Biased Wavelet Neural Network and Its Application to Streamflow Forecast

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Abstract. Long leading-time streamflow forecast is a complex non-linear procedure. Traditional methods are easy to get slow convergence and low efficiency. The biased wavelet neural network (BWNN) based on BP learning algorithm is proposed and used to forecast monthly streamlfow. It inherits the multiresolution capability of wavelets analysis and the nonlinear input-output mapping trait of artificial neural networks. With the new set of biased wavelets, BWNN can effectively cut down the redundancy from multiresolution calculation. The learning rate and momentum coefficient are employed in BP algorithm to accelerate convergence and avoid falling into local minimum. BWNN is applied to Fengtan reservoir as case study. Its simulation performance is compared with the results obtained from autoregressive integrated moving average, genetic algorithm, feedforward neural network and traditional wavelet neural network models. It is shown that BWNN has high model efficiency index, low computing redundancy and provides satisfying forecast precision.

1 Introduction

Streamflow forecast is very important in exploring and optimizing water resources management. Long leading- time forecast with high accuracy gives more scientific and efficient instructions to flood prevention, reservoir regulation and drainage basin management. Due to its complex non-linear process, such forecast is generally built on qualitative analysis, since the corresponding quantitative analysis has greater errors especially for extreme values of streamflow.

Methods have been adopted to solve this problem. Statistics forecast is used most [1]. Its basic principle is to seek and analyze the change rules of hydrology ingredients and the relations with other factors by statistics. Regression analysis [2] and time series analysis [3] are main forms of such statistics method, but they have the disadvantage of amplifying frequency noise in the data when differencing. Threshold regression model [4] and projection pursuit model [5] are developed by setting restrictions on variables, yet they can not fully explain many complex hydrological datasets with inherent static feature. Recent researches reveal that artificial neural networks (ANNs) have been widely used for water resources variables modeling. As ANNs are nonlinear data-driven methods, they suit well to nonlinear input-output mapping techniques.

J. Wang et al. (Eds.): ISNN 2006, LNCS 3971, pp. 880 – 888, 2006.

However, there inevitably exists low convergence and local optimum problems when streamflow forecasting.

Wavelet analysis theory is regarded as a great progress of Fourier analysis. By dilations and translations, wavelet transform can extract the detail information of signals with multiresolution capability. In 1992, Zhang [6] explicated the concept and algorithm of wavelet neural network (WNN). WNN inherits the merits of wavelet transform and artificial neural networks. It implements wavelet transform by adjusting the shape of wavelet basis during training period and gets the excellent approximation and pattern classification [7]. Some adaptive WNN [8] was presented to meliorate WNN. Robert [9] illustrated the adaptively biased wavelets expansions. The biased wavelets are a new set of nonzero-mean functions, and show good performance when working with a “large number” functions from multiresolution calculations.

In this paper, we give the topology of biased wavelet neural network (BWNN) and attempt to apply it to forecast monthly streamflow. Section 2 introduces the basic information of wavelet neural network. In Section 3, biased wavelets, topology of BWNN and BP-based learning algorithm are explained. In Section 4, the monthly streamflow forecast of Fengtan reservoir using BWNN is employed as case study, and results from the modeling experiment are reported and discussed with other algorithms. Finally, conclusions and suggestions are obtained in Section 5.

2 Wavelet Neural Network

The wavelet transform of a signal $f(x)$ is defined as the integral transformation [10]:

$$W_f(a,b) = \int_R f(x)\overline{\Psi}_{ab}(x)dx \tag{1}$$

in which, $\overline{\Psi}_{ab}(x) = |a|^{1/2}\Psi(\frac{x-b}{a})$, $\overline{\Psi}_{ab}(x)$ is called wavelet function, $\Psi(x)$ is the mother wavelet. a and b are dilation and translation parameters respectively.

