1 Growth model

1.1 Planner Solution

- A representative household chooses how much to consume and how much to invest in order to maximize $\sum_{t=0}^{\infty} \beta^t u(c_t)$.

- The final good is produced using capital$K$ according to the production function $Y = K^\alpha$. The agent needs to decide how much of that product to consume and how much to use for production of new capital goods.

- **Adjustment costs:** In order to produce I_t new capital goods the representative agent needs to sacrifice $\frac{1}{2} I_t^2$ final goods.

- We express the problem in terms of choosing the saving rate s such that consumption is $C_t = (1 - s_t) Y_t$ and investment is defined by the equation $\frac{1}{2} I_t^2 = s_t Y_t$. Thus:

$$I_t = \sqrt{2 s_t Y_t}$$

- The recursive formulation of the problem is:

$$\begin{aligned} V_t\left(K_t\right) = \max_{s_t, K_{t+1}} & \; U(C_t) + \beta \mathbb{E}_t V_{t+1}(k_{t+1}) \qquad \text{s.t.} \\ & C_t = Y_t(1 - s_t) \\ & Y_t = K_t^\alpha \\ & K_{t+1} \leq (1 - \delta) K_t + \sqrt{2 s_t Y_t} \end{aligned}$$

where $U(C_t)$ is the utility of consumption. Since we have a convex adjustment cost and decreasing returns to scale, we can use linear utility function $U(C) = C$

- The lagrangian of the problem for firm i is:

$$\mathcal{L}_t = \mathbb{E}_t \sum_{r=0}^{\infty} \beta^r \left((1 - s_{t+r}) K_{t+r}^\alpha + Q_{t+r} \left[(1 - \delta) K_{t+r} + \sqrt{2 s_{t+r} K_{t+r}^\alpha} - K_{t+r+1} \right] \right)$$

- The first order conditions are:

$$[s_t] \qquad Q_t = \sqrt{2 s_t K_t^{\alpha}} = I_t \tag{1.1}$$

$$[K_{t+1}] \qquad Q_t = \beta(1 - s_{t+1})\alpha K_{t+1}^{\alpha-1} + \beta Q_{t+1}\left[(1-\delta) + \frac{s_{t+1}\alpha K_{t+1}^{\alpha-1}}{\sqrt{2 s_{t+1} K_{t+1}^{\alpha}}}\right] \tag{1.2}$$

1.2 Steady State

- In the steady state, we have that $\sqrt{2sK^{\alpha}} = \delta K$ and so $sK^{\alpha-1} = \delta^2 K/2$.
- By replacing the first F.O.C. in the second F.O. an, getting rid of time subscript we get

$$\sqrt{2sK^{\alpha}} = \beta\alpha K^{\alpha-1} - \beta\alpha s K^{\alpha-1} + \beta\sqrt{2sK^{\alpha}}(1-\delta) + \beta\alpha s K^{\alpha-1} \tag{1.3}$$

- Collecting terms and replacing $\sqrt{2sK^{\alpha}} = \delta K$ we get:

$$\delta K = \beta\alpha K^{\alpha-1} + \beta\delta K(1-\delta) \tag{1.4}$$

- Finally, solving for K we get:

$$K = \left[\frac{\beta\alpha}{\delta[1-\beta(1-\delta)]}\right]^{\frac{1}{2-\alpha}} \tag{1.5}$$

1.3 Market Solution

- Now we have agents, the final good agent, which purchases capital goods and consume the final good, and the capital good firm, which produces the capital good subject to a quadratic cost.

1.3.1 Final Good firm

- The production function is:

$$Y = K^{\alpha}$$

where $Z_{i,t}$ is the TFP shock and follows a stochastic process $Z_{i,t+1}^f \sim \mathcal{P}(Z_{i,t}^f)$.

– Profits in each period are

$$\pi^f_{i,t} = p^c_{i,t} Y_{i,t} - \sum_{j=1}^{N^c} p^k_{j,t} I_{i,j,t}$$

– The firm is risk-adverse and it maximizes the discounted sum of utility over profits $U(\pi_t) = \frac{\pi^{1-\gamma^f}}{1-\gamma^f}$.

