1

PROPERTIES OF ANTENNAS

One approach to an antenna book starts with a discussion of how antennas radiate. Beginning with Maxwell's equations, we derive electromagnetic waves. After that lengthy discussion, which contains a lot of mathematics, we discuss how these waves excite currents on conductors. The second half of the story is that currents radiate and produce electromagnetic waves. You may already have studied that subject, or if you wish to further your background, consult books on electromagnetics. The study of electromagnetics gives insight into the mathematics describing antenna radiation and provides the rigor to prevent mistakes. We skip the discussion of those equations and move directly to practical aspects.

It is important to realize that antennas radiate from currents. Design consists of controlling currents to produce the desired radiation distribution, called its *pattern*. In many situations the problem is how to prevent radiation from currents, such as in circuits. Whenever a current becomes separated in distance from its return current, it radiates. Simply stated, we design to keep the two currents close together, to reduce radiation. Some discussions will ignore the current distribution and instead, consider derived quantities, such as fields in an aperture or magnetic currents in a slot or around the edges of a microstrip patch. You will discover that we use any concept that provides insight or simplifies the mathematics.

An antenna converts bound circuit fields into propagating electromagnetic waves and, by reciprocity, collects power from passing electromagnetic waves. Maxwell's equations predict that any time-varying electric or magnetic field produces the opposite field and forms an electromagnetic wave. The wave has its two fields oriented orthogonally, and it propagates in the direction normal to the plane defined by the perpendicular electric and magnetic fields. The electric field, the magnetic field, and the direction of propagation form a right-handed coordinate system. The propagating wave field intensity decreases by $1/R$ away from the source, whereas a static field

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drops off by $1/R^2$. Any circuit with time-varying fields has the capability of radiating to some extent.

We consider only time-harmonic fields and use phasor notation with time dependence $e^{j\omega t}$. An outward-propagating wave is given by $e^{-j(kR-\omega t)}$, where k, the wave number, is given by $2\pi/\lambda$. λ is the wavelength of the wave given by c/f, where c is the velocity of light (3×10^8 m/s in free space) and f is the frequency. Increasing the distance from the source decreases the phase of the wave.

Consider a two-wire transmission line with fields bound to it. The currents on a single wire will radiate, but as long as the ground return path is near, its radiation will nearly cancel the other line's radiation because the two are 180° out of phase and the waves travel about the same distance. As the lines become farther and farther apart, in terms of wavelengths, the fields produced by the two currents will no longer cancel in all directions. In some directions the phase delay is different for radiation from the current on each line, and power escapes from the line. We keep circuits from radiating by providing close ground returns. Hence, high-speed logic requires ground planes to reduce radiation and its unwanted crosstalk.

1-1 ANTENNA RADIATION

Antennas radiate spherical waves that propagate in the radial direction for a coordinate system centered on the antenna. At large distances, spherical waves can be approximated by plane waves. Plane waves are useful because they simplify the problem. They are not physical, however, because they require infinite power.

The Poynting vector describes both the direction of propagation and the power density of the electromagnetic wave. It is found from the vector cross product of the electric and magnetic fields and is denoted **S**:

$$\mathbf{S} = \mathbf{E} \times \mathbf{H}^* \qquad \mathrm{W/m^2}$$

Root mean square (RMS) values are used to express the magnitude of the fields. $\mathbf{H}^*$ is the complex conjugate of the magnetic field phasor. The magnetic field is proportional to the electric field in the far field. The constant of proportion is η, the impedance of free space ($\eta = 376.73\,\Omega$):

$$|\mathbf{S}| = S = \frac{|\mathbf{E}|^2}{\eta} \qquad \mathrm{W/m^2} \tag{1-1}$$

Because the Poynting vector is the vector product of the two fields, it is orthogonal to both fields and the triplet defines a right-handed coordinate system: (**E, H, S**).

Consider a pair of concentric spheres centered on the antenna. The fields around the antenna decrease as $1/R$, $1/R^2$, $1/R^3$, and so on. Constant-order terms would require that the power radiated grow with distance and power would not be conserved. For field terms proportional to $1/R^2$, $1/R^3$, and higher, the power density decreases with distance faster than the area increases. The energy on the inner sphere is larger than that on the outer sphere. The energies are not radiated but are instead concentrated around the antenna; they are near-field terms. Only the $1/R^2$ term of the Poynting vector ($1/R$ field terms) represents radiated power because the sphere area grows as R^2 and gives a constant product. All the radiated power flowing through the inner sphere will propagate to the outer sphere. The sign of the input reactance depends on the near-field predominance of field type: electric (capacitive) or magnetic (inductive). At resonance (zero reactance) the stored energies due to the near fields are equal. Increasing the stored fields increases the circuit Q and narrows the impedance bandwidth.

Far from the antenna we consider only the radiated fields and power density. The power flow is the same through concentric spheres:

$$4\pi R_1^2 S_{1,\text{avg}} = 4\pi R_2^2 S_{2,\text{avg}}$$

The average power density is proportional to $1/R^2$. Consider differential areas on the two spheres at the same coordinate angles. The antenna radiates only in the radial direction; therefore, no power may travel in the θ or ϕ direction. Power travels in flux tubes between areas, and it follows that not only the average Poynting vector but also every part of the power density is proportional to $1/R^2$:

$$S_1 R_1^2 \sin\theta \, d\theta \, d\phi = S_2 R_2^2 \sin\theta \, d\theta \, d\phi$$

Since in a radiated wave S is proportional to $1/R^2$, E is proportional to $1/R$. It is convenient to define radiation intensity to remove the $1/R^2$ dependence:

$$U(\theta, \phi) = S(R, \theta, \phi) R^2 \qquad \text{W/solid angle}$$

Radiation intensity depends only on the direction of radiation and remains the same at all distances. A probe antenna measures the relative radiation intensity (pattern) by moving in a circle (constant R) around the antenna. Often, of course, the antenna rotates and the probe is stationary.

Some patterns have established names. Patterns along constant angles of the spherical coordinates are called either *conical* (constant θ) or *great circle* (constant ϕ). The great circle cuts when $\phi = 0^\circ$ or $\phi = 90^\circ$ are the principal plane patterns. Other named cuts are also used, but their names depend on the particular measurement positioner, and it is necessary to annotate these patterns carefully to avoid confusion between people measuring patterns on different positioners. Patterns are measured by using three scales: (1) linear (power), (2) square root (field intensity), and (3) decibels (dB). The dB scale is used the most because it reveals more of the low-level responses (sidelobes).

Figure 1-1 demonstrates many characteristics of patterns. The half-power beamwidth is sometimes called just the beamwidth. The tenth-power and null beamwidths are used in some applications. This pattern comes from a parabolic reflector whose feed is moved off the axis. The vestigial lobe occurs when the first sidelobe becomes joined to the main beam and forms a shoulder. For a feed located on the axis of the parabola, the first sidelobes are equal.

1-2 GAIN

Gain is a measure of the ability of the antenna to direct the input power into radiation in a particular direction and is measured at the peak radiation intensity. Consider the

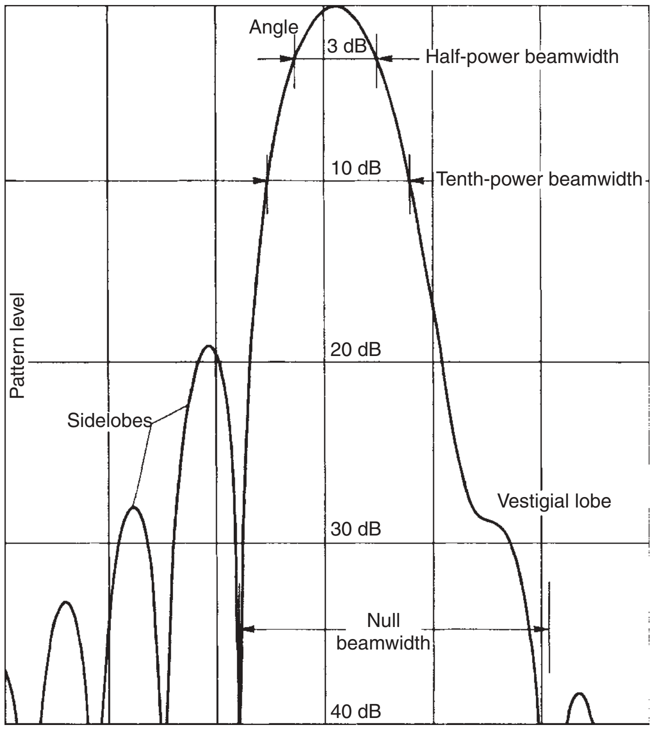


FIGURE 1-1 Antenna pattern characteristics.

power density radiated by an isotropic antenna with input power P_0 at a distance R: $S = P_0/4\pi R^2$. An isotropic antenna radiates equally in all directions, and its radiated power density S is found by dividing the radiated power by the area of the sphere $4\pi R^2$. The isotropic radiator is considered to be 100% efficient. The gain of an actual antenna increases the power density in the direction of the peak radiation:

$$S = \frac{P_0 G}{4\pi R^2} = \frac{|\mathbf{E}|^2}{\eta} \quad \text{or} \quad |\mathbf{E}| = \frac{1}{R}\sqrt{\frac{P_0 G \eta}{4\pi}} = \sqrt{S\eta} \tag{1-2}$$

Gain is achieved by directing the radiation away from other parts of the radiation sphere. In general, gain is defined as the gain-biased pattern of the antenna:

$$\begin{aligned} S(\theta,\phi) &= \frac{P_0 G(\theta,\phi)}{4\pi R^2} && \text{power density} \\ U(\theta,\phi) &= \frac{P_0 G(\theta,\phi)}{4\pi} && \text{radiation intensity} \end{aligned} \tag{1-3}$$

The surface integral of the radiation intensity over the radiation sphere divided by the input power P_0 is a measure of the relative power radiated by the antenna, or the antenna efficiency:

$$\frac{P_r}{P_0} = \int_0^{2\pi}\int_0^{\pi} \frac{G(\theta,\phi)}{4\pi} \sin\theta \, d\theta \, d\phi = \eta_e \qquad \text{efficiency}$$

where P_r is the radiated power. Material losses in the antenna or reflected power due to poor impedance match reduce the radiated power. In this book, integrals in the equation above and those that follow express concepts more than operations we perform during design. Only for theoretical simplifications of the real world can we find closed-form solutions that would call for actual integration. We solve most integrals by using numerical methods that involve breaking the integrand into small segments and performing a weighted sum. However, it is helpful that integrals using measured values reduce the random errors by averaging, which improves the result.

In a system the transmitter output impedance or the receiver input impedance may not match the antenna input impedance. Peak gain occurs for a receiver impedance conjugate matched to the antenna, which means that the resistive parts are the same and the reactive parts are the same magnitude but have opposite signs. Precision gain measurements require a tuner between the antenna and receiver to conjugate-match the two. Alternatively, the mismatch loss must be removed by calculation after the measurement. Either the effect of mismatches is considered separately for a given system, or the antennas are measured into the system impedance and mismatch loss is considered to be part of the efficiency.

Example Compute the peak power density at 10 km of an antenna with an input power of 3 W and a gain of 15 dB.

First convert dB gain to a ratio: $G = 10^{15/10} = 31.62$. The power spreads over the sphere area with radius 10 km or an area of $4\pi(10^4)^2$ m^2. The power density is

$$S = \frac{(3\,\text{W})(31.62)}{4\pi \times 10^8\ \text{m}^2} = 75.5\,\text{nW/m}^2$$

We calculate the electric field intensity using Eq. (1-2):

$$|\mathbf{E}| = \sqrt{S\eta} = \sqrt{(75.5 \times 10^{-9})(376.7)} = 5333\,\mu\text{V/m}$$

Although gain is usually relative to an isotropic antenna, some antenna gains are referred to a $\lambda/2$ dipole with an isotropic gain of 2.14 dB.

If we approximate the antenna as a point source, we compute the electric field radiated by using Eq. (1-2):

$$E(\theta, \phi) = \frac{e^{-jkR}}{R}\sqrt{\frac{P_0 G(\theta, \phi)\eta}{4\pi}} \tag{1-4}$$

This requires only that the antenna be small compared to the radial distance R. Equation (1-4) ignores the direction of the electric field, which we define as *polarization*. The units of the electric field are volts/meter. We determine the far-field pattern by multiplying Eq. (1-4) by R and removing the phase term e^{-jkR} since phase has meaning only when referred to another point in the far field. The far-field electric field E_{ff} unit is volts:

$$E_{\text{ff}}(\theta, \phi) = \sqrt{\frac{P_0 G(\theta, \phi)\eta}{4\pi}} \quad \text{or} \quad G(\theta, \phi) = \frac{1}{P_0}\left[E_{\text{ff}}(\theta, \phi)\sqrt{\frac{4\pi}{\eta}}\right]^2 \tag{1-5}$$

During analysis, we often normalize input power to 1 W and can compute gain easily from the electric field by multiplying by a constant $\sqrt{4\pi/\eta} = 0.1826374$.

1-3 EFFECTIVE AREA

Antennas capture power from passing waves and deliver some of it to the terminals. Given the power density of the incident wave and the effective area of the antenna, the power delivered to the terminals is the product

$$P_d = SA_{\text{eff}} \tag{1-6}$$

For an aperture antenna such as a horn, parabolic reflector, or flat-plate array, effective area is physical area multiplied by aperture efficiency. In general, losses due to material, distribution, and mismatch reduce the ratio of the effective area to the physical area. Typical estimated aperture efficiency for a parabolic reflector is 55%. Even antennas with infinitesimal physical areas, such as dipoles, have effective areas because they remove power from passing waves.