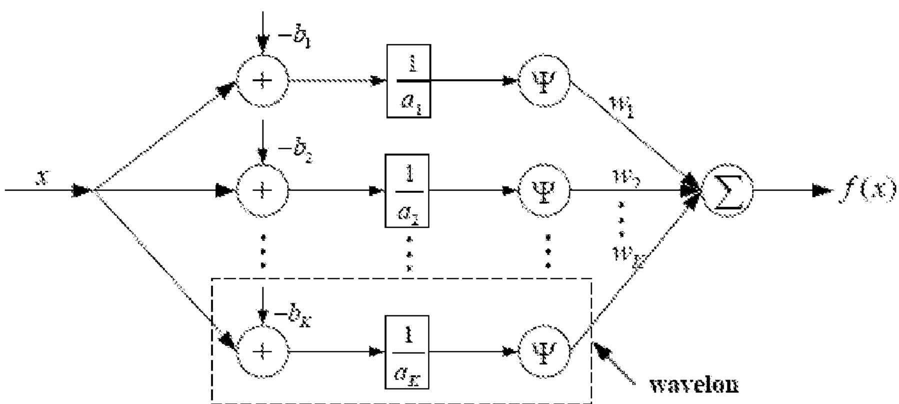

Fig. 1. The Topology of Wavelet Neural Network for approximation

According to [6], a signal $f(x)$ can be approximated from a series of wavelets, which can be obtained by dilating and translating the mother wavelet as follows:

$$f(x)=\sum_{k=1}^{K} w_k \Psi(\frac{x-b_k}{a_k}) . \tag{2}$$

where w_k are weight coefficients, K is the number of wavelets and $f(x)$ is the approximation output of WNN for the signal. Fig.1 gives the topology of the network to formula (2).

3 Biased Wavelet Neural Network

3.1 Biased Wavelets

A set H of biased wavelets employed in this paper is defined in [9] as:

$$H=\{h_{a,b,c}(x)=|a|^{1/2}\left[\Psi(\frac{x-b}{a})+c\Phi(\frac{x-b}{a})\right], a\in R^{+}, b,c\in R\} \tag{3}$$

in which $\Psi\in L^2(R)$ is a mother wavelet and the function Φ satisfies the following terms:

1) $\Phi\in L^2(R)$
2) $\tilde{\Phi}(0)\neq 0$
3) $\Psi(x)$ is rapidly decreasing to zero when $|x|\to\infty$
4) $\tilde{\Psi}(\omega)$ is rapidly decreasing to zero when $|\omega|\to\infty$

It is obvious that the proposed biased wavelets do not fit the "admissibility condition" (for $\tilde{\Phi}(0)\neq 0$), but using them can efficiently reduce the redundancy in wavelet neural network, which is analyzed in detail hereafter.

3.2 Biased Wavelet Neural Network

Let biased wavelets substitute the mother wavelet Ψ in equation (2), and $S_{a,b,c}(x)$ is the corresponding summation. We can write that:

$$\begin{aligned} S_{a,b,c}(x) &= \sum_{k=1}^{K} w_k \cdot H = \sum_{k=1}^{K} w_i \cdot [\Psi(\frac{x-b_k}{a_k})+c_k \cdot \Phi(\frac{x-b_k}{a_k})] \\ &= \sum_{k=1}^{K} w_k \cdot \Psi(\frac{x-b_k}{a_k}) + \sum_{k=1}^{K} w_k \cdot c_k \cdot \Phi(\frac{x-b_k}{a_k}) \\ &= f(x)+L(x) \end{aligned} \tag{4}$$

From equation (4), when $c=0$, $S_{a,b,c=0}(x)=f(x)$. That is to say, $f(x) \subset S(x)$, and if the bias part $L(x)$ is added to $f(x)$, the signal is desired to be approximated and expressed as:

$$f(x)=\sum_{k=1}^{K} w_k \cdot [\Psi(\frac{x-b_k}{a_k})+c_k \cdot \Phi(\frac{x-b_k}{a_k})] \tag{5}$$

Based on the biased wavelets and equation (5), Fig 2 is the topology of the presented biased wavelet neural network (BWNN). It is constructed by two important parts, traditional part and bias part. The original part is the same as that in the traditional wavelet neural network (WNN), while in the bias part, the function Φ is multiplied by coefficient c.

