Top-Down Parsing (Objectives)

- Given a grammar, the student will be able to convert the grammar to LL(1) form if possible.
- Given an LL(1) grammar, the student will be able to construct a corresponding predictive parser for the grammar

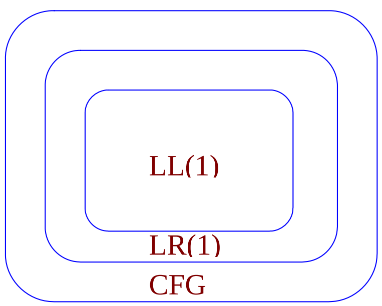
Context-Free Grammars

- A context-free grammar G is a quadruple, (N,T,P,S) where N is a set of nonterminals, T is a set of terminals, P is a set of productions and $S \in N$ is the start symbol.
- Example

E → T + E
  | T - E
  | T
T → F * T
  | F / T
  | F
F → num
  | id
  | id [E]

N = {E,T,F}
T = {+,-,*,/,num,id}
S = E

Types of Context-Free Grammars



Top-Down Parsers

- Start at the root of the parse tree and fill in the children
  - expand the grammar from the start symbol
- pick a production and try to match the input token
- may require backtracking
  - if the picked production doesn't match the input at some point
- predictive recursive descent parsers do not require backtracking
- predictive parsers need LL(k) grammars
  - Left-to-right scan, Leftmost derivation, k symbols of lookahead
  - we will look at LL(1) grammars

Top-down Parsers w/o Lookahead

- To build a parse tree start with the root of the parse tree labeled with the start symbol
- Repeat the following steps until the left-to-right ordering of the leaves matches the input string
  1. At a node labeled A select a production with A on its lhs and for each symbol on its rhs construct a parse tree.
  2. When a terminal is added to a leaf of the parse tree that does not match the input string, backtrack up the tree to where a different choice could have been made that may lead to a correct derivation.
  3. Find the next node to be expanded and go to step 1.
- **Key**: select the right production in step 1.

Example Grammar

$G \rightarrow E$	(1)
$E \rightarrow E + T$	(2)
$\mid E - T$	(3)
$\mid T$	(4)
$T \rightarrow T * F$	(5)
$\mid T / F$	(6)
$\mid F$	(7)
$F \rightarrow$ num	(8)
$\mid$ id	(9)
$\mid$ id [E]	(10)

- Parse the string x - 2

Problems with No Lookahead

- Termination
  - should not depend on the choice of production
  - parsers should always terminate
- Determinism
  - parsers should be deterministic
  - there should be only one choice at each step of the parse
- Solution
  - add lookahead to the parsing algorithm
  - fix grammar so that infinite loops are not possible

Writing an LL(1) Grammar

- eliminate infinite loops
  - make it impossible to recursively expand a grammar symbol either immediately or through a chain of expansions
  - eliminate left recursion
- eliminate ambiguity
  - make at most one choice for expanding each grammar symbol based upon the next input character
  - left factor the grammar

Eliminating Left Recursion

- A grammar is left recursive if

  $\exists A \in N \mid A \Rightarrow^* A\alpha$

  for some string α
- Eliminating immediate left recursion

  $A \to A\alpha \mid \beta$ becomes $A \to \beta A'$, $A' \to \alpha A' \mid \varepsilon$
- We must eliminate indirect left recursion too.

Algorithm

// grammar must have production $S' \to S$
arrange non-terminals in some order
for *i* = 1 to *n* **do**
 for *j* = 1 to *i*-1 **do** // make sure no A_j in rhs
  replace each $A_i \to A_j\gamma$ by $A_i \to \delta_1\gamma \mid \delta_2\gamma \mid \ldots \mid \delta_k\gamma$
   where $A_j \to \delta_1 \mid \delta_2 \mid \ldots \mid \delta_k$
 end // only immediate recursion left for A_i
 eliminate immediate left recursion for A_i
end

Example

S → Aa
A → Bb
B → Sc | c

G → E
E → E + T
 | E - T
 | T
T → T * F
 | T / F
 | F
F → num
 | id
 | id [E]

FIRST Sets

- For a production $A \to \alpha \mid \beta$ we would like a distinct way to choose the correct production.
  - Answer: use lookahead
- FIRST sets
  - For some rhs of a production α, FIRST(α) is the set of tokens that appear as the first symbol in some string derived from α

    $x \in \text{FIRST}(\alpha) \Leftrightarrow \exists x \in \Sigma \mid \alpha \Rightarrow^* x\gamma$

