HEAD START MATHS

Graphs of Quadratic & Other Special Functions

CEMTL
Regional Centre for Excellence in Mathematics Teaching and Learning

centre for excellence
sigma Σ
in mathematics & statistics support

Foreword

The Regional Centre for Excellence in Mathematics Teaching and Learning (CEMTL) is collaboration between the Shannon Consortium Partners: University of Limerick, Institute of Technology, Limerick; Institute of Technology, Tralee and Mary Immaculate College of Education, Limerick., and is driven by the Mathematics Learning Centre (MLC) and The Centre for Advancement in Mathematics Education and Technology (CAMET) at the University of Limerick.

CEMTL is committed to providing high quality educational resources for both students and teachers of mathematics. To that end this package has been developed to a high standard and has been peer reviewed by faculty members from the University of Limericks Department of Mathematics and Statistics and sigma, the UK based Centre for Excellence in Teaching and Learning (CETL). Through its secondment programme, sigma provided funding towards the creation of these workbooks.

Please be advised that the material contained in this document is for information purposes only and is correct and accurate at the time of publishing. CEMTL will endeavour to update the information contained in this document on a regular basis.

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7 Graphs of Quadratic and Other Special Functions

7.1 Graphs of Quadratic Functions

In our last workshop we learned how to draw graphs given the equation of a line and a set of coordinated points. We ended up with a straight line each time. In this section we are going to look at other types of functions and their graphs.

As we have already seen a function of the form $y = ax^2 + bx + c$, where a, b and c are constants is called a **Quadratic Function**.

If the a value (x^2) is positive (+), we get a $\bigcup$ shape if we sketch the graph of the function.

If the a value is negative (-), we get a $\bigcap$ shape if we sketch the graph of the function.

Example 1

Sketch the graph of the function $y = x^2 - 3x - 1$ in the domain $-2 \leq x \leq 4$.

We make out a table of ordered pairs as follows:

x =	-2	-1	0	1	2	3	4
x^2	4	1	0	1	4	9	16
-3x	6	3	0	-3	-6	-9	-12
-1	-1	-1	-1	-1	-1	-1	-1
y =	9	3	-1	-3	-3	-1	3

Points for graph: $(-2, 9), (-1, 3), (0, -1), (1, -3), (2, -3), (3, -1), (4, 3)$.

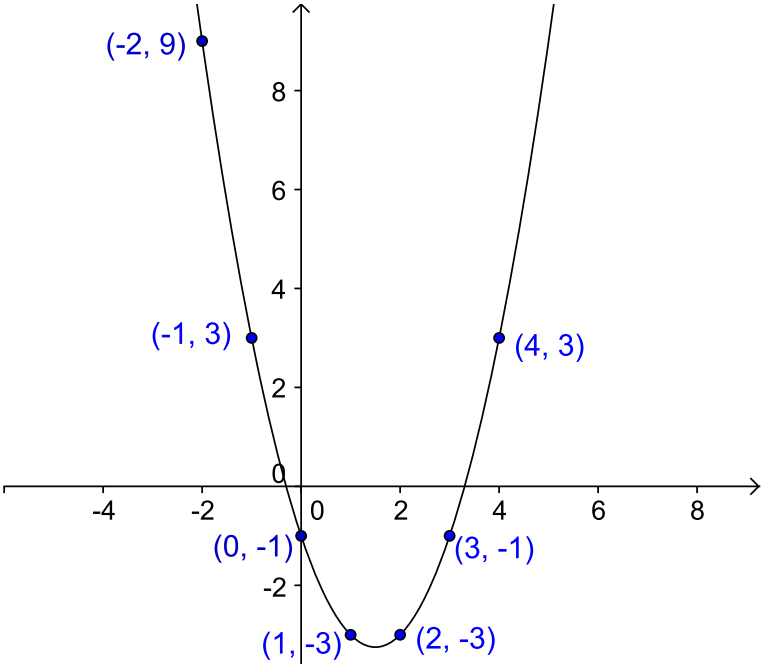


Figure 1: $y = x^2 - 3x - 1$

Example 2

Draw a graph of the function $y = 4 + 2x - x^2$ in the domain - $2 \leq x \leq 4$

x =	-2	-1	0	1	2	3	4
4	4	4	4	4	4	4	4
+ 2x	-4	-2	0	2	4	6	8
- x^2	-4	-1	0	-1	-4	-9	-16
y =	-4	1	4	5	4	1	-4

Points for graph: $(-2, -4), (-1, 1), (0, 4), (1, 5), (2, 4), (3, 1), (4, -4)$.

