HEAD START MATHS

Solving Problems

CEMTL
Regional Centre for Excellence in Mathematics Teaching and Learning

centre for excellence
sigma
in mathematics & statistics support

Foreword

The Regional Centre for Excellence in Mathematics Teaching and Learning (CEMTL) is collaboration between the Shannon Consortium Partners: University of Limerick, Institute of Technology, Limerick; Institute of Technology, Tralee and Mary Immaculate College of Education, Limerick., and is driven by the Mathematics Learning Centre (MLC) and The Centre for Advancement in Mathematics Education and Technology (CAMET) at the University of Limerick.

CEMTL is committed to providing high quality educational resources for both students and teachers of mathematics. To that end this package has been developed to a high standard and has been peer reviewed by faculty members from the University of Limericks Department of Mathematics and Statistics and sigma, the UK based Centre for Excellence in Teaching and Learning (CETL). Through its secondment programme, sigma provided funding towards the creation of these workbooks.

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8.1 Introduction to Problem Solving

At this stage we have learned to solve linear, quadratic and simultaneous equations. We have also learned how to solve equations involving fractions. Now we are going to learn how to use these methods to solve various mathematical problems. The first thing we need to do when solving any practical problem is to translate all the given information into mathematical terms.

Example 1

When 8 is taken from $12x$, the result is 16. Find x

8 is taken from (i.e. subtracted from) $12x \rightarrow 12x - 8$

8 is taken from $12x$, the result is $16 \rightarrow 12x - 8 = 16$

We solve for x as normal.

Example 2

When 11 is added to 6 times x, the result is 107. Find x.

6 times $x \rightarrow 6x$

11 is added to 6 times $x \rightarrow 11 + 6x$

11 is added to 6 times x, the result is $107 \rightarrow 11 + 6x = 107$

Solve as normal for x.

8.2 Problems with Linear Equations

Generally when we are solving problems, we use some letter, for example x, to represent the unknown value we are trying to figure out.

Example 3

When 38 is added to twice a certain number, the result is 80. Find the number.

We will called the 'certain number' x.

- Twice a certain number $\rightarrow 2x$
- 38 is added $\rightarrow 2x + 38$
- Result is $80 \rightarrow 2x + 38 = 80$

Solve as normal to find x

(Ans: x = 21).

Example 4

When 8 is taken from 5 times a certain number, the result is the same as adding 6 to 3 times this number. Find the number.

We have 2 situations here:

- 8 is taken from 5 times a certain number
- Adding 6 to 3 times this number

We are also told that both of these situations are the same. In other words, they are equal to each other.

Therefore:

8 is taken from 5 times a certain number = Adding 6 to 3 times this number

Again we let the 'certain number' = x

Case 1

5 times a certain number = $5x$

8 is taken from 5 times a certain number = $5x - 8$

Case 2

3 times this number = $3x$

Adding 6 to 3 times this number = $3x + 6$

So $5x - 8 = 3x + 6$

We solve for x as before.

(Ans: x = 7).

Example 5

A father is 32 years older than his son. If the sum of their ages is 80, what age is the son now?

Let son's age = x.

We do not know the exact age of the father yet either but we know he is 32 years older than the son i.e. 32 years more than x

$\longrightarrow$ Father's age = $x + 32$.

The sum of both ages together is 80

$\longrightarrow$ Son's age + Father's age = 80

$\longrightarrow x + (x + 32) = 80$

We now solve for x as normal.

(Ans: x = 24).

Example 6

Paul is four times as old as Luke. In sixteen years time Paul will be twice Luke's age. What age are they both now?

Let Luke's age = x.

Paul is four times Luke's age i.e. Luke's age $\times 4$

Therefore, Paul's age = $4x$

Sometimes when we are given quite a lot of information like this, it is convenient to use a table as follows:

	Now	In sixteen years
Luke	x	x + 16
Paul	$4x$	$4x$ + 16

In sixteen years time Paul's age will be twice Luke's age i.e. Paul's age in sixteen years = $2\times$ Luke's age in sixteen years

i.e. $4x + 16 = 2(x + 16)$

$4x + 16 = 2x + 32$

We continue as before to solve for x, Luke's age, from which we can easily deduce Paul's age.

(Ans: 8, 32).

