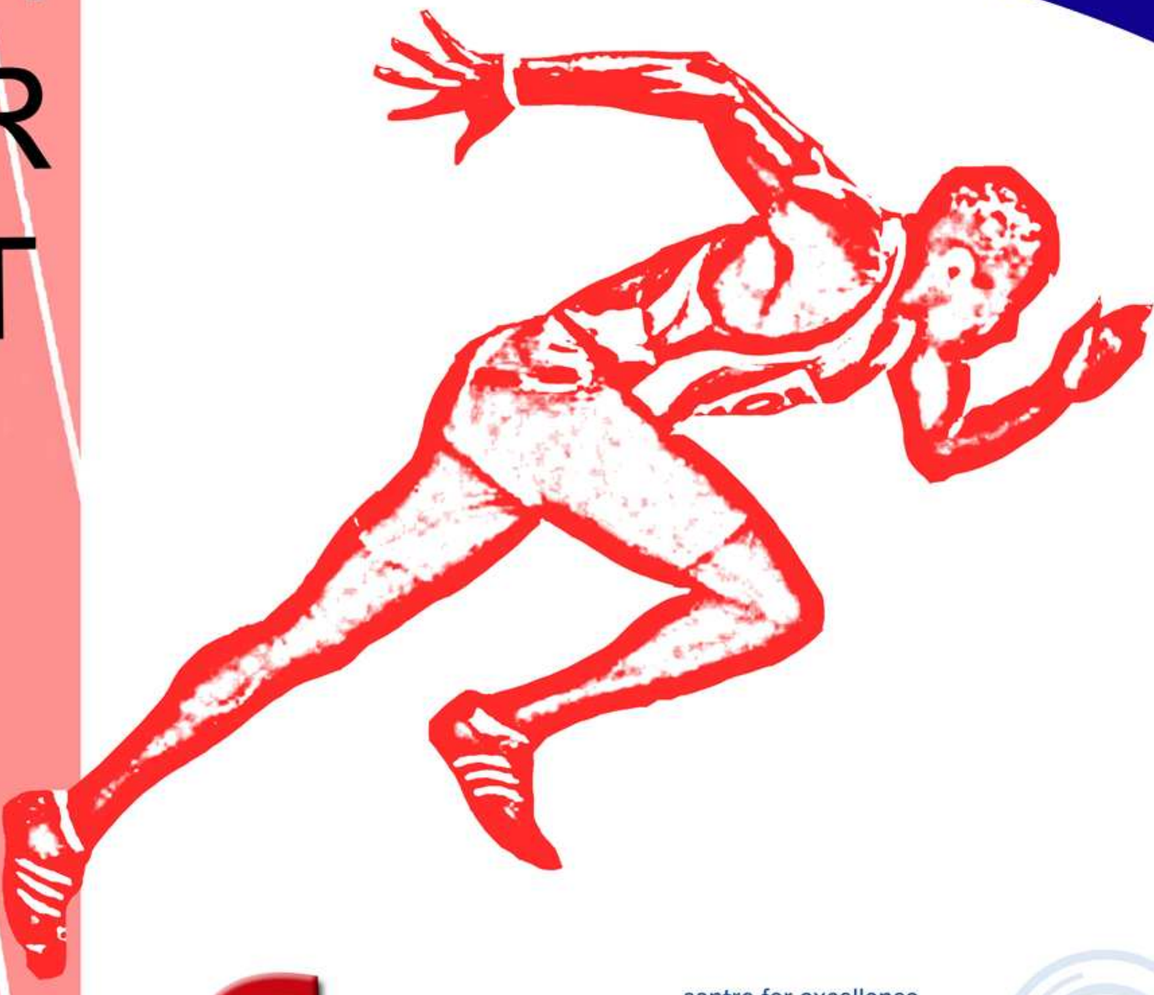
HEAD START MATHS

Indices & Logarithms



CEMTL
Regional Centre for Excellence in Mathematics Teaching and Learning

centre for excellence
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Foreword

The Regional Centre for Excellence in Mathematics Teaching and Learning (CEMTL) is collaboration between the Shannon Consortium Partners: University of Limerick, Institute of Technology, Limerick; Institute of Technology, Tralee and Mary Immaculate College of Education, Limerick., and is driven by the Mathematics Learning Centre (MLC) and The Centre for Advancement in Mathematics Education and Technology (CAMET) at the University of Limerick.

