Kinematik und Dynamik

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Contents

Chapter 1

Vorwort

Dieses Kapitel basiert auf der Arbeit von Openstax Wolfe et al. (2015) und wurde von Ben Kasel übersetzt und an das luxemburgische Schulsystem angepasst.

Chapter 2

Kinematik

2.1 Einleitung

Figure 2.1: Die Bewegung eines Falken durch die Luft kann mit der Position, Geschwindigkeit und Beschleunigung des Vogels beschrieben werden. Bewegt er sich geradlinig, spricht man von einer ein-dimensionalen Bewegung. (credit: Vince Maidens, Wikimedia Commons)

Wo man auch hinsieht, beobachtet man Körper in Bewegung, von einem Tennismatch zum Vorbeiflug einer Raumsonde am Neptun. Selbst wenn man ruhig sitzt, bewegt das Herz Blut durch unsere Blutgefässe und jedes einzelne Atom und Molekül vibriert und bewegt sich kontinuierlich. Welche Fragen könnte man sich zu Bewegungen stellen? *Wann wird eine bestimmte Raumsonde den Mars erreichen? Wie fliegt ein Fussball wenn er in einem bestimmten Winkel getreten wird?*

Die Analyse von Bewegungen liefert uns nicht nur Antworten auf diese Fragen, sondern bildet auch die Grundlage für weiterführende Konzepte in der Physik. So kann man *Kräfte* nur schwer begreifen wenn man das Konzept der *Beschleunigung* nicht versteht. Diese Verbindung zwischen Kräften und Beschleunigung ist nötig um die *Grundidee 3* zu verstehen.

Um Bewegungen quantitativ zu beschreibungen, wird in dieser Einheit das Thema des *Bezugsystems* behandelt. Wenn Sie bereits von einer Freundin am Bahnhofsteig verabschiedet haben, verstehen Sie diese Idee bestimmt. Sie sehen wie Ihre Freundin sich von Ihnen entfernt, obwohl sie sich für die anderen Passagiere im Ruhezustand befindet. Das Bezugsystem beeinflusst die Beobachtung und dieser Zusammenhang muss für ein grundlegendes Verständnis von 3.A und Essential Knowledge 3.A.1. klar sein.

Das formale Studium der Physik beginnt mit der **Kinematik** die als *Lehre der Bewegung ohne deren Ursachen* definiert ist. Man untersucht also zum Beispiel die ein- oder zwei-dimensionale Bewegung eines Fussballs ohne sich zu Fragen welche Kräfte die Bewegung verursacht haben. In diesem Kapitel beschränken wir uns auf die einfachsten Bewegungen, solche die eindimensional oder geradlinig sind. Die gelernten Konzepte wenden wir dann im nächsten Kapitel auf Bewegungen mit kurvenförmigen Bahnkurven oder zweidimensionale Bewegungen an, zum Beispiel ein Auto in einer Kurve.

Der Inhalt dieses Kapitels unterstützt

Grundidee 3 Wechselwirkungen zwischen Körpern können mit Kräften beschrieben werden.

dauerhaftes Verständnis 3.A Beobachter in Inertialsystemen sind sich einig über die Charakteristiken von Kräften.

Essential Knowledge 3.A.1 Ein Beobachter in einem bestimmten Bezugsystem kann die Bewegung eines Körpers andhand der Grössen Position, Positionsänderung, Strecke, Geschwindigkeit (Vektor oder Betrag) und Beschleunigung beschreiben.

2.2 Positionsänderung

2.2.1 Lernziele

Nach diesem Kapitel solltest du folgendes können:

- Position, Positionsänderung, Strecke, zurückgelegte Strecke in einem bestimmten Bezugsystem definieren.
- Den Zusammenhang zwischen Possition und Positionsänderung erklären.
- Den Unterschied zwischen Positionsänderung und zurückgelegter Strecke kennen.
- Die Positionsänderung, und Strecke anhand der Anfangs- und Endposition, sowie der Bahnkurve zwischen beiden Punkten

Figure 2.2: Diese Fahrradfahrer in Vietnam können anhand ihrer Position in Bezug auf die Gebäude oder den Kanal beschrieben werden. Ihre Bewegung kann mit der Änderung der Position in diesem Bezugsystem beschrieben werden. (credit: Suzan Black, Fotopedia)

Die Informationen in diesem Kapitel unterstützen folgende Punkte des AP® Kurriculums:

- **3.A.1.1** Der Schüler kann die Bewegung eines Körpers sprachlich, mathematisch und und grafisch beschreiben. **(S.P. 1.5, 2.1, 2.2)**
- **3.A.1.3** Der Schüler kann Versuchsergebnisse analysieren und diese dazu nutzen die Bewegung des untersuchten Körpers sprachlich, mathematisch und grafisch darstellen. **(S.P. 5.1)**

2.2.2 Position

Bovir man sich der Beschreibung der Bewegung widmen kann, muss man zuvor die Position oder den Ort eines Körpers zu einem bestimmten Zeitpunkt zu bestimmen — Wo befindet sich der Körper, oder genauer gefragt, wo im Bezug zu einem bestimmten Beobachter. Die Erde wird oft als Bezugsystem benutzt und wir beschreiben die Bewegung eines Körpers in Bezug auf Körper die in diesem Bezugsystem ruhen. Zum Beispiel wird der Start einer Rakete in Bezug auf die gesamte Erde beschrieben, während die Position des Lehrers in Bezug auf die Tafel beschrieben wird. (Abbildung **?**{import-auto-id2972079} link.) In other cases, we use reference frames that are not stationary but are in motion relative to the Earth. To describe the position of a person in an airplane, for example, we use the airplane, not the Earth, as the reference frame. (See link.)

2.2.3 Displacement

If an object moves relative to a reference frame (for example, if a professor moves to the right relative to a white board or a passenger moves toward the rear of an airplane), then the object's position changes. This change in position is known as displacement. The word "displacement" implies that an object has moved, or has been displaced.

Displacement

Displacement is the *change in position* of an object:

$$\Delta x = x_{\mathrm{f}} - x_0,$$

where Δx is displacement, x_{f} is the final position, and x_0 is the initial position.

In this text the upper case Greek letter Δ (delta) always means "change in" whatever quantity follows it; thus, Δx means *change in position*. Always solve for displacement by subtracting initial position x_0 from final position x_{f}.

Note that the SI unit for displacement is the meter (m) (see Physical Quantities and Units), but sometimes kilometers, miles, feet, and other units of length are used. Keep in mind that when units other than the meter are used in a problem, you may need to convert them into meters to complete the calculation.

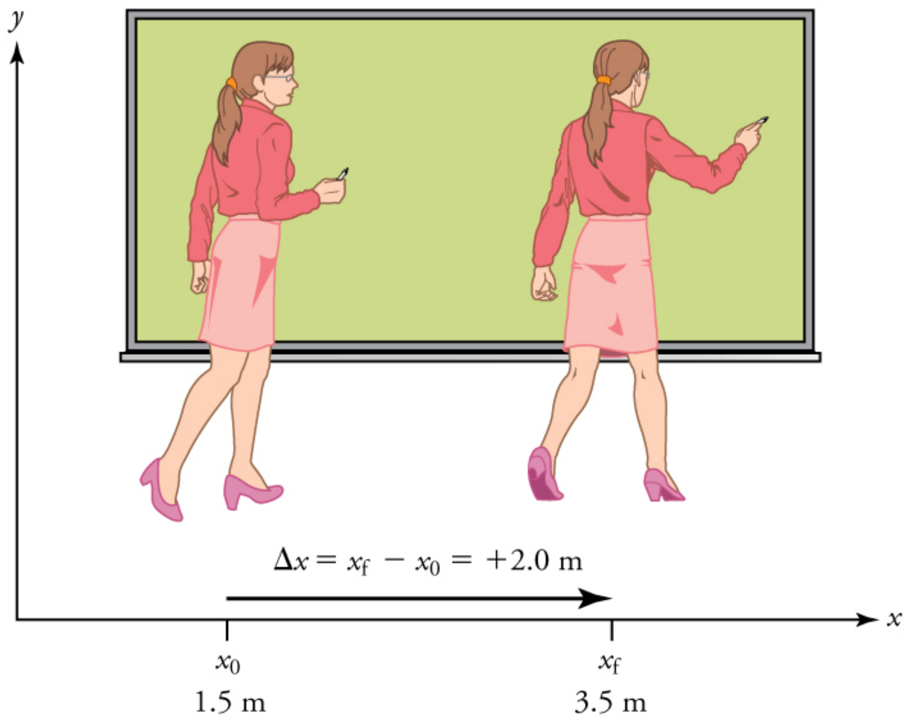


Figure 2.3: A professor paces left and right while lecturing. Her position relative to the blackboard is given by x. The $+2.0$ m displacement of the professor relative to the blackboard is represented by an arrow pointing to the right.

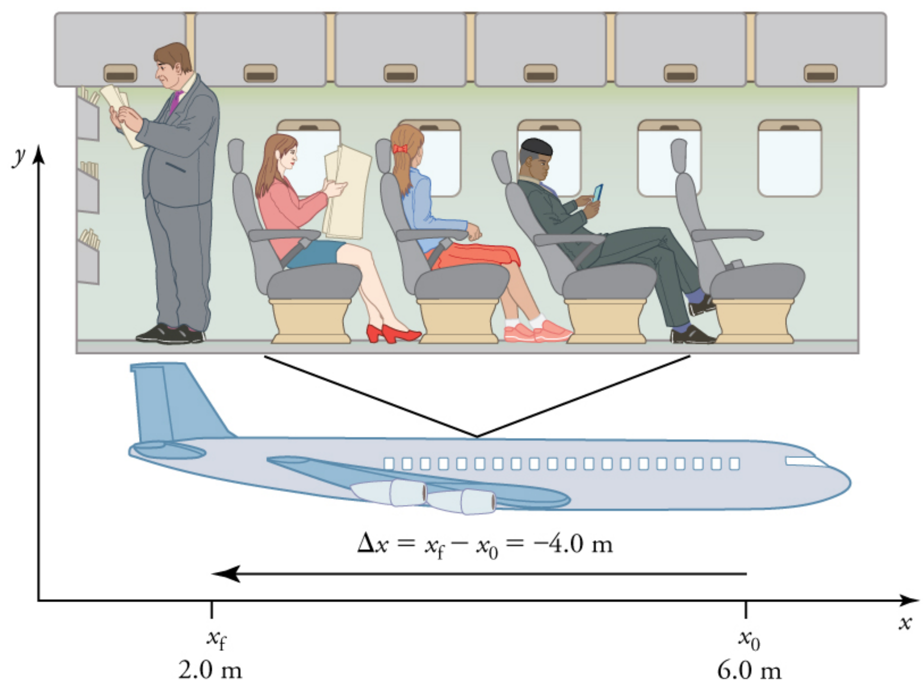


Figure 2.4: A passenger moves from his seat to the back of the plane. His location relative to the airplane is given by x. The -4 m displacement of the passenger relative to the plane is represented by an arrow toward the rear of the plane. Notice that the arrow representing his displacement is twice as long as the arrow representing the displacement of the professor (he moves twice as far) in link.

Note that displacement has a direction as well as a magnitude. The professor's displacement is 2.0 m to the right, and the airline passenger's displacement is 4.0 m toward the rear. In one-dimensional motion, direction can be specified with a plus or minus sign. When you begin a problem, you should select which direction is positive (usually that will be to the right or up, but you are free to select positive as being any direction). The professor's initial position is $x_0 = 1.5$ m and her final position is $x_\mathrm{f} = 3.5$ m. Thus her displacement is

$$\Delta x = x_f - x_0 = 3.5\text{ m} - 1.5\ m = +2.0\text{ m}.$$

In this coordinate system, motion to the right is positive, whereas motion to the left is negative. Similarly, the airplane passenger's initial position is $x_0 = 6.0\ m$ and his final position is $x_f = 2.0\ m$, so his displacement is

$$\Delta x = x_f - x_0 = 2.0\ m - 6.0\ m = -4.0\ m.$$

His displacement is negative because his motion is toward the rear of the plane, or in the negative x direction in our coordinate system.

2.2.4 Distance

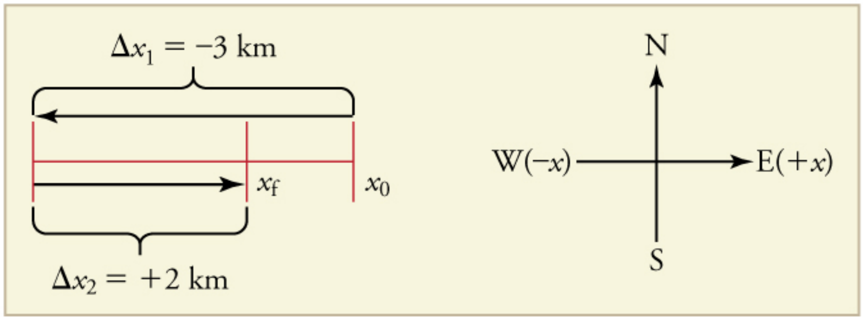
Although displacement is described in terms of direction, distance is not. Distance is defined to be *the magnitude or size of displacement between two positions*. Note that the distance between two positions is not the same as the distance traveled between them. Distance traveled is *the total length of the path traveled between two positions*. Distance has no direction and, thus, no sign. For example, the distance the professor walks is 2.0 m. The distance the airplane passenger walks is 4.0 m.

Misconception Alert: Distance Traveled vs. Magnitude of Displacement

It is important to note that the *distance traveled*, however, can be greater than the magnitude of the displacement (by magnitude, we mean just the size of the displacement without regard to its direction; that is, just a number with a unit). For example, the professor could pace back and forth many times, perhaps walking a distance of 150 m during a lecture, yet still end up only 2.0 m to the right of her starting point. In this case her displacement would be +2.0 m, the magnitude of her displacement would be 2.0 m, but the distance she traveled would be 150 m. In kinematics we nearly always deal with displacement and magnitude of displacement, and almost never with distance traveled. One way to think about this is to assume you marked the start of the motion and the end of the motion. The displacement is simply the difference in the position of the two marks and is independent of the path taken in traveling between the two marks. The distance traveled, however, is the total length of the path taken between the two marks.

Check Your Understanding

A cyclist rides 3 km west and then turns around and rides 2 km east. (a) What is her displacement? (b) What distance does she ride? (c) What is the magnitude of her displacement?



a The rider's displacement is $\Delta x = x_f - x_0 = -1$ km. (The displacement is negative because we take east to be positive and west to be negative.)

b The distance traveled is 3 km + 2 km = 5 km.

c The magnitude of the displacement is 1 *km*.

2.2.5 Section Summary

- Kinematics is the study of motion without considering its causes. In this chapter, it is limited to motion along a straight line, called one-dimensional motion.
- Displacement is the change in position of an object.
- In symbols, displacement Δx is defined to be ::: {#fs-id2929393 data-type="equation"}

  $$\Delta x = x_f - x_0,$$

  :::

  where x_0 is the initial position and x_f is the final position. In this text, the Greek letter Δ (delta) always means "change in" whatever quantity follows it. The SI unit for displacement is the meter (m). Displacement has a direction as well as a magnitude.
- When you start a problem, assign which direction will be positive.
- Distance is the magnitude of displacement between two positions.
- Distance traveled is the total length of the path traveled between two positions.

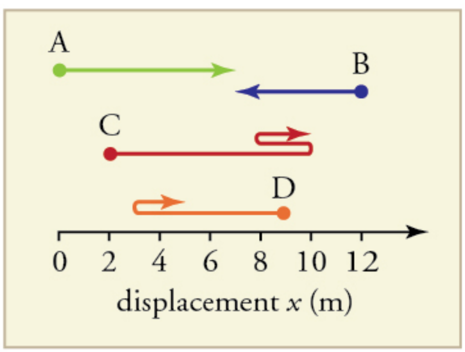
2.2.6 Conceptual Questions

Give an example in which there are clear distinctions among distance traveled, displacement, and magnitude of displacement. Specifically identify each quantity in your example.

Under what circumstances does distance traveled equal magnitude of displacement? What is the only case in which magnitude of displacement and distance are exactly the same?

Bacteria move back and forth by using their flagella (structures that look like little tails). Speeds of up to 50 m/s $\left(50 \times 10^{-6}\ \text{m/s}\right)$ have been observed. The total distance traveled by a bacterium is large for its size, while its displacement is small. Why is this?

2.2.7 Problems & Exercises



Find the following for path A in link: (a) The distance traveled. (b) The magnitude of the displacement from start to finish. (c) The displacement from start to finish.

a 7 m

b 7 m

c +7 *m*

Find the following for path B in link: (a) The distance traveled. (b) The magnitude of the displacement from start to finish. (c) The displacement from start to finish.

Find the following for path C in link: (a) The distance traveled. (b) The magnitude of the displacement from start to finish. (c) The displacement from start to finish.

a 13 m

b 9 m

c $+9\ m$

Find the following for path D in link: (a) The distance traveled. (b) The magnitude of the displacement from start to finish. (c) The displacement from start to finish.

2.2.8 Test Prep for AP Courses

Which of the following statements comparing position, distance traveled, and displacement is correct?

a. An object may record a distance traveled of zero while recording a non-zero displacement.
b. An object may record a non-zero distance traveled while recording a displacement of zero.
c. An object may record a non-zero distance traveled while maintaining a position of zero.
d. An object may record a non-zero displacement while maintaining a position of zero.

a

2.2.9 Glossary

kinematics the study of motion without considering its causes

position the location of an object at a particular time

displacement the change in position of an object

distance the magnitude of displacement between two positions

distance traveled the total length of the path traveled between two positions

2.3 Vectors, Scalars, and Coordinate Systems

2.3.1 Learning Objectives

By the end of this section, you will be able to:

- Define and distinguish between scalar and vector quantities.
- Assign a coordinate system for a scenario involving one-dimensional motion.

The information presented in this section supports the following AP® learning objectives and science practices:

- **3.A.1.2** The student is able to design an experimental investigation of the motion of an object.

Figure 2.5: The motion of this Eclipse Concept jet can be described in terms of the distance it has traveled (a scalar quantity) or its displacement in a specific direction (a vector quantity). In order to specify the direction of motion, its displacement must be described based on a coordinate system. In this case, it may be convenient to choose motion toward the left as positive motion (it is the forward direction for the plane), although in many cases, the x-coordinate runs from left to right, with motion to the right as positive and motion to the left as negative. (credit: Armchair Aviator, Flickr)

What is the difference between distance and displacement? Whereas displacement is defined by both direction and magnitude, distance is defined only by magnitude. Displacement is an example of a vector quantity. Distance is an example of a scalar quantity. A vector is any quantity with both *magnitude and direction.* Other examples of vectors include a velocity of 90 km/h east and a force of 500 newtons straight down.

The direction of a vector in one-dimensional motion is given simply by a plus (+) or minus (−) sign. Vectors are represented graphically by arrows. An arrow used to represent a vector has a length proportional to the vector's magnitude (e.g., the larger the magnitude, the longer the length of the vector) and points in the same direction as the vector.

Some physical quantities, like distance, either have no direction or none is specified. A scalar is any quantity that has a magnitude, but no direction. For example, a 20ºC temperature, the 250 kilocalories (250 Calories) of energy in a candy bar, a 90 km/h speed limit, a person's 1.8 m height, and a distance of 2.0 m are all scalars—quantities with no specified direction. Note, however, that a scalar can be negative, such as a −20ºC temperature. In this case, the minus sign indicates a point on a scale rather than a direction. Scalars are never represented by arrows.

2.3.2 Coordinate Systems for One-Dimensional Motion

In order to describe the direction of a vector quantity, you must designate a coordinate system within the reference frame. For one-dimensional motion, this is a simple coordinate system consisting of a one-dimensional coordinate line. In general, when describing horizontal motion, motion to the right is usually considered positive, and motion to the left is considered negative. With vertical motion, motion up is usually positive and motion down is negative. In some cases, however, as with the jet in link, it can be more convenient to switch the positive and negative directions. For example, if you are analyzing the motion of falling objects, it can be useful to define downwards as the positive direction. If people in a race are running to the left, it is useful to define left as the positive direction. It does not matter as long as the system is clear and consistent. Once you assign a positive direction and start solving a problem, you cannot change it.

Check Your Understanding

A person's speed can stay the same as he or she rounds a corner and changes direction. Given this information, is speed a scalar or a vector quantity? Explain.

Speed is a scalar quantity. It does not change at all with direction changes; therefore, it has magnitude only. If it were a vector quantity, it would change as direction changes (even if its magnitude remained constant).

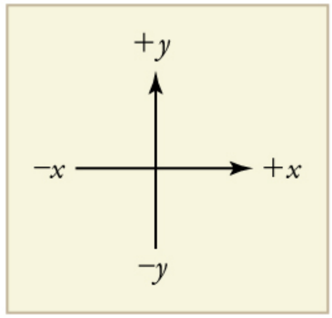


Figure 2.6: It is usually convenient to consider motion upward or to the right as positive (+) and motion downward or to the left as negative (−).

2.3.3 Section Summary

- A vector is any quantity that has magnitude and direction.
- A scalar is any quantity that has magnitude but no direction.
- Displacement and velocity are vectors, whereas distance and speed are scalars.
- In one-dimensional motion, direction is specified by a plus or minus sign to signify left or right, up or down, and the like.

2.3.4 Conceptual Questions

A student writes, "*A bird that is diving for prey has a speed of* $-10\,m/s$." What is wrong with the student's statement? What has the student actually described? Explain.

What is the speed of the bird in link?

Acceleration is the change in velocity over time. Given this information, is acceleration a vector or a scalar quantity? Explain.

A weather forecast states that the temperature is predicted to be $-5C$ the following day. Is this temperature a vector or a scalar quantity? Explain.

Switching Reference Frames

A fundamental tenet of physics is that information about an event can be gathered from a variety of reference frames. For example, imagine that you are a passenger walking toward the front of a bus. As you walk, your motion is observed by a fellow bus passenger and by an observer standing on the sidewalk.

Both the bus passenger and sidewalk observer will be able to collect information about you. They can determine how far you moved and how much time it took you to do so. However, while you moved at a consistent pace, both observers will get different results. To the passenger sitting on the bus, you moved forward at what one would consider a normal pace, something similar to how quickly you would walk outside on a sunny day. To the sidewalk observer though, you will have moved much quicker. Because the bus is also moving forward, the distance you move forward against the sidewalk each second increases, and the sidewalk observer must conclude that you are moving at a greater pace.

To show that you understand this concept, you will need to create an event and think of a way to view this event from two different frames of reference. In order to ensure that the event is being observed simultaneously from both frames, you will need an assistant to help out. An example of a possible event is to have a friend ride on a skateboard while tossing a ball. How will your friend observe the ball toss, and how will those observations be different from your own?

Your task is to describe your event and the observations of your event from both frames of reference. Answer the following questions below to demonstrate your understanding. For assistance, you can review the information given in the 'Position' paragraph at the start of Section 2.1.

1. What is your event? What object are both you and your assistant observing?
2. What do *you* see as the event takes place?
3. What does *your assistant* see as the event takes place?
4. How do your reference frames cause you and your assistant to have two different sets of observations?

2.3.5 Test Prep for AP Courses

A student is trying to determine the acceleration of a feather as she drops it to the ground. If the student is looking to achieve a positive velocity and positive acceleration, what is the *most sensible* way to set up her coordinate system?

a. Her hand should be a coordinate of zero and the upward direction should be considered positive.
b. Her hand should be a coordinate of zero and the downward direction should be considered positive.
c. The floor should be a coordinate of zero and the upward direction should be considered positive.
d. The floor should be a coordinate of zero and the downward direction should be considered positive.

2.3.6 Glossary

scalar a quantity that is described by magnitude, but not direction

vector a quantity that is described by both magnitude and direction

Figure 2.7: The motion of these racing snails can be described by their speeds and their velocities. (credit: tobitasflickr, Flickr)

2.4 Time, Velocity, and Speed

2.4.1 Learning Objectives

By the end of this section, you will be able to:

- Explain the relationships between instantaneous velocity, average velocity, instantaneous speed, average speed, displacement, and time.
- Calculate velocity and speed given initial position, initial time, final position, and final time.
- Derive a graph of velocity vs. time given a graph of position vs. time.
- Interpret a graph of velocity vs. time.

The information presented in this section supports the following AP® learning objectives and science practices:

- **3.A.1.1** The student is able to express the motion of an object using narrative, mathematical, and graphical representations. **(S.P. 1.5, 2.1, 2.2)**
- **3.A.1.3** The student is able to analyze experimental data describing the motion of an object and is able to express the results of the analysis using narrative, mathematical, and graphical representations. **(S.P. 5.1)**

There is more to motion than distance and displacement. Questions such as, "How long does a foot race take?" and "What was the runner's speed?" cannot be answered without an understanding of other concepts. In this section we add definitions of time, velocity, and speed to expand our description of motion.

2.4.2 Time

As discussed in Physical Quantities and Units, the most fundamental physical quantities are defined by how they are measured. This is the case with time. Every measurement of time involves measuring a change in some physical quantity. It may be a number on a digital clock, a heartbeat, or the position of the Sun in the sky. In physics, the definition of time is simple—time is *change*, or the interval over which change occurs. It is impossible to know that time has passed unless something changes.

The amount of time or change is calibrated by comparison with a standard. The SI unit for time is the second, abbreviated s. We might, for example, observe that a certain pendulum makes one full swing every 0.75 s. We could then use the pendulum to measure time by counting its swings or, of course, by connecting the pendulum to a clock mechanism that registers time on a dial. This allows us to not only measure the amount of time, but also to determine a sequence of events.

How does time relate to motion? We are usually interested in elapsed time for a particular motion, such as how long it takes an airplane passenger to get from his seat to the back of the plane. To find elapsed time, we note the time at the beginning and end of the motion and subtract the two. For example, a lecture may start at 11:00 A.M. and end at 11:50 A.M., so that the elapsed time would be 50 min. Elapsed time Δt is the difference between the ending time and beginning time,

$$\Delta t = t_f - t_0,$$

where Δt is the change in time or elapsed time, t_f is the time at the end of the motion, and t_0 is the time at the beginning of the motion. (As usual, the delta symbol, Δ, means the change in the quantity that follows it.)

Life is simpler if the beginning time t_0 is taken to be zero, as when we use a stopwatch. If we were using a stopwatch, it would simply read zero at the start of the lecture and 50 min at the end. If $t_0 = 0$, then $\Delta t = t_f \equiv t$.

In this text, for simplicity's sake,

- motion starts at time equal to zero ($t_0 = 0$)
- the symbol t is used for elapsed time unless otherwise specified ($\Delta t = t_f \equiv t$)

2.4.3 Velocity

Your notion of velocity is probably the same as its scientific definition. You know that if you have a large displacement in a small amount of time you have a large velocity, and that velocity has units of distance divided by time, such as miles per hour or kilometers per hour.

Average Velocity

Average velocity is *displacement (change in position) divided by the time of travel*,

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_0}{t_f - t_0},$$

where $\bar{v}$ is the *average* (indicated by the bar over the v) velocity, Δx is the change in position (or displacement), and x_f and x_0 are the final and beginning positions at times t_f and t_0, respectively. If the starting time t_0 is taken to be zero, then the average velocity is simply

$$\bar{v} = \frac{\Delta x}{t}.$$

Notice that this definition indicates that *velocity is a vector because displacement is a vector.* It has both magnitude and direction. The SI unit for velocity is meters per second or m/s, but many other units, such as km/h, mi/h (also written as mph), and cm/s, are in common use. Suppose, for example, an airplane passenger took 5 seconds to move −4 m (the minus sign indicates that displacement is toward the back of the plane). His average velocity would be

$$\bar{v} = \frac{\Delta x}{t} = \frac{-4 \text{ m}}{5 \text{ s}} = -0.8 \text{ m/s}.$$

The minus sign indicates the average velocity is also toward the rear of the plane.

The average velocity of an object does not tell us anything about what happens to it between the starting point and ending point, however. For example, we cannot tell from average velocity whether the airplane passenger stops momentarily or backs up before he goes to the back of the plane. To get more details, we must consider smaller segments of the trip over smaller time intervals.

The smaller the time intervals considered in a motion, the more detailed the information. When we carry this process to its logical conclusion, we are left with an infinitesimally small interval. Over such an interval, the average velocity becomes the *instantaneous velocity* or the *velocity at a specific instant.* A car's speedometer, for example, shows the magnitude (but not the direction) of the instantaneous velocity of the car. (Police give tickets based on instantaneous

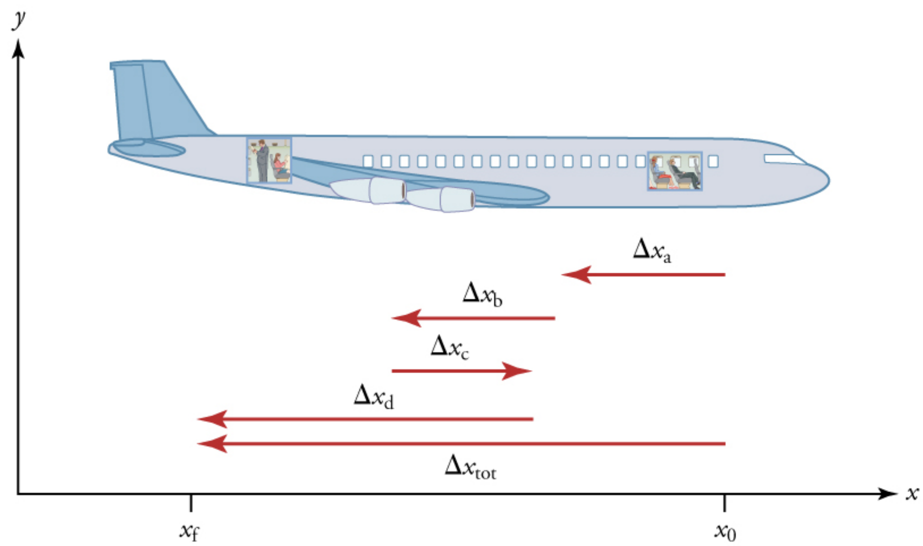


Figure 2.8: A more detailed record of an airplane passenger heading toward the back of the plane, showing smaller segments of his trip.

velocity, but when calculating how long it will take to get from one place to another on a road trip, you need to use average velocity.) Instantaneous velocity v is the average velocity at a specific instant in time (or over an infinitesimally small time interval).

Mathematically, finding instantaneous velocity, v, at a precise instant t can involve taking a limit, a calculus operation beyond the scope of this text. However, under many circumstances, we can find precise values for instantaneous velocity without calculus.

2.4.4 Speed

In everyday language, most people use the terms "speed" and "velocity" interchangeably. In physics, however, they do not have the same meaning and they are distinct concepts. One major difference is that speed has no direction. Thus *speed is a scalar*. Just as we need to distinguish between instantaneous velocity and average velocity, we also need to distinguish between instantaneous speed and average speed.

Instantaneous speed is the magnitude of instantaneous velocity. For example, suppose the airplane passenger at one instant had an instantaneous velocity of −3.0 m/s (the minus meaning toward the rear of the plane). At that same time his instantaneous speed was 3.0 m/s. Or suppose that at one time during a shopping trip your instantaneous velocity is 40 km/h due north. Your instantaneous speed at that instant would be 40 km/h—the same magnitude but without a direction. Average speed, however, is very different from average velocity. Average speed is the distance traveled divided by elapsed time.

We have noted that distance traveled can be greater than displacement. So average speed can be greater than average velocity, which is displacement divided by time. For example, if you drive to a store and return home in half an hour, and your car's odometer shows the total distance traveled was 6 km, then your average speed was 12 km/h. Your average velocity, however, was zero, because your displacement for the round trip is zero. (Displacement is change in position and, thus, is zero for a round trip.) Thus average speed is *not* simply the magnitude of average velocity.

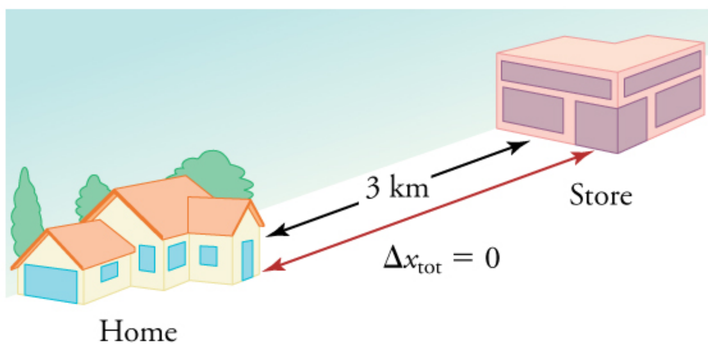


Figure 2.9: During a 30-minute round trip to the store, the total distance traveled is 6 km. The average speed is 12 km/h. The displacement for the round trip is zero, since there was no net change in position. Thus the average velocity is zero.

Another way of visualizing the motion of an object is to use a graph. A plot of position or of velocity as a function of time can be very useful. For example, for this trip to the store, the position, velocity, and speed-vs.-time graphs are displayed in link. (Note that these graphs depict a very simplified model of the trip. We are assuming that speed is constant during the trip, which is unrealistic given that we'll probably stop at the store. But for simplicity's sake, we will model it with no stops or changes in speed. We are also assuming that the route between the store and the house is a perfectly straight line.)

Making Connections: Take-Home Investigation—Getting a Sense of Speed

If you have spent much time driving, you probably have a good sense of speeds between about 10 and 70 miles per hour. But what are these in meters per second? What do we mean when we say that something is moving at 10 m/s? To get a better sense of what these values really mean, do some observations and calculations on your own:

- calculate typical car speeds in meters per second

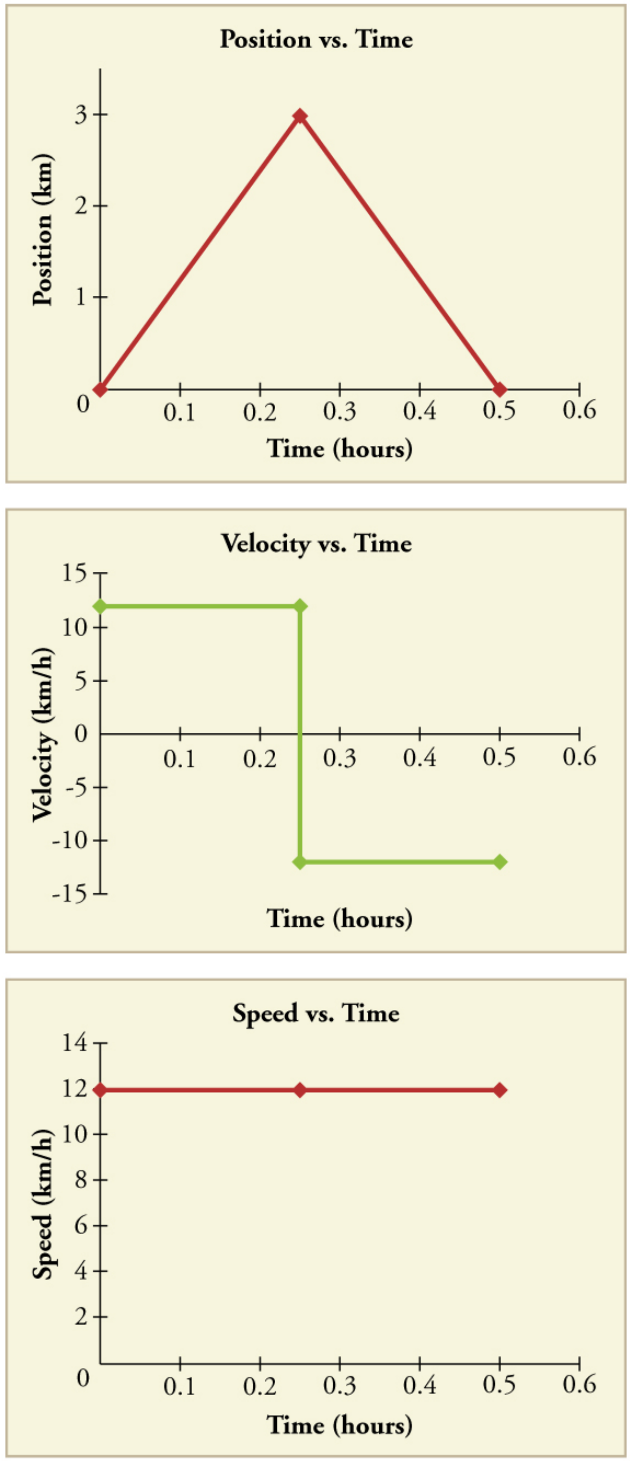


Figure 2.10: Position vs. time, velocity vs. time, and speed vs. time on a trip. Note that the velocity for the return trip is negative.

- estimate jogging and walking speed by timing yourself; convert the measurements into both m/s and mi/h
- determine the speed of an ant, snail, or falling leaf

Check Your Understanding

A commuter train travels from Baltimore to Washington, DC, and back in 1 hour and 45 minutes. The distance between the two stations is approximately 40 miles. What is (a) the average velocity of the train, and (b) the average speed of the train in m/s?

a The average velocity of the train is zero because $x_f = x_0$; the train ends up at the same place it starts.

b The average speed of the train is calculated below. Note that the train travels 40 miles one way and 40 miles back, for a total distance of 80 miles.

$$\frac{\text{distance}}{\text{time}} = \frac{80 \text{ miles}}{105 \text{ minutes}}$$

$$\frac{80 \text{ miles}}{105 \text{ minutes}} \times \frac{5280 \text{ feet}}{1 \text{ mile}} \times \frac{1 \text{ meter}}{3.28 \text{ feet}} \times \frac{1 \text{ minute}}{60 \text{ seconds}} = 20 \text{ m/s}$$

2.4.5 Section Summary

- Time is measured in terms of change, and its SI unit is the second (s). Elapsed time for an event is ::: {#import-auto-id4146177 data-type="equation"}

  $$\Delta t = t_f - t_0,$$

  :::

  where t_f is the final time and t_0 is the initial time. The initial time is often taken to be zero, as if measured with a stopwatch; the elapsed time is then just t.

- Average velocity $\bar{v}$ is defined as displacement divided by the travel time. In symbols, average velocity is ::: {#import-auto-id4086565 data-type="equation"}

  $$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_\mathrm{f} - x_0}{t_\mathrm{f} - t_0}.$$

  :::

- The SI unit for velocity is m/s.

- Velocity is a vector and thus has a direction.

- Instantaneous velocity v is the velocity at a specific instant or the average velocity for an infinitesimal interval.

- Instantaneous speed is the magnitude of the instantaneous velocity.
- Instantaneous speed is a scalar quantity, as it has no direction specified.
- Average speed is the total distance traveled divided by the elapsed time. (Average speed is *not* the magnitude of the average velocity.) Speed is a scalar quantity; it has no direction associated with it.

2.4.6 Conceptual Questions

Give an example (but not one from the text) of a device used to measure time and identify what change in that device indicates a change in time.

There is a distinction between average speed and the magnitude of average velocity. Give an example that illustrates the difference between these two quantities.

Does a car's odometer measure position or displacement? Does its speedometer measure speed or velocity?

If you divide the total distance traveled on a car trip (as determined by the odometer) by the time for the trip, are you calculating the average speed or the magnitude of the average velocity? Under what circumstances are these two quantities the same?

How are instantaneous velocity and instantaneous speed related to one another? How do they differ?

2.4.7 Problems & Exercises

a Calculate Earth's average speed relative to the Sun. (b) What is its average velocity over a period of one year?

a 3.0×10^4 m/s

b 0 m/s

A helicopter blade spins at exactly 100 revolutions per minute. Its tip is 5.00 m from the center of rotation. (a) Calculate the average speed of the blade tip in the helicopter's frame of reference. (b) What is its average velocity over one revolution?

The North American and European continents are moving apart at a rate of about 3 cm/y. At this rate how long will it take them to drift 500 km farther apart than they are at present?

2×10^7 years

Land west of the San Andreas fault in southern California is moving at an average velocity of about 6 cm/y northwest relative to land east of the fault. Los Angeles is west of the fault and may thus someday be at the same latitude as San Francisco, which is east of the fault. How far in the future will this occur if the displacement to be made is 590 km northwest, assuming the motion remains constant?

On May 26, 1934, a streamlined, stainless steel diesel train called the Zephyr set the world's nonstop long-distance speed record for trains. Its run from Denver to Chicago took 13 hours, 4 minutes, 58 seconds, and was witnessed by more than a million people along the route. The total distance traveled was 1633.8 km. What was its average speed in km/h and m/s?

34.689 m/s = 124.88 km/h

Tidal friction is slowing the rotation of the Earth. As a result, the orbit of the Moon is increasing in radius at a rate of approximately 4 cm/year. Assuming this to be a constant rate, how many years will pass before the radius of the Moon's orbit increases by $3.84 \times 10^6\ m$ (1%)?

A student drove to the university from her home and noted that the odometer reading of her car increased by 12.0 km. The trip took 18.0 min. (a) What was her average speed? (b) If the straight-line distance from her home to the university is 10.3 km in a direction 25.0 south of east, what was her average velocity? (c) If she returned home by the same path 7 h 30 min after she left, what were her average speed and velocity for the entire trip?

a 40.0 km/h

b 34.3 km/h, 25º S of E.

c average speed = 3.20 km/h, $\bar{v} = 0$.

The speed of propagation of the action potential (an electrical signal) in a nerve cell depends (inversely) on the diameter of the axon (nerve fiber). If the nerve cell connecting the spinal cord to your feet is 1.1 m long, and the nerve impulse speed is 18 m/s, how long does it take for the nerve signal to travel this distance?

Conversations with astronauts on the lunar surface were characterized by a kind of echo in which the earthbound person's voice was so loud in the astronaut's space helmet that it was picked up by the astronaut's microphone and transmitted back to Earth. It is reasonable to assume that the echo time equals the time necessary for the radio wave to travel from the Earth to the Moon and back (that is, neglecting any time delays in the electronic equipment). Calculate the distance from Earth to the Moon given that the echo time was 2.56 s and that radio waves travel at the speed of light (3.00×10^8 m/s).

384,000 km

A football quarterback runs 15.0 m straight down the playing field in 2.50 s. He is then hit and pushed 3.00 m straight backward in 1.75 s. He breaks the tackle and runs straight forward another 21.0 m in 5.20 s. Calculate his average velocity (a) for each of the three intervals and (b) for the entire motion.

The planetary model of the atom pictures electrons orbiting the atomic nucleus much as planets orbit the Sun. In this model you can view hydrogen, the simplest atom, as having a single electron in a circular orbit 1.06×10^{-10} m in diameter. (a) If the average speed of the electron in this orbit is known to be 2.20×10^{6} m/s, calculate the number of revolutions per second it makes about the nucleus. (b) What is the electron's average velocity?

a 6.61×10^{15} rev/s

b 0 m/s

2.4.8 Test Prep for AP Courses

A group of students has two carts, A and B, with wheels that turn with negligible friction. The two carts travel along a straight horizontal track and eventually collide. Before the collision, cart A travels to the right and cart B is initially at rest. After the collision, the carts stick together.

a. Describe an experimental procedure to determine the velocities of the carts before and after the collision, including all the additional equipment you would need. You may include a labeled diagram of your setup to help in your description. Indicate what measurements you would take and how you would take them. Include enough detail so that another student could carry out your procedure.
b. There will be sources of error in the measurements taken in the experiment both before and after the collision. Which velocity will be more greatly affected by this error: the velocity prior to the collision or the velocity after the collision? Or will both sets of data be affected equally? Justify your answer.

a. Use tape to mark off two distances on the track — one for cart A before the collision and one for the combined carts after the collision. Push cart A to give it an initial speed. Use a stopwatch to measure the time it takes for the cart(s) to cross the marked distances. The speeds are the distances divided by the times.
b. If the measurement errors are of the same magnitude, they will have a greater effect after the collision. The speed of the combined carts will be less than the initial speed of cart A. As a result, these errors will be a greater percentage of the actual velocity value after the collision occurs. (Note: Other arguments could properly be made for 'more error before the collision' and error that 'equally affects both sets of measurement.')

2.4.9 Glossary

average speed distance traveled divided by time during which motion occurs

average velocity displacement divided by time over which displacement occurs

instantaneous velocity velocity at a specific instant, or the average velocity over an infinitesimal time interval

instantaneous speed magnitude of the instantaneous velocity

time change, or the interval over which change occurs

model simplified description that contains only those elements necessary to describe the physics of a physical situation

elapsed time the difference between the ending time and beginning time

2.5 Acceleration

Figure 2.11: A plane decelerates, or slows down, as it comes in for landing in St. Maarten. Its acceleration is opposite in direction to its velocity. (credit: Steve Conry, Flickr)

2.5.1 Learning Objectives

By the end of this section, you will be able to:

- Define and distinguish between instantaneous acceleration and average acceleration.
- Calculate acceleration given initial time, initial velocity, final time, and final velocity.

The information presented in this section supports the following AP® learning objectives and science practices:

- **3.A.1.1** The student is able to express the motion of an object using narrative, mathematical, and graphical representations. **(S.P. 1.5, 2.1, 2.2)**
- **3.A.1.3** The student is able to analyze experimental data describing the motion of an object and is able to express the results of the analysis using narrative, mathematical, and graphical representations. **(S.P. 5.1)**

In everyday conversation, to accelerate means to speed up. The accelerator in a car can in fact cause it to speed up. The greater the acceleration, the greater the change in velocity over a given time. The formal definition of acceleration is consistent with these notions, but more inclusive.

Average Acceleration

Average Acceleration is *the rate at which velocity changes*,

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_0}{t_f - t_0},$$

where $\bar{a}$ is average acceleration, v is velocity, and t is time. (The bar over the a means *average* acceleration.)

Because acceleration is velocity in m/s divided by time in s, the SI units for acceleration are $\mathrm{m/s}^2$, meters per second squared or meters per second per second, which literally means by how many meters per second the velocity changes every second.

Recall that velocity is a vector—it has both magnitude and direction. This means that a change in velocity can be a change in magnitude (or speed), but it can also be a change in *direction*. For example, if a car turns a corner at constant speed, it is accelerating because its direction is changing. The quicker you turn, the greater the acceleration. So there is an acceleration when velocity changes either in magnitude (an increase or decrease in speed) or in direction, or both.

Acceleration as a Vector

Acceleration is a vector in the same direction as the *change* in velocity, Δv. Since velocity is a vector, it can change either in magnitude or in direction. Acceleration is therefore a change in either speed or direction, or both.

Keep in mind that although acceleration is in the direction of the *change* in velocity, it is not always in the direction of *motion*. When an object's acceleration is in the same direction of its motion, the object will speed up. However, when an object's acceleration is opposite to the direction of its motion, the object will slow down. Speeding up and slowing down should not be confused with a positive and negative acceleration. The next two examples should help to make this distinction clear.

Figure 2.12: A subway train in Sao Paulo, Brazil, decelerates as it comes into a station. It is accelerating in a direction opposite to its direction of motion. (credit: Yusuke Kawasaki, Flickr)

Making Connections: Car Motion

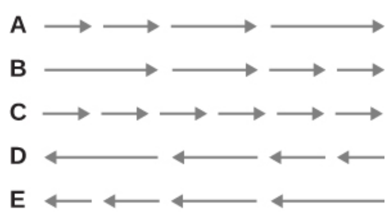


Figure 2.13: Above are arrows representing the motion of five cars (A–E). In all five cases, the positive direction should be considered to the right of the page.

Consider the acceleration and velocity of each car in terms of its direction of travel.

Because the positive direction is considered to the right of the paper, Car A is moving with a positive velocity. Because it is speeding up while moving with a positive velocity, its acceleration is also considered positive.

Because the positive direction is considered to the right of the paper, Car B

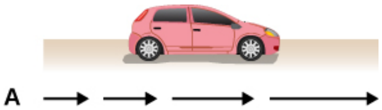


Figure 2.14: Car A is speeding up.

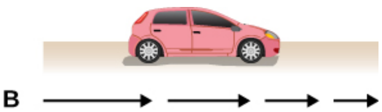


Figure 2.15: Car B is slowing down.

is also moving with a positive velocity. However, because it is slowing down while moving with a positive velocity, its acceleration is considered negative. (This can be viewed in a mathematical manner as well. If the car was originally moving with a velocity of +25 m/s, it is finishing with a speed less than that, like +5 m/s. Because the change in velocity is negative, the acceleration will be as well.)

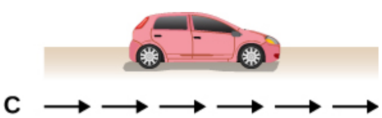


Figure 2.16: Car C has a constant speed.

Because the positive direction is considered to the right of the paper, Car C is moving with a positive velocity. Because all arrows are of the same length, this car is not changing its speed. As a result, its change in velocity is zero, and its acceleration must be zero as well.

Because the car is moving opposite to the positive direction, Car D is moving with a negative velocity. Because it is speeding up while moving in a negative direction, its acceleration is negative as well.

Because it is moving opposite to the positive direction, Car E is moving with a negative velocity as well. However, because it is slowing down while moving in a negative direction, its acceleration is actually positive. As in example B, this may be more easily understood in a mathematical sense. The car is originally moving with a large negative velocity (−25 m/s) but slows to a final velocity that is less negative (−5 m/s). This change in velocity, from −25 m/s to −5 m/s, is actually a positive change ($v_f - v_i = -5$ m/s $- -25$ m/s of 20 m/s. Because the change in velocity is positive, the acceleration must also be positive.

Making Connection - Illustrative Example

The three graphs below are labeled A, B, and C. Each one represents the position

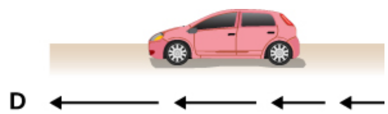


Figure 2.17: Car D is speeding up in the opposite direction of Cars A, B, C.

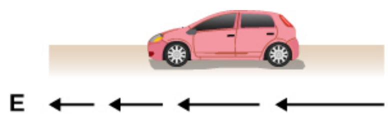


Figure 2.18: Car E is slowing down in the same direction as Car D and opposite of Cars A, B, C.

of a moving object plotted against time.

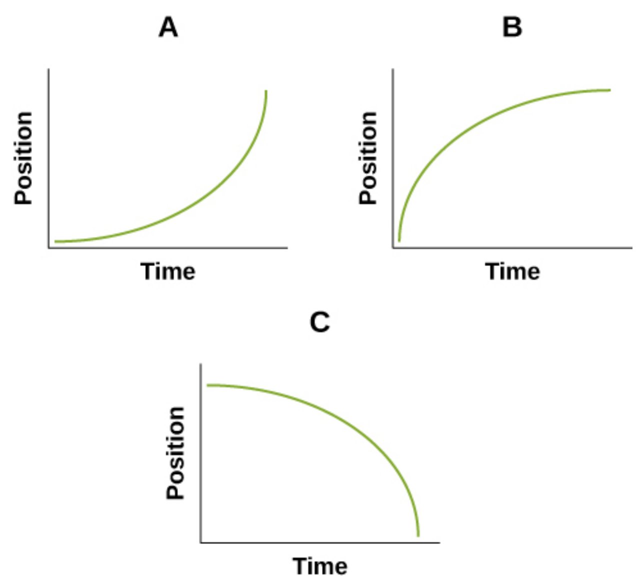


Figure 2.19: Three position and time graphs: A, B, and C.

As we did in the previous example, let's consider the acceleration and velocity of each object in terms of its direction of travel.

Object A is continually increasing its position in the positive direction. As a result, its velocity is considered positive.

During the first portion of time (shaded grey) the position of the object does not change much, resulting in a small positive velocity. During a later portion of time (shaded green) the position of the object changes more, resulting in a

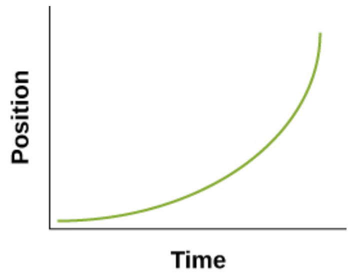


Figure 2.20: Graph A of Position (y axis) vs. Time (x axis).

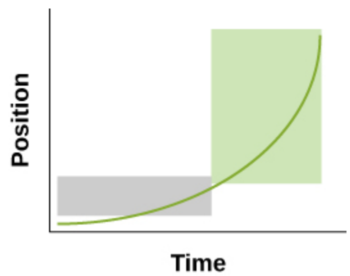


Figure 2.21: Breakdown of Graph A into two separate sections.

larger positive velocity. Because this positive velocity is increasing over time, the acceleration of the object is considered positive.

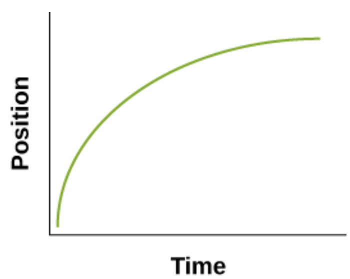


Figure 2.22: Graph B of Position (y axis) vs. Time (x axis).

As in case A, Object B is continually increasing its position in the positive direction. As a result, its velocity is considered positive.

During the first portion of time (shaded grey) the position of the object changes a large amount, resulting in a large positive velocity. During a later portion of time (shaded green) the position of the object does not change as much, resulting in a smaller positive velocity. Because this positive velocity is decreasing over time, the acceleration of the object is considered negative.

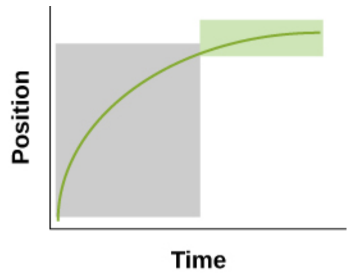


Figure 2.23: Breakdown of Graph B into two separate sections.

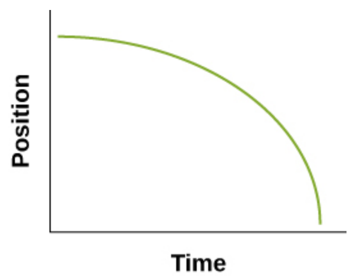


Figure 2.24: Graph C of Position (y axis) vs. Time (x axis).

Object C is continually decreasing its position in the positive direction. As a result, its velocity is considered negative.

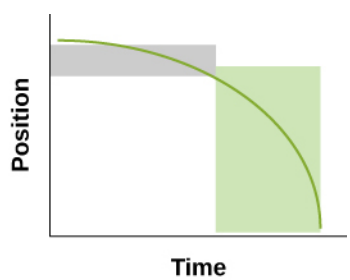


Figure 2.25: Breakdown of Graph C into two separate sections.

During the first portion of time (shaded grey) the position of the object does not change a large amount, resulting in a small negative velocity. During a later portion of time (shaded green) the position of the object changes a much larger amount, resulting in a larger negative velocity. Because the velocity of the object is becoming more negative during the time period, the change in velocity is negative. As a result, the object experiences a negative acceleration.

Calculating Acceleration: A Racehorse Leaves the Gate

A racehorse coming out of the gate accelerates from rest to a velocity of 15.0 m/s due west in 1.80 s. What is its average acceleration?

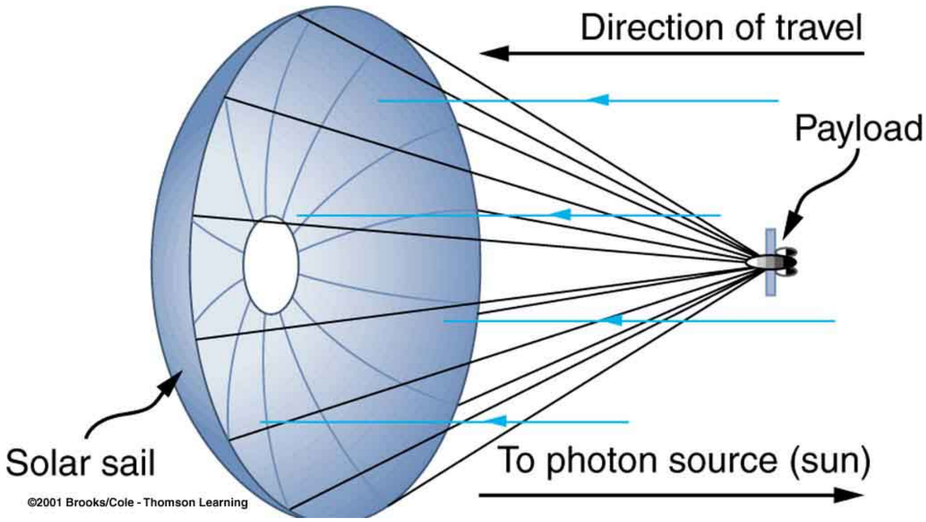
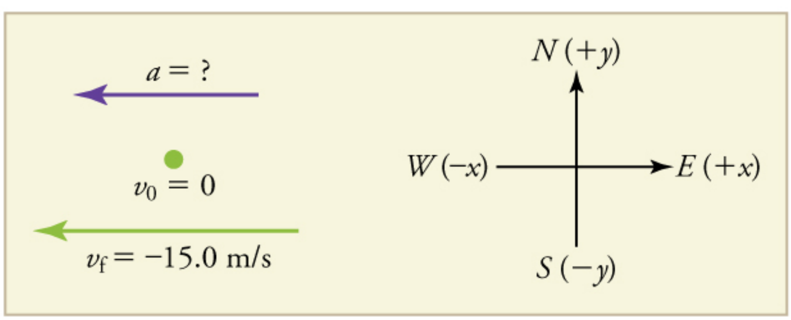


Figure 2.26: (credit: Jon Sullivan, PD Photo.org)

Strategy

First we draw a sketch and assign a coordinate system to the problem. This is a simple problem, but it always helps to visualize it. Notice that we assign east as positive and west as negative. Thus, in this case, we have negative velocity.



We can solve this problem by identifying Δv and Δt from the given information and then calculating the average acceleration directly from the equation $\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_0}{t_f - t_0}$.

Solution

1. Identify the knowns. $v_0 = 0$, $v_f = -15.0$ m/s (the minus sign indicates direction toward the west), $\Delta t = 1.80$ s.

2. Find the change in velocity. Since the horse is going from zero to −15.0 m/s, its change in velocity equals its final velocity: $\Delta v = v_f = -15.0$ m/s.

3. Plug in the known values (Δv and Δt) and solve for the unknown $\bar{a}$.

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{-15.0 \text{ m/s}}{1.80 \text{ s}} = -8.33 \text{ m/s}^2.$$

Discussion

The minus sign for acceleration indicates that acceleration is toward the west. An acceleration of 8.33 m/s^2 due west means that the horse increases its velocity by 8.33 m/s due west each second, that is, 8.33 meters per second per second, which we write as 8.33 m/s^2. This is truly an average acceleration, because the ride is not smooth. We shall see later that an acceleration of this magnitude would require the rider to hang on with a force nearly equal to his weight.

2.5.2 Instantaneous Acceleration

Instantaneous acceleration a, or the *acceleration at a specific instant in time*, is obtained by the same process as discussed for instantaneous velocity in Time, Velocity, and Speed—that is, by considering an infinitesimally small interval of time. How do we find instantaneous acceleration using only algebra? The answer is that we choose an average acceleration that is representative of the motion. link shows graphs of instantaneous acceleration versus time for two very different motions. In link(a), the acceleration varies slightly and the average over the entire interval is nearly the same as the instantaneous acceleration at any time. In this case, we should treat this motion as if it had a constant acceleration equal to the average (in this case about 1.8 m/s^2). In link(b), the acceleration varies drastically over time. In such situations it is best to consider smaller time intervals and choose an average acceleration for each. For example, we could consider motion over the time intervals from 0 to 1.0 s and from 1.0 to 3.0 s as separate motions with accelerations of +3.0 m/s^2 and –2.0 m/s^2, respectively.

The next several examples consider the motion of the subway train shown in link. In (a) the shuttle moves to the right, and in (b) it moves to the left. The examples are designed to further illustrate aspects of motion and to illustrate some of the reasoning that goes into solving problems.

Calculating Displacement: A Subway Train

What are the magnitude and sign of displacements for the motions of the subway train shown in parts (a) and (b) of link?

Strategy

A drawing with a coordinate system is already provided, so we don't need to make a sketch, but we should analyze it to make sure we understand what it

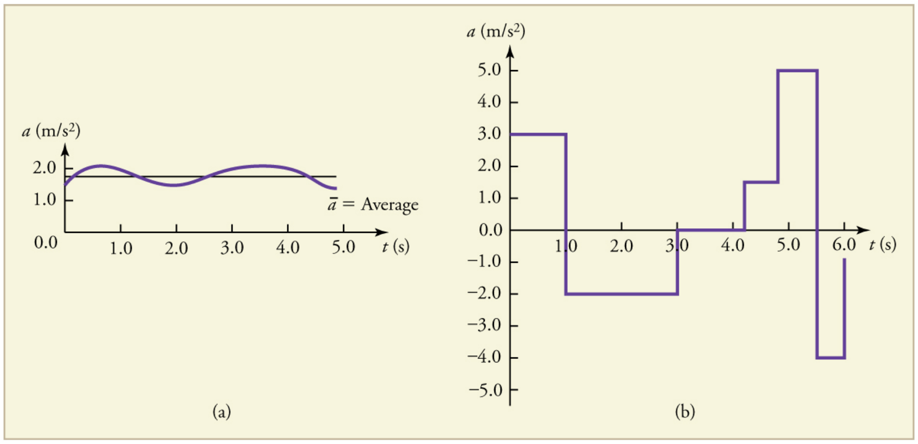


Figure 2.27: Graphs of instantaneous acceleration versus time for two different one-dimensional motions. (a) Here acceleration varies only slightly and is always in the same direction, since it is positive. The average over the interval is nearly the same as the acceleration at any given time. (b) Here the acceleration varies greatly, perhaps representing a package on a post office conveyor belt that is accelerated forward and backward as it bumps along. It is necessary to consider small time intervals (such as from 0 to 1.0 s) with constant or nearly constant acceleration in such a situation.

is showing. Pay particular attention to the coordinate system. To find displacement, we use the equation $\Delta x = x_f - x_0$. This is straightforward since the initial and final positions are given.

Solution

1. Identify the knowns. In the figure we see that x_f =6.70 km and x_0 =4.70 km for part (a), and $x'_f = 3.75$ km and $x'_0 = 5.25$ km for part (b).

2. Solve for displacement in part (a).

$$\Delta x = x_f - x_0 = 6.70 \text{ km} - 4.70 \text{ km} = +2.00 \text{ km}$$

3. Solve for displacement in part (b).

$$\Delta x' = x'_f - x'_0 = 3.75 \text{ km} - 5.25 \text{ km} = -1.50 \text{ km}$$

Discussion

The direction of the motion in (a) is to the right and therefore its displacement has a positive sign, whereas motion in (b) is to the left and thus has a minus

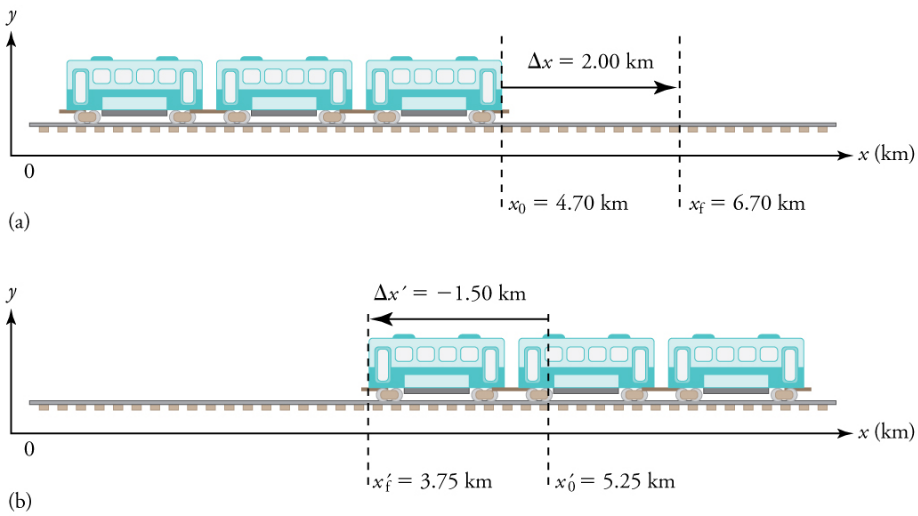


Figure 2.28: One-dimensional motion of a subway train considered in link, link, link, link, link, and link. Here we have chosen the x-axis so that + means to the right and − means to the left for displacements, velocities, and accelerations. (a) The subway train moves to the right from x_0 to x_f. Its displacement Δx is +2.0 km. (b) The train moves to the left from x'_0 to x'_f. Its displacement $\Delta x'$ is −1.5 km. (Note that the prime symbol ( ) is used simply to distinguish between displacement in the two different situations. The distances of travel and the size of the cars are on different scales to fit everything into the diagram.)

sign.

Comparing Distance Traveled with Displacement: A Subway Train

What are the distances traveled for the motions shown in parts (a) and (b) of the subway train in link?

Strategy

To answer this question, think about the definitions of distance and distance traveled, and how they are related to displacement. Distance between two positions is defined to be the magnitude of displacement, which was found in link. Distance traveled is the total length of the path traveled between the two positions. (See Displacement.) In the case of the subway train shown in link, the distance traveled is the same as the distance between the initial and final positions of the train.

Solution

1. The displacement for part (a) was +2.00 km. Therefore, the distance between the initial and final positions was 2.00 km, and the distance traveled was 2.00 km.

2. The displacement for part (b) was −1.5 km. Therefore, the distance between the initial and final positions was 1.50 km, and the distance traveled was 1.50 km.

Discussion

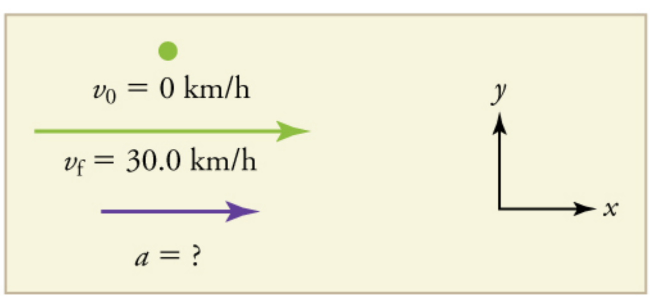
Distance is a scalar. It has magnitude but no sign to indicate direction.

Calculating Acceleration: A Subway Train Speeding Up

Suppose the train in link(a) accelerates from rest to 30.0 km/h in the first 20.0 s of its motion. What is its average acceleration during that time interval?

Strategy

It is worth it at this point to make a simple sketch:



This problem involves three steps. First we must determine the change in velocity, then we must determine the change in time, and finally we use these values to calculate the acceleration.

Solution

1. Identify the knowns. $v_0 = 0$ (the trains starts at rest), $v_f = 30.0$ km/h, and $\Delta t = 20.0$ s.

2. Calculate Δv. Since the train starts from rest, its change in velocity is $\Delta v = +30.0$ km/h, where the plus sign means velocity to the right.

3. Plug in known values and solve for the unknown, $\bar{a}$.

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{+30.0 \text{ km/h}}{20.0 \ s}$$

4. Since the units are mixed (we have both hours and seconds for time), we need to convert everything into SI units of meters and seconds. (See Physical Quantities and Units for more guidance.)

$$\bar{a} = \left(\frac{+30 \text{ km/h}}{20.0 \text{ s}}\right)\left(\frac{10^3 \text{ m}}{1 \text{ km}}\right)\left(\frac{1 \text{ h}}{3600 \text{ s}}\right) = 0.417 \text{ m/s}^2$$

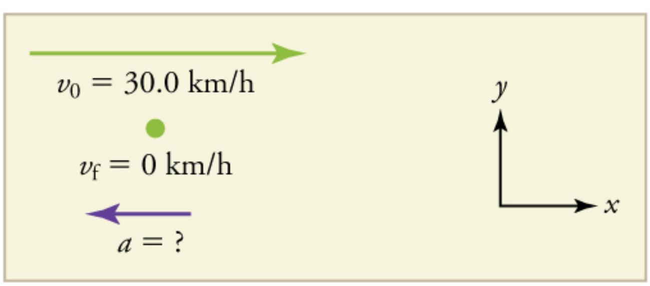
Discussion

The plus sign means that acceleration is to the right. This is reasonable because the train starts from rest and ends up with a velocity to the right (also positive). So acceleration is in the same direction as the *change* in velocity, as is always the case.

Calculate Acceleration: A Subway Train Slowing Down

Now suppose that at the end of its trip, the train in link(a) slows to a stop from a speed of 30.0 km/h in 8.00 s. What is its average acceleration while stopping?

Strategy



In this case, the train is decelerating and its acceleration is negative because it is toward the left. As in the previous example, we must find the change in velocity and the change in time and then solve for acceleration.

Solution

1. Identify the knowns. $v_0 = 30.0$ km/h, $v_f = 0\ km/h$ (the train is stopped, so its velocity is 0), and Δt =8.00 s.

2. Solve for the change in velocity, Δv.

$$\Delta v = v_f - v_0 = 0 - 30.0\ \text{km/h} = -30.0\ \text{km/h}$$

3. Plug in the knowns, Δv and Δt, and solve for $\bar{a}$.

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{-30.0\ \text{km/h}}{8.00\ \text{s}}$$

4. Convert the units to meters and seconds.

$$\bar{a} = \frac{\Delta v}{\Delta t} = \left(\frac{-30.0\ \text{km/h}}{8.00\ \text{s}}\right)\left(\frac{10^3\ \text{m}}{1\ \text{km}}\right)\left(\frac{1\ \text{h}}{3600\ \text{s}}\right) = -1.04\ \text{m/s}^2.$$

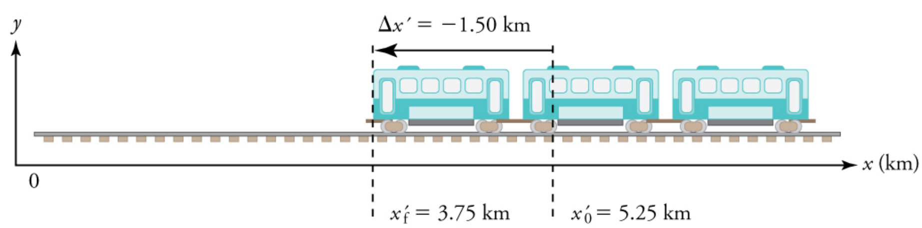
Discussion

The minus sign indicates that acceleration is to the left. This sign is reasonable because the train initially has a positive velocity in this problem, and a negative acceleration would oppose the motion. Again, acceleration is in the same direction as the *change* in velocity, which is negative here. This acceleration can be called a deceleration because it has a direction opposite to the velocity.

The graphs of position, velocity, and acceleration vs. time for the trains in link and link are displayed in link. (We have taken the velocity to remain constant from 20 to 40 s, after which the train decelerates.)

Calculating Average Velocity: The Subway Train

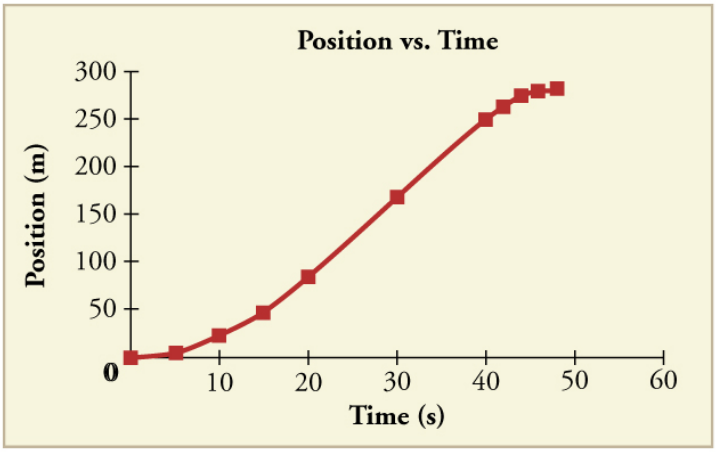
What is the average velocity of the train in part b of link, and shown again below, if it takes 5.00 min to make its trip?

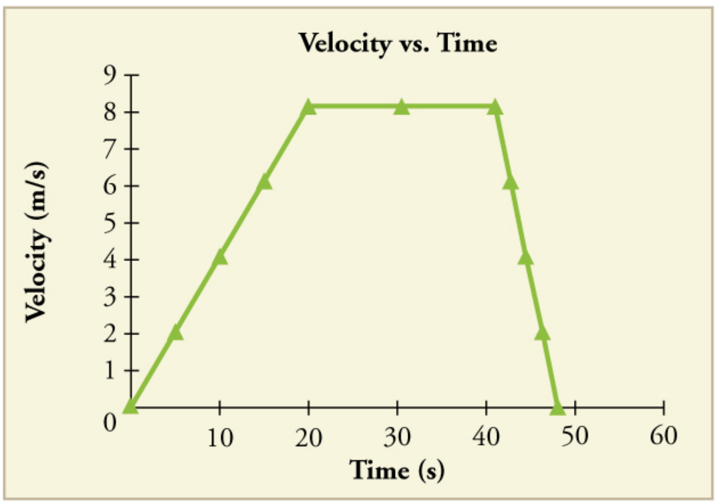


Strategy

Average velocity is displacement divided by time. It will be negative here, since the train moves to the left and has a negative displacement.

Solution





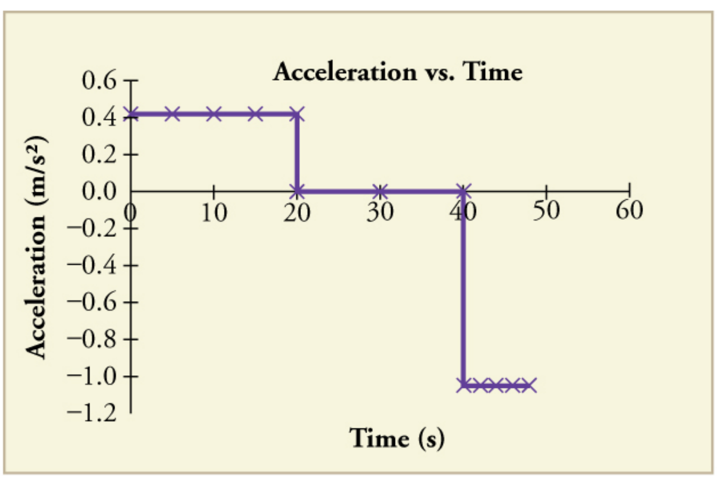


Figure 2.29: (a) Position of the train over time. Notice that the train's position changes slowly at the beginning of the journey, then more and more quickly as it picks up speed. Its position then changes more slowly as it slows down at the end of the journey. In the middle of the journey, while the velocity remains constant, the position changes at a constant rate. (b) Velocity of the train over time. The train's velocity increases as it accelerates at the beginning of the journey. It remains the same in the middle of the journey (where there is no acceleration). It decreases as the train decelerates at the end of the journey. (c) The acceleration of the train over time. The train has positive acceleration as

1. Identify the knowns. $x'_f = 3.75$ km, x'_0 =5.25 km, Δt =5.00 min.
2. Determine displacement, $\Delta x'$. We found $\Delta x'$ to be -1.5 km in link.
3. Solve for average velocity.

$$\bar{v} = \frac{\Delta x'}{\Delta t} = \frac{-1.50 \text{ km}}{5.00 \text{ min}}$$

4. Convert units.

$$\bar{v} = \frac{\Delta x'}{\Delta t} = \left(\frac{-1.50 \text{ km}}{5.00 \text{ min}}\right)\left(\frac{60 \text{ min}}{1\ h}\right) = -18.0 \text{ km/h}$$

Discussion

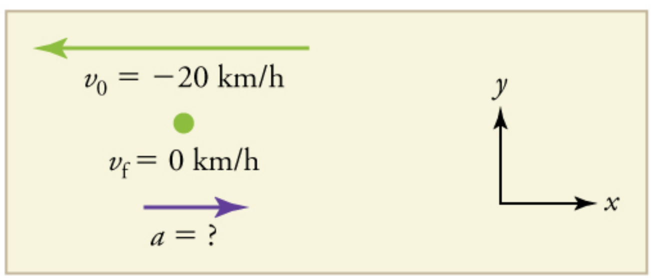
The negative velocity indicates motion to the left.

Calculating Deceleration: The Subway Train

Finally, suppose the train in link slows to a stop from a velocity of 20.0 km/h in 10.0 s. What is its average acceleration?

Strategy

Once again, let's draw a sketch:



As before, we must find the change in velocity and the change in time to calculate average acceleration.

Solution

1. Identify the knowns. $v_0 = -20$ km/h, $v_f = 0\ km/h$, $\Delta t = 10.0\ s$.
2. Calculate Δv. The change in velocity here is actually positive, since

$$\Delta v = v_f - v_0 = 0 - (-20 \text{ km/h}) = +20 \text{ km/h}.$$

3. Solve for $\bar{a}$.

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{+20.0 \text{ km/h}}{10.0 \ s}$$

4. Convert units.

$$\bar{a} = \left(\frac{+20.0 \text{ km/h}}{10.0 \text{ s}}\right)\left(\frac{10^3 \ m}{1 \ km}\right)\left(\frac{1 \ h}{3600 \text{ s}}\right) = +0.556 \text{ m/s}^2$$

Discussion

The plus sign means that acceleration is to the right. This is reasonable because the train initially has a negative velocity (to the left) in this problem and a positive acceleration opposes the motion (and so it is to the right). Again, acceleration is in the same direction as the *change* in velocity, which is positive here. As in link, this acceleration can be called a deceleration since it is in the direction opposite to the velocity.

2.5.3 Sign and Direction

Perhaps the most important thing to note about these examples is the signs of the answers. In our chosen coordinate system, plus means the quantity is to the right and minus means it is to the left. This is easy to imagine for displacement and velocity. But it is a little less obvious for acceleration. Most people interpret negative acceleration as the slowing of an object. This was not the case in link, where a positive acceleration slowed a negative velocity. The crucial distinction was that the acceleration was in the opposite direction from the velocity. In fact, a negative acceleration will *increase* a negative velocity. For example, the train moving to the left in link is sped up by an acceleration to the left. In that case, both v and a are negative. The plus and minus signs give the directions of the accelerations. If acceleration has the same sign as the velocity, the object is speeding up. If acceleration has the opposite sign as the velocity, the object is slowing down.

Check Your Understanding

An airplane lands on a runway traveling east. Describe its acceleration.

If we take east to be positive, then the airplane has negative acceleration, as it is accelerating toward the west. It is also decelerating: its acceleration is opposite in direction to its velocity.

Moving Man Simulation

Learn about position, velocity, and acceleration graphs. Move the little man back and forth with the mouse and plot his motion. Set the position, velocity, or acceleration and let the simulation move the man for you.

2.5.4 Section Summary

- Acceleration is the rate at which velocity changes. In symbols, average acceleration $\bar{a}$ is ::: {#import-auto-id2412659 data-type="equation"}

  $$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_0}{t_f - t_0}.$$

  :::
- The SI unit for acceleration is m/s^2.
- Acceleration is a vector, and thus has a both a magnitude and direction.
- Acceleration can be caused by either a change in the magnitude or the direction of the velocity.
- Instantaneous acceleration a is the acceleration at a specific instant in time.
- Deceleration is an acceleration with a direction opposite to that of the velocity.

2.5.5 Conceptual Questions

Is it possible for speed to be constant while acceleration is not zero? Give an example of such a situation.

Is it possible for velocity to be constant while acceleration is not zero? Explain.

Give an example in which velocity is zero yet acceleration is not.

If a subway train is moving to the left (has a negative velocity) and then comes to a stop, what is the direction of its acceleration? Is the acceleration positive or negative?

Plus and minus signs are used in one-dimensional motion to indicate direction. What is the sign of an acceleration that reduces the magnitude of a negative velocity? Of a positive velocity?

2.5.6 Problems & Exercises

A cheetah can accelerate from rest to a speed of 30.0 m/s in 7.00 s. What is its acceleration?

$4.29\ m/s^2$

Professional Application

Dr. John Paul Stapp was U.S. Air Force officer who studied the effects of extreme deceleration on the human body. On December 10, 1954, Stapp rode a rocket sled, accelerating from rest to a top speed of 282 m/s (1015 km/h) in 5.00 s, and was brought jarringly back to rest in only 1.40 s! Calculate his (a) acceleration and (b) deceleration. Express each in multiples of g (9.80 m/s^2) by taking its ratio to the acceleration of gravity.

A commuter backs her car out of her garage with an acceleration of 1.40 m/s^2. (a) How long does it take her to reach a speed of 2.00 m/s? (b) If she then brakes to a stop in 0.800 s, what is her deceleration?

a 1.43 s

b -2.50 m/s^2

Assume that an intercontinental ballistic missile goes from rest to a suborbital speed of 6.50 km/s in 60.0 s (the actual speed and time are classified). What is its average acceleration in m/s^2 and in multiples of g (9.80 m/s^2)?

2.5.7 Test Prep for AP Courses

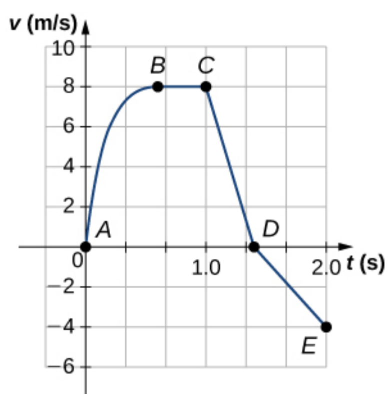


Figure 2.30: Graph showing Velocity vs. Time of a cart.

A cart is constrained to move along a straight line. A varying net force along the direction of motion is exerted on the cart. The cart's velocity v as a function of time t is shown in the graph. The five labeled points divide the graph into four sections.

Which of the following correctly ranks the magnitude of the average acceleration of the cart during the four sections of the graph?

a. $a_{CD} > a_{AB} > a_{BC} > a_{DE}$
b. $a_{BC} > a_{AB} > a_{CD} > a_{DE}$
c. $a_{AB} > a_{BC} > a_{DE} > a_{CD}$
d. $a_{CD} > a_{AB} > a_{DE} > a_{BC}$

Push a book across a table and observe it slow to a stop.

Draw graphs showing the book's position vs. time and velocity vs. time if the direction of its motion is considered positive.

Draw graphs showing the book's position vs. time and velocity vs. time if the direction of its motion is considered negative.

The position vs. time graph should be represented with a positively sloped line whose slope steadily decreases to zero. The y-intercept of the graph may be any value. The line on the velocity vs. time graph should have a positive y-intercept and a negative slope. Because the final velocity of the book is zero, the line should finish on the x-axis.

The position vs. time graph should be represented with a negatively sloped line whose slope steadily decreases to zero. The y-intercept of the graph may be any value. The line on the velocity vs. time graph should have a negative y-intercept and a positive slope. Because the final velocity of the book is zero, the line should finish on the x-axis.]

2.5.8 Glossary

acceleration the rate of change in velocity; the change in velocity over time

average acceleration the change in velocity divided by the time over which it changes

instantaneous acceleration acceleration at a specific point in time

deceleration acceleration in the direction opposite to velocity; acceleration that results in a decrease in velocity

2.6 Motion Equations for Constant Acceleration in One Dimension

2.6.1 Learning Objectives

By the end of this section, you will be able to:

- Calculate displacement of an object that is not accelerating, given initial position and velocity.
- Calculate final velocity of an accelerating object, given initial velocity, acceleration, and time.
- Calculate displacement and final position of an accelerating object, given initial position, initial velocity, time, and acceleration.

The information presented in this section supports the following AP® learning objectives and science practices:

- **3.A.1.1** The student is able to express the motion of an object using narrative, mathematical, or graphical representations. **(S.P. 1.5, 2.1,**

Figure 2.31: Kinematic equations can help us describe and predict the motion of moving objects such as these kayaks racing in Newbury, England. (credit: Barry Skeates, Flickr)

2.2)

- **3.A.1.3** The student is able to analyze experimental data describing the motion of an object and is able to express the results of the analysis using narrative, mathematical, and graphical representations. **(S.P. 5.1)**

We might know that the greater the acceleration of, say, a car moving away from a stop sign, the greater the displacement in a given time. But we have not developed a specific equation that relates acceleration and displacement. In this section, we develop some convenient equations for kinematic relationships, starting from the definitions of displacement, velocity, and acceleration already covered.

2.6.2 Notation: *t*, *x*, *v*, *a*

First, let us make some simplifications in notation. Taking the initial time to be zero, as if time is measured with a stopwatch, is a great simplification. Since elapsed time is $\Delta t = t_f - t_0$, taking $t_0 = 0$ means that $\Delta t = t_f$, the final time on the stopwatch. When initial time is taken to be zero, we use the subscript 0 to denote initial values of position and velocity. That is, x_0 *is the initial position* and v_0 *is the initial velocity.* We put no subscripts on the final values. That is, t *is the final time*, x *is the final position*, and v *is the final velocity.* This gives a simpler expression for elapsed time—now, $\Delta t = t$. It also simplifies the expression for displacement, which is now $\Delta x = x - x_0$. Also, it simplifies the expression for change in velocity, which is now $\Delta v = v - v_0$. To summarize, using the simplified notation, with the initial time taken to be zero,

$$\left.\begin{array}{lcl}\Delta t & = & t \\ \Delta x & = & x - x_0 \\ \Delta v & = & v - v_0\end{array}\right\}$$

where *the subscript 0 denotes an initial value and the absence of a subscript denotes a final value* in whatever motion is under consideration.

