Q1 Probability, Part I

16 Points

Below is a table listing the probabilities of three binary random variables. Fill in the correct values for each marginal or conditional probability below.

X_0	X_1	X_2	$P(X_0, X_1, X_2)$
0	0	0	0.160
1	0	0	0.100
0	1	0	0.120
1	1	0	0.040
0	0	1	0.180
1	0	1	0.200
0	1	1	0.120
1	1	1	0.080

Q1.1

8 Points

$P(X_0 = 1, X_1 = 0, X_2 = 1)$

0.200

$P(X_0 = 0, X_1 = 1)$

0.240

$P(X_2 = 0)$

0.420

Q1.2

8 Points

$P(X_1 = 0 \mid X_0 = 1)$

0.7143

$P(X_0 = 1, X_1 = 0 \mid X_2 = 1)$

0.3448

$P(X_0 = 1 \mid X_1 = 0, X_2 = 1)$

0.5263

Q2 Probability, Part II

16 Points

You are given the prior distribution $P(X)$, and two conditional distributions $P(Y \mid X)$ and $P(Z \mid Y)$ as below (you are also given the fact that Z is independent from X given Y). All variables are binary variables. Compute the following joint distributions based on the chain rule.

X	$P(X)$
0	0.500
1	0.500

Y	X	$P(Y\|X)$
0	0	0.600
1	0	0.400
0	1	0.900
1	1	0.100

Z	Y	$P(Z\|Y)$
0	0	0.100
1	0	0.900
0	1	0.700
1	1	0.300

Q2.1

8 Points

$P(X = 0, Y = 0)$

.3

$P(X = 1, Y = 0)$

.45

$P(X = 0, Y = 1)$

.2

$P(X = 1, Y = 1)$

.05

Q2.2

8 Points

$P(X = 0, Y = 0, Z = 0)$

.03

$P(X = 1, Y = 1, Z = 0)$

.035

$P(X = 1, Y = 0, Z = 1)$

.405

$P(X = 1, Y = 1, Z = 1)$

.015

Q3 Probability, Part III

16 Points

For each of the following four subparts, you are given three joint probability distribution tables. For each distribution, please identify if the given independence / conditional independence assumption is true or false.

For your convenience, we have also provided some marginal and conditional probability distribution tables that could assist you in solving this problem.

X	Y	$P(X,Y)$
0	0	0.240
1	0	0.160
0	1	0.360
1	1	0.240

X	$P(X)$
0	0.600
1	0.400

Y	$P(Y)$
0	0.400
1	0.600

X is independent from Y.

- (•) True
- ( ) False

X	Y	$P(X,Y)$
0	0	0.540
1	0	0.360
0	1	0.060
1	1	0.040

X	$P(X)$
0	0.600
1	0.400

X	Y	$P(X\|Y)$
0	0	0.600
1	0	0.400
0	1	0.600
1	1	0.400

X is independent from Y.

- (selected) True
- False

X	Y	Z	$P(X,Y,Z)$
0	0	0	0.280
1	0	0	0.070
0	1	0	0.210
1	1	0	0.140
0	0	1	0.060
1	0	1	0.060
0	1	1	0.030
1	1	1	0.150

X	Z	$P(X\|Z)$
0	0	0.700
1	0	0.300
0	1	0.300
1	1	0.700

Y	Z	$P(Y\|Z)$
0	0	0.500
1	0	0.500
0	1	0.400
1	1	0.600

X	Y	Z	$P(X,Y\|Z)$
0	0	0	0.400
1	0	0	0.100
0	1	0	0.300
1	1	0	0.200
0	0	1	0.200
1	0	1	0.200
0	1	1	0.100
1	1	1	0.500

X is independent from Y given Z.

○ True

◉ False

X	Y	Z	$P(X,Y,Z)$
0	0	0	0.140
1	0	0	0.140
0	1	0	0.060
1	1	0	0.060
0	0	1	0.048
1	0	1	0.192
0	1	1	0.072
1	1	1	0.288

X	Z	$P(X\|Z)$
0	0	0.500
1	0	0.500
0	1	0.200
1	1	0.800

Y	Z	$P(Y\|Z)$
0	0	0.700
1	0	0.300
0	1	0.400
1	1	0.600

X	Y	Z	$P(X,Y\|Z)$
0	0	0	0.350
1	0	0	0.350
0	1	0	0.150
1	1	0	0.150
0	0	1	0.080
1	0	1	0.320
0	1	1	0.120
1	1	1	0.480

X is independent from Y given Z.

