Numerical Methods- Homework 2

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March 1, 2016

1 Model

Use Dynare to compute a basic Real Business Cycle model with a few extra and meaningful shocks.

Answer

[1]

In the following sections, I follow the simplest case from Dynare User Guide step by step:

- Households maximize utility over consumption, c_t and leisure, $1 - l_t$, where l_t is labor input:

$$\max_{c_t, l_t, k_{t+1}} E \sum_{t=0}^{\infty} \beta[\log c_t + \psi \log(1 - l_t)]$$

- and subject to the budget constraint:

$$c_t + k_{t+1} = w_t l_t + r_t k_t + (1 - \delta) k_t$$

  where k_t is capital stock, w_t represents real wages, r_t real interest rates or cost of capital and δ is depreciation rate.

[1] I follow the following notes as a start up to learn Dynare:

- UserGuide
- "A guide to Specifying Observation Equations for the estimation of DSGE models",written by Johannes Pfeifer. https://sites.google.com/site/pfeiferecon/dynare

- By maximizing the household problem with respect to consumption, leisure and capital stock, we can get Euler equation in consumption, capturing intertemporal consumption tradeoff, and the labor supply equation linking labor positively to wages and negatively to consumption

$$\frac{1}{c_t} = \beta E_t[\frac{1}{c_{t+1}}(1 + r_{t+1} - \delta)]$$
$$\psi \frac{c_t}{1 - l_t} = w_t$$

- Firm side: CRS production function,

$$y_t = k_t^{\alpha}(e^{z_t} l_t)^{1-\alpha}$$

where z_t captures technology which folows AR(1),i.e $z_t = \rho z_{t-1} + \varepsilon_t$ and ε_t $N(0, \sigma^2)$

- At equilibrium, the optimal pricing conditions yields the following equations

$$w_t = (1 - \alpha)\frac{y_t}{l_t}\frac{\epsilon - 1}{\epsilon}$$
$$r_t = \alpha \frac{y_t}{k_t}\frac{\epsilon - 1}{\epsilon}$$

The output figure is shown in the figure 1

MATRIX OF COVARIANCE OF EXOGENOUS SHOCKS

Variables	e
e	0.000109

POLICY AND TRANSITION FUNCTIONS

	y	c	k	i	l	
Constant	0.892102	0.707970	8.004287	0.184132	0.302742	2.9
(correction)	0.000018	-0.000016	0.000034	0.000034	0.000009	-0.
k(-1)	0.018513	0.045723	0.949791	-0.027209	-0.009252	
z(-1)	0.843223	0.201336	0.641887	0.641887	0.139496	
e	0.887604	0.211933	0.675671	0.675671	0.146838	
k(-1),k(-1)	-0.001497	-0.001103	-0.000394	-0.000394	0.000449	-0
z(-1),k(-1)	0.031259	0.006755	0.024504	0.024504	0.002706	0
z(-1),z(-1)	0.323596	0.049646	0.273950	0.273950	-0.005810	0
e,e	0.358555	0.055009	0.303545	0.303545	-0.006437	0
k(-1),e	0.032904	0.007111	0.025794	0.025794	0.002849	0
z(-1),e	0.681254	0.104518	0.576736	0.576736	-0.012231	0

MOMENTS OF SIMULATED VARIABLES

VARIABLE	MEAN	STD. DEV.	VARIANCE	SKEWNESS	KURTOSIS
y	0.894113	0.039822	0.001586	0.514727	0.458031
c	0.709366	0.024395	0.000595	0.654764	0.807058
k	8.030531	0.391105	0.152963	0.733092	0.969515
i	0.184747	0.019645	0.000386	0.123164	0.066725
l	0.302734	0.003864	0.000015	-0.250450	0.107752
y_l	2.952804	0.108662	0.011807	0.624036	0.714252
w	1.780541	0.065523	0.004293	0.624036	0.714252
r	0.033089	0.000937	0.000001	-0.370214	0.301992
z	0.000982	0.038186	0.001458	0.285845	0.175350

