UNPACKING THE NATURE OF DISCOURSE in Mathematics Classrooms

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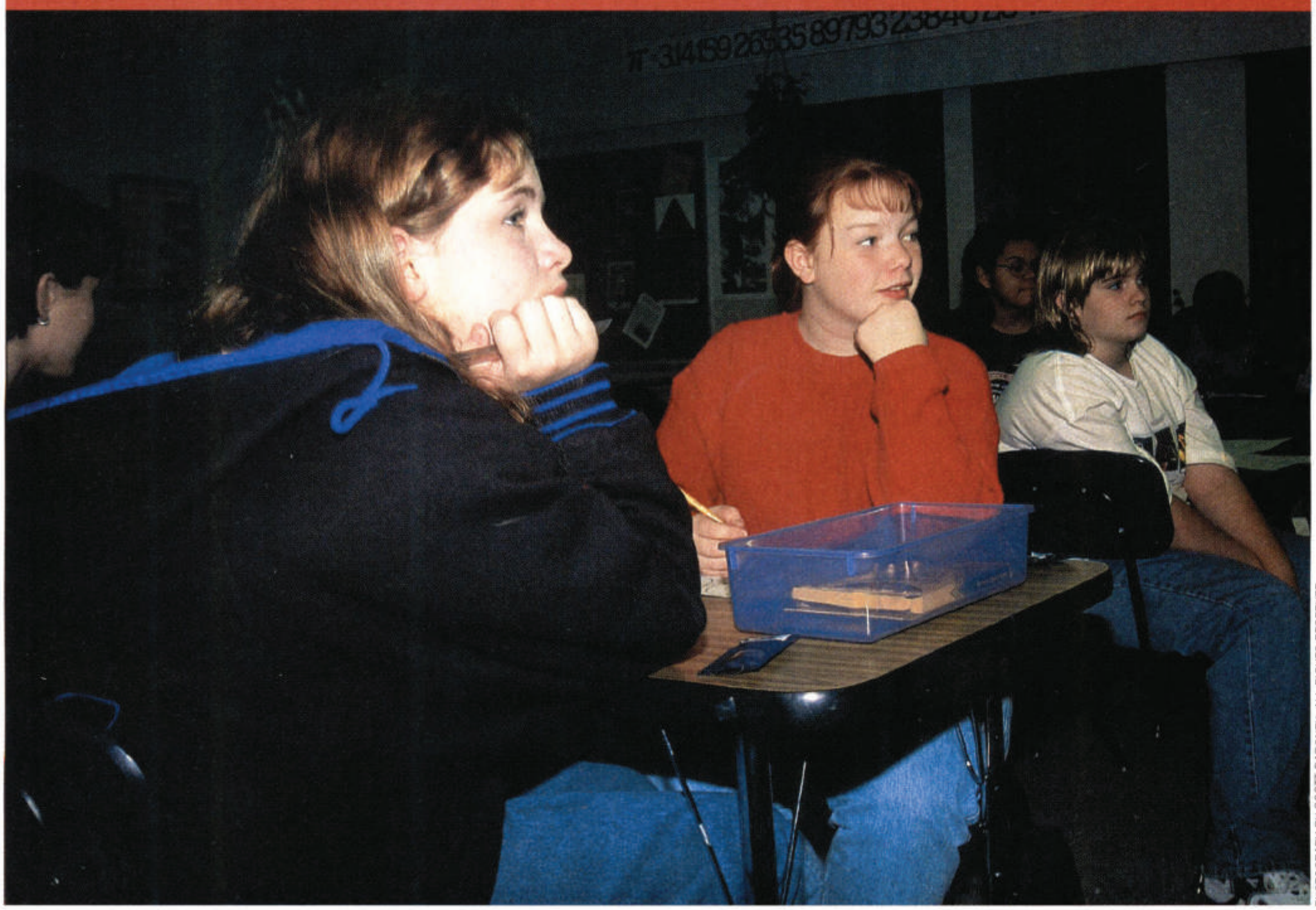

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THE ROLE OF DISCOURSE, ALTHOUGH always central in education and learning, is receiving increased attention in classrooms today as mathematics educators strive to better understand the factors that lead to increased learning. Indeed, scholars have argued for—and reform initiatives underscore—the importance of teachers' and students' engaging in discourse of various kinds (e.g., Ball 1991; NCTM 1991, 2000; Steinbring, Bussi, and Sierpinska 1998). These calls for more meaningful discourse are grounded in the social nature of mathematics learning, a vision of school mathematics practices that reflects both the essence of practices in the discipline itself and the need for students to be able to communicate their mathematical knowledge in a technological society.