- (selected) True
- False

Q4 Chain Rule

16 Points

Select all expressions that are equivalent to the specified probability using the given independence assumptions.

Given no independence assumptions, $P(A, B \mid C)$ =

- [ ] $\frac{P(C|A)P(A|B)P(B)}{P(C)}$
- [ ] $\frac{P(B,C|A)P(A)}{P(B,C)}$
- [x] $P(A \mid B, C)P(B \mid C)$
- [ ] $\frac{P(A|C)P(B,C)}{P(C)}$

Given that A is independent of B given C, $P(A, B \mid C)$ =

- [ ] $\frac{P(C|A)P(A|B)P(B)}{P(C)}$
- [ ] $\frac{P(B,C|A)P(A)}{P(B,C)}$
- [x] $P(A \mid B, C)P(B \mid C)$
- [x] $\frac{P(A|C)P(B,C)}{P(C)}$

Given no independence assumptions, $P(A \mid B, C)$ =

- [ ] $\frac{P(C|A)P(A|B)P(B)}{P(C)}$
- [x] $\frac{P(B,C|A)P(A)}{P(B,C)}$
- [ ] $\frac{P(A|C)P(C|B)P(B)}{P(B,C)}$
- [x] $\frac{P(C|A,B)P(B|A)P(A)}{P(B|C)P(C)}$

Given that A is independent of B given C, $P(A \mid B, C)$ =

- [ ] $\frac{P(C|A)P(A|B)P(B)}{P(C)}$
- [x] $\frac{P(B,C|A)P(A)}{P(B,C)}$
- [x] $\frac{P(A|C)P(C|B)P(B)}{P(B,C)}$
- [x] $\frac{P(C|A,B)P(B|A)P(A)}{P(B|C)P(C)}$

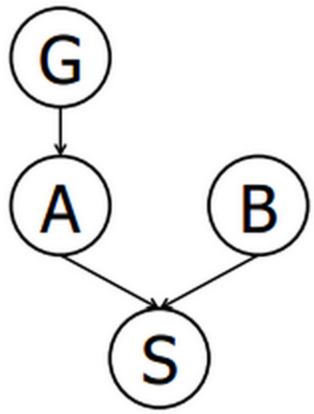
Q5 Bayes' Nets and Probability

18 Points

Suppose that a patient can have a symptom (S) that can be caused by two different diseases (A and B). It is known that the variation of gene G plays a big role in the manifestation of disease A. The Bayes' Net and corresponding probability tables for this situation are shown below.

G	$P(G)$
g	0.10
$\neg g$	0.90

G	A	$P(A \mid G)$
g	a	1.00
g	$\neg a$	0.00
$\neg g$	a	0.10
$\neg g$	$\neg a$	0.90



B	$P(B)$
b	0.40
$\neg b$	0.60

A	B	S	$P(S \mid A, B)$
a	b	s	1.00
a	b	$\neg s$	0.00
a	$\neg b$	s	0.90
a	$\neg b$	$\neg s$	0.10
$\neg a$	b	s	0.80
$\neg a$	b	$\neg s$	0.20
$\neg a$	$\neg b$	s	0.10
$\neg a$	$\neg b$	$\neg s$	0.90

Q5.1

12 Points

Compute P(g, a, b, s).

0.04

What is the probability that a patient has disease A?

0.19

What is the probability that a patient has disease A given that they have disease B?

0.19

What is the probability that a patient has disease A given that they have symptom S and disease B?

.2267

Q5.2

6 Points

What is the probability that a patient has the disease carrying gene variation G given that they have disease A?

.5263

What is the probability that a patient has the disease carrying gene variation G given that they have disease B?

