# Regression for Microbiome Data: Moving From Diversity to Inference



Brendan J. Kelly, MD, MS

Updated: 18 June 2020

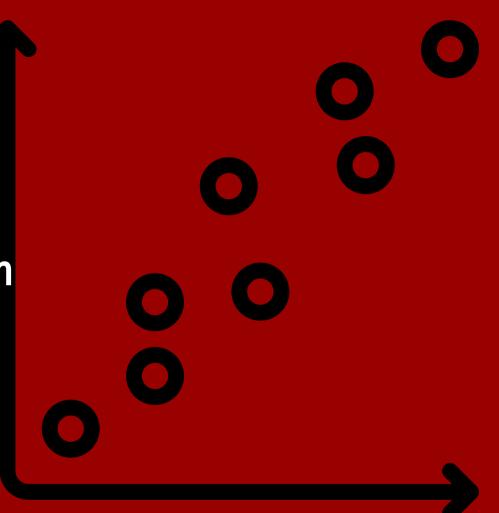
Review  $\alpha$ -diversity

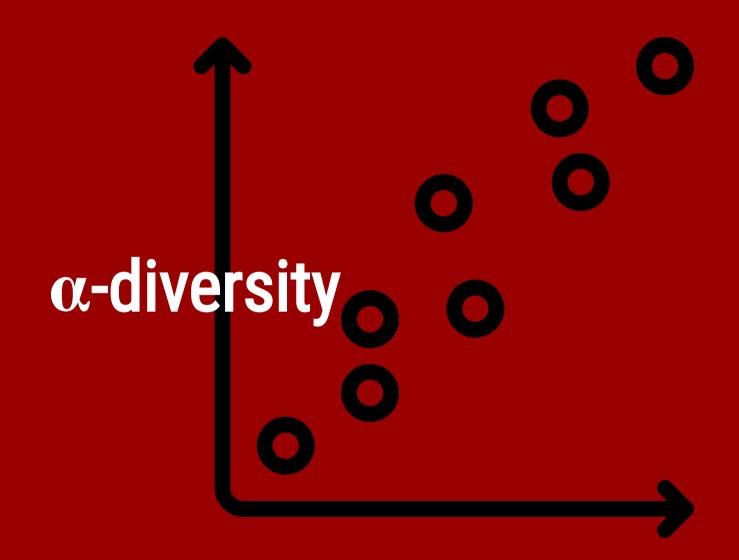
**Review β-diversity** 

 $\alpha/\beta$ -diversity  $\rightarrow$  linear regression

**β-diversity** → PERMANOVA

Compositional data?





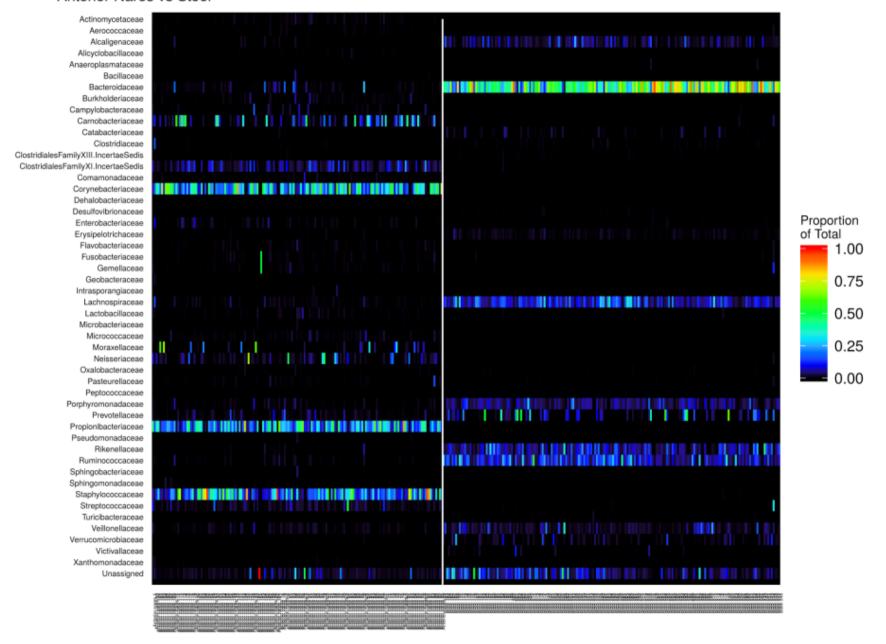
#### **High Dimensional Microbiome Data**

##		700013549	700014386	700014403	700014409	700014412	700014415
##	OTU_97.1	0	0	0	0	0	0
##	OTU_97.10	0	0	6	4	1	5
##	OTU_97.100	0	0	133	7	1	4
##	OTU_97.1000	0	0	0	0	0	0
##	OTU_97.10000	0	0	0	0	0	0
##	OTU_97.10001	0	0	0	0	0	1
##	OTU_97.10002	0	0	0	0	0	0
##	OTU_97.10003	0	0	0	0	0	0
##	OTU_97.10004	0	0	0	0	0	0
##	OTU_97.10005	0	0	0	0	0	0
##	OTU_97.10006	0	0	0	0	0	0
##	OTU_97.10007	0	0	0	0	0	0
##	OTU_97.10008	0	1	0	0	0	0
##	OTU_97.10009	0	0	1	0	0	0
##	OTU_97.1001	0	0	0	0	0	0
##	OTU_97.10010	0	0	0	0	0	0

