Last century:

NFA-ə   to   DFA
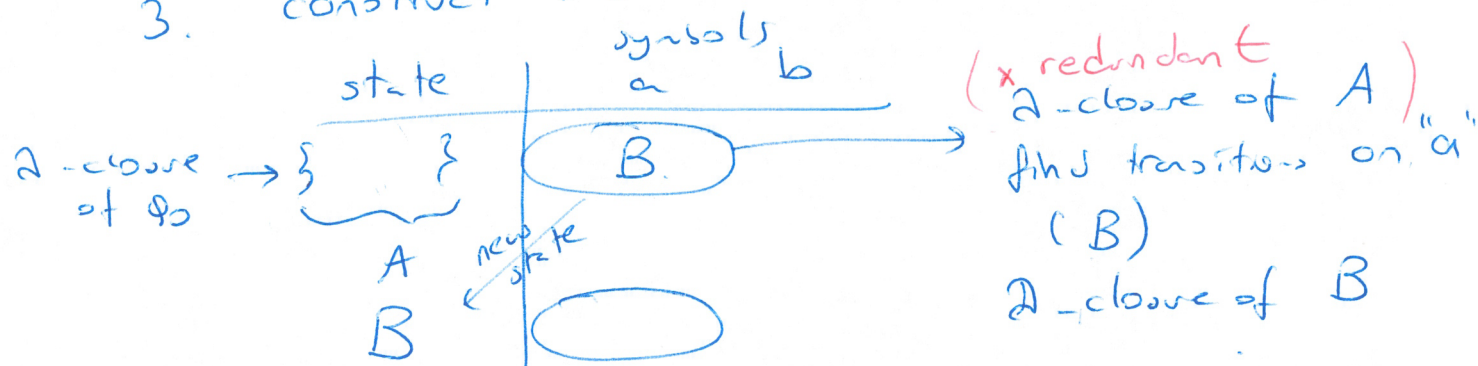(N)   (M)

steps:

1. write the transition function of N
   include a column for each state's
   ə-closure.

$$\text{symbols}$$

state	a	b	ə-closure
q_0			$\{q_0\} \cup$
q_1			$\{q_1\} \cup$

2. define the start state of M:
   ə-closure of the start state of N.

3. construct the transition function of N.

$$\text{symbols}$$

state	a	b
ə-closure → $\{ \underbrace{\quad}_{A} \}$	$\overset{\text{new state}}{\cancel{\;}}$ $\boxed{B.}$	
A		
B ←		

ə-closure
of q_0

($\times$ redundant ə-closure of A) "a"

finds transitions on "a"

(B)

ə-closure of B

$r_i = (a \cup b)^* bc$

w_1 _____

w_2 _____
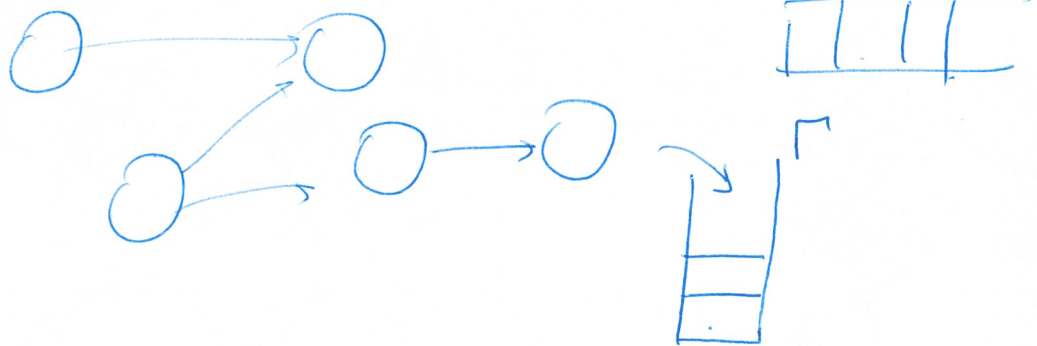
_____

_____

w_n _____

1. convert r_i to NFA-∂

2. convert NFA-∂ to DFA.

