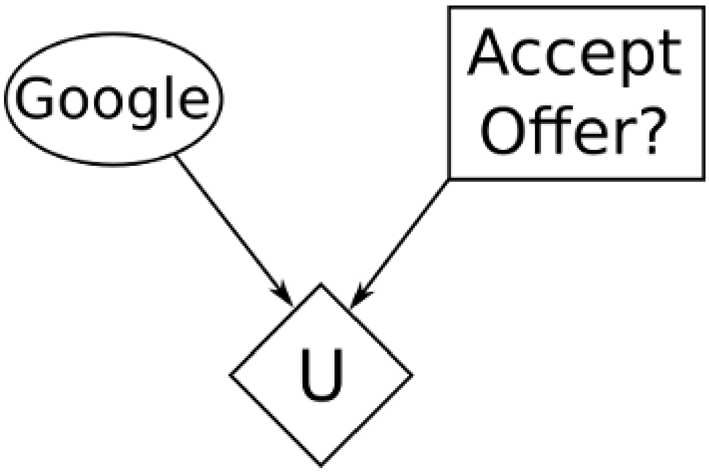
# **Q1** Decisions

15 Points

You've been job hunting, and you've narrowed your options to two companies: Acme and Google. You already have an offer from Acme, but it expires today, and you are still waiting for a response from Google. You are faced with the dilemma of whether or not to accept the offer from Acme, which is modeled by the following decision diagram:




Your prior belief about whether Google will hire you and utility over possible outcomes are as follows:

| Google outcome | P(Google outcome) |
|---|---|
| hired | 0.25 |
| not hired | 0.75 |

| Action | Google outcome | U |
|---|---|---|
| accept Acme offer | hired | 2000 |
| accept Acme offer | not hired | 8000 |
| reject Acme offer | hired | 10000 |
| reject Acme offer | not hired | 0 |

## **Q1.1**

5 Points

What is the expected utility of each action? (Note: throughout this problem answers will be evaluated to whole-number precision, so your answer should differ by no more than 1 from the exact answer.)

Action: accept Acme offer

6500

Action: reject Acme offer

2500

Which action should you take?

- ○ reject
- ◉ accept

**EXPLANATION**

EU(accept) = P(hired)U(accept, hired) + P(not hired)U(accept, not hired) = 6500

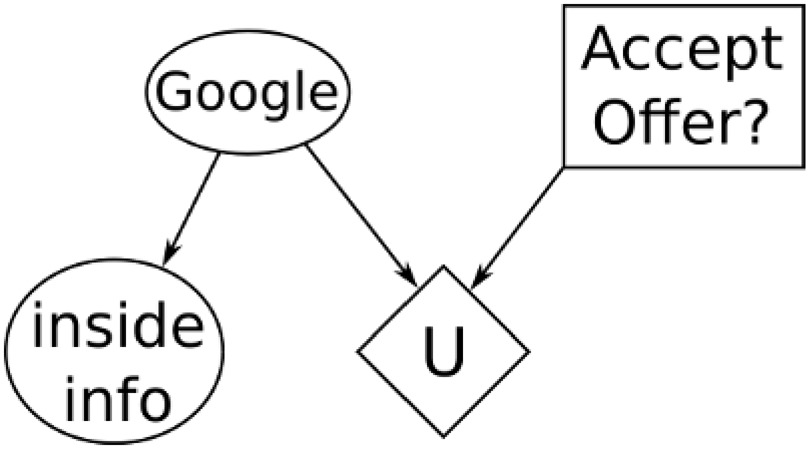
EU(decline) = P(hired)U(decline, hired) + P(not hired)U(decline, not hired) = 2500

You should take the "accept" action, because it gives more expected utility.

## Q1.2

10 Points

Suddenly, the phone rings. It's your uncle, who works at Google. Your uncle tells you he has some inside information about the status of your application. Your uncle won't tell you what the information is yet, but he might be willing to divulge it for the right price. You model the new situation by adding a new node to your decision diagram:




You create a CPT to model the relationship between the inside information and Google's future hiring decision:

| info | Google outcome | P(info \| Google outcome) |
|---|---|---|
| good news | hired | 0.7 |
| bad news | hired | 0.3 |
| good news | not hired | 0.1 |
| bad news | not hired | 0.9 |

We'll help grind through the probabilistic inference. The resulting distributions are:

| info | P(info) |
|---|---|
| good news | 0.25 |
| bad news | 0.75 |

| Google outcome | info | P(Google outcome \| info) |
|---|---|---|
| hired | good news | 0.7 |
| not hired | good news | 0.3 |
| hired | bad news | 0.1 |
| not hired | bad news | 0.9 |

Fill in the expected utilities for each action, for each possible type of information we could be given:

EU(accept Acme offer | good news)

3800

EU(reject Acme offer | good news)

7000

EU(accept Acme offer | bad news)

7400

EU(reject Acme offer | bad news)

1000

What is the maximum expected utility for each type of information we could be given?

MEU(good news)

7000

MEU(bad news)

7400

If we are given the inside information, what is the expected value of MEU?

7300

What is the value of perfect information of the random variable Inside Info?