#### High Dimensional Microbiome Data

- How to deal with high-dimensional microbiome data?
- Descriptive (e.g., heatmaps and stacked barplots)
- Test a priori hypotheses regarding specific OTUs/taxa
- Reduce dimensions:
  - single summary statistic (alpha diversity)
  - pairwise distances (beta diversity) with PCoA or PERMANOVA
  - community types (mixture modeling)

#### Anterior Nares vs Stool



#### High Dimensional Microbiome Data

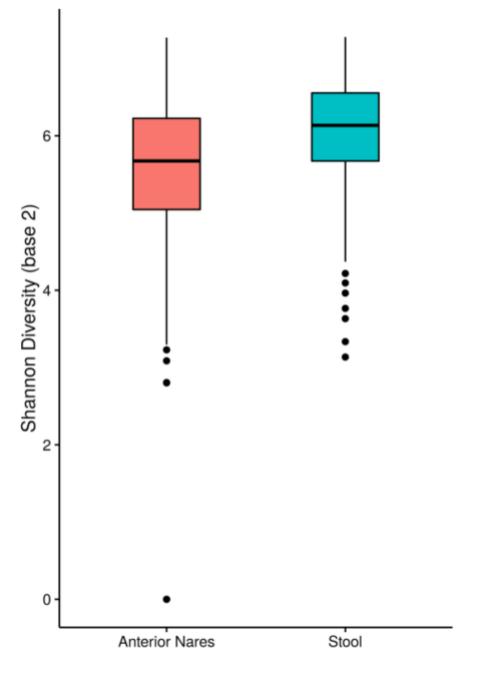
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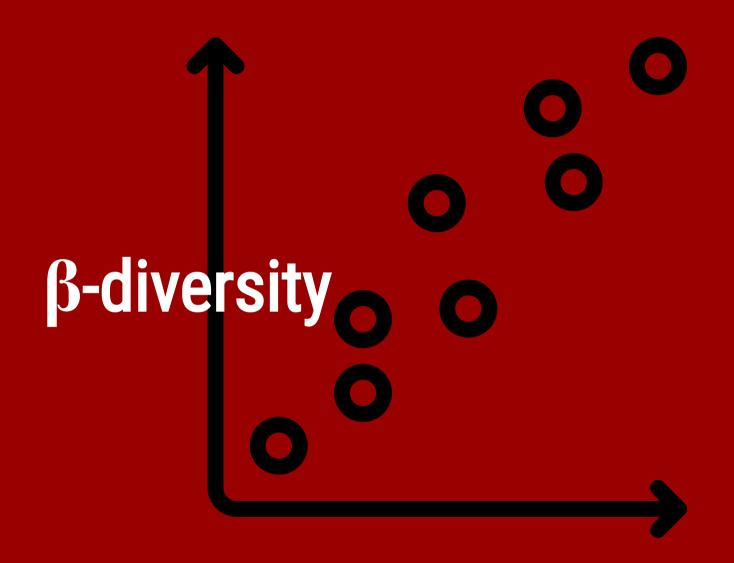
# **Shannon Diversity**

- Richness & evenness
- Shannon diversity:

```
$H' = -\sum_{i} * \log_{b}{(p_{i})}
```

• "The uncertainty contained in a probability distribution is the average logprobability of an event." (McElreath *Statistical Rethinking, 2nd* 2020)



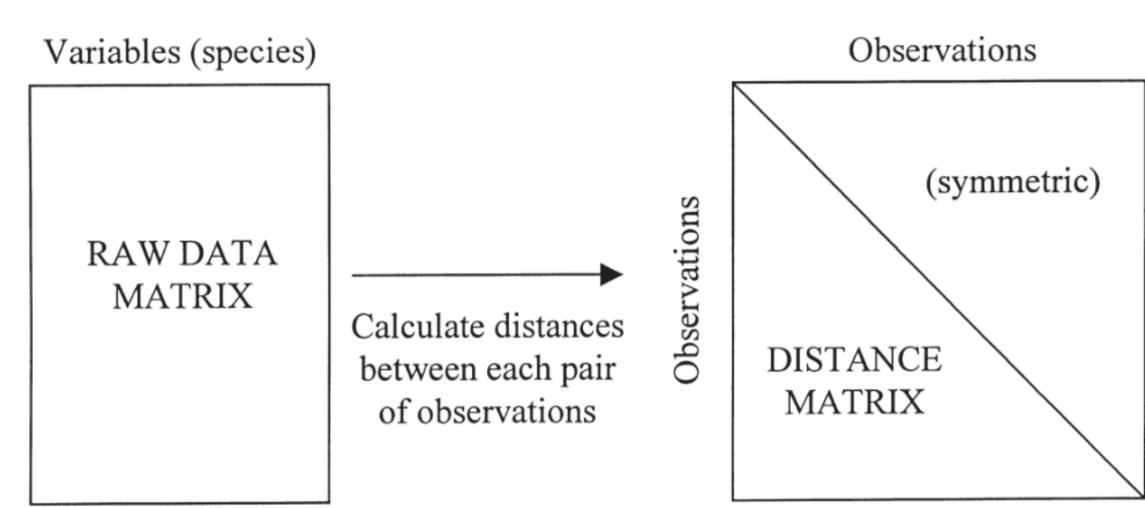


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#### **Beta Diversity as Dimension Reduction**

