# HW4 R

### February 16, 2020

```
[63]: set.seed(123)
    library(margins)
    library(repr)
    library(ggplot2)
    library(rsample)
    library(dplyr)
    library(purrr)
    library(splines)
    library(msmtools)
    options(repr.plot.width=5, repr.plot.height=4)
[77]: gss_train = read.csv('data/gss_train.csv')
    gss_test = read.csv('data/gss_test.csv')
```

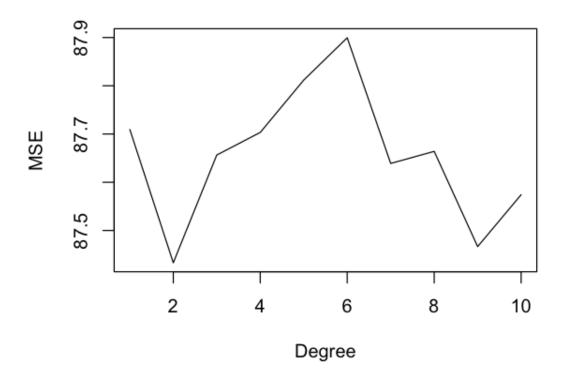
## 0.1 Egalitarianism and Income

### 0.1.1 1. Polynomial Regression

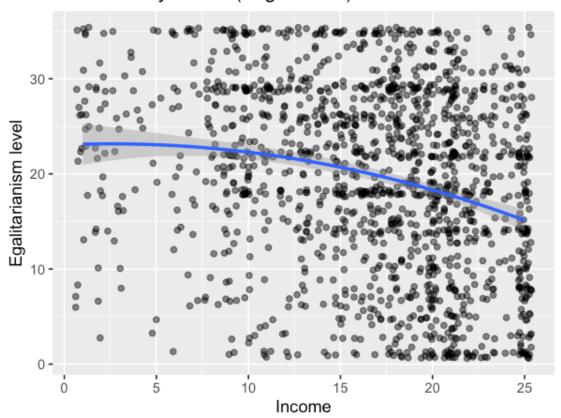
```
[4]: k = 10
fold = sample(k, nrow(gss_train), replace = TRUE)

mse = c()
for(j in 1:k){
    sub_mse = c()
    for(i in 1:k){
        test_set = gss_train[fold==i,]
            train_set = gss_train[fold!=i,]
        model = lm(egalit_scale ~ poly(income06, j), data = train_set)
        pred = predict(model, test_set)
        sub_mse = c(sub_mse, mean((pred - test_set$egalit_scale)^2))
    }
    mse = c(mse, mean(sub_mse))
}
```

```
[4]: plot(mse, type = 'l', xlab = 'Degree', ylab = 'MSE')
```



# Best Fit Polynomial (degree of 2)



```
[134]: best_model = lm(egalit_scale ~ stats::poly(income06, 2), data = train_set)
       summary(margins(best_model))
          factor | AME
                            SE
                                                                lower
                                                                           upper
                            0.04976301 -8.887136 6.270432e-19
       income06
                 -0.4422506
                                                               -0.5397843
                                                                           -0.3447169
[57]: cplot(best_model, 'income06', what = 'prediction')
         xvals
                  yvals
                            upper
                                     lower
             1 22.70467 24.93877 20.47057
      1
      2
             2 22.78141 24.72831 20.83451
```

