Kraus

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In this project we formalize parts of the article *Unbounded length minimal synchronizing words for quantum channels over qutrits* and notions in quantum information theory.

In particular we formalize the notion of a measure-once one-way general quantum finite automaton (MO-1gQFA) introduced by Li, Qiu, Zou, Li, Wu, Mateus [1] in 2012. As an alternative to "MO-1gQFA" we refer to these simply as Kraus operator automata. In particular we formalize cut-point languages as in [2].

Definition 1. The operation $\rho \mapsto \sum_i K_i \rho K_i^*$.

Lemma 2. *Each projective is PSD.*

Lemma 3. *A probability mass function on two orthogonal projections $P, P^\perp$, with measure equal to the trace of the density matrix ρ times the projection.*

Definition 4. Transition function δ^* corresponding to a word over an alphabet, where each symbol is mapped to a completely positive map in Kraus form, of rank at most r.

Lemma 5. *A projection-valued measure, with measure equal to the trace of the density matrix ρ times the projection.*

Lemma 6. *If A and B are PSD then AB has nonnegative trace.*

Lemma 7. *If P is a projection and ρ is PSD then the trace of $P\rho$ is nonnegative.*

Definition 8. Projection-valued measure.

Definition 9. Quantum channel.

Lemma 10. *Kraus operators preserve the PSD property.*

Lemma 11. *An automaton based on a quantum channel maps density matrices to density matrices while reading a single letter.*

Lemma 12. *An automaton based on a quantum channel maps density matrices to density matrices while reading a word.*

Lemma 13. *A basis state density matrix has trace one.*

Lemma 14. *A pure state is PSD.*

Definition 15. The projection-valued measure corresponding to a word belonging to the measure-once language of the KOA $\mathcal{K}$.

Definition 16. We accept a word if starting in e_0 we end up in e_1 with probability at least 1/2.

References

[1] Lvzhou Li, Daowen Qiu, Xiangfu Zou, Lvjun Li, Lihua Wu, and Paulo Mateus. Characterizations of one-way general quantum finite automata. *Theoret. Comput. Sci.*, 419:73–91, 2012.

[2] Abuzer Yakaryılmaz and A. C. Cem Say. Unbounded-error quantum computation with small space bounds. *Inform. and Comput.*, 209(6):873–892, 2011.