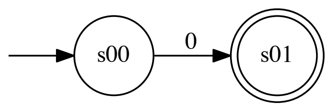
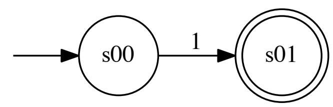
Construct a minimal DFA for regular expression 1(10|01)*1 following steps below:

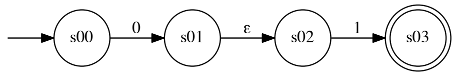
(a) Construct an equivalent NFS using Thompson's construction.



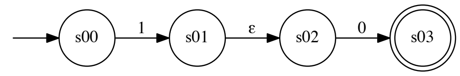
NFA Term: 0



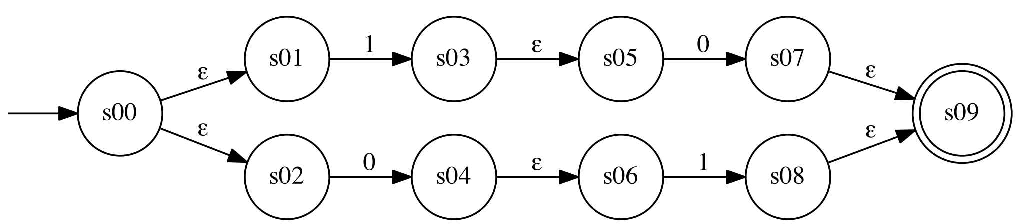
NFA Term: 1



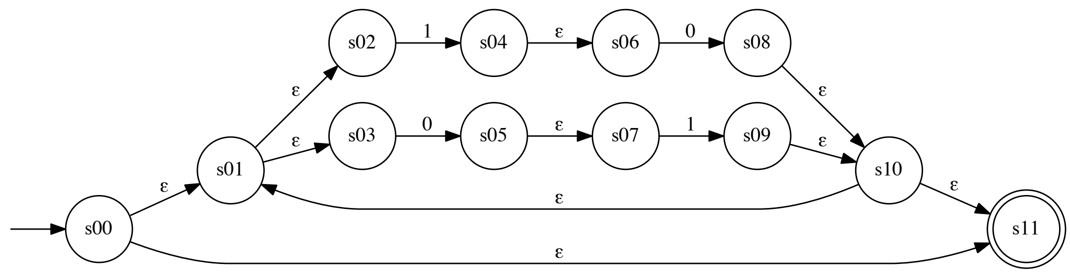
NFA Term: 01



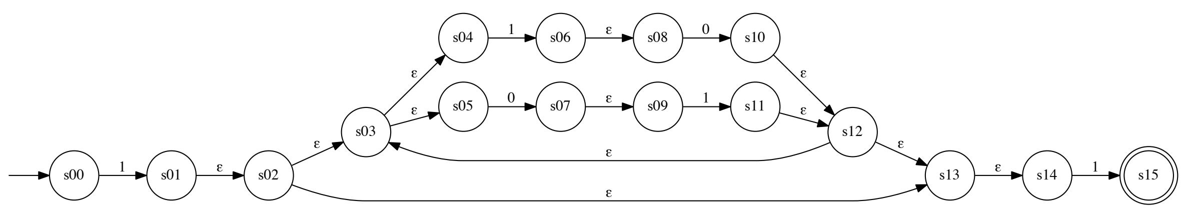
NFA Term: 10



NFA Term: 10|01



NFA Term: (10|01)*



NFA Term: 1(10|01)*1

(b) Convert the NFA to an equivalent DFA.

$q_0 = \varepsilon\text{-closure}(\{s_{00}\}) = \{s_{00}\}$ # Start state.

$T[q_0, 0] = \varepsilon\text{-closure}(\{\}) = q_e$
$T[q_0, 1] = \varepsilon\text{-closure}(\{s_{01}\}) = \{s_{01}, s_{02}, s_{03}, s_{04}, s_{05}, s_{13}, s_{14}\} = q_1$

$T[q_1, 0] = \varepsilon\text{-closure}(\{s_{07}\}) = \{s_{07}, s_{09}\} = q_2$
$T[q_1, 1] = \varepsilon\text{-closure}(\{s_{06}, s_{15}\}) = \{s_{06}, s_{08}, s_{15}\} = q_3$ # Final state.