- Summarize each sample's relationship to other samples:
  - pairwise distances
  - OTU table → square matrix
- Many beta diversity metrics:
  - just counts versus counts + phylogeny
  - weighted versus unweighted

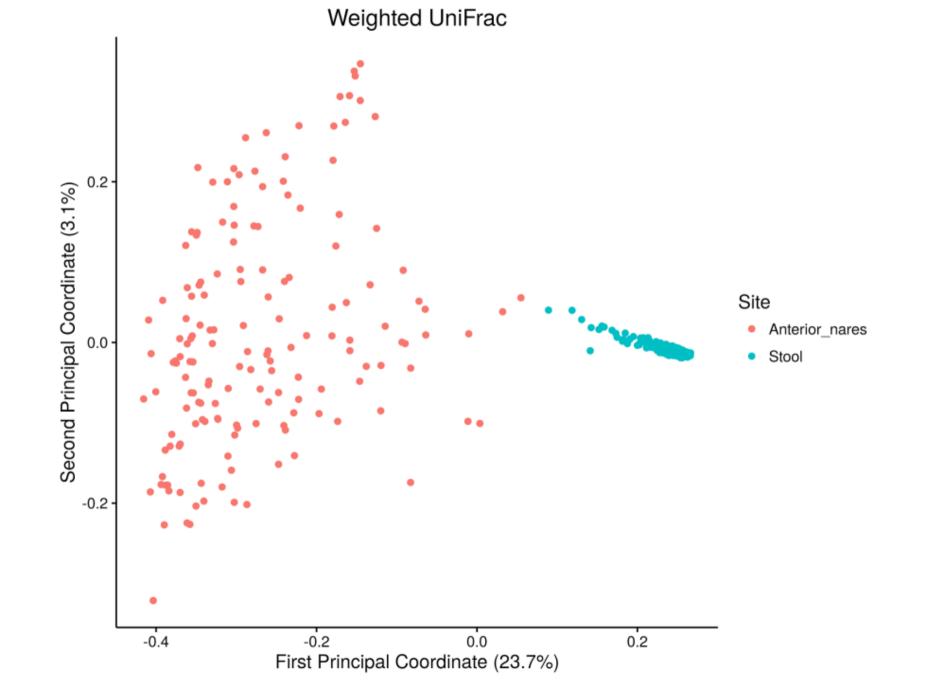


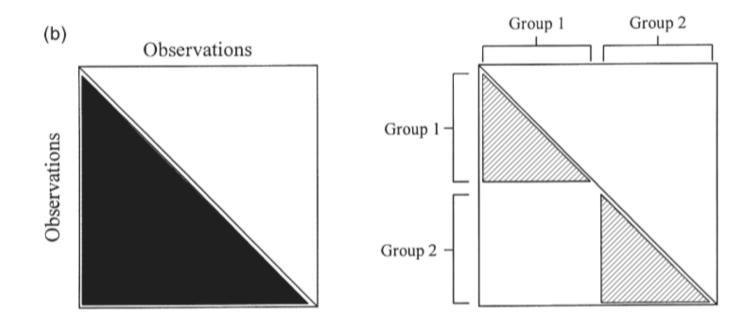
#### **Distance Metrics for Beta Diversity**

- Just counts versus counts + phylogeny:
  - Jaccard:  $\langle J(A,B) = \frac{A \cap B}{A \cup B} \rangle & \langle d_{J}(A,B) = 1 J(A,B) \rangle$
  - UniFrac: fraction of unique branch length in tree
- Weighted versus unweighted:
  - weighted: counts matter
  - unweighted: binary (presence-absence)

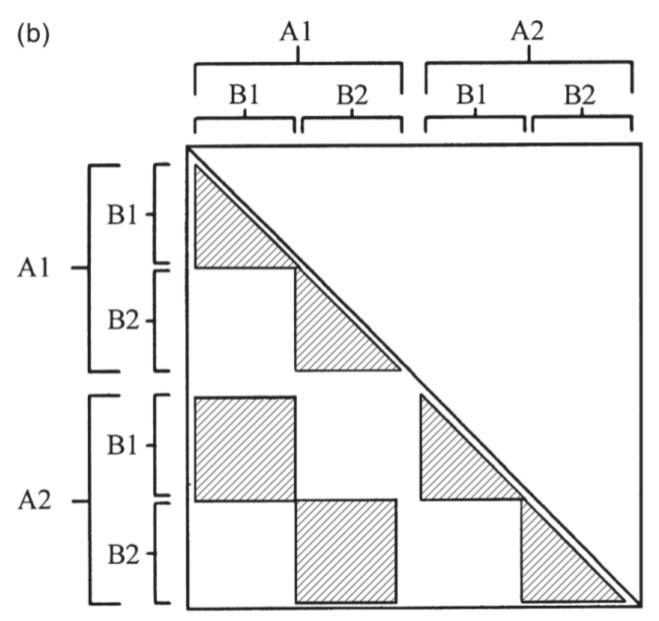
#### Pairwise Distances → PCoA

- PCoA: principal coordinate analysis
  - any metric distance, even if non-Euclidean
  - like PCA, eigenvalue decomposition (maximum variance) but mediated by distance function (no original descriptors)
  - unlike PCA, does not allow projection of original descriptors in reduceddimension space





