



Further evidence on the relationship between spot and futures prices[☆]



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ARTICLE INFO

Article history:

Received 22 February 2016

Received in revised form

13 July 2016

Accepted 13 July 2016

Available online 26 July 2016

JEL classification:

Q02

L72

C13

Keywords:

Theory of storage

Interest-adjusted basis

ABSTRACT

Based on a theoretical model, Tilton et al. (2011) concluded that spot and futures prices should be highly correlated during periods of strong contango and much less correlated during periods of weak contango and backwardation. More recently, Gulley and Tilton (2014) found empirical support of this hypothesis for copper data during the period of 1994–2011.

In this note, we show that Gulley and Tilton's findings can be rationalized by the theory of storage, as periods of contango and backwardation can be singled out by the sign of the interest-adjusted basis (i. e., storage cost rate minus convenience yield). Our estimation results for the six base metals of the London Metal Exchange show that a stronger association between futures and spot returns during periods of high stocks (i.e., positive interest-adjusted basis) holds only contemporaneously. Indeed, Granger causality, especially from futures to spot returns, may take place regardless of stock levels.

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Tilton et al. (2011) studied the association between futures and spot prices during periods of contango and backwardation. Based on a theoretical model, the authors asserted that higher futures prices, brought about by a surge in investor demand, would have a comparable effect on the spot price during (strong) contango, but a much lesser effect on the latter during backwardation.² Consequently, spot and futures prices should be highly correlated during periods of strong contango and much less correlated during periods of weak contango and backwardation.

More recently, Gulley and Tilton (2014) conducted an empirical test of the above hypothesis for copper data on the basis of the cost-of-carry model. Based on a sample of daily observations for the period of 1994–2011, the authors concluded that the correlation between the spot and futures returns was higher during strong contango. Their finding was robust to the convenience yield value, the futures contract maturity, and to the subsample period.

In this note we show that Gulley and Tilton's findings can be rationalized by the theory of storage³, as periods of contango and backwardation can be singled out by the sign of the interest-adjusted basis (i. e., warehousing cost minus convenience yield). Specifically, the cost-of-carry model states that, in the absence of arbitrage, the relation between the futures price at t for delivery at T , $F_{t,T}$, and the spot price at t , S_t , is given by

$$F_{t,T} = S_t \exp \left[(r_t + u_t - y_t) \tau \right] \quad (1)$$

where $(r_t + u_t - y_t)$ represents the cost of carry, in that r_t , u_t , and y_t are, respectively, the risk-free rate, the storage cost rate, and the convenience yield at time t , and $\tau \equiv (T - t)$ is the time remaining until contract maturity.

From Eq. (1) it follows that

$$\ln(F_{t,T}/S_t) - r_t \tau = (u_t - y_t) \tau \equiv iab_t \quad (2)$$

where iab_t represents the interest-adjusted basis at time t .

At low stock levels, the convenience yield exceeds the storage cost rate (i.e., $y_t > u_t$) and $iab_t < 0$; at high stock levels, the convenience yield falls toward zero, and iab_t increases toward the storage cost rate at a decreasing rate. Fama and French (1988) argued that an implication of the above is that a permanent

[☆]Funds from Fondecyt Grant no. 1130003 are greatly acknowledged.

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² Tilton et al. argue that if investors demand drives up the futures price, this will encourage investors to buy on the spot market and sell forward on the futures market. This inter-temporal arbitrage will continue until the price difference between the spot and the futures markets returns to an amount that just covers the cost of holding stock. On the other hand, when the spot and futures markets are in backwardation or weak contango, the association between spot and futures prices is much weaker because an inter-temporal arbitrage from futures to spot markets is unfeasible. (Tilton et al. refer to strong contango as the case in which the futures price is well-above the spot price so that to cover the cost of holding stocks.)

