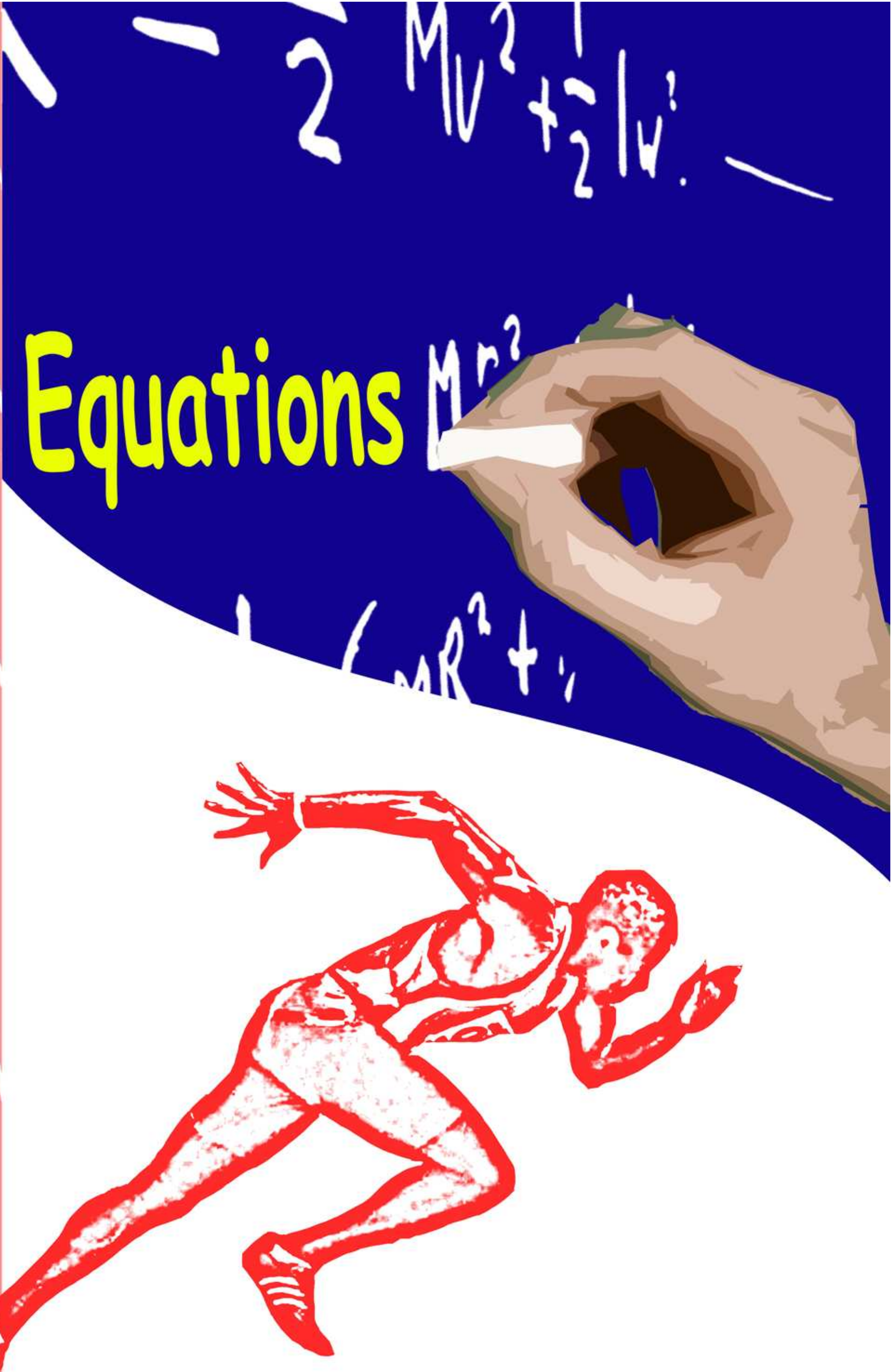
HEAD START MATHS

Equations



CEMTL
Regional Centre for Excellence in Mathematics Teaching and Learning

centre for excellence
sigma
in mathematics & statistics support

Foreword

The Regional Centre for Excellence in Mathematics Teaching and Learning (CEMTL) is collaboration between the Shannon Consortium Partners: University of Limerick, Institute of Technology, Limerick; Institute of Technology, Tralee and Mary Immaculate College of Education, Limerick., and is driven by the Mathematics Learning Centre (MLC) and The Centre for Advancement in Mathematics Education and Technology (CAMET) at the University of Limerick.

CEMTL is committed to providing high quality educational resources for both students and teachers of mathematics. To that end this package has been developed to a high standard and has been peer reviewed by faculty members from the University of Limericks Department of Mathematics and Statistics and sigma, the UK based Centre for Excellence in Teaching and Learning (CETL). Through its secondment programme, sigma provided funding towards the creation of these workbooks.

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4.1 Introduction to Equations

Take the statement $x + 5 = 8$

This mathematical 'statement' contains an equal to sign (=) and hence is called an **equation**.

In an equation, the left hand side of the equal to sign must always equal the right hand side of the equal to sign.

In the above statement the letter x represents an unknown value or **variable**.

We **solve** equations to find the value of the unknown(s)/variable(s).

In this particular example, we know that x must represent the number 3 ... because $3 + 5 = 8$.

It is not always this obvious though.

When solving equations we carry out operations (add, subtract, multiply, divide etc.) to find values for unknowns.