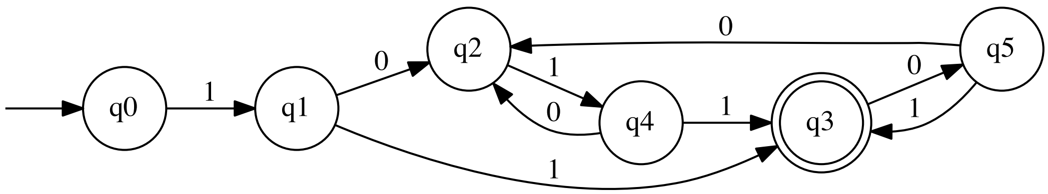
$T[q_2, 0] = \varepsilon\text{-closure}(\{\}) = q_e$
$T[q_2, 1] = \varepsilon\text{-closure}(\{s_{11}\}) = \{s_{03}, s_{04}, s_{05}, s_{11}, s_{12}, s_{13}, s_{14}\} = q_4$

$T[q_3, 0] = \varepsilon\text{-closure}(\{s_{10}\}) = \{s_{03}, s_{04}, s_{05}, s_{10}, s_{12}, s_{13}, s_{14}\} = q_5$
$T[q_3, 1] = \varepsilon\text{-closure}(\{\}) = q_e$

$T[q_4, 0] = \varepsilon\text{-closure}(\{s_{07}\}) = q_2$ # Already computed.
$T[q_4, 1] = \varepsilon\text{-closure}(\{s_{06}, s_{15}\}) = q_3$ # Already computed.

$T[q_5, 0] = \varepsilon\text{-closure}(\{s_{07}\}) = q_2$ # Already computed.
$T[q_5, 1] = \varepsilon\text{-closure}(\{s_{06}, s_{15}\}) = q_3$ # Already computed.

	0	1
q_0	q_e	q_1
q_1	q_2	q_3
q_2	q_e	q_4
q_3	q_5	q_e
q_4	q_2	q_3
q_5	q_4	q_3



Equivalent DFA.

(c) Minimize the DFA in step (b).

Final state:
$p_0 = \{q_3\}$

Other states:
$p_1 = \{q_0, q_1, q_2, q_4, q_5\}$

p_1	0
q_0	p_e
q_1	p_1
q_2	p_e
q_4	p_1
q_5	p_1

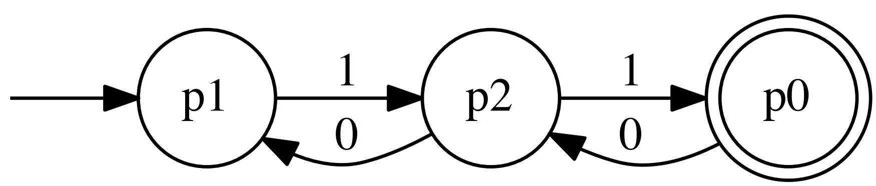
Split p_1 into 2 partitions:
$p_1 = \{q_0, q_2\}$
$p_2 = \{q_1, q_4, q_5\}$

Now,
$p_0 = \{q_3\}$
$p_1 = \{q_0, q_2\}$
$p_2 = \{q_1, q_4, q_5\}$

p_1	0	1
q_0	p_e	p_2
q_2	p_e	p_2

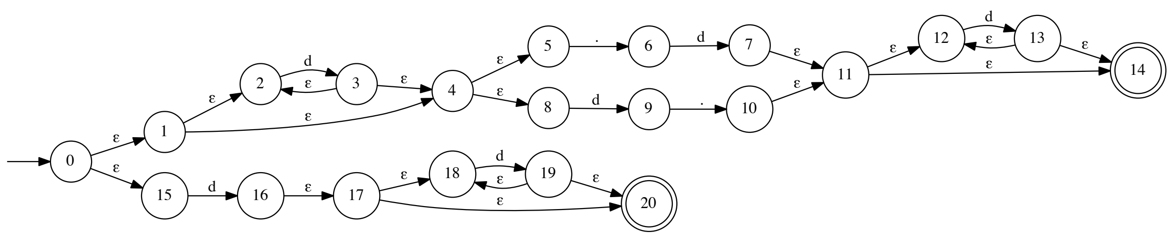
p_2	0	1
q_1	p_1	p_0
q_4	p_1	p_0
q_5	p_1	p_0

No change.