³ The theory of storage, developed by Brennan (1958), Telser (1958) and Working (1949), states that the convenience yield (i.e., benefit of holding a physical commodity) falls at a decreasing rate as a commodity stock level increases.

demand shock has a large impact on spot prices when stock levels are low, but a smaller effect on futures prices because the market anticipates future demand and supply responses. Therefore, spot and futures returns should be less correlated when the interest-adjusted basis is negative.

We consider two tests to assess the degree of association between spot and futures markets, depending on the sign of the interest-adjusted basis. The first one is suitable to determine whether two Pearson correlations differ in magnitude statistically. This test is utilized to gauge the difference in correlation between spot and futures returns when the interest-adjusted basis is positive or negative. The second test is a Granger causality test which accommodates the existence of cointegration between spot and futures prices. This test is utilized to assess whether feed-back effects between spot and futures prices are stronger when the interest-adjusted basis is positive.

Specifically, under the assumption of two independent samples, T_1 and T_2 , a statistical test for the null hypothesis that the correlations between the spot and futures log-returns during periods of positive (ρ_1) and negative (ρ_2) interest-adjusted basis are equal is given by

$$\left(\frac{1}{2} \ln \left(\frac{1 + \hat{\rho}_1}{1 - \hat{\rho}_1} \right) - \frac{1}{2} \ln \left(\frac{1 + \hat{\rho}_2}{1 - \hat{\rho}_2} \right) \right) / \sqrt{\frac{1}{T_1 - 3} + \frac{1}{T_2 - 3}} \approx N(0, 1) \quad (3)$$

where $\hat{\rho}_1$ and $\hat{\rho}_2$ are the sample Pearson correlation coefficients based on T_1 and T_2 observations, respectively (see, for instance, Miller and Miller, 2012, chapter 14).⁴

Testing for linear spillover effects from the futures (spot) log-return to the spot (futures) log-return is based on Granger causality. In the absence of cointegration between spot and futures (log) prices, Granger causality detects feed-back effects from spot (futures) to futures (spot) log-returns. When spot and futures (log) prices are cointegrated, linkages between spot and futures (log) prices are gauged on the basis of an error-correction model.⁵

Our sample consists of daily observations of last (close) spot and futures prices of the six non-ferrous metals traded on the LME—aluminum, copper, lead, nickel, tin, and zinc—available for Bloomberg subscribers at <http://www.bloomberg.com/markets/commodities/futures/metals>. The futures prices correspond with 3- and 15-month contracts. The risk-free rate is approximated by the 1-month Eurodollar deposit rate (London), available at the Federal Reserve Economic Data (FRED), <http://research.stlouisfed.org/fred2/>.⁶ The sample period is January 1991–May 2015 which yields an approximate number of 6150 daily observations for each metal.

On the basis of the spot and futures prices and the proxy for the risk-free rate, the interest-adjusted basis (*iab*) is computed for each metal over the sample period. The mean and the standard deviation of the *iab* for 3- and 15-month futures are computed for the full sample and for periods of positive and negative *iab*. The figures shown in Table 1 suggest that the *iab* tends to be more volatile when it is negative, that is, when the convenience yield exceeds the storage cost rate. This is an implication of the theory of storage, as shocks produce more independent variation in spot and futures prices when stocks levels are low (see, for instance, Fama

Table 1

Descriptive statistics of the interest-adjusted basis: January 1991–May 2015.

	3-month futures			15-month futures		
	Full sample	<i>iab</i> > 0	<i>iab</i> < 0	Full sample	<i>iab</i> > 0	<i>iab</i> < 0
Aluminum						
Mean	0.006	0.010	−0.010	−0.071	0.017	−0.058
S.E.	0.012	0.006	0.011	0.050	0.013	0.043
Copper						
Mean	−0.012	0.005	−0.022	−0.110	0.012	−0.096
S.E.	0.022	0.005	0.022	0.087	0.011	0.086
Lead						
Mean	0.000	0.012	−0.022	−0.105	0.009	−0.085
S.E.	0.021	0.007	0.019	0.077	0.007	0.073
Nickel						
Mean	−0.007	0.004	−0.018	−0.119	0.002	−0.086
S.E.	0.020	0.004	0.023	0.100	0.001	0.095
Tin						
Mean	−0.006	0.002	−0.011	−0.057	0.005	−0.053
S.E.	0.010	0.002	0.010	0.039	0.007	0.039
Zinc						
Mean	0.003	0.011	−0.018	−0.107	0.012	−0.077
S.E.	0.019	0.006	0.027	0.075	0.008	0.070

