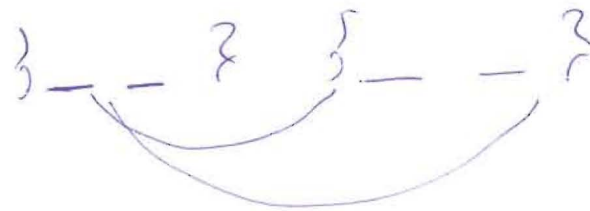


Last time:

concatenation of sets

$$X Y = \{ xy \mid x \in X, y \in Y \}$$



$$X^0 = \{ \lambda \}$$

$$X^0 \neq \{ \cdot \}$$

$$X^1 = X$$

$$X^2 = XX$$

The **Kleene star** of a set X is defined as

$$X^* = \bigcup_{i=0}^{\infty} X^i \quad \rightarrow \text{can contain longer strings.}$$

Σ^* = all strings you can generate from Σ (alphabet)

always contains λ because $X^0 = \{ \lambda \}$

$$X^+ = X X^* = \bigcup_{i=1}^{\infty} X^i$$

set operations: $\cup, \cap, -, \times, *$
language: a subset of Σ^* $\left\{ \begin{array}{l} \text{concatenation} \\ \text{Cartesian product} \end{array} \right.$

recursive definitions are a way of defining the "rules" for the strings in a language.

Regular set: is a language that is defined using only three set operations:
 $\cup, \cdot, *$

Let Σ be an alphabet. The **regular sets** over Σ are defined as:

- basis: $\emptyset, \{ \lambda \}, \{ a \}$ for every $a \in \Sigma$
are regular sets over Σ .

- recursive step: Let X and Y be regular sets, then

$$X \cup Y$$

$$XY$$

$$X^*$$

are regular sets.

- closure

all strings over $\Sigma = \{a, b\}$, that start with an "a".

$$L = \{a\} \cdot \Sigma^*$$

$$\hookrightarrow \{\lambda, a, b, aa, \dots\}$$

$$a \cdot \underbrace{\Sigma^*}_{\text{anything including } \lambda}$$

Example: all strings of even length over $\Sigma = \{a\}$

$$L_1 = \{\lambda, aa, aaaa, \dots\}$$

recursive definition:

basis: $\lambda \in L_1$

rec. step: if $w \in L_1$, then $waaw \in L_1$

regular set:

$$\{a\}^* = \{\lambda, a, aa, aaaa, \dots\}$$

$$\{ \{a\}^* \}^*$$

$$\{aa\}^* = \{\lambda, aa, aaaa, \dots\}$$

$$(\{a\} \{a\})^* \\ \{aa\}^*$$

Example: set of all strings that L_2 start with an "a" and has even length. (4)

$$\Sigma = \{a, b\}$$

$$\{aa\}^* \cup \{ab\}^*$$

$$aaaa \cup ababab$$

$$abbb$$

rec. defn

basis: $aa, ab \in L_2$

rec step: if $w \in L_2$ then $\left. \begin{matrix} waa \\ wab \\ wba \\ wbb \end{matrix} \right\} \in L_2$

$$\{aa, ab\} \cup \{aa, ab, ba, bb\}^*$$

bb does not start with a

$$\{a, aa, ab, ba, bb\}$$

$$x^2 \left\{ \begin{matrix} aaaa, aaab, aaba, aabb, \\ abaa, abab, abba, abbb \end{matrix} \right.$$

$$x^3 \mid$$

$$\overline{\in X} \quad \overline{\in X} \quad \overline{\in X}$$

$\sim 10^6$ times

Note: $X \times Y$, $X \cap Y$, $X - Y$

these are not regular sets.

Regular expressions

basis: \emptyset , λ , a are regular expressions over Σ , $a \in \Sigma$.

recursive step: if X and Y are regular expressions then

$X \cup Y$, $X \cdot Y$ and X^* are regular expressions.

$X | Y$
"or"

example: strings of even number of "a"s:

$(aa)^*$ \neq Unix aa^*

example: all strings that begin with an a and has even length. $\Sigma = \{a, b\}$

$(aa \cup ab)(aa \cup ab \cup ba \cup bb)^*$
 $(aa | ab)(aa | ab | ba | bb)^*$

example set of all strings that contain substring bb over $\Sigma = \{a, b\}$

$$(a \cup b)^* bb (a \cup b)^*$$

$$(a \cup b)^* bb (a \cup b)^*$$

allow ab also

$$\left((a \cup b)^* bb (a \cup b)^* \right) \cup ab$$

$$\{a, b\}^* \{bb\} \{a, b\}^*$$

example over $\{a, b\}$. The set of strings containing substring bb or aa.

$$\left((a \cup b)^* bb (a \cup b)^* \right) \cup \left((a \cup b)^* aa (a \cup b)^* \right)$$

$$(a \cup b)^* (aa \cup bb) (a \cup b)^*$$