3 22.82438 24.51346 21.13530

4 22.83360 24.29562 21.37159

5 22.80906 24.07630 21.54182 6 22.75076 23.85695 21.64457

7 22.65870 23.63828 21.67911

8 22.53288 23.41943 21.64632

9 22.37329 23.19701 21.54957 10 22.17995 22.96501 21.39489

11 21.95285 22.71570 21.19000

3

4

5

6 7

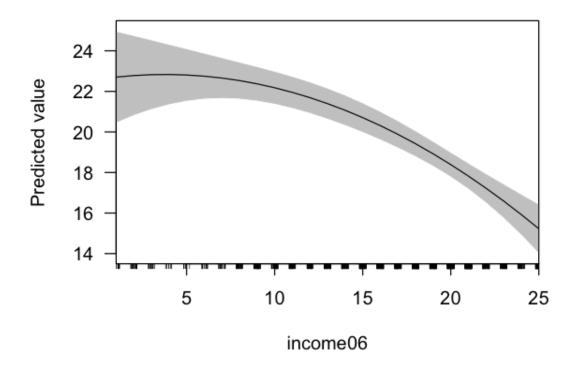
8

9

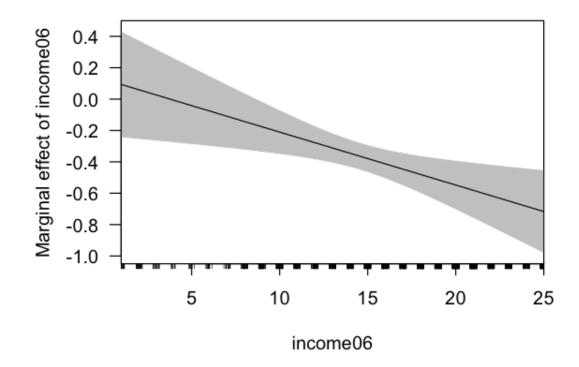
10

11

```
12
      12 21.69199 22.44122 20.94276
13
      13 21.39737 22.13490 20.65984
14
      14 21.06899 21.79188 20.34610
15
      15 20.70685 21.40932 20.00439
16
      16 20.31095 20.98645 19.63546
      17 19.88130 20.52486 19.23773
17
      18 19.41788 20.02914 18.80661
18
      19 18.92070 19.50777 18.33363
19
20
      20 18.38976 18.97336 17.80616
```



```
[56]: cplot(best_model, 'income06', what = 'effect')
```



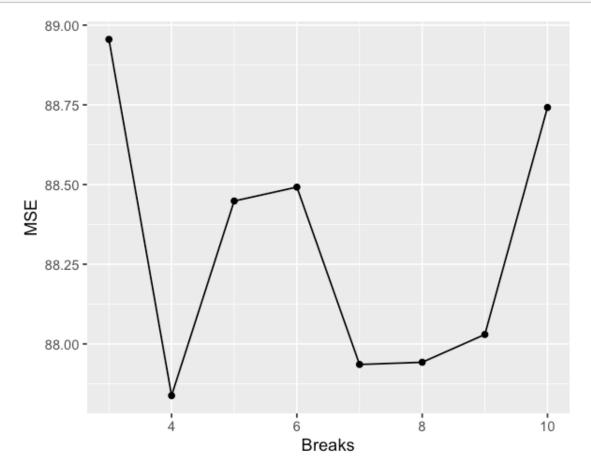
As a conclusion, the best polynomial model has 2 degrees, although the increase of MSE is not monotonic as the degree increases. Also, as income06 increases, its marginal effect decreases from positive to negative. The AME for the model of 2 degrees is -0.4369. This suggests that on average, one unit increase in income decreases the level of egalitarianism by about 0.4 unit. In another word, the richer people tend to be care less about equal distribution of the wealth.

### 0.1.2 2. Step Function

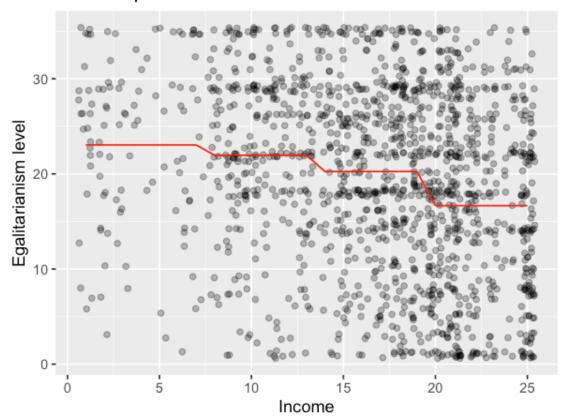
```
model = lm(egalit_scale~cut(income06,unique(breaks)), data = train_set)
    pred = predict(model, test_set)

sub_mse = c(sub_mse, mean((pred - test_set$egalit_scale)^2))
}
mse = c(mse, mean(sub_mse))
}
```

```
[15]: step_sum = data.frame('Breaks' = 3:k, "MSE"=mse)
ggplot(step_sum, aes(Breaks, MSE))+ geom_line()+geom_point()
```