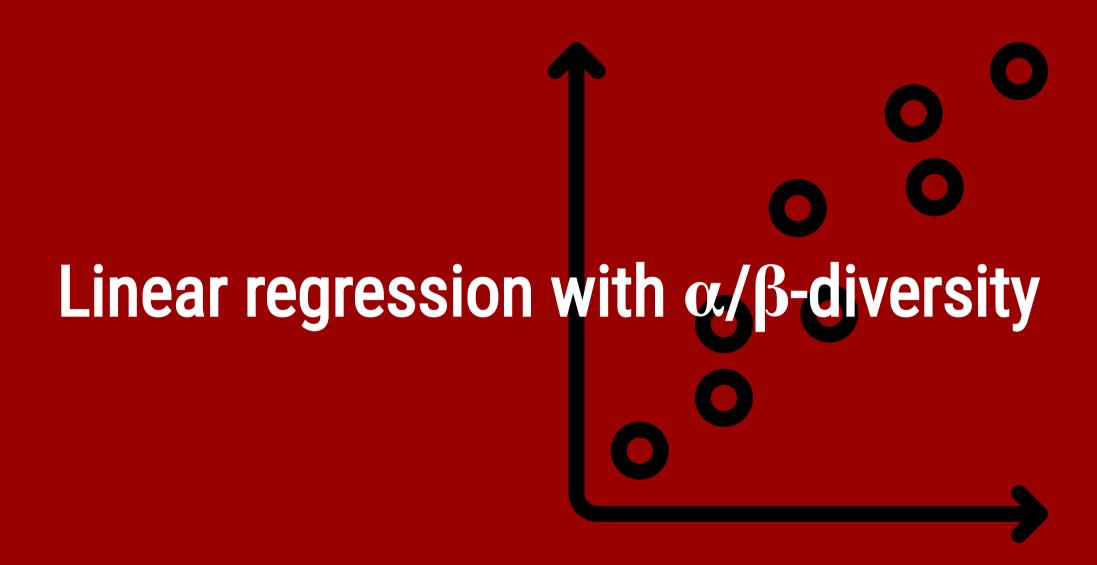
$$F = \frac{SS_A/(a-1)}{SS_W/(N-a)}$$



$$R^2=1-rac{ ext{SS}_{ ext{W}}}{ ext{SS}_{ ext{W}}+ ext{SS}_{ ext{A}}}=rac{ ext{SS}_{ ext{A}}}{ ext{SS}_{ ext{T}}} \qquad \qquad \omega^2=rac{ ext{SS}_{ ext{A}}-(a-1)rac{ ext{SS}_{ ext{W}}}{N-a}}{ ext{SS}_{ ext{T}}+rac{ ext{SS}_{ ext{W}}}{N-a}}$$

**Table 1.** Effect sizes observed from various exposures/interventions in studies of various microbiome sampling sites are shown as measured by omega-squared ( $\omega^2$ ) statistics, together with the *P*-values from PERMANOVA test

Site	Comparison groups		$\omega^2/P$ -value					
	Control	Exposure	Weighted UniFrac	Unweighted UniFrac	Weighted Jaccard	Unweighted Jaccard	Reference	
Nares	Non-smoker (33)	Smoker (29)	0.042/0.001	0.009/0.001	0.023/0.001	0.007/0.001	Charlson et al. (2010)	
Oral	Non-smoker (33)	Smoker (29)	0.032/0.001	0.008/0.001	0.024/0.001	0.007/0.001	Charlson et al. (2010)	
Gut	Before feeding (10)	After feeding (10)	0.056/0.138	0.013/0.986	0/0.989	0.014/0.985	Wu et al. (2011)	
Oral	No azithromycin (42)	Azithromycin (6)	0.063/0.01	0.039/0.001	0.099/0.004	0.032/0.001	Charlson et al. (2012)	
Lung	No azithromycin (34)	Azithromycin (6)	0.065/0.005	0.038/0.001	0.019/0.089	0.033/0.001	Charlson et al. (2012)	
Skin	Left retroauricular (186)	Right retroauricular (187)	0.000/0.828	0.0001/0.327	0.000/0.986	0.000/1.000	HMP Consortium (2012b)	
Human	Anterior nares (161)	Stool (187)	0.567/0.001	0.201/0.001	0.230/0.001	0.117/0.001	HMP Consortium (2012b)	



```
# install.packages("tidyverse")
library(tidyverse)

# install.packages("vegan")
library(vegan)

# install.packages("ape")
library(ape)

set.seed(16)

otu_tab <- read_rds(
"./data/HMP_OTU_table_matrix_stool_nares.rds"
)

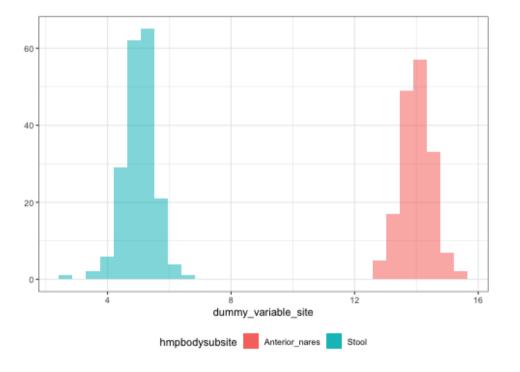
otu_tab %>%
    str(vec.len = 3)
```

```
## num [1:43140, 1:10] 0 0 0 0 0 0 0 0 0 ...
## - attr(*, "dimnames")=List of 2
## ..$ : chr [1:43140] "OTU_97.1" "OTU_97.10" "OTU_97.100" ...
## ..$ : chr [1:10] "700014718" "700014767" "700014923" ...
```