CEMTL is committed to providing high quality educational resources for both students and teachers of mathematics. To that end this package has been developed to a high standard and has been peer reviewed by faculty members from the University of Limericks Department of Mathematics and Statistics and sigma, the UK based Centre for Excellence in Teaching and Learning (CETL). Through its secondment programme, sigma provided funding towards the creation of these workbooks.

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9 Indices and Logarithms

9.1 Indices

We have already seen that $2 \times 2 \times 2 \times 2 \times 2$ can be written in the shorthand form 2^5.

2^5 is pronounced 2 **to the power of** 5.

2 is referred to as the **Base** and 5 is referred to as the **Power** or the **Index**.

There are a number of rules which we adhere to when working with indices or powers.

Rule 1:

$x^m \times x^n = x^{m+n}$

Add the powers when multiplying numbers with the same base.

Example:

$2^4 \times 2^3 = 2^{4+3} = 2^7$.

$3^5 \times 3^6 = 3^{5+6} = 3^{11}$.

Rule 2:

$\dfrac{x^m}{x^n} = x^{m-n}$

Subtract the powers when dividing 2 numbers with the same base.

Example:

$\dfrac{4^5}{4^3} = 4^{5-3} = 4^2$.

$\dfrac{3^7}{3} = 3^{7-1} = 3^6$.

Rule 3:

$(x^m)^n = x^{mn}$

Multiply the powers when raising one power to another power.

Example:

$(x^3)^2 = x^6.$

$(2^4)^3 = 2^{12}.$

Rule 4:

$(xy)^m = x^m y^m$

If a product is raised to a power, each factor is raised to that power.

Example:

$(xy)^2 = x^2y^2.$

$(ab)^3 = a^3b^3.$

Rule 5:

$$\left(\frac{x}{y}\right)^m = \frac{x^m}{y^n}$$

If a quotient is raised to a power, both numerator (top number) and denominator (bottom number) are raised to that power.

Example:

$$\left(\frac{3}{4}\right)^2 = \frac{3^2}{4^2} = \frac{9}{16}.$$

$$\left(\frac{a}{b}\right)^3 = \frac{a^3}{b^3}.$$

Rule 6:

$x^0 = 1$

Any number raised to the power of zero is 1.

Example:

$5^0 = 1$

$1,000^0 = 1$

Rule 7:

$$x^{-m} = \frac{1}{x^m} \quad \text{or} \quad \frac{1}{x^{-m}} = x^m$$

A number with a negative power is equal to its reciprocal with a positive power.

Example:

$$4^{-2} = \frac{1}{4^2} = \frac{1}{16}.$$

$$5^{-3} = \frac{1}{5^3} = \frac{1}{125}.$$

$$\frac{1}{2^{-5}} = 2^5 = 32.$$

$$\frac{1}{10^{-2}} = 10^2 = 100.$$

Rule 8:

$$x^{\frac{m}{n}} = \left(x^{\frac{1}{n}}\right)^m = (\sqrt[n]{x})^m$$

Example:

$16^{\frac{3}{4}} = (16^{\frac{1}{4}})^3 = (\sqrt[4]{16})^3 = 2^3 = 8.$

$125^{\frac{2}{3}} = (125^{\frac{1}{3}})^2 = (\sqrt[3]{125})^2 = 5^2 = 25.$

$81^{\frac{1}{2}} = (81^{\frac{1}{2}})^1 = (\sqrt{81})^1 = 9.$

Exercises 1

Simplify Each of the Following Using the Rules You Have Just Learned:

1. $(243)^{\frac{4}{5}}$

2. $10,000^0$

3. $\dfrac{1}{7^{-2}}$

4. $\left(\dfrac{3}{4}\right)^3$

5. $(121)^{\frac{3}{2}}$

6. $(64)^{\frac{4}{3}}$

7. $\left(\dfrac{49}{100}\right)^{\frac{1}{2}}$

8. 2^{-7}

9. $\dfrac{4}{4^{-4}}$

10. $(27)^{\frac{2}{3}}$

11. $\left(\dfrac{3}{10}\right)^3$

12. $\left(\dfrac{9}{16}\right)^{\frac{3}{2}}$

13. $\left(\frac{3}{7}\right)^{-2}$

14. $\left(\frac{8}{125}\right)^{\frac{2}{3}}$

15. $\left(\frac{1}{2}\right)^{-5}$

9.2 Logarithms

A **Logarithm** (or **Log** for short) is another name for a power or index.

In other words, the log of a particular number is the power that the base must be raised to yield the particular number.

Example

$10^2 = 100$

2 is called the **log** (or **power** or **index**) to which 10, the **base**, must be raised to get the value 100.

We write this as $\log_{10} 100 = 2$

How can we calculate logs?