1-4 PATH LOSS [1, p. 183]

We combine the gain of the transmitting antenna with the effective area of the receiving antenna to determine delivered power and path loss. The power density at the receiving antenna is given by Eq. (1-3), and the received power is given by Eq. (1-6). By combining the two, we obtain the path loss:

$$\frac{P_d}{P_t} = \frac{A_2 G_1(\theta, \phi)}{4\pi R^2}$$

Antenna 1 transmits, and antenna 2 receives. If the materials in the antennas are linear and isotropic, the transmitting and receiving patterns are identical (reciprocal) [2, p. 116]. When we consider antenna 2 as the transmitting antenna and antenna 1 as the receiving antenna, the path loss is

$$\frac{P_d}{P_t} = \frac{A_1 G_2(\theta, \phi)}{4\pi R^2}$$

Since the responses are reciprocal, the path losses are equal and we can gather and eliminate terms:

$$\frac{G_1}{A_1} = \frac{G_2}{A_2} = \text{constant}$$

Because the antennas were arbitrary, this quotient must equal a constant. This constant was found by considering the radiation between two large apertures [3]:

$$\frac{G}{A} = \frac{4\pi}{\lambda^2} \tag{1-7}$$

We substitute this equation into path loss to express it in terms of the gains or effective areas:

$$\frac{P_d}{P_t} = G_1 G_2 \left(\frac{\lambda}{4\pi R}\right)^2 = \frac{A_1 A_2}{\lambda^2 R^2} \tag{1-8}$$

We make quick evaluations of path loss for various units of distance R and for frequency f in megahertz using the formula

$$\text{path loss(dB)} = K_U + 20\log(fR) - G_1(\text{dB}) - G_2(\text{dB}) \tag{1-9}$$

where K_U depends on the length units:

Unit	K_U
km	32.45
nm	37.80
miles	36.58
m	−27.55
ft	−37.87

Example Compute the gain of a 3-m-diameter parabolic reflector at 4 GHz assuming 55% aperture efficiency.

Gain is related to effective area by Eq. (1-7):

$$G = \frac{4\pi A}{\lambda^2}$$

We calculate the area of a circular aperture by $A = \pi(D/2)^2$. By combining these equations, we have

$$G = \left(\frac{\pi D}{\lambda}\right)^2 \eta_a = \left(\frac{\pi D f}{c}\right)^2 \eta_a \tag{1-10}$$

where D is the diameter and η_a is the aperture efficiency. On substituting the values above, we obtain the gain:

$$G = \left[\frac{3\pi(4 \times 10^9)}{0.3 \times 10^9}\right]^2 (0.55) = 8685 \quad (39.4\,\text{dB})$$

Example Calculate the path loss of a 50-km communication link at 2.2 GHz using a transmitter antenna with a gain of 25 dB and a receiver antenna with a gain of 20 dB.

$$\text{Path loss} = 32.45 + 20\log[2200(50)] - 25 - 20 = 88.3\,\text{dB}$$

What happens to transmission between two apertures as the frequency is increased? If we assume that the effective area remains constant, as in a parabolic reflector, the transmission increases as the square of frequency:

$$\frac{P_d}{P_t} = \frac{A_1 A_2}{R^2}\frac{1}{\lambda^2} = \frac{A_1 A_2}{R^2}\left(\frac{f}{c}\right)^2 = Bf^2$$

where B is a constant for a fixed range. The receiving aperture captures the same power regardless of frequency, but the gain of the transmitting antenna increases as the square of frequency. Hence, the received power also increases as frequency squared. Only for antennas, whose gain is a fixed value when frequency changes, does the path loss increase as the square of frequency.

1-5 RADAR RANGE EQUATION AND CROSS SECTION

Radar operates using a double path loss. The radar transmitting antenna radiates a field that illuminates a target. These incident fields excite surface currents that also radiate to produce a second field. These fields propagate to the receiving antenna, where they are collected. Most radars use the same antenna both to transmit the field and to collect the signal returned, called a *monostatic* system, whereas we use separate antennas for *bistatic* radar. The receiving system cannot be detected in a bistatic system because it does not transmit and has greater survivability in a military application.

We determine the power density illuminating the target at a range R_T by using Eq. (1-2):

$$S_{\text{inc}} = \frac{P_T G_T(\theta, \phi)}{4\pi R_T^2} \tag{1-11}$$

The target's radar cross section (RCS), the scattering area of the object, is expressed in square meters or dBm^2: 10 log(square meters). The RCS depends on both the incident and reflected wave directions. We multiply the power collected by the target with its receiving pattern by the gain of the effective antenna due to the currents induced:

$$\text{RCS} = \sigma = \frac{\text{power}_{\text{reflected}}}{\text{power density incident}} = \frac{P_s(\theta_r, \phi_r, \theta_i, \phi_i)}{P_T G_T / 4\pi R_T^2} \tag{1-12}$$

In a communication system we call P_s the *equivalent isotropic radiated power* (EIRP), which equals the product of the input power and the antenna gain. The target becomes the transmitting source and we apply Eq. (1-2) to find the power density at the receiving antenna at a range R_R from the target. Finally, the receiving antenna collects the power density with an effective area A_R. We combine these ideas to obtain the power delivered to the receiver:

$$P_{\text{rec}} = S_R A_R = \frac{A_R P_T G_T \sigma(\theta_r, \phi_r, \theta_i, \phi_i)}{(4\pi R_T^2)(4\pi R_R^2)}$$

We apply Eq. (1-7) to eliminate the effective area of the receiving antenna and gather terms to determine the bistatic radar range equation:

$$\frac{P_{\text{rec}}}{P_T} = \frac{G_T G_R \lambda^2 \sigma(\theta_r, \phi_r, \theta_i, \phi_i)}{(4\pi)^3 R_T^2 R_R^2} \tag{1-13}$$

We reduce Eq. (1-13) and collect terms for monostatic radar, where the same antenna is used for both transmitting and receiving:

$$\frac{P_{\text{rec}}}{P_T} = \frac{G^2 \lambda^2 \sigma}{(4\pi)^3 R^4}$$

Radar received power is proportional to $1/R^4$ and to G^2.

We find the approximate RCS of a flat plate by considering the plate as an antenna with an effective area. Equation (1-11) gives the power density incident on the plate that collects this power over an area A_R:

$$P_C = \frac{P_T G_T(\theta, \phi)}{4\pi R_T^2} A_R$$

The power scattered by the plate is the power collected, P_C, times the gain of the plate as an antenna, G_P:

$$P_s = P_C G_P = \frac{P_T G_T(\theta_i, \phi_i)}{4\pi R_T^2} A_R G_P(\theta_r, \phi_r)$$

This scattered power is the effective radiated power in a particular direction, which in an antenna is the product of the input power and the gain in a particular direction. We calculate the plate gain by using the effective area and find the scattered power in terms of area:

$$P_s = \frac{P_T G_T 4\pi A_R^2}{4\pi R_T^2 \lambda^2}$$

We determine the RCS σ by Eq. (1-12), the scattered power divided by the incident power density:

$$\sigma = \frac{P_s}{P_T G_T / 4\pi R_T^2} = \frac{4\pi A_R^2}{\lambda^2} = \frac{G_R(\theta_i, \phi_i) G_R(\theta_r, \phi_r)\lambda^2}{4\pi} \tag{1-14}$$

The right expression of Eq. (1-14) divides the gain into two pieces for bistatic scattering, where the scattered direction is different from the incident direction. Monostatic scattering uses the same incident and reflected directions. We can substitute any object for the flat plate and use the idea of an effective area and its associated antenna gain. An antenna is an object with a unique RCS characteristic because part of the power received will be delivered to the antenna terminals. If we provide a good impedance match to this signal, it will not reradiate and the RCS is reduced. When we illuminate an antenna from an arbitrary direction, some of the incident power density will be scattered by the structure and not delivered to the antenna terminals. This leads to the division of antenna RCS into the antenna mode of reradiated signals caused by terminal mismatch and the structural mode, the fields reflected off the structure for incident power density not delivered to the terminals.

1-6 WHY USE AN ANTENNA?

We use antennas to transfer signals when no other way is possible, such as communication with a missile or over rugged mountain terrain. Cables are expensive and take a long time to install. Are there times when we would use antennas over level ground? The large path losses of antenna systems lead us to believe that cable runs are better.

Example Suppose that we must choose between using a low-loss waveguide run and a pair of antennas at 3 GHz. Each antenna has 10 dB of gain. The low-loss waveguide has only 19.7 dB/km loss. Table 1-1 compares losses over various distances. The waveguide link starts out with lower loss, but the antenna system soon overtakes it. When the path length doubles, the cable link loss also doubles in decibels, but an antenna link

TABLE 1-1 Losses Over Distance

Distance (km)	Waveguide Loss (dB)	Antenna Path Loss (dB)
2	39.4	88
4	78.8	94
6	118.2	97.6
10	197	102

increases by only 6 dB. As the distance is increased, radiating between two antennas eventually has lower losses than in any cable.

Example A 200-m outside antenna range was set up to operate at 2 GHz using a 2-m-diameter reflector as a source. The receiver requires a sample of the transmitter signal to phase-lock the local oscillator and signal at a 45-MHz difference. It was proposed to run an RG/U 115 cable through the power and control cable conduit, since the run was short. The cable loss was 36 dB per 100 m, giving a total cable loss of 72 dB. A 10-dB coupler was used on the transmitter to pick off the reference signal, so the total loss was 82 dB. Since the source transmitted 100 mW (20 dBm), the signal was −62 dBm at the receiver, sufficient for phase lock.

A second proposed method was to place a standard-gain horn (15 dB of gain) within the beam of the source on a small stand out of the way of the measurement and next to the receiver. If we assume that the source antenna had only 30% aperture efficiency, we compute gain from Eq. (1-10) ($\lambda = 0.15$ m):

$$G = \left(\frac{2\pi}{0.15}\right)^2 (0.3) = 526 \quad (27.2\,\text{dB})$$

The path loss is found from Eq. (1-9) for a range of 0.2 km:

$$32.45 + 20\log[2000(0.2)] - 27.2 - 15 = 42.3\,\text{dB}$$

The power delivered out of the horn is 20 dBm − 42.3 dB = −22.3 dBm. A 20-dB attenuator must be put on the horn to prevent saturation of the receiver (−30 dBm). Even with a short run, it is sometimes better to transmit the signal between two antennas instead of using cables.

1-7 DIRECTIVITY

Directivity is a measure of the concentration of radiation in the direction of the maximum:

$$\text{directivity} = \frac{\text{maximum radiation intensity}}{\text{average radiation intensity}} = \frac{U_{\max}}{U_0} \tag{1-15}$$

Directivity and gain differ only by the efficiency, but directivity is easily estimated from patterns. Gain—directivity times efficiency—must be measured.

The average radiation intensity can be found from a surface integral over the radiation sphere of the radiation intensity divided by 4π, the area of the sphere in steradians:

$$\text{average radiation intensity} = \frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi} U(\theta,\phi)\sin\theta\, d\theta\, d\phi = U_0 \tag{1-16}$$

This is the radiated power divided by the area of a unit sphere. The radiation intensity $U(\theta,\phi)$ separates into a sum of co- and cross-polarization components:

$$U_0 = \frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi} [U_C(\theta,\phi) + U_\times(\theta,\phi)]\sin\theta\, d\theta\, d\phi \tag{1-17}$$

Both co- and cross-polarization directivities can be defined:

$$\text{directivity}_{\text{C}} = \frac{U_{\text{C,max}}}{U_0} \qquad \text{directivity}_{\times} = \frac{U_{\times,\text{max}}}{U_0} \tag{1-18}$$

Directivity can also be defined for an arbitrary direction $D(\theta, \phi)$ as radiation intensity divided by the average radiation intensity, but when the coordinate angles are not specified, we calculate directivity at U_{max}.

1-8 DIRECTIVITY ESTIMATES

Because a ratio of radiation intensities is used to calculate directivity, the pattern may be referred to any convenient level. The most accurate estimate is based on measurements at equal angle increments over the entire radiation sphere. The average may be found from coarse measurements by using numerical integration, but the directivity measured is affected directly by whether the maximum is found. The directivity of antennas with well-behaved patterns can be estimated from one or two patterns. Either the integral over the pattern is approximated or the pattern is approximated with a function whose integral is found exactly.

1-8.1 Pencil Beam

By estimating the integral, Kraus [4] devised a method for pencil beam patterns with its peak at $\theta = 0^\circ$. Given the half-power beamwidths of the principal plane patterns, the integral is approximately the product of the beamwidths. This idea comes from circuit theory, where the integral of a time pulse is approximately the pulse width (3-dB points) times the pulse peak: $U_0 = \theta_1\theta_2/4\pi$, where θ_1 and θ_2 are the 3-dB beamwidths, in radians, of the principal plane patterns:

$$\text{directivity} = \frac{4\pi}{\theta_1\theta_2}(\text{rad}) = \frac{41{,}253}{\theta_1\theta_2}(\text{deg}) \tag{1-19}$$

Example Estimate the directivity of an antenna with E- and H-plane (principal plane) pattern beamwidths of 24° and 36°.

$$\text{Directivity} = \frac{41{,}253}{24(36)} = 47.75 \quad (16.8\,\text{dB})$$

An analytical function, $\cos^{2N}(\theta/2)$, approximates a broad pattern centered on $\theta = 0^\circ$ with a null at $\theta = 180^\circ$:

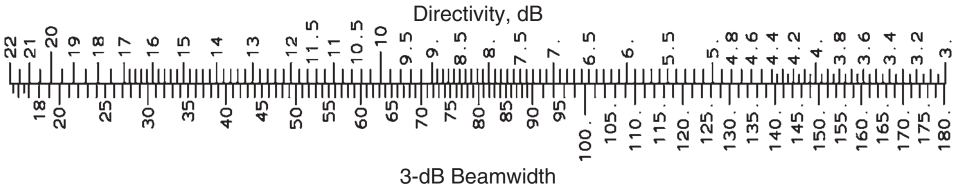
$$U(\theta) = \cos^{2N}(\theta/2) \quad \text{or} \quad E = \cos^{N}(\theta/2)$$

The directivity of this pattern can be computed exactly. The characteristics of the approximation are related to the beamwidth at a specified level, Lvl(dB):

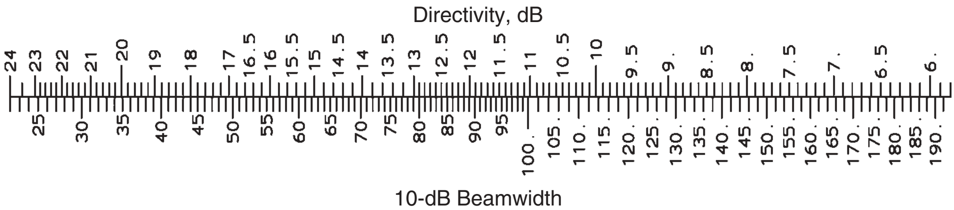
$$\text{beamwidth [Lvl(dB)]} = 4\cos^{-1}(10^{-\text{Lvl(dB)}/20N}) \tag{1-20a}$$

$$N = \frac{-\text{Lvl(dB)}}{20\log[\cos(\text{beamwidth}_{\text{Lvl(dB)}}/4)]} \tag{1-20b}$$

$$\text{directivity} = N + 1 \quad \text{(ratio)} \tag{1-20c}$$



SCALE 1-1 3-dB beamwidth and directivity relationship for $\cos^{2N}(\theta/2)$ pattern.