# **Best Step Function**



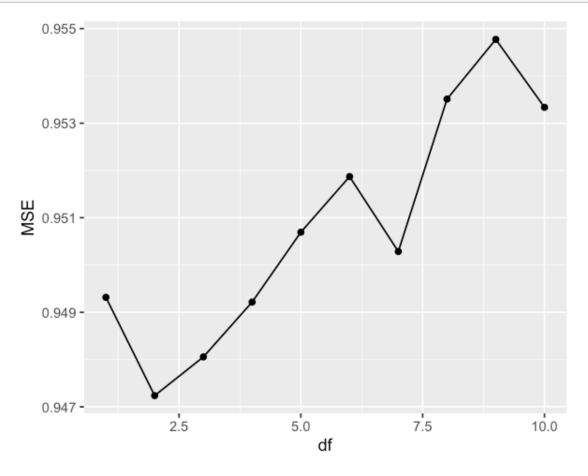
As a summary, the best step function has 4 breaks (5 intervals). Still, as income increases, the predicted level of egalitarianism decreases step by step.

### 0.1.3 Natural Regression Spline

```
[59]: mse = c()
    for(j in 1:k){
        sub_mse = c()
        for(i in 1:k){
            test_set = gss_train[fold==i,]
            train_set = gss_train[fold!=i,]
            model = lm(egalit_scale ~ ns(income06, df = j), data = train_set)
```

```
pred = predict(model, test_set)
    sub_mse = c(sub_mse, mean((pred - test_set$egalit_scale)^2))
}
mse = c(mse, mean(sub_mse))
}
```

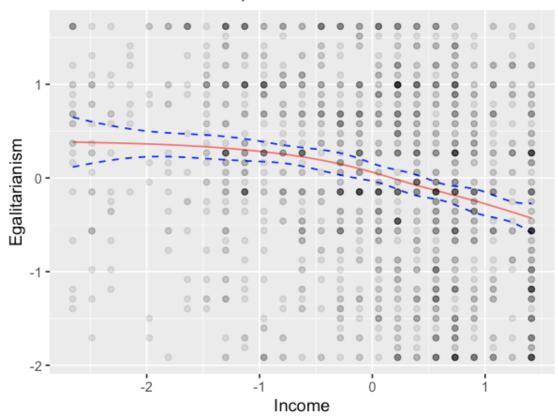
```
[60]: step_sum = data.frame('df' = 1:k, "MSE"=mse)
ggplot(step_sum, aes(df, MSE))+ geom_line()+geom_point()
## best knots is 2 (df = 5)
```



#### theme(legend.position = "none")

```
xvals
                 yvals
                           upper
                                     lower
  -2.655726 0.3829889 0.6488579 0.1171198
  -2.614648 0.3819858 0.6370112 0.1269603
  -2.573571 0.3809774 0.6253548 0.1366000
  -2.532494 0.3799584 0.6139188 0.1459980
  -2.491416 0.3789235 0.6027349 0.1551122
  -2.450339 0.3778675 0.5918363 0.1638987
  -2.409262 0.3767850 0.5812575 0.1723125
  -2.368185 0.3756707 0.5710341 0.1803073
9 -2.327107 0.3745193 0.5612028 0.1878359
10 -2.286030 0.3733256 0.5518001 0.1948512
11 -2.244953 0.3720843 0.5428621 0.2013064
12 -2.203875 0.3707900 0.5344232 0.2071567
13 -2.162798 0.3694374 0.5265143 0.2123605
14 -2.121721 0.3680213 0.5191614 0.2168813
15 -2.080644 0.3665364 0.5123831 0.2206897
16 -2.039566 0.3649774 0.5061890 0.2237658
17 -1.998489 0.3633389 0.5005770 0.2261009
18 -1.957412 0.3616158 0.4955323 0.2276992
19 -1.916334 0.3598026 0.4910261 0.2285790
20 -1.875257 0.3578941 0.4870157 0.2287725
```

# Best Natural Cubic Spline



As shown from the two figures above, the best natural regression spline has 2 knots (df = 3). This model is smoother compared to the previous two models and basically demonstrates the same information that, as income increases, people tend to be less egalitarian.

