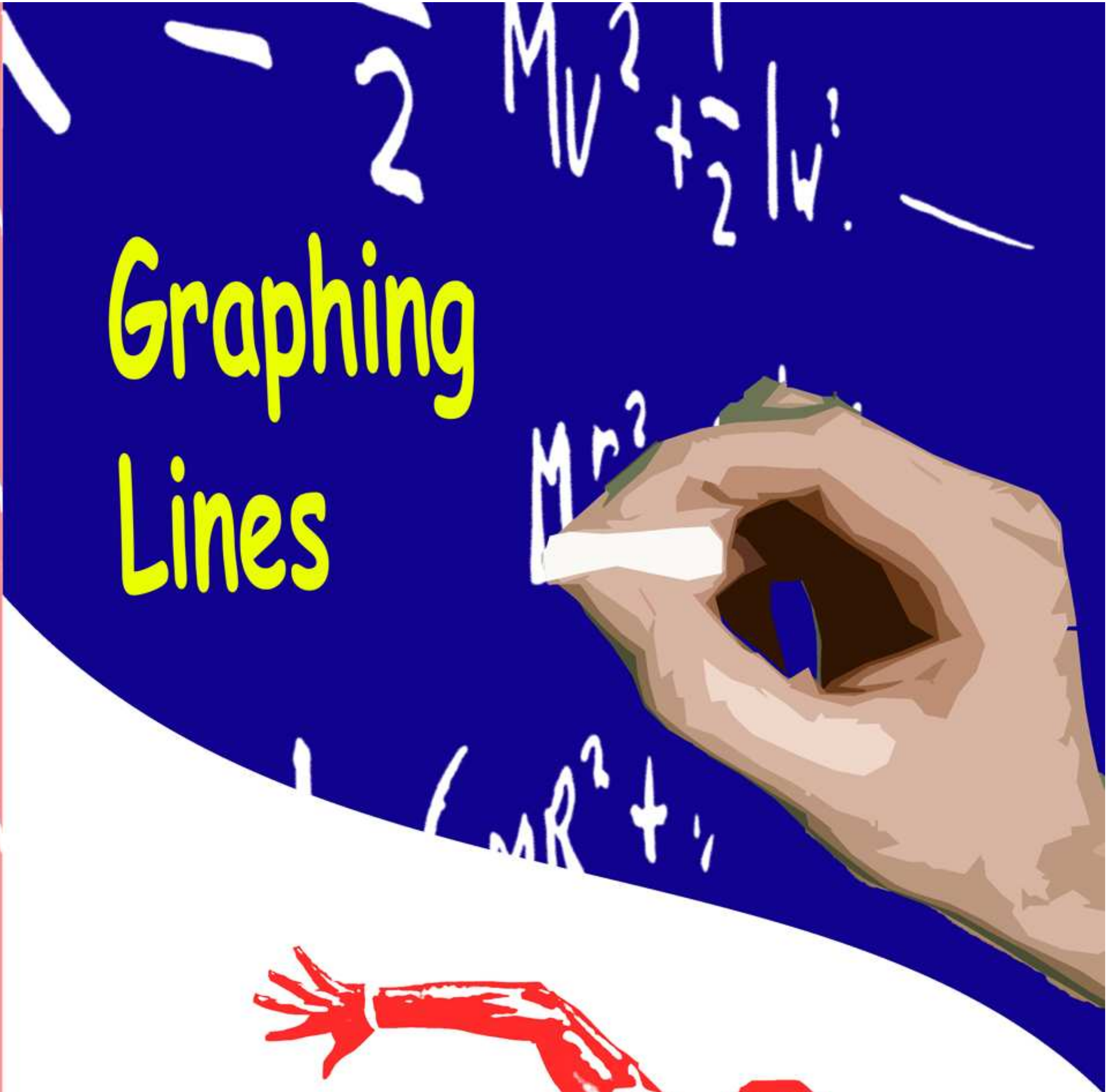
HEAD MATHS

START

Graphing Lines



CEMTL
Regional Centre for Excellence in Mathematics Teaching and Learning

centre for excellence
sigma
in mathematics & statistics support

Foreword

The Regional Centre for Excellence in Mathematics Teaching and Learning (CEMTL) is collaboration between the Shannon Consortium Partners: University of Limerick, Institute of Technology, Limerick; Institute of Technology, Tralee and Mary Immaculate College of Education, Limerick., and is driven by the Mathematics Learning Centre (MLC) and The Centre for Advancement in Mathematics Education and Technology (CAMET) at the University of Limerick.

CEMTL is committed to providing high quality educational resources for both students and teachers of mathematics. To that end this package has been developed to a high standard and has been peer reviewed by faculty members from the University of Limericks Department of Mathematics and Statistics and sigma, the UK based Centre for Excellence in Teaching and Learning (CETL). Through its secondment programme, sigma provided funding towards the creation of these workbooks.

Please be advised that the material contained in this document is for information purposes only and is correct and accurate at the time of publishing. CEMTL will endeavour to update the information contained in this document on a regular basis.

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6.1 The Coordinated Plane

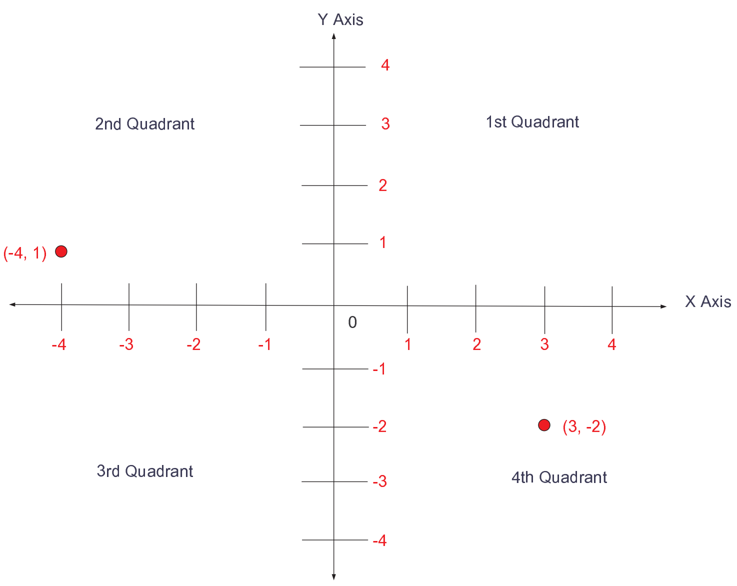
The diagram below is called the **Coordinated Plane** or the **Cartesian Plane**, named after the French mathematician Rene Descartes. Each quarter section of the plane is called a **Quadrant.** x (horizontal) and y (vertical) axes are used to identify and situate points in a coordinated plane

