

## Section 4.7 Removal of Direct Left Recursion

(1)

A rule with direct left recursion looks like

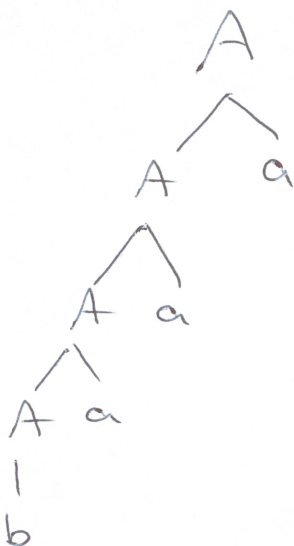
this

$$A \rightarrow A w$$

the variable on the left hand side appears as the first character on the right hand side.

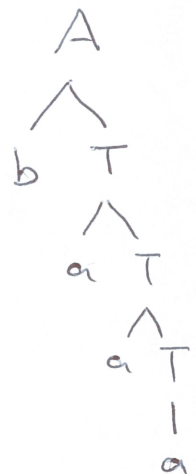
$$\begin{array}{l} A \rightarrow w A b \\ B \rightarrow w A b \end{array} \quad \left. \begin{array}{l} \text{recursive but not} \\ \text{not directly left} \\ \text{recursive.} \end{array} \right\}$$

$$A \rightarrow A a | b \quad \} \quad b a^*$$



$$\begin{array}{l} S \rightarrow b A \\ A \rightarrow a A | a \end{array}$$

$$\begin{array}{l} A \rightarrow b | b T \\ T \rightarrow a T | a \end{array}$$



$$A \rightarrow \overbrace{Aa}^{u_1} \mid \overbrace{Ab}^{u_2} \mid \overbrace{c}^{v_1} \mid \overbrace{d}^{v_2}$$



$$A \rightarrow Au_1 \mid Au_2 \mid \dots \mid Au_j \mid v_1 \mid v_2 \mid \dots \mid v_k$$

replace this:

$$A \rightarrow v_1 \mid v_2 \mid \dots \mid v_k \mid v_1 T \mid v_2 T \mid \dots \mid v_k T$$

$$T \rightarrow u_1 \mid u_2 \mid \dots \mid u_j \mid u_1 T \mid u_2 T \mid \dots \mid u_j T$$

$$A \rightarrow \overbrace{Aa}^{u_1} \mid \overbrace{Ab}^{u_2} \mid \overbrace{c}^{v_1} \mid \overbrace{d}^{v_2}$$

$$A \rightarrow c \mid d \mid cT \mid dT$$

$$T \rightarrow a \mid b \mid aT \mid bT$$

conditions: S is not recursive  
 $S \rightarrow a$  is the only 2-rule  
 no rules of the form  $A \rightarrow A$

### Example 4.7.1

$$A \rightarrow \overbrace{Aa}^{u_1} \mid \overbrace{Aab}^{u_2} \mid \overbrace{bb}^{v_1} \mid \overbrace{b}^{v_2}$$

$$A \rightarrow b \mid bb \mid bT \mid bbT$$

$$T \rightarrow a \mid ab \mid aT \mid abT$$

$$A \rightarrow \underline{Aa} \mid \underline{b} \mid \underline{cB} \mid \underline{aAb} \mid \underline{ABA} \mid \underline{AA} \quad (3)$$

$u_1 \quad v_1 \quad v_2 \quad v_3 \quad u_2 \quad u_3$

$$B \rightarrow dB \mid d \quad \left. \begin{array}{l} \text{? will stay because there is} \\ \text{no direct left recursion.} \end{array} \right\}$$

$$A \rightarrow b \mid cB \mid aAb \mid bT \mid cBT \mid aAbT \quad \left. \begin{array}{l} \text{? v's} \end{array} \right\}$$

$$T \rightarrow a \mid BA \mid aT \mid BAT \mid AT$$

Chapter 18

$$S \rightarrow \textcircled{a}S \mid bS \mid T$$

$$T \rightarrow cT \mid c$$

accc

bc

$$S \Rightarrow \textcircled{a}S$$