### 0.1.4 4. Egalitarianism and Everything

```
[44]: options(warn=-1)
```

 $\label{eq:decomposition} \textbf{Data Pre-Processing} \quad \text{see reference https://stackoverflow.com/questions/23619188/r-scaling-numeric-values-only-in-a-dataframe-with-mixed-types/36819351\#36819351$ 

```
[81]: standardize = function(data){
    df = data %>%
        mutate_if(is.matrix, as.numeric) %>%
        mutate_if(is.numeric, c) %>%
        mutate_if(is.numeric, scale)
```

```
a = as.character(df$relig)
a[a %in% c('CATHOLIC', 'PROTESTANT', 'CHRISTIAN')] = 1
a[!(a %in% c('CATHOLIC', 'PROTESTANT', 'CHRISTIAN'))] = 0
df$relig = as.numeric(a)
a = as.character(df$marital)
a[a == 'Married'] = 1
a[a != 'Married'] = 0
df$marital = as.numeric(a)
a = as.character(df$attend)
a[a == 'Never'] = 0
a[(a == '<0nce/yr')|(a == '0nce/yr')] = 1
a[a == 'Sev times/yr'] = 2
a[a == 'Once/mo'] = 3
a[a == '2-3 times /mo'] = 4
a[(a == 'Every wk')|(a == '>Once/wk')|(a == 'Nrly evry wk')] = 5
df$attend = as.numeric(a)
a = as.character(df$polviews)
a[a == 'ExtrmLib'] = 0
a[a == 'Liberal'] = 1
a[a == 'SlghtLib'] = 2
a[a == 'Moderate'] = 3
a[a == 'SlghtCons'] = 4
a[a == 'Conserv'] = 5
a[a == 'ExtrmCons'] = 6
df$polviews = as.numeric(a)
a = as.character(df$degree)
a[a == '<HS'] = 0
a[a == 'HS'] = 1
a[a == 'Junior Coll'] = 2
a[a == 'Bachelor deg'] = 3
a[a == 'Graduate deg'] = 4
df$degree = as.numeric(a)
a = as.character(df$news)
a[a == 'NEVER'] = 0
a[a == 'LESS THAN ONCE WK'] = 1
a[a == 'ONCE A WEEK'] = 2
a[a == 'FEW TIMES A WEEK'] = 3
a[a == 'EVERYDAY'] = 4
df$news = as.numeric(a)
```

```
a = as.character(df$pray)
a[a == 'NEVER'] = 0
a[a == 'ONCE A WEEK'] = 1
a[a == 'LT ONCE A WEEK'] = 2
a[a == 'SEVERAL TIMES A WEEK'] = 3
a[a == 'ONCE A DAY'] = 4
a[a == 'SEVERAL TIMES A DAY'] = 5
df$pray = as.numeric(a)

df
}

gss_train = standardize(gss_train)
gss_test = standardize(gss_test)
```