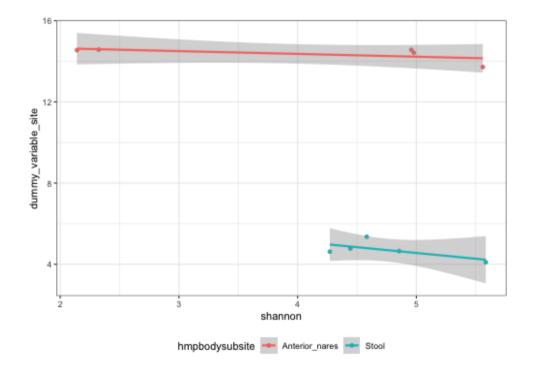
```
## # A tibble: 431,400 x 3
     otu id
                  specimen_id read_count
     <chr>
                  <chr>
                                   <dbl>
                  700014718
## 1 OTU 97.1
   2 OTU 97.10
                 700014718
   3 OTU 97.100 700014718
   4 OTU 97.1000 700014718
## 5 OTU_97.10000 700014718
## 6 OTU 97.10001 700014718
## 7 OTU_97.10002 700014718
   8 OTU_97.10003 700014718
   9 OTU 97.10004 700014718
## 10 OTU 97.10005 700014718
## # ... with 431,390 more rows
```

```
read_tsv(file = "./data/v13_map_uniquebyPSN.txt.bz2") %>%
  rename_all(.funs = ~ gsub("#","",tolower(.x))) %>%
  rename(specimen_id = sampleid) %>%
  distinct() -> specimen data
specimen data %>%
  group_by(hmpbodysubsite) %>%
  mutate(dummy_variable_site =
           rnorm(n = length(hmpbodysubsite),
                 mean = nchar(unique(hmpbodysubsite)),
                 sd = 0.5)) %>%
  ungroup() %>%
  filter(hmpbodysubsite %in%
           c("Anterior_nares", "Stool")) %>%
  select(specimen_id,
         hmpbodysubsite,
         dummy_variable_site) %>%
  mutate(specimen_id = as.character(specimen_id)) %>%
  distinct() -> specimen_data
specimen_data
```

```
## # A tibble: 361 x 3
      specimen_id hmpbodysubsite dummy_variable_site
      <chr>>
                  <chr>>
                                                <dbl>
   1 700013549
                  Stool
                                                 5.87
   2 700014386
                  Stool
                                                 4.68
   3 700014445
                  Anterior nares
                                                14.2
   4 700014488
                  Stool
                                                 4.95
    5 700014497
                  Stool
                                                 5.36
   6 700014527
                                                13.9
                  Anterior nares
   7 700014555
                  Stool
                                                 4.39
                  Stool
                                                 4.10
    8 700014718
    9 700014767
                  Anterior nares
                                                14.5
## 10 700014797
                                                13.3
                  Anterior nares
## # ... with 351 more rows
```

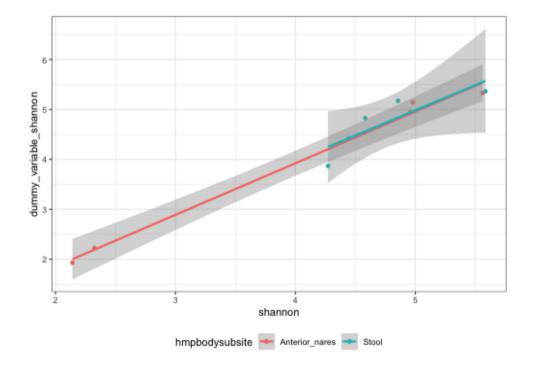


```
## # A tibble: 10 x 5
      specimen_id shannon hmpbodysubsite dummy_variable_site dummy_var
      <chr>>
                    <dbl> <chr>
                                                       <dbl>
   1 700014718
                     5.58 Stool
                                                        4.10
   2 700014767
                     2.14 Anterior nares
                                                       14.5
   3 700014923
                     2.32 Anterior nares
                                                       14.6
   4 700016920
                     4.96 Anterior nares
                                                       14.6
   5 700023706
                     4.98 Anterior nares
                                                       14.4
                                                       13.7
   6 700038343
                     5.56 Anterior nares
   7 700095956
                     4.86 Stool
                                                        4.65
   8 700105834
                     4.58 Stool
                                                        5.36
   9 700107189
                     4.27 Stool
                                                        4.62
## 10 700109383
                     4.44 Stool
                                                        4.78
```



```
shannon_summary %>%
    Im(formula = dummy_variable_site ~ shannon,
        data = .) %>%
    summary()
```

```
##
## Call:
## lm(formula = dummy_variable_site ~ shannon, data = .)
##
## Residuals:
               10 Median
                                     Max
      Min
## -5.0672 -4.0586 -0.9973 4.8409 6.0355
## Coefficients:
              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 16.328
                           6.301
                                   2.591
                                            0.032 *
## shannon
                -1.555
                           1.395 -1.115
                                            0.297
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 5.041 on 8 degrees of freedom
## Multiple R-squared: 0.1345, Adjusted R-squared: 0.02627
## F-statistic: 1.243 on 1 and 8 DF, p-value: 0.2973
```

