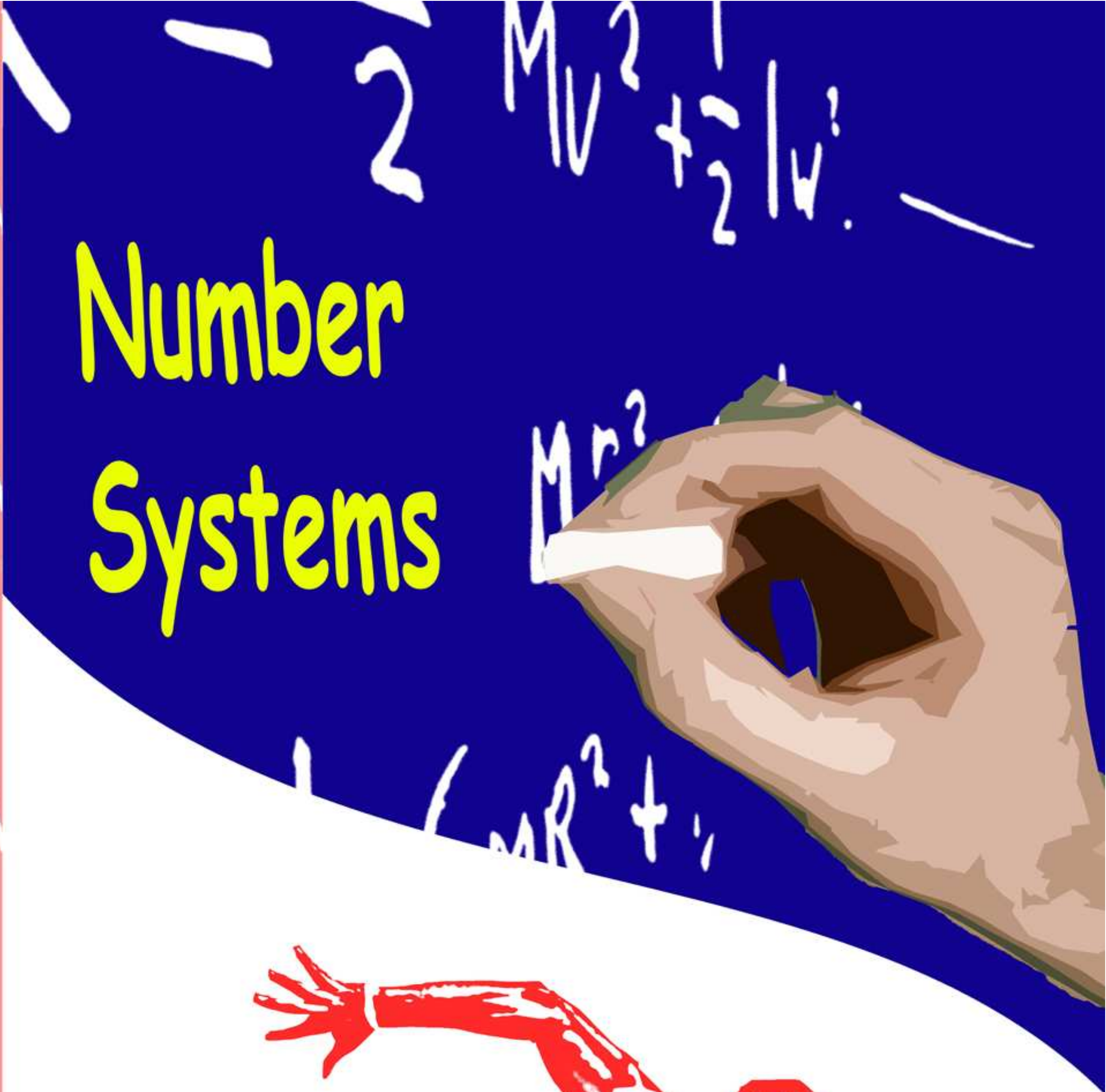
HEAD START MATHS

Number Systems



CEMTL
Regional Centre for Excellence in Mathematics Teaching and Learning

centre for excellence
sigma Σ
in mathematics & statistics support

Foreword

The Regional Centre for Excellence in Mathematics Teaching and Learning (CEMTL) is collaboration between the Shannon Consortium Partners: University of Limerick, Institute of Technology, Limerick; Institute of Technology, Tralee and Mary Immaculate College of Education, Limerick., and is driven by the Mathematics Learning Centre (MLC) and The Centre for Advancement in Mathematics Education and Technology (CAMET) at the University of Limerick.

CEMTL is committed to providing high quality educational resources for both students and teachers of mathematics. To that end this package has been developed to a high standard and has been peer reviewed by faculty members from the University of Limericks Department of Mathematics and Statistics and sigma, the UK based Centre for Excellence in Teaching and Learning (CETL). Through its secondment programme, sigma provided funding towards the creation of these workbooks.

Please be advised that the material contained in this document is for information purposes only and is correct and accurate at the time of publishing. CEMTL will endeavour to update the information contained in this document on a regular basis.

Table of Contents

1.1	Introduction to Number Systems	1
1.2	Natural Numbers	2
1.3	Integers	3
1.4	Real Numbers	4
1.5	Factors/ Divisors	5
1.6	Prime Numbers/ Composite Numbers	6
1.7	Multiples	7
1.8	The Order of Numbers	8
1.9	Inequalities on the Number Line	9
1.10	Indices	13
1.11	Rules of Indices	14
1.12	Order of Operations	18
1.13	Rules for Addition and Subtraction	20
1.14	Multiplication of Integers	22
1.15	Division of Integers	23
1.16	Answers	26

1 Number Systems

In this workshop we are going to look at various number systems we will encounter when studying mathematics. Mathematics is a language so we need to be able to speak it and to write it. For this reason, you will be introduced to various mathematical vocabulary and notation as we proceed through this workbook.