– The problem of producers is:

$$\mathcal{V}^f_t\left(\{K_{i,j,t}\}_j,\, Z^f_t\right) = \max_{\{K_{i,j,t+1},\, I_{i,j,t}\}_j, L^f_{i,t}} \frac{\left(\pi^f_{i,t}\right)^{1-\gamma^f}}{1-\gamma^f} + \beta_f E_t \mathcal{V}^f_{t+1}(\{K_{i,jt+1}\}_j,\, Z^f_{t+1}) \quad \text{s.t.}$$
$$K_{i,j,t+1} \leq (1-\delta) K_{i,j,t} + I_{i,j,t} \quad \text{for} \quad j \in [1, ..., N^c]$$

– The lagrangian of the problem for firm i is:

$$\mathcal{L}_{i,t} = \mathbb{E}_t \sum_{r=0}^{\infty} \beta^r_f \left(\frac{\left(\pi^f_{i,t+r}\right)^{1-\gamma^f}}{1-\gamma^f} + \sum_{j=1}^{N^c} (Q_{i,j,t+r}[(1-\delta)K_{i,j,t+r} + I_{i,j,t+r} - K_{i,j,t+r+1}]) \right)$$

– The first order conditions are:

$$[K_{i,j,t+1}] \qquad Q_{i,j,t} = \beta_f \mathbb{E}_t \left[\left(\pi^f_{i,t+1}\right)^{-\gamma^f} p^c_{i,t+1} \frac{\alpha^F}{N^f} \frac{Y_{i,t+1}}{K_{i,k,t+1}} + Q_{i,j,t+1}(1-\delta) \right] \tag{1.6}$$

$$\left[I^f_{i,j,t}\right] \qquad Q_{i,j,t} = \left(\pi^f_{i,t}\right)^{-\gamma^f} p^k_{j,t} \tag{1.7}$$

– This sector adds the following equations to the system of equations that describe the economy:

$$Y_i = Z_{i,t}^f \left(\sum_{j=1}^{N^c} K_{i,j}^{1/N^c} \right)^{\alpha^f} \tag{1.8}$$

$$\mathcal{M}_{i,t} = \mathbb{E}_t \left[\beta_f \left(\frac{\pi_{i,t+1}}{\pi_{i,t}} \right)^{-\gamma^f} \right] \tag{1.9}$$

$$K_{i,j,t+1} = (1-\delta) K_{i,j,t} + I_{i,j,t}^f \tag{1.10}$$

$$p_{j,t}^k = \mathbb{E}_t \left[\mathcal{M}_{i,t} \left(p_{i,t}^c \frac{\alpha^F}{N^f} \frac{Y_{i,t+1}}{K_{i,j,t+1}} + p_{j,t+1}^k (1-\delta) \right) \right] \tag{1.11}$$

$$Z_{i,t+1}^f \sim \mathcal{P}(Z_{i,t}^f) \tag{1.12}$$

1.3.2 Capital good firms (superscript c)

- Capital good firms maximize the discounted sum of profits. Profits in each period are

$$\pi_{j,t}^c = p_{j,t}^k I_{j,t}^c - \frac{1}{2} \left(I_{j,t}^c \right)^2$$

- The lagrangian of the problem for firm j is:

$$\mathcal{L}_{j,t} = \mathbb{E}_t \sum_{r=0}^{\infty} \beta_f^r \left(p_{j,t+r}^k I_{j,t+r}^c - \frac{1}{2} I_{j,t+r}^2 \right)$$

- The first order conditions is:

$$[i_{j,t}] \qquad\qquad p_t^K = I_{j,t} \tag{1.13}$$

- For $N^f = 1$, $N^c = 1$, The steady state is:

$$\begin{aligned} Y &= K^{\alpha^f} \\ I &= \delta K \\ I &= p^k \\ p^k &= \frac{\beta_f}{1-(1-\delta)\beta_f} \alpha^f K^{\alpha^f - 1} \end{aligned}$$

- Solution:

$$
\begin{aligned}
Y &= K^{\alpha^f} \\
I &= \delta K \\
I &= p^k \\
K &= \left(\frac{\beta_f}{1-(1-\delta)\beta_f} \frac{\alpha^f}{\delta} \right)^{\frac{1}{2-\alpha^f}}
\end{aligned}
$$

2 Two sector model

- This document describes a two sector economy with N^f final good firms and N^c capital goods firms.

- Final good firms use capital goods and labor to produce the consumption good. Capital good firms uses land and labor to produce the capital goods.

- In addition to the two types of firms, a representative Household supply labor to both sectors and consume.

- We first present a competitive equilibrium model with representative agents.

- We then write the economy in strategic game form, which makes it suitable for learning based optimization.

2.1 Structure of the economy

- There are N^f final good firms indexed by i and N^c capital good firms indexed by j.

