Generation of decay of SM particles into millicharged particles

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1 Introduction

In this document we discuss the MC generation of decays and the calculation of branching ratios for processes of the type $A \to \zeta^+\zeta^- X$ where A is a SM particle, ζ is a millicharged particle and X are other SM particles. We start from $A \to e^+e^- X$, calculate the branching ratio for the decay into ζ's (if kinematically allowed) and then discussed the methods to generate the actual decays.

We do not consider $Z \to \zeta\zeta$ since we expect that the Drell Yan process should be dealt separately through an external MG model. This is because the coupling of the ζ to the Z are not given by a simple rescaling of the SM Zee vertex by Q^2, where Q is the charge of the ζ in units of e.

The generation of the SM A particles is not discussed here.

2 Processes

The main processes for $A \to e^+e^- X$ at the LHC are:

- $\pi^0 \to e^+e^-\gamma$ (BR=1.17%, Dalitz decay)
- $\eta \to e^+e^-\gamma$ (BR=0.7%, Dalitz decay)
- $\eta' \to e^+e^-\gamma$ (BR=5e-4, Dalitz decay)
- $\omega \to \pi^0 e^+e^-$ (BR=8e-4, Dalitz decay)

- $\eta' \to \omega e^+e^-$ (BR=2e-4, Dalitz Decay)
- $\eta' \to \pi^+\pi^-e^+e^-\gamma$ (BR=2e-3, sort of Dalitz, skip for now)
- $V \to e^+e^-$ (V = onia, ϕ, ρ, ω)

Note that if we are only interested on ζ masses above 100 MeV, Dalitz decays of π^0 and decays of η' into ω do not contribute. In all cases the BR for $A \to \zeta^+\zeta^-X$ can be obtained by rescaling $A \to e^+e^-X$ by a factor of Q^2 times an additional mass-dependent factor. In general this factor consistes of a phase space piece $\sqrt{1-(2m_\zeta/m_A)^2}$ plus an additional piece that arises from the matrix element.

3 Branching Ratios

3.1 Dalitz BR

The partial width for $A \to e^+e^-\gamma$ can be written as[1, 2]:

$$\frac{d\Gamma}{dq^2} = \frac{2\alpha}{3\pi q^2} \;\; (1+\frac{2m_e^2}{q^2}) \;\; \sqrt{1-\frac{4m_e^2}{q^2}} \;\; (1-\frac{q^2}{m_A^2})^3 \;\; |F(q^2)|^2 \;\; \Gamma(A \to \gamma\gamma) \quad (1)$$

where m_A is the mass of A, q^2 is the mass-squared of the e^+e^- pair, and $F(q^2)$ is a form factor. This form factor is such that $F(0) = 1$ and for pions is usually parametrized near $q^2 = 0$ as $F(q^2) = 1 + a\frac{q^2}{m_\pi^2}$ with $a \approx 0.03$. The form factor can also be estimated in the Vector Dominance Model (VDM) as

$$|F(q^2)|^2 = \frac{m_\rho^4 + m_\rho^2\Gamma_\rho^2}{(m_\rho^2 - q^2)^2 + m_\rho^2\Gamma_\rho^2} \quad (2)$$

where m_ρ and Γ_ρ are the mass and width of the ρ meson. The VDM model assumes that the decay proceeds through $\pi^0 \to \gamma V^*, V^* \to e^+e^-$ and $V = \rho$ or ω; equation 2 neglects the difference between ρ and ω.

In the case of $A \to e^+e^-X$, when X is not γ, the partial width can be written as

$$\begin{aligned} \frac{d\Gamma}{dq^2} = & \frac{\alpha}{3\pi q^2} \; (1+\frac{2m_e^2}{q^2}) \; \sqrt{1-\frac{4m_e^2}{q^2}} \; \cdot \\ & \left[(1+\frac{q^2}{m_A^2-m_X^2})^2 - \frac{4m_A^2 q^2}{(m_A^2-m_X^2)^2}\right]^{3/2} \quad |F_{AX}(q^2)|^2 \;\; \Gamma(A \to X\gamma) \end{aligned} \tag{3}$$

where m_X is the mass of X, and the transition form factor F_{AX} can also be approximated as in equation 2.

For millicharged particles, Dalitz decays branching ratios can be obtained by integrating equation 1 or 3 from $q^2 = 4m_\zeta^2$ to the kinematical limit, substituting m_ζ for m_e, and rescaling by Q^2. Some numerical results, for $Q = 1$ are given in Table 1. Note the sharp drop in branching ratios with mass, especially for the π^0. The calculations with the electron and muon masses are in good agreeemnt with the PDG.

3.2 Vector meson branching ratios

At lowest order the SM decay rate for $V \to \ell\ell$ is given by the Van Royen-Weisskopf formula[3, 4]:

$$\Gamma(V \to \ell\ell) = 4\pi\alpha^2 \frac{f_V^2}{m_V} Q_q^2 (1-4x_\ell^2)^{1/2}(1+2x_\ell^2) \tag{4}$$

where f_V is the vector decay constant, m_V is the vector mass, Q_q is the charge of the quark that makes up the meson, $x_\ell = m_\ell/m_V$, and m_ℓ is the lepton mass.
Thus the ratio of BR for $V \to \zeta\zeta$ to $V \to ee$ is given by

$$\frac{\Gamma(V \to \zeta\zeta)}{\Gamma(V \to ee)} = Q^2 \frac{(1-4x_\zeta^2)^{1/2}(1+2x_\zeta^2)}{(1-4x_\ell^2)^{1/2}(1+2x_\ell^2)} \tag{5}$$

where $x_\zeta = m_\ell/m_V$, and m_ζ is the mass of ζ.