1.1 Introduction to Number Systems

$\mathbb{N}$ - Natural Numbers

Natural numbers are the counting numbers we first learned as children. They are the set of non-negative, whole numbers. The dots below indicate that it is an infinite set of numbers.
$0, 1, 2, 3, 4, \ldots$.

$\mathbb{Z}$ - The Integers

Often times we need to deal with negative numbers, for example, temperature scales or even bank account balances! For this reason we have the set of integers. They are the set of positive and negative whole numbers and 0 (which is neither positive nor negative!).
$\ldots - 3, -2, -1, 0, 1, 2, 3 \ldots$

$\mathbb{Q}$ - The Rationals

Everyday we deal with a certain group of numbers that are not whole numbers called fractions. For example, a half price sale, division of a sum of money into quarters etc. This set of numbers is called Rational numbers. Any number that can be written in the form $\frac{a}{b}$ is a Rational number (where a and b are integers and b is not equal to 0). We will deal with this set of numbers in greater detail in the next workshop.

$\mathbb{R}$ - The Reals

The set of Real numbers is the set of all positive and negative, whole and fractional numbers. In other words, it includes all of the above number systems and more besides.

1.2 $\mathbb{N}$ - Natural Numbers

In this section we are going to learn a little bit more about Natural numbers. As stated earlier, Natural numbers are all the non-negative (+) whole/full numbers. It is an infinite set as there are an infinite number of Natural numbers

We say 4 'is an element of' the set $\mathbb{N}$ or 4 is a Natural number.

Mathematically we write this as

$4 \in \mathbb{N}$

Natural numbers are 'ordered' i.e. they progress from small to large. They can be classified as 'Odd' or 'Even'.

Even Natural Numbers

$0, 2, 4, 6, 8, \ldots$

These are Natural numbers that can be divided exactly by 2 i.e. with no remainder.

