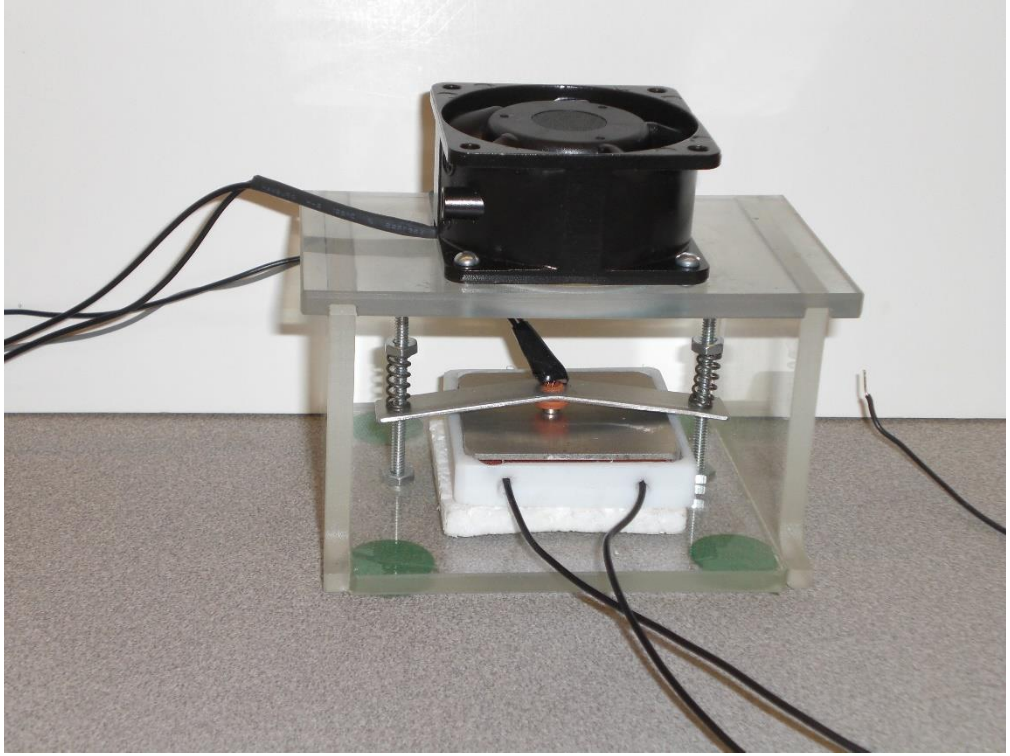
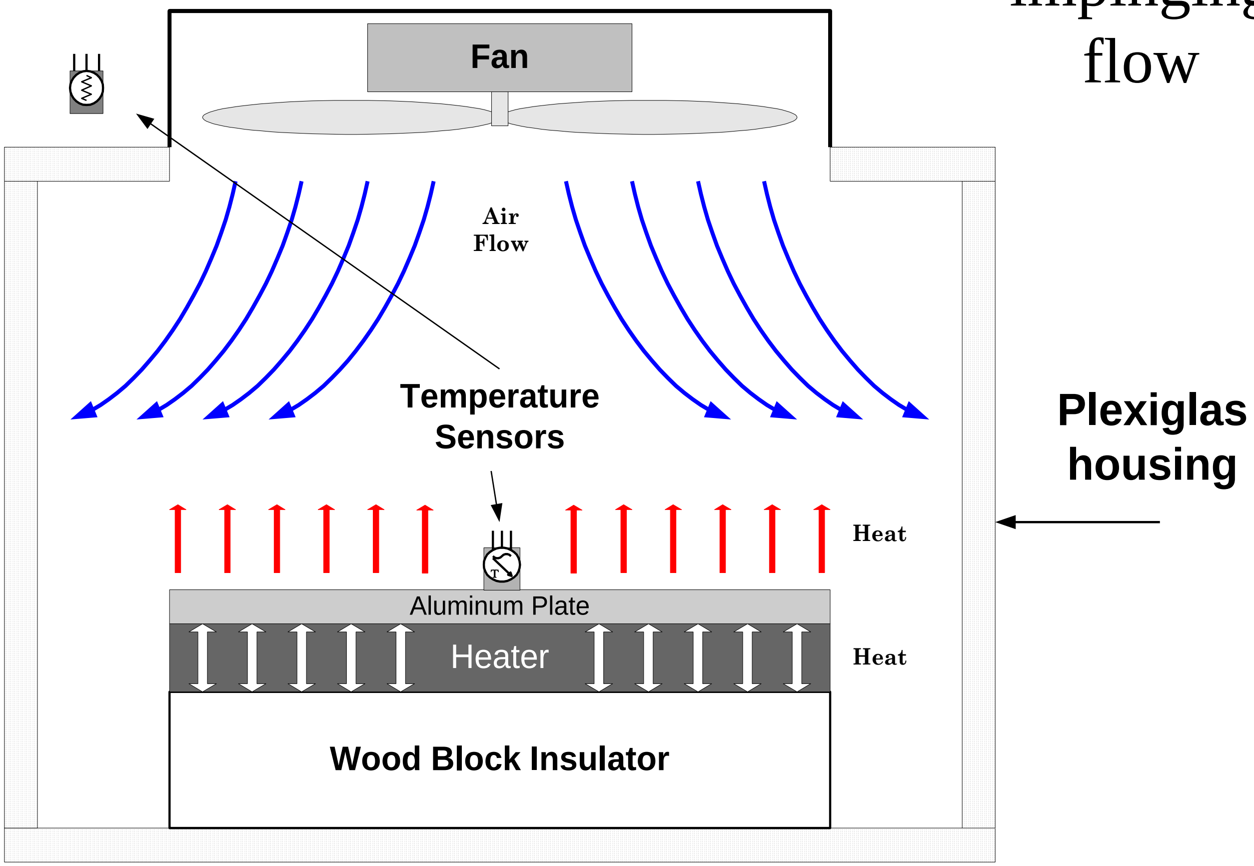
Modeling Thermal Systems

System Schematic



System Schematic

impinging flow



Heater

Specifications	Value
Manufacturer	Omega Engineering Inc
Model Number	SRMU100202P
Heater Resistance	350 ohms
Heater Area	4 in^2
Heater Thickness	0.1 in

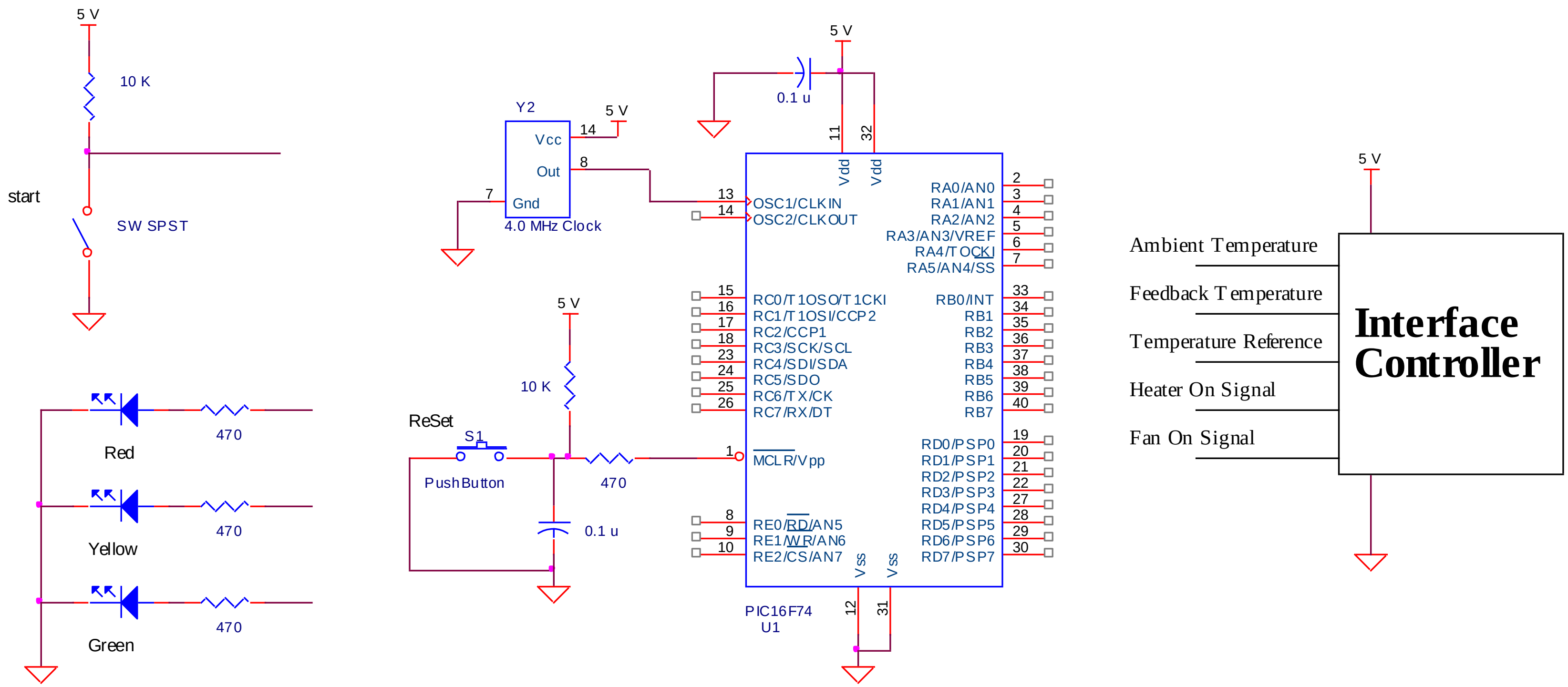
Aluminum Plate

Property	Value
Melting Point	775^0 K
Density, ρ	2770 kg/m^3
Specific Heat, cp	875 J / (kg - ^{0}K)
Thermal Conductivity, k	177 W / (m - ^{0}K)

Sensors

Specification	AD590	AD22100
Rated Temperature Range	-55° C to 150° C	0° C to 100° C
Nominal Output at 0° C	273.2 µA	1.126 V
Nominal Output at 25° C	298.2 µA	1.836 V
Temperature Coefficient	1 µA / ° C	18.43 mV / ° C
Absolute Error	± 5.5° C	± 2.0° C
Nonlinearity	0.8° C	0.5° C