8.3 Problems with Simultaneous Equations

We use simultaneous equations when we need to solve for two unknowns.

We will use x and y in these situations to represent the unknown values we wish to find.

Example 7

Two coffees and three scones cost €7.05, while three coffees and a scone cost €6.90. Find the cost of a coffee and the cost of a scone.

Solution

The first thing we will do is allocate the unknowns x and y

Let the price of a coffee = x and the price of a scone = y

Using the information we were given, we create two equations in x and y as follows:

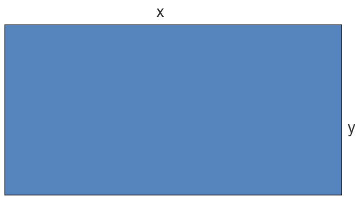
Two coffees and three scones cost €$3.85 \longrightarrow 2x + 3y = 705$

Three coffees and a scone cost €$5.45 \longrightarrow 3x + y = 690$

We proceed to solve the two simultaneous equations to find x and y as before.

Example 8

Twice the length of a rectangle and five times its width measures 29cm. When six times the width is subtracted from three times the length the result is 3cm. Find the length and the width of this rectangle.



Solution

Let the length be x cm and the width be y cm.
Twice the length ($2x$) and (+) five times the width ($5y$) measures 29cm:

$\longrightarrow 2x + 5y = 29$

Six times the width ($6y$) is subtracted from (-) three times the length ($3x$) the result is 3cm:

$\longrightarrow 3x - 6y = 3$

We have our two simultaneous equations so can proceed to solve to find x and y.

8.4 Problems with Quadratic Equations

In problems with quadratic equations, we will find x^2 or the product (multiplication) of two expressions involving x.

Example 9

One number is 5 greater than another number. If their product is 234, find the two numbers.

Solution

We have two numbers. Let us call the first number x.

The second number is 5 greater than the first $\longrightarrow x + 5$.

We are told that the product of these two numbers is $234 \longrightarrow x(x + 5) = 234$.

Multiplying out the bracket and subtracting 234 from both sides we get:

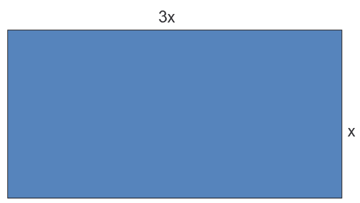
$x^2 + 5x - 234 = 0$.

Using either the guide number method or formula we can proceed to solve for x.

Example 10

The length of a rectangular shaped lawn is 3 times as long as its width. If its area is 432 m^2. Find the length and width of the lawn.

Solution



If the width of the lawn is x metres, then the length is 3 times this i.e. $3x$.

We know that the area of a rectangle is calculated by multiplying the length by the width

$x(3x) = 432m^2$

Multiplying out the bracket and subtracting 432 from both sides we get:

$3x^2 - 432 = 0.$

We can now proceed as normal.

Exercises 1

1. When six times a certain number is increased by 130, the result is 94. Find the number.

2. Paul has €22 more than Shane. If Shane gave Paul another €2, Paul would have 3 times as much money as Shane. How much money does each have at present?

3. Two numbers differ by 4. The sum of their squares is 346. Find the two numbers.

4. The width of a rectangle is 15cm less than its length. The area of the rectangle is 286cm^2. Find the dimensions of the rectangle.

5. If 250 standing tickets and 600 seated tickets are sold for a rugby game in Thomond Park, the takings will amount to €$49,000$. However if 500 standing tickets are sold and 300 seated tickets are sold, the takings will amount to €$39,500$. How much does a seated ticket cost?How much does a standing ticket cost?

6. Four bottles of beer and five packets of chips cost €20.40 while three bottles of beer and six packets of chips cost €17.19. How much would a bottle of beer and two packets of chips cost?

8.5 Answers

Example 8: Coffee = €1.95, Scone = €1.05
Example 9: $x = 7cm; y = 3cm$
Example 10: $x = 18$ or $x = 13$ or $x = -18$ or $x = -13$
Example 11:$x = 12m$
Exercises 1:

1. $x = -6$
2. Shane has €15 and Paul had €37
3. 11 and 15 or -11 and -15
4. $11cm$ and $26cm$
5. Standing €40; Seated €65
6. €5.73

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