Odd Natural Numbers

$1, 3, 5, 7, 9, \ldots$

These are Natural numbers that cannot be divided exactly by 2.

Note:

There has been much debate in the mathematics world as to whether 0 is included in the set of Natural numbers or not. For the purposes of this course 0 is included.

1.3 $\mathbb{Z}$ - Integers

The set of Integers is the set of all positive (+) and negative (-) whole/full numbers including 0.

We say - 6 'is an element of' $\mathbb{Z}$ or -6 is an Integer.

Mathematically we write this as

- $6 \in \mathbb{Z}$

ALL Natural numbers are Integers but NOT ALL Integers are Natural numbers.

Example

$12 \in \mathbb{Z}$ and $12 \in \mathbb{N}$

-$6 \in \mathbb{Z}$ but -$6 \notin \mathbb{N}$

We say - 6 'is not an element of' the set $\mathbb{N}$ because -6 is a negative number.

1.4 $\mathbb{R}$ - Real Numbers

The set of Real numbers contains all Natural numbers, Integers and Rational numbers.

$\ldots, -3, -2.27, 0, 1, 4.9216, \frac{11}{2}, \ldots$

The diagram below illustrates this:

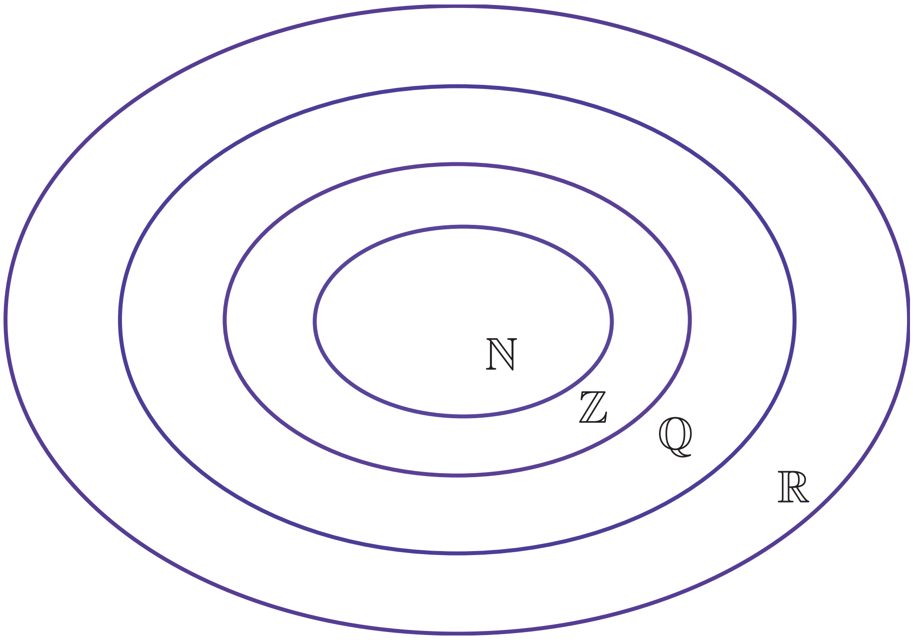


Figure 1: Number Systems

1.5 Factors/ Divisors

The factors of 24 are all the numbers that divide exactly into 24 with no remainder;

$$24 \div 1 = 24$$
$$24 \div 2 = 12$$
$$24 \div 3 = 8$$
$$24 \div 4 = 6$$
$$24 \div 6 = 4$$
$$24 \div 8 = 3$$
$$24 \div 12 = 2$$
$$24 \div 24 = 1$$

So we can list the factors of 24 as $1, 2, 3, 4, 6, 8, 12$ and 24.
Similarly the factors of 18 are $1, 2, 3, 6, 9$ and 18.

We can see that $1, 2, 3$ and 6 are factors of both 24 and 18 i.e. they are common to both.
They are called **Common Factors** or **Common Divisors** of 24 and 18.