- In each period, final good firm i prduces the consumption good Y_i using labor L_i^f and capital good $K_{i,j}$, where j indexes capital goods sold by firm j. The production function is:

$$
Y_i = Z_{i,t}^f \left(\sum_{j=1}^{N^c} K_{i,j}^{1/N^c} \right)^{\alpha^f} \left(L_i^f \right)^{1-\alpha^f}
$$

- Capital goods firm j uses land X_j and labor L_j^f to produced new capital good I_j. The production function is

$$
I_j^c = X_j^{\alpha^c} \left(L_j^c \right)^{1-\alpha^C}
$$

.

- The representative household decides how much to consume of each final good, C_i, and how many hours to work L^h. The household have utility function

$$U(C, L^h) = \frac{1}{1-\gamma_h}\left(C - \phi\frac{\left(L^h\right)^{1+\eta}}{1+\eta}\right)^{1-\gamma}$$

where C is a bundle with constant elasticity of substitution:

$$C = \left(\sum_{i=1}^{N^f} C_i^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$$

2.2 Final Good firm (superscript f)

- The production function is:

$$Y_i = Z_{i,t}^f \left(\sum_{j=1}^{N^c} K_{i,j}^{1/N^c}\right)^{\alpha^f} \left(L_i^f\right)^{1-\alpha^f}$$

where $Z_{i,t}$ is the TFP shock and follows a stochastic process $Z_{i,t+1}^f \sim \mathcal{P}(Z_{i,t}^f)$.

- Profits in each period are

$$\pi_{i,t}^f = p_{i,t}^c Y_{i,t} - w_t L_{i,t}^f - \sum_{j=1}^{N^c} p_{j,t}^k I_{i,j,t}$$

- The firm is risk-adverse and it maximizes the discounted sum of utility over profits $U(\pi_t) = \frac{\pi^{1-\gamma^f}}{1-\gamma^f}$.
- The problem of producers is:

$$\mathcal{V}_t^f\left(\{K_{i,j,t}\}_j,\, Z_t^f\right) = \max_{\{K_{i,j,t+1}, I_{i,j,t}\}_j, L_{i,t}^f} \frac{\left(\pi_{i,t}^f\right)^{1-\gamma^f}}{1-\gamma^f} + \beta_f E_t \mathcal{V}_{t+1}^f(\{K_{i,jt+1}\}_j,\, Z_{t+1}^f) \quad \text{s.t.}$$

$$K_{i,j,t+1} \leq (1-\delta)K_{i,j,t} + I_{i,j,t} \quad \text{for} \quad j \in [1, ..., N^c]$$

- The lagrangian of the problem for firm i is:

$$\mathcal{L}_{i,t} = \mathbb{E}_t \sum_{r=0}^{\infty} \beta_f^r \left(\frac{\left(\pi_{i,t+r}^f\right)^{1-\gamma^f}}{1-\gamma^f} + \sum_{j=1}^{N^c} \left(Q_{i,j,t+r}[(1-\delta)K_{i,j,t+r} + I_{i,j,t+r} - K_{i,j,t+r+1}]\right)\right)$$

- The first order conditions are:

$$[K_{i,j,t+1}] \qquad Q_{i,j,t} = \beta_f \mathbb{E}_t \left[\left(\pi^f_{i,t+1} \right)^{-\gamma^f} p^c_{i,t+1} \frac{\alpha^F}{N^f} \frac{Y_{i,t+1}}{K_{i,k,t+1}} + Q_{i,j,t+1}(1-\delta) \right] \tag{2.1}$$

$$\left[I^f_{i,j,t} \right] \qquad Q_{i,j,t} = \left(\pi^f_{i,t} \right)^{-\gamma^f} p^k_{j,t} \tag{2.2}$$

$$\left[L^f_{i,t} \right] \qquad w_t = p^c_{i,t}(1-\alpha^f) \frac{Y_{i,t}}{L^f_{i,t}} \tag{2.3}$$