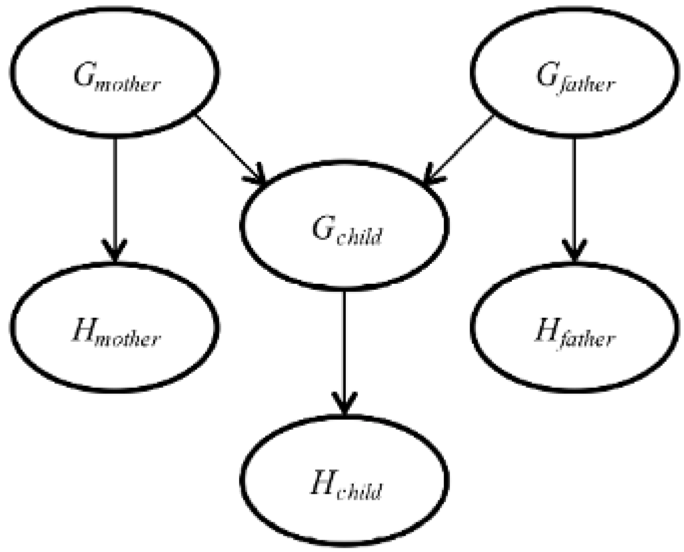
.1

Q6 Bayes' Nets Independence

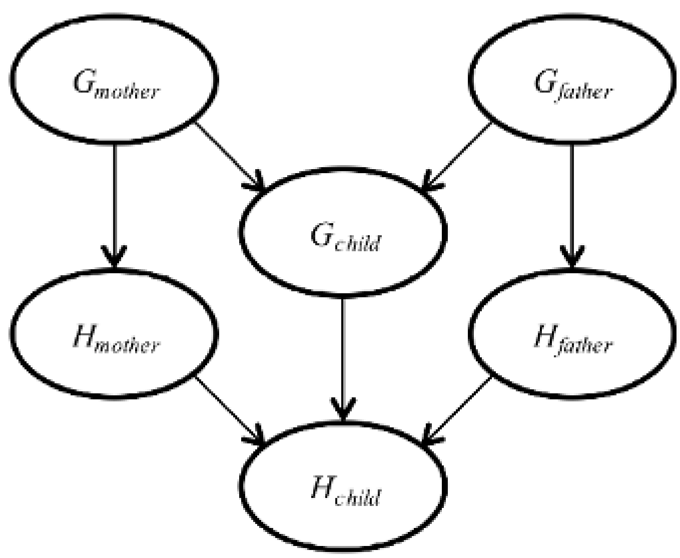
18 Points

Let H_x be a random variable denoting the handedness of an individual x, with possible values l or r. A common hypothesis is that left- or right-handedness is inherited by a simple mechanism; that is, perhaps there is a gene G_x, also with values l or r, and perhaps actual handedness turns out mostly the same (with some probability s) as the gene an individual possesses. Furthermore, perhaps the gene itself is equally likely to be inherited from either of an individual's parents, with a small nonzero probability m of a random mutation flipping the handedness.

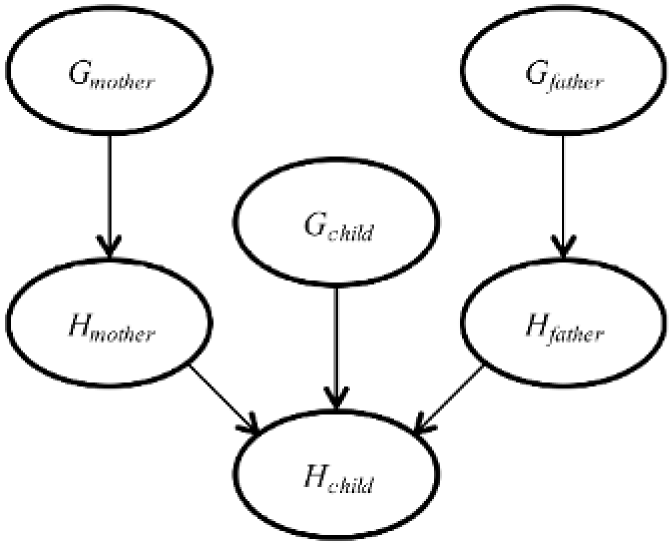
The following three images are possible models involving the genes G and handednesses H.