CORRELATION OF SIMULATED VARIABLES

VARIABLE	y	c	k	i	l	y_l	w	r	z
y	1.0000	0.9239	0.8385	0.8799	0.6950	0.9684	0.9684	0.1855	0.9914
c	0.9239	1.0000	0.9831	0.6310	0.3673	0.9901	0.9901	-0.2041	0.8677
k	0.8385	0.9831	1.0000	0.4789	0.1915	0.9479	0.9479	-0.3789	0.7623
i	0.8799	0.6310	0.4789	1.0000	0.9527	0.7336	0.7336	0.6296	0.9322
l	0.6950	0.3673	0.1915	0.9527	1.0000	0.4940	0.4940	0.8352	0.7807
y_l	0.9684	0.9901	0.9479	0.7336	0.4940	1.0000	1.0000	-0.0650	0.9287
w	0.9684	0.9901	0.9479	0.7336	0.4940	1.0000	1.0000	-0.0650	0.9287
r	0.1855	-0.2041	-0.3789	0.6296	0.8352	-0.0650	-0.0650	1.0000	0.3087
z	0.9914	0.8677	0.7623	0.9322	0.7807	0.9287	0.9287	0.3087	1.0000

AUTOCORRELATION OF SIMULATED VARIABLES

VARIABLE	1	2	3	4	5
y	0.9723	0.9431	0.9141	0.8873	0.8641
c	0.9951	0.9887	0.9810	0.9722	0.9626
k	0.9985	0.9956	0.9910	0.9850	0.9775
i	0.9333	0.8649	0.7995	0.7419	0.6952
l	0.9180	0.8340	0.7537	0.6840	0.6281
y_l	0.9890	0.9766	0.9631	0.9495	0.9364
w	0.9890	0.9766	0.9631	0.9495	0.9364
r	0.9278	0.8539	0.7833	0.7217	0.6723
z	0.9620	0.9223	0.8835	0.8485	0.8192

2 Estimation

In this section, I follow Iourii Manovskii's labor productivity [2] and I use `CPS_ye_aw_all` as productivity, which is derived by dividing output `BEA_y_all` by `CPS_e_aw_all`.

Here is the complete .mod file for the estimation of my super simple basic model, and I store the data as `q2_RBC_data`, in which `CPS_ye_aw_all` as productivity i.e. y. [3]

Matlab output

```
parameters
        prior mean   post. mean        90% HPD interval    prior       pstdev

alpha        0.350       0.3803      0.3780      0.3825    beta         0.0200
beta         0.990       0.9929      0.9929      0.9930    beta         0.0020
delta        0.025       0.0261      0.0258      0.0264    beta         0.0030
psi          1.750       1.8725      1.8651      1.8798    gamma        0.1000
rho          0.950       1.0000      1.0000      1.0000    beta         0.0500
epsilon       10.000      10.3085     10.2607     10.3712    gamma        0.5000

standard deviation of shocks
        prior mean   post. mean        90% HPD interval    prior       pstdev

e            0.010       0.0012      0.0012      0.0012    invg           Inf
Total computing time : 0h42m23s
```

[4]

[2] http://economics.sas.upenn.edu/~manovski/papers/labor_productivity.xls and its data description http://economics.sas.upenn.edu/~manovski/papers/Productivity_Data_Description.pdf

[3] Refer User Guide section "Launching the estimation" for the details.

[4] Please refer - An Introduction to Graphs in Dynare by Johannes Pfeifer

Reference

Matlab/Dynare code for Section1

The following is the details in `q1_RBC.mod` [5]

Matlab

```
addpath c:\dynare\4.4.3\matlab
cd c:\dynare\work
dynare q1_RBC
```

Dynare

```
//  The stochastic case
// Refer User Guide Chapter 3
// You can find the complete code : 3.9.1 The stochastic model

var y c k i l y_l w r z;
varexo e;  // with shocks

parameters beta psi delta alpha rho sigma epsilon;
alpha = 0.33;
beta = 0.99;
delta = 0.023;
psi = 1.75;
rho = 0.95;
sigma = (0.007/(1-alpha));
epsilon = 10;


/*3.5.1 Model in Dynare notation*/
/*Just in case you need a hint or two to recognize these equations, heres
a brief description: the rst equation is the Euler equation in consumption.
The second the labor supply function. The third the accounting identity. The
fourth is the production function. The fth and sixth are the marginal cost
equal to markup equations. The seventh is the investment equality. The
eighth an identity that may be useful and the last the equation of motion of
technology.*/

model;
(1/c) = beta*(1/c(+1))*(1+r(+1)-delta);
psi*c/(1-l) = w;
c+i = y;
y = (k(-1)^alpha)*(exp(z)*l)^(1-alpha);
```