A similar method using lines of Longitude and Latitude i.e. degrees West, North etc. are used to indicate positioning on the globe. For example, Limerick city is situated at $52^o 40' N, 08^o 37' W$

Plotting Points

Every point in the plane has an x coordinate (where it is situated in relation to the horizontal X axis) and a y coordinate (where it is situated in relation to the vertical Y axis). The x and y coordinates are often referred to as **ordered pairs**.

Remember: x comes before y (like in the alphabet) so we move along the x axis first before we move along the y axis.



Exercise 1

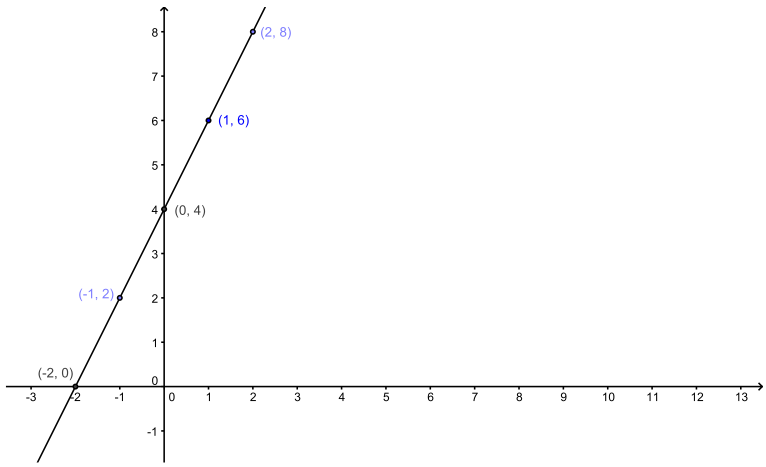
Draw a Coordinated Plane and Plot the Following Points

$(3, 4)$, $(1, 6)$, $(-4, 2)$, $(6, -1)$, $(-2, -5)$, $(2, 0)$, $(0, -3)$, $(-1, 0)$, $(0, 1)$, $(-3, -3)$

6.2 Equation of a line

The relationship between the x and y coordinates of every point on a line determines what we call the **Equation of the Line**.

For example, $y = 2x + 4$ is the equation of the line drawn below:



For every point on this line, the sum of 2 times the x coordinate plus 4 will always be equal to the y coordinate. Try it and see!

$(0, 4) \rightarrow x = 0, y = 4$
$(4) = 2(0) + 4$
$4 = 0 + 4$
$4 = 4$

6.3 Slope of a Line

In the above graph when $x = 0, y = 4$. If we increase the x value to 1, the y value is now 6. When x is increased to 2, the y value is now 8 and so on. In other words, for every unit increase in x, there is a 2 unit increase in y. This is caused by the x coefficient, 2. It is referred to as the **slope** or **gradient** of the line.

Notice also that the graph cuts the y - axis at the point $(0, 4)$ i.e. when $x = 0, y = 4$. This is called the y **- axis intercept**.

If we rewrite the above equation as $y = 2x + 4$, we can immediately determine the slope of the line (the x coefficient) and the y - axis intercept (the constant).

Note:

$y = mx + c$ is called the standard form of the equation of a line, where m is the slope or the gradient of the line, and c is the y - axis intercept.

6.4 Graphing Lines

Example 1

Sketch the line $3x + y = 9$

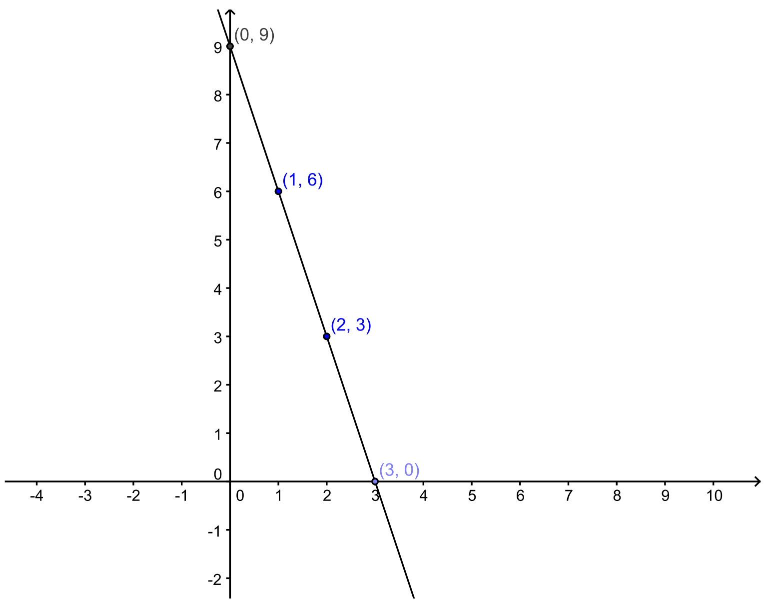
Solution

In order to plot the line we need to calculate the coordinates of a few points on the line by putting $x = 0, 1, 2, 3, 4$ in turn. The easiest way to do this is construct a table of x and y values.