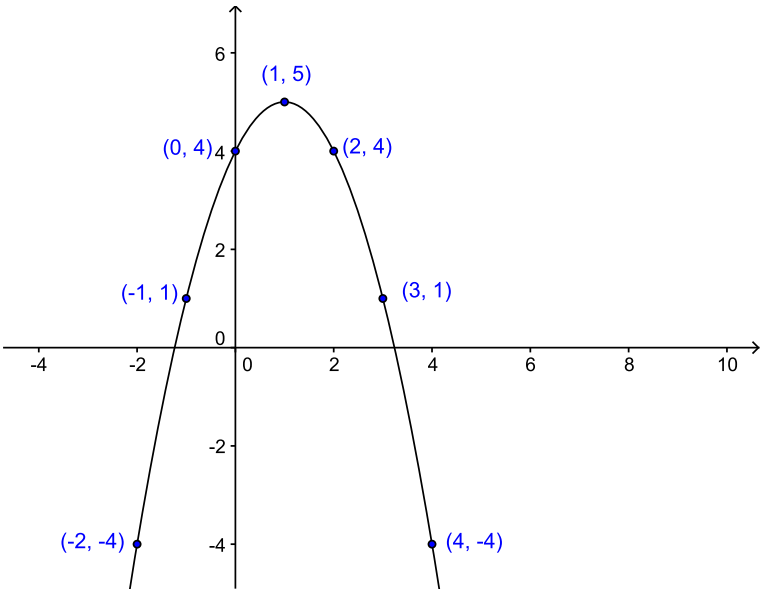
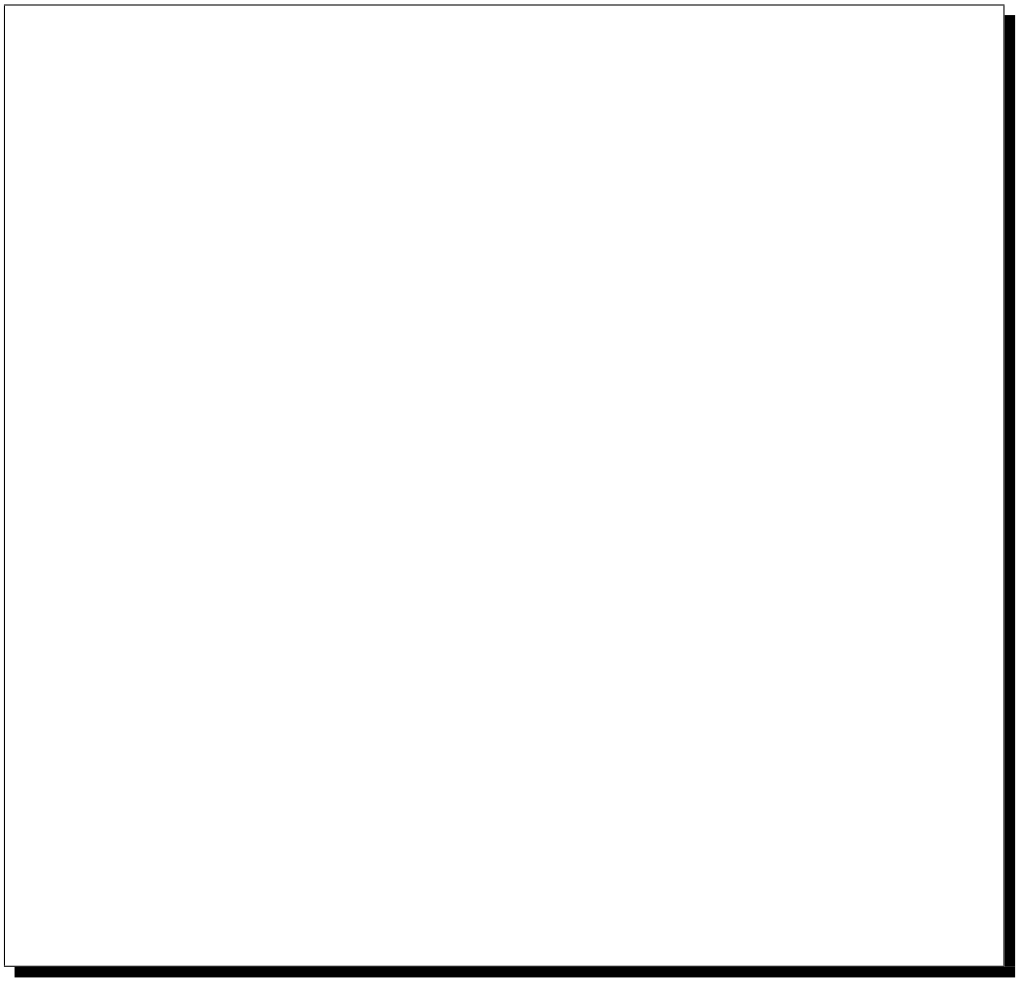


