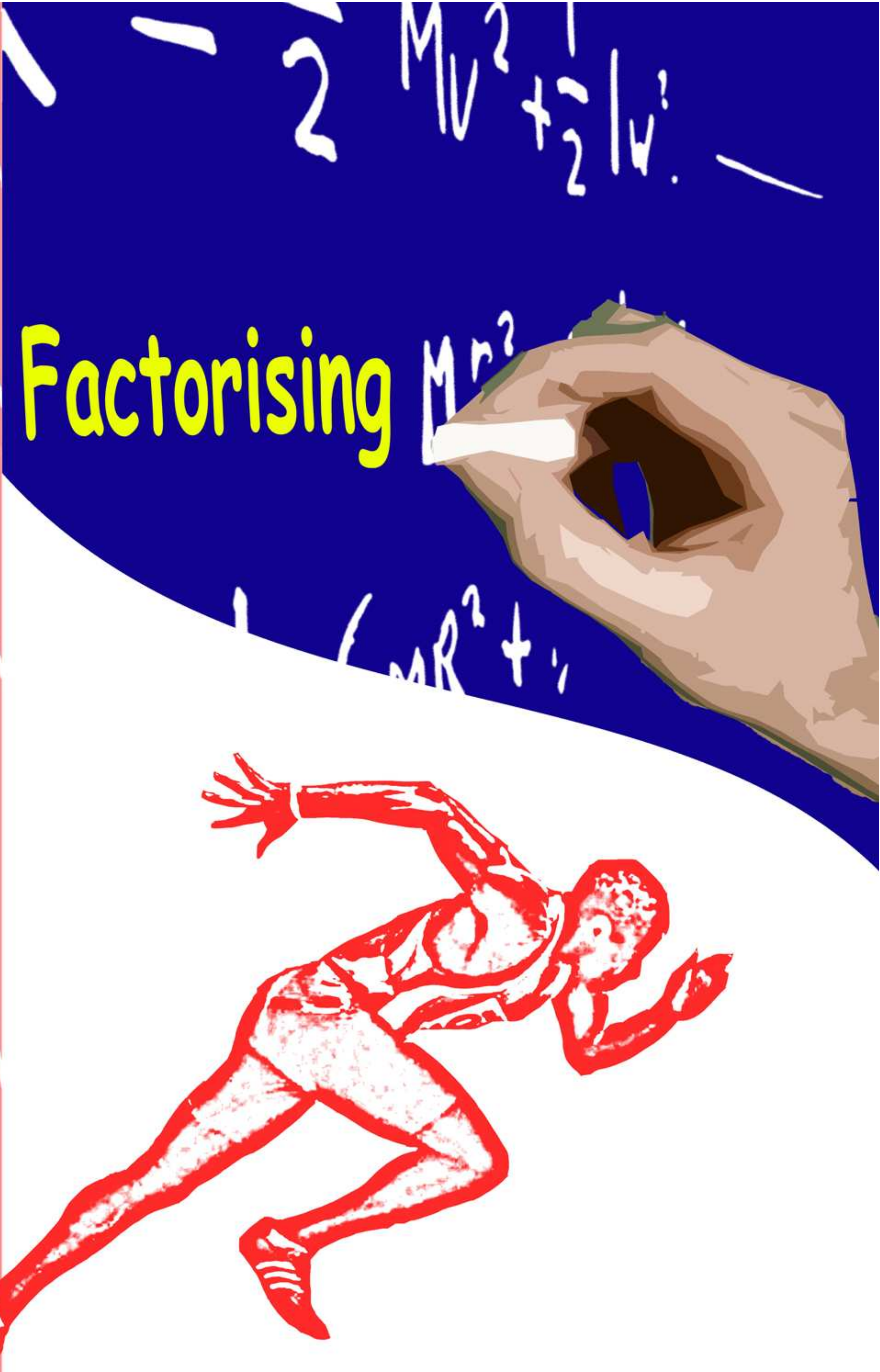
HEAD START MATHS

Factorising



CEMTL
Regional Centre for Excellence in Mathematics Teaching and Learning

centre for excellence
sigma Σ
in mathematics & statistics support

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Foreword

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Table of Contents

5 Factorising

5.1 Factors

In our first workshop on Number Systems we learned about factors.

For example: $1 \times 6 = 6$ and $2 \times 3 = 6$

Therefore $1, 2, 3$, and 6 are all factors of 6.

In other words, all the numbers that divide exactly into 6.

In our Algebra workshop we learned how to multiply out brackets

We saw that $5(a + b) = 5a + 5b$

Therefore 5 and $(a + b)$ are factors of $5a + 5b$.