SCALE 1-2 10-dB beamwidth and directivity relationship for $\cos^{2N}(\theta/2)$ pattern.

Scales 1-1 and 1-2, which give the relationship between beamwidth and directivity using Eq. (1-20), are useful for quick conversion between the two properties. You can use the two scales to estimate the 10-dB beamwidth given the 3-dB beamwidth. For example, an antenna with a 90° 3-dB beamwidth has a directivity of about 7.3 dB. You read from the lower scale that an antenna with 7.3-dB directivity has a 159.5° 10-dB beamwidth. Another simple way to determine the beamwidths at different pattern levels is the square-root factor approximation:

$$\frac{\mathrm{BW}[\mathrm{Lvl}\ 2(\mathrm{dB})]}{\mathrm{BW}[\mathrm{Lvl}\ 1(\mathrm{dB})]} = \sqrt{\frac{\mathrm{Lvl}\ 2(\mathrm{dB})}{\mathrm{Lvl}\ 1(\mathrm{dB})}}$$

By this factor, $\text{beamwidth}_{10\,\text{dB}} = 1.826\ \text{beamwidth}_{3\,\text{dB}}$; an antenna with a 90° 3-dB beamwidth has a $(1.826)90° = 164.3°$ 10-dB beamwidth.

This pattern approximation requires equal principal plane beamwidths, but we use an elliptical approximation with unequal beamwidths:

$$U(\theta,\phi) = \cos^{2N_e}(\theta/2)\cos^2\phi + \cos^{2N_h}(\theta/2)\sin^2\phi \tag{1-21}$$

where N_e and N_h are found from the principal plane beamwidths. We combine the directivities calculated in the principal planes by the simple formula

$$\text{directivity (ratio)} = \frac{2\cdot \text{directivity}_e \cdot \text{directivity}_h}{\text{directivity}_e + \text{directivity}_h} \tag{1-22}$$

Example Estimate the directivity of an antenna with E- and H-plane pattern beamwidths of 98° and 140°.

From the scale we read a directivity of 6.6 dB in the E-plane and 4.37 dB in the H-plane. We convert these to ratios and apply Eq. (1-22):

$$\text{directivity (ratio)} = \frac{2(4.57)(2.74)}{4.57+2.74} = 3.426 \quad \text{or} \quad 10\log(3.426) = 5.35\,\text{dB}$$

Many analyses of paraboloidal reflectors use a feed pattern approximation limited to the front hemisphere with a zero pattern in the back hemisphere:

$$U(\theta) = \cos^{2N}\theta \quad \text{or} \quad E = \cos^{N}\theta \qquad \text{for } \theta \leq \pi/2(90^\circ)$$

The directivity of this pattern can be found exactly, and the characteristics of the approximation are

$$\text{beamwidth [Lvl(dB)]} = 2\cos^{-1}(10^{-\text{Lvl(dB)}/20N}) \tag{1-23a}$$

$$N = \frac{-\text{Lvl(dB)}}{20\log[\cos(\text{beamwidth}_{\text{Lvl(dB)}}/2)]} \tag{1-23b}$$

$$\text{directivity} = 2(2N+1) \quad \text{(ratio)} \tag{1-23c}$$

We use the elliptical model [Eq. (1-21)] with this approximate pattern and use Eq. (1-22) to estimate the directivity when the E- and H-plane beamwidths are different.

1-8.2 Butterfly or Omnidirectional Pattern

Many antennas have nulls at $\theta = 0^\circ$ with rotational symmetry about the z-axis (Figure 1-2). Neither of the directivity estimates above can be used with these patterns because they require the beam peak to be at $\theta = 0^\circ$. We generate this type of antenna pattern by using mode 2 log-periodic conical spirals, shaped reflectors, some higher-order-mode waveguide horns, biconical horns, and traveling-wave antennas. A formula similar to Kraus's can be found if we assume that all the power is between the 3-dB beamwidth angles θ_1 and θ_2:

$$U_0 = \frac{1}{2}\int_{\theta_1}^{\theta_2} \sin\theta \, d\theta = \frac{\cos\theta_1 - \cos\theta_2}{2}$$

Rotational symmetry eliminates integration over ϕ:

$$\text{directivity} = \frac{U_{\max}}{U_0} = \frac{2}{\cos\theta_1 - \cos\theta_2} \tag{1-24}$$

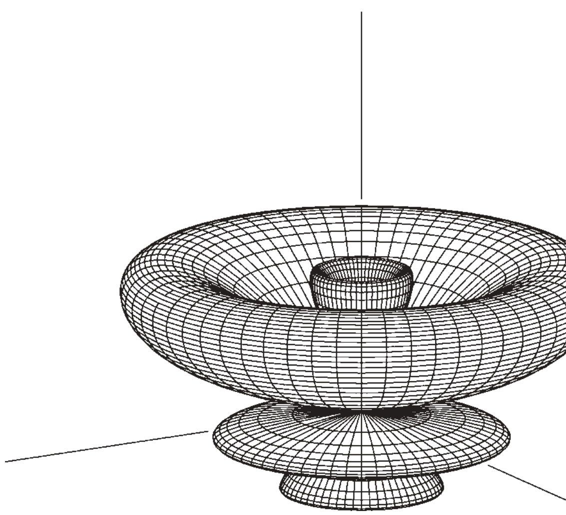
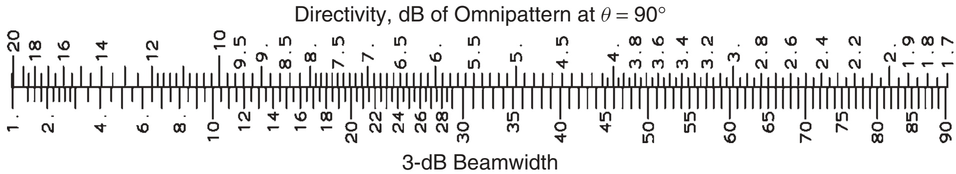


FIGURE 1-2 Omnidirectional antenna pattern with sidelobes scanned above the horizon.



SCALE 1-3 Relationship between 3-dB beamwidth of omnidirectional pattern and directivity.

Example A pattern with rotational symmetry has half-power points at 35° and 75°. Estimate the directivity.

$$\text{Directivity} = \frac{2}{\cos 35^\circ - \cos 75^\circ} = 3.57 \quad (5.5\,\text{dB})$$

If the pattern also has symmetry about the $\theta = 90^\circ$ plane, the integral for the average radiation intensity has limits from 0 to $\pi/2$. Equation (1-24) reduces to directivity $= 1/\cos\theta_1$.

Example A rotationally symmetric pattern with a maximum at 90° has a 45° beamwidth. Estimate the directivity.
$\theta_1 = 90^\circ - 45^\circ/2 = 67.5^\circ$, so

$$\text{directivity} = \frac{1}{\cos 67.5^\circ} = 2.61 \quad (4.2\,\text{dB})$$

The pattern can be approximated by the function

$$U(\theta) = B\sin^{2M}(\theta/2)\cos^{2N}(\theta/2)$$

but the directivity estimates found by integrating this function show only minor improvements over Eq. (1-24). Nevertheless, we can use the expression for analytical patterns. Given beam edges θ_L and θ_U at a level Lvl(dB), we find the exponential factors.

$$AA = \frac{\ln[\cos(\theta_U/2)] - \ln[\cos(\theta_L/2)]}{\ln[\sin(\theta_L/2)] - \ln[\sin(\theta_U/2)]} \quad \text{and} \quad TM_2 = \tan^{-1}\sqrt{AA}$$

$$N = \frac{-|\text{Lvl(dB)}|/8.68589}{AA\{\ln[\sin(\theta_L/2)] - \ln(\sin TM_2)\} + \ln(\cos(\theta_L/2)) - \ln(\cos TM_2)}$$

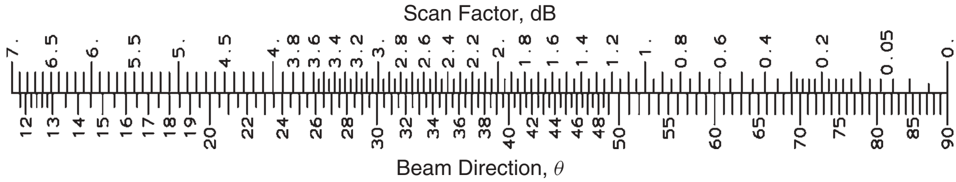
$$M = AA(N)$$

A second pattern model of an omnidirectional pattern based on the pattern function with minor sidelobes and a beam peak at θ_0 measured from the symmetry axis is

$$\frac{\sin[b(\theta_0 - \theta)]}{b(\theta_0 - \theta)}$$

We estimate the directivity from the half-power beamwidth (HPBW) and the beam peak θ_0 [5]:

$$\text{directivity(dB)} = 10\log\frac{101}{(\text{HPBW} - 0.0027\text{HPBW}^2)\sin\theta_0} \tag{1-25}$$



SCALE 1-4 Additional directivity of omnidirectional pattern when scanned into conical pattern.

Scale 1-3 evaluates this formula for a beam at $\theta_0 = 90^\circ$ given HPBW, and Scale 1-4 gives the additional gain when the beam peak scans toward the axis.

The directivity of butterfly patterns with unequal beamwidths in the principal planes cannot be estimated directly from the foregoing formulas. Similarly, some pencil beam patterns have large sidelobes which decrease the directivity and cannot be estimated accurately from Eq. (1-19). Both problems are solved by considering the directivity as an estimate of the average radiation intensity.

Example A butterfly pattern peak is at 50° in both principal planes, but the beamwidths are 20° and 50°. Estimate the directivity.

The 3-dB pattern points are given by:

Cut 1 (40° and 60°):

$$U_{01} = \frac{\cos 40^\circ - \cos 60^\circ}{2} = 0.133$$

Cut 2 (25° and 75°):

$$U_{02} = \frac{\cos 25^\circ - \cos 75^\circ}{2} = 0.324$$

Average the two pattern integral estimates:

$$U_0 = \frac{0.133 + 0.324}{2} = 0.228$$

$$\text{directivity} = \frac{U_{\max}}{U_0} = \frac{1}{0.228} = 4.38 \quad (6.4\,\text{dB})$$

Suppose that the beams are at different levels on the same pattern. For example, the lobe on the right of the first pattern is the peak and the left lobe is reduced by 3 dB. The peaks of the second pattern are reduced by 1 dB. We can average on one pattern alone. Each lobe contributes $U_{\max}(\cos\theta_1 - \cos\theta_2)/4$ to the integral. The integral of the first pattern is approximated by

$$\frac{0.266 + 0.266 \times 10^{-3/10}}{4} = 0.100$$

The integral of the second pattern is reduced 1 dB from the peak. The average radiation intensity is found by averaging the two pattern averages:

$$U_0 = \frac{0.100 + 0.324 \times 10^{-0.1}}{2} = 0.178$$

$$\text{directivity} = \frac{1}{U_0} = 5.602 \quad (7.5\,\text{dB})$$

Pencil beam patterns with large sidelobes can be averaged in a similar manner: $U_p = 1/\text{directivity}$. By using Eq. (1-19) and assuming equal beamwidths, we have $U_p = \text{HPBW}^2/41,253$, where U_p is the portion of the integral due to the pencil beam and HPBW is the beamwidth in degrees.

Example Consider a pencil beam antenna with pattern beamwidths of 50° and 70° in the principal planes. The second pattern has a sidelobe at $\theta = 60^\circ$ down 5 dB from the peak and a 30° beamwidth below the 5 dB. What is the effect of the sidelobe on the directivity estimate?

Without the sidelobe the directivity estimate is

$$\text{directivity} = \frac{41253}{50(70)} = 11.79 \quad (10.7\,\text{dB})$$

Consider each pattern separately:

$$U_{\text{P1}} = \frac{50^2}{41{,}253} = 0.0606 \qquad U_{\text{P2}} = \frac{70^2}{41{,}253} = 0.1188$$

The sidelobe adds to the second integral:

$$U_{\text{PS2}} = \frac{(\cos 45^\circ - \cos 75^\circ)10^{-5/10}}{4} = 0.0354$$

Averaging the integrals of the parts gives us 0.1074:

$$\text{directivity} = \frac{1}{U_0} = 9.31 \quad (9.7\,\text{dB})$$

If there had been a sidelobe on each side, each would have added to the integral. Estimating integrals in this manner has limited value. Remember that these are only approximations. More accurate results can be obtained by digitizing the pattern and performing numerical integration on each pattern by using Eq. (1-16) or (1-17).

1-9 BEAM EFFICIENCY

Radiometer system designs [6, p. 31–6] specify the antenna in terms of beam efficiency. For a pencil beam antenna with the boresight at $\theta = 0$, the beam efficiency is the ratio (or percent) of the pattern power within a specified cone centered on the boresight to the total radiated power. In terms of the radiation intensity U,

$$\text{beam efficiency} = \frac{\displaystyle\int_0^{\theta_1}\int_0^{2\pi} U(\theta,\phi)\sin\theta\,d\theta\,d\phi}{\displaystyle\int_0^{\pi}\int_0^{2\pi} U(\theta,\phi)\sin\theta\,d\theta\,d\phi} \tag{1-26}$$

where U includes both polarizations if necessary. Extended noise sources, such as radiometry targets, radiate noise into sidelobes of the antenna. Beam efficiency measures the probability of the detected target being located within the main beam ($\theta \le \theta_1$).

