Computational Physics HW 8

Ben Johnston

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1 Problem 1

The first part of this quesiton involved importing two sets of data corresponding to the waveforms of single notes played by a piano and a trumpet and plotting each of them - this can be shown in *Figure 1* below:

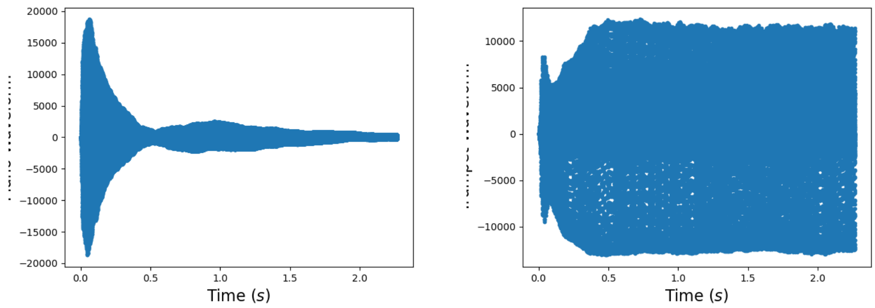

Figure 1: Left: Piano waveform. Right: Trumpet waveform

Following this the program calculated the discrete Fourier transform (DFT) for each of the waveforms using SciPy's fast Fourier transform (FFT) algorithm scipy.fft.rfft, the results of which were plotted for the first 10,000 coefficients. This can be seen below in *Figure 2*:

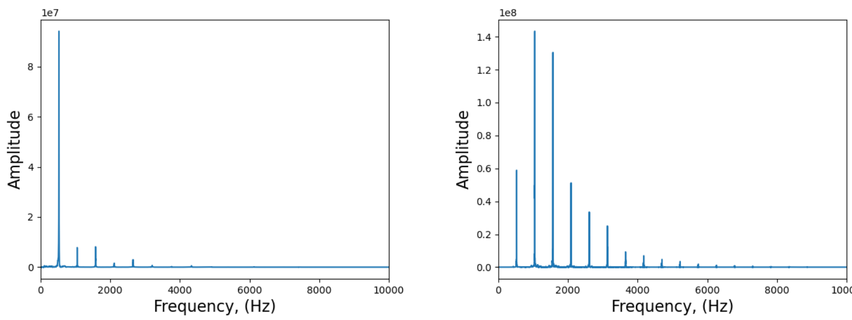

Figure 2: Left: Piano FFT. Right: Trumpet FFT

From the plots seen above it can be said that when the piano is played there is generally only one specific note being played (due to the one main peak), however for the trumpet there are many peaks that are large in amplitude and so it can be concluded that the trumpet produces resonances when played that cause other frequencies to be relatively prevalent.

The final part of this problem aimed to determine the notes that each of the instruments were playing (given that middle C is 261Hz). This was done by determining the frequency at which the maximum frequency in the Fourier spectrum occurred - these were:

$$f_{piano} = 525.231\text{Hz}$$

$$f_{trumpet} = 1043.85743857\text{Hz}$$

Given that an increase in one octave corresponds to a doubling in frequency it can be said that the piano was playing a C one octave above middle C and the trumpet was playing a C two octaves above middle C.

2 Problem 2

This question involved solving the set of well-known differential equations known as the Lorentz equations. These equations demonstrated one of the first incontrovertible examples of deterministic chaos, which is the notion of underlying patterns with in a noticeably chaotic system. The equations are as follows:

$$\frac{dx}{dt} = \sigma(y - x) \tag{1}$$

$$\frac{dy}{dt} = rx - y - xz \tag{2}$$

$$\frac{dz}{dt} = xy - bz \tag{3}$$

For the purposes of this problem we were to set $\sigma = 10$, $r = 28$ and $b = \frac{8}{3}$ initial values $(x, y, z) = (0, 1, 0)$. This set of differential equations were solved using SciPy's initial value problem solver. The first plot that was produced was a plot of y as a function of time, which can be seen in *Figure 3*:

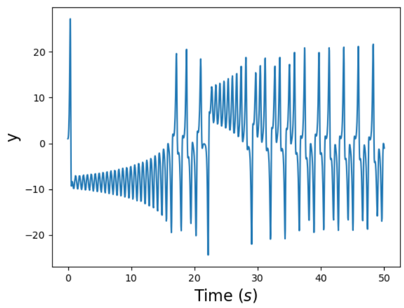

Figure 3: y as a function of time

Finally, a plot of z as a function of x was plotted and can be seen in *Figure 4*:

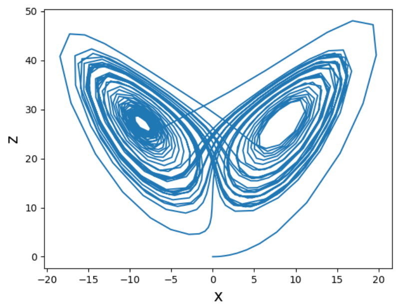

Figure 4: z as a funciton of x

This second plot is known as the Lorentz attractor, which is known as a "strange attractor" i.e. a plot that never repeats itself.