We also saw how $(x + 5)(x + 3) = x^2 + 8x + 15$

Therefore $(x + 5)$ and $(x + 3)$ are factors of $x^2 + 8x + 15$.

And so on...

When we find the factors of an algebraic expression (called a **polynomial**)we say that the expression has been **factorised**.

In this workshop we will learn how to factorise many different types of polynomial.

5.2 Taking out a Common Factor

Example 1

Factorise $100d + 150e + 200f$

We can see that 50 divides into all 3 terms. So we take 50 outside a bracket and divide every term by 50

$$50\left(\frac{100d}{50}+\frac{150e}{50}+\frac{200f}{50}\right)$$

$50(2d + 3e + 4f)$

Example 2

Factorise $4a + 24b - 16c$

We can see that 4 divides into all 3 terms.

$$4\left(\frac{4a}{4}+\frac{24b}{4}-\frac{16c}{4}\right)$$

$4(a + 6b - 4c)$

Example 3

Factorise $3\beta^3 - 6\beta^2 + 9\beta$

Here we can divide all terms by 3β i.e. take 3β outside the bracket

$$3\beta\left(\frac{3\beta^3}{3\beta}-\frac{6\beta^2}{3\beta}+\frac{9\beta}{3\beta}\right)$$

$3\beta(\beta^2 - 2\beta + 3)$

Example 4

Factorise $12s^2t + 8st^2 - 6s^2t^2$

Here we can divide all terms by $2st$

$$2st\left(\frac{12s^2t}{2st}+\frac{8st^2}{2st}+\frac{6s^2t^2}{2st}\right)$$

$2st(6s + 4t - 3st)$

Exercises 1

Factorise Each of the Following

1. $6s + 30t$

2. $21u - 42v$

3. $-ab + ac$

4. $25\alpha\beta - 35\beta\delta$

5. $21\theta^3 - 24\theta$

6. $33q^4 - 22q^2$

7. $-25\alpha\beta^2\gamma + 50\alpha^2\beta^2\gamma^2$

8. $-4x^5 - 16x^4z - 8xy^3$

9. $abc + a^3b^4c^2 - a^6b^{10}c^2$

10. $l^2mn^2 - m^2n + ln^2$

5.3 Factorising by Grouping

Look at the expression

$ax + ay + 6x + 6y$

It is clear that there is no factor common to all four terms.

However if we look closely we see that a is common to the first 2 terms and 6 is common to the second 2 terms.

When this occurs we use a method called **Factorising by Grouping** to factorise such expressions.

What this means is we group terms together in pairs where there is a common factor and then factorise like before.

In this particular example, the first 2 terms have a in common and the second 2 terms have 6 in common.

$(ax + ay) + (6x + 6y)$

= $a(x + y) + 6(x + y)$

As you can see the terms inside both brackets are the same so . . .

We can write the expression as $(a + 6)(x + y)$

Therefore our factors are $(a + 6)$ and $(x + y)$.

Example

Factorise $4ab + 4ac + 5xb + 5xc$

$(4ab + 4ac) + (5xb + 5xc)$

= $4a(b + c) + 5x(b + c)$

= $(4a + 5x)(b + c)$

The factors are $(4a + 5x)$ and $(b + c)$

Exercises 2

Factorise the Following

1. $qx + qy + 4x + 4y$

2. $2bc - 2bd + 17c - 17d$

3. $\pi\rho + \kappa\rho + \pi\lambda + \kappa\lambda$

4. $5as - 10bs + 2aq - 4bq$

5. $tp - 2t + 16p - 32$

6. $2l\alpha + \alpha\delta - 2l\phi - \delta\phi$

7. $uv - 2cd - vc + 2ud$

8. $2\mu - 3\lambda\kappa - 6\lambda + \kappa\mu$

9. $ih - 4ij + 8gj - 2gh$

10. $16s + 2r - rs - 32$

5.4 Factorising Quadratics

A mathematical expression of the form

$ax^2 + bx + c$ (where a, b, c are constants) is called a **Quadratic** expression.

If we multiply out the following:

$(a + 2)(a + 7)$

We get

$a(a + 7) + 2(a + 7)$

= $a^2 + 7a + 2a + 14$

= $a^2 + 9a + 14$

So $a^2 + 9a + 14 = (a + 2)(a + 7)$

In other words when we factorise $a^2 + 9a + 14$ we get$(a + 2)(a + 7)$.

In this section we will learn how to factorise (find the factors of) quadratic expressions. To do this we can use a method called **The Guide Number Method.**

Example 1

Factorise $x^2 + 7x + 12$

Step 1: Find the Guide Number (GN)

We find the guide number by multiplying the x^2 coefficient (in this case 1) by the constant (in this case 12)
GN = 1×12 = 12.

