CS3311
Oct 12, 2012
Friday ①

Last time: context-free grammers
CFGs

$A \to aA \mid \lambda$ $\qquad a^*$

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$A \to aA \mid a$ $\qquad a^+$

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$\{a^i b^i \mid i \geq 0\}$

$S \to aSb \mid \lambda$

$S \Rightarrow a\underline{S}b \Rightarrow a\ \underline{aSb}\ b \Rightarrow aaaSbbb$

$\Rightarrow \underbrace{aaabbb}$
equal number of "a"s and "b"s, in order
(a's precede b's)

abbaab

Formally, a context-free grammar (CFG) is a ~~quadruple~~ quadruple $(V, \Sigma, P, S)$

$S$: starting symbol $S \in V$

$P$: set of production rules

e.g. $P = \{S \to aSb \mid \lambda\}$

$= \{S \to aSb, S \to \lambda\}$

$\Sigma$: alphabet (terminals, symbols)

$V$: set of variables (intended to be replaced)

~~$A \to (V \cup$~~

$V \to X$

"context-free" can replace without looking at the left or right of the variable.

$A \to X$

$A \in V$ $\quad X \in (V \cup \Sigma)^*$

$aA \to bcd$ } context-sensitive

has context

read

A null production is of the form $A \to \lambda$

$A \in V$

Let $G = (V, \Sigma, P, S)$ be a CFG and $v \in (V \cup \Sigma)^*$. The set of strings derivable from $v$ is defined as follows:

basis: $v$ is derivable from $v$

recursive step: if $u = xAy$ is derivable from $v$ and $A \to w \in P$, then $xwy$ is derivable from $v$.

$S \to aSb \mid c$

$S \Rightarrow \underbrace{\overset{x}{a}\boxed{S}\overset{y}{b}}_{v} \Rightarrow \overset{x}{a}\,\overset{w}{\boxed{aSb}}\,\overset{y}{b} \Rightarrow aacbb$

$S \to abc \in P$

The language of a grammar, $G$, denoted by $L(G)$ ~~is the~~ is all the strings derivable from the start symbol $S$.

$A \overset{*}{\Rightarrow} w$ derivable in 0 or more steps

$A \overset{+}{\Rightarrow} w$ derivable in 1 or more steps.

$L(G) = \{ w \mid w \in \Sigma^*, S \overset{*}{\Rightarrow} w \}$

* $L = \{ a^n b^m c^k \mid n, m, k \geq 0 \}$

$a^* \; b^* \; c^*$

$S \rightarrow A B C$

$A \rightarrow a A \mid \lambda$

$B \rightarrow b B \mid \lambda$

$C \rightarrow c C \mid \lambda$

shrinks, contracts

$S \Rightarrow ABC \Rightarrow aABC \Rightarrow aaABC \Rightarrow aaBC$

$\Rightarrow aabBC \Rightarrow aabC \Rightarrow aab$

$S \rightarrow aSb$

* $L = \{ a^n b^m c^k \mid n, m, k > 0 \}$

$S \rightarrow ABC$ $\quad B \rightarrow bB \mid b$ $\quad C \rightarrow cC \mid c$

$A \rightarrow aA \mid a$

* $L = \{ a^n b^m c^n \mid n, m \geq 0 \}$

$S \rightarrow aSc$ $\quad aSBc$

$B \rightarrow bB \mid \lambda$

$S \Rightarrow aSBc \Rightarrow a\,aSBc\,c$

$S \rightarrow aSc \mid T$

$T \rightarrow bT \mid \lambda$ $\quad \{b^*$

$S \Rightarrow aSc \Rightarrow aaScc \Rightarrow aaTcc \Rightarrow$

$aabTcc \Rightarrow aabcc$

* $L = \{a^n b^{2n} \mid n \geq 0\}$

$S \to aA \mid$

abb

aabbbb

aaabbbbbb

a

a bb

a bb bb

$S \to aSbb \mid \lambda$

* $L = \{a^n b^m c^m d^{2n} \mid n \geq 0, m > 0\}$

$S \to aSdd \mid T$ $\qquad \lambda \notin L$

$T \to bTc \mid bc$

* $L = \{a^n b^m c^m d^{2n} \mid n \geq 0, m \geq 0\}$

$S \to aSdd \mid T \qquad ; \lambda$

$T \to bTc \mid \lambda \qquad S \Rightarrow \lambda$

$S \Rightarrow T \Rightarrow \lambda \mid S \Rightarrow a\boxed{S}dd \Rightarrow a\boxed{aSdd}dd$

$\Rightarrow aaTdddd$

$S \to aTdd$

$T \to bTc \mid bc$