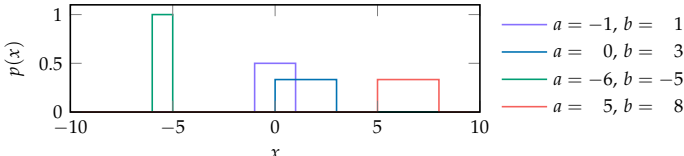# B Probability Distributions

This appendix summarizes several different families of probability distributions relevant to the topics introduced in this book.[1] The distributions are represented by either probability mass functions or probability density functions, and the relevant functions are provided along with the parameters that govern each distribution. Plots show how the various parameters influence the distribution. The index includes page references to where these distributions are used in the body of the book. Some distributions are *univariate*, meaning they are distributions over a scalar varable; others are *multivariate*, meaning they are distributions over multiple variables.

Name	Parameters		Distribution Function
Uniform $\mathcal{U}(a,b)$	a b	lower bound upper bound	$p(x) = \frac{1}{b-a}$ with $x \in (a,b)$

Name	Parameters		Distribution Function
Gaussian (univariate) $\mathcal{N}(\mu, \sigma^2)$	μ σ^2	mean variance	$p(x) = \frac{1}{\sigma}\phi\left(\frac{x-\mu}{\sigma}\right)$ where $\phi(x) = \frac{1}{\sqrt{2\pi}}\exp(-x^2/2)$ with $x \in \mathbb{R}$

| **Beta** | $\alpha > 0$ | shape | $p(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}$ |
| Beta(α, β) | $\beta > 0$ | shape | with $x \in (0,1)$ |

Gaussian	$\boldsymbol{\mu}$	mean	$p(\mathbf{x}) = \frac{1}{(2\pi)^{n/2}	\boldsymbol{\Sigma}	^{1/2}} \exp\left(-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^{\top}\boldsymbol{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu})\right)$ where $n = \dim(\mathbf{x})$
(multivariate)	$\boldsymbol{\Sigma}$	covariance	with $\mathbf{x} \in \mathbb{R}^n$
$\mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$			

$\boldsymbol{\mu} = [0,0], \boldsymbol{\Sigma} = [1\,0;\,0\,1]$ $\boldsymbol{\mu} = [0,5], \boldsymbol{\Sigma} = [3\,0;\,0\,3]$ $\boldsymbol{\mu} = [3,3], \boldsymbol{\Sigma} = [4\,2;\,2\,4]$

| **Dirichlet** | $\boldsymbol{\alpha} > 0$ | concentration | $p(\mathbf{x}) = \frac{\Gamma(\alpha_0)}{\prod_{i=1}^{n}\Gamma(\alpha_i)} \prod_{i=1}^{n} x_i^{\alpha_i-1}$ where $\alpha_0 = \sum_i \alpha_i$ |
| Dir$(\boldsymbol{\alpha})$ | | | with $x_i \in (0,1)$ and $\sum_i x_i = 1$ |

$\boldsymbol{\alpha} = [1,1,1]$ $\boldsymbol{\alpha} = [5,5,5]$ $\boldsymbol{\alpha} = [2,1,5]$