Because of the equal to sign, whatever operations we carry out on the left hand side, we must always do exactly the same on the right hand side.

When solving equations, our aim is to get x (or whatever letter the unknown value is represented by) on its own on one side of the equal to sign.

Example 1

$x + 5 = 27$

In order to get x on its own here we must subtract 5 from the left hand side.
If we do this, we must also subtract 5 from the right hand side.

$x + 5 - 5 = 27 - 5$

Solution: $x = 22$.

Example 2

$3\theta = 99$

We know that 3θ means θ multiplied by 3.
To get θ on its own here we need to divide the left hand side by 3....and of course the right hand side!

$$\frac{3\theta}{3} = \frac{99}{3}$$

Solution: $\theta = 33$.

Example 3

$2x + 10 = 32$

Subtract 10 from both sides

$2x + 10 - 10 = 32 - 10$

$2x = 22$

Divide both sides by 2 to find x

$$\frac{2x}{2} = \frac{22}{2}$$

Solution: $x = 11$.

Example 4

$2p - 1 = 27$

To get $2p$ on its own we need to add one to both sides

$2p - 1 + 1 = 27 + 1$

$2p = 28.$

Finally we divide both sides by 2 to get p on its own i.e. to solve for p

$$\frac{2p}{2} = \frac{28}{2}$$

Solution: $p = 14.$

Example 5

$4m + 6 = m + 27$

The first thing we need to do is subtract m from both sides

$4m + 6 - m = m + 27 - m$

$3m + 6 = 27$

The next thing we do, in order to get $3m$ on its own, is subtract 6 from both sides

$3m + 6 - 6 = 27 - 6$

$3m = 21$

Finally we divide both sides by 3 to solve for m

$$\frac{3m}{3} = \frac{21}{3}$$

Solution: $m = 7.$

Example 6

$2(\beta + 4) = 3(2\beta - 12)$

First we must remove the brackets

$2\beta + 8 = 6\beta - 36$

We then subtract 6β from both sides

$2\beta + 8 - 6\beta = 6\beta - 36 - 6\beta$

-$4\beta + 8 = -36$

Next we subtract 8 from both sides

-$4\beta + 8 - 8 = -36 - 8$

-$4\beta = -44$.