The first thing we do is rewrite the equation of the line as $y = 9 - 3x$.
We then substitute the chosen values for x into the equation to find the corresponding y values as in the table below

x	$y = 6 - 3x$	y
0	y = 9 - 3(0)	9
1	y = 9 - 3(1)	6
2	y = 9 - 3(2)	3
3	y = 9 - 3(3)	0
4	y = 9 - 3(4)	-3

The points we need to graph our line are: $(0, 9)$, $(1, 6)$, $(2, 3)$, $(3, 0)$, $(4, -3)$



Note 1: In actual fact two points are sufficient to draw any line. If we just plotted two of the points above, we would still get the same graph.
Note 2: Notice how a negative slope (m = -3 in this case) changes the orientation of the graph.

Example 2

Sketch the line $2x - y = 4$

Solution

The first thing we do is rewrite the equation of the line as $-y = 4 - 2x$

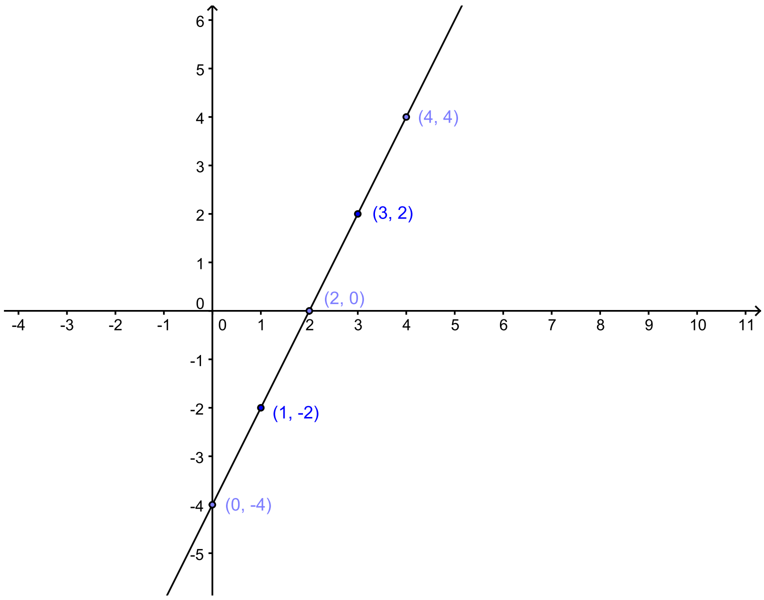
Changing the signs throughout we get $y = 2x - 4$

We then substitute the given values for x into the equation to find the corresponding y values as in the table below.

x	y = 2x - 4	y
0	y = 2(0) - 4	-4
1	y = 2(1) - 4	-2
2	y = 2(2) - 4	0
3	y = 2(3) - 4	2
4	y = 2(4) - 4	4

The points we need to draw our line are:

$(0, -4), (1, -2), (2, 0), (3, 2), (4, 4)$



Exercises 2

Graph the Following Lines and Write Down the Slope and Y-Axis Intercept:

1. $x + y = 7$

2. $2x + y = 10$

3. $3x - y = 6$

4. $3x - 2y = 6$

5. $y = -4x - 1$

6. $y = 5 - x$

7. $y = 7x$

8. $x + 6y = 12$

6.5 A Quicker Way!!

Sometimes you will be asked to draw a graph but you may not be given values for x. When this occurs you can pick your own.

The quickest and easiest thing to do is:

let $x = 0$ and find y

then

let $y = 0$ and find x.

This will give us two points which is sufficient to draw any line!!

Example

Graph the line $3x + 4y = 12$.

Let $x = 0$,

$3(0) + 4y = 12$

$0 + 4y = 12$

$4y = 12$

$\frac{4y}{4} = \frac{12}{4}$

$y = 3$

Our first point is $(0, 3)$.

Now let $y = 0$,

$3x + 4(0) = 12$

$3x + 0 = 12$

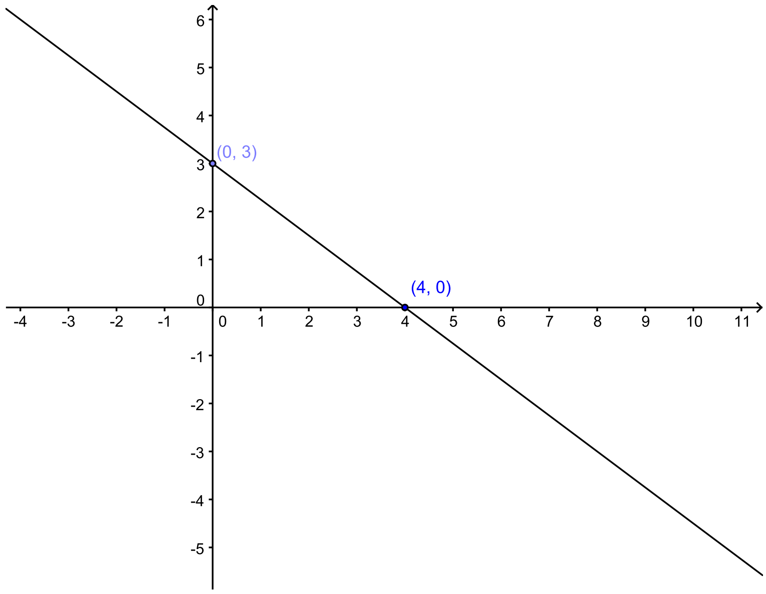
$3x = 12$

$\frac{3x}{3} = \frac{12}{3}$

$x = 4$.

Our second point is $(4, 0)$.

We can plot these two points and then use a ruler to draw the line through them.



Exercises 3

Graph the Following Lines:

1. $x + 5y = 10$

2. $2x + y = -12$

3. $2x - y = 18$

4. -$3x - y = 9$

5. $-x + 4y = 8$

6.6 Simultaneous Equations

When we solve two equations at the same time, we are looking for the point at which the 2 lines intersect i.e. the point (x,y) that lies on both lines.

In our Equations workshop we learned how to do this algebraically (i.e. using algebra). Now we will learn how to do it graphically (i.e. using graphs).