800

## **Q2** Value of Perfect Information

10 Points

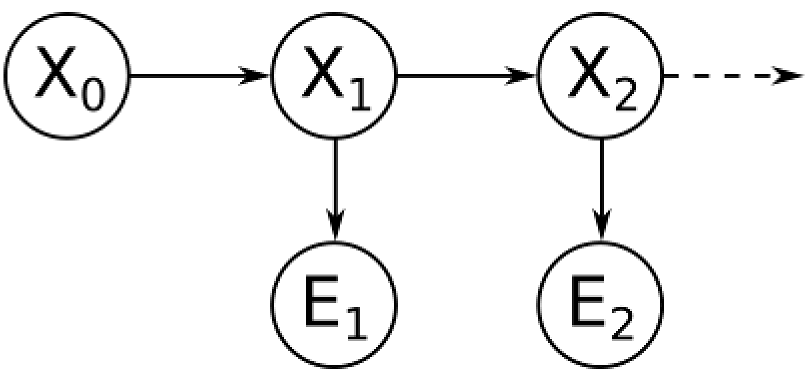
Consider the value of perfect information (VPI) of observing some node in an arbitrary decision network. Which of the following are true statements?

- [ ] VPI is guaranteed to be positive ($> 0$).
- [x] VPI is guaranteed to be nonnegative ($\geq 0$).
- [ ] VPI is guaranteed to be nonzero.
- [x] The MEU after observing a node could potentially be less than the MEU before observing that node.
- [ ] For any two nodes $X$ and $Y$, $\mathrm{VPI}(X) + \mathrm{VPI}(Y) \geq \mathrm{VPI}(X, Y)$. That is, the sum of individual VPI's for two nodes is always greater than or equal to the VPI of observing both nodes.
- [x] VPI is guaranteed to be exactly zero for any node that is conditionally independent (given the evidence so far) of all parents of the utility node.

## **Q3** HMMs, Part I

15 Points

Consider the HMM shown below.




The prior probability $P(X_0)$, dynamics model $P(X_{t+1} \mid X_t)$, and sensor model $P(E_t \mid X_t)$ are as follows:

| $X_0$ | $P(X_0)$ |
|---|---|
| 0 | 0.15 |
| 1 | 0.85 |

| $X_{t+1}$ | $X_t$ | $P(X_{t+1}\|X_t)$ |
|---|---|---|
| 0 | 0 | 0.6 |
| 1 | 0 | 0.4 |
| 0 | 1 | 0.9 |
| 1 | 1 | 0.1 |

| $E_t$ | $X_t$ | $P(E_t\|X_t)$ |
|---|---|---|
| a | 0 | 0.8 |
| b | 0 | 0.15 |
| c | 0 | 0.05 |
| a | 1 | 0.35 |
| b | 1 | 0.05 |
| c | 1 | 0.6 |

We perform a first dynamics update, and fill in the resulting belief distribution $B'(X_1)$.

| $X_1$ | $B'(X_1)$ |
|---|---|
| 0 | 0.855 |
| 1 | 0.145 |

We incorporate the evidence $E_1 = c$. We fill in the evidence-weighted distribution $P(E_1 = c \mid X_1)B'(X_1)$, and the (normalized) belief distribution $B(X_1)$.

| $X_1$ | $P(E_1 = c\|X_1)B'(X_1)$ |
|---|---|
| 0 | 0.04275 |
| 1 | 0.087 |

| $X_1$ | $B(X_1)$ |
|---|---|
| 0 | 0.329479768786 |
| 1 | 0.670520231214 |

You get to perform the second dynamics update. Fill in the resulting belief distribution $B'(X_2)$.

$B'(X_2 = 0)$

.801156

$B'(X_2 = 1)$

.198844

Now incorporate the evidence $E_2 = c$. Fill in the evidence-weighted distribution $P(E_2 = c \mid X_2)B'(X_2)$, and the (normalized) belief distribution $B(X_2)$.

$P(E_2 = c \mid X_2)B'(X_2)$ when $X_2 = 0$

.0400578

$P(E_2 = c \mid X_2)B'(X_2)$ when $X_2 = 1$

.1193064

$B(X_2 = 0)$

.25136009

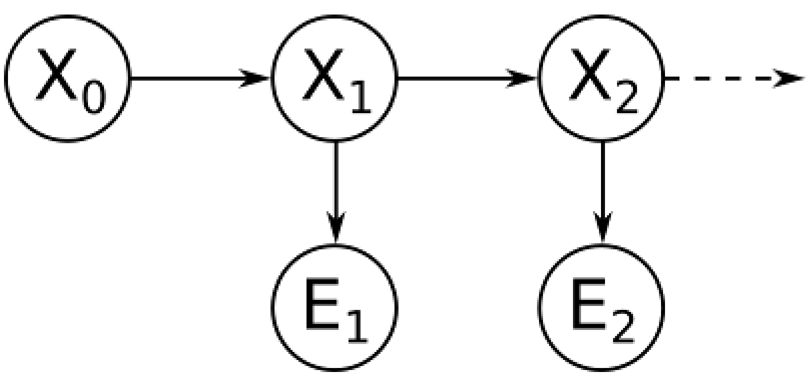
$B(X_2 = 1)$

.74863991

## **Q4** HMMs, Part II

15 Points

Consider the same HMM (but with different probabilities).