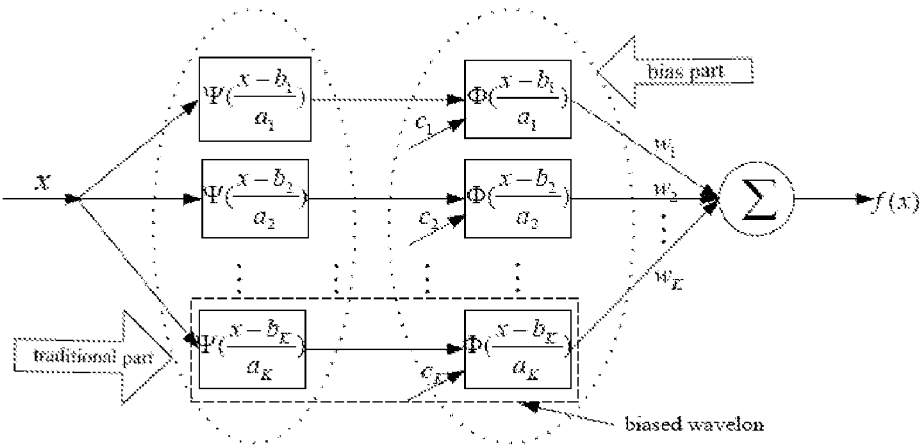

Fig. 2. The Topology of Biased Wavelet Neural Network for approximation. The dashed rectangular refers to integration of biased wavelon and the dotted ellipses indicate the important two parts of BWNN, traditional part and bias part, respectively.

The essence of wavelet transform is mapping the 1-D signal $f(x)$ isometrily to the 2-D wavelets space made up of the set $W_f(a,b)$. In WNN, the wavelet transform of $f(x)$ and its inverse transform are not corresponding one by one. As the wavelet functions $\overline{\Psi}_{ab}(x)$ are super-complete, they are not linearly independent and there exist some relations, which result in the redundancy. For BWNN, it is the bias part that has the redundancy cut down. BWNN is an adaptive style of WNN, and according to the different selection of c and Φ, $\overline{h}_{abc}(x)$ have more freedom, and the time-frequency window has flexible changes in the whole time area without fixed values, which indicates that the shape of the wavelet adapts to a particular problem, not parameters of fixed-shaped wavelet.

3.3 The Learning Algorithm of BWNN Based on BP

In this Section, the proposed BWNN is to be trained with BP algorithm, and the weights w_k , scales a_k , translations b_k and bias parameters c_k are adjusted adaptively. Suppose that $y_m(m=1\cdots M)$ is the observed value of the mth signal and $f(x_m)$ is the calculated output of BWNN. Let $t'=\dfrac{t-b_k}{a_k}$, and the cost function is defined as:

$$E=\frac{1}{2}\sum_{m=1}^{M}[y_m-f(x_m)]^2 \tag{6}$$

The partial derivatives of E to each parameter are computed:

$$\begin{aligned}
\frac{\partial E}{\partial w_k}&=-(y-f)\cdot(\Psi(t')+c_k\cdot\Phi(t'))\\
\frac{\partial E}{\partial b_k}&=\sum_{m=1}^{M}(y-f)\cdot w_k\cdot(\Psi'(t')+c_k\cdot\Phi'(t'))/a_k\\
\frac{\partial E}{\partial a_k}&=\sum_{m=1}^{M}(y-f)\cdot w_k\cdot(\Psi'(t')+c_k\cdot\Phi'(t'))\cdot(x-b_k)/a_k^2\\
\frac{\partial E}{\partial c_k}&=\sum_{m=1}^{M}(y-f)\cdot w_k\cdot\Phi'(t')
\end{aligned} \tag{7}$$

Learning rate η and momentum coefficient μ are adopted in revising parameters to avoid falling to local minimum and improve the convergence of BWNN. The iterative equations are:

$$\begin{aligned}
w_k(m+1)&=w_k(m)-\eta\cdot\frac{\partial E}{\partial w_k}+\alpha\cdot\Delta w_k(m)\\
b_k(m+1)&=b_k(m)-\eta\cdot\frac{\partial E}{\partial b_k}+\alpha\cdot\Delta b_k(m)\\
a_k(m+1)&=a_k(m)-\eta\cdot\frac{\partial E}{\partial a_k}+\alpha\cdot\Delta a_k(m)\\
c_k(m+1)&=c_k(m)-\eta\cdot\frac{\partial E}{\partial c_k}+\alpha\cdot\Delta c_k(m)
\end{aligned} \tag{8}$$

In the following section, BWNN shows its ability in monthly streamflow forecast. Mexican hat wavelet used as “mother wavelet” in the case study is:

$$\Psi(x)=(1-x^2)\exp(-\frac{x^2}{2}) \tag{9}$$

The employed bias function is Gaussian function which exhibits good resolution in both time and frequency domain, and given in:

$$\Phi(x) = \exp(-\frac{x^2}{2}) \tag{10}$$

4 Case Study

Fengtan reservoir located in the domain of Yuanling county down of You river, and 45 kilometers away from the center of Yuanling. The reservoir drainage area is 17500 square kilometer and occupies 94.4% of the whole You river area. Fengtan reservoir belongs to seasonal regulation reservoir and its basic information is listed in Table 1. The water supply of Yuanling county area depends highly on Fengtan reservoir and the monthly streamflow forecast helps the operators to well prepare for the future streamflow and make proper decisions.

