DUBLIN INSTITUTE OF TECHNOLOGY
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DT228 BSc Computer Science
DT211 BSc Computing
DT263 Higher Certificate in Computing

YEAR I

Supplemental Semester I Examination 2011-12

Mathematics

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Date:
Time:

Answer <u>Question 1</u> and any <u>2</u> other questions

Mathematical Tables and Graph paper are available

Q1

(a) Let A be the set of characters appearing in the string"$device$", B be the set of characters appearing in the string "$digital$" and C be the set of characters in the string"$capacity$". List the elements of the following sets:

(i) $(A \cup B)$ (ii) $(A \cap B)$ (iii) $(A \cup B) \backslash C$ (iv) $(B \cup C) \cap (A \cup C)$

[5 marks]

(b) Use the properties of logarithms to evaluate the following:

(i) $4log_3(2187) - 2log_4\sqrt{2} + log_5\left(\frac{1}{125}\right)$

(ii) $-4log_2(512) - 7log_7\left(\frac{1}{343}\right) - 3log_6\sqrt{216}$

[5 marks]

(c) Find the inverse of the matrix $\begin{pmatrix} 2 & 4 \\ -2 & -1 \end{pmatrix}$.

Hence or otherwise, solve the following system of equations:

$$2x + 4y = 10$$
$$-2x - y = -1$$

[5 marks]

(d) Test the following binary relation R on the given set S for reflexivity, symmetry and transitivity

$$S = \boldsymbol{N}, \quad R = \{(a, b): ab \ is \ even\}$$

[5 marks]

(e) Find the mean, median and variance of the following set of data:

$$6, 14, 16.6, 12.9, 10.2, 14$$

[5 marks]

(f) Let $f: \boldsymbol{N} \to \boldsymbol{N}$ be given by$f(x) = \sqrt{x + 2}$

Let $g: \boldsymbol{N} \to \boldsymbol{N}$ be given by $g(x) = 2x + 7$

Calculate:

(i) $(f \circ g)(x)$

(ii) $(g \circ f)(2)$

(iii) $(f \circ f)(7)$

[5 marks]

(g) Let $U = \{1, 2, 3, 4, 5, 6\}$ be the universal set. Represent the set $A = \{1, 3, 5\}$ with bit string representation.

[5 marks]

(h) Use Euclid's Algorithm to find the hcf of 2,542 and 6,286.

[5 marks]

Q2

(a) In computer graphics the rotation of the plane counterclockwise about the origin (0,0) through an angle θ radians is given by the matrix

$$R_\theta = \begin{pmatrix} cos\theta & -sin\theta & 0 \\ sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Show that the inverse matrix R_θ^{-1} is given by the matrix

$$R_\theta^{-1} = \begin{pmatrix} cos\theta & sin\theta & 0 \\ -sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

[12 marks]

(b) A rectangle having vertices A, B, C and D given in homogenous coordinates

$$A = \begin{pmatrix} 12 \\ -10 \\ 1 \end{pmatrix}, B = \begin{pmatrix} -15 \\ -10 \\ 1 \end{pmatrix}, C = \begin{pmatrix} -15 \\ 20 \\ 1 \end{pmatrix}, D = \begin{pmatrix} 12 \\ 20 \\ 1 \end{pmatrix}$$

is represented by the matrix

$$M = \begin{pmatrix} 12 & -15 & -15 & 12 \\ -10 & -10 & 20 & 20 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

Find the image of this rectangle under the rotation of the plane through an angle of $\frac{2\pi}{5}$ radians clockwise about the origin.

[12 marks]

(c) Let $A = \begin{pmatrix} 3 & -4 \\ 0 & 2 \\ 7 & 7 \end{pmatrix}, B = \begin{pmatrix} 5 & -1 & 6 \\ 1 & 3 & -8 \end{pmatrix}, C = \begin{pmatrix} 5 & 0 \\ -1 & 3 \end{pmatrix}$ and $D = \begin{pmatrix} 5 & -4 & -6 \\ 7 & 10 & -1 \\ 0 & 3 & 7 \end{pmatrix}$.

Evaluate (if possible)

(i) $2AD$

(ii) C^{-1}

(iii) $2D + (5C)^T$

[6 marks]

Q3

(a) Let $A = \{3, 4, 5\}, B = \{x, y, z\}$ and $C = \{y, z, w\}$ be sets. List the elements of the following sets

(i) The *power set of A,* $P(A)$.

(ii) The symmetric difference of B and C, $B\Delta C$

(iii) The Cartesian product of A and B, $A \times B$.

[10 marks]

(b) Let $U = \{10, 11, 12, 13, 14, 15, 16, 17\}$ be the universal set. Let $A = \{11, 13, 15, 16, 17\}$ and $B = \{10, 11, 12, 13, 14, 15\}$ be sets. Use bit string representation to find the following sets:

(i) $\bar{B}$
(ii) $A \cap B$
(iii) $A \cup B$

[10 marks]

(c) Use a truth table to verify if the following are equivalent formulas:

(i) $F \vee (G \wedge H) \sim (F \vee G) \vee (F \wedge H)$
(ii) $\neg(F \vee G) \sim \neg F \wedge \neg G$ (De Morgan's Law)

[10 marks]

Q4

(a) Write out the operational tables for $\boldsymbol{Z}_6$.
Use Fermat's Little Theorem to find the inverses of 1 and 5 modulo 6. Check your answers against the multiplication table for $\boldsymbol{Z}_6$.

[12 marks]

(b) Find the multiplicative inverse of 43 in $\boldsymbol{Z}_{261}^*$.

[10 marks]

(c) Use ***prime factorisation*** to calculate $hcf(291060, 646800)$.

[8 marks]