The prior probability $P(X_0)$, dynamics model $P(X_{t+1} \mid X_t)$, and sensor model $P(E_t \mid X_t)$ are as follows:

| $X_0$ | $P(X_0)$ |
|---|---|
| 0 | 0.2 |
| 1 | 0.8 |

| $X_{t+1}$ | $X_t$ | $P(X_{t+1}\|X_t)$ |
|---|---|---|
| 0 | 0 | 0.3 |
| 1 | 0 | 0.7 |
| 0 | 1 | 0.05 |
| 1 | 1 | 0.95 |

| $E_t$ | $X_t$ | $P(E_t\|X_t)$ |
|---|---|---|
| a | 0 | 0.3 |
| b | 0 | 0.15 |
| c | 0 | 0.55 |
| a | 1 | 0.1 |
| b | 1 | 0.45 |
| c | 1 | 0.45 |

In this question we'll assume the sensor is broken and we get no more evidence readings after $E_2$. We are forced to rely on dynamics updates only going forward. In the limit as $t \to \infty$, our belief about $X_t$ should converge to a stationary distribution $\tilde{B}(X_\infty)$ defined as follows:

$$\tilde{B}(X_\infty) := \lim_{t\to\infty} P(X_t \mid E_1, E_2)$$

## Q4.1

10 Points

Recall that the stationary distribution satisfies the equation

$$\tilde{B}(X_\infty) = \sum_{X_\infty} P(X_{t+1} \mid X_t)\tilde{B}(X_\infty)$$

for all values in the domain of $X$.

In the case of this problem, we can write these relations as a set of linear equations of the form

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} \tilde{B}(X_\infty = 0) \\ \tilde{B}(X_\infty = 1) \end{bmatrix} = \begin{bmatrix} \tilde{B}(X_\infty = 0) \\ \tilde{B}(X_\infty = 1) \end{bmatrix}$$

In the spaces below, fill in the coefficients of the linear system.

**a**

0.3

**b**

0.05

**c**

0.7

**d**

0.95

### Q4.2

5 Points

The system you have written has many solutions (consider (0,0), for example), but to get a probability distribution we want the solution that sums to one. Fill in your solution below.

$\tilde{B}(X_\infty = 0)$

0.067

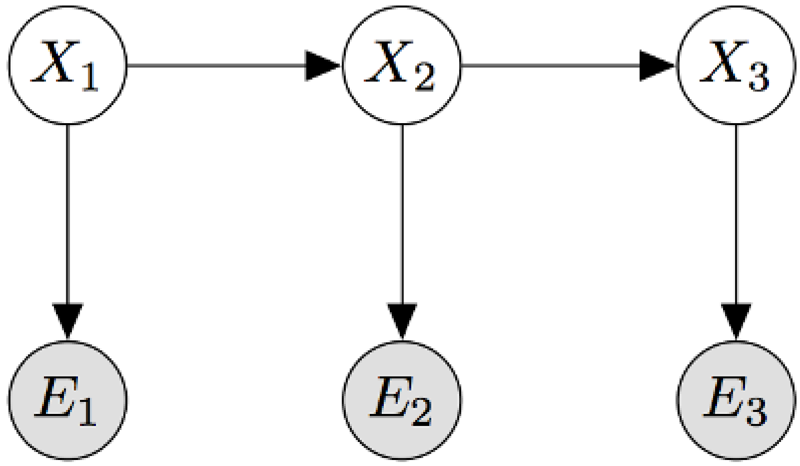
$\tilde{B}(X_\infty = 1)$

.933

## **Q5** Modified HMM Update Equations

15 Points

Consider the HMM graph structure shown below.




Recall the Forward algorithm is a two step iterative algorithm used to approximate the probability distribution

$P(X_t \mid e_1, ..., e_t)$.

The two steps of the algorithm are as follows:

Elapse Time:

$P(X_t \mid e_{1...t-1}) = \sum_{x_{t-1}} P(X_t \mid x_{t-1}) P(x_{t-1} \mid e_{1...t-1})$

Observe:

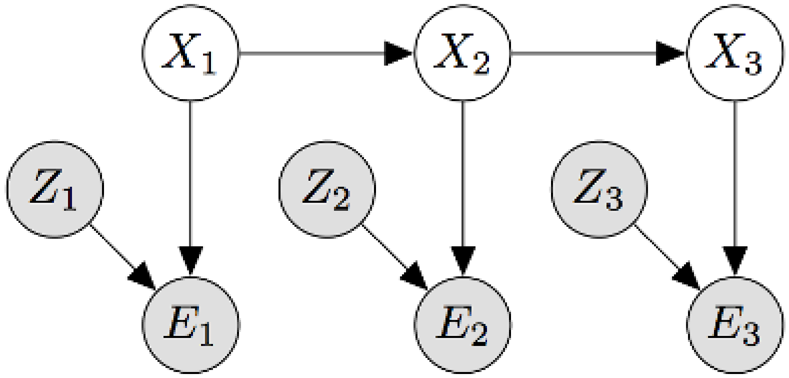
$P(X_t \mid e_{1...t}) = \frac{P(e_t|X_t)P(X_t|e_{1...t-1})}{\sum_{x_t} P(e_t|x_t)P(x_t|e_{1...t-1})}$

For this problem we will consider modifying the forward algorithm as the HMM graph structure changes. Our goal will continue to be to create an iterative algorithm which is able to compute the distribution of states, $X_t$, given all available evidence from time 0 to time $t$.

