9.1.7.

a) $2x + y + 5z = 0$ ↕
$x - 3z = 1$
$7x + 2y + 2z = 1$

$x - 3z = 1$
$2x + y + 5z = 0$
$7x + 2y + 2z = 1$

II − 2·I

$x - 3z = 1$
$y + 11z = -2$
$7x + 2y + 2z = 1$

III − 7·I

$x - 3z = 1$
$y + 11z = -2$
$2y + 23z = -6$

III − 2·II

$x - 3z = 1$
$y + 11z = -2$
$z = -2$

$x = 1 + 3z = -5$
$y = -2 - 11z = 20$
$z = -2$

---

b) $2x + y + 5z = 0$
$x - 3z = 1$
$7x + 2y + z = 3$

I ↔ II
II − 2·I

$x - 3z = 1$
$y + 11z = -2$
$7x + 2y + z = 3$

III − 7·I

$x - 3z = 1$
$y + 11z = -2$
$2y + 22z = -4$

III − 2·II

$x - 3z = 1$
$y + 11z = -2$
$0 = 0$

$x = 1 + 3z$
$y = -2 - 11z$

1 Sasitan: $(1,-2,0)+z(3,-11,1)$

$x = 1 + 3z$

$y = -2 - 11z$

$z = 0 + 1z$

$$(x,y,z) = (1+3z, -2-11z, 0+1z)$$
$$= (1,-2,0) + (3z,-11z,1z)$$
$$= (1,-2,0) + z(3,-11,1)$$

---

c) $3x + 8y - 2z = 1$ ↔
$x + 2y = 1$
$4x + 3y + 5z = 9$

$x + 2y = 1$
$3x + 8y - 2z = 1$
$4x + 3y + 5z = 9$

II − 3·I

$x + 2y = 1$
$2y - 2z = -2$
$4x + 3y + 5z = 9$

III − 4·I

$x + 2y = 1$
$2y - 2z = -2$
$-5y + 5z = 5$

II · $\frac{1}{2}$

$x + 2y = 1$
$y - z = -1$
$-5y + 5z = 5$

III + 5·II

$x + 2y = 1$
$y - z = -1$
$0 = 0$

$x = 1 - 2y = 1 - 2\cdot(-1+z) = 3 - 2z$

$y = -1 + z$

$$x = 1 - 2y$$
$$-z = -1 - y$$
$$z = 1 + y$$

$$x = 1 - 2y$$
$$y = 0 + y$$
$$z = 1 + y$$

$$(1, 0, 1) + y(-2, 1, 1)$$

$$x = 3 - 2z$$
$$y = -1 + z$$
$$z = z$$

$$(3, -1, 0) + z(-2, 1, 1)$$

9.1.5 c

$$\frac{1}{2}x_0 + 2x_1 - \frac{1}{2}x_2 + 4x_3 = 10$$
$$2x_0 - \frac{1}{2}x_1 - x_3 = 0$$

I · 2

$$x_0 + 4x_1 - x_2 + 8x_3 = 20$$
$$2x_0 - \frac{1}{2}x_1 - x_3 = 0$$

II · 2

$$x_0 + 4x_1 - x_2 + 8x_3 = 20$$
$$4x_0 - x_1 - 2x_3 = 0$$

II − 4·I

$$x_0 + 4x_1 - x_2 + 8x_3 = 20$$
$$-17x_1 + 4x_2 - 34x_3 = -80$$

$$x_0 + 4x_1 - x_2 + 8x_3 = 20$$
$$-17x_1 + 4x_2 - 34x_3 = -80$$

II $\cdot \frac{1}{-17}$

$$x_0 + 4x_1 - x_2 + 8x_3 = 20$$
$$x_1 - \frac{4}{17}x_2 + 2x_3 = \frac{80}{17}$$

I $-4 \cdot$ II

$$x_0 \quad (-1+\tfrac{16}{17})x_2 = 20 - 4 \cdot \frac{80}{17}$$
$$x_1 - \frac{4}{17}x_2 + 2x_3 = \frac{80}{17}$$

$$x_0 - \frac{1}{17}x_2 = \frac{340}{17} - \frac{320}{17} = \frac{20}{17}$$
$$x_1 - \frac{4}{17}x_2 + 2x_3 = \frac{80}{17}$$

$$x_0 = \frac{20}{17} + \frac{1}{17}x_2 + 0\,x_3$$
$$x_1 = \frac{80}{17} + \frac{4}{17}x_2 - 2x_3$$
$$x_2 = 0 + 1 \cdot x_2 + 0 \cdot x_3$$
$$x_3 = 0 + 0 \cdot x_2 + 1 \cdot x_3$$

$$\left(\frac{20}{17}, \frac{80}{17}, 0, 0\right) + x_2\left(\frac{1}{17}, \frac{4}{17}, 1, 0\right) + x_3(0, -2, 0, 1)$$

$$(x_0, x_1, x_2, x_3) = \left(\frac{20}{17} + \frac{1}{17}x_2, \frac{80}{17} + \frac{4}{17}x_2 - 2x_3, x_2, x_3\right)$$
$$= \left(\frac{20}{17}, \frac{80}{17}, 0, 0\right) + \left(\frac{1}{17}x_2, \frac{4}{17}x_2, x_2, 0\right) + (0, -2x_3, 0, x_3)$$