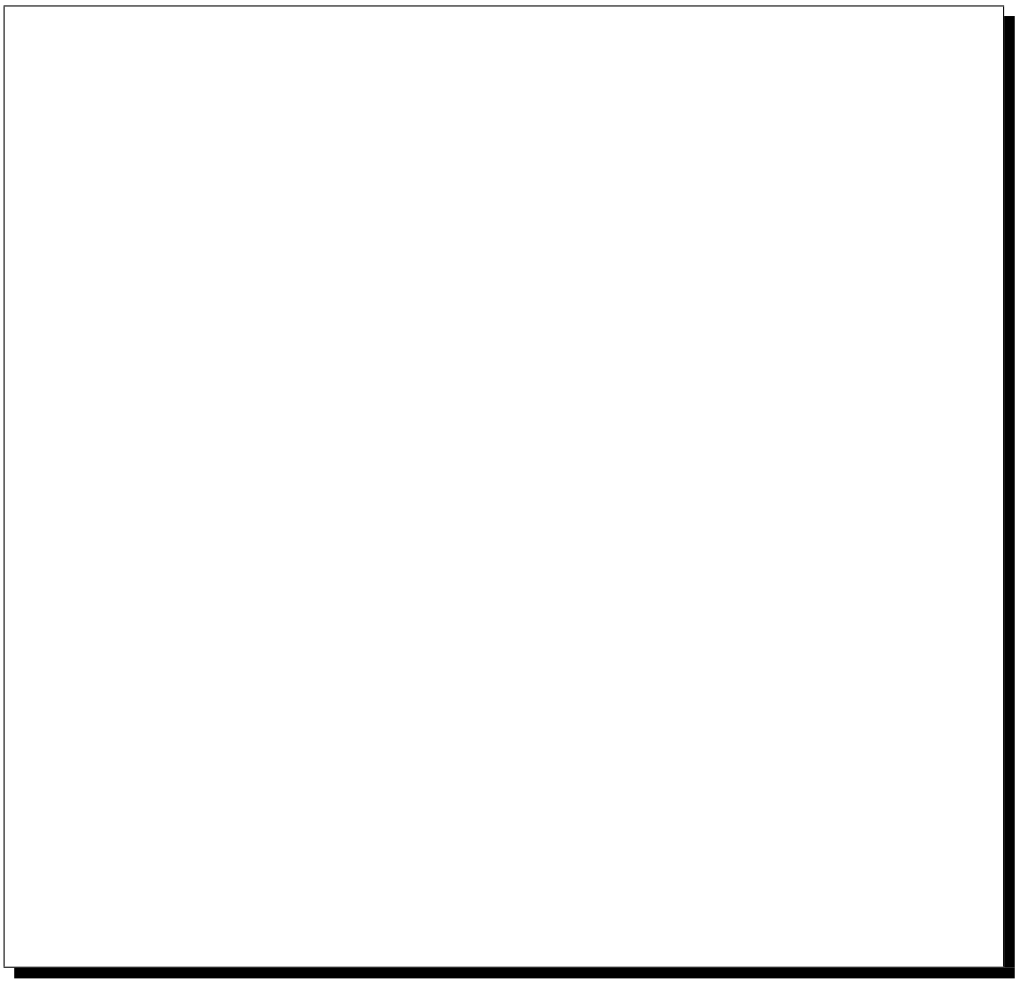
Figure 2: $y = 4 + 2x - x^2$

Exercises 1

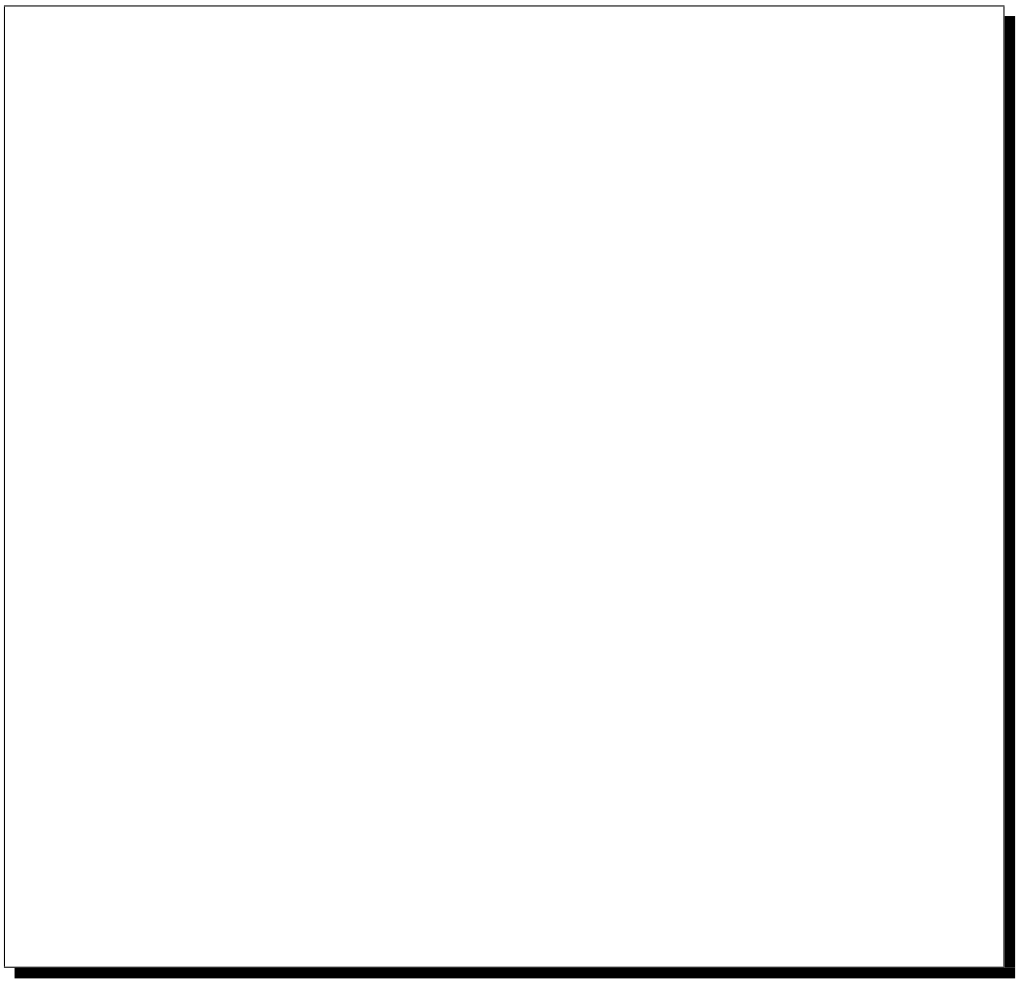
1. Sketch the function $y = 2x^2 - 5x - 2$ in the domain - $3 \leq x \leq 3$.



2. Sketch the function $y = 10 + x - 2x^2$ in the domain - $2 \leq x \leq 2$.



3. Sketch the function $y = 3x - x^2$ in the domain $0 \leq x \leq 5$.



7.2 Special Functions

1. The Exponential Function

$y = e^x$ is called the **Exponential Function** where e = 2.7182818...

Using a calculator we will complete the following table by evaluating the decimal value of y:

x =	-2	-1.5	-1	-0.5	0	0.5	1
y =	e^{-2}	$e^{-1.5}$	e^{-1}	$e^{-0.5}$	e^{0}	$e^{0.5}$	e^{1}
y =	0.135	0.223	0.368	0.606	1	1.648	2.718

Points for graph:
$(-2, 0.135), (-1.5, 0.223), (-1, 0.368), (-0.5, 0.606), (0, 1), (0.5, 1.648), (1, 2.718)$

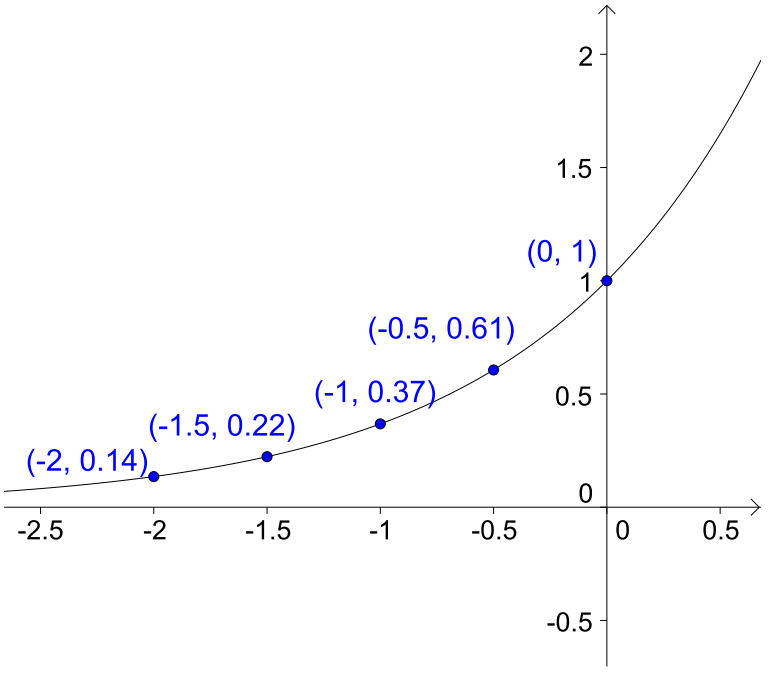


Figure 3: $y = e^x$

2. The Logarithmic Function

The **Logarithmic Function** is defined by $y = \log x$

Using a calculator we will complete the following table to get the ordered pairs needed to draw the graph:

x =	0.1	0.3	0.6	1.0	1.3	1.6	3.0
y =	$\log(0.1)$	$\log(0.3)$	$\log(0.6)$	$\log(1.0)$	$\log(1.3)$	$\log(1.6)$	$\log(3.0)$
y =	-1	-0.523	-0.222	0	0.114	0.204	0.477

Points for graph:
$(0.1, -1), (0.3, -0.523), (0.6, -0.222), (1, 0), (1.3, 0.114), (1.6, 0.204), (3, 0.477)$