Basically we graph both lines and see at which point they intersect.

Example:

Find the point of intersection of the lines $3x + y = 9$ and $2x - y = 6$.

Solution

We will start by finding 2 points on each line.

$3x + y = 9$

Let $x = 0$

$3(0) + y = 9$

$0 + y = 9$

$y = 9$

First point $(0, 9)$

Let $y = 0$

$3x + 0 = 9$

$3x = 9$

$\frac{3x}{3} = \frac{9}{3}$

$x = 3$

Second point $(3, 0)$.

$2x - y = 6$

Let $x = 0$,

$2(0) - y = 6$

$0 - y = 6$

$y = -6$.

First point $(0, -6)$.

Let $y = 0$,

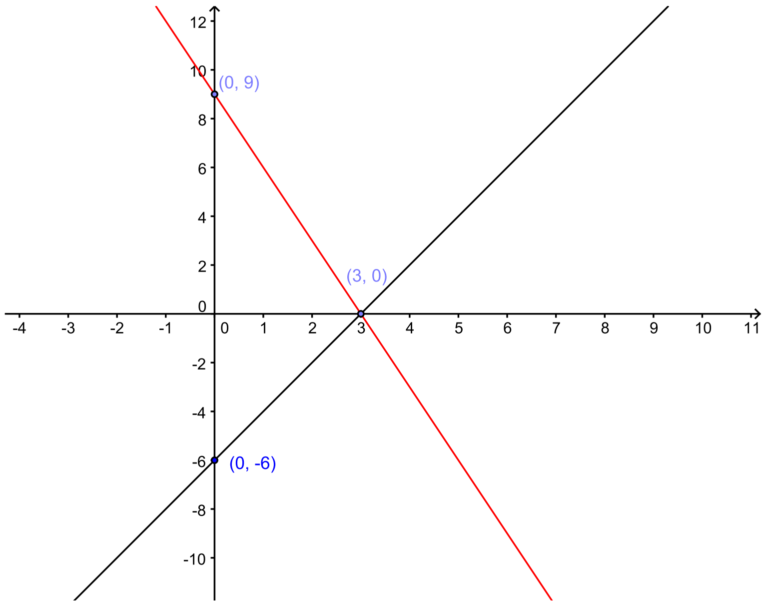
$2x - 0 = 6$

$2x = 6$

$\frac{2x}{2} = \frac{6}{2}$

$x = 3$.

Second point $(3, 0)$.
We can see that the lines intersect at the point (3,0) i.e. (3,0) is the only point that is on both lines.



Exercises 4

Graph the Following Pairs of Lines and Find Their Points of Intersection

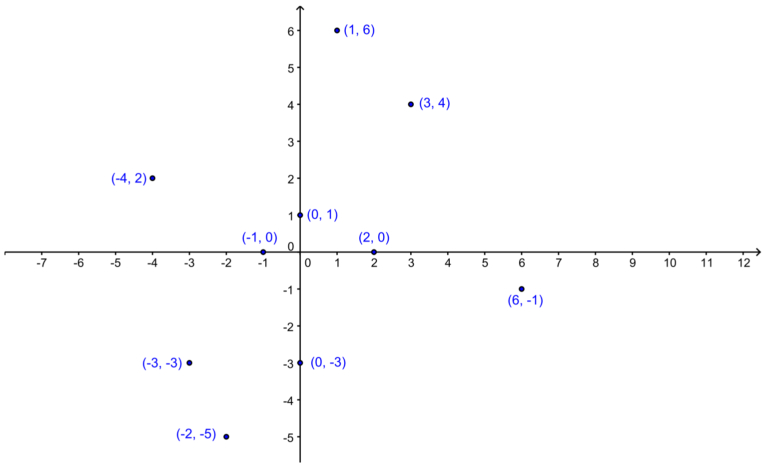
1. $3x - y = -7$
   $4x + y = 0$

2. $5x - y = 10$
   $2x + y = 4$

3. $-4x + y = -8$
   $4x + y = 0$

6.7 Answers

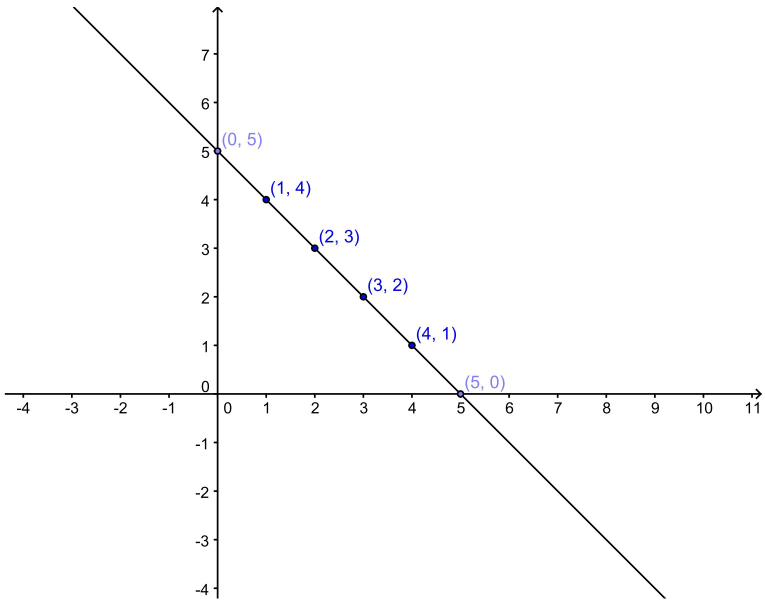
Exercise 1:



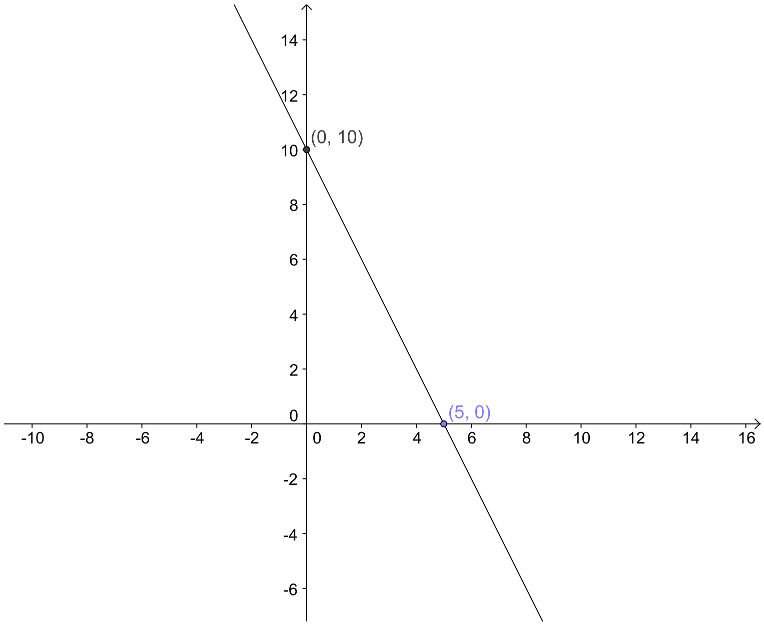
Exercise 2:

1 : $x + y = 7$

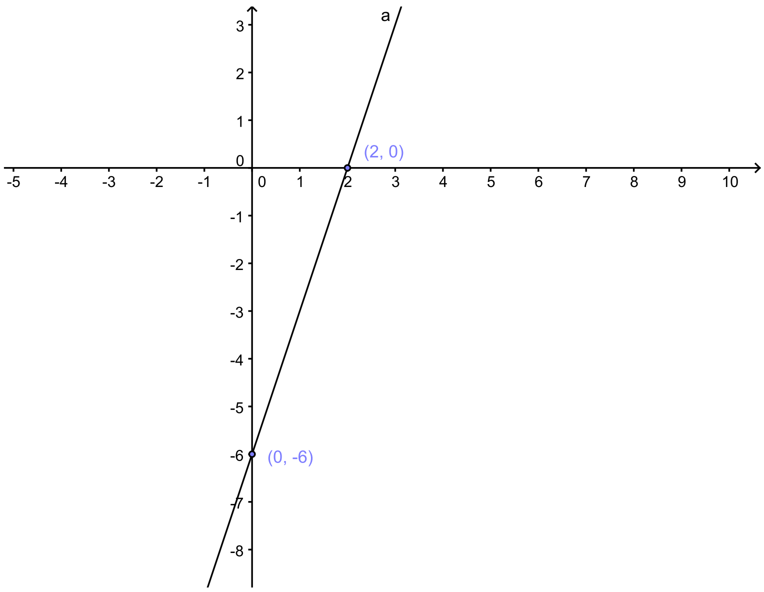
$m = -1$ y - axis intercept = $(0, 7)$



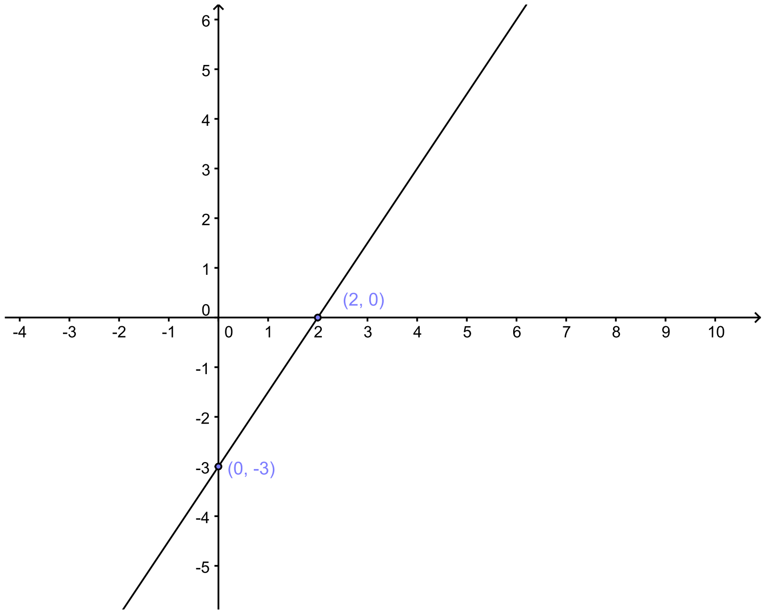
$2:\quad 2x + y = 10$
$m = -2 \quad y$ - axis intercept = $(0, 10)$



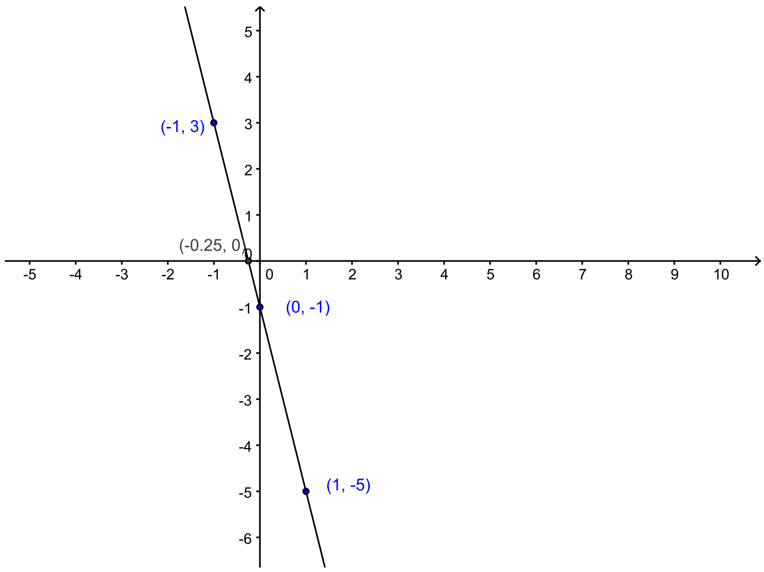
$3:\quad 3x - y = 6$
$m = 3 \quad y$ - axis intercept = $(0, -6)$