Minimized DFA.

Problem 2.5 in Section 2.6 at page 104.



NFA.

Note: I use the NFA in the textbook for decimals. The textbook constructs "concatenation" in a slightly different way than my notes.

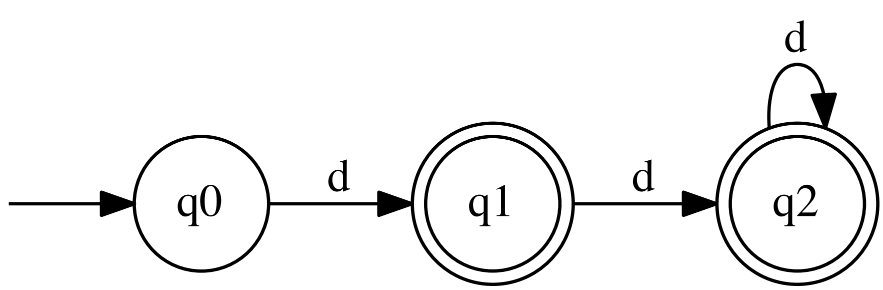
NFA → DFA

Here I try to reuse the results for decimals. So I assume the NFA → DFA → minimal DFA steps for decimals are done.

Below we construct minimal DFA for integers.

$q_0 = \varepsilon\text{-closure}(\{s_{15}\}) = \{s_{15}\}$

$T[q_0, d] = \varepsilon\text{-closure}(\{s_{16}\}) = \{s_{16}, s_{17}, s_{18}, s_{20}\} = q_1$ # Final state
$T[q_1, d] = \varepsilon\text{-closure}(\{s_{19}\}) = \{s_{18}, s_{19}, s_{20}\} = q_2$ # Final state
$T[q_2, d] = \varepsilon\text{-closure}(\{s_{19}\}) = q_2$