FIRST Sets

- KEY PROPERTYS: Whenever two productions $A \rightarrow \alpha \mid \beta$ both appear in the same grammar
  1. $FIRST(\alpha) \cap FIRST(\beta) = \emptyset$
  2. At most one of α and β derives ε
  3. If $\beta \Rightarrow^* \varepsilon$ then α does not derive any string beginning with a terminal in FOLLOW(A). (Similar for α)

  This allows the parser to make the right choice of productions with one symbol of lookahead
- The text book define predict set:
  - predict($A \rightarrow \alpha$) = FIRST(α) $\cup$ (if $\alpha \Rightarrow^* \varepsilon$ then Follow(A) else $\emptyset$)
- Use left factoring to try to obtain this property

Computing First Sets

- To build FIRST(X)

  if X is a terminal then
  FIRST(X) = {X}
  else if $X \rightarrow \varepsilon$ then
  FIRST(X) $\cup$= $\{\varepsilon\}$
  else if $X \rightarrow Y_1Y_2 \ldots Y_k$ then
  FIRST(X) $\cup$= FIRST(Y_1)
  $\forall i \mid \varepsilon \in$ FIRST(Y_j), $1 \leq j < i$, FIRST(X) $\cup$= FIRST(Y_i)

Example

- Compute FIRST sets for the expression grammar

FOLLOW Sets

- To construct FOLLOW sets:
  1. place \$ in FOLLOW(S')
  2. for $A \rightarrow \alpha B\beta$ add FIRST(β)-$\{\varepsilon\}$ to FOLLOW(B)
  3. for $A \rightarrow \alpha B$ add FOLLOW(A) to FOLLOW(B)
  4. for $A \rightarrow \alpha B\beta$ if $\varepsilon \in$ FIRST(β) add FOLLOW(A) to FOLLOW(B)

Example

- Compute FOLLOW sets for the expression grammar

Left Factoring

- Restructure the grammar so FIRST sets of possible productions do not intersect

while a common prefix for alternatives exists for some A
  do
    find the longest prefix α common to two or more of
      A's alternatives
    replace productions $A \rightarrow \alpha\beta_1 \mid \alpha\beta_2 \mid \ldots \mid \alpha\beta_n$ with
      $A \rightarrow \alpha L$
      $L \rightarrow \beta_1 \mid \beta_2 \mid \ldots \mid \beta_n$
  end

Example

- Left factor the expression grammar

Left Recursion and Left Factoring

$G \rightarrow E$
$E \rightarrow E + T$
  $\mid E - T$
  $\mid T$
$T \rightarrow T * F$
  $\mid T / F$
  $\mid F$
$F \rightarrow num$
  $\mid id$
  $\mid id\ [E]$

$G \rightarrow E$
$E \rightarrow T\ E'$
$E' \rightarrow +\ T\ E'$
  $\mid -\ T\ E'$
  $\mid \varepsilon$
$T \rightarrow F\ T'$
$T' \rightarrow *\ F\ T'$
  $\mid /\ F\ T'$
  $\mid \varepsilon$
$F \rightarrow num$
  $\mid id\ \ F'$
$F' \rightarrow [E]$
  $\mid \varepsilon$

First and Follow Sets

	First	Follow
G	{num, id}	{$}
E	{num, id}	{$,]}
E'	{+, -, ε}	{$,]}
T	{num, id}	{$, + , -,]}
T'	{*, /, ε}	{$, + , -,]}
F	{num, id}	{$, + , -, *, /,]}
F'	{ [, ε}	{$, + , -, *, /,]}

LL(1)?

- Show that the converted grammar is LL(1)

Predictive Parser

- If left factoring can be done and left recursion is removed, the grammar is LL(1)
- Construct a top-down predictive parser
  - Each non-terminal becomes a function
  - When a terminal is the first symbol on the rhs of a production, match that terminal with the next input symbol

Predictive Parser

```
G() {
    token = NextToken()
    if (E() == ERROR) then
      return ERROR;
    }
E() {
    if (T() == ERROR)
      return ERROR;
    else return E'();
    }
T() {
    if (F() == ERROR) then
      return ERROR
    else return T'();
    }
```

```
F() {
    if (token == NUM ||
        token == ID) then {
           ptoken = token;
           token = NextToken();
           if (ptoken == NUM) then
              return OK;
           else return F'(); }
    else return ERROR;}
E'() {
    if (token == PLUS ||
        token == MINUS) then {
      token = NextToken();
      if (T() == ERROR) then
        return ERROR
      else
        return E'();}
    else return OK;
    }
// T' is similar
```