If we know the base and the power we can work out the value.

For example, $3^4 = 3 \times 3 \times 3 \times 3 = 81$

If we know the base and the value then we can work out the $\log$.

For example, base 3, value 81, log ?

$\log_3 81$ . . . we ask ourselves what is the power to which 3 must be raised to get 81?

$$\begin{aligned} 3^? &= 81 \\ 3^4 &= 81 \end{aligned}$$

Therefore $\log_3 81 = 4$

So, in general

$$\text{If} \quad a^b = c \quad \text{then} \quad \log_a c = b$$

Example:

$10^4 = 10000 \quad \longrightarrow \quad \log_{10} 10000 = 4$

$4^5 = 1024 \quad \longrightarrow \quad \log_4 1024 = 5$

$8^3 = 512 \quad \longrightarrow \quad \log_8 512 = 3$

Exercises 2

Evaluate the Following

1. $\log_3 81$

2. $\log_8 64$

3. $\log_5 125$

4. $\log_2 16$

5. $\log_{16} 2$

6. $\log_4 2$

7. $\log_7 7$

8. $\log_2 128$

9. $\log_3 729$

10. $\log_{10} 1$

Rules of Logs

Like indices, we have a number of rules we must adhere to when dealing with logs.

(NOTE: Logs are only defined for positive numbers.)

Rule 1:

$\log_a m + \log_a n = \log_a mn$

Example:

$\log_2 5 + \log_2 7 = \log_2(5 \times 7) = \log_2 35$

$\log_{10} 3 + \log_{10} 20 = \log_{10}(3 \times 20) = \log_{10} 60$

Rule 2:

$\log_a m - \log_a n = \log_a \left(\frac{m}{n}\right)$

Example:

$\log_2 15 - \log_2 5 = \log_2 \left(\frac{15}{5}\right) = \log_2 3$

$\log_4 120 - \log_4 20 = \log_4 \left(\frac{120}{20}\right) = \log_4 6$

NB: In rules 1 and 2 the base of the numbers MUST be the same.

Rule 3:

$\log_a m^n = n \log_a m$

Example:

$\log_4 5^2 = 2 \log_4 5$

$\log_{10} x^3 = 3 \log_{10} x$

Rule 4:

$\log_n m = \dfrac{\log_a m}{\log_a n}$ (change of base)

Example:

$$\log_{32} 16 = \frac{\log_2 16}{\log_2 32} = \frac{4}{5}$$
$$\log_2 64 = \frac{\log_2 64}{\log_2 2} = \frac{6}{1} = 6$$

Rule 5:

$\log_a a = 1$

Example:

$\log_{10} 10 = 1$ (because $10^1 = 10$)

$\log_4 4 = 1$ (because $4^1 = 4$)

Rule 6:

$\log_a 1 = 0$

Example:

$\log_{100} 1 = 0$ (because $100^0 = 1 \ldots$ rule no. 6 of indices)

$\log_x 1 = 0$ (because $x^0 = 1 \ldots$ rule no. 6 of indices)

Exercises 3

Simplify the Following to a Single Log Expression

1. $\log_2 8 + \log_2 6$

2. $\log_3 25 - \log_3 5$

3. $2\log_{10} 5 + \log_{10} 3$

4. $\log_5 100 - \log_5 20$

5. $\log_4 36 + \log_4 2$

6. $\log_{10} 7 + \log_{10} 3$

7. $\log_{10} 6 + 3\log_{10} 2$

8. $\dfrac{\log_2 18}{\log_2 9}$

9. $\log x + \log 3x$

10. $\log_2 12 - \log_2 6 + \log_2 4$

9.3 Answers

Exercises 1:

1). 81	2). 1	3). 49	4). $\frac{27}{64}$
5). 1331	6). 256	7). $\frac{7}{10}$	8). $\frac{1}{128}$
9). 1024	10). 9	11). $\frac{27}{1000}$	12). $\frac{27}{64}$
13). $5\frac{4}{9}$	14). $\frac{4}{25}$	15). 32	

Exercises 2:

1). 4	2). 2	3). 3	4). 4
5). $\frac{1}{4}$	6). $\frac{1}{2}$	7). 1	8). 7
9). 6	10). 0		

Exercises 3:

1). $\log_2 48$	2). $\log_3 5$	3). $\log_{10} 75$	4). $\log_5 5$
5). $\log_4 72$	6). $\log_{10} 21$	7). $\log_{10} 48$	8). $\log_9 18$
9). $\log 3x^2$	10). $\log_2 8$		

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