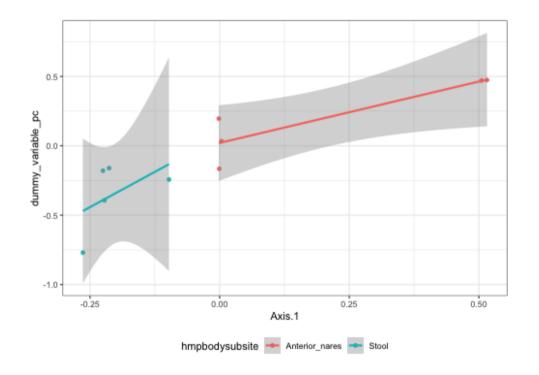


```
shannon_summary %>%
    Im(formula = dummy_variable_shannon ~ shannon,
        data = .) %>%
summary()
```

```
##
## Call:
## lm(formula = dummy_variable_shannon ~ shannon, data = .)
##
## Residuals:
       Min
                10 Median
                                         Max
## -0.34993 -0.18375 0.01602 0.14710 0.34814
## Coefficients:
              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -0.20539
                         0.30165 -0.681
                                           0.515
## shannon
              1.03619
                         0.06678 15.515 2.96e-07 ***
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 0.2413 on 8 degrees of freedom
## Multiple R-squared: 0.9678, Adjusted R-squared: 0.9638
## F-statistic: 240.7 on 1 and 8 DF, p-value: 2.965e-07
```

```
##
## Call:
## lm(formula = dummy_variable_shannon ~ shannon + hmpbodysubsite.
      data = .)
##
##
## Residuals:
      Min
               10 Median
                                     Max
## -0.3688 -0.1690 0.0134 0.1674 0.3323
## Coefficients:
                      Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                      -0.20055
                                 0.32224 -0.622
                                                    0.553
## shannon
                      1.03088
                                 0.07540 13.672 2.64e-06 ***
## hmpbodysubsiteStool 0.03673
                                 0.17233
                                           0.213
                                                    0.837
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.2572 on 7 degrees of freedom
## Multiple R-squared: 0.968, Adjusted R-squared: 0.9589
## F-statistic: 106 on 2 and 7 DF, p-value: 5.834e-06
```

```
## # A tibble: 10 x 8
                           Axis.2 shannon hmpbodysubsite dummy_varial
      specimen_id
                   Axis.1
      <chr>>
                    <dbl>
                            <dbl>
                                    <dbl> <chr>
                 -2.64e-1
   1 700014718
                           0.222
                                     5.58 Stool
   2 700014767
                  5.16e-1 0.174
                                     2.14 Anterior nares
   3 700014923
                 5.05e-1
                           0.183
                                     2.32 Anterior nares
   4 700016920
                  3.50e-3 -0.434
                                     4.96 Anterior nares
   5 700023706
                 -8.28e-4 -0.290
                                     4.98 Anterior nares
                 -1.54e-3 -0.435
   6 700038343
                                     5.56 Anterior nares
   7 700095956
                 -2.22e-1 0.189
                                     4.86 Stool
                 -2.25e-1
                           0.174
                                     4.58 Stool
   8 700105834
   9 700107189
                 -9.78e-2 0.0363
                                     4.27 Stool
## 10 700109383
                 -2.13e-1 0.180
                                     4.44 Stool
## # ... with 2 more variables: dummy_variable_shannon <dbl>,
      dummy_variable_pc <dbl>
```