[5] I paste the code here just for my own reference in case I forget how I did that.

```
w = y*((epsilon-1)/epsilon)*(1-alpha)/l;
r = y*((epsilon-1)/epsilon)*alpha/k(-1);
i = k-(1-delta)*k(-1);
y_l = y/l;
z = rho*z(-1)+e;
end;

/* 3.6.1 Stochastic models and steady states*/

varobs y;

initval;
k = 9;
c = 0.76;
l = 0.3;
w = 2.07;
r = 0.03;
z = 0;
e = 0;
end;

// Calculate steady state and simulation
steady;
check;

/* 3.7.3 Stochastic models*/
shocks;
var e = sigma^2;
end;

stoch_simul(periods=2100);
// end of simulation

/* User Guide chapter 5
// Estimation


estimated params;
alpha, beta pdf, 0.35, 0.02;
beta, beta pdf, 0.99, 0.002;
delta, beta pdf, 0.025, 0.003;
psi, gamma pdf, 1.75, 0.1;
rho, beta pdf, 0.95, 0.05;
epsilon, gamma pdf, 10, 0.5;
stderr e, inv gamma pdf, 0.01, inf;
end;
```

```
// End of estimation

estimation(datafile=simuldataRBC,nobs=200,first obs=500,
mh replic=2000,mh nblocks=2,mh drop=0.45,mh jscale=0.8,
mode compute=4);
```

Matlab code for Section2

Matlab Code:

```
addpath c:\dynare\4.4.3\matlab
cd c:\dynare\work
dynare q2_RBC_Est
```

Dynare code: `q2_RBC_Est.mod`

```
var y c k i l y_l w r z;
varexo e;

parameters beta psi delta alpha rho gamma sigma epsilon;
alpha = 0.33;
beta = 0.99;
delta = 0.023;
psi = 1.75;
rho = 0.95;
sigma = (0.007/(1-alpha));
epsilon = 10;

model;
(1/c) = beta*(1/c(+1))*(1+r(+1)-delta);
psi*c/(1-l) = w;
c+i = y;
y = (k(-1)^alpha)*(exp(z)*l)^(1-alpha);
w = y*((epsilon-1)/epsilon)*(1-alpha)/l;
r = y*((epsilon-1)/epsilon)*alpha/k(-1);
i = k-(1-delta)*k(-1);
y_l = y/l;
z = rho*z(-1)+e;
end;

varobs y;

initval;
k = 9;
c = 0.76;
l = 0.3;
```

```
w = 2.07;
r = 0.03;
z = 0;
e = 0;
end;

// Calculate steady state and simulation
steady;
check;

shocks;
var e = sigma^2;
end;

stoch_simul(periods=2100);
// end of simulation


// Estimation
estimated_params;
alpha, beta_pdf, 0.35, 0.02;
beta, beta_pdf, 0.99, 0.002;
delta, beta_pdf, 0.025, 0.003;
psi, gamma_pdf, 1.75, 0.1;
rho, beta_pdf, 0.95, 0.05;
epsilon, gamma_pdf, 10, 0.5;
stderr e, inv_gamma_pdf, 0.01, inf;
end;
// End of estimation

estimation(datafile=q2_RBC_data,mh_replic=20000,mh_nblocks=2,mh_drop=0.45,mh_jscale=0.8
```