Microcontroller Circuit

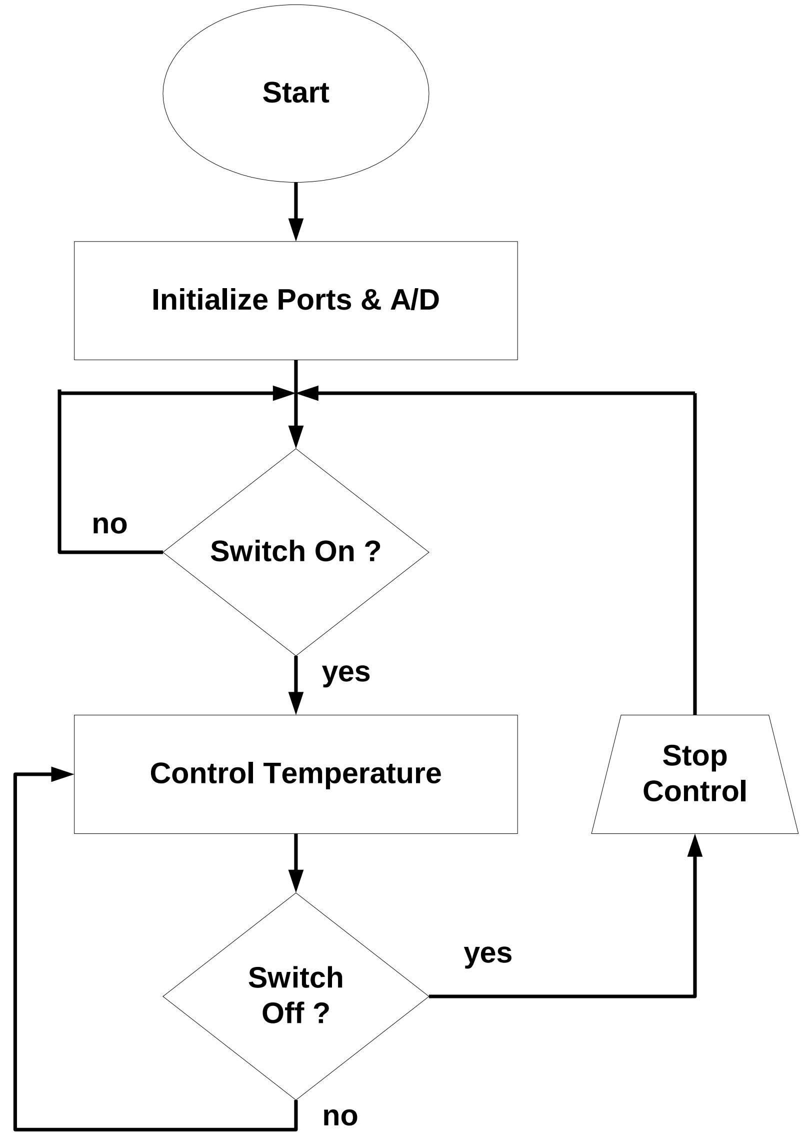


red LED $\Rightarrow$ above temperature band

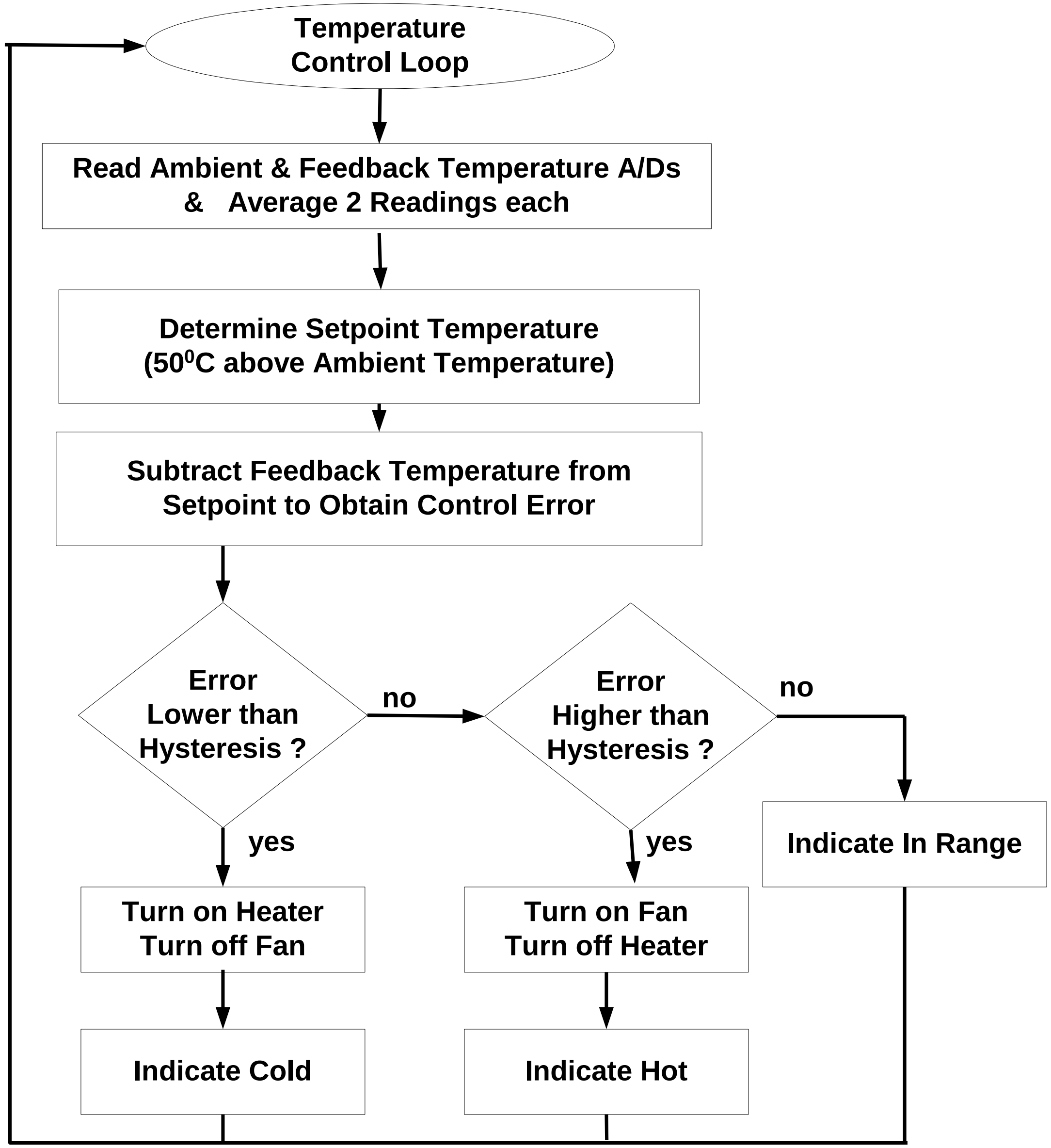
yellow LED $\Rightarrow$ within temperature band