We now make the important assumption that *acceleration is constant.* This assumption allows us to avoid using calculus to find instantaneous acceleration. Since acceleration is constant, the average and instantaneous accelerations are equal. That is,

$$\bar{a} = a = \text{constant},$$

so we use the symbol a for acceleration at all times. Assuming acceleration to be constant does not seriously limit the situations we can study nor degrade the accuracy of our treatment. For one thing, acceleration *is* constant in a great number of situations. Furthermore, in many other situations we can accurately describe motion by assuming a constant acceleration equal to the average acceleration for that motion. Finally, in motions where acceleration changes drastically, such as a car accelerating to top speed and then braking to a stop, the motion can be considered in separate parts, each of which has its own constant acceleration.

Solving for Displacement (Δx) and Final Position (x) from Average Velocity when Acceleration (a) is Constant

To get our first two new equations, we start with the definition of average velocity:

$$\bar{v} = \frac{\Delta x}{\Delta t}.$$

Substituting the simplified notation for Δx and Δt yields

$$\bar{v} = \frac{x - x_0}{t}.$$

Solving for x yields

$$x = x_0 + \bar{v}t,$$

where the average velocity is

$$\bar{v} = \frac{v_0 + v}{2} \text{ (constant } a\text{).}$$

The equation $\bar{v} = \frac{v_0+v}{2}$ reflects the fact that, when acceleration is constant, v is just the simple average of the initial and final velocities. For example, if you steadily increase your velocity (that is, with constant acceleration) from 30 to 60 km/h, then your average velocity during this steady increase is 45 km/h. Using the equation $\bar{v} = \frac{v_0+v}{2}$ to check this, we see that

$$\bar{v} = \frac{v_0 + v}{2} = \frac{30 \text{ km/h} + 60 \text{ km/h}}{2} = 45 \text{ km/h},$$

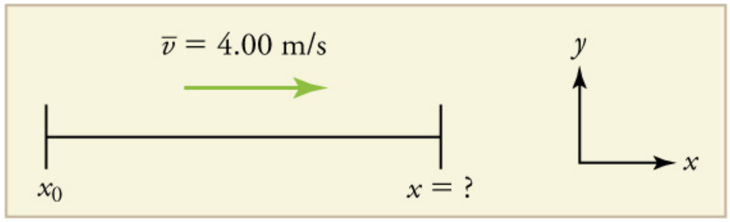
which seems logical.

Calculating Displacement: How Far does the Jogger Run?

A jogger runs down a straight stretch of road with an average velocity of 4.00 m/s for 2.00 min. What is his final position, taking his initial position to be zero?

Strategy

Draw a sketch.



The final position x is given by the equation

$$x = x_0 + \bar{v}t.$$

To find x, we identify the values of x_0, $\bar{v}$, and t from the statement of the problem and substitute them into the equation.

Solution

1. Identify the knowns. $\bar{v} = 4.00$ m/s, $\Delta t = 2.00$ min, and $x_0 = 0$ m.

2. Enter the known values into the equation.

$$x = x_0 + \bar{v}t = 0 + (4.00 \text{ m/s})(120 \text{ s}) = 480 \text{ m}$$

Discussion

Velocity and final displacement are both positive, which means they are in the same direction.

The equation $x = x_0 + \bar{v}t$ gives insight into the relationship between displacement, average velocity, and time. It shows, for example, that displacement is a linear function of average velocity. (By linear function, we mean that displacement depends on $\bar{v}$ rather than on $\bar{v}$ raised to some other power, such as $\bar{v}^2$. When graphed, linear functions look like straight lines with a constant slope.) On a car trip, for example, we will get twice as far in a given time if we average 90 km/h than if we average 45 km/h.

Solving for Final Velocity

We can derive another useful equation by manipulating the definition of acceleration.

$$a = \frac{\Delta v}{\Delta t}$$

Substituting the simplified notation for Δv and Δt gives us

$$a = \frac{v - v_0}{t} \text{ (constant } a\text{)}.$$

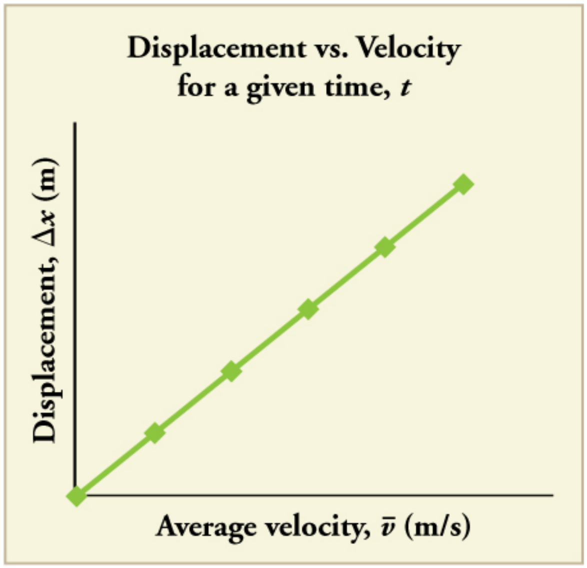


Figure 2.32: There is a linear relationship between displacement and average velocity. For a given time t, an object moving twice as fast as another object will move twice as far as the other object.

Solving for v yields

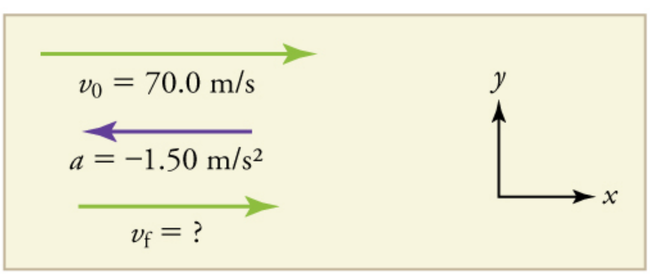
$$v = v_0 + \text{at (constant } a\text{)}.$$

Calculating Final Velocity: An Airplane Slowing Down after Landing

An airplane lands with an initial velocity of 70.0 m/s and then decelerates at 1.50 m/s^2 for 40.0 s. What is its final velocity?

Strategy

Draw a sketch. We draw the acceleration vector in the direction opposite the velocity vector because the plane is decelerating.



Solution

1. Identify the knowns. $v_0 = 70.0$ m/s, $a = -1.50$ m/s^2, $t = 40.0$ s.

2. Identify the unknown. In this case, it is final velocity, $v_{f.}$

3. Determine which equation to use. We can calculate the final velocity using the equation $v = v_0 + \mathrm{at}$.

4. Plug in the known values and solve.

$$v = v_0 + \mathrm{at} = 70.0 \text{ m/s} + \left(-1.50 \text{ m/s}^2\right)(40.0 \text{ s}) = 10.0 \text{ m/s}$$

Discussion

The final velocity is much less than the initial velocity, as desired when slowing down, but still positive. With jet engines, reverse thrust could be maintained long enough to stop the plane and start moving it backward. That would be indicated by a negative final velocity, which is not the case here.

Figure 2.33: The airplane lands with an initial velocity of 70.0 m/s and slows to a final velocity of 10.0 m/s before heading for the terminal. Note that the acceleration is negative because its direction is opposite to its velocity, which is positive.

In addition to being useful in problem solving, the equation $v = v_0 + \mathrm{at}$ gives us insight into the relationships among velocity, acceleration, and time. From it we can see, for example, that

- final velocity depends on how large the acceleration is and how long it lasts
- if the acceleration is zero, then the final velocity equals the initial velocity ($v = v_0$), as expected (i.e., velocity is constant)
- if a is negative, then the final velocity is less than the initial velocity

(All of these observations fit our intuition, and it is always useful to examine basic equations in light of our intuition and experiences to check that they do indeed describe nature accurately.)

Making Connections: Real-World Connection

Figure 2.34: The Space Shuttle *Endeavor* blasts off from the Kennedy Space Center in February 2010. (credit: Matthew Simantov, Flickr)

An intercontinental ballistic missile (ICBM) has a larger average acceleration than the Space Shuttle and achieves a greater velocity in the first minute or two of flight (actual ICBM burn times are classified—short-burn-time missiles are more difficult for an enemy to destroy). But the Space Shuttle obtains a greater final velocity, so that it can orbit the earth rather than come directly back down as an ICBM does. The Space Shuttle does this by accelerating for a longer time.

Solving for Final Position When Velocity is Not Constant ($a \neq 0$)

We can combine the equations above to find a third equation that allows us to calculate the final position of an object experiencing constant acceleration. We start with

$$v = v_0 + \text{at}.$$

Adding v_0 to each side of this equation and dividing by 2 gives

$$\frac{v_0 + v}{2} = v_0 + \frac{1}{2}\text{at}.$$

Since $\frac{v_0+v}{2} = \bar{v}$ for constant acceleration, then

$$\bar{v} = v_0 + \frac{1}{2}\text{at}.$$

Now we substitute this expression for $\bar{v}$ into the equation for displacement, $x = x_0 + \bar{v}t$, yielding

$$x = x_0 + v_0 t + \frac{1}{2}\text{at}^2 \text{ (constant } a\text{)}.$$

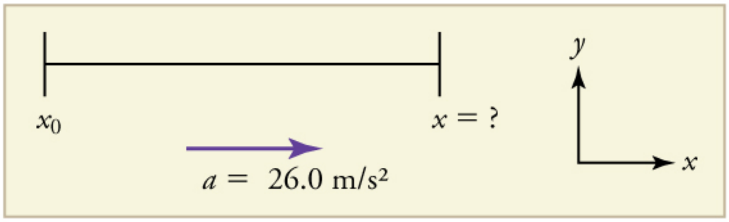
Calculating Displacement of an Accelerating Object: Dragsters

Dragsters can achieve average accelerations of 26.0 m/s^2. Suppose such a dragster accelerates from rest at this rate for 5.56 s. How far does it travel in this time?

Figure 2.35: U.S. Army Top Fuel pilot Tony "The Sarge" Schumacher begins a race with a controlled burnout. (credit: Lt. Col. William Thurmond. Photo Courtesy of U.S. Army.)

Strategy

Draw a sketch.



We are asked to find displacement, which is x if we take x_0 to be zero. (Think about it like the starting line of a race. It can be anywhere, but we call it 0 and measure all other positions relative to it.) We can use the equation $x = x_0 + v_0 t + \frac{1}{2}\text{at}^2$once we identify v_0, a, and t from the statement of the problem.

Solution

1. Identify the knowns. Starting from rest means that $v_0 = 0$, a is given as $26.0\ \text{m/s}^2$ and t is given as 5.56 s.

2. Plug the known values into the equation to solve for the unknown x:

$$x = x_0 + v_0 t + \frac{1}{2}\text{at}^2.$$

Since the initial position and velocity are both zero, this simplifies to

$$x = \frac{1}{2}\text{at}^2.$$

Substituting the identified values of a and t gives

$$x = \frac{1}{2}\left(26.0\ \text{m/s}^2\right)(5.56\ \text{s})^2,$$

yielding

$$x = 402\ \text{m}.$$

Discussion

If we convert 402 m to miles, we find that the distance covered is very close to one quarter of a mile, the standard distance for drag racing. So the answer is reasonable. This is an impressive displacement in only 5.56 s, but top-notch dragsters can do a quarter mile in even less time than this.

What else can we learn by examining the equation $x = x_0 + v_0 t + \frac{1}{2}\text{at}^2$? We see that:

- displacement depends on the square of the elapsed time when acceleration is not zero. In link, the dragster covers only one fourth of the total distance in the first half of the elapsed time
- if acceleration is zero, then the initial velocity equals average velocity ($v_0 = \bar{v}$) and $x = x_0 + v_0 t + \frac{1}{2}\mathrm{at}^2$ becomes $x = x_0 + v_0 t$

Solving for Final Velocity when Velocity Is Not Constant ($a \neq 0$)

A fourth useful equation can be obtained from another algebraic manipulation of previous equations.

If we solve $v = v_0 + \mathrm{at}$ for t, we get

$$t = \frac{v - v_0}{a}.$$

Substituting this and $\bar{v} = \frac{v_0 + v}{2}$ into $x = x_0 + \bar{v}t$, we get

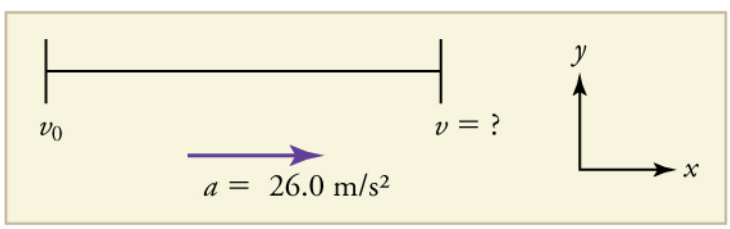
$$v^2 = v_0^2 + 2a\,(x - x_0)\ \ (\mathrm{constant}a).$$

Calculating Final Velocity: Dragsters

Calculate the final velocity of the dragster in link without using information about time.

Strategy

Draw a sketch.



The equation $v^2 = v_0^2 + 2a(x - x_0)$ is ideally suited to this task because it relates velocities, acceleration, and displacement, and no time information is required.

Solution

1. Identify the known values. We know that $v_0 = 0$, since the dragster starts from rest. Then we note that $x - x_0 = 402$ m (this was the answer in link). Finally, the average acceleration was given to be $a = 26.0\ \mathrm{m/s}^2$.

2. Plug the knowns into the equation $v^2 = v_0^2 + 2a(x - x_0)$ and solve for v.

$$v^2 = 0 + 2\left(26.0\ \mathrm{m/s}^2\right)(402\ \mathrm{m}).$$

Thus

$$v^2 = 2.09 \times 10^4 \ \mathrm{m}^2/\mathrm{s}^2.$$

To get v, we take the square root:

$$v = \sqrt{2.09 \times 10^4 \ \mathrm{m}^2/\mathrm{s}^2} = 145 \ \mathrm{m/s}.$$

Discussion

145 m/s is about 522 km/h or about 324 mi/h, but even this breakneck speed is short of the record for the quarter mile. Also, note that a square root has two values; we took the positive value to indicate a velocity in the same direction as the acceleration.

An examination of the equation $v^2 = v_0^2 + 2a(x - x_0)$ can produce further insights into the general relationships among physical quantities:

- The final velocity depends on how large the acceleration is and the distance over which it acts
- For a fixed deceleration, a car that is going twice as fast doesn't simply stop in twice the distance—it takes much further to stop. (This is why we have reduced speed zones near schools.)

2.6.3 Putting Equations Together

In the following examples, we further explore one-dimensional motion, but in situations requiring slightly more algebraic manipulation. The examples also give insight into problem-solving techniques. The box below provides easy reference to the equations needed.

Summary of Kinematic Equations (constant a)

$$x = x_0 + \bar{v}t$$

$$\bar{v} = \frac{v_0 + v}{2}$$

$$v = v_0 + \mathrm{at}$$

$$x = x_0 + v_0 t + \frac{1}{2}\mathrm{at}^2$$

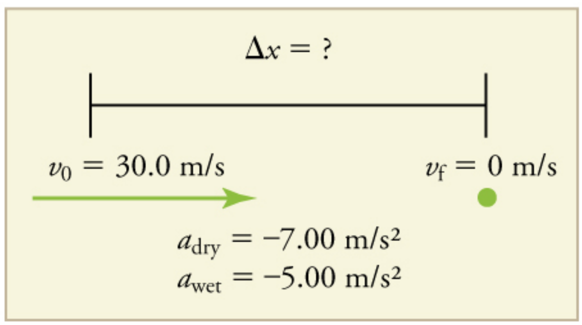
$$v^2 = v_0^2 + 2a\left(x - x_0\right)$$

Calculating Displacement: How Far Does a Car Go When Coming to a Halt?

On dry concrete, a car can decelerate at a rate of 7.00 m/s^2, whereas on wet concrete it can decelerate at only 5.00 m/s^2. Find the distances necessary to stop a car moving at 30.0 m/s (about 110 km/h) (a) on dry concrete and (b) on wet concrete. (c) Repeat both calculations, finding the displacement from the point where the driver sees a traffic light turn red, taking into account his reaction time of 0.500 s to get his foot on the brake.

Strategy

Draw a sketch.



In order to determine which equations are best to use, we need to list all of the known values and identify exactly what we need to solve for. We shall do this explicitly in the next several examples, using tables to set them off.

Solution for (a)

1. Identify the knowns and what we want to solve for. We know that $v_0 = 30.0$ m/s; $v = 0$; $a = -7.00$ m/s^2 (a is negative because it is in a direction opposite to velocity). We take x_0 to be 0. We are looking for displacement Δx, or $x - x_0$.

2. Identify the equation that will help up solve the problem. The best equation to use is

$$v^2 = v_0^2 + 2a\,(x - x_0).$$

This equation is best because it includes only one unknown, x. We know the values of all the other variables in this equation. (There are other equations that would allow us to solve for x, but they require us to know the stopping time, t, which we do not know. We could use them but it would entail additional calculations.)

3. Rearrange the equation to solve for x.

$$x - x_0 = \frac{v^2 - v_0^2}{2a}$$

4. Enter known values.

$$x - 0 = \frac{0^2 - (30.0 \text{ m/s})^2}{2\left(-7.00 \text{ m/s}^2\right)}$$

Thus,

$$x = 64.3 \text{ m on dry concrete.}$$

Solution for (b)

This part can be solved in exactly the same manner as Part A. The only difference is that the deceleration is -5.00 m/s^2. The result is

$$x_{\text{wet}} = 90.0 \text{ m on wet concrete.}$$

Solution for (c)

Once the driver reacts, the stopping distance is the same as it is in Parts A and B for dry and wet concrete. So to answer this question, we need to calculate how far the car travels during the reaction time, and then add that to the stopping time. It is reasonable to assume that the velocity remains constant during the driver's reaction time.

1. Identify the knowns and what we want to solve for. We know that $\bar{v} = 30.0$ m/s; $t_{\text{reaction}} = 0.500\,\text{s}$; $a_{\text{reaction}} = 0$. We take $x_{0-\text{reaction}}$ to be 0. We are looking for x_{reaction}.

2. Identify the best equation to use.

$x = x_0 + \bar{v}t$ works well because the only unknown value is x, which is what we want to solve for.

3. Plug in the knowns to solve the equation.

$$x = 0 + (30.0 \text{ m/s})(0.500 \text{ s}) = 15.0 \text{ m.}$$

This means the car travels 15.0 m while the driver reacts, making the total displacements in the two cases of dry and wet concrete 15.0 m greater than if he reacted instantly.

4. Add the displacement during the reaction time to the displacement when braking.

$$x_{\text{braking}} + x_{\text{reaction}} = x_{\text{total}}$$

a. 64.3 m + 15.0 m = 79.3 m when dry
b. 90.0 m + 15.0 m = 105 m when wet

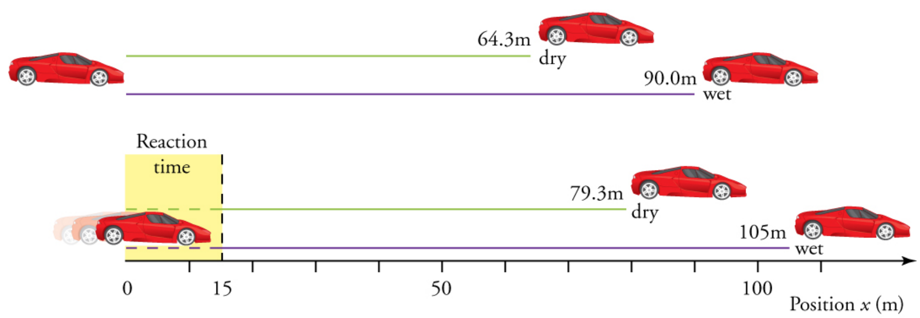


Figure 2.36: The distance necessary to stop a car varies greatly, depending on road conditions and driver reaction time. Shown here are the braking distances for dry and wet pavement, as calculated in this example, for a car initially traveling at 30.0 m/s. Also shown are the total distances traveled from the point where the driver first sees a light turn red, assuming a 0.500 s reaction time.

Discussion

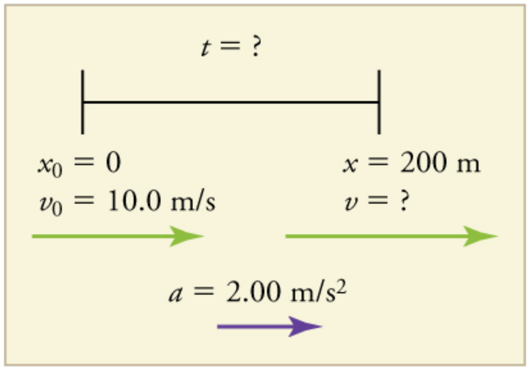
The displacements found in this example seem reasonable for stopping a fast-moving car. It should take longer to stop a car on wet rather than dry pavement. It is interesting that reaction time adds significantly to the displacements. But more important is the general approach to solving problems. We identify the knowns and the quantities to be determined and then find an appropriate equation. There is often more than one way to solve a problem. The various parts of this example can in fact be solved by other methods, but the solutions presented above are the shortest.

Calculating Time: A Car Merges into Traffic

Suppose a car merges into freeway traffic on a 200-m-long ramp. If its initial velocity is 10.0 m/s and it accelerates at 2.00 m/s^2, how long does it take to travel the 200 m up the ramp? (Such information might be useful to a traffic engineer.)

Strategy

Draw a sketch.



We are asked to solve for the time t. As before, we identify the known quantities in order to choose a convenient physical relationship (that is, an equation with one unknown, t).

Solution

1. Identify the knowns and what we want to solve for. We know that $v_0 = 10$ m/s; $a = 2.00\ \text{m/s}^2$; and $x = 200$ m.

2. We need to solve for t. Choose the best equation. $x = x_0 + v_0 t + \frac{1}{2}\text{at}^2$ works best because the only unknown in the equation is the variable t for which we need to solve.

3. We will need to rearrange the equation to solve for t. In this case, it will be easier to plug in the knowns first.

$$200\ \text{m} = 0\ \text{m} + (10.0\ \text{m/s})t + \frac{1}{2}\left(2.00\ \text{m/s}^2\right) t^2$$

4. Simplify the equation. The units of meters (m) cancel because they are in each term. We can get the units of seconds (s) to cancel by taking $t = t$ s, where t is the magnitude of time and s is the unit. Doing so leaves

$$200 = 10t + t^2.$$

5. Use the quadratic formula to solve for t.

a Rearrange the equation to get 0 on one side of the equation.

$$t^2 + 10t - 200 = 0$$

This is a quadratic equation of the form

$$\text{at}^2 + \text{bt} + c = 0,$$

where the constants are $a = 1.00$, $b = 10.0$, and $c = -200$.

b Its solutions are given by the quadratic formula:

$$t = \frac{-b \pm \sqrt{b^2 - 4\mathrm{ac}}}{2a}.$$

This yields two solutions for t, which are

$$t = 10.0 \text{ and} -20.0.$$

In this case, then, the time is $t = t$ in seconds, or

$$t = 10.0 \text{ s and} - 20.0 \text{ s}.$$

A negative value for time is unreasonable, since it would mean that the event happened 20 s before the motion began. We can discard that solution. Thus,

$$t = 10.0 \text{ s}.$$

Discussion

Whenever an equation contains an unknown squared, there will be two solutions. In some problems both solutions are meaningful, but in others, such as the above, only one solution is reasonable. The 10.0 s answer seems reasonable for a typical freeway on-ramp.

With the basics of kinematics established, we can go on to many other interesting examples and applications. In the process of developing kinematics, we have also glimpsed a general approach to problem solving that produces both correct answers and insights into physical relationships. Problem-Solving Basics discusses problem-solving basics and outlines an approach that will help you succeed in this invaluable task.

Making Connections: Take-Home Experiment—Breaking News

We have been using SI units of meters per second squared to describe some examples of acceleration or deceleration of cars, runners, and trains. To achieve a better feel for these numbers, one can measure the braking deceleration of a car doing a slow (and safe) stop. Recall that, for average acceleration, $\bar{a} = \Delta v / \Delta t$. While traveling in a car, slowly apply the brakes as you come up to a stop sign. Have a passenger note the initial speed in miles per hour and the time taken (in seconds) to stop. From this, calculate the deceleration in miles per hour per second. Convert this to meters per second squared and compare with other decelerations mentioned in this chapter. Calculate the distance traveled in braking.

Check Your Understanding

A manned rocket accelerates at a rate of 20 m/s^2 during launch. How long does it take the rocket reach a velocity of 400 m/s?

To answer this, choose an equation that allows you to solve for time t, given only a, v_0, and v.

$$v = v_0 + \text{at}$$

Rearrange to solve for t.

t = v − v 0 a = 400 m/s − 0 m/s 20 m/s 2 = 20 s t = v − v 0 a = 400 m/s − 0 m/s 20 m/s 2 = 20 s size 12{t= { {v - v”” lSub { size 8{0} } } over {a} } = { {”400 m/s” - ”0 m/s”} over {”20 m/s” rSup { size 8{2} } } } =”20 s”} {}

2.6.4 Section Summary

- To simplify calculations we take acceleration to be constant, so that $\bar{a} = a$ at all times.
- We also take initial time to be zero.
- Initial position and velocity are given a subscript 0; final values have no subscript. Thus, ::: {#import-auto-id2168257 data-type="equation"}

$$\left.\begin{array}{ccc}\Delta t & = & t \\ \Delta x & = & x - x_0 \\ \Delta v & = & v - v_0\end{array}\right\}$$

:::

- The following kinematic equations for motion with constant a are useful: ::: {#import-auto-id2175197 data-type="equation"}

$$x = x_0 + \bar{v}t$$

:::

$$\bar{v} = \frac{v_0 + v}{2}$$

$$v = v_0 + \text{at}$$

$$x = x_0 + v_0 t + \frac{1}{2}\text{at}^2$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

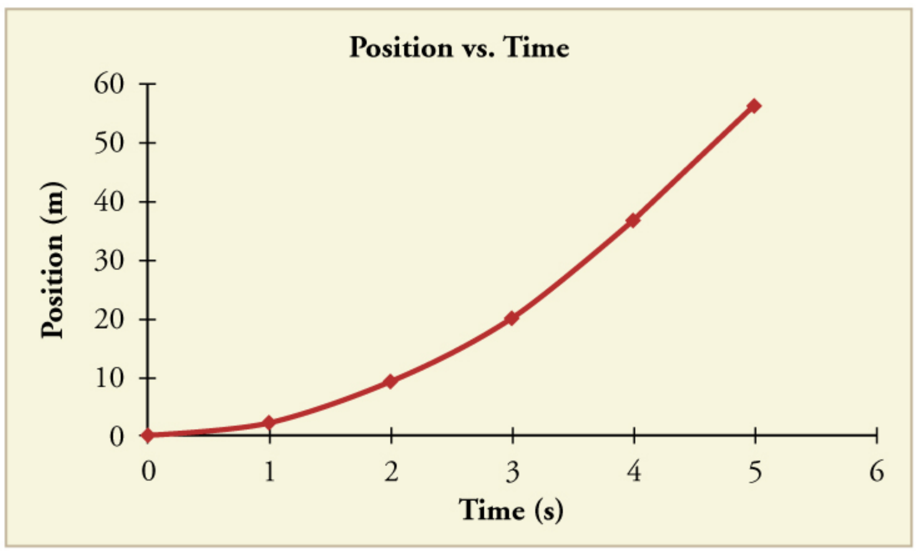
- In vertical motion, y is substituted for x.

2.6.5 Problems & Exercises

An Olympic-class sprinter starts a race with an acceleration of 4.50 m/s^2. (a) What is her speed 2.40 s later? (b) Sketch a graph of her position vs. time for this period.

a 10.8 m/s

b



A well-thrown ball is caught in a well-padded mitt. If the deceleration of the ball is 2.10×10^4 m/s^2, and 1.85 ms (1 ms $= 10^{-3}$ s) elapses from the time the ball first touches the mitt until it stops, what was the initial velocity of the ball?

38.9 m/s (about 87 miles per hour)

A bullet in a gun is accelerated from the firing chamber to the end of the barrel at an average rate of 6.20×10^5 m/s^2 for 8.10×10^{-4} s. What is its muzzle velocity (that is, its final velocity)?

a A light-rail commuter train accelerates at a rate of 1.35 m/s^2. How long does it take to reach its top speed of 80.0 km/h, starting from rest? (b) The same train ordinarily decelerates at a rate of 1.65 m/s^2. How long does it take to come to a stop from its top speed? (c) In emergencies the train can decelerate more rapidly, coming to rest from 80.0 km/h in 8.30 s. What is its emergency deceleration in m/s^2?

a 16.5 s

b 13.5 s

c $-2.68\ m/s^2$

While entering a freeway, a car accelerates from rest at a rate of $2.40\ m/s^2$ for 12.0 s. (a) Draw a sketch of the situation. (b) List the knowns in this problem. (c) How far does the car travel in those 12.0 s? To solve this part, first identify the unknown, and then discuss how you chose the appropriate equation to solve for it. After choosing the equation, show your steps in solving for the unknown, check your units, and discuss whether the answer is reasonable. (d) What is the car's final velocity? Solve for this unknown in the same manner as in part (c), showing all steps explicitly.

At the end of a race, a runner decelerates from a velocity of 9.00 m/s at a rate of $2.00\ m/s^2$. (a) How far does she travel in the next 5.00 s? (b) What is her final velocity? (c) Evaluate the result. Does it make sense?

a 20.0 m

b -1.00 m/s

c This result does not really make sense. If the runner starts at 9.00 m/s and decelerates at $2.00\ m/s^2$, then she will have stopped after 4.50 s. If she continues to decelerate, she will be running backwards.

Professional Application:

Blood is accelerated from rest to 30.0 cm/s in a distance of 1.80 cm by the left ventricle of the heart. (a) Make a sketch of the situation. (b) List the knowns in this problem. (c) How long does the acceleration take? To solve this part, first identify the unknown, and then discuss how you chose the appropriate equation to solve for it. After choosing the equation, show your steps in solving for the unknown, checking your units. (d) Is the answer reasonable when compared with the time for a heartbeat?

In a slap shot, a hockey player accelerates the puck from a velocity of 8.00 m/s to 40.0 m/s in the same direction. If this shot takes 3.33×10^{-2} s, calculate the distance over which the puck accelerates.

0.799 m

A powerful motorcycle can accelerate from rest to 26.8 m/s (100 km/h) in only 3.90 s. (a) What is its average acceleration? (b) How far does it travel in that time?

Freight trains can produce only relatively small accelerations and decelerations. (a) What is the final velocity of a freight train that accelerates at a rate of

$0.0500\ \mathrm{m/s}^2$ for 8.00 min, starting with an initial velocity of 4.00 m/s? (b) If the train can slow down at a rate of $0.550\ \mathrm{m/s}^2$, how long will it take to come to a stop from this velocity? (c) How far will it travel in each case?

a 28.0 m/s

b 50.9 s

c 7.68 km to accelerate and 713 m to decelerate

A fireworks shell is accelerated from rest to a velocity of 65.0 m/s over a distance of 0.250 m. (a) How long did the acceleration last? (b) Calculate the acceleration.

A swan on a lake gets airborne by flapping its wings and running on top of the water. (a) If the swan must reach a velocity of 6.00 m/s to take off and it accelerates from rest at an average rate of $0.350\ \mathrm{m/s}^2$, how far will it travel before becoming airborne? (b) How long does this take?

a 51.4 m

b 17.1 s

Professional Application:

A woodpecker's brain is specially protected from large decelerations by tendon-like attachments inside the skull. While pecking on a tree, the woodpecker's head comes to a stop from an initial velocity of 0.600 m/s in a distance of only 2.00 mm. (a) Find the acceleration in $\mathrm{m/s}^2$ and in multiples of g $\left(g = 9.80\ \mathrm{m/s}^2\right)$. (b) Calculate the stopping time. (c) The tendons cradling the brain stretch, making its stopping distance 4.50 mm (greater than the head and, hence, less deceleration of the brain). What is the brain's deceleration, expressed in multiples of g?

An unwary football player collides with a padded goalpost while running at a velocity of 7.50 m/s and comes to a full stop after compressing the padding and his body 0.350 m. (a) What is his deceleration? (b) How long does the collision last?

a $-80.4\ \mathrm{m/s}^2$

b 9.33×10^{-2} s

In World War II, there were several reported cases of airmen who jumped from their flaming airplanes with no parachute to escape certain death. Some fell about 20,000 feet (6000 m), and some of them survived, with few life-threatening injuries. For these lucky pilots, the tree branches and snow drifts on the ground allowed their deceleration to be relatively small. If we assume that a pilot's speed upon impact was 123 mph (54 m/s), then what was his deceleration? Assume that the trees and snow stopped him over a distance of 3.0 m.

Consider a grey squirrel falling out of a tree to the ground. (a) If we ignore air resistance in this case (only for the sake of this problem), determine a squirrel's velocity just before hitting the ground, assuming it fell from a height of 3.0 m. (b) If the squirrel stops in a distance of 2.0 cm through bending its limbs, compare its deceleration with that of the airman in the previous problem.

a 7.7 m/s

b -15×10^2 m/s^2. This is about 3 times the deceleration of the pilots, who were falling from thousands of meters high!

An express train passes through a station. It enters with an initial velocity of 22.0 m/s and decelerates at a rate of 0.150 m/s^2 as it goes through. The station is 210 m long. (a) How long is the nose of the train in the station? (b) How fast is it going when the nose leaves the station? (c) If the train is 130 m long, when does the end of the train leave the station? (d) What is the velocity of the end of the train as it leaves?

Dragsters can actually reach a top speed of 145 m/s in only 4.45 s—considerably less time than given in link and link. (a) Calculate the average acceleration for such a dragster. (b) Find the final velocity of this dragster starting from rest and accelerating at the rate found in (a) for 402 m (a quarter mile) without using any information on time. (c) Why is the final velocity greater than that used to find the average acceleration? *Hint*: Consider whether the assumption of constant acceleration is valid for a dragster. If not, discuss whether the acceleration would be greater at the beginning or end of the run and what effect that would have on the final velocity.

a 32.6 m/s^2

b 162 m/s

c $v > v_{\text{max}}$, because the assumption of constant acceleration is not valid for a dragster. A dragster changes gears, and would have a greater acceleration in first gear than second gear than third gear, etc. The acceleration would be greatest at the beginning, so it would not be accelerating at 32.6 m/s^2 during the last few meters, but substantially less, and the final velocity would be less than 162 m/s.

A bicycle racer sprints at the end of a race to clinch a victory. The racer has an initial velocity of 11.5 m/s and accelerates at the rate of 0.500 m/s^2 for 7.00 s. (a) What is his final velocity? (b) The racer continues at this velocity to the finish line. If he was 300 m from the finish line when he started to accelerate, how much time did he save? (c) One other racer was 5.00 m ahead when the winner started to accelerate, but he was unable to accelerate, and traveled at 11.8 m/s until the finish line. How far ahead of him (in meters and in seconds) did the winner finish?

In 1967, New Zealander Burt Munro set the world record for an Indian motorcycle, on the Bonneville Salt Flats in Utah, with a maximum speed of 183.58 mi/h.

The one-way course was 5.00 mi long. Acceleration rates are often described by the time it takes to reach 60.0 mi/h from rest. If this time was 4.00 s, and Burt accelerated at this rate until he reached his maximum speed, how long did it take Burt to complete the course?

104 s

a A world record was set for the men's 100-m dash in the 2008 Olympic Games in Beijing by Usain Bolt of Jamaica. Bolt "coasted" across the finish line with a time of 9.69 s. If we assume that Bolt accelerated for 3.00 s to reach his maximum speed, and maintained that speed for the rest of the race, calculate his maximum speed and his acceleration. (b) During the same Olympics, Bolt also set the world record in the 200-m dash with a time of 19.30 s. Using the same assumptions as for the 100-m dash, what was his maximum speed for this race?

a $v = 12.2$ m/s; $a = 4.07$ m/s^2

b $v = 11.2$ m/s

2.6.6 Test Prep for AP Courses

A group of students is attempting to determine the average acceleration of a marble released from the top of a long ramp. Below is a set of data representing the marble's position with respect to time.

Position (cm)	Time (s)
0.0	0.0
0.3	0.5
1.25	1.0
2.8	1.5
5.0	2.0
7.75	2.5
11.3	3.0

Use the data table above to construct a graph determining the acceleration of the marble. Select a set of data points from the table and plot those points on the graph. Fill in the blank column in the table for any quantities you graph other than the given data. Label the axes and indicate the scale for each. Draw a best-fit line or curve through your data points.

Using the best-fit line, determine the value of the marble's acceleration.

Figure 2.37: Problem-solving skills are essential to your success in Physics. (credit: scui3asteveo, Flickr)

2.7 Problem-Solving Basics for One Dimensional Kinematics

2.7.1 Learning Objectives

By the end of this section, you will be able to:

- Apply problem-solving steps and strategies to solve problems of one-dimensional kinematics.
- Apply strategies to determine whether or not the result of a problem is reasonable, and if not, determine the cause.

Problem-solving skills are obviously essential to success in a quantitative course in physics. More importantly, the ability to apply broad physical principles, usually represented by equations, to specific situations is a very powerful form of knowledge. It is much more powerful than memorizing a list of facts. Analytical skills and problem-solving abilities can be applied to new situations, whereas a list of facts cannot be made long enough to contain every possible circumstance. Such analytical skills are useful both for solving problems in this text and for applying physics in everyday and professional life.

2.7.2 Problem-Solving Steps

While there is no simple step-by-step method that works for every problem, the following general procedures facilitate problem solving and make it more meaningful. A certain amount of creativity and insight is required as well.

2.7.2.1 Step 1

Examine the situation to determine which physical principles are involved. It often helps to *draw a simple sketch* at the outset. You will also need to decide which direction is positive and note that on your sketch. Once you have identified the physical principles, it is much easier to find and apply the equations representing those principles. Although finding the correct equation is essential, keep in mind that equations represent physical principles, laws of nature, and relationships among physical quantities. Without a conceptual understanding of a problem, a numerical solution is meaningless.

2.7.2.2 Step 2

Make a list of what is given or can be inferred from the problem as stated (identify the knowns). Many problems are stated very succinctly and require some inspection to determine what is known. A sketch can also be very useful at this point. Formally identifying the knowns is of particular importance in applying physics to real-world situations. Remember, “stopped” means velocity is zero, and we often can take initial time and position as zero.

2.7.2.3 Step 3

Identify exactly what needs to be determined in the problem (identify the unknowns). In complex problems, especially, it is not always obvious what needs to be found or in what sequence. Making a list can help.

2.7.2.4 Step 4

Find an equation or set of equations that can help you solve the problem. Your list of knowns and unknowns can help here. It is easiest if you can find equations that contain only one unknown—that is, all of the other variables are known, so you can easily solve for the unknown. If the equation contains more than one unknown, then an additional equation is needed to solve the problem. In some problems, several unknowns must be determined to get at the one needed most. In such problems it is especially important to keep physical principles in mind to avoid going astray in a sea of equations. You may have to use two (or more) different equations to get the final answer.

2.7.2.5 Step 5

Substitute the knowns along with their units into the appropriate equation, and obtain numerical solutions complete with units. This step produces the numerical answer; it also provides a check on units that can help you find errors. If the units of the answer are incorrect, then an error has been made. However, be warned that correct units do not guarantee that the numerical part of the answer is also correct.

2.7.2.6 Step 6

Check the answer to see if it is reasonable: Does it make sense? This final step is extremely important—the goal of physics is to accurately describe nature. To see if the answer is reasonable, check both its magnitude and its sign, in addition to its units. Your judgment will improve as you solve more and more physics problems, and it will become possible for you to make finer and finer judgments regarding whether nature is adequately described by the answer to a problem. This step brings the problem back to its conceptual meaning. If you can judge whether the answer is reasonable, you have a deeper understanding of physics than just being able to mechanically solve a problem.

When solving problems, we often perform these steps in different order, and we also tend to do several steps simultaneously. There is no rigid procedure that will work every time. Creativity and insight grow with experience, and the basics of problem solving become almost automatic. One way to get practice is to work out the text's examples for yourself as you read. Another is to work as many end-of-section problems as possible, starting with the easiest to build confidence and progressing to the more difficult. Once you become involved in physics, you will see it all around you, and you can begin to apply it to situations you encounter outside the classroom, just as is done in many of the applications in this text.

2.7.3 Unreasonable Results

Physics must describe nature accurately. Some problems have results that are unreasonable because one premise is unreasonable or because certain premises are inconsistent with one another. The physical principle applied correctly then produces an unreasonable result. For example, if a person starting a foot race accelerates at 0.40 m/s^2 for 100 s, his final speed will be 40 m/s (about 150 km/h)—clearly unreasonable because the time of 100 s is an unreasonable premise. The physics is correct in a sense, but there is more to describing nature than just manipulating equations correctly. Checking the result of a problem to see if it is reasonable does more than help uncover errors in problem solving—it also builds intuition in judging whether nature is being accurately described.

Use the following strategies to determine whether an answer is reasonable and, if it is not, to determine what is the cause.

2.7.3.1 Step 1

Solve the problem using strategies as outlined and in the format followed in the worked examples in the text. In the example given in the preceding paragraph, you would identify the givens as the acceleration and time and use the equation below to find the unknown final velocity. That is,

$$v = v_0 + \text{at} = 0 + \left(0.40\ \text{m/s}^2\right)(100\ \text{s}) = 40\ \text{m/s}.$$

2.7.3.2 Step 2

Check to see if the answer is reasonable. Is it too large or too small, or does it have the wrong sign, improper units, …? In this case, you may need to convert meters per second into a more familiar unit, such as miles per hour.

$$\left(\frac{40\ \text{m}}{s}\right)\left(\frac{3.28\ \text{ft}}{m}\right)\left(\frac{1\ \text{mi}}{5280\ \text{ft}}\right)\left(\frac{60\ \text{s}}{\text{min}}\right)\left(\frac{60\ \text{min}}{1\ \text{h}}\right) = 89\ mph$$

This velocity is about four times greater than a person can run—so it is too large.

2.7.3.3 Step 3

If the answer is unreasonable, look for what specifically could cause the identified difficulty. In the example of the runner, there are only two assumptions that are suspect. The acceleration could be too great or the time too long. First look at the acceleration and think about what the number means. If someone accelerates at $0.40\ \text{m/s}^2$, their velocity is increasing by 0.4 m/s each second. Does this seem reasonable? If so, the time must be too long. It is not possible for someone to accelerate at a constant rate of $0.40\ \text{m/s}^2$ for 100 s (almost two minutes).

2.7.4 Section Summary

- *The six basic problem solving steps for physics are:*

  Step 1. Examine the situation to determine which physical principles are involved.

  Step 2. Make a list of what is given or can be inferred from the problem as stated (identify the knowns).

  Step 3. Identify exactly what needs to be determined in the problem (identify the unknowns).

  Step 4. Find an equation or set of equations that can help you solve the problem.

Step 5. Substitute the knowns along with their units into the appropriate equation, and obtain numerical solutions complete with units.

Step 6. Check the answer to see if it is reasonable: Does it make sense?

2.7.5 Conceptual Questions

What information do you need in order to choose which equation or equations to use to solve a problem? Explain.

What is the last thing you should do when solving a problem? Explain.

2.8 Falling Objects

2.8.1 Learning Objectives

By the end of this section, you will be able to:

- Describe the effects of gravity on objects in motion.
- Describe the motion of objects that are in free fall.
- Calculate the position and velocity of objects in free fall.

The information presented in this section supports the following AP® learning objectives and science practices:

- **3.A.1.1** The student is able to express the motion of an object using narrative, mathematical, or graphical representations. **(S.P. 1.5, 2.1, 2.2)**
- **3.A.1.2** The student is able to design an experimental investigation of the motion of an object. **(S.P. 4.2)**
- **3.A.1.3** The student is able to analyze experimental data describing the motion of an object and is able to express the results of the analysis using narrative, mathematical, and graphical representations. **(S.P. 5.1)**

Falling objects form an interesting class of motion problems. For example, we can estimate the depth of a vertical mine shaft by dropping a rock into it and listening for the rock to hit the bottom. By applying the kinematics developed so far to falling objects, we can examine some interesting situations and learn much about gravity in the process.

2.8.2 Gravity

The most remarkable and unexpected fact about falling objects is that, if air resistance and friction are negligible, then in a given location all objects fall toward the center of Earth with the *same constant acceleration*, *independent of their mass*. This experimentally determined fact is unexpected, because we are so accustomed to the effects of air resistance and friction that we expect light objects to fall slower than heavy ones.

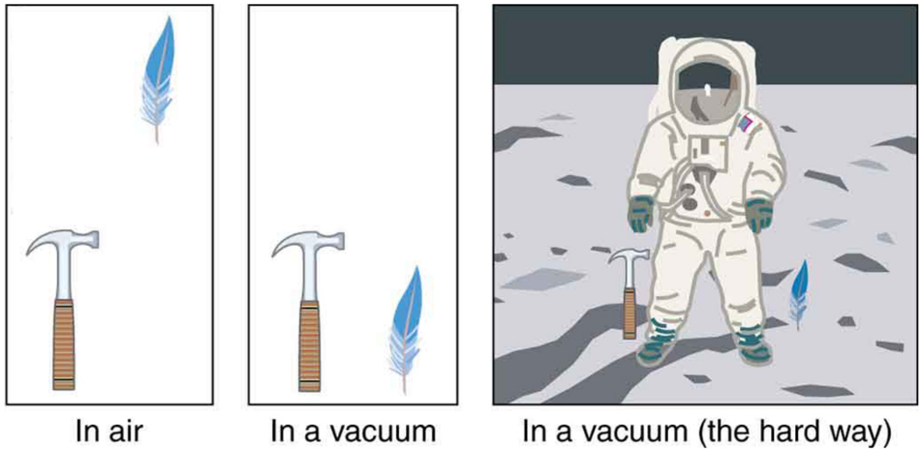


Figure 2.38: A hammer and a feather will fall with the same constant acceleration if air resistance is considered negligible. This is a general characteristic of gravity not unique to Earth, as astronaut David R. Scott demonstrated on the Moon in 1971, where the acceleration due to gravity is only 1.67 $\mathrm{m/s}^2$.

In the real world, air resistance can cause a lighter object to fall slower than a heavier object of the same size. A tennis ball will reach the ground after a hard baseball dropped at the same time. (It might be difficult to observe the difference if the height is not large.) Air resistance opposes the motion of an object through the air, while friction between objects—such as between clothes and a laundry chute or between a stone and a pool into which it is dropped—also opposes motion between them. For the ideal situations of these first few chapters, an object *falling without air resistance or friction* is defined to be in free-fall.

The force of gravity causes objects to fall toward the center of Earth. The acceleration of free-falling objects is therefore called the acceleration due to gravity. The acceleration due to gravity is *constant*, which means we can apply the kinematics equations to any falling object where air resistance and friction are negligible. This opens a broad class of interesting situations to us. The acceleration due to gravity is so important that its magnitude is given its own symbol, g. It is constant at any given location on Earth and has the average value

$$g = 9.80\ \mathrm{m/s}^2.$$

Although g varies from 9.78 $\mathrm{m/s}^2$ to 9.83 $\mathrm{m/s}^2$, depending on latitude, altitude, underlying geological formations, and local topography, the average value of 9.80 $\mathrm{m/s}^2$ will be used in this text unless otherwise specified. The direction of the acceleration due to gravity is *downward (towards the center of Earth)*. In fact, its direction *defines* what we call vertical. Note that whether the acceleration a in the kinematic equations has the value $+g$ or $-g$ depends on how we define our coordinate system. If we define the upward direction as positive, then $a = -g = -9.80\ \text{m/s}^2$, and if we define the downward direction as positive, then $a = g = 9.80\ \text{m/s}^2$.

2.8.3 One-Dimensional Motion Involving Gravity

The best way to see the basic features of motion involving gravity is to start with the simplest situations and then progress toward more complex ones. So we start by considering straight up and down motion with no air resistance or friction. These assumptions mean that the velocity (if there is any) is vertical. If the object is dropped, we know the initial velocity is zero. Once the object has left contact with whatever held or threw it, the object is in free-fall. Under these circumstances, the motion is one-dimensional and has constant acceleration of magnitude g. We will also represent vertical displacement with the symbol y and use x for horizontal displacement.

Kinematic Equations for Objects in Free-Fall where Acceleration = -g

$$v = v_0 - \text{gt}$$

$$y = y_0 + v_0 t - \frac{1}{2}\text{gt}^2$$

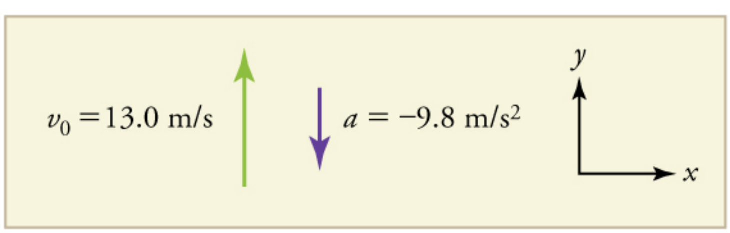
$$v^2 = v_0^2 - 2g\left(y - y_0\right)$$

Calculating Position and Velocity of a Falling Object: A Rock Thrown Upward

A person standing on the edge of a high cliff throws a rock straight up with an initial velocity of 13.0 m/s. The rock misses the edge of the cliff as it falls back to Earth. Calculate the position and velocity of the rock 1.00 s, 2.00 s, and 3.00 s after it is thrown, neglecting the effects of air resistance.

Strategy

Draw a sketch.



We are asked to determine the position y at various times. It is reasonable to take the initial position y_0 to be zero. This problem involves one-dimensional motion in the vertical direction. We use plus and minus signs to indicate direction, with up being positive and down negative. Since up is positive, and the rock is thrown upward, the initial velocity must be positive too. The acceleration due to gravity is downward, so a is negative. It is crucial that the initial velocity and the acceleration due to gravity have opposite signs. Opposite signs indicate that the acceleration due to gravity opposes the initial motion and will slow and eventually reverse it.

Since we are asked for values of position and velocity at three times, we will refer to these as y_1 and v_1; y_2 and v_2; and y_3 and v_3.

Solution for Position y_1

1. Identify the knowns. We know that $y_0 = 0$; $v_0 = 13.0$ m/s; $a = -g = -9.80$ m/s^2; and $t = 1.00$ s.

2. Identify the best equation to use. We will use $y = y_0 + v_0 t + \frac{1}{2}\text{at}^2$ because it includes only one unknown, y (or y_1, here), which is the value we want to find.

3. Plug in the known values and solve for y_1.

y 1 = 0 + 13 . 0 m/s 1 . 00 s + 1 2 − 9 . 80 m/s 2 1 . 00 s 2 = 8 . 10 m y 1 = 0 + 13 . 0 m/s 1 . 00 s + 1 2 − 9 . 80 m/s 2 1 . 00 s 2 = 8 . 10 m size 12{y"" lSub { size 8{1} } =0+ left ("13" "." "0 m/s" right) left (1 "." "00 s" right)+ { {1} over {2} } left (- 9 "." "80"" m/s" rSup { size 8{2} } right) left (1 "." "00 s" right) rSup { size 8{2} } =8 "." "10"'m} {}

Discussion

The rock is 8.10 m above its starting point at $t = 1.00$ s, since $y_1 > y_0$. It could be *moving* up or down; the only way to tell is to calculate v_1 and find out if it is positive or negative.

Solution for Velocity v_1

1. Identify the knowns. We know that $y_0 = 0$; $v_0 = 13.0$ m/s; $a = -g = -9.80$ m/s^2; and $t = 1.00$ s. We also know from the solution above that $y_1 = 8.10$ m.

2. Identify the best equation to use. The most straightforward is $v = v_0 - \text{gt}$ (from $v = v_0 + \text{at}$, where a = gravitational acceleration = $-g$).

3. Plug in the knowns and solve.

$$v_1 = v_0 - \text{gt} = 13.0 \text{ m/s} - \left(9.80 \text{ m/s}^2\right)(1.00 \text{ s}) = 3.20 \text{ m/s}$$

Discussion

The positive value for v_1 means that the rock is still heading upward at $t = 1.00$ s. However, it has slowed from its original 13.0 m/s, as expected.

Solution for Remaining Times

The procedures for calculating the position and velocity at $t = 2.00$ s and 3.00 s are the same as those above. The results are summarized in link and illustrated in link.

Table 2.2: Results

Time, t	Position, y	Velocity, v	Acceleration, a
1.00 s	8.10 m	3.20 m/s	-9.80 m/s^2
2.00 s	6.40 m	-6.60 m/s	-9.80 m/s^2
3.00 s	-5.10 m	-16.4 m/s	-9.80 m/s^2

Graphing the data helps us understand it more clearly.

Discussion

The interpretation of these results is important. At 1.00 s the rock is above its starting point and heading upward, since y_1 and v_1 are both positive. At 2.00 s, the rock is still above its starting point, but the negative velocity means it is moving downward. At 3.00 s, both y_3 and v_3 are negative, meaning the rock is below its starting point and continuing to move downward. Notice that when the rock is at its highest point (at 1.5 s), its velocity is zero, but its acceleration is still -9.80 m/s^2. Its acceleration is -9.80 m/s^2 for the whole trip—while it is moving up and while it is moving down. Note that the values for y are the positions (or displacements) of the rock, not the total distances traveled. Finally, note that free-fall applies to upward motion as well as downward. Both have the same acceleration—the acceleration due to gravity, which remains constant the entire time. Astronauts training in the famous Vomit Comet, for example, experience free-fall while arcing up as well as down, as we will discuss in more detail later.

Making Connections: Take-Home Experiment—Reaction Time

A simple experiment can be done to determine your reaction time. Have a friend hold a ruler between your thumb and index finger, separated by about 1 cm. Note the mark on the ruler that is right between your fingers. Have your friend drop the ruler unexpectedly, and try to catch it between your two fingers. Note the new reading on the ruler. Assuming acceleration is that due to gravity, calculate your reaction time. How far would you travel in a car (moving at 30 m/s) if the time it took your foot to go from the gas pedal to the brake was twice this reaction time?

Calculating Velocity of a Falling Object: A Rock Thrown Down

What happens if the person on the cliff throws the rock straight down, instead of straight up? To explore this question, calculate the velocity of the rock when

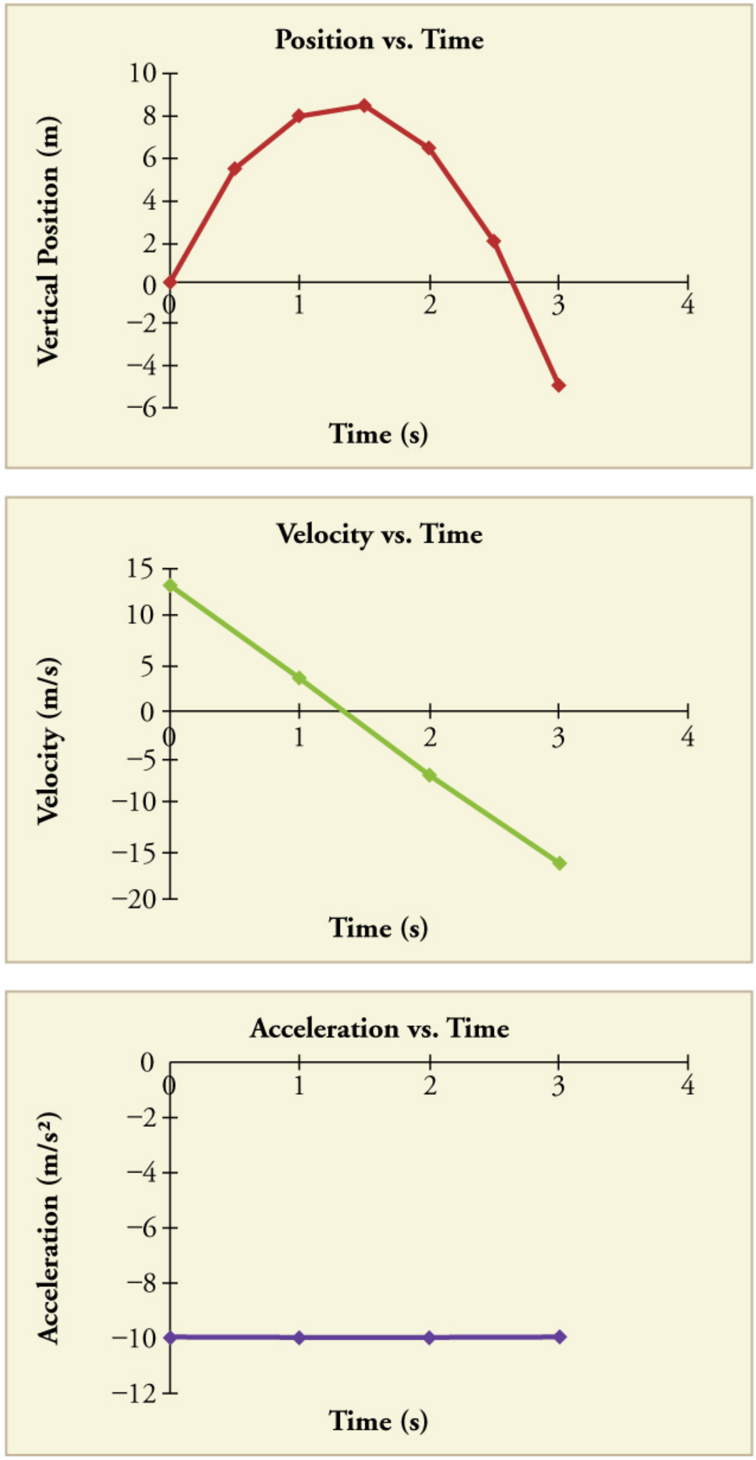
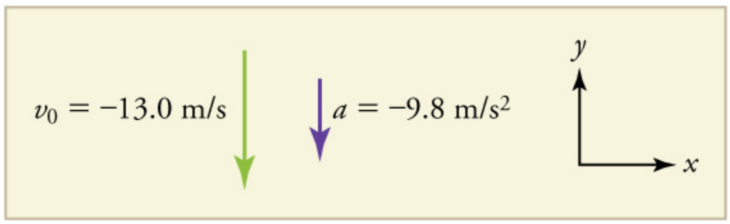


Figure 2.39: Vertical position, vertical velocity, and vertical acceleration vs. time for a rock thrown vertically up at the edge of a cliff. Notice that velocity changes linearly with time and that acceleration is constant. *Misconception Alert!* Notice that the position vs. time graph shows vertical position only. It is easy to get the impression that the graph shows some horizontal motion—the shape of the graph looks like the path of a projectile. But this is not the case; the horizontal axis is *time*, not space. The actual path of the rock in space is straight up, and straight down.

it is 5.10 m below the starting point, and has been thrown downward with an initial speed of 13.0 m/s.

Strategy

Draw a sketch.



Since up is positive, the final position of the rock will be negative because it finishes below the starting point at $y_0 = 0$. Similarly, the initial velocity is downward and therefore negative, as is the acceleration due to gravity. We expect the final velocity to be negative since the rock will continue to move downward.

Solution

1. Identify the knowns. $y_0 = 0$; $y_1 = -5.10$ m; $v_0 = -13.0$ m/s; $a = -g = -9.80$ m/s^2.

2. Choose the kinematic equation that makes it easiest to solve the problem. The equation $v^2 = v_0^2 + 2a(y - y_0)$ works well because the only unknown in it is v. (We will plug y_1 in for y.)

3. Enter the known values

$$v^2 = (-13.0 \text{ m/s})^2 + 2\left(-9.80 \text{ m/s}^2\right)(-5.10 \text{ m} - 0 \text{ m}) = 268.96 \text{ m}^2/\text{s}^2,$$

where we have retained extra significant figures because this is an intermediate result.

Taking the square root, and noting that a square root can be positive or negative, gives

$$v = \pm 16.4 \text{ m/s}.$$

The negative root is chosen to indicate that the rock is still heading down. Thus,

$$v = -16.4 \text{ m/s}.$$

Discussion

Note that *this is exactly the same velocity the rock had at this position when it was thrown straight upward with the same initial speed.* (See link and link(a).) This is not a coincidental result. Because we only consider the acceleration due to gravity in this problem, the *speed* of a falling object depends only on its initial speed and its vertical position relative to the starting point. For example, if the velocity of the rock is calculated at a height of 8.10 m above the starting point (using the method from link) when the initial velocity is 13.0 m/s straight up, a result of ± 3.20 m/s is obtained. Here both signs are meaningful; the positive value occurs when the rock is at 8.10 m and heading up, and the negative value occurs when the rock is at 8.10 m and heading back down. It has the same *speed* but the opposite direction.

Another way to look at it is this: In link, the rock is thrown up with an initial velocity of 13.0 m/s. It rises and then falls back down. When its position is $y = 0$ on its way back down, its velocity is -13.0 m/s. That is, it has the same speed on its way down as on its way up. We would then expect its velocity at a position of $y = -5.10$ m to be the same whether we have thrown it upwards at $+13.0$ m/s or thrown it downwards at -13.0 m/s. The velocity of the rock on its way down from $y = 0$ is the same whether we have thrown it up or down to start with, as long as the speed with which it was initially thrown is the same.

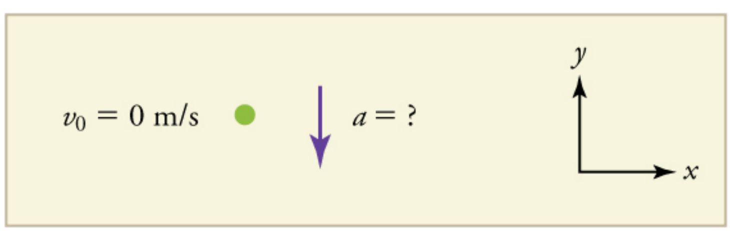
Find g from Data on a Falling Object

The acceleration due to gravity on Earth differs slightly from place to place, depending on topography (e.g., whether you are on a hill or in a valley) and subsurface geology (whether there is dense rock like iron ore as opposed to light rock like salt beneath you.) The precise acceleration due to gravity can be calculated from data taken in an introductory physics laboratory course. An object, usually a metal ball for which air resistance is negligible, is dropped and the time it takes to fall a known distance is measured. See, for example, link. Very precise results can be produced with this method if sufficient care is taken in measuring the distance fallen and the elapsed time.

Suppose the ball falls 1.0000 m in 0.45173 s. Assuming the ball is not affected by air resistance, what is the precise acceleration due to gravity at this location?

Strategy

Draw a sketch.



We need to solve for acceleration a. Note that in this case, displacement is

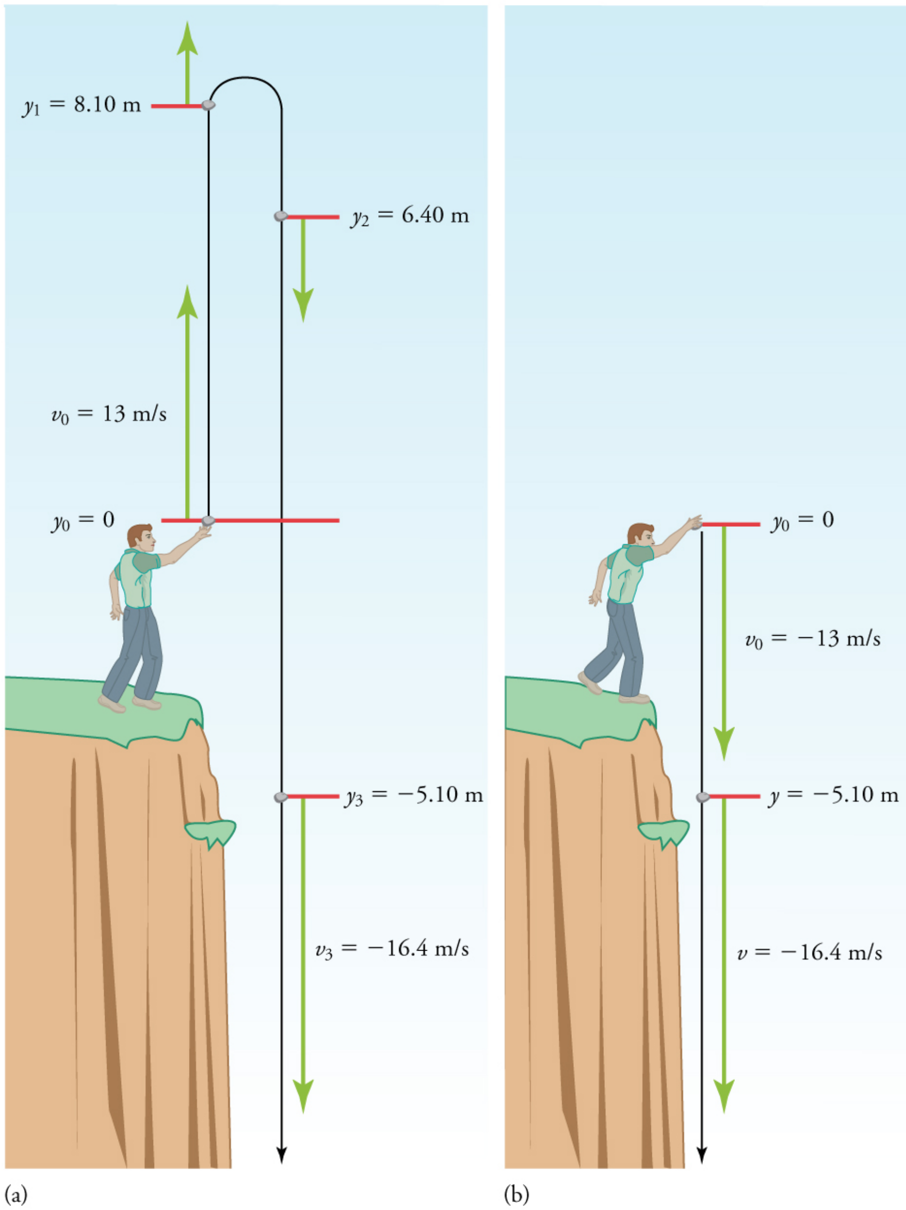
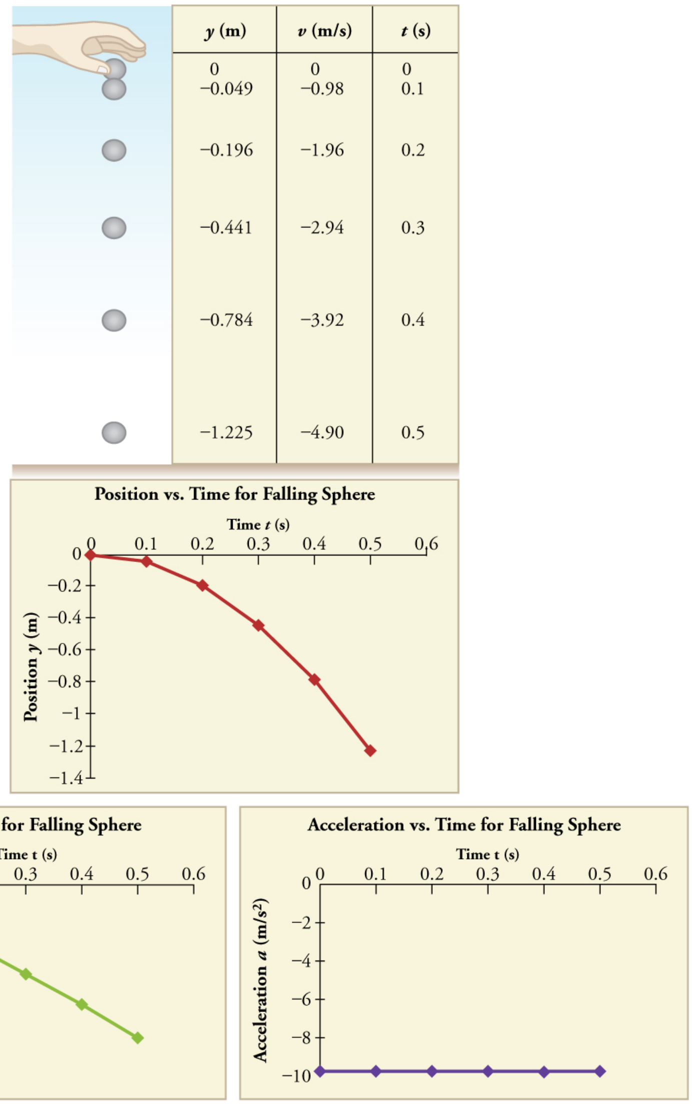


Figure 2.40: (a) A person throws a rock straight up, as explored in link. The arrows are velocity vectors at 0, 1.00, 2.00, and 3.00 s. (b) A person throws a rock straight down from a cliff with the same initial speed as before, as in link. Note that at the same distance below the point of release, the rock has the same velocity in both cases.



y (m)	v (m/s)	t (s)
0	0	0
−0.049	−0.98	0.1
−0.196	−1.96	0.2
−0.441	−2.94	0.3
−0.784	−3.92	0.4
−1.225	−4.90	0.5

Figure 2.41: Positions and velocities of a metal ball released from rest when air resistance is negligible. Velocity is seen to increase linearly with time while displacement increases with time squared. Acceleration is a constant and is equal to gravitational acceleration.

downward and therefore negative, as is acceleration.

Solution

1. Identify the knowns. $y_0 = 0$; $y = -1.0000$ m; $t = 0.45173$; $v_0 = 0$.

2. Choose the equation that allows you to solve for a using the known values.

$$y = y_0 + v_0 t + \frac{1}{2}\text{at}^2$$

3. Substitute 0 for v_0 and rearrange the equation to solve for a. Substituting 0 for v_0 yields

$$y = y_0 + \frac{1}{2}\text{at}^2.$$

Solving for a gives

$$a = \frac{2\,(y - y_0)}{t^2}.$$

4. Substitute known values yields

$$a = \frac{2(-1.0000\text{ m} - 0)}{(0.45173\text{ s})^2} = -9.8010\text{ m/s}^2,$$

so, because $a = -g$ with the directions we have chosen,

$$g = 9.8010\text{ m/s}^2.$$

Discussion

The negative value for a indicates that the gravitational acceleration is downward, as expected. We expect the value to be somewhere around the average value of 9.80 m/s^2, so 9.8010 m/s^2 makes sense. Since the data going into the calculation are relatively precise, this value for g is more precise than the average value of 9.80 m/s^2; it represents the local value for the acceleration due to gravity.

Applying the Science Practices: Finding Acceleration Due to Gravity

While it is well established that the acceleration due to gravity is quite nearly 9.8 m/s^2 at all locations on Earth, you can verify this for yourself with some basic materials.

Your task is to find the acceleration due to gravity at your location. Achieving an acceleration of precisely 9.8 m/s^2 will be difficult. However, with good preparation and attention to detail, you should be able to get close. Before you begin working, consider the following questions.

What measurements will you need to take in order to find the acceleration due to gravity?

What relationships and equations found in this chapter may be useful in calculating the acceleration?

What variables will you need to hold constant?

What materials will you use to record your measurements?

Upon completing these four questions, record your procedure. Once recorded, you may carry out the experiment. If you find that your experiment cannot be carried out, you may revise your procedure.

Once you have found your experimental acceleration, compare it to the assumed value of 9.8 m/s^2. If error exists, what were the likely sources of this error? How could you change your procedure in order to improve the accuracy of your findings?

Check Your Understanding

A chunk of ice breaks off a glacier and falls 30.0 meters before it hits the water. Assuming it falls freely (there is no air resistance), how long does it take to hit the water?

We know that initial position $y_0 = 0$, final position $y = -30.0$ m, and $a = -g = -9.80\ \text{m/s}^2$. We can then use the equation $y = y_0 + v_0 t + \frac{1}{2}\text{at}^2$ to solve for t. Inserting $a = -g$, we obtain

$$\begin{aligned} y &= 0 + 0 - \tfrac{1}{2}\text{gt}^2 \\ t^2 &= \tfrac{2y}{-g} \\ t &= \pm\sqrt{\tfrac{2y}{-g}} = \pm\sqrt{\tfrac{2(-30.0\ \text{m})}{-9.80\ m/\text{s}^2}} = \pm\sqrt{6.12\ s^2} = 2.47\ \text{s} \approx 2.5\ \text{s} \end{aligned}$$

where we take the positive value as the physically relevant answer. Thus, it takes about 2.5 seconds for the piece of ice to hit the water.

Equation Grapher

Learn about graphing polynomials. The shape of the curve changes as the constants are adjusted. View the curves for the individual terms (e.g. $y = \text{bx}$) to see how they add to generate the polynomial curve.

2.8.4 Section Summary

- An object in free-fall experiences constant acceleration if air resistance is negligible.

- On Earth, all free-falling objects have an acceleration due to gravity g, which averages ::: {#import-auto-id3547826 data-type="equation"}

$$g = 9.80 \text{ m/s}^2.$$

:::

- Whether the acceleration a should be taken as $+g$ or $-g$ is determined by your choice of coordinate system. If you choose the upward direction as positive, $a = -g = -9.80 \text{ m/s}^2$ is negative. In the opposite case, $a = +g = 9.80 \text{ m/s}^2$ is positive. Since acceleration is constant, the kinematic equations above can be applied with the appropriate $+g$ or $-g$ substituted for a.

- For objects in free-fall, up is normally taken as positive for displacement, velocity, and acceleration.

2.8.5 Conceptual Questions

What is the acceleration of a rock thrown straight upward on the way up? At the top of its flight? On the way down?

An object that is thrown straight up falls back to Earth. This is one-dimensional motion. (a) When is its velocity zero? (b) Does its velocity change direction? (c) Does the acceleration due to gravity have the same sign on the way up as on the way down?

Suppose you throw a rock nearly straight up at a coconut in a palm tree, and the rock misses on the way up but hits the coconut on the way down. Neglecting air resistance, how does the speed of the rock when it hits the coconut on the way down compare with what it would have been if it had hit the coconut on the way up? Is it more likely to dislodge the coconut on the way up or down? Explain.

If an object is thrown straight up and air resistance is negligible, then its speed when it returns to the starting point is the same as when it was released. If air resistance were not negligible, how would its speed upon return compare with its initial speed? How would the maximum height to which it rises be affected?

The severity of a fall depends on your speed when you strike the ground. All factors but the acceleration due to gravity being the same, how many times higher could a safe fall on the Moon be than on Earth (gravitational acceleration on the Moon is about 1/6 that of the Earth)?

How many times higher could an astronaut jump on the Moon than on Earth if his takeoff speed is the same in both locations (gravitational acceleration on the Moon is about 1/6 of g on Earth)?

2.8.6 Problems & Exercises

Assume air resistance is negligible unless otherwise stated.

Calculate the displacement and velocity at times of (a) 0.500, (b) 1.00, (c) 1.50, and (d) 2.00 s for a ball thrown straight up with an initial velocity of 15.0 m/s. Take the point of release to be $y_0 = 0$.

a $y_1 = 6.28$ m; $v_1 = 10.1$ m/s

b $y_2 = 10.1$ m; $v_2 = 5.20$ m/s

c $y_3 = 11.5 \ \ m$; $v_3 = 0.300$ m/s

d $y_4 = 10.4$ m; $v_4 = -4.60$ m/s

Calculate the displacement and velocity at times of (a) 0.500, (b) 1.00, (c) 1.50, (d) 2.00, and (e) 2.50 s for a rock thrown straight down with an initial velocity of 14.0 m/s from the Verrazano Narrows Bridge in New York City. The roadway of this bridge is 70.0 m above the water.

A basketball referee tosses the ball straight up for the starting tip-off. At what velocity must a basketball player leave the ground to rise 1.25 m above the floor in an attempt to get the ball?

$v_0 = 4.95$ m/s

A rescue helicopter is hovering over a person whose boat has sunk. One of the rescuers throws a life preserver straight down to the victim with an initial velocity of 1.40 m/s and observes that it takes 1.8 s to reach the water. (a) List the knowns in this problem. (b) How high above the water was the preserver released? Note that the downdraft of the helicopter reduces the effects of air resistance on the falling life preserver, so that an acceleration equal to that of gravity is reasonable.

A dolphin in an aquatic show jumps straight up out of the water at a velocity of 13.0 m/s. (a) List the knowns in this problem. (b) How high does his body rise above the water? To solve this part, first note that the final velocity is now a known and identify its value. Then identify the unknown, and discuss how you chose the appropriate equation to solve for it. After choosing the equation, show your steps in solving for the unknown, checking units, and discuss whether the answer is reasonable. (c) How long is the dolphin in the air? Neglect any effects due to his size or orientation.

a $a = -9.80$ m/s^2; $v_0 = 13.0$ m/s; $y_0 = 0$ m

b $v = 0$m/s. Unknown is distance y to top of trajectory, where velocity is zero. Use equation $v^2 = v_0^2 + 2a\left(y - y_0\right)$ because it contains all known values except for y, so we can solve for y. Solving for y gives

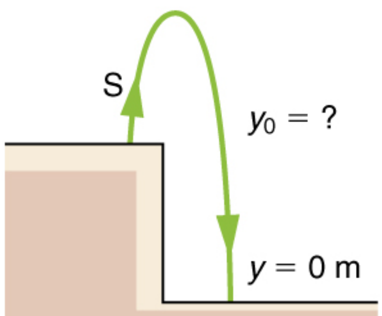
$$\begin{aligned} v^2 - v_0^2 &= 2a\left(y - y_0\right) \\ \frac{v^2 - v_0^2}{2a} &= y - y_0 \\ y &= y_0 + \frac{v^2 - v_0^2}{2a} = 0\ m + \frac{\left(0\ \mathrm{m/s}\right)^2 - \left(13.0\ \mathrm{m/s}\right)^2}{2\left(-9.80\ \mathrm{m/s}^2\right)} = 8.62\ \mathrm{m} \end{aligned}$$

Dolphins measure about 2 meters long and can jump several times their length out of the water, so this is a reasonable result.

c 2.65 s

A swimmer bounces straight up from a diving board and falls feet first into a pool. She starts with a velocity of 4.00 m/s, and her takeoff point is 1.80 m above the pool. (a) How long are her feet in the air? (b) What is her highest point above the board? (c) What is her velocity when her feet hit the water?

a Calculate the height of a cliff if it takes 2.35 s for a rock to hit the ground when it is thrown straight up from the cliff with an initial velocity of 8.00 m/s. (b) How long would it take to reach the ground if it is thrown straight down with the same speed?



a 8.26 m

b 0.717 s

A very strong, but inept, shot putter puts the shot straight up vertically with an initial velocity of 11.0 m/s. How long does he have to get out of the way if the shot was released at a height of 2.20 m, and he is 1.80 m tall?

You throw a ball straight up with an initial velocity of 15.0 m/s. It passes a tree branch on the way up at a height of 7.00 m. How much additional time will pass before the ball passes the tree branch on the way back down?

1.91 s

A kangaroo can jump over an object 2.50 m high. (a) Calculate its vertical speed when it leaves the ground. (b) How long is it in the air?

Standing at the base of one of the cliffs of Mt. Arapiles in Victoria, Australia, a hiker hears a rock break loose from a height of 105 m. He can't see the rock right away but then does, 1.50 s later. (a) How far above the hiker is the rock when he can see it? (b) How much time does he have to move before the rock hits his head?

a 94.0 m

b 3.13 s

An object is dropped from a height of 75.0 m above ground level. (a) Determine the distance traveled during the first second. (b) Determine the final velocity at which the object hits the ground. (c) Determine the distance traveled during the last second of motion before hitting the ground.

There is a 250-m-high cliff at Half Dome in Yosemite National Park in California. Suppose a boulder breaks loose from the top of this cliff. (a) How fast will it be going when it strikes the ground? (b) Assuming a reaction time of 0.300 s, how long will a tourist at the bottom have to get out of the way after hearing the sound of the rock breaking loose (neglecting the height of the tourist, which would become negligible anyway if hit)? The speed of sound is 335 m/s on this day.

a -70.0 m/s (downward)

b 6.10 s

A ball is thrown straight up. It passes a 2.00-m-high window 7.50 m off the ground on its path up and takes .310 s to go past the window. What was the ball's initial velocity?

Suppose you drop a rock into a dark well and, using precision equipment, you measure the time for the sound of a splash to return. (a) Neglecting the time required for sound to travel up the well, calculate the distance to the water if the sound returns in 2.0000 s. (b) Now calculate the distance taking into account the time for sound to travel up the well. The speed of sound is 332.00 m/s in this well.

a 19.6 m

b 18.5 m

A steel ball is dropped onto a hard floor from a height of 1.50 m and rebounds to a height of 1.45 m. (a) Calculate its velocity just before it strikes the floor. (b) Calculate its velocity just after it leaves the floor on its way back up. (c) Calculate its acceleration during contact with the floor if that contact lasts 0.0800 ms (8.00×10^{-5} s). (d) How much did the ball compress during its collision with the floor, assuming the floor is absolutely rigid?

A coin is dropped from a hot-air balloon that is 300 m above the ground and rising at 10.0 m/s upward. For the coin, find (a) the maximum height reached, (b) its position and velocity 4.00 s after being released, and (c) the time before it hits the ground.

a 305 m

b 262 m, -29.2 m/s

c 8.91 s

A soft tennis ball is dropped onto a hard floor from a height of 1.50 m and rebounds to a height of 1.10 m. (a) Calculate its velocity just before it strikes the floor. (b) Calculate its velocity just after it leaves the floor on its way back up. (c) Calculate its acceleration during contact with the floor if that contact lasts 3.50 ms (3.50×10^{-3} s). (d) How much did the ball compress during its collision with the floor, assuming the floor is absolutely rigid?

2.8.7 Test Prep for AP Courses

Observing a spacecraft land on a distant asteroid, scientists notice that the craft is falling at a rate of 5 m/s. When it is 100 m closer to the surface of the asteroid, the craft reports a velocity of 8 m/s. According to their data, what is the approximate gravitational acceleration on this asteroid?

a. 0 m/s^2
b. 0.03 m/s^2
c. 0.20 m/s^2
d. 0.65 m/s^2
e. 33 m/s^2

c

2.8.8 Glossary

free-fall the state of movement that results from gravitational force only

acceleration due to gravity acceleration of an object as a result of gravity

2.9 Graphical Analysis of One Dimensional Motion

2.9.1 Learning Objectives

By the end of this section, you will be able to:

- Describe a straight-line graph in terms of its slope and y-intercept.
- Determine average velocity or instantaneous velocity from a graph of position vs. time.
- Determine average or instantaneous acceleration from a graph of velocity vs. time.
- Derive a graph of velocity vs. time from a graph of position vs. time.
- Derive a graph of acceleration vs. time from a graph of velocity vs. time.

A graph, like a picture, is worth a thousand words. Graphs not only contain numerical information; they also reveal relationships between physical quantities. This section uses graphs of position, velocity, and acceleration versus time to illustrate one-dimensional kinematics.

2.9.2 Slopes and General Relationships

First note that graphs in this text have perpendicular axes, one horizontal and the other vertical. When two physical quantities are plotted against one another in such a graph, the horizontal axis is usually considered to be an independent variable and the vertical axis a dependent variable. If we call the horizontal axis the x-axis and the vertical axis the y-axis, as in link, a straight-line graph has the general form

$$y = \text{mx}+b.$$

Here m is the slope, defined to be the rise divided by the run (as seen in the figure) of the straight line. The letter b is used for the y-intercept, which is the point at which the line crosses the vertical axis.

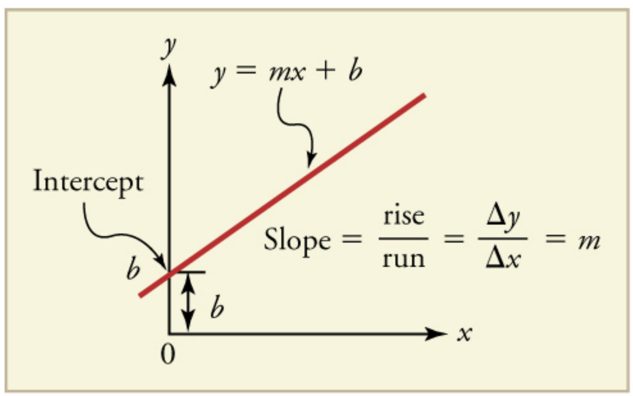


Figure 2.42: A straight-line graph. The equation for a straight line is $y = \text{mx} + b$
.

2.9.3 Graph of Position vs. Time ($a = 0$, so v is constant)

Time is usually an independent variable that other quantities, such as position, depend upon. A graph of position versus time would, thus, have x on the vertical axis and t on the horizontal axis. link is just such a straight-line graph. It shows a graph of position versus time for a jet-powered car on a very flat dry lake bed in Nevada.