- This sector adds the following equations to the system of equations that describe the economy:

$$Y_i = Z^f_{i,t} \left(\sum_{j=1}^{N^c} K^{1/N^c}_{i,j} \right)^{\alpha^f} \left(L^f_i \right)^{1-\alpha^f} \tag{2.4}$$

$$\mathcal{M}_{i,t} = \mathbb{E}_t \left[\beta_f \left(\frac{\pi_{i,t+1}}{\pi_{i,t}} \right)^{-\gamma^f} \right] \tag{2.5}$$

$$K_{i,j,t+1} = (1-\delta) K_{i,j,t} + I^f_{i,j,t} \tag{2.6}$$

$$p^k_{j,t} = \mathbb{E}_t \left[\mathcal{M}_{i,t} \left(p^c_{i,t} \frac{\alpha^F}{N^f} \frac{Y_{i,t+1}}{K_{i,j,t+1}} + p^k_{j,t+1}(1-\delta) \right) \right] \tag{2.7}$$

$$w_t = p^c_{i,t}(1-\alpha^f) \frac{Y_{i,t}}{L^f_{i,t}} \tag{2.8}$$

$$Z^f_{i,t+1} \sim \mathcal{P}(Z^f_{i,t}) \tag{2.9}$$

2.3 Capital good firms (superscript c)

- Capital good firms maximize the discounted sum of profits. Profits in each period are

$$\pi^c_{j,t} = p^k_{j,t} I^c_{j,t} - w_t L^c_{j,t} - p^x_t X_{j,t}$$

- The lagrangian of the problem for firm j is:

$$\mathcal{L}_{j,t} = \mathbb{E}_t \sum_{r=0}^{\infty} \beta^r_f \left(p^k_{j,t+r} I^c_{j,t+r} - w_{t+r} L^c_{j,t+r} - p^x_{t+r} X_{j,t+r} \right)$$

- The first order conditions are:

$$[X_{j,t}] \qquad p_t^x = p_{j,t}^k \alpha^c \frac{I_{j,t}}{X_{j,t}} \tag{2.10}$$

$$\left[L_{j,t}^c\right] \qquad w_t = p_{j,t}^k (1-\alpha^c) \frac{I_{j,t}}{L_{j,t}^c} \tag{2.11}$$

2.4 Household (superscript h)

- Te household consume a bundle:

$$C = \left(\sum_{i=1}^{N^f} C_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

- The household maximize intertemporal utility of consumption and leisure:

$$U(C, L^h) = \frac{1}{1-\gamma_h} \left(C - \phi \frac{\left(L^h\right)^{1+\eta}}{1+\eta} \right)^{1-\gamma}$$

- The lagrangian is

$$\begin{aligned}\mathcal{L} = \mathbb{E}_t \sum_{r=0}^{\infty} \beta_h^r \Bigg\{ & \frac{1}{1-\gamma} \left(C_{t+r} - \phi \frac{\left(L_{t+r}^h\right)^{1+\eta}}{1+\eta} \right)^{1-\gamma} + \\ & \mu_{t+r} \left[w_{t+r} L_{t+r}^h + p_{t+r}^x X_{t+r}^g + \sum_{i=1}^{N^f} \pi_{i,t+r}^f + \sum_{j=1}^{N^c} \pi_{j,t+r}^c - \sum_{i=1}^{N^f} p_{i,t+r}^c C_{i,t+r} \right] \Bigg\}\end{aligned}$$

- The first order conditions are

$$\left[C_{i,t}^S\right] \qquad \left(C_t - \phi \frac{\left(L_t^h\right)^{1+\eta}}{1+\eta} \right)^{-\gamma} \left(\sum_{i=1}^{N^f} C_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{1}{\sigma-1}} C_i^{\frac{-1}{\sigma}} - \mu_t p_{i,t}^c = 0 \tag{2.12}$$

$$\left[L_t^h\right] \qquad \left(C_t - \phi \frac{\left(L_t^h\right)^{1+\eta}}{1+\eta} \right)^{-\gamma} \phi \left(L_t^h\right)^{\eta} - \mu_t w_t = 0 \tag{2.13}$$

- We can summarize the conditions as:

$$\frac{p_{i,t}^c}{w_t} = \frac{\left(\sum_{i=1}^{N^f} C_i^{\frac{\sigma-1}{\sigma}}\right)^{\frac{1}{\sigma-1}} C_i^{\frac{-1}{\sigma}}}{\phi\left(L_t^h\right)^{\eta}} \quad \text{for} \quad i \in [1, ..., N^f] \tag{2.14}$$

2.5 Market Clearing

- Final good: $Y_{i,t} = C_{i,t}$ for $i \in [1, ..., N^f]$
- Capital Goods: $\sum_{i=1}^{N^f} I_{i,j,t}^f = I_{j,t}^c$ for $j \in [0, ..., N^c]$
- Labor Market: $L_t^s = \sum_{i=1}^{N^f} L_{i,t}^f + \sum_{j=1}^{N^c} L_{j,t}^c$
- Land: $1 = \sum_{j=1}^{N^c} X_{j,t}$