Sometimes we can calculate directivity more easily than the pattern everywhere required by the denominator of Eq. (1-26): for example, a paraboloidal reflector. We use Eqs. (1-15) and (1-16) to calculate the denominator integral:

$$\int_0^{\pi}\int_0^{2\pi} U(\theta,\phi)\sin\theta\, d\theta\, d\phi = \frac{4\pi U_{\max}}{\text{directivity}}$$

This reduces Eq. (1-26) to

$$\text{beam efficiency} = \frac{\text{directivity}\displaystyle\int_0^{\theta_1}\int_0^{2\pi} U(\theta,\phi)\sin\theta\, d\theta\, d\phi}{4\pi U_{\max}} \tag{1-27}$$

Equation (1-27) greatly reduces the pattern calculation requirements to compute beam efficiency when the directivity can be found without pattern evaluation over the entire radiation sphere.

1-10 INPUT-IMPEDANCE MISMATCH LOSS

When we fail to match the impedance of an antenna to its input transmission line leading from the transmitter or to the receiver, the system degrades due to reflected power. The input impedance is measured with respect to some transmission line or source characteristic impedance. When the two are not the same, a voltage wave is reflected, ρV, where ρ is the voltage reflection coefficient:

$$\rho = \frac{Z_A - Z_0}{Z_A + Z_0} \tag{1-28}$$

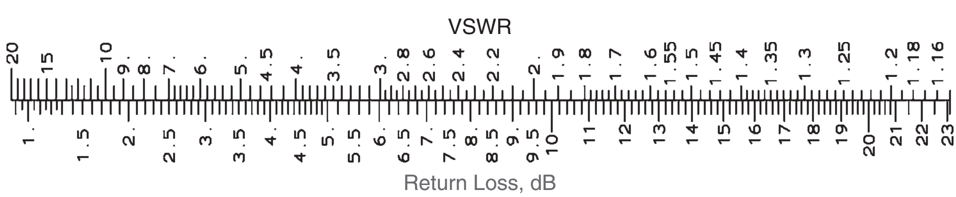
Z_A is the antenna impedance and Z_0 is the measurement characteristic impedance. On a transmission line the two traveling waves, incident and reflected, produce a standing wave:

$$V_{\max} = (1+|\rho|)V_i \qquad V_{\min} = (1-|\rho|)V_i \tag{1-29}$$

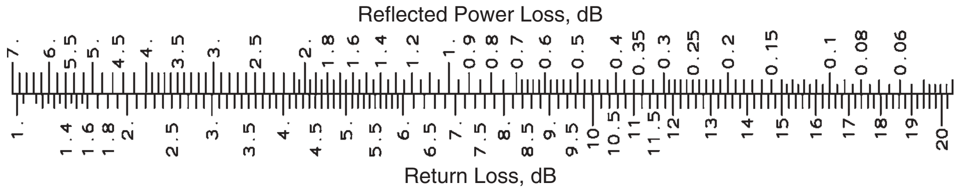
$$\text{VSWR} = \frac{V_{\max}}{V_{\min}} = \frac{1+|\rho|}{1-|\rho|} \tag{1-30}$$

VSWR is the voltage standing-wave ratio. We use the magnitude of ρ, a complex phasor, since all the terms in Eq. (1-28) are complex numbers. The reflected power is given by $V_i^2|\rho|^2/Z_0$. The incident power is V_i^2/Z_0. The ratio of the reflected power to the incident power is $|\rho|^2$. It is the returned power ratio. Scale 1-5 gives the conversion between return loss and VSWR:

$$\text{return loss} = -20\log|\rho| \tag{1-31}$$



SCALE 1-5 Relationship between return loss and VSWR.



SCALE 1-6 Reflected power loss due to antenna impedance mismatch.

The power delivered to the antenna is the difference between the incident and the reflected power. Normalized, it is expressed as

$$1 - |\rho|^2 \text{ or reflected power loss(dB)} = 10\log(1 - |\rho|^2) \tag{1-32}$$

The source impedance to achieve maximum power transfer is the complex conjugate of the antenna impedance [7, p. 94]. Scale 1-6 computes the power loss due to antenna impedance mismatch.

If we open-circuit the antenna terminals, the reflected voltage equals the incident voltage. The standing wave doubles the voltage along the transmission line compared to the voltage present when the antenna is loaded with a matched load. We consider the effective height of an antenna, the ratio of the open-circuit voltage to the input field strength. The open-circuit voltage is twice that which appears across a matched load for a given received power. We can either think of this as a transmission line with a mismatch that doubled the incident voltage or as a Thévenin equivalent circuit with an open-circuit voltage source that splits equally between the internal resistor and the load when it is matched to the internal resistor. Path loss analysis predicts the power delivered to a matched load. The mathematical Thévenin equivalent circuit containing the internal resistor does not say that half the power received by the antenna is either absorbed or reradiated; it only predicts the circuit characteristics of the antenna load under all conditions.

Possible impedance mismatch of the antenna requires that we derate the feed cables. The analysis above shows that the maximum voltage that occurs on the cable is twice that present when the cable impedance is matched to the antenna. We compute the maximum voltage given the VSWR using Eq. (1-29) for the maximum voltage:

$$V_{\max} = \frac{2\ \text{VSWR}(V_i)}{\text{VSWR} + 1} = \frac{2V_i}{1 + 1/\text{VSWR}} \tag{1-33}$$

1-11 POLARIZATION

The polarization of a wave is the direction of the electric field. We handle all polarization problems by using vector operations on a two-dimensional space using the far-field radial vector as the normal to the plane. This method is systematic and reduces chance of error. The spherical wave in the far field has only θ and ϕ components of the electric field: $\mathbf{E} = E_\theta\hat{\theta} + E_\phi\hat{\phi}$. E_θ and E_ϕ are phasor components in the direction of the unit vectors $\hat{\theta}$ and $\hat{\phi}$. We can also express the direction of the electric field in terms of a plane wave propagating along the z-axis: $\mathbf{E} = E_x\hat{\mathbf{x}} + E_y\hat{\mathbf{y}}$. The direction of propagation confines the electric field to a plane. Polarization is concerned with methods

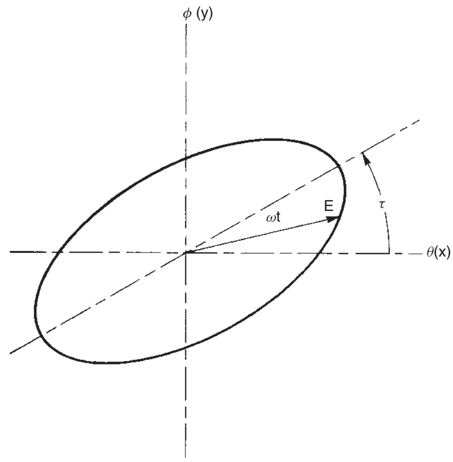


FIGURE 1-3 Polarization ellipse.

of describing this two-dimensional space. Both of the above are linear polarization expansions. We can rewrite them as

$$\mathbf{E} = E_\theta(\hat{\theta} + \hat{\rho}_L\hat{\phi}) \qquad \hat{\rho}_L = \frac{E_\phi}{E_\theta}$$

$$\mathbf{E} = E_x(\hat{\mathbf{x}} + \hat{\rho}_L\hat{\mathbf{y}}) \qquad \hat{\rho}_L = \frac{E_y}{E_x} \tag{1-34}$$

where $\hat{\rho}_L$ is the linear polarization ratio, a complex constant. If time is inserted into the expansions, and the tip of the electric field traced in space over time, it appears as an ellipse with the electric field rotating either clockwise (CW) or counter clockwise (CCW) (Figure 1-3). τ is the tilt of the polarization ellipse measured from the x-axis ($\phi = 0$) and the angle of maximum response. The ratio of the maximum to minimum linearly polarized responses on the ellipse is the axial ratio.

If $\hat{\rho}_L = e^{\pm j\pi/2}$, the ellipse expands to a circle and gives the special case of circular polarization. The electric field is constant in magnitude but rotates either CW (left hand) or CCW (right hand) at the rate ωt for propagation perpendicular to the page.

1-11.1 Circular Polarization Components

The two circular polarizations also span the two-dimensional space of polarization. The right- and left-handed orthogonal unit vectors defined in terms of linear components are

$$\hat{\mathbf{R}} = \frac{1}{\sqrt{2}}(\hat{\theta} - j\hat{\phi}) \quad \text{or} \quad \hat{\mathbf{R}} = \frac{1}{\sqrt{2}}(\hat{\mathbf{x}} - j\hat{\mathbf{y}}) \tag{1-35a}$$

$$\hat{\mathbf{L}} = \frac{1}{\sqrt{2}}(\hat{\theta} + j\hat{\phi}) \quad \text{or} \quad \hat{\mathbf{L}} = \frac{1}{\sqrt{2}}(\hat{\mathbf{x}} + j\hat{\mathbf{y}}) \tag{1-35b}$$

The electric field in the polarization plane can be expressed in terms of these new unit vectors:

$$\mathbf{E} = E_L\hat{\mathbf{L}} + E_R\hat{\mathbf{R}}$$

When projecting a vector onto one of these unit vectors, it is necessary to use the complex conjugate in the scalar (dot) product:

$$E_L = \mathbf{E} \cdot \hat{\mathbf{L}}^* \qquad E_R = \mathbf{E} \cdot \hat{\mathbf{R}}^*$$

When we project $\hat{\mathbf{R}}$ onto itself, we obtain

$$\hat{\mathbf{R}} \cdot \hat{\mathbf{R}}^* = \tfrac{1}{2}(\hat{\theta} - j\hat{\phi}) \cdot (\hat{\theta} + j\hat{\phi}) = \tfrac{1}{2}(1 - j \cdot j) = 1$$

Similarly,

$$\hat{\mathbf{L}} \cdot \hat{\mathbf{R}}^* = \tfrac{1}{2}(\hat{\theta} + j\hat{\phi}) \cdot (\hat{\theta} + j\hat{\phi}) = \tfrac{1}{2}(1 + j \cdot j) = 0$$

The right- and left-handed circular (RHC and LHC) components are orthonormal.

A circular polarization ratio can be defined from the equation

$$\mathbf{E} = E_L(\hat{\mathbf{L}} + \hat{\rho}_c\hat{\mathbf{R}}) \qquad \hat{\rho}_c = \frac{E_R}{E_L} = \rho_c e^{j\delta_c}$$

Let us look at a predominately left-handed circularly polarized wave when time and space combine to a phase of zero for E_L. We draw the polarization as two circles (Figure 1-4). The circles rotate at the rate ωt in opposite directions (Figure 1-5), with the center of the right-handed circular polarization circle moving on the end of the vector of the left-handed circular polarization circle. We calculate the phase of the circular polarization ratio $\hat{\rho}_c$ from the complex ratio of the right- and left-handed circular components. Maximum and minimum electric fields occur when the circles

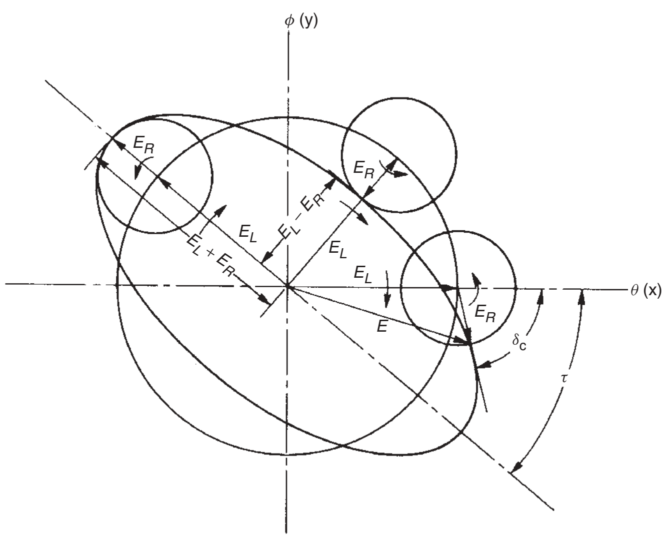


FIGURE 1-4 Polarization ellipse LHC and RHC components. (After J. S. Hollis, T. J. Lyons, and L. Clayton, *Microwave Antenna Measurements,* Scientific Atlanta, 1969, pp. 3–6. Adapted by permission.)

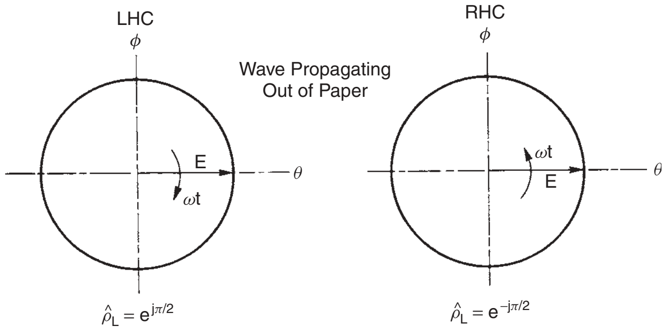
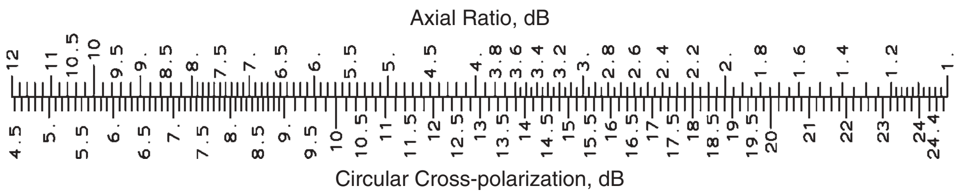


FIGURE 1-5 Circular polarization components. (After J. S. Hollis, T. J. Lyons, and L. Clayton, *Microwave Antenna Measurements,* Scientific Atlanta, 1969, pp. 3–5. Adapted by permission.)