green LED $\Rightarrow$ below temperature band

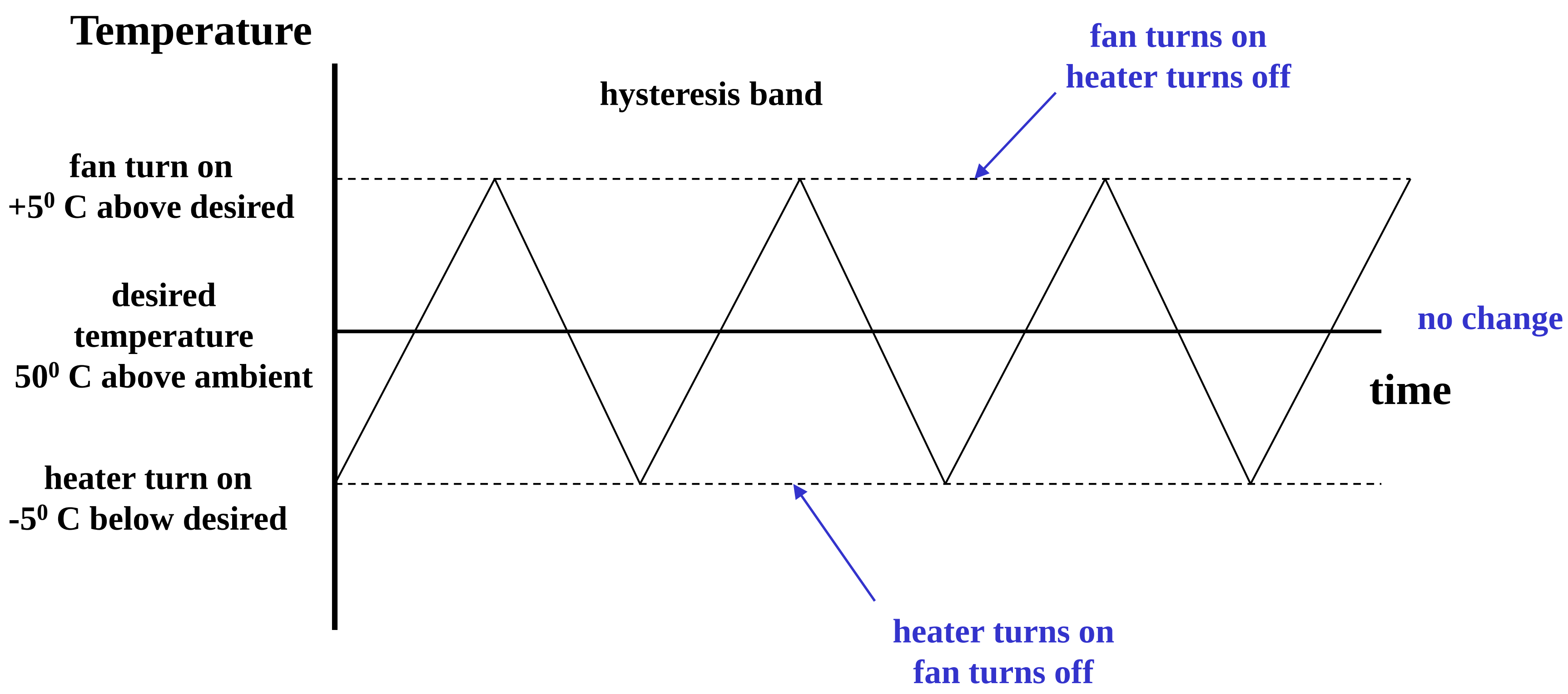
Control Program



Control Program FlowChart



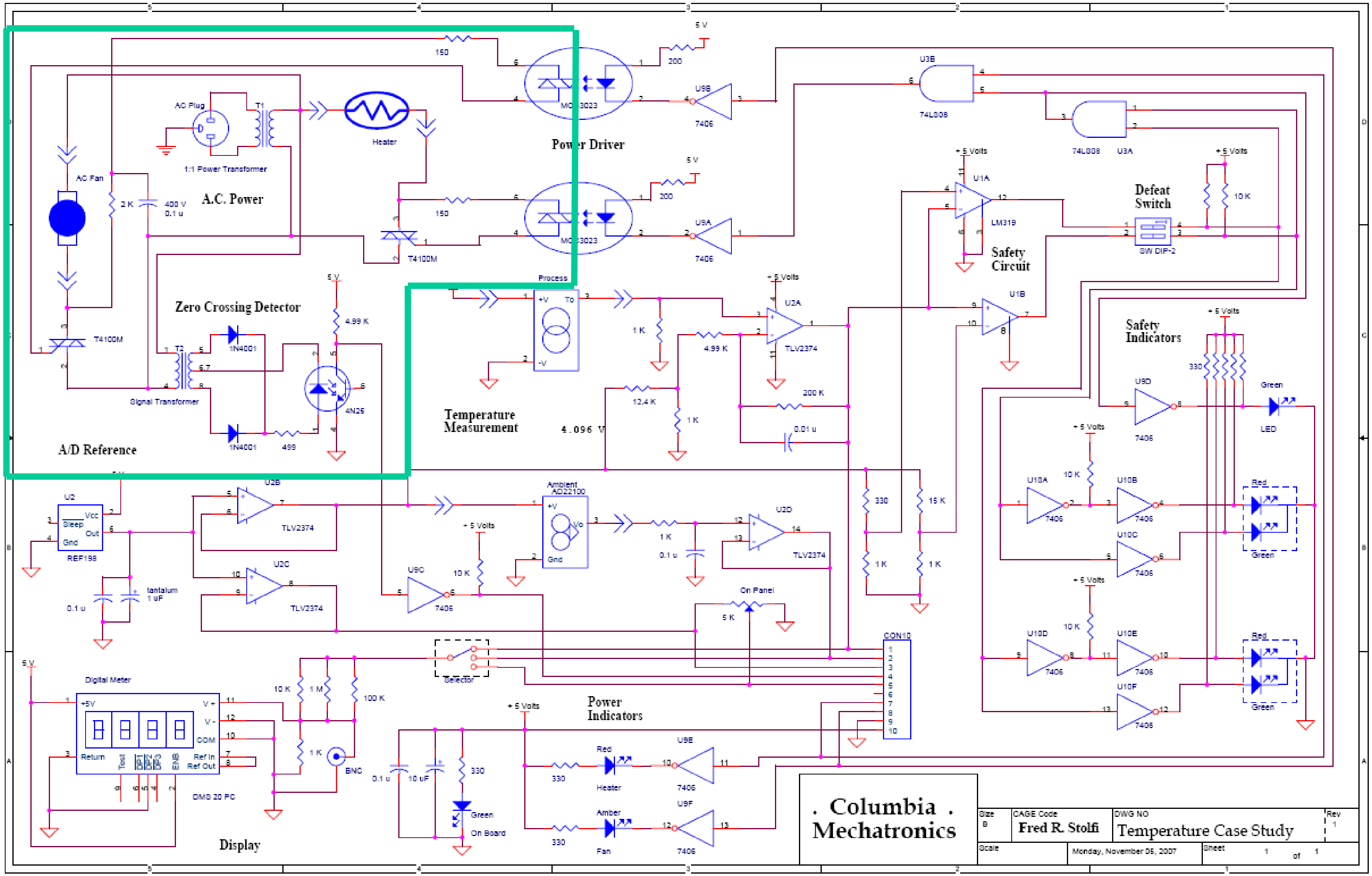
Temperature Signal



Note: hysteresis temperature 5^0 C

Note: to turn on the heater & fan, the signal input must go **LO**

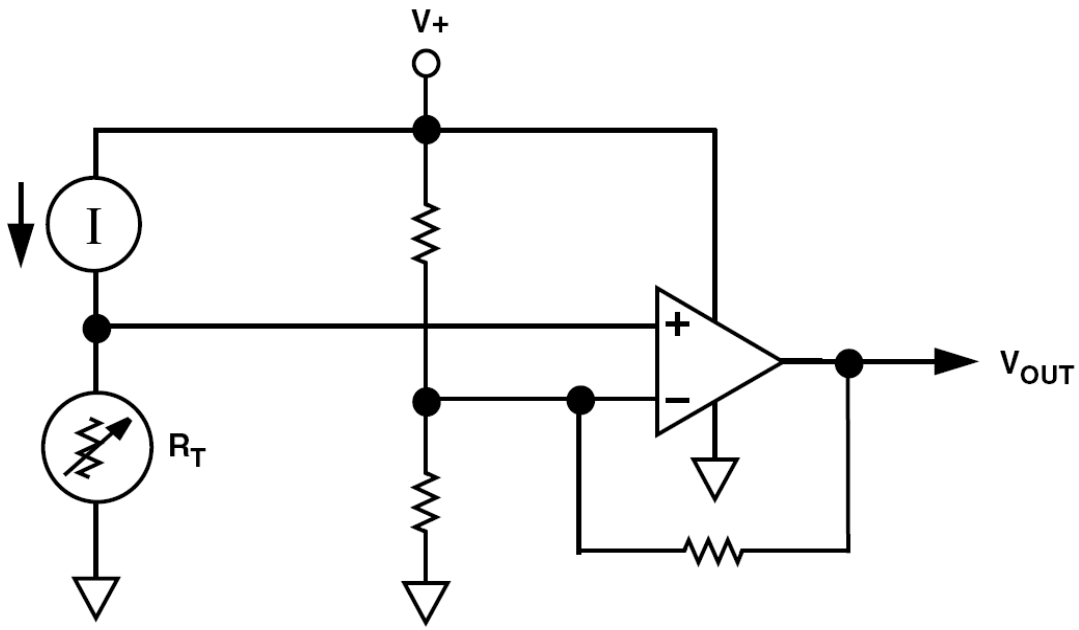
Thermal Control Circuit



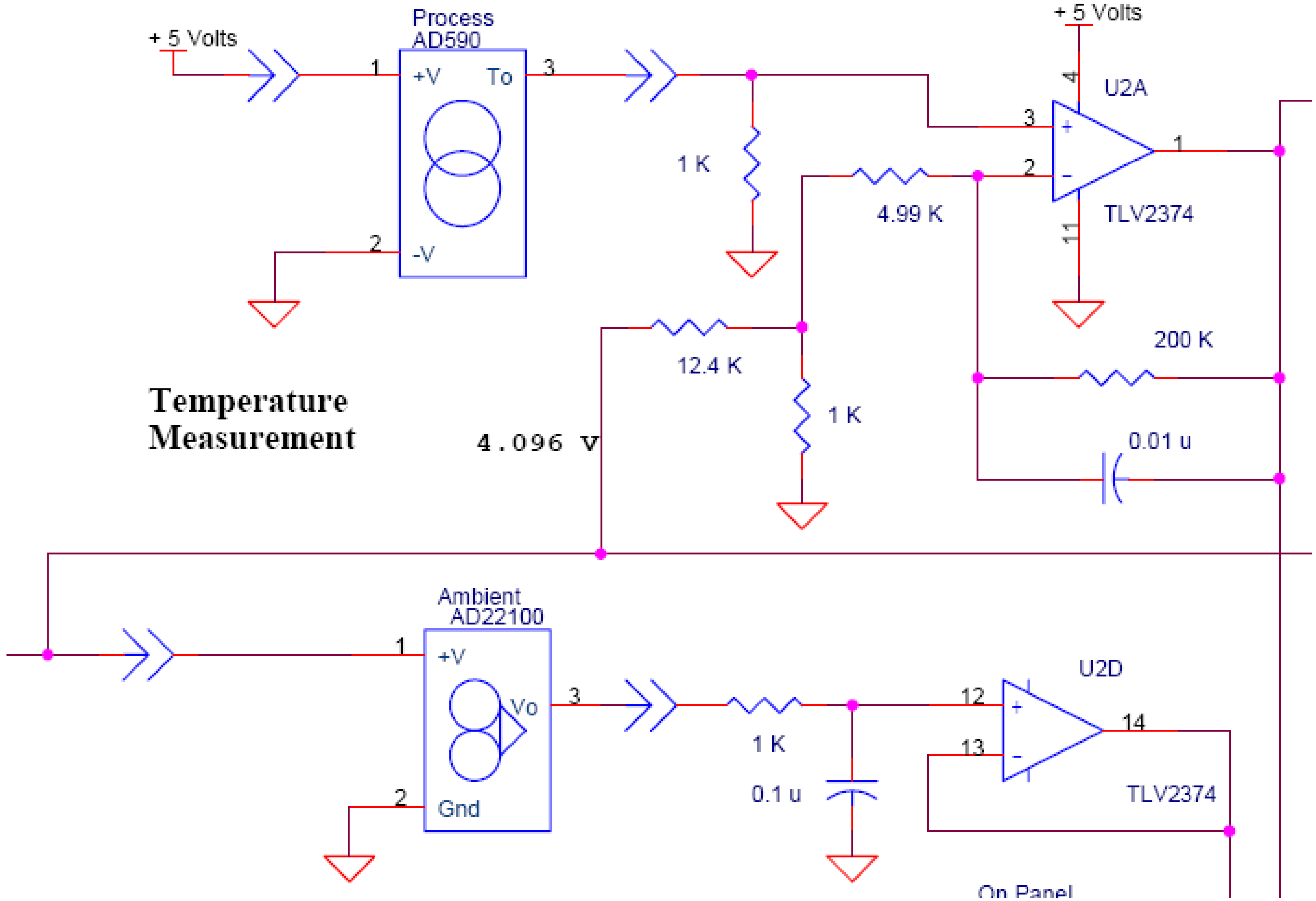
Sensors

Specification	**AD590**	**AD22100**
Rated Temperature Range	-55° C to 150° C	0° C to 100° C
Nominal Output at 0° C	273.2 µA	1.126 V
Nominal Output at 25° C	298.2 µA	1.836 V
Temperature Coefficient	1 µA / ° C	18.43 mV / ° C
Absolute Error	± 5.5° C	± 2.0° C
Nonlinearity	0.8° C	0.5° C

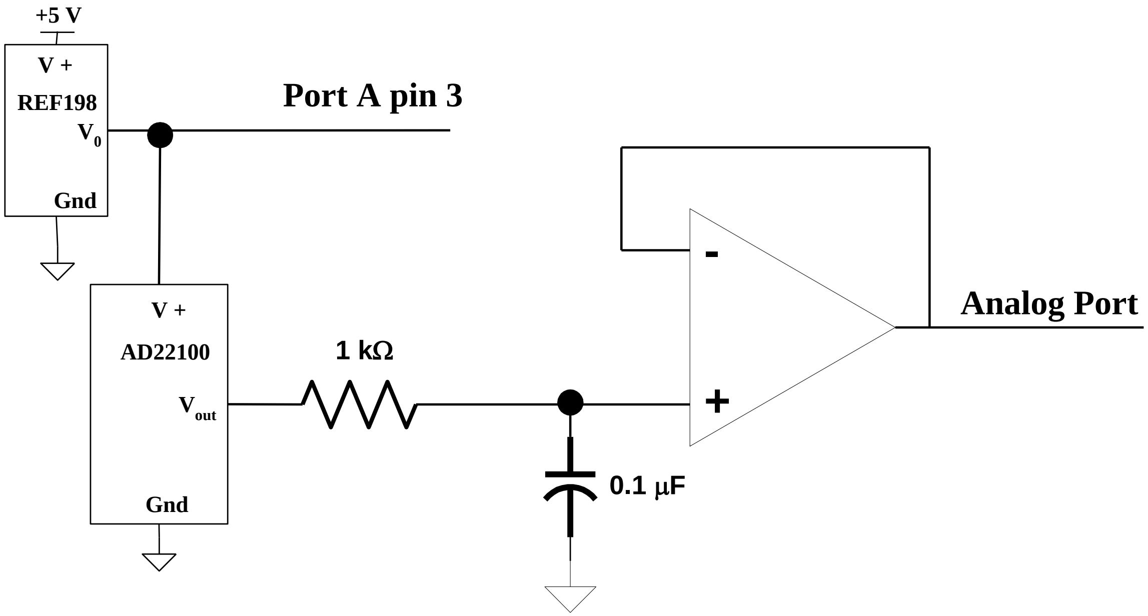
AD22100 Temperature Sensor Schematic



Temperature Measurement

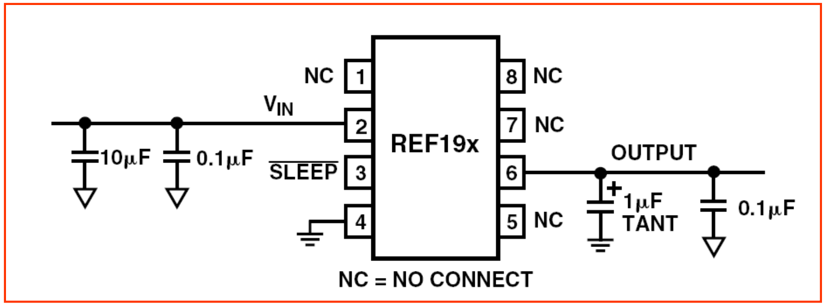


Ambient Sensor Circuit



- Reference for sensor & A/D converter provided by voltage reference
- Passive filter & OpAmp buffer