Using the relationship between dependent and independent variables, we see that the slope in the graph above is average velocity $\bar{v}$ and the intercept is

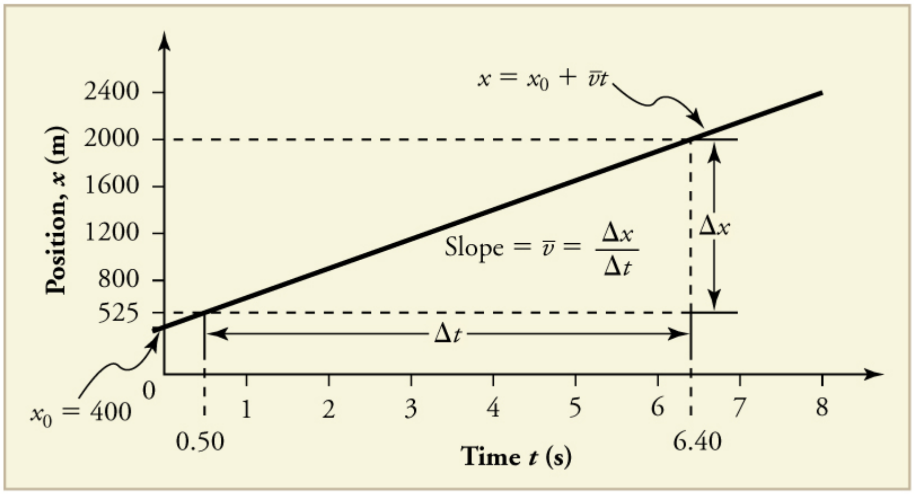


Figure 2.43: Graph of position versus time for a jet-powered car on the Bonneville Salt Flats.

position at time zero—that is, x_0. Substituting these symbols into $y = \mathrm{mx} + b$ gives

$$x = \bar{v}t + x_0$$

or

$$x = x_0 + \bar{v}t.$$

Thus a graph of position versus time gives a general relationship among position, velocity, and time, as well as giving detailed numerical information about a specific situation.

The Slope of x vs. t

The slope of the graph of position x vs. time t is velocity v.

$$\text{slope} = \frac{\Delta x}{\Delta t} = v$$

Notice that this equation is the same as that derived algebraically from other motion equations in Motion Equations for Constant Acceleration in One Dimension.

From the figure we can see that the car has a position of 400 m at time 0.650 m at $t = 1.0$ s, and so on. Its position at times other than those listed in the table can be read from the graph; furthermore, information about its velocity and acceleration can also be obtained from the graph.

Determining Average Velocity from a Graph of Position versus Time: Jet Car

Find the average velocity of the car whose position is graphed in link.

Strategy

The slope of a graph of x vs. t is average velocity, since slope equals rise over run. In this case, rise = change in position and run = change in time, so that

$$\text{slope} = \frac{\Delta x}{\Delta t} = \bar{v}.$$

Since the slope is constant here, any two points on the graph can be used to find the slope. (Generally speaking, it is most accurate to use two widely separated points on the straight line. This is because any error in reading data from the graph is proportionally smaller if the interval is larger.)

Solution

1. Choose two points on the line. In this case, we choose the points labeled on the graph: (6.4 s, 2000 m) and (0.50 s, 525 m). (Note, however, that you could choose any two points.)

2. Substitute the x and t values of the chosen points into the equation. Remember in calculating change (Δ) we always use final value minus initial value.

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{2000\text{ m} - 525\text{ m}}{6.4\text{ s} - 0.50\text{ s}},$$

yielding

$$\bar{v} = 250\text{ m/s}.$$

Discussion

This is an impressively large land speed (900 km/h, or about 560 mi/h): much greater than the typical highway speed limit of 60 mi/h (27 m/s or 96 km/h), but considerably shy of the record of 343 m/s (1234 km/h or 766 mi/h) set in 1997.

2.9.4 Graphs of Motion when a is constant but $a \neq 0$

The graphs in link below represent the motion of the jet-powered car as it accelerates toward its top speed, but only during the time when its acceleration is constant. Time starts at zero for this motion (as if measured with a stopwatch), and the position and velocity are initially 200 m and 15 m/s, respectively.

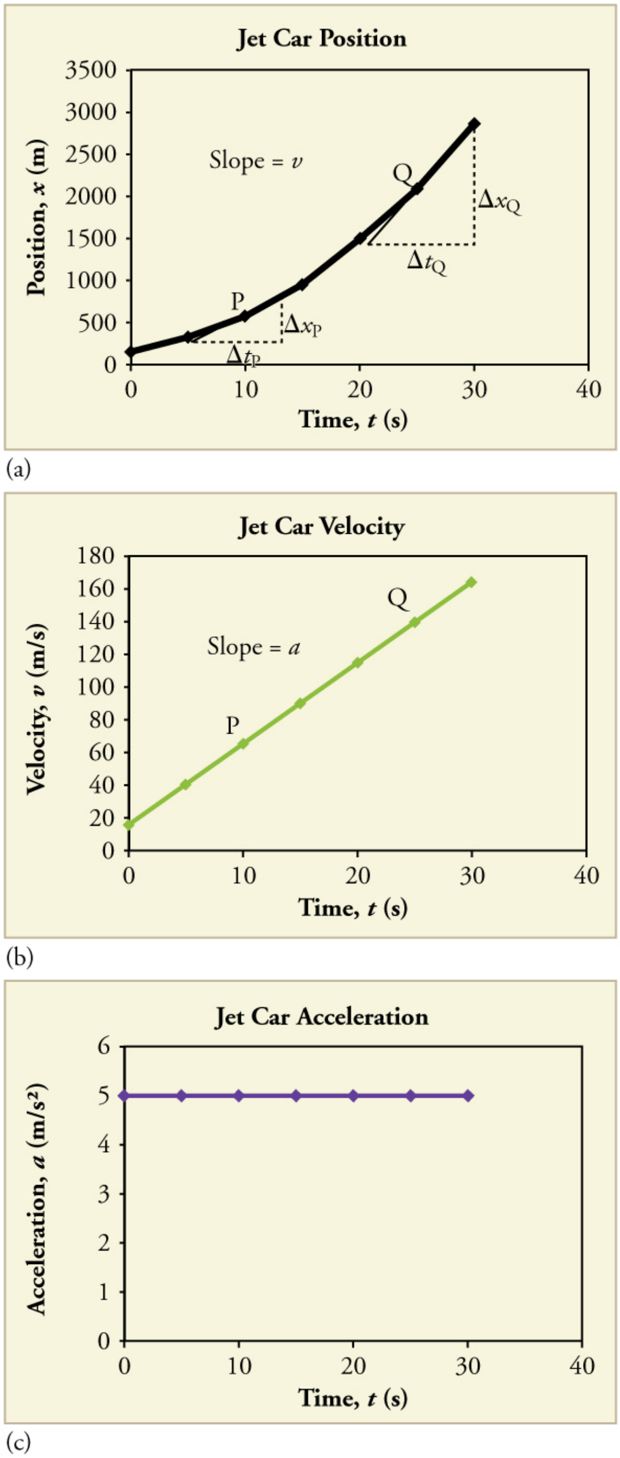


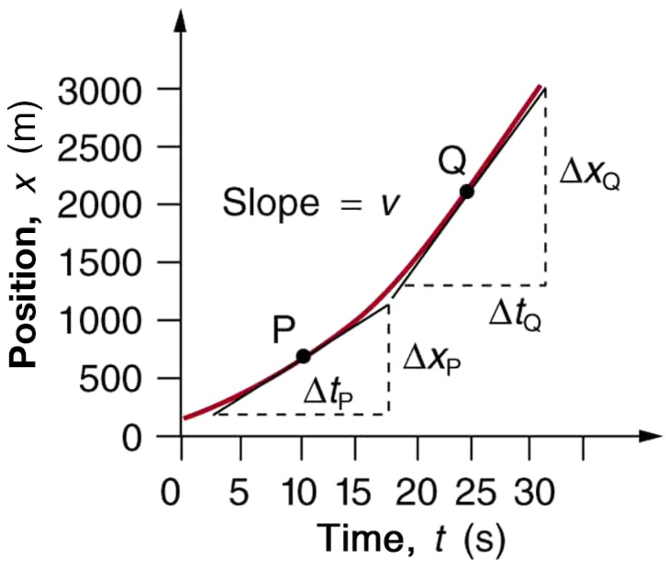
Figure 2.44: Graphs of motion of a jet-powered car during the time span when its acceleration is constant. (a) The slope of an x vs. t graph is velocity. This is shown at two points, and the instantaneous velocities obtained are plotted in the next graph. Instantaneous velocity at any point is the slope of the tangent at that point. (b) The slope of the v vs. t graph is constant for this part of the motion, indicating constant acceleration. (c) Acceleration has the constant value of 5.0 m/s^2 over the time interval plotted.

Figure 2.45: A U.S. Air Force jet car speeds down a track. (credit: Matt Trostle, Flickr)

The graph of position versus time in link(a) is a curve rather than a straight line. The slope of the curve becomes steeper as time progresses, showing that the velocity is increasing over time. The slope at any point on a position-versus-time graph is the instantaneous velocity at that point. It is found by drawing a straight line tangent to the curve at the point of interest and taking the slope of this straight line. Tangent lines are shown for two points in link(a). If this is done at every point on the curve and the values are plotted against time, then the graph of velocity versus time shown in link(b) is obtained. Furthermore, the slope of the graph of velocity versus time is acceleration, which is shown in link(c).

Determining Instantaneous Velocity from the Slope at a Point: Jet Car

Calculate the velocity of the jet car at a time of 25 s by finding the slope of the x vs. t graph in the graph below.



t (s)	x (m)
0	200
5	338
10	600
15	988
20	1500
25	2138
30	2900

Figure 2.46: The slope of an x vs. t graph is velocity. This is shown at two points. Instantaneous velocity at any point is the slope of the tangent at that point.

Strategy

The slope of a curve at a point is equal to the slope of a straight line tangent to the curve at that point. This principle is illustrated in link, where Q is the point at $t = 25$ s.

Solution

1. Find the tangent line to the curve at $t = 25$ s.

2. Determine the endpoints of the tangent. These correspond to a position of 1300 m at time 19 s and a position of 3120 m at time 32 s.

3. Plug these endpoints into the equation to solve for the slope, v.

$$\text{slope} = v_Q = \frac{\Delta x_Q}{\Delta t_Q} = \frac{(3120\text{ m} - 1300\text{ m})}{(32\text{ s} - 19\text{ s})}$$

Thus,

$$v_Q = \frac{1820\text{ m}}{13\text{ s}} = 140\text{ m/s}.$$

Discussion

This is the value given in this figure's table for v at $t = 25$ s. The value of 140 m/s for v_Q is plotted in link. The entire graph of v vs. t can be obtained in this fashion.

Carrying this one step further, we note that the slope of a velocity versus time graph is acceleration. Slope is rise divided by run; on a v vs. t graph, rise = change in velocity Δv and run = change in time Δt.

The Slope of v vs. t

The slope of a graph of velocity v vs. time t is acceleration a.

$$\text{slope} = \frac{\Delta v}{\Delta t} = a$$

Since the velocity versus time graph in link(b) is a straight line, its slope is the same everywhere, implying that acceleration is constant. Acceleration versus time is graphed in link(c).

Additional general information can be obtained from link and the expression for a straight line, $y = \text{mx} + b$.

In this case, the vertical axis y is V, the intercept b is v_0, the slope m is a, and the horizontal axis x is t. Substituting these symbols yields

$$v = v_0 + \text{at}.$$

A general relationship for velocity, acceleration, and time has again been obtained from a graph. Notice that this equation was also derived algebraically from other motion equations in Motion Equations for Constant Acceleration in One Dimension.

It is not accidental that the same equations are obtained by graphical analysis as by algebraic techniques. In fact, an important way to *discover* physical relationships is to measure various physical quantities and then make graphs of one quantity against another to see if they are correlated in any way. Correlations imply physical relationships and might be shown by smooth graphs such as those above. From such graphs, mathematical relationships can sometimes be postulated. Further experiments are then performed to determine the validity of the hypothesized relationships.

2.9.5 Graphs of Motion Where Acceleration is Not Constant

Now consider the motion of the jet car as it goes from 165 m/s to its top velocity of 250 m/s, graphed in link. Time again starts at zero, and the initial position and velocity are 2900 m and 165 m/s, respectively. (These were the final position and velocity of the car in the motion graphed in link.) Acceleration gradually decreases from 5.0 m/s^2 to zero when the car hits 250 m/s. The slope of the x vs. t graph increases until $t = 55$ s, after which time the slope is constant. Similarly, velocity increases until 55 s and then becomes constant, since acceleration decreases to zero at 55 s and remains zero afterward.

Calculating Acceleration from a Graph of Velocity versus Time

Calculate the acceleration of the jet car at a time of 25 s by finding the slope of the v vs. t graph in link(b).

Strategy

The slope of the curve at $t = 25$ s is equal to the slope of the line tangent at that point, as illustrated in link(b).

Solution

Determine endpoints of the tangent line from the figure, and then plug them into the equation to solve for slope, a.

$$\text{slope} = \frac{\Delta v}{\Delta t} = \frac{(260\ \text{m/s} - 210\ \text{m/s})}{(51\ \text{s} - 1.0\ s)}$$

$$a = \frac{50\ \text{m/s}}{50\ \text{s}} = 1.0\ m/\text{s}^2.$$

Discussion

Note that this value for a is consistent with the value plotted in link(c) at $t = 25$ s.

A graph of position versus time can be used to generate a graph of velocity versus time, and a graph of velocity versus time can be used to generate a

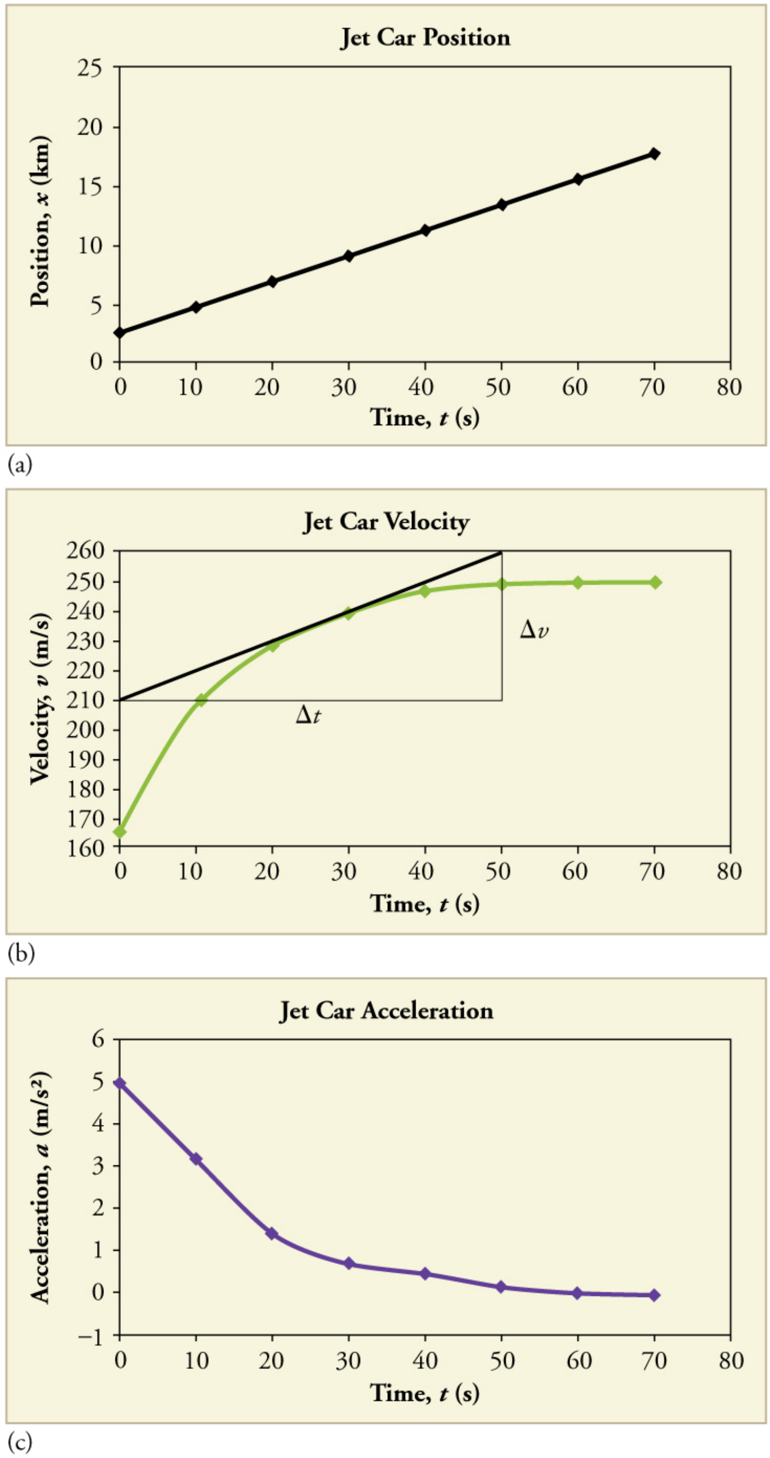
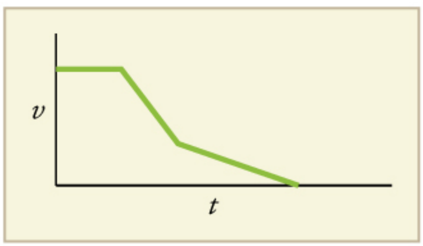


Figure 2.47: Graphs of motion of a jet-powered car as it reaches its top velocity. This motion begins where the motion in link ends. (a) The slope of this graph is velocity; it is plotted in the next graph. (b) The velocity gradually approaches its top value. The slope of this graph is acceleration; it is plotted in the final graph. (c) Acceleration gradually declines to zero when velocity becomes constant.

graph of acceleration versus time. We do this by finding the slope of the graphs at every point. If the graph is linear (i.e., a line with a constant slope), it is easy to find the slope at any point and you have the slope for every point. Graphical analysis of motion can be used to describe both specific and general characteristics of kinematics. Graphs can also be used for other topics in physics. An important aspect of exploring physical relationships is to graph them and look for underlying relationships.

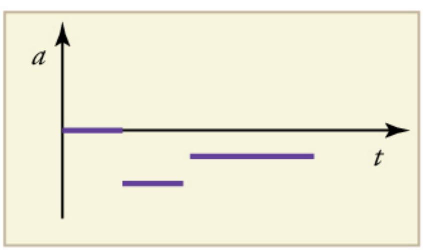
Check Your Understanding

A graph of velocity vs. time of a ship coming into a harbor is shown below. (a) Describe the motion of the ship based on the graph. (b)What would a graph of the ship's acceleration look like?



a The ship moves at constant velocity and then begins to decelerate at a constant rate. At some point, its deceleration rate decreases. It maintains this lower deceleration rate until it stops moving.

b A graph of acceleration vs. time would show zero acceleration in the first leg, large and constant negative acceleration in the second leg, and constant negative acceleration.

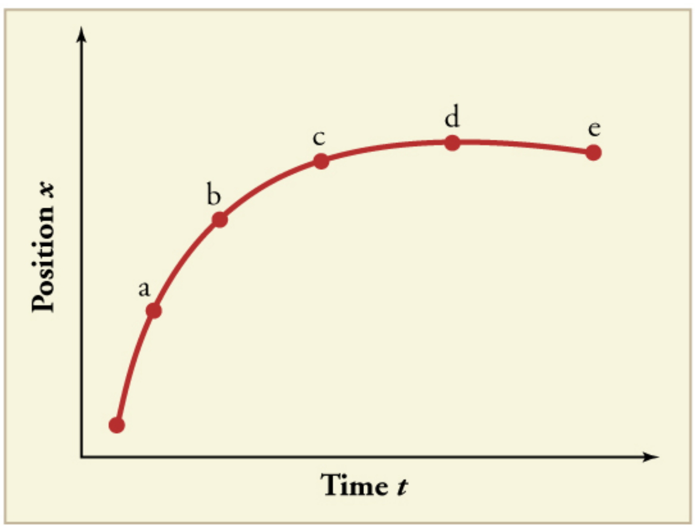


2.9.6 Section Summary

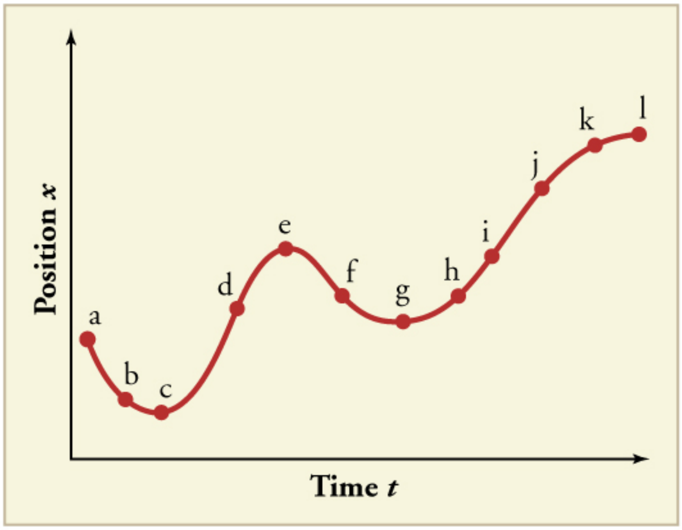
- Graphs of motion can be used to analyze motion.
- Graphical solutions yield identical solutions to mathematical methods for deriving motion equations.
- The slope of a graph of position x vs. time t is velocity v.
- The slope of a graph of velocity v vs. time t graph is acceleration a.
- Average velocity, instantaneous velocity, and acceleration can all be obtained by analyzing graphs.

2.9.7 Conceptual Questions

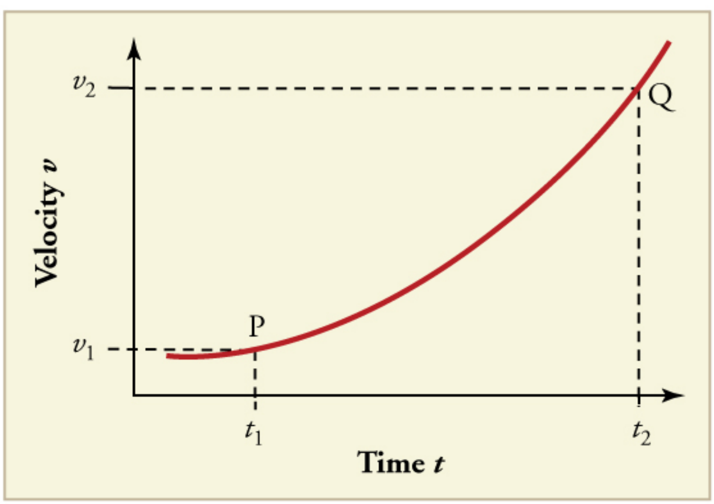
a Explain how you can use the graph of position versus time in link to describe the change in velocity over time. Identify (b) the time (t_a, t_b, t_c, t_d, or t_e) at which the instantaneous velocity is greatest, (c) the time at which it is zero, and (d) the time at which it is negative.



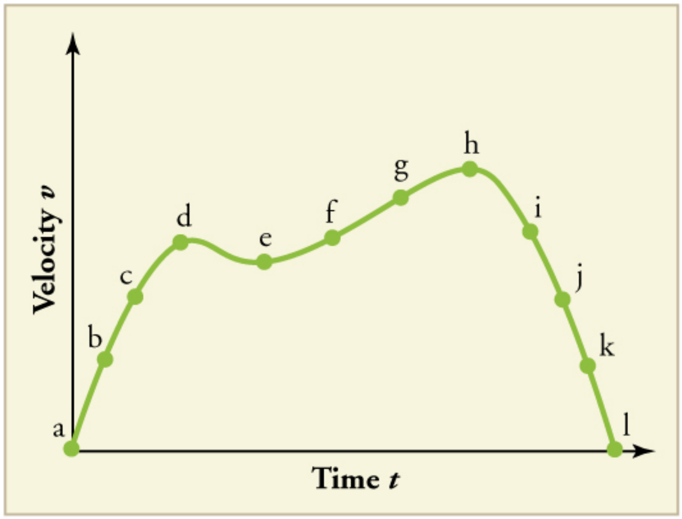
a Sketch a graph of velocity versus time corresponding to the graph of position versus time given in link. (b) Identify the time or times (t_a, t_b, t_c, etc.) at which the instantaneous velocity is greatest. (c) At which times is it zero? (d) At which times is it negative?



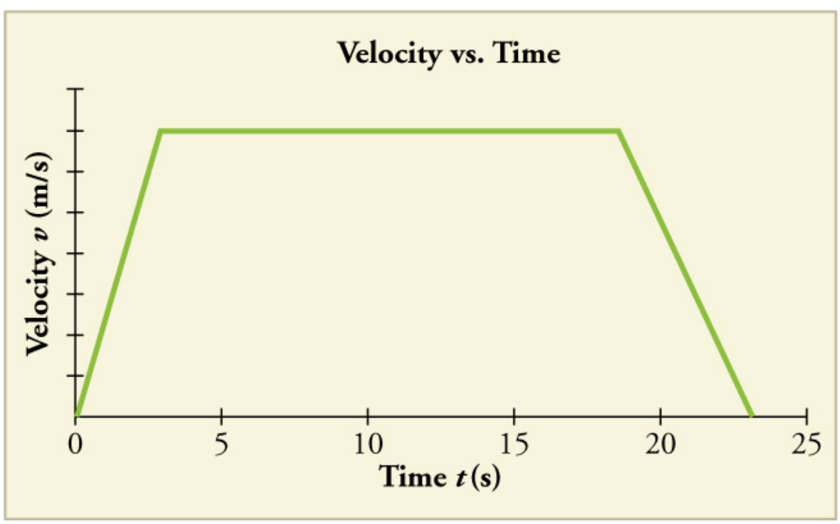
a Explain how you can determine the acceleration over time from a velocity versus time graph such as the one in link. (b) Based on the graph, how does acceleration change over time?



a Sketch a graph of acceleration versus time corresponding to the graph of velocity versus time given in link. (b) Identify the time or times (t_a, t_b, t_c, etc.) at which the acceleration is greatest. (c) At which times is it zero? (d) At which times is it negative?



Consider the velocity vs. time graph of a person in an elevator shown in link. Suppose the elevator is initially at rest. It then accelerates for 3 seconds, maintains that velocity for 15 seconds, then decelerates for 5 seconds until it stops. The acceleration for the entire trip is not constant so we cannot use the equations of motion from Motion Equations for Constant Acceleration in One Dimension for the complete trip. (We could, however, use them in the three individual sections where acceleration is a constant.) Sketch graphs of (a) position vs. time and (b) acceleration vs. time for this trip.

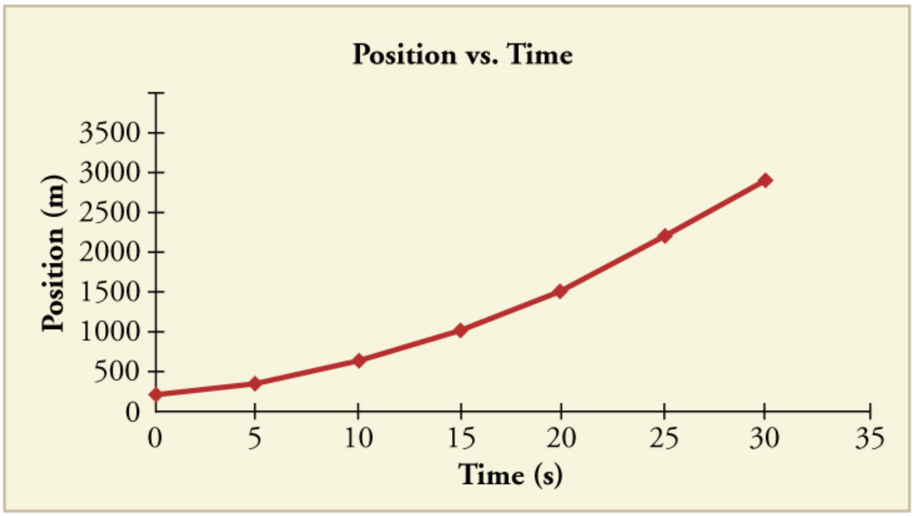


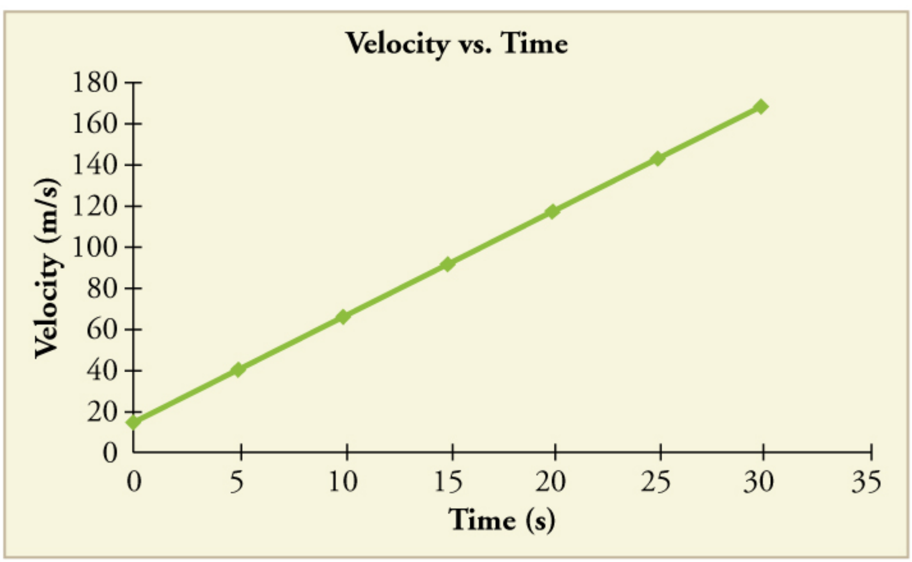
A cylinder is given a push and then rolls up an inclined plane. If the origin is the starting point, sketch the position, velocity, and acceleration of the cylinder vs. time as it goes up and then down the plane.

2.9.8 Problems & Exercises

Note: There is always uncertainty in numbers taken from graphs. If your answers differ from expected values, examine them to see if they are within data extraction uncertainties estimated by you.

a By taking the slope of the curve in link, verify that the velocity of the jet car is 115 m/s at $t = 20$ s. (b) By taking the slope of the curve at any point in link, verify that the jet car's acceleration is 5.0 m/s^2.

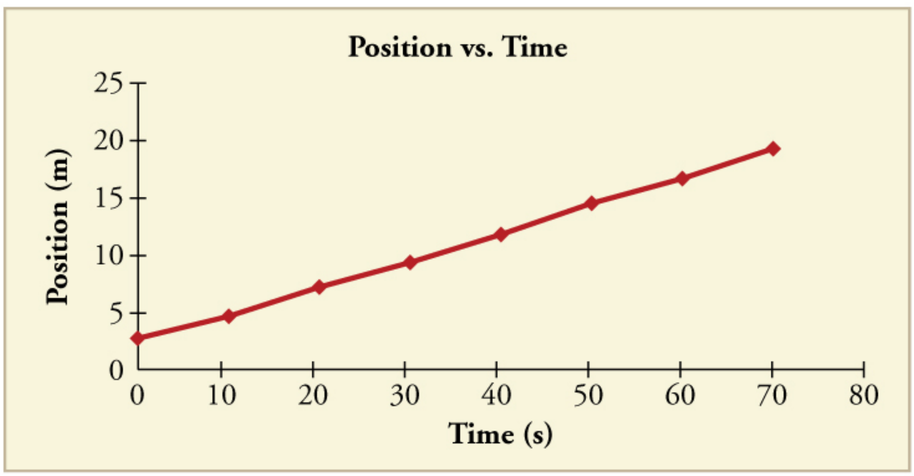




a 115 m/s

b 5.0 m/s^2

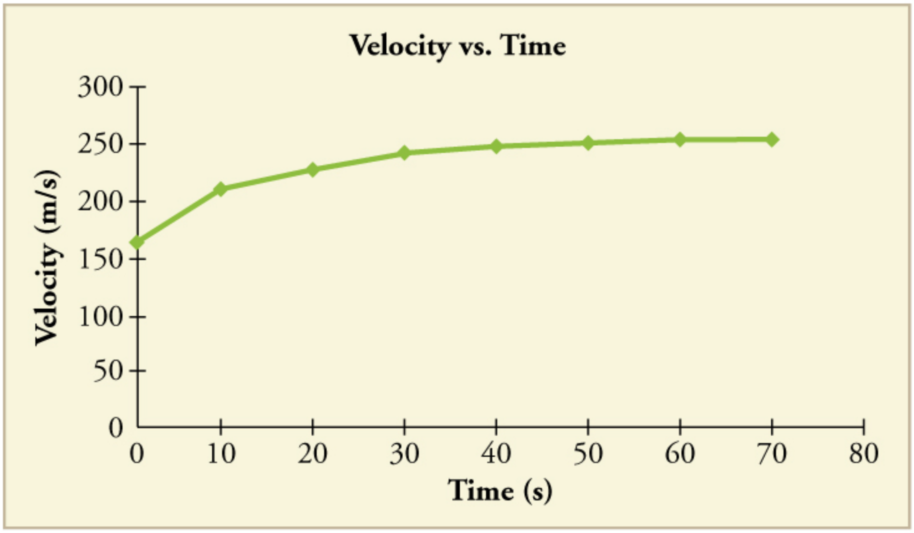
Using approximate values, calculate the slope of the curve in link to verify that the velocity at $t = 10.0$ s is 0.208 m/s. Assume all values are known to 3 significant figures.



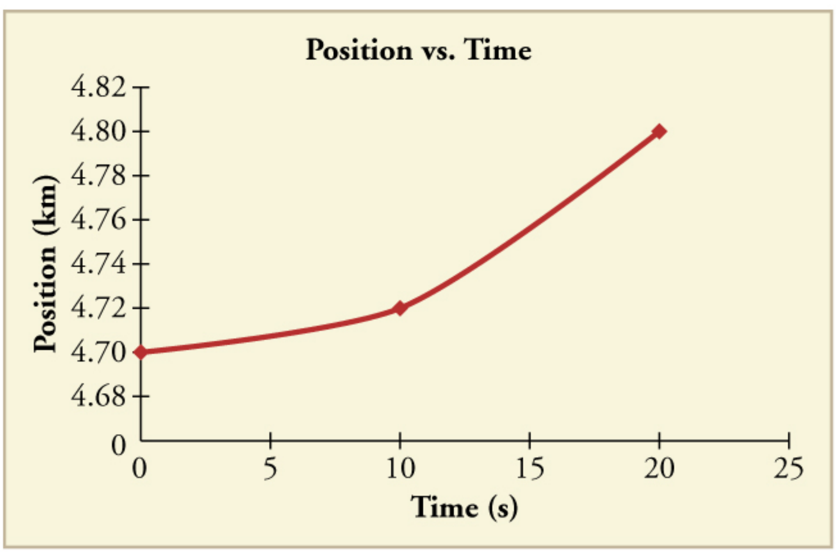
Using approximate values, calculate the slope of the curve in link to verify that the velocity at $t = 30.0$ s is 0.238 m/s. Assume all values are known to 3 significant figures.

$v = \frac{(11.7 - 6.95) \times 10^3 \text{ m}}{(40.0 - 20.0) \text{ s}} = 238 \text{ m/s}$

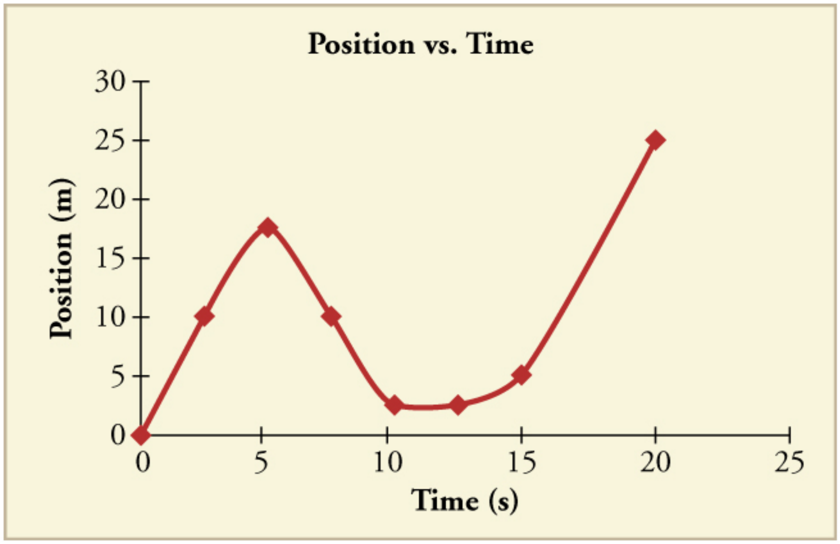
By taking the slope of the curve in link, verify that the acceleration is approximately 3.2 m/s^2 at $t = 10$ s.

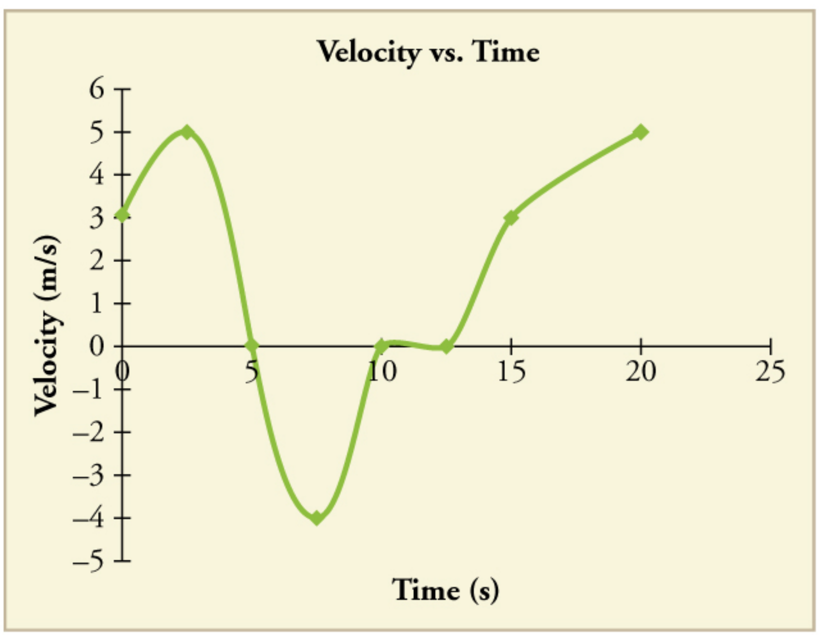


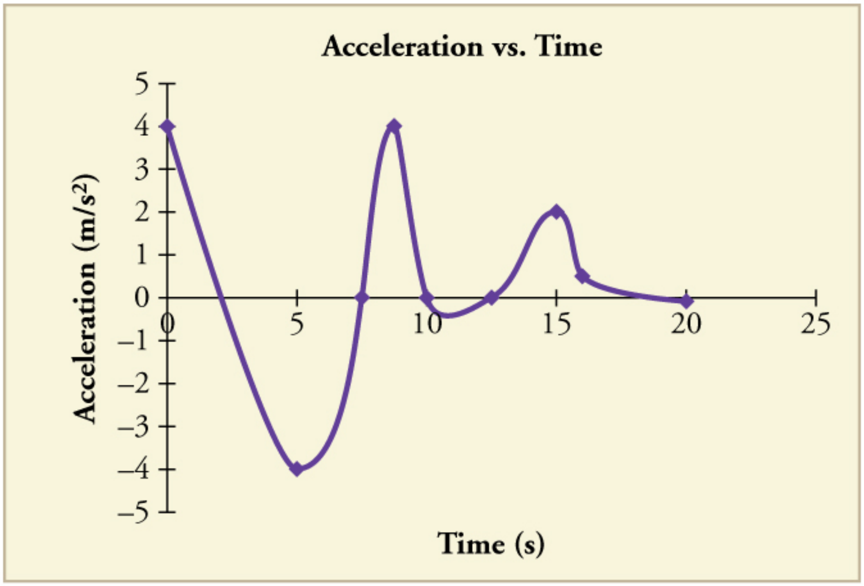
Construct the position graph for the subway shuttle train as shown in link(a). Your graph should show the position of the train, in kilometers, from t = 0 to 20 s. You will need to use the information on acceleration and velocity given in the examples for this figure.



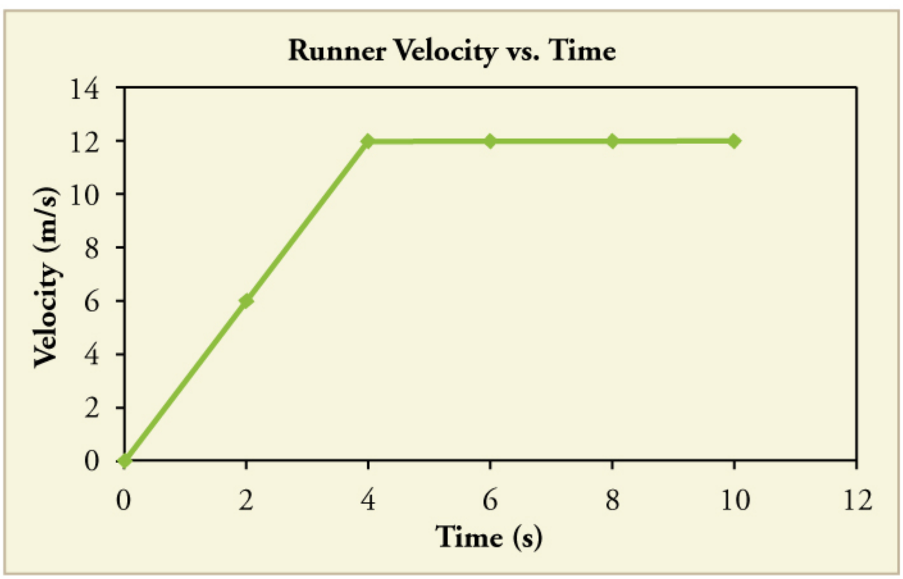
a Take the slope of the curve in link to find the jogger's velocity at $t = 2.5\ s$. (b) Repeat at 7.5 s. These values must be consistent with the graph in link.


Position vs. Time
Position (m)
30
25
20
15
10
5
0
0
5
10
15
20
25
Time (s)


Velocity vs. Time
Velocity (m/s)
6
5
4
3
2
1
0
−1
−2
−3
−4
−5
0
5
10
15
20
25
Time (s)



A graph of $v(t)$ is shown for a world-class track sprinter in a 100-m race. (See link). (a) What is his average velocity for the first 4 s? (b) What is his instantaneous velocity at $t = 5\ s$? (c) What is his average acceleration between 0 and 4 s? (d) What is his time for the race?



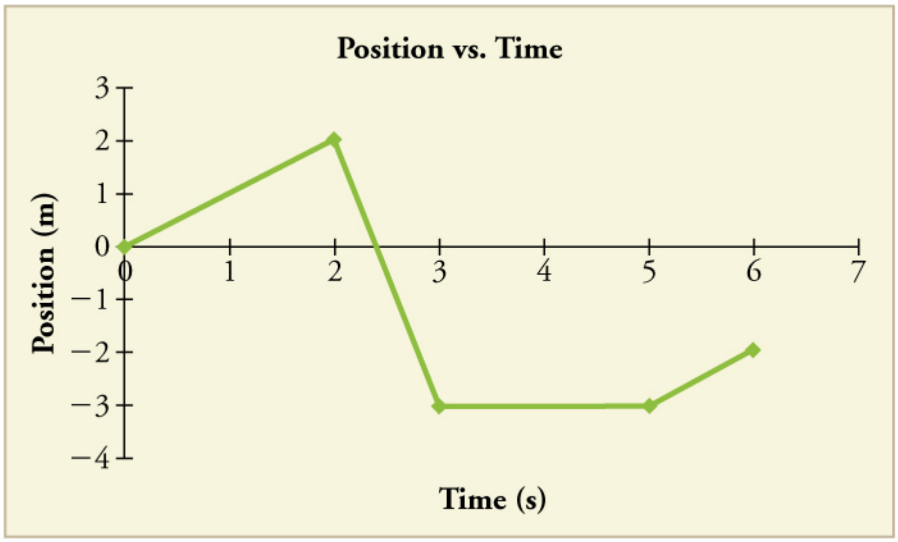
a 6 m/s

b 12 m/s

c 3 m/s^2

d 10 s

link shows the position graph for a particle for 6 s. (a) Draw the corresponding Velocity vs. Time graph. (b) What is the acceleration between 0 s and 2 s? (c) What happens to the acceleration at exactly 2 s?



2.9.9 Glossary

independent variable the variable that the dependent variable is measured with respect to; usually plotted along the x-axis

dependent variable the variable that is being measured; usually plotted along the y-axis

slope the difference in y-value (the rise) divided by the difference in x-value (the run) of two points on a straight line

y-intercept the y-value when $x = 0$, or when the graph crosses the y-axis

Chapter 3

Two-Dimensional Kinematics

3.1 Connection for AP® Courses

class="introduction" class="section-summary" title="Section Summary" class="conceptual-questions" title="Conceptual Questions" class="problems-exercises" title="Problems & Exercises" class="ap-test-prep" title="Test Prep for AP Courses"

Most instances of motion in everyday life involve changes in displacement and velocity that occur in more than one direction. For example, when you take a long road trip, you drive on different roads in different directions for different amounts of time at different speeds. How can these motions all be combined to determine information about the trip such as the total displacement and average velocity? If you kick a ball from ground level at some angle above the horizontal, how can you describe its motion? To what maximum height does the object rise above the ground? How long is the object in the air? How much horizontal distance is covered before the ball lands? To answer questions such as these, we need to describe motion in two dimensions.

Examining two-dimensional motion requires an understanding of both the scalar and the vector quantities associated with the motion. You will learn how to combine vectors to incorporate both the magnitude and direction of vectors into your analysis. You will learn strategies for simplifying the calculations involved by choosing the appropriate reference frame and by treating each dimension of the motion separately as a one-dimensional problem, but you will also see that the motion itself occurs in the same way regardless of your chosen reference frame (Essential Knowledge 3.A.1).

This chapter lays a necessary foundation for examining interactions of objects

Figure 3.1: Everyday motion that we experience is, thankfully, rarely as tortuous as a rollercoaster ride like this—the Dragon Khan in Spain's Universal Port Aventura Amusement Park. However, most motion is in curved, rather than straight-line, paths. Motion along a curved path is two- or three-dimensional motion, and can be described in a similar fashion to one-dimensional motion. (credit: Boris23/Wikimedia Commons)

described by forces (Big Idea 3). Changes in direction result from acceleration, which necessitates force on an object. In this chapter, you will concentrate on describing motion that involves changes in direction. In later chapters, you will apply this understanding as you learn about how forces cause these motions (Enduring Understanding 3.A). The concepts in this chapter support:

Big Idea 3 The interactions of an object with other objects can be described by forces.

Enduring Understanding 3.A All forces share certain common characteristics when considered by observers in inertial reference frames.

Essential Knowledge 3.A.1 An observer in a particular reference frame can describe the motion of an object using such quantities as position, displacement, distance, velocity, speed, and acceleration.

3.2 Kinematics in Two Dimensions: An Introduction

3.2.1 Learning Objectives

By the end of this section, you will be able to:

- Observe that motion in two dimensions consists of horizontal and vertical components.
- Understand the independence of horizontal and vertical vectors in two-dimensional motion.

The information presented in this section supports the following AP® learning objectives and science practices:

- **3.A.1.1** The student is able to express the motion of an object using narrative, mathematical, and graphical representations. **(S.P. 1.5, 2.1, 2.2)**
- **3.A.1.2** The student is able to design an experimental investigation of the motion of an object. **(S.P. 4.2)**
- **3.A.1.3** The student is able to analyze experimental data describing the motion of an object and is able to express the results of the analysis using narrative, mathematical, and graphical representations. **(S.P. 5.1)**

3.2.2 Two-Dimensional Motion: Walking in a City

Suppose you want to walk from one point to another in a city with uniform square blocks, as pictured in link.

The straight-line path that a helicopter might fly is blocked to you as a pedestrian, and so you are forced to take a two-dimensional path, such as the one

Figure 3.2: Walkers and drivers in a city like New York are rarely able to travel in straight lines to reach their destinations. Instead, they must follow roads and sidewalks, making two-dimensional, zigzagged paths. (credit: Margaret W. Carruthers)

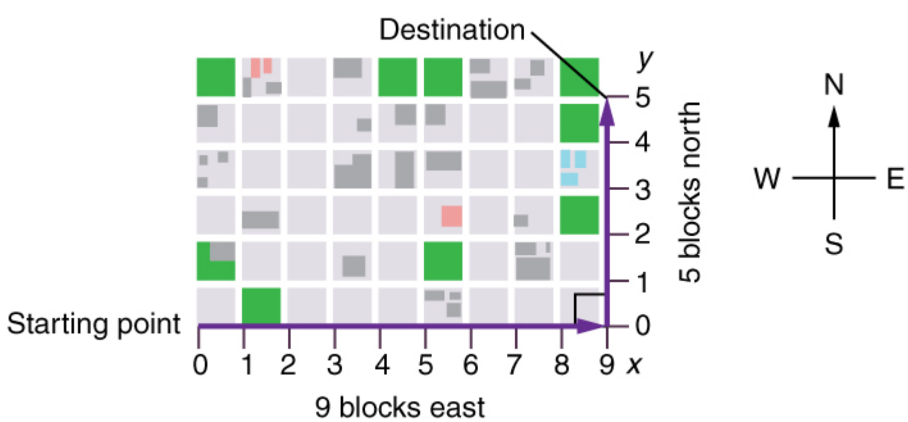


Figure 3.3: A pedestrian walks a two-dimensional path between two points in a city. In this scene, all blocks are square and are the same size.

shown. You walk 14 blocks in all, 9 east followed by 5 north. What is the straight-line distance?

An old adage states that the shortest distance between two points is a straight line. The two legs of the trip and the straight-line path form a right triangle, and so the Pythagorean theorem, $a^2 + b^2 = c^2$, can be used to find the straight-line distance.

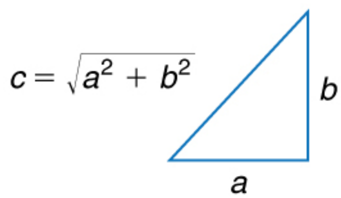


Figure 3.4: The Pythagorean theorem relates the length of the legs of a right triangle, labeled a and b, with the hypotenuse, labeled c. The relationship is given by: $a^2 + b^2 = c^2$. This can be rewritten, solving for c : $c = \sqrt{a^2 + b^2}$.

The hypotenuse of the triangle is the straight-line path, and so in this case its length in units of city blocks is $\sqrt{(9 \text{ blocks})^2 + (5 \text{ blocks})^2} = 10.3$ blocks, considerably shorter than the 14 blocks you walked. (Note that we are using three significant figures in the answer. Although it appears that "9" and "5" have only one significant digit, they are discrete numbers. In this case "9 blocks" is the same as "9.0 or 9.00 blocks." We have decided to use three significant figures in the answer in order to show the result more precisely.)

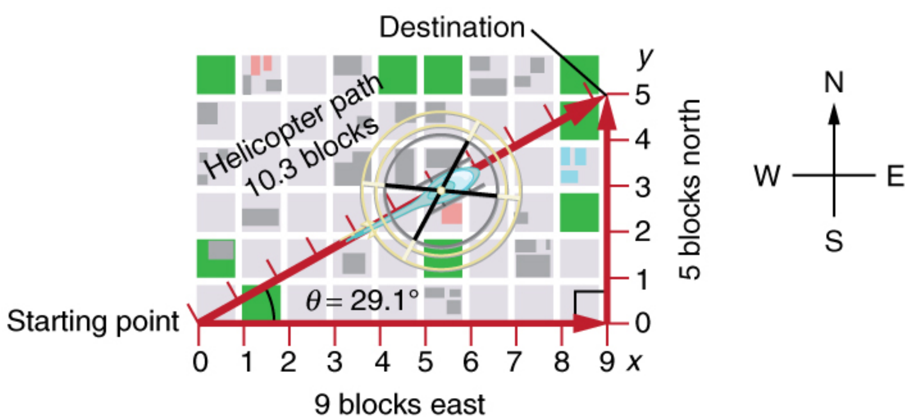


Figure 3.5: The straight-line path followed by a helicopter between the two points is shorter than the 14 blocks walked by the pedestrian. All blocks are square and the same size.

The fact that the straight-line distance (10.3 blocks) in link is less than the total distance walked (14 blocks) is one example of a general characteristic of vectors. (Recall that vectors are quantities that have both magnitude and direction.)

As for one-dimensional kinematics, we use arrows to represent vectors. The length of the arrow is proportional to the vector's magnitude. The arrow's length is indicated by hash marks in link and link. The arrow points in the same direction as the vector. For two-dimensional motion, the path of an object can be represented with three vectors: one vector shows the straight-line path between the initial and final points of the motion, one vector shows the horizontal component of the motion, and one vector shows the vertical component of the motion. The horizontal and vertical components of the motion add together to give the straight-line path. For example, observe the three vectors in link. The first represents a 9-block displacement east. The second represents a 5-block displacement north. These vectors are added to give the third vector, with a 10.3-block total displacement. The third vector is the straight-line path between the two points. Note that in this example, the vectors that we are adding are perpendicular to each other and thus form a right triangle. This means that we can use the Pythagorean theorem to calculate the magnitude of the total displacement. (Note that we cannot use the Pythagorean theorem to add vectors that are not perpendicular. We will develop techniques for adding vectors having any direction, not just those perpendicular to one another, in Vector Addition and Subtraction: Graphical Methods and Vector Addition and Subtraction: Analytical Methods.)

3.2.3 The Independence of Perpendicular Motions

The person taking the path shown in link walks east and then north (two perpendicular directions). How far he or she walks east is only affected by his or her motion eastward. Similarly, how far he or she walks north is only affected by his or her motion northward.

Independence of Motion

The horizontal and vertical components of two-dimensional motion are independent of each other. Any motion in the horizontal direction does not affect motion in the vertical direction, and vice versa.

This is true in a simple scenario like that of walking in one direction first, followed by another. It is also true of more complicated motion involving movement in two directions at once. For example, let's compare the motions of two baseballs. One baseball is dropped from rest. At the same instant, another is thrown horizontally from the same height and follows a curved path. A stroboscope has captured the positions of the balls at fixed time intervals as they fall.

Applying the Science Practices: Independence of Horizontal and Vertical Motion or Maximum Height and Flight Time

Choose one of the following experiments to design:

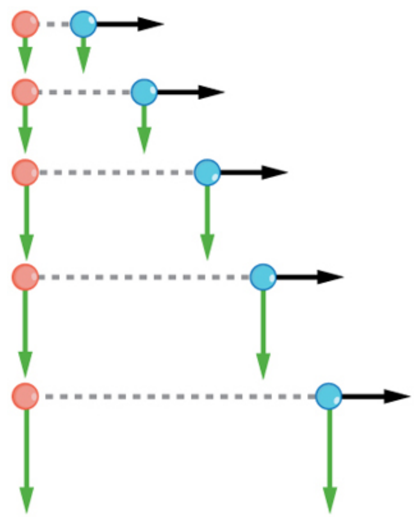


Figure 3.6: This shows the motions of two identical balls—one falls from rest, the other has an initial horizontal velocity. Each subsequent position is an equal time interval. Arrows represent horizontal and vertical velocities at each position. The ball on the right has an initial horizontal velocity, while the ball on the left has no horizontal velocity. Despite the difference in horizontal velocities, the vertical velocities and positions are identical for both balls. This shows that the vertical and horizontal motions are independent.

Design an experiment to confirm what is shown in Figure 3.6, that the vertical motion of the two balls is independent of the horizontal motion. As you think about your experiment, consider the following questions:

- How will you measure the horizontal and vertical positions of each ball over time? What equipment will this require?
- How will you measure the time interval between each of your position measurements? What equipment will this require?
- If you were to create separate graphs of the horizontal velocity for each ball versus time, what do you predict it would look like? Explain.
- If you were to compare graphs of the vertical velocity for each ball versus time, what do you predict it would look like? Explain.
- If there is a significant amount of air resistance, how will that affect each of your graphs?

Design a two-dimensional ballistic motion experiment that demonstrates the relationship between the maximum height reached by an object and the object's time of flight. As you think about your experiment, consider the following questions:

- How will you measure the maximum height reached by your object?
- How can you take advantage of the symmetry of an object in ballistic motion launched from ground level, reaching maximum height, and returning to ground level?
- Will it make a difference if your object has no horizontal component to its velocity? Explain.
- Will you need to measure the time at multiple different positions? Why or why not?
- Predict what a graph of travel time versus maximum height will look like. Will it be linear? Parabolic? Horizontal? Explain the shape of your predicted graph qualitatively or quantitatively.
- If there is a significant amount of air resistance, how will that affect your measurements and your results?

It is remarkable that for each flash of the strobe, the vertical positions of the two balls are the same. This similarity implies that the vertical motion is independent of whether or not the ball is moving horizontally. (Assuming no air resistance, the vertical motion of a falling object is influenced by gravity only, and not by any horizontal forces.) Careful examination of the ball thrown horizontally shows that it travels the same horizontal distance between flashes. This is due to the fact that there are no additional forces on the ball in the horizontal direction after it is thrown. This result means that the horizontal velocity is constant, and affected neither by vertical motion nor by gravity (which is vertical). Note that this case is true only for ideal conditions. In the real world, air resistance will affect the speed of the balls in both directions.

The two-dimensional curved path of the horizontally thrown ball is composed of two independent one-dimensional motions (horizontal and vertical). The key to analyzing such motion, called **projectile motion**, is to **resolve** (break) it into motions along perpendicular directions. Resolving two-dimensional motion into perpendicular components is possible because the components are independent. We shall see how to resolve vectors in Vector Addition and Subtraction: Graphical Methods and Vector Addition and Subtraction: Analytical Methods. We will find such techniques to be useful in many areas of physics.

Ladybug Motion 2D

Learn about position, velocity and acceleration vectors. Move the ladybug by setting the position, velocity or acceleration, and see how the vectors change. Choose linear, circular or elliptical motion, and record and playback the motion to analyze the behavior.

3.2.4 Test Prep for AP Courses

A ball is thrown at an angle of 45 degrees above the horizontal. Which of the following best describes the acceleration of the ball from the instant after it leaves the thrower's hand until the time it hits the ground?

a. Always in the same direction as the motion, initially positive and gradually dropping to zero by the time it hits the ground
b. Initially positive in the upward direction, then zero at maximum height, then negative from there until it hits the ground
c. Always in the opposite direction as the motion, initially positive and gradually dropping to zero by the time it hits the ground
d. Always in the downward direction with the same constant value

d

In an experiment, a student launches a ball with an initial horizontal velocity at an elevation 2 meters above ground. The ball follows a parabolic trajectory until it hits the ground. Which of the following accurately describes the graph of the ball's vertical acceleration versus time (taking the downward direction to be negative)?

a. A negative value that does not change with time
b. A gradually increasing negative value (straight line)
c. An increasing rate of negative values over time (parabolic curve)
d. Zero at all times since the initial motion is horizontal

A student wishes to design an experiment to show that the acceleration of an object is independent of the object's velocity. To do this, ball A is launched horizontally with some initial speed at an elevation 1.5 meters above the ground, ball B is dropped from rest 1.5 meters above the ground, and ball C is launched vertically with some initial speed at an elevation 1.5 meters above the ground. What information would the student need to collect about each ball in order to test the hypothesis?

We would need to know the horizontal and vertical positions of each ball at several times. From that data, we could deduce the velocities over several time intervals and also the accelerations (both horizontal and vertical) for each ball over several time intervals.

3.2.5 Summary

- The shortest path between any two points is a straight line. In two dimensions, this path can be represented by a vector with horizontal and vertical components.
- The horizontal and vertical components of a vector are independent of one another. Motion in the horizontal direction does not affect motion in the vertical direction, and vice versa.

3.2.6 Glossary

vector a quantity that has both magnitude and direction; an arrow used to represent quantities with both magnitude and direction

3.3 Vector Addition and Subtraction: Graphical Methods

3.3.1 Learning Objectives

By the end of this section, you will be able to:

- Understand the rules of vector addition, subtraction, and multiplication.
- Apply graphical methods of vector addition and subtraction to determine the displacement of moving objects.

The information presented in this section supports the following AP® learning objectives and science practices:

- **3.A.1.1** The student is able to express the motion of an object using narrative, mathematical, and graphical representations. **(S.P. 1.5, 2.1, 2.2)**
- **3.A.1.3** The student is able to analyze experimental data describing the motion of an object and is able to express the results of the analysis using narrative, mathematical, and graphical representations. **(S.P. 5.1)**

3.3.2 Vectors in Two Dimensions

A vector is a quantity that has magnitude and direction. Displacement, velocity, acceleration, and force, for example, are all vectors. In one-dimensional, or straight-line, motion, the direction of a vector can be given simply by a plus or minus sign. In two dimensions (2-d), however, we specify the direction of a vector relative to some reference frame (i.e., coordinate system), using an

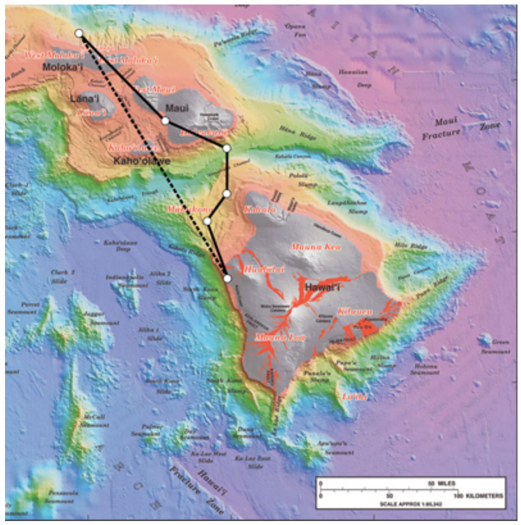


Figure 3.7: Displacement can be determined graphically using a scale map, such as this one of the Hawaiian Islands. A journey from Hawai'i to Moloka'i has a number of legs, or journey segments. These segments can be added graphically with a ruler to determine the total two-dimensional displacement of the journey. (credit: US Geological Survey)

arrow having length proportional to the vector's magnitude and pointing in the direction of the vector.

link shows such a *graphical representation of a vector*, using as an example the total displacement for the person walking in a city considered in Kinematics in Two Dimensions: An Introduction. We shall use the notation that a boldface symbol, such as D, stands for a vector. Its magnitude is represented by the symbol in italics, D, and its direction by θ.

Vectors in this Text

In this text, we will represent a vector with a boldface variable. For example, we will represent the quantity force with the vector F, which has both magnitude and direction. The magnitude of the vector will be represented by a variable in italics, such as F, and the direction of the variable will be given by an angle θ.

3.3.3 Vector Addition: Head-to-Tail Method

The head-to-tail method is a graphical way to add vectors, described in link below and in the steps following. The tail of the vector is the starting point of the vector, and the head (or tip) of a vector is the final, pointed end of the arrow.

***Step 1.* Draw an arrow to represent the first vector (9 blocks to the**

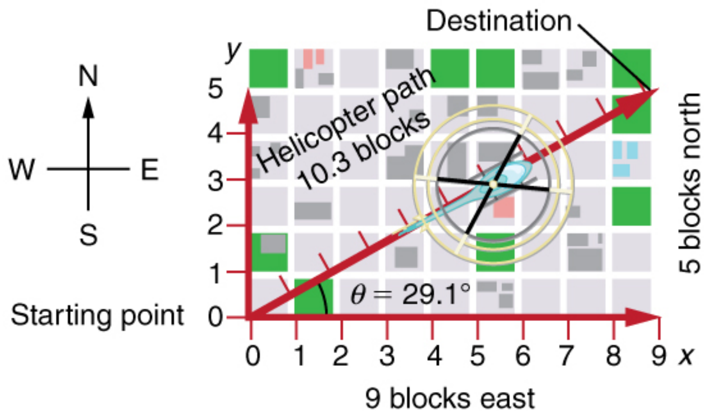


Figure 3.8: A person walks 9 blocks east and 5 blocks north. The displacement is 10.3 blocks at an angle 29.1° north of east.

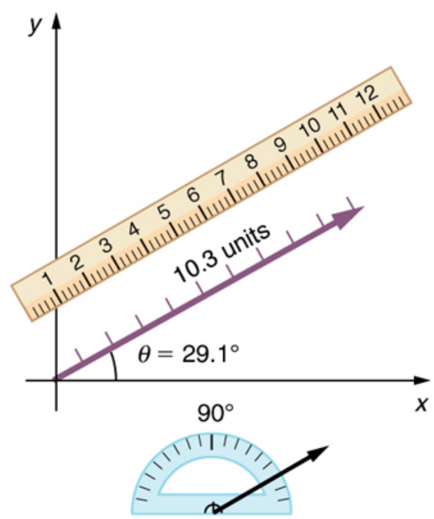


Figure 3.9: To describe the resultant vector for the person walking in a city considered in link graphically, draw an arrow to represent the total displacement vector D. Using a protractor, draw a line at an angle θ relative to the east-west axis. The length D of the arrow is proportional to the vector's magnitude and is measured along the line with a ruler. In this example, the magnitude D of the vector is 10.3 units, and the direction θ is 29.1° north of east.

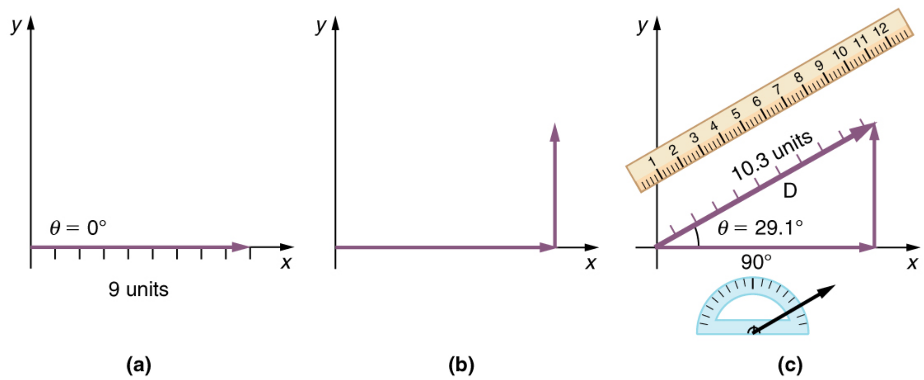
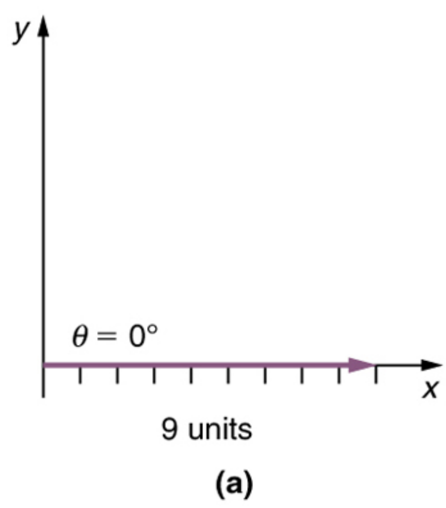
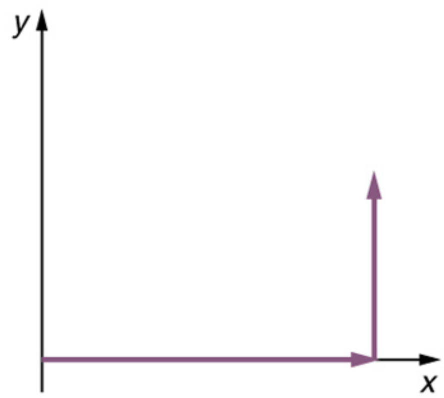


Figure 3.10: **Head-to-Tail Method:** The head-to-tail method of graphically adding vectors is illustrated for the two displacements of the person walking in a city considered in link. (a) Draw a vector representing the displacement to the east. (b) Draw a vector representing the displacement to the north. The tail of this vector should originate from the head of the first, east-pointing vector. (c) Draw a line from the tail of the east-pointing vector to the head of the north-pointing vector to form the sum or resultant vector D. The length of the arrow D is proportional to the vector's magnitude and is measured to be 10.3 units . Its direction, described as the angle with respect to the east (or horizontal axis) θ is measured with a protractor to be 29.1°.

east) using a ruler and protractor.



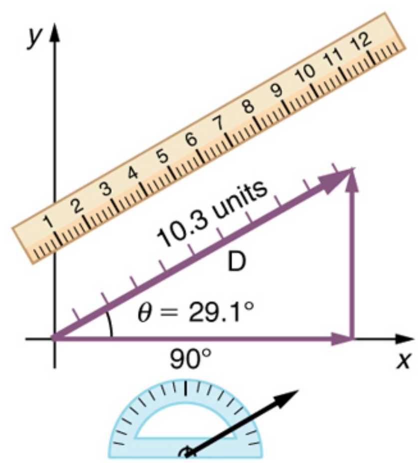
Step 2. Now draw an arrow to represent the second vector (5 blocks to the north). *Place the tail of the second vector at the head of the first vector.*



(b)

Step 3. *If there are more than two vectors, continue this process for each vector to be added. Note that in our example, we have only two vectors, so we have finished placing arrows tip to tail.*

Step 4. *Draw an arrow from the tail of the first vector to the head of the last vector.* This is the resultant, or the sum, of the other vectors.



(c)

Step 5. To get the magnitude of the resultant, *measure its length with a ruler. (Note that in most calculations, we will use the Pythagorean theorem to determine this length.)*

Step 6. To get the direction of the resultant, **measure the angle it makes with the reference frame using a protractor. (Note that in most calculations, we will use trigonometric relationships to determine this**

angle.)

The graphical addition of vectors is limited in accuracy only by the precision with which the drawings can be made and the precision of the measuring tools. It is valid for any number of vectors.

Adding Vectors Graphically Using the Head-to-Tail Method: A Woman Takes a Walk

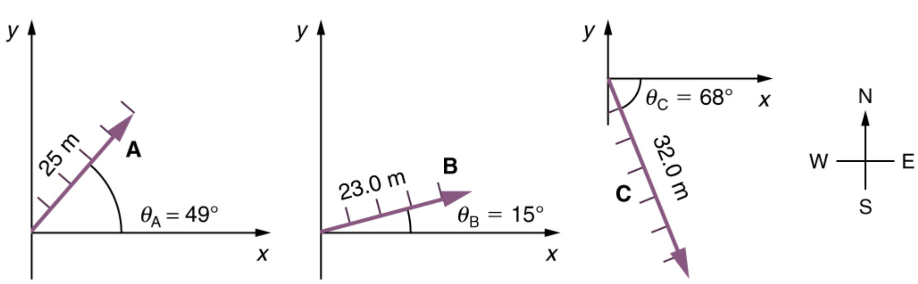
Use the graphical technique for adding vectors to find the total displacement of a person who walks the following three paths (displacements) on a flat field. First, she walks 25.0 m in a direction 49.0° north of east. Then, she walks 23.0 m heading 15.0° north of east. Finally, she turns and walks 32.0 m in a direction 68.0° south of east.

Strategy

Represent each displacement vector graphically with an arrow, labeling the first A, the second B, and the third C, making the lengths proportional to the distance and the directions as specified relative to an east-west line. The head-to-tail method outlined above will give a way to determine the magnitude and direction of the resultant displacement, denoted **R**.

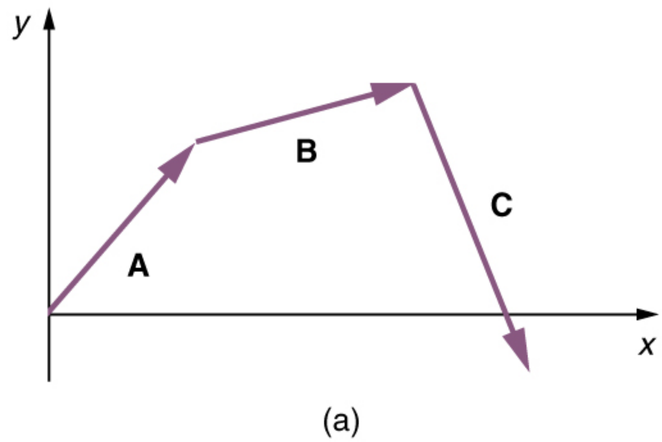
Solution

1 Draw the three displacement vectors.

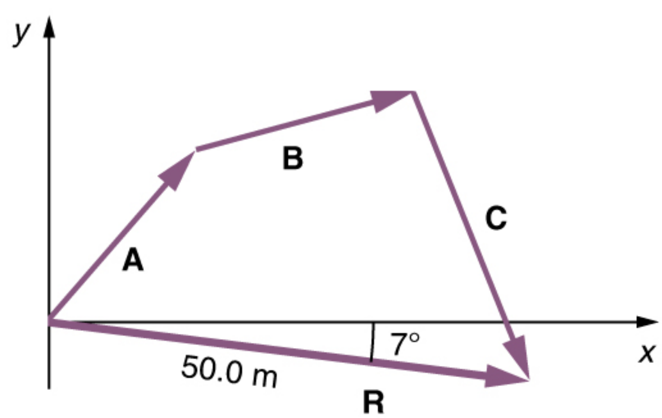


(a)

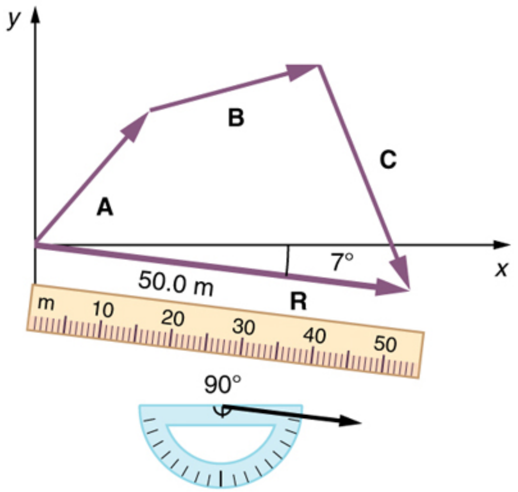
2 Place the vectors head to tail retaining both their initial magnitude and direction.



3 Draw the resultant vector, R.



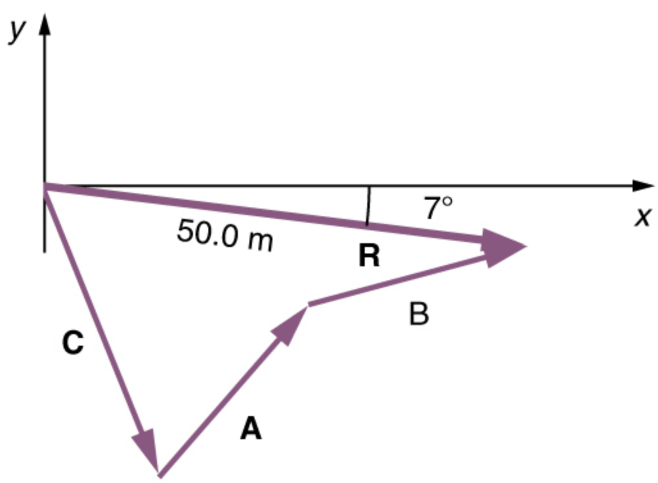
4 Use a ruler to measure the magnitude of **R**, and a protractor to measure the direction of R. While the direction of the vector can be specified in many ways, the easiest way is to measure the angle between the vector and the nearest horizontal or vertical axis. Since the resultant vector is south of the eastward pointing axis, we flip the protractor upside down and measure the angle between the eastward axis and the vector.



In this case, the total displacement **R** is seen to have a magnitude of 50.0 m and to lie in a direction 7.0° south of east. By using its magnitude and direction, this vector can be expressed as $R = 50.0$ m and $\theta = 7.0°$ south of east.

Discussion

The head-to-tail graphical method of vector addition works for any number of vectors. It is also important to note that the resultant is independent of the order in which the vectors are added. Therefore, we could add the vectors in any order as illustrated in link and we will still get the same solution.



Here, we see that when the same vectors are added in a different order, the result is the same. This characteristic is true in every case and is an important characteristic of vectors. Vector addition is commutative. Vectors can be added in any order.

$$\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}.$$

(This is true for the addition of ordinary numbers as well—you get the same result whether you add $\mathbf{2} + \mathbf{3}$ or $\mathbf{3} + \mathbf{2}$, for example).

3.3.4 Vector Subtraction

Vector subtraction is a straightforward extension of vector addition. To define subtraction (say we want to subtract **B** from **A** , written **A**–**B** , we must first define what we mean by subtraction. The *negative* of a vector **B** is defined to be –**B**; that is, graphically *the negative of any vector has the same magnitude but the opposite direction*, as shown in link. In other words, **B** has the same length as –**B**, but points in the opposite direction. Essentially, we just flip the vector so it points in the opposite direction.

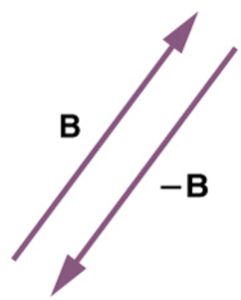


Figure 3.11: The negative of a vector is just another vector of the same magnitude but pointing in the opposite direction. So **B** is the negative of –**B**; it has the same length but opposite direction.

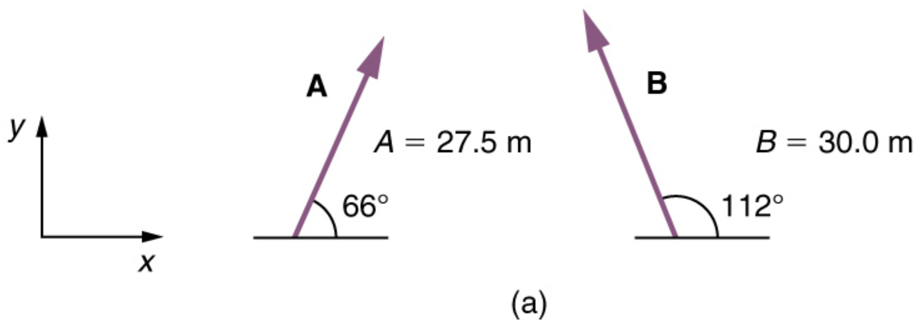
The **subtraction** of vector **B** from vector **A** is then simply defined to be the addition of –**B** to **A**. Note that vector subtraction is the addition of a negative vector. The order of subtraction does not affect the results.

$$\text{A} - \text{B} = \text{A} + (-\text{B}).$$

This is analogous to the subtraction of scalars (where, for example, $5 - 2 = 5 + (-2)$). Again, the result is independent of the order in which the subtraction is made. When vectors are subtracted graphically, the techniques outlined above are used, as the following example illustrates.

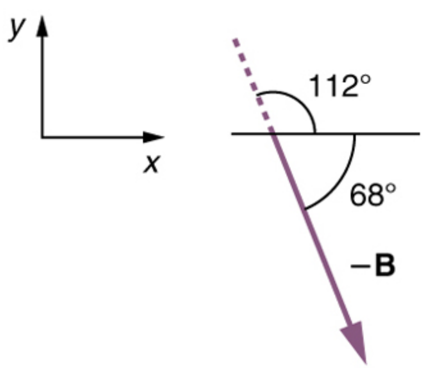
Subtracting Vectors Graphically: A Woman Sailing a Boat

A woman sailing a boat at night is following directions to a dock. The instructions read to first sail 27.5 m in a direction 66.0° north of east from her current location, and then travel 30.0 m in a direction 112° north of east (or 22.0° west of north). If the woman makes a mistake and travels in the **opposite** direction for the second leg of the trip, where will she end up? Compare this location with the location of the dock.



Strategy

We can represent the first leg of the trip with a vector **A**, and the second leg of the trip with a vector **B**. The dock is located at a location **A** + **B**. If the woman mistakenly travels in the *opposite* direction for the second leg of the journey, she will travel a distance B (30.0 m) in the direction 180°–112° = 68° south of east. We represent this as –**B**, as shown below. The vector –**B** has the same magnitude as **B** but is in the opposite direction. Thus, she will end up at a location **A**+(–**B**), or **A**–**B**.



We will perform vector addition to compare the location of the dock, A + **B**, with the location at which the woman mistakenly arrives, A + (–B).

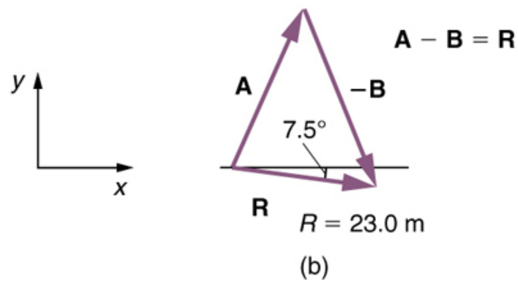
Solution

1 To determine the location at which the woman arrives by accident, draw vectors **A** and –**B**.

2 Place the vectors head to tail.

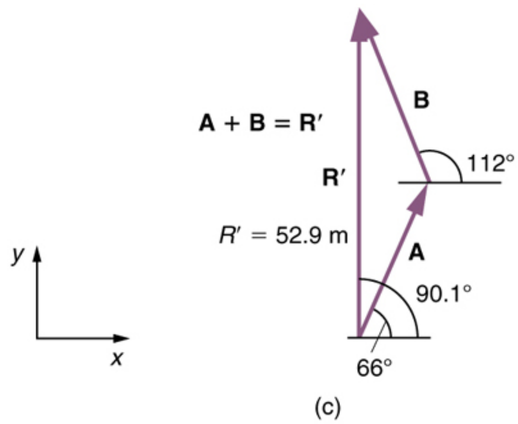
3 Draw the resultant vector **R**.

4 Use a ruler and protractor to measure the magnitude and direction of **R**.



In this case, $R = 23.0$ m and $\theta = 7.5°$ south of east.

5 To determine the location of the dock, we repeat this method to add vectors **A** and **B**. We obtain the resultant vector $\mathbf{R}'$:



In this case $R = 52.9$ m and $\theta = 90.1°$ north of east.

We can see that the woman will end up a significant distance from the dock if she travels in the opposite direction for the second leg of the trip.

Discussion

Because subtraction of a vector is the same as addition of a vector with the opposite direction, the graphical method of subtracting vectors works the same as for addition.

3.3.5 Multiplication of Vectors and Scalars

If we decided to walk three times as far on the first leg of the trip considered in the preceding example, then we would walk 3×27.5 m, or 82.5 m, in a direction 66.0° north of east. This is an example of multiplying a vector by a positive scalar. Notice that the magnitude changes, but the direction stays the same.

If the scalar is negative, then multiplying a vector by it changes the vector's magnitude and gives the new vector the **opposite** direction. For example, if you multiply by –2, the magnitude doubles but the direction changes. We can summarize these rules in the following way: When vector **A** is multiplied by a scalar c,

- the magnitude of the vector becomes the absolute value of cA,
- if c is positive, the direction of the vector does not change,
- if c is negative, the direction is reversed.

In our case, $c = 3$ and $A = 27.5\ m$. Vectors are multiplied by scalars in many situations. Note that division is the inverse of multiplication. For example, dividing by 2 is the same as multiplying by the value (1/2). The rules for multiplication of vectors by scalars are the same for division; simply treat the divisor as a scalar between 0 and 1.

3.3.6 Resolving a Vector into Components

In the examples above, we have been adding vectors to determine the resultant vector. In many cases, however, we will need to do the opposite. We will need to take a single vector and find what other vectors added together produce it. In most cases, this involves determining the perpendicular componentsof a single vector, for example the **x-** *and* **y**-components, or the north-south and east-west components.

For example, we may know that the total displacement of a person walking in a city is 10.3 blocks in a direction 29.0° north of east and want to find out how many blocks east and north had to be walked. This method is called **finding the components (or parts)** of the displacement in the east and north directions, and it is the inverse of the process followed to find the total displacement. It is one example of finding the components of a vector. There are many applications in physics where this is a useful thing to do. We will see this soon in Projectile Motion, and much more when we cover **forces** in Dynamics: Newton's Laws of Motion. Most of these involve finding components along perpendicular axes (such as north and east), so that right triangles are involved. The analytical techniques presented in Vector Addition and Subtraction: Analytical Methods are ideal for finding vector components.

Maze Game

Learn about position, velocity, and acceleration in the "Arena of Pain". Use the green arrow to move the ball. Add more walls to the arena to make the game more difficult. Try to make a goal as fast as you can.

3.3.7 Test Prep for AP Courses

A ball is launched vertically upward. The vertical position of the ball is recorded at various points in time in the table shown.

Table 3.1:

Height (m)	Time (sec)
0.490	0.1
0.882	0.2
1.176	0.3
1.372	0.4
1.470	0.5
1.470	0.6
1.372	0.7

Which of the following correctly describes the graph of the ball's vertical velocity versus time?

a. Always positive, steadily decreasing
b. Always positive, constant
c. Initially positive, steadily decreasing, becoming negative at the end
d. Initially zero, steadily getting more and more negative

Table 3.2:

Height (m)	Time (sec)
0.490	0.1
0.882	0.2
1.176	0.3
1.372	0.4
1.470	0.5
1.470	0.6
1.372	0.7

A ball is launched at an angle of 60 degrees above the horizontal, and the vertical position of the ball is recorded at various points in time in the table shown, assuming the ball was at a height of 0 at time t = 0.

a. Draw a graph of the ball's vertical velocity versus time.
b. Describe the graph of the ball's horizontal velocity.
c. Draw a graph of the ball's vertical acceleration versus time.

The graph of the ball's vertical velocity over time should begin at 4.90 m/s during the time interval 0 - 0.1 sec (there should be a data point at t = 0.05 sec, v = 4.90 m/s). It should then have a slope of -9.8 m/s^2, crossing through v = 0 at t = 0.55 sec and ending at v = -0.98 m/s at t = 0.65 sec.

The graph of the ball's horizontal velocity would be a constant positive value, a flat horizontal line at some positive velocity from t = 0 until t = 0.7 sec.