Step 2: Write out all the factors of the Guide Number

Factors of 12 are:
1×12, -1 $\times$ -12
2×6, -2 $\times$ -6
3×4, -3 $\times$ -4

Step 3: Pick the factors of 12 which *add* to give you the x coefficient in the original quadratic (in this case 7)

3×4

Step 4: Split the x term in the original quadratic into two terms using the values found in Step 3

$x^2 + 7x + 12$
= $x^2 + 4x + 3x + 12$

Step 5: Proceed by factorising by grouping

$(x^2 + 4x) + (3x + 12)$
= $x(x + 4) + 3(x + 4)$

Factors: $(x + 4)(x + 3)$.

Example 2

Factorise $a^2 + 9a + 20$

Step 1: Find the Guide Number (GN)

We find the guide number by multiplying the x^2 coefficient (in this case 1) by the constant (in this case 20)
GN = 1×20 = 20.

Step 2: Write out all the factors of the Guide Number

Factors of 20 are:
$1 \times 20, \quad -1 \times -20$
$2 \times 10, \quad -2 \times -10$
$4 \times 5, \quad -4 \times -5$

Step 3: Pick the factors of 20 which *add* to give you the a coefficient in the original quadratic (in this case 9)

4×5

Step 4: Split the a term in the original quadratic into two terms using the values found in Step 3

$a^2 + 9a + 20$
= $a^2 + 4a + 5a + 20$

Step 5: Proceed by factorising by grouping

$(a^2 + 4a) + (5a + 20)$
= $a(a + 4) + 5(a + 4)$

Factors: $(a + 4)(a + 5)$.

Example 3

Factorise $3x^2 + 7x + 2$.

Step 1: Find the Guide Number (GN)

We find the guide number by multiplying the x^2 coefficient (in this case 3) by the constant (in this case 2)
GN = $3 \times 2 = 6$.

Step 2: Write out all the factors of the Guide Number

Factors of 6 are:
$1 \times 6, \quad -1 \times -6$
$2 \times 3, \quad -2 \times -3$

Step 3: Pick the factors of 6 which *add* to give you the x coefficient in the original quadratic (in this case 7)

1×6

Step 4: Split the x term in the original quadratic into two terms using the values found in Step 3

$3x^2 + 7x + 2$
= $3x^2 + x + 6x + 2$

Step 5: Proceed by factorising by grouping

$(3x^2 + x) + (6x + 2)$
= $x(3x + 1) + 2(3x + 1)$

Factors: $(3x + 1)(x + 2)$.

Example 4

Factorise $x^2 - 7x + 10$

Step 1: Find the Guide Number (GN)

We find the guide number by multiplying the x^2 coefficient (in this case 1) by the constant (in this case 10)
GN = $1 \times 10 = 10$

Step 2: Write out all the factors of the Guide Number

Factors of 10 are:
1×10, -1 $\times -10$
2×5, -2 $\times -5$

Step 3: Pick the factors of 10 which *add* to give you the x coefficient in the original quadratic (in this case -7)

-2 $\times -5$

Step 4: Split the x term in the original quadratic into two terms using the values found in Step 3

$x^2 - 7x + 10$
= $x^2 - 2x - 5x + 10$

Step 5: Proceed by factorising by grouping

$(x^2 - 2x) + (-5x + 10)$
= $x(x - 2) - 5(x - 2)$
(We took out -5 here so that the terms in both brackets would be the same).

Factors: $(x - 5)(x - 2)$.

Example 5

Factorise $3x^2 - 10x - 8$

Step 1: Find the Guide Number (GN)

We find the guide number by multiplying the x^2 coefficient (in this case 3)by the constant (in this case -8)

GN = $3 \times -8 = -24$

Step 2: Write out all the factors of the Guide Number

Factors of -24 are:

1×-24, -1 $\times$ 24
2×-12, -2 $\times$ 12
3×-8, -3 $\times$ 8
4×-6, -4 $\times$ 6

Step 3: Pick the factors of -24 which *add* to give you the x coefficient in the original quadratic (in this case -10)

2×-12

Step 4: Split the x term in the original quadratic into two terms using the values found in Step 3

$3x^2 - 10x - 8$
= $3x^2 + 2x - 12x - 8$

Step 5: Proceed by factorising by grouping

$(3x^2 + 2x) + (-12x - 8)$
= $x(3x + 2) - 4(3x + 2)$

Factors: $(3x + 2)(x - 4)$.