6 is called the **Highest Common Factor** (HCF) OR
Highest Common Divisor (HCD) of 24 AND 18.

Exercise 1

Find the HCF of 32 and 48

1.6 Prime Numbers/ Composite Numbers

Prime numbers are numbers that have only 2 factors: themselves and 1.

Example

The factors of 7 are 7 (itself) and 1.

The factors of 13 are 13 and 1.

All numbers that are not prime numbers are called **Composite Numbers**.

Exercise 2

Can you think of any other prime numbers?

1.7 Multiples

The multiples of 2 are $2, 4, 6, 8, 10, 12,$ .... i.e. the numbers that 2 divides into exactly (i.e. with no remainder).

The multiples of 7 are $7, 14, 21, 28, 35,$ .... i.e. the numbers that 7 divides into exactly (i.e. with no remainder).

And so on!

The multiples of 10 are $10, 20, \underline{30}, 40, 50, \underline{60}, 70, 80, \underline{90}$ ....

The multiples of 15 are $15, \underline{30}, 45, \underline{60}, 75, \underline{90},$ .....

$30, 60$ and 90 etc. are **Common Multiples** but 30 is called the
Lowest Common Multiple (LCM) of 10 and 15.

Exercise 3

Find the LCM of 6 and 21

1.8 The Order of Numbers

Numbers are 'ordered'. By that we mean that they progress from small to large. On a number line, if we move to the right the numbers get bigger, as we move to the left the numbers get smaller.

We know that 10 is greater (bigger) than 4.

In mathematical terms we write:

$10 > 4$ (10 is greater than 4) or $4 < 10$ (4 is less than 10).

The symbol $\leq$ means 'less than or equal to'.

The symbol $\geq$ means 'greater than or equal to'.

Example

The first three integers > 2 are $3, 4, 5$ (Does **not** include 2).

The first three numbers ≥ 3 are $3, 4, 5$ (**Does** include 3).

Exercise 4

Say which of the following are true or false:

1. $5 > 4$
2. $6 > 9$
3. -4 < -2
4. -8 < 8

1.9 Inequalities on the Number Line

Suppose we wish to represent a set of numbers on a number line. For example, the set of Natural numbers greater than 1.

We can describe this set mathematically as $\{\, x \mid x > 1, x \in \mathbb{N}\}$

We read this as 'the set of all elements x, such that, x is greater than 1, where x is a Natural number'. The expression $x > 1$ where x is a Natural number ($x \in \mathbb{N}$) means that x may be any Natural number which is greater than 1.

The values of x are: $2, 3, 4, 5, 6,$ .....

Remember this set of numbers does not include 1 as we are only looking for numbers that are 'greater than' 1.
We represent it on the numberline as follows:

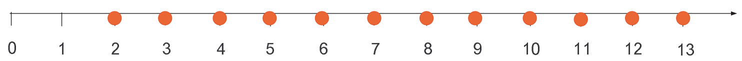


Figure 2: $\{x \mid x > 1, x \in \mathbb{N}\}$

The arrow to the right indicates that the set is infinite . . . it goes on forever.

The set of numbers x, for which $x \geq 3$ and $x \leq 8$, where $x \in \mathbb{N}$, is

$\{\, 3, 4, 5, 6, 7, 8 \,\}$

We can write this set of numbers as:

$3 \leq x \leq 8, x \in \mathbb{N}$

or

$\{\, x \mid 3 \leq x \leq 8, x \in \mathbb{N} \,\}$

On the numberline it looks like this:

Figure 3: $\{x \mid 3 \leq x \leq 8, x \in \mathbb{N}\}$

Example

Graph the following on the numberline:

$\{x \mid -7 < x \leq -2, x \in \mathbb{Z}\ \}$

Solution

This describes the set of whole numbers from -7 up to -2. Note how we exclude -7 but include -2.