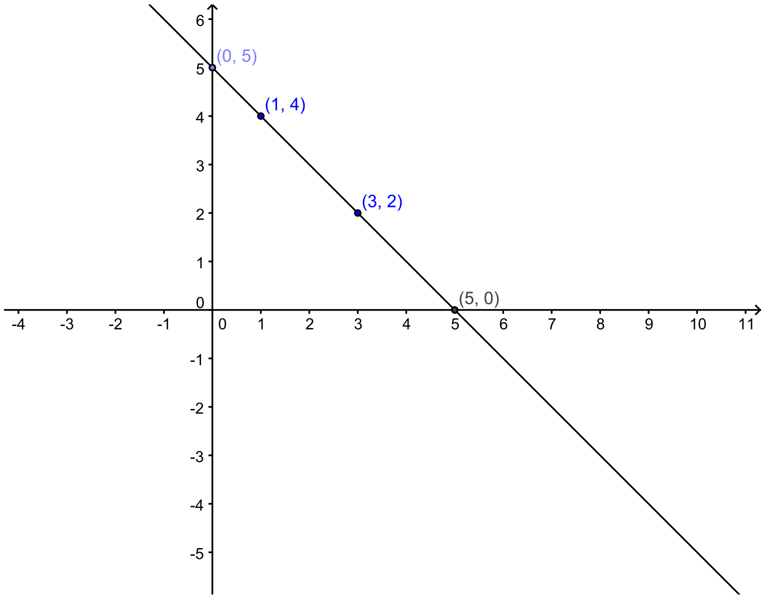
4 : $3x - 2y = 6$
$m = \frac{3}{2}$ y - axis intercept = $(0, -3)$



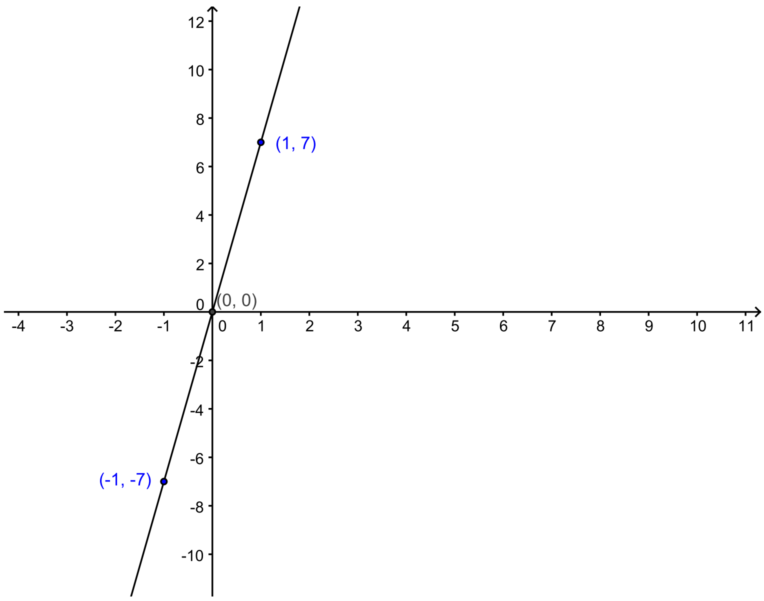
5 : $y = -4x - 1$
$m = -4$ y - axis intercept = $(0, -1)$



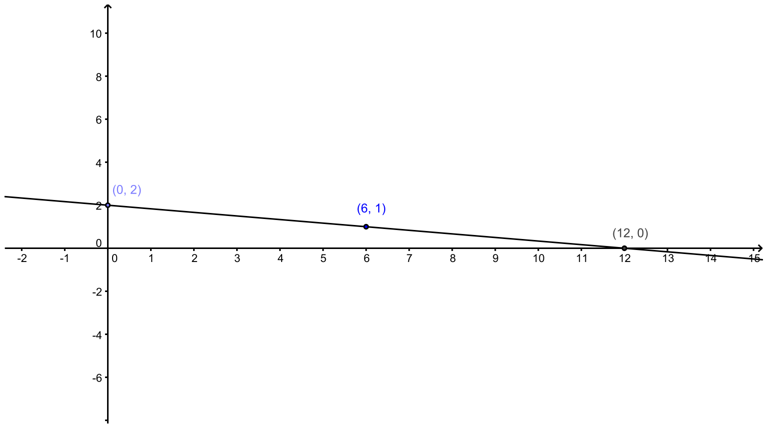
$6: \quad y = 5 - x$
$m = -1 \quad y$ - axis intercept = $(0, 5)$



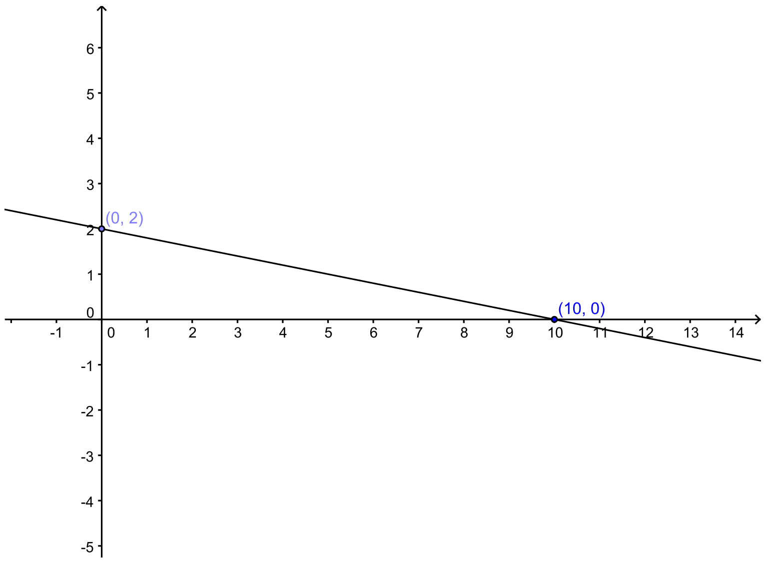
$7: \quad y = 7x$
$m = 7 \quad y$ - axis intercept = $(0, 0)$