As we move into the twenty-first century, efforts to enhance school mathematics teaching and learning continue. Building on research findings, previous reform recommendations, and lessons learned from past reform efforts, the NCTM's *Principles and Standards for School Mathematics* (2000) offers a renewed vision of school mathematics. Fostering meaningful mathematical discourse in classroom settings continues to be a central focus. This article describes a framework for examining mathematical discourse and shows how to apply this framework to appreciate the complex relationship between discourse and understanding in mathematics. In particular, we focus our attention on the role of discourse and the different types of discourse that emerge as students solve a particular task.

A Framework for Examining Discourse in a Mathematics Classroom

WE HAVE WORKED WITH CLASSROOM TEACHERS over the last four years in a longitudinal professional development project funded by the Colorado Commission of Higher Education. Our focus has been to help teachers better understand and implement reform-based mathematics instruction and assessment. One of our central goals was to help teachers foster more meaningful discourse in their classrooms. We soon realized, however, the difficulties involved in discussing "meaningful mathematical discourse." We drew on the work of Yuri Lotman to develop a framework to help us make sense of the different roles that discourse plays.

Lotman (1988) suggested that all discourse is distinguished by two very different functions: to convey meaning and to generate meaning. Wertsch (1991) used the terms ***univocal*** and ***dialogic,*** respectively, to represent these two functions. Univocal discourse is characterized by communication in which the listener receives the "exact" message that the speaker intends for the listener to receive. Once the speaker's intended meaning has been conveyed, the episode of univocal communication is considered to be successfully finished. Dialogic discourse, in contrast, is characterized by give-and-take communication in which the listener ***initially*** receives the "exact" message sent by the speaker. At this point, univocal discourse ends, but dialogic discourse has just begun. Dialogic discourse generates meaning by using dialogue as a "thinking device" (Lotman 1988, p. 36). The visions of reform-based mathematics education embody dialogic discourse in which both teachers and students are responsible for contributing to discussions.

The classroom vignettes in the next section portray the same teacher using the same task to foster both types of classroom discussions. These vignettes, which illustrate the interactions between a teacher and her students and the interactions among the students, are based on classroom observations of mathematics teachers participating in our professional development project and on our experiences as former secondary school mathematics teachers. Each vignette describes identical solution approaches, but the teacher's and students' responses are different and illustrate distinctions between the two types of discourse. Our primary focus is on the nature of the discourse demonstrated in each vignette.

The Dual Role of Discourse in Mathematics

Different types of discourse emerge as students solve different tasks

STUDENTS IN MS. BEE'S SEVENTH-grade mathematics class had spent the previous two weeks working with patterns. One of the goals of this work was for students to develop the ability to generalize relationships found in the patterns. For this activity, the students were challenged to find the sum of the first one hundred positive integers (i.e., $1 + 2 + 3 + \cdots + 100$). After introducing the problem, Ms. Bee asked the students to work on determining a solution in small groups. The following vignettes illustrate two possible ways that this activity could unfold.

An example of univocal discourse

(1) "What number are you on, Helen?" asked Andy as he stopped entering numbers into his calculator.

(2) "Thirty-two," she replied.

(3) "Tell me what your total is when you get to 41, so we can check our answers."

(4) Andy waited while Helen continued to enter the numbers on her calculator. The two other students in the group, Barney and Gomer, were working together to look for possible patterns. They compared the sum of the first ten numbers ($1 + 2 + \cdots + 10 = 55$) with the sum of the next ten numbers ($11 + 12 + \cdots + 20 = 155$) and made a conjecture that the sum of the numbers 21 through 30 would also increase by 100. At this point, Ms. Bee walked up to the group.

(5) "How are the four of you coming along?"

Univocal discourse focuses on sending an exact message

(6) Andy quickly spoke up, "We're almost halfway there. I've added the first forty-one numbers and am waiting to check my total with Helen's."

(7) "How about the two of you? Are you also using the calculator to find the sum?"

(8) "No, we've been looking for a pattern, and I think we found one," Gomer responded.