Note: The interest-adjusted basis (*iab*) is computed as $\ln(F_{t,T}/S_t) - r_t\tau$, where $F_{t,T}$ is the futures price at t for a contract with maturity at T , S_t is the spot price at t , r_t is the proxy for the risk-free rate at t , and τ is the time remaining until maturity, $(T - t)$. The data source is Bloomberg (www.bloomberg.com) and Federal Reserve Economic Data (FRED).

and French, 1988, Table 1).

As mentioned earlier, Gulley and Tilton (2014) concluded that the correlation between the copper spot and futures returns was higher during strong contango (i.e., when the futures price is well-above the spot price) during the period of 1994–2011. In the authors' view, their evidence lent support to the inter-temporal arbitrage discussion provided by Tilton et al. (2011).

However, a simpler explanation to Gulley and Tilton's findings may be found in the theory of storage, as periods of contango and backwardation can be singled out by the sign of the interest-adjusted basis, which in turn signals stocks metal shortage/abundance. Notice from Eq. (2), however, that $iab_t > 0$ is more stringent than $F_{t,T} > S_t$ (contango) because iab_t could be negative, for a sufficiently large interest rate, even when $F_{t,T} > S_t$. Hence, under high interest rates scenarios, it is likely to observe a positive interest-adjusted basis when the futures prices is much greater than the spot price (i.e., strong contango as defined by Gulley and Tilton).

To illustrate the above statement, Fig. 1 depicts the evolution of the log-ratio of the fitted 3-month futures price to the spot price, the log-ratio of the actual 3-month futures price to the spot price, and the interest-adjusted basis of copper. A positive (negative) value of the log-ratio indicates that the actual (fitted) 3-month futures price is greater (less) than the spot price. From Eq. (1), the fitted futures price is computed as $\hat{F}_{t,T} = S_t \exp[(r_t + u_t - y_t)\tau]$, where $u_t \hat{=} y_t$ in turn is the fitted value of $(u_t - y_t)$ from a regression of the interest-adjusted basis on a proxy for the convenience yield (further details on this methodology can be found in Fernandez, 2016).⁷ As can be seen from Fig. 1, May 2000 is an example where, despite the fact that the actual (and fitted) futures price exceeds

⁴ It is worth pointing out that Gulley and Tilton's (2014) analysis is based exclusively on a visual inspection of the magnitudes of the correlations between spot and futures returns. No statistical test is provided to assess the correlation difference during contango and backwardation.

⁵ A testable cointegration regression model from (1) is given by $\ln(F_{t,T}) = \beta_0 + \beta_1 \ln(S_t) + \beta_2(r_t + u_t - y_t)\tau + \eta_t$, where the non-observable convenience yield (y_t) can be obtained by Heaney's (2002) approximation, and r_t and u_t can be approximated by a Treasury bond rate and a LME warehousing rate available for each base metal, respectively.

⁶ This specific series is chosen to make the estimation results comparable to Gulley and Tilton's (2014).

⁷ This sort of an approximation is in line with Gulley and Tilton's (2014) approach. Specifically, the authors set alternative values for the convenience yield and approximate the storage cost by the London Metal Exchange average warehouse rent to compute the futures price from expression (1). Our estimation show that the correlation between the actual and fitted 3-month futures prices is close to 1 during January 1991–December 2014. However, the correlation of the log-ratios is much lower (0.48) during the same time period. This is apparent from Fig. 1.