3.3.8 Summary

- The **graphical method of adding vectors A** and **B** involves drawing vectors on a graph and adding them using the head-to-tail method. The resultant vector **R** is defined such that $\mathbf{A} + \mathbf{B} = \mathbf{R}$. The magnitude and direction of **R** are then determined with a ruler and protractor, respectively.
- The **graphical method of subtracting vector B** from **A** involves adding the opposite of vector **B**, which is defined as $-\mathbf{B}$. In this case, A–**B** = **A**+(–B)= R. Then, the head-to-tail method of addition is followed in the usual way to obtain the resultant vector **R**.
- Addition of vectors is commutative such that $\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$.
- The head-to-tail method of adding vectors involves drawing the first vector on a graph and then placing the tail of each subsequent vector at the head of the previous vector. The resultant vector is then drawn from the tail of the first vector to the head of the final vector.
- If a vector **A** is multiplied by a scalar quantity c, the magnitude of the product is given by cA. If c is positive, the direction of the product points in the same direction as **A**; if c is negative, the direction of the product points in the opposite direction as **A**.

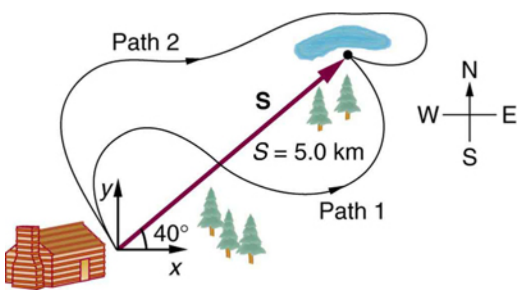
3.3.9 Conceptual Questions

Which of the following is a vector: a person's height, the altitude on Mt. Everest, the age of the Earth, the boiling point of water, the cost of this book, the Earth's population, the acceleration of gravity?

Give a specific example of a vector, stating its magnitude, units, and direction.

What do vectors and scalars have in common? How do they differ?

Two campers in a national park hike from their cabin to the same spot on a lake, each taking a different path, as illustrated below. The total distance traveled along Path 1 is 7.5 km, and that along Path 2 is 8.2 km. What is the final displacement of each camper?



If an airplane pilot is told to fly 123 km in a straight line to get from San Francisco to Sacramento, explain why he could end up anywhere on the circle shown in link. What other information would he need to get to Sacramento?

Suppose you take two steps **A** and **B** (that is, two nonzero displacements). Under what circumstances can you end up at your starting point? More generally, under what circumstances can two nonzero vectors add to give zero? Is the maximum distance you can end up from the starting point **A** + **B** the sum of the lengths of the two steps?

Explain why it is not possible to add a scalar to a vector.

If you take two steps of different sizes, can you end up at your starting point? More generally, can two vectors with different magnitudes ever add to zero? Can three or more?

3.3.10 Problems & Exercises

Use graphical methods to solve these problems. You may assume data taken from graphs is accurate to three digits.

Find the following for path A in link: (a) the total distance traveled, and (b) the magnitude and direction of the displacement from start to finish.

a 480 m

b 379 m, 18.4° east of north

Find the following for path B in link: (a) the total distance traveled, and (b) the magnitude and direction of the displacement from start to finish.

Find the north and east components of the displacement for the hikers shown in link.

north component 3.21 km, east component 3.83 km

Suppose you walk 18.0 m straight west and then 25.0 m straight north. How

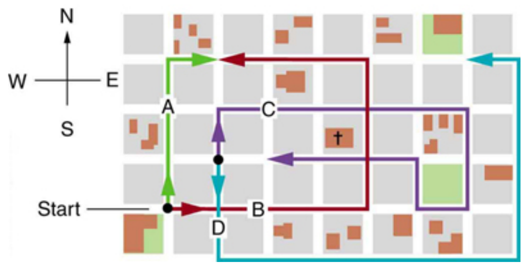


Figure 3.12: The various lines represent paths taken by different people walking in a city. All blocks are 120 m on a side.

far are you from your starting point, and what is the compass direction of a line connecting your starting point to your final position? (If you represent the two legs of the walk as vector displacements **A** and **B**, as in link, then this problem asks you to find their sum $\mathbf{R} = \mathbf{A} + \mathbf{B}$.)

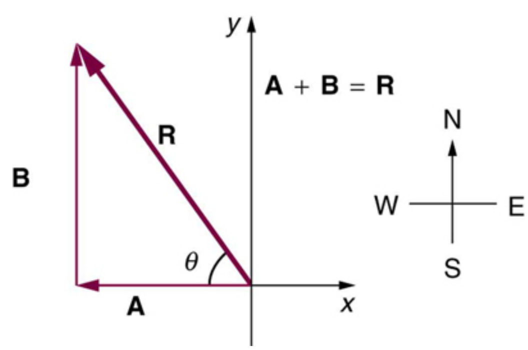
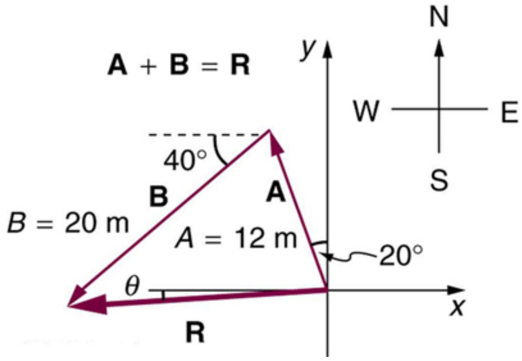


Figure 3.13: The two displacements **A** and **B** add to give a total displacement **R** having magnitude R and direction θ.

Suppose you first walk 12.0 m in a direction 20° west of north and then 20.0 m in a direction 40.0° south of west. How far are you from your starting point, and what is the compass direction of a line connecting your starting point to your final position? (If you represent the two legs of the walk as vector displacements **A** and **B**, as in link, then this problem finds their sum R = A + B.)



19.5 m, 4.65° south of west

Repeat the problem above, but reverse the order of the two legs of the walk; show that you get the same final result. That is, you first walk leg **B**, which is 20.0 m in a direction exactly 40° south of west, and then leg **A**, which is 12.0 m in a direction exactly 20° west of north. (This problem shows that $\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$.)

a Repeat the problem two problems prior, but for the second leg you walk 20.0 m in a direction 40.0° north of east (which is equivalent to subtracting **B** from **A** —that is, to finding $\mathbf{R}' = \mathbf{A} - \mathbf{B}$). (b) Repeat the problem two problems prior, but now you first walk 20.0 m in a direction 40.0° south of west and then 12.0 m in a direction 20.0° east of south (which is equivalent to subtracting **A** from **B** —that is, to finding $\mathbf{R}'' = \mathbf{B} - \mathbf{A} = -\mathbf{R}'$). Show that this is the case.

a 26.6 m, 65.1° north of east

b 26.6 m, 65.1° south of west

Show that the **order** of addition of three vectors does not affect their sum. Show this property by choosing any three vectors **A**, **B**, and **C**, all having different lengths and directions. Find the sum A + B + C then find their sum when added in a different order and show the result is the same. (There are five other orders in which **A**, **B**, and **C** can be added; choose only one.)

Show that the sum of the vectors discussed in link gives the result shown in link.

52.9 m, 90.1° with respect to the x-axis.

Find the magnitudes of velocities v_{A} and v_{B} in link

Find the components of v_{tot} along the **x-** and **y-**axes in link.

x-component 4.41 m/s

y-component 5.07 m/s

Find the components of v_{tot} along a set of perpendicular axes rotated 30° counterclockwise relative to those in link.

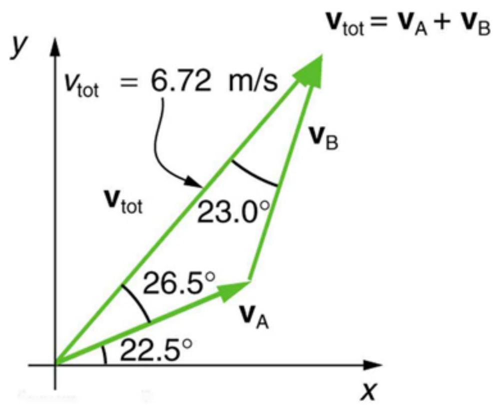


Figure 3.14: The two velocities $\mathbf{v}_\mathrm{A}$ and $\mathbf{v}_\mathrm{B}$ add to give a total $\mathbf{v}_\mathrm{tot}$.

3.3.11 Glossary

component (of a 2-d vector) a piece of a vector that points in either the vertical or the horizontal direction; every 2-d vector can be expressed as a sum of two vertical and horizontal vector components

commutative refers to the interchangeability of order in a function; vector addition is commutative because the order in which vectors are added together does not affect the final sum

direction (of a vector) the orientation of a vector in space

head (of a vector) the end point of a vector; the location of the tip of the vector's arrowhead; also referred to as the "tip"

head-to-tail method a method of adding vectors in which the tail of each vector is placed at the head of the previous vector

magnitude (of a vector) the length or size of a vector; magnitude is a scalar quantity

resultant the sum of two or more vectors

resultant vector the vector sum of two or more vectors

scalar a quantity with magnitude but no direction

tail the start point of a vector; opposite to the head or tip of the arrow

3.4 Vector Addition and Subtraction: Analytical Methods

3.4.1 Learning Objectives

By the end of this section, you will be able to:

- Understand the rules of vector addition and subtraction using analytical methods.
- Apply analytical methods to determine vertical and horizontal component vectors.
- Apply analytical methods to determine the magnitude and direction of a resultant vector.

The information presented in this section supports the following AP® learning objectives and science practices:

- **3.A.1.1** The student is able to express the motion of an object using narrative, mathematical, and graphical representations. **(S.P. 1.5, 2.1, 2.2)**

Analytical methods of vector addition and subtraction employ geometry and simple trigonometry rather than the ruler and protractor of graphical methods. Part of the graphical technique is retained, because vectors are still represented by arrows for easy visualization. However, analytical methods are more concise, accurate, and precise than graphical methods, which are limited by the accuracy with which a drawing can be made. Analytical methods are limited only by the accuracy and precision with which physical quantities are known.

3.4.2 Resolving a Vector into Perpendicular Components

Analytical techniques and right triangles go hand-in-hand in physics because (among other things) motions along perpendicular directions are independent. We very often need to separate a vector into perpendicular components. For example, given a vector like **A** in link, we may wish to find which two perpendicular vectors, $\mathbf{A}_x$ and $\mathbf{A}_y$, add to produce it.

$\mathbf{A}_x$ and $\mathbf{A}_y$ are defined to be the components of **A** along the **x**- and **y**-axes. The three vectors **A**, $\mathbf{A}_x$, and $\mathbf{A}_y$ form a right triangle:

$$\mathbf{A}_x + \mathrm{A}_y = \mathrm{A}.$$

Note that this relationship between vector components and the resultant vector holds only for vector quantities (which include both magnitude and direction). The relationship does not apply for the magnitudes alone. For example, if $\mathbf{A}_x = 3\ m$ east, $\mathbf{A}_y = 4\ m$ north, and $\mathbf{A} = 5\ m$ north-east, then it is true that the vectors $\mathbf{A}_x + \mathrm{A}_y = \mathrm{A}$. However, it is **not** true that the sum of the magnitudes of the vectors is also equal. That is,

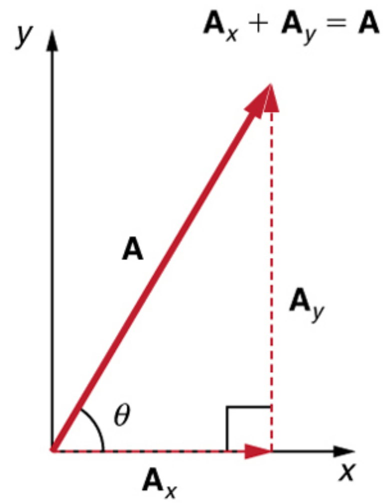


Figure 3.15: The vector **A**, with its tail at the origin of an x, y-coordinate system, is shown together with its x- and y-components, $\mathbf{A}_x$ and $\mathbf{A}_y$. These vectors form a right triangle. The analytical relationships among these vectors are summarized below.

3 m+4 m   5 m 3 m+4 m   5 m alignl { stack { size 12{"3 M + 4 M " <> " 5 M"} {} # {} } } {}

Thus,

$$A_x + A_y \neq A$$

If the vector **A** is known, then its magnitude A (its length) and its angle θ (its direction) are known. To find A_x and A_y, its **x-** and y**-components, we use the following relationships for a right triangle.

$$A_x = A \cos \theta$$

and

$$A_y = A \sin \theta.$$

Suppose, for example, that **A** is the vector representing the total displacement of the person walking in a city considered in Kinematics in Two Dimensions: An Introduction and Vector Addition and Subtraction: Graphical Methods.

Then $A = 10.3$ blocks and $\theta = 29.1$, so that

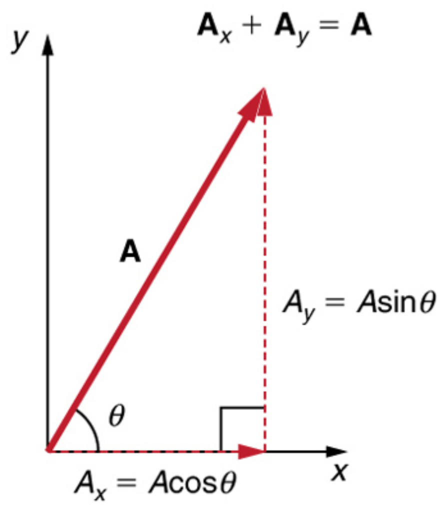


Figure 3.16: The magnitudes of the vector components $\mathbf{A}_x$ and $\mathbf{A}_y$ can be related to the resultant vector $\mathbf{A}$ and the angle θ with trigonometric identities. Here we see that $A_x = A \cos \theta$ and $A_y = A \sin \theta$.

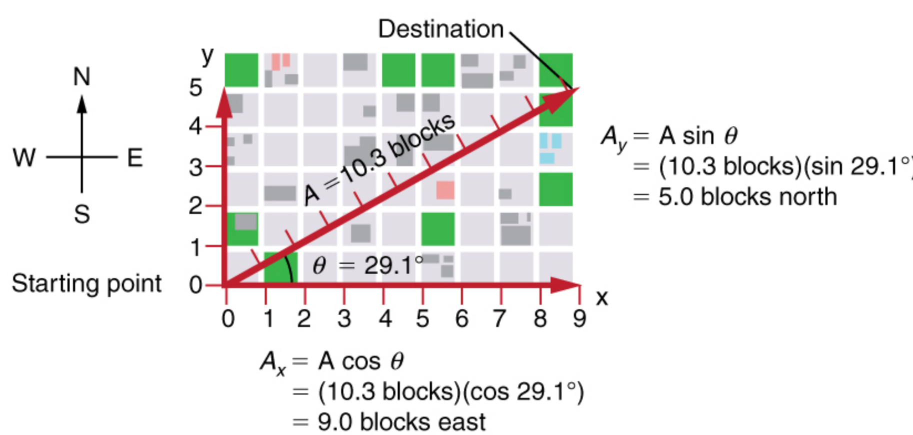


Figure 3.17: We can use the relationships $A_x = A \cos \theta$ and $A_y = A \sin \theta$ to determine the magnitude of the horizontal and vertical component vectors in this example.

$$A_x = A \cos \theta = (10.3 \text{ blocks}) (\cos 29.1) = 9.0 \text{ blocks}$$

$$A_y = A \sin \theta = (10.3 \text{ blocks}) (\sin 29.1) = 5.0 \text{ blocks}.$$

3.4.3 Calculating a Resultant Vector

If the perpendicular components $\mathbf{A}_x$ and $\mathbf{A}_y$ of a vector $\mathbf{A}$ are known, then $\mathbf{A}$ can also be found analytically. To find the magnitude A and direction θ of a vector from its perpendicular components $\mathbf{A}_x$ and $\mathbf{A}_y$, we use the following relationships:

$$A = \sqrt{A_{x^2} + A_{y^2}}$$

$$\theta = \tan^{-1}(A_y/A_x).$$

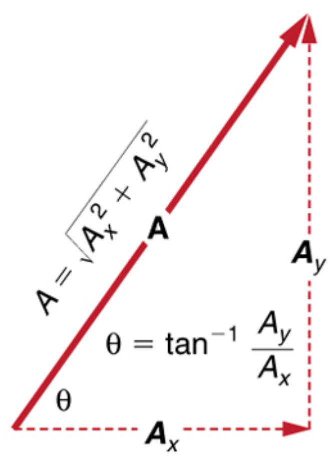


Figure 3.18: The magnitude and direction of the resultant vector can be determined once the horizontal and vertical components A_x and A_y have been determined.

Note that the equation $A = \sqrt{A_x^2 + A_y^2}$ is just the Pythagorean theorem relating the legs of a right triangle to the length of the hypotenuse. For example, if A_x and A_y are 9 and 5 blocks, respectively, then $A = \sqrt{9^2{+}5^2}{=}10.3$ blocks, again consistent with the example of the person walking in a city. Finally, the direction is $\theta = \tan^{-1}(5/9) = 29.1$, as before.

Determining Vectors and Vector Components with Analytical Methods

Equations $A_x = A \cos\theta$ and $A_y = A \sin\theta$ are used to find the perpendicular components of a vector—that is, to go from A and θ to A_x and A_y. Equations $A = \sqrt{A_x^2 + A_y^2}$ and $\theta = \tan^{-1}(A_y/A_x)$ are used to find a vector from its perpendicular components—that is, to go from A_x and A_y to A and θ. Both processes are crucial to analytical methods of vector addition and subtraction.

3.4.4 Adding Vectors Using Analytical Methods

To see how to add vectors using perpendicular components, consider link, in which the vectors **A** and **B** are added to produce the resultant **R**.

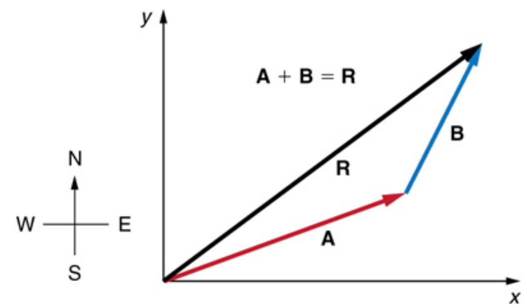


Figure 3.19: Vectors **A** and **B** are two legs of a walk, and **R** is the resultant or total displacement. You can use analytical methods to determine the magnitude and direction of **R**.

If **A** and **B** represent two legs of a walk (two displacements), then **R** is the total displacement. The person taking the walk ends up at the tip of **R**. There are many ways to arrive at the same point. In particular, the person could have walked first in the x-direction and then in the y-direction. Those paths are the x- and y-components of the resultant, $\mathbf{R}_x$ and $\mathbf{R}_y$. If we know $\mathbf{R}_x$ and $\mathbf{R}_y$, we can find R and θ using the equations $A = \sqrt{{A_x}^2 + {A_y}^2}$ and $\theta = \tan^{-1}(A_y/A_x)$. When you use the analytical method of vector addition, you can determine the components or the magnitude and direction of a vector.

Step 1. Identify the x- and y-axes that will be used in the problem. Then, find the components of each vector to be added along the chosen perpendicular axes. Use the equations $A_x = A \cos\theta$ and $A_y = A \sin\theta$ to find the components. In link, these components are A_x, A_y, B_x, and B_y. The angles that vectors **A** and **B** make with the **x**-axis are θ_{A} and θ_{B}, respectively.

Step 2. Find the components of the resultant along each axis by adding the components of the individual vectors along that axis. That is, as shown in link,

$$R_x = A_x + B_x$$

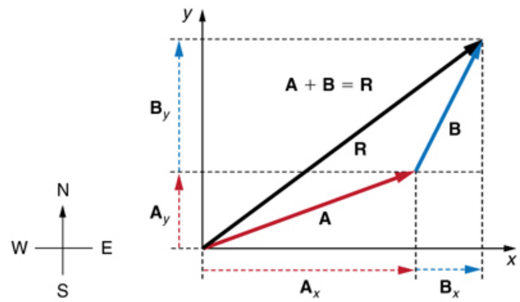


Figure 3.20: To add vectors **A** and **B**, first determine the horizontal and vertical components of each vector. These are the dotted vectors $\mathbf{A}_x$, $\mathbf{A}_y$, $\mathbf{B}_x$ and $\mathbf{B}_y$ shown in the image.

and

$$R_y = A_y + B_y.$$

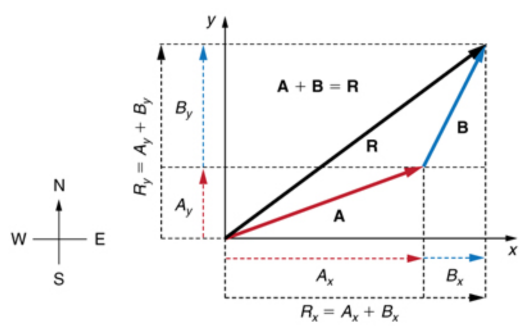


Figure 3.21: The magnitude of the vectors $\mathbf{A}_x$ and $\mathbf{B}_x$ add to give the magnitude R_x of the resultant vector in the horizontal direction. Similarly, the magnitudes of the vectors $\mathbf{A}_y$ and $\mathbf{B}_y$ add to give the magnitude R_y of the resultant vector in the vertical direction.

Components along the same axis, say the **x**-axis, are vectors along the same line and, thus, can be added to one another like ordinary numbers. The same is true for components along the **y**-axis. (For example, a 9-block eastward walk could be taken in two legs, the first 3 blocks east and the second 6 blocks east, for a total of 9, because they are along the same direction.) So resolving vectors into components along common axes makes it easier to add them. Now that the components of **R** are known, its magnitude and direction can be found.

Step 3. To get the magnitude R of the resultant, use the Pythagorean theorem:**

$$R = \sqrt{R_x^2 + R_y^2}.$$

Step 4. To get the direction of the resultant:**

$$\theta = \tan^{-1}(R_y/R_x).$$

The following example illustrates this technique for adding vectors using perpendicular components.

Adding Vectors Using Analytical Methods

Add the vector **A** to the vector **B** shown in link, using perpendicular components along the **x-** and **y**-axes. The **x-** and **y**-axes are along the east–west and north–south directions, respectively. Vector **A** represents the first leg of a walk in which a person walks 53.0 m in a direction 20.0º north of east. Vector **B** represents the second leg, a displacement of 34.0 m in a direction 63.0º north of east.

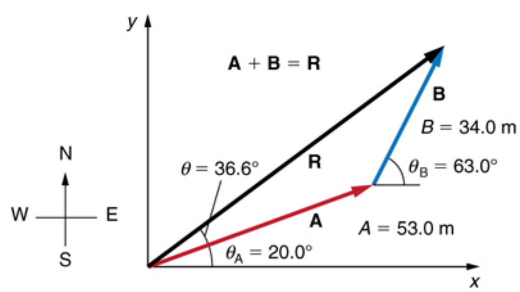


Figure 3.22: Vector **A** has magnitude 53.0 m and direction 20.0 north of the x-axis. Vector **B** has magnitude 34.0 m and direction 63.0º north of the x-axis. You can use analytical methods to determine the magnitude and direction of **R**.

Strategy

The components of **A** and **B** along the **x-** and **y**-axes represent walking due east and due north to get to the same ending point. Once found, they are combined to produce the resultant.

Solution

Following the method outlined above, we first find the components of **A** and **B** along the **x-** and **y**-axes. Note that $A = 53.0\ m$, $\theta_\mathrm{A} = 20.0$, $B = 34.0\ m$, and $\theta_\mathrm{B} = 63.0$. We find the **x**-components by using $A_x = A \cos \theta$, which gives

$$\begin{array}{rcl} A_x & = & A \cos \theta_A = (53.0\ m)(\cos 20.0^{\circ}) \\ & = & (53.0\ m)(0.940) = 49.8\ m \end{array}$$

and

$$\begin{aligned} B_x &= B \cos \theta_B = (34.0\ m)(\cos 63.0^{o}) \\ &= (34.0\ m)(0.454) = 15.4\ m. \end{aligned}$$

Similarly, the y-components are found using $A_y = A \sin \theta_A$:

$$\begin{aligned} A_y &= A \sin \theta_A = (53.0\ m)(\sin 20.0^{o}) \\ &= (53.0\ m)(0.342) = 18.1\ m \end{aligned}$$

and

$$\begin{aligned} B_y &= B \sin \theta_B = (34.0\ m)(\sin 63.0) \\ &= (34.0\ m)(0.891) = 30.3\ m. \end{aligned}$$

The **x-** and **y-**components of the resultant are thus

$$R_x = A_x + B_x = 49.8\ m{+}15.4\ m{=}\ 65.2\ m$$

and

$$R_y = A_y + B_y = 18.1\ m{+}30.3\ m{=}\ 48.4\ m.$$

Now we can find the magnitude of the resultant by using the Pythagorean theorem:

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(65.2)^2 + (48.4)^2 \text{ m}}$$

so that

$$R = 81.2\ m.$$

Finally, we find the direction of the resultant:

$$\theta = \tan^{-1}(R_y/R_x){=}{+}\tan^{-1}(48.4/65.2).$$

Thus,

$$\theta = \tan^{-1}(0.742) = 36.6.$$

Discussion

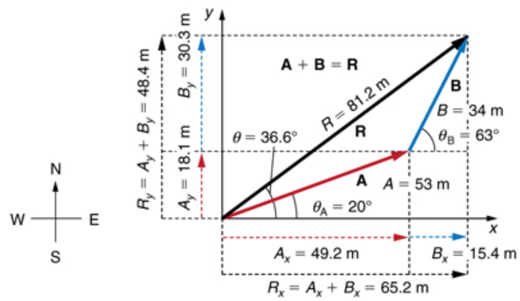


Figure 3.23: Using analytical methods, we see that the magnitude of **R** is 81.2 m and its direction is 36.6 north of east.

This example illustrates the addition of vectors using perpendicular components. Vector subtraction using perpendicular components is very similar—it is just the addition of a negative vector.

Subtraction of vectors is accomplished by the addition of a negative vector. That is, $\mathbf{A} - \mathbf{B} \equiv \mathbf{A} + (-\mathbf{B})$. Thus, *the method for the subtraction of vectors using perpendicular components is identical to that for addition.* The components of $-\mathbf{B}$ are the negatives of the components of **B**. The x- and **y**-components of the resultant $\mathbf{A} - \mathrm{B} = \mathrm{R}$ are thus

$$R_x = A_x + (-B_x)$$

and

$$R_y = A_y + \left(-B_y\right)$$

and the rest of the method outlined above is identical to that for addition. (See link.)

Analyzing vectors using perpendicular components is very useful in many areas of physics, because perpendicular quantities are often independent of one another. The next module, Projectile Motion, is one of many in which using perpendicular components helps make the picture clear and simplifies the physics.

Vector Addition

Learn how to add vectors. Drag vectors onto a graph, change their length and angle, and sum them together. The magnitude, angle, and components of each vector can be displayed in several formats.

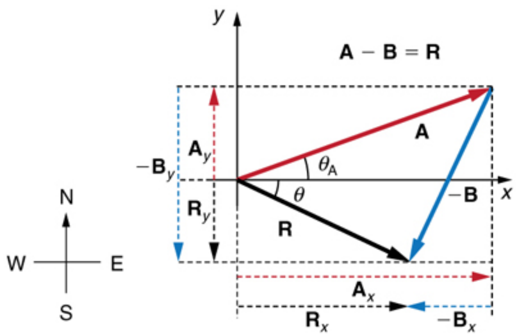


Figure 3.24: The subtraction of the two vectors shown in link. The components of $-\mathbf{B}$ are the negatives of the components of $\mathbf{B}$. The method of subtraction is the same as that for addition.

3.4.5 Summary

- The analytical method of vector addition and subtraction involves using the Pythagorean theorem and trigonometric identities to determine the magnitude and direction of a resultant vector.
- The steps to add vectors **A** and **B** using the analytical method are as follows:

  Step 1: Determine the coordinate system for the vectors. Then, determine the horizontal and vertical components of each vector using the equations

$$\begin{aligned} A_x &= A \cos\theta \\ B_x &= B \cos\theta \end{aligned}$$

  and

$$\begin{aligned} A_y &= A \sin\theta \\ B_y &= B \sin\theta. \end{aligned}$$

  Step 2: Add the horizontal and vertical components of each vector to determine the components R_x and R_y of the resultant vector, $\mathbf{R}$:

$$R_x = A_x + B_x$$

  and

$$R_y = A_y + B_{y.}$$

Step 3: Use the Pythagorean theorem to determine the magnitude, R, of the resultant vector $\mathbf{R}$:

$$R = \sqrt{R_x^2 + R_y^2}.$$

Step 4: Use a trigonometric identity to determine the direction, θ, of $\mathbf{R}$:

$$\theta = \tan^{-1}(R_y/R_x).$$

3.4.6 Conceptual Questions

Suppose you add two vectors $\mathbf{A}$ and $\mathbf{B}$. What relative direction between them produces the resultant with the greatest magnitude? What is the maximum magnitude? What relative direction between them produces the resultant with the smallest magnitude? What is the minimum magnitude?

Give an example of a nonzero vector that has a component of zero.

Explain why a vector cannot have a component greater than its own magnitude.

If the vectors $\mathbf{A}$ and $\mathbf{B}$ are perpendicular, what is the component of $\mathbf{A}$ along the direction of $\mathbf{B}$? What is the component of $\mathbf{B}$ along the direction of $\mathbf{A}$?

3.4.7 Problems & Exercises

Find the following for path C in link: (a) the total distance traveled and (b) the magnitude and direction of the displacement from start to finish. In this part of the problem, explicitly show how you follow the steps of the analytical method of vector addition.

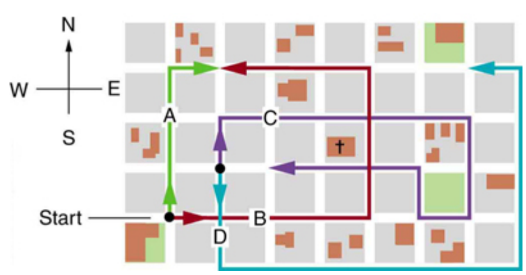


Figure 3.25: The various lines represent paths taken by different people walking in a city. All blocks are 120 m on a side.

a 1.56 km

b 120 m east

Find the following for path D in link: (a) the total distance traveled and (b) the magnitude and direction of the displacement from start to finish. In this part of the problem, explicitly show how you follow the steps of the analytical method of vector addition.

Find the north and east components of the displacement from San Francisco to Sacramento shown in link.

North-component 87.0 km, east-component 87.0 km

Solve the following problem using analytical techniques: Suppose you walk 18.0 m straight west and then 25.0 m straight north. How far are you from your starting point, and what is the compass direction of a line connecting your starting point to your final position? (If you represent the two legs of the walk as vector displacements $\mathbf{A}$ and $\mathbf{B}$, as in link, then this problem asks you to find their sum $\mathbf{R} = \mathbf{A} + \mathbf{B}$.)

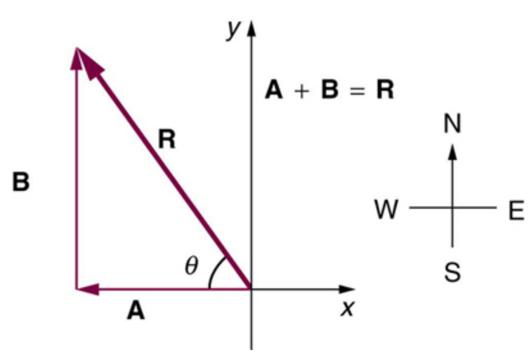


Figure 3.26: The two displacements $\mathbf{A}$ and $\mathbf{B}$ add to give a total displacement $\mathbf{R}$ having magnitude R and direction θ.

Note that you can also solve this graphically. Discuss why the analytical technique for solving this problem is potentially more accurate than the graphical technique.

Repeat link using analytical techniques, but reverse the order of the two legs of the walk and show that you get the same final result. (This problem shows that adding them in reverse order gives the same result—that is, $\mathbf{B} + \mathbf{A} = \mathbf{A} + \mathbf{B}$.) Discuss how taking another path to reach the same point might help to overcome an obstacle blocking you other path.

30.8 m, 35.8 west of north

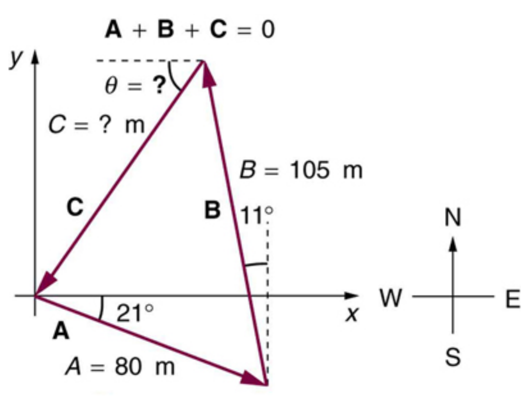
You drive 7.50 km in a straight line in a direction 15 east of north. (a) Find the distances you would have to drive straight east and then straight north to arrive at the same point. (This determination is equivalent to find the components of the displacement along the east and north directions.) (b) Show that you still arrive at the same point if the east and north legs are reversed in order.

Do link again using analytical techniques and change the second leg of the walk to 25.0 m straight south. (This is equivalent to subtracting $\mathbf{B}$ from $\mathbf{A}$ —that is, finding $\mathbf{R}' = \mathbf{A} - \mathbf{B}$) (b) Repeat again, but now you first walk 25.0 m north and then 18.0 m east. (This is equivalent to subtract $\mathbf{A}$ from $\mathbf{B}$ —that is, to find $\mathbf{A} = \mathbf{B} + \mathbf{C}$. Is that consistent with your result?)

a 30.8 m, 54.2 south of west

b 30.8 m, 54.2 north of east

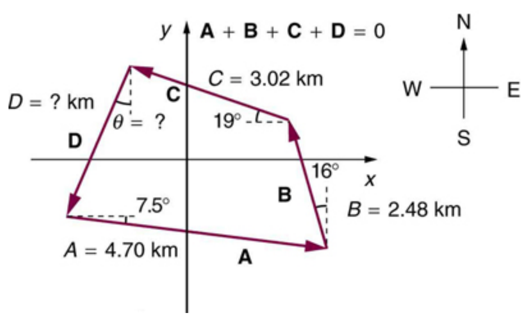
A new landowner has a triangular piece of flat land she wishes to fence. Starting at the west corner, she measures the first side to be 80.0 m long and the next to be 105 m. These sides are represented as displacement vectors $\mathbf{A}$ from $\mathbf{B}$ in link. She then correctly calculates the length and orientation of the third side C. What is her result?



You fly 32.0 km in a straight line in still air in the direction 35.0 south of west. (a) Find the distances you would have to fly straight south and then straight west to arrive at the same point. (This determination is equivalent to finding the components of the displacement along the south and west directions.) (b) Find the distances you would have to fly first in a direction 45.0 south of west and then in a direction 45.0 west of north. These are the components of the displacement along a different set of axes—one rotated 45.

18.4 km south, then 26.2 km west(b) 31.5 km at 45.0 south of west, then 5.56 km at 45.0 west of north

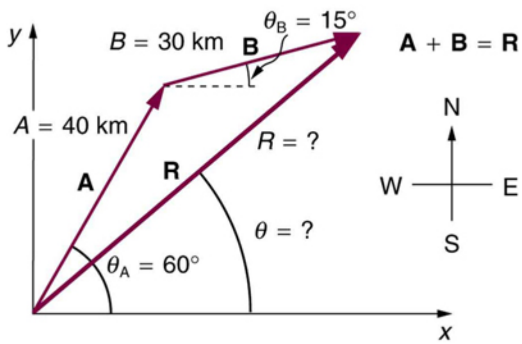
A farmer wants to fence off his four-sided plot of flat land. He measures the first three sides, shown as **A**, **B**, and **C** in link, and then correctly calculates the length and orientation of the fourth side **D**. What is his result?



In an attempt to escape his island, Gilligan builds a raft and sets to sea. The wind shifts a great deal during the day, and he is blown along the following straight lines: 2.50 km 45.0 north of west; then 4.70 km 60.0 south of east; then 1.30 km 25.0 south of west; then 5.10 km straight east; then 1.70 km 5.00 east of north; then 7.20 km 55.0 south of west; and finally 2.80 km 10.0 north of east. What is his final position relative to the island?

7.34 km, 63.5 south of east

Suppose a pilot flies 40.0 km in a direction 60º north of east and then flies 30.0 km in a direction 15º north of east as shown in link. Find her total distance R from the starting point and the direction θ of the straight-line path to the final position. Discuss qualitatively how this flight would be altered by a wind from the north and how the effect of the wind would depend on both wind speed and the speed of the plane relative to the air mass.



3.4.8 Glossary

analytical method the method of determining the magnitude and direction of a resultant vector using the Pythagorean theorem and trigonometric identities

3.5 Projectile Motion

3.5.1 Learning Objectives

By the end of this section, you will be able to:

- Identify and explain the properties of a projectile, such as acceleration due to gravity, range, maximum height, and trajectory.
- Determine the location and velocity of a projectile at different points in its trajectory.
- Apply the principle of independence of motion to solve projectile motion problems.

The information presented in this section supports the following AP® learning objectives:

- **3.A.1.1** The student is able to express the motion of an object using narrative, mathematical, and graphical representations. **(S.P. 1.5, 2.1, 2.2)**
- **3.A.1.3** The student is able to analyze experimental data describing the motion of an object and is able to express the results of the analysis using narrative, mathematical, and graphical representations. **(S.P. 5.1)**

Projectile motion is the motion of an object thrown or projected into the air, subject to only the acceleration of gravity. The object is called a projectile, and its path is called its trajectory. The motion of falling objects, as covered in Problem-Solving Basics for One-Dimensional Kinematics, is a simple one-dimensional type of projectile motion in which there is no horizontal movement. In this section, we consider two-dimensional projectile motion, such as that of a football or other object for which air resistance **is negligible**.

The most important fact to remember here is that **motions along perpendicular axes are independent** and thus can be analyzed separately. This fact was discussed in Kinematics in Two Dimensions: An Introduction, where vertical and horizontal motions were seen to be independent. The key to analyzing two-dimensional projectile motion is to break it into two motions, one along the horizontal axis and the other along the vertical. (This choice of axes is the most sensible, because acceleration due to gravity is vertical—thus, there will be no acceleration along the horizontal axis when air resistance is negligible.) As is customary, we call the horizontal axis the **x**-axis and the vertical axis the **y**-axis. link illustrates the notation for displacement, where **s** is defined to be the total displacement and **x** and **y** are its components along the horizontal and vertical axes, respectively. The magnitudes of these vectors are **s**, **x**, and **y**. (Note that in the last section we used the notation **A** to represent a vector with components $\mathbf{A}_x$ and $\mathbf{A}_y$. If we continued this format, we would call displacement **s** with components $\mathbf{s}_x$ and $\mathbf{s}_y$. However, to simplify the notation, we will simply represent the component vectors as **x** and **y**.)

Of course, to describe motion we must deal with velocity and acceleration, as well as with displacement. We must find their components along the **x**- and **y**-axes, too. We will assume all forces except gravity (such as air resistance and friction, for example) are negligible. The components of acceleration are then very simple: $a_y = -g = -9.80\ m/\mathrm{s}^2$. (Note that this definition assumes that the upwards direction is defined as the positive direction. If you arrange the coordinate system instead such that the downwards direction is positive, then acceleration due to gravity takes a positive value.) Because gravity is vertical, $a_x = 0$. Both accelerations are constant, so the kinematic equations can be used.

Review of Kinematic Equations (constant a)

$$x = x_0 + \bar{v}t$$

$$\bar{v} = \frac{v_0 + v}{2}$$

$$v = v_0 + \mathrm{at}$$

$$x = x_0 + v_0 t + \frac{1}{2}\mathrm{at}^2$$

$$v^2 = v_0^2 + 2a(x - x_0).$$

Given these assumptions, the following steps are then used to analyze projectile motion:

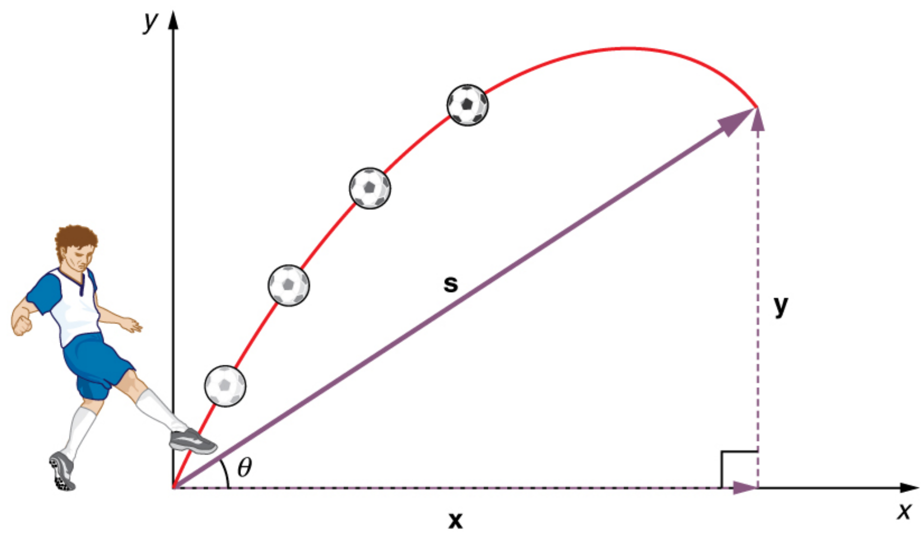


Figure 3.27: The total displacement **s** of a soccer ball at a point along its path. The vector **s** has components **x** and **y** along the horizontal and vertical axes. Its magnitude is s, and it makes an angle θ with the horizontal.

Step 1. *Resolve or break the motion into horizontal and vertical components along the x- and y-axes.* These axes are perpendicular, so $A_x = A \cos\theta$ and $A_y = A \sin\theta$ are used. The magnitude of the components of displacement **s** along these axes are x and y. The magnitudes of the components of the velocity **v** are $v_x = v \cos\theta$ and $v_y = v \sin\theta$, where v is the magnitude of the velocity and θ is its direction, as shown in link. Initial values are denoted with a subscript 0, as usual.

Step 2. *Treat the motion as two independent one-dimensional motions, one horizontal and the other vertical.* The kinematic equations for horizontal and vertical motion take the following forms:

$$\text{Horizontal Motion}(a_x = 0)$$

$$x = x_0 + v_x t$$

$$v_x = v_{0x} = v_x = \text{velocity is a constant.}$$

$$\text{Vertical Motion}\left(\text{assuming positive is up}\ \ a_y = -g = -9.80\ \text{m/s}^2\right)$$

$$y = y_0 + \frac{1}{2}(v_{0y} + v_y)t$$

$$v_y = v_{0y} - \mathrm{gt}$$

$$y = y_0 + v_{0y}t - \frac{1}{2}gt^2$$

$$v_y^2 = v_{0y}^2 - 2g(y - y_0).$$

Step 3. *Solve for the unknowns in the two separate motions—one horizontal and one vertical.* Note that the only common variable between the motions is time t. The problem solving procedures here are the same as for one-dimensional kinematics and are illustrated in the solved examples below.

Step 4. *Recombine the two motions to find the total displacement* **s** *and velocity* **v**. Because the x - and y -motions are perpendicular, we determine these vectors by using the techniques outlined in the Vector Addition and Subtraction: Analytical Methods and employing $A = \sqrt{A_x^2 + A_y^2}$ and $\theta = \tan^{-1}(A_y/A_x)$ in the following form, where θ is the direction of the displacement **s** and θ_v is the direction of the velocity **v**:

Total displacement and velocity

$$s = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1}(y/x)$$

$$v = \sqrt{v_x^2 + v_y^2}$$

$$\theta_v = \tan^{-1}(v_y/v_x).$$

A Fireworks Projectile Explodes High and Away

During a fireworks display, a shell is shot into the air with an initial speed of 70.0 m/s at an angle of 75.0° above the horizontal, as illustrated in link. The fuse is timed to ignite the shell just as it reaches its highest point above the ground. (a) Calculate the height at which the shell explodes. (b) How much time passed between the launch of the shell and the explosion? (c) What is the horizontal displacement of the shell when it explodes?

Strategy

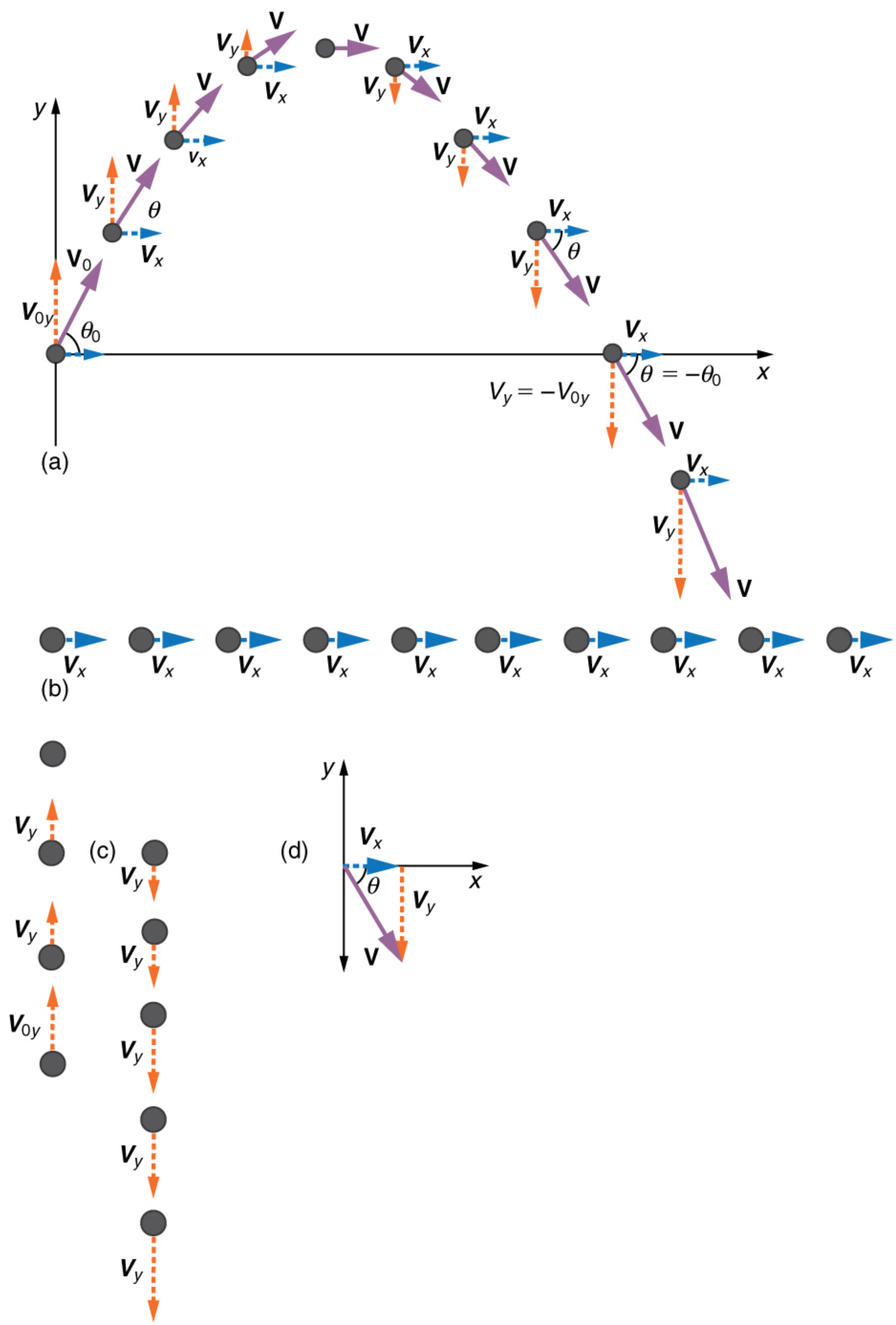


Figure 3.28: (a) We analyze two-dimensional projectile motion by breaking it into two independent one-dimensional motions along the vertical and horizontal axes. (b) The horizontal motion is simple, because $a_x = 0$ and v_x is thus constant. (c) The velocity in the vertical direction begins to decrease as the object rises; at its highest point, the vertical velocity is zero. As the object falls towards the Earth again, the vertical velocity increases again in magnitude but points in the opposite direction to the initial vertical velocity. (d) The x - and y -motions are recombined to give the total velocity at any given point on the trajectory.

Because air resistance is negligible for the unexploded shell, the analysis method outlined above can be used. The motion can be broken into horizontal and vertical motions in which $a_x = 0$ and $a_y = -g$. We can then define x_0 and y_0 to be zero and solve for the desired quantities.

Solution for (a)

By "height" we mean the altitude or vertical position y above the starting point. The highest point in any trajectory, called the apex, is reached when $v_y = 0$. Since we know the initial and final velocities as well as the initial position, we use the following equation to find y:

$$v_y^2 = v_{0y}^2 - 2g(y - y_0).$$

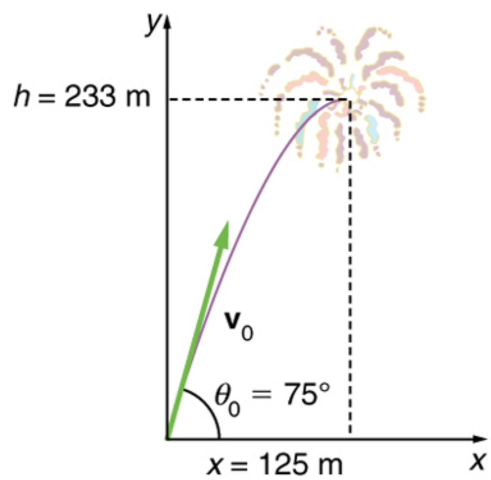


Figure 3.29: The trajectory of a fireworks shell. The fuse is set to explode the shell at the highest point in its trajectory, which is found to be at a height of 233 m and 125 m away horizontally.

Because y_0 and v_y are both zero, the equation simplifies to

$$0 = v_{0y}^2 - 2\text{gy}.$$

Solving for y gives

$$y = \frac{v_{0y}^2}{2g}.$$

Now we must find v_{0y}, the component of the initial velocity in the y-direction. It is given by v0y=v0sin v0y=v0sin size 12{v rSub { size 8{0y rSup} =v rSub

{0 rSup size 12{"sin" }} {}, where v_{0y} is the initial velocity of 70.0 m/s, and $\theta_0 = 75.0°$ is the initial angle. Thus,

$$v_{0y} = v_0 \sin \theta_0 = (70.0 \text{ m/s})(\sin 75°) = 67.6 \text{ m/s}.$$

and y is

$$y = \frac{(67.6 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)},$$

so that

$$y = 233 \text{ m}.$$

Discussion for (a)

Note that because up is positive, the initial velocity is positive, as is the maximum height, but the acceleration due to gravity is negative. Note also that the maximum height depends only on the vertical component of the initial velocity, so that any projectile with a 67.6 m/s initial vertical component of velocity will reach a maximum height of 233 m (neglecting air resistance). The numbers in this example are reasonable for large fireworks displays, the shells of which do reach such heights before exploding. In practice, air resistance is not completely negligible, and so the initial velocity would have to be somewhat larger than that given to reach the same height.

Solution for (b)

As in many physics problems, there is more than one way to solve for the time to the highest point. In this case, the easiest method is to use $y = y_0 + \frac{1}{2}(v_{0y} + v_y)t$. Because y_0 is zero, this equation reduces to simply

$$y = \frac{1}{2}(v_{0y} + v_y)t.$$

Note that the final vertical velocity, v_y, at the highest point is zero. Thus,

$$\begin{aligned} t &= \frac{2y}{(v_{0y}+v_y)} = \frac{2(233 \text{ m})}{(67.6 \text{ m/s})} \\ &= 6.90 \text{ s}. \end{aligned}$$

Discussion for (b)

This time is also reasonable for large fireworks. When you are able to see the launch of fireworks, you will notice several seconds pass before the shell explodes. (Another way of finding the time is by using $y = y_0 + v_{0y}t - \frac{1}{2}\text{gt}^2$, and solving the quadratic equation for t.)

Solution for (c)

Because air resistance is negligible, $a_x = 0$ and the horizontal velocity is constant, as discussed above. The horizontal displacement is horizontal velocity multiplied by time as given by $x = x_0 + v_x t$, where x_0 is equal to zero:

$$x = v_x t,$$

where v_x is the **x**-component of the velocity, which is given by $v_x = v_0 \cos \theta_0$. Now,

$$v_x = v_0 \cos \theta_0 = (70.0\ m/s)(\cos 75.0°) = 18.1\ m/s.$$

The time t for both motions is the same, and so x is

$$x = (18.1\ m/s)(6.90\ \text{s}) = 125\ \text{m}.$$

Discussion for (c)

The horizontal motion is a constant velocity in the absence of air resistance. The horizontal displacement found here could be useful in keeping the fireworks fragments from falling on spectators. Once the shell explodes, air resistance has a major effect, and many fragments will land directly below.

In solving part (a) of the preceding example, the expression we found for y is valid for any projectile motion where air resistance is negligible. Call the maximum height $y = h$; then,

$$h = \frac{v_{0y}^2}{2g}.$$

This equation defines the **maximum height of a projectile** and depends only on the vertical component of the initial velocity.

Defining a Coordinate System

It is important to set up a coordinate system when analyzing projectile motion. One part of defining the coordinate system is to define an origin for the x and y positions. Often, it is convenient to choose the initial position of the object as the origin such that $x_0 = 0$ and $y_0 = 0$. It is also important to define the positive and negative directions in the x and y directions. Typically, we define the positive vertical direction as upwards, and the positive horizontal direction is usually the direction of the object's motion. When this is the case, the vertical acceleration, g, takes a negative value (since it is directed downwards towards the Earth). However, it is occasionally useful to define the coordinates differently. For example, if you are analyzing the motion of a ball thrown downwards from the top of a cliff, it may make sense to define the positive direction downwards since the motion of the ball is solely in the downwards direction. If this is the case, g takes a positive value.

Calculating Projectile Motion: Hot Rock Projectile

Kilauea in Hawaii is the world's most continuously active volcano. Very active volcanoes characteristically eject red-hot rocks and lava rather than smoke and ash. Suppose a large rock is ejected from the volcano with a speed of 25.0 m/s and at an angle 35.0° above the horizontal, as shown in link. The rock strikes the side of the volcano at an altitude 20.0 m lower than its starting point. (a) Calculate the time it takes the rock to follow this path. (b) What are the magnitude and direction of the rock's velocity at impact?

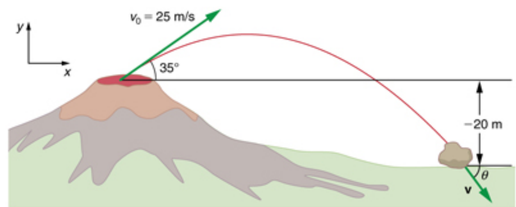


Figure 3.30: The trajectory of a rock ejected from the Kilauea volcano.

Strategy

Again, resolving this two-dimensional motion into two independent one-dimensional motions will allow us to solve for the desired quantities. The time a projectile is in the air is governed by its vertical motion alone. We will solve for t first. While the rock is rising and falling vertically, the horizontal motion continues at a constant velocity. This example asks for the final velocity. Thus, the vertical and horizontal results will be recombined to obtain v and θ_v at the final time t determined in the first part of the example.

Solution for (a)

While the rock is in the air, it rises and then falls to a final position 20.0 m lower than its starting altitude. We can find the time for this by using

$$y = y_0 + v_{0y}t - \frac{1}{2}\text{gt}^2.$$

If we take the initial position y_0 to be zero, then the final position is $y = -20.0\ \text{m}$. Now the initial vertical velocity is the vertical component of the initial velocity, found from $v_{0y} = v_0 \sin \theta_0 = (25.0\ \text{m/s})(\sin 35.0°) = 14.3\ \text{m/s}$. Substituting known values yields

$$-20.0\ m = (14.3\ m/s)t - \left(4.90\ \text{m/s}^2\right)t^2.$$

Rearranging terms gives a quadratic equation in t:

$$\left(4.90\ \mathrm{m/s}^2\right) t^2 - (14.3\ \mathrm{m/s})t - (20.0\ \mathrm{m}) = 0.$$

This expression is a quadratic equation of the form $at^2 + bt + c = 0$, where the constants are $a = 4.90$, $b = -14.3$, and $c = -20.0$. Its solutions are given by the quadratic formula:

$$t = \frac{-b \pm \sqrt{b^2 - 4\mathrm{ac}}}{2\mathrm{a}}.$$

This equation yields two solutions: $t = 3.96$ and $t = -1.03$. (It is left as an exercise for the reader to verify these solutions.) The time is $t = 3.96$ s or –1.03 s. The negative value of time implies an event before the start of motion, and so we discard it. Thus,

$$t = 3.96\ \mathrm{s}.$$

Discussion for (a)

The time for projectile motion is completely determined by the vertical motion. So any projectile that has an initial vertical velocity of 14.3 m/s and lands 20.0 m below its starting altitude will spend 3.96 s in the air.

Solution for (b)

From the information now in hand, we can find the final horizontal and vertical velocities v_x and v_y and combine them to find the total velocity v and the angle θ_0 it makes with the horizontal. Of course, v_x is constant so we can solve for it at any horizontal location. In this case, we chose the starting point since we know both the initial velocity and initial angle. Therefore:

$$v_x = v_0 \cos\theta_0 = (25.0\ m/s)(\cos 35°) = 20.5\ m/s.$$

The final vertical velocity is given by the following equation:

$$v_y = v_{0y} - \mathrm{gt},$$

where v_{0y} was found in part (a) to be 14.3 m/s. Thus,

$$v_y = 14.3\ m/s–(9.80\ \mathrm{m/s}^2)(3.96\ \mathrm{s})$$

so that

$$v_y = -24.5\ m/s.$$

To find the magnitude of the final velocity v we combine its perpendicular components, using the following equation:

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(20.5\ m/s)^2 + (-24.5\ m/s)^2},$$

which gives

$$v = 31.9\ m/s.$$

The direction θ_v is found from the equation:

$$\theta_v = \tan^{-1}(v_y/v_x)$$

so that

$$\theta_v = \tan^{-1}(-24.5/20.5) = \tan^{-1}(-1.19).$$

Thus,

$$\theta_v = -50.1^\circ.$$

Discussion for (b)

The negative angle means that the velocity is 50.1° below the horizontal. This result is consistent with the fact that the final vertical velocity is negative and hence downward—as you would expect because the final altitude is 20.0 m lower than the initial altitude. (See link.)

One of the most important things illustrated by projectile motion is that vertical and horizontal motions are independent of each other. Galileo was the first person to fully comprehend this characteristic. He used it to predict the range of a projectile. On level ground, we define range to be the horizontal distance R traveled by a projectile. Galileo and many others were interested in the range of projectiles primarily for military purposes—such as aiming cannons. However, investigating the range of projectiles can shed light on other interesting phenomena, such as the orbits of satellites around the Earth. Let us consider projectile range further.

How does the initial velocity of a projectile affect its range? Obviously, the greater the initial speed v_0, the greater the range, as shown in link(a). The initial angle θ_0 also has a dramatic effect on the range, as illustrated in link(b).

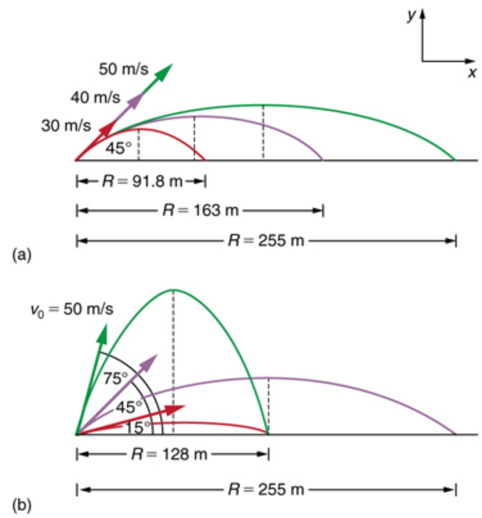


Figure 3.31: Trajectories of projectiles on level ground. (a) The greater the initial speed v_0, the greater the range for a given initial angle. (b) The effect of initial angle θ_0 on the range of a projectile with a given initial speed. Note that the range is the same for 15° and 75°, although the maximum heights of those paths are different.

For a fixed initial speed, such as might be produced by a cannon, the maximum range is obtained with $\theta_0 = 45°$. This is true only for conditions neglecting air resistance. If air resistance is considered, the maximum angle is approximately 38°. Interestingly, for every initial angle except 45°, there are two angles that give the same range—the sum of those angles is 90°. The range also depends on the value of the acceleration of gravity g. The lunar astronaut Alan Shepherd was able to drive a golf ball a great distance on the Moon because gravity is weaker there. The range R of a projectile on **level ground** for which air resistance is negligible is given by

$$R = \frac{v_0^2 \sin 2\theta_0}{g},$$

where v_0 is the initial speed and θ_0 is the initial angle relative to the horizontal. The proof of this equation is left as an end-of-chapter problem (hints are given), but it does fit the major features of projectile range as described.

When we speak of the range of a projectile on level ground, we assume that R is very small compared with the circumference of the Earth. If, however, the range is large, the Earth curves away below the projectile and acceleration of gravity changes direction along the path. The range is larger than predicted by the range equation given above because the projectile has farther to fall than it would on level ground. (See link.) If the initial speed is great enough, the projectile goes into orbit. This possibility was recognized centuries before it could be accomplished. When an object is in orbit, the Earth curves away from underneath the object at the same rate as it falls. The object thus falls continuously but never hits the surface. These and other aspects of orbital motion, such as the rotation of the Earth, will be covered analytically and in greater depth later in this text.

Once again we see that thinking about one topic, such as the range of a projectile, can lead us to others, such as the Earth orbits. In Addition of Velocities, we will examine the addition of velocities, which is another important aspect of two-dimensional kinematics and will also yield insights beyond the immediate topic.

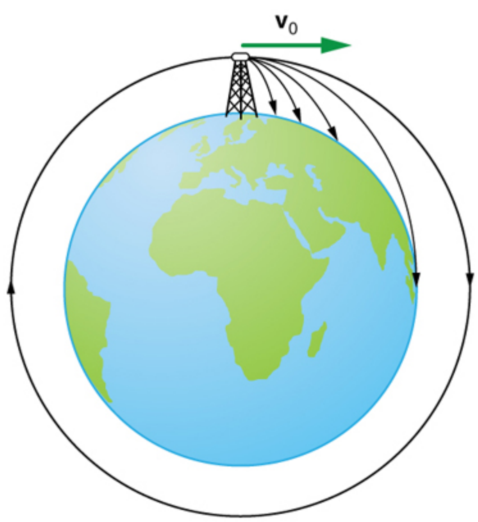


Figure 3.32: Projectile to satellite. In each case shown here, a projectile is launched from a very high tower to avoid air resistance. With increasing initial speed, the range increases and becomes longer than it would be on level ground because the Earth curves away underneath its path. With a large enough initial speed, orbit is achieved.

Projectile Motion

Blast a Buick out of a cannon! Learn about projectile motion by firing various objects. Set the angle, initial speed, and mass. Add air resistance. Make a game out of this simulation by trying to hit a target.

3.5.2 Test Prep for AP Courses

In an experiment, a student launches a ball with an initial horizontal velocity of 5.00 meters/sec at an elevation 2.00 meters above ground. Draw and clearly label with appropriate values and units a graph of the ball's horizontal velocity vs. time and the ball's vertical velocity vs. time. The graph should cover the motion from the instant after the ball is launched until the instant before it hits the ground. Assume the downward direction is negative for this problem.

"{=html}

3.5.3 Summary

- Projectile motion is the motion of an object through the air that is subject only to the acceleration of gravity.
- To solve projectile motion problems, perform the following steps:

  1. Determine a coordinate system. Then, resolve the position and/or velocity of the object in the horizontal and vertical components. The components of position **s** are given by the quantities x and y, and the components of the velocity **v** are given by $v_x = v \cos \theta$ and $v_y = v \sin \theta$, where v is the magnitude of the velocity and θ is its direction.
  2. Analyze the motion of the projectile in the horizontal direction using the following equations: ::: {#eip-898 data-type="equation"}

     $$\text{Horizontal motion}(a_x = 0)$$

     :::

     $$x = x_0 + v_x t$$

     $$v_x = v_{0x} = \mathbf{v}_{\mathrm{x}} = \text{velocity is a constant.}$$

  3. Analyze the motion of the projectile in the vertical direction using the following equations: ::: {#import-auto-id1939084 data-type="equation"}

     Vertical motion(Assuming positive direction is up; $a_y = -g = -9.80\ \text{m/s}^2$)

     :::

     $$y = y_0 + \frac{1}{2}(v_{0y} + v_y)t$$

     $$v_y = v_{0y} - \text{gt}$$

$$y = y_0 + v_{0y}t - \frac{1}{2}\mathrm{gt}^2$$

$$v_y^2 = v_{0y}^2 - 2g(y - y_0).$$

4. Recombine the horizontal and vertical components of location and/or velocity using the following equations: ::: {#import-auto-id2092332 data-type="equation"}

$$s = \sqrt{x^2 + y^2}$$

:::

$$\theta = \tan^{-1}(y/x)$$

$$v = \sqrt{v_x^2 + v_y^2}$$

$$\theta_{\mathrm{v}} = \tan^{-1}(v_y/v_x).$$

- The maximum height h of a projectile launched with initial vertical velocity v_{0y} is given by ::: {#import-auto-id1534227 data-type="equation"}

$$h = \frac{v_{0y}^2}{2g}.$$

:::

- The maximum horizontal distance traveled by a projectile is called the **range**. The range R of a projectile on level ground launched at an angle θ_0 above the horizontal with initial speed v_0 is given by ::: {#import-auto-id1951750 data-type="equation"}

$$R = \frac{v_0^2 \sin 2\theta_0}{g}.$$

:::

3.5.4 Conceptual Questions

Answer the following questions for projectile motion on level ground assuming negligible air resistance (the initial angle being neither 0° nor 90°): (a) Is the velocity ever zero? (b) When is the velocity a minimum? A maximum? (c) Can the velocity ever be the same as the initial velocity at a time other than at $t = 0$? (d) Can the speed ever be the same as the initial speed at a time other than at $t = 0$?

Answer the following questions for projectile motion on level ground assuming negligible air resistance (the initial angle being neither 0° nor 90°): (a) Is the acceleration ever zero? (b) Is the acceleration ever in the same direction as a component of velocity? (c) Is the acceleration ever opposite in direction to a component of velocity?

For a fixed initial speed, the range of a projectile is determined by the angle at which it is fired. For all but the maximum, there are two angles that give the same range. Considering factors that might affect the ability of an archer to hit a target, such as wind, explain why the smaller angle (closer to the horizontal) is preferable. When would it be necessary for the archer to use the larger angle? Why does the punter in a football game use the higher trajectory?

During a lecture demonstration, a professor places two coins on the edge of a table. She then flicks one of the coins horizontally off the table, simultaneously nudging the other over the edge. Describe the subsequent motion of the two coins, in particular discussing whether they hit the floor at the same time.

3.5.5 Problems & Exercises

A projectile is launched at ground level with an initial speed of 50.0 m/s at an angle of 30.0° above the horizontal. It strikes a target above the ground 3.00 seconds later. What are the x and y distances from where the projectile was launched to where it lands?

$$\begin{aligned} x &= 1.30 \text{ m} \times 10^2 \\ y &= 30.9 \text{ m}. \end{aligned}$$

A ball is kicked with an initial velocity of 16 m/s in the horizontal direction and 12 m/s in the vertical direction. (a) At what speed does the ball hit the ground? (b) For how long does the ball remain in the air? (c)What maximum height is attained by the ball?

A ball is thrown horizontally from the top of a 60.0-m building and lands 100.0 m from the base of the building. Ignore air resistance. (a) How long is the ball in the air? (b) What must have been the initial horizontal component of the velocity? (c) What is the vertical component of the velocity just before the ball hits the ground? (d) What is the velocity (including both the horizontal and vertical components) of the ball just before it hits the ground?

a 3.50 s

b 28.6 m/s (c) 34.3 m/s

d 44.7 m/s, 50.2° below horizontal

a A daredevil is attempting to jump his motorcycle over a line of buses parked end to end by driving up a 32° ramp at a speed of 40.0 m/s (144 km/h). How many buses can he clear if the top of the takeoff ramp is at the same height as the bus tops and the buses are 20.0 m long? (b) Discuss what your answer implies about the margin of error in this act—that is, consider how much greater the range is than the horizontal distance he must travel to miss the end of the last bus. (Neglect air resistance.)

An archer shoots an arrow at a 75.0 m distant target; the bull's-eye of the target is at same height as the release height of the arrow. (a) At what angle must the arrow be released to hit the bull's-eye if its initial speed is 35.0 m/s? In this part of the problem, explicitly show how you follow the steps involved in solving projectile motion problems. (b) There is a large tree halfway between the archer and the target with an overhanging horizontal branch 3.50 m above the release height of the arrow. Will the arrow go over or under the branch?

a 18.4°

b The arrow will go over the branch.

A rugby player passes the ball 7.00 m across the field, where it is caught at the same height as it left his hand. (a) At what angle was the ball thrown if its initial speed was 12.0 m/s, assuming that the smaller of the two possible angles was used? (b) What other angle gives the same range, and why would it not be used? (c) How long did this pass take?

Verify the ranges for the projectiles in link(a) for $\theta = 45°$ and the given initial velocities.

R=v02 sin2 0gFor =45°, R=v02gR=v02 sin2 0gFor =45°, R=v02galignl { stack { size 12{R= { {v rSub { size 8{0} rSup { size 8{2} } } "sin"2 rSub { size 8{0} } } over {g} } } {} # "For " ="45"°: {} # R= { {v rSub { size 8{0} rSup { size 8{2} } } } over {g} } {} } } {}

$R = 91.8$ m for $v_0 = 30$ m/s; $R = 163$ m for $v_0 = 40$ m/s; $R = 255$ m for $v_0 = 50$ m/s.

Verify the ranges shown for the projectiles in link(b) for an initial velocity of 50 m/s at the given initial angles.

The cannon on a battleship can fire a shell a maximum distance of 32.0 km. (a) Calculate the initial velocity of the shell. (b) What maximum height does it reach? (At its highest, the shell is above 60% of the atmosphere—but air resistance is not really negligible as assumed to make this problem easier.) (c) The ocean is not flat, because the Earth is curved. Assume that the radius of the Earth is 6.37×10^3 km. How many meters lower will its surface be 32.0 km from the ship along a horizontal line parallel to the surface at the ship? Does your answer imply that error introduced by the assumption of a flat Earth in projectile motion is significant here?

a 560 m/s

b 8.00×10^3 m

c 80.0 m. This error is not significant because it is only 1% of the answer in part (b).

An arrow is shot from a height of 1.5 m toward a cliff of height H. It is shot with a velocity of 30 m/s at an angle of 60° above the horizontal. It lands on the top edge of the cliff 4.0 s later. (a) What is the height of the cliff? (b) What is the maximum height reached by the arrow along its trajectory? (c) What is the arrow's impact speed just before hitting the cliff?

In the standing broad jump, one squats and then pushes off with the legs to see how far one can jump. Suppose the extension of the legs from the crouch position is 0.600 m and the acceleration achieved from this position is 1.25 times the acceleration due to gravity, g. How far can they jump? State your assumptions. (Increased range can be achieved by swinging the arms in the direction of the jump.)

1.50 m, assuming launch angle of 45°

The world long jump record is 8.95 m (Mike Powell, USA, 1991). Treated as a projectile, what is the maximum range obtainable by a person if he has a take-off speed of 9.5 m/s? State your assumptions.

Serving at a speed of 170 km/h, a tennis player hits the ball at a height of 2.5 m and an angle θ below the horizontal. The baseline from which the ball is served is 11.9 m from the net, which is 0.91 m high. What is the angle θ such that the ball just crosses the net? Will the ball land in the service box, which has an outermost service line is 6.40 m from the net?

$\theta = 6.1^\circ$

yes, the ball lands at 5.3 m from the net

A football quarterback is moving straight backward at a speed of 2.00 m/s when he throws a pass to a player 18.0 m straight downfield. (a) If the ball is thrown at an angle of 25° relative to the ground and is caught at the same height as it is released, what is its initial speed relative to the ground? (b) How long does it take to get to the receiver? (c) What is its maximum height above its point of release?

Gun sights are adjusted to aim high to compensate for the effect of gravity, effectively making the gun accurate only for a specific range. (a) If a gun is sighted to hit targets that are at the same height as the gun and 100.0 m away, how low will the bullet hit if aimed directly at a target 150.0 m away? The muzzle velocity of the bullet is 275 m/s. (b) Discuss qualitatively how a larger muzzle velocity would affect this problem and what would be the effect of air resistance.

a −0.486 m

b The larger the muzzle velocity, the smaller the deviation in the vertical direction, because the time of flight would be smaller. Air resistance would have the effect of decreasing the time of flight, therefore increasing the vertical deviation.

An eagle is flying horizontally at a speed of 3.00 m/s when the fish in her talons wiggles loose and falls into the lake 5.00 m below. Calculate the velocity of the fish relative to the water when it hits the water.

An owl is carrying a mouse to the chicks in its nest. Its position at that time is 4.00 m west and 12.0 m above the center of the 30.0 cm diameter nest. The owl is flying east at 3.50 m/s at an angle 30.0° below the horizontal when it accidentally drops the mouse. Is the owl lucky enough to have the mouse hit the nest? To answer this question, calculate the horizontal position of the mouse when it has fallen 12.0 m.

4.23 m. No, the owl is not lucky; he misses the nest.

Suppose a soccer player kicks the ball from a distance 30 m toward the goal. Find the initial speed of the ball if it just passes over the goal, 2.4 m above the ground, given the initial direction to be 40° above the horizontal.

Can a goalkeeper at her/ his goal kick a soccer ball into the opponent's goal without the ball touching the ground? The distance will be about 95 m. A goalkeeper can give the ball a speed of 30 m/s.

No, the maximum range (neglecting air resistance) is about 92 m.

The free throw line in basketball is 4.57 m (15 ft) from the basket, which is 3.05 m (10 ft) above the floor. A player standing on the free throw line throws the ball with an initial speed of 8.15 m/s, releasing it at a height of 2.44 m (8 ft) above the floor. At what angle above the horizontal must the ball be thrown to exactly hit the basket? Note that most players will use a large initial angle rather than a flat shot because it allows for a larger margin of error. Explicitly show how you follow the steps involved in solving projectile motion problems.

In 2007, Michael Carter (U.S.) set a world record in the shot put with a throw of 24.77 m. What was the initial speed of the shot if he released it at a height of 2.10 m and threw it at an angle of 38.0° above the horizontal? (Although the maximum distance for a projectile on level ground is achieved at 45° when air resistance is neglected, the actual angle to achieve maximum range is smaller; thus, 38° will give a longer range than 45° in the shot put.)

15.0 m/s

A basketball player is running at 5.00 m/s directly toward the basket when he jumps into the air to dunk the ball. He maintains his horizontal velocity. (a) What vertical velocity does he need to rise 0.750 m above the floor? (b) How far from the basket (measured in the horizontal direction) must he start his jump to reach his maximum height at the same time as he reaches the basket?

A football player punts the ball at a 45.0° angle. Without an effect from the wind, the ball would travel 60.0 m horizontally. (a) What is the initial speed of the ball? (b) When the ball is near its maximum height it experiences a brief gust of wind that reduces its horizontal velocity by 1.50 m/s. What distance does the ball travel horizontally?

a 24.2 m/s

b The ball travels a total of 57.4 m with the brief gust of wind.

Prove that the trajectory of a projectile is parabolic, having the form $y = \mathrm{ax} + \mathrm{bx}^2$. To obtain this expression, solve the equation $x = v_{0x}t$ for t and substitute it into the expression for $y = v_{0y}t–(1/2)\mathrm{gt}^2$ (These equations describe the x and y positions of a projectile that starts at the origin.) You should obtain an equation of the form $y = \mathrm{ax} + \mathrm{bx}^2$ where a and b are constants.

Derive $R = \frac{v_0^2 \text{ sin } 2\theta_0}{g}$ for the range of a projectile on level ground by finding the time t at which y becomes zero and substituting this value of t into the expression for $x - x_0$, noting that $R = x - x_0$

$y - y_0 = 0 = v_{0y}t - \frac{1}{2}gt^2 = (v_0 \text{ sin } \theta)t - \frac{1}{2}gt^2,$

so that $t = \frac{2(v_0 \text{ sin } \theta)}{g}$

$x - x_0 = v_{0x}t = (v_0 \text{ cos } \theta)t = R$, and substituting for t gives:

$R = v_0 \text{ cos } \theta\left(\frac{2v_0 \text{ sin } \theta}{g}\right) = \frac{2v_0^2 \text{ sin } \theta \text{ cos } \theta}{g}$

since $2 \text{ sin } \theta \text{ cos } \theta = \text{sin } 2\theta$, the range is:

$R = \frac{{v_0}^2 \text{ sin } 2\theta}{g}.$

Unreasonable Results (a) Find the maximum range of a super cannon that has a muzzle velocity of 4.0 km/s. (b) What is unreasonable about the range you found? (c) Is the premise unreasonable or is the available equation inapplicable? Explain your answer. (d) If such a muzzle velocity could be obtained, discuss the effects of air resistance, thinning air with altitude, and the curvature of the Earth on the range of the super cannon.

Construct Your Own Problem Consider a ball tossed over a fence. Construct a problem in which you calculate the ball's needed initial velocity to just clear the fence. Among the things to determine are; the height of the fence, the distance to the fence from the point of release of the ball, and the height at which the ball is released. You should also consider whether it is possible to choose the initial speed for the ball and just calculate the angle at which it is thrown. Also examine the possibility of multiple solutions given the distances and heights you have chosen.

3.5.6 Glossary

air resistance a frictional force that slows the motion of objects as they travel through the air; when solving basic physics problems, air resistance is assumed to be zero

kinematics the study of motion without regard to mass or force

motion displacement of an object as a function of time

projectile an object that travels through the air and experiences only acceleration due to gravity

projectile motion the motion of an object that is subject only to the acceleration of gravity

range the maximum horizontal distance that a projectile travels

trajectory the path of a projectile through the air

3.6 Addition of Velocities

3.6.1 Learning Objectives

By the end of this section, you will be able to:

- Apply principles of vector addition to determine relative velocity.
- Explain the significance of the observer in the measurement of velocity.

The information presented in this section supports the following AP® learning objectives and science practices:

- **3.A.1.1** The student is able to express the motion of an object using narrative, mathematical, and graphical representations. **(S.P. 1.5, 2.1, 2.2)**
- **3.A.1.3** The student is able to analyze experimental data describing the motion of an object and is able to express the results of the analysis using narrative, mathematical, and graphical representations. **(S.P. 5.1)**

3.6.2 Relative Velocity

If a person rows a boat across a rapidly flowing river and tries to head directly for the other shore, the boat instead moves **diagonally** relative to the shore, as in link. The boat does not move in the direction in which it is pointed. The reason, of course, is that the river carries the boat downstream. Similarly, if a small airplane flies overhead in a strong crosswind, you can sometimes see that the plane is not moving in the direction in which it is pointed, as illustrated in link. The plane is moving straight ahead relative to the air, but the movement of the air mass relative to the ground carries it sideways.

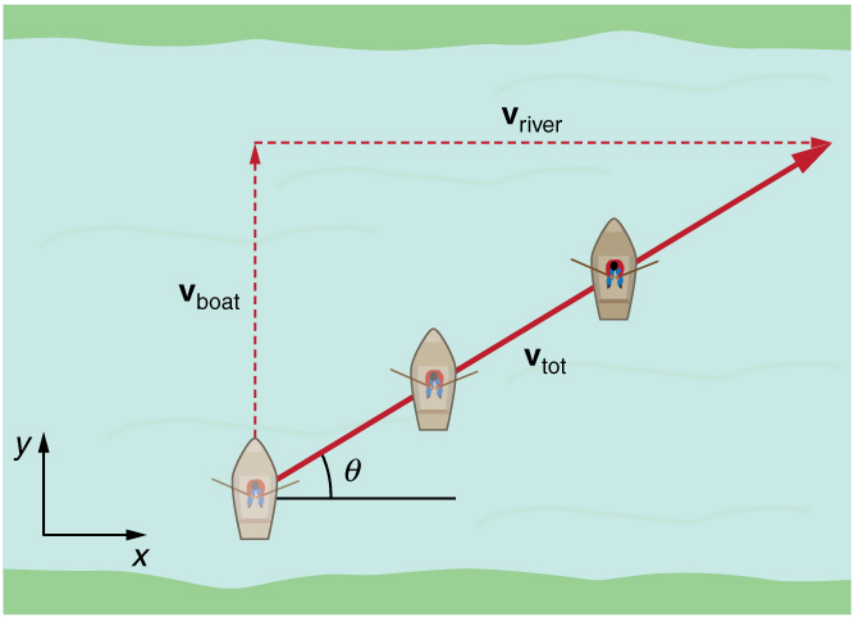


Figure 3.33: A boat trying to head straight across a river will actually move diagonally relative to the shore as shown. Its total velocity (solid arrow) relative to the shore is the sum of its velocity relative to the river plus the velocity of the river relative to the shore.

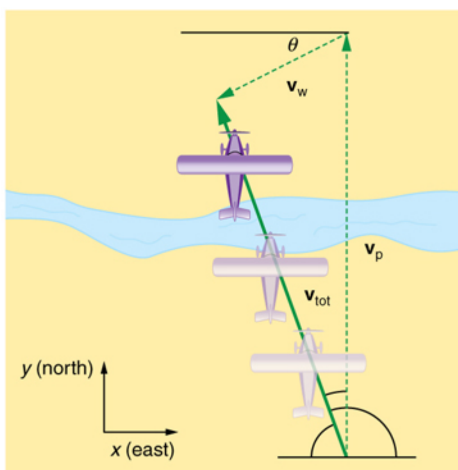


Figure 3.34: An airplane heading straight north is instead carried to the west and slowed down by wind. The plane does not move relative to the ground in the direction it points; rather, it moves in the direction of its total velocity (solid arrow).

In each of these situations, an object has a velocity relative to a medium (such as a river) and that medium has a velocity relative to an observer on solid ground. The velocity of the object **relative to the observer** is the sum of these velocity vectors, as indicated in link and link. These situations are only two of many in which it is useful to add velocities. In this module, we first re-examine how to add velocities and then consider certain aspects of what relative velocity means.

How do we add velocities? Velocity is a vector (it has both magnitude and direction); the rules of vector addition discussed in Vector Addition and Subtraction: Graphical Methods and Vector Addition and Subtraction: Analytical Methods apply to the addition of velocities, just as they do for any other vectors. In one-dimensional motion, the addition of velocities is simple—they add like ordinary numbers. For example, if a field hockey player is moving at 5 m/s straight toward the goal and drives the ball in the same direction with a velocity of 30 m/s relative to her body, then the velocity of the ball is 35 m/s relative to the stationary, profusely sweating goalkeeper standing in front of the goal.

In two-dimensional motion, either graphical or analytical techniques can be used to add velocities. We will concentrate on analytical techniques. The following equations give the relationships between the magnitude and direction of velocity (v and θ) and its components (v_x and v_y) along the **x-** and **y-**axes of an appropriately chosen coordinate system:

$$v_x = v \cos \theta$$

$$v_y = v \sin \theta$$

$$v = \sqrt{v_x^2 + v_y^2}$$

$$\theta = \tan^{-1}(v_y/v_x).$$

These equations are valid for any vectors and are adapted specifically for velocity. The first two equations are used to find the components of a velocity when its magnitude and direction are known. The last two are used to find the magnitude and direction of velocity when its components are known.

Take-Home Experiment: Relative Velocity of a Boat

Fill a bathtub half-full of water. Take a toy boat or some other object that floats in water. Unplug the drain so water starts to drain. Try pushing the boat from one side of the tub to the other and perpendicular to the flow of water. Which way do you need to push the boat so that it ends up immediately opposite?

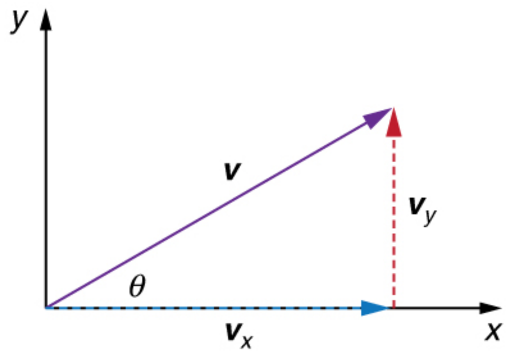


Figure 3.35: The velocity, v, of an object traveling at an angle θ to the horizontal axis is the sum of component vectors $\mathbf{v}_x$ and $\mathbf{v}_y$.

Compare the directions of the flow of water, heading of the boat, and actual velocity of the boat.