3. Simulate the DFA on each w_i.

   if DFA accepts $w_i \in r_i$

   if DFA rejects $w_i \notin r_i$
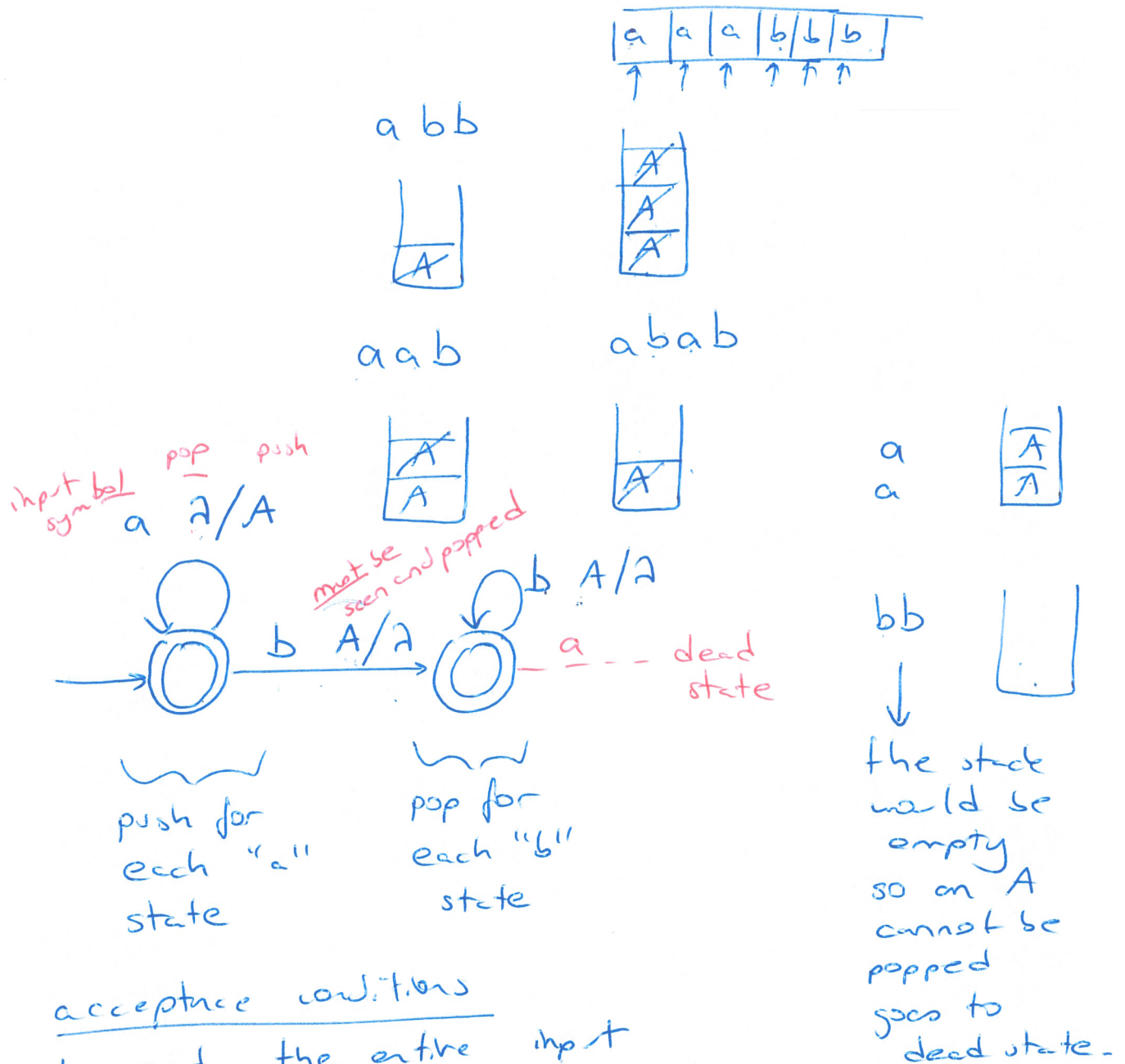
Section 7.1 Pushdown Automata.