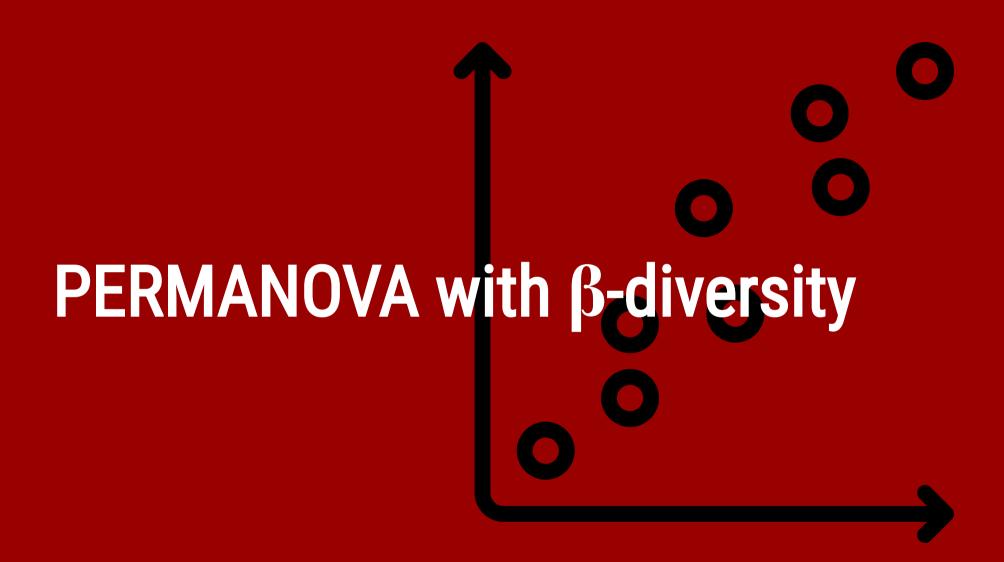


```
pc_summary %>%
    lm(formula = dummy_variable_pc ~ Axis.1,
        data = .) %>%
    summary()
```

```
##
## Call:
## lm(formula = dummy_variable_pc ~ Axis.1, data = .)
##
## Residuals:
       Min
                 10 Median
                                          Max
## -0.38811 -0.05846 -0.04998 0.14369 0.27116
## Coefficients:
              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -0.07387
                         0.06180 -1.195 0.266241
## Axis.1
               1.16777
                         0.22633
                                   5.160 0.000864 ***
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 0.1954 on 8 degrees of freedom
## Multiple R-squared: 0.7689, Adjusted R-squared:
## F-statistic: 26.62 on 1 and 8 DF, p-value: 0.0008641
```

```
pc_summary %>%
    Im(formula = dummy_variable_pc ~ Axis.2,
        data = .) %>%
    summary()
```

```
##
## Call:
## lm(formula = dummy_variable_pc ~ Axis.2, data = .)
##
## Residuals:
       Min
                 10 Median
                                          Max
## -0.63113 -0.17200 -0.04452 0.10175 0.59896
## Coefficients:
              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -0.07387
                         0.12574 -0.587
                                            0.573
## Axis.2
              -0.29223
                         0.48408 -0.604
                                            0.563
## Residual standard error: 0.3976 on 8 degrees of freedom
## Multiple R-squared: 0.04357, Adjusted R-squared: -0.07598
## F-statistic: 0.3644 on 1 and 8 DF, p-value: 0.5628
```



```
otu_tab %>%
  t() %>% # TRANSPOSE
  vegdist(x = ., method = "jaccard") -> otu_dist
otu_dist %>%
  str(vec.len = 2)
```

```
## 'dist' num [1:45] 1 0.996 ...
## - attr(*, "Size")= int 10
## - attr(*, "Labels")= chr [1:10] "700014718" "700014767" ...
## - attr(*, "Diag")= logi FALSE
## - attr(*, "Upper")= logi FALSE
## - attr(*, "method")= chr "jaccard"
## - attr(*, "call")= language vegdist(x = ., method = "jaccard")
```

```
labels(otu_dist) %>% #match order from dist
  enframe(value = "specimen_id") %>%
  select(specimen_id) %>%
  left_join(pc_summary, by = "specimen_id") %>%
  mutate(dummy_category = Axis.1 > mean(Axis.1)) %>%
  distinct() -> sorted_summary
sorted_summary
```

```
## # A tibble: 10 x 9
                           Axis.2 shannon hmpbodysubsite dummy_varial
      specimen_id
                   Axis.1
      <chr>>
                     <dbl>
                            <dbl>
                                     <dbl> <chr>
                  -2.64e-1
   1 700014718
                           0.222
                                     5.58 Stool
   2 700014767
                  5.16e-1
                           0.174
                                     2.14 Anterior nares
   3 700014923
                  5.05e-1
                           0.183
                                     2.32 Anterior nares
   4 700016920
                  3.50e-3 -0.434
                                     4.96 Anterior nares
   5 700023706
                  -8.28e-4 -0.290
                                     4.98 Anterior nares
                  -1.54e-3 -0.435
   6 700038343
                                     5.56 Anterior nares
   7 700095956
                  -2.22e-1 0.189
                                     4.86 Stool
                  -2.25e-1
                                     4.58 Stool
   8 700105834
                           0.174
   9 700107189
                  -9.78e-2 0.0363
                                     4.27 Stool
## 10 700109383
                 -2.13e-1 0.180
                                     4.44 Stool
## # ... with 3 more variables: dummy_variable_shannon <dbl>,
      dummy_variable_pc <dbl>, dummy_category <lql>
```

```
##
## Call:
## adonis(formula = otu_dist ~ hmpbodysubsite, data = sorted_summary)
##
## Permutation: free
## Number of permutations: 999
##
## Terms added sequentially (first to last)
##
                Df SumsOfSqs MeanSqs F.Model R2 Pr(>F)
## hmpbodysubsite 1 0.7036 0.70364 1.5719 0.16422 0.014 *
## Residuals
                 8 3.5811 0.44764
                                          0.83578
## Total
            9 4.2847
                               1.00000
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
```