Adding Velocities: A Boat on a River

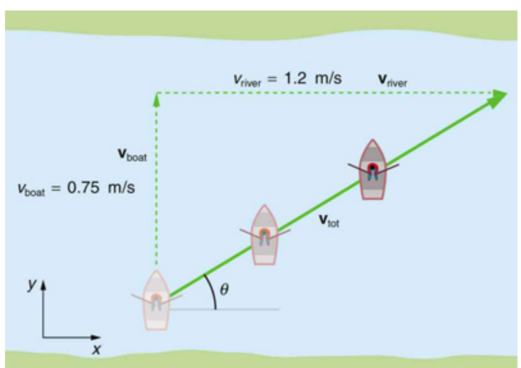


Figure 3.36: A boat attempts to travel straight across a river at a speed 0.75 m/s. The current in the river, however, flows at a speed of 1.20 m/s to the right. What is the total displacement of the boat relative to the shore?

Refer to link, which shows a boat trying to go straight across the river. Let us calculate the magnitude and direction of the boat's velocity relative to an observer on the shore, $\mathbf{v}_{\text{tot}}$. The velocity of the boat, $\mathbf{v}_{\text{boat}}$, is 0.75 m/s in the y-direction relative to the river and the velocity of the river, $\mathbf{v}_{\text{river}}$, is 1.20 m/s to the right.

Strategy

We start by choosing a coordinate system with its x-axis parallel to the velocity of the river, as shown in link. Because the boat is directed straight toward the other shore, its velocity relative to the water is parallel to the y-axis and perpendicular to the velocity of the river. Thus, we can add the two velocities by using the equations $v_{\text{tot}} = \sqrt{v_x^2 + v_y^2}$ and $\theta = \tan^{-1}(v_y/v_x)$ directly.

Solution

The magnitude of the total velocity is

$$v_{\text{tot}} = \sqrt{v_x^2 + v_y^2},$$

where

$$v_x = v_{\text{river}} = 1.20 \text{ m/s}$$

and

$$v_y = v_{\text{boat}} = 0.750 \text{ m/s}.$$

Thus,

$$v_{\text{tot}} = \sqrt{(1.20 \text{ m/s})^2 + (0.750 \text{ m/s})^2}$$

yielding

$$v_{\text{tot}} = 1.42 \text{ m/s}.$$

The direction of the total velocity θ is given by:

$$\theta = \tan^{-1}(v_y/v_x) = \tan^{-1}(0.750/1.20).$$

This equation gives

$$\theta = 32.0.$$

Discussion

Both the magnitude v and the direction θ of the total velocity are consistent with link. Note that because the velocity of the river is large compared with the velocity of the boat, it is swept rapidly downstream. This result is evidenced by the small angle (only 32.0) the total velocity has relative to the riverbank.

Calculating Velocity: Wind Velocity Causes an Airplane to Drift

Calculate the wind velocity for the situation shown in link. The plane is known to be moving at 45.0 m/s due north relative to the air mass, while its velocity relative to the ground (its total velocity) is 38.0 m/s in a direction 20.0º west of north.

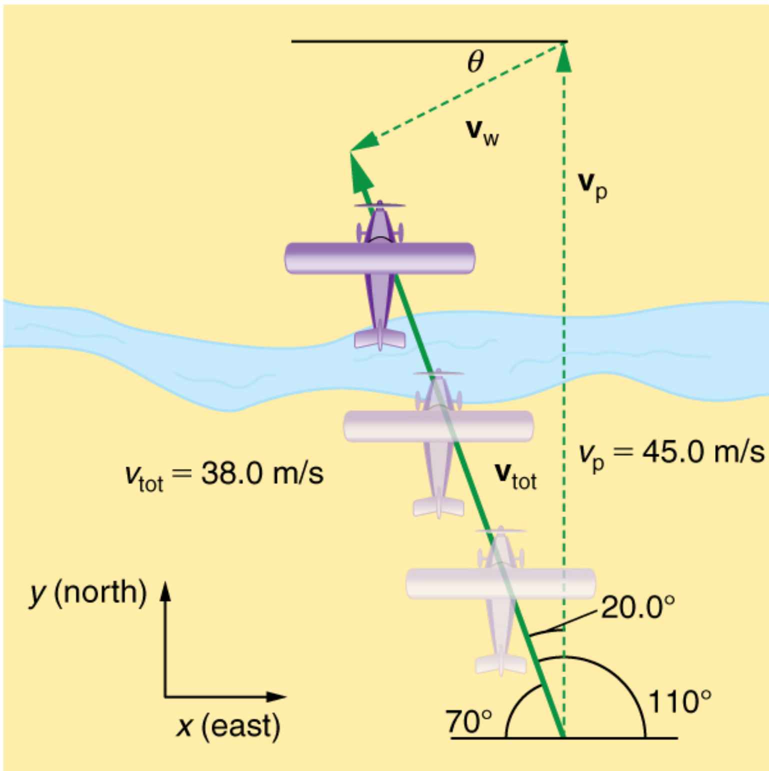


Figure 3.37: An airplane is known to be heading north at 45.0 m/s, though its velocity relative to the ground is 38.0 m/s at an angle west of north. What is the speed and direction of the wind?

Strategy

In this problem, somewhat different from the previous example, we know the total velocity $\mathbf{v}_{tot}$ and that it is the sum of two other velocities, $\mathbf{v}_w$ (the wind) and $\mathbf{v}_p$ (the plane relative to the air mass). The quantity $\mathbf{v}_p$ is known, and we are asked to find $\mathbf{v}_w$. None of the velocities are perpendicular, but it is possible to find their components along a common set of perpendicular axes. If we can find the components of $\mathbf{v}_w$, then we can combine them to solve for its magnitude and direction. As shown in link, we choose a coordinate system with its x-axis due east and its y-axis due north (parallel to $\mathbf{v}_p$). (You may wish to look back at the discussion of the addition of vectors using perpendicular components in Vector Addition and Subtraction: Analytical Methods.)

Solution

Because $\mathbf{v}_{\text{tot}}$ is the vector sum of the $\mathbf{v}_{\text{w}}$ and $\mathbf{v}_{\text{p}}$, its x- and y-components are the sums of the x- and y-components of the wind and plane velocities. Note that the plane only has vertical component of velocity so $v_{px} = 0$ and $v_{py} = v_{\text{p}}$. That is,

$$v_{\text{tot}x} = v_{\text{w}x}$$

and

$$v_{\text{tot}y} = v_{\text{w}y} + v_{\text{p}}.$$

We can use the first of these two equations to find $v_{\text{w}x}$:

$$v_{\text{w}x} = v_{\text{tot}x} = v_{\text{tot}}\text{cos } 110^{\text{o}}.$$

Because $v_{\text{tot}} = 38.0\ m/\text{s}$ and cos 110^{o} = –0.342 we have

$$v_{\text{w}x} = (38.0\ \text{m/s})(–0.342)=–13.0\ \text{m/s}.$$

The minus sign indicates motion west which is consistent with the diagram.

Now, to find v_{wy} we note that

$$v_{\text{tot}y} = v_{\text{w}y} + v_{\text{p}}$$

Here $v_{\text{tot}y}$ $=v_{\text{tot}}\text{sin } 110^{\text{o}}$; thus,

$$v_{\text{w}y} = (38.0\ m/s)(0.940) - 45.0\ m/s = -9.29\ \text{m/s}.$$

This minus sign indicates motion south which is consistent with the diagram.

Now that the perpendicular components of the wind velocity $v_{\text{w}x}$ and $v_{\text{w}y}$ are known, we can find the magnitude and direction of $\mathbf{v}_{\text{w}}$. First, the magnitude is

$$\begin{aligned} v_{\text{w}} &= \sqrt{v_{\text{w}x}^2 + v_{\text{w}y}^2} \\ &= \sqrt{(-13.0\ m/s)^2 + (-9.29\ \text{m/s})^2} \end{aligned}$$

so that

$$v_{\text{w}} = 16.0\ m/s.$$

The direction is:

$$\theta = \tan^{-1}(v_{\mathrm{w}y}/v_{\mathrm{w}x}) = \tan^{-1}(-9.29/-13.0)$$

giving

$$\theta = 35.6.$$

Discussion

The wind's speed and direction are consistent with the significant effect the wind has on the total velocity of the plane, as seen in link. Because the plane is fighting a strong combination of crosswind and head-wind, it ends up with a total velocity significantly less than its velocity relative to the air mass as well as heading in a different direction.

Note that in both of the last two examples, we were able to make the mathematics easier by choosing a coordinate system with one axis parallel to one of the velocities. We will repeatedly find that choosing an appropriate coordinate system makes problem solving easier. For example, in projectile motion we always use a coordinate system with one axis parallel to gravity.

3.6.3 Relative Velocities and Classical Relativity

When adding velocities, we have been careful to specify that the **velocity is relative to some reference frame**. These velocities are called relative velocities. For example, the velocity of an airplane relative to an air mass is different from its velocity relative to the ground. Both are quite different from the velocity of an airplane relative to its passengers (which should be close to zero). Relative velocities are one aspect of relativity, which is defined to be the study of how different observers moving relative to each other measure the same phenomenon.

Nearly everyone has heard of relativity and immediately associates it with Albert Einstein (1879–1955), the greatest physicist of the 20th century. Einstein revolutionized our view of nature with his **modern** theory of relativity, which we shall study in later chapters. The relative velocities in this section are actually aspects of classical relativity, first discussed correctly by Galileo and Isaac Newton. Classical relativity is limited to situations where speeds are less than about 1% of the speed of light—that is, less than 3,000 km/s. Most things we encounter in daily life move slower than this speed.

Let us consider an example of what two different observers see in a situation analyzed long ago by Galileo. Suppose a sailor at the top of a mast on a moving ship drops his binoculars. Where will it hit the deck? Will it hit at the base of the mast, or will it hit behind the mast because the ship is moving forward? The answer is that if air resistance is negligible, the binoculars will hit at the base of the mast at a point directly below its point of release. Now let us consider what two different observers see when the binoculars drop. One observer is on the ship and the other on shore. The binoculars have no horizontal velocity relative to the observer on the ship, and so he sees them fall straight down the mast. (See link.) To the observer on shore, the binoculars and the ship have the **same** horizontal velocity, so both move the same distance forward while the binoculars are falling. This observer sees the curved path shown in link. Although the paths look different to the different observers, each sees the same result—the binoculars hit at the base of the mast and not behind it. To get the correct description, it is crucial to correctly specify the velocities relative to the observer.

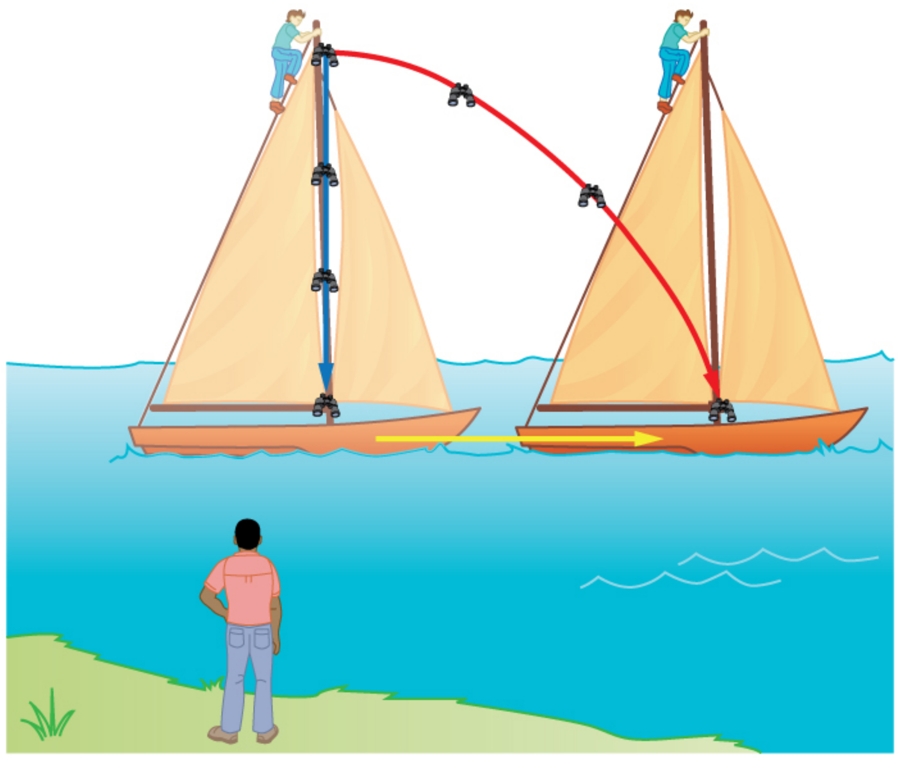


Figure 3.38: Classical relativity. The same motion as viewed by two different observers. An observer on the moving ship sees the binoculars dropped from the top of its mast fall straight down. An observer on shore sees the binoculars take the curved path, moving forward with the ship. Both observers see the binoculars strike the deck at the base of the mast. The initial horizontal velocity is different relative to the two observers. (The ship is shown moving rather fast to emphasize the effect.)

Calculating Relative Velocity: An Airline Passenger Drops a Coin

An airline passenger drops a coin while the plane is moving at 260 m/s. What is the velocity of the coin when it strikes the floor 1.50 m below its point of release: (a) Measured relative to the plane? (b) Measured relative to the Earth?

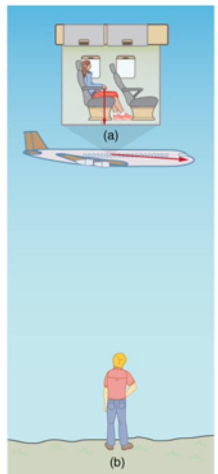


Figure 3.39: The motion of a coin dropped inside an airplane as viewed by two different observers. (a) An observer in the plane sees the coin fall straight down. (b) An observer on the ground sees the coin move almost horizontally.

Strategy

Both problems can be solved with the techniques for falling objects and projectiles. In part (a), the initial velocity of the coin is zero relative to the plane, so the motion is that of a falling object (one-dimensional). In part (b), the initial velocity is 260 m/s horizontal relative to the Earth and gravity is vertical, so this motion is a projectile motion. In both parts, it is best to use a coordinate system with vertical and horizontal axes.

Solution for (a)

Using the given information, we note that the initial velocity and position are zero, and the final position is 1.50 m. The final velocity can be found using the equation:

$$v_y{}^2 = v_{0,y}{}^2 - 2g(y - y_0).$$

Substituting known values into the equation, we get

$$v_y{}^2 = 0^2 - 2(9.80 \text{ m/s}^2)(-1.50 \text{ m} - 0\ m) = 29.4 \text{ m}^2/\text{s}^2$$

yielding

$$v_y = -5.42 \text{ m/s}.$$

We know that the square root of 29.4 has two roots: 5.42 and -5.42. We choose the negative root because we know that the velocity is directed downwards, and we have defined the positive direction to be upwards. There is no initial horizontal velocity relative to the plane and no horizontal acceleration, and so the motion is straight down relative to the plane.

Solution for (b)

Because the initial vertical velocity is zero relative to the ground and vertical motion is independent of horizontal motion, the final vertical velocity for the coin relative to the ground is $v_y = -5.42$ m/s, the same as found in part (a). In contrast to part (a), there now is a horizontal component of the velocity. However, since there is no horizontal acceleration, the initial and final horizontal velocities are the same and $v_x = 260$ m/s. The x- and y-components of velocity can be combined to find the magnitude of the final velocity:

$$v = \sqrt{{v_x}^2 + {v_y}^2}.$$

Thus,

$$v = \sqrt{(260 \text{ m/s})^2 + (-5.42 \text{ m/s})^2}$$

yielding

$$v = 260.06 \text{ m/s}.$$

The direction is given by:

$$\theta = \tan^{-1}(v_y/v_x) = \tan^{-1}(-5.42/260)$$

so that

$$\theta = \tan^{-1}(-0.0208) = -1.19^{\text{o}}.$$

Discussion

In part (a), the final velocity relative to the plane is the same as it would be if the coin were dropped from rest on the Earth and fell 1.50 m. This result fits our experience; objects in a plane fall the same way when the plane is flying horizontally as when it is at rest on the ground. This result is also true in moving cars. In part (b), an observer on the ground sees a much different motion for the coin. The plane is moving so fast horizontally to begin with that its final velocity is barely greater than the initial velocity. Once again, we see that in two dimensions, vectors do not add like ordinary numbers—the final velocity v in part (b) is **not** (260 – 5.42) m/s; rather, it is 260.06 m/s. The velocity's magnitude had to be calculated to five digits to see any difference from that of the airplane. The motions as seen by different observers (one in the plane and one on the ground) in this example are analogous to those discussed for the binoculars dropped from the mast of a moving ship, except that the velocity of the plane is much larger, so that the two observers see **very** different paths. (See link.) In addition, both observers see the coin fall 1.50 m vertically, but the one on the ground also sees it move forward 144 m (this calculation is left for the reader). Thus, one observer sees a vertical path, the other a nearly horizontal path.

Making Connections: Relativity and Einstein

Because Einstein was able to clearly define how measurements are made (some involve light) and because the speed of light is the same for all observers, the outcomes are spectacularly unexpected. Time varies with observer, energy is stored as increased mass, and more surprises await.

Motion in 2D

Try the "Ladybug Motion 2D" simulation. Learn about position, velocity, and acceleration vectors. Move the ball with the mouse or let the simulation move the ball in four types of motion (2 types of linear, simple harmonic, circle).

3.6.4 Summary

- Velocities in two dimensions are added using the same analytical vector techniques, which are rewritten as ::: {#import-auto-id2094926 data-type="equation"}

  $$v_x = v \cos \theta$$

  :::

  $$v_y = v \sin \theta$$

  $$v = \sqrt{v_x^2 + v_y^2}$$

  $$\theta = \tan^{-1}(v_y/v_x).$$

- Relative velocity is the velocity of an object as observed from a particular reference frame, and it varies dramatically with reference frame.

- **Relativity** is the study of how different observers measure the same phenomenon, particularly when the observers move relative to one another. **Classical relativity** is limited to situations where speed is less than about 1% of the speed of light (3000 km/s).

3.6.5 Conceptual Questions

What frame or frames of reference do you instinctively use when driving a car? When flying in a commercial jet airplane?

A basketball player dribbling down the court usually keeps his eyes fixed on the players around him. He is moving fast. Why doesn't he need to keep his eyes on the ball?

If someone is riding in the back of a pickup truck and throws a softball straight backward, is it possible for the ball to fall straight down as viewed by a person standing at the side of the road? Under what condition would this occur? How would the motion of the ball appear to the person who threw it?

The hat of a jogger running at constant velocity falls off the back of his head. Draw a sketch showing the path of the hat in the jogger's frame of reference. Draw its path as viewed by a stationary observer.

A clod of dirt falls from the bed of a moving truck. It strikes the ground directly below the end of the truck. What is the direction of its velocity relative to the truck just before it hits? Is this the same as the direction of its velocity relative to ground just before it hits? Explain your answers.

3.6.6 Problems & Exercises

Bryan Allen pedaled a human-powered aircraft across the English Channel from the cliffs of Dover to Cap Gris-Nez on June 12, 1979. (a) He flew for 169 min at an average velocity of 3.53 m/s in a direction 45º south of east. What was his total displacement? (b) Allen encountered a headwind averaging 2.00 m/s almost precisely in the opposite direction of his motion relative to the Earth. What was his average velocity relative to the air? (c) What was his total displacement relative to the air mass?

a 35.8 km, 45º south of east

b 5.53 m/s, 45º south of east

c 56.1 km, 45º south of east

A seagull flies at a velocity of 9.00 m/s straight into the wind. (a) If it takes the bird 20.0 min to travel 6.00 km relative to the Earth, what is the velocity of the wind? (b) If the bird turns around and flies with the wind, how long will he take to return 6.00 km? (c) Discuss how the wind affects the total round-trip time compared to what it would be with no wind.

Near the end of a marathon race, the first two runners are separated by a distance of 45.0 m. The front runner has a velocity of 3.50 m/s, and the second a velocity of 4.20 m/s. (a) What is the velocity of the second runner relative to the first? (b) If the front runner is 250 m from the finish line, who will win the race, assuming they run at constant velocity? (c) What distance ahead will the winner be when she crosses the finish line?

a 0.70 m/s faster

b Second runner wins

c 4.17 m

Verify that the coin dropped by the airline passenger in the link travels 144 m horizontally while falling 1.50 m in the frame of reference of the Earth.

A football quarterback is moving straight backward at a speed of 2.00 m/s when he throws a pass to a player 18.0 m straight downfield. The ball is thrown at an angle of 25.0 relative to the ground and is caught at the same height as it is released. What is the initial velocity of the ball *relative to the quarterback* ?

17.0 m/s, 22.1

A ship sets sail from Rotterdam, The Netherlands, heading due north at 7.00 m/s relative to the water. The local ocean current is 1.50 m/s in a direction 40.0º north of east. What is the velocity of the ship relative to the Earth?

a A jet airplane flying from Darwin, Australia, has an air speed of 260 m/s in a direction 5.0 south of west. It is in the jet stream, which is blowing at 35.0 m/s in a direction 15 south of east. What is the velocity of the airplane relative to the Earth? (b) Discuss whether your answers are consistent with your expectations for the effect of the wind on the plane's path.

a 230 m/s, 8.0 south of west

b The wind should make the plane travel slower and more to the south, which is what was calculated.

a In what direction would the ship in link have to travel in order to have a velocity straight north relative to the Earth, assuming its speed relative to the water remains 7.00 m/s? (b) What would its speed be relative to the Earth?

a Another airplane is flying in a jet stream that is blowing at 45.0 m/s in a direction 20º south of east (as in link). Its direction of motion relative to the Earth is 45.0 south of west, while its direction of travel relative to the air is 5.00 south of west. What is the airplane's speed relative to the air mass? (b) What is the airplane's speed relative to the Earth?

a 63.5 m/s

b 29.6 m/s

A sandal is dropped from the top of a 15.0-m-high mast on a ship moving at 1.75 m/s due south. Calculate the velocity of the sandal when it hits the deck of the ship: (a) relative to the ship and (b) relative to a stationary observer on shore. (c) Discuss how the answers give a consistent result for the position at which the sandal hits the deck.

The velocity of the wind relative to the water is crucial to sailboats. Suppose a sailboat is in an ocean current that has a velocity of 2.20 m/s in a direction 30.0 east of north relative to the Earth. It encounters a wind that has a velocity of 4.50 m/s in a direction of 50.0 south of west relative to the Earth. What is the velocity of the wind relative to the water?

6.68 m/s, 53.3 south of west

The great astronomer Edwin Hubble discovered that all distant galaxies are receding from our Milky Way Galaxy with velocities proportional to their distances. It appears to an observer on the Earth that we are at the center of an expanding universe. link illustrates this for five galaxies lying along a straight line, with the Milky Way Galaxy at the center. Using the data from the figure, calculate the velocities: (a) relative to galaxy 2 and (b) relative to galaxy 5. The results mean that observers on all galaxies will see themselves at the center of the expanding universe, and they would likely be aware of relative velocities, concluding that it is not possible to locate the center of expansion with the given information.

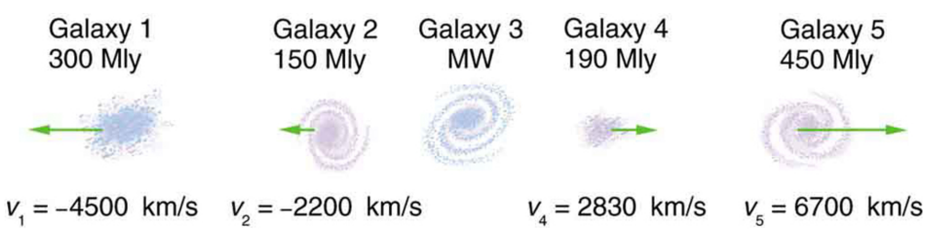


Figure 3.40: Five galaxies on a straight line, showing their distances and velocities relative to the Milky Way (MW) Galaxy. The distances are in millions of light years (Mly), where a light year is the distance light travels in one year. The velocities are nearly proportional to the distances. The sizes of the galaxies are greatly exaggerated; an average galaxy is about 0.1 Mly across.

a Use the distance and velocity data in link to find the rate of expansion as a function of distance.

b If you extrapolate back in time, how long ago would all of the galaxies have been at approximately the same position? The two parts of this problem give you some idea of how the Hubble constant for universal expansion and the time back to the Big Bang are determined, respectively.

a $H_{\text{average}} = 14.9 \frac{\text{km/s}}{\text{Mly}}$

b 20.2 billion years

An athlete crosses a 25-m-wide river by swimming perpendicular to the water current at a speed of 0.5 m/s relative to the water. He reaches the opposite side at a distance 40 m downstream from his starting point. How fast is the water in the river flowing with respect to the ground? What is the speed of the swimmer with respect to a friend at rest on the ground?

A ship sailing in the Gulf Stream is heading 25.0 west of north at a speed of 4.00 m/s relative to the water. Its velocity relative to the Earth is 4.80 m/s 5.00 west of north. What is the velocity of the Gulf Stream? (The velocity obtained is typical for the Gulf Stream a few hundred kilometers off the east coast of the United States.)

1.72 m/s, 42.3 north of east

An ice hockey player is moving at 8.00 m/s when he hits the puck toward the goal. The speed of the puck relative to the player is 29.0 m/s. The line between the center of the goal and the player makes a 90.0 angle relative to his path as shown in link. What angle must the puck's velocity make relative to the player (in his frame of reference) to hit the center of the goal?

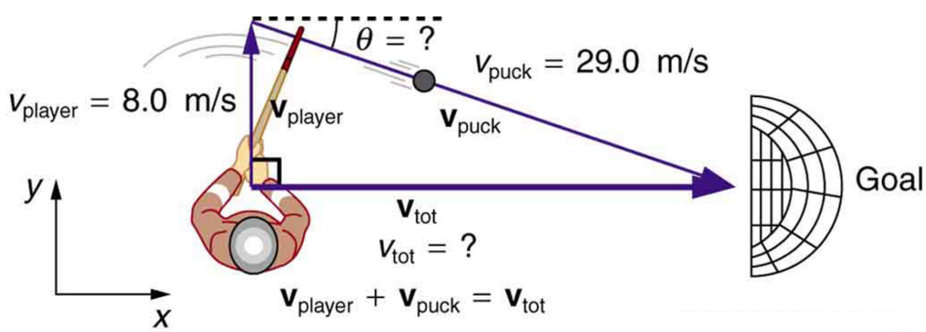


Figure 3.41: An ice hockey player moving across the rink must shoot backward to give the puck a velocity toward the goal.

Unreasonable Results Suppose you wish to shoot supplies straight up to astronauts in an orbit 36,000 km above the surface of the Earth. (a) At what velocity must the supplies be launched? (b) What is unreasonable about this velocity? (c) Is there a problem with the relative velocity between the supplies and the astronauts when the supplies reach their maximum height? (d) Is the premise unreasonable or is the available equation inapplicable? Explain your answer.

Unreasonable Results A commercial airplane has an air speed of 280 m/s due east and flies with a strong tailwind. It travels 3000 km in a direction 5 south of east in 1.50 h. (a) What was the velocity of the plane relative to the ground?

(b) Calculate the magnitude and direction of the tailwind's velocity. (c) What is unreasonable about both of these velocities? (d) Which premise is unreasonable?

Construct Your Own Problem Consider an airplane headed for a runway in a cross wind. Construct a problem in which you calculate the angle the airplane must fly relative to the air mass in order to have a velocity parallel to the runway. Among the things to consider are the direction of the runway, the wind speed and direction (its velocity) and the speed of the plane relative to the air mass. Also calculate the speed of the airplane relative to the ground. Discuss any last minute maneuvers the pilot might have to perform in order for the plane to land with its wheels pointing straight down the runway.

3.6.7 Glossary

classical relativity the study of relative velocities in situations where speeds are less than about 1% of the speed of light—that is, less than 3000 km/s

relative velocity the velocity of an object as observed from a particular reference frame

relativity the study of how different observers moving relative to each other measure the same phenomenon

velocity speed in a given direction

vector addition the rules that apply to adding vectors together

Chapter 4

Dynamics: Force and Newton's Laws of Motion

4.1 Connection for AP® Courses

class="introduction" class="section-summary" title="Section Summary" class="conceptual-questions" title="Conceptual Questions" class="problems-exercises" title="Problems & Exercises" class="ap-test-prep" title="Test Prep for AP Courses"

Motion draws our attention. Motion itself can be beautiful, causing us to marvel at the forces needed to achieve spectacular motion, such as that of a jumping dolphin, a leaping pole vaulter, a bird in flight, or an orbiting satellite. The study of motion is kinematics, but kinematics only *describes* the way objects move—their velocity and their acceleration. Dynamics considers the forces that affect the motion of moving objects and systems. Newton's laws of motion are the foundation of dynamics. These laws provide an example of the breadth and simplicity of principles under which nature functions. They are also universal laws in that they apply to situations on Earth as well as in space.

Isaac Newton's (1642–1727) laws of motion were just one part of the monumental work that has made him legendary. The development of Newton's laws marks the transition from the Renaissance into the modern era. This transition was characterized by a revolutionary change in the way people thought about the physical universe. For many centuries natural philosophers had debated the nature of the universe based largely on certain rules of logic, with great weight given to the thoughts of earlier classical philosophers such as Aristotle (384–322 BC). Among the many great thinkers who contributed to this change were Newton and Galileo Galilei (1564–1647).

Galileo was instrumental in establishing *observation* as the absolute determinant

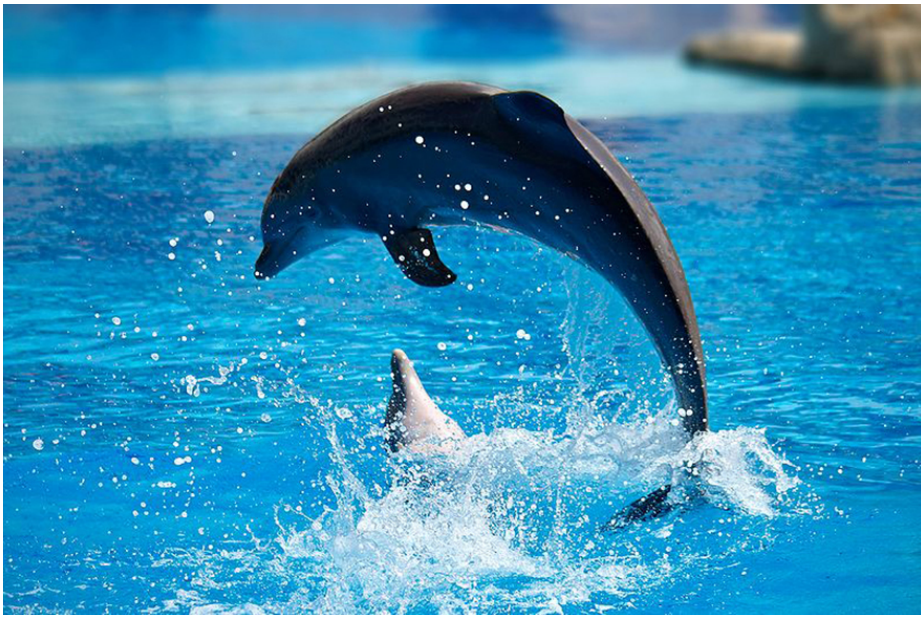
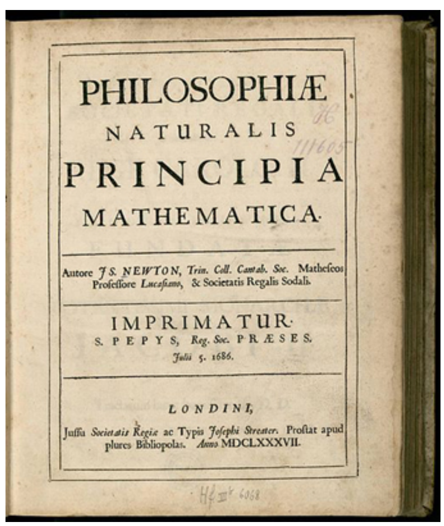


Figure 4.1: Newton's laws of motion describe the motion of the dolphin's path. (credit: Jin Jang)



PHILOSOPHIÆ
NATURALIS
PRINCIPIA
MATHEMATICA.

Autore *J S. NEWTON*, *Trin. Coll. Cantab. Soc.* Mathefeos Profeffore *Lucafiano*, & Societatis Regalis Sodali.

IMPRIMATUR.
S. PEPYS, *Reg. Soc.* PRÆSES.
Julii 5. 1686.

LONDINI,
Juffu *Societatis Regiæ* ac Typis *Jofephi Streater*. Proftat apud plures Bibliopolas. *Anno* MDCLXXXVII.

Figure 4.2: Isaac Newton's monumental work, *Philosophiae Naturalis Principia Mathematica*, was published in 1687. It proposed scientific laws that are still used today to describe the motion of objects. (credit: Service commun de la documentation de l'Université de Strasbourg)

of truth, rather than "logical" argument. Galileo's use of the telescope was his most notable achievement in demonstrating the importance of observation. He discovered moons orbiting Jupiter and made other observations that were inconsistent with certain ancient ideas and religious dogma. For this reason, and because of the manner in which he dealt with those in authority, Galileo was tried by the Inquisition and punished. He spent the final years of his life under a form of house arrest. Because others before Galileo had also made discoveries by *observing* the nature of the universe and because repeated observations verified those of Galileo, his work could not be suppressed or denied. After his death, his work was verified by others, and his ideas were eventually accepted by the church and scientific communities.

Galileo also contributed to the formulation of what is now called Newton's first law of motion. Newton made use of the work of his predecessors, which enabled him to develop laws of motion, discover the law of gravity, invent calculus, and make great contributions to the theories of light and color. It is amazing that many of these developments were made by Newton working alone, without the benefit of the usual interactions that take place among scientists today.

Newton's laws are introduced along with Big Idea 3, that interactions can be described by forces. These laws provide a theoretical basis for studying motion depending on interactions between the objects. In particular, Newton's laws are applicable to all forces in inertial frames of references (Enduring Understanding 3.A). We will find that all forces are vectors; that is, forces always have both a magnitude and a direction (Essential Knowledge 3.A.2). Furthermore, we will learn that all forces are a result of interactions between two or more objects (Essential Knowledge 3.A.3). These interactions between any two objects are described by Newton's third law, stating that the forces exerted on these objects are equal in magnitude and opposite in direction to each other (Essential Knowledge 3.A.4).

We will discover that there is an empirical cause-effect relationship between the net force exerted on an object of mass m and its acceleration, with this relationship described by Newton's second law (Enduring Understanding 3.B). This supports Big Idea 1, that inertial mass is a property of an object or a system. The mass of an object or a system is one of the factors affecting changes in motion when an object or a system interacts with other objects or systems (Essential Knowledge 1.C.1). Another is the net force on an object, which is the vector sum of all the forces exerted on the object (Essential Knowledge 3.B.1). To analyze this, we use free-body diagrams to visualize the forces exerted on a given object in order to find the net force and analyze the object's motion (Essential Knowledge 3.B.2).

Thinking of these objects as systems is a concept introduced in this chapter, where a system is a collection of elements that could be considered as a single object without any internal structure (Essential Knowledge 5.A.1). This will support Big Idea 5, that changes that occur to the system due to interactions are governed by conservation laws. These conservation laws will be the focus of later chapters in this book. They explain whether quantities are conserved in the given system or change due to transfer to or from another system due to interactions between the systems (Enduring Understanding 5.A).

Furthermore, when a situation involves more than one object, it is important to define the system and analyze the motion of a whole system, not its elements, based on analysis of external forces on the system. This supports Big Idea 4, that interactions between systems cause changes in those systems. All kinematics variables in this case describe the motion of the center of mass of the system (Essential Knowledge 4.A.1, Essential Knowledge 4.A.2). The internal forces between the elements of the system do not affect the velocity of the center of mass (Essential Knowledge 4.A.3). The velocity of the center of mass will change only if there is a net external force exerted on the system (Enduring Understanding 4.A).

We will learn that some of these interactions can be explained by the existence of fields extending through space, supporting Big Idea 2. For example, any object that has mass creates a gravitational field in space (Enduring Understanding 2.B). Any material object (one that has mass) placed in the gravitational field will experience gravitational force (Essential Knowledge 2.B.1).

Forces may be categorized as contact or long-distance (Enduring Understanding 3.C). In this chapter we will work with both. An example of a long-distance force is gravitation (Essential Knowledge 3.C.1). Contact forces, such as tension, friction, normal force, and the force of a spring, result from interatomic electric forces at the microscopic level (Essential Knowledge 3.C.4).

It was not until the advent of modern physics early in the twentieth century that it was discovered that Newton's laws of motion produce a good approximation to motion only when the objects are moving at speeds much, much less than the speed of light and when those objects are larger than the size of most molecules (about 10^{-9} m in diameter). These constraints define the realm of classical mechanics, as discussed in Introduction to the Nature of Science and Physics. At the beginning of the twentieth century, Albert Einstein (1879–1955) developed the theory of relativity and, along with many other scientists, quantum theory. Quantum theory does not have the constraints present in classical physics. All of the situations we consider in this chapter, and all those preceding the introduction of relativity in **Special Relativity**, are in the realm of classical physics.

The development of special relativity and empirical observations at atomic scales led to the idea that there are four basic forces that account for all known phenomena. These forces are called fundamental (Enduring Understanding 3.G). The properties of gravitational (Essential Knowledge 3.G.1) and electromagnetic (Essential Knowledge 3.G.2) forces are explained in more detail.

Big Idea 1 Objects and systems have properties such as mass and charge. Systems may have internal structure.

Essential Knowledge 1.C.1 Inertial mass is the property of an object or a system that determines how its motion changes when it interacts with other objects or systems.

Big Idea 2 Fields existing in space can be used to explain interactions.

Enduring Understanding 2.A A field associates a value of some physical quantity with every point in space. Field models are useful for describing interactions that occur at a distance (long-range forces) as well as a variety of other physical phenomena.

Essential Knowledge 2.A.1 A vector field gives, as a function of position (and perhaps time), the value of a physical quantity that is described by a vector.

Essential Knowledge 2.A.2 A scalar field gives the value of a physical quantity.

Enduring Understanding 2.B A gravitational field is caused by an object with mass.

Essential Knowledge 2.B.1 A gravitational field g at the location of an object with mass m causes a gravitational force of magnitude mg to be exerted on the object in the direction of the field.

Big Idea 3 The interactions of an object with other objects can be described by forces.

Enduring Understanding 3.A All forces share certain common characteristics when considered by observers in inertial reference frames.

Essential Knowledge 3.A.2 Forces are described by vectors.

Essential Knowledge 3.A.3 A force exerted on an object is always due to the interaction of that object with another object.

Essential Knowledge 3.A.4 If one object exerts a force on a second object, the second object always exerts a force of equal magnitude on the first object in the opposite direction.

Enduring Understanding 3.B Classically, the acceleration of an object interacting with other objects can be predicted by using $a = \sum F/m$.

Essential Knowledge 3.B.1 If an object of interest interacts with several other objects, the net force is the vector sum of the individual forces.

Essential Knowledge 3.B.2 Free-body diagrams are useful tools for visualizing the forces being exerted on a single object and writing the equations that represent a physical situation.

Enduring Understanding 3.C At the macroscopic level, forces can be categorized as either long-range (action-at-a-distance) forces or contact forces.

Essential Knowledge 3.C.1 Gravitational force describes the interaction of one object that has mass with another object that has mass.

Essential Knowledge 3.C.4 Contact forces result from the interaction of one object touching another object, and they arise from interatomic electric forces. These forces include tension, friction, normal, spring (Physics 1), and buoyant (Physics 2).

Enduring Understanding 3.G Certain types of forces are considered fundamental.

Essential Knowledge 3.G.1 Gravitational forces are exerted at all scales and dominate at the largest distance and mass scales.

Essential Knowledge 3.G.2 Electromagnetic forces are exerted at all scales and can dominate at the human scale.

Big Idea 4 Interactions between systems can result in changes in those systems.

Enduring Understanding 4.A The acceleration of the center of mass of a system is related to the net force exerted on the system, where $a = \sum F/m$.

Essential Knowledge 4.A.1 The linear motion of a system can be described by the displacement, velocity, and acceleration of its center of mass.

Essential Knowledge 4.A.2 The acceleration is equal to the rate of change of velocity with time, and velocity is equal to the rate of change of position with time.

Essential Knowledge 4.A.3 Forces that systems exert on each other are due to interactions between objects in the systems. If the interacting objects are parts of the same system, there will be no change in the center-of-mass velocity of that system.

Big Idea 5 Changes that occur as a result of interactions are constrained by conservation laws.

Enduring Understanding 5.A Certain quantities are conserved, in the sense that the changes of those quantities in a given system are always equal to the transfer of that quantity to or from the system by all possible interactions with other systems.

Essential Knowledge 5.A.1 A system is an object or a collection of objects. The objects are treated as having no internal structure.

4.2 Development of Force Concept

4.2.1 Learning Objectives

By the end of this section, you will be able to:

- Understand the definition of force.

The information presented in this section supports the following AP® learning objectives and science practices:

- **3.A.2.1** The student is able to represent forces in diagrams or mathematically using appropriately labeled vectors with magnitude, direction, and units during the analysis of a situation. **(S.P. 1.1)**
- **3.A.3.2** The student is able to challenge a claim that an object can exert a force on itself. **(S.P. 6.1)**
- **3.A.3.3** The student is able to describe a force as an interaction between two objects and identify both objects for any force. **(S.P. 1.4)**
- **3.B.2.1** The student is able to create and use free-body diagrams to analyze physical situations to solve problems with motion qualitatively and quantitatively. **(S.P. 1.1, 1.4, 2.2)**

Dynamics is the study of the forces that cause objects and systems to move. To understand this, we need a working definition of force. Our intuitive definition of force—that is, a push or a pull—is a good place to start. We know that a push or pull has both magnitude and direction (therefore, it is a vector quantity) and can vary considerably in each regard. For example, a cannon exerts a strong force on a cannonball that is launched into the air. In contrast, Earth exerts only a tiny downward pull on a flea. Our everyday experiences also give us a good idea of how multiple forces add. If two people push in different directions on a third person, as illustrated in link, we might expect the total force to be in the direction shown. Since force is a vector, it adds just like other vectors, as illustrated in link(a) for two ice skaters. Forces, like other vectors, are represented by arrows and can be added using the familiar head-to-tail method or by trigonometric methods. These ideas were developed in Two-Dimensional Kinematics.

By definition, force is always the result of an interaction of two or more objects. No object possesses force on its own. For example, a cannon does not possess force, but it can exert force on a cannonball. Earth does not possess force on its own, but exerts force on a football or on any other massive object. The skaters in Figure 4.3 exert force on one another as they interact.

No object can exert force on itself. When you clap your hands, one hand exerts force on the other. When a train accelerates, it exerts force on the track and vice versa. A bowling ball is accelerated by the hand throwing it; once the hand is no longer in contact with the bowling ball, it is no longer accelerating the bowling ball or exerting force on it. The ball continues moving forward due to inertia.

link(b) is our first example of a free-body diagram, which is a technique used to illustrate all the external forces acting on a body. The body is represented by a single isolated point (or free body), and only those forces acting *on* the body from the outside (external forces) are shown. (These forces are the only ones shown, because only external forces acting on the body affect its motion. We can ignore any internal forces within the body.) Free-body diagrams are very useful in analyzing forces acting on a system and are employed extensively in the study and application of Newton's laws of motion.

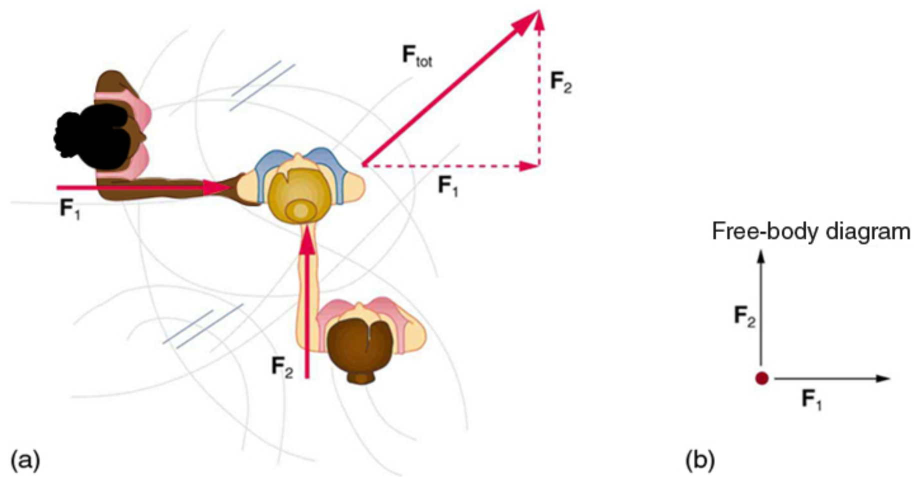


Figure 4.3: Part (a) shows an overhead view of two ice skaters pushing on a third. Forces are vectors and add like other vectors, so the total force on the third skater is in the direction shown. In part (b), we see a free-body diagram representing the forces acting on the third skater.

A more quantitative definition of force can be based on some standard force, just as distance is measured in units relative to a standard distance. One possibility is to stretch a spring a certain fixed distance, as illustrated in link, and use the force it exerts to pull itself back to its relaxed shape—called a *restoring force*—as a standard. The magnitude of all other forces can be stated as multiples of this standard unit of force. Many other possibilities exist for standard forces. (One that we will encounter in Magnetism is the magnetic force between two wires carrying electric current.) Some alternative definitions of force will be given later in this chapter.

Take-Home Experiment: Force Standards

To investigate force standards and cause and effect, get two identical rubber bands. Hang one rubber band vertically on a hook. Find a small household item that could be attached to the rubber band using a paper clip, and use this item as a weight to investigate the stretch of the rubber band. Measure the amount of stretch produced in the rubber band with one, two, and four of these (identical) items suspended from the rubber band. What is the relationship between the number of items and the amount of stretch? How large a stretch would you expect for the same number of items suspended from two rubber bands? What happens to the amount of stretch of the rubber band (with the weights attached) if the weights are also pushed to the side with a pencil?

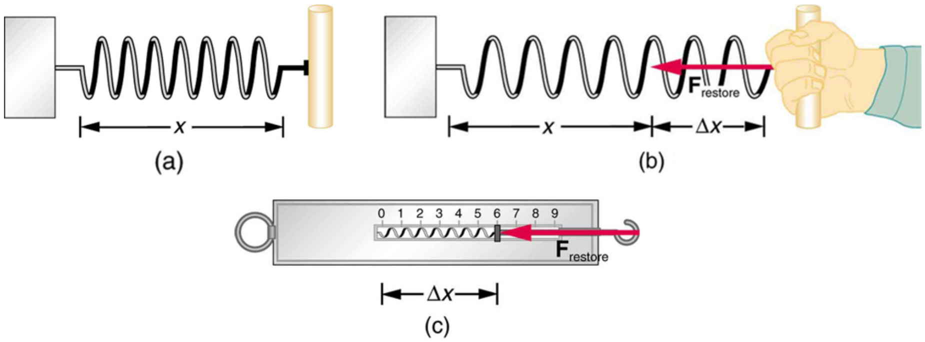
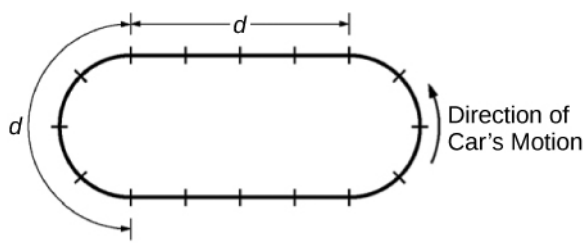


Figure 4.4: The force exerted by a stretched spring can be used as a standard unit of force. (a) This spring has a length x when undistorted. (b) When stretched a distance Δx, the spring exerts a restoring force, $\mathbf{F}_{\text{restore}}$, which is reproducible. (c) A spring scale is one device that uses a spring to measure force. The force $\mathbf{F}_{\text{restore}}$ is exerted on whatever is attached to the hook. Here $\mathbf{F}_{\text{restore}}$ has a magnitude of 6 units in the force standard being employed.

4.2.2 Test Prep for AP Courses



The figure above represents a racetrack with semicircular sections connected by straight sections. Each section has length d, and markers along the track are spaced $d/4$ apart. Two people drive cars counterclockwise around the track, as shown. Car X goes around the curves at constant speed v_c, increases speed at constant acceleration for half of each straight section to reach a maximum speed of $2v_c$, then brakes at constant acceleration for the other half of each straight section to return to speed v_c. Car Y also goes around the curves at constant speed v_c, increases its speed at constant acceleration for one-fourth of each straight section to reach the same maximum speed $2v_c$, stays at that speed for half of each straight section, then brakes at constant acceleration for the remaining fourth of each straight section to return to speed v_c.

a On the figures below, draw an arrow showing the direction of the net force on each of the cars at the positions noted by the dots. If the net force is zero at any position, label the dot with 0.

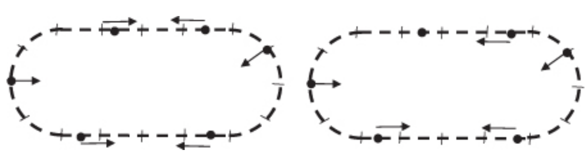
The position of the six dots on the Car Y track on the right are as follows:

- The first dot on the left center of the track is at the same position as it is on the Car X track.
- The second dot is just slight to the right of the Car X dot (less than a dash) past three perpendicular hash marks moving to the right.
- The third dot is about one and two-thirds perpendicular hash marks to the right of the center top perpendicular has mark.
- The fourth dot is in the same position as the Car X figure (one perpendicular hash mark above the center right perpendicular hash mark).
- The fifth dot is about one and two-third perpendicular hash marks to the right of the center bottom perpendicular hash mark.
- The sixth dot is in the same position as the Car Y dot (one and two third perpendicular hash marks to the left of the center bottom hash mark).

b

i. Indicate which car, if either, completes one trip around the track in less time, and justify your answer qualitatively without using equations.

ii. Justify your answer about which car, if either, completes one trip around the track in less time quantitatively with appropriate equations.



Car X is shown on the left, and Car Y is shown on the right.

i.

Car X takes longer to accelerate and does not spend any time traveling at top speed. Car Y accelerates over a shorter time and spends time going at top speed. So Car Y must cover the straightaways in a shorter time. Curves take the same time, so Car Y must overall take a shorter time.

ii.

The only difference in the calculations for the time of one segment of linear acceleration is the difference in distances. That shows that Car X takes longer to accelerate. The equation $\frac{d}{4v_c} = t_c$ corresponds to Car Y traveling for a time at top speed.

Substituting $a = \frac{v_c}{t_1}$ into the displacement equation in part (b) ii gives $D = \frac{3}{2}v_c t_1$.

This shows that a car takes less time to reach its maximum speed when it accelerates over a shorter distance. Therefore, Car Y reaches its maximum speed more quickly, and spends more time at its maximum speed than Car X does, as argued in part (b) i.

Which of the following is an example of a body exerting a force on itself?

a. a person standing up from a seated position
b. a car accelerating while driving
c. both of the above
d. none of the above

A hawk accelerates as it glides in the air. Does the force causing the acceleration come from the hawk itself? Explain.

A body cannot exert a force on itself. The hawk may accelerate as a result of several forces. The hawk may accelerate toward Earth as a result of the force due to gravity. The hawk may accelerate as a result of the additional force exerted on it by wind. The hawk may accelerate as a result of orienting its body to create less air resistance, thus increasing the net force forward.

What causes the force that moves a boat forward when someone rows it?

a. The force is caused by the rower's arms.
b. The force is caused by an interaction between the oars and gravity.
c. The force is caused by an interaction between the oars and the water the boat is traveling in.
d. The force is caused by friction.

4.2.3 Section Summary

- Dynamics is the study of how forces affect the motion of objects.
- Force is a push or pull that can be defined in terms of various standards, and it is a vector having both magnitude and direction.
- External forces are any outside forces that act on a body. A free-body diagram is a drawing of all external forces acting on a body.

4.2.4 Conceptual Questions

Propose a force standard different from the example of a stretched spring discussed in the text. Your standard must be capable of producing the same force repeatedly.

What properties do forces have that allow us to classify them as vectors?

4.2.5 Glossary

dynamics the study of how forces affect the motion of objects and systems

external force a force acting on an object or system that originates outside of the object or system

free-body diagram a sketch showing all of the external forces acting on an object or system; the system is represented by a dot, and the forces are represented by vectors extending outward from the dot

force a push or pull on an object with a specific magnitude and direction; can be represented by vectors; can be expressed as a multiple of a standard force

4.3 Newton's First Law of Motion: Inertia

4.3.1 Learning Objectives

By the end of this section, you will be able to:

- Define mass and inertia.
- Understand Newton's first law of motion.

Experience suggests that an object at rest will remain at rest if left alone, and that an object in motion tends to slow down and stop unless some effort is made to keep it moving. What Newton's first law of motion states, however, is the following:

Newton's First Law of Motion

There exists an inertial frame of reference such that a body at rest remains at rest, or, if in motion, remains in motion at a constant velocity unless acted on by a net external force.

Note the repeated use of the verb "remains." We can think of this law as preserving the status quo of motion.

The first law of motion postulates the existence of at least one frame of reference which we call an inertial reference frame, relative to which the motion of an object not subject to forces is a straight line at a constant speed. An inertial reference frame is any reference frame that is not itself accelerating. A car traveling at constant velocity is an inertial reference frame. A car slowing down for a stoplight, or speeding up after the light turns green, will be accelerating and is not an inertial reference frame. Finally, when the car goes around a turn, which is due to an acceleration changing the direction of the velocity vector, it is not an inertial reference frame. Note that Newton's laws of motion are only valid for inertial reference frames.

Rather than contradicting our experience, Newton's first law of motion states that there must be a *cause* (which is a net external force) *for there to be any change in velocity (either a change in magnitude or direction)* in an inertial reference frame. We will define *net external force* in the next section. An object sliding across a table or floor slows down due to the net force of friction acting on the object. If friction disappeared, would the object still slow down?

The idea of cause and effect is crucial in accurately describing what happens in various situations. For example, consider what happens to an object sliding along a rough horizontal surface. The object quickly grinds to a halt. If we spray the surface with talcum powder to make the surface smoother, the object slides farther. If we make the surface even smoother by rubbing lubricating oil on it, the object slides farther yet. Extrapolating to a frictionless surface, we can imagine the object sliding in a straight line indefinitely. Friction is thus the *cause* of the slowing (consistent with Newton's first law). The object would not slow down at all if friction were completely eliminated. Consider an air hockey table. When the air is turned off, the puck slides only a short distance before friction slows it to a stop. However, when the air is turned on, it creates a nearly frictionless surface, and the puck glides long distances without slowing down. Additionally, if we know enough about the friction, we can accurately predict how quickly the object will slow down. Friction is an external force.

Newton's first law is completely general and can be applied to anything from an object sliding on a table to a satellite in orbit to blood pumped from the heart. Experiments have thoroughly verified that any change in velocity (speed or direction) must be caused by an external force. The idea of *generally applicable or universal laws* is important not only here—it is a basic feature of all laws of physics. Identifying these laws is like recognizing patterns in nature from which further patterns can be discovered. The genius of Galileo, who first developed the idea for the first law, and Newton, who clarified it, was to ask the fundamental question, "What is the cause?" Thinking in terms of cause and effect is a worldview fundamentally different from the typical ancient Greek approach when questions such as "Why does a tiger have stripes?" would have been answered in Aristotelian fashion, "That is the nature of the beast." True perhaps, but not a useful insight.

4.3.2 Mass

The property of a body to remain at rest or to remain in motion with constant velocity is called inertia. Newton's first law is often called the law of inertia. As we know from experience, some objects have more inertia than others. It is obviously more difficult to change the motion of a large boulder than that of a basketball, for example. The inertia of an object is measured by its mass.

An object with a small mass will exhibit less inertia and be more affected by other objects. An object with a large mass will exhibit greater inertia and be less affected by other objects. This inertial mass of an object is a measure of how difficult it is to alter the uniform motion of the object by an external force.

Roughly speaking, mass is a measure of the amount of "stuff" (or matter) in something. The quantity or amount of matter in an object is determined by the numbers of atoms and molecules of various types it contains. Unlike weight, mass does not vary with location. The mass of an object is the same on Earth, in orbit, or on the surface of the Moon. In practice, it is very difficult to count and identify all of the atoms and molecules in an object, so masses are not often determined in this manner. Operationally, the masses of objects are determined by comparison with the standard kilogram.

Check Your Understanding

Which has more mass: a kilogram of cotton balls or a kilogram of gold?

They are equal. A kilogram of one substance is equal in mass to a kilogram of another substance. The quantities that might differ between them are volume and density.

4.3.3 Section Summary

- Newton's first law of motion states that in an inertial frame of reference a body at rest remains at rest, or, if in motion, remains in motion at a constant velocity unless acted on by a net external force. This is also known as the law of inertia.
- Inertia is the tendency of an object to remain at rest or remain in motion. Inertia is related to an object's mass.
- Mass is the quantity of matter in a substance.

4.3.4 Conceptual Questions

How are inertia and mass related?

What is the relationship between weight and mass? Which is an intrinsic, unchanging property of a body?

4.3.5 Glossary

inertia the tendency of an object to remain at rest or remain in motion

law of inertia see Newton's first law of motion

mass the quantity of matter in a substance; measured in kilograms

Newton's first law of motion in an inertial frame of reference, a body at rest remains at rest, or, if in motion, remains in motion at a constant velocity unless acted on by a net external force; also known as the law of inertia

4.4 Newton's Second Law of Motion: Concept of a System

4.4.1 Learning Objectives

By the end of this section, you will be able to:

- Define net force, external force, and system.
- Understand Newton's second law of motion.
- Apply Newton's second law to determine the weight of an object.

Newton's second law of motion is closely related to Newton's first law of motion. It mathematically states the cause and effect relationship between force and changes in motion. Newton's second law of motion is more quantitative and is used extensively to calculate what happens in situations involving a force. Before we can write down Newton's second law as a simple equation giving the exact relationship of force, mass, and acceleration, we need to sharpen some ideas that have already been mentioned.

First, what do we mean by a change in motion? The answer is that a change in motion is equivalent to a change in velocity. A change in velocity means, by definition, that there is an acceleration. Newton's first law says that a net external force causes a change in motion; thus, we see that a *net external force causes acceleration*.

Another question immediately arises. What do we mean by an external force? An intuitive notion of external is correct—an external force acts from outside the system of interest. For example, in link(a) the system of interest is the wagon plus the child in it. The two forces exerted by the other children are external forces. An internal force acts between elements of the system. Again looking at link(a), the force the child in the wagon exerts to hang onto the wagon is an internal force between elements of the system of interest. Only external forces affect the motion of a system, according to Newton's first law. (The internal forces actually cancel, as we shall see in the next section.) *You must define the boundaries of the system before you can determine which forces are external.* Sometimes the system is obvious, whereas other times identifying the boundaries of a system is more subtle. The concept of a system is fundamental to many areas of physics, as is the correct application of Newton's laws. This concept will be revisited many times on our journey through physics.

When we describe the acceleration of a system, we are modeling the system as a single point which contains all of the mass of that system. The point we choose for this is the point about which the system's mass is evenly distributed. For example, in a rigid object, this center of mass is the point where the object will stay balanced even if only supported at this point. For a sphere or disk made of homogenous material, this point is of course at the center. Similarly, for a rod made of homogenous material, the center of mass will be at the midpoint.

For the rider in the wagon in Figure 4.5, the center of mass is probably between the rider's hips. Due to internal forces, the rider's hand or hair may accelerate slightly differently, but it is the acceleration of the system's center of mass that interests us. This is true whether the system is a vehicle carrying passengers, a bowl of grapes, or a planet. When we draw a free-body diagram of a system, we represent the system's center of mass with a single point and use vectors to indicate the forces exerted on that center of mass. (See Figure 4.5.)

Now, it seems reasonable that acceleration should be directly proportional to and in the same direction as the net (total) external force acting on a system. This assumption has been verified experimentally and is illustrated in link. In part (a), a smaller force causes a smaller acceleration than the larger force illustrated in part (c). For completeness, the vertical forces are also shown; they are assumed to cancel since there is no acceleration in the vertical direction. The vertical forces are the weight $\mathbf{w}$ and the support of the ground $\mathbf{N}$, and the horizontal force $\mathbf{f}$ represents the force of friction. These will be discussed in more detail in later sections. For now, we will define friction as a force that opposes the motion past each other of objects that are touching. link(b) shows how vectors representing the external forces add together to produce a net force, $\mathbf{F}_{\text{net}}$.

To obtain an equation for Newton's second law, we first write the relationship of acceleration and net external force as the proportionality

$$\mathbf{a}\propto\mathbf{F}_{\text{net}},$$

where the symbol $\propto$ means "proportional to," and $\mathbf{F}_{\text{net}}$ is the net external force. (The net external force is the vector sum of all external forces and can be determined graphically, using the head-to-tail method, or analytically, using components. The techniques are the same as for the addition of other vectors, and are covered in Two-Dimensional Kinematics.) This proportionality states what we have said in words—*acceleration is directly proportional to the net external force*. Once the system of interest is chosen, it is important to identify the external forces and ignore the internal ones. It is a tremendous simplification not to have to consider the numerous internal forces acting between objects within the system, such as muscular forces within the child's body, let alone the myriad of forces between atoms in the objects, but by doing so, we can easily solve some very complex problems with only minimal error due to our simplification

Now, it also seems reasonable that acceleration should be inversely proportional to the mass of the system. In other words, the larger the mass (the inertia), the smaller the acceleration produced by a given force. And indeed, as illustrated in link, the same net external force applied to a car produces a much smaller acceleration than when applied to a basketball. The proportionality is written as

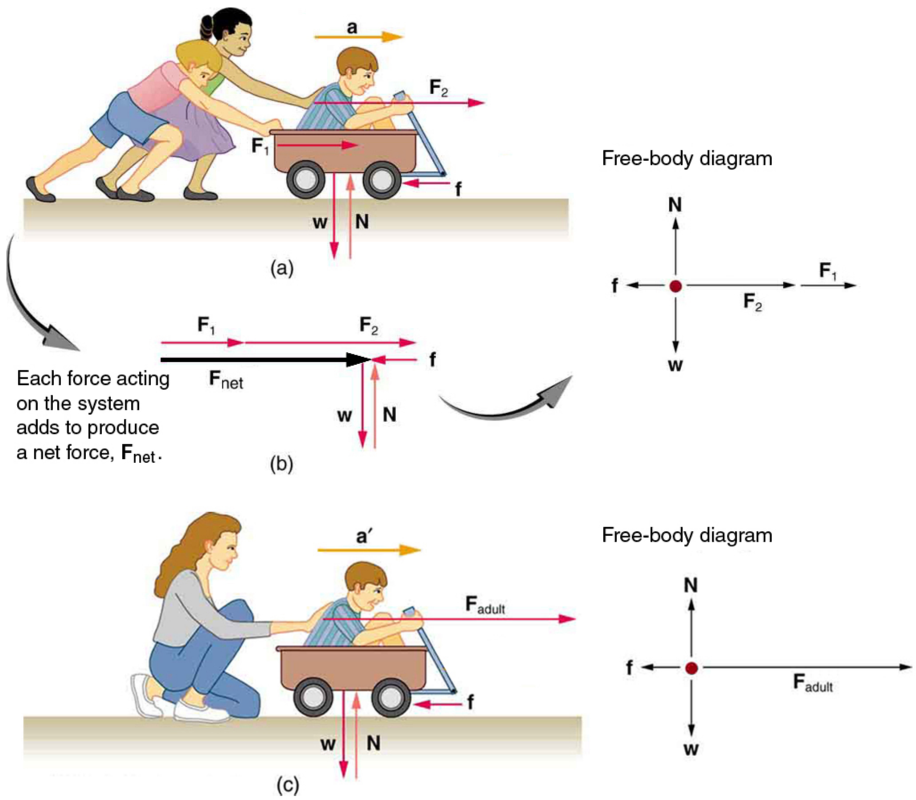


Figure 4.5: Different forces exerted on the same mass produce different accelerations. (a) Two children push a wagon with a child in it. Arrows representing all external forces are shown. The system of interest is the wagon and its rider. The weight **w** of the system and the support of the ground **N** are also shown for completeness and are assumed to cancel. The vector **f** represents the friction acting on the wagon, and it acts to the left, opposing the motion of the wagon. (b) All of the external forces acting on the system add together to produce a net force, $\mathbf{F}_{\text{net}}$. The free-body diagram shows all of the forces acting on the system of interest. The dot represents the center of mass of the system. Each force vector extends from this dot. Because there are two forces acting to the right, we draw the vectors collinearly. (c) A larger net external force produces a larger acceleration (**a'** $>$ **a**) when an adult pushes the child.

$$\mathbf{a} \propto \frac{1}{m}$$

where m is the mass of the system. Experiments have shown that acceleration is exactly inversely proportional to mass, just as it is exactly linearly proportional to the net external force.

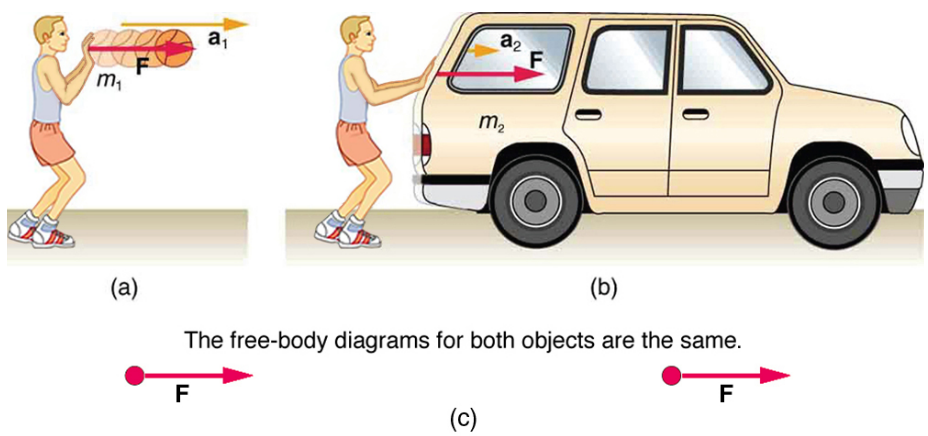


Figure 4.6: The same force exerted on systems of different masses produces different accelerations. (a) A basketball player pushes on a basketball to make a pass. (The effect of gravity on the ball is ignored.) (b) The same player exerts an identical force on a stalled SUV and produces a far smaller acceleration (even if friction is negligible). (c) The free-body diagrams are identical, permitting direct comparison of the two situations. A series of patterns for the free-body diagram will emerge as you do more problems.

Both of these proportionalities have been experimentally verified repeatedly and consistently, for a broad range of systems and scales. Thus, it has been **experimentally** found that the acceleration of an object depends *only* on the net external force and the mass of the object. Combining the two proportionalities just given yields Newton's second law of motion.

Applying the Science Practices: Testing the Relationship Between Mass, Acceleration, and Force

Plan three simple experiments using objects you have at home to test relationships between mass, acceleration, and force.

a Design an experiment to test the relationship between mass and acceleration. What will be the independent variable in your experiment? What will be the dependent variable? What controls will you put in place to ensure force is constant?

b Design a similar experiment to test the relationship between mass and force. What will be the independent variable in your experiment? What will be the dependent variable? What controls will you put in place to ensure acceleration is constant?

c Design a similar experiment to test the relationship between force and acceleration. What will be the independent variable in your experiment? What will be the dependent variable? Will you have any trouble ensuring that the mass is constant?

What did you learn?

Newton's Second Law of Motion

The acceleration of a system is directly proportional to and in the same direction as the net external force acting on the system, and inversely proportional to its mass.

In equation form, Newton's second law of motion is

$$\mathbf{a} = \frac{\mathbf{F}_{\text{net}}}{m}.$$

This is often written in the more familiar form

$$\mathbf{F}_{\text{net}} = m\mathbf{a}.$$

When only the magnitude of force and acceleration are considered, this equation is simply

$$F_{\text{net}} = \text{ma}.$$

Although these last two equations are really the same, the first gives more insight into what Newton's second law means. The law is a *cause and effect relationship* among three quantities that is not simply based on their definitions. The validity of the second law is completely based on experimental verification.

Applying the Science Practices: Systems and Free-Body Diagrams

First, consider a person on a sled sliding downhill. What is the system in this situation? Try to draw a free-body diagram describing this system, labeling all the forces and their directions. Which of the forces are internal? Which are external?

Next, consider a person on a sled being pushed along level ground by a friend. What is the system in this situation? Try to draw a free-body diagram describing this system, labelling all the forces and their directions. Which of the forces are internal? Which are external?

4.4.2 Units of Force

$\mathbf{F}_{\text{net}} = m\mathbf{a}$ is used to define the units of force in terms of the three basic units for mass, length, and time. The SI unit of force is called the newton (abbreviated N) and is the force needed to accelerate a 1-kg system at the rate of 1m/s^2. That is, since $\mathbf{F}_{\text{net}} = m\mathbf{a}$,

$$1\ \text{N} = 1\ \text{kg} \cdot \text{m/s}^2.$$

While almost the entire world uses the newton for the unit of force, in the United States the most familiar unit of force is the pound (lb), where 1 N = 0.225 lb.

4.4.3 Weight and the Gravitational Force

When an object is dropped, it accelerates toward the center of Earth. Newton's second law states that a net force on an object is responsible for its acceleration. If air resistance is negligible, the net force on a falling object is the gravitational force, commonly called its weight **w**. Weight can be denoted as a vector **w** because it has a direction; *down* is, by definition, the direction of gravity, and hence weight is a downward force. The magnitude of weight is denoted as w. Galileo was instrumental in showing that, in the absence of air resistance, all objects fall with the same acceleration g. Using Galileo's result and Newton's second law, we can derive an equation for weight.

Consider an object with mass m falling downward toward Earth. It experiences only the downward force of gravity, which has magnitude w. Newton's second law states that the magnitude of the net external force on an object is $F_{\text{net}} = \text{ma}$.

Since the object experiences only the downward force of gravity, $F_{\text{net}} = w$. We know that the acceleration of an object due to gravity is g, or $a = g$. Substituting these into Newton's second law gives

Weight

This is the equation for *weight*—the gravitational force on a mass m:

$$w = \text{mg}.$$

Since g =9.80 m/s^2 on Earth, the weight of a 1.0 kg object on Earth is 9.8 N, as we see:

$$w = \text{mg} = (1.0\ \text{kg})(9.80\ \text{m/s}^2) = 9.8\ \text{N}.$$

Recall that g can take a positive or negative value, depending on the positive direction in the coordinate system. Be sure to take this into consideration when solving problems with weight.

When the net external force on an object is its weight, we say that it is in free-fall. That is, the only force acting on the object is the force of gravity. In the real world, when objects fall downward toward Earth, they are never truly in free-fall because there is always some upward force from the air acting on the object.

The acceleration due to gravity g varies slightly over the surface of Earth, so that the weight of an object depends on location and is not an intrinsic property of the object. Weight varies dramatically if one leaves Earth's surface. On the Moon, for example, the acceleration due to gravity is only 1.67 m/s^2. A 1.0-kg mass thus has a weight of 9.8 N on Earth and only about 1.7 N on the Moon.

The broadest definition of weight in this sense is that *the weight of an object is the gravitational force on it from the nearest large body*, such as Earth, the Moon, the Sun, and so on. This is the most common and useful definition of weight in physics. It differs dramatically, however, from the definition of weight used by NASA and the popular media in relation to space travel and exploration. When they speak of "weightlessness" and "microgravity," they are really referring to the phenomenon we call "free-fall" in physics. We shall use the above definition of weight, and we will make careful distinctions between free-fall and actual weightlessness.

It is important to be aware that weight and mass are very different physical quantities, although they are closely related. Mass is the quantity of matter (how much "stuff") and does not vary in classical physics, whereas weight is the gravitational force and does vary depending on gravity. It is tempting to equate the two, since most of our examples take place on Earth, where the weight of an object only varies a little with the location of the object. Furthermore, the terms *mass* and *weight* are used interchangeably in everyday language; for example, our medical records often show our "weight" in kilograms, but never in the correct units of newtons.

Common Misconceptions: Mass vs. Weight

Mass and weight are often used interchangeably in everyday language. However, in science, these terms are distinctly different from one another. Mass is a measure of how much matter is in an object. The typical measure of mass is the kilogram (or the "slug" in English units). Weight, on the other hand, is a measure of the force of gravity acting on an object. Weight is equal to the mass of an object (m) multiplied by the acceleration due to gravity (g). Like any other force, weight is measured in terms of newtons (or pounds in English units).

Assuming the mass of an object is kept intact, it will remain the same, regardless of its location. However, because weight depends on the acceleration due to gravity, the weight of an object *can change* when the object enters into a region with stronger or weaker gravity. For example, the acceleration due to gravity on the Moon is 1.67 m/s^2 (which is much less than the acceleration due to gravity on Earth, 9.80 m/s^2). If you measured your weight on Earth and then measured your weight on the Moon, you would find that you "weigh" much less, even though you do not look any skinnier. This is because the force of gravity is weaker on the Moon. In fact, when people say that they are "losing weight," they really mean that they are losing "mass" (which in turn causes them to weigh less).

Take-Home Experiment: Mass and Weight

What do bathroom scales measure? When you stand on a bathroom scale, what happens to the scale? It depresses slightly. The scale contains springs that compress in proportion to your weight—similar to rubber bands expanding when pulled. The springs provide a measure of your weight (for an object which is not accelerating). This is a force in newtons (or pounds). In most countries, the measurement is divided by 9.80 to give a reading in mass units of kilograms. The scale measures weight but is calibrated to provide information about mass. While standing on a bathroom scale, push down on a table next to you. What happens to the reading? Why? Would your scale measure the same "mass" on Earth as on the Moon?

What Acceleration Can a Person Produce when Pushing a Lawn Mower?

Suppose that the net external force (push minus friction) exerted on a lawn mower is 51 N (about 11 lb) parallel to the ground. The mass of the mower is 24 kg. What is its acceleration?

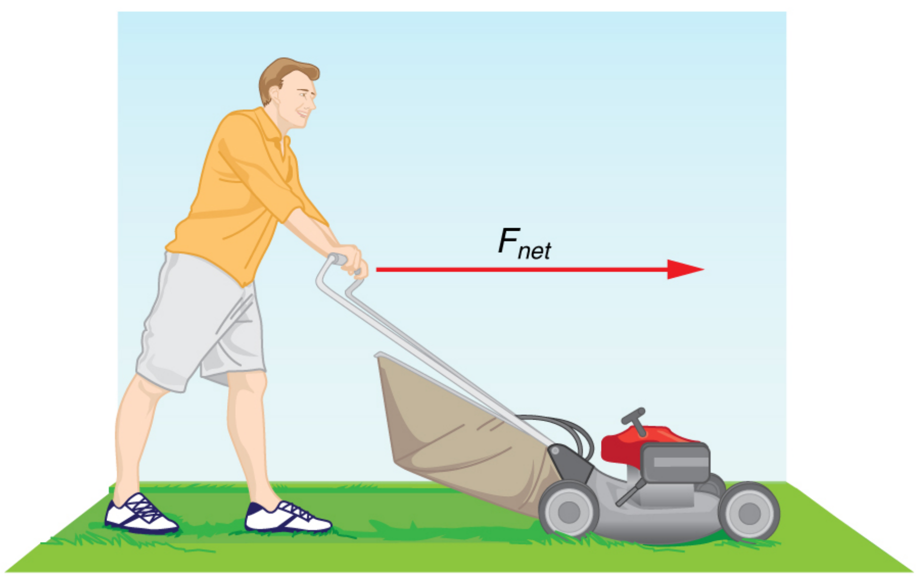


Figure 4.7: The net force on a lawn mower is 51 N to the right. At what rate does the lawn mower accelerate to the right?

Strategy

Since $\mathbf{F}_{\text{net}}$ and m are given, the acceleration can be calculated directly from Newton's second law as stated in $\mathbf{F}_{\text{net}} = m\mathbf{a}$.

Solution

The magnitude of the acceleration a is $a = \frac{F_{\text{net}}}{m}$. Entering known values gives

$$a = \frac{51 \text{ N}}{24 \text{ kg}}$$

Substituting the units $\text{kg} \cdot \text{m/s}^2$ for N yields

$$a = \frac{51 \text{ kg} \cdot \text{m/s}^2}{24 \text{ kg}} = 2.1 \text{ m/s}^2.$$

Discussion

The direction of the acceleration is the same direction as that of the net force, which is parallel to the ground. There is no information given in this example about the individual external forces acting on the system, but we can say something about their relative magnitudes. For example, the force exerted by the person pushing the mower must be greater than the friction opposing the motion (since we know the mower moves forward), and the vertical forces must cancel if there is to be no acceleration in the vertical direction (the mower is moving only horizontally). The acceleration found is small enough to be reasonable for a person pushing a mower. Such an effort would not last too long because the person's top speed would soon be reached.

What Rocket Thrust Accelerates This Sled?

Prior to manned space flights, rocket sleds were used to test aircraft, missile equipment, and physiological effects on human subjects at high speeds. They consisted of a platform that was mounted on one or two rails and propelled by several rockets. Calculate the magnitude of force exerted by each rocket, called its thrust $\mathbf{T}$, for the four-rocket propulsion system shown in link. The sled's initial acceleration is 49 m/s^2, the mass of the system is 2100 kg, and the force of friction opposing the motion is known to be 650 N.

Strategy

Although there are forces acting vertically and horizontally, we assume the vertical forces cancel since there is no vertical acceleration. This leaves us with only horizontal forces and a simpler one-dimensional problem. Directions are indicated with plus or minus signs, with right taken as the positive direction. See the free-body diagram in the figure.

Solution

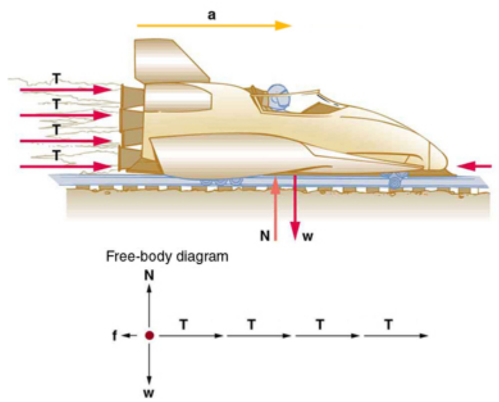


Figure 4.8: A sled experiences a rocket thrust that accelerates it to the right. Each rocket creates an identical thrust **T**. As in other situations where there is only horizontal acceleration, the vertical forces cancel. The ground exerts an upward force **N** on the system that is equal in magnitude and opposite in direction to its weight, **w**. The system here is the sled, its rockets, and rider, so none of the forces *between* these objects are considered. The arrow representing friction (**f**) is drawn larger than scale.

Since acceleration, mass, and the force of friction are given, we start with Newton's second law and look for ways to find the thrust of the engines. Since we have defined the direction of the force and acceleration as acting "to the right," we need to consider only the magnitudes of these quantities in the calculations. Hence we begin with

$$F_{\text{net}} = \text{ma},$$

where F_{net} is the net force along the horizontal direction. We can see from link that the engine thrusts add, while friction opposes the thrust. In equation form, the net external force is

$$F_{\text{net}} = 4T - f.$$

Substituting this into Newton's second law gives

$$F_{\text{net}} = \text{ma} = 4T - f.$$

Using a little algebra, we solve for the total thrust $4T$:

$$4T = \text{ma} + f.$$

Substituting known values yields

$$4T = \text{ma} + f = (2100\ \text{kg})(49\ \text{m/s}^2) + 650\ \text{N}.$$

So the total thrust is

$$4T = 1.0\times10^5\ \text{N},$$

and the individual thrusts are

$$T = \frac{1.0\times10^5\ \text{N}}{4} = 2.6 \times 10^4\ \text{N}.$$

Discussion

The numbers are quite large, so the result might surprise you. Experiments such as this were performed in the early 1960s to test the limits of human endurance and the setup designed to protect human subjects in jet fighter emergency ejections. Speeds of 1000 km/h were obtained, with accelerations of 45 g's. (Recall that g, the acceleration due to gravity, is $9.80\ \text{m/s}^2$. When we say that an acceleration is 45 g's, it is $45 \times 9.80\ \text{m/s}^2$, which is approximately $440\ \text{m/s}^2$.) While living subjects are not used any more, land speeds of 10,000 km/h have been obtained with rocket sleds. In this example, as in the preceding one, the system of interest is obvious. We will see in later examples that choosing the system of interest is crucial—and the choice is not always obvious.

Newton's second law of motion is more than a definition; it is a relationship among acceleration, force, and mass. It can help us make predictions. Each of those physical quantities can be defined independently, so the second law tells us something basic and universal about nature. The next section introduces the third and final law of motion.

Applying the Science Practices: Sums of Forces

Recall that forces are vector quantities, and therefore the net force acting on a system should be the vector sum of the forces.

a Design an experiment to test this hypothesis. What sort of a system would be appropriate and convenient to have multiple forces applied to it? What features of the system should be held constant? What could be varied? Can forces be arranged in multiple directions so that, while the hypothesis is still tested, the resulting calculations are not too inconvenient?

b Another group of students has done such an experiment, using a motion capture system looking down at an air hockey table to measure the motion of the 0.10-kg puck. The table was aligned with the cardinal directions, and a compressed air hose was placed in the center of each side, capable of varying levels of force output and fixed so that it was aimed at the center of the table.

Table 4.1:

Forces	Measured acceleration (magnitudes)
3 N north, 4 N west	48 ± 4 m/s^2
5 N south, 12 N east	132 ± 6 m/s^2
6 N north, 12 N east, 4 N west	99 ± 3 m/s^2

Given the data in the table, is the hypothesis confirmed? What were the directions of the accelerations?

4.4.3.1 Section Summary

- Acceleration, $\mathbf{a}$, is defined as a change in velocity, meaning a change in its magnitude or direction, or both.
- An external force is one acting on a system from outside the system, as opposed to internal forces, which act between components within the system.
- Newton's second law of motion states that the acceleration of a system is directly proportional to and in the same direction as the net external force acting on the system, and inversely proportional to its mass.
- In equation form, Newton's second law of motion is $\mathbf{a} = \frac{\mathbf{F}_{\text{net}}}{m}$.
- This is often written in the more familiar form: $\mathbf{F}_{\text{net}} = m\mathbf{a}$.
- The weight $\mathbf{w}$ of an object is defined as the force of gravity acting on an object of mass m. The object experiences an acceleration due to gravity $\mathbf{g}$: ::: {#eip-id1171442332762 data-type="equation"}

  $$\mathbf{w} = m\mathbf{g}.$$

  :::
- If the only force acting on an object is due to gravity, the object is in free fall.
- Friction is a force that opposes the motion past each other of objects that are touching.

4.4.3.2 Conceptual Questions

Which statement is correct? (a) Net force causes motion. (b) Net force causes change in motion. Explain your answer and give an example.

Why can we neglect forces such as those holding a body together when we apply Newton's second law of motion?

Explain how the choice of the "system of interest" affects which forces must be considered when applying Newton's second law of motion.

Describe a situation in which the net external force on a system is not zero, yet its speed remains constant.

A system can have a nonzero velocity while the net external force on it *is* zero. Describe such a situation.

A rock is thrown straight up. What is the net external force acting on the rock when it is at the top of its trajectory?

a Give an example of different net external forces acting on the same system to produce different accelerations. (b) Give an example of the same net external force acting on systems of different masses, producing different accelerations. (c) What law accurately describes both effects? State it in words and as an equation.

If the acceleration of a system is zero, are no external forces acting on it? What about internal forces? Explain your answers.

If a constant, nonzero force is applied to an object, what can you say about the velocity and acceleration of the object?

The gravitational force on the basketball in link is ignored. When gravity *is* taken into account, what is the direction of the net external force on the basketball—above horizontal, below horizontal, or still horizontal?

4.4.3.3 Problem Exercises

You may assume data taken from illustrations is accurate to three digits.

A 63.0-kg sprinter starts a race with an acceleration of 4.20 $\mathrm{m/s^2}$. What is the net external force on him?

265 N

If the sprinter from the previous problem accelerates at that rate for 20 m, and then maintains that velocity for the remainder of the 100-m dash, what will be his time for the race?

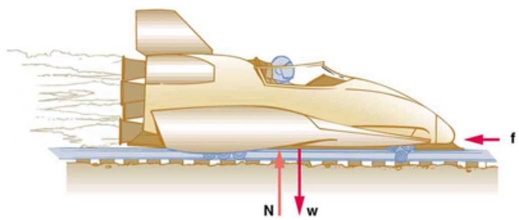
A cleaner pushes a 4.50-kg laundry cart in such a way that the net external force on it is 60.0 N. Calculate the magnitude of its acceleration.

13.3 $\mathrm{m/s^2}$

Since astronauts in orbit are apparently weightless, a clever method of measuring their masses is needed to monitor their mass gains or losses to adjust diets. One way to do this is to exert a known force on an astronaut and measure the acceleration produced. Suppose a net external force of 50.0 N is exerted and the astronaut's acceleration is measured to be 0.893 $\mathrm{m/s^2}$. (a) Calculate her mass. (b) By exerting a force on the astronaut, the vehicle in which they orbit experiences an equal and opposite force. Discuss how this would affect the measurement of the astronaut's acceleration. Propose a method in which recoil of the vehicle is avoided.