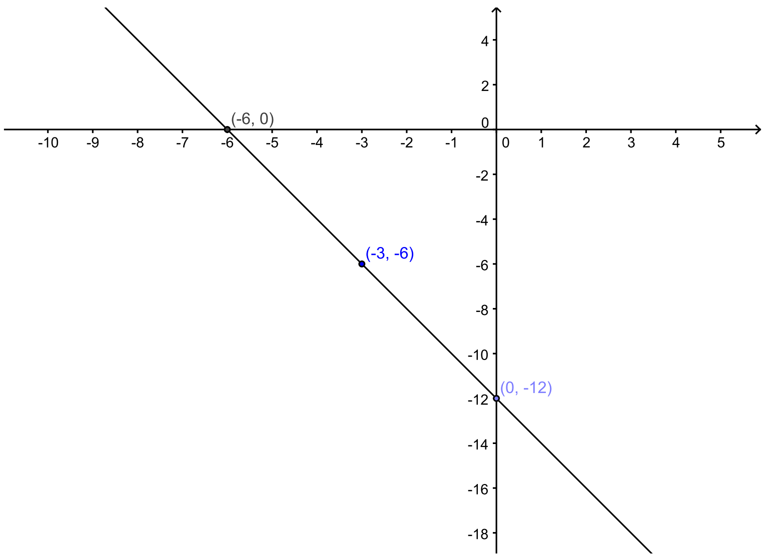
$8:\quad x+6y=12$
$m=-\frac{1}{6}\quad y$ - axis intercept = $(0,2)$