9.1.84)

$$\begin{aligned} x \quad\quad + z &= 5 \\ 2x - y + 3z &= 12 \\ 2y - 5z &= -4 \end{aligned}$$

II − 2·I

$$\begin{aligned} x \quad\quad + z &= 5 \\ -y + z &= 2 \\ 2y - 5z &= -4 \end{aligned}$$

III − 2II

$$\begin{aligned} x \quad\quad + z &= 5 \\ y - z &= -2 \\ -3z &= 0 \end{aligned}$$

−1·II

$$\begin{aligned} x \quad\quad + z &= 5 \\ y - z &= -2 \\ 2y - 5z &= -4 \end{aligned}$$

III · $\frac{1}{-3}$

$$\begin{aligned} x \quad\quad + z &= 5 \\ y - z &= -2 \\ z &= 0 \end{aligned}$$

$$\begin{aligned} x &= 5 \\ y &= -2 \\ z &= 0 \end{aligned}$$

$\underline{(5, -2, 0)}$

---

c)

$$\begin{aligned} x - 2y \quad\quad &= -1 \\ 4x - 5y - z &= -2 \end{aligned}$$

II − 4·I

$$\begin{aligned} x - 2y \quad\quad &= -1 \\ 3y - z &= 2 \end{aligned}$$

$x = -1 + 2y$

$z = -2 + 3y$

$(-1, 0, -2) + y(2, 1, 3)$

$x = -1 + 2y = -1 + 2\left(\frac{2}{3} + \frac{1}{3}z\right) = \frac{1}{3} + \frac{2}{3}z = x$

$y = \frac{2}{3} + \frac{1}{3}z$

$\frac{2}{3} + \frac{1}{3}z = y$

$0 + 1z = z$

$\left(\frac{1}{3}, \frac{2}{3}, 0\right) + z\left(\frac{2}{3}, \frac{1}{3}, 1\right)$

9.1.9.

$$\begin{aligned} 3y + 2z - w &= 1 \\ x - y + 2z + w &= 0 \\ 5y - 4z - 3w &= 5 \\ -3x + 4y + 2z - 2w &= -3 \end{aligned}$$

$$\begin{aligned} x - y + 2z + w &= 0 \\ 3y + 2z - w &= 1 \\ 5y - 4z - 3w &= 5 \\ -3x + 4y + 2z - 2w &= -3 \end{aligned}$$

IV + 3·I

$$\begin{aligned} x - y + 2z + w &= 0 \\ 3y + 2z - w &= 1 \\ 5y - 4z - 3w &= 5 \\ y + 8z + w &= -3 \end{aligned}$$

$$\begin{aligned} x - y + 2z + w &= 0 \\ y + 8z + w &= -3 \\ 5y - 4z - 3w &= 5 \\ 3y + 2z - w &= 1 \end{aligned}$$

III - 5·II
IV - 3·II

$$\begin{aligned} x - y + 2z + w &= 0 \\ y + 8z + w &= -3 \\ -44z - 8w &= 20 \\ -22z - 4w &= 10 \end{aligned}$$

III - 2·IV

$$\begin{aligned} x - y + 2z + w &= 0 \\ y + 8z + w &= -3 \\ 0 &= 0 \\ -22z - 4w &= 10 \end{aligned}$$

III · $-\frac{1}{22}$

$$\begin{aligned} x - y + 2z + w &= 0 \\ y + 8z + w &= -3 \\ z + \frac{2}{11}w &= \frac{-5}{11} \\ 0 &= 0 \end{aligned}$$

Practice Problems LinAlg

① a)
$$\begin{aligned} x_1 + 4x_2 - 2x_3 + 8x_4 &= 12 \\ x_2 - 7x_3 + 2x_4 &= 7 \\ 5x_3 - x_4 &= 7 \\ x_3 + 3x_4 &= -5 \end{aligned}$$

Neste steg: $III - 5 \cdot IV$

Alt: $IV - \frac{1}{5} III$

b)
$$\begin{aligned} x_1 - 3x_2 + 5x_3 - 2x_4 &= 0 \\ x_2 + 8x_3 &= -4 \\ 2x_3 &= 3 \qquad III \cdot \tfrac{1}{2} \\ x_4 &= 1 \end{aligned}$$

③ Er $(3, 4, -2)$ en løsning av

$$\begin{aligned} 5x_1 - x_2 + 2x_3 &= 7 \\ -2x_1 + 6x_2 + 9x_3 &= 0 \\ -7x_1 + 5x_2 - 3x_3 &= -7 \quad ? \end{aligned}$$

$5 \cdot 3 - 4 + 2 \cdot (-2) = 7$ ✓

$-2 \cdot 3 + 6 \cdot 4 + 9 \cdot (-2) = 0$ ✓

$-7 \cdot 3 + 5 \cdot 4 - 3 \cdot (-2) = 5 \neq -7$ ✗

~~Ex~~ 1. For hvilke verdier h og k er systemet konsistent?

