

9.1.7.

$$\begin{array}{l} \text{a)} \quad 2x + y + 5z = 0 \\ \quad \quad x \quad -3z = 1 \\ \quad \quad 7x + 2y + 2z = 1 \end{array}$$

$$\begin{array}{l} x \quad -3z = 1 \\ 2x + y + 5z = 0 \\ 7x + 2y + 2z = 1 \end{array}$$

II - 2 · I

$$\begin{array}{l} x \quad -3z = 1 \\ \quad y + 11z = -2 \\ 7x + 2y + 2z = 1 \end{array}$$

III - 7 · I

$$\begin{array}{l} x \quad -3z = 1 \\ \quad y + 11z = -2 \\ 2y + 23z = -6 \end{array}$$

III - 2 · II

$$\begin{array}{l} x \quad -3z = 1 \\ \quad y + 11z = -2 \\ \quad \quad z = -2 \end{array}$$

$$\begin{array}{l} x = 1 + 3z = -5 \\ y = -2 - 11z = 20 \\ z = -2 \end{array}$$

$$\begin{array}{l} \text{b)} \quad 2x + y + 5z = 0 \\ \quad \quad x \quad -3z = 1 \\ \quad \quad 7x + 2y + z = 3 \end{array}$$

I + III
II - 2 · I

$$\begin{array}{l} x \quad -3z = 1 \\ \quad y + 11z = -2 \\ 7x + 2y + z = 3 \end{array}$$

III - 7 · I

$$\begin{array}{l} x \quad -3z = 1 \\ \quad y + 11z = -2 \\ 2y + 22z = -4 \end{array}$$

III - 2 · II

$$\begin{array}{l} x \quad -3z = 1 \\ \quad y + 11z = -2 \\ \quad \quad 0 = 0 \end{array}$$

$$\begin{array}{l} x = 1 + 3z \\ y = -2 - 11z \end{array}$$

①

1. Sasitan: $(1, -2, 0) + z(3, -11, 1)$

$$\begin{aligned} x &= 1 + 3z \\ y &= -2 - 11z \\ z &= 0 + 1z \end{aligned}$$

$$\begin{aligned} (x, y, z) &= (1 + 3z, -2 - 11z, 0 + 1z) \\ &= (1, -2, 0) + (3z, -11z, 1z) \\ &= (1, -2, 0) + z(3, -11, 1) \end{aligned}$$

c)
$$\begin{array}{lcl} 3x + 8y - 2z = 1 & \text{I} & x + 2y = 1 \\ x + 2y = 1 & \text{II} & 3x + 8y - 2z = 1 \\ 4x + 3y + 5z = 9 & \text{III} & 4x + 3y + 5z = 9 \end{array}$$

II - 3 · I

$$\begin{array}{lcl} x + 2y & = & 1 \\ 2y - 2z & = & -2 \\ (4x) + 3y + 5z & = & 9 \end{array}$$

III - 4 · I

$$\begin{array}{lcl} x + 2y & = & 1 \\ 2y - 2z & = & -2 \\ -5y + 5z & = & 5 \end{array}$$

II · $\frac{1}{2}$

$$\begin{array}{lcl} x + 2y & = & 1 \\ y - z & = & -1 \\ (-5y) + 5z & = & 5 \end{array}$$

III + 5 · II

$$\begin{array}{lcl} x + 2y & = & 1 \\ y - z & = & -1 \\ 0 & = & 0 \end{array}$$

$$\begin{aligned} x &= 1 - 2y = 1 - 2(-1 + z) = 3 - 2z \\ y &= -1 + z \end{aligned}$$