## 0.1.5 Linear Regression

There is no need to tune anything for linear regression. The average of 10-fold CV MSE is 0.6623, and the whole model MSE is 0.8918. When all predictors are considered, egalitarianism is significantly related to political views: the more liberal, the more egalitarinian. Younger and poorer people, aside from political views, are also more egalitarinian.

```
[42]: library(caret)
[45]: folds = trainControl(method = "cv", number = 10)
      gss_lm = train(
          egalit_scale ~ ., data = gss_train, method = "lm",
          trControl = folds,
          tuneLength = 10)
      gss_lm
     Linear Regression
     1481 samples
       44 predictor
     No pre-processing
     Resampling: Cross-Validated (10 fold)
     Summary of sample sizes: 1334, 1333, 1332, 1333, 1334, 1334, ...
     Resampling results:
                  Rsquared
       RMSE
                             MAE
       0.8137494 0.3448641 0.6456499
```

```
[46]: gss_lm_test = train(
         egalit_scale ~ ., data = gss_test, method = "lm")
     gss_lm_test
     Linear Regression
     493 samples
      44 predictor
     No pre-processing
     Resampling: Bootstrapped (25 reps)
     Summary of sample sizes: 493, 493, 493, 493, 493, ...
     Resampling results:
      RMSE
                 Rsquared
                            MAE
       0.9298948 0.2100389 0.7352836
     Tuning parameter 'intercept' was held constant at a value of TRUE
[47]: 0.8138173**2
     0.9443879**2
     0.66229859777929
     0.89186850566641
[48]: model = lm(egalit_scale ~ ., data = gss_train)
     pred = predict(model, gss_test)
     print(mean((pred - gss_test$egalit_scale)^2))
     summary(model)
     [1] 0.7035309
     Call:
     lm(formula = egalit_scale ~ ., data = gss_train)
     Residuals:
                   1Q
                        Median
                                    3Q
                                            Max
     -2.91394 -0.50116 -0.00216 0.53007 2.11940
     Coefficients: (2 not defined because of singularities)
                                Estimate Std. Error t value Pr(>|t|)
     (Intercept)
                                0.7613198 0.2981538
                                                      2.553 0.01077 *
     age
                               -0.0012061 0.0133492 -0.090 0.92802
     attend
     authoritarianism
                               0.0046642 0.0247906 0.188 0.85079
     blackYes
                               0.0995314 0.0717092 1.388 0.16536
     bornYES
                               -0.0310722 0.0701998 -0.443 0.65810
```

```
childs
                            0.0545790
                                       0.0246579
                                                   2.213 0.02703 *
colathNOT ALLOWED
                            0.0476824
                                       0.0618851
                                                   0.770 0.44113
colracNOT ALLOWED
                           -0.0028243
                                                  -0.050 0.95974
                                       0.0559383
colcomNOT FIRED
                                                  -0.022 0.98272
                           -0.0012327
                                       0.0569077
colmilNOT ALLOWED
                           -0.1218429
                                       0.0582975
                                                  -2.090 0.03679 *
colhomoNOT ALLOWED
                            0.0922454
                                       0.0802521
                                                   1.149 0.25057
colmslmYes, allowed
                           -0.0185229
                                       0.0579924
                                                  -0.319 0.74947
con_govt
                           -0.0333960
                                       0.0226076
                                                  -1.477 0.13984
                                       0.0227422 -1.869 0.06184 .
degree
                           -0.0425028
evangelicalLow
                            0.0962189
                                       0.1153185
                                                   0.834 0.40421
                            0.0786875
                                       0.0815192
                                                   0.965 0.33458
evangelicalMod
                                                  -4.394 1.20e-05 ***
grassNOT LEGAL
                           -0.2487933
                                       0.0566247
happyPRETTY HAPPY
                                                  -0.810 0.41785
                           -0.0515772
                                       0.0636449
happyVERY HAPPY
                           -0.1059799
                                       0.0707509
                                                  -1.498 0.13437
hispanic_2Yes
                            0.0035464
                                       0.0728445
                                                   0.049 0.96118
                            0.0088042
                                                   0.055 0.95585
homosexALWAYS WRONG
                                       0.1589927
homosexNOT WRONG AT ALL
                           -0.0238809
                                       0.1598742
                                                  -0.149 0.88128
homosexSOMETIMES WRONG
                            0.0002982
                                       0.1796307
                                                   0.002 0.99868
income06
                           -0.0652442
                                       0.0255296
                                                  -2.556 0.01070 *
marital
                                                      NΑ
                                                               NA
                                   NA
                                              NΑ
modeOVER THE PHONE
                            0.0361827
                                       0.0647565
                                                   0.559
                                                          0.57642
                           -0.0148825
                                       0.0146332
                                                  -1.017
                                                          0.30931
news
owngunREFUSED
                           -0.0593114
                                       0.1917475
                                                  -0.309 0.75712
                                                  -1.628 0.10382
owngunYES
                           -0.0838512
                                       0.0515163
partyid_3Ind
                           -0.2238890
                                       0.0538095
                                                  -4.161 3.36e-05 ***
partyid_3Rep
                                       0.0790325
                                                  -4.761 2.13e-06 ***
                           -0.3762475
                                                  -9.697
polviews
                           -0.1754156
                                       0.0180892
                                                          < 2e-16 ***
pornlaw2Not illegal to all -0.0633641
                                       0.0609583
                                                  -1.039 0.29877
                                                  -0.683 0.49451
                           -0.0108180
                                       0.0158312
pray
pres080bama
                            0.4182243
                                       0.0671503
                                                   6.228 6.21e-10 ***
                            0.0589227
                                       0.0957148
                                                   0.616 0.53825
reborn_rYes
                                              NA
                                                      NA
relig
                                   NA
                                                               NA
science_quiz
                           -0.0140464
                                       0.0283595
                                                  -0.495
                                                          0.62047
                           -0.1375263
                                       0.0498760
                                                  -2.757 0.00590 **
sexMale
                                       0.0232703
                                                   1.971 0.04887 *
sibs
                            0.0458757
                                                   0.128 0.89780
social_connect
                            0.0029566
                                       0.0230152
social cons3Liberal
                           -0.0288207
                                       0.0803269
                                                  -0.359 0.71980
social cons3Mod
                            0.0322244
                                       0.0631606
                                                   0.510 0.60999
southSouth
                                                  -0.771 0.44056
                           -0.0366951
                                       0.0475652
spend3Liberal
                            0.1792603
                                       0.0514608
                                                   3.483 0.00051 ***
                            0.0781263
                                                   1.374 0.16968
spend3Mod
                                       0.0568628
                                                   0.844 0.39903
teensexALWAYS WRONG
                            0.0578139
                                       0.0685319
teensexNOT WRONG AT ALL
                                                   0.102 0.91914
                            0.0116106
                                       0.1143490
                                                   0.547 0.58433
teensexSOMETIMES WRONG
                            0.0487270
                                       0.0890484
tolerance
                           -0.0930304
                                       0.0376209
                                                  -2.473 0.01352 *
tvhours
                            0.0511016
                                       0.0229663
                                                   2.225 0.02623 *
vetyearsLESS THAN 2 YRS
                            0.0875443
                                       0.1593179
                                                   0.549 0.58275
vetyearsMORE THAN 4 YRS
                            0.0572697
                                       0.1448891
                                                   0.395 0.69271
```

```
vetyearsNONE
                           0.0706486 0.0987147
                                                 0.716 0.47430
                                                 0.508 0.61181
wordsum
                           0.0133033 0.0262077
zodiacARIES
                          -0.1760371 0.1055921 -1.667 0.09571 .
zodiacCANCER
                          -0.0418149 0.1053861 -0.397 0.69159
                          -0.0562682 0.1039193 -0.541 0.58828
zodiacCAPRICORN
zodiacGEMINI
                          -0.1443622 0.1006603 -1.434 0.15175
zodiacLE0
                          -0.0848368 0.0974235 -0.871 0.38401
zodiacLIBRA
                          -0.1235218  0.0996629  -1.239  0.21540
zodiacPISCES
                          -0.1420679 0.1046778 -1.357 0.17494
zodiacSAGITTARIUS
                          -0.1221534 0.1035906 -1.179 0.23852
zodiacSCORPIO
                          -0.1733468 0.1044019 -1.660 0.09706 .
zodiacTAURUS
                          -0.0160679 0.1007930 -0.159 0.87336
zodiacVIRGO
                          0.0197235 0.1038762
                                                 0.190 0.84943
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 0.7944 on 1416 degrees of freedom
Multiple R-squared: 0.3962, Adjusted R-squared: 0.3689
F-statistic: 14.52 on 64 and 1416 DF, p-value: < 2.2e-16
```