(9) "Great. First of all, Andy, why don't you tell me what sum you have found?" asked Ms. Bee.

(10) Andy replied, "1720."

(11) "That seems a little high. Have you found your sum yet, Helen?"

(12) "No, I had to start over because I hit a wrong key."

(13) "Let's think about what the sum must be less than. For example, suppose we were adding forty-one 41s, then the sum would be 41 times 41. Barney or Gomer, can you use your calculator to find that product?"

(14) Barney entered the numbers into his calculator and responded, "The answer is 1681."

(15) "You can see that your sum, Andy, is greater than this number. You might want to double-check your calculations. That's one disadvantage of using the calculator; it's easy to make an entry mistake. I'd like to see both you and Helen try to use some of what we've been learning about patterns in the past couple of weeks to find the sum. The two of you might check in with Barney and Gomer to see what they are doing."

(16) Ms. Bee next turned her attention to Barney and Gomer. "Now, Barney, you said that the two of you thought you may have found a pattern. Tell me what you're doing." At this point, both Andy and Helen have gone back to entering numbers on their calculators.

(17) "We found the sum of the first ten numbers, then the next ten numbers. We were just getting ready to check the sum of the next ten numbers when you came up."

(18) "It sounds as if you are trying to find a pattern by looking at the sums of different groups of ten numbers. What you might try is a similar approach of breaking down the one hundred numbers into smaller sums, but start by looking at the sum of the first two numbers, then the first three numbers, and so on. See if this results in some kind of pattern."

(19) Gomer looked a bit puzzled. "Do you mean do $1 + 2$, then $1 + 2 + 3$, and keep going like that?"

(20) "Right. Remember how we found some of the patterns for the problems last week? Try that same approach, and see what you come up with."

(21) "OK. You want us to make a chart showing each total, then look for a pattern in the chart," Barney responded.

(22) "Right," Ms. Bee replied as she began to move toward another group of students. Barney and Gomer each pulled a new sheet of paper from their notebooks and began to work on the approach that Ms. Bee suggested.

Throughout this episode, Ms. Bee attempts to both hear what the students are doing (lines 5, 7, and 16) and to direct them to solve the problem in a specific way that she has emphasized for similar problems (lines 15 and 18). Ms. Bee first acknowledges the calculator approach used by both Andy and Helen, but she does not give them an opportunity to make sense of Andy's apparent miscalculation. Rather, she comments that his sum for the first forty-one numbers "seems a little high" and suggests that the students think about an upper limit for the sum (line 13). In asking the students to consider what the upper limit might be ("Let's think"), she encourages them to treat her suggestion as a "thinking device"; yet she suggests a procedure for determining the limit—perhaps in an effort to make sure that the students' understanding coincides with her own. Ms. Bee does not ask the students to consider her suggestion or to explain their understanding of the relationship of her calculation to Andy's miscalculation. Finally, Ms. Bee directs the students toward the method that she would like for them to use (last two sentences in line 15) instead of allowing them to generate their own approach.

Ms. Bee also elicits the approach that Barney and Gomer have been using (line 16). Her initial response (first sentence of line 18) verifies the accuracy of the received message (line 17). Next, she attempts to reorient the students toward her preferred approach (second sentence of line 18). This exchange is univocal because Ms. Bee does not try to understand the students' method—she does not hear what she wants to hear, thus she redirects the students. In Lotman's (1988) terms, she noted a difference between the expected message (her solution approach) and the received message (the students' solution approach) and perceives the discrepancy as a "defect in the communications channel" (p. 36). Gomer's question (line 19) indicates that he received her message as intended, and Ms. Bee confirms that their understandings match. To ensure that her message has been adequately conveyed, she then attempts to strengthen the match by providing a shared reference point: "Remember how we found some of the patterns for the problems last week?" Finally, Barney recognizes this shared reference by further explaining how he and Gomer will follow Ms. Bee's suggestion by using a chart (line 21). Ms. Bee then acknowledges that the students understand what she intended to communicate and that her message has been successfully conveyed (line 22).

This passage is primarily univocal because the teacher's intention is to convey the message that the students should use a particular approach. Ms. Bee does her best to move the students toward her solution method as she strives to align the students' thinking about the problem with her own. She makes sure that her intended message for this particular lesson, as well as for their work during the preceding two weeks, is adequately conveyed. Her focus is on how well everyone understands her perspective rather than on making sense of the students' unique approaches to the problem. In contrast, the following vignette highlights the role of the task, the students' solution approaches, and the teacher's comments as generators of meaning—the essence of dialogic discourse.