# multivariable possible...

```
adonis(otu_dist ~ hmpbodysubsite + dummy_category.
       data = sorted_summary)
##
## Call:
## adonis(formula = otu_dist ~ hmpbodysubsite + dummy_category, data = sorted_summary)
##
## Permutation: free
## Number of permutations: 999
##
## Terms added sequentially (first to last)
##
                Df SumsOfSqs MeanSqs F.Model
                                               R2 Pr(>F)
## hmpbodysubsite 1
                      0.7036 0.70364 1.6100 0.16422 0.002 **
## dummy_category 1 0.5218 0.52180 1.1939 0.12178 0.125
                7 3.0593 0.43704
## Residuals
                                           0.71400
## Total
           9 4.2847
                                1.00000
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
```

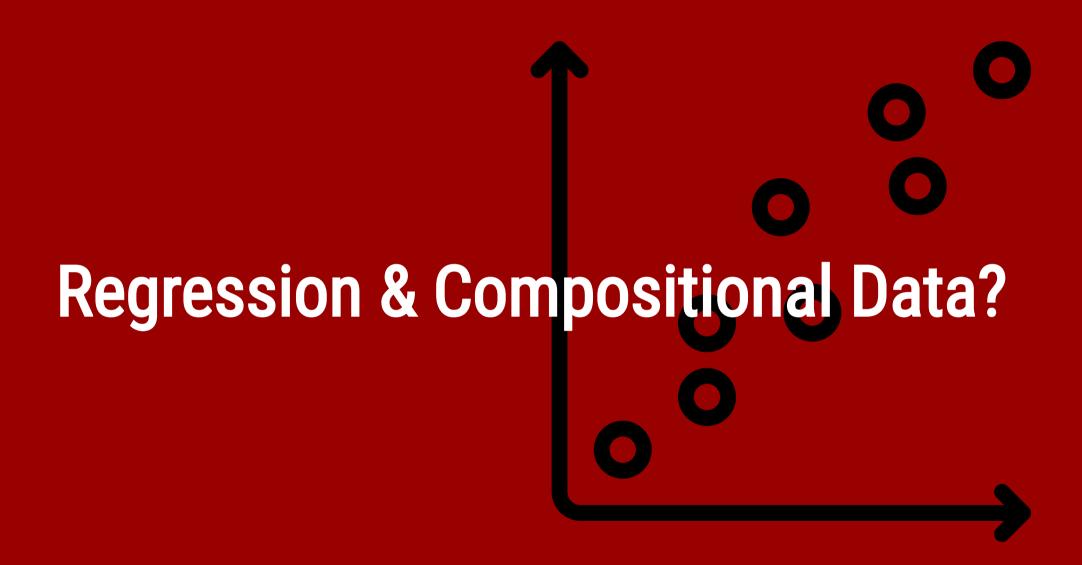
# ... but order matters!!!

```
adonis(otu_dist ~ dummy_category + hmpbodysubsite,
       data = sorted_summary)
##
## Call:
## adonis(formula = otu_dist ~ dummy_category + hmpbodysubsite, data = sorted_summary)
##
## Permutation: free
## Number of permutations: 999
##
## Terms added sequentially (first to last)
##
                Df SumsOfSqs MeanSqs F.Model
                                                R2 Pr(>F)
                      0.6329 0.63293 1.4482 0.14772 0.018 *
## dummy_category 1
## hmpbodysubsite 1 0.5925 0.59251 1.3557 0.13828 0.036 *
                7 3.0593 0.43704
## Residuals
                                           0.71400
## Total
           9 4.2847
                                    1.00000
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
```

```
##
## Call:
## adonis(formula = otu_dist ~ dummy_category, data = sorted_summary,
                                                                        strata = sorted_summary$hmpbodysubsite)
##
## Blocks: strata
## Permutation: free
## Number of permutations: 999
##
## Terms added sequentially (first to last)
##
                 Df SumsOfSqs MeanSqs F.Model
                                                  R2 Pr(>F)
## dummy_category 1
                       0.6329 0.63293 1.3865 0.14772 0.291
## Residuals
                  8 3.6518 0.45648
                                             0.85228
## Total
                  9 4.2847
                                            1.00000
```

```
adonis(otu_dist ~ dummy_category / hmpbodysubsite.
       data = sorted_summary)
##
## Call:
## adonis(formula = otu_dist ~ dummy_category/hmpbodysubsite, data = sorted_summary)
##
## Permutation: free
## Number of permutations: 999
##
## Terms added sequentially (first to last)
##
                               Df SumsOfSqs MeanSqs F.Model R2 Pr(>F)
                                    0.6329 0.63293 1.4482 0.14772 0.009 **
## dummy_category
## dummy_category:hmpbodysubsite 1 0.5925 0.59251 1.3557 0.13828 0.023 *
## Residuals
                                7 3.0593 0.43704
                                                          0.71400
## Total
                                9 4.2847
                                                    1.00000
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
```

# ... or do you mean nestedness?



#### Regression & Compositional Data?

Compositional data approaches correct OTU dependency:

```
o e.g., compositions::clr() or philr::philr()
```

- p >> n challenges persist
- Must pair compositional transform with regularization:
  - glmnet::glmnet for LASSO/ridge/elastic net
  - Bayesian methods



# Thank you!

Slides available: github.com/bjklab

brendank@pennmedicine.upenn.edu