REF198 Temperature Reference



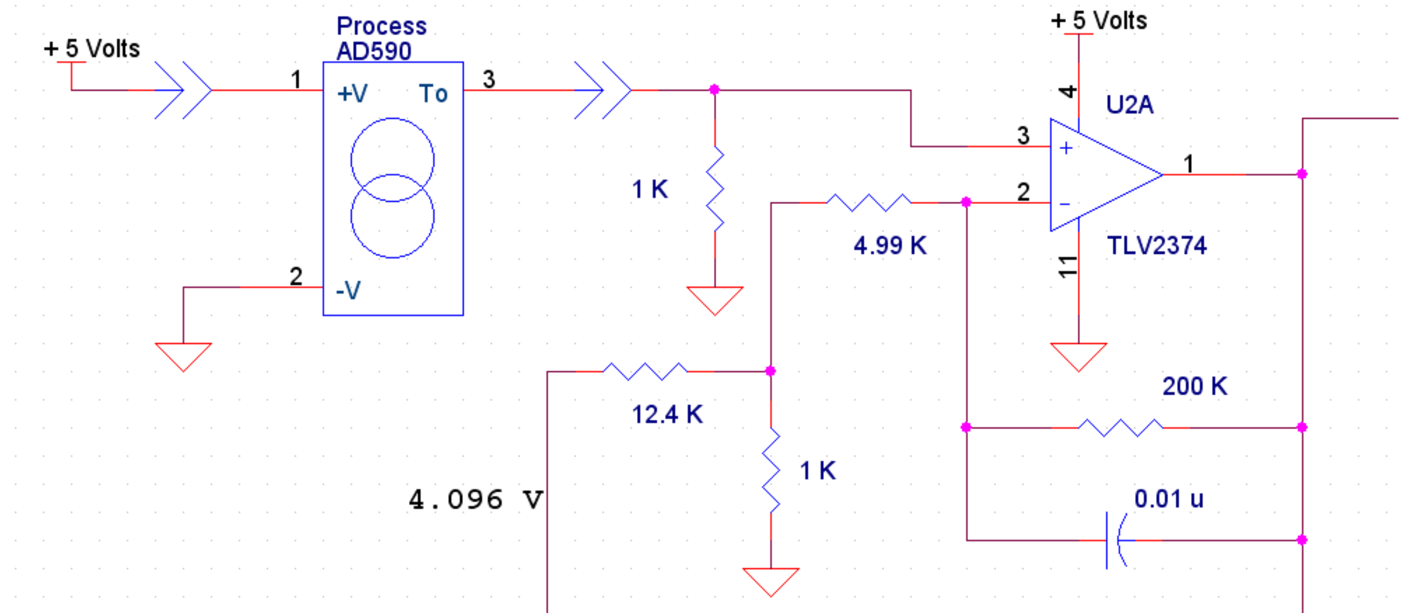
REF198–SPECIFICATIONS

ELECTRICAL CHARACTERISTICS (@ V_S = 5.0 V, T_A = 25°C, unless otherwise noted.)

Parameter	Symbol	Condition	Min	Typ	Max	Unit
INITIAL ACCURACY[1]						
E Grade	V_O	I_{OUT} = 0 mA	4.094	4.096	4.098	V
F Grade			4.091		4.101	V
G Grade			4.086		4.106	V
LINE REGULATION[2]						
E Grade	$\Delta V_O / \Delta V_{IN}$	4.5 V ≤ V_S ≤ 15 V, I_{OUT} = 0 mA		2	4	ppm/V
F and G Grades				4	8	ppm/V
LOAD REGULATION[2]						
E Grade	$\Delta V_O / \Delta V_{LOAD}$	V_S = 5.4 V, 0 mA ≤ I_{OUT} ≤ 30 mA		2	4	ppm/mA
F and G Grades				4	8	ppm/mA
DROPOUT VOLTAGE	$V_S - V_O$	V_S = 4.6 V, I_{LOAD} = 10 mA			0.50	V
		V_S = 5.4 V, I_{LOAD} = 30 mA			1.30	V
LONG-TERM STABILITY[3]	DV_O	1,000 Hours @ 125°C		1.2		mV
NOISE VOLTAGE	e_N	0.1 Hz to 10 Hz		40		μV p-p

NOTES
[1]Initial accuracy includes temperature hysteresis effect.
[2]Line and load regulation specifications include the effect of self-heating.
[3]Long-term drift is guaranteed by 1,000 hours life test performed on three independent wafer lots at 125°C, with an LTPD of 1.3.
Specifications subject to change without notice.

Feedback Sensor Circuit



- Scales reading to span A/D converter
- Offsets reading to eliminate ambient temperature
- Filters reading for anti-aliasing

Defining Equations

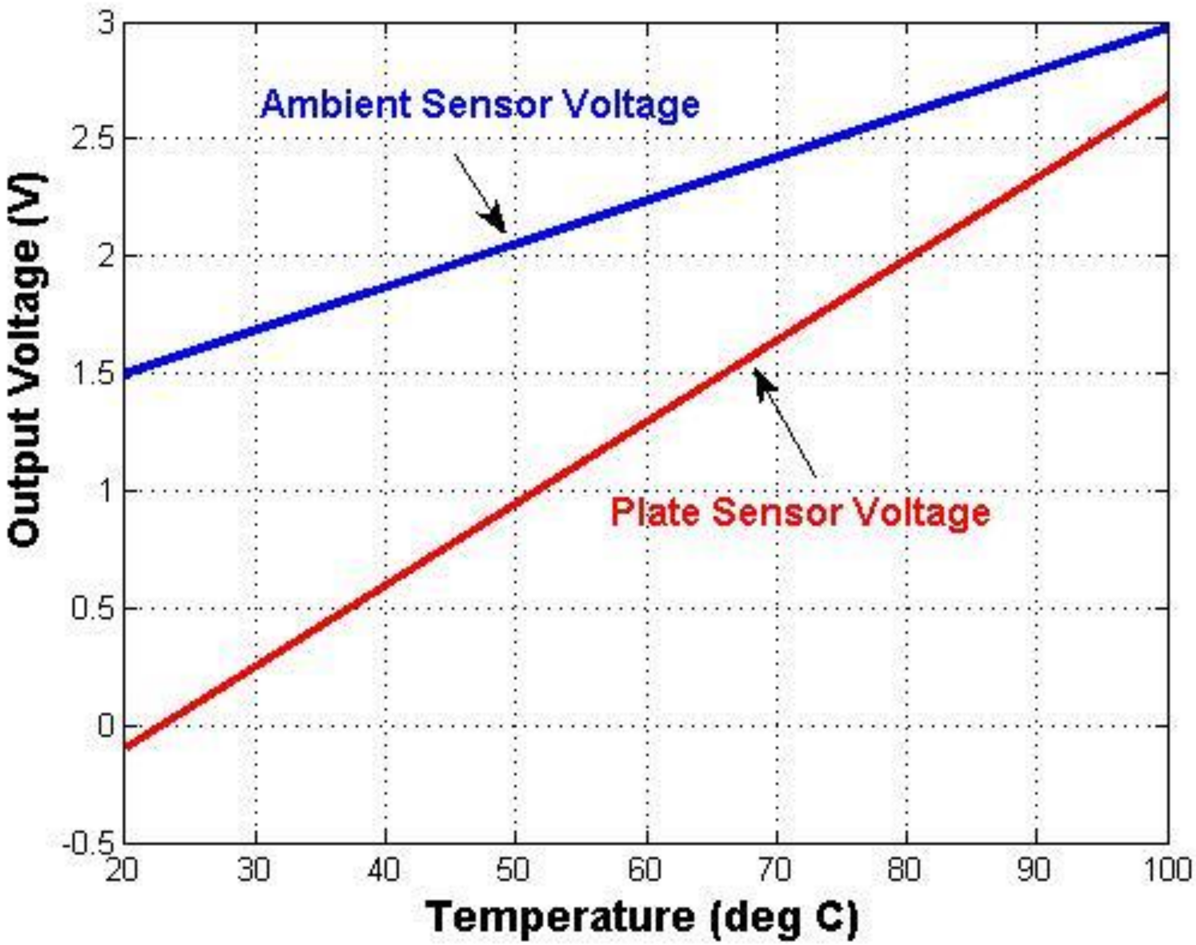
- Feedback Sensor

$$V_{feedback} = \left(34.75 \ \frac{mV}{{}^{0}K} \right) \left(\text{Temperature } ({}^{0}K) \right) - 10.29 \text{ V}$$

- Ambient Sensor

$$V_{ambient} = \left(18.43 \ \frac{mV}{{}^{0}C} \right) \left(\text{Temperature } ({}^{0}C) \right) + 1.126 \text{ V}$$

Temperature Sensor Plot



Hint to Simplify Program

$$\textbf{Gain} = \frac{\mathbf{34.75}}{\mathbf{18.43}} = \mathbf{1.9} \approx \mathbf{2}$$

Assume ambient temperature = 25^0 C

$\Rightarrow$ Desired temperature = 75^0 C

$\Rightarrow$ Heater turn on temperature = 70^0 C

$\Rightarrow$ Fan turn on temperature = 80^0 C

Program

Values

- Ambient Sensor

$$V_{ambient} = \left(18.43 \ \frac{mV}{{}^{0}C}\right)\left(\text{Temperature} \ (25\ {}^{0}C)\right) + 1.126\,V = 1.59\,V$$

$$ADRES = \frac{1.59}{4.096}\ 255 = 99 \ \text{(in binary)}$$

- Feedback Sensor

$$V_{feedback} = \left(34.75 \ \frac{mV}{{}^{0}K}\right)\left(\text{Temperature} \ ((70 + 273.16)\ {}^{0}K)\right) - 10.29\,V = 1.64\,V$$

$$ADRES = \frac{1.64}{4.096}\ 255 = 102$$

Values

- Feedback Sensor

$$V_{feedback} = \left(34.75 \ \frac{mV}{{}^{0}K} \right) \left(Temperature \ \left((80 + 273.16)\ {}^{0}K \right) \right) - 10.29\,V \ = 1.98\,V$$

$$ADRES = \frac{1.98}{4.096}\ 255 \ = 123$$

Code:

If ambient value is 99:

If plate is less than 102 – turn off fan & turn on heater

If greater than 123 – turn on fan & turn off heater

If between 102 & 123 – do NOTHING

2 Byte Addition & Subtraction

- Two Byte Add

```
MOVF     ArgA1,W
ADDWF    ArgB1,F
MOVF     ArgA0,W
BTFSC    STATUS,C
INCFSZ   ArgA0,W
ADDWF    ARGB0,F
```

- Two Byte Subtract

```
MOVF     ArgA1,W
SUBWF    ArgB1,F
MOVF     ArgA0,W
BTFSS    STATUS,C
INCFSZ   ArgA0,W
SUBWF    ArgB0,F
```

Subtract SUBWF Command

SUBWF — Subtract W from f

Syntax:	[*label*] SUBWF f,d
Operands:	$0 \le f \le 127$ $d \in [0,1]$
Operation:	(f) - (W) → destination
Status Affected:	C, DC, Z
Encoding:	`00` `0010` `dfff` `ffff`
Description:	Subtract (2's complement method) W register from register 'f'. If 'd' is 0 the result is stored in the W register. If 'd' is 1 the result is stored back in register 'f'.
Words:	1
Cycles:	1