PDA

A PDA is an NFA-∂ with a stack.

Σ

$\{ a^i b^i \mid i \geq 0 \}$

If we have a PDA, how can we recognize strings from

$$\{ a^i b^i \mid i \geq 0 \}$$

```
| a | a | a | b | b | b |
  ↑   ↑   ↑   ↑   ↑   ↑
```

a bb

aab   abab

input symbol   pop   push
        a   ā/A

b A/a   must be seen and popped   b A/a

b A/a

a - - - dead state

push for each "a" state

pop for each "b" state

a
a

bb
↓

the stack would be empty so on A cannot be popped goes to dead state.

b A/A

acceptance conditions

1. read the entire input

2. must be at an accepting state

3. the stack must be empty

Definition 7.1.1 A pushdown automaton (PDA)
is a sextuple $M = (Q, \Sigma, \Gamma, \delta, q_0, F)$

$F \subseteq Q$
set of accepting states

initial state

transition function

stack alphabet

input alphabet

set of states

example transition
$$(Q_1, a, A) = (Q_2, B)$$

$Q_1 \xrightarrow{a \; A|B} Q_2$

before

stack = Q_1

input... a b a

stack

A
B

after

state = Q_2

input: a b a

B
B

Example transitions involving a

a   A/a
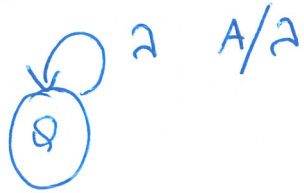
does not read
the input
only pops, does
not push

a   a/A
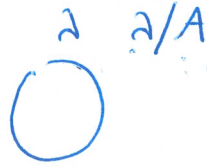
does not read
the input
does not pop,
only pushes

a   a/a

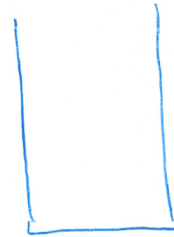does not
read the
input,
does not pop,
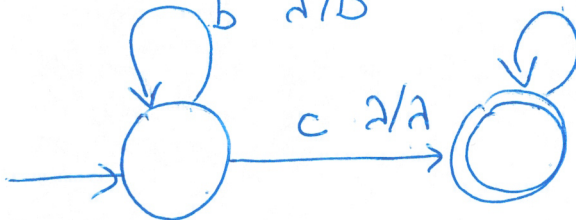does not push.

$$\{ w c w^R \mid w \in \{a, b\}^* \}$$

abcba   ✓
abbcbb   ✗
abbcbab   ✗

push until the separator
pop until empty

a   a/A
b   a/B

b   B/a
a   A/a

c   a/a
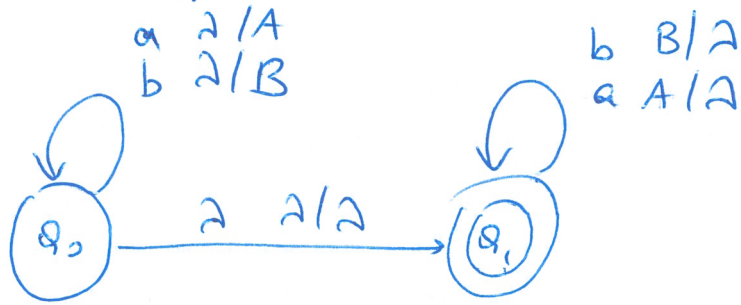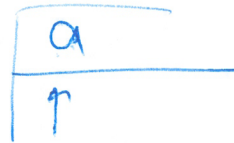

recognizing
the part before
the separator

recognizing
(matching)
the part after the separator

$\{ ww^R \mid w \in \{a,b\}^* \}$

q_0 self-loop:
a a/A
b a/B

q_1 self-loop:
b B/a
a A/a

$q_0 \xrightarrow{a \ a/a} q_1$

a

a
↑

A

abb   bba

a b b b a
↑ ↑↑↑ ↑

B
B
A