SCALE 1-7 Circular cross-polarization/axial ratio.

alternately add and subtract as shown in Figure 1-4. Scale 1-7 shows the relationship between circular cross-polarization and axial ratio:

$$E_{\max} = (|E_L| + |E_R|)/\sqrt{2} \qquad E_{\min} = (|E_L| - |E_R|)/\sqrt{2}$$

$$\text{axial ratio} = \begin{cases} \dfrac{E_{\max}}{E_{\min}} = \dfrac{|E_L| + |E_R|}{|E_L| - |E_R|} = \dfrac{1 + |\hat{\rho}_c|}{1 - |\hat{\rho}_c|} & \text{LHC} \\ \dfrac{E_{\max}}{E_{\min}} = \dfrac{|E_R| + |E_L|}{|E_R| - |E_L|} = \dfrac{|\hat{\rho}_c| + 1}{|\hat{\rho}_c| - 1} & \text{RHC} \end{cases} \tag{1-36}$$

$$0 \le \begin{cases} |\hat{\rho}_c| < 1 & \text{LHC} \\ \left|\dfrac{1}{\hat{\rho}_c}\right| < 1 & \text{RHC} \end{cases}$$

$$\text{axial ratio(dB)} = 20 \log \frac{E_{\max}}{E_{\min}}$$

The tilt angle of the polarization ellipse τ is one-half δ_c, the phase of $\hat{\rho}_c$. Imagine time moving forward in Figure 1-5. When the LHC vector has rotated $\delta_c/2$ CW, the RHC vector has rotated $\delta_c/2$ CCW and the two align for a maximum.

1-11.2 Huygens Source Polarization

When we project the currents induced on a paraboloidal reflector to an aperture plane, Huygens source radiation induces aligned currents that radiate zero cross-polarization in the principal planes. We separate feed antenna radiation into orthogonal Huygens sources for this case. To calculate the far-field pattern of a paraboloid reflector, we can skip the step involving currents and integrate over the Huygens source fields in the aperture plane directly. We transform the measured fields of the feed into orthogonal Huygens sources by

$$\begin{bmatrix} E_\mathrm{c} \\ E_\mathrm{x} \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} E_{\theta f} \\ E_{\phi f} \end{bmatrix} \tag{1-37}$$

where E_c is the $\phi = 0$ direction of polarization in the feed pattern and E_x is the $\phi = 90^\circ$ polarization. This division corresponds to Ludwig's third definition of cross-polarization [8]. The following matrix converts the Huygens source polarizations to the normal far-field components of spherical coordinates:

$$\begin{bmatrix} E_\theta \\ E_\phi \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} E_\mathrm{c} \\ E_\mathrm{x} \end{bmatrix} \tag{1-38}$$

1-11.3 Relations Between Bases

In problems with antennas at arbitrary orientations, circularly polarized components have an advantage over linear components. When the coordinate system is rotated, both the amplitude and phase change for $\hat{\rho}_L$, the linear polarization ratio, whereas the circular polarization ratio $\hat{\rho}_c$ magnitude is constant under rotations and only the phase changes. In other words, the ratio of the diameters of the circles (Figure 1-4) is constant.

The circular components can be found from linear polarization components by projection.

$$E_R = (E_\theta\hat{\theta} + E_\phi\hat{\phi}) \cdot \hat{\mathbf{R}}^* = \frac{1}{\sqrt{2}}(E_\theta\hat{\theta} + E_\phi\hat{\phi}) \cdot (\hat{\theta} + j\hat{\phi})$$

$$E_R = \frac{1}{\sqrt{2}}(E_\theta + jE_\phi) \tag{1-39}$$

Similarly,

$$E_L = \frac{1}{\sqrt{2}}(E_\theta - jE_\phi)$$

The linear polarizations can be found in terms of the circular components in the same manner:

$$E_\theta = \frac{1}{\sqrt{2}}(E_L + E_R) \qquad E_\phi = \frac{j}{\sqrt{2}}(E_L - E_R)$$

These relations enable the conversion between polarizations.

Good circularly polarized antennas over a wide bandwidth are difficult to build, but good linearly polarized antennas are obtained easily. After we measure the phase and amplitude of E_θ and E_ϕ component phasors, we compute the circular components from Eq. (1-39), the axial ratio by using Eq. (1-36), and the polarization ellipse tilt τ from one-half the phase of E_R/E_L. We employ a leveled phase-locked source to record two patterns with orthogonal linear sources (or the same linear source is rotated between patterns). Afterward, we use the equations given above to convert polarization to any desired polarization components. We calculate the maximum and minimum linear components by projecting the linear components into the rotated coordinate system of the polarization ellipse:

$$E_{\max} = E_\theta \cos\tau + E_\phi \sin\tau$$
$$E_{\min} = -E_\theta \sin\tau + E_\phi \cos\tau$$

1-11.4 Antenna Polarization Response

The path loss formulas assume that the two antennas have matched polarizations. Polarization mismatch adds an extra loss. We determine polarization efficiency by applying the scalar (dot) product between normalized polarization vectors. An antenna transmitting in the z-direction has the linear components

$$\mathbf{E}_a = E_1(\hat{\mathbf{x}} + \hat{\rho}_{L1}\hat{\mathbf{y}})$$

The incident wave on the antenna is given by

$$\mathbf{E}_i = E_2(\hat{\mathbf{x}} + \hat{\rho}_{L2}\hat{\mathbf{y}})$$

where the wave is expressed in the coordinates of the source antenna. The z-axis of the source is in the direction opposite that of the antenna. It is necessary to rotate the coordinates of the receiving antenna wave. Rotating about the x-axis is equivalent to changing the sign of the tilt angle or taking the complex conjugate of $\mathbf{E}_a$.

The measurement antenna projects the incident wave polarization onto the antenna polarization. The antenna measures the incident field, but we need to normalize the antenna polarization to a unit vector to calculate polarization efficiency:

$$\mathbf{E}_2 \cdot \mathbf{E}_1^* = \frac{E_2 E_1^*(1 + \hat{\rho}_{L2}\hat{\rho}_{L1}^*)}{\sqrt{1 + |\hat{\rho}_{L1}|^2}}$$

We normalize both the incident wave and antenna responses to determine loss due to polarization mismatch:

$$\frac{\mathbf{E}_i}{|\mathbf{E}_i|} = \frac{\hat{\mathbf{x}} + \hat{\rho}_{L2}\hat{\mathbf{y}}}{\sqrt{1 + \hat{\rho}_{L2}^*\hat{\rho}_{L2}}} \qquad \frac{\mathbf{E}_a^*}{|\mathbf{E}_a|} = \frac{\hat{\mathbf{x}} + \hat{\rho}_{L1}^*\hat{\mathbf{y}}}{\sqrt{1 + \hat{\rho}_{L1}^*\hat{\rho}_{L1}}}$$

The normalized voltage response is

$$\frac{\mathbf{E}_i \cdot \mathbf{E}_a^*}{|\mathbf{E}_i||\mathbf{E}_a|} = \frac{1 + \hat{\rho}_{L1}^*\hat{\rho}_{L2}}{\sqrt{1 + \hat{\rho}_{L1}\hat{\rho}_{L1}^*}\sqrt{1 + \hat{\rho}_{L2}\hat{\rho}_{L2}^*}} \tag{1-40}$$

When we express it as a power response, we obtain the polarization efficiency Γ:

$$\Gamma = \frac{|\mathbf{E}_i \cdot \mathbf{E}_a^*|^2}{|\mathbf{E}_i|^2|\mathbf{E}_a|^2} = \frac{1 + |\hat{\rho}_{L1}|^2|\hat{\rho}_{L2}|^2 + 2|\hat{\rho}_{L1}||\hat{\rho}_{L2}|\cos(\delta_1 - \delta_2)}{(1 + |\hat{\rho}_{L1}|^2)(1 + |\hat{\rho}_{L2}|^2)} \tag{1-41}$$

This is the loss due to polarization mismatch. Given that δ_1 and δ_2 are the phases of the polarization ratios of the antenna and the incident wave. As expressed in terms of linear polarization ratios, the formula is awkward because when the antenna is rotated to determine the peak response, both the amplitudes and phases change. A formula using circular polarization ratios would be more useful, because only phase changes under rotation.

Two arbitrary polarizations are orthogonal ($\Gamma = 0$) only if

$$|\hat{\rho}_1| = \frac{1}{|\hat{\rho}_2|} \quad \text{and} \quad \delta_1 - \delta_2 = \pm 180^\circ \tag{1-42}$$

This can be expressed as vectors by using unit vectors: $\mathbf{a}_1 \cdot \mathbf{a}_2^* = 0$; $\mathbf{a}_1$ and $\mathbf{a}_2$ are the orthonormal generalized basis vectors for polarization. We can define polarization in terms of this basis with a polarization ratio ρ. By paralleling the analysis above for linear polarizations, we obtain the polarization efficiency for an arbitrary orthonormal polarization basis:

$$\Gamma = \frac{1 + |\hat{\rho}_1|^2|\hat{\rho}_2|^2 + 2|\hat{\rho}_1||\hat{\rho}_2|\cos(\delta_1 - \delta_2)}{(1 + |\hat{\rho}_1|^2)(1 + |\hat{\rho}_2|^2)} \tag{1-43}$$

It has the same form as Eq. (1-41) derived for linear polarizations.

We can use Eq. (1-43) with circular polarizations whose polarization ratio ρ_c magnitudes are constant with rotations of the antenna. The maximum and minimum polarization efficiencies occur when $\delta_1 - \delta_2$ equals 0° and 180°, respectively. The polarization efficiency becomes

$$\Gamma_{\max/\min} = \frac{(1 \pm |\hat{\rho}_1||\hat{\rho}_2|)^2}{(1 + |\hat{\rho}_1|^2)(1 + |\hat{\rho}_2|^2)} \tag{1-44}$$

In all other vector pair bases for polarization, the magnitude of the polarization ratio ρ changes under rotations.

Figure 1-6 expresses Eq. (1-44) as a nomograph. If we have fixed installations, we can rotate one antenna until the maximum response is obtained and realize minimum polarization loss. In transmission between mobile antennas such as those mounted on missiles or satellites, the orientation cannot be controlled and the maximum polarization loss must be used in the link analysis. Circularly polarized antennas are used in these cases.

Example A satellite telemetry antenna is RHC with an axial ratio of 7 dB. The ground station is RHC with a 1.5-dB axial ratio. Determine the polarization loss.

Because the orientation of the satellite is unknown, we must use the maximum polarization loss. To find it, use the RHC ends of the scales in Figure 1-6. Draw a line from 7 on the leftmost scale to 1.5 on the center scale. Read the loss on the scale between: 0.9 dB. The measured cross-polarization response of a linearly polarized antenna is the reciprocal of the axial ratio, the same absolute magnitude in decibels.

Example Suppose that the linear cross-polarization responses of two antennas in a stationary link are given as 10 and 20 dB. Compute the minimum polarization loss.

We rotate one of the antennas until the maximum response is found. The specification of cross-polarization response does not state whether an antenna is predominately left- or right-handed circularly polarized. It must be one or the other. Suppose that the 20-dB cross-polarization antenna is LHC. If the other antenna also is LHC, we use a line drawn from the lower portion of the center scale in Figure 1-6 to the rightmost LHC scale and read 0.2 dB of loss on the scale between the two. The second possibility is

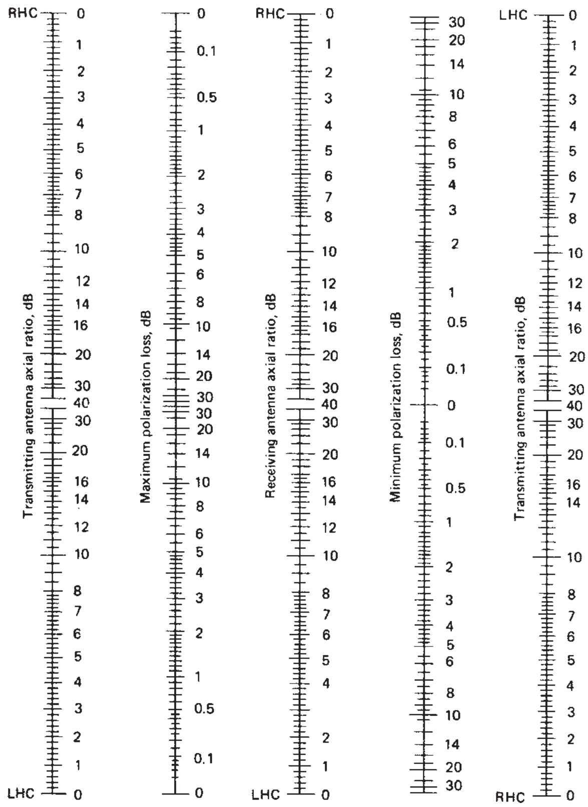


FIGURE 1-6 Maximum and minimum polarization loss. (After A. C. Ludwig, A simple graph for determining polarization loss, *Microwave Journal,* vol. 19, no. 9, September 1976, p. 63.)

that the antenna could be predominately RHC. On drawing a line to the RHC (lower) scale, we read 0.7 dB on the center scale. When polarization is expressed in terms of linearly polarized components, it is ambiguous to give only magnitudes and no information of the circular polarization sense.

1-11.5 Phase Response of Rotating Antennas

The polarization sense of an antenna can be determined from the phase slope of a rotating antenna. Before starting the phase measurement, determine that the setup is proper. Some older phase–amplitude receivers are ambiguous, depending on whether the local oscillator frequency was above or below the signal frequency. We use the convention that increased distance between antennas gives decreased phase. Move the antenna away from the source and observe decreasing phase or correct the setup. A rotating linearly polarized source field is given by

$$E_s = E_2(\cos\alpha\hat{\mathbf{x}} + \sin\alpha\hat{\mathbf{y}})$$

where α is clockwise rotation viewed from the direction of propagation (forward). A horizontally polarized linear antenna has the response $E_a = E_1\hat{\mathbf{x}}$. It responds to the rotating linear source field, $E_1E_2\cos\alpha$. The phase is constant under rotation until the null is passed and it flips 180° through the null.