Finally we divide both sides by -4 to solve for β

$$\frac{-4\beta}{-4} = \frac{-44}{-4}$$

Solution: $\beta = 11$.

Exercises 1

Solve the Following Equations

1. $x + 27 = 15$

2. $3\sigma - 3 = 9$

3. $9 = 7\chi - 5$

4. $3m + 7 = m - 3$

5. $5x + 20 = 4x + 30$

6. $3 - 2k = 7 - 3k$

7. $3x - 18 = 112 - 2x$

8. $18 = 18\lambda - 18$

9. $4(\delta - 2) = 124$

10. $5(x-1) = 125$

11. $7(2\alpha - 1) = 3(5\alpha + 4)$

12. $3(a+6) = -2(3a)$

13. $2(x + 4) - 3(2x + 1) = 801$

14. $4(3k + 1) + 2(-5k - 1) = 16$

15. $4(\omega + 1) - 3(2\omega - 4) = 3(\omega - 3)$

4.2 Equations Involving Fractions

- Find the common denominator,
- Multiply every term in the equation by the common denominator,
- Solve as normal.

Example

Solve for x.

$$\frac{3x}{4} - \frac{2x}{3} = \frac{1}{6}$$

The common denominator here is 12 so we multiply each term by 12.

$$12(\frac{3x}{4}) - 12(\frac{2x}{3}) = 12(\frac{1}{6})$$

The denominators all divide exactly into 12 so we can cancel down to get

$$3(3x) - 4(2x) = 2(1)$$

Multiply to get;

$$9x - 8x = 2$$

Solution: $x = 2$.

Exercises 2

Solve the Following Equations

1. $\dfrac{4\delta}{9} = \dfrac{4}{3}$

2. $\dfrac{2x}{5} = \dfrac{1}{2}$

3. $\dfrac{\eta}{2} + \dfrac{\eta}{3} = \dfrac{5}{2}$

4. $\dfrac{3\lambda}{2} + \dfrac{\lambda + 4}{8} = 7$

5. $\dfrac{2x - 1}{9} = 45$

6. $\dfrac{2\theta + 1}{3} + \dfrac{\theta}{4} = \dfrac{13}{6}$

7. $\frac{\phi + 3}{7} + \frac{\phi - 1}{2} = \frac{13}{7}$

8. $\frac{-x - 2}{12} + \frac{7x + 1}{3} = \frac{x}{4}$

9. $\frac{k + 3}{4} - \frac{k + 2}{3} = 1$

10. $\dfrac{3s+1}{5} - \dfrac{2s+2}{4} = \dfrac{1}{2}$

4.3 Solving Simultaneous Equations

Take the following two equations in two unknowns, x and y:

Equation 1: $x + y = 6$

Equation 2: $3x - y = 2$

For equation 1 there are an infinite amount of values of x and y that will satisfy this equation i.e. values we can substitute for x and y that will make the statement true (i.e. the left hand side will equal the right hand side).

For example:

$x = 0$ and $y = 6$ or $x = 1$ and $y = 5$, or $x = -2$ and $y = 8$ etc.

Similarly for equation 2, we have a infinite amount of values for x and y that will satisfy this equation.

For example:

$x = 5$ and $y = 13$ or $x = 0$ and $y = -2$ and so on...

However, there is only one set of values for x and y that will satisfy both equation 1 and equation 2 at the same time . . . the values are $x = 2$, $y = 4$.

The method for finding these values is called **solving simultaneous equations** i.e. solving two equations **at the same time.**

Example 1

Solve the following simultaneous equations:

$2x + y = 1$

$-6x - y = 3$

Step 1

Get the *same coefficients for either* x ***or*** y

We see that in both equations the y coefficients are 1.

Step 2

Make sure the chosen coefficients have *opposite signs* (i.e. + and -)

Again we can see that the y coefficients have different signs.