$$2x_1 - x_2 = h$$
$$-6x_1 + 3x_2 = k$$

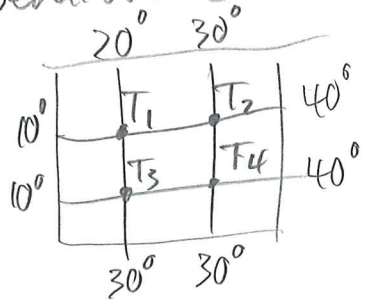
II + 3·I

$$2x_1 - x_2 = h$$
$$0 = k + 3h$$

$$\boxed{k = -3h}$$

Løsningsforslag 33/34 Lineær Algebra:

33) Hver temperatur er gitt som gjennomsnittet av temperaturene rundt



Vi får:

$$T_1 = \frac{10^\circ + 20^\circ + T_2 + T_3}{4} \Rightarrow 4T_1 = 10^\circ + 20^\circ + T_2 + T_3$$

$$\Rightarrow 4T_1 - T_2 - T_3 = 30^\circ$$

$$T_2 = \frac{T_1 + 30^\circ + 40^\circ + T_4}{4} \Rightarrow 4T_2 = T_1 + 70^\circ + T_4$$

$$\Rightarrow -T_1 + 4T_2 - T_4 = 70^\circ$$

$$T_3 = \frac{10^\circ + T_1 + T_4 + 30^\circ}{4} \Rightarrow 4T_3 = 40^\circ + T_1 + T_4$$

$$\Rightarrow -T_1 + 4T_3 - T_4 = 40^\circ$$

$$T_4 = \frac{T_3 + T_2 + 40^\circ + 30^\circ}{4} \Rightarrow 4T_4 = T_3 + T_2 + 70^\circ$$

$$\Rightarrow -T_2 - T_3 + 4T_4 = 70^\circ$$

Vi har:

$$\begin{aligned} 4T_1 - T_2 - T_3 &= 30 \\ -T_1 + 4T_2 - T_4 &= 70 \\ -T_1 + 4T_3 - T_4 &= 40 \\ -T_2 - T_3 + 4T_4 &= 70 \end{aligned}$$

34) skal løse

$$\begin{aligned} 4T_1 - T_2 - T_3 &= 30 \\ -T_1 + 4T_2 - T_4 &= 70 \\ -T_1 + 4T_3 - T_4 &= 40 \\ -T_2 - T_3 + 4T_4 &= 70 \end{aligned}$$

I ⇔ II

(-1)·I

$$\begin{aligned} T_1 - 4T_2 + T_4 &= -70 \\ 4T_1 - T_2 - T_3 &= 30 \\ -T_1 + 4T_3 - T_4 &= 40 \\ -T_2 - T_3 + 4T_4 &= 70 \end{aligned}$$

II − 4I

III + I

$$\begin{aligned} T_1 - 4T_2 + T_4 &= -70 \\ 15T_2 - T_3 - 4T_3 &= 310 \\ -4T_2 + 4T_3 &= -30 \\ -T_2 - T_3 + 4T_4 &= 70 \end{aligned}$$

II ⇔ IV

(-1)·II

$$\begin{aligned} T_1 - 4T_2 + T_4 &= -70 \\ T_2 + T_3 - 4T_4 &= -70 \\ -4T_2 + 4T_3 &= -30 \\ 15T_2 - T_3 - 4T_3 &= 310 \end{aligned}$$

III + 4II
IV − 15II

$$T_1 - 4T_2 + T_4 = -70$$
$$T_2 + T_3 - 4T_4 = -70$$
$$8T_3 - 16T_4 = -310$$
$$-16T_3 + 56T_4 = 1360$$

$\frac{1}{8}\cdot$ III

$$T_1 - 4T_2 + T_4 = -70$$
$$T_2 + T_3 - 4T_4 = -70$$
$$T_3 - 2T_4 = -\frac{155}{4}$$
$$-16T_3 + 56T_4 = 1360$$

IV + 16·III

$$T_1 - 4T_2 + T_4 = -70$$
$$T_2 + T_3 - 4T_4 = -70$$
$$T_3 - 2T_4 = -\frac{155}{4}$$
$$24T_4 = 740$$

$\frac{1}{24}\cdot$ IV

$$T_1 - 4T_2 + T_4 = -70$$
$$T_2 + T_3 - 4T_4 = -70$$
$$T_3 - 2T_4 = -\frac{155}{4}$$
$$T_4 = \frac{185}{6} = 30.8\overline{3}$$

$$T_3 = -\frac{155}{4} + 2\cdot\frac{185}{6} = \frac{275}{12} = 22.91\overline{6}$$

$$T_2 = -70 - \frac{275}{12} + 4\cdot\frac{185}{6} = \frac{365}{12} = 30.41\overline{6}$$

$$T_1 = -70 + 4\cdot\frac{365}{12} - \frac{185}{6} = \frac{125}{6} = 20.8\overline{3}$$