectively.

1. Calculate the total energy trapped in the capacitors as $t \rightarrow \infty$.
2. What is the equivalent circuit of the two series-connected capacitors?



From $v(t) = \frac{1}{C}\int_{t_0}^{t} i(\tau)d\tau + v(t_0)$, t_0=0, we know:

$$v_1 = 0.5 \times 10^6 \times \int_0^t 240 \times 10^{-6} e^{-10x}\,dx - 10 = -12e^{-10t} + 2\text{ V}$$

$$v_2 = 0.125 \times 10^6 \times \int_0^t 240 \times 10^{-6} e^{-10x}\,dx - 5 = -3e^{-10t} - 2\text{ V}$$

$$v_1(\infty) = 2\ V, v_2(\infty) = -2\ V,$$

$$w_1 = \frac{1}{2}C_1 v_1(\infty)^2 = \frac{1}{2}(2)(4) \times 10^{-6} = 4\ \mu J$$

$$w_2 = \frac{1}{2}C_2 v_2(\infty)^2 = \frac{1}{2}(8)(4) \times 10^{-6} = 16\ \mu J$$

$$w = 4 + 16 = 20\ \mu J$$

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = \frac{2 \times 8}{2 + 8} = 1.6\ \mu F$$

$$v_0(0) = -10 - 5 = -15\ V$$

Summary

1. An inductor does not permit an instantaneous change in its terminal current.
2. An inductor permits an instantaneous change in its terminal voltage.
3. An inductor behaves as short circuit in the presence of a constant terminal current.

Inductance is a linear circuit parameter that relates the voltage induced by a time-varying magnetic field to the current producing the filed.

Inductors

$$v = L\frac{di}{dt} \quad \text{(V)}$$

$$i = \frac{1}{L}\int_{t_0}^{t} v\, d\tau + i(t_0) \quad \text{(A)}$$

$$p = vi = Li\frac{di}{dt} \quad \text{(W)}$$

$$w = \frac{1}{2}Li^2 \quad \text{(J)}$$

Series-Connected

$$L_{eq} = L_1 + L_2 + \cdots + L_n$$

Parallel-Connected

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \cdots + \frac{1}{L_n}$$

$$i(t_0) = i_1(t_0) + i_2(t_0) + \cdots + i_n(t_0)$$

Summary

1. An capacitor does not permit an instantaneous change in its terminal voltage.
2. An capacitor permits an instantaneous change in its terminal current.
3. An capacitor behaves as open circuit in the presence of a constant terminal voltage.

Capacitance is a linear circuit parameter that relates current induced by a time-varying electric field to the voltage producing the filed.

Capacitors

$$v = \frac{1}{C}\int_{t_0}^{t} i\,d\tau + v(t_0) \quad \text{(V)}$$

$$i = C\frac{dv}{dt} \quad \text{(A)}$$

$$p = vi = Cv\frac{dv}{dt} \quad \text{(W)}$$

$$w = \frac{1}{2}Cv^2 \quad \text{(J)}$$

Series-Connected

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_n}$$

$$v(t_0) = v_1(t_0) + v_2(t_0) + \cdots + v_n(t_0)$$

Parallel-Connected

$$C_{eq} = C_1 + C_2 + \cdots + C_n$$