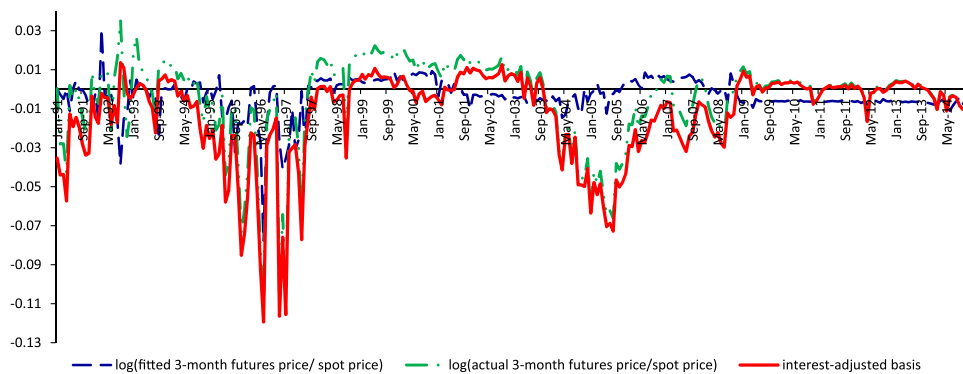


Fig. 1. Interest-adjusted basis and log-ratio of 3-month futures price to spot price of copper: January 1991–December 2014. *Note:* The fitted futures price is computed as $\hat{F}_{t,T} = S_t \exp[(r_t + u_t - y_t)\tau]$, where $u_t \hat{=} y_t$ (i.e., storage cost rate minus convenience yield) in turn is the fitted value from a regression of the interest-adjusted basis on a proxy for the convenience yield.

Table 2

Bivariate correlations of daily spot and futures log returns for a positive and negative interest-adjusted basis: January 1991–May 2015.

Metal	3-month futures				15-month futures							
	$iab > 0$ ρ_1	$iab < 0$ ρ_2	t -Stat $\rho_1 = \rho_2$	p -Value	$iab > 0$ σ_F/σ_S ratio	$iab < 0$ σ_F/σ_S ratio	$iab > 0$ ρ_1	$iab < 0$ ρ_2	t -Stat $\rho_1 = \rho_2$	p -Value	$iab > 0$ σ_F/σ_S ratio	$iab < 0$ σ_F/σ_S ratio
Aluminum	0.954 (4674)	0.899 (1485)	13.67	0.000	0.90	0.80	0.950 (3450)	0.886 (2709)	16.47	0.000	0.91	0.83
Copper	0.959 (2383)	0.893 (3785)	18.87	0.000	0.95	0.80	0.967 (1710)	0.891 (4458)	21.60	0.000	0.95	0.82
Lead	0.953 (3948)	0.933 (2210)	7.07	0.000	0.87	0.81	0.940 (3101)	0.866 (3057)	16.54	0.000	0.90	0.80
Nickel	0.963 (3174)	0.970 (2979)	−0.81	0.423	0.95	0.91	0.988 (1719)	0.923 (4434)	33.36	0.000	0.98	0.91
Tin	0.970 (2177)	0.956 (3976)	6.49	0.000	0.99	0.91	0.937 (1531)	0.934 (4622)	0.83	0.405	0.98	0.92
Zinc	0.973 (4565)	0.931 (1586)	16.19	0.000	0.92	0.80	0.980 (3429)	0.892 (2722)	33.52	0.000	0.93	0.81

Notes: (1) Number of observations in parenthesis. (2) p -Values based on the Fisher transformation are for a two-sided test: $H_0: \rho_1 = \rho_2$ versus $H_1: \rho_1 \neq \rho_2$. (3) σ_F/σ_S is the ratio of the futures return volatility over the spot return volatility. Figures in bold indicates that the null hypothesis of equal volatilities cannot be rejected at a 5% significance level. (4) The data source is Bloomberg (www.bloomberg.com) and Federal Reserve Economic Data (FRED).