Exercises 3

Factorise the Following

1. $x^2 + 10x + 24$

2. $a^2 + 12a + 27$

3. $2\beta^2 + 9\beta + 4$

4. $3\theta^2 + 22\theta - 16$

5. $2t^2 + 5t - 3$

6. $b^2 - b - 12$

7. $3\alpha^2 - 13\alpha - 10$

8. $4z^2 - 13z + 3$

9. $2q^2 - 9q + 7$

10. $w^2 - 16w + 64$

5.5 Solving Quadratic Equations

Method 1: Solving by Factorisation

Example 1

Solve the following:

$x^2 + 8x + 15 = 0.$

Factorise using the guide number method to get

$(x + 3)(x + 5) = 0$

The product of two values is zero only if at least one of the values is zero.

So we have $(x + 3) = 0 \quad$ or $\quad (x + 5) = 0$

So we solve to get $x = -3 \quad$ or $\quad x = -5$

Example 2

Solve the following:

$3x^2 + x - 2 = 0.$

Factorise using the guide number method to get

$(3x - 2)(x + 1) = 0$

At least one of these values is zero.

So we have $(3x - 2) = 0 \quad$ or $\quad (x + 1) = 0$

So we solve to get $x = \dfrac{2}{3} \quad$ or $\quad x = -1$

Method 2: Solving by Formula

Not all quadratic equations can be factorised.

For example, solve $2x^2 + 8x + 3 = 0$

The guide number method will not work here. Why?

The guide number in this instance works out to be $2 \times 3 = 6$.

There are no factors of 6 which add up to 8 so the equation does not factorise. For such equations there is a formula we can use. Given a quadratic equation of the form $ax^2 + bx + c = 0$, $a \neq 0$, the following formula can be used to find the values of x that solve the quadratic:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example

Solve $2x^2 + 8x + 3 = 0$

$a = 2$ (x^2 coefficient),

$b = 8$ (x coefficient),

$c = 3$ (constant).

Substitute the values for a, b and c into the formula to solve for x

$$x = \frac{-8 \pm \sqrt{(8)^2 - 4(2)(3)}}{2(2)}$$
$$x = \frac{-8 \pm \sqrt{64 - 24}}{4}$$
$$x = \frac{-8 \pm \sqrt{40}}{4}$$
$$x = \frac{-8 \pm 6.32}{4}$$

So

$$x = \frac{-8 + 6.32}{4} \quad \text{or} \quad x = \frac{-8 - 6.32}{4}$$
$$x = \frac{-1.68}{4} \quad \text{or} \quad x = \frac{-14.32}{4}$$

$x = -0.42$ or $x = -3.58$

Exercises 4

Solve the Following Quadratic Equations

1. $x^2 - x - 6 = 0$ (By Guide Number method)

2. $2k^2 - 3k - 2 = 0$ (By Guide Number method)

3. $\delta^2 - 3\delta - 10 = 0$ (By formula)

4. $3\theta^2 - 4\theta + 1 = 0$ (By formula)

5.6 Answers

Exercises 1:

1). $6(s+5t)$
2). $21(u-2v)$
3). $a(-b+c)$
4). $5\beta(5\alpha-7\delta)$
5). $3\theta(7\theta^2-8)$
6). $11q^2(3q^2-2)$
7). $-25\alpha\beta^2\gamma(1-2\alpha\gamma)$
8). $-4x(x^4+4x^3z+2y^{3)}$
9). $abc(1+a^2b^3c-a^5b^9c)$
10). $n(l^2mn-m^2+ln)$

Exercises 2:

1). $(q+4)(x+y)$
2). $(2b+17)(c-d)$
3). $(\rho+\lambda)(\pi+\kappa)$
4). $(5s+2q)(a-2b)$
5). $(t+16)(p-2)$
6). $(\alpha-\phi)(2l+\delta)$
7). $(u-c)(v+2d)$
8). $(2+\kappa)(\mu-3\lambda)$
9). $(h-4j)(i-2g)$
10). $(16-r)(s-2)$

Exercises 3:

1). $(x+6)(x+4)$
2). $(a+9)(a+3)$
3). $(\beta+4)(2\beta+1)$
4). $(3\theta-2)(\theta+8)$
5). $(2t-1)(t+3)$
6). $(b+3)(b-4)$
7). $(3\alpha+2)(\alpha-5)$
8). $(z-3)(4z-1)$
9). $(q-1)(2q-7)$
10). $(w-8)^2$

Exercises 4:

1. $x=3$ or $x=-2$
2. $k=-\frac{1}{2}$ or $k=2$
3. $\delta=5$ or $\delta=-2$
4. $\theta=1$ or $\theta=\frac{1}{3}$

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