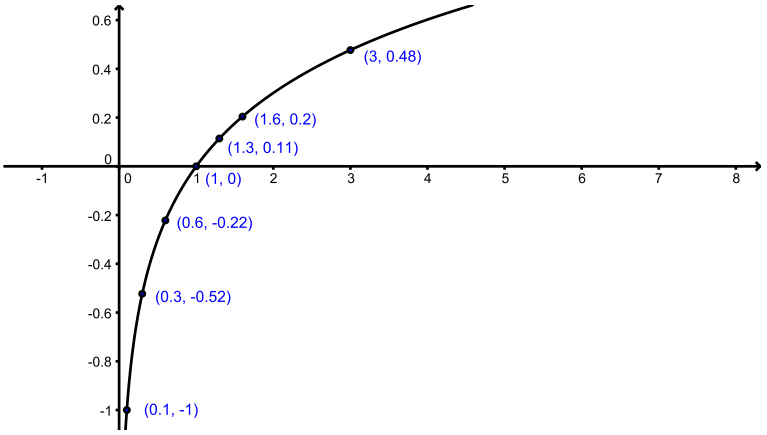
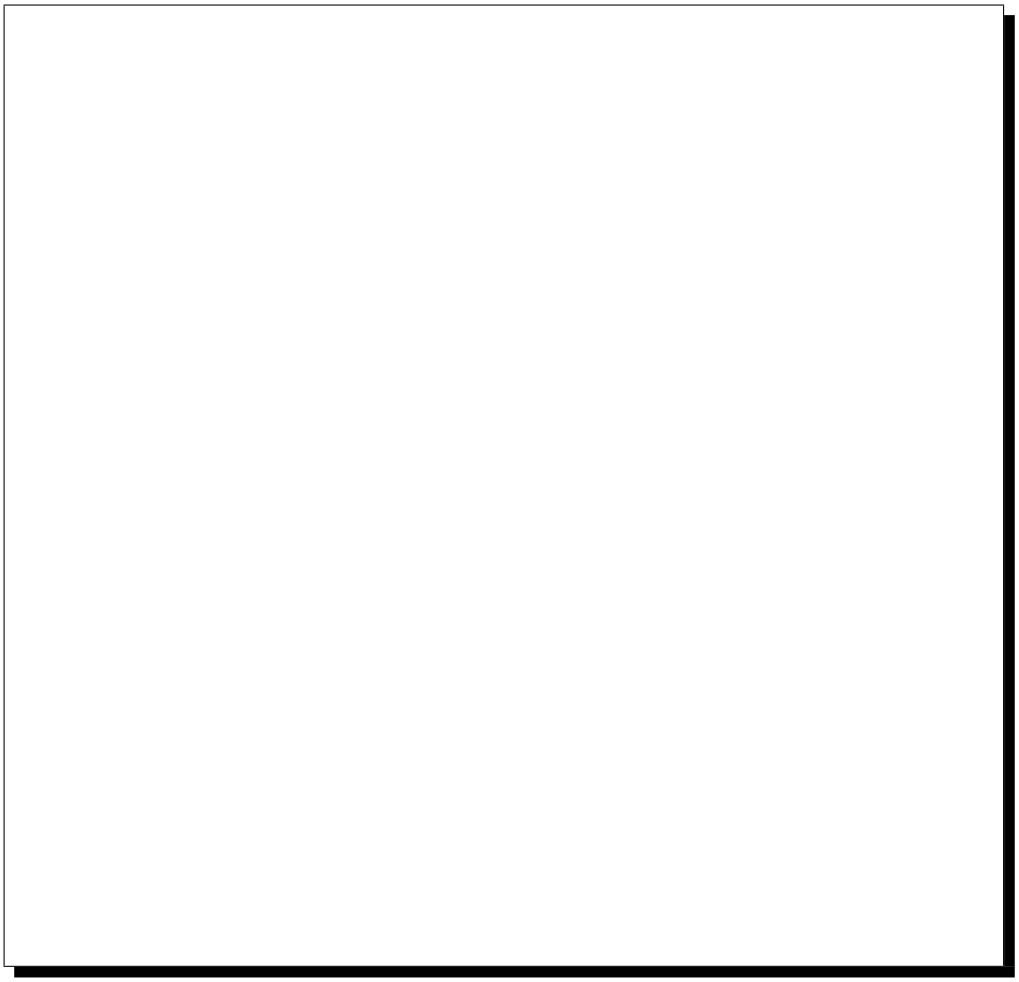


Figure 4: $y = logx$

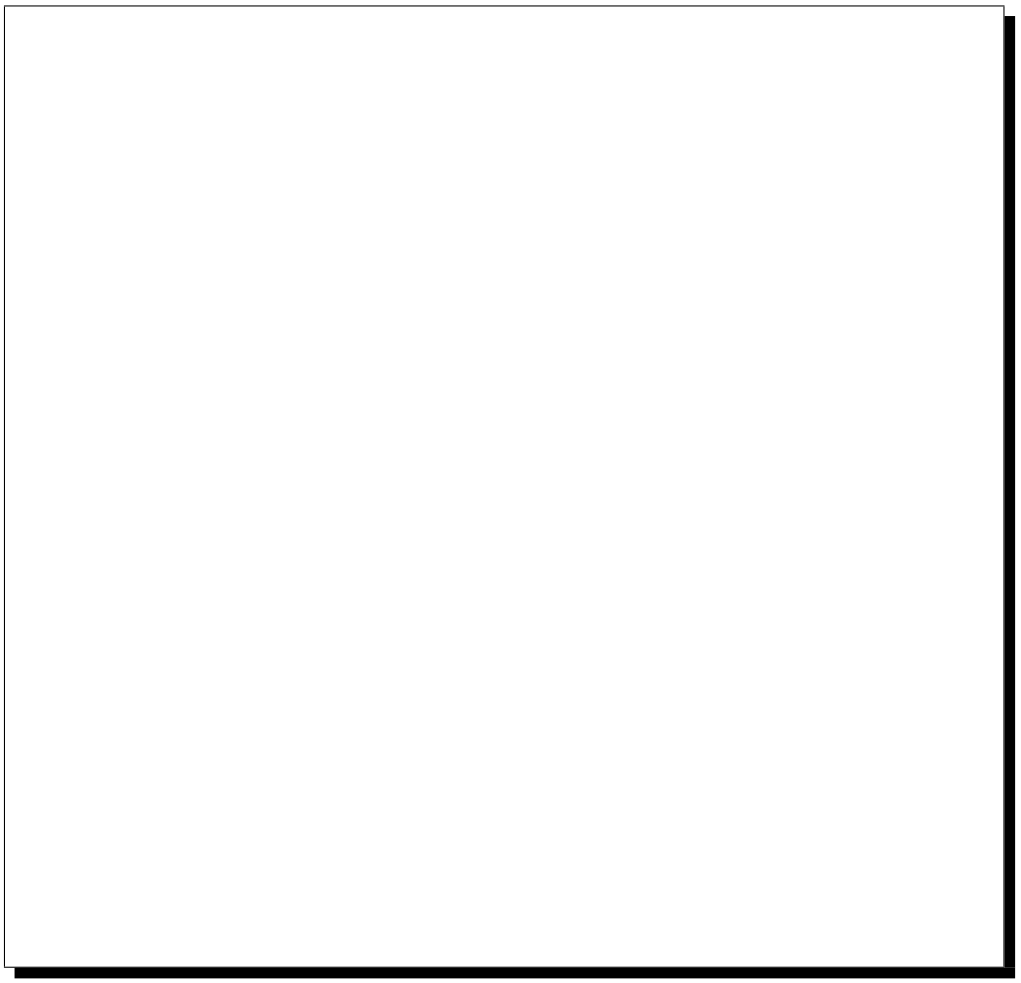
Exercises 2

Graph the Following Functions:

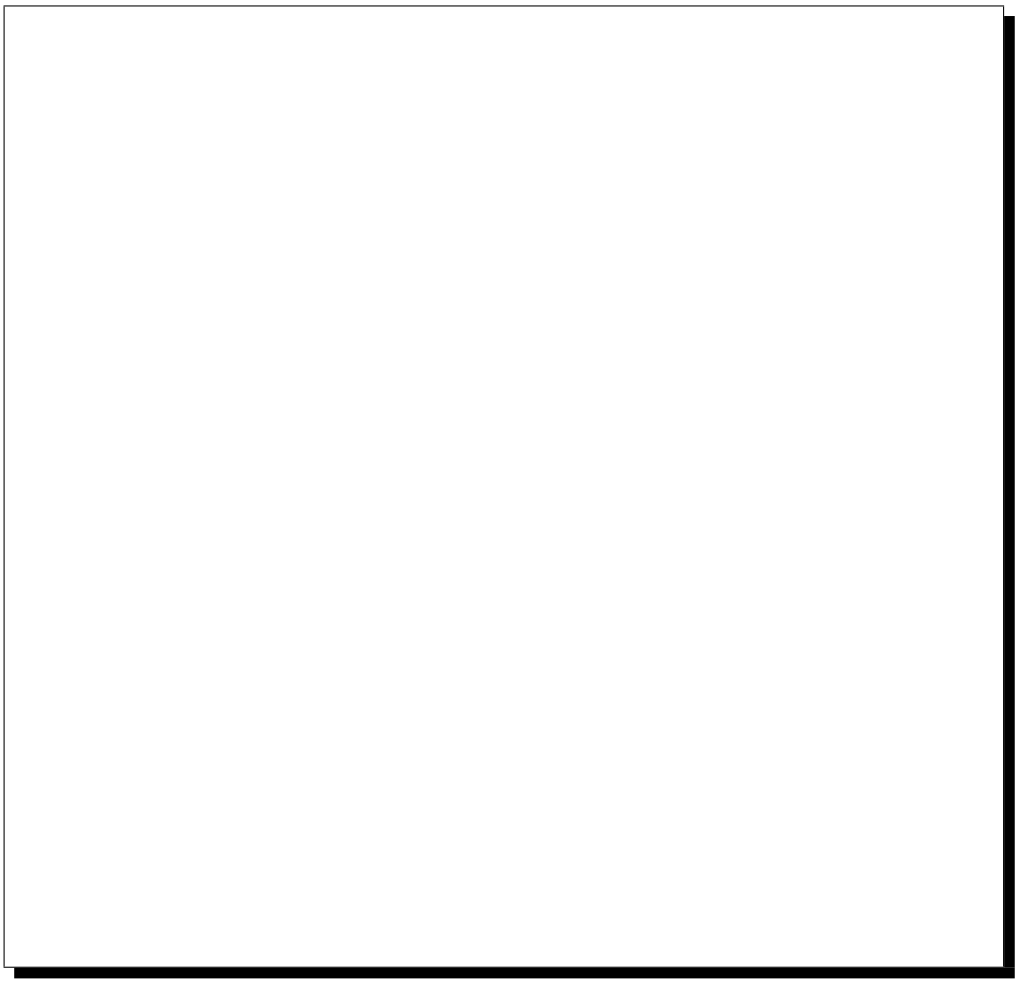
1. $y = \log(2x)$



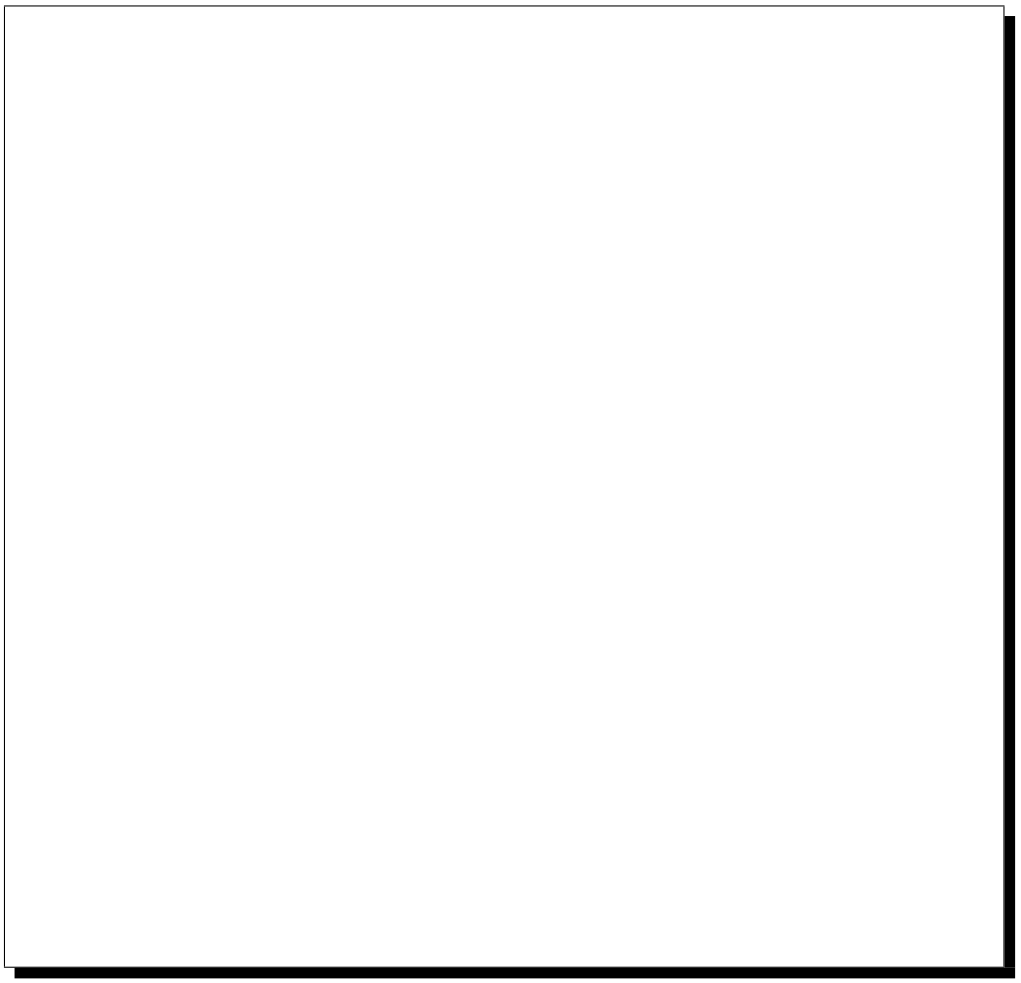
2. $y = 5 \log x$



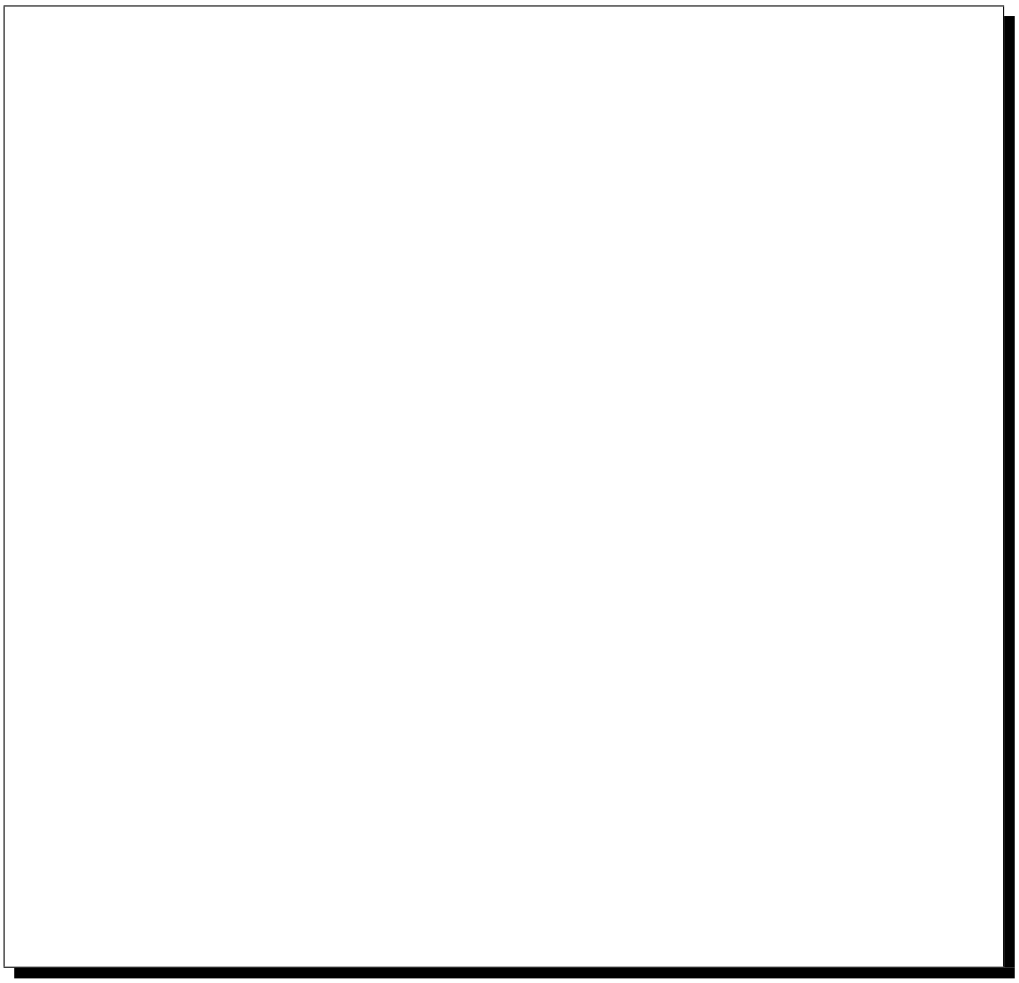
3. $y = e^{2x}$



4. $y = 4e^x$



5. $y = e^{-x}$