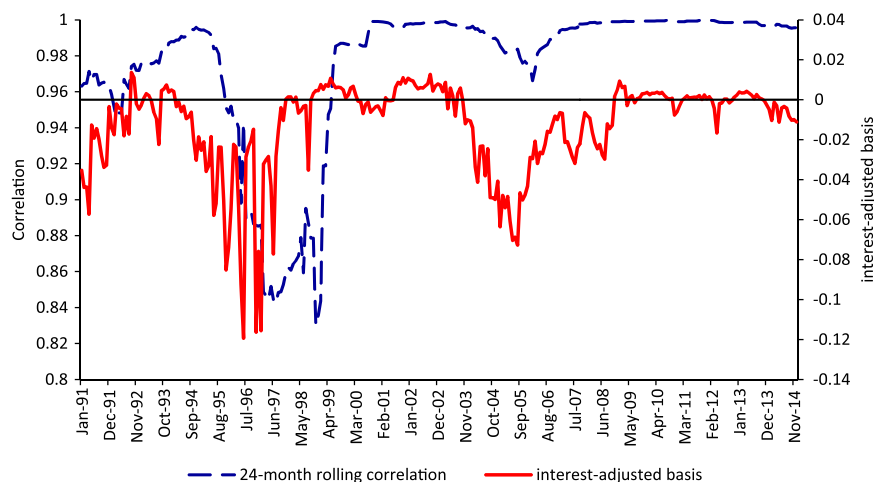


Fig. 2. Copper spot and 3-month futures returns association: January 1991–December 2014. *Note:* The bivariate-rolling correlation and the interest-adjusted basis are computed on the basis of monthly figures of spot and futures prices.

Table 3
Granger causality between spot and futures log returns: January 1991–May 2015.

(i) From spot (S) to futures (F) log returns									
Metal	Coint.	3-month maturity			Coint.	15-month maturity			
		$S \rightarrow F$	$S \rightarrow F$	$S \rightarrow F$		$S \rightarrow F$	$S \rightarrow F$	$S \rightarrow F$	
		FS	$iab > 0$	$iab < 0$		FS	$iab > 0$	$iab < 0$	
Aluminum	Yes	0.017	0.038	0.024	No	0.859	0.393	0.935	
Copper	Yes	0.713	0.262	0.970	No	0.023	0.012	0.077	
Lead	Yes	0.128	0.191	0.091	No	0.535	0.296	0.153	
Nickel	Yes	0.312	0.027	0.864	No	0.945	0.158	0.893	
Tin	Yes	0.440	0.006	0.130	Yes	0.000	0.000	0.007	
Zinc	Yes	0.245	0.106	0.638	Yes	0.356	0.245	0.863	
(ii) From futures (F) to spot (S) log returns									
Metal	Coint.	3-month maturity			Coint.	15-month maturity			
		$F \rightarrow S$	$F \rightarrow S$	$F \rightarrow S$		$F \rightarrow S$	$F \rightarrow S$	$F \rightarrow S$	
		FS	$iab > 0$	$iab < 0$		FS	$iab > 0$	$iab < 0$	
Aluminum	Yes	0.000	0.000	0.002	No	0.893	0.701	0.115	
Copper	Yes	0.000	0.910	0.000	No	0.465	0.126	0.319	
Lead	Yes	0.000	0.000	0.000	No	0.478	0.613	0.187	
Nickel	Yes	0.000	0.000	0.020	No	0.250	0.899	0.254	
Tin	Yes	0.000	0.001	0.000	Yes	0.642	0.021	0.561	
Zinc	Yes	0.002	0.000	0.341	Yes	0.343	0.382	0.669	

Notes: (1) FS: full sample; $iab > 0$ and $iab < 0$: positive and negative interest-adjusted basis, respectively. (2) p -Values less than or equal to 10% in bold. (3) 'Coint' indicates the existence of cointegration in the full sample at the significance level of 10% or less under Engle–Granger test. If cointegration exists, Granger causality is tested by using the error-correction model (ECM). (4) The optimal lag length is determined by Schwartz Information criterion (SIC).

the spot price, the interest-adjusted basis is negative.⁸ Notice that one advantage of utilizing the interest-adjusted basis to identify periods of low/high metal stocks, as opposed to the log-ratio of the fitted futures price to the spot price, is that the former is not subject to measurement error.