In link, the net external force on the 24-kg mower is stated to be 51 N. If the force of friction opposing the motion is 24 N, what force F (in newtons) is the person exerting on the mower? Suppose the mower is moving at 1.5 m/s when the force F is removed. How far will the mower go before stopping?

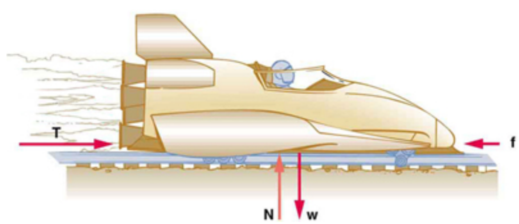
The same rocket sled drawn in link is decelerated at a rate of 196 m/s^2. What force is necessary to produce this deceleration? Assume that the rockets are off. The mass of the system is 2100 kg.



a If the rocket sled shown in link starts with only one rocket burning, what is the magnitude of its acceleration? Assume that the mass of the system is 2100 kg, the thrust T is 2.4×10^4 N, and the force of friction opposing the motion is known to be 650 N. (b) Why is the acceleration not one-fourth of what it is with all rockets burning?

a 12 m/s^2.

b The acceleration is not one-fourth of what it was with all rockets burning because the frictional force is still as large as it was with all rockets burning.

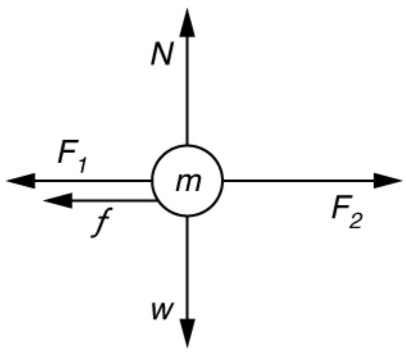


What is the deceleration of the rocket sled if it comes to rest in 1.1 s from a speed of 1000 km/h? (Such deceleration caused one test subject to black out and have temporary blindness.)

Suppose two children push horizontally, but in exactly opposite directions, on a third child in a wagon. The first child exerts a force of 75.0 N, the second a force of 90.0 N, friction is 12.0 N, and the mass of the third child plus wagon is 23.0 kg. (a) What is the system of interest if the acceleration of the child in the wagon is to be calculated? (b) Draw a free-body diagram, including all forces acting on the system. (c) Calculate the acceleration. (d) What would the acceleration be if friction were 15.0 N?

a The system is the child in the wagon plus the wagon.

(b



c $a = 0.130\ \text{m/s}^2$ in the direction of the second child's push.

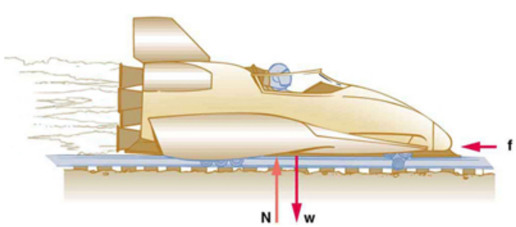
d $a = 0.00\ \text{m/s}^2$

A powerful motorcycle can produce an acceleration of $3.50\ \text{m/s}^2$ while traveling at 90.0 km/h. At that speed the forces resisting motion, including friction and air resistance, total 400 N. (Air resistance is analogous to air friction. It always opposes the motion of an object.) What is the magnitude of the force the motorcycle exerts backward on the ground to produce its acceleration if the mass of the motorcycle with rider is 245 kg?

The rocket sled shown in link accelerates at a rate of $49.0\ \text{m/s}^2$. Its passenger has a mass of 75.0 kg. (a) Calculate the horizontal component of the force the seat exerts against his body. Compare this with his weight by using a ratio. (b) Calculate the direction and magnitude of the total force the seat exerts against his body.

a 3.68×10^3 N . This force is 5.00 times greater than his weight.

b 3750 N; 11.3º above horizontal



Repeat the previous problem for the situation in which the rocket sled decelerates at a rate of $201\ \text{m/s}^2$. In this problem, the forces are exerted by the seat and restraining belts.

The weight of an astronaut plus his space suit on the Moon is only 250 N. How much do they weigh on Earth? What is the mass on the Moon? On Earth?

1.5×10^3 N, 150 kg, 150 kg

Suppose the mass of a fully loaded module in which astronauts take off from the Moon is 10,000 kg. The thrust of its engines is 30,000 N. (a) Calculate its the magnitude of acceleration in a vertical takeoff from the Moon. (b) Could it lift off from Earth? If not, why not? If it could, calculate the magnitude of its acceleration.

4.4.4 Glossary

acceleration the rate at which an object's velocity changes over a period of time

free-fall a situation in which the only force acting on an object is the force due to gravity

friction a force past each other of objects that are touching; examples include rough surfaces and air resistance

net external force the vector sum of all external forces acting on an object or system; causes a mass to accelerate

Newton's second law of motion the net external force $\mathbf{F}_{\text{net}}$ on an object with mass m is proportional to and in the same direction as the acceleration of the object, $\mathbf{a}$, and inversely proportional to the mass; defined mathematically as $\mathbf{a} = \frac{\mathbf{F}_{\text{net}}}{m}$

system defined by the boundaries of an object or collection of objects being observed; all forces originating from outside of the system are considered external forces

weight the force $\mathbf{w}$due to gravity acting on an object of mass m; defined mathematically as: $\boldsymbol{w} = m\boldsymbol{g}$, where $\mathbf{g}$ is the magnitude and direction of the acceleration due to gravity

4.5 Newton's Third Law of Motion: Symmetry in Forces

4.5.1 Learning Objectives

By the end of this section, you will be able to:

- Understand Newton's third law of motion.
- Apply Newton's third law to define systems and solve problems of motion.

The information presented in this section supports the following AP® learning objectives and science practices:

- **3.A.2.1** The student is able to represent forces in diagrams or mathematically using appropriately labeled vectors with magnitude, direction, and units during the analysis of a situation. **(S.P. 1.1)**
- **3.A.3.1** The student is able to analyze a scenario and make claims (develop arguments, justify assertions) about the forces exerted on an object by other objects for different types of forces or components of forces. **(S.P. 6.4, 7.2)**
- **3.A.3.3** The student is able to describe a force as an interaction between two objects and identify both objects for any force. **(S.P. 1.4)**
- **3.A.4.1** The student is able to construct explanations of physical situations involving the interaction of bodies using Newton's third law and the representation of action-reaction pairs of forces. **(S.P. 1.4, 6.2)**
- **3.A.4.2** The student is able to use Newton's third law to make claims and predictions about the action-reaction pairs of forces when two objects interact. **(S.P. 6.4, 7.2)**
- **3.A.4.3** The student is able to analyze situations involving interactions among several objects by using free-body diagrams that include the application of Newton's third law to identify forces. **(S.P. 1.4)**
- **3.B.2.1** The student is able to create and use free-body diagrams to analyze physical situations to solve problems with motion qualitatively and quantitatively. **(S.P. 1.1, 1.4, 2.2)**
- **4.A.2.1** The student is able to make predictions about the motion of a system based on the fact that acceleration is equal to the change in velocity per unit time, and velocity is equal to the change in position per unit time. **(S.P. 6.4)**
- **4.A.2.2** The student is able to evaluate using given data whether all the forces on a system or whether all the parts of a system have been identified. **(S.P. 5.3)**
- **4.A.3.1** The student is able to apply Newton's second law to systems to calculate the change in the center-of-mass velocity when an external force is exerted on the system. **(S.P. 2.2)**

There is a passage in the musical *Man of la Mancha* that relates to Newton's third law of motion. Sancho, in describing a fight with his wife to Don Quixote, says, "Of course I hit her back, Your Grace, but she's a lot harder than me and you know what they say, 'Whether the stone hits the pitcher or the pitcher hits the stone, it's going to be bad for the pitcher.' " This is exactly what happens whenever one body exerts a force on another—the first also experiences a force (equal in magnitude and opposite in direction). Numerous common experiences, such as stubbing a toe or throwing a ball, confirm this. It is precisely stated in Newton's third law of motion.

Newton's Third Law of Motion

Whenever one body exerts a force on a second body, the first body experiences a force that is equal in magnitude and opposite in direction to the force that it exerts.

This law represents a certain *symmetry in nature*: Forces always occur in pairs, and one body cannot exert a force on another without experiencing a force itself. We sometimes refer to this law loosely as "action-reaction," where the force exerted is the action and the force experienced as a consequence is the reaction. Newton's third law has practical uses in analyzing the origin of forces and understanding which forces are external to a system.

We can readily see Newton's third law at work by taking a look at how people move about. Consider a swimmer pushing off from the side of a pool, as illustrated in link. She pushes against the pool wall with her feet and accelerates in the direction *opposite* to that of her push. The wall has exerted an equal and opposite force back on the swimmer. You might think that two equal and opposite forces would cancel, but they do not *because they act on different systems*. In this case, there are two systems that we could investigate: the swimmer or the wall. If we select the swimmer to be the system of interest, as in the figure, then $\mathbf{F}_{\text{wall on feet}}$ is an external force on this system and affects its motion. The swimmer moves in the direction of $\mathbf{F}_{\text{wall on feet}}$. In contrast, the force $\mathbf{F}_{\text{feet on wall}}$ acts on the wall and not on our system of interest. Thus $\mathbf{F}_{\text{feet on wall}}$ does not directly affect the motion of the system and does not cancel $\mathbf{F}_{\text{wall on feet}}$. Note that the swimmer pushes in the direction opposite to that in which she wishes to move. The reaction to her push is thus in the desired direction.

Similarly, when a person stands on Earth, the Earth exerts a force on the person, pulling the person toward the Earth. As stated by Newton's third law of motion, the person also exerts a force that is equal in magnitude, but opposite in direction, pulling the Earth up toward the person. Since the mass of the Earth is so great, however, and $F = ma$, the acceleration of the Earth toward the person is not noticeable.

Other examples of Newton's third law are easy to find. As a professor paces in front of a whiteboard, she exerts a force backward on the floor. The floor exerts a reaction force forward on the professor that causes her to accelerate forward. Similarly, a car accelerates because the ground pushes forward on the drive wheels in reaction to the drive wheels pushing backward on the ground. You can see evidence of the wheels pushing backward when tires spin on a gravel road and throw rocks backward. In another example, rockets move forward by expelling gas backward at high velocity. This means the rocket exerts a large backward force on the gas in the rocket combustion chamber, and the gas therefore exerts a large reaction force forward on the rocket. This reaction force is called thrust. It is a common misconception that rockets propel themselves by pushing on the ground or on the air behind them. They actually work better in a vacuum, where they can more readily expel the exhaust gases. Helicopters similarly create lift by pushing air down, thereby experiencing an upward reaction force. Birds and airplanes also fly by exerting force on air in a direction opposite to that of whatever force they need. For example, the wings of a bird force air

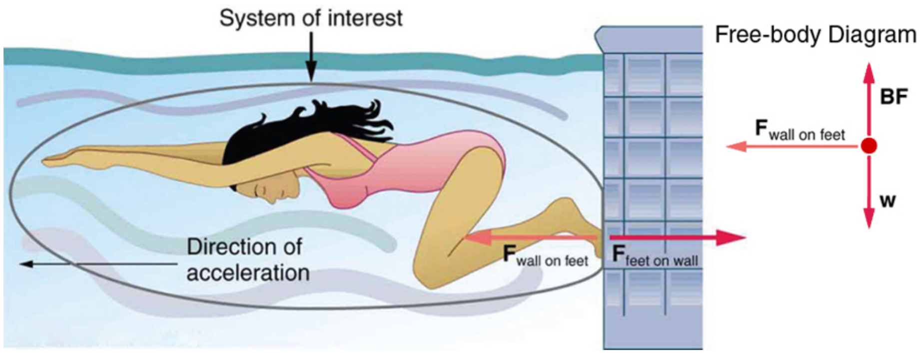


Figure 4.9: When the swimmer exerts a force $\mathbf{F}_{\text{feet on wall}}$ on the wall, she accelerates in the direction opposite to that of her push. This means the net external force on her is in the direction opposite to $\mathbf{F}_{\text{feet on wall}}$. This opposition occurs because, in accordance with Newton's third law of motion, the wall exerts a force $\mathbf{F}_{\text{wall on feet}}$ on her, equal in magnitude but in the direction opposite to the one she exerts on it. The line around the swimmer indicates the system of interest. Note that $\mathbf{F}_{\text{feet on wall}}$ does not act on this system (the swimmer) and, thus, does not cancel $\mathbf{F}_{\text{wall on feet}}$. Thus the free-body diagram shows only $\mathbf{F}_{\text{wall on feet}}$, $\mathbf{w}$, the gravitational force, and $\mathbf{BF}$, the buoyant force of the water supporting the swimmer's weight. The vertical forces $\mathbf{w}$ and $\mathbf{BF}$ cancel since there is no vertical motion.

downward and backward in order to get lift and move forward. An octopus propels itself in the water by ejecting water through a funnel from its body, similar to a jet ski. In a situation similar to Sancho's, professional cage fighters experience reaction forces when they punch, sometimes breaking their hand by hitting an opponent's body.

Getting Up To Speed: Choosing the Correct System

A physics professor pushes a cart of demonstration equipment to a lecture hall, as seen in link. Her mass is 65.0 kg, the cart's is 12.0 kg, and the equipment's is 7.0 kg. Calculate the acceleration produced when the professor exerts a backward force of 150 N on the floor. All forces opposing the motion, such as friction on the cart's wheels and air resistance, total 24.0 N.

Strategy

Since they accelerate as a unit, we define the system to be the professor, cart, and equipment. This is System 1 in link. The professor pushes backward with a force $\mathbf{F}_{\text{foot}}$ of 150 N. According to Newton's third law, the floor exerts a forward reaction force $\mathbf{F}_{\text{floor}}$ of 150 N on System 1. Because all motion is horizontal, we can assume there is no net force in the vertical direction. The problem is therefore one-dimensional along the horizontal direction. As noted, $\mathbf{f}$ opposes the motion and is thus in the opposite direction of $\mathbf{F}_{\text{floor}}$. Note that we do not

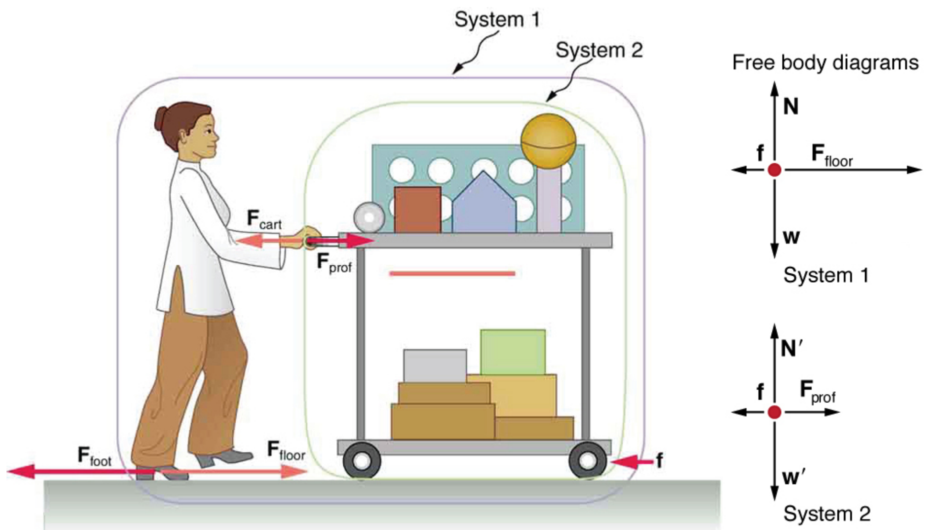


Figure 4.10: A professor pushes a cart of demonstration equipment. The lengths of the arrows are proportional to the magnitudes of the forces (except for **f**, since it is too small to draw to scale). Different questions are asked in each example; thus, the system of interest must be defined differently for each. System 1 is appropriate for this example, since it asks for the acceleration of the entire group of objects. Only $\mathbf{F}_{\text{floor}}$ and **f** are external forces acting on System 1 along the line of motion. All other forces either cancel or act on the outside world. System 2 is chosen for link so that $\mathbf{F}_{\text{prof}}$ will be an external force and enter into Newton's second law. Note that the free-body diagrams, which allow us to apply Newton's second law, vary with the system chosen.

include the forces $\mathbf{F}_{\text{prof}}$ or $\mathbf{F}_{\text{cart}}$ because these are internal forces, and we do not include $\mathbf{F}_{\text{foot}}$ because it acts on the floor, not on the system. There are no other significant forces acting on System 1. If the net external force can be found from all this information, we can use Newton’s second law to find the acceleration as requested. See the free-body diagram in the figure.

Solution

Newton’s second law is given by

$$a = \frac{F_{\text{net}}}{m}.$$

The net external force on System 1 is deduced from link and the discussion above to be

$$F_{\text{net}} = F_{\text{floor}} - f = 150\ \text{N} - 24.0\ \text{N} = 126\ \text{N}.$$

The mass of System 1 is

$$m = (65.0 + 12.0 + 7.0)\ \text{kg} = 84\ \text{kg}.$$

These values of F_{net} and m produce an acceleration of

a = F net m , a = 1 26 N 84 kg = 1 . 5 m/s 2 . a = F net m , a = 1 26 N 84 kg = 1 . 5 m/s 2 . alignl { stack { size 12{a= { {F rSub { size 8{”net”} } } over {m} } ,} {} # a = { {1”26 N”} over {”84”” kg”} } =” 1” ”.” ”5 m/s” rSup { size 8{2} } ”.” {} } } {}

Discussion

None of the forces between components of System 1, such as between the professor’s hands and the cart, contribute to the net external force because they are internal to System 1. Another way to look at this is to note that forces between components of a system cancel because they are equal in magnitude and opposite in direction. For example, the force exerted by the professor on the cart results in an equal and opposite force back on her. In this case both forces act on the same system and, therefore, cancel. Thus internal forces (between components of a system) cancel. Choosing System 1 was crucial to solving this problem.

Force on the Cart—Choosing a New System

Calculate the force the professor exerts on the cart in link using data from the previous example if needed.

Strategy

If we now define the system of interest to be the cart plus equipment (System 2 in link), then the net external force on System 2 is the force the professor exerts on the cart minus friction. The force she exerts on the cart, $\mathbf{F}_{\text{prof}}$, is an external force acting on System 2. $\mathbf{F}_{\text{prof}}$ was internal to System 1, but it is external to System 2 and will enter Newton's second law for System 2.

Solution

Newton's second law can be used to find $\mathbf{F}_{\text{prof}}$. Starting with

$$a = \frac{F_{\text{net}}}{m}$$

and noting that the magnitude of the net external force on System 2 is

$$F_{\text{net}} = F_{\text{prof}} - f,$$

we solve for F_{prof}, the desired quantity:

$$F_{\text{prof}} = F_{\text{net}} + f.$$

The value of f is given, so we must calculate net F_{net}. That can be done since both the acceleration and mass of System 2 are known. Using Newton's second law we see that

$$F_{\text{net}} = \text{ma},$$

where the mass of System 2 is 19.0 kg (m= 12.0 kg + 7.0 kg) and its acceleration was found to be $a = 1.5\ \text{m/s}^2$ in the previous example. Thus,

$$F_{\text{net}} = \text{ma},$$

$$F_{\text{net}} = (19.0\ \text{kg})(1.5\ \text{m/s}^2) = 29\ \text{N}.$$

Now we can find the desired force:

$$F_{\text{prof}} = F_{\text{net}} + f,$$

$$F_{\text{prof}} = 29\ \text{N}+24.0\ \text{N} = 53\ \text{N}.$$

Discussion

It is interesting that this force is significantly less than the 150-N force the professor exerted backward on the floor. Not all of that 150-N force is transmitted to the cart; some of it accelerates the professor.

The choice of a system is an important analytical step both in solving problems and in thoroughly understanding the physics of the situation (which is not necessarily the same thing).

Gravity Force Lab

Visualize the gravitational force that two objects exert on each other. Change properties of the objects in order to see how it changes the gravity force.

4.5.2 Test Prep for AP Courses

What object or objects commonly exert forces on the following objects in motion? (a) a soccer ball being kicked, (b) a dolphin jumping, (c) a parachutist drifting to Earth.

a A soccer player, gravity, air, and friction commonly exert forces on a soccer ball being kicked.

b Gravity and the surrounding water commonly exert forces on a dolphin jumping. (The dolphin moves its muscles to exert a force on the water. The water exerts an equal force on the dolphin, resulting in the dolphin's motion.)

c Gravity and air exert forces on a parachutist drifting to Earth.

A ball with a mass of 0.25 kg hits a gym ceiling with a force of 78.0 N. What is the net force on the ball?

a. 2.50 N downward
b. 75.5 N downward
c. 78.0 N downward
d. 80.5 N downward

Which of the following is true?

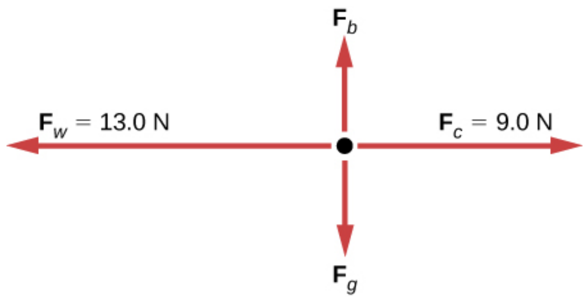
a. Earth exerts a force due to gravity on your body, and your body exerts a smaller force on the Earth, because your mass is smaller than the mass of the Earth.
b. The Moon orbits the Earth because the Earth exerts a force on the Moon and the Moon exerts a force equal in magnitude and direction on the Earth.
c. A rocket taking off exerts a force on the Earth equal to the force the Earth exerts on the rocket.
d. An airplane cruising at a constant speed is not affected by gravity.

c

Stationary skater A pushes stationary skater B, who then accelerates at 5.0 m/s^2. Skater A does not move. Since forces act in action-reaction pairs, explain why

Skater A did not move?

The current in a river exerts a force of 9.0 N on a balloon floating in the river. A wind exerts a force of 13.0 N on the balloon in the opposite direction. Draw a free-body diagram to show the forces acting on the balloon. Use your free-body diagram to predict the effect on the balloon.



The diagram consists of a black dot in the center and two small red arrows pointing up (Fb) and down (Fg) and two long red arrows pointing right (Fc = 9.0 N) and left (Fw=13.0 N).

In the diagram, F_{g} represents the force due to gravity on the balloon, and F_b represents the buoyant force. These two forces are equal in magnitude and opposite in direction. F_{c} represents the force of the current. F_{w} represents the force of the wind. The net force on the balloon will be $F_w - F_c = 4.0$ N and the balloon will accelerate in the direction the wind is blowing.

A force is applied to accelerate an object on a smooth icy surface. When the force stops, which of the following will be true? (Assume zero friction.)

a. The object's acceleration becomes zero.
b. The object's speed becomes zero.
c. The object's acceleration continues to increase at a constant rate.
d. The object accelerates, but in the opposite direction.

A parachutist's fall to Earth is determined by two opposing forces. A gravitational force of 539 N acts on the parachutist. After 2 s, she opens her parachute and experiences an air resistance of 615 N. At what speed is the parachutist falling after 10 s?

Since $m = F/a$, the parachutist has a mass of $539\,\mathrm{N}/9.8\ \mathrm{km/s}^2 = 55\,\mathrm{kg}$.

For the first 2 s, the parachutist accelerates at 9.8 m/s^2.

$$\begin{aligned} v &= at \\ &= 9.8\tfrac{\mathrm{m}}{\mathrm{s}^2} \bullet 2\mathrm{s} \\ &= 19.6\ \ \tfrac{\mathrm{m}}{\mathrm{s}} \end{aligned}$$

Her speed after 2 s is 19.6 m/s.

From 2 s to 10 s, the net force on the parachutist is 539 N – 615 N, or 76 N upward.

$$\begin{aligned} a &= \frac{F}{m} \\ &= \frac{-76\,\text{N}}{55\,\text{kg}} \\ &= -1.4\,\frac{\text{m}}{\text{s}^2} \end{aligned}$$

Since $v = v_0 + at$, $v = 19.6\,\text{m/s}^2 + (-1.4\,\text{m/s}^2)(8\text{s}) = 8.4\,\text{m/s}^2$.

At 10 s, the parachutist is falling to Earth at 8.4 m/s.

A flight attendant pushes a cart down the aisle of a plane in flight. In determining the acceleration of the cart relative to the plane, which factor do you not need to consider?

a. The friction of the cart's wheels.
b. The force with which the flight attendant's feet push on the floor.
c. The velocity of the plane.
d. The mass of the items in the cart.

A landscaper is easing a wheelbarrow full of soil down a hill. Define the system you would analyze and list all the forces that you would need to include to calculate the acceleration of the wheelbarrow.

The system includes the gardener and the wheelbarrow with its contents. The following forces are important to include: the weight of the wheelbarrow, the weight of the gardener, the normal force for the wheelbarrow and the gardener, the force of the gardener pushing against the ground and the equal force of the ground pushing back against the gardener, and any friction in the wheelbarrow's wheels.

Two water-skiers, with masses of 48 kg and 61 kg, are preparing to be towed behind the same boat. When the boat accelerates, the rope the skiers hold onto accelerates with it and exerts a net force of 290 N on the skiers. At what rate will the skiers accelerate?

a. 10.8 m/s^2
b. 2.7 m/s^2
c. 6.0 m/s^2 and 4.8 m/s^2
d. 5.3 m/s^2

A figure skater has a mass of 40 kg and her partner's mass is 50 kg. She pushes against the ice with a force of 120 N, causing her and her partner to move forward. Calculate the pair's acceleration. Assume that all forces opposing the motion, such as friction and air resistance, total 5.0 N.

The system undergoing acceleration is the two figure skaters together.

Net force $= 120\,\text{N} - 5.0\,\text{N} = 115\,\text{N}$.

Total mass $= 40\,\text{kg}$ + 50 kg = 90 kg.

Using Newton's second law, we have that

$$\begin{aligned} a &= \frac{F}{m} \\ &= \frac{115\,\text{N}}{90\,\text{kg}} \\ &= 1.28\,\frac{\text{m}}{\text{s}^2} \end{aligned}$$

The pair accelerates forward at 1.28 m/s^2.

4.5.3 Section Summary

- Newton's third law of motion represents a basic symmetry in nature. It states: Whenever one body exerts a force on a second body, the first body experiences a force that is equal in magnitude and opposite in direction to the force that the first body exerts.
- A thrust is a reaction force that pushes a body forward in response to a backward force. Rockets, airplanes, and cars are pushed forward by a thrust reaction force.

4.5.4 Conceptual Questions

When you take off in a jet aircraft, there is a sensation of being pushed back into the seat. Explain why you move backward in the seat—is there really a force backward on you? (The same reasoning explains whiplash injuries, in which the head is apparently thrown backward.)

A device used since the 1940s to measure the kick or recoil of the body due to heart beats is the "ballistocardiograph." What physics principle(s) are involved here to measure the force of cardiac contraction? How might we construct such a device?

Describe a situation in which one system exerts a force on another and, as a consequence, experiences a force that is equal in magnitude and opposite in direction. Which of Newton's laws of motion apply?

Why does an ordinary rifle recoil (kick backward) when fired? The barrel of a recoilless rifle is open at both ends. Describe how Newton's third law applies when one is fired. Can you safely stand close behind one when it is fired?

An American football lineman reasons that it is senseless to try to out-push the opposing player, since no matter how hard he pushes he will experience an equal and opposite force from the other player. Use Newton's laws and draw a free-body diagram of an appropriate system to explain how he can still out-push the opposition if he is strong enough.

Newton's third law of motion tells us that forces always occur in pairs of equal and opposite magnitude. Explain how the choice of the "system of interest" affects whether one such pair of forces cancels.

4.5.5 Problem Exercises

What net external force is exerted on a 1100-kg artillery shell fired from a battleship if the shell is accelerated at 2.40×10^4 m/s^2? What is the magnitude of the force exerted on the ship by the artillery shell?

Force on shell: 2.64×10^7 N

Force exerted on ship = -2.64×10^7 N, by Newton's third law

A brave but inadequate rugby player is being pushed backward by an opposing player who is exerting a force of 800 N on him. The mass of the losing player plus equipment is 90.0 kg, and he is accelerating at 1.20 m/s^2 backward. (a) What is the force of friction between the losing player's feet and the grass? (b) What force does the winning player exert on the ground to move forward if his mass plus equipment is 110 kg? (c) Draw a sketch of the situation showing the system of interest used to solve each part. For this situation, draw a free-body diagram and write the net force equation.

4.5.6 Glossary

Newton's third law of motion whenever one body exerts a force on a second body, the first body experiences a force that is equal in magnitude and opposite in direction to the force that the first body exerts

thrust a reaction force that pushes a body forward in response to a backward force; rockets, airplanes, and cars are pushed forward by a thrust reaction force

4.6 Normal, Tension, and Other Examples of Force

4.6.1 Learning Objectives

By the end of this section, you will be able to:

- Define normal and tension forces.
- Apply Newton's laws of motion to solve problems involving a variety of forces.
- Use trigonometric identities to resolve weight into components.

The information presented in this section supports the following AP® learning objectives and science practices:

- **2.B.1.1** The student is able to apply $F = mg$ to calculate the gravitational force on an object with mass m in a gravitational field of strength g in the context of the effects of a net force on objects and systems. **(S.P. 2.2, 7.2)**

- **3.A.2.1** The student is able to represent forces in diagrams or mathematically using appropriately labeled vectors with magnitude, direction, and units during the analysis of a situation. **(S.P. 1.1)**
- **3.A.3.1** The student is able to analyze a scenario and make claims (develop arguments, justify assertions) about the forces exerted on an object by other objects for different types of forces or components of forces. **(S.P. 6.4, 7.2)**
- **3.A.3.3** The student is able to describe a force as an interaction between two objects and identify both objects for any force. **(S.P. 1.4)**
- **3.A.4.1** The student is able to construct explanations of physical situations involving the interaction of bodies using Newton's third law and the representation of action-reaction pairs of forces. **(S.P. 1.4, 6.2)**
- **3.A.4.2** The student is able to use Newton's third law to make claims and predictions about the action-reaction pairs of forces when two objects interact. **(S.P. 6.4, 7.2)**
- **3.A.4.3** The student is able to analyze situations involving interactions among several objects by using free-body diagrams that include the application of Newton's third law to identify forces. **(S.P. 1.4)**
- **3.B.1.3** The student is able to re-express a free-body diagram representation into a mathematical representation and solve the mathematical representation for the acceleration of the object. **(S.P. 1.5, 2.2)**
- **3.B.2.1** The student is able to create and use free-body diagrams to analyze physical situations to solve problems with motion qualitatively and quantitatively. **(S.P. 1.1, 1.4, 2.2)**

Forces are given many names, such as push, pull, thrust, lift, weight, friction, and tension. Traditionally, forces have been grouped into several categories and given names relating to their source, how they are transmitted, or their effects. The most important of these categories are discussed in this section, together with some interesting applications. Further examples of forces are discussed later in this text.

4.6.2 Normal Force

Weight (also called force of gravity) is a pervasive force that acts at all times and must be counteracted to keep an object from falling. You definitely notice that you must support the weight of a heavy object by pushing up on it when you hold it stationary, as illustrated in link(a). But how do inanimate objects like a table support the weight of a mass placed on them, such as shown in link(b)? When the bag of dog food is placed on the table, the table actually sags slightly under the load. This would be noticeable if the load were placed on a card table, but even rigid objects deform when a force is applied to them. Unless the object is deformed beyond its limit, it will exert a restoring force much like a deformed spring (or trampoline or diving board). The greater the deformation, the greater the restoring force. So when the load is placed on the table, the table sags until the restoring force becomes as large as the weight of the load. At this point the net external force on the load is zero. That is the situation when the load is stationary on the table. The table sags quickly, and the sag is slight so we do not notice it. But it is similar to the sagging of a trampoline when you climb onto it.

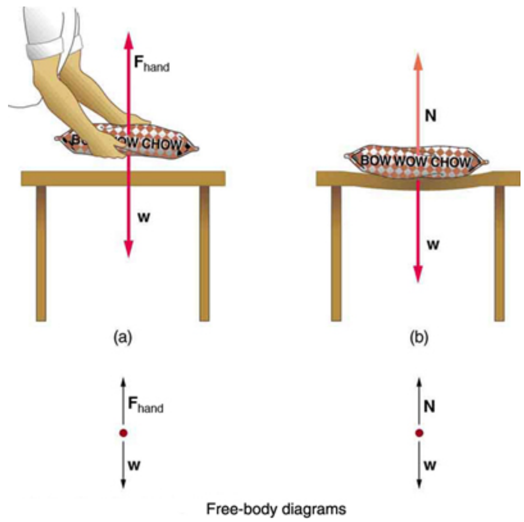


Figure 4.11: (a) The person holding the bag of dog food must supply an upward force $\mathbf{F}_{\text{hand}}$ equal in magnitude and opposite in direction to the weight of the food $\mathbf{w}$. (b) The card table sags when the dog food is placed on it, much like a stiff trampoline. Elastic restoring forces in the table grow as it sags until they supply a force $\mathbf{N}$ equal in magnitude and opposite in direction to the weight of the load.

We must conclude that whatever supports a load, be it animate or not, must supply an upward force equal to the weight of the load, as we assumed in a few of the previous examples. If the force supporting a load is perpendicular to the surface of contact between the load and its support, this force is defined to be a normal force and here is given the symbol $\mathbf{N}$. (This is not the unit for force N.) The word *normal* means perpendicular to a surface. The normal force can be less than the object's weight if the object is on an incline, as you will see in the next example.

Common Misconception: Normal Force (N) vs. Newton (N)

In this section we have introduced the quantity normal force, which is represented by the variable $\mathbf{N}$. This should not be confused with the symbol for the newton, which is also represented by the letter N. These symbols are particularly important to distinguish because the units of a normal force ($\mathbf{N}$) happen to be newtons (N). For example, the normal force $\mathbf{N}$ that the floor exerts on a chair might be $\mathbf{N} = 100$ N. One important difference is that normal force is a vector, while the newton is simply a unit. Be careful not to confuse these letters in your calculations! You will encounter more similarities among variables and units as you proceed in physics. Another example of this is the quantity work (W) and the unit watts (W).

Weight on an Incline, a Two-Dimensional Problem

Consider the skier on a slope shown in link. Her mass including equipment is 60.0 kg. (a) What is her acceleration if friction is negligible? (b) What is her acceleration if friction is known to be 45.0 N?

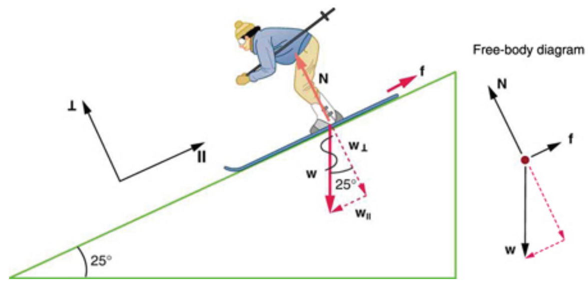


Figure 4.12: Since motion and friction are parallel to the slope, it is most convenient to project all forces onto a coordinate system where one axis is parallel to the slope and the other is perpendicular (axes shown to left of skier). **N** is perpendicular to the slope and **f** is parallel to the slope, but **w** has components along both axes, namely $\mathbf{w}_\perp$ and $\mathbf{w}_\parallel$. **N** is equal in magnitude to $\mathbf{w}_\perp$, so that there is no motion perpendicular to the slope, but f is less than $w_\parallel$, so that there is a downslope acceleration (along the parallel axis).

Strategy

This is a two-dimensional problem, since the forces on the skier (the system of interest) are not parallel. The approach we have used in two-dimensional kinematics also works very well here. Choose a convenient coordinate system and project the vectors onto its axes, creating *two* connected *one*-dimensional problems to solve. The most convenient coordinate system for motion on an incline is one that has one coordinate parallel to the slope and one perpendicular to the slope. (Remember that motions along mutually perpendicular axes are independent.) We use the symbols $\perp$ and $\parallel$ to represent perpendicular and parallel, respectively. This choice of axes simplifies this type of problem, because there is no motion perpendicular to the slope and because friction is always parallel to the surface between two objects. The only external forces acting on the system are the skier's weight, friction, and the support of the slope, respectively labeled **w**, **f**, and **N** in link. **N** is always perpendicular to the slope, and **f** is parallel to it. But **w** is not in the direction of either axis, and so the first step we take is to project it into components along the chosen axes, defining $w_\parallel$ to be the component of weight parallel to the slope and $w_\perp$ the component of weight perpendicular to the slope. Once this is done, we can consider the two separate problems of forces parallel to the slope and forces perpendicular to the slope.

Solution

The magnitude of the component of the weight parallel to the slope is $w_{\parallel} = w \sin{(25^o)} = mg \sin{(25^o)}$, and the magnitude of the component of the weight perpendicular to the slope is $w_{\perp} = w \cos{(25^o)} = mg \cos{(25^o)}$.

a Neglecting friction. Since the acceleration is parallel to the slope, we need only consider forces parallel to the slope. (Forces perpendicular to the slope add to zero, since there is no acceleration in that direction.) The forces parallel to the slope are the amount of the skier's weight parallel to the slope $w_{\parallel}$ and friction f. Using Newton's second law, with subscripts to denote quantities parallel to the slope,

$$a_{\parallel} = \frac{F_{\text{net}\parallel}}{m}$$

where $F_{\text{net}\parallel} = w_{\parallel} = \text{mg} \sin{(25^o)}$, assuming no friction for this part, so that

$$a_{\parallel} = \frac{F_{\text{net}\parallel}}{m} = \frac{\text{mg} \sin{(25^o)}}{m} = g \sin{(25^o)}$$

$$(9.80 \text{ m/s}^2)(0.4226) = 4.14 \text{ m/s}^2$$

is the acceleration.

b Including friction. We now have a given value for friction, and we know its direction is parallel to the slope and it opposes motion between surfaces in contact. So the net external force is now

$$F_{\text{net}\parallel} = w_{\parallel} - f,$$

and substituting this into Newton's second law, $a_{\parallel} = \frac{F_{\text{net}\parallel}}{m}$, gives

$$a_{\parallel} = \frac{F_{\text{net}||}}{m} = \frac{w_{\parallel} - f}{m} = \frac{\text{mg} \sin{(25^o)} - f}{m}.$$

We substitute known values to obtain

$$a_{\parallel} = \frac{(60.0 \text{ kg})(9.80 \text{ m/s}^2)(0.4226) - 45.0 \text{ N}}{60.0 \text{ kg}},$$

which yields

$$a_{\parallel} = 3.39 \text{ m/s}^2,$$

which is the acceleration parallel to the incline when there is 45.0 N of opposing friction.

Discussion

Since friction always opposes motion between surfaces, the acceleration is smaller when there is friction than when there is none. In fact, it is a general result that if friction on an incline is negligible, then the acceleration down the incline is $a = g \sin\theta$, *regardless of mass.* This is related to the previously discussed fact that all objects fall with the same acceleration in the absence of air resistance. Similarly, all objects, regardless of mass, slide down a frictionless incline with the same acceleration (if the angle is the same).

Resolving Weight into Components

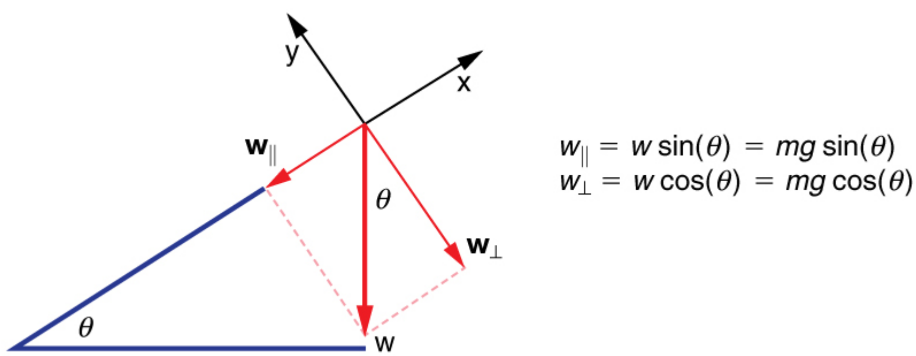


Figure 4.13: An object rests on an incline that makes an angle with the horizontal.

When an object rests on an incline that makes an angle θ with the horizontal, the force of gravity acting on the object is divided into two components: a force acting perpendicular to the plane, $\mathbf{w}_{\perp}$**,** and a force acting parallel to the plane, $\mathbf{w}_{\parallel}$. The perpendicular force of weight, $\mathbf{w}_{\perp}$**,** is typically equal in magnitude and opposite in direction to the normal force, **N**. The force acting parallel to the plane, $\mathbf{w}_{\parallel}$, causes the object to accelerate down the incline. The force of friction, **f**, opposes the motion of the object, so it acts upward along the plane.

It is important to be careful when resolving the weight of the object into components. If the angle of the incline is at an angle θ to the horizontal, then the magnitudes of the weight components are

$$w_{\|} = w \sin(\theta) = \text{mg} \sin(\theta)$$

and

$$w_{\perp} = w \cos(\theta) = \text{mg} \cos(\theta).$$

Instead of memorizing these equations, it is helpful to be able to determine them from reason. To do this, draw the right triangle formed by the three weight vectors. Notice that the angle θ of the incline is the same as the angle formed between $\mathbf{w}$ and $\mathbf{w}_{\perp}$. Knowing this property, you can use trigonometry to determine the magnitude of the weight components:

$$\begin{aligned} \cos(\theta) &= \tfrac{w_{\perp}}{w} \\ w_{\perp} &= w \cos(\theta) = \text{mg} \cos(\theta) \end{aligned}$$

$$\begin{aligned} \sin(\theta) &= \tfrac{w_{\|}}{w} \\ w_{\|} &= w \sin(\theta) = \text{mg} \sin(\theta) \end{aligned}$$

Take-Home Experiment: Force Parallel

To investigate how a force parallel to an inclined plane changes, find a rubber band, some objects to hang from the end of the rubber band, and a board you can position at different angles. How much does the rubber band stretch when you hang the object from the end of the board? Now place the board at an angle so that the object slides off when placed on the board. How much does the rubber band extend if it is lined up parallel to the board and used to hold the object stationary on the board? Try two more angles. What does this show?

4.6.3 Tension

A tension is a force along the length of a medium, especially a force carried by a flexible medium, such as a rope or cable. The word "tension" comes from a Latin word meaning"to stretch." Not coincidentally, the flexible cords that carry muscle forces to other parts of the body are called *tendons*. Any flexible connector, such as a string, rope, chain, wire, or cable, can exert pulls only parallel to its length; thus, a force carried by a flexible connector is a tension with direction parallel to the connector. It is important to understand that tension is a pull in a connector. In contrast, consider the phrase: "You can't push a rope." The tension force pulls outward along the two ends of a rope.

Consider a person holding a mass on a rope as shown in link.

Tension in the rope must equal the weight of the supported mass, as we can prove using Newton's second law. If the 5.00-kg mass in the figure is stationary,

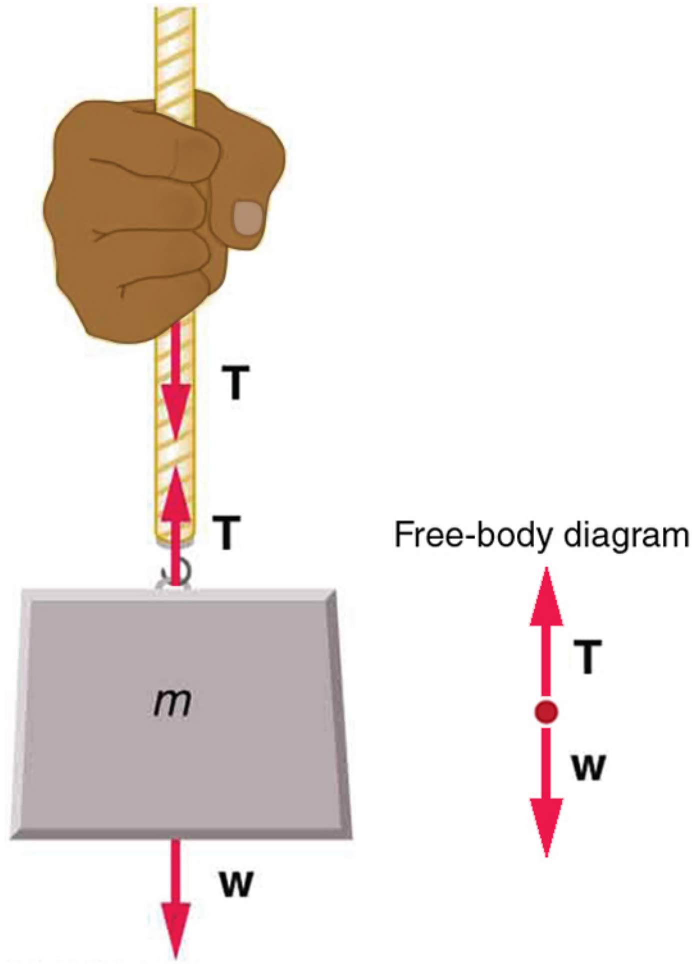


Figure 4.14: When a perfectly flexible connector (one requiring no force to bend it) such as this rope transmits a force **T**, that force must be parallel to the length of the rope, as shown. The pull such a flexible connector exerts is a tension. Note that the rope pulls with equal force but in opposite directions on the hand and the supported mass (neglecting the weight of the rope). This is an example of Newton's third law. The rope is the medium that carries the equal and opposite forces between the two objects. The tension anywhere in the rope between the hand and the mass is equal. Once you have determined the tension in one location, you have determined the tension at all locations along the rope.

then its acceleration is zero, and thus $\mathbf{F}_{\text{net}} = 0$. The only external forces acting on the mass are its weight $\mathbf{w}$ and the tension $\mathbf{T}$ supplied by the rope. Thus,

$$F_{\text{net}} = T - w = 0,$$

where T and w are the magnitudes of the tension and weight and their signs indicate direction, with up being positive here. Thus, just as you would expect, the tension equals the weight of the supported mass:

$$T = w = \text{mg}.$$

For a 5.00-kg mass, then (neglecting the mass of the rope) we see that

$$T = \text{mg} = (5.00 \text{ kg})(9.80 \text{ m/s}^2) = 49.0 \text{ N}.$$

If we cut the rope and insert a spring, the spring would extend a length corresponding to a force of 49.0 N, providing a direct observation and measure of the tension force in the rope.

Flexible connectors are often used to transmit forces around corners, such as in a hospital traction system, a finger joint, or a bicycle brake cable. If there is no friction, the tension is transmitted undiminished. Only its direction changes, and it is always parallel to the flexible connector. This is illustrated in link (a) and (b).

What Is the Tension in a Tightrope?

Calculate the tension in the wire supporting the 70.0-kg tightrope walker shown in link.

Strategy

As you can see in the figure, the wire is not perfectly horizontal (it cannot be!), but is bent under the person's weight. Thus, the tension on either side of the person has an upward component that can support his weight. As usual, forces are vectors represented pictorially by arrows having the same directions as the forces and lengths proportional to their magnitudes. The system is the tightrope walker, and the only external forces acting on him are his weight $\mathbf{w}$ and the two tensions $\mathbf{T}_{\text{L}}$ (left tension) and $\mathbf{T}_{\text{R}}$ (right tension), as illustrated. It is reasonable to neglect the weight of the wire itself. The net external force is zero since the system is stationary. A little trigonometry can now be used to find the tensions. One conclusion is possible at the outset—we can see from part (b) of the figure that the magnitudes of the tensions T_{L} and T_{R} must be equal. This is because there is no horizontal acceleration in the rope, and the only forces acting to the left and right are T_{L} and T_R. Thus, the magnitude of those forces must be equal so that they cancel each other out.

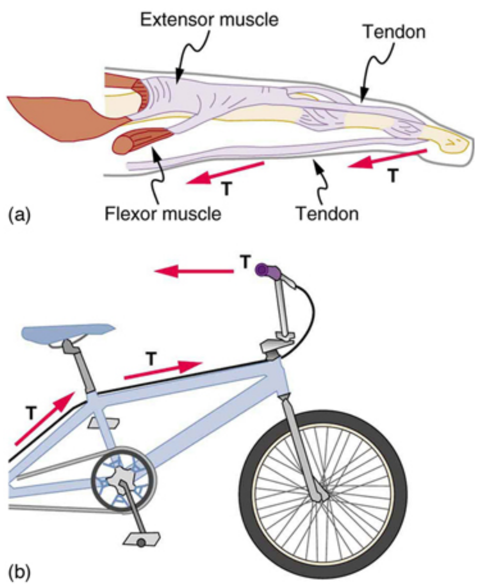


Figure 4.15: (a) Tendons in the finger carry force **T** from the muscles to other parts of the finger, usually changing the force's direction, but not its magnitude (the tendons are relatively friction free). (b) The brake cable on a bicycle carries the tension **T** from the handlebars to the brake mechanism. Again, the direction but not the magnitude of **T** is changed.

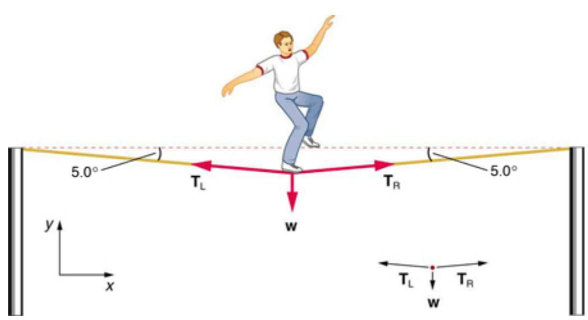


Figure 4.16: The weight of a tightrope walker causes a wire to sag by 5.0 degrees. The system of interest here is the point in the wire at which the tightrope walker is standing.

Whenever we have two-dimensional vector problems in which no two vectors are parallel, the easiest method of solution is to pick a convenient coordinate system and project the vectors onto its axes. In this case the best coordinate system has one axis horizontal and the other vertical. We call the horizontal the x-axis and the vertical the y-axis.

Solution

First, we need to resolve the tension vectors into their horizontal and vertical components. It helps to draw a new free-body diagram showing all of the horizontal and vertical components of each force acting on the system.

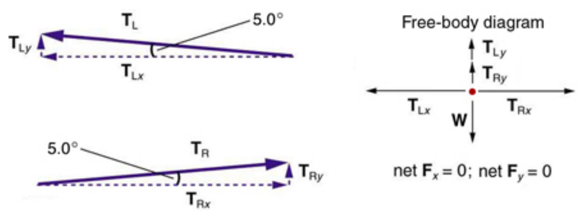


Figure 4.17: When the vectors are projected onto vertical and horizontal axes, their components along those axes must add to zero, since the tightrope walker is stationary. The small angle results in T being much greater than w.

Consider the horizontal components of the forces (denoted with a subscript x):

$$F_{\mathrm{net}x} = T_{\mathrm{L}x} - T_{\mathrm{R}x}.$$

The net external horizontal force $F_{\mathrm{net}x} = 0$, since the person is stationary. Thus,

$$\begin{aligned} F_{\mathrm{net}x} = 0 &= T_{\mathrm{L}x} - T_{\mathrm{R}x} \\ T_{\mathrm{L}x} &= T_{\mathrm{R}x}. \end{aligned}$$

Now, observe link. You can use trigonometry to determine the magnitude of T_{L} and T_{R}. Notice that:

$$\begin{aligned} \cos(5.0) &= \frac{T_{\mathrm{L}x}}{T_{\mathrm{L}}} \\ T_{\mathrm{L}x} &= T_{\mathrm{L}} \cos(5.0) \\ \cos(5.0) &= \frac{T_{\mathrm{R}x}}{T_{\mathrm{R}}} \\ T_{\mathrm{R}x} &= T_{\mathrm{R}} \cos(5.0). \end{aligned}$$

Equating $T_{\mathrm{L}x}$ and $T_{\mathrm{R}x}$:

$$T_{\mathrm{L}} \cos(5.0) = T_{\mathrm{R}} \cos(5.0).$$

Thus,

$$T_{\mathrm{L}} = T_{\mathrm{R}} = T,$$

as predicted. Now, considering the vertical components (denoted by a subscript y), we can solve for T. Again, since the person is stationary, Newton's second law implies that net $F_y = 0$. Thus, as illustrated in the free-body diagram in link,

$$F_{\mathrm{net}y} = T_{\mathrm{L}y} + T_{\mathrm{R}y} - w = 0.$$

Observing link, we can use trigonometry to determine the relationship between $T_{\mathrm{L}y}$, $T_{\mathrm{R}y}$, and T. As we determined from the analysis in the horizontal direction, $T_{\mathrm{L}} = T_{\mathrm{R}} = T$:

$$\begin{array}{lcl} \sin\,(5.0) & = & \frac{T_{\mathrm{L}y}}{T_{\mathrm{L}}} \\ T_{\mathrm{L}y} = T_{\mathrm{L}}\,\sin\,(5.0) & = & T\,\sin\,(5.0) \\ \sin\,(5.0) & = & \frac{T_{\mathrm{R}y}}{T_{\mathrm{R}}} \\ T_{\mathrm{R}y} = T_{\mathrm{R}}\,\sin\,(5.0) & = & T\,\sin\,(5.0). \end{array}$$

Now, we can substitute the values for $T_{\mathrm{L}y}$ and $T_{\mathrm{R}y}$, into the net force equation in the vertical direction:

$$\begin{array}{lcl} F_{\mathrm{net}y} & = & T_{\mathrm{L}y} + T_{\mathrm{R}y} - w = 0 \\ F_{\mathrm{net}y} & = & T\,\sin\,(5.0) + T\,\sin\,(5.0) - w = 0 \\ 2\,T\,\sin\,(5.0) - w & = & 0 \\ 2\,T\,\sin\,(5.0) & = & w \end{array}$$

and

$$T = \frac{w}{2\,\sin\,(5.0)} = \frac{\mathrm{mg}}{2\,\sin\,(5.0)},$$

so that

$$T = \frac{(70.0\ \mathrm{kg})(9.80\ \mathrm{m/s}^2)}{2(0.0872)},$$

and the tension is

$$T = 3900\ \mathrm{N}.$$

Discussion

Note that the vertical tension in the wire acts as a normal force that supports the weight of the tightrope walker. The tension is almost six times the 686-N weight of the tightrope walker. Since the wire is nearly horizontal, the vertical component of its tension is only a small fraction of the tension in the wire. The large horizontal components are in opposite directions and cancel, and so most of the tension in the wire is not used to support the weight of the tightrope walker.

If we wish to *create* a very large tension, all we have to do is exert a force perpendicular to a flexible connector, as illustrated in link. As we saw in the last example, the weight of the tightrope walker acted as a force perpendicular to the rope. We saw that the tension in the roped related to the weight of the tightrope walker in the following way:

$$T = \frac{w}{2 \sin(\theta)}.$$

We can extend this expression to describe the tension T created when a perpendicular force ($\mathbf{F}_{\perp}$) is exerted at the middle of a flexible connector:

$$T = \frac{F_{\perp}}{2 \sin(\theta)}.$$

Note that θ is the angle between the horizontal and the bent connector. In this case, T becomes very large as θ approaches zero. Even the relatively small weight of any flexible connector will cause it to sag, since an infinite tension would result if it were horizontal (i.e., $\theta = 0$ and $\sin\theta = 0$). (See link.)

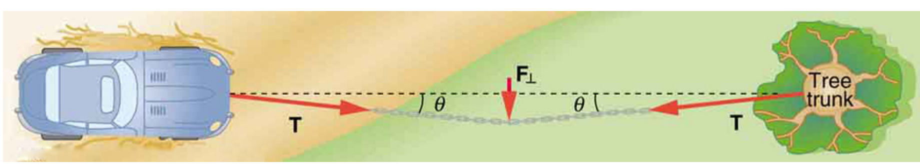


Figure 4.18: We can create a very large tension in the chain by pushing on it perpendicular to its length, as shown. Suppose we wish to pull a car out of the mud when no tow truck is available. Each time the car moves forward, the chain is tightened to keep it as nearly straight as possible. The tension in the chain is given by $T = \frac{F_{\perp}}{2 \sin(\theta)}$; since θ is small, T is very large. This situation is analogous to the tightrope walker shown in link, except that the tensions shown here are those transmitted to the car and the tree rather than those acting at the point where $\mathbf{F}_{\perp}$ is applied.

4.6.4 Extended Topic: Real Forces and Inertial Frames

There is another distinction among forces in addition to the types already mentioned. Some forces are real, whereas others are not. *Real forces* are those that

Figure 4.19: Unless an infinite tension is exerted, any flexible connector—such as the chain at the bottom of the picture—will sag under its own weight, giving a characteristic curve when the weight is evenly distributed along the length. Suspension bridges—such as the Golden Gate Bridge shown in this image—are essentially very heavy flexible connectors. The weight of the bridge is evenly distributed along the length of flexible connectors, usually cables, which take on the characteristic shape. (credit: Leaflet, Wikimedia Commons)

have some physical origin, such as the gravitational pull. Contrastingly, *fictitious forces* are those that arise simply because an observer is in an accelerating frame of reference, such as one that rotates (like a merry-go-round) or undergoes linear acceleration (like a car slowing down). For example, if a satellite is heading due north above Earth's northern hemisphere, then to an observer on Earth it will appear to experience a force to the west that has no physical origin. Of course, what is happening here is that Earth is rotating toward the east and moves east under the satellite. In Earth's frame this looks like a westward force on the satellite, or it can be interpreted as a violation of Newton's first law (the law of inertia). An inertial frame of reference is one in which all forces are real and, equivalently, one in which Newton's laws have the simple forms given in this chapter.

Earth's rotation is slow enough that Earth is nearly an inertial frame. You ordinarily must perform precise experiments to observe fictitious forces and the slight departures from Newton's laws, such as the effect just described. On the large scale, such as for the rotation of weather systems and ocean currents, the effects can be easily observed.

The crucial factor in determining whether a frame of reference is inertial is whether it accelerates or rotates relative to a known inertial frame. Unless stated otherwise, all phenomena discussed in this text are considered in inertial frames.

All the forces discussed in this section are real forces, but there are a number of other real forces, such as lift and thrust, that are not discussed in this section. They are more specialized, and it is not necessary to discuss every type of force. It is natural, however, to ask where the basic simplicity we seek to find in physics is in the long list of forces. Are some more basic than others? Are some different manifestations of the same underlying force? The answer to both questions is yes, as will be seen in the next (extended) section and in the treatment of modern physics later in the text.

Forces in 1 Dimension

Explore the forces at work when you try to push a filing cabinet. Create an applied force and see the resulting friction force and total force acting on the cabinet. Charts show the forces, position, velocity, and acceleration vs. time. View a free-body diagram of all the forces (including gravitational and normal forces).

4.6.5 Test Prep for AP Courses

An archer shoots an arrow straight up with a force of 24.5 N. The arrow has a mass of 0.4 kg. What is the force of gravity on the arrow?

a. $9.8\ \mathrm{m/s^2}$
b. 9.8 N
c. 61.25 N
d. 3.9 N

A cable raises a mass of 120.0 kg with an acceleration of 1.3 m/s2. What force of tension is in the cable?

The force of tension must equal the force of gravity plus the force necessary to accelerate the mass. $F = mg$ can be used to calculate the first, and $F = ma$ can be used to calculate the second.

For gravity:

$$F = mg$$
$$= (120.0\,\mathrm{kg})(9.8\,\mathrm{m/s}^2)$$
$$= 1205.4\,\mathrm{N}$$

For acceleration:

$$F = ma$$
$$= (120.0\,\mathrm{kg})(1.3\,\mathrm{m/s}^2)$$
$$= 159.9\,\mathrm{N}$$

The total force of tension in the cable is 1176 N + 156 N = 1332 N.

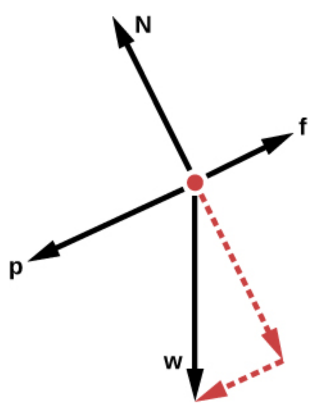
A child pulls a wagon along a grassy field. Define the system, the pairs of forces at work, and the results.

Two teams are engaging in a tug–of-war. The rope suddenly snaps. Which statement is true about the forces involved?

a. The forces exerted by the two teams are no longer equal; the teams will accelerate in opposite directions as a result.
b. The forces exerted by the players are no longer balanced by the force of tension in the rope; the teams will accelerate in opposite directions as a result.
c. The force of gravity balances the forces exerted by the players; the teams will fall as a result
d. The force of tension in the rope is transferred to the players; the teams will accelerate in opposite directions as a result.

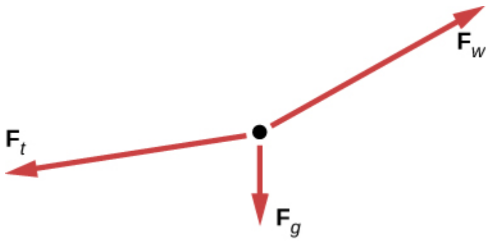
b

The following free-body diagram represents a toboggan on a hill. What acceleration would you expect, and why?



a. Acceleration down the hill; the force due to being pushed, together with the downhill component of gravity, overcomes the opposing force of friction.
b. Acceleration down the hill; friction is less than the opposing component of force due to gravity.
c. No movement; friction is greater than the force due to being pushed.
d. No movement; friction is greater than the sum of the downhill forces.

Draw a free-body diagram to represent the forces acting on a kite on a string that is floating stationary in the air. Label the forces in your diagram.



The diagram has a black dot and three solid red arrows pointing away from the dot. Arrow Ft is long and pointing to the left and slightly down. Arrow Fw is also long and is a bit below a diagonal line halfway between pointing up and pointing to the right. A short arrow Fg is pointing down.

$\mathbf{F}_{\mathrm{g}}$ is the force on the kite due to gravity.

$\mathbf{F}_{w}$ is the force exerted on the kite by the wind.

$\mathbf{F}_{\mathrm{t}}$ is the force of tension in the string holding the kite. It must balance the vector sum of the other two forces for the kite to float stationary in the air.

A car is sliding down a hill with a slope of 20°. The mass of the car is 965 kg. When a cable is used to pull the car up the slope, a force of 4215 N is applied. What is the car's acceleration, ignoring friction?

4.6.6 Section Summary

- When objects rest on a surface, the surface applies a force to the object that supports the weight of the object. This supporting force acts perpendicular to and away from the surface. It is called a normal force, **N**.
- When objects rest on a non-accelerating horizontal surface, the magnitude of the normal force is equal to the weight of the object: ::: {#eip-id1331551 data-type="equation"}

  $$N = \mathrm{mg}.$$

  :::
- When objects rest on an inclined plane that makes an angle θ with the horizontal surface, the weight of the object can be resolved into components that act perpendicular ($\mathbf{w}_{\perp}$) and parallel ($\mathbf{w}_{\|}$) to the surface of the plane. These components can be calculated using: ::: {#eip-id2419526 data-type="equation"}

  $$w_{\|} = w \sin(\theta) = \mathrm{mg} \sin(\theta)$$

  :::

  $$w_{\perp} = w \cos(\theta) = \mathrm{mg} \cos(\theta).$$

- The pulling force that acts along a stretched flexible connector, such as a rope or cable, is called tension, **T**. When a rope supports the weight of an object that is at rest, the tension in the rope is equal to the weight of the object: ::: {#eip-id1166246 data-type="equation"}

$$T = \text{mg}.$$

:::

- In any inertial frame of reference (one that is not accelerated or rotated), Newton's laws have the simple forms given in this chapter and all forces are real forces having a physical origin.

4.6.7 Conceptual Questions

If a leg is suspended by a traction setup as shown in link, what is the tension in the rope?

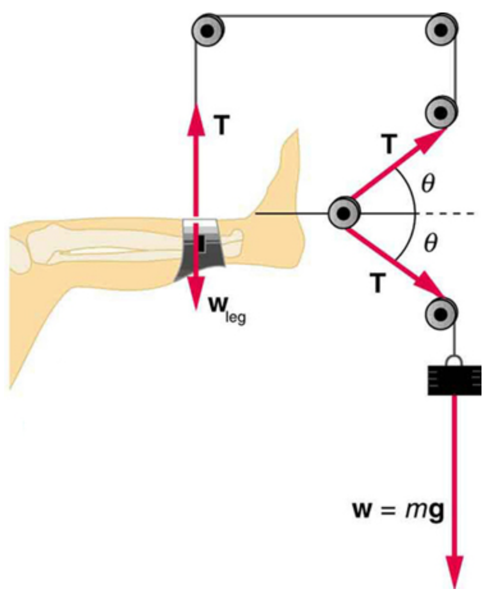


Figure 4.20: A leg is suspended by a traction system in which wires are used to transmit forces. Frictionless pulleys change the direction of the force T without changing its magnitude.

In a traction setup for a broken bone, with pulleys and rope available, how might we be able to increase the force along the tibia using the same weight? (See link.) (Note that the tibia is the shin bone shown in this image.)

4.6.8 Problem Exercises

Two teams of nine members each engage in a tug of war. Each of the first team's members has an average mass of 68 kg and exerts an average force of 1350 N horizontally. Each of the second team's members has an average mass of 73 kg and exerts an average force of 1365 N horizontally. (a) What is magnitude of the acceleration of the two teams? (b) What is the tension in the section of rope between the teams?

a. $0.11\ \mathrm{m/s}^2$
b. 1.2×10^4 N

What force does a trampoline have to apply to a 45.0-kg gymnast to accelerate her straight up at $7.50\ \mathrm{m/s}^2$? Note that the answer is independent of the velocity of the gymnast—she can be moving either up or down, or be stationary.

a Calculate the tension in a vertical strand of spider web if a spider of mass 8.00×10^{-5} kg hangs motionless on it. (b) Calculate the tension in a horizontal strand of spider web if the same spider sits motionless in the middle of it much like the tightrope walker in link. The strand sags at an angle of 12º below the horizontal. Compare this with the tension in the vertical strand (find their ratio).

a 7.84×10^{-4} N

b 1.89×10^{-3} N . This is 2.41 times the tension in the vertical strand.

Suppose a 60.0-kg gymnast climbs a rope. (a) What is the tension in the rope if he climbs at a constant speed? (b) What is the tension in the rope if he accelerates upward at a rate of $1.50\ \mathrm{m/s}^2$?

Show that, as stated in the text, a force $\mathbf{F}_\perp$ exerted on a flexible medium at its center and perpendicular to its length (such as on the tightrope wire in link) gives rise to a tension of magnitude $T = \frac{F_\perp}{2\ \sin\ (\theta)}$.

Newton's second law applied in vertical direction gives

$$F_y = F - 2T\ \sin\ \theta = 0$$

$$F = 2T\ \sin\ \theta$$

$$T = \frac{F}{2\ \sin\ \theta}.$$

Consider the baby being weighed in link. (a) What is the mass of the child and basket if a scale reading of 55 N is observed? (b) What is the tension T_1 in the cord attaching the baby to the scale? (c) What is the tension T_2 in the cord attaching the scale to the ceiling, if the scale has a mass of 0.500 kg? (d) Draw a sketch of the situation indicating the system of interest used to solve each part. The masses of the cords are negligible.

4.6.9 Glossary

inertial frame of reference a coordinate system that is not accelerating; all forces acting in an inertial frame of reference are real forces, as opposed to fictitious forces that are observed due to an accelerating frame of reference

normal force the force that a surface applies to an object to support the weight of the object; acts perpendicular to the surface on which the object rests

tension the pulling force that acts along a medium, especially a stretched flexible connector, such as a rope or cable; when a rope supports the weight of an object, the force on the object due to the rope is called a tension force

4.7 Problem-Solving Strategies

4.7.1 Learning Objectives

By the end of this section, you will be able to:

- Apply a problem-solving procedure to solve problems using Newton's laws of motion

The information presented in this section supports the following AP® learning objectives and science practices:

- **3.A.2.1** The student is able to represent forces in diagrams or mathematically using appropriately labeled vectors with magnitude, direction, and units during the analysis of a situation. **(S.P. 1.1)**
- **3.A.3.3** The student is able to describe a force as an interaction between two objects and identify both objects for any force. **(S.P. 1.4)**
- **3.B.1.1** The student is able to predict the motion of an object subject to forces exerted by several objects using an application of Newton's second law in a variety of physical situations with acceleration in one dimension. **(S.P. 6.4, 7.2)**
- **3.B.1.3** The student is able to re-express a free-body diagram representation into a mathematical representation and solve the mathematical representation for the acceleration of the object. **(S.P. 1.5, 2.2)**
- **3.B.2.1** The student is able to create and use free-body diagrams to analyze physical situations to solve problems with motion qualitatively and quantitatively. **(S.P. 1.1, 1.4, 2.2)**

Success in problem solving is obviously necessary to understand and apply physical principles, not to mention the more immediate need of passing exams. The

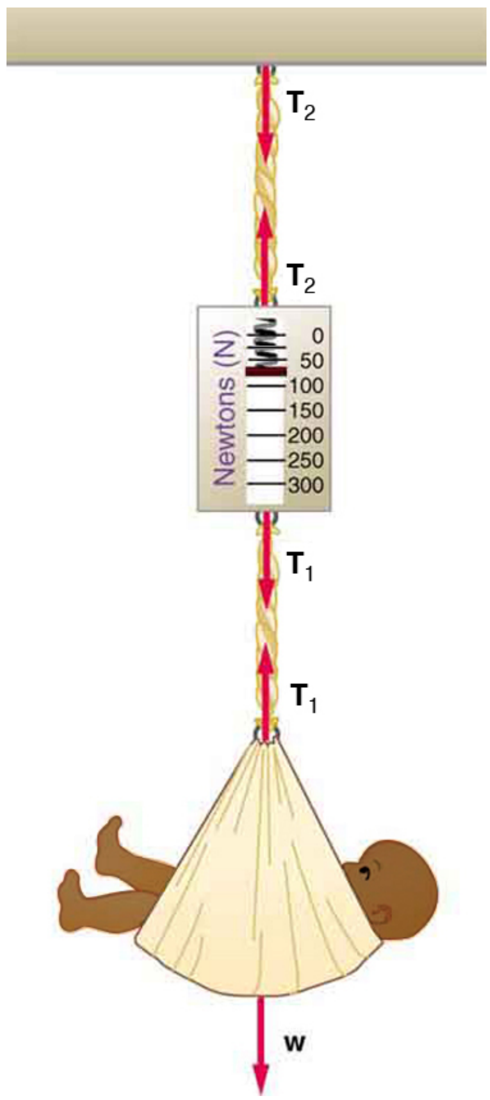


Figure 4.21: A baby is weighed using a spring scale.

basics of problem solving, presented earlier in this text, are followed here, but specific strategies useful in applying Newton's laws of motion are emphasized. These techniques also reinforce concepts that are useful in many other areas of physics. Many problem-solving strategies are stated outright in the worked examples, and so the following techniques should reinforce skills you have already begun to develop.

4.7.2 Problem-Solving Strategy for Newton's Laws of Motion

Step 1. As usual, it is first necessary to identify the physical principles involved. *Once it is determined that Newton's laws of motion are involved (if the problem involves forces), it is particularly important to draw a careful sketch of the situation.* Such a sketch is shown in link(a). Then, as in link(b), use arrows to represent all forces, label them carefully, and make their lengths and directions correspond to the forces they represent (whenever sufficient information exists).

Step 2. Identify what needs to be determined and what is known or can be inferred from the problem as stated. That is, make a list of knowns and unknowns. *Then carefully determine the system of interest.* This decision is a crucial step, since Newton's second law involves only external forces. Once the system of interest has been identified, it becomes possible to determine which forces are external and which are internal, a necessary step to employ Newton's second law. (See link(c).) Newton's third law may be used to identify whether forces are exerted between components of a system (internal) or between the system and something outside (external). As illustrated earlier in this chapter, the system of interest depends on what question we need to answer. This choice becomes easier with practice, eventually developing into an almost unconscious process. Skill in clearly defining systems will be beneficial in later chapters as well.

A diagram showing the system of interest and all of the external forces is called a free-body diagram. Only forces are shown on free-body diagrams, not acceleration or velocity. We have drawn several of these in worked examples. link(c) shows a free-body diagram for the system of interest. Note that no internal forces are shown in a free-body diagram.

Step 3. Once a free-body diagram is drawn, *Newton's second law can be applied to solve the problem.* This is done in link(d) for a particular situation. In general, once external forces are clearly identified in free-body diagrams, it should be a straightforward task to put them into equation form and solve for the unknown, as done in all previous examples. If the problem is one-dimensional—that is, if all forces are parallel—then they add like scalars. If the problem is two-dimensional, then it must be broken down into a pair of one-dimensional problems. This is done by projecting the force vectors onto a set of axes chosen for convenience. As seen in previous examples, the choice of axes can simplify the problem. For example, when an incline is involved, a set of axes with one axis parallel to the incline and one perpendicular to it is most convenient. It is

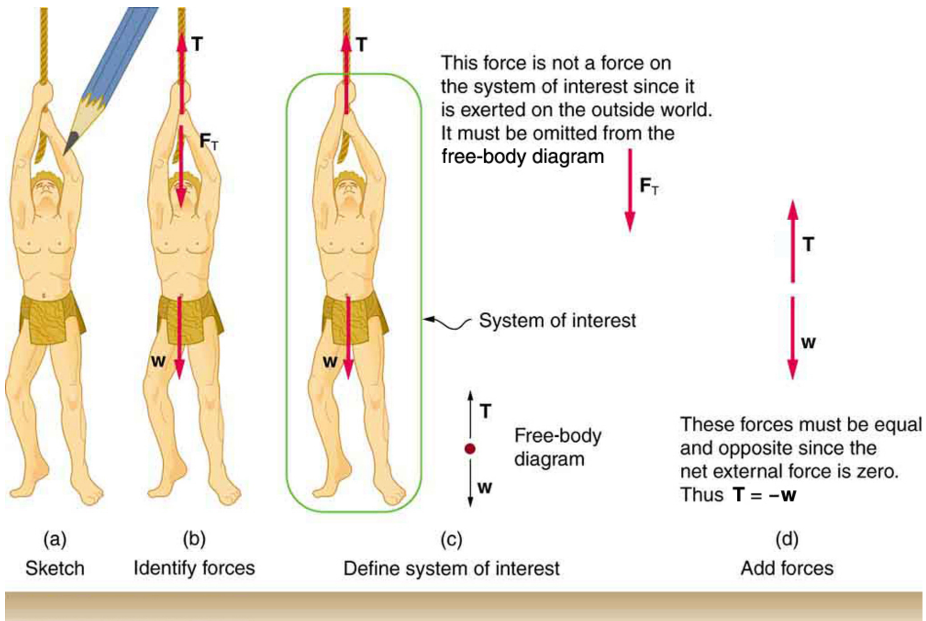


Figure 4.22: (a) A sketch of Tarzan hanging from a vine. (b) Arrows are used to represent all forces. $\mathbf{T}$ is the tension in the vine above Tarzan, $\mathbf{F}_\mathrm{T}$ is the force he exerts on the vine, and $\mathbf{w}$ is his weight. All other forces, such as the nudge of a breeze, are assumed negligible. (c) Suppose we are given the ape man's mass and asked to find the tension in the vine. We then define the system of interest as shown and draw a free-body diagram. $\mathbf{F}_\mathrm{T}$ is no longer shown, because it is not a force acting on the system of interest; rather, $\mathbf{F}_\mathrm{T}$ acts on the outside world. (d) Showing only the arrows, the head-to-tail method of addition is used. It is apparent that $\mathbf{T} = -\mathbf{w}$, if Tarzan is stationary.

almost always convenient to make one axis parallel to the direction of motion, if this is known.

Applying Newton's Second Law

Before you write net force equations, it is critical to determine whether the system is accelerating in a particular direction. If the acceleration is zero in a particular direction, then the net force is zero in that direction. Similarly, if the acceleration is nonzero in a particular direction, then the net force is described by the equation: $F_{\text{net}} = \text{ma}$.

For example, if the system is accelerating in the horizontal direction, but it is not accelerating in the vertical direction, then you will have the following conclusions:

$$F_{\text{net }x} = \text{ma},$$

$$F_{\text{net }y} = 0.$$

You will need this information in order to determine unknown forces acting in a system.

Step 4. As always, *check the solution to see whether it is reasonable.* In some cases, this is obvious. For example, it is reasonable to find that friction causes an object to slide down an incline more slowly than when no friction exists. In practice, intuition develops gradually through problem solving, and with experience it becomes progressively easier to judge whether an answer is reasonable. Another way to check your solution is to check the units. If you are solving for force and end up with units of m/s, then you have made a mistake.

4.7.3 Test Prep for AP Courses

A toboggan with two riders has a total mass of 85.0 kg. A third person is pushing with a horizontal force of 42.5 N on a toboggan moving on a horizontal surface at the top of a hill that has a downward angle of 15°. The force of friction on the toboggan is 31.0 N. Which statement describes an accurate free-body diagram to represent the situation?

a. An arrow of magnitude 10.5 N points down the slope of the hill.
b. An arrow of magnitude 833 N points straight down.
c. An arrow of magnitude 833 N points perpendicular to the slope of the hill.
d. An arrow of magnitude 73.5 N points down the slope of the hill.

b

A mass of 2.0 kg is suspended from the ceiling of an elevator by a rope. What is the tension in the rope when the elevator (i) accelerates upward at 1.5 m/s^2? (ii) accelerates downward at 1.5 m/s^2?

a. *i* 22.6 N; (ii) 16.6 N
b. Because the mass is hanging from the elevator itself, the tension in the rope will not change in either case.
c. *i* 22.6 N; (ii) 19.6 N
d. *i* 16.6 N; (ii) 19.6 N

Which statement is true about drawing free-body diagrams?

a. Drawing a free-body diagram should be the last step in solving a problem about forces.
b. Drawing a free-body diagram helps you compare forces quantitatively.
c. The forces in a free-body diagram should always balance.
d. Drawing a free-body diagram can help you determine the net force.

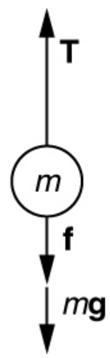
d

4.7.4 Section Summary

- To solve problems involving Newton's laws of motion, follow the procedure described:
  1. Draw a sketch of the problem.
  2. Identify known and unknown quantities, and identify the system of interest. Draw a free-body diagram, which is a sketch showing all of the forces acting on an object. The object is represented by a dot, and the forces are represented by vectors extending in different directions from the dot. If vectors act in directions that are not horizontal or vertical, resolve the vectors into horizontal and vertical components and draw them on the free-body diagram.
  3. Write Newton's second law in the horizontal and vertical directions and add the forces acting on the object. If the object does not accelerate in a particular direction (for example, the x-direction) then $F_{\text{net }x} = 0$. If the object does accelerate in that direction, $F_{\text{net }x} = \text{ma}$.
  4. Check your answer. Is the answer reasonable? Are the units correct?

4.7.5 Problem Exercises

A 5.00×10^5-kg rocket is accelerating straight up. Its engines produce 1.250×10^7 N of thrust, and air resistance is 4.50×10^6 N. What is the rocket's acceleration? Explicitly show how you follow the steps in the Problem-Solving Strategy for Newton's laws of motion.



Using the free-body diagram:

$F_{\text{net}} = T - f - mg = \text{ma},$

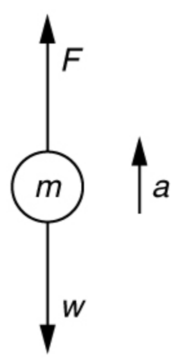
so that

$a = \frac{T - f - \text{mg}}{m} = \frac{1.250 \times 10^7 \text{ N} - 4.50 \times 10^6 \text{ } N - (5.00 \times 10^5 \text{ kg})(9.80 \text{ m/s}^2)}{5.00 \times 10^5 \text{ kg}} = 6.20 \text{ m/s}^2.$

The wheels of a midsize car exert a force of 2100 N backward on the road to accelerate the car in the forward direction. If the force of friction including air resistance is 250 N and the acceleration of the car is 1.80 m/s^2, what is the mass of the car plus its occupants? Explicitly show how you follow the steps in the Problem-Solving Strategy for Newton's laws of motion. For this situation, draw a free-body diagram and write the net force equation.

Calculate the force a 70.0-kg high jumper must exert on the ground to produce an upward acceleration 4.00 times the acceleration due to gravity. Explicitly show how you follow the steps in the Problem-Solving Strategy for Newton's laws of motion.

1. Use Newton's laws of motion.



2. Given : $a = 4.00g = (4.00)(9.80 \text{ m/s}^2) = 39.2 \text{ m/s}^2$; $m = 70.0$ kg,

   Find: F.

3. $\sum F{=}{+}F - w = \text{ma}$, so that $F = \text{ma} + w = \text{ma} + \text{mg} = m(a+g)$.

   $F = (70.0\ \text{kg})[(39.2\ \text{m/s}^2) + (9.80\ \text{m/s}^2)] = 3.43 \times 10^3\text{N}$. The force exerted by the high-jumper is actually down on the ground, but F is up from the ground and makes him jump.

4. This result is reasonable, since it is quite possible for a person to exert a force of the magnitude of 10^3 N.

When landing after a spectacular somersault, a 40.0-kg gymnast decelerates by pushing straight down on the mat. Calculate the force she must exert if her deceleration is 7.00 times the acceleration due to gravity. Explicitly show how you follow the steps in the Problem-Solving Strategy for Newton's laws of motion.

A freight train consists of two 8.00×10^4-kg engines and 45 cars with average masses of 5.50×10^4 kg . (a) What force must each engine exert backward on the track to accelerate the train at a rate of $5.00{\times}10^{-2}\ \text{m/s}^2$ if the force of friction is 7.50×10^5 N, assuming the engines exert identical forces? This is not a large frictional force for such a massive system. Rolling friction for trains is small, and consequently trains are very energy-efficient transportation systems. (b) What is the force in the coupling between the 37th and 38th cars (this is the force each exerts on the other), assuming all cars have the same mass and that friction is evenly distributed among all of the cars and engines?

a 4.41×10^5 N

b 1.50×10^5 N

Commercial airplanes are sometimes pushed out of the passenger loading area by a tractor. (a) An 1800-kg tractor exerts a force of 1.75×10^4 N backward on the pavement, and the system experiences forces resisting motion that total 2400 N. If the acceleration is $0.150\ \text{m/s}^2$, what is the mass of the airplane? (b) Calculate the force exerted by the tractor on the airplane, assuming 2200 N of the friction is experienced by the airplane. (c) Draw two sketches showing the systems of interest used to solve each part, including the free-body diagrams for each.

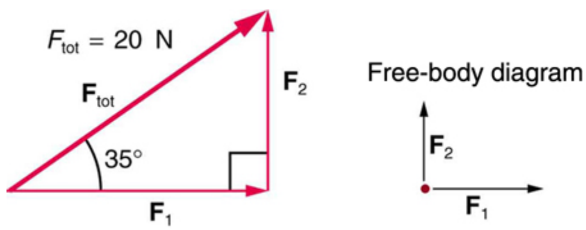
A 1100-kg car pulls a boat on a trailer. (a) What total force resists the motion of the car, boat, and trailer, if the car exerts a 1900-N force on the road and produces an acceleration of $0.550\ \text{m/s}^2$? The mass of the boat plus trailer is 700 kg. (b) What is the force in the hitch between the car and the trailer if 80% of the resisting forces are experienced by the boat and trailer?

a 910 N

b 1.11×10^3 N

a Find the magnitudes of the forces $\mathbf{F}_1$ and $\mathbf{F}_2$ that add to give the total force $\mathbf{F}_{\text{tot}}$ shown in link. This may be done either graphically or by using trigonometry.

(b) Show graphically that the same total force is obtained independent of the order of addition of $\mathbf{F}_1$ and $\mathbf{F}_2$. (c) Find the direction and magnitude of some other pair of vectors that add to give $\mathbf{F}_{\text{tot}}$. Draw these to scale on the same drawing used in part (b) or a similar picture.



Two children pull a third child on a snow saucer sled exerting forces $\mathbf{F}_1$ and $\mathbf{F}_2$ as shown from above in link. Find the acceleration of the 49.00-kg sled and child system. Note that the direction of the frictional force is unspecified; it will be in the opposite direction of the sum of $\mathbf{F}_1$ and $\mathbf{F}_2$.

$a = 0.139$ m/s, $\theta = 12.4$ north of east

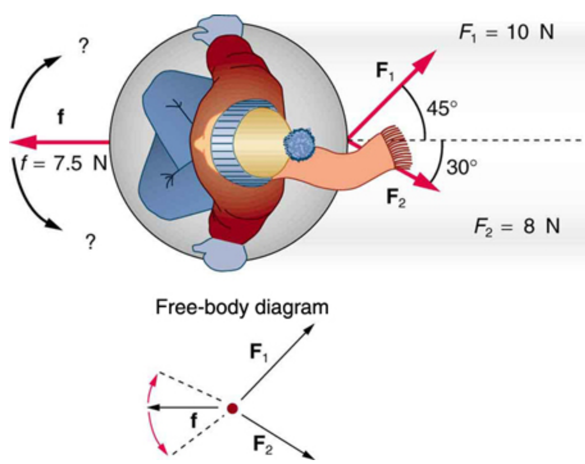
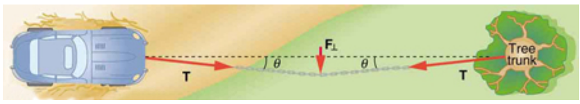


Figure 4.23: An overhead view of the horizontal forces acting on a child's snow saucer sled.

