# CS 3311 Formal Models of Computation — Sample Exam 2

**Question 1**. *(20 points)* Give a regular expression for the following sets:

**Part a.** $L = \{w \mid w \in \{a, b\}^*$ and $w$ begins and ends with $aa$ or $bb\}$

**Part b.** $L = \{w \mid w \in \{a, b\}^*$ and the number of $a$'s in $w$ is divisible by three $\}$

**Question 2**. *(10 points)* Consider the following grammar $G$:

$$\begin{aligned} S &\rightarrow aSb \mid A \\ A &\rightarrow cAd \mid B \\ B &\rightarrow eBf \mid \lambda \end{aligned}$$

**Part a.** Give a derivation for a terminal string such that the $S \rightarrow aSb$ rule is used exactly twice, and the $A \rightarrow cAd$ rule is used exactly once during the derivation.

**Part b.** Give a derivation for a terminal string such that the $S \rightarrow aSb$ rule is used exactly once, the $A \rightarrow cAd$ rule is not used, and the $B \rightarrow eBf$ rule is used exactly twice during the derivation.

**Part c.** Use set notation to define the language generated by the grammar.

**Question 3**. *(10 points)* Construct a **context-free grammar** over $\{a, b, c\}$ whose language is $\{a^n b^m c^{2n+m} \mid n, m > 0\}$. Explain how you construct the grammar.

**Question 4**. *(10 points)*

**Part a.** Construct a **context-free grammar** over $\{a, b, c\}$ whose language is $\{a^i b^j c^k \mid i = j$ or $j = k$ where $i, j, k \geq 0\}$. Explain how you construct the grammar.

**Part b.** Show that your grammar is ambiguous. Present a comprehensive proof with complete sentences.

**Question 5**. *(10 points)* Give a **context-free grammar** for the set of strings over $\{a, b\}$ where each string has an odd length, and contains exactly one '$b$'. Explain how the grammar generates the strings.

**Question 6.** *(10 points)* Consider the CFG $G$ defined by the following productions. Prove by induction that $L(G) = \{a^n b^m \mid 0 \leq n < m\}$. In other words, prove by induction that every string in $L(G)$ has the form $a^n b^m$ where $0 \leq n < m$.

$$S \to aSb \mid B$$
$$B \to bB \mid b$$

**Question 7**. *(20 points)* Consider the following grammar $G$ with a non-recursive start symbol ($S$):

$$\begin{array}{lll} S \to T & A \to aA \mid BC & B \to bB \mid \lambda \\ T \to ABC \mid aBC & & C \to cC \mid \lambda \end{array}$$

**Part a.** Show the set of nullable variables in $G$.

**Part b.** Construct an essentially noncontracting grammar $G_L$ (with a non-recursive start symbol) equivalent to $G$.