An example of dialogic discourse

(1) "What number are you on, Helen?" asked Andy as he stopped entering numbers into his calculator.

(2) "Thirty-two," she replied.

(3) "Tell me what your total is when you get to 41, so we can check our answers."

(4) Andy waited while Helen continued to enter the numbers on her calculator. The two other students in the group, Barney and Gomer, were working together to look for possible patterns. They compared the sum of the first ten numbers ($1 + 2 + \cdots + 10 = 55$) with the sum of the next ten numbers ($11 + 12 + \cdots + 20 = 155$) and made a conjecture that the sum of the next ten numbers, 21 through 30, would also increase by 100. At this point, Ms. Bee walked up to the group.

Dialogic discourse focuses on two-way communication

(5) "How are the four of you coming along?"

(6) Andy quickly spoke up, "We're almost halfway there. I've added the first forty-one numbers and am waiting to check my total with Helen's."

(7) "How about the two of you? Are you also using the calculator to find the sum?"

(8) "No, we've been looking for a pattern, and I think we found one," Gomer responded.

(9) "Great. First of all, Andy, why don't you tell me what sum you have found?" asked Ms. Bee.

(10) Andy replied, "1720."

(11) "That seems a little high. Have you found your sum yet, Helen?"

(12) "No, I had to start over because I hit a wrong key."

(13) Addressing the entire group, Ms. Bee asked, "Why do you think I said that Andy's sum seems a little high?"

(14) "Because you already know the answer," Helen suggested, smiling.

(15) "Well, let's think about what the sum must be less than. Any ideas on how we might determine an upper limit for the sum? Barney and Gomer, what do you think?"

(16) "It's probably less than a million," Gomer suggested.

(17) "OK, I'd agree with that. Can we think of a way, using mathematics, to find an upper limit?"

(18) "What if we only added half of the numbers and then multiplied it by 2?" suggested Helen.

(19) "That would not be an upper limit because the upper half of the numbers is greater than the lower half of the numbers," argued Barney.

(20) "So what if we took the upper half of the numbers and multiplied that total by 2?" Andy asked.

Communication mismatch can be a point of departure for inquiry

(21) At this point, Ms. Bee decided to involve the rest of the class in this discussion. She asked for the students' attention and described what this group was attempting to figure out. After a brief discussion, Ms. Bee turned her attention back to Barney and Gomer and asked about the pattern that they claimed to have found.

(22) "Now, Barney, you said that the two of you thought you may have found a pattern. Tell me what you're doing." At this point, both Andy and Helen have gone back to entering numbers into their calculators.

(23) Barney described their approach: "Well, we found the sum of the first ten numbers, then the sum of the next ten numbers. We were just getting ready to check the sum of the next ten numbers when you came up."

(24) "So it sounds as if you are trying to find a pattern by looking at the sums of different groups of ten numbers. So what is it you are looking for with these sets of numbers?" Ms. Bee asked.

(25) "We are checking for a possible pattern. Since the sum of the second set of ten numbers went up by 100 compared with the sum of the first set of ten numbers, we think that the sum of the next set of ten numbers will also go up by 100."

(26) "So what will that tell you?"

(27) Barney began to respond, "If it works . . . ," when Gomer interrupted, "We'll know what the pattern is, then it will be easy to find the final total."

(28) "Cool. I haven't seen that approach before in any of my other classes. It will be interesting to try to figure out why that works." At this point, Ms. Bee moved on to another group of students.

A number of aspects of this dialogue embody characteristics of dialogic discourse. Again, Ms. Bee first attempts to hear what the students are doing (lines 5, 7, 9, and 22), but then she continues to listen and uses the students' discourse to generate meaning for both herself (lines 24 and 26) and her students (lines 13, 15, and 17). Ms. Bee again acknowledges the calculator approach used by both Andy and Helen; however, she takes advantage of a learning opportunity by using Andy's apparent miscalculation as a point of departure to look for an upper limit. In Ms. Bee's mind, Andy's error offered an unexpected, yet appropriate, avenue to further explore important mathematics. Rather than tell Andy and Helen her procedure for finding an upper limit, Ms. Bee turns to the whole class for further inquiry and discussion. She prompts students three times to treat her suggestion as a thinking device and gives them repeated opportunities to consider her suggestions (lines 13, 15, and 17). She also directs students to use mathematics (line 17), at which point we begin to see students generate their own meanings by treating one another's statements as thinking devices (lines 18 through 20).