G_{mother}
G_{father}
G_{child}
H_{mother}
H_{father}
H_{child}

(a)


G_{mother}
G_{father}
G_{child}
H_{mother}
H_{father}
H_{child}

(b)



(c)

Which of the three networks above claim that $P(G_{father}, G_{mother}, G_{child}) = P(G_{father})P(G_{mother})P(G_{child})$?

- [ ] (a)
- [ ] (b)
- [x] (c)

Which of the three networks make independence claims that are consistent with the hypothesis about the inheritance of handedness?

- [x] (a)
- [x] (b)
- [ ] (c)

Which of the three networks is the best description of the hypothesis?

- (●) (a)
- ( ) (b)
- ( ) (c)

Q7 Combining Factors

16 Points

Given the factors $P(A \mid C)$ and $P(B \mid A, C)$ what is the resulting factor after joining over C?

- ○ $P(A, B, C)$
- ○ $P(A \mid B, C)$
- ◉ $P(A, B \mid C)$
- ○ None of the above.

Given the factors $P(A|B)$ and $P(B|C)$ and $P(C)$ which factor will be created after joining on C and summing out over C?

- ○ $P(B, C)$
- ◉ $P(B)$
- ○ $P(C)$
- ○ None of the above

Given the factors $P(A|C)$ and $P(B|A, C)$ what is the resulting factor after joining over A and summing over A?

- ○ $P(C)$
- ○ $P(B)$
- ○ $P(B, C)$
- ○ $P(A \mid C)$
- ◉ $P(B \mid C)$
- ○ None of the above.

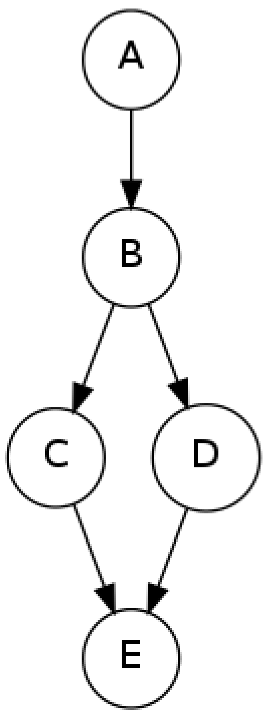
Given the factors $P(C|A)$, $P(D|A, B, C)$, $P(B|A, C)$, what is the resulting factor after joining over C and summing over C?

- ○ $P(D \mid A)$
- ○ $P(C, D \mid A)$
- ○ $P(B, C, D \mid A)$
- ◉ $P(B, D \mid A)$
- ○ $P(C, B \mid A, D) * P(A \mid D)$
- ○ None of the above.

Q8 Variable Elimination Tables

20 Points

Assume the following Bayes Net and corresponding CPTs. In this exercise, we are given the query $P(C \mid e = 1)$, and we will complete the tables for each factor generated during the elimination process.



After introducing evidence, we have the following probability tables.

A	$P(A)$
0	0.100
1	0.900

B	A	$P(B\|A)$
0	0	0.500
1	0	0.500
0	1	0.400
1	1	0.600

C	B	$P(C\|B)$
0	0	0.400
1	0	0.600
0	1	0.300
1	1	0.700

D	B	$P(D\|B)$
0	0	0.600
1	0	0.400
0	1	0.900
1	1	0.100

C	D	$P(e=1\|C,D)$
0	0	0.600
1	0	0.200
0	1	0.600
1	1	0.200

Q8.1

15 Points

Three steps are required for elimination, with the resulting factors listed below:

Step 1: eliminate A. We get the factor $f_1(B) = \sum_a P(a)P(B|a)$

Step 2: eliminate B. We get the factor $f_2(C, D) = \sum_b P(C|b)P(D|b)f_1(b)$

Step 3: eliminate D. We get the factor $f_3(C, e = 1) = \sum_d P(e = 1|C, d)f_2(C, d)$.

Fill in the missing quantities. (some of the quantities are computed for you)

$f_1(B = 0) =$

.41

$f_1(B = 1) =$

.59

$f_2(C = 0, D = 0) =$

.2577

$f_2(C = 1, D = 0) =$

.5193

$f_2(C = 0, D = 1) = 0.083$

$f_2(C = 1, D = 1) = 0.14$

$f_3(C = 0, e = 1) =$

.20442

$f_3(C = 1, e = 1) = 0.132$

Q8.2

5 Points

After getting the final factor $f_3(C, e = 1)$, a final renormalization step needs to be carried out to obtain the conditional probability $P(C|e = 1)$. Fill in the final conditional probabilities below.