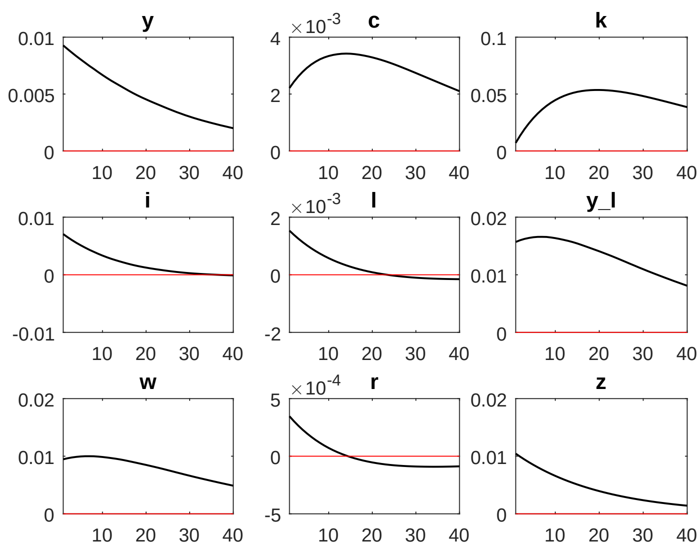

Figure 1: Impulse response function to technology shock

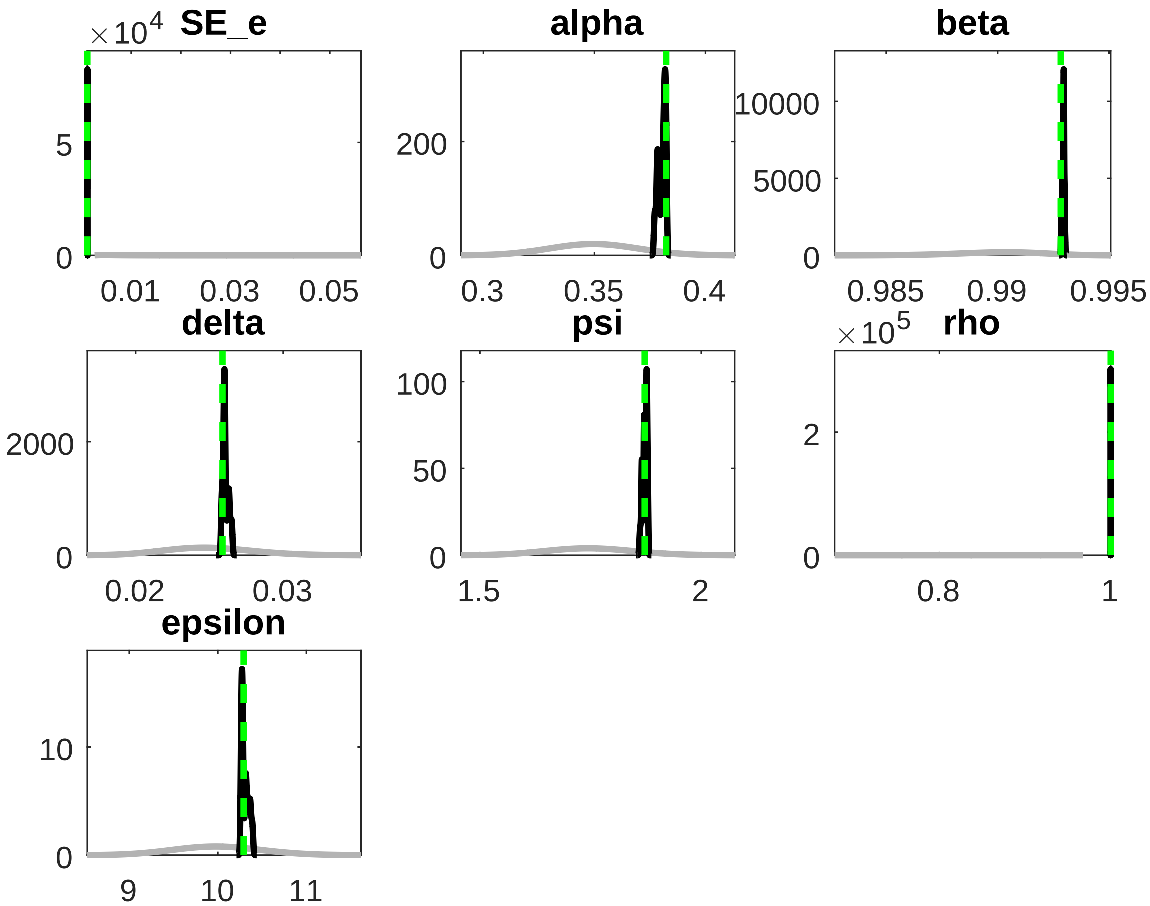

Figure 2: **Prior vs Posterior density comparison** : The x-axis displays part of the support of the prior distribution, while the y-axis displays the corresponding density. The grey line shows the prior density, while the black line shows the density of the posterior distribution.