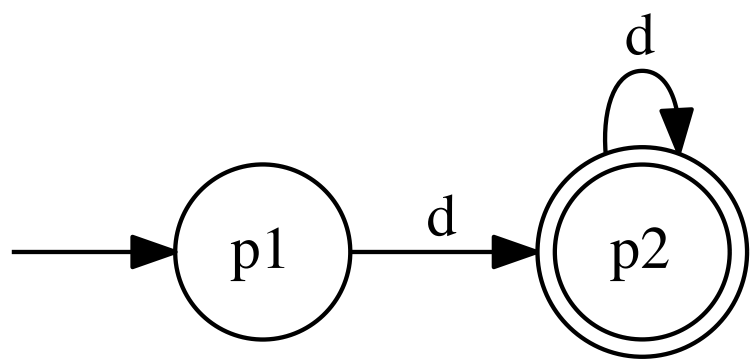
DFA → minimal DFA for integer

$p_1 = \{q_0\}$, $p_2 = \{q_1, q_2\}$

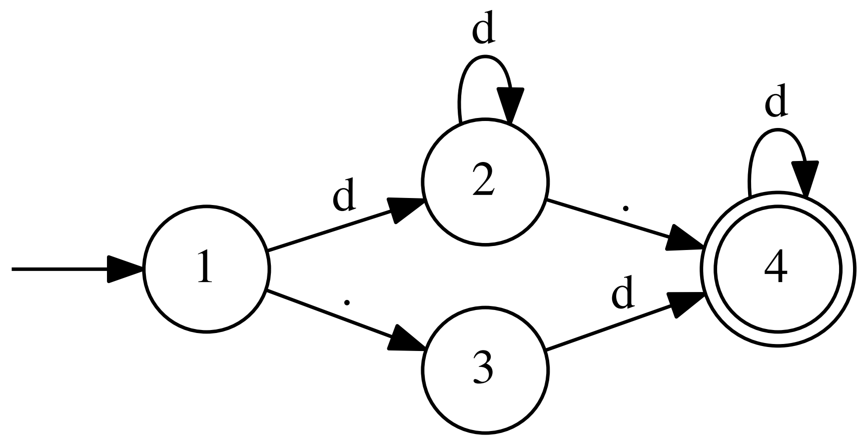
Try p_2,

	d
q_1	p_2
q_2	p_2

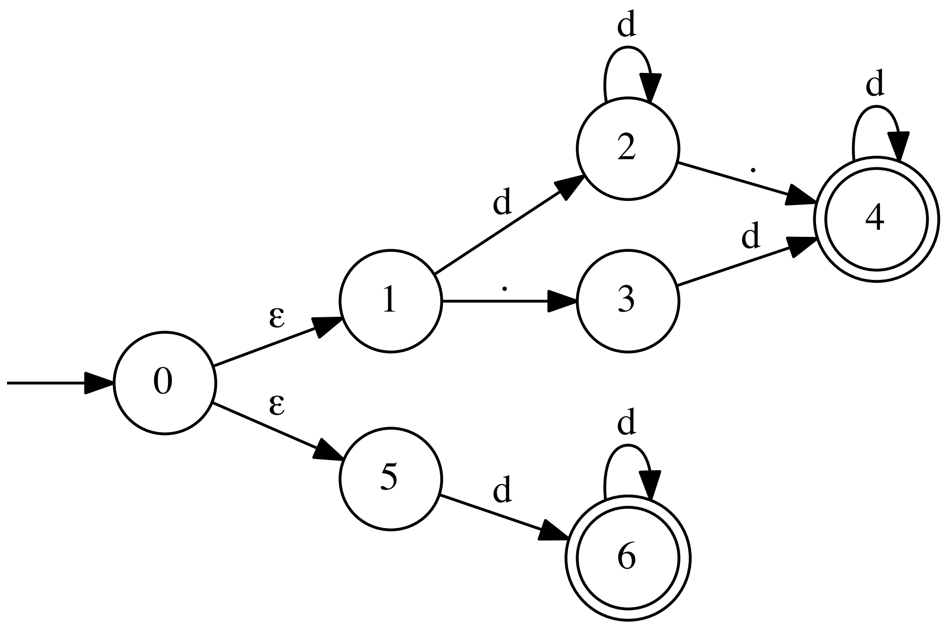
no further partitioning needed, we get:



The textbook gives the minimal DFA for decimals as follows:



Combine both, we have NFA below:



State t_4 accepts decimals.
State t_6 accepts integers.

Note that it is still an NFA. We now transform it to DFA and then minimal DFA.

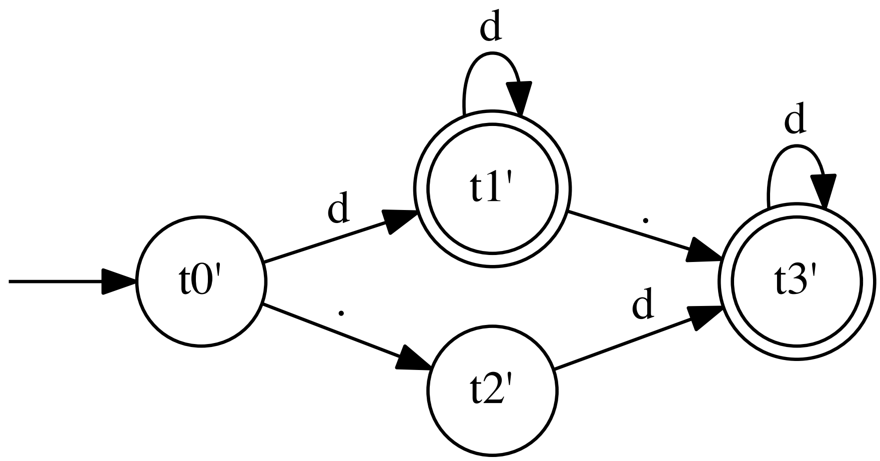
$t'_0 = \varepsilon\text{-closure}(\{t_0\}) = \{t_0, t_1, t_5\}$ # Start state.