In this part, BWNN is verified by the real data: monthly streamflow time series recorded in Fengtan reservoir from Jan. 1952 to Dec. 2002 shown in Fig. 3. The monthly streamflow series are computed by summing each average daily flow and dividing by the number of days corresponding to that particular month. It is periodical, time dependant and shows significant linkage in frequency domain, which leads to the nonstationary condition. Data of Jan. 1952~ Nov. 1997 are chosen to train the BWNN, and the others from Dec.1997~ Dec.2002 for test.

The constructed BWNN is single input, sixteen biased wavelons and single output. The initializing procedure of parameters a_k and b_k resembles that in [11]. The values of w_k and c_k are uniformly sampling in [-1, 1], and usually c_k begins with zero. As that mentioned above, proper selection of learning rate η and momentum coefficient μ avail BWNN of performance. In the beginning, $\eta = 0.18, \alpha = 0.72$, and they are adjusted manually during convergence. Including the defined cost function E, model efficiency index R^2 is also adhibited to evaluate the model performance, and estimate the approximation precision.

$$R^2 = 1 - \frac{\sum_{m=1}^{M} (y_m - f(x_m))^2}{\sum_{m=1}^{M} (f(x_m) - \bar{f})^2} \tag{11}$$

y_m and $f(x_m)$ are the observed and calculated values, and $\bar{f}$ is mean of the streamflow time series. In general, $R^2 > 0.8$ indicates a good model.

To further assess the performance of the proposed BWNN, it is compared to other models listed in Table 2 using cost function E and model efficiency index R^2. BWNN has a R^2 index value of 0.94 indicating a very satisfying forecasts, while autoregressive integrated moving average (ARIMA) model are unsatisfactory with largest E. Since the monthly series are still nonstationary after differenced, ARIMA model is hard to draw good results in which the first- and second-order moments depend only on time differences. The model of genetic algorithm (GA) is a compromise. Although it has much lower E than ARIMA, it is the least efficient model in the four evolutionary algorithms. Feedforward neural network (FNN) and traditional wavelet neural network (WNN) models in column four and five respectively are managed likewise BWNN: the same network construction, BP-based learning algorithm and Mexican hat wavelet except for sigmoid function in FNN. They both have effective indexes values, and provide good (or acceptable) forecast results with better R^2 index values, but not enough when more accuracy results are required in flood season. During the validation period, Fengtan reservoir had complex inflow cases, such that two months in sequence may run from the maximum to the minimum (or inverse) sharply. FNN and WNN can not find these suddenly changes (peak or trough) at some points, which lead to the worse statistics indexes than BWNN.

Fig. 4 exhibits forecasting results of BWNN and WNN. For the nonstationary monthly streamflow series, BWNN shows the superiorities in lessening signal noises and cutting down redundancy with adaptive bias coefficient and bias function. In streamflow forecast, generally speaking, it is permitted to supply slightly bigger value in peak and smaller in trough, because the worse condition may gain more preparations. The maximum value in Jul. 1999 during test period is detected by BWNN and so does the minimum. However, WNN loses several peak values and even gets the wrong trend of series for certain months. In Fengtan drainage area, the flood season is from June to September, but there unexpectly comes inundation around November at times without any signs in advance, even precipitations. Such peak value is difficult to forecast, while in this case study, BWNN supplies the close result of Nov. 2000. Although WNN leaves out part of extreme values, it gains the overall annual streamflow of the abundant water years and dried years.