$$x = 1 - 2y$$

$$-z = -1 - y$$

$$z = 1 + y$$

$$\begin{array}{l} x = 1 - 2y \\ y = 0 + y \\ z = 1 + y \end{array}$$

$$x = 3 - 2z$$

$$y = -1 + z$$

$$z = z$$

$$(1, 0, 1) + y(-2, 1, 1)$$

$$(3, -1, 0) + z(-2, 1, 1)$$

9.1.5 c

$$\frac{1}{2}x_0 + 2x_1 - \frac{1}{2}x_2 + 4x_3 = 10$$

$$2x_0 - \frac{1}{2}x_1 - x_3 = 0$$

I · 2

$$x_0 + 4x_1 - x_2 + 8x_3 = 20$$

$$2x_0 - \frac{1}{2}x_1 - x_3 = 0$$

II · 2

$$x_0 + 4x_1 - x_2 + 8x_3 = 20$$

$$4x_0 - x_1 - 2x_3 = 0$$

II - 4 · I

$$x_0 + 4x_1 - x_2 + 8x_3 = 20$$

$$-17x_1 + 4x_2 - 34x_3 = -80$$

②

$$x_0 + 4x_1 - x_2 + 8x_3 = 20$$

$$-17x_1 + 4x_2 - 34x_3 = -80$$

$$\text{II} \cdot \frac{1}{-17}$$

$$x_0 + 4x_1 - x_2 + 8x_3 = 20$$

$$x_1 - \frac{4}{17}x_2 + 2x_3 = \frac{80}{17}$$

$$\text{I} - 4 \cdot \text{II}$$

$$x_0 \quad (-1 + \frac{16}{17})x_2 \quad = 20 - 4 \cdot \frac{80}{17}$$

$$x_1 - \frac{4}{17}x_2 + 2x_3 = \frac{80}{17}$$

$$x_0 \quad - \frac{1}{17}x_2 \quad = \frac{340}{17} - \frac{320}{17} = \frac{20}{17}$$

$$x_1 - \frac{4}{17}x_2 + 2x_3 = \frac{80}{17}$$

$$\begin{cases} x_0 = \frac{20}{17} + \frac{1}{17}x_2 + 0x_3 \\ x_1 = \frac{80}{17} + \frac{4}{17}x_2 - 2x_3 \\ x_2 = 0 + 1 \cdot x_2 + 0 \cdot x_3 \\ x_3 = 0 + 0 \cdot x_2 + 1 \cdot x_3 \end{cases}$$

$$\left(\frac{20}{17}, \frac{80}{17}, 0, 0 \right) + x_2 \left(\frac{1}{17}, \frac{4}{17}, 1, 0 \right) + x_3 (0, -2, 0, 1)$$

$$(x_0, x_1, x_2, x_3) \in \left(\frac{20}{17} + \frac{1}{17}x_2, \frac{80}{17} + \frac{4}{17}x_2 - 2x_3, x_2, x_3 \right)$$

$$= \left(\frac{20}{17}, \frac{80}{17}, 0, 0 \right) + \left(\frac{1}{17}x_2, \frac{4}{17}x_2, x_2, 0 \right) + (0, -2x_3, 0, x_3)$$

9.1.86)

$$\begin{array}{rcl} x & + z & = 5 \\ 2x - y + 3z & = & 12 \\ 2y - 5z & = & -4 \end{array}$$

II - 2 · I

$$\begin{array}{rcl} x & + z & = 5 \\ -y + z & = & 2 \\ 2y - 5z & = & 4 \end{array}$$

-1 · II

$$\begin{array}{rcl} x & + z & = 5 \\ y - z & = & -2 \\ 2y - 5z & = & -4 \end{array}$$

III - 2II

$$\begin{array}{rcl} x & + z & = 5 \\ y - z & = & -2 \\ -3z & = & 0 \end{array}$$

III · $\frac{1}{-3}$

$$\begin{array}{rcl} x & + z & = 5 \\ y - z & = & -2 \\ z & = & 0 \end{array}$$

$$\begin{array}{rcl} x & = & 5 \\ y & = & -2 \\ z & = & 0 \end{array}$$

$$\underline{\underline{(5, -2, 0)}}$$

c)

$$\begin{array}{rcl} x - 2y & = & -1 \\ 4x - 5y - z & = & -2 \end{array} \quad \begin{array}{rcl} \text{II} - 4 \cdot \text{I} \\ x - 2y & = & -1 \\ 3y - z & = & 2 \end{array}$$

$$\begin{array}{l} x = -1 + 2y \\ z = -2 + 3y \end{array}$$

$$(-1, 0, -2) + y(2, 1, 3)$$

$$x = -1 + 2y = -1 + 2\left(\frac{2}{3} + \frac{1}{3}z\right) = \frac{1}{3} + \frac{2}{3}z = x$$

$$y = \frac{2}{3} + \frac{1}{3}z$$

$$\frac{2}{3} + \frac{1}{3}z = y$$

$$0 + 1z = z$$

$$\left(\frac{1}{3}, \frac{2}{3}, 0\right) + z\left(\frac{2}{3}, \frac{1}{3}, 1\right)$$

②

9.1.9.

