

Last time: context-free grammars
CFGs

$$A \rightarrow aA \mid \epsilon \quad a^*$$

$$A \rightarrow aA \mid a \quad a^+$$

$$\{ a^i b^i \mid i \geq 0 \}$$

$$S \rightarrow aSb \mid \epsilon$$

$$S \Rightarrow a \underline{S} b \Rightarrow a \underline{a S b} b \Rightarrow a a a S b b b$$

$$\Rightarrow \underbrace{a a a b b b}_{\text{equal number of "a"s and "b"s, in order}} \\ (\text{a's precede b's})$$

$$a b b a a b$$

(2)

Formally, a context-free grammar (CFG)

is a ~~quadruple~~
quadruple

(V, Σ, P, S)

starting symbol
 $S \in V$

set of production rules
e.g. $P = \{ S \rightarrow aSb \mid a \}$
 $= \{ S \rightarrow aSb, S \rightarrow a \}$

alphabet (terminals, symbols)

set of variables
(intended to be replaced)

$A \rightarrow (V \cup \Sigma)$

$V \rightarrow X$

\downarrow

"context-free"

$A \rightarrow x$
 $\downarrow \quad \downarrow$
 $A \in V \quad x \in (V \cup \Sigma)^*$

can replace without
looking at the left
or right of the
variable.

$aA \rightarrow bcd$ } context-sensitive
has context read

A null production is of the form $A \rightarrow \lambda$
 $A \in V$

(3)

Let $G = (V, \Sigma, P, S)$ be a CFG and
 $v \in (V \cup \Sigma)^*$. The set of strings
 derivable from v is defined as follows:

basis: v is derivable from v

recursive step: if $u = xAy$ is derivable
 from v and $A \rightarrow w \in P$,
 then xwy is derivable from v .

$$S \rightarrow aSb \mid c$$

$$S \Rightarrow \underbrace{a}_{\substack{x \\ v}} \underbrace{Sb}_{\substack{y \\ w}} \Rightarrow a \underbrace{a}_{\substack{x \\ w}} \underbrace{Sb}_{\substack{y \\ w}} \Rightarrow aacbb$$

$$S \rightarrow abc \in P$$

The language of a grammar, G , denoted by
 $L(G)$ is ~~the set of~~ all the strings derivable
 from the start symbol S .

$A \xRightarrow{*} w$ derivable in 0 or more steps

$A \xRightarrow{+} w$ derivable in 1 or more steps.

$$L(G) = \{ w \mid w \in \Sigma^*, S \xRightarrow{*} w \}$$

$$* L = \{ a^n b^m c^k \mid n, m, k \geq 0 \}$$

(4)

$$\underline{a^*} \underline{b^*} \underline{c^*}$$

$$S \rightarrow ABC$$

$$A \rightarrow aA \mid \lambda$$

$$B \rightarrow bB \mid \lambda$$

$$C \rightarrow cC \mid \lambda$$

$$S \Rightarrow ABC \Rightarrow aABC \Rightarrow aaABC \xrightarrow{\text{shrinks, contracts}} aaBC \\ \Rightarrow aabBC \Rightarrow aabC \Rightarrow aab$$

$$S \rightarrow aSb$$

$$* L = \{ a^n b^m c^k \mid n, m, k \geq 0 \}$$

$$S \rightarrow ABC$$

$$A \rightarrow aA \mid a$$

$$B \rightarrow bB \mid b$$

$$C \rightarrow cC \mid c$$

$$* L = \{ a^n b^m c^n \mid n, m \geq 0 \}$$

$$S \rightarrow aSbc$$

$$B \rightarrow bB \mid \lambda$$

$$S \Rightarrow aSbc \Rightarrow aaSbcc$$

$$S \rightarrow aSc \mid T$$

$$T \rightarrow bT \mid \lambda \quad \{ b^* \}$$

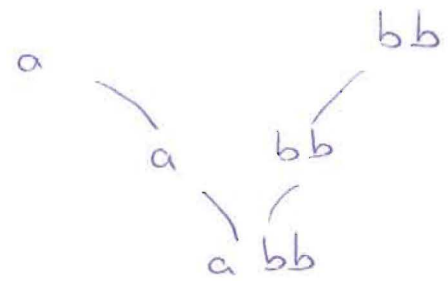
$$S \Rightarrow aSc \Rightarrow aaSc \Rightarrow aaTcc \Rightarrow \dots \\ aa bTcc \Rightarrow aa bcc$$

$$* L = \{ a^n b^{2n} \mid n \geq 0 \}$$

(5)

$$S \rightarrow aA \mid$$

abb
 $aa bbbb$
 $aaa bbbbbbb$



$$S \rightarrow aSbb \mid \lambda$$

$$* L = \{ a^n \underbrace{b^m c^m}_{\text{red bracket}} d^{2n} \mid n \geq 0, m > 0 \}$$

$$S \rightarrow aSdd \mid T \quad \text{2} \notin L$$

$$T \rightarrow bTc \mid bc$$

$$* L = \{ a^n b^m c^m d^{2n} \mid n \geq 0, m \geq 0 \}$$

$$S \rightarrow aSdd \mid T \quad ; \lambda$$

$$T \rightarrow bTc \mid \lambda$$

$$S \Rightarrow T \Rightarrow \lambda \quad \left| \quad \begin{array}{l} S \Rightarrow \lambda \\ S \Rightarrow a[S]dd \Rightarrow a[aSdd]dd \\ \Rightarrow aaTdddd \end{array} \right.$$

$$\begin{array}{l}
 S \rightarrow aTdd \\
 T \rightarrow bTc \mid bc
 \end{array}$$