Linear Algebra

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Contents

1 Exercises on the geometry of linear equation

Source

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1.1 Problem 1.1

Problem 1.1

1.3 #4. *Introduction to Linear Algebra*: Strang. Find a combination $x_1\vec{w_1} + x_2\vec{w_2}$ that gives the zero vector

$$\vec{w_1} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \vec{w_2} = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \vec{w_3} = \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix}$$

Those vectors are (independent/dependent) The three vectors lie in a [BLANK] The matrix W with those columns is not invertible

We must find scalars x_1, x_2, x_3 such that

$$x_1\vec{w_1} + x_2\vec{w_2} + x_3\vec{w_3} = \vec{0}$$

or written differently...

$$x_1 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + x_2 \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} + x_3 \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

The obvious (and trivial) solution is to let every scalar be equal to 0

$$x_1 = 0$$
$$x_2 = 0$$
$$x_3 = 0$$

Another solution could be

$$x_1 = 1$$
$$x_2 = -2$$
$$x_3 = 1$$

which would give us

$$1(1) + -2(4) + 1(7) = 0$$
$$1(2) + -2(5) + 1(8) = 0$$
$$1(3) + -2(6) + 1(9) = 0$$

These vectors are **dependent** because there is a combination of the vectors that gives us the 0 vector. Because the vectors are dependent, they must lie on the same plane

1.2 Problem 1.2

Problem 1.2

Multiply:

$$\begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 3 \\ 4 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 3 \\ 4 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix} = 3 \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix} + -2 \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} + 1 \begin{bmatrix} 0 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 3(1) + -2(2) + 1(0) \\ 3(2) + -2(0) + 1(3) \\ 3(4) + -2(1) + 1(1) \end{bmatrix} = \begin{bmatrix} -1 \\ 9 \\ 11 \end{bmatrix}$$

1.3 Problem 1.3

Problem 1.3

True or False. If matrix $A = 3 \times 2$, and matrix $B = 2 \times 3$, their product will equal a 3×3 matrix AB. If this is false, write a similar sentence which is correct

This is **true**. You can only multiply two matrices if the columns of the first one equal the rows of the second one. Their product will be a matrix with the same number of rows as the first matrix, and the same number of columns as the second matrix

$$A(m \text{ by } n) \cdot B(n \text{ by } p) = AB(m \text{ by } p)$$