Q Cycle Activity:

Q1	Q2	Q3	Q4
Decode	Read register 'f'	Process data	Write to destination

Example 1: `SUBWF REG1,1`

Case 1: Before Instruction

REG1= 3
W = 2
C = x
Z = x

After Instruction

REG1= 1
W = 2
C = 1 ; result is positive
Z = 0

Case 2: Before Instruction

REG1= 2
W = 2
C = x
Z = x

After Instruction

REG1= 0
W = 2
C = 1 ; result is zero
Z = 1

Case 3: Before Instruction

REG1= 1
W = 2
C = x
Z = x

After Instruction

REG1= 0xFF
W = 2
C = 0 ; result is negative
Z = 0

Add ADDWF Command

ADDWF — Add W and f

Syntax:	[*label*] ADDWF f,d
Operands:	$0 \le f \le 127$ $d \in [0,1]$
Operation:	$(W) + (f) \rightarrow$ destination
Status Affected:	C, DC, Z
Encoding:	`00` | `0111` | `dfff` | `ffff`
Description:	Add the contents of the W register with register 'f'. If 'd' is 0 the result is stored in the W register. If 'd' is 1 the result is stored back in register 'f'.
Words:	1
Cycles:	1
Q Cycle Activity:	

Q1	Q2	Q3	Q4
Decode	Read register 'f'	Process data	Write to destination

Subtract SUBLW Command

SUBLW — Subtract W from Literal

Syntax:	[*label*] SUBLW k
Operands:	$0 \leq k \leq 255$
Operation:	$k - (W) \rightarrow W$
Status Affected:	C, DC, Z
Encoding:	11 | 110x | kkkk | kkkk
Description:	The W register is subtracted (2's complement method) from th literal 'k'. The result is placed in the W register.
Words:	1
Cycles:	1
Q Cycle Activity:	

Q1	Q2	Q3	Q4
Decode	Read literal 'k'	Process data	Write to W register

Example 1: `SUBLW 0x02`

Case 1: Before Instruction

W = 0x01
C = x
Z = x

After Instruction

W = 0x01
C = 1 ; result is positive
Z = 0

Case 2: Before Instruction

W = 0x02
C = x
Z = x

After Instruction

W = 0x00
C = 1 ; result is zero
Z = 1

Case 3: Before Instruction

W = 0x03
C = x
Z = x

After Instruction

W = 0xFF
C = 0 ; result is negative
Z = 0

Example 2 `SUBLW MYREG`

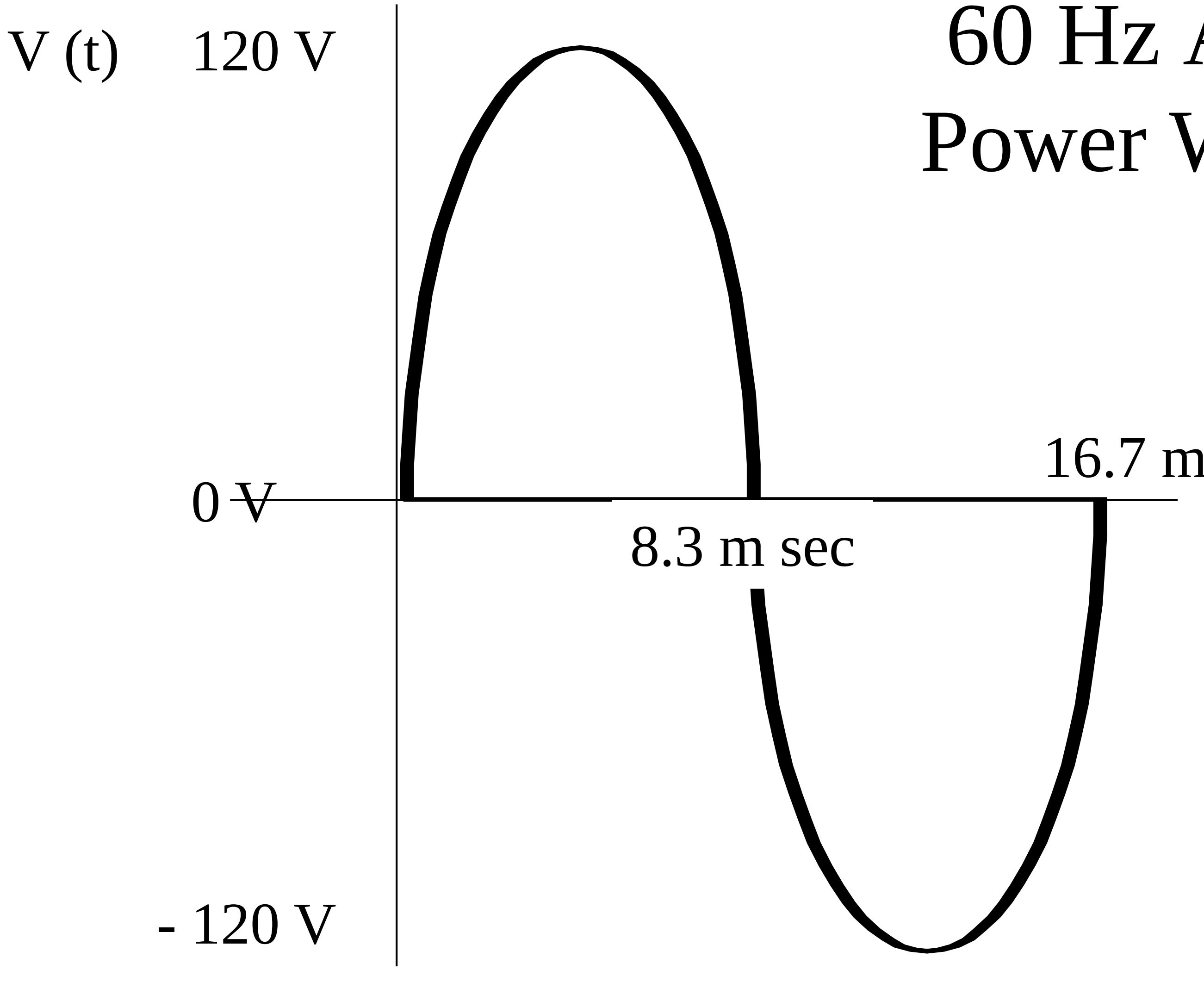
Before Instruction

W = 0x10
Address of `MYREG` † = 0x37
† `MYREG` is a symbol for a data memory location

After Instruction

W = 0x27
C = 1 ; result is positive

60 Hz A.C. Power Wave



A.C. Power

- The heat loss in the resistive heater is independent of the direction of the current.
- If the current is sinusoidal, the power loss is proportional to the rms (root mean square) current. That is, the rms current of an a. c. source is equivalent to the d. c. current of a d. c. source in terms of power dissipated across the resistive element.
- The power dissipated across the resistive heater with a sinusoidal source is:

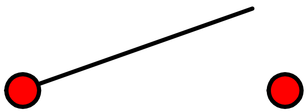
$$Q_{heater} = P_{heater} = R_{heater}\ Irms_{heater}^{\ 2}$$

$$Irms_{heater} = \sqrt{\frac{1}{T}\int_T \left(I_{max}\cos(\omega t + \phi)\right)^2\ dt}$$

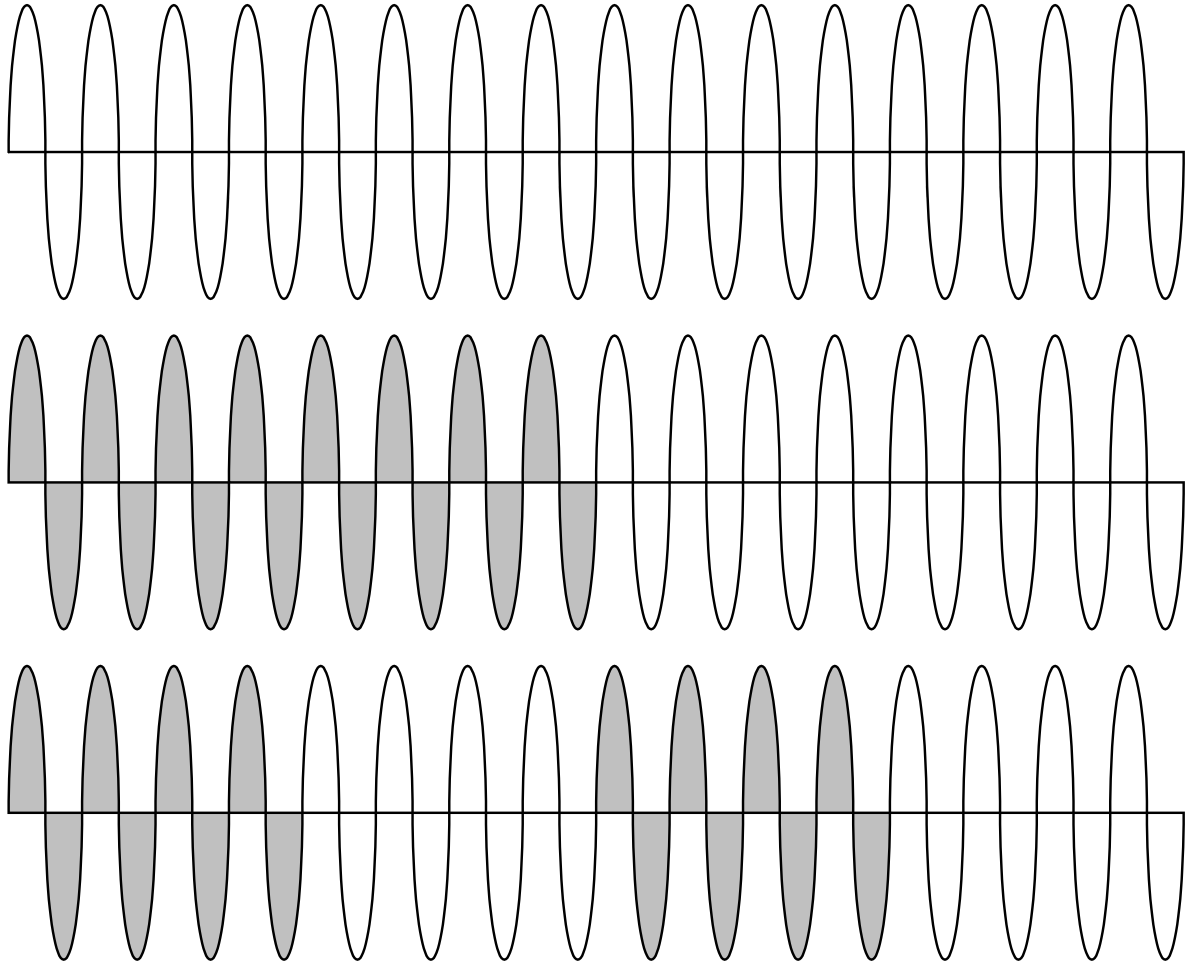
$$= \sqrt{I_{max}^{\ 2}\ \frac{1}{T}\int_T \left(\frac{1}{2} + \frac{1}{2}\cos(2\omega t + 2\phi)\right) dt}$$