## Q5.1

5 Points

Consider the graph below where new observed variables, $Z_i$, are introduced and influence the evidence.




What will the modified elapse time update be?

$P(X_t \mid e_{1\ldots t-1}, z_{1\ldots t-1})$ =

- ○ $\sum_{x_{t-1}} P(X_t \mid z_{1\ldots t-1})P(x_{t-1} \mid e_{1\ldots t-1}, z_{1\ldots t-1})$
- ◉ $\sum_{x_{t-1}} P(X_t \mid x_{t-1})P(x_{t-1} \mid e_{1\ldots t-1}, z_{1\ldots t-1})$
- ○ $\sum_{x_{t-1}} P(X_t \mid e_{1\ldots t-1}, z_{1\ldots t-1})P(x_{t-1} \mid x_{t-1}, z_{1\ldots t-1})$
- ○ $\sum_{x_{t-1}} P(X_t \mid x_{t-1})P(x_{t-1} \mid e_{1\ldots t-1})$ (no change)

What will the modified observed update be?

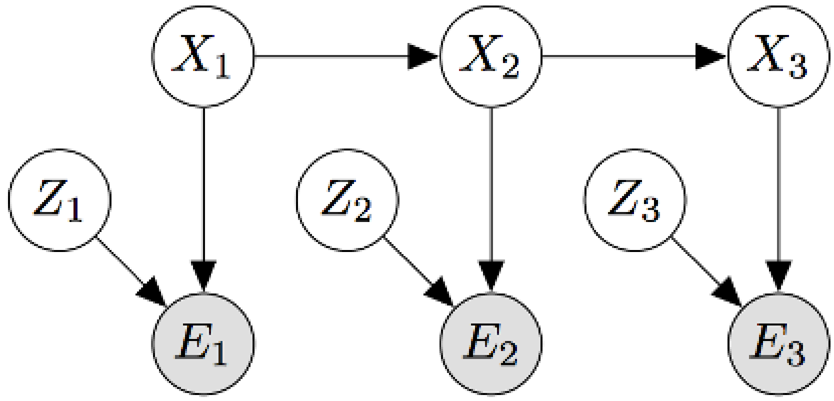
$P(X_t \mid e_{1\ldots t}, z_{1\ldots t})$ =

- ○ $\frac{P(e_t|X_t,z_t)P(X_t|e_{1\ldots t-1},z_{1\ldots t-1})}{\sum_{x_t,z_t} P(e_t|x_t,z_t)P(x_t|e_{1\ldots t-1},z_{1\ldots t-1})}$
- ◉ $\frac{P(e_t|X_t,z_t)P(X_t|e_{1\ldots t-1},z_{1\ldots t-1})}{\sum_{x_t} P(e_t|x_t,z_t)P(x_t|e_{1\ldots t-1},z_{1\ldots t-1})}$
- ○ $\frac{P(e_t|X_t)P(X_t|e_{1\ldots t-1})}{\sum_{z_t} P(e_t|x_t,z_t)P(x_t|e_{1\ldots t-1},z_{1\ldots t-1})}$
- ○ $\frac{P(e_t|X_t,z_t)P(X_t|e_{1\ldots t-1},z_{1\ldots t-1})}{\sum_{z_t} P(e_t|x_t,z_t)P(x_t|e_{1\ldots t-1},z_{1\ldots t-1})}$
- ○ $\frac{P(e_t|X_t)P(X_t|e_{1\ldots t-1})}{\sum_{x_t} P(e_t|x_t)P(x_t|e_{1\ldots t-1})}$ (no change)

## Q5.2

5 Points

Next, consider the graph below where the $Z_i$ variables are unobserved.




What will the modified elapse time update be?

$P(X_t \mid e_{1\dots t-1})$ =

- ○ $\sum_{x_{t-1}} P(X_t \mid z_{1\dots t-1}) P(x_{t-1} \mid e_{1\dots t-1}, z_{1\dots t-1})$
- ○ $\sum_{x_{t-1}} P(X_t \mid x_{t-1}) P(x_{t-1} \mid e_{1\dots t-1}, z_{1\dots t-1})$
- ○ $\sum_{x_{t-1}} P(X_t \mid e_{1\dots t-1}, z_{1\dots t-1}) P(x_{t-1} \mid x_{t-1}, z_{1\dots t-1})$
- ◉ $\sum_{x_{t-1}} P(X_t \mid x_{t-1}) P(x_{t-1} \mid e_{1\dots t-1})$ (no change)

What will the modified observed update be?