$$\begin{aligned} 3y + 2z - w &= 1 \quad \downarrow \\ x - y + 2z + w &= 0 \end{aligned}$$

$$\begin{aligned} 5y - 4z - 3w &= 5 \\ -3x + 4y + 2z - 2w &= -3 \end{aligned}$$

$$\begin{aligned} x - y + 2z + w &= 0 \\ 3y + 2z - w &= 1 \\ 5y - 4z - 3w &= 5 \\ (-3x + 4y + 2z - 2w) &= -3 \end{aligned}$$

IV + 3·I

$$\begin{aligned} x - y + 2z + w &= 0 \\ 3y + 2z - w &= 1 \quad \uparrow \\ 5y - 4z - 3w &= 5 \quad \downarrow \\ y + 8z + w &= -3 \end{aligned}$$

$$\begin{aligned} x - y + 2z + w &= 0 \\ y + 8z + w &= -3 \\ 5y - 4z - 3w &= 5 \\ 3y + 2z - w &= 1 \end{aligned}$$

III - 5 · II

IV - 3 · II

$$x - y + 2z + w = 0$$

$$y + 8z + w = -3$$

$$-44z - 8w = 20$$

$$-22z - 4w = 10$$

III - 2IV

$$x - y + 2z + w = 0$$

$$y + 8z + w = -3$$

$$0 = 0$$

$$-22z - 4w = 10$$

$$\text{III} \cdot -\frac{1}{22}$$

$$x - y + 2z + w = 0$$

$$y + 8z + w = -3$$

$$z + \frac{2}{11}w = -\frac{5}{11}$$

$$0 = 0$$

Practice Problems LinAlg

①
a) $x_1 + 4x_2 - 2x_3 + 8x_4 = 12$

$$x_2 - 7x_3 + 2x_4 = 7$$

$$5x_3 - x_4 = 7$$

$$x_3 + 3x_4 = -5$$

Next steps: $\text{III} - 5 \cdot \text{IV}$

Alt: $\text{IV} - \frac{1}{5} \text{III}$

b) $x_1 - 3x_2 + 5x_3 - 2x_4 = 0$

$$x_2 + 8x_3 = -4$$

$$2x_3 = 3$$

$$x_4 = 1$$

$$\text{III} \cdot \frac{1}{2}$$

③ Er $(3, 4, -2)$ en løsning av

$$5x_1 - x_2 + 2x_3 = 7$$

$$-2x_1 + 6x_2 + 9x_3 = 0$$

$$-7x_1 + 5x_2 - 3x_3 = -7 \quad ?$$

$$5 \cdot 3 - 4 + 2 \cdot (-2) = 7 \quad \checkmark$$

$$-2 \cdot 3 + 6 \cdot 4 + 9 \cdot (-2) = 0 \quad \checkmark$$

$$-7 \cdot 3 + 5 \cdot 4 - 3 \cdot (-2) = 5 \neq -7 \quad \times$$

#1 For hvilke verdier h og k er systemet konsistent?

$$\begin{aligned} 2x_1 - x_2 &= h \\ -6x_1 + 3x_2 &= k \end{aligned}$$

II + 3 · I

$$2x_1 - x_2 = h$$

$$0 = k + 3h$$

$$\boxed{k = -3h}$$

Løsning 1.1.33 og 1.1.34, oppdatert.

1.1.33

Hver av temperatuorene er gitt ved
gjennomsnittet av temperatuorene rundt seg

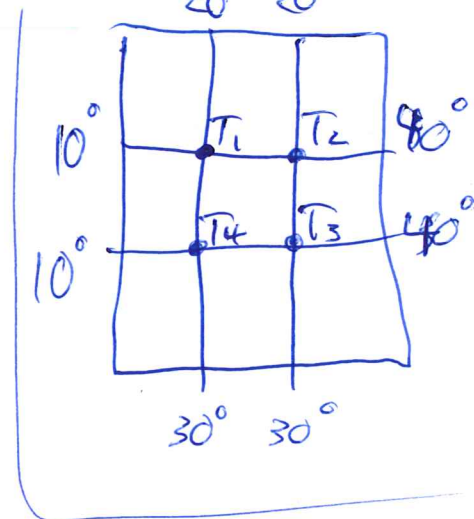
selv. Vi får derfor:

$$T_1 = \frac{10 + 20 + T_2 + T_4}{4}$$

$$T_2 = \frac{20 + 40 + T_1 + T_3}{4}$$

$$T_3 = \frac{40 + 30 + T_2 + T_4}{4}$$

$$T_4 = \frac{30 + 10 + T_1 + T_3}{4}$$



Vi ganger alle likningene med 4 og får:

$$4T_1 = 30 + T_2 + T_4$$

$$4T_2 = 60 + T_1 + T_3$$

$$4T_3 = 70 + T_2 + T_4$$

$$4T_4 = 40 + T_1 + T_3$$

V: Flytter alle de ukjente over på venstre side og får:

$$\begin{aligned}4T_1 - T_2 - T_4 &= 30 \\-T_1 + 4T_2 - T_3 &= 60 \\-T_2 + 4T_3 - T_4 &= 70 \\-T_1 - T_3 + 4T_4 &= 40\end{aligned}$$

Disse likningene vil bestemme (estimatet av) temperaturene T_1, T_2, T_3, T_4 .

~~W345~~

~~Vi skal løse~~

1.1.34/ Vi skal løse:

$$\begin{array}{lcl} \textcircled{I} & 4T_1 - T_2 & -T_4 = 30 \\ \textcircled{II} & -T_1 + 4T_2 - T_3 & = 60 \\ \textcircled{III} & -T_2 + 4T_3 - T_4 & = 70 \\ \textcircled{IV} & -T_3 + 4T_4 & = 40 \end{array}$$

$\textcircled{I} \leftrightarrow \textcircled{IV}$

$$\begin{array}{lcl} -T_1 & -T_3 + 4T_4 & = 40 \\ -T_1 + 4T_2 - T_3 & & = 60 \\ -T_2 + 4T_3 - T_4 & & = 70 \\ 4T_1 - T_2 & -T_4 & = 30 \end{array}$$

$-1 \cdot \textcircled{I}$

$$\begin{array}{lcl} T_1 & +T_3 - 4T_4 & = -40 \\ -T_1 + 4T_2 - T_3 & & = 60 \\ -T_2 + 4T_3 - T_4 & & = 70 \\ 4T_1 - T_2 & -T_4 & = 30 \end{array}$$

$\textcircled{II} + \textcircled{I}$

$$T_1 + T_3 - 4T_4 = -40$$

$\textcircled{IV} - 4\textcircled{I}$

$$4T_2 - 4T_4 = 20$$

$$-T_2 + 4T_3 - T_4 = 70$$

$$-T_2 - 4T_3 + 15T_4 = 190$$

$$\frac{1}{4} \cdot \textcircled{\text{II}} \quad T_1 \quad + T_3 - 4T_4 = -40$$

$$T_2 \quad - T_4 = 5$$

$$-T_2 + 4T_3 - T_4 = 70$$

$$-T_2 - 4T_3 + 15T_4 = 190$$

$$\textcircled{\text{III}} + \textcircled{\text{II}}$$

$$T_1 \quad + T_3 - 4T_4 = -40$$

$$\textcircled{\text{IV}} + \textcircled{\text{I}}$$

$$T_2 \quad - T_4 = 5$$

$$4T_3 - 2T_4 = 75$$

$$-4T_3 + 14T_4 = 195$$

$$\textcircled{\text{IV}} + \textcircled{\text{III}}$$

$$T_1 \quad + T_3 - 4T_4 = -40$$

$$T_2 \quad - T_4 = 5$$

$$4T_3 - 2T_4 = 75$$

$$12T_4 = 270$$

$$\textcircled{\text{IV}} \cdot \frac{1}{12}$$

$$T_1$$

$$+ T_3 - 4T_4 = -40$$

$$T_2$$

$$- T_4 = 5$$

$$4T_3 - 2T_4 = 75$$

$$T_4 = 22.5$$

V: Sär:

$$T_4 = 22.5$$

$$4T_3 - 2 \cdot 22.5 = 75 \Rightarrow 4T_3 = 75 + 45 = 120$$

$$\Rightarrow T_3 = 30$$

$$T_2 - 22.5 = 5 \Rightarrow T_2 = 22.5 + 5 = 27.5$$

$$T_1 + 30 - 4 \cdot 22.5 = -40$$

$$\Rightarrow T_1 = -40 + 90 - 30 = 20$$

Sår

$$T_1 = 20$$

$$T_2 = 27.5$$

$$T_3 = 30$$

$$T_4 = 22.5$$
