Student Work:
Suppose that the population of bacteria in a test tube t hours after 8am is given by $B = N(t)$, where

$$B = N(t) = 200 + 12\sqrt[3]{t}$$

(a) Find a formula for $N^{-1}(B)$.

$$\begin{aligned} B &= 200 + 12\sqrt[3]{t} \\ \frac{B}{200} &= 12\sqrt[3]{t} \\ \frac{B}{200 \cdot 12} &= \sqrt[3]{t} \\ t &= \left(\frac{B}{2400}\right)^3 \end{aligned}$$

So the inverse is $\boxed{N^{-1}(B) = \left(\frac{B}{2400}\right)^3}$

(b) Calculate and interpret $N^{-1}(254)$. *Write your interpretation in a complete sentence with units.*

$$\begin{aligned} 254 &= 200 + 12\sqrt[3]{t} \\ \frac{254}{200} &= 12\sqrt[3]{t} \\ \frac{254}{200 \cdot 12} &= \sqrt[3]{t} \\ t &= \left(\frac{254}{2400}\right)^3 \approx 0.001185 \end{aligned}$$

The number of bacteria after 0.001185 *hours is 254.*

Rubric 1:
Suppose that the population of bacteria in a test tube t hours after 8am is given by $B = N(t)$, where

$$B = N(t) = 200 + 12\sqrt[3]{t}$$

(a) Find a formula for $N^{-1}(B)$.

Solution:

$$N^{-1}(B) = \left(\frac{B - 200}{12}\right)^3$$

- 1 point for setting the equation equal to B
- 2 points for correctly solving for t
- 2 point for writing this as an inverse function using the appropriate variable.

(b) Calculate and interpret $N^{-1}(254)$. *Write your interpretation in a complete sentence with units.*

Solution:

$$N^{-1}(254) = 91.125$$

- 1 point for plugging in 254 into $N^{-1}(B)$.
- 2 points for correct work and getting the above answer
- 2 point for a complete sentence with units

Student Reflection:

(a) *The inverse function always has its inputs and outputs swapped from the original function. This is because with the inverse function, the inputs become the output and output becomes the input. Given* $B = N(t)$*, the output variable is* B*, and the input variable is* t*, so by solving for* t*, we get* $t = N^{-1}(B)$ *as the inverse function, where the output is now* t *and the input is now* B*.*

(b) *The input to* $N^{-1}(B)$ *is* B*, the output of* $N(t)$*, so I can just redo my work from part (a) to find* t *when* $B = 254$*.* B *represents the outputs of* $B = N(t)$*, (the number of bacteria), and* t *represents the inputs (the number of hours). So the number of hours it takes to get get* 254 *bacteria is whatever* t *is when I solve for it.*

Discussion Questions:

1. What do you think the goals of the problem that the student solved?

2. Now that you've read the students reflection, look back to their work. Do you think they have met the goals of this problem?

3. Do you think **Rubric 1** accurately assessed whether the student has met the goals of the problem? Identify parts of the rubric to support your answer.

Rubric 2:
Suppose that the population of bacteria in a test tube t hours after 8am is given by $B = N(t)$, where

$$B = N(t) = 200 + 12\sqrt[3]{t}$$

(a) Find a formula for $N^{-1}(B)$.

Solution:

$$N^{-1}(B) = \left(\frac{B - 200}{12}\right)^3$$

There are a few solution paths but grading should be roughly equivalent to

- 1 point for setting the equation equal to B
- 1 point for an attempt to solve in terms of t
- 1 point for a correct solution for t
- 2 points for writing this as an inverse function using the appropriate variable.

(b) Calculate and interpret $N^{-1}(254)$. *Write your interpretation in a complete sentence with units.*

Solution:

$$N^{-1}(254) = 91.125$$

- 1 point for setting $B = 254$ in either $B = N(t)$ or $N^{-1}(B)$.
- 2 points for correctly solving for t based on their answer to part (a)
- 2 point for a complete sentence with units