### 0.1.6 Elastic Net Regression

With 10 fold elastic net, the best alpha is 0.6 and the best lambda is 0.03345934. The train CV MSE is 0.68 and the final MSE is 0.74. This is much better than the linear model.

```
glmnet
```

493 samples
44 predictor

```
No pre-processing
Resampling: Bootstrapped (25 reps)
Summary of sample sizes: 493, 493, 493, 493, 493, 493, ...
Resampling results:

RMSE Rsquared MAE
0.8589117 0.2740568 0.6873249

Tuning parameter 'alpha' was held constant at a value of 0.7
Tuning
parameter 'lambda' was held constant at a value of 0.03345934
```

```
[51]: mean(gss_elnet$results$RMSE^2) elnet_best$results$RMSE^2
```

0.676020168272908

0.737729330083205

### 0.1.7 Principal Component Regression

The best number of components is 10, the train CV MSE is 0.78 and the final MSE is 0.78. This means that PCR performs slightly worse than the elastic net model because our dataset has lots of categorical variables. But it is still better than the linear model because the linear model does not consider the covariance between predictors.

```
[52]: folds = trainControl(method = "cv", number = 10)
gss_pcr = train(
    egalit_scale ~ ., data = gss_train, method = "pcr",
    trControl = folds,
    tuneLength = 10)
```

```
[53]: gss_pcr
```

Principal Component Analysis

```
1481 samples 44 predictor
```

No pre-processing

Resampling: Cross-Validated (10 fold)

Summary of sample sizes: 1334, 1333, 1333, 1332, 1334, 1334, ...