$T[t'_0, d] = \varepsilon\text{-closure}(\{t_2, t_6\}) = \{t_2, t_6\} = t'_1$ # Final state for integer
$T[t'_0, .] = \varepsilon\text{-closure}(\{t_3\}) = \{t_3\} = t'_2$

$T[t'_1, d] = \varepsilon\text{-closure}(\{t_2, t_6\}) = t'_1$
$T[t'_1, .] = \varepsilon\text{-closure}(\{t_4\}) = \{t_4\} = t'_3$ # Final state for decimal

$T[t'_2, d] = \varepsilon\text{-closure}(\{t_4\}) = t'_3$
$T[t'_2, .] = \emptyset$ # error.

$T[t'_3, d] = \varepsilon\text{-closure}(\{t_4\}) = t'_3$
$T[t'_3, .] = \emptyset$



DFA → minimal DFA

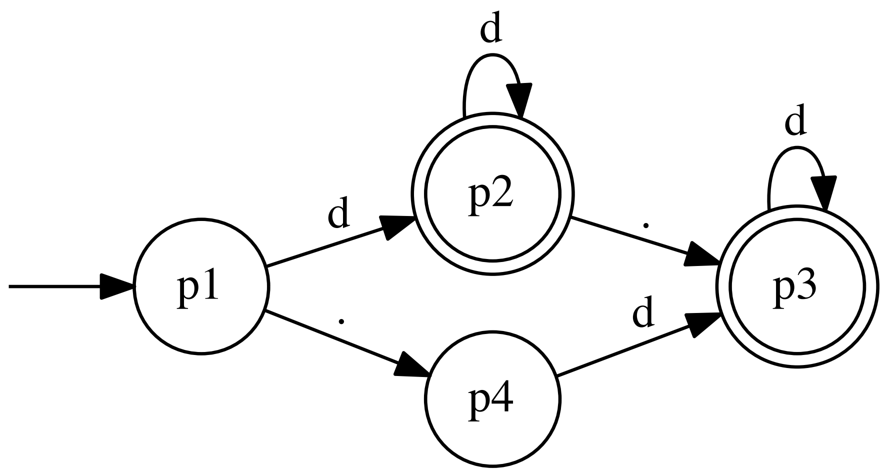
$p_1 = \{t'_0, t'_2\}$, $p_2 = \{t'_1\}$, $p_3 = \{t'_3\}$

Note that we put t'_1, and t'_3 into two different partitions to distinguish integer from decimal.

Try p_1,

	d
t'_0	p_2
t'_2	p_3

split p_1 to $p_1 = \{t'_0\}$, $p_4 = \{t'_2\}$.


d
d
p2
.
d
p1
p3
.
d
p4

Minimize the DFA.

$P_1 = \{q_0, q_1, q_3\}$
$P_2 = \{q_2, q_4\}$

Try P_1.

	a
q_0	p_1
q_1	p_2
q_3	p_2

Partition P_1 by "a" to $P_1 = \{q_0\}$, $P_3 = \{q_1, q_3\}$.
Now $P_1 = \{q_0\}$, $P_2 = \{q_2, q_4\}$, $P_3 = \{q_1, q_3\}$.

Try P_2.

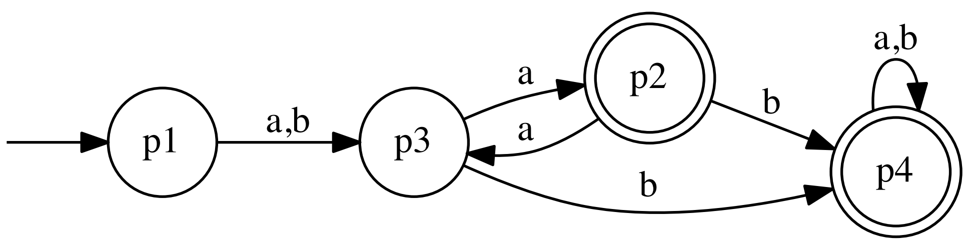
	a
q_2	p_3
q_4	p_2

Split P_2 by "a" to $P_2 = \{q_2\}$, $P_4 = \{q_4\}$.
Now $P_1 = \{q_0\}$, $P_2 = \{q_2\}$, $P_3 = \{q_1, q_3\}$, $P_4 = \{q_4\}$.

Try P_3.

	a	b
q_1	p_2	p_4
q_3	p_2	p_4

No further partitioning needed.



Minimized DFA.