$P(C = 0 \mid e = 1) =$

.60936

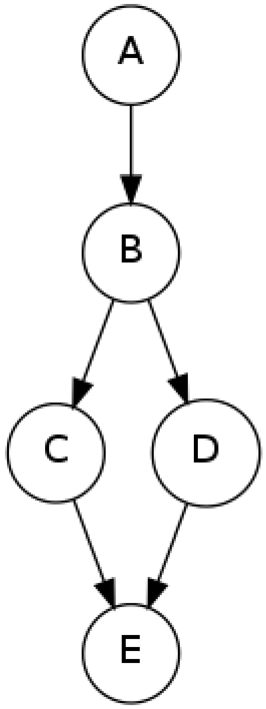
$P(C = 1 \mid e = 1) =$

.39237

Q9 Rejection Sampling

16 Points

We will work with a Bayes' net of the following structure.



In this question, we will perform rejection sampling to estimate $P(C = 1 \mid B = 1, E = 1)$. Perform one round of rejection sampling, using the random samples given in the table below. Variables are sampled in the order A, B, C, D, E. In the boxes below, choose the value (0 or 1) that each variable gets assigned to. **Note that the sampling attempt should stop as soon as you discover that the sample will be rejected. In that case mark the assignment of that variable and write $\texttt{none}$ for the rest of the variables.**

When generating random samples, use as many values as needed from the table below, which we generated independently and uniformly at random from $[0, 1)$. Use numbers from left to right. To sample a binary variable W with probability $P(W = 0) = p$ and $P(W = 1) = 1 - p$ using a value a from the table, choose $W = 0$ if $a < p$ and $W = 1$ if $a \geq p$.

0.320	0.037	0.303	0.318	0.032	0.969	0.018	0.058	0.908	0.249

A	$P(A)$
0	0.200
1	0.800

B	A	$P(B\|A)$
0	0	0.800
1	0	0.200
0	1	0.400
1	1	0.600

C	B	$P(C\|B)$
0	0	0.600
1	0	0.400
0	1	0.400
1	1	0.600

D	B	$P(D\|B)$
0	0	0.800
1	0	0.200
0	1	0.600
1	1	0.400

E	C	D	$P(E\|C, D)$
0	0	0	0.800
1	0	0	0.200
0	1	0	0.600
1	1	0	0.400
0	0	1	0.400
1	0	1	0.600
0	1	1	0.400
1	1	1	0.600

Enter either a 0 or 1 for each variable that you assign a value to. Upon rejecting a sample, enter its assigned value, and enter `none` for the remaining variables. For example, if C gets rejected, fill in `none` for D and E.

A:

1

B:

0

C:

none

D:

none

E:

none

Which variable will get rejected?

- ○ A
- ◉ B
- ○ C
- ○ D
- ○ E
- ○ None of the variables will get rejected

Q10 Estimating Probabilities from Samples

16 Points

Below are a set of samples obtained by running rejection sampling for the Bayes' net from the previous question. Use them to estimate $P(C = 1 \mid B = 1, E = 1)$. The estimation cannot be made whenever all samples were rejected. In this case, input -1 into the box below.

Sample 1

	0	**1**	**rejected**
A		x	
B	x		x
C			
D			
E			

Sample 2

	0	**1**	**rejected**
A	x		
B		x	
C	x		
D	x		
E		x	

Sample 3

	0	**1**	**rejected**
A		x	
B		x	
C		x	
D	x		
E		x	

Sample 4

	0	**1**	**rejected**
A		x	
B	x		x
C			
D			
E			

Sample 5

	0	**1**	**rejected**
A	x		
B		x	
C		x	
D	x		
E	x		x

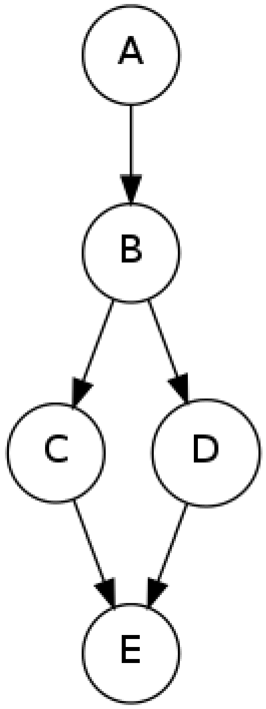
Estimation:

.5

Q11 Likelihood Weighting

16 Points

We will work with a Bayes' net of the following structure.



