October 24, 2012
CS3311

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exam 01
exam 02
exam 03
exam 04

} lecture notes directories

Sample exam question 6

$S \to aSb \mid B$ $\qquad a^n b^m \quad 0 \le n < m$

$B \to bB \mid b$

guarantees $a^i b^i \quad i \geq 0$

$a^i S b^i \Rightarrow a^i B b^i$

guarantees at least one "b"

inductive proof

basis: prove for shortest string (shortest derivation steps)

$S \Rightarrow B \Rightarrow b$ ✓

$S \Rightarrow aSb \Rightarrow aBb \Rightarrow abb$ ✓

inductive hypothesis (IH)

n derivation steps to generate a string

$S \overset{n}{\Rightarrow} w$

For any string w in L(G), generated using n steps of derivation assume that w is of the form $a^n b^m \quad 0 \le n < m$.

inductive step

Show that for any string u derived using $n+1$ steps ~~then~~ u is of the form $a^n b^m$ $0 \le n < m$.

$$\underbrace{S \Rightarrow \text{- - - - - -} \Rightarrow w}_{n} \qquad a^n b^m$$

$$\underbrace{S \Rightarrow \text{- - - - - -} \Rightarrow u}_{n+1}$$

if n then $n+1$

example

3 $S \Rightarrow aSb \Rightarrow aBb \Rightarrow asb$

4 $S \Rightarrow aSb \Rightarrow aaSbb \Rightarrow aaBbb \Rightarrow aabbb$

4 $S \Rightarrow aSb \Rightarrow aBb \Rightarrow abBb \Rightarrow abbb$

number of 'a's = number of 'b's

$S \Rightarrow aSb \Rightarrow \underbrace{v_1 S v_2}$ $\Rightarrow v_1 B v_2 \Rightarrow$

$S \to aSb$ $\qquad$ $S \to B$ $\qquad$ $B \to bB$

$v_3 B v_4 \Rightarrow v_3 b v_4$

$a^i S b^i$ $\qquad$ $a^i B b^i$

$a^{i+1} S b^{i+1}$

if we obtain $n+1$ steps by adding another $S \to aSb$ rule

1 or more b's from $S \to B$

if we obtain $n+1$ steps by adding another ~~$S \to$~~ $B \to bB$ rule

Order matters (for TERM and REACH)

$S \to a \mid AB$

$A \to b$

"correct" order = TERM, REACH

$TERM = \{S, A\}$

$V - TERM = \{B\}$

$S \to a$

$A \to b$

$REACH = \{S\}$

$V - REACH = \{A\}$

$S \to a$ $\quad L(G) = \{a\}$

"incorrect" order REACH, TERM

$S \to a \mid AB$

$A \to b$

$REACH = \{S, A, B\}$

$V - REACH = \phi$

$TERM = \{S, A\}$ $\quad V - TERM = \{B\}$

$S \to a$

$A \to b$