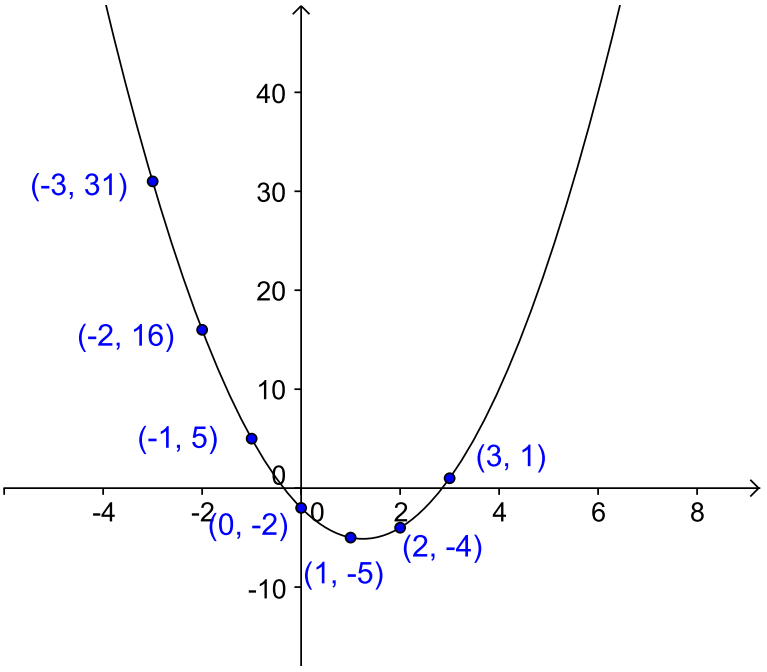


7.3 Answers

Exercise 1:

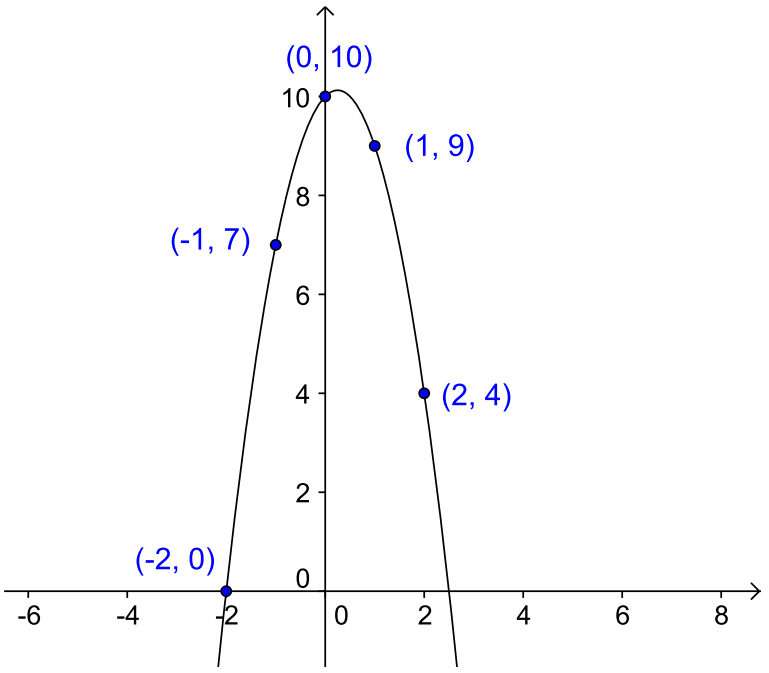
1: $y = 2x^2 - 5x - 2$

$x =$	-3	-2	-1	0	1	2	3
$2x^2$	18	8	2	0	2	8	18
$-5x$	15	10	5	0	-5	-10	-15
-2	-2	-2	-2	-2	-2	-2	-2
$y =$	31	16	5	-2	-5	-4	1