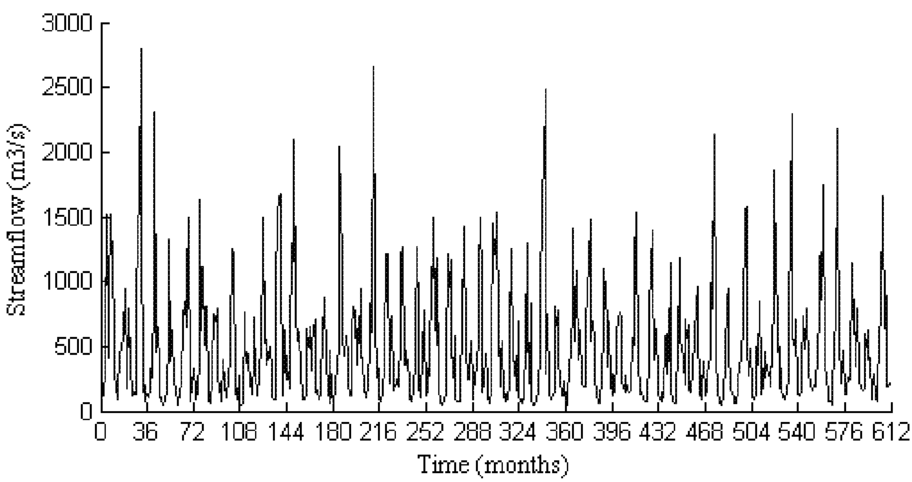

Fig. 3. Observed monthly streamflow time series of Fengtan Reservoir from year 1952~2002, totally 612 months

Table 1. Basic Information for Fengtan Reservoir

Average Multi-years precipitation	Average streamflow of multi-years	Maximum instantaneous streamflow	Minimum instantaneous streamflow	Total reservoir capacity
1415 mm	15.9 billion km^3	16900 km^3/s	40 km^3/s	1.73 billion km^3

Table 2. Model validation statistics for the forecasts

	ARIMA	GA	FNN	WNN	BWNN
E	670	203.7	186.9	115.2	38.1
R^2	0.64	0.82	0.85	0.87	0.94

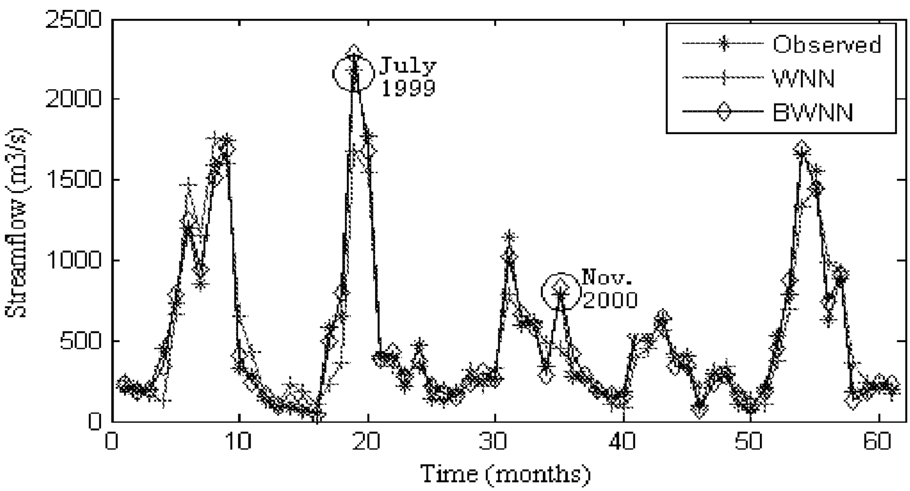

Fig. 4. Forecasting results of biased wavelet neural network (BWNN) and traditional wavelet neural network (WNN) based on BP algorithm. The circled points at Jul. 1999 and Nov. 2000 are the maximum streamflow and inundation outside flood season respectively, which are hard to forecast.

5 Conclusions and Suggestions

The BP-based biased wavelet neural network (BWNN) is presented and applied to monthly streamflow forecast in this paper. The monthly streamflow is stochastic, periodic and influenced by many factors. As its complexity, traditional statistics approaches are difficult to reflect its nonlinear characteristics. BWNN is an improvement of traditional wavelet neural network (WNN). Acceding to the capability of multiresolution analysis and nonlinear mapping, BWNN can decrease effectively the computing redundancy with flexible changes of time-frequency window. In the monthly streamflow forecast simulation of Fengtan reservoir, BWNN shows its effectiveness and superiorities in convergence and efficiency to ARIMA, GA, FNN and WNN. As the limited study for BWNN, suggestions are listed below:

1) For monthly series, due to the typically periodicity, certain delay of the signals could be added as the input nodes of BWNN.
2) The frame pattern of how to establish the biased wavelets is also expected. The bias function used in case study is Gaussian function. It is a compromise between time and frequency domain. Other bias functions meet the definition may be employed to reflect the essence of signals.
3) During the learning process, network convergence shows nonlinearity and may lead to oscillation and local values, which comes from the inherent deficiency of BP algorithm. The selection of learning rate and momentum coefficient may meliorate this phenomenon, but better leaning algorithms are still preferred.

Acknowledgements. This research is supported by Nature Science Foundation of China under grant No. 50579022.

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