2.6 Deterministic Steady-State

- There are x unkowns: prices w, p^x, p^k, quantities Y, L^f, L^c, I, K.
- Given these X variables we have the following conditions:

$$\begin{aligned}
Y_i &= \left(\sum_{j=1}^{N^c} K_{i,j}^{1/N^c}\right)^{\alpha^f} \left(L_i^f\right)^{1-\alpha^f} \quad \text{for } i \in [1, ..., N^f] \\
I_{i,j}^f &= \delta K_{i,j} \quad \text{for } i \in [1, ..., N^f] \text{ and } j \in [1, ..., N^c] \\
I_j^c &= X_j^{\alpha^c} \left(L_j^c\right)^{1-\alpha^c} \quad \text{for } j \in [1, ..., N^c] \\
p_j^k &= \beta_f \left(p_i^c \frac{\alpha^F}{N^f} \frac{Y_i}{K_{i,j}} + p_j^k(1-\delta)\right) \quad \text{for } i \in [1, ..., N^f] \text{ and } j \in [1, ..., N^c] \\
w &= p_i^c(1-\alpha^f)\frac{Y_i}{L_i^f} \quad \text{for } i \in [1, ..., N^f] \\
p^x &= p_j^k \alpha^c \frac{I_j^c}{X_j} \quad \text{for } j \in [1, ..., N^c] \\
w &= p_j^k(1-\alpha^c)\frac{I_j^c}{L_j^c} \quad \text{for } j \in [1, ..., N^j] \\
\frac{p_i^c}{w} &= \frac{\left(\sum_{i=1}^{N^f} C_i^{\frac{\sigma-1}{\sigma}}\right)^{\frac{1}{\sigma-1}} C_i^{\frac{-1}{\sigma}}}{\phi\left(L_t^h\right)^{\eta}} \quad \text{for} \quad i \in [1, ..., N^f] \\
\sum_{i=1}^{N^f} p_i^c C_i &= wL^h + p^x
\end{aligned}$$

2.7 The $N^f = 1$ and $N^c = 1$ case

- We assume that land is fixed. The first order condition of the capital goods sector now are:

$$[X_{j,t}] \qquad p_t^x = p_{j,t}^k \alpha^c \frac{I_{j,t}}{X_{j,t}} \tag{2.15}$$

$$\left[L_{j,t}^c\right] \qquad w_t = p_{j,t}^k (1-\alpha^c) \frac{I_{j,t}}{L_{j,t}^c} \tag{2.16}$$

- We can take out many variables from the steady state

$$\begin{aligned}
Y &= K^{\alpha^f} \left(L^f\right)^{1-\alpha^f} \\
w &= \phi \left(L^f + L^c\right)^{\eta} \\
p^x &= p^k \alpha^c \delta K \\
I &= \delta K \\
I &= (L^c)^{1-\alpha^c} \\
p^k K &= \frac{\beta_f}{1-(1-\delta)\beta_f} \alpha^f K^{\alpha^f} \left(L^f\right)^{1-\alpha^f}
\end{aligned}$$

- The steady state is

$$\begin{aligned}
\phi \left(L^f + L^c\right)^{\eta} L^f &= (1-\alpha^f) \left(\delta^{-1} (L^c)^{1-\alpha^c}\right)^{\alpha^f} \left(L^f\right)^{1-\alpha^f} \\
\phi \left(L^f + L^c\right)^{\eta} L^c &= (1-\alpha^c)\delta \frac{\beta_f}{1-(1-\delta)\beta_f} \alpha^f \left(\delta^{-1} (L^c)^{1-\alpha^c}\right)^{\alpha^f} \left(L^f\right)^{1-\alpha^f}
\end{aligned}$$

- We can confirm our solution by confirming that the goods market clear:

$$Y = w \left(L^f + L^c\right) + \pi^c + \pi^f + p^x$$

- The results are have the following directory:
- Figures:

3 Model with no labor and exogenous cost curve

3.1 Final Good firm (superscript f)

- The production function is:

$$Y_i = Z^f_{i,t} \left(\sum_{j=1}^{N^c} K_{i,j}^{1/N^c} \right)^{\alpha^f}$$

where $Z_{i,t}$ is the TFP shock and follows a stochastic process $Z^f_{i,t+1} \sim \mathcal{P}(Z^f_{i,t})$.