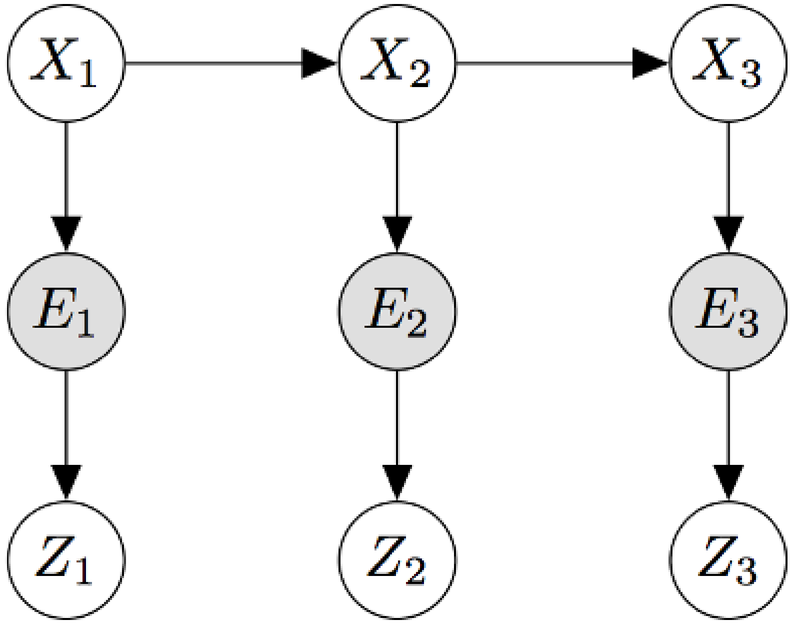
$P(X_t \mid e_{1\dots t})$ =

- ○ $\frac{P(X_t|e_{1\dots t-1})P(z_t)P(e_t|X_t,z_t)}{\sum_{z_t} P(x_t|e_{1\dots t-1})P(e_t|x_t,z_t)P(z_t)}$
- ○ $\frac{P(X_t|e_{1\dots t-1})\sum_{z_t} P(z_t)P(e_t|X_t,z_t)}{P(x_t|e_{1\dots t-1})\sum_{z_t} P(e_t|x_t,z_t)P(z_t)}$
- ○ $\frac{P(X_t|e_{1\dots t-1})P(z_t)P(e_t|X_t,z_t)}{\sum_{x_t} P(x_t|e_{1\dots t-1})P(e_t|x_t,z_t)P(z_t)}$
- ◉ $\frac{P(X_t|e_{1\dots t-1})\sum_{z_t} P(z_t)P(e_t|X_t,z_t)}{\sum_{x_t} P(x_t|e_{1\dots t-1})\sum_{z_t} P(e_t|x_t,z_t)P(z_t)}$
- ○ $\frac{P(e_t|X_t)P(X_t|e_{1\dots t-1})}{\sum_{x_t} P(e_t|x_t)P(x_t|e_{1\dots t-1})}$ (no change)

## Q5.3

5 Points

Finally, consider a graph where the newly introduced variables are unobserved and influenced by the evidence nodes.




What will the modified elapse time update be?

$P(X_t \mid e_{1 \ldots t-1})$ =

- ○ $\sum_{x_{t-1}} P(X_t \mid z_{1 \ldots t-1}) P(x_{t-1} \mid e_{1 \ldots t-1}, z_{1 \ldots t-1})$
- ○ $\sum_{x_{t-1}} P(X_t \mid x_{t-1}) P(x_{t-1} \mid e_{1 \ldots t-1}, z_{1 \ldots t-1})$
- ○ $\sum_{x_{t-1}} P(X_t \mid e_{1 \ldots t-1}, z_{1 \ldots t-1}) P(x_{t-1} \mid x_{t-1}, z_{1 \ldots t-1})$
- ⦿ $\sum_{x_{t-1}} P(X_t \mid x_{t-1}) P(x_{t-1} \mid e_{1 \ldots t-1})$ (no change)

What will the modified observed update be?

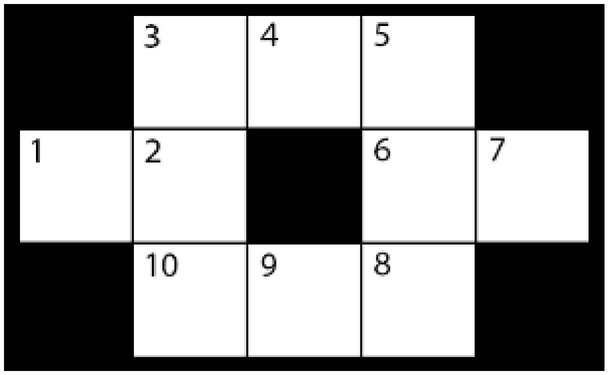
$P(X_t \mid e_{1 \ldots t})$ =

- ○ $\frac{P(e_t|X_t, z_t) P(X_t|e_{1 \ldots t-1}, z_{1 \ldots t-1})}{\sum_{z_t} P(e_t|x_t, z_t) P(x_t|e_{1 \ldots t-1}, z_{1 \ldots t-1})}$
- ○ $\frac{P(e_t|X_t, z_t) P(X_t|e_{1 \ldots t-1}, z_{1 \ldots t-1})}{\sum_{x_t} P(e_t|x_t, z_t) P(x_t|e_{1 \ldots t-1}, z_{1 \ldots t-1})}$
- ○ $\frac{P(e_t|X_t) P(X_t|e_{1 \ldots t-1})}{\sum_{x_t, z_t} P(e_t|x_t, z_t) P(x_t|e_{1 \ldots t-1}, z_{1 \ldots t-1})}$
- ⦿ $\frac{P(e_t|X_t) P(X_t|e_{1 \ldots t-1})}{\sum_{x_t} P(e_t|x_t) P(x_t|e_{1 \ldots t-1})}$ (no change)

## **Q6** Particle Filtering

20 Points

In this question, we will use a particle filter to track the state of a robot that is lost in the small map below:




The robot's state is represented by an integer $1 \leq X_t \leq 10$ corresponding to its location in the map at time $t$. We will approximate our belief over this state with $N = 8$ particles.