In this question, we will perform likelihood weighting to estimate $P(C = 1 \mid B = 1, E = 1)$. Generate a sample and its weight, using the random samples given in the table below. Variables are sampled in the order A, B, C, D, E. In the table below, select the assignments to the variables you sampled.

When generating random samples, use as many values as needed from the table below, which we generated independently and uniformly at random from $[0, 1)$. Use numbers from left to right. To sample a binary variable W with probability $P(W = 0) = p$ and $P(W = 1) = 1 - p$ using a value a from the table, choose $W = 0$ if $a < p$ and $W = 1$ if $a \geq p$.

0.249	0.052	0.299	0.773	0.715	0.550	0.703	0.105	0.236	0.153

A	$P(A)$
0	0.200
1	0.800

B	A	$P(B\|A)$
0	0	0.400
1	0	0.600
0	1	0.200
1	1	0.800

C	B	$P(C\|B)$
0	0	0.600
1	0	0.400
0	1	0.600
1	1	0.400

D	B	$P(D\|B)$
0	0	0.800
1	0	0.200
0	1	0.600
1	1	0.400

E	C	D	$P(E\|C,D)$
0	0	0	0.200
1	0	0	0.800
0	1	0	0.600
1	1	0	0.400
0	0	1	0.800
1	0	1	0.200
0	1	1	0.800
1	1	1	0.200

Input Answers Here

A:

1

B:

1

C:

0

D:

0

E:

1

What is the weight for the sample you obtained above?

.64

Q12 Estimating Probabilities from Weighted Samples

16 Points

Below are a set of weighted samples obtained by running likelihood weighting for the Bayes' net from the previous question. Use them to estimate $P(C = 1 \mid B = 1, E = 1)$. Input -1 in the box below if the estimation cannot be made.

Sample 1

	0	1
A		x
B		x
C		x
D		x
E		x

Weight = 0.64

Sample 2

	0	1
A		x
B		x
C		x
D		x
E		x

Weight = 0.64

Sample 3

	0	1
A		x
B		x
C	x	
D		x
E		x

Weight = 0.32

Sample 4

	0	1
A		x
B		x
C	x	
D	x	
E		x

Weight = 0.16

Sample 5

	0	1
A	x	
B		x
C		x
D		x
E		x

Weight = 0.48

Estimation:

.7857

HW 4 (Electronic Component)

GRADED

STUDENT

Benjamin Cheung

TOTAL POINTS

200 / 200 pts

QUESTION 1

Probability, Part I — **16** / 16 pts

1.1 (no title) — **8** / 8 pts

1.2 (no title) — **8** / 8 pts

QUESTION 2

Probability, Part II — **16** / 16 pts

2.1 (no title) — **8** / 8 pts

2.2 (no title) — **8** / 8 pts

QUESTION 3

Probability, Part III **16** / 16 pts

QUESTION 4

Chain Rule **16** / 16 pts

QUESTION 5

Bayes' Nets and Probability **18** / 18 pts

5.1 (no title) **12** / 12 pts

5.2 (no title) **6** / 6 pts

QUESTION 6

Bayes' Nets Independence **18** / 18 pts

QUESTION 7

Combining Factors **16** / 16 pts

QUESTION 8

Variable Elimination Tables **20** / 20 pts

8.1 (no title) **15** / 15 pts

8.2 (no title) **5** / 5 pts

QUESTION 9

Rejection Sampling **16** / 16 pts

QUESTION 10

Estimating Probabilities from Samples **16** / 16 pts

QUESTION 11

Likelihood Weighting **16** / 16 pts

QUESTION 12

Estimating Probabilities from Weighted Samples **16** / 16 pts