The set is {-6, $-5, -4, -3, -2$ }

On the numberline it looks like this:

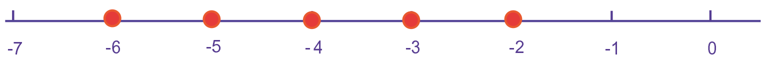


Figure 4: $\{x \mid -7 < x \leq -2, x \in \mathbb{Z}\}$

Example

Graph the following on the numberline:

$\{x \mid -2 < x \leq 1, x \in \mathbb{R}\ \}$

Solution

This describes the set of real numbers from -2 up to 1. Note how we exclude -2 (hence the open circle) but include 1 (hence the closed circle).

It is not possible to list all the numbers in this set as there are an infinite amount of numbers. We represent this fact with a thick line on the numberline as follows:

Figure 5: $\{x \mid -2 < x \leq 1, x \in \mathbb{R}\}$

Exercises 5

Graph the Following Sets on the Numberline

1. $\{x \mid x \geq 3, x \in \mathbb{N}\}$

2. $\{x \mid x \leq 0, x \in \mathbb{R}\}$

3. $\{x \mid x > -3, x \in \mathbb{Z}\}$

4. $\{x \mid x < 6, x \in \mathbb{N}\}$

5. $\{x \mid 3 \leq x \leq 5, x \in \mathbb{N}\}$

6. $\{x \mid -1 < x \leq 4, x \in \mathbb{Z}\}$

7. $\{x \mid 4 < x < 9, x \in \mathbb{R}\}$

8. $\{x \mid -3 \leq x \leq -2, x \in \mathbb{Z}\}$

1.10 Indices

$3 \times 3 \times 3 \times 3 \times 3 \times 3$ can also be written in the shorter form of

$$3^6$$

We read this as 3 **to the power of** 6. (There are 6 threes multiplied together).

6 is the **Power** or the **Index** of 3. **Indices** are the plural of Index.

Other Examples

$5 \times 5 \times 5 = 5^3$

We call this 5 to the power of 3 or, more commonly, 5 **cubed**.

$10 \times 10 = 10^2$

We call this 10 to the power of 2 or, more commonly, 10 **squared**.

1.11 Rules of Indices

We will deal with indices in detail in a later workshop. For now there are two rules of indices we must familiarise ourselves with. We will illustrate these rules by some examples.

Rule 1

When multiplying numbers with the same base (the number which is being raised to a power), we add the powers.

e.g. $2^3 \times 2^4 = 2^7$

$2^3 \times 2^4 = (2 \times 2 \times 2) \quad \times \quad (2 \times 2 \times 2 \times 2)$

$= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$= 2^7$

$\Rightarrow 2^3 \times 2^4 = 2^7$

Rule 2

When dividing numbers with the same base, we subtract the powers.

e.g. $2^9 \div 2^4 = 2^5$

$$2^9 \div 2^4 = \frac{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}{2 \times 2 \times 2 \times 2}$$

$$= \frac{\cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{2} \times 2 \times 2 \times 2 \times 2 \times 2}{\cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{2}}$$

$= 2 \times 2 \times 2 \times 2 \times 2$

$= 2^5$

$\Rightarrow 2^9 \div 2^4 = 2^5$

Note 1

If the index of a number is not written, this means the number is raised to the power of 1, which is itself.

Example:

$2 = 2^1$

$1000 = 1000^1$

$8 \times 8^3 = 8^1 \times 8^3 = 8^4$

$9^5 \div 9 = 9^5 \div 9^1 = 9^4$

Note 2

Any number raised to the power of 0 is equal to one.