Resampling results across tuning parameters:

```
ncomp RMSE Rsquared MAE

1 0.9897603 0.02926820 0.8241770

2 0.9684009 0.06483337 0.8079843
```

```
3
      0.9707907 0.06163400 0.8084389
4
      0.8410155 0.29335015 0.6855206
5
      0.8406428 0.29384154 0.6841887
6
      0.8411283 0.29293655 0.6846652
7
      0.8416448 0.29208661 0.6850168
8
      0.8421907 0.29111912 0.6848049
9
      0.8424655 0.29066026 0.6851048
10
      0.8422671 0.29084255 0.6854361
```

RMSE was used to select the optimal model using the smallest value. The final value used for the model was ncomp = 5.

Principal Component Analysis

```
493 samples
44 predictor
```

No pre-processing

Resampling: Bootstrapped (25 reps)

Summary of sample sizes: 493, 493, 493, 493, 493, 493, ...

Resampling results:

```
RMSE Rsquared MAE 0.8826469 0.2142523 0.7196249
```

Tuning parameter 'ncomp' was held constant at a value of 5

```
[55]: mean(gss_pcr$results$RMSE^2)
pcr_best$results$RMSE^2
```

0.78181575682232

0.779065551997975

### 0.1.8 Partial least squares regression

The best number of components is 6, the train CV MSE is 0.67 and the final MSE is 0.79. This means that PLS performs worse than the previous two models but better than the linear model.

```
[56]: folds = trainControl(method = "cv", number = 10)
gss_pls = train(
```

```
egalit_scale ~ ., data = gss_train, method = "pls",
         trControl = folds,
         tuneLength = 10)
      gss_pls
     Partial Least Squares
     1481 samples
       44 predictor
     No pre-processing
     Resampling: Cross-Validated (10 fold)
     Summary of sample sizes: 1333, 1333, 1332, 1332, 1333, 1334, ...
     Resampling results across tuning parameters:
       ncomp RMSE
                         Rsquared
                                    MAE
              0.8681898 0.2474046 0.7087913
        1
        2
              0.8308068 0.3122735 0.6740116
        3
              0.8229471 0.3256703 0.6634647
        4
              0.8127777 0.3437296 0.6462019
        5
              0.8047999 0.3566256 0.6395816
        6
              0.8059920 0.3548967 0.6386655
        7
              0.8082478 0.3514184 0.6397831
        8
              0.8088984 0.3505123 0.6392299
        9
              0.8096682 0.3493316 0.6402650
       10
              0.8104453 0.3481965 0.6412029
     RMSE was used to select the optimal model using the smallest value.
     The final value used for the model was ncomp = 5.
[57]: pls_best = train(
          egalit_scale ~ ., data = gss_test, method = "pls",
         tuneGrid = expand.grid(ncomp = gss_pls$bestTune$ncomp))
      pls best
     Partial Least Squares
     493 samples
      44 predictor
     No pre-processing
     Resampling: Bootstrapped (25 reps)
     Summary of sample sizes: 493, 493, 493, 493, 493, 493, ...
     Resampling results:
```

RMSE

Rsquared

0.8884938 0.2415459 0.7077435

MAE

Tuning parameter 'ncomp' was held constant at a value of 5

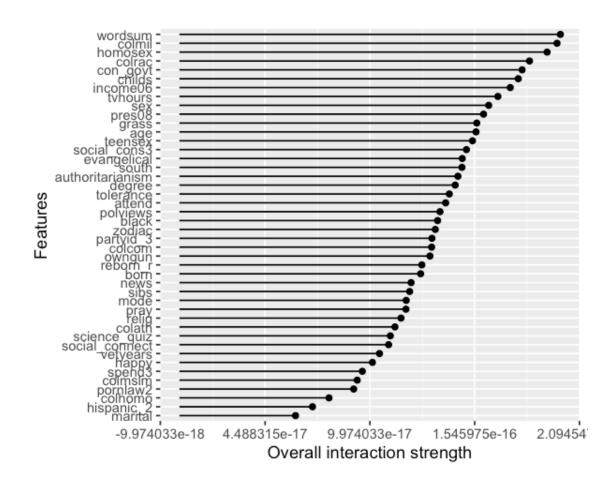
```
[58]: mean(gss_pls$results$RMSE^2)
pls_best$results$RMSE^2
```