An RHC polarized antenna has the response $E_1(\hat{\mathbf{x}} - j\hat{\mathbf{y}})$. It responds to the rotating linear source field,

$$E_1E_2(\cos\alpha - j\sin\alpha) = E_1E_2e^{-j\alpha}$$

The magnitude remains constant, but the phase decreases with rotation. Phase increases when the antenna is LHC. By observing the phase slope, the sense of the predominant polarization can be determined: RHC = negative phase slope; LHC = positive phase slope. It is easily remembered by considering the basis vectors of circular polarization:

$$\hat{\mathbf{R}} = \frac{1}{\sqrt{2}}(\hat{\mathbf{x}} - j\hat{\mathbf{y}})$$

In rotation from the x-axis to the y-axis, the phase decreases 90°.

1-11.6 Partial Gain

If we measure the antenna gain to one polarization (e.g., RHC) and operate it in a link with an antenna also measured to one polarization, Eq. (1-44) fails to predict the response. Polarization efficiency assumes that the antenna gain was measured using a source field with matched polarization. Gains referred to a single polarization are partial gains. If we align the two polarization ellipses of the two antennas, the response increases. Similarly, when the ellipses are crossed, the link suffers polarization loss. To obtain the full gain, we add the factor

$$10\log(1 + |\rho|^2) \tag{1-45}$$

to the partial gain, an expression valid using ρ for either circular or linear polarization. In terms of axial ratio A for circular polarization, the conversion is

$$20\log\frac{\sqrt{2(1 + A^2)}}{1 + A}$$

When using measured partial gains for both antennas, the range of polarization efficiency is given by

$$\text{polarization efficiency } \Gamma = 20\log(1 \pm \rho_1\rho_2) \tag{1-46}$$

We can convert Eq. (1-46) to expressions that use the axial ratio of the two antennas:

$$\text{maximum polarization efficiency} = 20\log\frac{2(A_1A_2 + 1)}{(A_1 + 1)(A_2 + 1)}$$

$$\text{minimum polarization efficiency} = 20\log\frac{2(A_1 + A_2)}{(A_1 + 1)(A_2 + 1)}$$

1-11.7 Measurement of Circular Polarization Using Amplitude Only

The analyses given above assume that you can measure both amplitude and phase response of antennas, whereas in some cases only amplitude can be measured. If you do not know the sense of circular polarization, it will be necessary to build two antennas that are identical except for their circular polarization sense. For example, you can build two identically sized counter-wound helical wire antennas. You determine polarization sense by using both sources and comparing measured levels. Once you establish the polarization sense, mount a linearly polarized measurement antenna with low cross-polarization. For a given pointing direction of the antenna under test, rotate the source antenna and record the maximum and minimum levels. The ratio of the maximum to the minimum is the axial ratio.

To measure gain, rotate the measurement linearly polarized antenna to determine the peak response. Replace the antenna under test with a linearly polarized gain standard (horn) and perform a gain comparison measurement. Given the antenna axial ratio A, you adjust the linearly polarized gain by the correction factor:

$$\text{gain correction factor(dB)} = 20 \log \frac{A+1}{\sqrt{2}A} \tag{1-47}$$

We obtain the RHC and LHC response from

$$E_R = \frac{1}{\sqrt{2}}(E_{\max} + E_{\min}) \quad \text{and} \quad E_L = \frac{1}{\sqrt{2}}(E_{\max} - E_{\min})$$

assuming that the antenna is predominately RHC.

1-12 VECTOR EFFECTIVE HEIGHT

The vector effective height relates the open-circuit voltage response of an antenna to the incident electric field. Although we normally think of applying effective height to a line antenna, such as a transmitting tower, the concept can be applied to any antenna. For a transmitting tower, effective height is the physical height multiplied by the ratio of the average current to the peak current:

$$V_{\text{OC}} = \mathbf{E}_i \cdot \mathbf{h}^* \tag{1-48}$$

The vector includes the polarization properties of the antenna. Remember from our discussion of antenna impedance mismatch that the open-circuit voltage V_{OC} is twice that across a matched load Z_L for a given received power: $V_{\text{OC}} = 2\sqrt{P_{\text{rec}} Z_L}$. The received power is the product of the incident power density S and the effective area of the antenna, A_{eff}. Gathering terms, we determine the open-circuit voltage from the incident field strength E and a polarization efficiency Γ:

$$V_{\text{OC}} = 2E\sqrt{\frac{Z_L A_{\text{eff}} \Gamma}{\eta}}$$

We calculate polarization efficiency by using the scalar product between the normalized incident electric field and the normalized vector effective height:

$$\Gamma = \frac{|\mathbf{E}_i \cdot \mathbf{h}^*|^2}{|\mathbf{E}_i|^2 |\mathbf{h}|^2} \tag{1-49}$$

Equation (1-49) is equivalent to Eq. (1-41) because both involve the scalar product between the incident wave and the receiving polarization, but the expressions have different normalizations. You can substitute vector effective height of the transmitting antenna for the incident wave in Eq. (1-49) and calculate polarization efficiency between two antennas. When an antenna rotates, we rotate **h**. We could describe polarization calculations in terms of vector effective height, which would parallel and repeat the discussion given in Section 1-11. We relate the magnitude of the effective height h to the effective area A_{eff} and the load impedance Z_L:

$$h = 2\sqrt{\frac{Z_L A_{\text{eff}}}{\eta}} \tag{1-50}$$

The mutual impedance in the far field between two antennas can be found from the vector effective heights of both antennas [9, p. 6–9]. Given the input current I_1 to the first antenna, we find the open-circuit voltage of the second antenna:

$$Z_{12} = \frac{(V_2)_{\text{OC}}}{I_1} = \frac{jk\eta e^{-jkr}}{4\pi r}\mathbf{h}_1 \cdot \mathbf{h}_2^* \tag{1-51}$$

When we substitute Eq. (1-50) into Eq. (1-51) and gather terms, we obtain a general expression for the normalized mutual impedance of an arbitrary pair of antennas given the gain of each in the direction of the other antenna as a function of spacing r:

$$\frac{Z_{12}}{\sqrt{Z_{L1} Z_{L2}}} = \frac{j\sqrt{G_1 G_2}}{kr} e^{-jkr} \frac{\mathbf{h}_1 \cdot \mathbf{h}_2^*}{|\mathbf{h}_1||\mathbf{h}_2|} \tag{1-52}$$

The magnitude of mutual impedance increases when the gain increases or the distance decreases. Of course, Eq. (1-52) is based on a far-field equation and gives only an approximate answer, but it produces good results for dipoles spaced as close as 1λ. Figure 1-7 gives a plot of Eq. (1-52) for isotropic gain antennas with matched polarizations which shows the $1/R$ amplitude decrease with distance and that resistance and

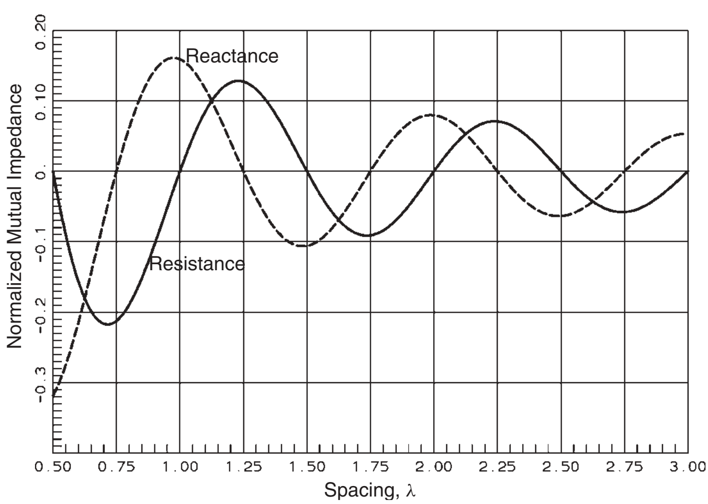


FIGURE 1-7 Normalized mutual impedance (admittance) from the vector effective length for two antennas with 0 dB gain along the line between them.

reactance curves are shifted out of phase. The cosine and sine factors of the complex exponential produce this effect. We multiply these curves by the product of the antenna gains, but the increased gain from larger antennas means that it is a greater distance to the far field. When we bring two antennas close together, the currents on each antenna radiate and excite additional currents on the other that modify the result given by Eq. (1-52). But as we increase the distance, these induced current effects fade. Equivalent height analysis can be repeated using magnetic currents (e.g., used with microstrip patches), and Eqs. (1-51) and (1-52) become mutual admittance. Figure 1-7 is also valid for these antennas when we substitute normalized mutual admittance for normalized mutual impedance. For antennas with pattern nulls directed toward each other, the mutual impedance decreases at the rate $1/R^2$, due to the polarization of current direction **h.**

1-13 ANTENNA FACTOR

The EMC community uses an antenna connected to a receiver such as a spectrum analyzer, a network analyzer, or an RF voltmeter to measure field strength E. Most of the time these devices have a load resistor Z_L that matches the antenna impedance. The incident field strength E_i equals antenna factor AF times the received voltage V_{rec}. We relate this to the antenna effective height:

$$\text{AF} = \frac{E_i}{V_{\text{rec}}} = \frac{2}{h} \tag{1-53}$$

AF has units meter^{-1} but is often given as dB(m^{-1}). Sometimes, antenna factor is referred to the open-circuit voltage and it would be one-half the value given by Eq. (1-53). We assume that the antenna is aligned with the electric field; in other words, the antenna polarization is the electric field component measured:

$$\text{AF} = \sqrt{\frac{\eta}{Z_L A_{\text{eff}}}} = \frac{1}{\lambda}\sqrt{\frac{4\pi}{Z_L G}}$$

This measurement may be corrupted by a poor impedance match to the receiver and any cable loss between the antenna and receiver that reduces the voltage and reduces the calculated field strength.

1-14 MUTUAL COUPLING BETWEEN ANTENNAS

The simplest approach for coupling between antennas is to start with a far-field approximation. We can modify Eq. (1-8) for path loss and add the phase term for the finite distance to determine the S-parameter coupling:

$$S_{21} = \sqrt{G_1 G_2}\frac{e^{-jkr}}{2kr}\frac{\mathbf{E}_1 \cdot \mathbf{E}_2^*}{|\mathbf{E}_1||\mathbf{E}_2|} \tag{1-54}$$

Equation (1-54) includes the polarization efficiency when the transmitted polarization does not match the receiving antenna polarization. We have an additional phase term because the signal travels from the radiation phase center along equivalent transmission lines to the terminals of each antenna. Equations (1-52) and (1-54) have the same accuracy except that Eq. (1-54) eliminates the need to solve the two-port circuit matrix equation for transmission loss. These formulas assume that antenna size is insignificant compared to the distance between the antennas, and each produces approximately uniform amplitude and phase fields over the second element.

We can improve on Eq. (1-54) when we use the current distribution on one of the two antennas and calculate the near-field fields radiated by the second antenna at the location of these currents. Since currents vary across the receiving antenna, we use vector current densities to include direction: $\mathbf{J}_r$ electric and $\mathbf{M}_r$ magnetic. Although magnetic current densities are fictitious, they simplify the representation of some antennas. We compute coupling from reactance, an integral across these currents [see Eq. (2-34)]:

$$S_{21} = \frac{j}{2\sqrt{P_r P_t}} \iiint (\mathbf{E}_t \cdot \mathbf{J}_r - \mathbf{H}_t \cdot \mathbf{M}_r)\, dV \tag{1-55}$$

The input power to the transmitting antenna P_t produces fields $\mathbf{E}_t$ and $\mathbf{H}_t$. The power P_r into the receiving antenna excites the currents. The scalar product between the incident fields and the currents includes polarization efficiency. If we know the currents on the transmitting antennas, we calculate the near-field pattern response from them at the location of the receiving antenna. Similar to many integrals, Eq. (1-55) is notional because we perform the integral operations only where currents exist. The currents could be on wire segments or surfaces. A practical implementation of Eq. (1-55) divides the currents into patches or line segments and performs the scalar products between the currents and fields on each patch and sums the result. A second form of the reactance [see Eq. (2-35)] involves an integral over a surface surrounding the receiving antenna. In this case each antenna radiates its field to this surface, which requires near-field pattern calculations for both. Equation (1-55) requires adding the phase length between the input ports and the currents, similar to using Eq. (1-54). When we use Eq. (1-55), we assume that radiation between the two antennas excites insignificant additional currents on each other. We improve the answer by using a few iterations of physical optics, which finds induced currents from incident fields (Chapter 2).

We improve on Eq. (1-55) by performing a moment method calculation between the two antennas. This involves subdividing each antenna into small elements excited with simple assumed current densities. Notice the similarity between Eqs. (1-52) and (1-54) and realize that Eq. (1-55) is a near-field version of Eq. (1-54). We use reactance to compute the mutual impedance Z_{21} between the small elements as well as their self-impedance. For the moment method we calculate a mutual impedance matrix with a row and column for each small current element. We formulate a matrix equation using the mutual impedance matrix and an excitation vector to reduce coupling to a circuit problem. This method includes the additional currents excited on each antenna due to the radiation of the other.