Example:

$10^0 = 1$

$9^5 \div 9^5 = 9^0 = 1$

Exercises 6

Rewrite the Following in the Form of a^b

1. $4^5 \times 4^6 \times 4^2$

2. 5×5^2

3. $6 \times 6 \times 6^2$

4. $8^5 \div 8^3$

5. $2^{10} \div 2$

6. $5^6 \times 5^4 \div 5^7$

7. 4×2^2

8. $\dfrac{3^4 \times 3^5}{3^9}$

9. $\dfrac{5^{11}}{5^2 \times 5^7}$

10. $\dfrac{3^3}{3 \times 3^2}$

1.12 Order of Operations

In mathematics there are various operations we may be required to carry out e.g. addition, multiplication, raising a number to a power or index etc. There is a specific order to how these operations are carried out. We do not always work from left to right like we would in English. In other words, there is a precedence and we can use the following rule to remind us which order we should follow: **BIDMAS**

- Brackets
- Indices
- Division
- Multiplication
- Addition
- Subtraction

If there are any brackets present, the work inside the brackets must be carried out first, followed by any indices that need to be evaluated. For division and multiplication we can work left to right. Addition and subtraction are carried out last, again working from left to right.

Example

Calculate the following:

$20 \div 4 + 7 \times 2^3 - (13 - 6)$

Solution

B	$20 \div 4 + 7 \times 2^3 - (13 - 6)$	Work out the bracket
I	$20 \div 4 + 7 \times 2^3 - 7$	Evaluate the index
D	$20 \div 4 + 7 \times 8 - 7$	Perform the division
M	$5 + 7 \times 8 - 7$	Carry out the multiplication
A	$5 + 56 - 7$	Do the addition
S	$61 - 7$	Calculate the final answer

Answer: 54

Exercises 7

Simplify the Following:

$$18 \div 6 + 2 \times 7 = \boxed{}$$

$$11 + 3 \times 7 - 28 \div 4 = \boxed{}$$

$$12 \times 3 - 84 \div 12 = \boxed{}$$

$$5^2 \div 5 + 56 \div (5 + 2) = \boxed{}$$

$$(5 + 6) \times 3^2 + 35 \div (25 - 20) = \boxed{}$$

$$5 \times (9 \times 3) \div 3 + 3^4 = \boxed{}$$

$$(56 - 45) \times 2 + 36 \div 18 = \boxed{}$$

$$(23 + 1) \div 6 + 17 \times 2^2 - 25 = \boxed{}$$

$$135 \div 15 - 3^2 + (6^3 - 1) = \boxed{}$$

$$(4 \times 4^2 - 4) - (4^3 - 4^2) = \boxed{}$$

1.13 Rules for Addition and Subtraction

When working with numbers we need to be careful when dealing with positive (+) and negative (-) numbers. In this section, we will learn how to add and subtract numbers. Check if the numbers you wish to add or subtract have same or different signs then follow the rules below:

Same Signs

- **Add as normal**
- **Keep the same sign**

Example

$+3 + 14 = +17$,

$-18 - 5 = -23$.

Different Signs

- **Take the smaller number from the bigger number**
- **Keep the sign of the bigger number in your answer**

Example

$+13 - 5 = +8$,

$-25 + 10 = -15$.

Exercises 8

Simplify the Following:

$$7+8 = \boxed{}$$

$$-3+9 = \boxed{}$$

$$15-20 = \boxed{}$$

$$-23-25 = \boxed{}$$

$$-14-15-19 = \boxed{}$$

$$-100+100 = \boxed{}$$

$$19-15+6 = \boxed{}$$

$$14-21-7+15 = \boxed{}$$

$$-7+15+20-50 = \boxed{}$$

$$-16+24-10-15 = \boxed{}$$

1.14 Multiplication of Integers

There are 4 possible cases when multiplying integers.

Case 1

$+ \times + = +$

e.g. $+9 \times +4 = +36$

Case 2

$+ \times - = -$

e.g. $+5 \times -6 = -30$

Case 3

$- \times + = -$

e.g. $-2 \times +10 = -20$

Case 4

$- \times - = +$

e.g. $-3 \times -8 = +24$

Remember....