Table 2 in turn reports correlations between spot and futures log-returns, and volatility-ratio tests of spot to futures returns (σ_F/σ_S) when the interest-adjusted basis varies in sign. The statistical difference between correlations is tested on the basis of expression (3). As can be seen from the table, except for nickel (3-month futures) and tin (15-month futures), spot and futures returns are statistically more closely correlated when the interest-adjusted basis is positive (i.e., high stock levels). This is Gulley and Tilton's finding for (strong) contango. For the sake of illustration, Fig. 2 depicts the 24-month rolling correlation of the spot and 3-month futures log-returns of copper, along with the corresponding interest-adjusted basis. As can be seen from the figure, the minimum correlation is observed around the time the interest-adjusted basis is the most negative. In general, a change in the sign of the interest-adjusted basis from positive to negative is associated with a correlation decrease.

Table 2 also shows that, when the interest-adjusted basis is positive, the spot and futures returns are about equally volatile,

particularly in the cases of copper, nickel, tin for both futures maturities. This is a prediction of the theory of storage: demand and supply shocks produce similar changes in spot and futures prices when stocks are high. On the contrary, when the interest-adjusted basis is negative (i.e., low stocks), spot prices are more sensitive to shocks than futures prices. For instance, the ratio of the futures return volatility over the spot return volatility (σ_F/σ_S) of copper is 0.80 when the interest-adjusted basis is negative, as opposed to 0.95 when it is positive.

One may wonder whether the stronger association between futures and spot prices observed at high stock levels occurs contemporaneously only. In other words, it seems relevant to test whether there are lagged feed-back effects between futures and spot markets, and whether such linkages depend on stock levels. Specifically, Table 3 shows the results of testing for Granger causality from spot to futures returns (Panel (i)) and from futures to spot returns (Panel (ii)) for the 3- and 15-month contract maturities. The tests are computed for the full sample and for periods of positive and negative interest-adjusted basis. In addition, for each maturity, the presence/absence of cointegration between the (log) futures price and the (log) spot price in the full sample is reported.

As Table 3 shows, cointegration holds for 3-month contracts mostly. Exceptions are 15-month tin and zinc futures. Moreover, feed-back effects from lagged futures returns to the current spot return are more frequent (Panel (ii)) than conversely (Panel (i)). In particular, such linkages are statistically significant for the full sample and periods of low and high stock levels for 3-month contracts (left-hand side of Panel (ii)). No feed-back effects from lagged futures returns to the current spot return are present for a 15-month maturity, with the exception of tin during periods of high stocks (right-hand side of Panel (ii)). In sum, the implication of the theory of storage as to a stronger association between returns on futures and spot prices during periods of high stocks finds empirical support only contemporaneously. Indeed, Granger causality, especially from futures to spot returns, may take place regardless of stock levels. From this perspective, Tilton et al.'s (2011) hypothesis would be less supported.

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⁸ In May 2000, the actual and fitted 3-month futures prices (F_1 and \hat{F}_1) exceeded the spot price (S) by USD 26.5 and USD 14, respectively. Along the period of January 1991–December 2014, the greatest value of $(F_1 - S)$ was USD 83 in September 1992, i.e., a period of strong contango as can be seen from Fig. 1. On the other hand, the smallest positive value of $(F_1 - S)$ was USD 0.69 in January 1991, i.e., a period of backwardation or weak contango according to Gulley and Tilton. Hence, in May 2000 the spot and futures prices lay somewhere between weak and strong contango.