The 2D Eulerian diffusion model may be written as

$$\phi_t = D(\phi_{xx} + \phi_{yy}) \tag{1}$$

Suppose there is no boundaries and the concentration is zero far away. The n-th moment in x-direction is

$$M_n(\phi) = \int x^n \phi dx \tag{2}$$

By using partial integration twice, the time derivative becomes

$$\frac{d}{dt} M_2(\phi) = \int x^2 D(\phi_{xx}) = DM_2(\phi_{xx}) = -2DM_1(\phi_x) = 2DM_0(\phi) \tag{3}$$

Demonstrating that the variance in x-direction grows like $2Dt$.

LADiM uses a random walk of type

$$X_i^+ = X_i + u_i \Delta t \tag{4}$$

where the random velocities u_i are identical and independent randomly distributed with mean zero and variance σ_u^2.

The first moment is unchanged. For the second moment

$$X_i^{+2} = X_i^2 + 2X_i u_i \Delta t + u_i^2 \Delta t^2 \tag{5}$$

Averaging using $\bar{u}_i = 0$, we have

$$M_2^{+2} = M_2^2 + \sigma_u^2 \Delta t^2 \tag{6}$$

Comparing the two approaches gives the same increase in the variance if

$$2D = \sigma_u^2 \Delta t \tag{7}$$

This can be demonstrated more stringently by the theory of stochastic differential equations, for example the book Kloeden and Platen 1995.

If the random distribution is to jump a distance $\pm R$, the standard deviation $\sigma = R$. If the random distribution is uniformly distributed between $-R$ and R the standard deviation is $\sigma = \sqrt{1/3}R$ as Gillybrand points out. In this case, $R^2 = 6D$ must be used.

LADiM uses a normal distribution with standard deviation of *displacement* $\sigma = \sqrt{2D\Delta t}$.