Same signs give plus +

Different signs give minus -

1.15 Division of Integers

Case 1

$+ \div + = +$

e.g. $+9 \div +3 = +3$

Case 2

$+ \div - = -$

e.g. $+15 \div -3 = -5$

Case 3

$- \div + = -$

e.g. $-12 \div +6 = -2$

Case 4

$- \div - = +$

e.g. $-30 \div -10 = +3$

Remember....

Same signs give plus $+$

Different signs give minus $-$

Exercises 9

Express Each of the Following as a Single Integer

$$-6 \times -9 = \boxed{}$$

$$15 \times -2 = \boxed{}$$

$$-16 \times -3 = \boxed{}$$

$$-20 \div -5 = \boxed{}$$

$$99 \div -11 = \boxed{}$$

$$-91 \div 7 = \boxed{}$$

$$\frac{-360}{-12} = \boxed{}$$

$$\frac{-160}{20} = \boxed{}$$

$$\frac{330}{-3} = \boxed{}$$

$$-15 \times -3 \times -2 = \boxed{}$$

$$12 \times 4 \times -1 = \boxed{}$$

$$0 \times -7 \times 10 = \boxed{}$$

$$\frac{-5 \times 6}{-10} = \boxed{}$$

$$\frac{25 \times -3}{-5} = \boxed{}$$

$$\frac{-7 \times -7}{-7} = \boxed{}$$

Exercises 10

Express Each of the Following as a Single Integer

$$15 + 12 \times -2 - 3 = \boxed{}$$

$$-8 - 12 \div -4 + 3^2 = \boxed{}$$

$$(-5)(-8 + 12) + 39 \div -13 = \boxed{}$$

$$6(2 - 3) - 7 - 4 = \boxed{}$$

$$-(2 + 13) \div 5 + 15 = \boxed{}$$

$$(-4)(-7) - 5(19 - 21) = \boxed{}$$

$$4^2 \div -8 - 6^2 - 20 = \boxed{}$$

$$-(8 \times -6) \div 12 + 20 = \boxed{}$$

$$(15 - 12)^4 - (9 \div 3^2) = \boxed{}$$

$$6^2 + 3^2 - (4^2 \times -7) = \boxed{}$$

1.16 Answers

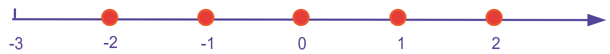
Exercise 1: 16
Exercise 3: 42
Exercise 4: Draw a numberline to see where these answers have come from.

1). True 2). False 3). True 4). True

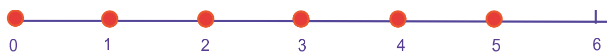
Exercise 5:
1).

2).

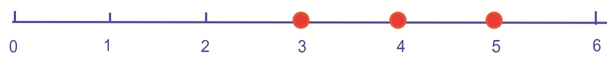
3).



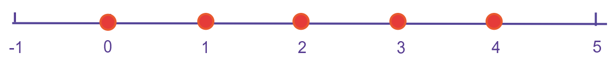
4).



5).

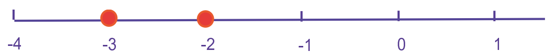


6).



7).

8).



Exercise 6:

1). 4^{13}	2). 5^3	3). 6^4	4). 8^2
5). 2^9	6). 5^3	7). 2^4	8). $3^0 = 1$
9). 5^2	10). $3^0 = 1$		

Exercise 7:

1). 17	2). 25	3). 29	4). 13
5). 106	6). 126	7). 24	8). 47
9). 215	10). 12		

Exercise 8:

1). 15	2). 6	3). -5	4). -48
5). -48	6). 0	7). 10	8). 1
9). -22	10). -17		

Exercise 9:

1). 54	2). -30	3). 48	4). 4
5). -9	6). -13	7). 30	8). -8
9). -110	10). -90	11). -48	12). 0
13). 3	14). 15	15). -7	

Exercise 10:

1). -12	2). 4	3). -23	4). -17
5). 12	6). 38	7). -58	8). 24
9). 80	10). 157		