$$= \frac{I_{max}}{\sqrt{2}}$$

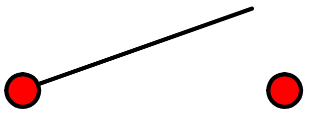
50 % A.C. Power



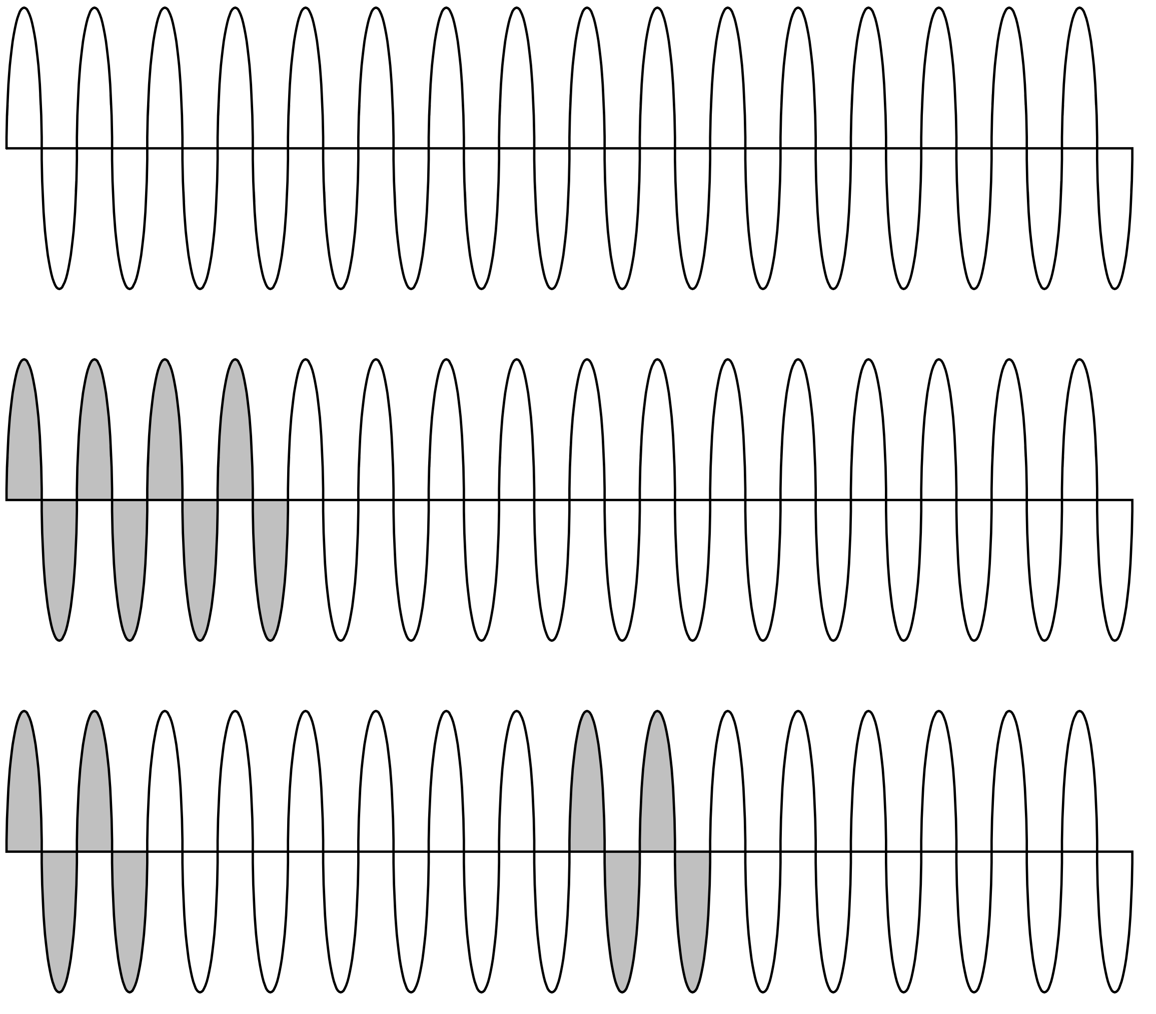
switch is
closed ½
the time



25 % A.C. Power

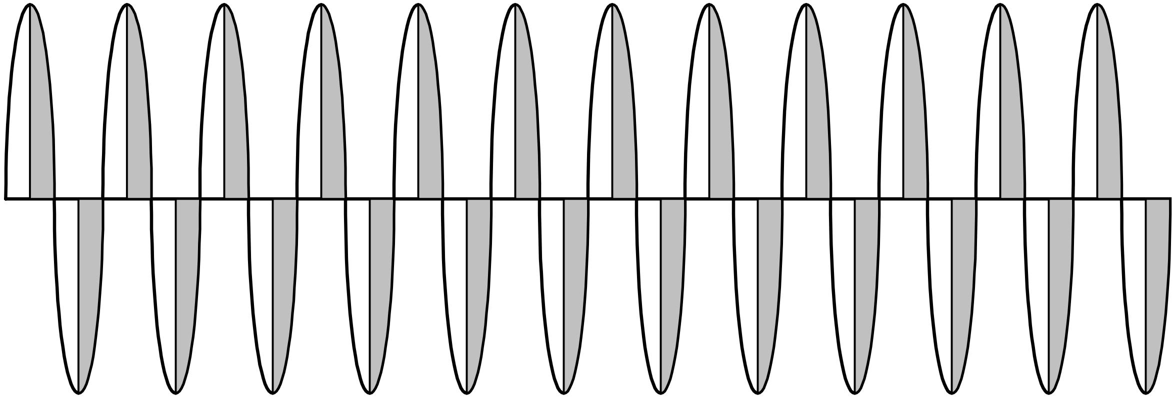


switch is closed ¼ the time

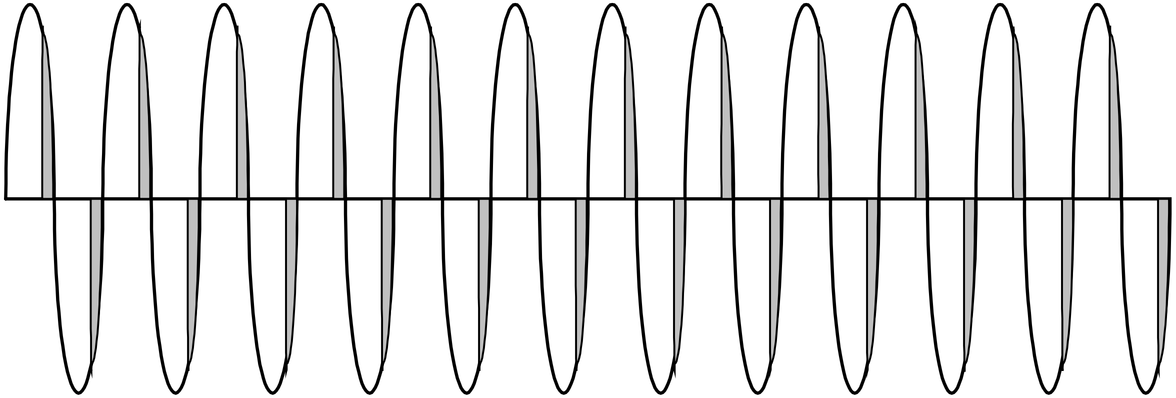


A.C. Power - Phase Control

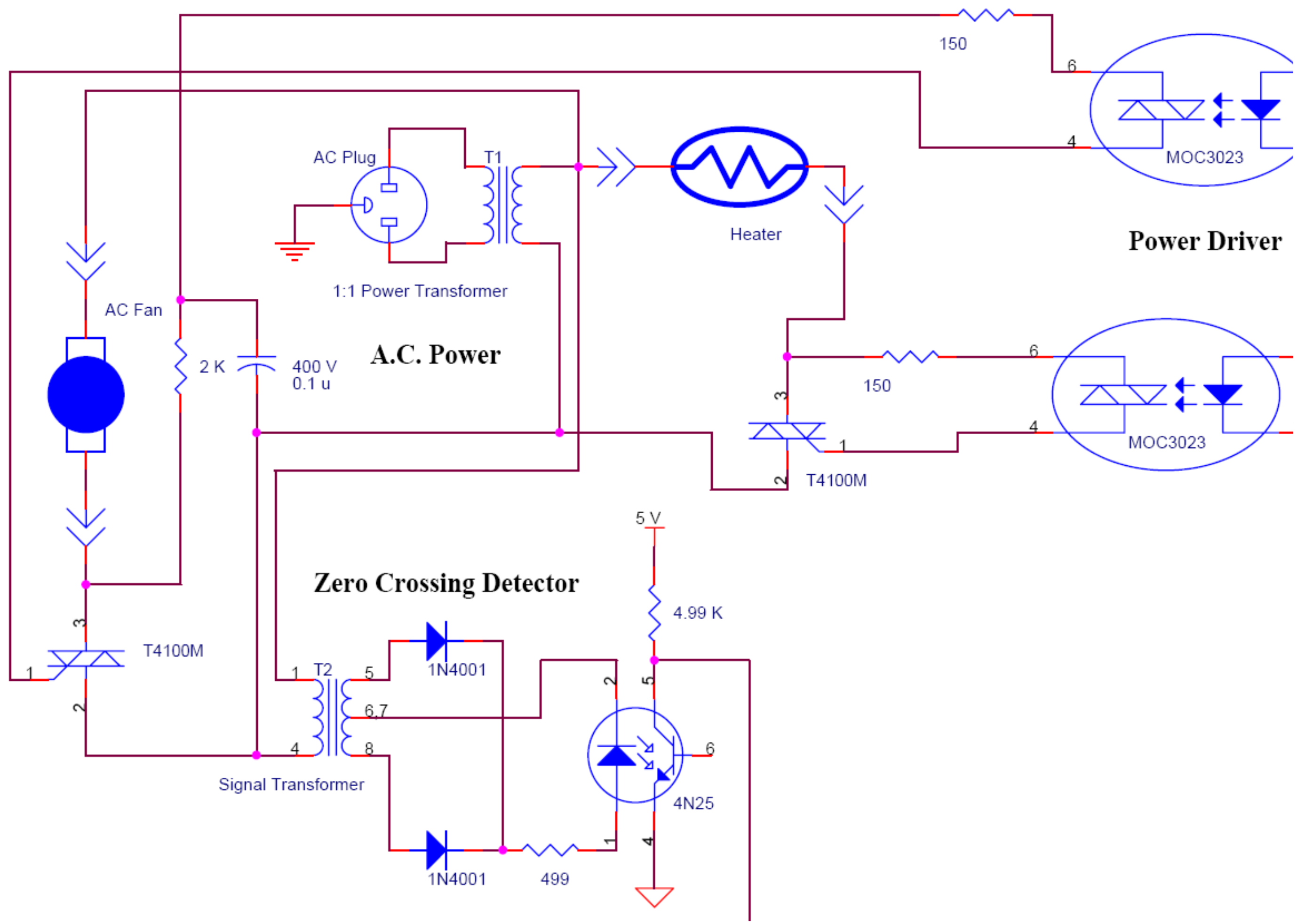
50 % Power



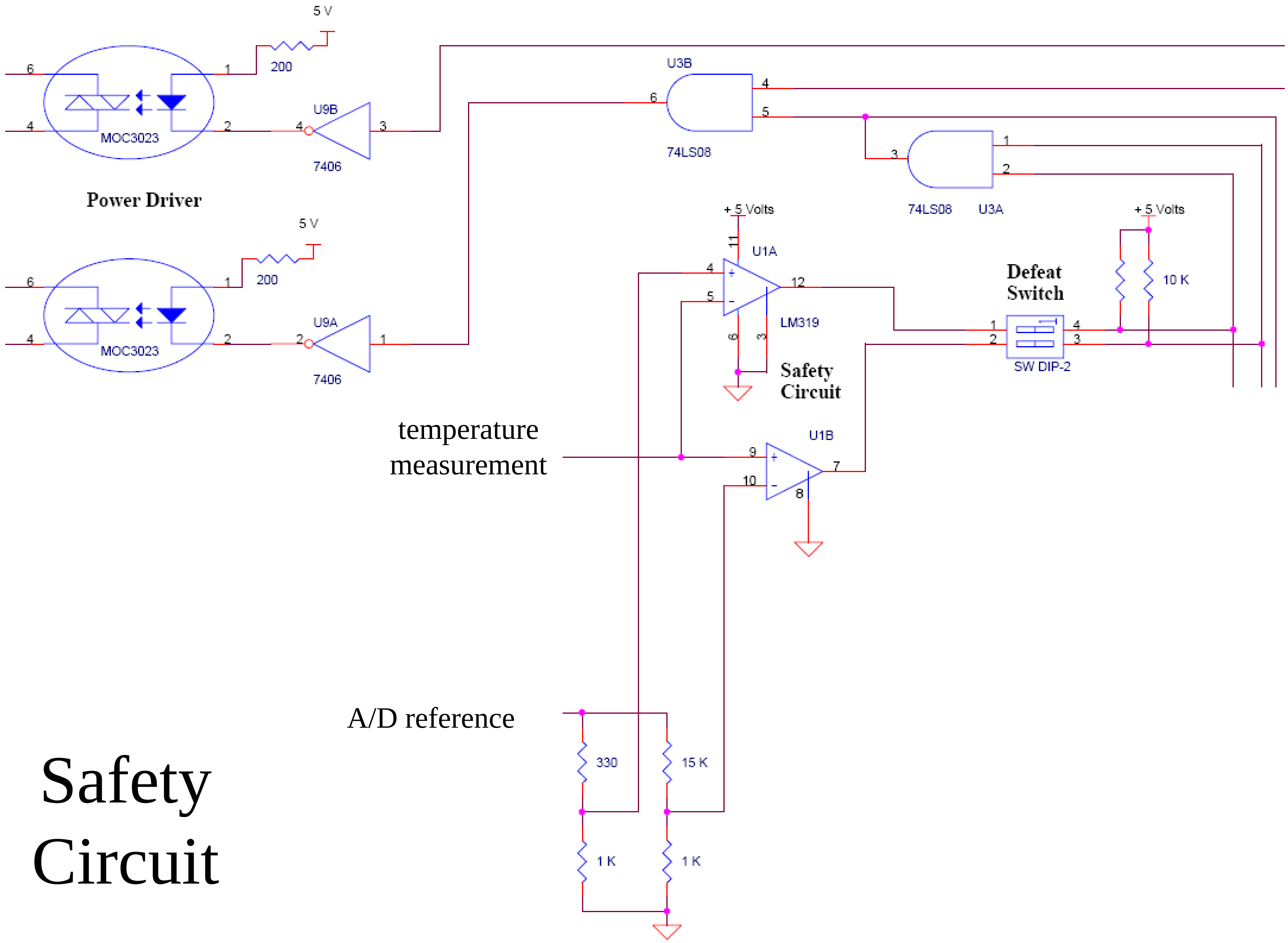
25 % Power



A.C. Power Driver



Safety Circuit



Connector

White	——	Heater Control
Yellow	——	Fan Control
Green	——	Plate Temperature
Blue	——	Ambient Temperature
Orange	——	Voltage Reference
Red	——	+ 5 V
Black	——	Ground
	wires	

Ports

Port A Pin 1 – Ambient Sensor

Port A Pin 2 – Plate Sensor

Port A Pin 3 – Reference Voltage

Port C Pin 5 – Toggle Switch

Port D Pin 0 – Red LED

Port D Pin 1 – Yellow LED

Port D Pin 2 – Green LED

Port D Pin 3 – Blue LED (on / off)

Port D Pin 6 – Heater (on -= low)

Port D Pin 7 – Fan (on -= low)

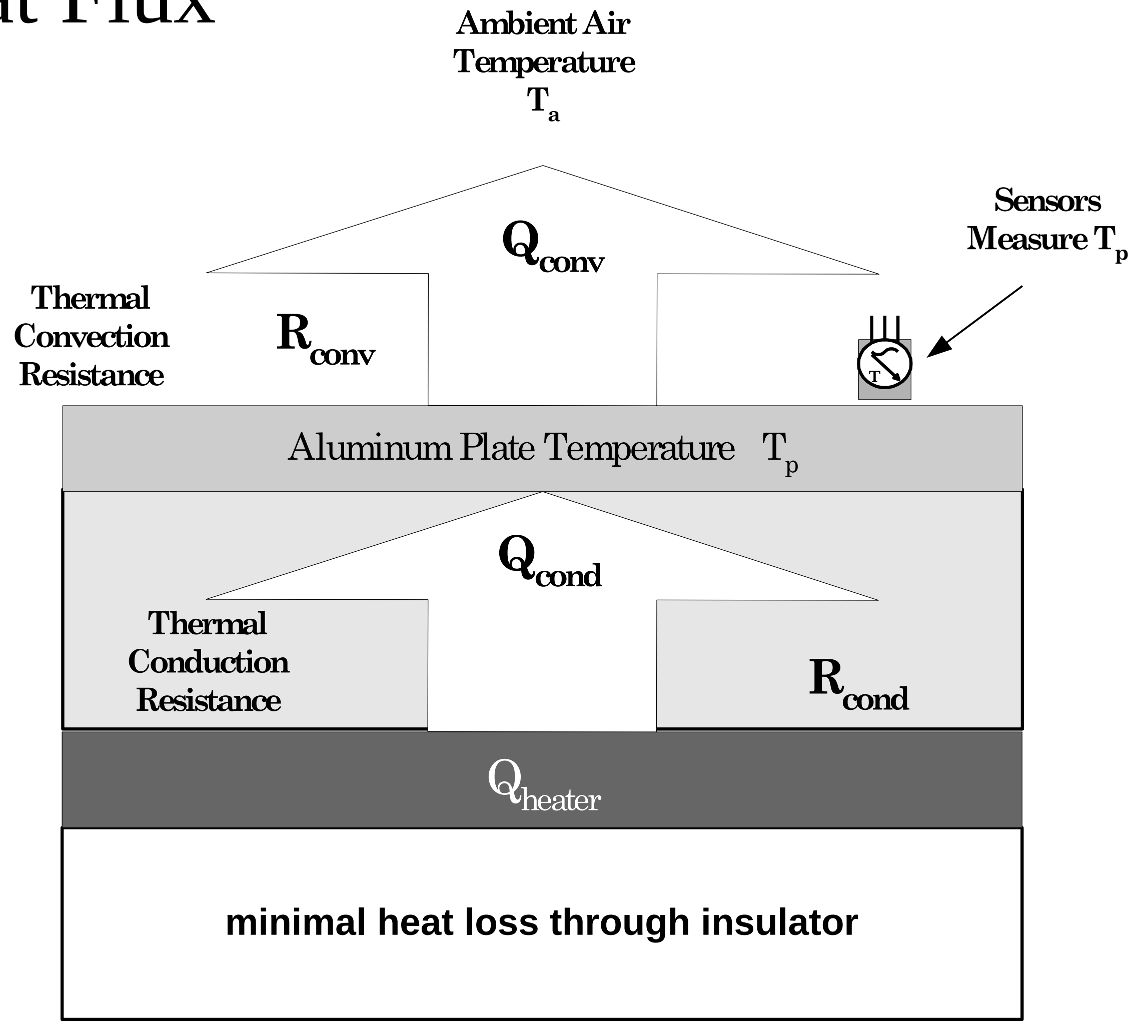
Simplifications in Modeling

Approximation	Mathematical Simplification
Neglect small effects	Reduces the number and complexity of the equations of motion
Assume the environment is independent of system operation	Reduces the number and complexity of the equations of motion
Replace distributed characteristics with appropriate lumped elements	Leads to ordinary (rather than partial) differential equations
Assume linear relationships	Makes equations linear; allows superposition of solutions
Assume constant parameters	Leads to constant coefficients in the differential equations
Neglect uncertainty and noise	Avoids statistical treatment

Simplifying Assumptions

- The temperature of the plate is uniform.
- There in no heat loss through the sides of the plate, i.e., heat conduction is one-dimensional through the plate.
- The thermal conductivity of the plate is constant, i.e., independent of time, temperature, space, or direction of heat flow.
- The heat loss due to radiation is negligible compared to the convective heat loss from the plate.
- The heat convection coefficient is constant and is evaluated at the operating temperature of the plate.
- The heat loss through the insulating (wood) layer is negligible, i.e., heat loss through the insulating layer, and subsequent convective heat loss from the bottom supporting Plexiglas plate, is negligible compared to the other heat losses in the system.
- The sensor dynamics are negligible, i.e., the sensor dynamics are very fast relative to the dynamics of the rest of the system. This includes the dynamics of the OpAmp interface circuit.
- Ambient air temperature is unaffected by the heat flux from the plate.