- Profits in each period are

$$\pi^f_{i,t} = p^c_{i,t} Y_{i,t} - \sum_{j=1}^{N^c} p^k_{j,t} I_{i,j,t}$$

- The firm is risk-adverse and it maximizes the discounted sum of utility over profits $U(\pi_t) = \frac{\pi^{1-\gamma^f}}{1-\gamma^f}$.
- The problem of producers is:

$$\mathcal{V}^f_t \left(\{K_{i,j,t}\}_j,\, Z^f_t \right) = \max_{\{K_{i,j,t+1}, I_{i,j,t}\}_j, L^f_{i,t}} \frac{\left(\pi^f_{i,t}\right)^{1-\gamma^f}}{1-\gamma^f} + \beta_f E_t \mathcal{V}^f_{t+1}(\{K_{i,jt+1}\}_j,\, Z^f_{t+1}) \quad \text{s.t.}$$
$$K_{i,j,t+1} \leq (1-\delta) K_{i,j,t} + I_{i,j,t} \quad \text{for} \quad j \in [1, ..., N^c]$$

- The lagrangian of the problem for firm i is:

$$\mathcal{L}_{i,t} = \mathbb{E}_t \sum_{r=0}^{\infty} \beta^r_f \left(\frac{\left(\pi^f_{i,t+r}\right)^{1-\gamma^f}}{1-\gamma^f} + \sum_{j=1}^{N^c} (Q_{i,j,t+r}[(1-\delta)K_{i,j,t+r} + I_{i,j,t+r} - K_{i,j,t+r+1}]) \right)$$

- The first order conditions are:

$$[K_{i,j,t+1}] \qquad Q_{i,j,t} = \beta_f \mathbb{E}_t \left[\left(\pi^f_{i,t+1}\right)^{-\gamma^f} p^c_{i,t+1} \frac{\alpha^F}{N^f} \frac{Y_{i,t+1}}{K_{i,k,t+1}} + Q_{i,j,t+1}(1-\delta) \right] \tag{3.1}$$

$$\left[I^f_{i,j,t}\right] \qquad Q_{i,j,t} = \left(\pi^f_{i,t}\right)^{-\gamma^f} p^k_{j,t} \tag{3.2}$$

- This sector adds the following equations to the system of equations that describe the economy:

$$Y_i = Z_{i,t}^f \left(\sum_{j=1}^{N^c} K_{i,j}^{1/N^c} \right)^{\alpha^f} \tag{3.3}$$

$$\mathcal{M}_{i,t} = \mathbb{E}_t \left[\beta_f \left(\frac{\pi_{i,t+1}}{\pi_{i,t}} \right)^{-\gamma^f} \right] \tag{3.4}$$

$$K_{i,j,t+1} = (1-\delta) K_{i,j,t} + I_{i,j,t}^f \tag{3.5}$$

$$p_{j,t}^k = \mathbb{E}_t \left[\mathcal{M}_{i,t} \left(p_{i,t}^c \frac{\alpha^F}{N^f} \frac{Y_{i,t+1}}{K_{i,j,t+1}} + p_{j,t+1}^k (1-\delta) \right) \right] \tag{3.6}$$

$$Z_{i,t+1}^f \sim \mathcal{P}(Z_{i,t}^f) \tag{3.7}$$

3.2 Capital good firms (superscript c)

- Capital good firms maximize the discounted sum of profits. Profits in each period are

$$\pi_{j,t}^c = p_{j,t}^k I_{j,t}^c - \frac{1}{2} \left(I_{j,t}^c \right)^2$$

- The lagrangian of the problem for firm j is:

$$\mathcal{L}_{j,t} = \mathbb{E}_t \sum_{r=0}^{\infty} \beta_f^r \left(p_{j,t+r}^k I_{j,t+r}^c - \frac{1}{2} I_{j,t+r}^2 \right)$$

- The first order conditions is:

$$[i_{j,t}] \qquad\qquad p_t^K = I_{j,t} \tag{3.8}$$

- For $N^f = 1$, $N^c = 1$, The steady state is:

$$\begin{aligned} Y &= K^{\alpha^f} \\ I &= \delta K \\ I &= p^k \\ p^k &= \frac{\beta_f}{1-(1-\delta)\beta_f} \alpha^f K^{\alpha^f - 1} \end{aligned}$$

- Solution:

$$
\begin{aligned}
Y &= K^{\alpha^f} \\
I &= \delta K \\
I &= p^k \\
K &= \left(\frac{\beta_f}{1-(1-\delta)\beta_f} \frac{\alpha^f}{\delta} \right)^{\frac{1}{2-\alpha^f}}
\end{aligned}
$$