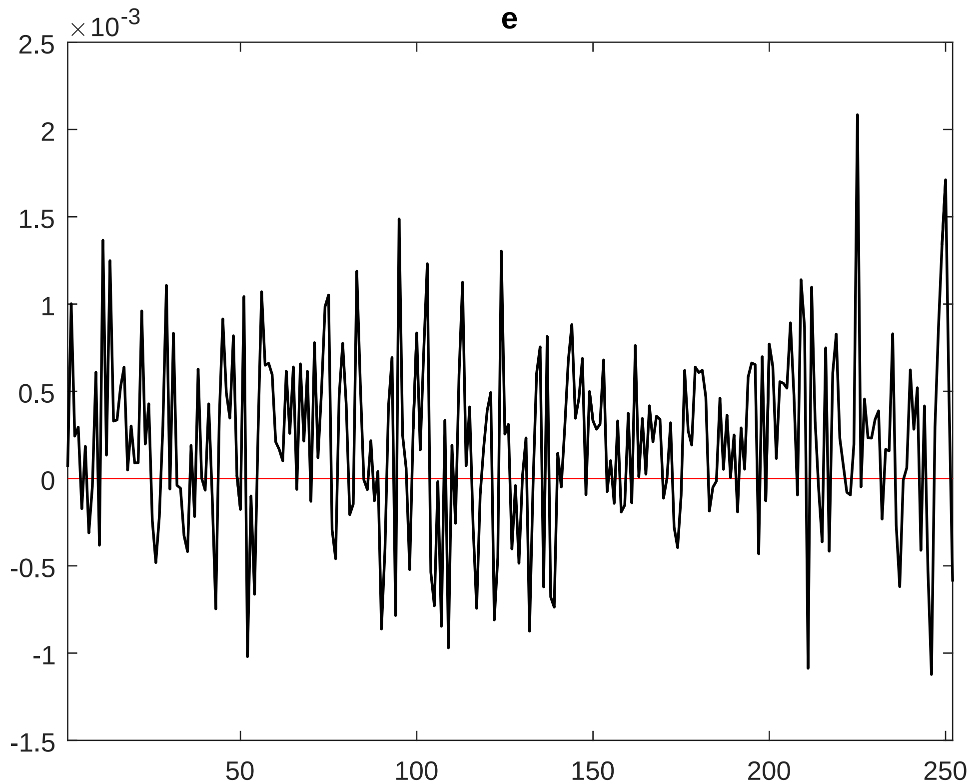

Figure 3: **Smoothed Shocks** Smoothed Shocks plot generated by the estimation- command when either maximum likelihood estimation (Maximum Likelihood (ML)) is used or Bayesian estimation without the smoother-option. It is stored in the main folder. The black line depicts the estimate of the smoothed structural shocks (best guess for the structural shocks given all observations), derived from the Kalman smoother at the posterior mode (ML) or posterior mean (Bayesian estimation).

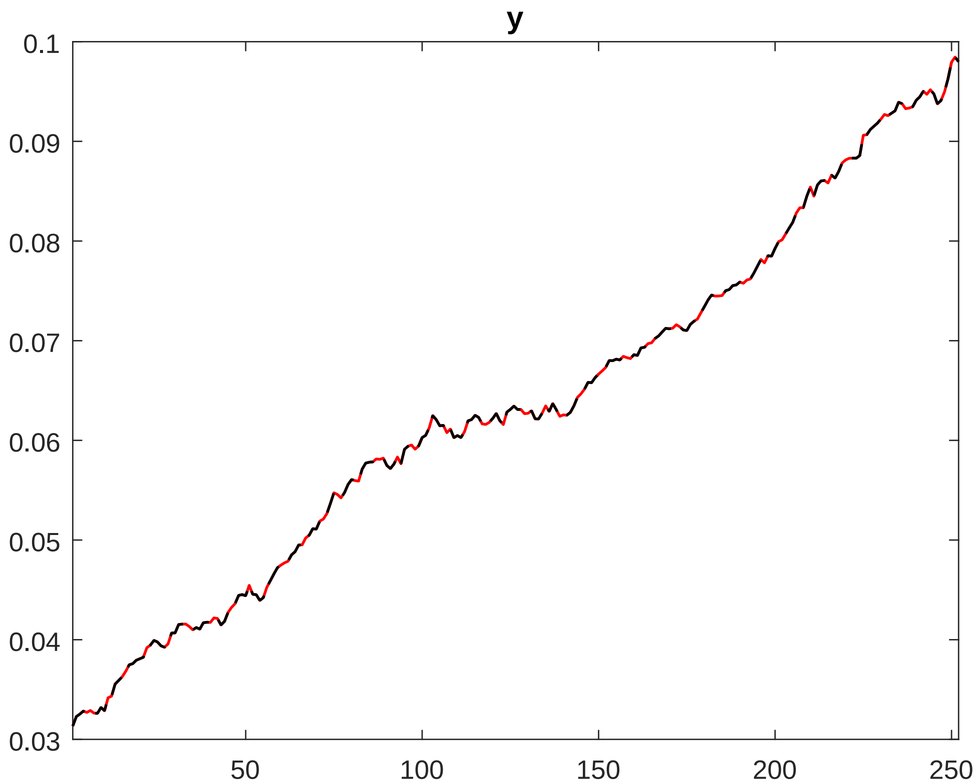

Figure 4: **Historical and smoothed variables** Historical and smoothed variables plot generated by the `estimation` command when either maximum likelihood estimation (ML) is used or Bayesian estimation without the smoother-option. It is stored in the main folder. The dotted black line depicts the actually observed data, while the red line depicts the estimate of the smoothed variable (best guess for the observed variable given all observations), derived from the Kalman smoother at the posterior mode (ML) or posterior mean (Bayesian estimation). In case of no measurement error, both series are identical as is the case in the upper panel.