You have no control over the robot's actions. At each timestep, the robot either stays in place, or moves to any one of its neighboring locations, all with equal probability. For example, if the robot starts in state $X_t = 7$, it will move to state $X_{t+1} = 6$ with probability $\frac{1}{2}$ or $X_{t+1} = 7$ with probability $\frac{1}{2}$. Similarly, if the robot starts in state $X_t = 2$, the next state $X_{t+1}$ can be any element of $\{1, 2, 3, 10\}$, and each occurs with probability $\frac{1}{4}$.

At each time step, a sensor on the robot gives a reading $E_t \in \{H, C, T, D\}$ corresponding to the *type* of state the robot is in. The possible types are:

- **Hallway (H)** for states bordered by two parallel walls (4,9).
- **Corner (C)** for states bordered by two orthogonal walls (3,5,8,10).
- **Tee (T)** for states bordered by one wall (2,6).
- **Dead End (D)** for states bordered by three walls (1,7).

The sensor is not very reliable: it reports the correct type with probability $\frac{1}{2}$, but gives erroneous readings the rest of the time, with probability $\frac{1}{6}$ for each of the three other possible readings.

## **Q6.1** Sensor Model

3 Points

Fill in the sensor model below:

P( Sensor Reading = H | State Type = H )

1/2

P( Sensor Reading = C | State Type = H )

1/6

P( Sensor Reading = T | State Type = H )

1/6

P( Sensor Reading = D | State Type = H )

1/6

P( Sensor Reading = H | State Type = C )

1/6

P( Sensor Reading = C | State Type = C )

1/2

P( Sensor Reading = T | State Type = C )

1/6

P( Sensor Reading = D | State Type = C )

1/6

P( Sensor Reading = H | State Type = T )

1/6

P( Sensor Reading = C | State Type = T )

1/6

P( Sensor Reading = T | State Type = T )

1/2

P( Sensor Reading = D | State Type = T )

1/6

P( Sensor Reading = H | State Type = D )

1/6

P( Sensor Reading = C | State Type = D )

1/6

P( Sensor Reading = T | State Type = D )

1/6

P( Sensor Reading = D | State Type = D )

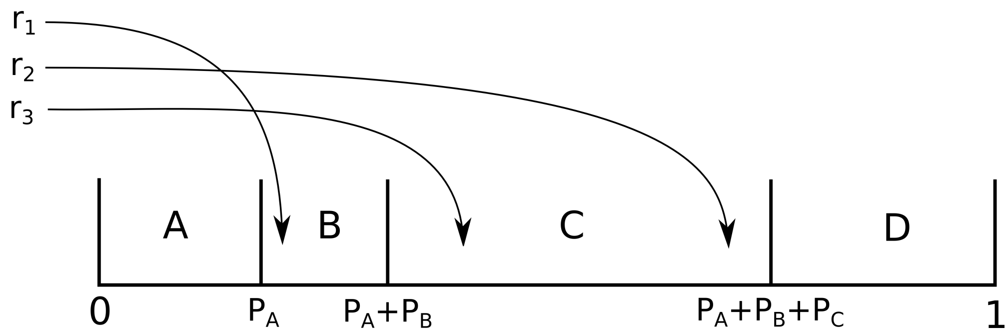
1/2

## **Q6.2** Sampling Review and Initial Belief State

3 Points

Suppose that we want to sample from a set of 4 events, $\{A, B, C, D\}$, which occur with corresponding probabilities $P_A, P_B, P_C, P_D$. First, we form the set of cumulative weights, given by
$\{0, P_A, P_A + P_B, P_A + P_B + P_C, 1\}$.
These weights partition the $[0, 1)$ interval into bins, as shown below. We then draw a number $r$ uniformly at random from $[0, 1)$ and pick $A, B, C,$ or $D$ based on which bin $r$ lands in. The process is illustrated in the diagram below. If $r_1$, uniformly chosen from $[0, 1)$,

lands in the interval $[P_A, P_A + P_B]$, then the resulting sample would be $B$. Similarly, if $r_2$ lands in $[P_A + P_B, P_A + P_B + P_C]$, the sample would be $C$, and $r_3$ landing in $[P_A + P_B, P_A + P_B + P_C]$ would also be $C$.




Now we will sample the starting positions for our particles at time $t = 0$. For each particle $p_i$, we have generated a random number $r_i$ sampled uniformly from $[0, 1)$. Your job is to use these numbers to sample a starting location for each particle. As a reminder, locations are integers from the range $[1, 10]$, as shown in the map. You should assume that the locations go in ascending order and that each location has equal probability. The random number generated for particle i, denoted by $r_i$, is provided. Please fill in the locations of the eight particles.