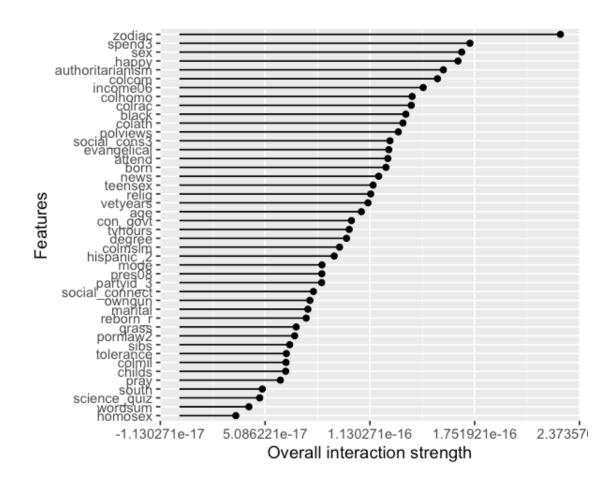
0.669913423225353

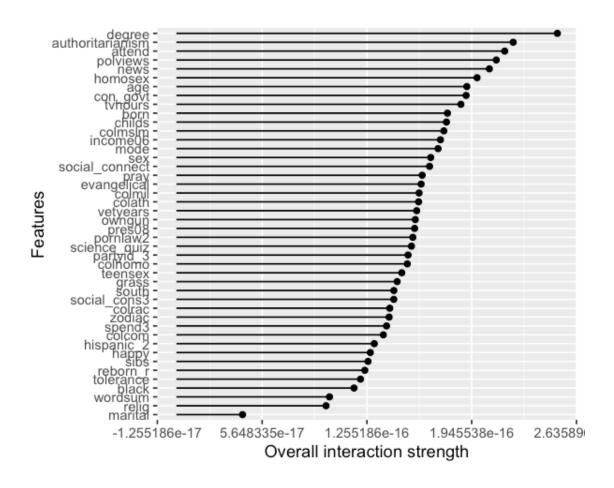
0.789421254989816

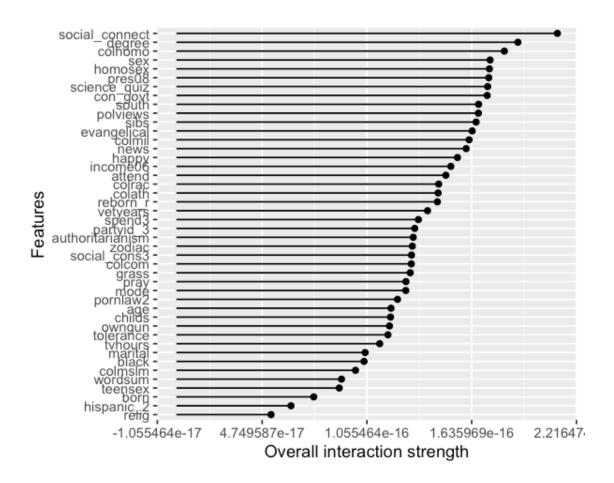
#### 0.1.9 5. Feature Interaction Plots

[70]: plot(Interaction\$new(predictor\_lm))









#### [85]: elnet\_best

glmnet

493 samples
44 predictor

No pre-processing

Resampling: Bootstrapped (25 reps)

Summary of sample sizes: 493, 493, 493, 493, 493, 493, ...

Resampling results:

RMSE Rsquared MAE 0.8589117 0.2740568 0.6873249

Tuning parameter 'alpha' was held constant at a value of 0.7  $\,$  Tuning

parameter 'lambda' was held constant at a value of 0.03345934

- wordsum, colmil, and homosex have the highest interaction strength in the linear model
- zodiac, spend3, and sex have the highest interaction strength in the elastic net model
- degree, authoritarianism, and attend have the highest interaction strength in the PCR model
- social\_connect, degree, and colhomo have the highest interaction strength in the PLS model
- Terms like polviews and degree tend to have relatively high interaction strength. This is pretty intuitive: political views and level of education clearly affect many other aspects of a person's character. For example, polviews can affect people's choice on whether grass is legal and whether homosexuality is acceptable—and therefore affect grass and homosex.degree, on the other hand, influences wordsum and a general character of how people observe information (news, for example).
- PCR yields the term with the highest overall interaction strength across all models.
- The difference of interaction strength among terms is the largest in the elastic net model: this probably means that lasso and ridge methods reduce the influence of some of the features.

[]:	