Last time:

concatenation of sets

$XY = \{xy \mid x \in X, y \in Y\}$

$\{\_\ \_\} \{\_\ \_\}$

$X^0 = \{\lambda\}$ $\qquad X^0 \neq \{\}$

$X^1 = X$

$X^2 = XX$

The Kleene star of a set X is defined as

$$X^* = \bigcup_{i=0}^{\infty} X^i$$ → can contain longer strings.

Σ^* = all strings you can generate from Σ (alphabet)

→ always contains λ because $X^0 = \{\lambda\}$

$$X^+ = XX^* = \bigcup_{i=1}^{\infty} X^i$$

set operations: $\cup, \cap, -, \times, {}^*, \cdot$

language: a subset of Σ^*

$\times$ ∟ Cartesian product

$\cdot$ concatenation

recursive definitions are a way of defining the "rules" for the strings in a language.

Regular set: is a language that is defined using only three set operations:

$\cup, \cdot, {}^*$

Let Σ be an alphabet. The regular sets over Σ are defined as:

- basis: $\emptyset, \{\lambda\}, \{a\}$ for every $a \in \Sigma$ are regular sets over Σ.
- recursive step: Let X and Y be regular sets, then

  $X \cup Y$

  XY

  X^*

  are regular sets.
- closure

... of all strings over $\Sigma = \{a, b\}$, that start with an "a".

$L = \{a\} . \Sigma^*$

$\hookrightarrow \{\lambda, a, b, aa, \dots\}$

$a \underbrace{\qquad\qquad}_{\text{anything including } \lambda}$

Σ^*

Example: all strings of even length over $\Sigma = \{a\}$

$L_1 = \{\lambda, aa, aaaa, \dots\}$

recursive definition:

basis: $\lambda \in L_1$

rec. step: if $w \in L_1$ then $waa \in L_1$

regular set:

$\{a\}^* = \{\lambda, a, aa, aaa, \dots\}$

$\{\{a\}^+\}^*$

$\{aa\}^* = \{\lambda, aa, aaaa, \dots\}$

$(\{a\}\{a\})^*$

$\{aa\}^*$

Example: set of all strings that start with an "a" and has even length. L_2

(4)

$\Sigma = \{a, b\}$

$\{aa\}^* \cup \{ab\}^*$

aaaa $\cup$ ababab

abbb

rec. defn

basis: $aa, ab \in L_2$

rec step: if $w \in L_2$ then $\left.\begin{matrix} waa \\ wab \\ wba \\ wbb \end{matrix}\right\} \in L_2$

$$\underbrace{\{aa, ab\} \cdot \overbrace{\{aa, ab, ba, bb\}^*}^{X}}$$

bb does not start with a

$\{\overset{X^0}{\lambda}, \overbrace{aa, ab, ba, bb}^{X^1},$

X^2 | aaaa, aaab, aaba, aabb, abaa, abab, abba, abbb

X^3 |

$\overline{}$ $\overline{}$

$\uparrow$ 1

$\overline{\in X}$ $\overline{\in X}$ $\overline{\in X}$

_ _ _ _ _ . . . . . _ $\} 10^6$ times

Note: $X \times Y$, $X \cap Y$, $X - Y$

these arenot regular sets.

Regular expressions

basis: $\emptyset$, λ, a are regular expressions over Σ, $a \in \Sigma$.

recursive step: if X and Y are regular expressions then $X \cup Y$, XY and X^* are regular expressions.

$X | Y$

"or"

example: string of even number of "a"s:

$(aa)^*$ $\neq$ unix aa^*

example: all strings that begin with a and has even length. $\Sigma = \{a, b\}$

$(aa \cup ab)(aa \cup ab \cup ba \cup bb)^*$

$(aa | ab)(aa | ab | ba | bb)^*$

example set of all strings that contain substrings bb over $\Sigma = \{a, b\}$

$$(a \cup b)^* \, bb \, (a \cup b)^*$$

$$\Sigma^*$$

$$(a \cup b)^* \, bb \, (a \cup b)^*$$

allow ab also

$$\left((a \cup b)^* \, bb \, (a \cup b)^* \right) \cup ab$$

$$\{a, b\}^* \{bb\} \{a, b\}^*$$

example over $\{a, b\}$. The set of strings containing substring bb or aa.

$$\left((a \cup b)^* \, bb \, (a \cup b)^* \right) \cup \left((a \cup b)^* \, aa \, (a \cup b)^* \right)$$

$$(a \cup b)^* (aa \cup bb)(a \cup b)^*$$