1.15 ANTENNA NOISE TEMPERATURE [10]

To a communication or radar system, an antenna contributes noise from two sources. The antenna receives noise power because it looks out on the sky and ground. The

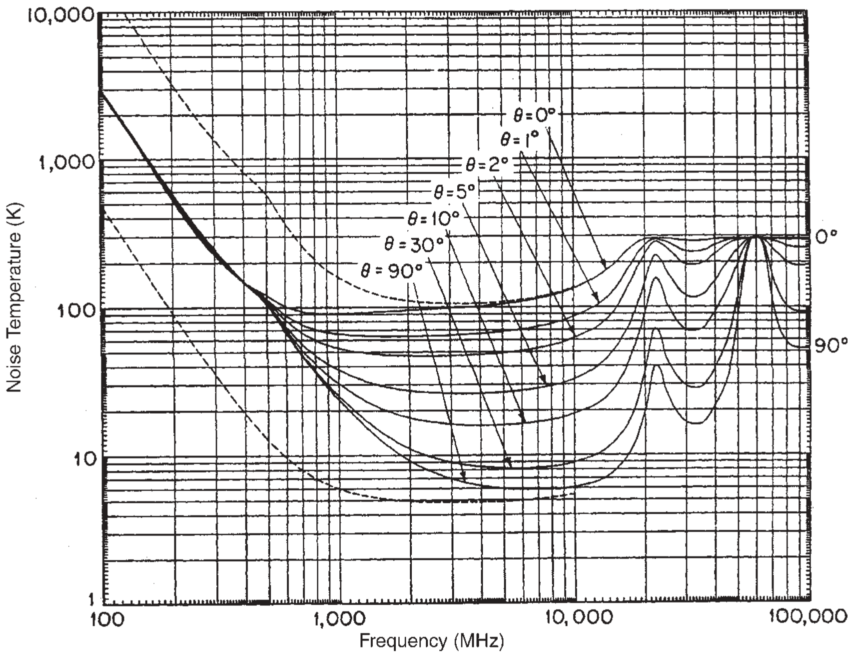


FIGURE 1-8 Antenna sky temperature. Noise temperature of an idealized antenna (lossless, no Earth-directed sidelobes) located at the Earth's surface, as a function of frequency, for a number of beam elevation angles. Solid curves are for geometric-mean galactic temperature, sun noise 10 times quiet level, sun in unity-gain sidelobe, cool temperate-zone troposphere, 2.7 K cosmic blackbody radiation, zero ground noise. The upper dashed curve is for maximum galactic noise (center of galaxy, narrow-beam antenna). Sun noise 100 times quiet level, zero elevation, other factors the same as solid curves. The lower dashed curve is for minimum galactic noise, zero sun noise, 90° elevation angle. (The bump in the curves at about 500 MHz is due to the sun-noise characteristic. The curves for low elevation angles lie below those for high angles at frequencies below 400 MHz because of reduction of galactic noise by atmospheric absorption. The maxima at 22.2 and 60 GHz are due to the water-vapor and oxygen absorption resonance.) (From L. V. Blake, A guide to basic pulse-radar maximum-range calculation, *Naval Research Laboratory Report 5868*, December 1962.)

ground generates noise because it is about 290 K and a portion of the antenna pattern falls on it. Similarly, the sky adds noise dependent on the elevation angle and the operating frequency. Figure 1-8 gives the sky temperature versus frequency and elevation angle. The frequency range of lowest noise occurs in the middle of microwave frequencies of 1 to 12 GHz. The graphs show a large variation between the dashed curves, which occurs because of antenna direction and the pointing relative to the galactic center. In the middle of microwaves the sky noise temperatures are around 50 K, whereas near zenith the temperature is under 10 K. Near the horizon it rises because of the noise from oxygen and water vapor. The exact value must be determined for each application. As frequency decreases below 400 MHz, the sky temperature rises rapidly and becomes independent of antenna pointing. The curve continues the rapid rise at the same slope for lower frequencies. Low-frequency sky temperatures are often given as decibels relative to 290 K.

An antenna receives this blackbody noise from the environment, but the value that affects the communication system depends both on the pattern shape and the direction of the main beam. We determine the antenna noise temperature by integrating the pattern times the environmental noise temperature distribution:

$$T_a = \frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi} G(\theta,\phi)T_s(\theta,\phi)\sin\theta\, d\theta\, d\phi \tag{1-56}$$

where $G(\theta,\phi)$ is the antenna gain pattern and $T_s(\theta,\phi)$ is the angle-dependent blackbody radiation of the environment. Changing the antenna pointing changes T_a. Equation (1-56) is a weighted average of the environment noise temperature, usually referred to as the sky temperature. The second source of noise in the antenna is that of components that have both dissipative losses and reflection losses that generate noise.

A receiving system needs to maximize the signal-to-noise ratio for given resources. System considerations, such as bit error rate, establish the required S/N ratio. We determine the noise power from the product

$$N = k_0 B_n T_e \tag{1-57}$$

where k_0 is Boltzmann's constant (1.38×10^{-23}W/K $\cdot$ Hz $= -228.6$ dB) and B_n is the receiver bandwidth (Hz). T_e is the effective noise temperature (K). When referring noise temperature to other parts of the network, we increase or decrease it by the gain or loss, since it represents power and not a true temperature. Antenna gain is a measure of the signal level, since we can increase gain independent of the noise temperature, although the gain pattern is a factor by Eq. (1-56).

The antenna conductor losses have an equivalent noise temperature:

$$T_e = (L-1)T_p \tag{1-58}$$

where T_p is the antenna physical temperature and L is the loss (a ratio > 1). From a systems point of view, we include the transmission line run to the first amplifier or mixer of the receiver. We do not include the current distribution losses (aperture efficiencies) that reduce gain in Eq. (1-58) because they are a loss of potential antenna gain and not noise-generating losses (random electrons). The antenna–receiver chain includes mismatch losses, but these do not generate random electrons, only reflected waves, and have a noise temperature of zero. We include them in a cascaded devices noise analysis as an element with loss only.

Noise characteristics of some receiver components are specified as the noise figure F_N (ratio), and cascaded devices' noise analysis can be analyzed using the noise figure, but we will use noise temperature. Convert the noise figures to noise temperature using

$$T_E = (F_N - 1)T_0 \tag{1-59}$$

T_0 is the standard reference temperature 290 K.

We calculate noise temperature for the entire receiver chain of devices at a particular point normally at the input to the first device. To calculate the S/N ratio we use the transmitter power, path loss (including antenna gain and polarization efficiency), and the gains (losses) of any devices for signal to the location in the receiver chain where noise temperature is being calculated. We characterize a given antenna by the ratio G/T, a measure independent of transmitter power and path loss, but including the receiver noise characteristics. Using the input of the first device as the noise reference point, we calculate the input noise temperature from component noise temperatures and gains:

$$T = T_1 + \frac{T_2}{G_1} + \frac{T_3}{G_1 G_2} + \frac{T_4}{G_1 G_2 G_3} + \cdots \tag{1-60}$$

Equation (1-60) merely states that noise temperatures are powers that decrease when we pass backward through a device with gain G. Each noise term is referred to the input of the device, and we pass backward to all previous devices and reduce noise temperature by $1/G$. If we decided to locate the noise reference point at the input to the second device, the noise initially referred to the chain input would increase by the gain of the first device. The system noise temperature becomes $T_{(2)}$:

$$T_{(2)} = T_1 G_1 + T_2 + \frac{T_3}{G_2} + \frac{T_4}{G_2 G_3} + \cdots$$

The signal also passes through the first device and the new gain at the input to the second device becomes GG_1. The gain and the noise temperature change by the same factor G_1 and produce a constant ratio. By extending these operations to any location in the receiver chain, we show that G/T is constant through the receiver device chain.

It is easiest to illustrate G/T noise calculations with an example. A ground station has a 5-m-diameter paraboloid reflector with 60% aperture efficiency with the system operating at 2.2 GHz ($\lambda = 0.136$ m). We compute antenna directivity using the physical area and aperture efficiency:

$$\text{directivity} = 0.60\left(\frac{\pi \cdot \text{Dia}}{\lambda}\right)^2 = 0.60\left(\frac{5\pi}{0.136}\right)^2 = 7972 \quad (39\,\text{dB})$$

The reflector feed loss is 0.2 dB and it has a VSWR of 1.5 : 1. The cable between the feed and the first amplifier (LNA) has a 0.5-dB loss. These are elements under control of the antenna designer. We calculate the noise temperature of these by using Eq. (1-58) when we use a physical antenna temperature of 37.7°C (100°F) (310.8 K).

Feed loss: $T_1 = (10^{0.2/10} - 1)310.8 = 14.65\,\text{K}$
Feed mismatch: $T_2 = 0\,\text{K}$
Cable: $T_3 = (10^{0.5/10} - 1)310.8 = 37.92\,\text{K}$

The gains of these devices are $G_1 = 10^{-0.2/10} = 0.955$ (feed loss), $G_2 = 10^{-0.18/10} = 0.959$ (reflected power loss for 1.5 : 1 VSWR), and $G_3 = 10^{-0.5/10} = 0.891$ (cable loss). The antenna sees the environment that generates noise due to blackbody radiation from the sky and ground. A typical value for the antenna pointed at 5° elevation is 50 K. This is not a physical temperature but represents an equivalent received power. Remember that the 60% aperture efficiency has no noise or loss contribution, because it only represents the loss of potential gain, since no random electrons are generated.

We must consider the rest of the receiver chain when calculating the total input noise temperature. For this example we assume that the LNA has a noise figure of

2 dB with 20 dB gain. The final portion of the receiver includes the mixer and IF of the receiver, which we assume has a 10-dB noise figure. We use Eq. (1-59) to convert noise figure to noise temperature.

$$\text{LNA noise temperature } T_4 = 290(10^{2/10} - 1) = 162.62\,\text{K}$$

$$\text{Receiver noise temperature } T_5 = 290(10^{10/10} - 1) = 2610\,\text{K}$$

The 20-dB (100) LNA gain greatly reduces the effect of the 2610-K receiver. We calculate the contribution of each device to the input noise temperature by applying Eq. (1-60) to each device. We pass the noise temperature of the receiver through the four devices, and its temperature is reduced by the gain of each device:

$$T_{e5} = \frac{T_5}{G_1G_2G_3G_4} = \frac{2610}{0.955(0.959)(0.891)(100)} = 31.98\,\text{K}$$

The gain of the LNA greatly reduced the effective noise of the receiver at the antenna input. This operation shows that cascading noise temperature involves passing each device's noise temperature through the gains of all preceding devices to the input and reducing it by the product of their gains. Similarly, we perform this operation on all the other noise temperatures.

$$\begin{aligned}
T_{e4} &= \frac{T_4}{G_1G_2G_3} = \frac{169.62}{0.955(0.959)(0.891)} = 207.86\,\text{K}\\
T_{e3} &= \frac{T_3}{G_1G_2} = \frac{37.92}{0.955(0.959)} = 41.40\,\text{K}\\
T_{e2} &= \frac{T_2}{G_1} = \frac{0}{0.955} = 0\\
T_{e1} &= T_1 = 14.65
\end{aligned}$$

These operations illustrate that the cascaded devices' noise temperature equation (1-60) is easily derived by considering the passage of noise temperature (power) through devices with gain to a common point where we can add the contributions.

The sky temperature is not an input noise temperature but the noise power delivered at the fictitious point called the *antenna directivity*, where gain = directivity. Since noise temperature represents power, we convert it to decibels and subtract it from directivity to compute G/T:

$$G/T(\text{dB}) = 39 - 10\log(345.9) = 13.6\,\text{dB}$$

This G/T is a measure of the antenna and receiver combined performance when the antenna is pointed to 5° elevation. Changing the pointing direction affects only the sky temperature added directly to the final result. We use G/T in the link budget of the communication system.

We can supply a single value for the antenna gain and noise temperature at the output port connected to the receiver. Recognize that the first three noise temperatures and the sky temperature are associated with the antenna. We moved the noise reference of each device to the input by dividing by the gain of the preceding devices. To move to the output of the antenna, we increase the noise temperature and the antenna gain by the product of the gain for the devices:

$$\begin{aligned} T &= (T_{\text{sky}} + T_{e1} + T_{e2} + T_{e3})G_1G_2G_3 \\ &= (50 + 14.65 + 0 + 37.92)10^{-0.88/10} = 83.7\,\text{K} \\ \text{gain(dB)} &= \text{directivity(dB)} - 0.88\,\text{dB} = 39 - 0.88 = 38.12\,\text{dB} \end{aligned}$$

This reduces the antenna to a single component similar to the directivity and sky temperature that started our analysis.

1-16 COMMUNICATION LINK BUDGET AND RADAR RANGE

We illustrate communication system design and path loss by considering a sample link budget example. The 5-m-diameter reflector is pointing at a satellite in an orbit 370 km above the Earth with a telemetry antenna radiating 10 W at 2.2 GHz. Since the antenna pattern has to cover the visible Earth, its performance is compromised. Considering the orbit geometry and antenna pointing is beyond the scope of this discussion. The range from a satellite at 370 km to a ground station pointing at 5° is 1720 km. The satellite antenna pointing angle from nadir is 70.3°, and a typical antenna for this application would have gain = −2 dBiC (RHC gain relative to an isotropic antenna) and an axial ratio of 6 dB. Assume that the ground station antenna has a 2-dB axial ratio. We apply the nomograph of Figure 1-6 to read the maximum polarization loss of 0.85 dB since we cannot control the orientation of the polarization ellipses. The link budget needs to show margin in the system, so we take worst-case numbers. When we apply Eq. (1-9) for path loss, we leave out the antenna gains and add them as separate terms in the link budget (Table 1-2):

$$\text{free-space path loss} = 32.45 + 20\log[2200(1720)] = 164\,\text{dB}$$

The link budget shows a 4.4-dB margin, which says that the communication link will be closed. This link budget is only one possible accounting scheme of the system parameters. Everyone who writes out a link budget will separate the parameters differently. This budget shows typical elements.

Radar systems have similar link budgets or detection budgets that consider S/N:

$$S/N = \frac{P_{\text{rec}}}{KTB} = \frac{P_T G_T(\text{directivity})\lambda^2\sigma}{(4\pi)^3 R^4 KTB} = \frac{(\text{EIRP})\lambda^2(G/T)\sigma}{(4\pi)^3 R^4 KB}$$

The radar has a required S/N value to enable it to process the information required, which leads to the maximum range equation:

$$R = \left[\frac{(\text{EIRP})\lambda^2(G/T)\sigma}{(4\pi)^3(S/N)_{\text{req}}KB}\right]^{1/4} \tag{1-61}$$

Equation (1-61) clearly shows the role of the transmitter, EIRP; the receiver and antenna noise; G/T; and the requirement for signal quality, S/N_{req}, on the radar range for a given target size σ.

