CS 3311 Formal Models of Computation
Sample Exam 3

Question 1. *(5 points)* Consider the following grammar G:

$$\begin{aligned}
S &\rightarrow TT \mid AT \mid BT \mid CT \\
T &\rightarrow aT \mid a \\
B &\rightarrow bB \mid bC \\
C &\rightarrow cB \\
A &\rightarrow dA \mid d
\end{aligned}$$

Part a. Construct the TERM set for G.

Part b. Use the TERM set to construct an equivalent grammar G_T that does not contain variables that do not generate strings of terminals.

Question 2. *(5 points)* Consider the following grammar G:

$$\begin{aligned}
S &\rightarrow AB \mid BC & \quad D &\rightarrow dD \mid d \\
A &\rightarrow aA \mid a & \quad E &\rightarrow eE \mid e \\
B &\rightarrow bB \mid b & \quad F &\rightarrow fF \mid f \\
C &\rightarrow DE & \quad H &\rightarrow hH \mid FH \mid h
\end{aligned}$$

Part a. Construct the REACH set for G.

Part b. Use the REACH set to construct an equivalent grammar G_U that does not contain unreachable variables.

Question 3. *(10 points)* Convert the following grammar G into Chomsky normal form. Show your steps clearly. Note that G already satisfies the conditions on the start symbol S, λ-rules, useless symbols, and chain rules.

$$\begin{aligned}
S &\rightarrow AACD \\
A &\rightarrow aAb \mid ab \\
C &\rightarrow aC \mid a \\
D &\rightarrow aDa \mid bDb \mid aa \mid bb
\end{aligned}$$

Question 4. *(15 points)* Consider the following grammar G. Note that G was obtained by transforming the grammar $\quad S \rightarrow bSb \mid aa \quad$ to Chomsky Normal Form.

$$\begin{aligned}
S &\rightarrow BR \mid AA \\
T &\rightarrow BR \mid AA \\
R &\rightarrow TB \\
A &\rightarrow a \\
B &\rightarrow b
\end{aligned}$$

Part 2a. *(10 points)* Give the upper diagonal matrix produced by the CYK algorithm when run with G and the input string $baab$. **Show all your work.**

Part 2b. *(5 points)* Is $baab \in L(G)$? Why? Provide the reason based on the upper diagonal matrix you constructed.

Question 5. *(15 points)* Consider the following grammar G. It is the same as the grammar as in Question 5. It is repeated for your convenience.

$$\begin{aligned}
&S \rightarrow BR \,|\, AA \\
&T \rightarrow BR \,|\, AA \\
&R \rightarrow TB \\
&A \rightarrow a \\
&B \rightarrow b
\end{aligned}$$

Part 3a. *(10 points)* Give the upper diagonal matrix produced by the CYK algorithm when run with G and the input string $bbaa$. **Show all your work.**

Part 3b. *(5 points)* Is $bbaa \in L(G)$? Why? Provide the reason based on the upper diagonal matrix you constructed.

Question 6. *(10 points)* Remove direct left recursion from the grammar shown below. Use the rule described in class. Work on one variable at a time and show the intermediate results.

$$\begin{aligned}
&S \rightarrow aE \\
&E \rightarrow EbT|T \\
&T \rightarrow TcF|F \\
&F \rightarrow dEd \,|\, e
\end{aligned}$$

Question 7. *(15 points)* Consider the following grammar G:

$$\begin{aligned}
&S \rightarrow aAbBc \,|\, BA \\
&A \rightarrow a \,|\, c \\
&B \rightarrow bb \,|\, bc
\end{aligned}$$

Part 5a. *(5 points)* Draw the graph of grammar G.

Part 5b. *(10 points)* Give the lookahead sets for each variable and rule of grammar G.

Question 8. *(15 points)* Give the state diagram of a **DFA** that accepts the set of strings of length 4 over $\{a, b\}$ that begin and end with the same symbol. Briefly explain how you construct the machine and **do not use nondeterminism**.

Question 9. *(10 points)* Use Theorem 5.5.3 and Example 6.1.1 to convert the regular expression
$(a \cup b)^*(ca \cup cb)^*$ into an NFA-λ. Apply the full steps and do not simplify the machine. Do not construct the machine directly.