Heat Flux



Ambient Air
Temperature
T_a
Q_{conv}
Sensors
Measure T_p
Thermal
Convection
Resistance
R_{conv}
T
Aluminum Plate Temperature T_p
Q_{cond}
Thermal
Conduction
Resistance
R_{cond}
Q_{heater}
minimal heat loss through insulator

Thermal Components

- There are only 2 types of thermal components
  - Thermal capacitance
  - Thermal resistance

Thermal Capacitance

- Heat flows into a body of solid, liquid, or gas, this thermal energy may show up in various forms such as mechanical work or changes in kinetic energy of a flowing fluid.
- Assume the addition of thermal energy does not cause significant mechanical work or kinetic energy change, the added energy shows up as stored internal energy and manifests itself as a rise in temperature of the body.
- Provided that there is no change of phase and that the range of temperatures is not excessive, the relationship between the temperature of the body and the heat stored can be considered to be linear.

Thermal Capacitance Equations

$Q_{in}(t) - Q_{out}(t)$ = net heat flow rate into body

$$\int_{t_0}^{t} \left[Q_{in}(\lambda) - Q_{out}(\lambda) \right] d\lambda \ = \ \text{net heat supplied between times } t_0 \text{ and } t$$

Assume that heat supplied during this time interval equals a constant **C** times **ΔT**.

$$C \ \Delta T = C\left[T(t) - T(t_0) \right] = \int_{t_0}^{t} \left[Q_{in}(\lambda) - Q_{out}(\lambda) \right] d\lambda$$

where **C** is the thermal capacitance (J / ^{0}C) and is equal to ***M* σ**, where ***M*** is the mass of the body (kg) and **σ** is the specific heat of the body (J / [kg – ^{0}C]). Differentiating the above equation results in:

$$\dot{T} = \frac{1}{C} \left[Q_{in}(t) - Q_{out}(t) \right]$$

Thermal Resistance

- Whenever two objects have different temperatures, there is a tendency for heat to be transferred from the hot region to the cold region in an attempt to equalize temperatures.
- For a given temperature difference, the rate of heat transfer varies depending on the thermal resistance of the path between the hot and cold region.
- The nature and magnitude of the thermal resistance depend on the modes of heat transfer involved:
  - Conduction
  - Convection
  - Radiation

Conduction

- In conduction, heat flows from one body to another through the medium connecting them at a rate proportional to the temperature difference between the points:

$$Q(t) = \frac{1}{R}\left[T_1(t) - T_2(t)\right]$$

- **R** is the thermal resistance ($[{}^0C - s] / J$ or ${}^0C / W$) which equals *L / Ak*, where *A* is the cross-sectional area of the heat flux path, *L* is the length of the path, and *k* is the thermal conductivity of the material ($W / [m - {}^0C]$).