Step 3

Add the two equations together

$2x + y = 1$

$\underline{-6x - y = 3}$

$-4x = 4$

Step 4

Solve for x

Divide both sides by -4 to solve for x

$\frac{-4x}{-4} = \frac{4}{-4}$

$x = -1.$

Step 5

Replace x value in <u>either</u> equation to solve for y

Take, for example, the first equation:

$2x + y = 1$

Substitute $x = -1$ into this equation and solve for y.

$2(-1) + y = 1$

-2 $+ y = 1$

Add 2 to both sides,

-2 $+ y + 2 = 1 + 2$

$y = 3$.

Our Solution Set is (-1, 3)

Example 2

Solve the following simultaneous equations:

$3a + 2b = 14$

$4a + b = 12$

Step 1

Get the *same coefficients for either a* ***<u>or</u>*** b

We can multiply the second equation by 2 to get the same b coefficients.

$3a + 2b = 14$

$8a + 2b = 24$

Step 2

Make sure they have *opposite signs* (i.e. + and -)

We need to multiply one of the equations by -1 to get opposite signs for the b coefficients. Let's take the second equation.

$3a + 2b = 14$

-$8a - 2b = -24$

Step 3

Add the two equations together.

$3a + 2b = 14$

$\underline{-8a - 2b = -24}$

-$5a = -10$

Step 4

Solve for a

Divide both sides by -5 to solve for a.

$$\frac{-5a}{-5} = \frac{-10}{-5}$$

$a = 2$

Step 5

Replace x value in <u>either</u> equation to solve for b

Take, for example, the first equation:

$3a + 2b = 14$

Substitute $a = 2$ into this equation and solve for b.

$3(2) + 2b = 14$

$6 + 2b = 14$

Subtract 6 from both sides.

$6 + 2b - 6 = 14 - 6$

$2b = 8$

Finally, we divide both sides by 2.

$$\frac{2b}{2} = \frac{8}{2}$$

$b = 4$

Our Solution Set is (2, 4).

Exercises 3

Solve the Following Pairs of Equations:

1. $2x - y = -4$
   $x + y = 1$

2. $a + 6b = 3$
   $4a + 6b = 12$

3. $3p + q = 8$
   -$2p + q = 3$

4. $f - 2g = 10$
   $f + g = -2$

5. $3Q_1 + 2Q_2 = -9$
   $2Q_1 + 3Q_2 = -11$

6. $3\alpha - \beta = 4$
   $3\alpha + 2\beta = 10$

7. $4\theta + \lambda = 4$
   $\theta - 2\lambda = -8$

8. $5\sigma + 2\tau = -22$
   $3\sigma + 7\tau = -48$

9. $6m - n = 59$
   $m + 15n = 25$

10. -$2r - s = -22$
    $4r + 9s = 30$

4.4 Answers

Exercises 1:

1). $x = -12$	2). $\sigma = 4$	3). $\chi = 2$	4). $m = -5$
5). $x = 10$	6). $k = 4$	7). $x = 26$	8). $\lambda = 2$
9). $\delta = 33$	10). $x = 26$	11). $\alpha = -19$	12). $a = -2$
13). $x = -199$	14). $k = 7$	15). $\omega = 5$	

Exercises 2:

1). $\delta = 3$	2). $x = \frac{5}{4}$	3). $\eta = 3$	4). $\lambda = 4$
5). $x = 203$	6). $\theta = 2$	7). $\phi = 3$	8). $x = -\frac{1}{12}$
9). $k = -11$	10). $s = 8$		

Exercises 3:

1). $x = -1; y = 2$	2). $a = 3; b = 0$	3). $p = 1; q = 5$
4). $f = 2; g = -4$	5). $Q_1 = -1; Q_2 = -3$	6). $\alpha = 2; \beta = 2$
7). $\theta = 0; \lambda = 4$	8). $\sigma = -2; \tau = -6$	9). $m = 10; n = 1$
10). $r = 12; s = -2$		