$r_1 = 0.914$

$p_1 =$

10

$r_2 = 0.473$

$p_2 =$

5

$r_3 = 0.679$

$p_3 =$

7

$r_4 = 0.879$

$p_4 =$

9

$r_5 = 0.212$

$p_5 =$

3

$r_6 = 0.024$

$p_6 =$

1

$r_7 = 0.458$

$p_7 =$

5

$r_8 = 0.154$

$p_8 =$

2

At this point, it is *highly recommended* that you copy down the starting locations for each particle as you will need them to answer Part 3.

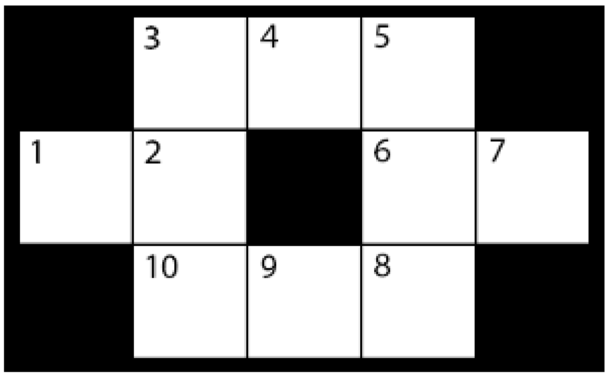
## **Q6.3** Time Update

3 Points

Now we'll perform a time update from $t = 0$ to $t = 1$ using the transition model. Stated again, the transition model is as follows: At each timestep, the robot either stays in place, or moves to any one of its neighboring locations, all with equal probability.

For each particle, take the starting position you found in Part 2, and perform the time update for that particle. You should again sample from the range $[0, 1)$, where the bins are the possible locations **sorted in ascending numerical order**. As an example, if $X_t = 2$, the next state can be one of $\{1, 2, 3, 10\}$, each with equal probability, so the $[0, 0.25)$ bin would be for $X_{t+1} = 1$, the $[0.25, 0.5)$ bin would be for $X_{t+1} = 2$, the $[0.5, 0.75)$ bin would be for $X_{t+1} = 3$, and the $[0.75, 1)$ bin would be for $X_{t+1} = 10$.

The map is shown again below:




$r_1 = 0.674$

$p_1 =$

10

$r_2 = 0.119$

$p_2 =$

4

$r_3 = 0.748$

$p_3 =$

7

$r_4 = 0.802$

$p_4 =$

10

$r_5 = 0.357$

$p_5 =$

3

$r_6 = 0.736$

$p_6 =$

2

$r_7 = 0.425$

$p_7 =$

5

$r_8 = 0.058$

$p_8 =$

1

At this point, it is **highly recommended** that you copy down the new locations for each particle as you will need them to answer Part 4, Part 5, and Part 6.

## **Q6.4** Probability Distribution Induced by the Particles

3 Points

Recall that a particle filter just keeps track of a list of particles, but at any given time, we can compute a probability distribution from these particles. Using the current newly updated set of particles (that you found in Part 3) , give the estimated probability that the robot is in each location.

$P(X_1 = 1)$

1/8

$P(X_1 = 2)$

1/8

$P(X_1 = 3)$

1/8

$P(X_1 = 4)$

1/8

$P(X_1 = 5)$

1/8

$P(X_1 = 6)$

0

$P(X_1 = 7)$

1/8

$P(X_1 = 8)$

0

$P(X_1 = 9)$

0

$P(X_1 = 10)$

1/4

## Q6.5 Incorporating Evidence

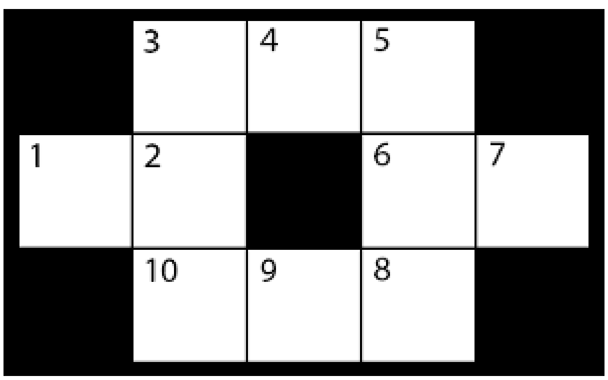
3 Points

The sensor reading at $t = 1$ is: $E_1 = D$

Using the sensor model you specified in Part 1, incorporate the evidence by reweighting the particles. Also enter the normalized and cumulative weights for each particle. The normalized weight for a specific particle can be calculated by taking that particle's weight and dividing by the sum of all the particle weights. The cumulative weight keeps track of a running sum of all the weights of the particles seen so far (meaning, particle $i$ will have a cumulative weight equal to the sum of the weights of all particles $j$ such that $j \leq i$ ).

Refer back to Part 3 to get the positions of your particles.

The map is shown again below:




Particle $p_1$ weight:

1/6

$p_1$ normalized weight:

1/12

$p_1$ cumulative weight:

1/12

Particle $p_2$ weight:

1/6

$p_2$ normalized weight:

1/12

$p_2$ cumulative weight:

1/6

Particle $p_3$ weight:

1/2

$p_3$ normalized weight:

1/4

$p_3$ cumulative weight:

5/12

Particle $p_4$ weight:

1/6

$p_4$ normalized weight:

1/12

$p_4$ cumulative weight:

1/2

Particle $p_5$ weight:

1/6

$p_5$ normalized weight:

1/12

$p_5$ cumulative weight:

7/12

Particle $p_6$ weight:

1/6

$p_6$ normalized weight:

1/12

$p_6$ cumulative weight:

2/3

Particle $p_7$ weight:

1/6

$p_7$ normalized weight:

1/12

$p_7$ cumulative weight:

3/4

Particle $p_8$ weight:

1/2

$p_8$ normalized weight:

1/4

$p_8$ cumulative weight:

1

## Q6.6 Resampling

3 Points

Finally, we'll resample the particles. This reallocates resources to the most relevant parts of the state space in the next time update step.