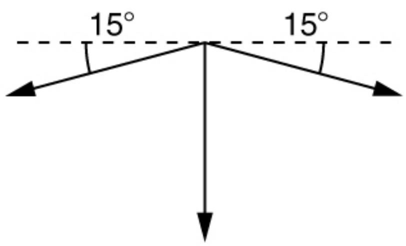
Suppose your car was mired deeply in the mud and you wanted to use the method illustrated in link to pull it out. (a) What force would you have to exert perpendicular to the center of the rope to produce a force of 12,000 N on the car if the angle is 2.00°? In this part, explicitly show how you follow the steps in the Problem-Solving Strategy for Newton's laws of motion. (b) Real ropes stretch under such forces. What force would be exerted on the car if the angle increases to 7.00° and you still apply the force found in part (a) to its center?



What force is exerted on the tooth in link if the tension in the wire is 25.0 N? Note that the force applied to the tooth is smaller than the tension in the wire, but this is necessitated by practical considerations of how force can be applied in the mouth. Explicitly show how you follow steps in the Problem-Solving Strategy for Newton's laws of motion.

1. Use Newton's laws since we are looking for forces.

2. Draw a force diagram:



3. The tension is given as $T = 25.0$ N. Find F_{app}. Using Newton's laws gives: $\Sigma\ \mathrm{F}_x = 0$, so that applied force is due to the y-components of the two tensions: $F_{\text{app}} = 2\ T\ \sin\ = 2(25.0\ \mathrm{N})\sin\left(15^{\mathrm{o}}\right) = 12.9\ \mathrm{N}$

   The x-components of the tension cancel. $\sum F_x = 0$.

4. This seems reasonable, since the applied tensions should be greater than the force applied to the tooth.

link shows Superhero and Trusty Sidekick hanging motionless from a rope. Superhero's mass is 90.0 kg, while Trusty Sidekick's is 55.0 kg, and the mass of the rope is negligible. (a) Draw a free-body diagram of the situation showing all forces acting on Superhero, Trusty Sidekick, and the rope. (b) Find the tension in the rope above Superhero. (c) Find the tension in the rope between Superhero and Trusty Sidekick. Indicate on your free-body diagram the system of interest used to solve each part.

A nurse pushes a cart by exerting a force on the handle at a downward angle 35.0^{o} below the horizontal. The loaded cart has a mass of 28.0 kg, and the force of friction is 60.0 N. (a) Draw a free-body diagram for the system of interest. (b) What force must the nurse exert to move at a constant velocity?

Construct Your Own Problem Consider the tension in an elevator cable during the time the elevator starts from rest and accelerates its load upward to some cruising velocity. Taking the elevator and its load to be the system of interest, draw a free-body diagram. Then calculate the tension in the cable. Among the

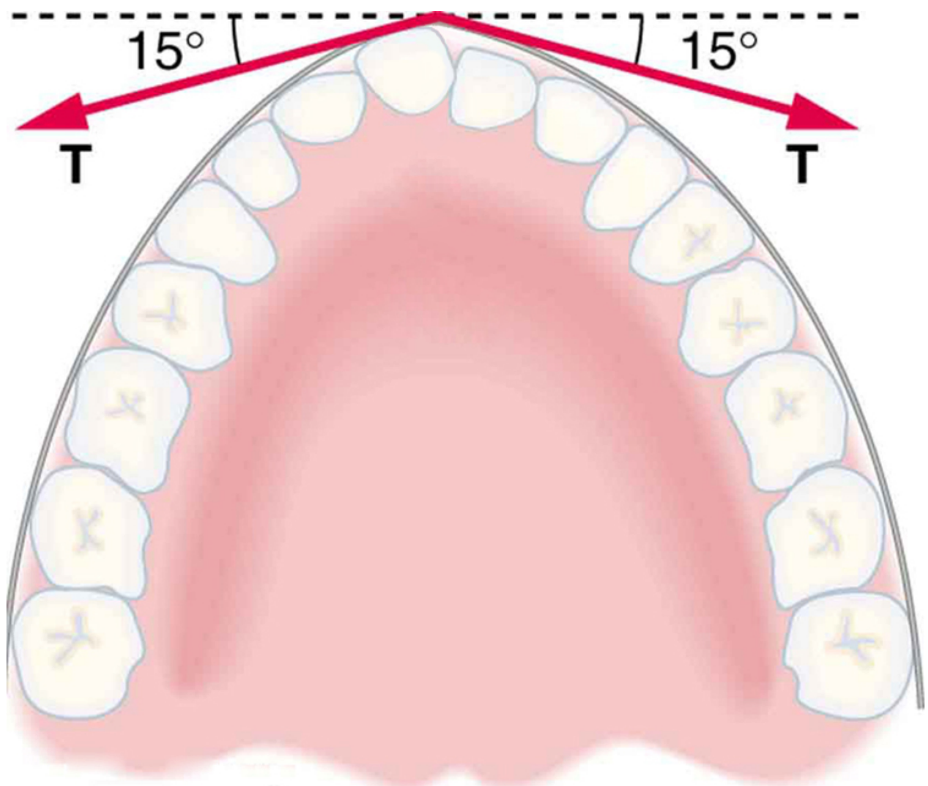


Figure 4.24: Braces are used to apply forces to teeth to realign them. Shown in this figure are the tensions applied by the wire to the protruding tooth. The total force applied to the tooth by the wire, $\mathbf{F}_{\text{app}}$, points straight toward the back of the mouth.

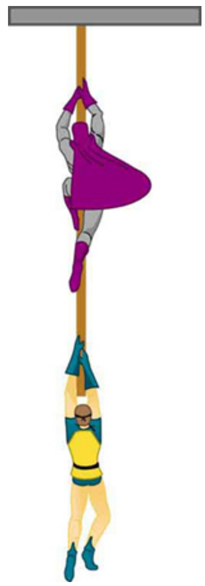


Figure 4.25: Superhero and Trusty Sidekick hang motionless on a rope as they try to figure out what to do next. Will the tension be the same everywhere in the rope?

things to consider are the mass of the elevator and its load, the final velocity, and the time taken to reach that velocity.

Construct Your Own Problem Consider two people pushing a toboggan with four children on it up a snow-covered slope. Construct a problem in which you calculate the acceleration of the toboggan and its load. Include a free-body diagram of the appropriate system of interest as the basis for your analysis. Show vector forces and their components and explain the choice of coordinates. Among the things to be considered are the forces exerted by those pushing, the angle of the slope, and the masses of the toboggan and children.

Unreasonable Results (a) Repeat link, but assume an acceleration of 1.20 m/s^2 is produced. (b) What is unreasonable about the result? (c) Which premise is unreasonable, and why is it unreasonable?

Unreasonable Results (a) What is the initial acceleration of a rocket that has a mass of 1.50×10^6 kg at takeoff, the engines of which produce a thrust of 2.00×10^6 N? Do not neglect gravity. (b) What is unreasonable about the result? (This result has been unintentionally achieved by several real rockets.) (c) Which premise is unreasonable, or which premises are inconsistent? (You may find it useful to compare this problem to the rocket problem earlier in this section.)

4.8 Further Applications of Newton's Laws of Motion

4.8.1 Learning Objectives

By the end of this section, you will be able to:

- Apply problem-solving techniques to solve for quantities in more complex systems of forces.
- Integrate concepts from kinematics to solve problems using Newton's laws of motion.

The information presented in this section supports the following AP® learning objectives and science practices:

- **3.A.2.1** The student is able to represent forces in diagrams or mathematically using appropriately labeled vectors with magnitude, direction, and units during the analysis of a situation. **(S.P. 1.1)**
- **3.A.3.1** The student is able to analyze a scenario and make claims (develop arguments, justify assertions) about the forces exerted on an object by other objects for different types of forces or components of forces. **(S.P. 6.4, 7.2)**
- **3.A.3.3** The student is able to describe a force as an interaction between two objects and identify both objects for any force. **(S.P. 1.4)**
- **3.B.1.1** The student is able to predict the motion of an object subject to forces exerted by several objects using an application of Newton's second law in a variety of physical situations with acceleration in one dimension. **(S.P. 6.4, 7.2)**
- **3.B.1.3** The student is able to re-express a free-body diagram representation into a mathematical representation and solve the mathematical representation for the acceleration of the object. **(S.P. 1.5, 2.2)**
- **3.B.2.1** The student is able to create and use free-body diagrams to analyze physical situations to solve problems with motion qualitatively and quantitatively. **(S.P. 1.1, 1.4, 2.2)**

There are many interesting applications of Newton's laws of motion, a few more of which are presented in this section. These serve also to illustrate some further subtleties of physics and to help build problem-solving skills.

Drag Force on a Barge

Suppose two tugboats push on a barge at different angles, as shown in link. The first tugboat exerts a force of 2.7×10^5 N in the x-direction, and the second tugboat exerts a force of 3.6×10^5 N in the y-direction.

If the mass of the barge is 5.0×10^6 kg and its acceleration is observed to be $7.5\times10^{-2}\ \mathrm{m/s^2}$ in the direction shown, what is the drag force of the water on the barge resisting the motion? (Note: drag force is a frictional force exerted by fluids, such as air or water. The drag force opposes the motion of the object.)

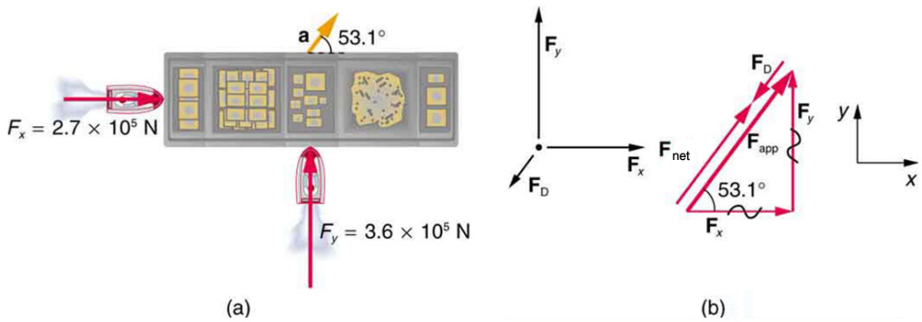


Figure 4.26: (a) A view from above of two tugboats pushing on a barge. (b) The free-body diagram for the ship contains only forces acting in the plane of the water. It omits the two vertical forces—the weight of the barge and the buoyant force of the water supporting it cancel and are not shown. Since the applied forces are perpendicular, the x- and y-axes are in the same direction as $\mathbf{F}_x$ and $\mathbf{F}_y$. The problem quickly becomes a one-dimensional problem along the direction of $\mathbf{F}_{\text{app}}$, since friction is in the direction opposite to $\mathbf{F}_{\text{app}}$.

Strategy

The directions and magnitudes of acceleration and the applied forces are given in link**(a)**. We will define the total force of the tugboats on the barge as $\mathbf{F}_{\text{app}}$ so that:

$$\mathbf{F}_{\text{app}}=\mathbf{F}_x+\mathbf{F}_y$$

Since the barge is flat bottomed, the drag of the water $\mathbf{F}_{\text{D}}$ will be in the direction opposite to $\mathbf{F}_{\text{app}}$, as shown in the free-body diagram in link(b). The system of interest here is the barge, since the forces on *it* are given as well as its acceleration. Our strategy is to find the magnitude and direction of the net applied force $\mathbf{F}_{\text{app}}$, and then apply Newton's second law to solve for the drag force $\mathbf{F}_{\text{D}}$.

Solution

Since $\mathbf{F}_x$ and $\mathbf{F}_y$ are perpendicular, the magnitude and direction of $\mathbf{F}_{\text{app}}$ are easily found. First, the resultant magnitude is given by the Pythagorean theorem:

$$\begin{aligned} F_{\text{app}} &= \sqrt{\mathrm{F}_x^2+\mathrm{F}_y^2} \\ F_{\text{app}} &= \sqrt{(2.7\times10^5\ \mathrm{N})^2+(3.6\times10^5\ \mathrm{N})^2} = 4.5\times10^5\ \mathrm{N}. \end{aligned}$$

The angle is given by

$$\begin{aligned}\theta &= \tan^{-1}\left(\frac{F_y}{F_x}\right)\\ \theta &= \tan^{-1}\left(\frac{3.6\times10^5\ \text{N}}{2.7\times10^5\ \text{N}}\right) = 53^{\text{o}},\end{aligned}$$

which we know, because of Newton's first law, is the same direction as the acceleration. $\mathbf{F}_{\text{D}}$ is in the opposite direction of $\mathbf{F}_{\text{app}}$, since it acts to slow down the acceleration. Therefore, the net external force is in the same direction as $\mathbf{F}_{\text{app}}$, but its magnitude is slightly less than $\mathbf{F}_{\text{app}}$. The problem is now one-dimensional. From link**(b)**, we can see that

$$F_{\text{net}} = F_{\text{app}} - F_{\text{D}}.$$

But Newton's second law states that

$$F_{\text{net}} = \text{ma}.$$

Thus,

$$F_{\text{app}} - F_{\text{D}} = \text{ma}.$$

This can be solved for the magnitude of the drag force of the water F_{D} in terms of known quantities:

$$F_{\text{D}} = F_{\text{app}} - \text{ma}.$$

Substituting known values gives

$$\text{F}_{\text{D}} = (4.5\times10^5\ \text{N}) - (5.0\times10^6\ \text{kg})(7.5\times10^{-2}\ \text{m/s}^2) = 7.5\times10^4\ \text{N}.$$

The direction of $\mathbf{F}_{\text{D}}$ has already been determined to be in the direction opposite to $\mathbf{F}_{\text{app}}$, or at an angle of 53^{o} south of west.

Discussion

The numbers used in this example are reasonable for a moderately large barge. It is certainly difficult to obtain larger accelerations with tugboats, and small speeds are desirable to avoid running the barge into the docks. Drag is relatively small for a well-designed hull at low speeds, consistent with the answer to this example, where F_{D} is less than 1/600th of the weight of the ship.

In the earlier example of a tightrope walker we noted that the tensions in wires supporting a mass were equal only because the angles on either side were equal.

Consider the following example, where the angles are not equal; slightly more trigonometry is involved.

Different Tensions at Different Angles

Consider the traffic light (mass 15.0 kg) suspended from two wires as shown in link. Find the tension in each wire, neglecting the masses of the wires.

Strategy

The system of interest is the traffic light, and its free-body diagram is shown in link(c). The three forces involved are not parallel, and so they must be projected onto a coordinate system. The most convenient coordinate system has one axis vertical and one horizontal, and the vector projections on it are shown in part (d) of the figure. There are two unknowns in this problem (T_1 and T_2), so two equations are needed to find them. These two equations come from applying Newton's second law along the vertical and horizontal axes, noting that the net external force is zero along each axis because acceleration is zero.

Solution

First consider the horizontal or x-axis:

$$F_{\mathrm{net}x} = T_{2x} - T_{1x} = 0.$$

Thus, as you might expect,

$$T_{1x} = T_{2x}.$$

This gives us the following relationship between T_1 and T_2:

$$T_1 \cos\,(30^{\mathrm{o}}) = T_2 \cos\,(45^{\mathrm{o}}).$$

Thus,

$$T_2 = (1.225)T_1.$$

Note that T_1 and T_2 are not equal in this case, because the angles on either side are not equal. It is reasonable that T_2 ends up being greater than T_1, because it is exerted more vertically than T_1.

Now consider the force components along the vertical or y-axis:

$$F_{\mathrm{net}\ y} = T_{1y} + T_{2y} - w = 0.$$

This implies

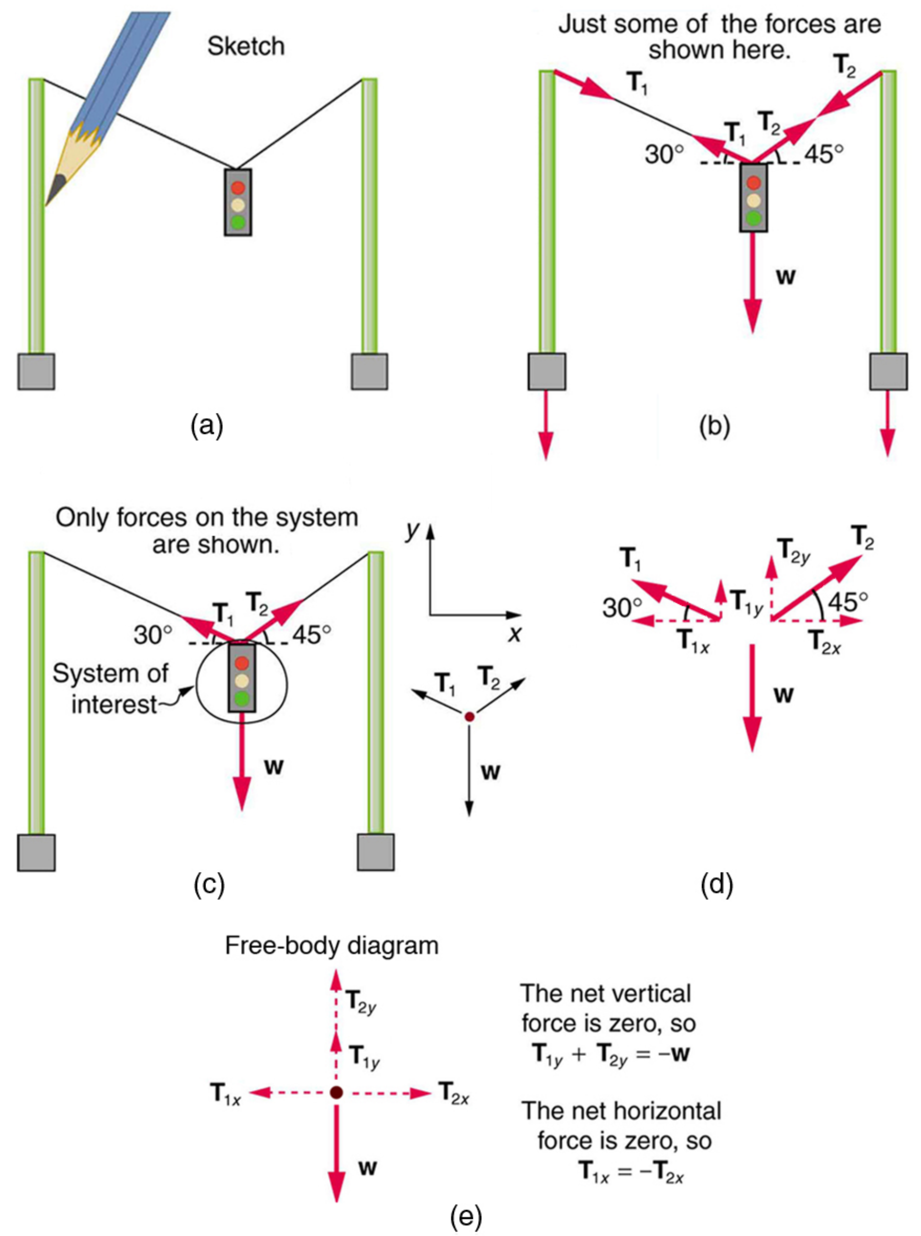


Figure 4.27: A traffic light is suspended from two wires. (b) Some of the forces involved. (c) Only forces acting on the system are shown here. The free-body diagram for the traffic light is also shown. (d) The forces projected onto vertical (y) and horizontal (x) axes. The horizontal components of the tensions must cancel, and the sum of the vertical components of the tensions must equal the weight of the traffic light. (e) The free-body diagram shows the vertical and horizontal forces acting on the traffic light.

$$T_{1y} + T_{2y} = w.$$

Substituting the expressions for the vertical components gives

$$T_1 \sin(30^{\mathrm{o}}) + T_2 \sin(45^{\mathrm{o}}) = w.$$

There are two unknowns in this equation, but substituting the expression for T_2 in terms of T_1 reduces this to one equation with one unknown:

$$T_1(0.500) + (1.225T_1)(0.707) = w = \mathrm{mg},$$

which yields

$$(1.366)\,T_1 = (15.0\text{ kg})(9.80\text{ m/s}^2).$$

Solving this last equation gives the magnitude of T_1 to be

$$T_1 = 108\text{ N}.$$

Finally, the magnitude of T_2 is determined using the relationship between them, $T_2 = 1.225\ T_1$, found above. Thus we obtain

$$T_2 = 132\text{ N}.$$

Discussion

Both tensions would be larger if both wires were more horizontal, and they will be equal if and only if the angles on either side are the same (as they were in the earlier example of a tightrope walker).

The bathroom scale is an excellent example of a normal force acting on a body. It provides a quantitative reading of how much it must push upward to support the weight of an object. But can you predict what you would see on the dial of a bathroom scale if you stood on it during an elevator ride? Will you see a value greater than your weight when the elevator starts up? What about when the elevator moves upward at a constant speed: will the scale still read more than your weight at rest? Consider the following example.

What Does the Bathroom Scale Read in an Elevator?

link shows a 75.0-kg man (weight of about 165 lb) standing on a bathroom scale in an elevator. Calculate the scale reading: (a) if the elevator accelerates upward at a rate of 1.20 m/s^2, and (b) if the elevator moves upward at a constant speed of 1 m/s.

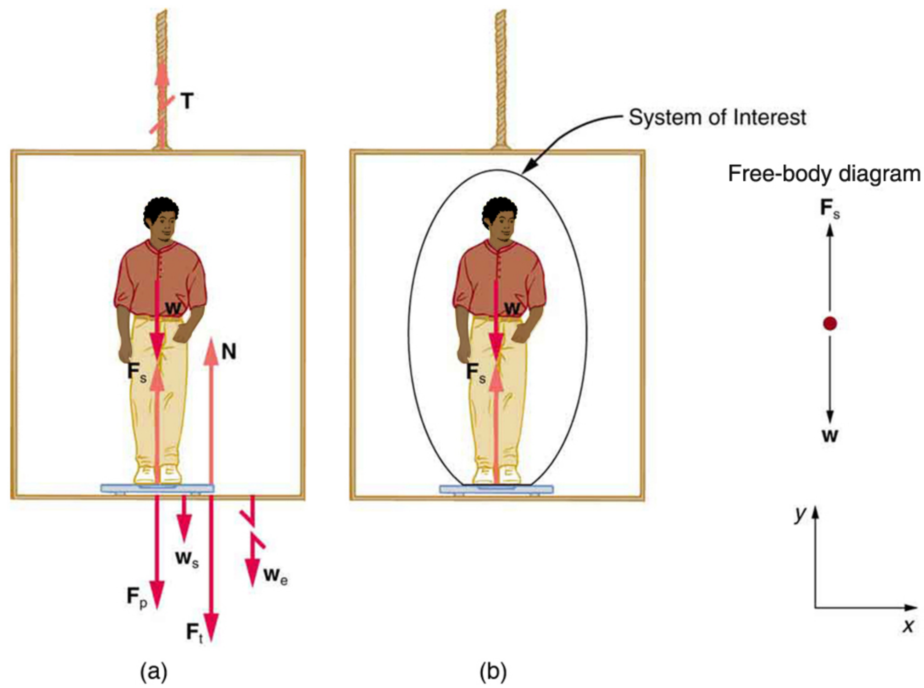


Figure 4.28: (a) The various forces acting when a person stands on a bathroom scale in an elevator. The arrows are approximately correct for when the elevator is accelerating upward—broken arrows represent forces too large to be drawn to scale. $\mathbf{T}$ is the tension in the supporting cable, $\mathbf{w}$ is the weight of the person, $\mathbf{w}_\mathrm{s}$ is the weight of the scale, $\mathbf{w}_\mathrm{e}$ is the weight of the elevator, $\mathbf{F}_\mathrm{s}$ is the force of the scale on the person, $\mathbf{F}_\mathrm{p}$ is the force of the person on the scale, $\mathbf{F}_\mathrm{t}$ is the force of the scale on the floor of the elevator, and $\mathbf{N}$ is the force of the floor upward on the scale. (b) The free-body diagram shows only the external forces acting on the designated system of interest—the person.

Strategy

If the scale is accurate, its reading will equal F_{p}, the magnitude of the force the person exerts downward on it. link(a) shows the numerous forces acting on the elevator, scale, and person. It makes this one-dimensional problem look much more formidable than if the person is chosen to be the system of interest and a free-body diagram is drawn as in link(b). Analysis of the free-body diagram using Newton's laws can produce answers to both parts (a) and (b) of this example, as well as some other questions that might arise. The only forces acting on the person are his weight **w** and the upward force of the scale $\mathbf{F}_{\mathrm{s}}$. According to Newton's third law $\mathbf{F}_{\mathrm{p}}$ and $\mathbf{F}_{\mathrm{s}}$ are equal in magnitude and opposite in direction, so that we need to find F_{s} in order to find what the scale reads. We can do this, as usual, by applying Newton's second law,

$$F_{\mathrm{net}} = \mathrm{ma}.$$

From the free-body diagram we see that $F_{\mathrm{net}} = F_{\mathrm{s}} - w$, so that

$$F_{\mathrm{s}} - w = \mathrm{ma}.$$

Solving for F_{s} gives an equation with only one unknown:

$$F_{\mathrm{s}} = \mathrm{ma} + w,$$

or, because $w = mg$, simply

$$F_{\mathrm{s}} = \mathrm{ma} + \mathrm{mg}.$$

No assumptions were made about the acceleration, and so this solution should be valid for a variety of accelerations in addition to the ones in this exercise.

Solution for (a)

In this part of the problem, $a = 1.20\ \mathrm{m/s}^2$, so that

$$F_{\mathrm{s}} = (75.0\ \mathrm{kg})(1.20\ \mathrm{m/s}^2) + (75.0\ \mathrm{kg})(9.80\ \mathrm{m/s}^2),$$

yielding

$$F_{\mathrm{s}} = 825\ \mathrm{N}.$$

Discussion for (a)

This is about 185 lb. What would the scale have read if he were stationary? Since his acceleration would be zero, the force of the scale would be equal to his weight:

$$\begin{aligned}
F_{\text{net}} &= \text{ma} = 0 = F_{\text{s}} - w \\
F_{\text{s}} &= w = \text{mg} \\
F_{\text{s}} &= (75.0\text{ kg})(9.80\text{ m/s}^2) \\
F_{\text{s}} &= 735\text{ N.}
\end{aligned}$$

So, the scale reading in the elevator is greater than his 735-N (165 lb) weight. This means that the scale is pushing up on the person with a force greater than his weight, as it must in order to accelerate him upward. Clearly, the greater the acceleration of the elevator, the greater the scale reading, consistent with what you feel in rapidly accelerating versus slowly accelerating elevators.

Solution for (b)

Now, what happens when the elevator reaches a constant upward velocity? Will the scale still read more than his weight? For any constant velocity—up, down, or stationary—acceleration is zero because $a = \frac{\Delta v}{\Delta t}$, and $\Delta v = 0$.

Thus,

$$F_{\text{s}} = \text{ma} + \text{mg} = 0 + \text{mg}.$$

Now

$$F_{\text{s}} = (75.0\text{ kg})(9.80\text{ m/s}^2),$$

which gives

$$F_{\text{s}} = 735\text{ N.}$$

Discussion for (b)

The scale reading is 735 N, which equals the person's weight. This will be the case whenever the elevator has a constant velocity—moving up, moving down, or stationary.

The solution to the previous example also applies to an elevator accelerating downward, as mentioned. When an elevator accelerates downward, a is negative, and the scale reading is *less* than the weight of the person, until a constant downward velocity is reached, at which time the scale reading again becomes equal to the person's weight. If the elevator is in free-fall and accelerating downward at g, then the scale reading will be zero and the person will *appear* to be weightless.

4.8.2 Integrating Concepts: Newton's Laws of Motion and Kinematics

Physics is most interesting and most powerful when applied to general situations that involve more than a narrow set of physical principles. Newton's laws of motion can also be integrated with other concepts that have been discussed previously in this text to solve problems of motion. For example, forces produce accelerations, a topic of kinematics, and hence the relevance of earlier chapters. When approaching problems that involve various types of forces, acceleration, velocity, and/or position, use the following steps to approach the problem:

Problem-Solving Strategy

Step 1. *Identify which physical principles are involved.* Listing the givens and the quantities to be calculated will allow you to identify the principles involved.
Step 2. *Solve the problem using strategies outlined in the text.* If these are available for the specific topic, you should refer to them. You should also refer to the sections of the text that deal with a particular topic. The following worked example illustrates how these strategies are applied to an integrated concept problem.

What Force Must a Soccer Player Exert to Reach Top Speed?

A soccer player starts from rest and accelerates forward, reaching a velocity of 8.00 m/s in 2.50 s. (a) What was his average acceleration? (b) What average force did he exert backward on the ground to achieve this acceleration? The player's mass is 70.0 kg, and air resistance is negligible.

Strategy

1. To solve an *integrated concept problem*, we must first identify the physical principles involved and identify the chapters in which they are found. Part (a) of this example considers *acceleration* along a straight line. This is a topic of *kinematics*. Part (b) deals with *force*, a topic of *dynamics* found in this chapter.
2. The following solutions to each part of the example illustrate how the specific problem-solving strategies are applied. These involve identifying knowns and unknowns, checking to see if the answer is reasonable, and so forth.

Solution for (a)

We are given the initial and final velocities (zero and 8.00 m/s forward); thus, the change in velocity is $\Delta v = 8.00\ m/s$. We are given the elapsed time, and so $\Delta t = 2.50\ s$. The unknown is acceleration, which can be found from its definition:

$$a = \frac{\Delta v}{\Delta t}.$$

Substituting the known values yields

$$\begin{aligned} a &= \frac{8.00\ m/s}{2.50\ s} \\ &= 3.20\ \text{m/s}^2. \end{aligned}$$

Discussion for (a)

This is an attainable acceleration for an athlete in good condition.

Solution for (b)

Here we are asked to find the average force the player exerts backward to achieve this forward acceleration. Neglecting air resistance, this would be equal in magnitude to the net external force on the player, since this force causes his acceleration. Since we now know the player's acceleration and are given his mass, we can use Newton's second law to find the force exerted. That is,

$$F_{\text{net}} = \text{ma}.$$

Substituting the known values of m and a gives

$$\begin{aligned} F_{\text{net}} &= (70.0\ \text{kg})(3.20\ \text{m/s}^2) \\ &= 224\ \text{N}. \end{aligned}$$

Discussion for (b)

This is about 50 pounds, a reasonable average force.

This worked example illustrates how to apply problem-solving strategies to situations that include topics from different chapters. The first step is to identify the physical principles involved in the problem. The second step is to solve for the unknown using familiar problem-solving strategies. These strategies are found throughout the text, and many worked examples show how to use them for single topics. You will find these techniques for integrated concept problems useful in applications of physics outside of a physics course, such as in your profession, in other science disciplines, and in everyday life. The following problems will build your skills in the broad application of physical principles.

4.8.3 Test Prep for AP Courses

A basketball player jumps as he shoots the ball. Describe the forces that are acting on the ball and on the basketball player. What are the results?

Two people push on a boulder to try to move it. The mass of the boulder is 825 kg. One person pushes north with a force of 64 N. The other pushes west with a force of 38 N. Predict the magnitude of the acceleration of the boulder. Assume that friction is negligible.

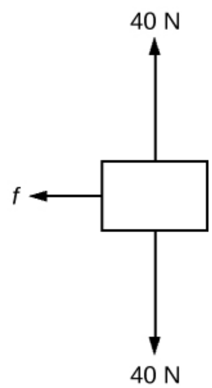
A free-body diagram would show a northward force of 64 N and a westward force of 38 N. The net force is equal to the sum of the two applied forces. It can be found using the Pythagorean theorem:

$$\begin{aligned} F_{\text{net}} = & \sqrt{{F_x}^2 + {F_y}^w} \\ = & \sqrt{(38\,\text{N})^2 + (64\,\text{N})^2} \\ = & 74.4\,\text{N} \end{aligned}$$

Since $a = \frac{F}{m}$,

$$\begin{aligned} a & = \tfrac{74.4\,\text{N}}{825\,\text{kg}} \\ & = 0.09\,\text{m/s}^2 \end{aligned}$$

The boulder will accelerate at 0.09 m/s^2.



The figure shows the forces exerted on a block that is sliding on a horizontal surface: the gravitational force of 40 N, the 40 N normal force exerted by the surface, and a frictional force exerted to the left. The coefficient of friction between the block and the surface is 0.20. The acceleration of the block is most nearly

a. 1.0 m/s^2 to the right
b. 1.0 m/s^2 to the left
c. 2.0 m/s^2 to the right
d. 2.0 m/s^2 to the left

4.8.4 Summary

- Newton's laws of motion can be applied in numerous situations to solve problems of motion.
- Some problems will contain multiple force vectors acting in different directions on an object. Be sure to draw diagrams, resolve all force vectors into horizontal and vertical components, and draw a free-body diagram.

Always analyze the direction in which an object accelerates so that you can determine whether $F_{\text{net}} = \text{ma}$ or $F_{\text{net}} = 0$.

- The normal force on an object is not always equal in magnitude to the weight of the object. If an object is accelerating, the normal force will be less than or greater than the weight of the object. Also, if the object is on an inclined plane, the normal force will always be less than the full weight of the object.
- Some problems will contain various physical quantities, such as forces, acceleration, velocity, or position. You can apply concepts from kinematics and dynamics in order to solve these problems of motion.

4.8.5 Conceptual Questions

To simulate the apparent weightlessness of space orbit, astronauts are trained in the hold of a cargo aircraft that is accelerating downward at g. Why will they appear to be weightless, as measured by standing on a bathroom scale, in this accelerated frame of reference? Is there any difference between their apparent weightlessness in orbit and in the aircraft?

A cartoon shows the toupee coming off the head of an elevator passenger when the elevator rapidly stops during an upward ride. Can this really happen without the person being tied to the floor of the elevator? Explain your answer.

4.8.6 Problem Exercises

A flea jumps by exerting a force of 1.20×10^{-5} N straight down on the ground. A breeze blowing on the flea parallel to the ground exerts a force of 0.500×10^{-6} N on the flea. Find the direction and magnitude of the acceleration of the flea if its mass is 6.00×10^{-7} kg. Do not neglect the gravitational force.

$10.2\ \text{m/s}^2$, 4.67º from vertical

Two muscles in the back of the leg pull upward on the Achilles tendon, as shown in link. (These muscles are called the medial and lateral heads of the gastrocnemius muscle.) Find the magnitude and direction of the total force on the Achilles tendon. What type of movement could be caused by this force?

A 76.0-kg person is being pulled away from a burning building as shown in link. Calculate the tension in the two ropes if the person is momentarily motionless. Include a free-body diagram in your solution.

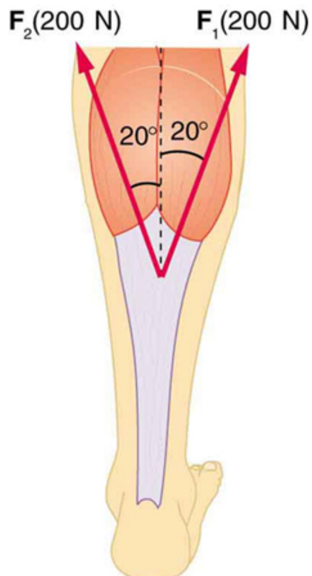
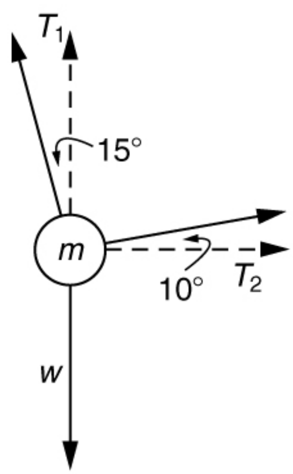


Figure 4.29: Achilles tendon



$T_1 = 736$ N

$T_2 = 194$ N

Integrated Concepts A 35.0-kg dolphin decelerates from 12.0 to 7.50 m/s in 2.30 s to join another dolphin in play. What average force was exerted to slow him if he was moving horizontally? (The gravitational force is balanced by the buoyant force of the water.)

Integrated Concepts When starting a foot race, a 70.0-kg sprinter exerts an

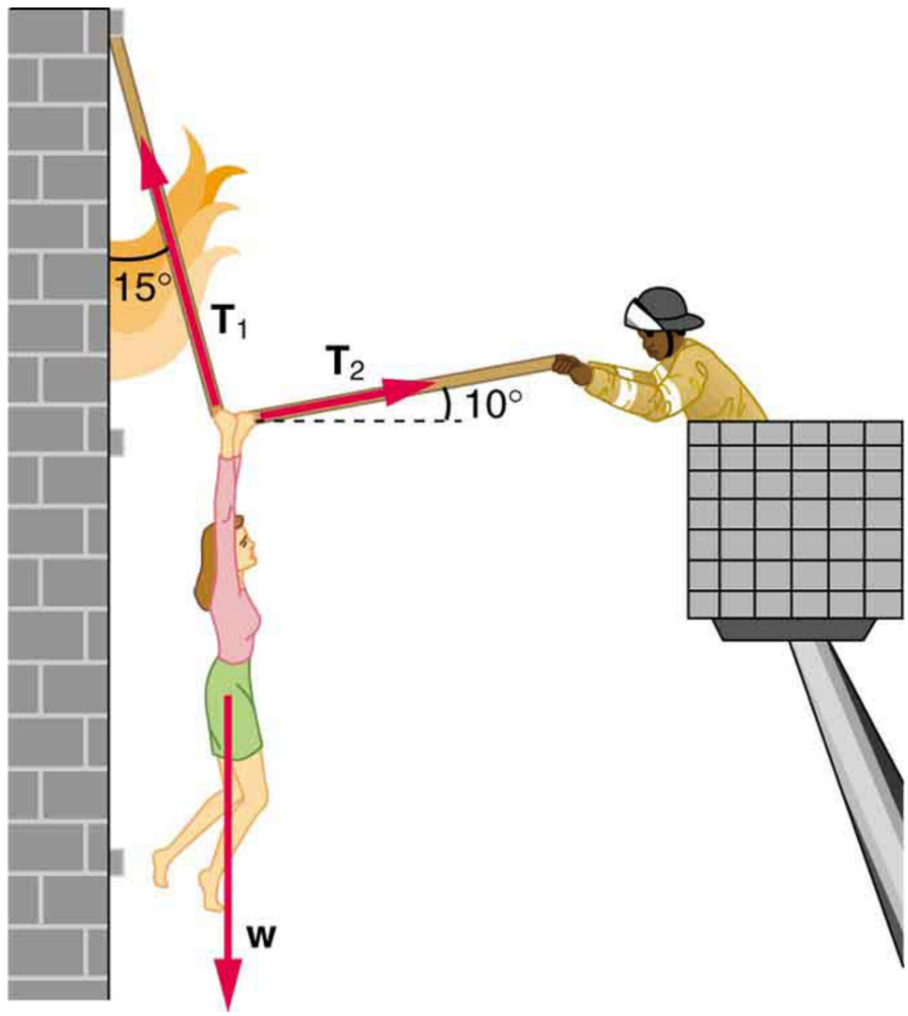


Figure 4.30: The force $\mathbf{T}_2$ needed to hold steady the person being rescued from the fire is less than her weight and less than the force $\mathbf{T}_1$ in the other rope, since the more vertical rope supports a greater part of her weight (a vertical force).

average force of 650 N backward on the ground for 0.800 s. (a) What is his final speed? (b) How far does he travel?

a 7.43 m/s

b 2.97 m

Integrated Concepts A large rocket has a mass of 2.00×10^6 kg at takeoff, and its engines produce a thrust of 3.50×10^7 N. (a) Find its initial acceleration if it takes off vertically. (b) How long does it take to reach a velocity of 120 km/h straight up, assuming constant mass and thrust? (c) In reality, the mass of a rocket decreases significantly as its fuel is consumed. Describe qualitatively how this affects the acceleration and time for this motion.

Integrated Concepts A basketball player jumps straight up for a ball. To do this, he lowers his body 0.300 m and then accelerates through this distance by forcefully straightening his legs. This player leaves the floor with a vertical velocity sufficient to carry him 0.900 m above the floor. (a) Calculate his velocity when he leaves the floor. (b) Calculate his acceleration while he is straightening his legs. He goes from zero to the velocity found in part (a) in a distance of 0.300 m. (c) Calculate the force he exerts on the floor to do this, given that his mass is 110 kg.

a 4.20 m/s

b 29.4 m/s^2

c 4.31×10^3 N

Integrated Concepts A 2.50-kg fireworks shell is fired straight up from a mortar and reaches a height of 110 m. (a) Neglecting air resistance (a poor assumption, but we will make it for this example), calculate the shell's velocity when it leaves the mortar. (b) The mortar itself is a tube 0.450 m long. Calculate the average acceleration of the shell in the tube as it goes from zero to the velocity found in (a). (c) What is the average force on the shell in the mortar? Express your answer in newtons and as a ratio to the weight of the shell.

Integrated Concepts Repeat link for a shell fired at an angle 10.0º from the vertical.

a 47.1 m/s

b 2.47×10^3 m/s^2

c 6.18×10^3 N . The average force is 252 times the shell's weight.

Integrated Concepts An elevator filled with passengers has a mass of 1700 kg. (a) The elevator accelerates upward from rest at a rate of 1.20 m/s^2 for 1.50 s. Calculate the tension in the cable supporting the elevator. (b) The elevator continues upward at constant velocity for 8.50 s. What is the tension in the cable during this time? (c) The elevator decelerates at a rate of 0.600 m/s^2 for

3.00 s. What is the tension in the cable during deceleration? (d) How high has the elevator moved above its original starting point, and what is its final velocity?

Unreasonable Results (a) What is the final velocity of a car originally traveling at 50.0 km/h that decelerates at a rate of 0.400 m/s^2 for 50.0 s? (b) What is unreasonable about the result? (c) Which premise is unreasonable, or which premises are inconsistent?

Unreasonable Results A 75.0-kg man stands on a bathroom scale in an elevator that accelerates from rest to 30.0 m/s in 2.00 s. (a) Calculate the scale reading in newtons and compare it with his weight. (The scale exerts an upward force on him equal to its reading.) (b) What is unreasonable about the result? (c) Which premise is unreasonable, or which premises are inconsistent?

4.9 Extended Topic: The Four Basic Forces—An Introduction

4.9.1 Learning Objectives

By the end of this section, you will be able to:

- Understand the four basic forces that underlie the processes in nature.

The information presented in this section supports the following AP® learning objectives and science practices:

- **3.C.4.1** The student is able to make claims about various contact forces between objects based on the microscopic cause of those forces. **(S.P. 6.1)**
- **3.C.4.2** The student is able to explain contact forces (tension, friction, normal, buoyant, spring) as arising from interatomic electric forces and that they therefore have certain directions. **(S.P. 6.2)**
- **3.G.1.1** The student is able to articulate situations when the gravitational force is the dominant force and when the electromagnetic, weak, and strong forces can be ignored. **(S.P. 7.1)**

One of the most remarkable simplifications in physics is that only four distinct forces account for all known phenomena. In fact, nearly all of the forces we experience directly are due to only one basic force, called the electromagnetic force. (The gravitational force is the only force we experience directly that is not electromagnetic.) This is a tremendous simplification of the myriad of *apparently* different forces we can list, only a few of which were discussed in the previous section. As we will see, the basic forces are all thought to act through the exchange of microscopic carrier particles, and the characteristics of the basic forces are determined by the types of particles exchanged. Action at a distance, such as the gravitational force of Earth on the Moon, is explained by the existence of a force field rather than by "physical contact."

The *four basic forces* are the gravitational force, the electromagnetic force, the weak nuclear force, and the strong nuclear force. Their properties are summarized in link. Since the weak and strong nuclear forces act over an extremely short range, the size of a nucleus or less, we do not experience them directly, although they are crucial to the very structure of matter. These forces determine which nuclei are stable and which decay, and they are the basis of the release of energy in certain nuclear reactions. Nuclear forces determine not only the stability of nuclei, but also the relative abundance of elements in nature. The properties of the nucleus of an atom determine the number of electrons it has and, thus, indirectly determine the chemistry of the atom. More will be said of all of these topics in later chapters.

Concept Connections: The Four Basic Forces

The four basic forces will be encountered in more detail as you progress through the text. The gravitational force is defined in Uniform Circular Motion and Gravitation, electric force in Electric Charge and Electric Field, magnetic force in Magnetism, and nuclear forces in Radioactivity and Nuclear Physics. On a macroscopic scale, electromagnetism and gravity are the basis for all forces. The nuclear forces are vital to the substructure of matter, but they are not directly experienced on the macroscopic scale.

Properties of the Four Basic Forces

$$footnote$$

The graviton is a proposed particle, though it has not yet been observed by scientists. See the discussion of gravitational waves later in this section. The particles W^+, W^-, and Z^0 are called vector bosons; these were predicted by theory and first observed in 1983. There are eight types of gluons proposed by scientists, and their existence is indicated by meson exchange in the nuclei of atoms.

Force

Approximate Relative Strengths

Range

Attraction/Repulsion

Carrier Particle

Gravitational

$$10^{-38}$$

$$\infty$$

attractive only

Graviton

Electromagnetic

10^{-2}

∞

attractive and repulsive

Photon

Weak nuclear

10^{-13}

$< 10^{-18}$m

attractive and repulsive

W^{+}, W^{-}, Z^{0}

Strong nuclear

1

$< 10^{-15}$m

attractive and repulsive

gluons

The gravitational force is surprisingly weak—it is only because gravity is always attractive that we notice it at all. Our weight is the gravitational force due to the *entire* Earth acting on us. On the very large scale, as in astronomical systems, the gravitational force is the dominant force determining the motions of moons, planets, stars, and galaxies. The gravitational force also affects the nature of space and time. As we shall see later in the study of general relativity, space is curved in the vicinity of very massive bodies, such as the Sun, and time actually slows down near massive bodies.

Take a good look at the ranges for the four fundamental forces listed in link. The range of the strong nuclear force, 10^{-15} m, is approximately the size of the nucleus of an atom; the weak nuclear force has an even shorter range. At scales on the order of 10−10 m, approximately the size of an atom, both nuclear forces are completely dominated by the electromagnetic force. Notice that this scale is still utterly tiny compared to our everyday experience. At scales that we do experience daily, electromagnetism tends to be negligible, due to its attractive and repulsive properties canceling each other out. That leaves gravity, which is usually the strongest of the forces at scales above ~10^{-4} m, and hence includes our everyday activities, such as throwing, climbing stairs, and walking.

Electromagnetic forces can be either attractive or repulsive. They are long-range forces, which act over extremely large distances, and they nearly cancel for macroscopic objects. (Remember that it is the *net* external force that is important.) If they did not cancel, electromagnetic forces would completely overwhelm the gravitational force. The electromagnetic force is a combination of electrical forces (such as those that cause static electricity) and magnetic forces (such as those that affect a compass needle). These two forces were thought to be quite distinct until early in the 19th century, when scientists began to discover that they are different manifestations of the same force. This discovery is a classical case of the *unification of forces*. Similarly, friction, tension, and all of the other classes of forces we experience directly (except gravity, of course) are due to electromagnetic interactions of atoms and molecules. It is still convenient to consider these forces separately in specific applications, however, because of the ways they manifest themselves.

Concept Connections: Unifying Forces

Attempts to unify the four basic forces are discussed in relation to elementary particles later in this text. By "unify" we mean finding connections between the forces that show that they are different manifestations of a single force. Even if such unification is achieved, the forces will retain their separate characteristics on the macroscopic scale and may be identical only under extreme conditions such as those existing in the early universe.

Physicists are now exploring whether the four basic forces are in some way related. Attempts to unify all forces into one come under the rubric of Grand Unified Theories (GUTs), with which there has been some success in recent years. It is now known that under conditions of extremely high density and temperature, such as existed in the early universe, the electromagnetic and weak nuclear forces are indistinguishable. They can now be considered to be different manifestations of one force, called the *electroweak* force. So the list of four has been reduced in a sense to only three. Further progress in unifying all forces is proving difficult—especially the inclusion of the gravitational force, which has the special characteristics of affecting the space and time in which the other forces exist.

While the unification of forces will not affect how we discuss forces in this text, it is fascinating that such underlying simplicity exists in the face of the overt complexity of the universe. There is no reason that nature must be simple—it simply is.

4.9.2 Action at a Distance: Concept of a Field

All forces act at a distance. This is obvious for the gravitational force. Earth and the Moon, for example, interact without coming into contact. It is also true for all other forces. Friction, for example, is an electromagnetic force between atoms that may not actually touch. What is it that carries forces between objects? One way to answer this question is to imagine that a force field surrounds whatever object creates the force. A second object (often called a *test object*) placed in this field will experience a force that is a function of location and other variables. The field itself is the "thing" that carries the force from one object to another. The field is defined so as to be a characteristic of the object creating it; the field does not depend on the test object placed in it. Earth's gravitational field, for example, is a function of the mass of Earth and the distance from its center, independent of the presence of other masses. The concept of a field is useful because equations can be written for force fields surrounding objects (for gravity, this yields $w = \mathrm{mg}$ at Earth's surface), and motions can be calculated from these equations. (See link.)

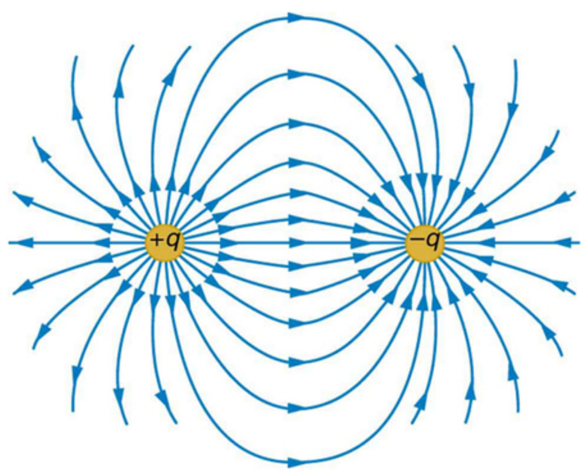


Figure 4.31: The electric force field between a positively charged particle and a negatively charged particle. When a positive test charge is placed in the field, the charge will experience a force in the direction of the force field lines.

Concept Connections: Force Fields

The concept of a **force field** is also used in connection with electric charge and is presented in Electric Charge and Electric Field. It is also a useful idea for all the basic forces, as will be seen in Particle Physics. Fields help us to visualize forces and how they are transmitted, as well as to describe them with precision and to link forces with subatomic carrier particles.

Making Connections: Vector and Scalar Fields

These fields may be either scalar or vector fields. Gravity and electromagnetism are examples of vector fields. A test object placed in such a field will have both the magnitude and direction of the resulting force on the test object completely defined by the object's location in the field. We will later cover examples of scalar fields, which have a magnitude but no direction.

The field concept has been applied very successfully; we can calculate motions and describe nature to high precision using field equations. As useful as the field concept is, however, it leaves unanswered the question of what carries the force. It has been proposed in recent decades, starting in 1935 with Hideki Yukawa's (1907–1981) work on the strong nuclear force, that all forces are transmitted by the exchange of elementary particles. We can visualize particle exchange as analogous to macroscopic phenomena such as two people passing a basketball back and forth, thereby exerting a repulsive force without touching one another. (See link.)

This idea of particle exchange deepens rather than contradicts field concepts. It is more satisfying philosophically to think of something physical actually moving between objects acting at a distance. link lists the exchange or carrier particles, both observed and proposed, that carry the four forces. But the real fruit of the particle-exchange proposal is that searches for Yukawa's proposed particle found it *and* a number of others that were completely unexpected, stimulating yet more research. All of this research eventually led to the proposal of quarks as the underlying substructure of matter, which is a basic tenet of GUTs. If successful, these theories will explain not only forces, but also the structure of matter itself. Yet physics is an experimental science, so the test of these theories must lie in the domain of the real world. As of this writing, scientists at the CERN laboratory in Switzerland are starting to test these theories using the world's largest particle accelerator: the Large Hadron Collider. This accelerator (27 km in circumference) allows two high-energy proton beams, traveling in opposite directions, to collide. An energy of 14 trillion electron volts will be available. It is anticipated that some new particles, possibly force carrier particles, will be found. (See link.) One of the force carriers of high interest that researchers hope to detect is the Higgs boson. The observation of its properties might tell us why different particles have different masses.

Tiny particles also have wave-like behavior, something we will explore more in a later chapter. To better understand force-carrier particles from another perspective, let us consider gravity. The search for gravitational waves has been going on for a number of years. Almost 100 years ago, Einstein predicted the existence of these waves as part of his general theory of relativity. Gravitational waves are created during the collision of massive stars, in black holes, or in supernova explosions—like shock waves. These gravitational waves will travel through space from such sites much like a pebble dropped into a pond sends out ripples—except these waves move at the speed of light. A detector apparatus has been built in the U.S., consisting of two large installations nearly 3000 km apart—one in Washington state and one in Louisiana! The facility is

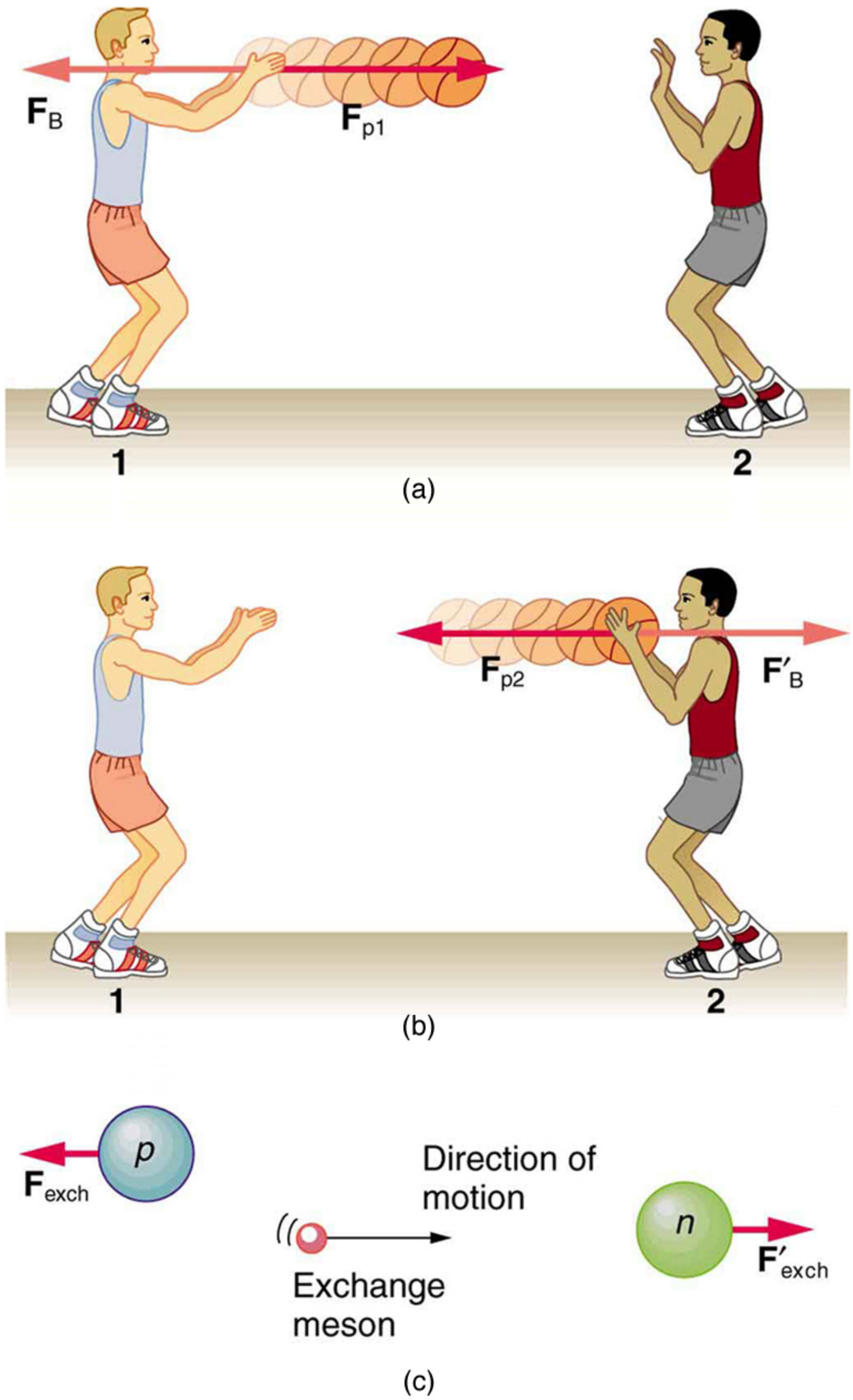


Figure 4.32: The exchange of masses resulting in repulsive forces. (a) The person throwing the basketball exerts a force $\mathbf{F}_{\mathrm{p1}}$ on it toward the other person and feels a reaction force $\mathbf{F}_{\mathrm{B}}$ away from the second person. (b) The person catching the basketball exerts a force $\mathbf{F}_{\mathrm{p2}}$ on it to stop the ball and feels a reaction force $\mathbf{F'}_{\mathrm{B}}$ away from the first person. (c) The analogous exchange of a meson between a proton and a neutron carries the strong nuclear forces $\mathbf{F}_{\mathrm{exch}}$ and $\mathbf{F'}_{\mathrm{exch}}$ between them. An attractive force can also be exerted by the exchange of a mass—if person 2 pulled the basketball away from the first person as he tried to retain it, then the force between them would be attractive.

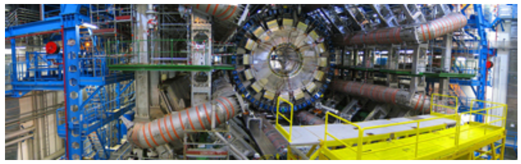


Figure 4.33: The world's largest particle accelerator spans the border between Switzerland and France. Two beams, traveling in opposite directions close to the speed of light, collide in a tube similar to the central tube shown here. External magnets determine the beam's path. Special detectors will analyze particles created in these collisions. Questions as broad as what is the origin of mass and what was matter like the first few seconds of our universe will be explored. This accelerator began preliminary operation in 2008. (credit: Frank Hommes)

called the Laser Interferometer Gravitational-Wave Observatory (LIGO). Each installation is designed to use optical lasers to examine any slight shift in the relative positions of two masses due to the effect of gravity waves. The two sites allow simultaneous measurements of these small effects to be separated from other natural phenomena, such as earthquakes. Initial operation of the detectors began in 2002, and work is proceeding on increasing their sensitivity. Similar installations have been built in Italy (VIRGO), Germany (GEO600), and Japan (TAMA300) to provide a worldwide network of gravitational wave detectors.

International collaboration in this area is moving into space with the joint EU/US project LISA (Laser Interferometer Space Antenna). Earthquakes and other Earthly noises will be no problem for these monitoring spacecraft. LISA will complement LIGO by looking at much more massive black holes through the observation of gravitational-wave sources emitting much larger wavelengths. Three satellites will be placed in space above Earth in an equilateral triangle (with 5,000,000-km sides) (link). The system will measure the relative positions of each satellite to detect passing gravitational waves. Accuracy to within 10% of the size of an atom will be needed to detect any waves. The launch of this project might be as early as 2018.

"I'm sure LIGO will tell us something about the universe that we didn't know before. The history of science tells us that any time you go where you haven't been before, you usually find something that really shakes the scientific paradigms of the day. Whether gravitational wave astrophysics will do that, only time will tell." —David Reitze, LIGO Input Optics Manager, University of Florida

The ideas presented in this section are but a glimpse into topics of modern physics that will be covered in much greater depth in later chapters.

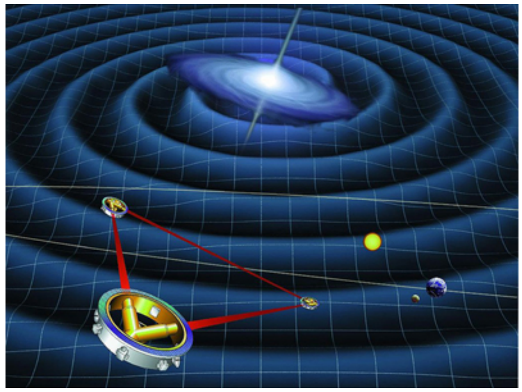


Figure 4.34: Space-based future experiments for the measurement of gravitational waves. Shown here is a drawing of LISA's orbit. Each satellite of LISA will consist of a laser source and a mass. The lasers will transmit a signal to measure the distance between each satellite's test mass. The relative motion of these masses will provide information about passing gravitational waves. (credit: NASA)

4.9.3 Test Prep for AP Courses

Which phenomenon correctly describes the direction and magnitude of normal forces?

a. electromagnetic attraction
b. electromagnetic repulsion
c. gravitational attraction
d. gravitational repulsion

b

Explain which of the four fundamental forces is responsible for a ball bouncing off the ground after it hits, and why this force has this effect.

Which of the basic forces best explains tension in a rope being pulled between two people? Is the acting force causing attraction or repulsion in this instance?

a. gravity; attraction
b. electromagnetic; attraction
c. weak and strong nuclear; attraction
d. weak and strong nuclear; repulsion

b

Explain how interatomic electric forces produce the normal force, and why it has the direction it does.

The gravitational force is the weakest of the four basic forces. In which case can the electromagnetic, strong, and weak forces be ignored because the gravitational force is so strongly dominant?

a. a person jumping on a trampoline
b. a rocket blasting off from Earth
c. a log rolling down a hill
d. all of the above

d

Describe a situation in which gravitational force is the dominant force. Why can the other three basic forces be ignored in the situation you described?

4.9.4 Summary

- The various types of forces that are categorized for use in many applications are all manifestations of the *four basic forces* in nature.
- The properties of these forces are summarized in link.
- Everything we experience directly without sensitive instruments is due to either electromagnetic forces or gravitational forces. The nuclear forces are responsible for the submicroscopic structure of matter, but they are not directly sensed because of their short ranges. Attempts are being made to show all four forces are different manifestations of a single unified force.
- A force field surrounds an object creating a force and is the carrier of that force.

4.9.5 Conceptual Questions

Explain, in terms of the properties of the four basic forces, why people notice the gravitational force acting on their bodies if it is such a comparatively weak force.

What is the dominant force between astronomical objects? Why are the other three basic forces less significant over these very large distances?

Give a detailed example of how the exchange of a particle can result in an **attractive** force. (For example, consider one child pulling a toy out of the hands of another.)

4.9.6 Problem Exercises

a What is the strength of the weak nuclear force relative to the strong nuclear force? (b) What is the strength of the weak nuclear force relative to the electromagnetic force? Since the weak nuclear force acts at only very short distances, such as inside nuclei, where the strong and electromagnetic forces also act, it might seem surprising that we have any knowledge of it at all. We have such knowledge because the weak nuclear force is responsible for beta decay, a type of nuclear decay not explained by other forces.

a 1×10^{-13}

b 1×10^{-11}

a What is the ratio of the strength of the gravitational force to that of the strong nuclear force? (b) What is the ratio of the strength of the gravitational force to that of the weak nuclear force? (c) What is the ratio of the strength of the gravitational force to that of the electromagnetic force? What do your answers imply about the influence of the gravitational force on atomic nuclei?

What is the ratio of the strength of the strong nuclear force to that of the electromagnetic force? Based on this ratio, you might expect that the strong force dominates the nucleus, which is true for small nuclei. Large nuclei, however, have sizes greater than the range of the strong nuclear force. At these sizes, the electromagnetic force begins to affect nuclear stability. These facts will be used to explain nuclear fusion and fission later in this text.

10^2

4.9.7 Glossary

carrier particle a fundamental particle of nature that is surrounded by a characteristic force field; photons are carrier particles of the electromagnetic force

force field a region in which a test particle will experience a force

Chapter 5

Further Applications of Newton's Laws: Friction, Drag, and Elasticity

5.1 Connection for AP® Courses

class="introduction" class="section-summary" title="Section Summary" class="conceptual-questions" title="Conceptual Questions" class="problems-exercises" title="Problems & Exercises" class="ap-test-prep" title="Test Prep for AP Courses"

Have you ever wondered why it is difficult to walk on a smooth surface like ice? The interaction between you and the surface is a result of forces that affect your motion. In the previous chapter, you learned Newton's laws of motion and examined how net force affects the motion, position and shape of an object. Now we will look at some interesting and common forces that will provide further applications of Newton's laws of motion.

The information presented in this chapter supports learning objectives covered under Big Idea 3 of the AP Physics Curriculum Framework, which refer to the nature of forces and their roles in interactions among objects. The chapter discusses examples of specific contact forces, such as friction, air or liquid drag, and elasticity that may affect the motion or shape of an object. It also discusses the nature of forces on both macroscopic and microscopic levels (Enduring Understanding 3.C and Essential Knowledge 3.C.4). In addition, Newton's laws are applied to describe the motion of an object (Enduring Understanding 3.B) and to examine relationships between contact forces and other forces exerted on an object (Enduring Understanding 3.A, 3.A.3 and Essential Knowledge 3.A.4). The examples in this chapter give you practice in using vector properties of

Figure 5.1: Total hip replacement surgery has become a common procedure. The head (or ball) of the patient's femur fits into a cup that has a hard plastic-like inner lining. (credit: National Institutes of Health, via Wikimedia Commons)

forces (Essential Knowledge 3.A.2) and free-body diagrams (Essential Knowledge 3.B.2) to determine net force (Essential Knowledge 3.B.1).

Big Idea 3 The interactions of an object with other objects can be described by forces.

Enduring Understanding 3.A All forces share certain common characteristics when considered by observers in inertial reference frames.

Essential Knowledge 3.A.2 Forces are described by vectors.

Essential Knowledge 3.A.3 A force exerted on an object is always due to the interaction of that object with another object.

Essential Knowledge 3.A.4 If one object exerts a force on a second object, the second object always exerts a force of equal magnitude on the first object in the opposite direction.

Enduring Understanding 3.B Classically, the acceleration of an object interacting with other objects can be predicted by using $\vec{a} = \frac{\sum \vec{F}}{m}$.

Essential Knowledge 3.B.1 If an object of interest interacts with several other objects, the net force is the vector sum of the individual forces.

Essential Knowledge 3.B.2 Free-body diagrams are useful tools for visualizing forces being exerted on a single object and writing the equations that represent a physical situation.

Enduring Understanding 3.C At the macroscopic level, forces can be categorized as either long-range (action-at-a-distance) forces or contact forces.

Essential Knowledge 3.C.4 Contact forces result from the interaction of one object touching another object, and they arise from interatomic electric forces. These forces include tension, friction, normal, spring (Physics 1), and buoyant (Physics 2).

5.2 Friction

5.2.1 Learning Objectives

By the end of this section, you will be able to:

- Discuss the general characteristics of friction.
- Describe the various types of friction.
- Calculate the magnitudes of static and kinetic frictional forces.

The information presented in this section supports the following AP® learning objectives and science practices:

- **3.C.4.1** The student is able to make claims about various contact forces between objects based on the microscopic cause of those forces. **(S.P.**

6.1)

- **3.C.4.2** The student is able to explain contact forces (tension, friction, normal, buoyant, spring) as arising from interatomic electric forces and that they therefore have certain directions. **(S.P. 6.2)**

Friction is a force that is around us all the time that opposes relative motion between systems in contact but also allows us to move (which you have discovered if you have ever tried to walk on ice). While a common force, the behavior of friction is actually very complicated and is still not completely understood. We have to rely heavily on observations for whatever understandings we can gain. However, we can still deal with its more elementary general characteristics and understand the circumstances in which it behaves.

Friction

Friction is a force that opposes relative motion between systems in contact.

One of the simpler characteristics of friction is that it is parallel to the contact surface between systems and always in a direction that opposes motion or attempted motion of the systems relative to each other. If two systems are in contact and moving relative to one another, then the friction between them is called kinetic friction. For example, friction slows a hockey puck sliding on ice. But when objects are stationary, static friction can act between them; the static friction is usually greater than the kinetic friction between the objects.

Kinetic Friction

If two systems are in contact and moving relative to one another, then the friction between them is called kinetic friction.

Imagine, for example, trying to slide a heavy crate across a concrete floor—you may push harder and harder on the crate and not move it at all. This means that the static friction responds to what you do—it increases to be equal to and in the opposite direction of your push. But if you finally push hard enough, the crate seems to slip suddenly and starts to move. Once in motion it is easier to keep it in motion than it was to get it started, indicating that the kinetic friction force is less than the static friction force. If you add mass to the crate, say by placing a box on top of it, you need to push even harder to get it started and also to keep it moving. Furthermore, if you oiled the concrete you would find it to be easier to get the crate started and keep it going (as you might expect).

link is a crude pictorial representation of how friction occurs at the interface between two objects. Close-up inspection of these surfaces shows them to be rough. So when you push to get an object moving (in this case, a crate), you must raise the object until it can skip along with just the tips of the surface hitting, break off the points, or do both. A considerable force can be resisted by friction with no apparent motion. The harder the surfaces are pushed together (such as if another box is placed on the crate), the more force is needed to move them. Part of the friction is due to adhesive forces between the surface molecules of the two objects, which explain the dependence of friction on the nature of the substances. Adhesion varies with substances in contact and is a complicated aspect of surface physics. Once an object is moving, there are fewer points of contact (fewer molecules adhering), so less force is required to keep the object moving. At small but nonzero speeds, friction is nearly independent of speed.

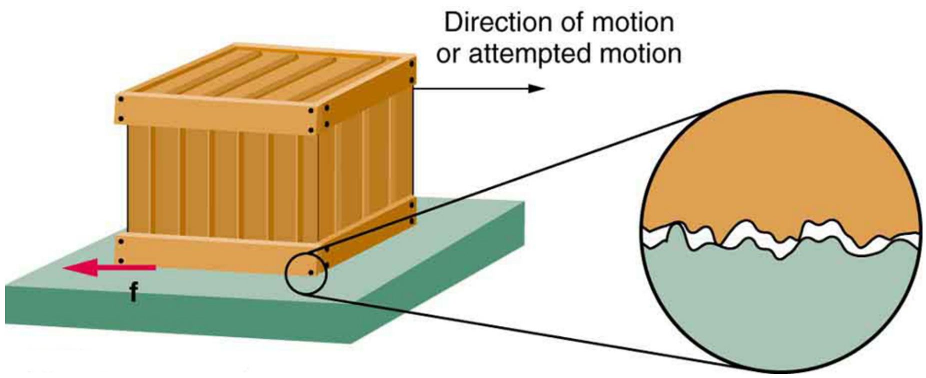


Figure 5.2: Frictional forces, such as f, always oppose motion or attempted motion between objects in contact. Friction arises in part because of the roughness of the surfaces in contact, as seen in the expanded view. In order for the object to move, it must rise to where the peaks can skip along the bottom surface. Thus a force is required just to set the object in motion. Some of the peaks will be broken off, also requiring a force to maintain motion. Much of the friction is actually due to attractive forces between molecules making up the two objects, so that even perfectly smooth surfaces are not friction-free. Such adhesive forces also depend on the substances the surfaces are made of, explaining, for example, why rubber-soled shoes slip less than those with leather soles.

The magnitude of the frictional force has two forms: one for static situations (static friction), the other for when there is motion (kinetic friction).

When there is no motion between the objects, the magnitude of static friction $\mathbf{f}_\mathrm{s}$ is

$$f_s \leq \mu_s N,$$

where μ_s is the coefficient of static friction and N is the magnitude of the normal force (the force perpendicular to the surface).

Magnitude of Static Friction

Magnitude of static friction f_s is

$$f_\mathrm{s} \leq \mu_\mathrm{s} N,$$

where μ_s is the coefficient of static friction and N is the magnitude of the normal force.

The symbol $\leq$ means *less than or equal to*, implying that static friction can have a minimum and a maximum value of $\mu_s N$. Static friction is a responsive force that increases to be equal and opposite to whatever force is exerted, up to its maximum limit. Once the applied force exceeds $f_{s(max)}$, the object will move. Thus

$$f_{\mathrm{s(max)}} = \mu_{\mathrm{s}} N.$$

Once an object is moving, the magnitude of kinetic friction $\mathbf{f}_\mathrm{k}$ is given by

$$f_\mathrm{k} = \mu_\mathrm{k} N,$$

where μ_k is the coefficient of kinetic friction. A system in which $f_k = \mu_k N$ is described as a system in which *friction behaves simply.*

Magnitude of Kinetic Friction

The magnitude of kinetic friction f_k is given by

$$f_\mathrm{k} = \mu_\mathrm{k} N,$$

where μ_k is the coefficient of kinetic friction.

As seen in link, the coefficients of kinetic friction are less than their static counterparts. That values of μ in link are stated to only one or, at most, two digits is an indication of the approximate description of friction given by the above two equations.

Table 5.1: Coefficients of Static and Kinetic Friction

System	Static frictionμ_s	Kinetic frictionμ_k
Rubber on dry concrete	1.0	0.7
Rubber on wet concrete	0.7	0.5
Wood on wood	0.5	0.3
Waxed wood on wet snow	0.14	0.1
Metal on wood	0.5	0.3
Steel on steel (dry)	0.6	0.3
Steel on steel (oiled)	0.05	0.03
Teflon on steel	0.04	0.04
Bone lubricated by synovial fluid	0.016	0.015
Shoes on wood	0.9	0.7
Shoes on ice	0.1	0.05
Ice on ice	0.1	0.03

System	Static frictionμ_s	Kinetic frictionμ_k
Steel on ice	0.04	0.02

The equations given earlier include the dependence of friction on materials and the normal force. The direction of friction is always opposite that of motion, parallel to the surface between objects, and perpendicular to the normal force. For example, if the crate you try to push (with a force parallel to the floor) has a mass of 100 kg, then the normal force would be equal to its weight, $W = mg = (100\text{ kg})(9.80\text{ m/s}^2) = 980\text{ N}$, perpendicular to the floor. If the coefficient of static friction is 0.45, you would have to exert a force parallel to the floor greater than $f_{s(max)} = \mu_s N = (0.45)\,(980\text{ N}) = 440\text{ N}$ to move the crate. Once there is motion, friction is less and the coefficient of kinetic friction might be 0.30, so that a force of only 290 N ($f_k = \mu_k N = (0.30)(980\text{ N}) = 290\text{ N}$) would keep it moving at a constant speed. If the floor is lubricated, both coefficients are considerably less than they would be without lubrication. Coefficient of friction is a unit less quantity with a magnitude usually between 0 and 1.0. The coefficient of the friction depends on the two surfaces that are in contact.

Take-Home Experiment

Find a small plastic object (such as a food container) and slide it on a kitchen table by giving it a gentle tap. Now spray water on the table, simulating a light shower of rain. What happens now when you give the object the same-sized tap? Now add a few drops of (vegetable or olive) oil on the surface of the water and give the same tap. What happens now? This latter situation is particularly important for drivers to note, especially after a light rain shower. Why?

Many people have experienced the slipperiness of walking on ice. However, many parts of the body, especially the joints, have much smaller coefficients of friction—often three or four times less than ice. A joint is formed by the ends of two bones, which are connected by thick tissues. The knee joint is formed by the lower leg bone (the tibia) and the thighbone (the femur). The hip is a ball (at the end of the femur) and socket (part of the pelvis) joint. The ends of the bones in the joint are covered by cartilage, which provides a smooth, almost glassy surface. The joints also produce a fluid (synovial fluid) that reduces friction and wear. A damaged or arthritic joint can be replaced by an artificial joint (link). These replacements can be made of metals (stainless steel or titanium) or plastic (polyethylene), also with very small coefficients of friction.

Other natural lubricants include saliva produced in our mouths to aid in the swallowing process, and the slippery mucus found between organs in the body, allowing them to move freely past each other during heartbeats, during breathing, and when a person moves. Artificial lubricants are also common in hospitals and doctor's clinics. For example, when ultrasonic imaging is carried out, the gel that couples the transducer to the skin also serves to lubricate the surface

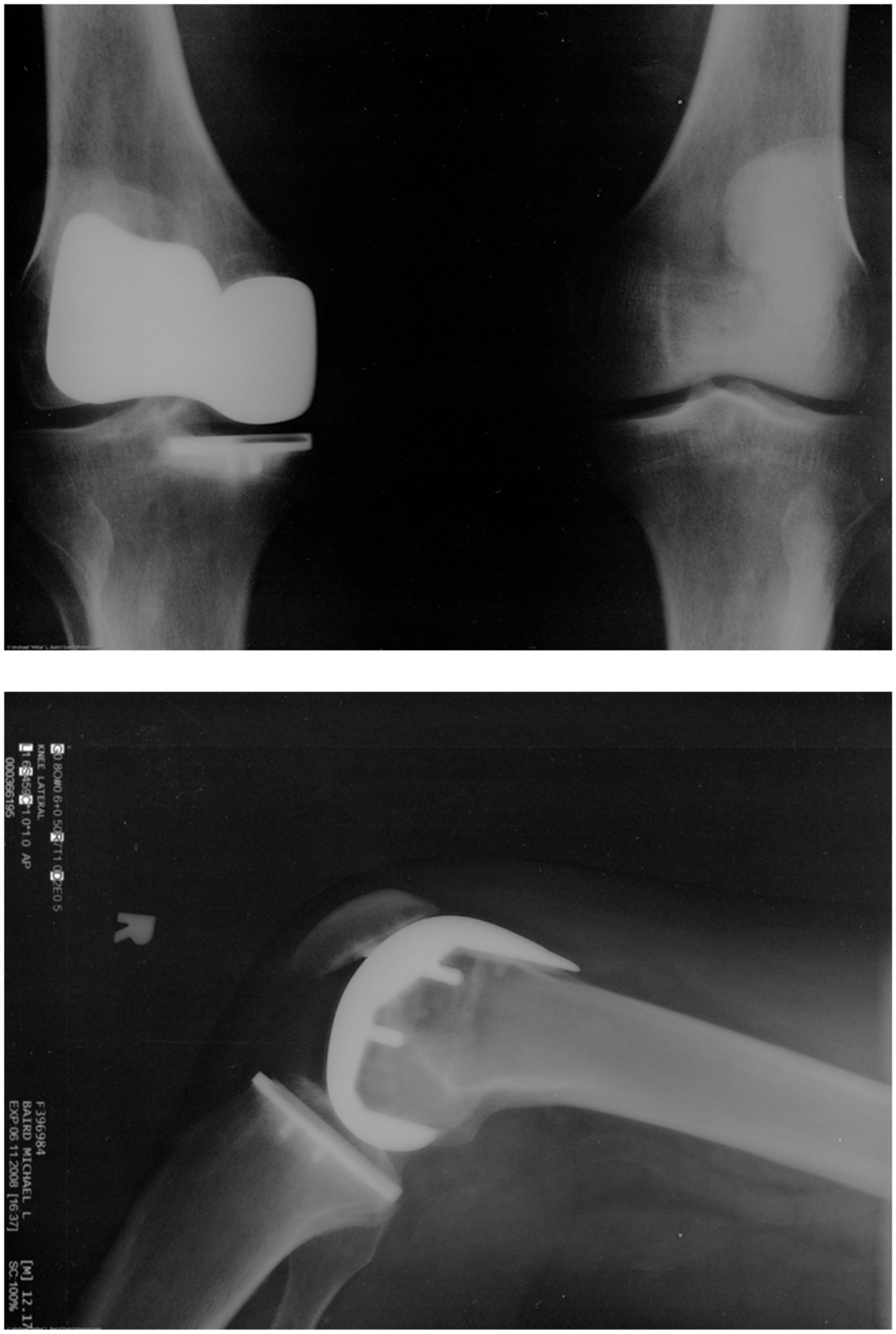


Figure 5.3: Artificial knee replacement is a procedure that has been performed for more than 20 years. In this figure, we see the post-op x rays of the right knee joint replacement. (credit: Mike Baird, Flickr)

between the transducer and the skin—thereby reducing the coefficient of friction between the two surfaces. This allows the transducer to mover freely over the skin.

Skiing Exercise

A skier with a mass of 62 kg is sliding down a snowy slope. Find the coefficient of kinetic friction for the skier if friction is known to be 45.0 N.

Strategy

The magnitude of kinetic friction was given in to be 45.0 N. Kinetic friction is related to the normal force N as $f_{\mathrm{k}} = \mu_{\mathrm{k}} N$; thus, the coefficient of kinetic friction can be found if we can find the normal force of the skier on a slope. The normal force is always perpendicular to the surface, and since there is no motion perpendicular to the surface, the normal force should equal the component of the skier's weight perpendicular to the slope. (See the skier and free-body diagram in link.)

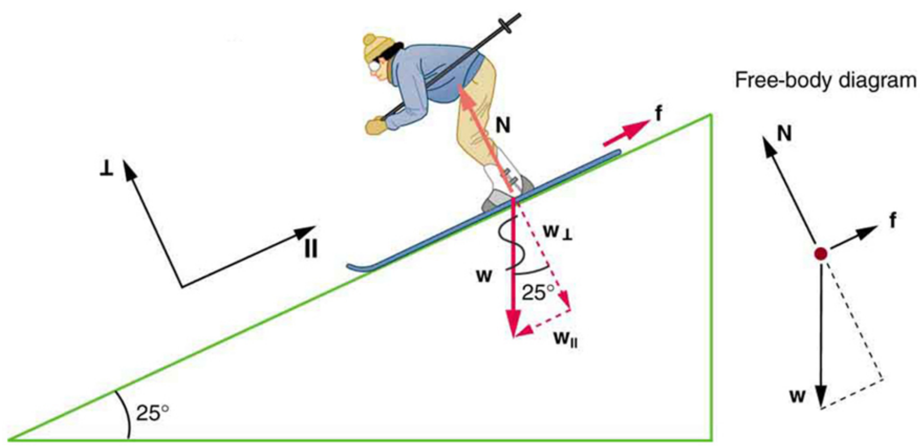


Figure 5.4: The motion of the skier and friction are parallel to the slope and so it is most convenient to project all forces onto a coordinate system where one axis is parallel to the slope and the other is perpendicular (axes shown to left of skier). **N** (the normal force) is perpendicular to the slope, and **f** (the friction) is parallel to the slope, but **w** (the skier's weight) has components along both axes, namely **w** and $\mathbf{W}_{//}$. N is equal in magnitude to w w , so there is no motion perpendicular to the slope. However, f is less than $W_{//}$ in magnitude, so there is acceleration down the slope (along the x-axis).

That is,

$$N = w_{\perp} = w \cos 25^{\mathrm{o}} = \mathrm{mg} \cos 25^{\mathrm{o}}.$$

Substituting this into our expression for kinetic friction, we get

$$f_k = \mu_k \text{mg} \cos 25^o,$$

which can now be solved for the coefficient of kinetic friction μ_k.

Solution

Solving for μ_k gives

$$\mu_k = \frac{f_k}{N} = \frac{f_k}{w \cos 25^o} = \frac{f_k}{\text{mg} \cos 25^o}.$$

Substituting known values on the right-hand side of the equation,

$$\mu_k = \frac{45.0 \text{ N}}{(62 \text{ kg})(9.80 \text{ m/s}^2)(0.906)} = 0.082.$$

Discussion

This result is a little smaller than the coefficient listed in link for waxed wood on snow, but it is still reasonable since values of the coefficients of friction can vary greatly. In situations like this, where an object of mass m slides down a slope that makes an angle θ with the horizontal, friction is given by $f_k = \mu_k \text{mg} \cos \theta$. All objects will slide down a slope with constant acceleration under these circumstances. Proof of this is left for this chapter's Problems and Exercises.

Take-Home Experiment

An object will slide down an inclined plane at a constant velocity if the net force on the object is zero. We can use this fact to measure the coefficient of kinetic friction between two objects. As shown in link, the kinetic friction on a slope $f_k = \mu_k \text{mg} \cos \theta$. The component of the weight down the slope is equal to mg $\sin \theta$ (see the free-body diagram in link). These forces act in opposite directions, so when they have equal magnitude, the acceleration is zero. Writing these out:

$$f_k = \text{Fg}_x$$

$$\mu_k \text{mg} \cos \theta = \text{mg} \sin \theta.$$

Solving for μ_k, we find that

$$\mu_k = \frac{\text{mg} \sin \theta}{\text{mg} \cos \theta} = \tan \theta.$$

Put a coin on a book and tilt it until the coin slides at a constant velocity down the book. You might need to tap the book lightly to get the coin to move. Measure the angle of tilt relative to the horizontal and find μ_{k}. Note that the coin will not start to slide at all until an angle greater than θ is attained, since the coefficient of static friction is larger than the coefficient of kinetic friction. Discuss how this may affect the value for μ_{k} and its uncertainty.

We have discussed that when an object rests on a horizontal surface, there is a normal force supporting it equal in magnitude to its weight. Furthermore, simple friction is always proportional to the normal force.