As a sanity check, we use equation 5 and BR($\psi(2S) \to ee$)=7.93e-3 to predict BR($\psi(2S) \to \tau\tau$) = 3.1e-3, in agreement with the PDG value of (3.1 $\pm$ 0.4)e-3.

m_ζ (MeV)	$\pi^0 \to \zeta\zeta\gamma$	$\eta \to \zeta\zeta\gamma$	$\eta' \to \zeta\zeta\gamma$	$\eta' \to \zeta\zeta\omega$	$\omega \to \zeta\zeta\pi^0$
0.511 (=m_e)	1.17 e-2	6.6 e-3	4.6 e-4	1.8 e-4	7.6 e-4
PDG for ee	(1.17 ± 0.04)e-2	(6.9 ± 0.4)e-4		(2.0 ± 0.4)e-4	(7.7 ± 0.6)e-4
10	2.8 e-3	2.9 e-3	2.5 e-4	5.7 e-5	3.7 e-4
30	3.5 e-4	1.6 e-3	1.8 e-4	1.7 e-5	2.3 e-4
50	1.2 e-5	1.0 e-3	1.4 e-4	4.3 e-6	1.6 e-4
60	2.7 e-7	8.2 e-4	1.3 e-4	1.7 e-6	1.4 e-4
90		4.3 e-4	1.0 e-4		9.2 e-5
105.7 (=m_μ)		3.0 e-4	9.2 e-5		7.4 e-5
PDG for $\mu\mu$		(3.1 ± 0.4) e-4	(1.1 ± 0.3) e-4		(1.3 ± 0.2) e-4
150		8.9 e-5	6.8 e-5		3.7 e-5
200		1.2 e-5	4.8 e-5		1.5 e-5
250		1.0 e-7	3.2 e-5		3.6 e-6
400			5.6 e-7		

Table 1: Branching ratios for different Dalitz decay modes as a function of m_ζ for $Q = 1$ calculated based on equations 1, 2, and 3. When possible we compare with the values from the 2019 PDG.

4 Generation of the decays

We provide functions that take as input the lab frame 4-vector of either pseudoscalar (P) or vector (V) meson, and return the 4 vectors of the two ζ's from the decay. We assume that the V's are unpolarized.

4.1 Generation of Dalitz decays

The implementation goes as follows (this should also work for the Dalitz decay of the ω as long as the ω is unpolarized):

- Rotate the 4-vector of P from the lab frame into frame S_1 such that P is traveling in the z-direction.
- Boost along z into frame S_2 where P is at rest.
- Pick a q^2 according to equation 1.
- Generate a decay $P \to X\gamma^*$ where γ^* is a particle of $m^2 = q^2$. The γ^* direction is random in ϕ and random in $\cos\theta$.

- Rotate the γ^* 4-vector into a frame S_3 such that the γ^* is traveling in the z-direction.
- Boost along z into frame S_4 where γ^* is at rest.
- Generate a decay $\gamma^* \to \zeta^+\zeta^-$ such that the angle ϕ of the ζ^+ is random and $\cos\theta$ is picked according to [5]

$$\frac{dN}{d\cos\theta} = 1 + \cos^2\theta + \frac{4m_\zeta^2}{q^2}\sin^2\theta \tag{6}$$

- Set the 3-vector of the ζ^- to be back-to-back with the ζ^+.
- Boost the 4-vectors of the ζ's from S_4 to S_3.
- Rotate the 4-vectors of the ζ's from S_3 to S_2.
- Boost the 4-vectors of the ζ's from S_2 to S_1.
- Rotate the 4-vectors of the ζ's from S_1 to the lab frame.

4.2 Generation of vector decays

The procedure is the following:

- Rotate the 4-vector of V from the lab frame into frame S_1 such that V is traveling in the z-direction.
- Boost along z into frame S_2 where V is at rest.
- Generate a decay $V \to \zeta^+\zeta^-$ such that both the angle ϕ and the $\cos\theta$ of the ζ^+ are random.
- Set the 3-vector of the ζ^- to be back-to-back with the ζ^+.
- Boost the 4-vectors of the ζ's from S_2 to S_1.
- Rotate the 4-vectors of the ζ's from S_1 to the lab frame.

5 Code

Code to calculate the branching ratios and to generate the decays can be found at https://github.com/bjmarsh/milliq_mcgen

References

[1] L. G. Landsberg, Phys. Rep. 128, 301 (1985).

[2] See for example http://cds.cern.ch/record/683210/files/soft-96-032.pdf.

[3] Aloni, D., Efrati, A., Grossman, Y. et al. J. High Energ. Phys. (2017) 2017: 19.

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[5] P. Adlarson et al., Phys. Rev. C 95, 025202 (2007).