Notice that your cumulative weights effectively tell you where the bins used in resampling the particles lie. For example, for particle 1, you calculated the cumulative weight to be some value, p. Then, on a random value draw, if a value between 0 and p was chosen, you would generate a new particle where particle 1 is. Use these bounds to resample the eight particles. In the "New Particle" row, enter the particle corresponding to the bin that the random value chose. In the "New Location" row, enter the location corresponding to this new particle. You may need to look back at Part 3 to get the locations of the particles.

$r_1 = 0.403$

New particle for $p_1$:

3

New location for $p_1$:

7

$r_2 = 0.218$

New particle for $p_2$:

3

New location for $p_2$:

7

$r_3 = 0.217$

New particle for $p_3$:

3

New location for $p_3$:

7

$r_4 = 0.826$

New particle for $p_4$:

8

New location for $p_4$:

1

$r_5 = 0.717$

New particle for $p_5$:

7

New location for $p_5$:

5

$r_6 = 0.460$

New particle for $p_6$:

4

New location for $p_6$:

10

$r_7 = 0.794$

New particle for $p_7$:

8

New location for $p_7$:

1

$r_8 = 0.016$

New particle for $p_8$:

1

New location for $p_8$:

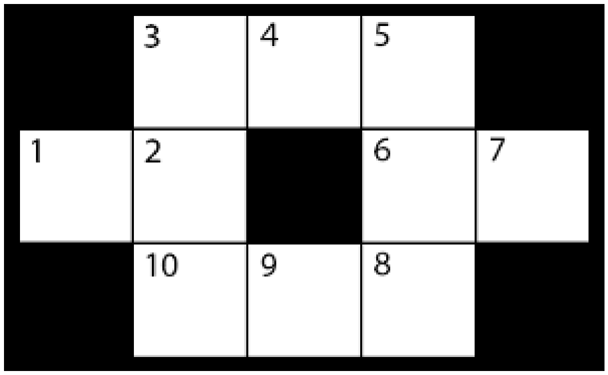
10

## **Q6.7** Analysis

2 Points

The sensor provided a reading $E_1 = D$. What fraction of the particles are now on a dead end?

The map is shown again below:




5/8

This completes everything for the first time step, $t = 0 \rightarrow t = 1$. Of course, we would now continue by repeating the time update, evidence incorporation by reweighting, and resampling. We'll leave that to the computers, though.

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## **Q7** Particle Filtering Implementation

10 Points

Consider the following particle filter implementations.

Default Implementation: Resample after Evidence Incorporation

1. Initialize particles by sampling from initial state distribution.
2. Repeat:
    1. Perform time update
    2. Weight according to evidence
    3. Resample according to weights

Alternative Implementation: Resample after Time Update

1. Initialize particles by sampling from initial state distribution and assigning uniform weights.
2. Repeat:
    1. Perform time update, retaining weights
    2. Resample according to weights
    3. Weight according to evidence

For each of the following statements about the two implementations, select whether they are true or false.

The default implementation will typically provide a better estimate of the distribution than the alternate implementation.

- [x] True
- [ ] False

If the observation model is deterministic then the default implementation will typically provide a better estimate of the distribution than the alternate implementation.

- [x] True
- [ ] False

If the transition model is deterministic then the default implementation will typically provide a better estimate of the distribution than the alternate implementation.

- [ ] True
- [x] False

---

## HW 5 (Electronic Component)

● GRADED

**STUDENT**

Benjamin Cheung

**TOTAL POINTS**

**100 / 100 pts**

**QUESTION 1**

Decisions **15** / 15 pts

1.1 (no title) **5** / 5 pts

✔ + 5 pts **Correct**

**+ 0 pts** Incorrect

1.2 (no title) **10** / 10 pts

QUESTION 2

Value of Perfect Information **10** / 10 pts

QUESTION 3

HMMs, Part I **15** / 15 pts

QUESTION 4

HMMs, Part II **15** / 15 pts

4.1 (no title) **10** / 10 pts

4.2 (no title) **5** / 5 pts

QUESTION 5

Modified HMM Update Equations **15** / 15 pts

5.1 (no title) **5** / 5 pts

5.2 (no title) **5** / 5 pts

5.3 (no title) **5** / 5 pts

QUESTION 6

Particle Filtering **20** / 20 pts

6.1 Sensor Model **3** / 3 pts

6.2 Sampling Review and Initial Belief State **3** / 3 pts

6.3 Time Update **3** / 3 pts

6.4 Probability Distribution Induced by the Particles **3** / 3 pts

6.5 Incorporating Evidence **3** / 3 pts

6.6 Resampling **3** / 3 pts

6.7 Analysis **2** / 2 pts

QUESTION 7

Particle Filtering Implementation **10** / 10 pts