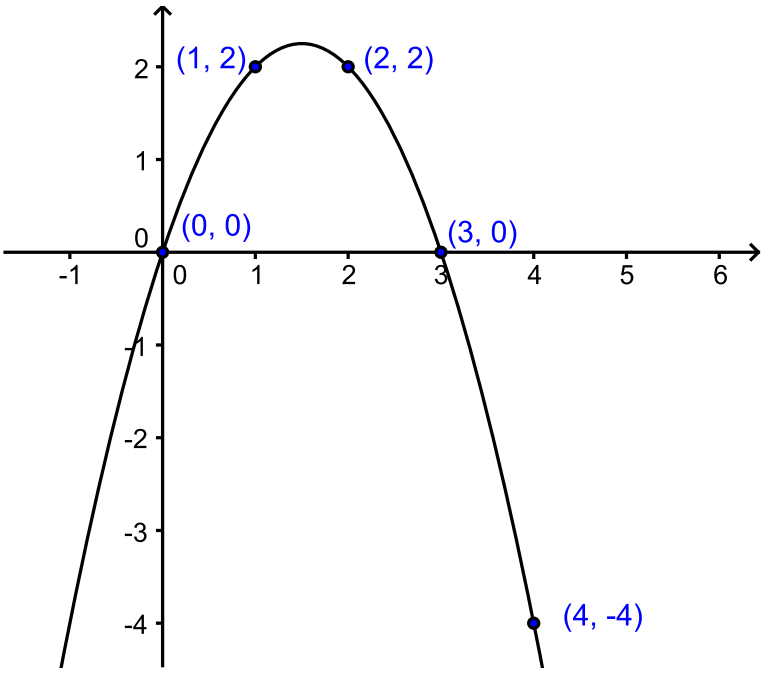
2: $y = 10 + x - 2x^2$

x =	-2	-1	0	1	2
10	10	10	0	10	10
$+x$	-2	-1	0	1	2
$-2x^2$	-8	-2	0	-2	-8
y =	0	7	10	9	4



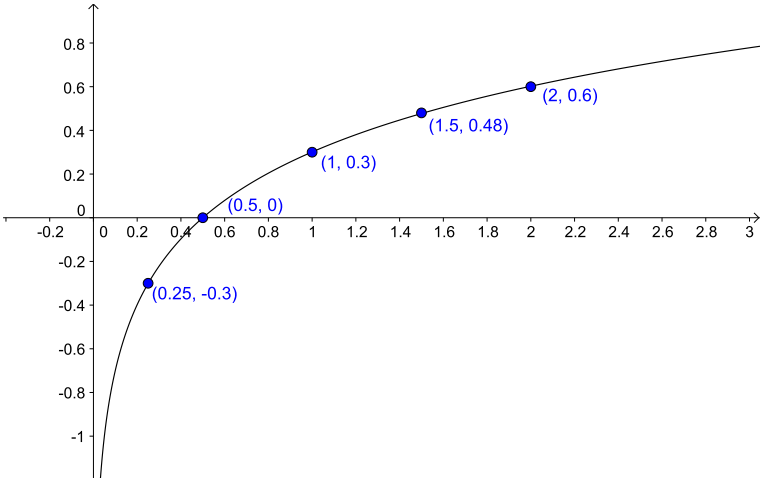
3: $y = 3x - x^2$

x =	0	1	2	3	4	5
$3x$	0	3	6	9	12	15
$-x^2$	0	-1	-4	-9	-16	-25
y =	0	2	2	0	-4	-10

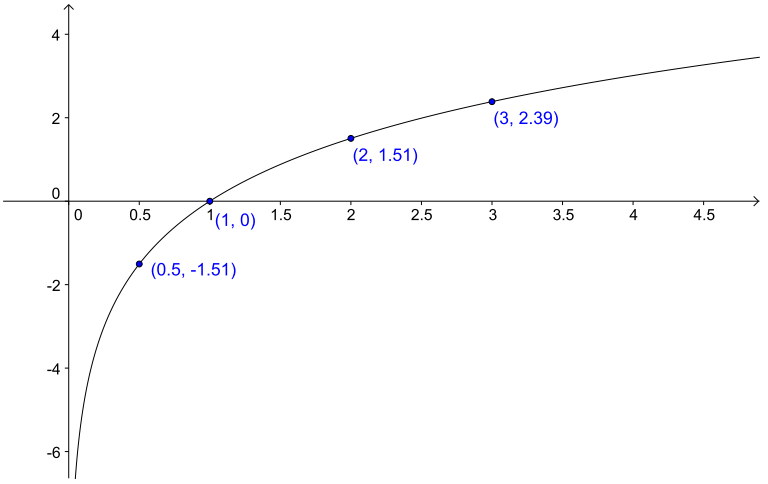


Exercise 2:

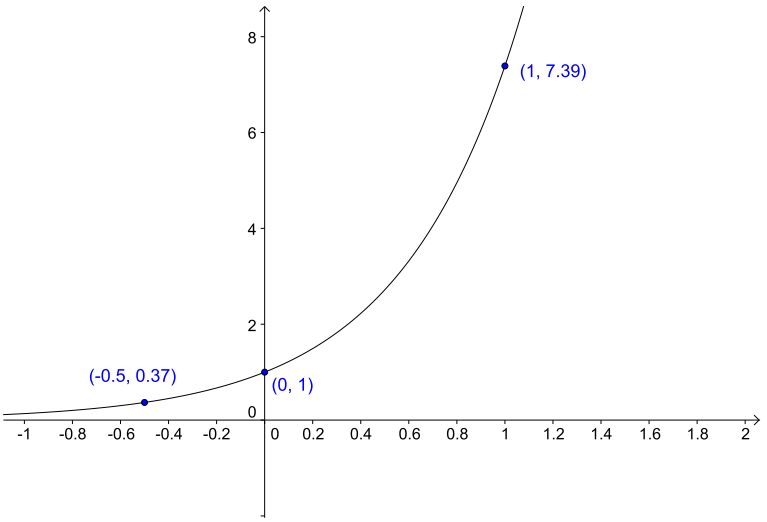
1: $y = log(2x)$



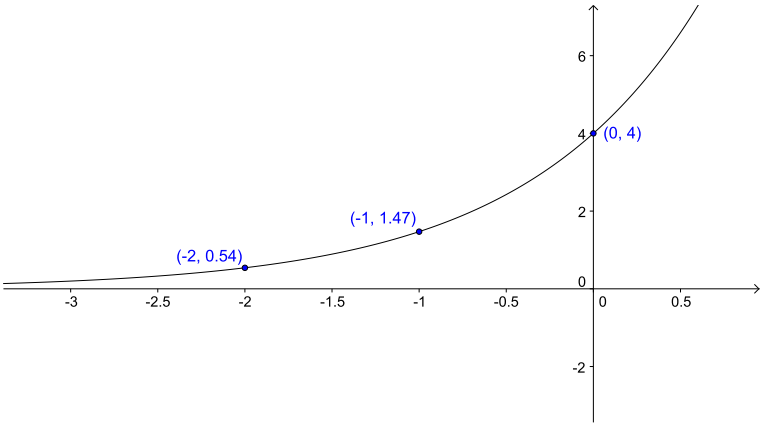
2: $y = 5logx$



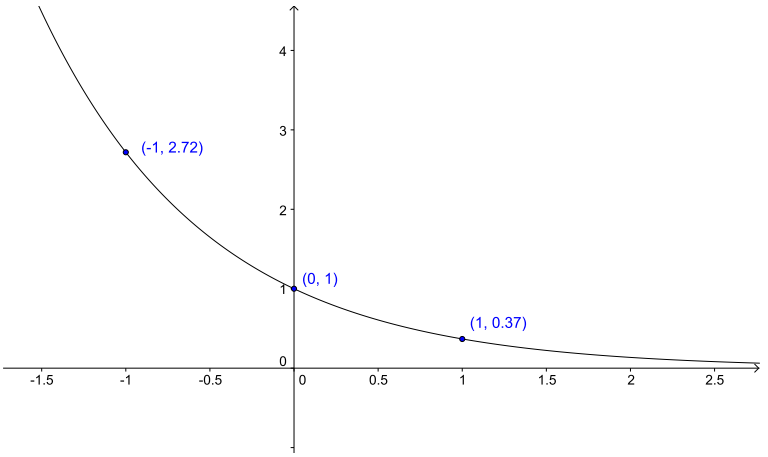
3: $y = e^{2x}$

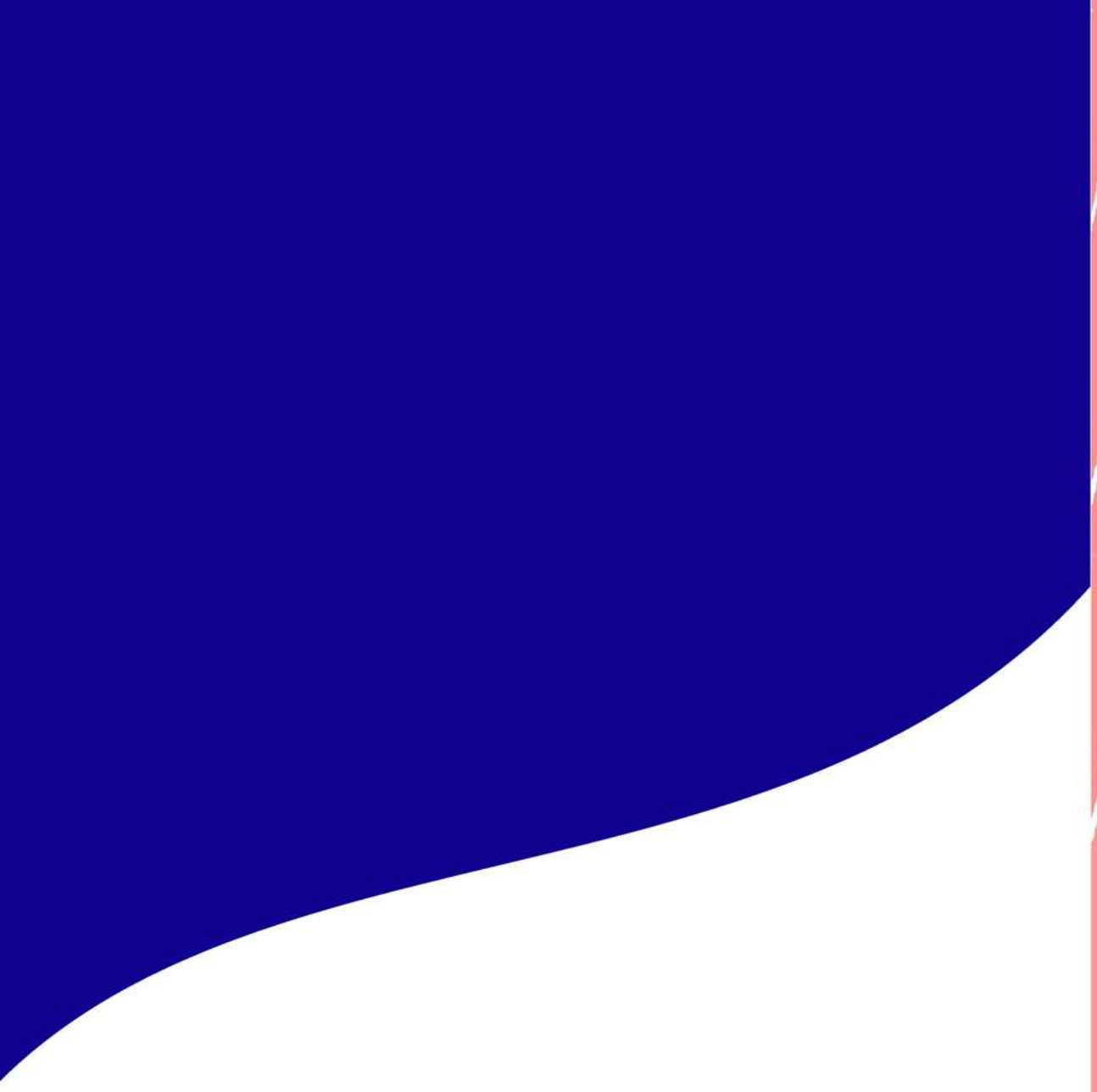

(1, 7.39)
(-0.5, 0.37)
(0, 1)
8
6
4
2
0
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2

4: $y = 4e^{x}$


(0, 4)
(-1, 1.47)
(-2, 0.54)
6
4
2
0
-2
-3
-2.5
-2
-1.5
-1
-0.5
0
0.5

5: $y = e^{-x}$



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