As in the previous episode, Ms. Bee again elicits the solution approach that Barney and Gomer have been working on (line 22). Her initial response (first sentence of line 24) verifies the accuracy of the received message (line 23)—an aspect of univocal discourse necessary for clear communication. Rather than guide Barney and Gomer to her solution method, however, Ms. Bee asks what they were looking for with their approach (line 24) and how their approach will help them solve the original problem (line 26). Ms. Bee is listening dialogically, using the students' responses as generators of meaning to better understand what they are thinking. In Lotman's terms, she notes a difference between the expected message (her solution approach) and the received message (the students' solution approach). She perceives this mismatch not as a "defect in the communications channel"

(Lotman 1988, p. 36) but rather as a point of departure to generate new meanings for herself and her students; thus, the students' and teacher's utterances function as thinking devices. When Ms. Bee leaves Barney and Gomer alone to explore how their pattern might be used to solve the problem, she and her students also seem comfortable in sharing the mathematical authority in the classroom (lines 27 and 28).

This passage is primarily dialogic because (*a*) the teacher intends to understand her students' thinking, (*b*) the teacher uses her students' statements as thinking devices, and (*c*) the students use Ms. Bee's suggestion and their classmates' statements as thinking devices. Ms. Bee does not attempt to convey a particular message, that is, to engage students in a specific approach. Instead, she is open to her students' ideas and is willing to pursue unexpected approaches to generate new mathematical understanding—the core of dialogic discourse.

Meaningful Mathematical Discourse in Classrooms

WE REALIZE THAT THE DISTINCTION BETWEEN univocal and dialogic discourse is at times difficult to discern. Indeed, in any social interaction involving spoken communication, each individual must both decipher what is said and generate his or her own meaning from it. Consequently, all discourse is, to some degree, both dialogic and univocal. In other words, discourse may be thought of as a continuum that is more or less dialogic or univocal. We find that most discourse, however, is characterized primarily by one of these functions. We often look to the speaker's intent—to transfer meaning or generate new meaning—to determine which function is more prevalent. In a similar fashion, we also examine the listener's intent in making sense of classroom discourse.

To recognize the dual role of discourse in classrooms, we as teachers must reflect on our instructional goals and how they relate to our intentions as speakers and listeners and to our students' intentions. In the first vignette, the goal in Ms. Bee's mind was for her students to arrive at a solution using what they had previously learned about patterns. Ms. Bee's students understood that she had a particular idea of what mathematical approach she wanted them to use. When she and her students engaged in discourse, the intentions of both teacher and students as they spoke and listened reflected this overall understanding. The vignette, therefore, reflects the goals of the teacher and, as a result, is mostly univocal. In the second vignette, Ms. Bee's goals were clearly different because she was more interested in pursuing students' ideas and seeing where those ideas led mathematically. The teacher and students understood that the classroom goals were much more dynamic, and although the focus was still on finding patterns, the speakers' and listeners' intentions had changed because the class was not searching for the teacher's way to solve the problem. The students in the second scenario were sharing the mathematical authority of the classroom, using the teacher's and one another's statements as thinking devices to generate new, and sometimes unexpected, mathematical meaning.

We are not arguing that one of these vignettes is necessarily better than the other. Both univocal and dialogic can be seen as appropriate forms of discourse, depending on the daily instructional goals. We also recognize the reality of a teacher's classroom, which includes the competing demands of depth versus breadth in content coverage, the presence of students of dissimilar abilities and interests, and time constraints. These factors often influence the classroom goals, which in turn influence the nature of discourse. Nevertheless, we recognize the need for students and teachers to engage in more dialogic discourse; students will acquire a deeper understanding of mathematics when they use their own statements, as well as those of their peers and teacher, as thinking devices. This goal speaks to the heart of reform-based mathematics instruction, which is the hope that the accompanying pedagogical approaches and strategies will lead students to acquire a deeper understanding of mathematics.

Understanding deepens when students use their own statements

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