Equation (1-61) applies to CW radar, whereas most radars use pulses. We increase radar performance by adding many pulses. We ignore the aspects of pulse train encoding

TABLE 1-2 Link Budget

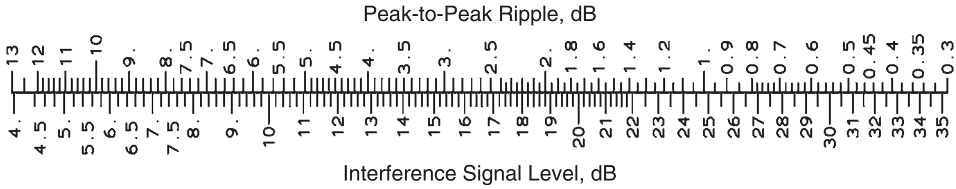
Frequency	2.2 GHz	Information only
Transmit power	10 dBW	10 log(10)
Transmit antenna gain	−2 dB	
EIRP (effective isotropic radiated power)	8 dBW	Transmit power dBW + antenna gain dB
Free-space path loss	164 dB	Isotropic antenna path loss
Polarization loss	0.85	Maximum for uncontrolled orientation
Atmospheric loss	0.30	5° elevation at 2.2 GHz
Rain loss	0.00	Little loss at this frequency
Pointing loss	0.00	
Receive antenna directivity	39 dB	Location in receiver chain for G/T calculation
G/T	13.6 dB	From preceding section
Boltzmann's constant	228.6 dB	
Carrier/noise (C/N) ratio (ignores bandwidth)	85 dB	EIRP + G/T − path loss − polarization loss −atmospheric loss − rain loss + 228.6
Bit rate: 8 Mb/s	69 dB	10 log(bit rate) bandwidth
E_b/N_0 (energy per bit/noise density)	16 dB	$E_b/N_0 = C/N - 10$ log(bit rate)
Implementation loss	2 dB	Groups extra system losses
E_b/N_0 required	9.6 dB	For bit error rate (BER) = 10^{-5} in QPSK
Margin	4.4 dB	E_b/N_0 − required E_b/N_0 − implementation loss

that allow coherent addition. Radar range is determined by the total energy contained in the pulses summed. We replace EIRP with G_T(energy) since $P_T \times$ time = energy. It is the total energy that illuminates the target that determines the maximum detection range. Using antennas in radar leads to speaking of the radiated energy correct for pulsed systems, but when we do not integrate pulse shape times time, the antenna radiates power. To be correct we should call radiation that we integrate over angular space to find power, "power density." To say "energy radiated in the sidelobes" is poor physics unless it is a radar system, because it is power.

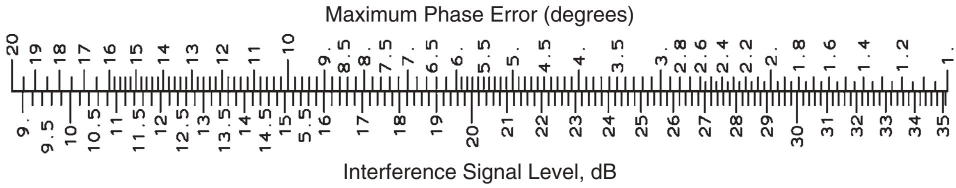
1-17 MULTIPATH

Multipath means that the field intensity at a particular point is the sum of a number of waves that arrive from different directions or from different sources. It arises from signal transmission paths such as edge reflections from the mounting structure around an antenna and general reflections from objects near the antenna. Nearby reflections only seem to modify the antenna pattern, while reflections from additional objects cause rapid ripple with changing pattern angle. In Section 3-1 we discuss how to use the ripple angular rate and pattern distribution to locate its source. Multipath causes degraded system performance or measurement errors. Of course, multipath can improve performance as well. In fact, we add nearby objects, such as ground planes, to improve antenna performance.

We specify pattern response in terms of the power response, but we add fields. An extra signal −20 dB relative to the main signal is 0.01 in power but 0.1 in field strength



SCALE 1-8 Signal peak-to-peak amplitude ripple due to multipath signal.



SCALE 1-9 Peak phase error due to multipath signal.

(voltage). Since the extra signal can have any phase relative to the main signal, it can add or subtract. Given an extra signal MP(dB), the pattern ripple is

$$\text{ripple(dB)} = 20\log\frac{1+10^{\text{MP}/20}}{1-10^{\text{MP}/20}} \tag{1-62}$$

where MP(dB) has a negative sign. Scale 1-8 gives the relationship between peak-to-peak amplitude ripple and the level of the multipath signal. Equation (1-62) is numerically the same as the relationship between return loss and 20 log(VSWR). The multipath signal can change the phase when summed with the main signal over a range given by

$$\text{maximum phase error} = \pm\tan^{-1}(10^{\text{MP}/20}) \tag{1-63}$$

Scale 1-9 calculates the peak phase error due to a multipath signal.

1-18 PROPAGATION OVER SOIL

When we position antennas over soil and propagate the signal any significant distance, it will reflect from soil or water and produce a large multipath signal. Soil is a conductive dielectric that reflects horizontally and vertically polarized signals differently. Typical ground constants are listed in Table 1-3. Given the grazing angle ψ measured between the reflected ray and ground, the voltage reflection coefficients are

$$\rho_h = \frac{\sin\psi - \sqrt{\varepsilon_r - jx - \cos^2\psi}}{\sin\psi + \sqrt{\varepsilon_r - jx - \cos^2\psi}} \quad \text{and} \quad \rho_v = \frac{(\varepsilon_r - jx)\sin\psi - \sqrt{\varepsilon_r - jx - \cos^2\psi}}{(\varepsilon_r - jx)\sin\psi + \sqrt{\varepsilon_r - jx - \cos^2\psi}} \tag{1-64}$$

where $x = \sigma/\omega\varepsilon_0 = 17,975\sigma/\text{frequency(MHz)}$.

Figure 1-9 gives the reflection coefficient for the two polarizations versus grazing angle. Horizontal polarization reflects from soil about the same as a metal surface. Vertical polarization reflection produces a more interesting curve. The graph shows that the reflection is low over a region of grazing angles. The minimum reflection direction is called the *Brewster angle*. At this angle the reflected wave is absorbed

TABLE 1-3 Typical Ground Constants

Surface	Dielectric Constant	Conductivity (S)
Dry ground	4–7	0.001
Average ground	15	0.005
Wet ground	25–30	0.020
Fresh water	81	0.010
Seawater	81	5.0

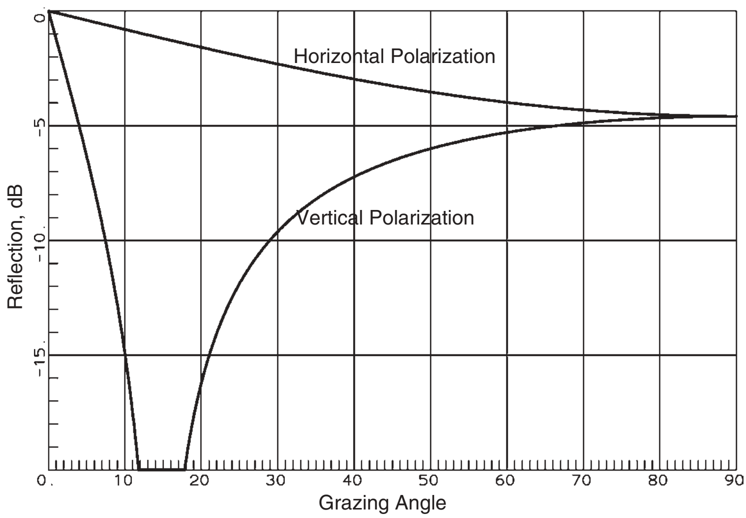


FIGURE 1-9 Average soil reflection for horizontal and vertical polarization.

into the soil. At high grazing angles ρ_h has a phase near 180° and ρ_v a phase of 0°. When the grazing angle decreases and becomes less than the Brewster angle, the vertical polarization reflection changes from 0° to 180°. Remember that for most general response nulls, the signal phase changes by 180° when passing through the transition. As the grazing angle approaches zero both reflection coefficients approach -1 and multipath is independent of polarization.

The electric field at the receiving antenna is the sum of the direct wave plus the reflected wave, which traveled along a longer path:

$$E = E_d[1 - \exp(-j\Delta\phi)] = E_d(1 - \cos\Delta\phi + j\sin\Delta\phi)$$

We compute the magnitude

$$|E| = |E_d|\sqrt{1 + \cos^2\Delta\phi + \sin^2\Delta\phi - 2\cos\Delta\phi} = 2|E_d|\sin(\Delta\phi/2)$$

for the small phase difference between the two equal-amplitude signals. The received power P_{rec} is proportional to E^2. The path loss for this multipath link is modified from the free-space equation:

$$P_{\text{rec}} = 4P_T\left(\frac{\lambda}{4\pi d}\right)^2 G_T G_R \sin^2\frac{2\pi h_T h_R}{\lambda d} \to P_T G_T G_R\left(\frac{h_T h_R}{d^2}\right)^2 \tag{1-65}$$

Equation (1-65) states that the power received is proportional to $1/d^4$ and increases by h^2 for either antenna. We can approximate the propagation over soil by a region for closely spaced antennas when the results consist of the free-space transmission with $1/d^2$ average transmission with significant variation due to multipath and a second region proportional to $1/d^4$ with small multipath variations. The breakpoint between the two models occurs at a distance $d = 4h_T h_R/\lambda$.

Experiments at mobile telephone frequencies showed that Eq. (1-65) overestimates the received power when the receiving antenna height is less than 30 m and a more correct model modifies the exponent of h_R [11, p. 38]:

$$P_{\text{rec}} = P_T G_T G_R \frac{h_T^2 h_R^C}{d^4} \tag{1-66}$$

Below 10 m, $C = 1$ and the exponent varies linearly between 10 and 30 m: $C = h_R/20 + \frac{1}{2}$.

On a narrow-beam terrestrial propagation path, scattering from an object along a path an odd multiple of $\lambda/2$ produces a signal that reduces the main path signal. Given an obstacle at a distance h radial from the direct ray path and located d_T from the transmitter and a distance d_R from the receiving antenna, we determine the differential path length as

$$\Delta = \frac{h^2}{2}\frac{d_T + d_R}{d_T d_R} = n\frac{\lambda}{2} \quad \text{or} \quad \text{clearance height } h = \sqrt{\frac{n\lambda d_T d_R}{d_T + d_R}} \tag{1-67}$$

We call these Fresnel clearance zones of order n. The direct path should clear obstacles by at least one clearance zone distance h to prevent the scattered signal from having a negative impact on the communication link. The first Fresnel zone touches ground when $d_T = 4h_T h_R/\lambda$ is the breakpoint distance between $1/d^2$ and $1/d^4$ propagation models.

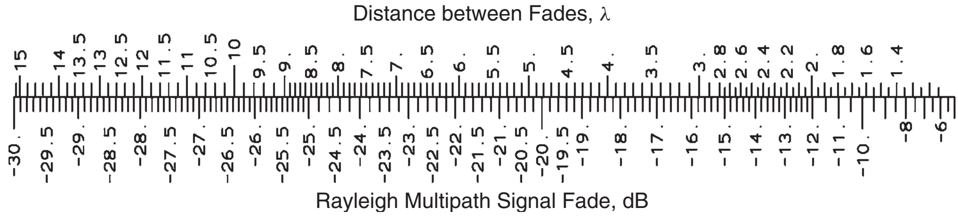
1-19 MULTIPATH FADING

Most mobile communication occurs when there is no direct path between the base station antennas and the mobile user. The signal reflects off many objects along the path between the two. This propagation follows a Rayleigh probability distribution about the mean signal level:

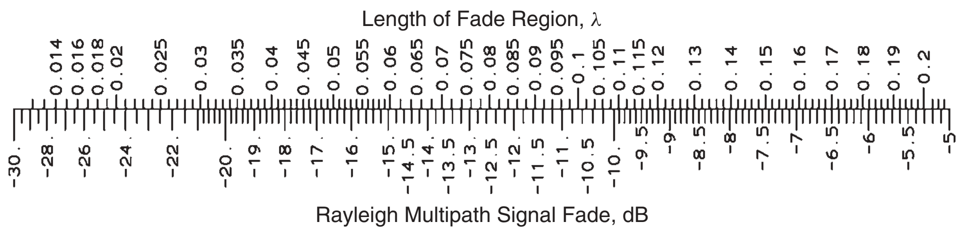
$$p_r(r) = \frac{r}{\alpha^2}\exp\left(-\frac{r^2}{2\alpha^2}\right) \quad \text{prob}[r < R] = P_R(R) = 1 - \exp\left(-\frac{R^2}{2\alpha^2}\right)$$

R is the signal level, α the value of the peak in the distribution, with mean $= \alpha\sqrt{\pi/2}$ and median $R_M = \alpha\sqrt{2\ln(2)} = 1.1774\alpha$. The median signal level is found by fitting measured data for various localities (town, small town, open country, etc.) into a prediction model. The signal will have large signal fades where the level drops rapidly. The Rayleigh model can be solved for the average distance between fades given the level. As a designer it is important to realize the magnitude of the problem [12, pp. 125–130]:

$$\text{average distance between fades} = \lambda \frac{2^{(R/R_M)^2}}{\sqrt{2\pi\ln(2)}(R/R_M)} \tag{1-68}$$



SCALE 1-10 Average distance between fades and depth of fade for a Rayleigh multipath.



SCALE 1-11 Average fade length and depth of fade for a Rayleigh multipath.

R is the fade level (ratio) and R_M is the median signal level found from a propagation model. Scale 1-10 shows the relationship between the average distance between fades and the depth of fade for Rayleigh multipath. A mobile channel operating at 1.85 GHz ($\lambda = 16.2$ cm) has a 15-dB fade every 2.75λ which equals 44.5 cm, while 10-dB fades occur every $1.62\lambda = 26.25$ cm. The communication system must overcome these fades. Fortunately, the deep fades occur over a short distance:

$$\text{average length of fade} = \lambda \frac{2^{(R/R_M)^2} - 1}{\sqrt{2\pi \ln(2)}(R/R_M)}$$

The signal fades and then recovers quickly for a moving user. Scale 1-11 shows the average fade length along a path given the depth of fade. For the 1.85-GHz channel the 15-dB fade occurs only over $0.06(16.2) = 0.97$ cm, and the 10-dB fade length is $0.109(16.2) = 1.76$ cm.

The solution to mobile communication multipath fading is found either in increasing the link margin with higher gain base station antennas or the application of diversity techniques. We use multiple paths between the user and the base station so that while one path experiences a fade, the other one does not. Diversity has no effect on the median signal level, but it reduces the effects of the nulls due to the Rayleigh distribution propagation.

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