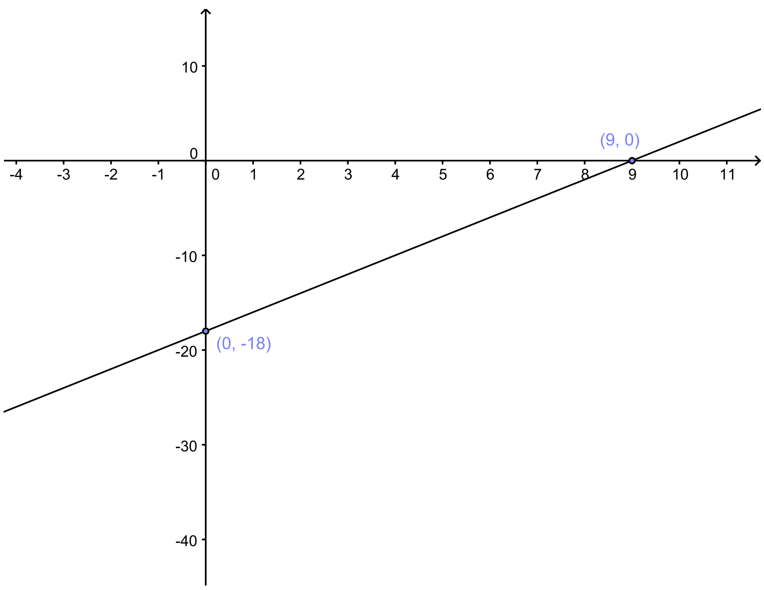
Exercise 3:
$1:\quad x+5y=10$



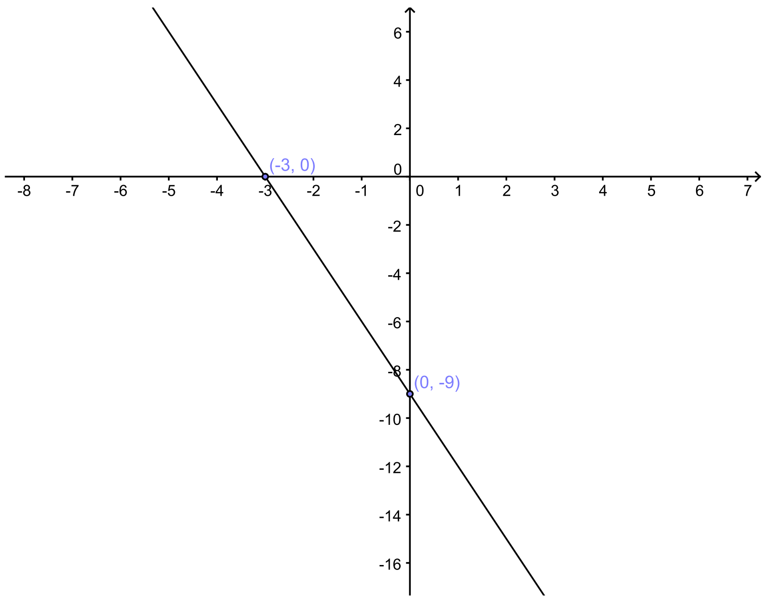
2 : $2x + y = -12$



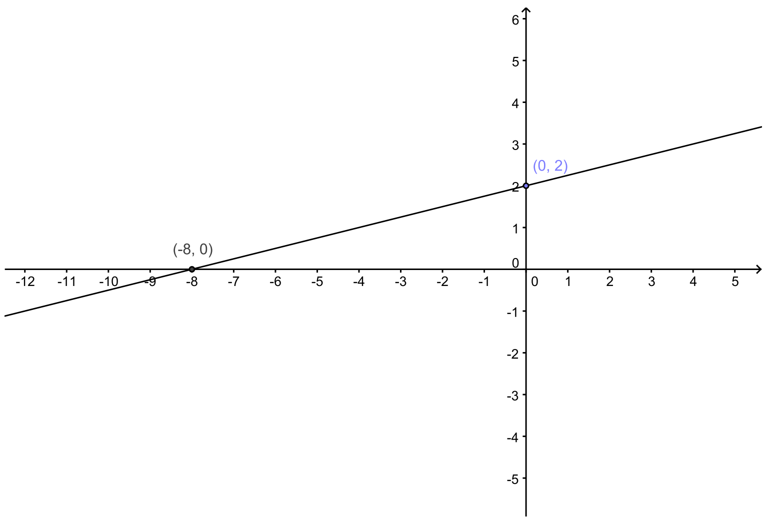
3 : $2x - y = 18$



4 : $-3x - y = 9$

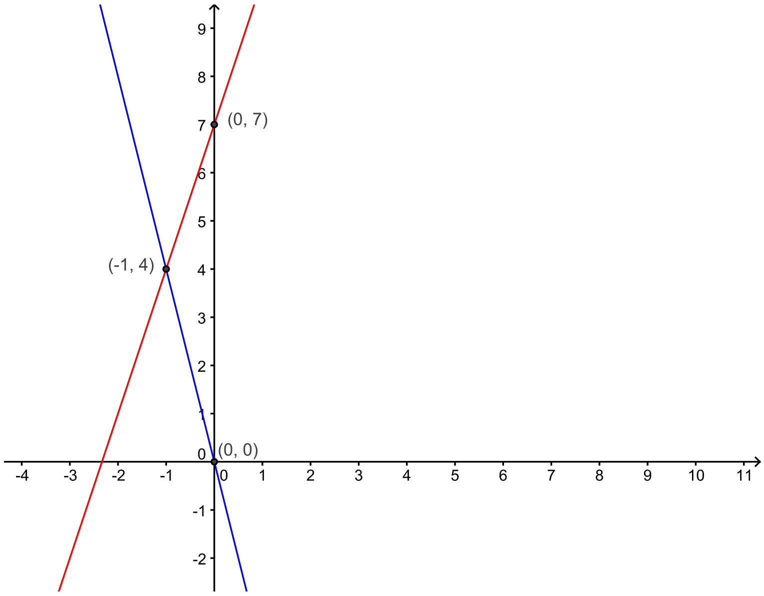


5 : $-x + 4y = 8$

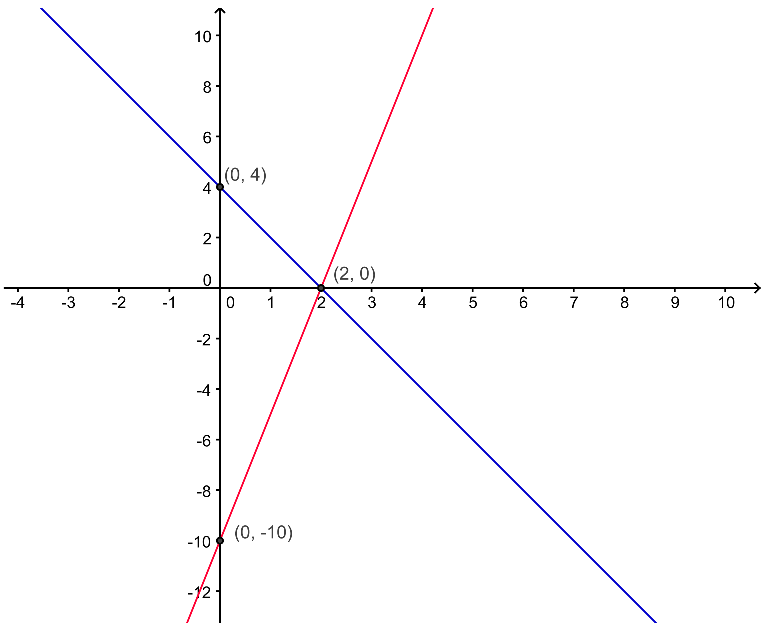


Exercise 4:

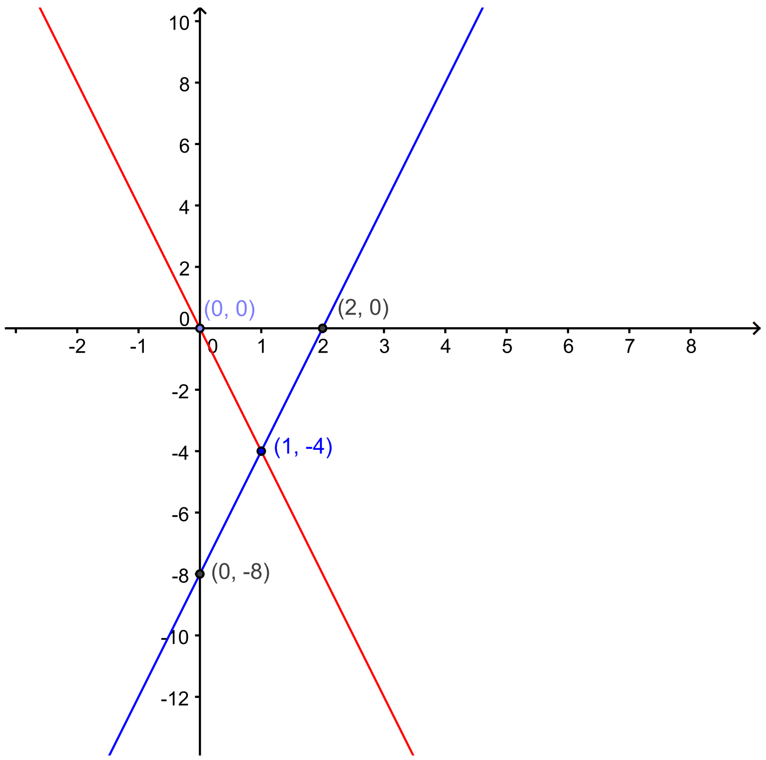
1 : $3x - y = -7, 4x + y = 0$



2 : $5x - y = 10, 2x + y = 4$



3 : $-4x + y = 8, 4x + y = 0$



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