Making Connections: Submicroscopic Explanations of Friction

The simpler aspects of friction dealt with so far are its macroscopic (large-scale) characteristics. Great strides have been made in the atomic-scale explanation of friction during the past several decades. Researchers are finding that the atomic nature of friction seems to have several fundamental characteristics. These characteristics not only explain some of the simpler aspects of friction—they also hold the potential for the development of nearly friction-free environments that could save hundreds of billions of dollars in energy which is currently being converted (unnecessarily) to heat.

link illustrates one macroscopic characteristic of friction that is explained by microscopic (small-scale) research. We have noted that friction is proportional to the normal force, but not to the area in contact, a somewhat counterintuitive notion. When two rough surfaces are in contact, the actual contact area is a tiny fraction of the total area since only high spots touch. When a greater normal force is exerted, the actual contact area increases, and it is found that the friction is proportional to this area.

But the atomic-scale view promises to explain far more than the simpler features of friction. The mechanism for how heat is generated is now being determined. In other words, why do surfaces get warmer when rubbed? Essentially, atoms are linked with one another to form lattices. When surfaces rub, the surface atoms adhere and cause atomic lattices to vibrate—essentially creating sound waves that penetrate the material. The sound waves diminish with distance and their energy is converted into heat. Chemical reactions that are related to frictional wear can also occur between atoms and molecules on the surfaces. link shows how the tip of a probe drawn across another material is deformed by atomic-scale friction. The force needed to drag the tip can be measured and is found to be related to shear stress, which will be discussed later in this chapter. The variation in shear stress is remarkable (more than a factor of 10^{12}) and difficult to predict theoretically, but shear stress is yielding a fundamental understanding of a large-scale phenomenon known since ancient times—friction.

Forces and Motion

Explore the forces at work when you try to push a filing cabinet. Create an applied force and see the resulting friction force and total force acting on the

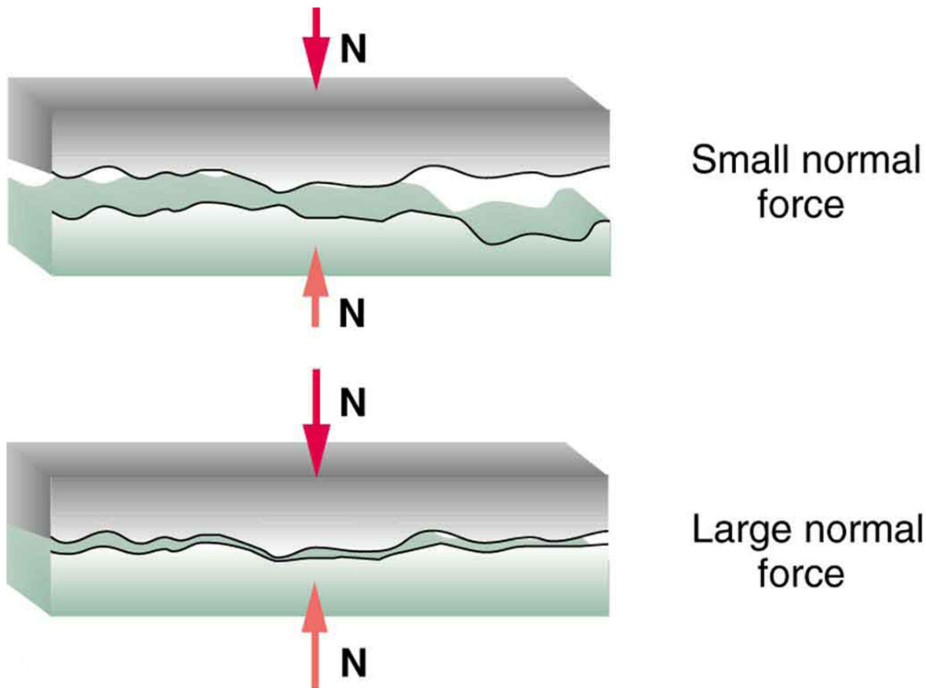


Figure 5.5: Two rough surfaces in contact have a much smaller area of actual contact than their total area. When there is a greater normal force as a result of a greater applied force, the area of actual contact increases as does friction.

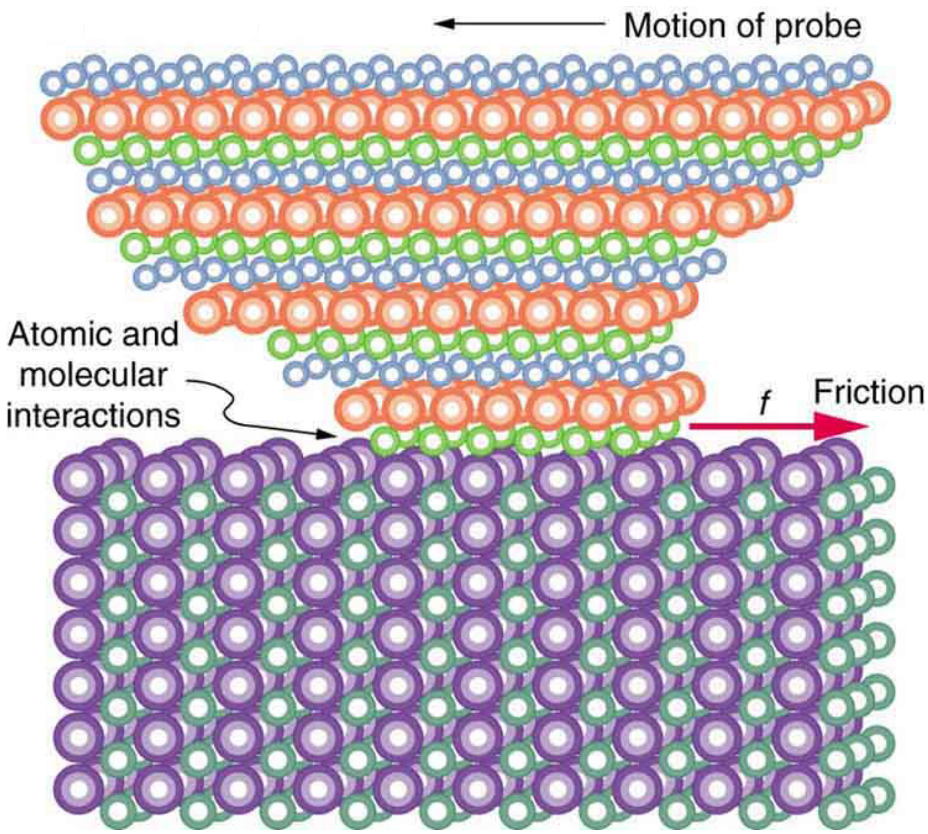


Figure 5.6: The tip of a probe is deformed sideways by frictional force as the probe is dragged across a surface. Measurements of how the force varies for different materials are yielding fundamental insights into the atomic nature of friction.

cabinet. Charts show the forces, position, velocity, and acceleration vs. time. Draw a free-body diagram of all the forces (including gravitational and normal forces).

5.2.2 Test Prep for AP Courses

When a force of 20 N is applied to a stationary box weighing 40 N, the box does not move. This means the coefficient of static friction

a. is equal to 0.5.
b. is greater than 0.5.
c. is less than 0.5.
d. cannot be determined.

b

A 2-kg block slides down a ramp which is at an incline of 25º. If the frictional force is 4.86 N, what is the coefficient of friction? At what incline will the box slide at a constant velocity? Assume g = 10 m/s^2.

A block is given a short push and then slides with constant friction across a horizontal floor. Which statement best explains the direction of the force that friction applies on the moving block?

a. Friction will be in the same direction as the block's motion because molecular interactions between the block and the floor will deform the block in the direction of motion.
b. Friction will be in the same direction as the block's motion because thermal energy generated at the interface between the block and the floor adds kinetic energy to the block.
c. Friction will be in the opposite direction of the block's motion because molecular interactions between the block and the floor will deform the block in the opposite direction of motion.
d. Friction will be in the opposite direction of the block's motion because thermal energy generated at the interface between the block and the floor converts some of the block's kinetic energy to potential energy.

c

A student pushes a cardboard box across a carpeted floor and afterwards notices that the bottom of the box feels warm. Explain how interactions between molecules in the cardboard and molecules in the carpet produced this heat.

5.2.3 Section Summary

- Friction is a contact force between systems that opposes the motion or attempted motion between them. Simple friction is proportional to the normal force N pushing the systems together. (A normal force is always perpendicular to the contact surface between systems.) Friction depends on both of the materials involved. The magnitude of static frictionf_s between systems stationary relative to one another is given by ::: {#eip-940 data-type="equation"}

$$f_\mathrm{s} \leq \mu_\mathrm{s} N,$$

:::

where μ_s is the coefficient of static friction, which depends on both of the materials.

- The kinetic friction force f_k between systems moving relative to one another is given by ::: {#eip-799 data-type="equation"}

  $$f_\mathrm{k} = \mu_\mathrm{k} N,$$

  :::

  where μ_k is the coefficient of kinetic friction, which also depends on both materials.

5.2.4 Conceptual Questions

Define normal force. What is its relationship to friction when friction behaves simply?

The glue on a piece of tape can exert forces. Can these forces be a type of simple friction? Explain, considering especially that tape can stick to vertical walls and even to ceilings.

When you learn to drive, you discover that you need to let up slightly on the brake pedal as you come to a stop or the car will stop with a jerk. Explain this in terms of the relationship between static and kinetic friction.

When you push a piece of chalk across a chalkboard, it sometimes screeches because it rapidly alternates between slipping and sticking to the board. Describe this process in more detail, in particular explaining how it is related to the fact that kinetic friction is less than static friction. (The same slip-grab process occurs when tires screech on pavement.)

5.2.5 Problems & Exercises

A physics major is cooking breakfast when he notices that the frictional force between his steel spatula and his Teflon frying pan is only 0.200 N. Knowing the coefficient of kinetic friction between the two materials, he quickly calculates the normal force. What is it?

5.00 *N*

a When rebuilding her car's engine, a physics major must exert 300 N of force to insert a dry steel piston into a steel cylinder. What is the magnitude of the normal force between the piston and cylinder? (b) What is the magnitude of the force would she have to exert if the steel parts were oiled?

a What is the maximum frictional force in the knee joint of a person who supports 66.0 kg of her mass on that knee? (b) During strenuous exercise it is possible to exert forces to the joints that are easily ten times greater than the weight being supported. What is the maximum force of friction under such conditions? The frictional forces in joints are relatively small in all circumstances except when the joints deteriorate, such as from injury or arthritis. Increased frictional forces can cause further damage and pain.

Suppose you have a 120-kg wooden crate resting on a wood floor. (a) What maximum force can you exert horizontally on the crate without moving it? (b) If you continue to exert this force once the crate starts to slip, what will the magnitude of its acceleration then be?

a 588 N

b 1.96 m/s^2

a If half of the weight of a small 1.00×10^3 kg utility truck is supported by its two drive wheels, what is the magnitude of the maximum acceleration it can achieve on dry concrete? (b) Will a metal cabinet lying on the wooden bed of the truck slip if it accelerates at this rate? (c) Solve both problems assuming the truck has four-wheel drive.

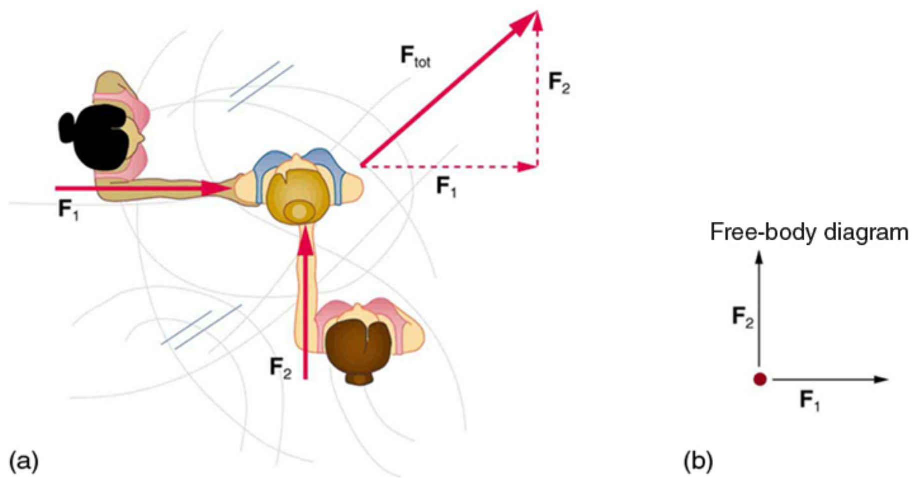
A team of eight dogs pulls a sled with waxed wood runners on wet snow (mush!). The dogs have average masses of 19.0 kg, and the loaded sled with its rider has a mass of 210 kg. (a) Calculate the magnitude of the acceleration starting from rest if each dog exerts an average force of 185 N backward on the snow. (b) What is the magnitude of the acceleration once the sled starts to move? (c) For both situations, calculate the magnitude of the force in the coupling between the dogs and the sled.

a 3.29 m/s^2

b 3.52 m/s^2

c 980 N; 945 N

Consider the 65.0-kg ice skater being pushed by two others shown in link. (a) Find the direction and magnitude of $\mathbf{F}_{\text{tot}}$, the total force exerted on her by the others, given that the magnitudes F_1 and F_2 are 26.4 N and 18.6 N, respectively. (b) What is her initial acceleration if she is initially stationary and wearing steel-bladed skates that point in the direction of $\mathbf{F}_{\text{tot}}$? (c) What is her acceleration assuming she is already moving in the direction of $\mathbf{F}_{\text{tot}}$? (Remember that friction always acts in the direction opposite that of motion or attempted motion between surfaces in contact.)



Show that the acceleration of any object down a frictionless incline that makes an angle θ with the horizontal is $a = g\ \sin\ \theta$. (Note that this acceleration is independent of mass.)

Show that the acceleration of any object down an incline where friction behaves simply (that is, where $f_{\mathrm{k}} = \mu_{\mathrm{k}} N$) is $a = g(\ \sin\ \theta - \mu_{\mathrm{k}}\cos\ \theta\,)$. Note that the acceleration is independent of mass and reduces to the expression found in the previous problem when friction becomes negligibly small ($\mu_{\mathrm{k}} = 0$).

Calculate the deceleration of a snow boarder going up a 5.0, slope assuming the coefficient of friction for waxed wood on wet snow. The result of link may be useful, but be careful to consider the fact that the snow boarder is going uphill. Explicitly show how you follow the steps in Problem-Solving Strategies.

$1.83\ \mathrm{m/s}^2$

a Calculate the acceleration of a skier heading down a 10.0 slope, assuming the coefficient of friction for waxed wood on wet snow. (b) Find the angle of the slope down which this skier could coast at a constant velocity. You can neglect air resistance in both parts, and you will find the result of link to be useful. Explicitly show how you follow the steps in the Problem-Solving Strategies.

If an object is to rest on an incline without slipping, then friction must equal the component of the weight of the object parallel to the incline. This requires greater and greater friction for steeper slopes. Show that the maximum angle of an incline above the horizontal for which an object will not slide down is $\theta = \tan^{-1}\ _{\mathrm{s}}$. You may use the result of the previous problem. Assume that $a = 0$ and that static friction has reached its maximum value.

Calculate the maximum deceleration of a car that is heading down a 6 slope (one that makes an angle of 6 with the horizontal) under the following road conditions. You may assume that the weight of the car is evenly distributed on all four tires and that the coefficient of static friction is involved—that is, the tires are not allowed to slip during the deceleration. (Ignore rolling.) Calculate for a car: (a) On dry concrete. (b) On wet concrete. (c) On ice, assuming that $\mu_s = 0.100$, the same as for shoes on ice.

Calculate the maximum acceleration of a car that is heading up a 4º slope (one that makes an angle of 4º with the horizontal) under the following road conditions. Assume that only half the weight of the car is supported by the two drive wheels and that the coefficient of static friction is involved—that is, the tires are not allowed to slip during the acceleration. (Ignore rolling.) (a) On dry concrete. (b) On wet concrete. (c) On ice, assuming that $_s =$ 0.100, the same as for shoes on ice.

a 4.20 m/s^2

b 2.74 m/s^2

c –0.195 m/s^2

Repeat link for a car with four-wheel drive.

A freight train consists of two 8.00×10^5-kg engines and 45 cars with average masses of 5.50×10^5 kg. (a) What force must each engine exert backward on the track to accelerate the train at a rate of 5.00×10^{-2} m/s^2 if the force of friction is 7.50×10^5 N, assuming the engines exert identical forces? This is not a large frictional force for such a massive system. Rolling friction for trains is small, and consequently trains are very energy-efficient transportation systems. (b) What is the magnitude of the force in the coupling between the 37th and 38th cars (this is the force each exerts on the other), assuming all cars have the same mass and that friction is evenly distributed among all of the cars and engines?

a 1.03×10^6 N

b 3.48×10^5 N

Consider the 52.0-kg mountain climber in link. (a) Find the tension in the rope and the force that the mountain climber must exert with her feet on the vertical rock face to remain stationary. Assume that the force is exerted parallel to her legs. Also, assume negligible force exerted by her arms. (b) What is the minimum coefficient of friction between her shoes and the cliff?

A contestant in a winter sporting event pushes a 45.0-kg block of ice across a frozen lake as shown in link(a). (a) Calculate the minimum force F he must exert to get the block moving. (b) What is the magnitude of its acceleration once it starts to move, if that force is maintained?

a 51.0 N

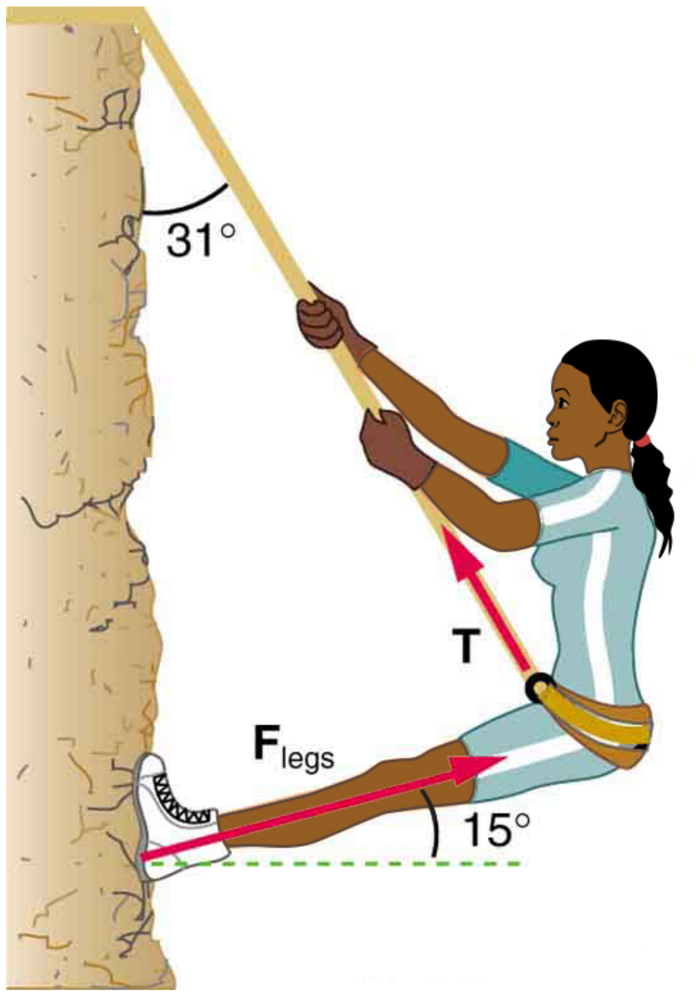


Figure 5.7: Part of the climber's weight is supported by her rope and part by friction between her feet and the rock face.

b 0.720 m/s^2

Repeat link with the contestant pulling the block of ice with a rope over his shoulder at the same angle above the horizontal as shown in link(b).

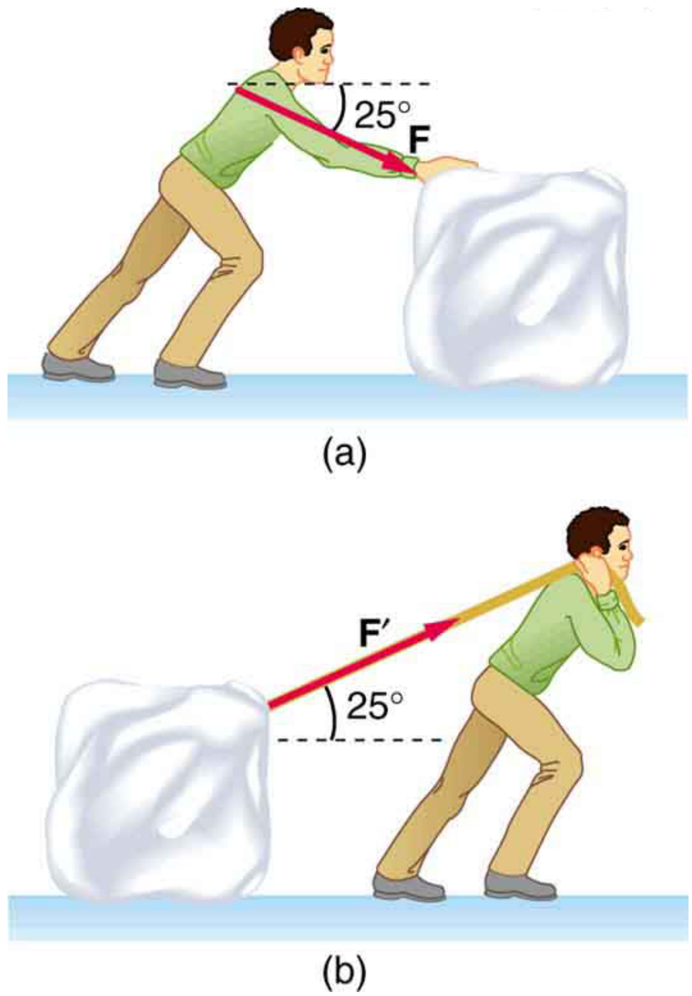


Figure 5.8: Which method of sliding a block of ice requires less force—(a) pushing or (b) pulling at the same angle above the horizontal?

5.2.6 Glossary

friction a force that opposes relative motion or attempts at motion between systems in contact

kinetic friction a force that opposes the motion of two systems that are in contact and moving relative to one another

static friction a force that opposes the motion of two systems that are in contact and are not moving relative to one another

magnitude of static friction $f_s \leq \mu_s\ N$, where μ_s is the coefficient of static friction and N is the magnitude of the normal force

magnitude of kinetic friction $f_k = \mu_k N$, where μ_k is the coefficient of kinetic friction

5.3 Drag Forces

5.3.1 Learning Objectives

By the end of this section, you will be able to:

- Define drag force and model it mathematically.
- Discuss the applications of drag force.
- Define terminal velocity.
- Perform calculations to find terminal velocity.

Another interesting force in everyday life is the force of drag on an object when it is moving in a fluid (either a gas or a liquid). You feel the drag force when you move your hand through water. You might also feel it if you move your hand during a strong wind. The faster you move your hand, the harder it is to move. You feel a smaller drag force when you tilt your hand so only the side goes through the air—you have decreased the area of your hand that faces the direction of motion. Like friction, the drag force always opposes the motion of an object. Unlike simple friction, the drag force is proportional to some function of the velocity of the object in that fluid. This functionality is complicated and depends upon the shape of the object, its size, its velocity, and the fluid it is in. For most large objects such as bicyclists, cars, and baseballs not moving too slowly, the magnitude of the drag force F_{D} is found to be proportional to the square of the speed of the object. We can write this relationship mathematically as $F_D \propto v^2$. When taking into account other factors, this relationship becomes

$$F_{\mathrm{D}} = \frac{1}{2} C \rho \mathrm{Av}^2,$$

where C is the drag coefficient, A is the area of the object facing the fluid, and ρ is the density of the fluid. (Recall that density is mass per unit volume.) This equation can also be written in a more generalized fashion as $F_{\mathrm{D}} = \mathrm{bv}^n$, where b is a constant equivalent to $0.5 C \rho A$. We have set the exponent for these equations as 2 because, when an object is moving at high velocity through air, the magnitude of the drag force is proportional to the square of the speed. As we shall see in a few pages on fluid dynamics, for small particles moving at low speeds in a fluid, the exponent is equal to 1.

Drag Force

Drag force F_{D} is found to be proportional to the square of the speed of the object. Mathematically

$$F_{\mathrm{D}} \propto v^2$$

$$F_{\mathrm{D}} = \frac{1}{2} C \rho \mathrm{Av}^2,$$

where C is the drag coefficient, A is the area of the object facing the fluid, and ρ is the density of the fluid.

Athletes as well as car designers seek to reduce the drag force to lower their race times. (See link). "Aerodynamic" shaping of an automobile can reduce the drag force and so increase a car's gas mileage.

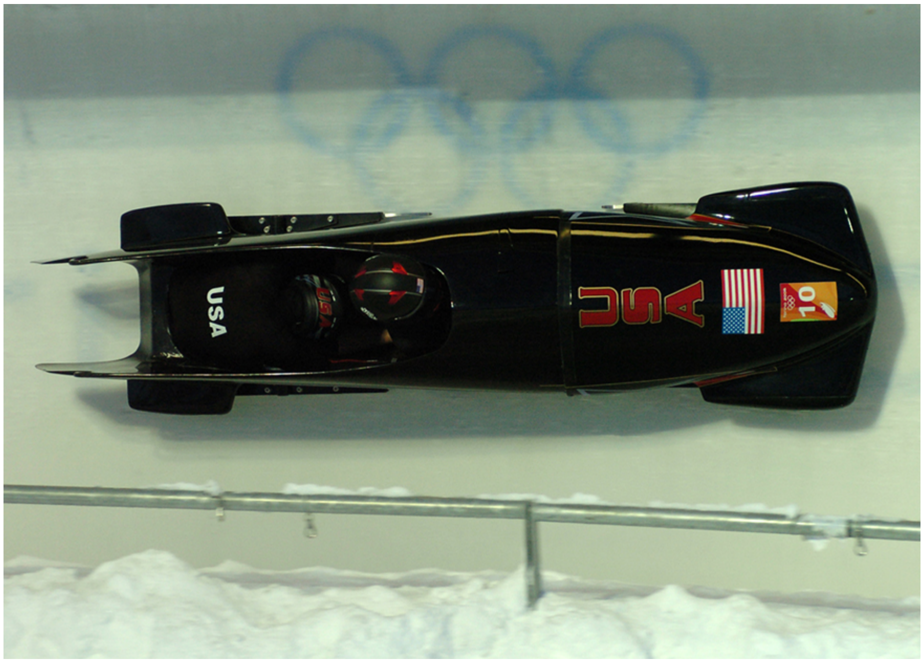


Figure 5.9: From racing cars to bobsled racers, aerodynamic shaping is crucial to achieving top speeds. Bobsleds are designed for speed. They are shaped like a bullet with tapered fins. (credit: U.S. Army, via Wikimedia Commons)

The value of the drag coefficient, C , is determined empirically, usually with the use of a wind tunnel. (See link).

The drag coefficient can depend upon velocity, but we will assume that it is a constant here. link lists some typical drag coefficients for a variety of objects.

Figure 5.10: NASA researchers test a model plane in a wind tunnel. (credit: NASA/Ames)

Notice that the drag coefficient is a dimensionless quantity. At highway speeds, over 50% of the power of a car is used to overcome air drag. The most fuel-efficient cruising speed is about 70–80 km/h (about 45–50 mi/h). For this reason, during the 1970s oil crisis in the United States, maximum speeds on highways were set at about 90 km/h (55 mi/h).

Table 5.2: Typical values of Drag Coefficient C

Object	C
Airfoil	0.05
Toyota Camry	0.28
Ford Focus	0.32
Honda Civic	0.36
Ferrari Testarossa	0.37
Dodge Ram pickup	0.43
Sphere	0.45
Hummer H2 SUV	0.64
Skydiver (feet first)	0.70
Bicycle	0.90
Skydiver (horizontal)	1.0
Circular flat plate	1.12

Substantial research is under way in the sporting world to minimize drag. The dimples on golf balls are being redesigned as are the clothes that athletes wear. Bicycle racers and some swimmers and runners wear full bodysuits. Australian Cathy Freeman wore a full body suit in the 2000 Sydney Olympics, and won the gold medal for the 400 m race. Many swimmers in the 2008 Beijing Olympics wore (Speedo) body suits; it might have made a difference in breaking many world records (See link). Most elite swimmers (and cyclists) shave their body hair. Such innovations can have the effect of slicing away milliseconds in a race, sometimes making the difference between a gold and a silver medal. One consequence is that careful and precise guidelines must be continuously developed to maintain the integrity of the sport.

Some interesting situations connected to Newton's second law occur when considering the effects of drag forces upon a moving object. For instance, consider a skydiver falling through air under the influence of gravity. The two forces acting on him are the force of gravity and the drag force (ignoring the buoyant force). The downward force of gravity remains constant regardless of the velocity at which the person is moving. However, as the person's velocity increases, the magnitude of the drag force increases until the magnitude of the drag force is equal to the gravitational force, thus producing a net force of zero. A zero net force means that there is no acceleration, as given by Newton's second law. At this point, the person's velocity remains constant and we say that the person has reached his *terminal velocity* (v_t). Since F_{D} is proportional to the speed, a

Figure 5.11: Body suits, such as this LZR Racer Suit, have been credited with many world records after their release in 2008. Smoother “skin” and more compression forces on a swimmer’s body provide at least 10% less drag. (credit: NASA/Kathy Barnstorff)

heavier skydiver must go faster for F_D to equal his weight. Let's see how this works out more quantitatively.

At the terminal velocity,

$$F_{net} = mg - F_D = ma = 0.$$

Thus,

$$mg = F_D.$$

Using the equation for drag force, we have

$$mg = \frac{1}{2}\rho CAv^2.$$

Solving for the velocity, we obtain

$$v = \sqrt{\frac{2mg}{\rho CA}}.$$

Assume the density of air is $\rho = 1.21\ \mathrm{kg/m^3}$. A 75-kg skydiver descending head first will have an area approximately $A = 0.18\ \mathrm{m^2}$ and a drag coefficient of approximately $C = 0.70$. We find that

$$\begin{aligned} v &= \sqrt{\frac{2(75\ \mathrm{kg})(9.80\ \mathrm{m/s^2})}{(1.21\ \mathrm{kg/m^3})(0.70)(0.18\ \mathrm{m^2})}} \\ &= 98\ \mathrm{m/s} \\ &= 350\ \mathrm{km/h}. \end{aligned}$$

This means a skydiver with a mass of 75 kg achieves a maximum terminal velocity of about 350 km/h while traveling in a pike (head first) position, minimizing the area and his drag. In a spread-eagle position, that terminal velocity may decrease to about 200 km/h as the area increases. This terminal velocity becomes much smaller after the parachute opens.

Take-Home Experiment

This interesting activity examines the effect of weight upon terminal velocity. Gather together some nested coffee filters. Leaving them in their original shape, measure the time it takes for one, two, three, four, and five nested filters to fall to the floor from the same height (roughly 2 m). (Note that, due to the way the filters are nested, drag is constant and only mass varies.) They obtain terminal velocity quite quickly, so find this velocity as a function of mass. Plot the terminal velocity v versus mass. Also plot v^2 versus mass. Which of these relationships is more linear? What can you conclude from these graphs?

A Terminal Velocity

Find the terminal velocity of an 85-kg skydiver falling in a spread-eagle position.

Strategy

At terminal velocity, $F_{\text{net}} = 0$. Thus the drag force on the skydiver must equal the force of gravity (the person's weight). Using the equation of drag force, we find $\text{mg} =\frac{1}{2}\ \text{CAv}^2$.

Thus the terminal velocity v_t can be written as

$$v_{\text{t}} = \sqrt{\frac{\text{2mg}}{\text{CA}}}.$$

Solution

All quantities are known except the person's projected area. This is an adult (85 kg) falling spread eagle. We can estimate the frontal area as

$$A = (2\ m)(0.35\ \text{m}) = 0.70\ \text{m}^2.$$

Using our equation for v_{t}, we find that

$$\begin{aligned} v_{\text{t}} &= \sqrt{\frac{2(85\ \text{kg})(9.80\ \text{m/s}^2)}{(1.21\ \text{kg/m}^3)(1.0)(0.70\ \text{m}^2)}} \\ &= 44\ \text{m/s}. \end{aligned}$$

Discussion

This result is consistent with the value for v_{t} mentioned earlier. The 75-kg skydiver going feet first had a $v = 98$ m/s. He weighed less but had a smaller frontal area and so a smaller drag due to the air.

The size of the object that is falling through air presents another interesting application of air drag. If you fall from a 5-m high branch of a tree, you will likely get hurt—possibly fracturing a bone. However, a small squirrel does this all the time, without getting hurt. You don't reach a terminal velocity in such a short distance, but the squirrel does.

The following interesting quote on animal size and terminal velocity is from a 1928 essay by a British biologist, J.B.S. Haldane, titled "On Being the Right Size."

To the mouse and any smaller animal,

gravity

presents practically no dangers. You can drop a mouse down a thousand-yard mine shaft; and, on arriving at the bottom, it gets a slight shock and walks away, provided that the ground is fairly soft. A rat is killed, a man is broken, and a horse splashes. For the resistance presented to movement by the air is proportional to the surface of the moving object. Divide an animal's length, breadth, and height each by ten; its weight is reduced to a thousandth, but its surface only to a hundredth. So the resistance to falling in the case of the small animal is relatively ten times greater than the driving force.

The above quadratic dependence of air drag upon velocity does not hold if the object is very small, is going very slow, or is in a denser medium than air. Then we find that the drag force is proportional just to the velocity. This relationship is given by Stokes' law, which states that

$$F_{\mathrm{s}} = 6\pi r\eta v,$$

where r is the radius of the object, η is the viscosity of the fluid, and v is the object's velocity.

Stokes' Law

$$F_{\mathrm{s}} = 6\pi r\eta v,$$

where r is the radius of the object, η is the viscosity of the fluid, and v is the object's velocity.

Good examples of this law are provided by microorganisms, pollen, and dust particles. Because each of these objects is so small, we find that many of these objects travel unaided only at a constant (terminal) velocity. Terminal velocities for bacteria (size about 1 m) can be about 2 m/s. To move at a greater speed, many bacteria swim using flagella (organelles shaped like little tails) that are powered by little motors embedded in the cell. Sediment in a lake can move at a greater terminal velocity (about 5 m/s), so it can take days to reach the bottom of the lake after being deposited on the surface.

If we compare animals living on land with those in water, you can see how drag has influenced evolution. Fishes, dolphins, and even massive whales are streamlined in shape to reduce drag forces. Birds are streamlined and migratory species that fly large distances often have particular features such as long necks. Flocks of birds fly in the shape of a spear head as the flock forms a streamlined pattern (see link). In humans, one important example of streamlining is the shape of sperm, which need to be efficient in their use of energy.

Galileo's Experiment

Galileo is said to have dropped two objects of different masses from the Tower of Pisa. He measured how long it took each to reach the ground. Since stopwatches weren't readily available, how do you think he measured their fall time? If the

Figure 5.12: Geese fly in a V formation during their long migratory travels. This shape reduces drag and energy consumption for individual birds, and also allows them a better way to communicate. (credit: Julo, Wikimedia Commons)

objects were the same size, but with different masses, what do you think he should have observed? Would this result be different if done on the Moon?

5.3.2 Section Summary

- Drag forces acting on an object moving in a fluid oppose the motion. For larger objects (such as a baseball) moving at a velocity v in air, the drag force is given by ::: {#eip-233 data-type="equation"}

  $$F_{\text{D}} = \frac{1}{2}C\rho Av^2,$$

  :::

  where C is the drag coefficient (typical values are given in link), A is the area of the object facing the fluid, and ρ is the fluid density.
- For small objects (such as a bacterium) moving in a denser medium (such as water), the drag force is given by Stokes' law, ::: {#eip-307 data-type="equation"}

  $$F_{\text{s}} = 6\ \ \text{rv},$$

  :::

  where r is the radius of the object, η is the fluid viscosity, and v is the object's velocity.

5.3.3 Conceptual Questions

Athletes such as swimmers and bicyclists wear body suits in competition. Formulate a list of pros and cons of such suits.

Two expressions were used for the drag force experienced by a moving object in a liquid. One depended upon the speed, while the other was proportional to the square of the speed. In which types of motion would each of these expressions be more applicable than the other one?

As cars travel, oil and gasoline leaks onto the road surface. If a light rain falls, what does this do to the control of the car? Does a heavy rain make any difference?

Why can a squirrel jump from a tree branch to the ground and run away undamaged, while a human could break a bone in such a fall?

5.3.4 Problems & Exercise

The terminal velocity of a person falling in air depends upon the weight and the area of the person facing the fluid. Find the terminal velocity (in meters per second and kilometers per hour) of an 80.0-kg skydiver falling in a pike (headfirst) position with a surface area of 0.140 m^2.

115 m/s; 414 km/hr

A 60-kg and a 90-kg skydiver jump from an airplane at an altitude of 6000 m, both falling in the pike position. Make some assumption on their frontal areas and calculate their terminal velocities. How long will it take for each skydiver to reach the ground (assuming the time to reach terminal velocity is small)? Assume all values are accurate to three significant digits.

A 560-g squirrel with a surface area of 930 cm^2 falls from a 5.0-m tree to the ground. Estimate its terminal velocity. (Use a drag coefficient for a horizontal skydiver.) What will be the velocity of a 56-kg person hitting the ground, assuming no drag contribution in such a short distance?

25 m/s; 9.9 m/s

To maintain a constant speed, the force provided by a car's engine must equal the drag force plus the force of friction of the road (the rolling resistance). (a) What are the magnitudes of drag forces at 70 km/h and 100 km/h for a Toyota Camry? (Drag area is 0.70 m^2) (b) What is the magnitude of drag force at 70 km/h and 100 km/h for a Hummer H2? (Drag area is 2.44 m^2) Assume all values are accurate to three significant digits.

By what factor does the drag force on a car increase as it goes from 65 to 110 km/h?

2.9

Calculate the speed a spherical rain drop would achieve falling from 5.00 km (a) in the absence of air drag (b) with air drag. Take the size across of the drop to be 4 mm, the density to be 1.00×10^3 kg/m^3, and the surface area to be πr^2.

Using Stokes' law, verify that the units for viscosity are kilograms per meter per second.

$[\eta] = \frac{[F_s]}{[r][v]} = \frac{\text{kg}\cdot\text{m/s}^2}{\text{m}\cdot\text{m/s}} = \frac{\text{kg}}{\text{m}\cdot\text{s}}$

Find the terminal velocity of a spherical bacterium (diameter 2.00 μm) falling in water. You will first need to note that the drag force is equal to the weight at terminal velocity. Take the density of the bacterium to be 1.10×10^3 kg/m^3.

Stokes' law describes sedimentation of particles in liquids and can be used to measure viscosity. Particles in liquids achieve terminal velocity quickly. One can measure the time it takes for a particle to fall a certain distance and then use Stokes' law to calculate the viscosity of the liquid. Suppose a steel ball bearing (density 7.8×10^3 kg/m^3, diameter 3.0 mm) is dropped in a container of motor oil. It takes 12 s to fall a distance of 0.60 m. Calculate the viscosity of the oil.

0.76 kg/m $\cdot\, s$

5.3.5 Glossary

drag force F_D, found to be proportional to the square of the speed of the object; mathematically ::: {#fs-id2600233 .unnumbered data-type="equation" data-label=""}

$$F_D \propto v^2$$

:::

$$F_D = \frac{1}{2} C \rho A v^2,$$

where C is the drag coefficient, A is the area of the object facing the fluid, and ρ is the density of the fluid

Stokes' law $F_s = 6\pi r\eta v$, where r is the radius of the object, η is the viscosity of the fluid, and v is the object's velocity

5.4 Elasticity: Stress and Strain

5.4.1 Learning Objectives

By the end of this section, you will be able to:

- State Hooke's law.
- Explain Hooke's law using graphical representation between deformation and applied force.
- Discuss the three types of deformations such as changes in length, sideways shear, and changes in volume.
- Describe with examples the Young's modulus, shear modulus, and bulk modulus.
- Determine the change in length given mass, length, and radius.

We now move from consideration of forces that affect the motion of an object (such as friction and drag) to those that affect an object's shape. If a bulldozer pushes a car into a wall, the car will not move but it will noticeably change shape. A change in shape due to the application of a force is a deformation. Even very small forces are known to cause some deformation. For small deformations, two important characteristics are observed. First, the object returns to its original shape when the force is removed—that is, the deformation is elastic for small deformations. Second, the size of the deformation is proportional to the force—that is, for small deformations, Hooke's law is obeyed. In equation form, Hooke's law is given by

$$F = k\Delta L,$$

where ΔL is the amount of deformation (the change in length, for example) produced by the force F, and k is a proportionality constant that depends on the shape and composition of the object and the direction of the force. Note that this force is a function of the deformation ΔL —it is not constant as a kinetic friction force is. Rearranging this to

$$\Delta L = \frac{F}{k}$$

makes it clear that the deformation is proportional to the applied force. link shows the Hooke's law relationship between the extension ΔL of a spring or of a human bone. For metals or springs, the straight line region in which Hooke's law pertains is much larger. Bones are brittle and the elastic region is small and the fracture abrupt. Eventually a large enough stress to the material will cause it to break or fracture.

Hooke's Law

$$F = k\Delta L,$$

where ΔL is the amount of deformation (the change in length, for example) produced by the force F, and k is a proportionality constant that depends on the shape and composition of the object and the direction of the force.

$$\Delta L = \frac{F}{k}$$

The proportionality constant k depends upon a number of factors for the material. For example, a guitar string made of nylon stretches when it is tightened, and the elongation ΔL is proportional to the force applied (at least for small deformations). Thicker nylon strings and ones made of steel stretch less for the same applied force, implying they have a larger k (see link). Finally, all three strings return to their normal lengths when the force is removed, provided the deformation is small. Most materials will behave in this manner if the deformation is less than about 0.1% or about 1 part in 10^3.

Stretch Yourself a Little

How would you go about measuring the proportionality constant k of a rubber band? If a rubber band stretched 3 cm when a 100-g mass was attached to it, then how much would it stretch if two similar rubber bands were attached to the same mass—even if put together in parallel or alternatively if tied together in series?

We now consider three specific types of deformations: changes in length (tension and compression), sideways shear (stress), and changes in volume. All deformations are assumed to be small unless otherwise stated.

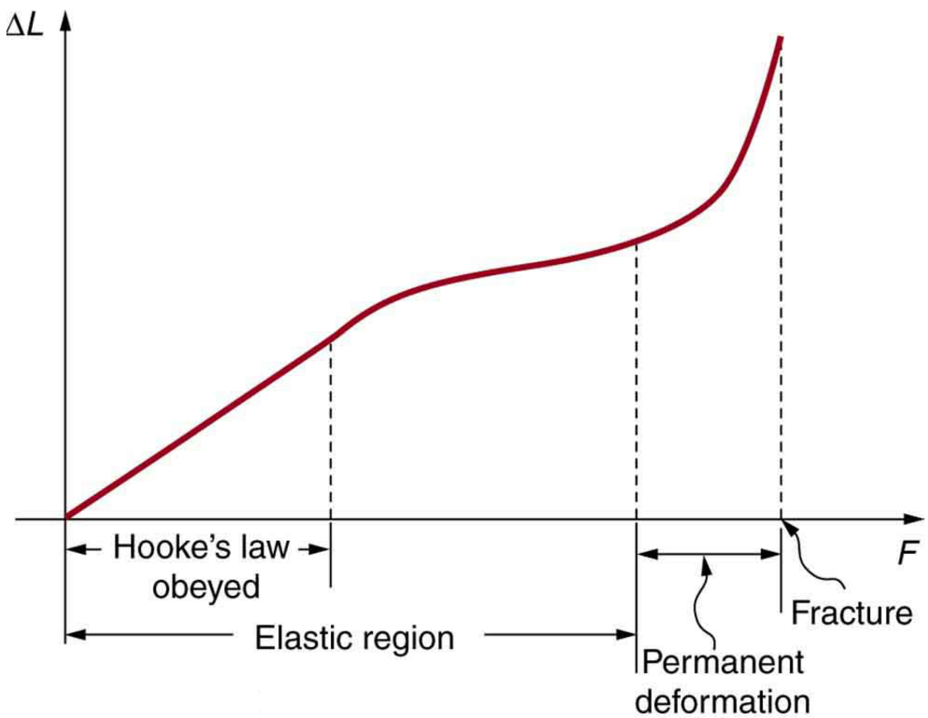


Figure 5.13: A graph of deformation ΔL versus applied force F. The straight segment is the linear region where Hooke's law is obeyed. The slope of the straight region is $\frac{1}{k}$. For larger forces, the graph is curved but the deformation is still elastic— ΔL will return to zero if the force is removed. Still greater forces permanently deform the object until it finally fractures. The shape of the curve near fracture depends on several factors, including how the force F is applied. Note that in this graph the slope increases just before fracture, indicating that a small increase in F is producing a large increase in L near the fracture.

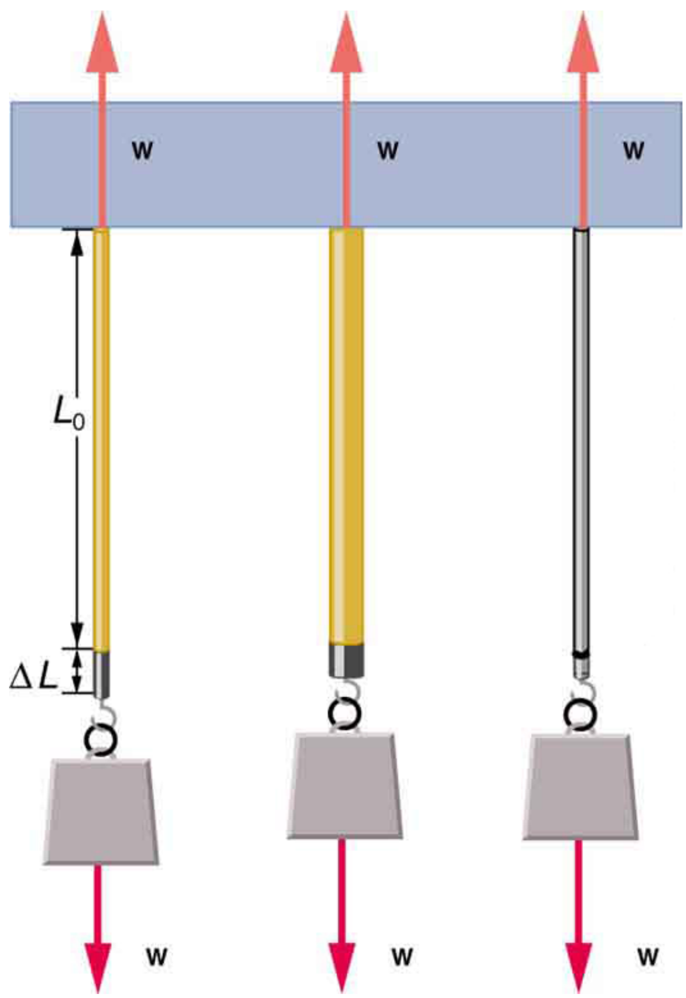


Figure 5.14: The same force, in this case a weight (w), applied to three different guitar strings of identical length produces the three different deformations shown as shaded segments. The string on the left is thin nylon, the one in the middle is thicker nylon, and the one on the right is steel.

5.4.2 Changes in Length—Tension and Compression: Elastic Modulus

A change in length ΔL is produced when a force is applied to a wire or rod parallel to its length L_0, either stretching it (a tension) or compressing it. (See link.)

Experiments have shown that the change in length (ΔL) depends on only a few variables. As already noted, ΔL is proportional to the force F and depends on the substance from which the object is made. Additionally, the change in length is proportional to the original length L_0 and inversely proportional to the cross-sectional area of the wire or rod. For example, a long guitar string will stretch more than a short one, and a thick string will stretch less than a thin one. We can combine all these factors into one equation for ΔL:

$$\Delta L = \frac{1}{Y}\frac{F}{A}L_0,$$

where ΔL is the change in length, F the applied force, Y is a factor, called the elastic modulus or Young's modulus, that depends on the substance, A is the cross-sectional area, and L_0 is the original length. link lists values of Y for several materials—those with a large Y are said to have a large tensile strength because they deform less for a given tension or compression.

Elastic Moduli

$$footnote$$

Approximate and average values. Young's moduli Y for tension and compression sometimes differ but are averaged here. Bone has significantly different Young's moduli for tension and compression.

Material

Young's modulus (tension–compression) Y (10^9 N/m^2)

Shear modulus S(10^9 N/m^2)

Bulk modulus B (10^9 N/m^2)

Aluminum

70

25

75

Bone – tension

16

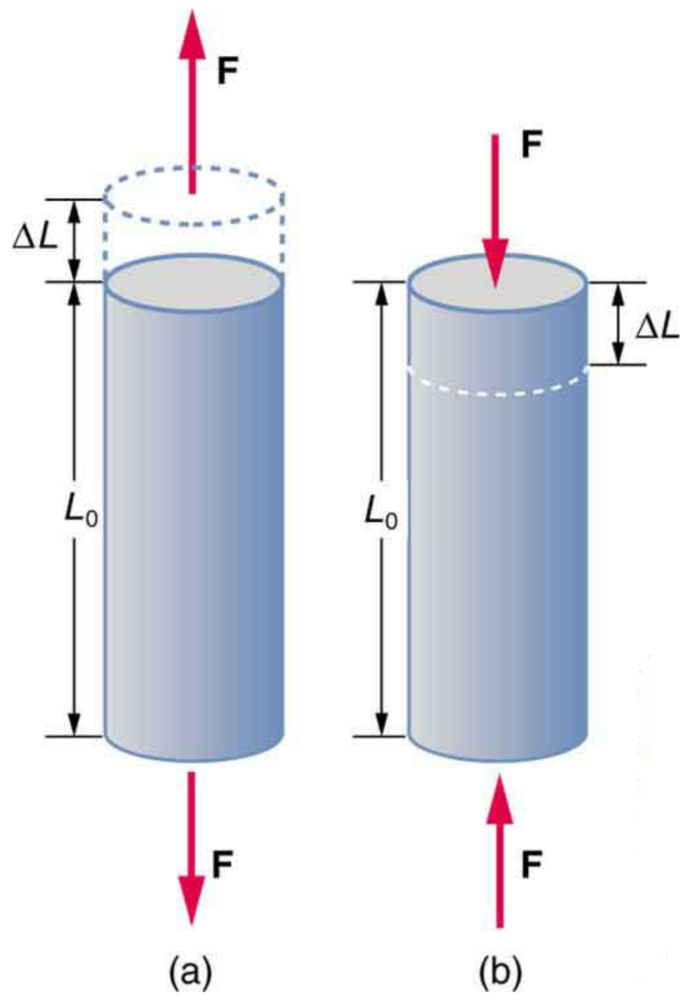


Figure 5.15: (a) Tension. The rod is stretched a length ΔL when a force is applied parallel to its length. (b) Compression. The same rod is compressed by forces with the same magnitude in the opposite direction. For very small deformations and uniform materials, ΔL is approximately the same for the same magnitude of tension or compression. For larger deformations, the cross-sectional area changes as the rod is compressed or stretched.

80

8

Bone – compression

9

Brass

90

35

75

Brick

15

Concrete

20

Glass

70

20

30

Granite

45

20

45

Hair (human)

10

Hardwood

15

10

Iron, cast

100

40

90

Lead

16

5

50

Marble

60

20

70

Nylon

5

Polystyrene

3

Silk

6

Spider thread

3

Steel

210

80

130

Tendon

1

Acetone

0.7

Ethanol

0.9

Glycerin

4.5

Mercury

25

Water

2.2

Young's moduli are not listed for liquids and gases in link because they cannot be stretched or compressed in only one direction. Note that there is an assumption that the object does not accelerate, so that there are actually two applied forces of magnitude F acting in opposite directions. For example, the strings in link are being pulled down by a force of magnitude w and held up by the ceiling, which also exerts a force of magnitude w.

The Stretch of a Long Cable

Suspension cables are used to carry gondolas at ski resorts. (See link) Consider a suspension cable that includes an unsupported span of 3020 m. Calculate the amount of stretch in the steel cable. Assume that the cable has a diameter of 5.6 cm and the maximum tension it can withstand is 3.0×10^6 N.

Figure 5.16: Gondolas travel along suspension cables at the Gala Yuzawa ski resort in Japan. (credit: Rudy Herman, Flickr)

Strategy

The force is equal to the maximum tension, or $F = 3.0 \times 10^6$ N. The cross-sectional area is $\pi r^2 = 2.46 \times 10^{-3}$ m^2. The equation $\Delta L = \frac{1}{Y}\frac{F}{A}L_0$ can be used to find the change in length.

Solution

All quantities are known. Thus,

$$\begin{aligned}\Delta L &= \left(\frac{1}{210\times 10^{9}\ \text{N/m}^{2}}\right)\left(\frac{3.0\times 10^{6}\ \text{N}}{2.46\times 10^{-3}\ \text{m}^{2}}\right)(3020\ \text{m})\\ &= 18\ \text{m}.\end{aligned}$$

Discussion

This is quite a stretch, but only about 0.6% of the unsupported length. Effects of temperature upon length might be important in these environments.

Bones, on the whole, do not fracture due to tension or compression. Rather they generally fracture due to sideways impact or bending, resulting in the bone shearing or snapping. The behavior of bones under tension and compression is important because it determines the load the bones can carry. Bones are classified as weight-bearing structures such as columns in buildings and trees. Weight-bearing structures have special features; columns in building have steel-reinforcing rods while trees and bones are fibrous. The bones in different parts of the body serve different structural functions and are prone to different stresses. Thus the bone in the top of the femur is arranged in thin sheets separated by marrow while in other places the bones can be cylindrical and filled with marrow or just solid. Overweight people have a tendency toward bone damage due to sustained compressions in bone joints and tendons.

Another biological example of Hooke's law occurs in tendons. Functionally, the tendon (the tissue connecting muscle to bone) must stretch easily at first when a force is applied, but offer a much greater restoring force for a greater strain. link shows a stress-strain relationship for a human tendon. Some tendons have a high collagen content so there is relatively little strain, or length change; others, like support tendons (as in the leg) can change length up to 10%. Note that this stress-strain curve is nonlinear, since the slope of the line changes in different regions. In the first part of the stretch called the toe region, the fibers in the tendon begin to align in the direction of the stress—this is called *uncrimping*. In the linear region, the fibrils will be stretched, and in the failure region individual fibers begin to break. A simple model of this relationship can be illustrated by springs in parallel: different springs are activated at different lengths of stretch. Examples of this are given in the problems at end of this chapter. Ligaments (tissue connecting bone to bone) behave in a similar way.

Unlike bones and tendons, which need to be strong as well as elastic, the arteries and lungs need to be very stretchable. The elastic properties of the arteries are essential for blood flow. The pressure in the arteries increases and arterial walls stretch when the blood is pumped out of the heart. When the aortic valve shuts, the pressure in the arteries drops and the arterial walls relax to maintain the blood flow. When you feel your pulse, you are feeling exactly this—the elastic behavior of the arteries as the blood gushes through with each pump of the heart. If the arteries were rigid, you would not feel a pulse. The heart is also an organ with special elastic properties. The lungs expand with muscular effort when we breathe in but relax freely and elastically when we breathe out.

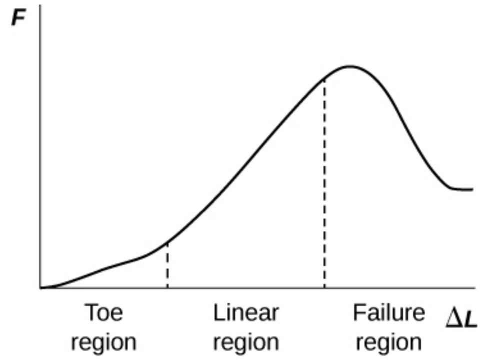


Figure 5.17: Typical stress-strain curve for mammalian tendon. Three regions are shown: (1) toe region (2) linear region, and (3) failure region.

Our skins are particularly elastic, especially for the young. A young person can go from 100 kg to 60 kg with no visible sag in their skins. The elasticity of all organs reduces with age. Gradual physiological aging through reduction in elasticity starts in the early 20s.

Calculating Deformation: How Much Does Your Leg Shorten When You Stand on It?

Calculate the change in length of the upper leg bone (the femur) when a 70.0 kg man supports 62.0 kg of his mass on it, assuming the bone to be equivalent to a uniform rod that is 40.0 cm long and 2.00 cm in radius.

Strategy

The force is equal to the weight supported, or

$$F = \text{mg} = (62.0\ kg)\left(9.80\ \text{m/s}^2\right) = 607.6\ N,$$

and the cross-sectional area is $\pi r^2 = 1.257{\times}10^{-3}\ \text{m}^2$. The equation $\Delta L = \frac{1}{Y}\frac{F}{A}L_0$ can be used to find the change in length.

Solution

All quantities except ΔL are known. Note that the compression value for Young's modulus for bone must be used here. Thus,

$$\begin{aligned}\Delta L &= \left(\frac{1}{9{\times}10^9\ \text{N/m}^2}\right)\left(\frac{607.6\ \text{N}}{1.257{\times}10^{-3}\ \text{m}^2}\right)(0.400\ \text{m})\\ &= 2{\times}10^{-5}\ \text{m}.\end{aligned}$$

Discussion

This small change in length seems reasonable, consistent with our experience that bones are rigid. In fact, even the rather large forces encountered during strenuous physical activity do not compress or bend bones by large amounts. Although bone is rigid compared with fat or muscle, several of the substances listed in link have larger values of Young's modulus Y. In other words, they are more rigid and have greater tensile strength.

The equation for change in length is traditionally rearranged and written in the following form:

$$\frac{F}{A} = Y\frac{\Delta L}{L_0}.$$

The ratio of force to area, $\frac{F}{A}$, is defined as stress (measured in $\mathrm{N/m}^2$), and the ratio of the change in length to length, $\frac{\Delta L}{L_0}$, is defined as strain (a unitless quantity). In other words,

$$\text{stress} = Y \times \text{strain}.$$

In this form, the equation is analogous to Hooke's law, with stress analogous to force and strain analogous to deformation. If we again rearrange this equation to the form

$$F = \text{YA}\frac{\Delta L}{L_0},$$

we see that it is the same as Hooke's law with a proportionality constant

$$k = \frac{\text{YA}}{L_0}.$$

This general idea—that force and the deformation it causes are proportional for small deformations—applies to changes in length, sideways bending, and changes in volume.

Stress

The ratio of force to area, $\frac{F}{A}$, is defined as stress measured in $\mathrm{N/m}^2$.

Strain

The ratio of the change in length to length, $\frac{\Delta L}{L_0}$, is defined as strain (a unitless quantity). In other words,

$$\text{stress} = Y \times \text{strain}.$$

5.4.3 Sideways Stress: Shear Modulus

link illustrates what is meant by a sideways stress or a *shearing force*. Here the deformation is called Δx and it is perpendicular to L_0, rather than parallel as with tension and compression. Shear deformation behaves similarly to tension and compression and can be described with similar equations. The expression for shear deformation is

$$\Delta x = \frac{1}{S}\frac{F}{A}L_0,$$

where S is the shear modulus (see link) and F is the force applied perpendicular to L_0 and parallel to the cross-sectional area A. Again, to keep the object from accelerating, there are actually two equal and opposite forces F applied across opposite faces, as illustrated in link. The equation is logical—for example, it is easier to bend a long thin pencil (small A) than a short thick one, and both are more easily bent than similar steel rods (large S).

Shear Deformation

$$\Delta x = \frac{1}{S}\frac{F}{A}L_0,$$

where S is the shear modulus and F is the force applied perpendicular to L_0 and parallel to the cross-sectional area A.

Examination of the shear moduli in link reveals some telling patterns. For example, shear moduli are less than Young's moduli for most materials. Bone is a remarkable exception. Its shear modulus is not only greater than its Young's modulus, but it is as large as that of steel. This is one reason that bones can be long and relatively thin. Bones can support loads comparable to that of concrete and steel. Most bone fractures are not caused by compression but by excessive twisting and bending.

The spinal column (consisting of 26 vertebral segments separated by discs) provides the main support for the head and upper part of the body. The spinal column has normal curvature for stability, but this curvature can be increased, leading to increased shearing forces on the lower vertebrae. Discs are better at withstanding compressional forces than shear forces. Because the spine is not vertical, the weight of the upper body exerts some of both. Pregnant women and people that are overweight (with large abdomens) need to move their shoulders back to maintain balance, thereby increasing the curvature in their spine and so increasing the shear component of the stress. An increased angle due to more curvature increases the shear forces along the plane. These higher shear forces increase the risk of back injury through ruptured discs. The lumbosacral disc (the wedge shaped disc below the last vertebrae) is particularly at risk because of its location.

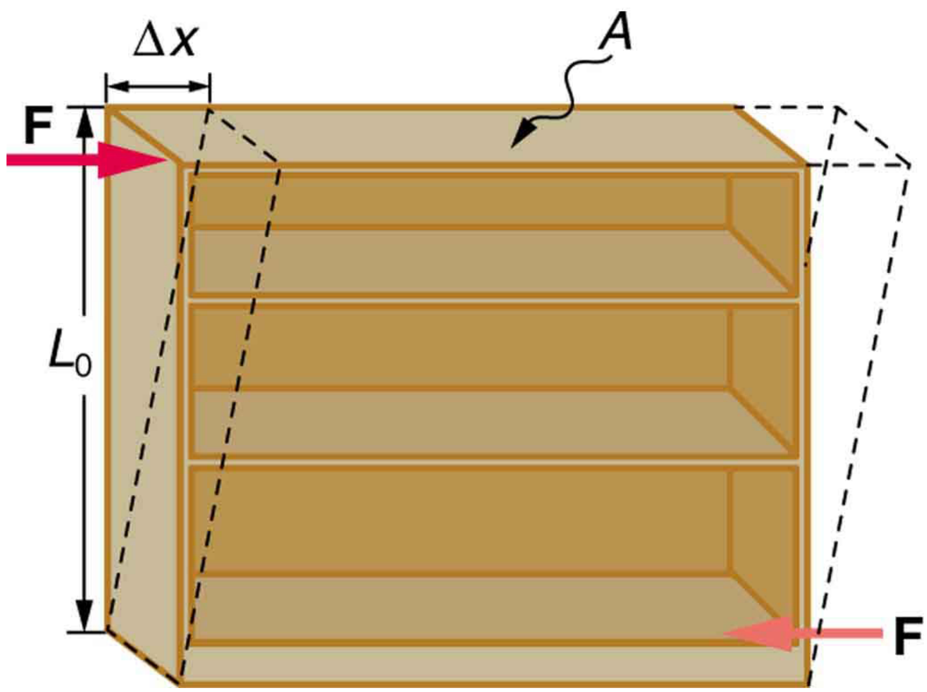


Figure 5.18: Shearing forces are applied perpendicular to the length L_0 and parallel to the area A, producing a deformation Δx. Vertical forces are not shown, but it should be kept in mind that in addition to the two shearing forces, **F**, there must be supporting forces to keep the object from rotating. The distorting effects of these supporting forces are ignored in this treatment. The weight of the object also is not shown, since it is usually negligible compared with forces large enough to cause significant deformations.

The shear moduli for concrete and brick are very small; they are too highly variable to be listed. Concrete used in buildings can withstand compression, as in pillars and arches, but is very poor against shear, as might be encountered in heavily loaded floors or during earthquakes. Modern structures were made possible by the use of steel and steel-reinforced concrete. Almost by definition, liquids and gases have shear moduli near zero, because they flow in response to shearing forces.

Calculating Force Required to Deform: That Nail Does Not Bend Much Under a Load

Find the mass of the picture hanging from a steel nail as shown in link, given that the nail bends only 1.80 µm. (Assume the shear modulus is known to two significant figures.)

Strategy

The force F on the nail (neglecting the nail's own weight) is the weight of the picture w. If we can find w, then the mass of the picture is just $\frac{w}{g}$. The equation $\Delta x = \frac{1}{S}\frac{F}{A}L_0$ can be solved for F.

Solution

Solving the equation $\Delta x = \frac{1}{S}\frac{F}{A}L_0$ for F, we see that all other quantities can be found:

$$F = \frac{SA}{L_0}\Delta x.$$

S is found in link and is $S = 80 \times 10^9\ \text{N/m}^2$. The radius r is 0.750 mm (as seen in the figure), so the cross-sectional area is

$$A = \pi r^2 = 1.77 \times 10^{-6}\ \text{m}^2.$$

The value for L_0 is also shown in the figure. Thus,

$$F = \frac{(80 \times 10^9\ \text{N/m}^2)(1.77 \times 10^{-6}\ \text{m}^2)}{(5.00 \times 10^{-3}\ \text{m})}(1.80 \times 10^{-6}\ \text{m}) = 51\ \text{N}.$$

This 51 N force is the weight w of the picture, so the picture's mass is

$$m = \frac{w}{g} = \frac{F}{g} = 5.2\ \text{kg}.$$

Discussion

This is a fairly massive picture, and it is impressive that the nail flexes only 1.80 µm—an amount undetectable to the unaided eye.

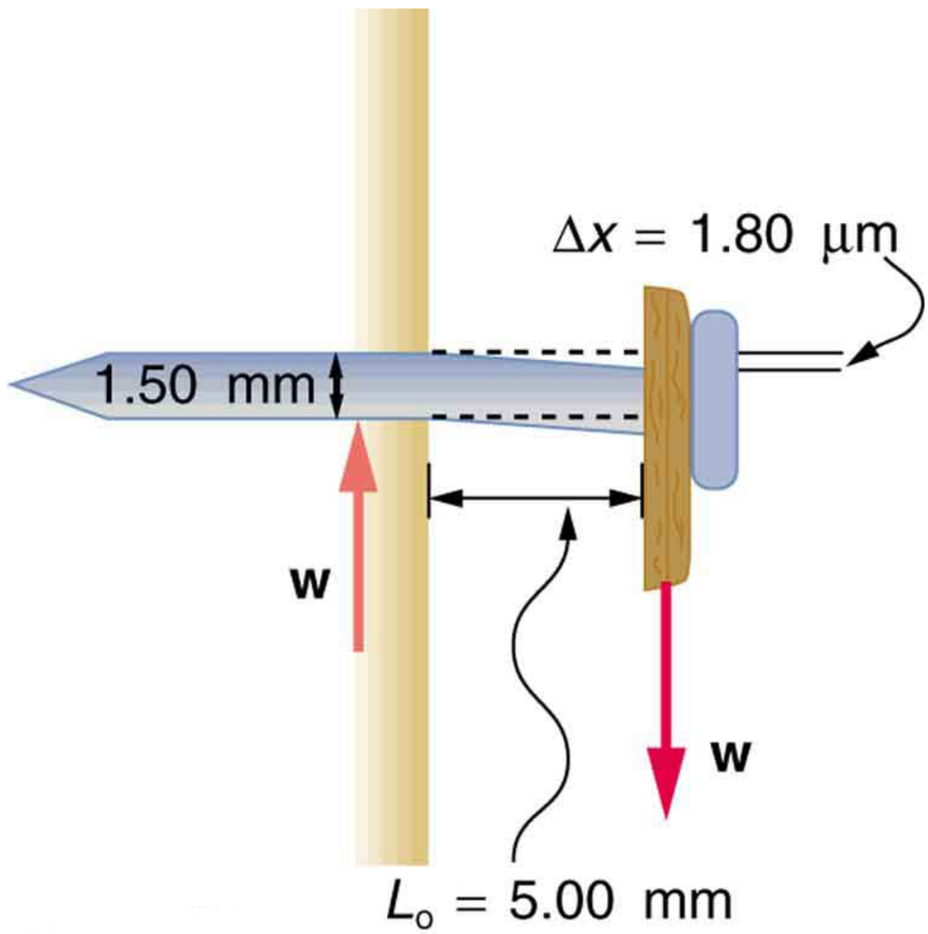


Figure 5.19: Side view of a nail with a picture hung from it. The nail flexes very slightly (shown much larger than actual) because of the shearing effect of the supported weight. Also shown is the upward force of the wall on the nail, illustrating that there are equal and opposite forces applied across opposite cross sections of the nail. See link for a calculation of the mass of the picture.

5.4.4 Changes in Volume: Bulk Modulus

An object will be compressed in all directions if inward forces are applied evenly on all its surfaces as in link. It is relatively easy to compress gases and extremely difficult to compress liquids and solids. For example, air in a wine bottle is compressed when it is corked. But if you try corking a brim-full bottle, you cannot compress the wine—some must be removed if the cork is to be inserted. The reason for these different compressibilities is that atoms and molecules are separated by large empty spaces in gases but packed close together in liquids and solids. To compress a gas, you must force its atoms and molecules closer together. To compress liquids and solids, you must actually compress their atoms and molecules, and very strong electromagnetic forces in them oppose this compression.

We can describe the compression or volume deformation of an object with an equation. First, we note that a force "applied evenly" is defined to have the same stress, or ratio of force to area $\frac{F}{A}$ on all surfaces. The deformation produced is a change in volume ΔV, which is found to behave very similarly to the shear, tension, and compression previously discussed. (This is not surprising, since a compression of the entire object is equivalent to compressing each of its three dimensions.) The relationship of the change in volume to other physical quantities is given by

$$\Delta V = \frac{1}{B}\frac{F}{A}V_0,$$

where B is the bulk modulus (see link), V_0 is the original volume, and $\frac{F}{A}$ is the force per unit area applied uniformly inward on all surfaces. Note that no bulk moduli are given for gases.

What are some examples of bulk compression of solids and liquids? One practical example is the manufacture of industrial-grade diamonds by compressing carbon with an extremely large force per unit area. The carbon atoms rearrange their crystalline structure into the more tightly packed pattern of diamonds. In nature, a similar process occurs deep underground, where extremely large forces result from the weight of overlying material. Another natural source of large compressive forces is the pressure created by the weight of water, especially in deep parts of the oceans. Water exerts an inward force on all surfaces of a submerged object, and even on the water itself. At great depths, water is measurably compressed, as the following example illustrates.

Calculating Change in Volume with Deformation: How Much Is Water Compressed at Great Ocean Depths?

Calculate the fractional decrease in volume ($\frac{\Delta V}{V_0}$) for seawater at 5.00 km depth, where the force per unit area is $5.00 \times 10^7\ N/m^2$.

Strategy

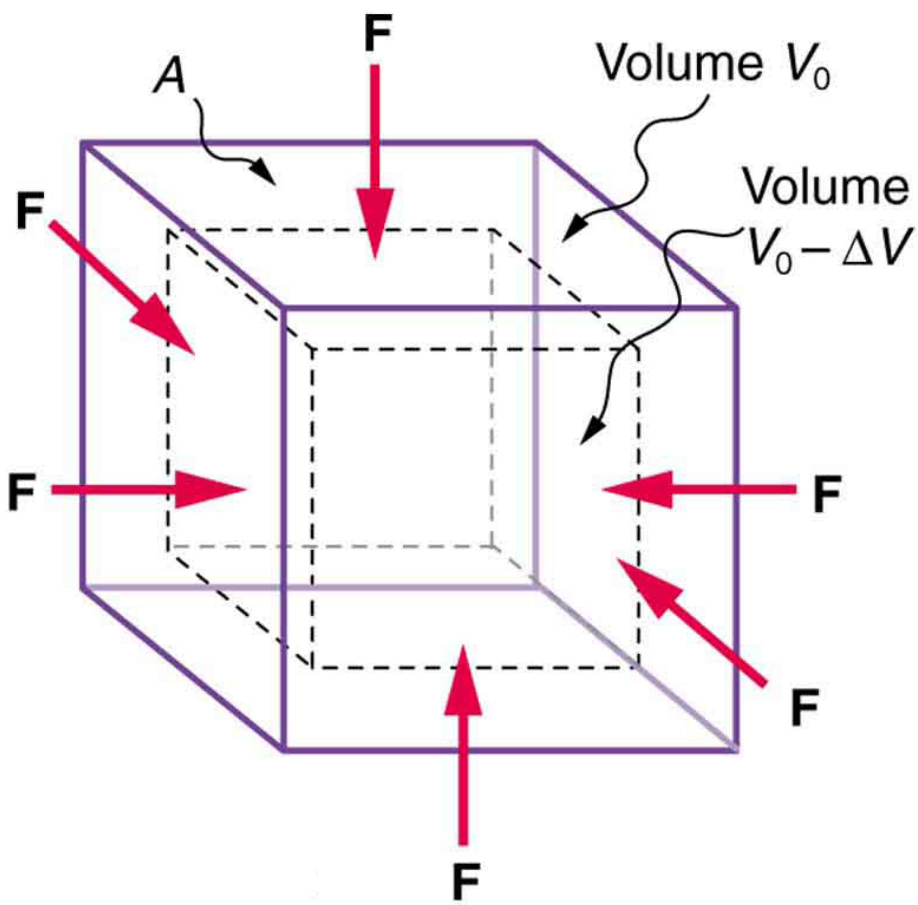


Figure 5.20: An inward force on all surfaces compresses this cube. Its change in volume is proportional to the force per unit area and its original volume, and is related to the compressibility of the substance.

Equation $\Delta V = \frac{1}{B}\frac{F}{A}V_0$ is the correct physical relationship. All quantities in the equation except $\frac{\Delta V}{V_0}$ are known.

Solution

Solving for the unknown $\frac{\Delta V}{V_0}$ gives

$$\frac{\Delta V}{V_0} = \frac{1}{B}\frac{F}{A}.$$

Substituting known values with the value for the bulk modulus B from link,

$$\begin{aligned}\frac{\Delta V}{V_0} &= \frac{5.00\times10^{7}\ \text{N/m}^2}{2.2\times10^{9}\ \text{N/m}^2}\\ &= 0.023 = 2.3\%.\end{aligned}$$

Discussion

Although measurable, this is not a significant decrease in volume considering that the force per unit area is about 500 atmospheres (1 million pounds per square foot). Liquids and solids are extraordinarily difficult to compress.

Conversely, very large forces are created by liquids and solids when they try to expand but are constrained from doing so—which is equivalent to compressing them to less than their normal volume. This often occurs when a contained material warms up, since most materials expand when their temperature increases. If the materials are tightly constrained, they deform or break their container. Another very common example occurs when water freezes. Water, unlike most materials, expands when it freezes, and it can easily fracture a boulder, rupture a biological cell, or crack an engine block that gets in its way.

Other types of deformations, such as torsion or twisting, behave analogously to the tension, shear, and bulk deformations considered here.

5.4.5 Section Summary

- Hooke's law is given by

  $$F = k\Delta L,$$

  where ΔL is the amount of deformation (the change in length), F is the applied force, and k is a proportionality constant that depends on the shape and composition of the object and the direction of the force. The relationship between the deformation and the applied force can also be written as

  $$\Delta L = \frac{1}{Y}\frac{F}{A}L_0,$$

where Y is *Young's modulus*, which depends on the substance, A is the cross-sectional area, and L_0 is the original length.

- The ratio of force to area, $\frac{F}{A}$, is defined as *stress*, measured in N/m^2.
- The ratio of the change in length to length, $\frac{\Delta L}{L_0}$, is defined as *strain* (a unitless quantity). In other words, ::: {#eip-98 data-type="equation"}

$$\text{stress} = Y \times \text{strain}.$$

:::

- The expression for shear deformation is

$$\Delta x = \frac{1}{S}\frac{F}{A}L_0,$$

where S is the shear modulus and F is the force applied perpendicular to L_0 and parallel to the cross-sectional area A.

- The relationship of the change in volume to other physical quantities is given by

$$\Delta V = \frac{1}{B}\frac{F}{A}V_0,$$

where B is the bulk modulus, V_0 is the original volume, and $\frac{F}{A}$ is the force per unit area applied uniformly inward on all surfaces.

5.4.6 Conceptual Questions

The elastic properties of the arteries are essential for blood flow. Explain the importance of this in terms of the characteristics of the flow of blood (pulsating or continuous).

What are you feeling when you feel your pulse? Measure your pulse rate for 10 s and for 1 min. Is there a factor of 6 difference?

Examine different types of shoes, including sports shoes and thongs. In terms of physics, why are the bottom surfaces designed as they are? What differences will dry and wet conditions make for these surfaces?

Would you expect your height to be different depending upon the time of day? Why or why not?

Would you expect a large or small stress to be required to deform a spider web? Why is this elasticity an important feature for a spider web?

Explain why pregnant women often suffer from back strain late in their pregnancy.

An old carpenter's trick to keep nails from bending when they are pounded into hard materials is to grip the center of the nail firmly with pliers. Why does this help?

When a glass bottle full of vinegar warms up, both the vinegar and the glass expand, but vinegar expands significantly more with temperature than glass. The bottle will break if it was filled to its tightly capped lid. Explain why, and also explain how a pocket of air above the vinegar would prevent the break. (This is the function of the air above liquids in glass containers.)

5.4.7 Problems & Exercises

During a circus act, one performer swings upside down hanging from a trapeze holding another, also upside-down, performer by the legs. If the upward force on the lower performer is three times her weight, how much do the bones (the femurs) in her upper legs stretch? You may assume each is equivalent to a uniform rod 35.0 cm long and 1.80 cm in radius. Her mass is 60.0 kg.

$$1.90 \times 10^{-3} \text{ cm}$$

During a wrestling match, a 150 kg wrestler briefly stands on one hand during a maneuver designed to perplex his already moribund adversary. By how much does the upper arm bone shorten in length? The bone can be represented by a uniform rod 38.0 cm in length and 2.10 cm in radius.

a The "lead" in pencils is a graphite composition with a Young's modulus of about $1{\times}10^9$ N/m^2 . Calculate the change in length of the lead in an automatic pencil if you tap it straight into the pencil with a force of 4.0 N. The lead is 0.50 mm in diameter and 60 mm long. (b) Is the answer reasonable? That is, does it seem to be consistent with what you have observed when using pencils?

(a)1 mm
(b) This does seem reasonable, since the lead does seem to shrink a little when you push on it.

TV broadcast antennas are the tallest artificial structures on Earth. In 1987, a 72.0-kg physicist placed himself and 400 kg of equipment at the top of one 610-m high antenna to perform gravity experiments. By how much was the antenna compressed, if we consider it to be equivalent to a steel cylinder 0.150 m in radius?

a By how much does a 65.0-kg mountain climber stretch her 0.800-cm diameter nylon rope when she hangs 35.0 m below a rock outcropping? (b) Does the answer seem to be consistent with what you have observed for nylon ropes? Would it make sense if the rope were actually a bungee cord?

(a)9 cm
(b)This seems reasonable for nylon climbing rope, since it is not supposed to

stretch that much.

A 20.0-m tall hollow aluminum flagpole is equivalent in strength to a solid cylinder 4.00 cm in diameter. A strong wind bends the pole much as a horizontal force of 900 N exerted at the top would. How far to the side does the top of the pole flex?

As an oil well is drilled, each new section of drill pipe supports its own weight and that of the pipe and drill bit beneath it. Calculate the stretch in a new 6.00 m length of steel pipe that supports 3.00 km of pipe having a mass of 20.0 kg/m and a 100-kg drill bit. The pipe is equivalent in strength to a solid cylinder 5.00 cm in diameter.

8.59 mm

Calculate the force a piano tuner applies to stretch a steel piano wire 8.00 mm, if the wire is originally 0.850 mm in diameter and 1.35 m long.

A vertebra is subjected to a shearing force of 500 N. Find the shear deformation, taking the vertebra to be a cylinder 3.00 cm high and 4.00 cm in diameter.

$$1.49 \times 10^{-7}\ \mathrm{m}$$

A disk between vertebrae in the spine is subjected to a shearing force of 600 N. Find its shear deformation, taking it to have the shear modulus of $1\times10^{9}\ \mathrm{N/m^2}$. The disk is equivalent to a solid cylinder 0.700 cm high and 4.00 cm in diameter.

When using a pencil eraser, you exert a vertical force of 6.00 N at a distance of 2.00 cm from the hardwood-eraser joint. The pencil is 6.00 mm in diameter and is held at an angle of 20.0º to the horizontal. (a) By how much does the wood flex perpendicular to its length? (b) How much is it compressed lengthwise?

$a\ 3.99 \times 10^{-7}\ m$

$b\ 9.67 \times 10^{-8}\ m$

To consider the effect of wires hung on poles, we take data from link, in which tensions in wires supporting a traffic light were calculated. The left wire made an angle 30.0º below the horizontal with the top of its pole and carried a tension of 108 N. The 12.0 m tall hollow aluminum pole is equivalent in strength to a 4.50 cm diameter solid cylinder. (a) How far is it bent to the side? (b) By how much is it compressed?

A farmer making grape juice fills a glass bottle to the brim and caps it tightly. The juice expands more than the glass when it warms up, in such a way that the volume increases by 0.2% (that is, $\Delta V/V_0 = 2 \times 10^{-3}$) relative to the space available. Calculate the magnitude of the normal force exerted by the juice per square centimeter if its bulk modulus is $1.8 \times 10^{9}\ \mathrm{N/m^2}$, assuming the bottle does not break. In view of your answer, do you think the bottle will survive?

4×10^6 N/m^2. This is about 36 atm, greater than a typical jar can withstand.

a When water freezes, its volume increases by 9.05% (that is, $\Delta V/V_0 = 9.05 \times 10^{-2}$). What force per unit area is water capable of exerting on a container when it freezes? (It is acceptable to use the bulk modulus of water in this problem.) (b) Is it surprising that such forces can fracture engine blocks, boulders, and the like?

This problem returns to the tightrope walker studied in link, who created a tension of 3.94×10^3 N in a wire making an angle 5.0º below the horizontal with each supporting pole. Calculate how much this tension stretches the steel wire if it was originally 15 m long and 0.50 cm in diameter.

1.4 cm

The pole in link is at a 90.0º bend in a power line and is therefore subjected to more shear force than poles in straight parts of the line. The tension in each line is 4.00×10^4 N, at the angles shown. The pole is 15.0 m tall, has an 18.0 cm diameter, and can be considered to have half the strength of hardwood. (a) Calculate the compression of the pole. (b) Find how much it bends and in what direction. (c) Find the tension in a guy wire used to keep the pole straight if it is attached to the top of the pole at an angle of 30.0º with the vertical. (Clearly, the guy wire must be in the opposite direction of the bend.)

5.4.8 Glossary

deformation change in shape due to the application of force

Hooke's law proportional relationship between the force F on a material and the deformation ΔL it causes, $F = k\Delta L$

tensile strength measure of deformation for a given tension or compression

stress ratio of force to area

strain ratio of change in length to original length

shear deformation deformation perpendicular to the original length of an object

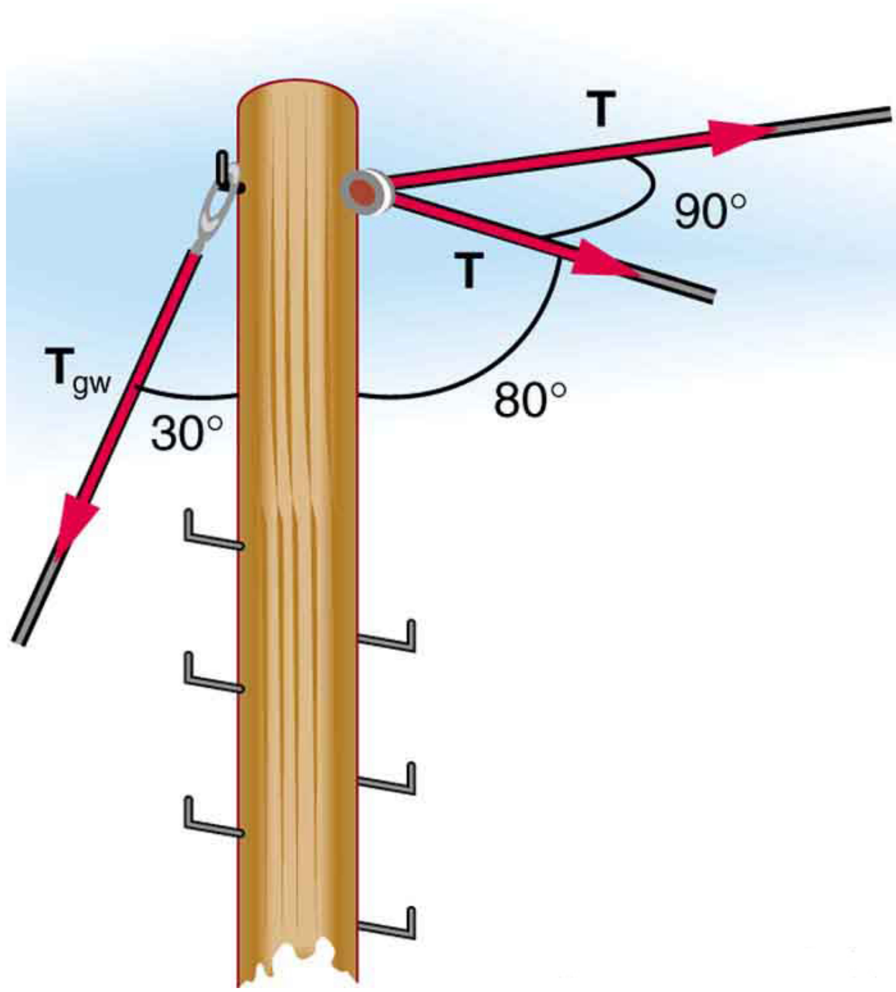


Figure 5.21: This telephone pole is at a 90º bend in a power line. A guy wire is attached to the top of the pole at an angle of 30º with the vertical.

Bibliography

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