- We can use this equation only when the body is being treated as a thermal resistance and does not store any heat.

Convection

- Many practical situations involve heat flow through fluid / solid interfaces by convection. In this case, heat flows by conduction through a thin layer of fluid (called the boundary layer) which adheres to the solid wall. At the interface between the boundary layer and the main body of fluid, the heat is carried away by the constantly moving fluid particles into the main stream.

$$Q(t) = h\,A\,\left[T_1(t) - T_2(t)\right]$$

- **h** is the film coefficient of heat transfer (J / [s – m^2 – ^{0}C] or W / [m^2 – ^{0}C]) and **A** is the surface area (m^2).
- While techniques exist to estimate **h**, in practice the value is often experimentally determined and usually varies with temperature. For the purpose of this case study, we will assume that **h** is a constant.

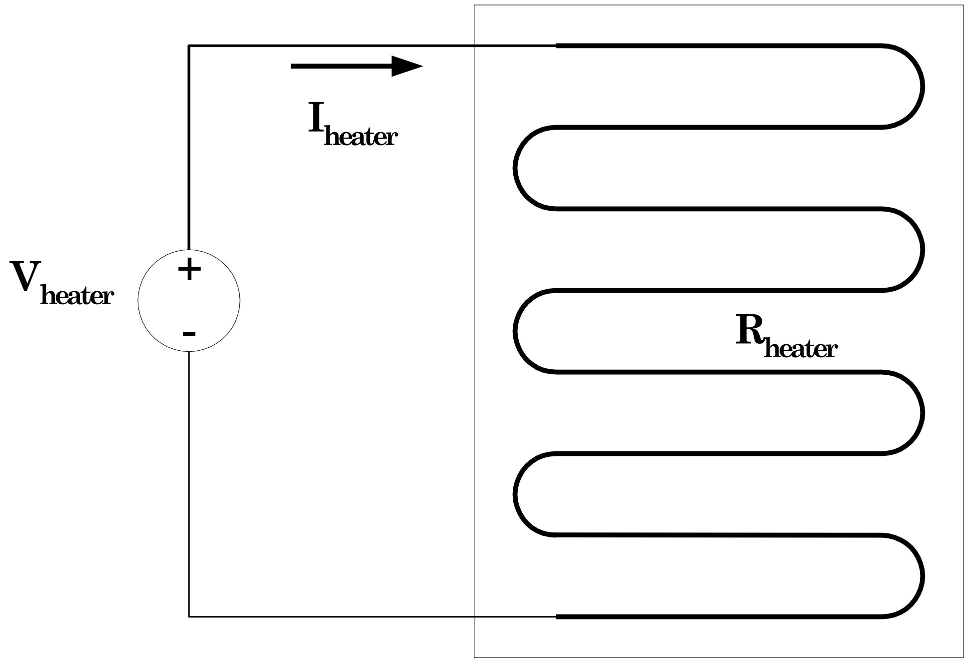
Radiation

- Two bodies can exchange thermal energy with no physical contact by the process of radiation. The rate of heat transfer depends on a surface property of each body called the emissivity, geometrical factors involving the portion of the emitted radiation from one body that actually strikes the other body, the surface areas involved, and the temperatures of the two bodies. For a typical configuration and materials, the defining equation takes the form:

$$Q(t) = \varepsilon A \left[T_1^4(t) - T_2^4(t) \right]$$

- For this equation, the temperatures are absolute.

Thermal Source



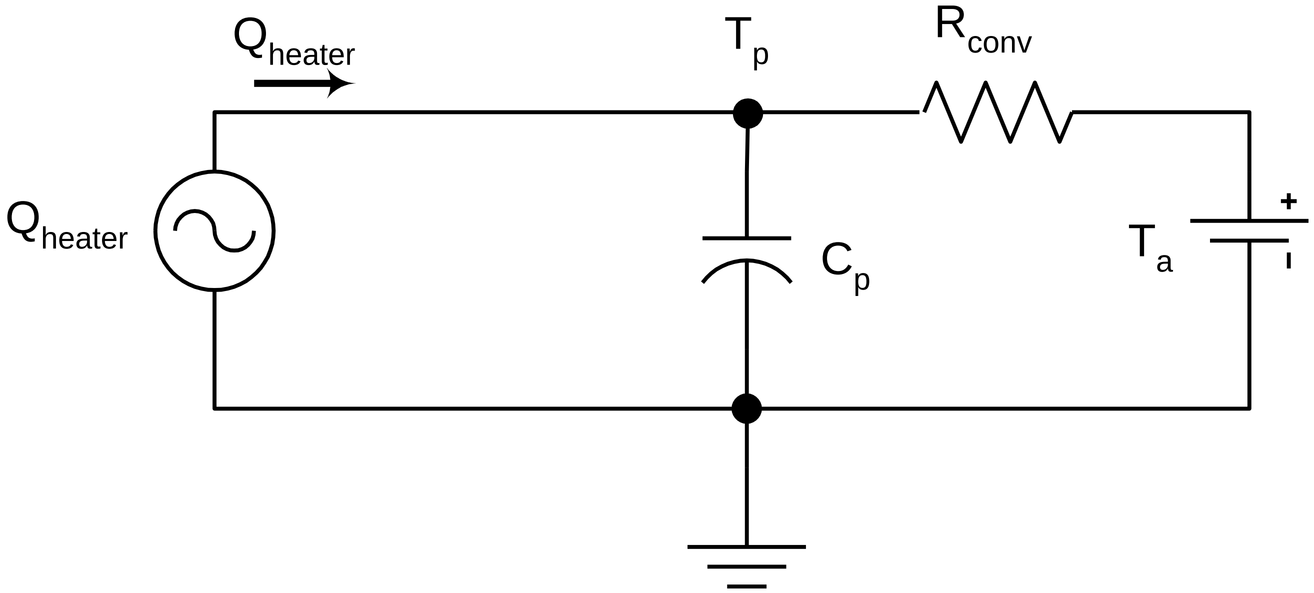
$\mathbf{R_{heater}}$ is the heater resistance
$\mathbf{I_{heater}}$ is the heater current
$\mathbf{V_{heater}}$ is the heater voltage.

Heat Source

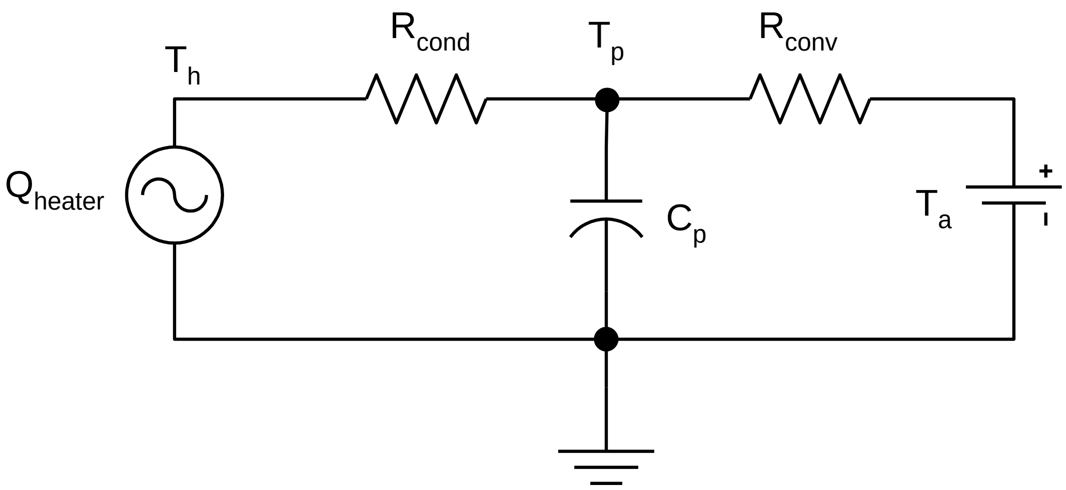
- If we assume 100% conversion efficiency, the heat flux produced by the resistive heater is equivalent to the power dissipated across the resistive element.
- The power dissipated across the resistive heater P_{heater} (J / s or W) is given by:

$$Q_{heater} = P_{heater} = V_{heater} \; I_{heater} = \frac{{V_{heater}}^2}{R_{heater}}$$

Thermal Model



Thermal Model



Since we are modeling a heat flux Q_{heater} rather than a temperature, the value of R_{cond} is not needed in the model.

Defining Equations

$$Q_{heater} = I_{heater}^{2} R_{heater}$$

$$Q_{conv}(t) = \frac{1}{R_{conv}} \left[T_p(t) - T_a(t) \right]$$

$$\dot{T}_p = \frac{1}{C_p} \left[Q_{heater}(t) - Q_{conv}(t) \right] = \frac{1}{C_p} \left[Q_{heater}(t) - \frac{1}{R_{conv}} \left[T_p(t) - T_a(t) \right] \right]$$

First Order Differential Equation

$$\dot{T}_p + \frac{1}{R_{conv} C_p} T_p = \frac{1}{C_p} Q_{heater} + \frac{1}{R_{conv} C_p} T_a$$

Operating Point

At the operating point:

$$T_p(t) = \overline{T}_p \qquad Q_{heater}(t) = \overline{Q}_{heater} \qquad \dot{T}_p = 0$$

$$\frac{1}{R_{conv}\ C_p}\ \overline{T}_p = \frac{1}{C_p}\ \overline{Q}_{heater} + \frac{1}{R_{conv}\ C_p}\ T_a$$

$$\overline{T}_p = R_{conv}\ \overline{Q}_{heater} + T_a$$

Incremental Variables

$$\hat{T}_p(t) = T_p(t) - \overline{T}_p \qquad \hat{Q}_{heater}(t) = Q_{heater}(t) - \overline{Q}_{heater}$$

$$\dot{\hat{T}}_p + \frac{1}{R_{conv}\, C_p}\left(\hat{T}_p + \overline{T}_p\right) = \frac{1}{C_p}\left(\hat{Q}_{heater} + \overline{Q}_{heater}\right) + \frac{1}{R_{conv}\, C_p} T_a$$

$$\dot{\hat{T}}_p + \frac{1}{R_{conv}\, C_p}\hat{T}_p = \frac{1}{C_p}\hat{Q}_{heater}$$

Observations

- If $\overline{Q}_{heater} > 0$ then $\overline{T}_p > T_a$ and the plate is being heated.
- If $\overline{Q}_{heater} < 0$ then $\overline{T}_p < T_a$ and the plate is being cooled.
- If $\overline{Q}_{heater} = 0$ then the nominal plate temperature is

$$\overline{T}_p = T_a \text{ and } \hat{Q}_{heater}(t) \approx Q_{heater}(t).$$

Transfer Function

$$s\ \hat{T}_p(s) + \frac{1}{R_{conv}\ C_p}\ \hat{T}_p(s) = \frac{1}{C_p}\ \hat{Q}_{heater}(s)$$

$$\frac{\hat{T}_p(s)}{\hat{Q}_{heater}(s)} = \frac{\frac{1}{C_p}}{s + \frac{1}{R_{conv}\ C_p}} = \frac{R_{conv}}{\left(R_{conv}\ C_p\right)\ s + 1} = \frac{R_{conv}}{\tau\, s + 1}$$